Simulation Studies of Foam for Enhanced Oil Recovery

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ABSTRACT

In gas-injection enhanced oil recovery (EOR), foam is a promising method to improve sweep. We studied foam for EOR through analytical modeling and simulations in two cases: the ability of foam to prevent gravity override over large distances and the effect of oil on foam processes.

Shan and Rossen (2004) present a simple model for SAG (surfactant-alternating-gas) foam process with vertical permeability $k_v$ equal to horizontal permeability $k_h$, but they did not extend the numerical calculations to large distances. Robert de Velde Harsenhorst (2012) extended the model to the case for $k_v < k_h$, which gives the surprising prediction that gravity override is worse as $k_v$ decreases. With large $k_v$, foam pushes gas downward in response to the pressure difference across the titled foam front. In the first part of this thesis, we extend the numerical calculations of de Velde Harsenhorst for the case $0 < k_v < k_h$. Then, we use an example from a North Sea field to test the simple model in simulation. The simulation result confirms that segregation would be insignificant over the large distances between wells, 6 km, as predicted by the simple model. Then, we compare sweep efficiency in the North Sea example with continuous foam injection. Complete segregation over a relatively short distance with continuous foam injection is confirmed with simulation. The segregation length in the simulation is similar to that in analytical model derived by Van der Bol (2007).

We also test the simple model with simulation under more demanding cases with more severe gravity segregation, using three sets of foam parameters: the North Sea example, model parameters fit to data of Persoff et al. (1991) and a foam model designed specifically to fit the assumptions of the simple model. Gravity segregation with SAG is worse as $k_v$ decreases in simulations in the latter two cases, which conforms to predictions of the simple model.

The effect of oil on foam is complex and not fully understood. In the second part of this thesis, we study the effect of oil on foam from 1D computer simulations of continuous foam injection with oil present. We attempt to relate and compare plots of gas mobility-reduction factor with foam, plotted on a ternary composition diagram, with the composition paths of displacements simulated using the same model parameters. We also use fractional-flow theory, which provides key insights into foam EOR displacements, to verify the foam simulation results where the simulation composition paths travel along the binary sides of the ternary phase diagram. In the preliminary results, the injection point is always within the region of strong foam, but the initial condition is either inside or outside this region. In all cases, regardless of the initial condition, oil was displaced ahead of the foam. These results, which need further study, suggest possible usefulness of drawing composition paths on a plot of gas mobility-reduction factor on the 3-phase diagram. However, a check of the process by which foam is first created near the inlet in these simulations suggests numerical artifacts may have affected the results. This possibility needs further research.
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CHAPTER 1: FOAM EOR THEORY AND RESEARCH BACKGROUND

1.1 Introduction

There are three stages of oil field development or oil recovery: primary, secondary and tertiary recovery as shown in Fig. 1. These three stages use various techniques to increase and maintain crude oil production at the optimum level. Primary oil recovery depends on the initial reservoir pressure which is high enough to force the oil to the well and then to the surface facilities. Primary recovery produces the first oil out which is commonly called the “easy” oil and it is the cheapest method of the oil recovery compared to the other two stages. Artificial lift (gas lift, electrical submersible pump, etc) is also applied in primary oil recovery when declining reservoir pressure reduces oil production. This method will bring down the bottom-hole pressure of the well bore so the oil flow can be increased. Typically on average, 10% of original oil in place is recovered using primary recovery (Tzimas et al., 2005).

During application of primary recovery in the oil field development, the reservoir pressure will decline by over time and there will be a period when the oil production is not economically viable anymore. Then secondary recovery is applied to increase the reservoir pressure in order to maintain oil production. In this stage, the reservoir is subjected to water flooding or gas injection to maintain reservoir pressure. The principle of water flooding is injecting large volumes of water under pressure into the reservoir through injector wells. As the water flows into the oil reservoir toward the production wells, the water sweeps some amount of oil. When the liquid reaches the surface facilities, the water is separated from the oil. Gas injection is another method used in the secondary recovery stage. A gas injected to the oil reservoir under high pressure to force the oil to the production wells. In this stage, approximately 15-50% of original oil in place can be recovered (Tzimas et al., 2005).

The third stage is tertiary recovery commonly called enhanced oil recovery (EOR). The definition of EOR is “the recovery of oil by injection of a fluid that is not native to the reservoir” (Lake, 1989). EOR is a means to extend the productive life of an otherwise depleted and uneconomic oil field. Different EOR methods exist to increase the oil recovery from depleted oil fields as shown in Fig. 1. In the thermal recovery approach, various methods are used to heat the crude oil in the reservoir which allows it to flow more easily to the production well. There are several thermal methods, with the two main ones being steam soak/cyclic and steam drive/flood. Both are most commonly applied to oil reservoirs that are relatively shallow, permeable and thick. The main mechanism in thermal recovery by steam is the reduction in the viscosity of oil.
The gas-injection (miscible/immiscible) process uses gases such as carbon dioxide, nitrogen, flue gas and hydrocarbon gas to dissolve in the oil to reduce its viscosity and increase its flow rate. CO₂ gas injection is commonly used due to its relatively low minimum miscibility pressure (MMP) and is applicable in a broad range of reservoirs. Gas injection can be combined with foam. The application of foam can reduce gas mobility and thereby increase sweep efficiency (Rossen, 1996). Foam is made from a mixture of surfactant, water and gas. Another EOR method is chemical (micellar-polymer, alkaline) injection which can increase capillary number and thereby reduce residual oil saturation.

Not all reservoirs are susceptible to EOR. There are several factors that influence the performance of EOR, e.g. the initial oil and water saturations at the beginning of the EOR process, permeability and rock type. For example, foam can collapse if it is in the presence of high oil saturation or low water saturation. Since the EOR method is relatively very expensive compared to primary and secondary recovery, it is very important to identify suitable candidate methods of EOR for each reservoir.

1.2 Foam EOR in Porous Media
Foam is divided into two categories, foam in bulk and foam in porous media. A bulk foam is a foam in a container much larger than individual bubbles, a common example is shaving...
cream. In the other hand, foam in porous media is defined as a dispersion of gas in liquid such that the liquid phase is interconnected and at least some of the gas flow paths through the pore space are blocked by lamellae, i.e. thin liquid films (Falls et al., 1988). The foam is a dispersion of surfactant solution and gas. A gas displacement process can be improved if the mobility of gas can be reduced and this can be achieved by foam, which is very effective in controlling gas mobility.

There are two types of foams in porous media. The first is a continuous-gas foam, in which there exists at least one pathway for gas flow in the porous medium that is unblocked by lamellae. The second is a discontinuous-gas foam which all the gas phase is made discontinuous by lamellae. These two types of foam are shown in Fig. 2.

![Image](image.png)

*Figure 2. Schematic of continuous and discontinuous-gas foams (Falls et al., 1988)*

Foam flow can be divided into two regimes according to the foam quality, $f_g$. Foam quality is the gas volume fraction of the total fluid flow rate. In the low-quality regime, the pressure gradient is almost independent of liquid flow rate. The low-quality regime is shear thinning as a function of overall flow rate. It is understood that in this regime bubble size is fixed, but water saturation does change with flow rates (Rossen and Wang, 1999; Alvarez et al., 2001). In the high-quality regime, the pressure gradient is almost independent of gas flow rate. In this regime, capillary pressure remains at the “limiting capillary pressure” and water saturation remains at the water saturation corresponding to the limiting capillary pressure. Previous studies found that the two regimes are dominated by different mechanisms: the low-quality regime by bubble trapping and mobilization and the high-quality regime by bubble coalescence near the limiting capillary pressure (Khatib et al., 1988; Rossen and Wang, 1999; Alvarez et al., 2001). An important characteristic of foam is foam texture, which refers to size of foam bubbles. Finely textured foam with small bubbles is referred to as strong foam because it reduces gas mobility significantly. Coarsely textured foam, with large bubbles, is often referred to as weak foam, which reduces gas mobility less than strong foam.

Foam EOR will have a successful displacement under favorable reservoir conditions for stable foam lamellae. The stability of foam in porous media essentially relies on the stability of the foam lamellae. Foam can be weakened or even destroyed at high oil saturations or
low water saturations (high capillary pressure). Foam that is collapsed completely will allow the injected gas to break through immediately to the production well and will not increase oil recovery. Surfactant selection is also an important factor to produce stable foam in a certain reservoir. Experiments using real reservoir fluid can help to determine factors that can increase the foam stability.

1.2.1 Foam Generation and Lamellae Creation
Foam generation requires the creation of lamellae. Fundamentally, there are three mechanism of lamellae creation: lamellae division, snap-off and leave-behind (Ransohoff and Radke, 1988, Rossen, 1996). Lamellae division happens by subdividing existing foam bubbles (lamellae) when a moving lamella enters a pore body with several pore throats. Lamellae will stretch, break and deposit new lamellae in each unblocked pore throat. The snap-off mechanism creates lamellae when liquid accumulates in a narrow pore throat and then bridges the throat: a new interface is created in the throat. It requires high capillary pressure, so gas can occupy the throat, followed by low capillary pressure to allow liquid to reinvade the throat. Another mechanism of foam generation is leave-behind. It creates lamellae in the throat between adjacent pore bodies when the adjacent pore bodies are entered by gas from separate directions. It occurs only during drainage.

1.2.2 Foam Stability
Foam stability depends on the stability of foam films or lamellae. Disjoining pressure is one of the main mechanisms that affect foam stability (Farajzadeh et al., 2012). The disjoining pressure is the force (per film area) deriving from the variation of free energy (per film area) with the film thickness. At equilibrium, the disjoining pressure of a flat film, \( \Pi \), equals the capillary pressure as in Eq. 1. \( \Pi_{\text{VW}} \) is van der Waals component and \( \Pi_{\text{EL}} \) is electrostatic component. A positive value of disjoining pressure gives a stable film while a negative value produces an unstable film (Farajzadeh et al., 2012):

\[
\Pi = \Pi_{\text{EL}} - \Pi_{\text{VW}} = P_c - \frac{\Delta G}{A} = \frac{2\sigma}{r} - \frac{\Delta G}{A} = \frac{2\sigma}{r} - \frac{\Delta G}{2\pi r^2}
\]

The limiting capillary pressure, \( P^*_c \), is the maximum capillary pressure that a film can withstand – it is the maximum value of \( \Pi \). The limiting capillary pressure is a function of the chemistry of the surfactant solution, salinity, temperature, and rock morphology, among other factors (Khatib et al., 1988). In water-wet porous media, capillary pressure increases with decreasing water saturation. When the capillary pressure approaches the limiting capillary pressure, the lamellae throughout the foam become unstable and this can lead to destruction of foam.

1.2.3 Foam Destruction
The primary mechanism for foam destruction in porous media is capillary suction \( P_c \) (Kovscek and Radke, 1994). In porous media, capillary pressure depends on the wetting-liquid saturation and surface tension. The rate of foam coalescence increases as gas-water capillary
pressure approaches limiting capillary pressure. When $P^*_c$ is reached, no foam bubbles can be sustained and coalescence is catastrophic.

1.3 Effect of Oil on Foam

The stability of foam in porous media depends on the stability of wetting, asymmetric and symmetric films (lamellae) in the porous media containing oil as shown in Fig. 3. The wetting films of water on the rock can be stable only if the rock is water-wet. The lamellae are stabilized by the surfactant molecules at the interface.

![Figure 3. Foam in porous media with oil present (Farajzadeh et al., 2012)](image)

The application of foam for EOR relies on the stability of the foam films (lamellae) when the foam contacts oil. Oil influences foam performance as a de-foaming agent by affecting lamellae stability. The de-foaming effect of oil is usually explained by the surface activity of the oil or dewetting of the oil by the aqueous solution. In general, there are two possible mechanisms by which an oil phase interacts with foam films; oil penetrates into the foam film and destroys it by bridging, or the foam film slides over a film of water covering the oil where new interface is created, an asymmetric film (Farajzadeh et al., 2012).

Bridging of foam films by oil droplets occurs if the (non-spreading) oil droplet attaches to both film surfaces. This means that the oil will encounter other surface and create a bridge across the lamella. The stability of the lamella then depends on the angle contact between the oil droplet and the surfactant solution, $\theta_{ow}$. The film will rupture if $\theta_{ow}$ larger than $90^0$ as shown in Fig. 4.
Figure 4. Bridging mechanism of oil droplet across film (lamellae). The left plot illustrates a condition with $\theta_{ow} < 90^\circ$ and the right plot $\theta_{ow} > 90^\circ$ (Farajzadeh et al., 2012)

The effect of oil bridging on foam stability is quantified by the bridging coefficient, $B$ (Eq. 2). In this equation, $\sigma$ is the interfacial tension and subscripts $o$, $g$ and $w$ stand for oil, gas and water respectively. A stable film will have negative $B$ values while unstable film is when $B$ is positive or zero values. (Farajzadeh et al., 2012).

$$B = \sigma_{wg}^2 + \sigma_{ow}^2 - \sigma_{og}^2$$

Another mechanism that can break the film is when the oil enters the gas-water interface and oil contacts gas directly which is called the pinch-off mechanism. This happens when the bridging coefficient, $B$, is positive and the configuration of the asymmetric film becomes unstable as shown in Fig. 5. The two gas-water-oil three-phase contact lines will move and meet each other. At this stage, the foam film will pinch off and the bubbles coalesce into one bubble (Farajzadeh et al., 2012).

Figure 5. Pinch-off mechanism: a) asymmetric film is stable, b) asymmetric film is unstable and oil contacts to gas directly and c) lamella pinches off and bubbles coalesce (Farajzadeh et al., 2012)
1.4 Foam Modeling in STARS™

STARS™ is a widely used commercial foam simulator. It fits foam behavior in both the high- and low-quality foam-flow regimes reasonably well (Cheng et al., 2000). STARS™ includes a population-balance option that does not concern us here. It is also includes a local-equilibrium model (Rossen et al., 1999) which represent the effects of bubble size implicitly in factors for reducing gas mobility as a function of phase saturations, compositions and other properties. This is the version of STARS™ model we use here.

In STARS™, the foam model modifies the gas relative permeability for the effect of foam by multiplication by a factor $FM$ which is inversely proportional to several other factors as in Eqs. 3 and 4.

$$k_{rg}^f = k_{rg}^o (S_w) FM$$  \hspace{1cm} \text{Eq. 3}$$

$$FM = \frac{1}{(1 + f_{mmob} F1 F2 F3 F4 F5 F6 f_{dry})}$$  \hspace{1cm} \text{Eq. 4}$$

$f_{mmob}$ describes the normalized resistance to flow of a foam of minimum-size bubbles, in the absence of factors increasing bubble size. $k_{rg}^f$ is the gas relative permeability in the presence of foam and $k_{rg}^o$ is the gas relative permeability in the absence of foam. $F1$ (Eq. 5) represents the effects of surfactant concentration; surfactant mole fraction in the aqueous phase is divided by parameter $f_{msurf}$ and then raised to power $ep_{surf}$ (Eq.5).

$F2$ (Eq. 6) describes the effect of oil saturation. $f_{moil}$ is a critical oil saturation, $ep_{oil}$ is exponent for oil saturation and $f_{oil}$ is lower oil saturation value; foam is completely destroyed at oil saturations greater than $f_{moil}$, and there is no effect of oil on foam for oil saturations less than $f_{oil}$. $F3$ represents the shear-thinning effect of pressure gradient on gas mobility in the low-quality regime, as represented in Eqs. 7 and 8. $f_{mcap}$ is a reference capillary number value and $ep_{cap}$ is the exponent for the effect of capillary number on gas mobility. $N_{ca}$ is capillary number, $k$ is absolute permeability, $\nabla p$ is pressure gradient and $\sigma_{wg}$ is water-gas surface tension. $F4$, $F5$ and $F6$ were set equal to one throughout this study (Cheng et al., 2000, Computer Modeling Group Ltd, 2011).

$$F1 = \left(\frac{\text{surfactant mole fraction}}{f_{msurf}}\right)^{ep_{surf}}$$ \hspace{1cm} \text{Eq. 5}$$

$$F2 = \left(\frac{f_{moil} - \text{oil saturation}}{f_{moil} - f_{oil}}\right)^{ep_{oil}}$$ \hspace{1cm} \text{Eq. 6}$$

$$F3 = \left(\frac{f_{mcap}}{N_{ca}}\right)^{ep_{cap}}$$ \hspace{1cm} \text{Eq. 7}$$

$$N_{ca} = \left(\frac{k \nabla p}{\sigma_{wg}}\right)$$ \hspace{1cm} \text{Eq. 8}$$
The effect of foam dryout at the limiting capillary pressure is accounted for by $f_{dry}$. $f_{dry}$ as shown in Eq. 9 controls gas mobility as water saturation decreases in the vicinity of $s_{dry}$. $s_{bet}$ controls the abruptness of the increase of gas mobility in the vicinity of water saturation $s_{dry}$ (Cheng et al., 2000, Computer Modeling Group Ltd, 2011):

$$f_{dry} = 0.5 + \frac{\arctan(s_{bet} (S_w-s_{dry}))}{\pi}$$ \hspace{1cm} (9)

If $FM$ equals unity then it corresponds to a situation with no foam. This can be achieved when at least one of the foam interpolation factors ($F1-F6$ or $f_{dry}$) is equal to zero. Foam is at maximum strength when the value of $FM < 0$.

1.5 Research Background and Objective

This thesis is in two parts. The first part concerns modeling of gravity override with foam; the motivation is given in the next chapter. The second part concerns the effect of oil on foam. The effect of oil on foam is only partially understood, and usually modeled as in one of two ways: (a) foam collapses at a smaller capillary pressure (higher water saturation) if oil present, or (b) the properties of full-strength foam are altered by oil. Eq. 6 represents the second sort of model. It is not known what difference it makes how this effect is represented.

In the second part, this thesis involves computer simulations of foam processes with oil present, using the current models for foam and the effect of oil on foam, as embodied in the simulator STARS$^{TM}$ of the Computer Modeling Group (Liu et al., 2011). It attempts to incorporate the insights of fractional-flow modeling of these processes, without attempting the full, extremely complex fractional-flow solution (Namdar Zanganeh et al., 2011).

In particular, we attempt to relate a plot of gas mobility as a function of oil and water saturations to the displacement path on the ternary diagram. Specifically, we compare plots of gas mobility-reduction factor with foam, plotted on a ternary composition diagram, and with the composition paths of displacements simulated using the same model parameters. Our hypothesis is that a relatively simple correlation between the plot of gas mobility on the ternary diagram and the displacement path on the same diagram can be shown, without having to undertake the complexity of three-phase fractional-flow solutions.
CHAPTER 2: GRAVITY OVERRIDE IN SAG FOAM PROCESS

2.1 Introduction
This chapter covers the modeling of gravity override in SAG foam process. We begin by extending the numerical calculations using a simple model for gravity override developed by De Velde Harsenhorst (2012). This model in turn is an extension of the earlier model of Shan and Rossen (2004), which assumed equal vertical and horizontal permeabilities, to unequal permeabilities ($0 < k_v < k_h$). We find numerical problems in the case $0 < k_v < k_h$, where the shape of the gas front with foam becomes unstable. There are several numerical approaches carried out to attempt to keep this instability under control. These are explained in section 2.3.

We also illustrate the usefulness of this model using a reservoir from the North Sea, simulating a foam process in that reservoir using the STARS™ simulator. This example shows that foam could prevent gravity segregation in surfactant-alternating-gas (SAG) injection over a distance of 6 km, as predicted by the model. Furthermore, we compare gravity segregation with SAG to that with continuous foam injection and check whether distance to complete gravity segregation in the simulation is similar to the length predicted by formulas derived by Rossen and Shen (2007) and Van der Bol (2007).

We also test the model under more challenging circumstances where gravity segregation is more severe, by increasing the dimensionless thickness of the reservoir ($Z_D$). The fit is reasonably good but the model prediction that segregation is worse with lower vertical permeability is not confirmed by the simulations with the first foam model employed. We then test the simple model using foam parameters recently fit to laboratory foam data by Rossen and Boeije (2013) and obtain a better fit; not quantitative, but confirming a surprising prediction of the model about the effect of vertical permeability. Finally, we employ an unrealistic foam model that does, however, more perfectly fits the simple model assumptions and obtain a somewhat better fit to simple model. The failure of the simulator to fit the model predictions more perfectly reflects, at least in part, the difference between the STARS foam model and that assumed in the model of De Velde Harsenhorst (2012).

2.2 SAG Foam Process
Gravity override, with or without foam, is the result of competition between horizontal pressure gradient and gravity (and density difference). Foam is intended to increase the production of hydrocarbon in a reservoir by causing gas to sweep the entire vertical reservoir interval. There are two ways to inject the surfactant and gas into the reservoir to form the foam. The first method is continuous foam injection where the surfactant and gas are injected simultaneously. The second method is injecting alternating slugs of surfactant solution and gas, commonly called as surfactant-alternating-gas (SAG) injection.
Continuous foam injection should be used if one wants to create and maintain foam in the near-well region, or to ensure that both gas and liquid enter the same zones in a heterogeneous reservoir (Shan and Rossen, 2004). The only way to improve vertical sweep efficiency with continuous foam injection, however, is by raising injection well pressure (Rossen et al., 2010). In such a process, this could fracture the formation if the pressure exceeds the formation fracture pressure.

In SAG foam process, the fluids can be injected at fixed injection rates or fixed injection pressure. A foam process employing SAG injection at fixed injection pressure controls gravity override better than continuous foam injection or SAG at fixed injection rate according to numerical simulations (Shi and Rossen, 1998). A SAG foam process can prevent excessive injection pressure in the reservoir that can lead to formation damage or even fracturing. This is a major advantage of SAG foam process compared to continuous foam injection. In particular, a process with a single large slug of surfactant followed by a large slug of gas shows both good injectivity and success in overcoming gravity segregation in homogeneous formations (Shan and Rossen, 2004).

Shan and Rossen (2004) extend the study of gravity override in SAG displacements using fractional-flow analysis, which shows that most of the well-to-well pressure drop is focused near the displacement front, where sweep efficiency is determined. There are several assumptions made in the study, including incompressible phases, Newtonian mobilities, one-dimensional displacement, absence of dispersion, absence of gradients of capillary pressure, absence of viscous fingering and immediate attainment of local steady state. In fractional-flow analysis, one plots fractional-flow curves from the mobilities of the individual phases, and then time-distance diagrams for a given displacement.

![Figure 6. Fractional-flow curves for foams of two different surfactant concentrations and construction of the shock for gas injection in a SAG process with 0.02% surfactant in the aqueous phase (Shan and Rossen, 2004)](image-url)
Shan and Rossen (2004) show fractional-flow curves for foam at two surfactant concentrations based on a model from Fisher et al. (1990) as shown in Fig. 6. There is a shock front from 100% liquid saturation at the initial condition to a point at extremely low water fractional flow, \( f_w \). The dotted line represents this shock front for gas injection into the surfactant bank with 0.02% surfactant concentration.

In a SAG foam process, when the gas is injected into the reservoir, the water saturation decreases and total mobility increases as one move upstream from the shock toward the injection well. Shan and Rossen (2004) show the total relative mobility as a function of position after 0.2 PV gas injection into a large bank of surfactant in a cylindrical reservoir, using the foam model of Fisher et al. (1990) (Fig. 7). The horizontal axis is position, \( r/R_e \), with \( r \) as radial position and \( R_e \) outer radius of the cylindrical reservoir. The shock front is approximately at position \( r/R_e = 0.5 \), where the total mobility behind this point is very low. The single grid block with the extraordinarily low mobility is a numerical artifact (Rossen et al., 1999), but behind this is a spreading wave with low mobility at the front and increasing mobility back toward the well. The increasing total mobility toward the injection well helps to avoid fracturing the reservoir formation. A SAG foam process with fixed injection pressure gives relatively high mobility ahead of and behind the displacement front and focuses on the reservoir pressure drop at the displacement front to fight gravity override.

![Figure 7. Total relative mobility after 0.2 PV gas injection in a SAG process (Shan and Rossen, 2004)](image)

The time-distance diagram for a SAG foam process in Fig. 7 is shown in Fig. 8 with the initial reservoir condition of 100% liquid saturation in a 1D reservoir. The vertical axis is dimensionless position, \( x_d \) (for a rectangular reservoir, \( x/L \)), and for a cylindrical reservoir, \((r/R_e)^2\). \( L \) is the length of reservoir and \( x \) is horizontal position. The circled values in Fig. 8 are the total relative mobility in \((\text{Pa.s})^{-1}\) for the given characteristic (dotted line) and the boxed
value is the total relative mobility of the initial saturation of the reservoir, i.e. 100% water. The total-relative-mobility values are bit lower at the leading edge of the gas bank than in the water bank ahead but then increase significantly in the spreading wave behind that.

Figure 8. Time-distance diagram for SAG foam process based on Fisher et al. model with 0.02% surfactant in the aqueous phase (Shan and Rossen, 2004)

2.3 A Simplified Model for Gas Injection in SAG Process
Shan and Rossen created an idealized model for SAG displacements (2004) for a case where horizontal and vertical permeabilities are equal. Fig. 9 shows a schematic model of Shan and Rossen (2004) where several assumptions are made. In this model, the front is compressed to a thickness of \( \tau \) which is assumed negligible on the scale of the displacement; within this front gas relative mobility is \( \lambda_{rf} \).

The foam front moves at an angle \( \alpha \) to the horizontal, perpendicular to its current profile. The total relative mobility ahead and behind this front is infinite, and therefore pressure is at hydrostatic equilibrium in the gas phase behind the front and also in the water bank ahead of the front. The densities of water and gas are assumed to be uniform and constant. The water saturation is uniform at \( S_{wf} \) in the gas bank and 1 in the water bank. \( H \) is the thickness of the reservoir and \( z \) is the position in vertical direction relative to the top of the formation.

Fig. 10 shows a schematic of the total relative mobility profile in the idealized model of Shan and Rossen for SAG displacements, for comparison with Fig. 7. It assumes that only the narrow front has very low mobility.
Shan and Rossen (2004) derived their idealized model for SAG displacements assuming horizontal permeability, $k_h$, is equal to vertical permeability, $k_v$ (Fig. 9). This derivation is presented in Appendix A.1. De Velde Harsenhorst (2012) extended this derivation for the cases $0 < k_v < k_h$, $k_v = 0$ and also derived an analytical solution for case $k_v = 0$. These derivations are presented in Appendix A.2, A.3 and A.4. Then, the model calculations are
built in an MS Excel spreadsheet. The results show that the foam displacement front is more convex with increasing \( k_v \). Although there are instabilities in the case of \( k_v = k_h \) they can be smoothed out by using fewer points along the front, adding in points by linear interpolation when necessary and smoothing the front. Fig. 11 shows the analytical and numerical solution for the extreme cases, \( k_v = 0 \) and \( k_v = k_h \), taken from De Velde Harsenhorst et al. (2013).

![Figure 11. Analytical solution for case \( k_v = 0 \) and numerical solution for case \( k_v = k_h \) (De Velde Harsenhorst et al., 2013)](image)

### 2.4 Numerical Calculations for the Case \( 0 < k_v < k_h \)

We perform numerical calculations for case \( 0 < k_v < k_h \) in an MS Excel spreadsheet and apply several numerical approaches to keep instability under control. The numerical calculation method for case \( 0 < k_v < k_h \) is presented in Appendix B. One example spreadsheet is shown in Fig. B1.

Unlike the case of \( k_v = k_h \), The movement of the front is not perpendicular to its slope. There is new parameter, angle \( \beta \), the direction the given point on the front is moving relative to horizontal. Angle \( \alpha \) is perpendicular to the front (Fig. A1). We track numerically the movement of points initially at \( X_D = 0 \) and various values of \( 0 \leq Z_D \leq 1 \). At a given time, for a given point \( i \), we evaluate angle \( \alpha \) from the position of the given point and an adjacent point; we evaluate \( \beta \) from the movement of the given point in the previous time step. This approach could lead to numerical problems if one point is not in its correct position, which affects its own value of \( \alpha \) and those of adjacent points, and the error grows in time. We applied a number of numerical approaches to keep this instability under control in the numerical calculations. Fig. 13 shows the final result for case \( k_v/k_h = 0.01 \) and 0.1 after we had applied these numerical approaches.

There are several numerical approaches applied in this calculation:

1. \( X_D \) values for first time step are calculated from analytical solution as in Eq. A.38.
2. The top point \((Z_D \) near zero) does not remain at the \( Z_D = 0 \) but starts to move downward in each time step. This is because we use the point below the given point...
rather than the point above in determining \( \alpha \). This means that we add in a new top point in each time step. We calculate the position of the new top point by assuming that \( \frac{dX_D}{dt_D} \), \( S_D \) and \( X_D \) have the same values as for the top point in the previous time step, while \( \frac{dZ_D}{dt_D} \) and \( Z_D \) are zero for the new point. This is shown in column C, D, G, H and I with green background color in Fig. B1.

3. The angles \( \alpha \) and \( \beta \) for the top point are assumed to be zero. This is shown in column J and K with green background color in Fig. B1.

4. We also apply smoothing operations for \( X_D \), \( Z_D \), \( \alpha \) and \( \beta \) using the same technique from De Velde Harsenhorst (2012). Specifically, we check the trends in \( X_D \), \( Z_D \), \( \alpha \) and \( \beta \). If any of these trends are not monotonic at a given point, the values of \( X_D \), \( Z_D \), \( \alpha \) and \( \beta \) for this point are replaced by the average of the values of the points above and below the given point.

We used three ratios of vertical to horizontal permeability in the numerical calculations for cases with \( 0 < k_v < k_h \); \( k_v/k_h = 0.01, 0.1 \) and \( 0.33 \). The results for the foam front from the model are shown in Fig. 12 for dimensionless times up to \( t_D = 8 \) and dimensionless distance in \( x \) direction up to \( X_D = 4 \).

![Figure 12. Foam front resulting from numerical calculations for \( k_v/k_h = 0.01, 0.1 \) and \( 0.33 \)](image)

There are instabilities in the results, which are more severe for higher ratio \( k_v/k_h \) and dimensionless time. The instability can be caused by the incorrect angle \( \alpha \) at some point and
creates systematic error in the numerical calculation. We then further adjust the numerical approaches to fix the instability:

1. We add the top point only if $Z_D$ for the current top point has a value greater than 0.01.
2. For the specific case in $k_v/k_h = 0.1$, we make a second check for monotonicity in $\alpha$. If $\alpha$ is still not monotonic we do a second interpolation between the adjusted values of $X_D$, $Z_D$, $\alpha$ and $\beta$. We do not apply this for case $k_v/k_h = 0.01$ since $\alpha$ is still monotonic until $t_D = 8$.

The foam front calculated with these approaches is shown for various dimensionless times in Fig. 13. The instability is reduced with the new numerical approaches. We produced results for case $k_v/k_h = 0.01$ and 0.1. However, there is still instability for $k_v/k_h = 0.33$ but it is less severe compared to the same case in Fig. 12. We are unable to obtain stable solution for $k_v/k_h = 0.33$.

![Figure 13. Foam front resulting from numerical calculations for $k_v/k_h = 0.01$, 0.1 and 0.33 (new numerical approach)](image)

Fig. 14 compares foam fronts for $k_v/k_h = 0$, 0.01, 0.1 and 1 at dimensionless time $t_D = 8$. The results for $k_v/k_h = 0$ and 1, are taken from De Velde Harsenhorst (2012). One concludes from this model that SAG foam process with increasing of vertical permeability can reduce gravity override. The reason is that foam pushes gas downward as well as forward across the tilted front.
Figure 14. Comparison of foam fronts at dimensionless time $t_D = 8$ for $k_v/k_h = 0, 0.01, 0.1$ and 1

2.5 North Sea Model in STARS™

We use an example from a North Sea reservoir using STARS™ simulation software to test whether foam could prevent gravity segregation over a distance of several kilometers, and compare results to the prediction of the simple model. The reservoir is in sandstones in the Rogn formation of Late Jurassic age. The main reservoir has depth about 1900 m and is relatively homogeneous. The under-saturated light oil column has 30 m of thickness and the permeability is very high. CO₂ foam was proposed to improve sweep efficiency in gas flooding in this field. There is 6 km distance between injection and production well and the permeability ratio in this example is $k_v/k_h = 0.613$. The main reservoir and fluid properties are shown in Table 1.

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reservoir dimensions, m; grid</td>
<td>6000 x 600 x 30; 40 x 1 x 30</td>
</tr>
<tr>
<td>Initial pressure, bar</td>
<td>165.8</td>
</tr>
<tr>
<td>Temperature, °C</td>
<td>71</td>
</tr>
<tr>
<td>Horizontal permeability $k_h$, md</td>
<td>7000</td>
</tr>
<tr>
<td>Vertical permeability $k_v$, md</td>
<td>4289</td>
</tr>
<tr>
<td>Porosity</td>
<td>0.35</td>
</tr>
<tr>
<td>Oil viscosity, cp</td>
<td>0.69 (165 bar); 0.58 (252 bar)</td>
</tr>
<tr>
<td>Water viscosity, cp</td>
<td>0.45 (165 bar); 0.40 (252 bar)</td>
</tr>
<tr>
<td>CO₂ viscosity, cp</td>
<td>0.041 (165 bar); 0.045 (252 bar)</td>
</tr>
<tr>
<td>Oil density, kg/m³</td>
<td>747 (165 bar); 832 (252 bar)</td>
</tr>
<tr>
<td>Water density, kg/m³</td>
<td>1017 (165 bar); 981 (252 bar)</td>
</tr>
<tr>
<td>CO₂ density, kg/m³</td>
<td>514 (165 bar); 686 (252 bar)</td>
</tr>
</tbody>
</table>

There are 13 oil components in the STARS™ model for this field and the composition of the crude oil from this reservoir is shown in Table 2. The reservoir is isothermal with
temperature 71 °C. The well model used is the Peaceman well model (Computer Modeling Group, 2011). The maximum bottom-hole injection pressure is 252 bar (2.52 x 10^7 Pa) due to expected fracturing pressure, and the production-well bottom-hole pressure is set at 165 bar (1.65 x 10^7 Pa). Table 3 shows the set of Corey relative-permeability parameters used in the simulation: input residual saturations, Corey exponents and endpoint relative permeabilities.

Table 2. Composition of crude oil from North Sea reservoir

<table>
<thead>
<tr>
<th>Component</th>
<th>Mole %</th>
<th>Component</th>
<th>Mole %</th>
</tr>
</thead>
<tbody>
<tr>
<td>CO₂</td>
<td>1.09</td>
<td>C₇-C₈</td>
<td>14.62</td>
</tr>
<tr>
<td>C₁</td>
<td>16.67</td>
<td>C₉-C₁₂</td>
<td>11.86</td>
</tr>
<tr>
<td>C₂</td>
<td>4.76</td>
<td>C₁₃-C₁₈</td>
<td>10.9</td>
</tr>
<tr>
<td>C₃</td>
<td>9.08</td>
<td>C₁₉-C₂₄</td>
<td>5.4</td>
</tr>
<tr>
<td>C₄</td>
<td>8.52</td>
<td>C₂₅-C₃₄</td>
<td>2.89</td>
</tr>
<tr>
<td>C₅</td>
<td>6.56</td>
<td>C₃₅+</td>
<td>2.87</td>
</tr>
<tr>
<td>C₆</td>
<td>4.76</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 3. Residual saturations, Corey exponents and endpoint relative permeabilities

<table>
<thead>
<tr>
<th>Sₜₗ₀</th>
<th>nₜ₀</th>
<th>Kₗ₀</th>
<th>1.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1</td>
<td>1.85</td>
<td>Kₗ₀</td>
<td>1.0</td>
</tr>
<tr>
<td>0.18</td>
<td>3.20</td>
<td>Kₗ₀</td>
<td>1.0</td>
</tr>
<tr>
<td>0.1</td>
<td>1.0</td>
<td>Kₗ₀</td>
<td>1.0</td>
</tr>
<tr>
<td>0.0</td>
<td>1.0</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The foam model parameters are shown in Table 4. Although the fₘₐₚₒᵦ value used in the simulation is relatively small compared to some other studies (Namdar Zanganeh, 2011; Liu et al., 2011), the foam works very well as shown in the simulation result in the next section. The other foam parameters used are quite optimistic values, for example fₘₒᵦ = 0.5 (foam is completely destroyed only for oil saturation above 0.5). Moreover, the displacement is miscible, so oil is displaced ahead of the foam. Foam weakens in the vicinity of a water saturation of 0.25 (fₘₖᵦ).

Table 4. Foam model parameters in North Sea example

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Corresponding exponent</th>
</tr>
</thead>
<tbody>
<tr>
<td>fₘₐₚₒᵦ, maximum mobility reduction factor</td>
<td>100</td>
<td></td>
</tr>
<tr>
<td>fₘₛᵤᵦ, critical surfactant concentration, mole fraction</td>
<td>0.00005</td>
<td>1 (epsₘᵤᵦ)</td>
</tr>
<tr>
<td>fₘₖₑᵦ, reference capillary number</td>
<td>0.0002</td>
<td>1 (eₘₖₑᵦ)</td>
</tr>
<tr>
<td>fₘₒᵦ, critical oil saturation: no foam when Sₜ₀-fₘₒᵦ</td>
<td>0.5</td>
<td>1 (eₘₒᵦ)</td>
</tr>
<tr>
<td>fₘₖᵦ, critical water saturation: no foam when Sₘ₀-fₘₖᵦ</td>
<td>0.25</td>
<td>500 (eₘₖᵦ)</td>
</tr>
</tbody>
</table>
2.5.1 SAG Foam Injection

The reservoir is initially at oil saturation of 0.9 and an initial water saturation of 0.1. Waterflood is performed in the reservoir for secondary recovery to displace oil for 5 years and then followed by surfactant injection for 6 years. At this stage, the maximum oil saturation in the reservoir is 0.18. This can prevent an adverse effect of oil on foam and focus the simulation result on the issue of gravity segregation in the SAG foam process. CO$_2$ is injected from the same well for 15 years. We neglect many practical considerations, including the cost of surfactant and supply of gas for this process. The injection-well pressure is set at 252 bar (2.52 x 10$^7$ Pa) maximum bottom-hole pressure and the production well is set at 165 bar (1.65 x 10$^7$ Pa) minimum-bottom hole pressure. This gives a difference in pressure of 87 bar (8.7 x 10$^6$ Pa) between injector and producer. The density difference between brine and CO$_2$ is between 503 and 295 kg/m$^3$, with average value of 399 kg/m$^3$. The reservoir thickness is 30 m and well-to-well distance is 6000 m (6 km). Using Eq. A.19, we calculate the dimensionless thickness $Z_D = 0.0135$ and well-to-well distance $X_D = 2.70$. Based on the simple model (Fig. 11 and 13), a foam SAG process could prevent gravity segregation for any value of $k_v/k_h$.

Fig. 15 shows the simulation result from STARS$^\text{TM}$. A foam bank approaches the production well with a distance of 6 km from injection well without appreciable gravity segregation. This confirms the prediction from the simple model discussed in the previous section, i.e. that foam could prevent gravity segregation over a distance of several km.

![Figure 15. CO$_2$ SAG foam process simulated for North Sea reservoir. The distance foam has traveled here is almost 6 km.](image)

The CO$_2$-oil displacement process is miscible in the simulation. The assumption of the simple model that mobility at the leading edge of the gas bank is much less than that ahead of the
bank and near the well does not hold perfectly in the example. Total relative mobility of the water ahead of the foam bank is 1539 (Pa s)$^{-1}$, at the leading edge of the foam bank 99 (Pa s)$^{-1}$ and at the injection well after 15 years of CO$_2$ injection is 1700 (Pa s)$^{-1}$. Total relative mobility would be close to 25,000 (Pa s)$^{-1}$ near the well if foam were completely broken at the dry conditions at the well. Although this assumption does not hold, the model predicts accurately that gravity segregation is not significant over a distance of 6 km.

### 2.5.2 Continuous Foam Injection

Shan and Rossen (2004) showed that foam processes employing continuous foam injection control gravity override worse than SAG injection at fixed injection pressure; cf. Shi and Rossen (1998). Stone (1982) assumed that upon uniform co-injection of gas and water at steady state the reservoir divides into three regions of uniform saturation with sharp boundaries between them; an override zone with only gas flowing, a mixed zone with both gas and water flowing and an underride zone with only water flowing.

We check this finding using the North Sea model, simulating continuous foam injection in the reservoir using STARS$^\text{TM}$ as shown in Fig. 16. For simplicity in this case, the reservoir is initially at water saturation of 1 and no oil is present. The pressure difference is maintained at 87 bar. In the simulation, two separate wells inject water and gas at fixed injection pressure. Both injection wells were set at 252 bar bottom hole pressure. The production well was constrained to 165 bar bottom hole pressure. The ratio of gas to water volumetric injection rate is 4 : 1, and injection rate was uniform along each well. Complete segregation occurred at approximately at about 600 m (or possibly at 450 m (3 grid blocks)) at steady state.

![Figure 16. Continuous foam injection process simulated for North Sea reservoir](image-url)
Rossen and Shen (2007) derived a relation between injection pressure and distance to complete segregation in a cylindrical reservoir with continuous co-injection of water and gas. There are two extreme cases for the derivation. The first is where the height of the mixed zone decreases negligibly with increasing distance \((r \text{ or } x)\) from the well and superficial velocity \(U\) is proportional to \((1-(x/L_g))\), where \(L_g\) is the complete segregation distance. The other case is where the height of the mixed zone shrinks proportionally to total flow rate \(Q\) and superficial velocity \(U\) is uniform in the mixed zone. Van der Bol (2007) derived the relation between injection pressure and segregation length for the same two cases in a rectangular reservoir analogous to the derivation of Rossen and Shen (2007). The first case is applied to our study since the mobility of the mixed zone is very low. The relation of this case is shown in Eq. 10.

\[
p(L_w) - p(L_g) = \frac{k_z(\rho_w - \rho_g)g}{2Hk_h} L_g^2 \quad \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots (10)
\]

where \(p(L_w)\) is the pressure at well radius \(L_w\) and \(p(L_g)\) is the pressure at the segregation distance \(L_g\). The difference of these pressures is 87 bar \((8.7 \times 10^6 \text{ Pa})\) in this case. \(k_z\) is equal to \(k_v\) and \(H\) is the thickness of the reservoir, 30 m. Using the average density \(\text{CO}_2\) of 399 kg/m\(^3\), we calculate complete segregation distance of North Sea example with continuous foam injection from Eq. 25, as \(L_g = 466.5\) m.

There is 22% of difference in segregation length from the simulation result (depending on which grid block one selects as the segregation distance). Nonetheless, the model check for continuous foam injection in North Sea example using STARS\textsuperscript{TM} confirms complete segregation over a relatively short distance, compared to the success of constant-pressure SAG injection.

### 2.6 Model Checks for Large \(Z_D\)

In the previous simulations, the dimensionless reservoir thickness \(Z_D\) was relatively small \((0.0135)\) compared to that in Figs. 11 and 13, and the previous simulation results were analyzed qualitatively. In this section, we check the model by increasing the value of \(Z_D\) toward 1 in the reservoir and then compare the foam-front profile from STARS\textsuperscript{TM} with the result in Fig. 13. To do this we must change the well-to-well pressure difference \((\Delta P)\) and the distance between injector and producer \((x)\).

There are three set of simulations for this model check with large \(Z_D\). The main difference between these simulations is the relative-permeability model we use in the simulations. In the first set of simulations, we use the same relative-permeability model as in the previous simulations using the North Sea model parameters and the STARS\textsuperscript{TM} foam model. As noted previously, with a small mobility reduction with foam \((\text{fmmob} = 100)\), this fits the assumptions of the idealized model poorly. Then, we perform simulations with an alternate
set of foam parameters, fit to laboratory data of Persoff et al. (1991); we call this the Persoff model. Finally, we employ a relative-permeability model which we call the Frontal model. In the Frontal model, we incorporate a foam gas-mobility reduction directly into an alternate relative-permeability function, so that we fit the assumptions of the idealized model very well. The results of these simulations using STARS™ are compared below with the results we have from idealized model.

2.6.1 North Sea Model
In these simulations, we reduce the reservoir length in the x direction to 105 m and reduce the well-to-well pressure difference ($\Delta P$) to 1.5 bar ($1.5 \times 10^5$ Pa), which gives to a much larger value of $Z_D$. The thickness of reservoir is still the same, 30 m, but, because injection pressure is less, the average density difference between brine and CO$_2$ is larger $\Delta \rho = 474$ kg/m$^3$. The numbers of gridblocks in x, y and z direction are also the same, 40 x 1 x 30. These changes of parameters give new value of $Z_D = 0.93$ (previously $Z_D = 0.0135$) and $X_D = 3.25$ (previously $X_D = 2.70$).

The reservoir is initially at water saturation $S_w = 1$ and no oil is present. Initially, the surfactant is injected for 3 years and then followed by 2 years of CO$_2$ injection (as in a SAG foam process). Again, we neglect the cost of surfactant and any limitations in the CO$_2$ supply for this simulation. The injection-well pressure is set at 166.5 bar ($1.665 \times 10^7$ Pa) maximum bottom-hole pressure and the production well set at 165 bar ($1.65 \times 10^7$ Pa) minimum bottom-hole pressure. We use a value of horizontal permeability equal to 1000 md in the simulations. Fig. 17 shows the simulation results from STARS™ using the North Sea model parameters with four different ratio of vertical permeability to horizontal permeability, $k_v/k_h = 0, 0.01, 0.1$ and 1.

The foam-front profiles from the STARS™ simulations in Fig. 17 show different results with the foam-front profiles from the simple model. Specifically, the extent of gravity segregation is not reduced with increasing of vertical permeability $k_v$ as it is in the simple model. Water displacement by CO$_2$ injection for the ratio $k_v/k_h = 0$ is greater than for $k_v/k_h = 1$ in the simulations. The white line at the extreme cases, $k_v/k_h = 0$ and 1, describes the foam-fronts profile from the simple model taken from De Velde Harsenhorst (2012) for comparison at the same $X_D$ and $Z_D$ of the simulations.

The difference between foam-front profile from the STARS™ simulations and the simple model could be because of relative-permeability and foam models used in the simulations. The relative-permeability and foam models do not satisfy the assumptions of the simple SAG model (much lower mobility at the foam front than ahead of and behind the front), and this may have caused the deviation from model predictions. We therefore apply alternate relative-permeability model, which we call the Persoff model, which better matches those assumptions, as explained in the next section.
2.6.2 Persoff Model
Rossen and Boeije (2013) fit STARS™ foam parameters to the data of Persoff et al. (1991), in a fit designed specifically to represent a SAG flood. STARS™ foam parameters used in the simulations are \( f_{mmob} = 25000 \), \( f_{mdry} (sf_{dry}) = 0.37 \) and \( e_{dry} (sf_{bet}) = 2500 \). The residual water saturation is 0.2 and residual gas saturation is 0.2. The end-point values of the water and gas relative-permeability functions are 0.2 and 0.94. The Corey exponent for water and gas are 4.2 and 1.3, respectively. Fig. 18 describes the relative-permeability model from Persoff et al. and Fig. 19 describes the fractional-flow function as a function of water saturation. There is a shock (black line) from initial condition at \( S_w = 1 \) to \( S_w = 0.35 \) as shown in Fig. 19.
We use the same phase viscosities and densities as in the North Sea example. Total relative mobility of the water ahead of the foam bank is $444 \text{ (Pa s)}^{-1}$, at the leading edge of the foam bank $90 \text{ (Pa s)}^{-1}$ and at the injection well is $5136.6 \text{ (Pa s)}^{-1}$ in the end of simulation. The average density difference between brine and CO$_2$ is $\Delta \rho = 476 \text{ kg/m}^3$. The well-to-well difference pressure, $\Delta P = 1.5 \text{ bar (1.5 x 10}^5 \text{ Pa})$. The reservoir length in $x$ direction is 120 m and thickness of the reservoir is 30 m.

The number of gridblocks in $x$, $y$ and $z$ directions are 40 x 1 x 30. The gas injection well constraint is set at 166.5 bar ($1.665 \times 10^7 \text{ Pa}$) bottom-hole pressure and the production well is set at 165 bar ($165 \times 10^7 \text{ Pa}$) bottom-hole pressure. The reservoir is initially at water saturation $S_w = 1$ and there is no oil present. Using all these parameters, the new dimensionless position are $Z_D = 0.93$ and $X_D = 3.73$. Fig. 20 shows the simulations result from the Persoff model for various values of $k_v/k_h$.

Fig. 20 compares the foam front at the extreme cases between simple model (white line), which is taken from De Velde Harsenhorst (2012), and the simulations at the same $Z_D$ and $X_D$. One can conclude that the comparison gives better result than North Sea model. In particular, there is somewhat better sweep with $k_v = k_h$ than for $k_v = 0$.

2.6.3 Frontal Model

In the last model check, we use an alternate relative-permeability model as shown in Fig. 21, which we believe satisfies the assumptions of the simple SAG model well. The point in this case is not to match behavior of any particular foam, but to test the simple model against
simulations with a foam model that better fits its assumptions. We incorporate gas mobility reduction directly in the relative-permeability model and do not use the foam model in STARS™ in the simulations.

The residual water saturation is 0.1 and residual gas saturation is 0. The end point water saturations of the water and gas relative-permeability functions are 0.2 and 1. Fig. 22 describes the fractional-flow function as a function of water saturation. There is one point tangency (red circle), with a shock downstream to (1, 1) and a shock upstream to (0.2, 0), with a small difference in slope between them, 5%. Fig. 23 (left) presents the time-distance diagram with the mobilities of the various banks from 1D simulation, which was performed to validate the relative-permeability model prior to the 2D simulation as in Fig. 24. The width of the low-mobility bank is enlarged somewhat from the fractional-flow estimate by numerical dispersion. Fig. 23 (right) shows the water saturation profile just before the breakthrough of gas in the 1D simulation.

![Figure 21. Relative permeability used in the Frontal model. Figure 22. Fractional flow vs water saturation.](image1)

![Figure 23 (left). Time-distance diagram with mobilities of the banks in (Pa s)^{-1}. Figure 23 (right). Water saturation profile just before breakthrough of gas.](image2)

As the gas-mobility reduction is incorporated directly into the relative-permeability function, the simulations in STARS™ are treated simply as gas injection. \( fmmob \) is set to zero in the STARS™ input. The reservoir is initially at water saturation \( S_w = 1 \) and there is no oil present.
The average density difference between brine and CO₂, \( \Delta \rho = 476 \text{ kg/m}^3 \). The well-to-well difference pressure, \( \Delta P = 1.5 \text{ bar} (1.5 \times 10^5 \text{ Pa}) \).

The reservoir length in the \( x \) direction is 120 m and thickness of the reservoir is 30 m. The numbers of gridblocks in \( x \), \( y \) and \( z \) directions are 40 \( \times \) 1 \( \times \) 30. The gas-injection well constraint is set at 166.5 bar (1.665 \( \times \) 10\(^7 \) Pa) bottom-hole pressure and the production well is set at 165 bar (165 \( \times \) 10\(^7 \) Pa) bottom-hole pressure. Using all these parameters, we calculate the new dimensionless position, \( Z_D = 0.93 \) and \( X_D = 3.73 \). The simulations results with the Frontal model with four different ratios of \( k_v/k_h = 0, 0.01, 0.1 \) and 1 using STARS™ are shown in Fig. 24.

The white lines in Fig. 24 represents the foam front from the simple model, taken from De Velde Harsenhorst (2012), in the extreme cases \( k_v/k_h = 0 \) and 1, at the same \( Z_D \) and \( X_D \) as the simulations. Although the foam-front profiles of the STARS™ simulation results and simple model do not precisely match, we get a closer match than in Fig. 17 (the North Sea model). Gravity segregation is worse as \( k_v \) decreases in the STARS™ simulation results, as predicted in the simple model. The fit to the simple model with this custom-made foam model is only about as good as with the Persoff model (Fig. 20), however.

![Figure 24. Frontal model foam fronts profile with four different ratio, kv/kh = 0, 0.01, 0.1 and 1](image-url)
Namdar Zanganeh et al. (2011) analyzed foam flooding for completely immiscible water, gas and oil using the Method of Characteristic (MOC). Their results are presented as composition paths on the ternary phase diagram. On the same diagram were plotted the set of allowable composition paths from the MOC, which tend to reflect the region of strong foam (composition paths often made abrupt turns at the edge of the strong-foam region on the ternary diagram, for instance). The analysis of these floods using the MOC is extremely complex. In this project, we attempt to test the hypothesis that composition paths from simulations, plotted on the same sort of ternary diagram, and superimposed on a plot of gas mobility-reduction factor, could give insights into foam displacements with oil without the need for complicated MOC solutions.

3.1 1D Reservoir Model

The reservoir model used for the one-dimensional studies has the dimensions 400*1*1 m (400 x 1 x 1 gridblocks). It is a horizontal and isothermal reservoir. We set permeability to 1000 md in x, y and z directions. The initial reservoir pressure is 140 bar, the temperature is constant at 50°C and the porosity is 0.25. The injectors and producer are placed opposite ends of the reservoir. The well model used is the Peaceman well model (1983). The minimum bottom-hole pressure at the producer is set at 95 bar. The gas and water injector maximum bottom-hole pressure is set at 160 bar.

Unlike the simulation studies of gravity override in foam processes presented above, we chose to use nitrogen (N₂) instead of carbon dioxide (CO₂) as the gas in the simulation studies of the effect of oil on foam. The main reason of we use N₂ is because it is nearly completely immiscible with oil. We want to observe the effect of oil on foam in the foam model without complications of miscibility and mutual solubility.

For simplicity, we chose dodecane (100% C₁₂) to represent oil in the simulation instead of the 13 oil components that we used in the North Sea example. The viscosity of dodecane at reservoir conditions is five times the viscosity of water. Detailed fluid properties used in the simulation are listed in Table 5.

The relative-permeability model from Ashoori et al. (2010) is used for all 1D simulations. Table 6 shows the input residual saturations, Corey relative-permeability exponents and endpoint permeabilities for this model.
Table 5. Fluid parameters at 140 bar and 50°C

<table>
<thead>
<tr>
<th>Fluid</th>
<th>Parameters</th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$P_{\text{crit}}$ [kPa]</td>
<td>$T_{\text{crit}}$ [°C]</td>
<td>MW [kg/gmole]</td>
<td></td>
</tr>
<tr>
<td>Water</td>
<td>22030</td>
<td>373</td>
<td>0.018</td>
<td></td>
</tr>
<tr>
<td>Surfactant</td>
<td>144</td>
<td>527</td>
<td>0.23</td>
<td></td>
</tr>
<tr>
<td>C_{12}</td>
<td>1810</td>
<td>385.2</td>
<td>0.17</td>
<td></td>
</tr>
<tr>
<td>N_{2}</td>
<td>3400</td>
<td>-147</td>
<td>0.014</td>
<td></td>
</tr>
</tbody>
</table>

Table 6. Fluid properties

<table>
<thead>
<tr>
<th>Fluid</th>
<th>Density [kg/m$^3$]</th>
<th>Viscosity [cp]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Water</td>
<td>973.3</td>
<td>1</td>
</tr>
<tr>
<td>Surfactant</td>
<td>535.9</td>
<td>1</td>
</tr>
<tr>
<td>C_{12}</td>
<td>850</td>
<td>5</td>
</tr>
<tr>
<td>N_{2}</td>
<td>78</td>
<td>0.015</td>
</tr>
</tbody>
</table>

Table 6. Relative permeability input functions for 1D simulations

<table>
<thead>
<tr>
<th>$S_{\text{wr}}$</th>
<th>$n_{\text{w}}$</th>
<th>4.2</th>
<th>$K_{\text{rw}}$</th>
<th>0.2</th>
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<tbody>
<tr>
<td>$S_{\text{org}}$</td>
<td>$n_{\text{ow}}$</td>
<td>1.3</td>
<td>$K_{\text{ro}}$</td>
<td>0.94</td>
</tr>
<tr>
<td>$S_{\text{orw}}$</td>
<td>$n_{g}$</td>
<td>1.3</td>
<td>$K_{\text{rg}}$</td>
<td>0.94</td>
</tr>
<tr>
<td>$S_{\text{gr}}$</td>
<td>$n_{\text{og}}$</td>
<td>1.3</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The foam model parameters are shown in Table 7. We increased the $fmmob$ value from the North Sea example, which is 100, to 10000 in this simulation, to represent stronger foam. With the given values of foam parameters $fmoil$ and $epoil$, foam is completely destroyed for oil saturations $S_o > fmoil = 0.2$; the gas mobility-reduction factor is a linear function of oil saturation for $S_o < 0.2$ (i.e., $epoil = 1$). With the given values of $fmdry$ and $epdry$, foam weakens (but does not collapse completely) in the vicinity of a water saturation of 0.25. Please note that the names of parameters $fmdry$ and $epdry$ are changed to $sfdry$ and $sfbet$ in the current version of STARSTM (2011). In the simulation, the foam strength is affected only by oil and water saturations. We neglected the parameters representing the effects of surfactant concentration and capillary number in this simulation. Surfactant is present at uniform concentration in the aqueous phase everywhere in the simulations.

Table 7. Foam model parameters in 1D simulations

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Corresponding exponent</th>
</tr>
</thead>
<tbody>
<tr>
<td>$fmmob$, maximum mobility reduction factor</td>
<td>10000</td>
<td></td>
</tr>
<tr>
<td>$fmoil$, critical oil saturation: no foam when $S_o&gt;fmoil$</td>
<td>0.2</td>
<td>1 ($epoil$)</td>
</tr>
<tr>
<td>$fmdry$, critical water saturation</td>
<td>0.25</td>
<td>2000 ($epdry$)</td>
</tr>
</tbody>
</table>
3.2 Baker’s Linear Interpolation Model for Oil Relative Permeability

There are three interpolation methods for oil relative permeability in three-phase flow available in STARS™: Stone’s model I, Stone’s model II and Baker’s linear interpolation model. STARS™ use Stone’s model II as the default interpolation method for relative permeability, which we used in our previous study of gravity override in SAG foam in the North sea example.

Stone’s model II assumes that the relative permeability of water in the three-phase system is equal to the water relative permeability in the two-phase water-oil system, and is a function of only $S_w$. The relative permeability of gas in the three-phase system is equal to the gas relative permeability in the two-phase liquid-gas system, and is a function only of $S_g$.

With Stone’s model II, the constant-$k_{ro}$ contours (isoperms) between the two-phase water-oil and gas-liquid data tend to be curved toward the $S_o = 0$ boundary when plotted on a saturation ternary diagram such as in Fig. 25. For the parameters of Ashoori et al. (2009) for relative permeabilities, the $k_{ro}=0$ isoperm may cross the $S_o = 0$ boundary, resulting in a range of $S_w$ values where $k_{ro} > 0$ when $S_o = 0$. This condition is detected for individual blocks at the end of each time step in our simulation. Besides the physical impossibility of this condition, it reduces numerical stability. After several warnings displayed during the simulation, the simulator then flagged a fatal error, which stopped the simulation automatically.

![Figure 25. Geometric construction of Baker’s linear interpolation method in ternary diagram](Computer Modeling Group, 2011)
We therefore chose Baker’s linear interpolation method, which is also available in STARS™. The method assumes that points of equal oil relative permeability on the two binaries, that is, $k_{row}(S_w) = k_{rog}(S_g)$, are linked with a straight line to form a series of $k_{ro}$ isoperms as in Fig. 25. Within the zone of $S_w < S_{wc}$ and $S_g < S_{gr}$ the model assumes that the $k_{ro}$ isoperms are at constant $S_g$ or $S_w$, respectively. Then, within the region of three-phase flow it draws isoperms between equal $k_{ro}$ values on the lines for $S_w = S_{wc}$ and $S_g = S_{gr}$. These straight lines between corresponding equal permeabilities determine the oil permeability at any given saturation between the two-phase curves. For our parameters, the isoperm for $k_{ro} = 0$ (i.e., residual oil saturation), extends along the gas-water binary; the residual oil saturation is zero.

An advantage of this method that it eliminates the problem of nonzero $k_{ro}$ for $S_o = 0$ that can occur using Stone’s model II. In this method, $k_{row}$ and $k_{rog}$ have the same range (0 to $k_{rocw}$). $k_{ro}$ interpolation depends only on $k_{row}$ and $k_{rog}$ at any $S_w$ and $S_g$. The algorithm in STARS™, using our relative-permeability parameters, gave a residual oil saturation of zero ($k_{ro} \to 0$ as $S_o \to 0$) in our simulations. It remains to see if foam can form in the presence of oil and reduce oil saturation to its residual value.

### 3.3 Simulation Injection Strategy and Cases

In this study, different injection strategies were carried out to observe the effect of oil on foam. Two parameters are varied in these simulations: initial oil saturation ($S_{o,I}$) and injected water fractional flow ($f_{w,J}$). The foam is continuously injected by injecting water containing surfactant and $N_2$ into the reservoir, except for cases where $f_{w,J} = 0$, in which only gas is injected into the reservoir (following surfactant injection). The initial water saturation, which also contains surfactant (reflecting a preflush of surfactant ahead of the foam), depends on the value of $S_{o,I}$. Initial oil saturations values are chosen from 0.1 to 0.3 in increments of 0.1. These reflect oil saturation less than, equal to, or greater than the saturation at which foam is destroyed by oil in these simulations. The injection strategies tested are as follows (all the reservoir fluid properties remain the same as listed in Table 5):

- **Case 1**: $S_{o,I} = 0.2$ and $f_{w,J} = 0.1$
- **Case 2**: $S_{o,I} = 0.2$ and $f_{w,J} = 1.3 \times 10^{-5}$
- **Case 3**: $S_{o,I} = 0.2$ and $f_{w,J} = 0$
- **Case 4**: $S_{o,I} = 0.3$ and $f_{w,J} = 0.1$
- **Case 5**: $S_{o,I} = 0.3$ and $f_{w,J} = 1.3 \times 10^{-5}$
- **Case 6**: $S_{o,I} = 0.3$ and $f_{w,J} = 0$
- **Case 7**: $S_{o,I} = 0.1$ and $f_{w,J} = 0.1$
- **Case 8**: $S_{o,I} = 0.1$ and $f_{w,J} = 1.3 \times 10^{-5}$
- **Case 9**: $S_{o,I} = 0.1$ and $f_{w,J} = 0$
CHAPTER 4: SIMULATION RESULTS AND DISCUSSION

4.1 Fractional-flow Analysis

Fractional-flow methods are very useful in the analysis of displacement process for two-phase flow. Namdar Zanganeh (2011) used the fractional-flow method or method of characteristics (MOC), to model complex three-phase foam enhanced oil recovery processes with oil. In our case, fractional flow analysis is carried out to analyze two phase flow, gas and water, to compare the injection point on the displacement path derived from STARS™ simulation results with that from the fractional-flow method.

Fig. 26 (left) shows the semi-log plot of the two-phase fractional-flow curve for water and gas corresponding to the Ashoori et al. (2010) relative-permeability model from Table 6 and foam parameters from Table 7 (i.e., the fluid properties used in this study). The semi-log plot of fractional-flow water curve shows the fractional flow of water on the vertical axis with logarithmic scale and water saturation value is in the horizontal axis with linear scale. Fig. 26 (right) is the linear plot of Fig. 26 (left).

As we can see from Fig. 26 (left and right), the fractional-flow water is close to zero in the range of \( S_w = 0.1 \) to 0.23. A semi-log plot can be very helpful to find the water saturation corresponding to a given injected water fractional flow \( f_{w,J} \) at very small values of \( f_{w,J} \). Based on Fig. 26 (left), we can determine the value of \( S_w \) at each injected water fractional flow for the three values of \( f_{w,J} \) used in our cases:

- \( f_{w,J} = 0.1 \) corresponds to \( S_w = 0.31 \) (\( S_g = 0.69 \))
- \( f_{w,J} = 1.3 \times 10^{-5} \) corresponds to \( S_w = 0.2 \) (\( S_g = 0.8 \))
- \( f_{w,J} = 0 \) corresponds to \( S_w = 0.1 \), i.e. residual water saturation
Fig. 27 shows the semi-log plot of characteristic velocity \( \frac{df_w}{dS_w} \). The displacements to be simulated are three-phase displacements, for which shock construction is much more complicated than with simple gas-water flow (Namdar Zanganeh et al., 2011). Nonetheless if the three-phase displacement travels along the water-gas binary, then saturations would move with the velocities equal to \( \frac{df_w}{dS_w} \) in Fig. 27. Note that for water saturations between 0.1 and 0.2, the characteristic velocity is small compared to characteristic velocities of saturations outside the range. There is a steep increase in \( \frac{df_w}{dS_w} \) between \( S_w = 0.24 \) and 0.25 corresponding to the steep increase in \( f_w \) value (Fig. 26).

### 4.2 Surface Plot of Gas Mobility Reduction Factor

The effect of foam on gas mobility is represented in STARS\textsuperscript{TM} for simplicity as a change in gas relative permeability. Gas relative permeability without foam, \( k_{rg}^0 \), is reduced by a factor \( FM \) to gas relative permeability with foam, \( k_{rg}^f \), as in Eq. 3. \( FM \) is called the gas mobility-reduction factor. To see how \( FM \) changes with saturations of gas and oil, we created a surface plot in MS Excel using 101 x 101 grid values of gas and oil saturation.

Fig. 28 shows surface plot of \( FM \) as function of saturations using the Ashoori et al. (2010) relative-permeability model from Table 6 and foam parameters from Table 7. \( FM \) values are plotted as the negative of their (base-10) logarithm to show the region of strong and weak foam clearly. Higher elevations on this plot mean the stronger foam and vice versa. From Fig. 28, we can see that the foam collapses with oil saturation values above 0.2. We use the surface plot in 2D format in the next section for comparison with the composition paths of displacements from simulation.
4.3 Comparison of 2D Plots

In this section, we compare plots of gas mobility-reduction factor with foam in 2D format, plotted on a composition diagram, with the composition paths of displacements simulated using the same model parameters. Our hypothesis is that a relatively simple correlation might exist between the plot of gas mobility-reduction factor on the phase diagram and the displacement paths on the same diagram, without having to undertake the complexity of three-phase fractional-flow solutions of Namdar Zanganeh (2011).

4.3.1 Comparison Plots for Case #1

For case #1, we set the initial oil saturation \( S_{o,I} \) equal to 0.2, initial water saturation to 0.8 and injected fractional flow of water \( f_{w,J} \) to 0.1. Fig. 29 compares gas mobility-reduction factor and composition path for the displacement (red dots) for this case. The injection point \( J \) from the simulation is in two-phase (water and gas) edge, with gas saturation equal to 0.71, water saturation equal to 0.29, and no oil saturation. The white arrow shows the direction of the composition path.

There is a slightly different value of water saturation at the injection grid block from the two-phase fractional flow analysis in Fig. 26, where gas saturation is equal to 0.69 and water saturation 0.31. Nevertheless, in either case the injection point \( J \) is in the strong-foam
region while the initial condition (I) is just outside the foam region. Moreover, there is an oil bank ahead of the foam with oil saturation clearly in the no-foam region. The result shows that foam inside the strong-foam region displaces the oil even in an oil bank with saturation great enough to kill foam. The saturation and total mobility profile from simulation are shown in Fig. C1. All the oil is pushed ahead of the foam, which is possible because residual saturation is zero and because the gas mobility is so low.

![Gas Saturation vs. FM](image)

**Figure 29. Comparison plots for case #1**

### 4.3.2 Comparison Plots for Case #2

In this case, we set the initial oil saturation ($S_o,I$) equal to 0.2, initial water saturation to 0.8 and injected fractional flow of water ($f_{w,J}$) to $1.3 \times 10^{-5}$, an extremely low value. This value of $f_{w,J}$ is chosen, based on the fractional flow curve in Fig. 26 (left) to obtain a higher gas saturation at the injection condition (i.e., weaker foam compared to case #1). Fig. 30 shows comparison plots for case #2. The injection point (J) now is in a somewhat weaker foam region compared to case #1: gas saturation is 0.79, water saturation 0.21, and again there is no oil. The initial condition (I) is the same with previous case, at the edge of the strong-foam region.

As in Case #1, there is a slight difference in values of saturation seen in the injection grid block from that two-phase fractional flow analysis in Fig. 26 (left), where gas saturation is 0.8 and water saturation equal to 0.2. The oil bank is formed ahead of the foam bank with saturation greater than 0.2 but this did not break the foam. The oil is displaced completely by foam at a point J which is still within the strong-foam region. The saturation and total mobility profiles from the simulation are shown in Fig. C2.
4.3.3 Comparison Plots for Case #3

For the next case, we set the injected fractional flow of water \( (f_{w,J}) \) equal to 0. This means that we only inject gas into the reservoir. The foam is formed since the water initially in the reservoir contains surfactant. The initial oil saturation \( (S_{o,I}) \) equal to 0.2 and initial water saturation is 0.8. Our goal in this injection strategy is to get the injection point (J) in much weaker foam region compared to case #2 and to observe whether this change affects the composition path or oil displacement.

Fig. 31 shows comparison plots for case #3. Surprisingly, the injection point (J) from the simulation is not much different from that in case #2, with gas saturation is 0.79, water saturation 0.21 and no oil saturation. The fractional-flow curve in Fig. 26 (left) shows that the \( f_{w,J} = 0 \) corresponds to \( S_w = 0.1 \), i.e. residual water saturation. The different result between simulation and fractional-flow analysis is explained by the extremely small characteristic velocities (Fig. 27) with water saturations between 0.1 and 0.2. The extremely small characteristic velocity, which is between \( 10^{-4} \) and \( 10^{-8} \), means that it will take a very long time to see the water saturation equal to 0.1 in the first grid block in the simulation. Thus, we see water saturation equal to 0.21 at the injection point after 200 days of injection of gas in the simulation. From Fig. 31, we can see as in the other two cases that the composition path enters the strong-foam region of the diagram and the oil is again displaced entirely ahead of the strong-foam. The saturation and total mobility profiles from simulation are shown in Fig. C3.
4.3.4 Comparison Plots for Case #4

For case #4, we increased the initial oil saturation \( (S_{o,I}) \) to 0.3 to move the initial condition outside the foam region. With this value, \( S_{o,I} \) is sufficient to kill the foam \( (f_{moil} = 0.2) \). The initial water saturation is set to 0.7 and injected fractional flow of water is 0.1.
Fig. 32 shows comparison plots for case #4. It shows that the oil did not break the foam; instead foam displaced the oil. An oil bank is formed ahead of the foam bank and there is no oil left enough behind of the foam bank to break the foam. At J, gas saturation is 0.71, water saturation is 0.29 and no oil is present. As in case #1, there is a slight difference with the saturations at J from fractional-flow analysis in Fig. 26 (left), where gas saturation is 0.69 and water saturation is 0.31. The saturation and total mobility profiles from simulation are shown in Fig. C4.

4.3.5 Comparison Plots for Case #5
In this case, we reduced injected fractional flow of water ($f_{w,J}$) to the extremely low value of $1.3 \times 10^{-5}$. We set initial oil saturation ($S_{o,I}$) to 0.3 and initial water saturation to 0.7. Fig. 33 shows comparison plots for case #5. The result shows oil displacement by strong foam region without oil breaking the foam. At J point, the saturations gas and water are 0.79 and 0.21, respectively. These values are again similar to but not identical to the values from fractional-flow analysis ($S_w = 0.2$ and $S_g = 0.8$). The saturation and total mobility profiles from simulation are shown in Fig. C5.

4.3.6 Comparison Plots for Case #6
Similar to case #3, we injected only gas, i.e. $f_{w,J} = 0$. The initial oil saturation ($S_{o,I}$) is the same as the previous case, i.e. 0.3, and initial water saturation is 0.7. Fig. 34 shows comparison plots for case #6, which shows the injection point (I) from the simulation is not very different from case #5. We believe this is due to extremely small characteristic velocities in Fig. 26 (left), that mean it takes a very long time to see $S_w = 0.1$ in the injection grid block in the
simulation. The result shows that the oil is displaced by foam. The saturations at J are similar to case #5, where gas saturation 0.79 and water saturation 0.21. The saturation and total mobility profiles from simulation are shown in Fig. C6.

Figure 34. Comparison plots for case #6

4.3.7 Comparison Plots for Case #7

For case #7, we set the initial oil saturation ($S_{o,I}$) equal to 0.1, initial water saturation is 0.9 and injected fractional flow of water ($f_{w,I}$) is 0.1. This is the lowest $S_{o,I}$ in our simulations. The initial condition (I) is in the strong-foam region.

Fig. 35 shows comparison plots for case #7 where we can see that the oil is displaced by foam. An oil bank forms ahead of the foam with no oil left in the foam bank to weaken the foam. The injection point (J) from the simulation is in two-phase edge of the diagram (water and gas), with gas saturation equal to 0.71, water saturation 0.29 and no oil. There is slight difference with the saturations at J from two-phase fractional flow analysis in Fig. 26 (left), where gas saturation equal to 0.69 and water saturation is 0.31. The saturation and total mobility profiles from simulation are shown in Fig. C7.
4.3.8 Comparison Plots for Case #8

We set the initial oil saturation ($S_{o,I}$) equal to 0.1, initial water saturation 0.9 and injected fractional flow of water ($f_{w,I}$) to $1.3 \times 10^{-5}$ in this case. Fig. 36 shows comparison plots for case #8 where oil displaced from J point to strong foam region.
Similar to cases #2 and #5, the injection point (J) gas saturation is 0.79, water saturation 0.21 and there is no remaining oil saturation. The injection grid block has slightly different saturations from those in the two-phase fractional flow analysis in Fig. 26 (left), where gas saturation is equal to 0.8 and water saturation 0.2. The saturation and total mobility profiles from simulation are shown in Fig. C8.

4.3.9 Comparison Plots for Case #9
For the last case, we set the injected fractional flow of water \((f_{w,J})\) to 0. The initial oil saturation \((S_o,I)\) the same as in the previous case, i.e. 0.1, and initial water saturation 0.9. Fig. 37 shows comparison plots for case #9. As with cases #3 and #6, the injection point (J) from the simulation is not different from that in case #8 due to extremely small characteristic velocities as discussed above. The result shows that the oil is displaced by strong foam. The saturation and total mobility profiles from simulation are shown in Fig. C9.

![Figure 37. Comparison plots for case #9](image)
4.4 Numerical Problem in the Simulations

We tried to investigate why strong foam is able to form with such a large oil saturation initially in place ($S_{o,I} = 0.3$) through checking simulations for several grid blocks near injector at early times. We chose case #6 as an example with $f_{w,J} = 0$ and $S_{o,I} = 0.3$ (possibly the most difficult case for foam to form, with initial gas saturation large enough to kill foam and no surfactant injected with the gas). Fig. 38 shows a compilation of saturation and total-mobility profiles for several periods of gas injection (2, 3, 4, 5, 6, 8, 10 and 200 days) with time step size equal to $10^{-4}$ day in the simulation.

As can be seen in Fig. 38, gas enters grid blocks #1-10 and very slowly starts to drain down the oil. In grid block #5 particularly, for some reason, oil saturation falls below 0.2 ($FMOIL = 0.2$), and then foam starts to build. It pushes more oil out and the foam gets stronger day by day of gas injection. This foam does not propagate to the next grid block, but by day 4 foam is forming in grid block #7. There is still no foam in grid blocks #1, 2, 3, 4 or 6. At day 10 there is foam in grid blocks #1, 5, 7, 8 and 20, but not in the other grid blocks. Then, at day 200, there is foam in all the first 100 grid blocks and oil has been displaced in this region. Fig. 38 shows that this is not smooth progression from one state to another in the early period of the simulation.

We also tried to investigate further by lowering the time step size to $10^{-7}$ day. Fig. 39 shows the compilation of the saturation and total-mobility profiles with the same periods as in Fig. 38. The change of time step size to a much lower value in the simulation has no physical significance, but it changes where the foam erupts: by day 10, in grid blocks #1, 5, 6, 10, 13 and 14. We believe this may reflect some sort of numerical problems and may affect all our simulation results with oil. Namdar Zanganeh et al. (2011) also reported the numerical problem in her simulation which will be explained in the next section. Rossen (2013) notes that when oil at a saturation sufficient to destroy foam is displaced ahead of a foam bank in finite-difference simulations it may reflect imperfect representation of the displacement. This issue needs further investigation.
Figure 38. Compilation results of saturations profile and total mobility with time-step size equal to \(10^{-4}\) day for case #6
Figure 39. Compilation results of saturations profile and total mobility with time-step size equal to $10^{-7}$ day for case #6
4.5 Discussion of Results

The results include various initial oil saturations \( (S_o, I) \) and injected fractional flow of water \( (f_{w, J}) \). The injection point \( (J) \) is placed in the strong foam or somewhat weaker foam regions and the initial condition \( (I) \) is placed inside or outside of the foam region. All cases give a composition path through the strong-foam region and complete displacement of oil. Almost no oil is left behind the foam front. In cases #3, 6 and 9, where injected fractional flow of water \( (f_{w, J}) \) is 0, the apparent injection point \( (J) \) from the simulations is not much different from cases #2, 5 and 8. We believe this is caused by the extremely small characteristic velocities between \( S_w = 0.1 \) and 0.2 (Fig. 27), which means that it will take a very long time to see the water saturation equal to 0.1 in the first grid block in the simulation.

The effect of oil on foam is complex and not fully understood (Farajzadeh et al., 2012). Namdar Zanganeh et al. (2011), using a different set of foam parameters that when \( f_{w, J} = 0 \) foam was destroyed if the initial oil saturation lay outside the strong-foam region. Furthermore, she documented difficulties in obtaining a correct simulation result, as illustrated in Fig 40.

![Figure 40 (left). Composition path of displacements. Legend (different colored dots) represents different dimensionless times.](image)

![Figure 40 (right) Analytical solution with heavy black line for displacement using MOC (Namdar Zanganeh et al., 2011)](image)

It is too early to make general conclusions about the usefulness of our approach, i.e. plotting gas mobility-reduction factor on the three phase diagram and the displacement path from our simulations on the same diagram. We believe that our simulations are not free of the numerical problems as we investigated with different time-step size in the simulation in the previous section. This might be the same numerical problem found by Namdar Zanganeh et al. (2011) in her simulation. There are too few cases of simulation studies and a need for more cases, to further illustrate of the effect of oil on foam.
CHAPTER 5: CONCLUSION AND RECOMMENDATIONS

5.1 Conclusions

Several conclusions can be drawn from these studies:

1. The simple model for gas injection in SAG indicates a constant-pressure SAG foam process can overcome gravity override over large distances as shown for $0 < k_v < k_h$ in this study.

2. This simple model proves reasonably accurate compared to simulation result from STARS™. Thus, it is a useful tool in initial assessment of the feasibility of foam propagation over large distances in foam SAG process.

3. The simple model shows that the vertical sweep is somewhat better with increasing vertical permeability in homogeneous reservoirs; this allows the foam front to push downwards in response to the tilting foam front and the pressure difference between the foam bank and the resident fluids. This result is confirmed by simulations with the Persoff and Frontal models.

4. Complete segregation over a relatively short distance with continuous foam injection is confirmed with STARS™ simulation. The segregation length in the simulation is similar to that in analytical models.

5. In all simulations of the effect of oil on foam, we see foam displacement of oil ahead of foam, whether the initial oil saturation is within or outside the strong-foam region of the ternary diagram. In all our cases, the injection point (J) was in or on the edge of the strong-foam region, even when only gas was injected. The displacement path then proceeded further into the strong-foam region, where oil was displaced.

6. Our foam simulations with oil may have been affected by numerical artifacts. This issue needs further investigation.

7. It is too early to make general conclusions about the usefulness of plotting gas mobility-reduction factor on the three phase diagram and the displacement path on the same diagram. Nevertheless, these preliminary results illustrate possible usefulness of drawing composition paths and gas mobility-reduction factor on 3-phase diagram.

5.2 Recommendations

The usefulness of plotting composition paths on the ternary diagram along with gas mobility-reduction factor need further study. Our recommendations are:

1. Investigate the cause of the apparent numerical instabilities in the initial creation of foam in our simulations and develop simulation approaches that give reliable results.

2. Apply alternate relative permeability model in the simulations to prevent extremely small characteristic velocities, especially for $f_{w,j} = 0$.

3. Try more cases with other foam parameters to get more-general conclusions, for example by changing the value of $f_{m,oil}$.
APPENDIX A: DERIVATIONS OF IDEALIZED MODEL FOR SAG DISPLACEMENTS
(from de Velde Harsenhorst, 2013)

A.1 Derivation for Case $k_v = k_h$

Shan and Rossen (2004) made derivation for case $k_v = k_h$ by applying Darcy’s law to the low-mobility zone. The Darcy velocity of gas in this zone is:

$$u_g = k \lambda_{rf} |\nabla P| \quad (A.1)$$

where $\lambda_{rf}$ is the gas relative mobility and $|\nabla P|$ is the magnitude of the pressure gradient in the low-mobility zone. This pressure gradient is determined by the width of the front. The width of the front, $\tau$, is proportional to $s$, the distance the front has traveled.

$$|\nabla P| = \frac{\Delta P}{\tau} \quad \tau = \frac{\tau^* s}{L^*} \quad (A.2)$$

where $\tau^*$ is the width of the front when it has traveled a distance $L^*$.

The interstitial velocity $v$ of the displacement front is:

$$v = \frac{k \lambda_{rf} |\nabla P|}{(1 - S_{wf}) \phi} \quad (A.3)$$

The Darcy velocity $u_g$, interstitial velocity $v$ and pressure gradient across the low mobility zones (the displacement front) are perpendicular to the interface between the gas and water zones. We track the movement of the front by tracking the movement of individual points in the front at positions $[x(t), z(t)]$. The distance travelled for each point is $s(t)$. The angle $\alpha$ indicates the front movement direction (relative to horizontal) perpendicular to the current front at time $t$.

$$\alpha = \arctan \left( \frac{\partial x}{\partial z} \right)_t \quad (A.4)$$

where $\left( \frac{\partial x}{\partial z} \right)_t$ is the slope of the displacement front at time $t$ of each point. In this case, $\theta = \alpha$. The interstitial velocity split in the $x$ and $z$ directions.

$$v_x = \frac{dx}{dt} = \frac{k \lambda_{rf} |\nabla P|}{(1 - S_{wf}) \phi} \cos(\alpha) \quad v_z = \frac{dz}{dt} = \frac{k \lambda_{rf} |\nabla P|}{(1 - S_{wf}) \phi} \sin(\alpha) \quad (A.5)$$

$$\frac{ds}{dt} = \left( v_x^2 + v_z^2 \right)^{1/2} \quad |\nabla P| = \frac{\Delta P^0 - \Delta \rho g z}{\tau} = \frac{(\Delta P^0 - \Delta \rho g z) L^*}{\tau^* s} \quad (A.6)$$

$v_x$ and $v_z$ are the velocity in $x$- and $z$-directions, respectively at a given point. The initial conditions and boundary conditions are:

**IC:** at $t = 0, \ x = 0$ ; at $t = 0, \ s = 0$

**BC:** at $z = 0, \ \alpha = 0$
The differential equations are converted to dimensionless differential equations for the positions of individual points of the displacement front as follows.

\[
\frac{dX_D}{dt_D} = \frac{(1 - Z_D)}{S_D} \cos(\alpha) \\
\frac{dZ_D}{dt_D} = \frac{(1 - Z_D)}{S_D} \sin(\alpha) 
\]

\[
\frac{dS_D}{dt_D} = \sqrt{\left(\frac{dX_D}{dt_D}\right)^2 + \left(\frac{dZ_D}{dt_D}\right)^2} 
\]

where,

\[
\alpha = \arctan\left(-\frac{dX_D}{dZ_D}\right)_{t_D} \\
X_D = \frac{\Delta \rho g x}{\Delta P^0} \\
Z_D = \frac{\Delta \rho g z}{\Delta P^0} \\
t_D = \frac{k \lambda_T \Delta \rho g}{(1 - S_{wf}) \phi \tau} t \\
S_D = \frac{\Delta \rho s}{\Delta P^0} \\
L^* = \frac{\Delta P^0}{\Delta \rho g} 
\]

The initial and boundary conditions are:

IC: at \( t_D = 0 \), \( X_D = 0 \); at \( t_D = 0 \), \( S_D = 0 \)

BC: at \( Z_D = 0 \), \( \alpha = 0 \)

**A.2 Derivation for Case for case \( 0 < k_v < k_h \)**

De Velde Harsenhorst (2012) extended the model of Shan and Rossen (2004) for case \( 0 < k_v < k_h \). The derivation is similar to that for \( k_v = k_h \) case (Appendix A.1) but the movement of the front is not perpendicular to its slope. There is a new parameter of angle, \( \beta \), the direction the given point on the front is moving relative to horizontal. \( \alpha \) is angle perpendicular to the front and \( \tau \) is the front thickness. The given point on the front travels in the direction of \( s \) with an angle \( \beta \) to the horizontal. Fig. A1 illustrates the case.
The front thickness in other directions is given by,

\[ \tau_a = \tau \cos(\alpha - \beta) \quad \tau_x = \frac{\tau \cos(\alpha - \beta)}{\cos(\alpha)} \quad \tau_z = \frac{\tau \cos(\alpha - \beta)}{\sin(\alpha)} \]  \hspace{1cm} (A.12)

The pressure gradient across the front is determined by front thickness, the well-to-well pressure difference and the difference in hydrostatic pressures in the water and gas ahead of and behind the front (Fig. 9).

\[ |\nabla P| = \frac{\Delta P(z)}{\tau} = \frac{(\Delta P^0 - \Delta pgz)}{\tau} \]  \hspace{1cm} (A.13)

In the \(x\) and \(z\) directions,

\[ |\nabla P|_{xi} = \frac{\Delta P(z)}{\tau_x} = \frac{(\Delta P^0 - \Delta pgz) \cos(\alpha)}{\tau \cos(\alpha - \beta)} = \frac{(\Delta P^0 - \Delta pgz)L^*}{\tau^* s \cos(\alpha - \beta)} \]  \hspace{1cm} (A.14)

\[ |\nabla P|_{zi} = \frac{\Delta P(z)}{\tau_z} = \frac{(\Delta P^0 - \Delta pgz) \sin(\alpha)}{\tau \cos(\alpha - \beta)} = \frac{(\Delta P^0 - \Delta pgz)L^*}{\tau^* s \cos(\alpha - \beta)} \]  \hspace{1cm} (A.15)

\(\Delta P^0\) is pressure difference across the front at the top of the well. \(\tau^*\) is the width of the front when it has traveled a distance \(L^*\). \(s\) is the distance front has travelled. The components of velocity of a point \(i\) on the front, \(v_{xi,i}\) and \(v_{zi,i}\), depend on the pressure gradient in \(x\) and \(z\) directions and also on the differing vertical and horizontal permeabilities:

\[ v_{xi,i} = \frac{dx_i}{dt} = \frac{k_h \lambda_{rf} |\nabla P|_{xi}}{(1 - S_{wf}) \phi} \]  \hspace{1cm} (A.16)
where $\lambda_{rf}$ is the relative mobility of gas in the front and $\phi$ is the porosity of reservoir. The dimensionless position is defined based on hydrostatic pressure and the rise in injection pressure at top of the well $\Delta P^0$ (Shan and Rossen, 2004):

$$X_{D,i} = \frac{\Delta \rho g x_i}{\Delta P^0} \quad Z_{D,i} = \frac{\Delta \rho g z_i}{\Delta P^0} \quad (A.19)$$

$$\alpha_i = \arctan \left( -\frac{dX_D}{dZ_D} \right) \quad \beta_i = \arctan \left( -\frac{dZ_{D,i}}{dt} \right) \quad S_D = \frac{\Delta \rho g s_i}{\Delta P^0} \quad L^* = \frac{\Delta P^0}{\Delta \rho g} \quad (A.20)$$

Dimensionless time is based on hydrostatic pressure and properties of the foam front ($\tau^*/\lambda_{rf}$):

$$t_D = \frac{k_h \lambda_{rf} \Delta \rho g}{(1 - S_{wf}) \phi \tau^*} \quad (A.21)$$

The movement of the given point on the front is given by:

$$\frac{dX_{D,i}}{dt_D} = \frac{(1 - Z_{D,i})}{S_{D,i}} \cos(\alpha_i) \quad \frac{dZ_{D,i}}{dt_D} = \left( \frac{k_p}{k_h} \right) \frac{(1 - Z_{D,i})}{S_{D,i}} \frac{\sin(\alpha_i)}{\cos(\alpha_i - \beta_i)} \quad (A.22)$$

$$\frac{dS_{D,i}}{dt_D} = \sqrt{\left( \frac{dX_{D,i}}{dt_D} \right)^2 + \left( \frac{dZ_{D,i}}{dt_D} \right)^2} \quad (A.23)$$

The initial and boundary conditions for all the points $i$ are:

- At $t_D = 0, X_{D,i} = 0, \alpha_i = 0$ for all $i \quad (A.24)$
- At $t_D = 0, S_{D,i} = 0$ for all $i \quad (A.25)$
- At $Z_D = 0, \frac{dX_D}{dt_D} = \frac{1}{S_{D,i}}$ for all $i \quad (A.26)$

### A.3 Derivation for Case $k_v = 0$

De Velde Harsenhorst (2012) adjusted the derivation from Shan and Rossen (2004) for the extreme case, $k_v = 0$, $\beta = 0$. In this case, $v_z = 0$ and the equations are:
The initial and boundary conditions are:

**IC:** at \( t = 0, \ x = 0; \) at \( t = 0, \ s = 0 \)

**BC:** at \( z = 0, \ \alpha = 0 \)

The dimensionless differential equations are:

\[
\frac{dX_D}{dt_D} = \frac{(1 - Z_D)}{S_D} \quad \frac{dZ_D}{dt_D} = 0 \quad \frac{dS_D}{dt_D} = \frac{dX_D}{dt_D} \quad (A.29)
\]

where,

\[
\beta = 0 \quad X_D = \frac{\Delta \rho g x}{\Delta P^0} \quad Z_D = \frac{\Delta \rho g z}{\Delta P^0} \quad t_D = \frac{k \lambda_{rf} \Delta \rho g}{(1 - S_{wf}) \phi \tau^*} \quad (A.32)
\]

\[
S_D = \frac{\Delta \rho g s}{\Delta P^0} \quad L^* = \frac{\Delta P^0}{\Delta \rho g} \quad (A.33)
\]

The dimensionless initial and boundary conditions are:

**IC:** at \( t_D = 0, \ X_D = 0; \) at \( t_D = 0, \ s_D = 0 \)

**BC:** at \( Z_D = 0, \ \beta = 0 \)

**A.4 Analytical Solution for Case \( k_v = 0 \)**

De Velde Harsenhorst (2012) also solved case \( k_v = 0 \) analytically. The total distance travelled \( s_D \) of a point is equal to the distance travelled in the \( x \) direction \( X_D \).

\[
\frac{dX_D}{dt_D} = \frac{(1 - Z_D)}{S_D} = \frac{(1 - Z_D)}{X_D} \quad (A.34)
\]

\[
X_D \frac{dX_D}{dt_D} = 1 - Z_D \quad (A.35)
\]
At \( t_0 = 0, X_0 = 0 \) then the constant \( C \) is zero. Eq. A.37 is multiplied by two and taking square root gives:

\[
X_0 = \sqrt{2(1-Z_0)}t_0 \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (A.38)
\]

We use Eq. A.38 to generate the first set of \( X_0 \) points for numerical calculations in the case \( 0 < k_v < k_h \).

**APPENDIX B: NUMERICAL CALCULATIONS FOR CASE \( 0 < k_v < k_h \) IN MS EXCEL SPREADSHEET**

The numerical method used in the calculations is Euler’s method, where the derivative at a point is used to calculate the position at the next time. At \( t_D = 0 \), we use the analytical solution (Eq. A.38) for the first time step.

![Figure B1](image)

**Figure B1.** One of the example numerical calculation model for case \( 0 < k_v < k_h \)

Explanation of each column in Fig. B1:

- **Column A** = \( X_D = X_{D,\text{previous}} + \frac{dX_D}{dt_D} \cdot dt \) (except for the first time step)

- **Column B** = \( Z_D = Z_{D,\text{previous}} + \frac{dZ_D}{dt_D} \cdot dt \)

- **Column C** = \( \frac{dX_D}{dt_D} = \frac{(1-Z_D)}{S_D} \cdot \frac{\cos(\alpha)}{\cos(\alpha-\beta)} \)
Column D = \frac{dZ_D}{dt_D} = \left(\frac{k_v}{k_h}\right) \frac{(1 - Z_D)}{S_D} \frac{\sin(\alpha)}{\cos(\alpha - \beta)}

Column E = \alpha = \arctan\left(-\frac{dX_D}{dZ_D}\right)_{t_D}

Column F = \beta = \arctan\left(-\frac{dZ_D}{dt_D}\right)_{t_D-1}

Column G = S_D = S_{D,previous} + \left(\sqrt{\left(\frac{dX_D}{dt_D}\right)^2 + \left(\frac{dZ_D}{dt_D}\right)^2}\right) \times dt_D

Column H, I, J and K are smoothed values from Column A, B, E and F respectively

**APPENDIX C: SATURATION AND TOTAL MOBILITY PROFILES FROM 1D SIMULATIONS WITH OIL**

In Figs. C1 – C9, we show the saturation and total mobility profiles of the 1D simulations of the effect of oil on foam for cases #1 to #9 (Section 4.3). These profiles are taken after 200 days of injection in the simulation.
**Figure C2.** Saturation and total mobility profile for case #2

**Figure C3.** Saturation and total mobility profile for case #3
Figure C4. Saturation and total mobility profile for case #4

Figure C5. Saturation and total mobility profile for case #5
Figure C6. Saturation and total mobility profile for case #6

Figure C7. Saturation and total mobility profile for case #7
Figure C8. Saturation and total mobility profile for case #8

Figure C9. Saturation and total mobility profile for case #9
REFERENCES


