Generic Model Predictive Control Framework for Advanced Driver Assistance Systems

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"Life can only be understood backwards; but it must be lived forwards."
（只有回顾过去才能理解生活;但不断前进才能继续生活.）
Søren Kierkegaard
Preface

At the very first PhD progress meeting I had with my supervisors, Serge Hoogendoorn recommended two books to me, *Optimal Control* (F.L. Lewis) and *Dynamic Noncooperative Game Theory* (T. Basar and G.J. Olsder). It was not until one year later did I start reading the first book and finally during the last year of my PhD I realised the relevance of the latter to the research. This reflects the process of a civil engineer catching up with a highly mathematics-involved topic to some extent. Nevertheless, I am more than happy to complete a PhD thesis on control design for intelligent vehicles.

Despite a few setback moments that nearly every PhD student encounters, doing a PhD research has been quite a pleasant journey for me. I would like to take this opportunity to thank the people who have been important to me during my PhD life. Many thanks goes to my supervisors, Serge Hoogendoorn, Bart van Arem and Winnie Daamen. Your ideas, discussions, and critical comments have helped me in conducting research and publishing results, and are much appreciated. I enjoyed the freedom I got in setting up the research directions and in solving different problems. I am particularly grateful to Winnie for coaching me towards an independent researcher and reviewing every detail of my (lengthy) papers and to Serge for guiding me to this interesting topic and your great enthusiasm in my work.

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Delft, September 2014
# Contents

List of Figures ................................................................. ix
List of Tables ............................................................... xiv
List of Symbols .............................................................. xvii
List of Acronyms and Abbreviations ................................... xix

1 Introduction ................................................................. 1
   1.1 Background of Advanced Driver Assistance Systems ............ 1
   1.2 Challenges for designing and testing ADAS ......................... 3
   1.3 Research objectives and questions .................................. 4
   1.4 Research approach .................................................... 5
   1.5 Research scope ....................................................... 6
   1.6 Main contributions .................................................... 7
      1.6.1 Scientific contributions ........................................ 7
      1.6.2 Practical contributions ......................................... 8
   1.7 Outline of the thesis ................................................ 9

2 Current knowledge on ADAS control design and impact assessment 11
   2.1 Structure of the literature review .................................. 11
   2.2 Hierarchical functional architecture and example control concepts . 12
      2.2.1 Road-based layers ............................................. 12
      2.2.2 Vehicle-based layers .......................................... 13
      2.2.3 Summary ...................................................... 15
2.3 Control algorithms for ADAS at platoon and vehicle levels  . . . . . 16
  2.3.1 ACC algorithms  . . . . . . . . . . . . . . . . . . . . . . . . . . 16
  2.3.2 Cooperative ACC algorithms  . . . . . . . . . . . . . . . . . . 23
  2.3.3 EcoACC algorithms  . . . . . . . . . . . . . . . . . . . . . . . . 26
  2.3.4 Discussion  . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 26
2.4 Methods for ADAS impact assessment  . . . . . . . . . . . . . . . . 28
  2.4.1 Performance measures and indicators  . . . . . . . . . . . . . . 29
  2.4.2 Models for ADAS impact assessment on traffic operations . . 30
  2.4.3 Models for ADAS impacts on sustainability  . . . . . . . . . . . 34
  2.4.4 Summary on impact models  . . . . . . . . . . . . . . . . . . . 38
2.5 Impact studies of ADAS  . . . . . . . . . . . . . . . . . . . . . . . 39
  2.5.1 Impact of ACC systems  . . . . . . . . . . . . . . . . . . . . . . 39
  2.5.2 Impact of CACC systems  . . . . . . . . . . . . . . . . . . . . . 42
  2.5.3 Impact of EcoACC systems  . . . . . . . . . . . . . . . . . . . 43
  2.5.4 Summary and discussion  . . . . . . . . . . . . . . . . . . . . . 43
2.6 Conclusions  . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 44

3 Model predictive control framework for ADAS  . . . . . . . . . . . . 45
  3.1 Core design assumptions and control objectives  . . . . . . . . . . 46
  3.2 Control framework formulation  . . . . . . . . . . . . . . . . . . . 47
    3.2.1 Supported driving as an optimal control cycle  . . . . . . . . . 47
    3.2.2 Formulation of the control problem  . . . . . . . . . . . . . . . 48
    3.2.3 Worked example: linear ACC algorithm  . . . . . . . . . . . . 50
  3.3 Solution approaches to optimal control problems  . . . . . . . . . 51
    3.3.1 Dynamic programming solutions  . . . . . . . . . . . . . . . . 52
    3.3.2 Numerical methods based on Pontryagin’s Principle  . . . . . 53
    3.3.3 Direct numerical solutions to optimisation problems  . . . . . 56
    3.3.4 Closed-form solutions for specific problems  . . . . . . . . . . 61
    3.3.5 Discussions  . . . . . . . . . . . . . . . . . . . . . . . . . . . 62
  3.4 ADAS performance assessment framework  . . . . . . . . . . . . . . 65
CONTENTS

3.4.1 Performance indicators and criteria 66
3.4.2 Simulation study under representative scenarios 67
3.4.3 Analytical approach for state-feedback algorithms 68
3.5 Conclusions 68

4 ACC and EcoACC controllers with dynamic programming solution 71
4.1 Control objectives and design assumptions 72
4.2 ACC and EcoACC controller formulation 73
  4.2.1 Stochastic system dynamics 73
  4.2.2 Cost specification for ACC and EcoACC systems 75
4.3 Optimal acceleration for ACC and EcoACC systems 77
  4.3.1 Dynamic programming approach 77
  4.3.2 Numerical solution based on finite difference method 78
  4.3.3 Computing the optimal strategy off-line 78
  4.3.4 Bounded acceleration and explicit delay 79
4.4 Simulation experimental design 79
4.5 Results and discussion 80
  4.5.1 Microscopic performance of ACC and EcoACC controllers 80
  4.5.2 Collective behaviour of ACC and EcoACC vehicles 82
  4.5.3 Discussion on Eco-driving strategies 86
4.6 Conclusions 86

5 Refined ACC and C-ACC controllers with analytical solution 89
5.1 Controller design assumptions and control objectives 90
5.2 ACC and C-ACC controller formulation 91
  5.2.1 System dynamics model 91
  5.2.2 Cost function specification 92
  5.2.3 Analytical solution approach 93
  5.2.4 Derivation of ACC algorithm 94
  5.2.5 Derivation of C-ACC algorithm 97
5.3 Analytical approach for assessing controller characteristics 98
5.3.1 Relevant definitions on stability .......................... 99
5.3.2 Equilibrium flow-density relation ........................ 102
5.3.3 Linear stability analysis ................................. 103
5.4 Microscopic performance of the ACC controller ........... 108
5.5 Macroscopic flow characteristics of ACC and C-ACC vehicles .... 110
  5.5.1 Fundamental diagram .................................. 111
  5.5.2 String stability of the ACC controller ................. 112
  5.5.3 Stabilisation/destabilisation effect of cooperative systems ... 116
5.6 Conclusions .................................................. 117

6 Flexible ACC and C-ACC controllers with fast numerical solution 119
  6.1 Design assumptions for ACC and C-ACC systems .......... 120
  6.2 Controller design formulation ............................... 122
    6.2.1 Control problem formulation .......................... 122
    6.2.2 ACC controller with variable time gap ................ 123
    6.2.3 C-ACC-HP controller formulation ...................... 126
    6.2.4 C-ACC-MI controller formulation ...................... 127
  6.3 Derivation of optimal accelerations ....................... 128
    6.3.1 A numerical solution algorithm based on Pontryagin’s Minimum Principle ...................... 129
    6.3.2 Optimal accelerations for ACC controllers ........... 130
    6.3.3 Optimal accelerations for C-ACC-HP controllers ....... 131
    6.3.4 Optimal accelerations for C-ACC-MI controllers ....... 132
  6.4 Experimental design for ACC and C-ACC performance assessment ... 132
    6.4.1 Key research questions ............................... 132
    6.4.2 Experiments for assessing individual ACC performance ... 133
    6.4.3 Experiments for assessing C-ACC platoon performance ... 135
  6.5 Individual ACC performance under representative scenarios .... 136
    6.5.1 Free driving and emergency-braking behaviour .......... 136
    6.5.2 Follow-the-leader under speed disturbance .......... 138
  6.6 Platoon performance of cooperative controllers ............ 142
<table>
<thead>
<tr>
<th>Section</th>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>6.6.1</td>
<td>C-ACC-HP performance</td>
<td>143</td>
</tr>
<tr>
<td>6.6.2</td>
<td>C-ACC-MI performance</td>
<td>148</td>
</tr>
<tr>
<td>6.6.3</td>
<td>Computational experiments</td>
<td>152</td>
</tr>
<tr>
<td>6.7</td>
<td>Conclusions</td>
<td>153</td>
</tr>
<tr>
<td>7</td>
<td>Impacts of ADAS systems on traffic operations and sustainability</td>
<td>155</td>
</tr>
<tr>
<td>7.1</td>
<td>Assumptions and algorithms for ACC and C-ACC controllers</td>
<td>156</td>
</tr>
<tr>
<td>7.1.1</td>
<td>ACC controller and decentralised algorithm</td>
<td>156</td>
</tr>
<tr>
<td>7.1.2</td>
<td>C-ACC controller and distributed algorithm</td>
<td>157</td>
</tr>
<tr>
<td>7.2</td>
<td>Experimental set-up</td>
<td>159</td>
</tr>
<tr>
<td>7.2.1</td>
<td>Bottleneck and necessary modelling aspects</td>
<td>159</td>
</tr>
<tr>
<td>7.2.2</td>
<td>Simulation model and network settings</td>
<td>160</td>
</tr>
<tr>
<td>7.2.3</td>
<td>Experimental scenarios</td>
<td>160</td>
</tr>
<tr>
<td>7.2.4</td>
<td>Assessment indicators</td>
<td>161</td>
</tr>
<tr>
<td>7.3</td>
<td>Simulation results on dynamic traffic operations</td>
<td>163</td>
</tr>
<tr>
<td>7.3.1</td>
<td>Verification of the reference scenario</td>
<td>163</td>
</tr>
<tr>
<td>7.3.2</td>
<td>Impacts of ACC systems on flow characteristics</td>
<td>165</td>
</tr>
<tr>
<td>7.3.3</td>
<td>Impacts of C-ACC systems on flow characteristics</td>
<td>171</td>
</tr>
<tr>
<td>7.3.4</td>
<td>Discussion on changed flow characteristics</td>
<td>172</td>
</tr>
<tr>
<td>7.4</td>
<td>Conclusions</td>
<td>176</td>
</tr>
<tr>
<td>8</td>
<td>Integrated variable speed limit control system with ACC vehicles</td>
<td>179</td>
</tr>
<tr>
<td>8.1</td>
<td>Introduction</td>
<td>179</td>
</tr>
<tr>
<td>8.2</td>
<td>Control design of integrated VSL control with ACC systems</td>
<td>182</td>
</tr>
<tr>
<td>8.2.1</td>
<td>Assumptions of integrated control</td>
<td>182</td>
</tr>
<tr>
<td>8.2.2</td>
<td>VSL control algorithm: SPECIALIST</td>
<td>183</td>
</tr>
<tr>
<td>8.2.3</td>
<td>ACC algorithm with variable desired speeds</td>
<td>186</td>
</tr>
<tr>
<td>8.2.4</td>
<td>Implementation of integrated control with traffic simulation</td>
<td>186</td>
</tr>
<tr>
<td>8.3</td>
<td>Experimental design</td>
<td>187</td>
</tr>
<tr>
<td>8.3.1</td>
<td>Bottleneck setting</td>
<td>187</td>
</tr>
<tr>
<td>8.3.2</td>
<td>Deployment scenarios</td>
<td>187</td>
</tr>
</tbody>
</table>
8.3.3 Assessment indicators ........................................ 188
8.4 Simulation results ................................................. 188
  8.4.1 Tuned variables ............................................... 189
  8.4.2 Performance of the integrated control paradigm .......... 190
8.5 Conclusions ......................................................... 196

9 Findings, conclusions, implications and recommendations 197
  9.1 Findings ............................................................. 197
  9.2 Conclusions ........................................................ 202
  9.3 Implications for practice ........................................ 203
  9.4 Recommendations for future research .......................... 205

Appendices .................................................................. 207

Bibliography ................................................................. 210

Summary ................................................................. 227

Samenvatting ................................................................ 231

Summary in Chinese ..................................................... 235

TRAIL Thesis Series .................................................... 239

Curriculum Vitae ......................................................... 241
## List of Figures

1.1 Schematic relations of ITS, AHS and ADAS. ............................................. 3
1.2 Research steps. ...................................................................................... 6
1.3 Overview of thesis structure. ................................................................. 9

2.1 Schematic structure of the sections following a general design procedure. 12
2.2 Abstract representation of an ADAS controller. .................................... 17
2.3 Schematic illustration of (a) ACC, (b) Multi-anticipative ACC and (c) Looking-backward ACC controllers. ................................. 17

3.1 Abstract representation of model predictive ADAS controller (grey rectangle). .......................................................... 47
3.2 Schematic representation of rolling horizon implementation of optimal control. ................................................................. 48
3.3 Solution family of optimal control problems. ........................................... 52
3.4 Illustration of indirect single shooting methods. ..................................... 55
3.5 Piecewise constant parametrisation of a control function $u(t,q)$ and the integrated state $x(t,q)$ with $N = 5$. ........................................ 57
3.6 Illustration of the direct multiple shooting method with piecewise constant representation of control function $u(t,q)$ and integrated state $x(t,p,q)$. 59
3.7 Illustration of the direct collocation method with piecewise constant parametrisation of control input ($N = 5$). ................................. 60

4.1 Schematic representation of the human intervention mechanism. ........ 73
4.2 System state for a controlled vehicle following a leader. ................. 74
4.3 Comparison of speed, acceleration, and spatial $CO_2$ emission rate of the 1st and 5th vehicle in EcoACC platoon (blue line) and ACC platoon (red line) in free traffic conditions. ................................. 82
4.4 Comparison of speed, acceleration, and spatial CO₂ emission rate of the 1st and 5th vehicle in EcoACC platoon (blue line) and ACC platoon (red line) in congested traffic conditions.

4.5 Stationary flow-density relationships for ACC and EcoACC traffic.

4.6 Total CO₂ emission contour plot with stationary flow-density relationship.

5.1 Illustration of local and string stability and instability (reproduced after Pueboobpaphan & van Arem (2010)).

5.2 Illustration of (a) convective upstream instability and (b) absolute instability in the spatio-temporal (x-t) plane, with vehicles travelling in the direction of increasing x. (Reproduced after Treiber & Kesting (2011)).

5.3 (a) Contour of optimal acceleration when following a vehicle driving at 54 km/h; (b) Contour of optimal cost with a vehicle trajectory.

5.4 Real and imaginary parts of two roots for local stability of the ACC model with default parameters.

5.5 Equilibrium (a) speed-gap relationship and (b) flow-density relationship with \( t_d = 1.0 \) s and \( t_d = 1.5 \) s and other default parameters in Table 5.1.

5.6 Stability region in a two-dimensional parameter plane of \( c_1 \) and \( t_d \) with (a) different \( c_2 \) and (b) different \( \eta \), under equilibrium speed of 72 km/h. Other parameters are default values.

5.7 (a) Growth rate of the more unstable branch \( \gamma_+ \) as a function of wave number under \( v_e = 54 \) km/h; (b) phase and group velocity as a function of wave number under \( v_e = 54 \) km/h of the ACC algorithm with default parameters.

5.8 (a) Phase, group, signal velocities as a function of equilibrium speed and (b) phase, group, signal velocities as a function of equilibrium density and spatio-temporal evolution of initial disturbance at the equilibrium speeds of (c) 48 km/h and (d) 72 km/h of ACC model with default parameters. Driving direction in (c) and (d) is from top to bottom.

5.9 Stability plot with safety cost weight \( c_1 \) and equilibrium speed of (a) ACC model with default parameters of \( c_2 = 0.0019 \) s\(^{-4} \), \( v_0 = 33.3 \) m/s, \( t_d = 1.0 \) s and \( s_0 = 1 \) m; (b) C-ACC model with \( c_4 = 0 \), \( c_5 = 0.9 c_2 \); (c) C-ACC model with \( c_4 = c_1/2 \), \( c_5 = c_2/2 \); S: Stable region; U: region with convective Upstream instability; A: region with Absolute instability; D: region with convective Downstream instability.
6.1 Illustration of controllers: (a) two ACC follower with an exogenous leader; (b) ACC follower and a human-driven vehicle with an exogenous leader; (c) cooperative controller for cooperation with equipped vehicles with perfect knowledge of follower behaviour (C-ACC-HP) and (d) cooperative controller for cooperation of equipped vehicle and human-driven vehicle with imperfect knowledge of follower behaviour (C-ACC-MI). ............................................. 121

6.2 Experiment 1: spatial evolution of (a) speed and (b) acceleration in the free driving and emergency braking scenario with different $c_3$ of 0.005, 0.01 and 0.02. Other parameters are set as default values. ................. 138

6.3 Experiment 2: emergency-braking behaviour with different safety cost weight $c_1$. Other parameters are set as default values. ................. 139

6.4 Experiment 3: emergency-braking behaviour with different $t_{d,m}$ and $u_{min}$. Other parameters are set as default values. ......................... 139

6.5 Experiment 4: evolution of (a) efficiency and safety cost; (b) gap deviation from desired gap and relative speed; and (c) optimal acceleration with default parameters in the normal following scenario. ............ 140

6.6 Experiment 4: evolution of (a) gap deviation from desired gap; (b) relative speed; (c) speed; (d) acceleration of ACC vehicles in the normal following scenario with different $c_2$. Other parameters are set at default values. ......................................................... 141

6.7 Experiment 5: evolution of (a) gap; (b) relative speed; (c) speed and (d) acceleration of the ACC vehicle in the normal following scenario with different $c_1$. Other parameters are set at default values. ............. 142

6.8 Experiment 6: evolution of (a) incurred cost $J$, (c) speed and (e) acceleration of representative vehicles in the platoon with homogeneous ACC followers and (b) incurred cost, (d) speed and (f) acceleration of representative vehicles in the platoon with homogeneous C-ACC-HP followers in the decelerating scenario. .................. 144

6.9 Experiment 7: evolution of (a) incurred cost $J$, (c) speed and (e) acceleration of representative vehicles in the platoon with homogeneous ACC followers and (d) incurred cost, (d) speed and (f) acceleration of representative vehicles in the platoon with homogeneous C-ACC-HP followers in the accelerating scenario. .................. 146

6.10 Experiments 6 and 7: hysteresis loops in the average speed-gap $(v(t), s(t))$ plane for (a) ACC followers and C-ACC-HP followers with $w = 0$; (b) ACC followers and C-ACC-HP followers with $w = 1$. Arrows indicate the evolution direction of the loops. ................................. 148
6.11 Experiment 7: (a) incurred cost $J$, (c) speed and (e) acceleration of representative vehicles in the platoon with homogeneous ACC followers and (d) incurred cost, (d) speed and (f) acceleration of representative vehicles in the platoon with homogeneous C-ACC-HP followers with $w = 1$ in the accelerating scenario. ................................................................. 149

6.12 Experiments 8 and 9: (a) speed and (c) acceleration in the decelerating phase of mixed platoon with ACC vehicles; (b) speed and (d) acceleration in the decelerating phase of mixed platoon with C-ACC-MI vehicles; (e) speed and (g) acceleration in the accelerating phase of mixed platoon with ACC vehicles; (f) speed and (h) acceleration in the accelerating phase of mixed platoon with C-ACC-MI vehicles. ... 151

6.13 Experiments 10 and 11: hysteresis loops in the average speed-gap $(v(t), s(t))$ plane for ACC + human followers (red line) and C-ACC-MI + human followers (blue line) with $w = 1$. Arrows indicate the evolution direction of the loops. ................................. 152

6.14 Experiments 10 and 11: CPU time (Computation time) as a function of the number of followers using the iPMP algorithm for the ACC and C-ACC-HP platoons, simulation period of 100 seconds with $\alpha = 0.01$ and $\epsilon_{max} = 0.1$. .............................. 153

7.1 (a) Flow contour and (b) time mean speed contour plots, and (c) flow-density plots and (d) gap-speed plots in Scenario 1 with 100% human drivers. ................................................................. 164

7.2 Spatio-temporal evolution of flow and speed of ACC with different penetration rate (Scenarios 2 - 5) in one simulation run. ............... 168

7.3 Flow-density plots for ACC impact study with different penetration rate (Scenarios 2 - 5) in one simulation run. .......................... 169

7.4 Spatio-temporal evolution of flow and speed of CACC with different penetration rate (Scenarios 6 - 9) in one simulation run. ............. 173

7.5 Flow-density plots for CACC impact study with different penetration rate (Scenarios 6 - 8) in one simulation run. .......................... 174

8.1 Schematic representation of bi-level control problem. Dashed lines are not covered in this study. ................................................................. 181

8.2 According to the shock wave theory the propagation of the front between two traffic states in the left figure has the same slope as the line connecting the two states in the density-flow diagram in the right figure. The arrow indicates the travel direction. Flow and density values are for two lanes. ................................................................. 184
8.3 The four phases of the SPECIALIST algorithm. Phase I: The shock wave is detected. Phase II: Speed limits are turned on in areas 2, 3, and 4. The shock wave dissolves. Phase III: The speed-limited area (area 4) resolves and flows out efficiently. Phase IV: The remaining area 5 is a forward propagating high-speed high-flow wave. Flow and density values are for two lanes. .................................................. 185

8.4 Spatio-temporal plots of flow for VSL control with different traffic compositions. .......................................................... 192

8.5 Spatio-temporal plots of speed for VSL control with different traffic compositions. ......................................................... 193

8.6 Spatio-temporal plots of flow and speed in scenarios with the unresolved jam and new jams triggered by VSL control. .......... 194
# List of Tables

<table>
<thead>
<tr>
<th>Table</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.1</td>
<td>Comparison of different control layers</td>
<td>15</td>
</tr>
<tr>
<td>2.2</td>
<td>Comparison of ACC algorithms in following mode</td>
<td>23</td>
</tr>
<tr>
<td>2.3</td>
<td>Comparison of control design methods</td>
<td>26</td>
</tr>
<tr>
<td>2.4</td>
<td>Overview of models for traffic operations impacts of ADAS</td>
<td>30</td>
</tr>
<tr>
<td>2.5</td>
<td>Overview of fuel consumption and emission models</td>
<td>37</td>
</tr>
<tr>
<td>2.6</td>
<td>Overview of ADAS impact studies</td>
<td>40</td>
</tr>
<tr>
<td>3.1</td>
<td>Comparison of solution approaches of optimal control problems</td>
<td>63</td>
</tr>
<tr>
<td>4.1</td>
<td>Overall impact on traffic flow and CO\textsubscript{2} emissions</td>
<td>83</td>
</tr>
<tr>
<td>5.1</td>
<td>ACC/C-ACC controller parameters</td>
<td>97</td>
</tr>
<tr>
<td>6.1</td>
<td>The new ACC controller parameters</td>
<td>126</td>
</tr>
<tr>
<td>6.2</td>
<td>The iPMP solution algorithm</td>
<td>130</td>
</tr>
<tr>
<td>6.3</td>
<td>Overview of experimental setup</td>
<td>134</td>
</tr>
<tr>
<td>6.4</td>
<td>Indicators for different test platoons during two scenarios (experiments 6-9)</td>
<td>143</td>
</tr>
<tr>
<td>7.1</td>
<td>Experimental scenarios for impact study of ACC/C-ACC systems</td>
<td>161</td>
</tr>
<tr>
<td>7.2</td>
<td>Indicators for different scenarios averaged over ten simulation runs for each scenario</td>
<td>170</td>
</tr>
<tr>
<td>8.1</td>
<td>Experimental scenarios for testing integrated VSL control with ACC systems</td>
<td>187</td>
</tr>
<tr>
<td>8.2</td>
<td>SPECIALIST parameter settings for different scenarios</td>
<td>189</td>
</tr>
<tr>
<td>8.3</td>
<td>Indicators for different scenarios</td>
<td>195</td>
</tr>
</tbody>
</table>
List of Symbols

\( a \)
Vehicle acceleration

\( b_{\text{max}} \)
Maximum braking \( (b_{\text{max}} > 0) \)

\( c_i \)
Weight factors in running cost, with \( i = 1, 2, \ldots \)

\( c_p \)
Phase velocity in linear stability analysis

\( c_g \)
Group velocity in linear stability analysis

\( c_s \)
Signal velocity in linear stability analysis

\( f(x, u, t) \)
System dynamics function

\( g \)
Small perturbation on equilibrium speed in linear stability analysis

\( \dot{G}(x, t) \)
Terminal cost (function/functional)

\( h \)
Small perturbation on equilibrium gap in linear stability analysis

\( H(x, \lambda, u, t) \)
Hamiltonian

\( J(x, u, t) \)
(Generalised) predicted cost

\( k \)
Wave number in linear stability analysis

\( K_s, K_v, K_{\Delta v} \)
Feedback control gains related to gap, speed and relative speed

\( l \)
Vehicle length

\( L(x, u, t) \)
Running cost (function/functional)

\( q \)
Flow/volume

\( q_e \)
Equilibrium flow/volume

\( s \)
Distance gap with respect to the preceding vehicle (bumper-to-bumper) distance

\( s_0 \)
Gap at standstill conditions

\( s_d \)
Desired gap

\( s_e \)
Equilibrium gap

\( s_f \)
Gap threshold for distinguishing following mode and cruising mode of ACC, C-ACC and EcoACC systems

\( t_{\text{TTC}} \)
Time to collision

\( T_p \)
Prediction horizon

\( t_d \)
Desired time gap

\( u(t) \)
Vector of control variables/inputs

\( u^*(t) \)
Optimal control variables/inputs

\( u_{\text{max}} > 0 \)
Maximum acceleration

\( u_{\text{min}} \)
Minimum acceleration, \( u_{\text{min}} = -b_{\text{max}} \)

\( v \)
Vehicle speed

\( v_0 \)
Free/desired speed
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$v_e$</td>
<td>Equilibrium speed</td>
</tr>
<tr>
<td>$w$</td>
<td>Asymmetric acceleration factor</td>
</tr>
<tr>
<td>$W(x,t)$</td>
<td>Value function/ Optimal cost-to-go</td>
</tr>
<tr>
<td>$x(t)$</td>
<td>Vector of state variables</td>
</tr>
<tr>
<td>$x^*(t)$</td>
<td>Optimal state variables</td>
</tr>
<tr>
<td>$x_0$</td>
<td>Initial state conditions</td>
</tr>
<tr>
<td>$Z_\epsilon(x,t)$</td>
<td>Spatio-temporal evolution of an initial disturbance $\epsilon$</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>Weight factor for updating the costate in the iterative numerical solution approach based on Pontryagin’s Minimum Principle</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>Complex growth rate in linear stability analysis</td>
</tr>
<tr>
<td>$\Delta v$</td>
<td>Relative speed with respect to the preceding vehicle</td>
</tr>
<tr>
<td>$\eta$</td>
<td>Discount factor</td>
</tr>
<tr>
<td>$\Theta$</td>
<td>Heaviside function</td>
</tr>
<tr>
<td>$\lambda(x,u,t)$</td>
<td>Co-state vector</td>
</tr>
<tr>
<td>$\Lambda$</td>
<td>Intermediate co-state vector in the iterative numerical solution approach based on Pontryagin’s Minimum Principle</td>
</tr>
<tr>
<td>$\xi$</td>
<td>Gaussian noise</td>
</tr>
<tr>
<td>$\rho$</td>
<td>Density</td>
</tr>
<tr>
<td>$\rho_e$</td>
<td>Equilibrium density</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>Growth rate of oscillation amplitude in linear stability analysis</td>
</tr>
<tr>
<td>$\omega$</td>
<td>Angular frequency in linear stability analysis</td>
</tr>
</tbody>
</table>
# List of Acronyms and Abbreviations

<table>
<thead>
<tr>
<th>Acronym</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>ACC</td>
<td>Adaptive Cruise Control</td>
</tr>
<tr>
<td>ADAS</td>
<td>Advanced Driver Assistance Systems</td>
</tr>
<tr>
<td>AHS</td>
<td>Automated Highway Systems</td>
</tr>
<tr>
<td>AI</td>
<td>Artificial Intelligence</td>
</tr>
<tr>
<td>BVP</td>
<td>Boundary Value Problem</td>
</tr>
<tr>
<td>CACC</td>
<td>Cooperative Adaptive Cruise Control based on multi-anticipation strategy, equivalent to MACC</td>
</tr>
<tr>
<td>C-ACC</td>
<td>Cooperative Adaptive Cruise Control based on cooperative control strategy</td>
</tr>
<tr>
<td>C-ACC-HP</td>
<td>Cooperative Adaptive Cruise Control in Homogeneous platoon with Perfect knowledge of follower behaviour</td>
</tr>
<tr>
<td>C-ACC-MI</td>
<td>Cooperative Adaptive Cruise Control in Mixed platoon with Imperfect knowledge of follower behaviour</td>
</tr>
<tr>
<td>CARE</td>
<td>Continuous-time Algebraic Riccati differential Equation</td>
</tr>
<tr>
<td>CTG</td>
<td>Constant Time Gap</td>
</tr>
<tr>
<td>CTH</td>
<td>Constant Time Headway</td>
</tr>
<tr>
<td>CVIS</td>
<td>Cooperative Vehicle Infrastructure Systems</td>
</tr>
<tr>
<td>DSRC</td>
<td>Dedicated Short Range Communication</td>
</tr>
<tr>
<td>EcoACC</td>
<td>Ecological Adaptive Cruise Control</td>
</tr>
<tr>
<td>HJB</td>
<td>Hamilton-Jacobi-Bellman equation</td>
</tr>
<tr>
<td>I2I</td>
<td>Infrastructure-to-Infrastructure (communication)</td>
</tr>
<tr>
<td>IDM</td>
<td>Intelligent Driver Model</td>
</tr>
<tr>
<td>ITS</td>
<td>Intelligent Transportation Systems</td>
</tr>
<tr>
<td>IV</td>
<td>Intelligent Vehicle</td>
</tr>
<tr>
<td>IVHS</td>
<td>Intelligent Vehicle Highway Systems</td>
</tr>
<tr>
<td>IVP</td>
<td>Initial Value Problem</td>
</tr>
<tr>
<td>iPMP</td>
<td>iterative numerical scheme based on Pontryagin’s Minimum Principle</td>
</tr>
<tr>
<td>LQR</td>
<td>Linear Quadratic Regulator</td>
</tr>
<tr>
<td>MACC</td>
<td>Multianticipative Adaptive Cruise Control based on multi-anticipation strategy</td>
</tr>
<tr>
<td>MPC</td>
<td>Model Predictive Control</td>
</tr>
<tr>
<td>NLP</td>
<td>Non-Linear Programming</td>
</tr>
<tr>
<td>ODE</td>
<td>Ordinary Differential Equation</td>
</tr>
<tr>
<td>OVM</td>
<td>Optimal Velocity Model</td>
</tr>
<tr>
<td>Abbreviation</td>
<td>Description</td>
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<tr>
<td>--------------</td>
<td>-------------</td>
</tr>
<tr>
<td>PDE</td>
<td>Partial Differential Equation</td>
</tr>
<tr>
<td>PMP</td>
<td>Pontryagin’s Minimum Principle</td>
</tr>
<tr>
<td>SPECIALIST</td>
<td>SPEed ControllIng ALgorIthm using Shockwave Theory</td>
</tr>
<tr>
<td>TTS</td>
<td>Total Time Spent in the network</td>
</tr>
<tr>
<td>V2I</td>
<td>Vehicle-to-Infrastructure (communication)</td>
</tr>
<tr>
<td>V2V</td>
<td>Vehicle-to-Vehicle (communication)</td>
</tr>
<tr>
<td>VSL</td>
<td>Variable Speed Limits</td>
</tr>
<tr>
<td>VTG</td>
<td>Variable Time Gap</td>
</tr>
<tr>
<td>VTH</td>
<td>Variable Time Headway</td>
</tr>
<tr>
<td>VKMT</td>
<td>Vehicle Kilometre Travelled</td>
</tr>
</tbody>
</table>
Chapter 1

Introduction

In this chapter, we introduce research background and problems, followed by the research objectives, approach and scope. Then, the scientific and practical contributions are presented and the outline of the thesis is described in the end.

1.1 Background of Advanced Driver Assistance Systems

The growing number of vehicles in road traffic systems has caused many problems, e.g. congestion, accidents, noise, and air pollution. Many measures have been proposed to improve sustainability of traffic systems, which can be categorised into long-term, medium-term and short-term solutions according to the time needed to affect traffic systems. While long-term solutions such as spatial and transport planning are important, they are limited by the economic and environmental impacts of infrastructure constructions and are constrained in densely populated areas due to the capacity in spatial development (Litman & Burwell, 2006). In contrast, medium- and short-term solutions aimed at better utilisation of available infrastructure systems are preferable due to the relatively low cost and flexibility of implementation. Such solutions, often called dynamic traffic management, take effects through influencing travel behaviour (destination, mode, departure time, route choices) and driving behaviour (lane-changing and car-following). Examples of medium- and short-term solutions include road pricing, route guidance, dynamic speed limits and ramp metering.

Thanks to the fast development of information and communication technologies, the medium- and short-term solutions have been enhanced by the so-called Intelligent Transport Systems (ITS). ITS is a generic term for the integrated application of communications, control and information processing technologies to the transport system (Miles & Chen, 2005). ITS applications are of particular interest to dynamic traffic management, since they can more accurately estimate and predict the traffic flow operations and more effectively control vehicles on the road. Among many application domains of ITS, Advanced Driver Assistance Systems (ADAS) and Automated
Highway Systems (AHS) attract considerable attention, since they change the way that drivers control their vehicles nowadays by automating part of or all the driving tasks.

The research on ADAS and AHS started a few decades ago and reached a peak in the 1990s, with emphasis on automated driving systems and their potential in increasing road capacity (Bender, 1991; Fenton & Mayhan, 1991; Heinrich, 1991). Vast studies were reported on ADAS control design and assessment of the impacts of ADAS on traffic flow operations under the PATH (Partners for Advanced Transit and Highways) programme in the United States (Hancock & Parasuraman, 1992; Haynes & Li, 1993; Fenton, 1994), the PROMETHEUS (PROgraMme for European Traffic with Highest Efficiency and Unprecedented Safety) (Williams, 1988; Brusaglino, 1992), DRIVE I and II (Dedicated Road Infrastructure for Vehicle safety in Europe) programmes in Europe (Michon, 1993; Keller, 1994), and the SSVS (Super Smart Vehicle Systems) programme in Japan (Tsugawa et al., 2000).

Entering the 21st century, cooperative systems that link vehicles and road infrastructure with communication technologies receive more and more attention since they are assumed to bring more benefits in traffic safety and efficiency (Reichardt et al., 2002; Hartman & Strasser, 2005). As a result, the majority of research interests turned to cooperative (vehicle road) systems, with considerable funded initiatives including the Intelligent Vehicle Initiatives (IVI) (Hartman & Strasser, 2005) and Connected Vehicles projects (RITA, 2013) in the U.S., the Auto21 cooperative driving systems in Canada (Halle, 2005), CVIS, COOPERS, SAFESPOT and eCoMove in Europe (Kovacs et al., 2006; Richter, 2006; Schendzielorz & Bonnefoi, 2006; Eikelenberg et al., 2010) and new developments in SSVS in Japan (Tsugawa, 2008). The large investment resulted in new development of ADAS control concepts, algorithms and applications in relation to traffic operation as well as (ecological and environmental) sustainability (Shladover, 2012).

There are different interpretations of cooperative systems (Tsugawa et al., 2000; Reichardt et al., 2002; Hartman & Strasser, 2005; Kovacs et al., 2006). In this thesis, we define cooperative systems as cooperative ADAS that utilise Vehicle-to-Vehicle (V2V) and/or Vehicle-to-Infrastructure (V2I) communications to enhance vehicles’ awareness of the driving environment and/or to assist IVs in negotiating, collaborating and making manoeuvre decisions under a common goal, i.e. improving overall efficiency, safety or sustainability. On the contrary, IVs with non-cooperative or autonomous ADAS do not communicate with others and do not compromise their own situations to benefits others when making control decisions. The relations of relevant system terminologies are shown in Figure 1.1.

Adaptive Cruise Control (ACC) systems are benchmark autonomous ADAS due to relatively simple functional needs and they are included in state-of-the-art vehicles. An ACC equipped vehicle uses its on-board radars to estimate the distance and relative speed with respect to its direct predecessor and regulates the vehicle speed and the following distance according to the driver preference. A multi-anticipative ACC system is an example of cooperative ADAS where IVs receive information from preceding
IVs and hence can react to multiple predecessors (Vanderwerf et al., 2001; Van Arem et al., 2006; Schakel et al., 2010). This multi-anticipative strategy can lead to earlier reaction to disturbances and hence has potentials to improve traffic flow operations (Vanderwerf et al., 2002).

1.2 Challenges for designing and testing ADAS

While acknowledging the achievements on ADAS development for the past decades, there are several challenges in designing ADAS applications to improve traffic operations and sustainability and assessing their impacts. These include refinement of existing controllers, development of control algorithms to operationalise new control concepts, understanding the traffic flow and sustainability impacts of ADAS.

The first challenge is to refine existing ADAS controllers. The first generation of ADAS such as ACC systems is not perfect yet. The widely-used linear gap control strategy for ACC systems leads to deactivation of ACC systems in safety-critical conditions and congested conditions due to the discrepancy between the ACC vehicular performance and the human driver desires (Viti et al., 2008). This suggests needs for refining the existing ADAS algorithms to improve safety and user acceptability, cf. Section 2.3.

New control concepts of ADAS using V2V communications and under sustainability concerns have been proposed during the past decades, including cooperative sensing, cooperative manoeuvring, eco-driving support, and in-vehicle actuation of traffic control signals. Cooperative sensing entails equipped vehicles sharing information with each other via V2V communication to improve the awareness of the situation. Cooperative manoeuvring pertains to the cooperation process of negotiation, task distribution and coordinative manoeuvring using V2V communication. Eco-driving support requires ADAS to control IVs in a safe, efficient, and environment-friendly way.
In-vehicle actuation of traffic control signals refers to a range of applications where in-vehicle systems are used as actuators of road-based traffic control systems, using V2I communications to transmit the traffic control signals to IVs. It is challenging to operationalise these new concepts into implementable algorithms using a generic framework and test ADAS that have not existed yet, cf. Section 2.2, 2.3.

Abundant literature exists regarding the impacts of ADAS on traffic flow operations. However, different conclusions have been found on the impacts of ADAS on traffic operations. Some researchers report increases in traffic capacity and stabilisation effects on traffic flow due to ACC systems (Rao & Varaiya, 1993; VanderWerf et al., 2002; Li & Shrivastava, 2002), while others are more conservative on the collective effects of ACC systems, showing no significant or even negative effects on traffic capacity and stability (Minderhoud & Bovy, 1999; Darbha & Rajagopal, 1999; Marsden et al., 2001). In addition, the impacts of ADAS on sustainability have not been addressed sufficiently. The lack of consensus on aggregate impacts of ADAS on traffic operations and the lack of insights into the sustainability impacts of ADAS in literature call for systematic and rigorous re-examination into the impacts of existing ADAS and investigation of possible impacts of ADAS under new control concepts, cf. Sections 2.4 and 2.5.

The first two challenges pertain to control methodology of ADAS while the third one pertains to assessment of ADAS impacts. All three challenges motivate the PhD research described in the ensuing of this thesis.

1.3 Research objectives and questions

The main objectives of this study are to develop a model predictive control framework for both non-cooperative and cooperative ADAS and to apply the framework in deriving and testing ADAS control algorithms under new control concepts to improve efficiency, safety, comfort and sustainability\(^1\).

To achieve the research objectives, the following research questions will be answered, which are categorised into four groups:

- Questions on state-of-the-art:
  1. Which ADAS concepts to improve traffic flow operations and sustainability have been proposed and which algorithms have been developed to operationalise these concepts?
  2. What are the impacts of existing ADAS on traffic operations and sustainability?

- Questions on control methodology and algorithms derivation:

\(^1\)By sustainability, we focus on reduction of fuel consumption and emissions, cf. Section 1.5.
3. How to formulate the control for ADAS vehicles into an optimisation problem?

4. Which solution approaches can be used to solve optimal control problems?

5. Which indicators and methods can be used to assess the performance of ADAS and impacts on traffic and sustainability?

6. How to derive ACC, ecological ACC (EcoACC) and cooperative ACC (C-ACC) controllers and operational algorithms under the control framework?

- Questions on controller performance for autonomous and cooperative vehicles:

7. How do the proposed ACC, EcoACC and C-ACC algorithms perform under representative scenarios at vehicle and platoon levels?

8. What are the impacts of ACC, EcoACC and C-ACC systems on collective traffic operations and sustainability?

- Question on feasibility and effectiveness of vehicle-road cooperation:

9. How to integrate traffic control with ADAS and what are the benefits of the integration?

1.4 Research approach

A four-step approach is taken to answer research questions, as shown in Figure 1.2. In the first step of exploration, literature review is conducted to identify the state-of-the-art and knowledge gaps on ADAS controller design and their impacts on traffic operations and sustainability, serving as a basis for the following steps. This step answers research questions 1 and 2.

In the second step, a generic control framework for ADAS is developed using model predictive control theory. The framework recasts the supported driving process as a rolling horizon optimal control problem. Different numerical and analytical solution approaches are compared and a generic numerical solution algorithm is proposed. This step answers research questions 3, 4 and 5.

In the third step, the control framework is applied to derive and refine control algorithms for different ADAS concepts, and the performance and impacts of the proposed ADAS controllers are assessed using traffic flow theory, microscopic traffic simulations and fuel consumption and emission models in the fourth step. Refinement of the control framework and the proposed ADAS controllers requires feedback from test and evaluation of ADAS controllers, resulting in the iterative process as shown in Figure 1.2. The two steps answers questions 6-9.
1.5 Research scope

Firstly, we focus on ADAS applications on motorway traffic. One reason underpinning this choice is that motorway traffic has less interruptions (e.g. interruptions caused by traffic lights, crossings and roundabouts) and is thus more promising for the implementation of cooperative systems in the near future. While not excluding the applicability of the control framework in urban traffic scenarios, motorway traffic provides a more controllable environment with less disturbances, which makes the relation of the design choices and the resulting ADAS behaviour more prominent.

Secondly, this thesis deals with ADAS that automate the longitudinal driving tasks, while the lateral driving tasks are assumed under control of human drivers. The longitudinal driving task includes maintaining a desired/free speed in free driving conditions and following the leader in a desired gap in car-following conditions. We emphasise that the control framework is not restricted to ADAS controllers for longitudinal driving support.

Thirdly, we focus on ACC systems and their extensions, i.e. cooperative and ecological ACC systems, due to the early availability of ACC systems in the market and the potentials of ACC systems and their extensions on influencing traffic operations (Tampère, 2004; Van Arem et al., 2006). Many other ADAS are designed specifically for safety concerns, and are usually referred to as Intelligent Vehicle Safety Systems (IVSSs). Examples of IVSSs include electronic stability control system, adaptive headlights system, blind spot monitoring system, lane departure warning system, collision avoidance systems, emergency call system (Richter, 2006; iCarSupport, 2011). Although important, these systems are beyond the scope of this thesis.

Lastly, for sustainability, we focus on one of the three pillars of sustainability, i.e. ecological and environmental sustainability (United Nations, 1987; Richardson, 1999). Hence, reducing fuel consumption and emissions are the subject of sustainability in this thesis, and other aspects of sustainability such as equity, accessibility etc. are not discussed in this thesis.
1.6 Main contributions

In this section, we highlight the main contributions of the thesis. We distinguish contributions which are of a scientific nature (either theoretical or methodological) and those of a more applied nature.

1.6.1 Scientific contributions

This thesis provides a synthesis of current knowledge on ADAS control methods, algorithms and impacts on traffic operations (Chapter 2). Revisiting the existing work identifies knowledge gaps in ADAS control design, provides a better understanding of the controller properties and characteristics of ADAS systems and explains differences in literature on the impacts of ADAS on traffic operations.

A generic model predictive control framework is developed (Chapter 3). The framework is generic in several ways. First of all, the well-known controller can be derived from the framework (Section 3.2). Secondly, multiple control objectives can be addressed under the framework. Thirdly, both non-cooperative and cooperative ADAS control concepts can be operationalised into implementable algorithms under the framework.

An efficient numerical solution algorithm based on Pontryagin’s Minimum Principle is developed (Section 6.3.1). The proposed algorithm solves the optimal control problem efficiently and is applied for simultaneous control of large scale systems with many vehicles (Chapters 6, 7 and 8). In addition, it does not pose strict requirements on the form of the cost function and provides insights into the solution direction in the state-space. The control framework and the solution algorithm can be applied to model/control other dynamical systems, such as pedestrians and vessels.

Novel controllers that operationalise non-cooperative, cooperative and ecological ADAS control concepts are designed and tested. The benchmark ACC controller is derived (Section 4.2) and refined (Sections 5.2, 6.2), taking into account safety and admissible constraints and flexible desired gap settings. The EcoACC controller is designed by including ecological cost in the objective function (Section 4.2). Cooperative ACC controllers under the cooperative manoeuvring concept (Sections 5.2, 6.2) are derived by optimising a joint cost function. One unique feature of the cooperative ACC controller is that it is not restricted to cooperation between intelligent vehicles. When a cooperative IV is followed by a human-driven vehicle, it can still exhibit cooperative behaviour by predicting the human follower behaviour. This is of importance for introducing such systems, since it does not rely on V2V communication and can function even with very low penetration rates of IVs in traffic.

Insights into the impacts of the designed ADAS on traffic operations and sustainability are provided (Sections 4.5, 5.5, 6.6, 7.3). The resulting capacity and stability of ADAS
vehicular traffic is largely determined by the controller parameters. The EcoACC controllers lead to lower speeds of IVs in cruising mode, higher speeds in following mode, and smoother accelerating behaviour. C-ACC systems lead to smoother behaviour under decelerating disturbance and responsive behaviour under accelerating disturbance, which reduce the inflow during the decelerating transition and increase the outflow during the accelerating transition. This property implies potentials of the cooperative sensing and cooperative manoeuvring strategy in improving traffic flow stability when vehicles travel into congestion and in increasing queue discharge rate when vehicle move out of congestions. Nevertheless, one should be careful in formulating and tuning controllers to avoid undesirable effects on traffic flow (Sections 5.5, 6.6, 7.3).

A new control application that connects and integrates decentralised ADAS controllers with a centralised traffic controller is implemented and tested (Chapter 8). The application is based on a new concept under V2I communications, which integrates decentralised model predictive ACC controllers at the vehicle level and a centralised variable speed limit controller at the link level for resolving stop-and-go waves. The integrated control system is implemented and tested in a multi-lane simulation environment. The control commands from the centralised controller are used as variable parameters for the decentralised vehicle controllers. The stop-and-go waves are resolved more efficiently under the integrated control paradigm.

1.6.2 Practical contributions

The insights regarding the impacts of intelligent vehicles equipped with ADAS on traffic flow operations and sustainability discussed in this section also have practical relevance (Chapter 9). They can support the road operators to make decisions and relevant policies on future traffic management with intelligent vehicles. The integration of decentralised ADAS controllers with link-level controllers also provides road operators a new way to managing intelligent vehicles (Chapter 8).

For industry (including vehicle manufacturers, OEMs, service providers, etc.), the optimal control framework and assessment framework can be used as guidance for developing ADAS controllers. Our work shows the flexibility and the generality of the optimal framework and performance assessment framework, which are desired features for ADAS developers. The two frameworks make the iterative design process easier.

From a driver’s or user’s perspective, the ADAS controller examples that under multiple criteria and constraints of collision-free, resemblance of human-like behaviour and stability concerns have potentials to improve the user acceptability of the ADAS systems since they extend the operational range of the ADAS controllers. The conclusion that intelligent vehicles can be developed to enhance safety, comfort, efficiency and to reduce fuels attracts potential users.
1.7 Outline of the thesis

Figure 1.3 gives an overview of the structure of the thesis. Chapter 2 reviews existing works on ADAS control algorithms, assessment methods and provides a better understanding of the impacts of ADAS on traffic operations and sustainability. It answers research questions regarding the state-of-the-art of ADAS control design and supports the statement in Chapter 1, i.e. challenges for designing and testing ADAS, and underpins the design choices for the control framework and assessment framework used in Chapter 3.

Chapter 3 describes the generic control framework and impacts assessment framework for ADAS, building on the findings of the literature study. The longitudinal driving task is formulated as a rolling horizon optimal control problem, which entails determining accelerations to optimise a cost function reflecting control objectives. Different
solution approaches to the optimal control problem are discussed and compared. A performance assessment framework for evaluating the performance of ADAS controllers at both microscopic and macroscopic level is proposed. The solution approaches and the performance assessment method are used for deriving specific control algorithms in the following chapters.

Several controllers at the vehicle and platoon level are designed and tested in Chapters 4, 5, 6 and 7, with different design considerations, control concepts and objectives, and solution approaches. Chapter 4 derives the benchmark ACC algorithm and an EcoACC algorithm, using a well-known solution approach of numerically solving the well-known Hamilton-Jacobi-Bellman (HJB) equation. A human interference mechanism is included in the ACC and EcoACC controller to prevent collisions, and the EcoACC controller uses a macroscopic emission model to calculate the ecological cost in the cost function. Impacts of the ACC and EcoACC systems on traffic and environment are assessed by microscopic simulation on a single-lane ring road.

Chapter 5 provides a refined version of the benchmark ACC algorithm and a cooperative ACC algorithm. The refined ACC algorithm includes an explicit safety mechanism in the objective function, which relieves the drivers of continuously monitoring the system and preparing for overruling. The cooperative ACC controller captures the collaboration between two IVs using V2V communications. The controllers are formulated as an infinite horizon control problem, which enables analytical solutions. The impacts of the controllers are assessed analytically, including capacity, local and string stability, using a linear stability analysis framework for a single-lane scenario.

Chapter 6 further refines the benchmark, ecological and cooperative ACC controllers. The benchmark ACC controller is improved by taking admissible control inputs into consideration, which yields more comfortable decelerations. The EcoACC controller is formulated using a physical modal fuel consumption model, which captures fuel consumed and pollutant emitted at dynamic driving conditions, e.g. acceleration and decelerating situations. A multi-anticipative ACC algorithm is derived under the cooperative sensing concept, and the cooperative ACC algorithm is refined to avoid the sluggish behaviour under accelerating stimulus and to incorporate collaboration between IV and human-driven vehicles. The performance of the proposed controllers are compared and synthesised through simulation in a single-lane.

Chapter 7 tests the ACC and C-ACC controllers in a multi-lane scenario under a bottleneck induced by lower speed limits. Impacts of different penetration rate of equipped vehicles are investigated.

Chapter 8 integrates the link-level traffic controller with vehicle-level ADAS controller via Vehicle-to-Infrastructure (V2I) communication and tests the benefits of the integration.

The thesis summarises the findings and conclusions in Chapter 9, including implications for practice and suggestions on future research directions.
Chapter 2

Current knowledge on ADAS control design and impact assessment

This chapter reviews existing studies on Advanced Driver Assistance Systems (ADAS) control design and impacts assessment on traffic operations and sustainability and answers research questions regarding the state-of-the-art of ADAS control methods in Chapter 1. The findings and conclusions of this chapter support the problem statement in the previous chapter and underpin the design choices for the following chapters.

This chapter is organised as follows. Section 2.1 describes the structure and the focus of the literature review following a general design procedure. Section 2.2 revisits the functions of controllers at different levels and highlights new control concepts related to ADAS. Platoon and vehicle level ADAS algorithms are discussed in Section 2.3, followed by the main models for assessing ADAS impacts in Section 2.4. Section 2.5 reviews the impact studies of ADAS on traffic operations and sustainability and Section 2.6 concludes the findings of the literature study.

2.1 Structure of the literature review

In the previous chapter, we stated that there are several challenges in designing and testing ADAS, including development of algorithms to operationalise new control concepts, refinement of existing systems, understanding the traffic flow and sustainability impacts of ADAS. This chapter provides an exhaustive literature review on ADAS control design and performance evaluation. The findings of this chapter elaborate the challenges and support the statements in Chapter 1.

Designing ADAS applications to improve traffic flow quality and sustainability is not a stand-alone task, due to the complexity of traffic systems and the inter-dependency between individual vehicular behaviour and collective traffic flow operations (Treiber et al., 2000). The design procedure starts from identifying functional needs and building control concepts, choosing control methods and developing operational algorithms,
selecting an appropriate method to assess the performance, and finally evaluating the ADAS performance. Feedback from the evaluation results may be necessary to improve the controller functionalities, algorithms and assessment methods.

Literature provides a wide range of ADAS functions and concepts at the network, link, platoon and vehicle levels (cf. Section 2.2). The first task of this chapter is to identify the most relevant ADAS functions and control concepts to improve traffic operations and sustainability. Some of the control concepts have been operationalised with control algorithms. The second task of this chapter is to compare the existing ADAS algorithms and identify their (methodological) adequacy and deficiency. The assessment of ADAS impacts is of importance to both vehicle manufacturers and road operators. The third task aims at a synthesised overview of the existing methods for evaluating ADAS impacts, and the last task of this chapter is to summarise the current findings on the impacts and particularly to understand the cause of the contradictory conclusions on the impacts of ADAS.

Section 2.2 to 2.5 of this chapter elaborate these tasks respectively and are structured based on the general design procedure as shown in Figure 2.1.

Figure 2.1: Schematic structure of the sections following a general design procedure.

## 2.2 Hierarchical functional architecture and example control concepts

Several functional architectures have been proposed for control systems with IVs, most of which pertain to a hierarchical structure where road-based traffic management systems are designed on top of vehicle-based systems. The hierarchical architecture can be used for controller design at different levels. In this section, we discuss a few control concepts related to ADAS to improve traffic operations and sustainability at different levels in the hierarchical architecture.

### 2.2.1 Road-based layers

At higher levels in the functional architecture, road-based traffic management systems regulate the collective traffic flow dynamics (Varaiya & Shladover, 1991; Cremer, 1992; Halle, 2005; Tsugawa et al., 2000; Kovacs et al., 2006; Baskar, 2009). The
traffic management systems are further categorised into network layer and link layer (Varaiya & Shladover, 1991; Baskar, 2009).

The main control task of the network layer is distributing the traffic in the network and the typical control measure at this layer is route guidance. The control cycle ranges from one hour to several hours, depending on the network size and variations of traffic demand (Varaiya & Shladover, 1991). The network controller requires the link layer to send macroscopic traffic variables through Infrastructure-to-Infrastructure (I2I) communication or the vehicle-based controller to send route and destination information through V2I communication to estimate demand and aerial-wide traffic state. In turn, the network layer controller sends reference control commands or signals, such as link target speed $V_r$ and link target density $K_r$ to the link layer via I2I communications or route guidance instruction $R_r$ to vehicle-based controllers via V2I communications.

The control task of the link layer is regulating the traffic flow, density and speed on this particular link, using control measures such as speed control and ramp metering (Varaiya & Shladover, 1991; Cremer, 1992; Baskar, 2009). In case of a platoon controller prevails under the link layer, the link layer also controls the platoon size and trajectory (Varaiya & Shladover, 1991; Baskar, 2009). The control cycle ranges from a few minutes to one hour. The messages from the network controller (reference route, link speed, link density) are used as (external) inputs for link layer. The link layer receives mesoscopic or microscopic traffic data from platoon or vehicle layer via V2I communication and in turn sends reference speed, platoon size and trajectory to platoon or vehicle layer via V2I communication.

One control concept of road-based layers relevant for the thesis is in-vehicle actuation of traffic control signals. This concept refers to a range of applications where in-vehicle systems as used as actuators of road-based traffic management systems (Varaiya & Shladover, 1991; Cremer, 1992; Baskar, 2009; Kovacs et al., 2006; Hegyi et al., 2013). The control signals are transmitted via V2I communication, and the signals are used as commands to the IVs. The information flows unidirectionally from network or link layer to IVs. The in-vehicle actuation of traffic control signals concept entails a hierarchical collaboration, i.e. IVs are forced to execute the control commands from traffic management systems. In-vehicle route guidance, which gives reference routes generated by the network controller to IVs and in-vehicle speed limits, which gives reference speed to IVs according to the speed control schemes generated at link layer are typical applications under this control concept.

### 2.2.2 Vehicle-based layers

Vehicle-based layers deal with manoeuvres of vehicles. Several studies (Varaiya & Shladover, 1991; Tsugawa et al., 2000; Halle, 2005; Baskar, 2009) proposed a platoon layer on top of the vehicle layer. The control cycle of the platoon layer ranges from a few seconds to a few minutes. This layer is mainly concerned with inter-platoon
manoeuvres such as merges with other platoons, splits, and lane changes under the instructions from the road-based controllers (Baskar, 2009). The platoon leader receives individual vehicle data from the vehicles in the platoon and from other platoon leaders via V2V communication, and sends the reference speed and gap, as well as its own position, speed and acceleration to its followers via V2V communication, which will be used by the followers to compute their control signals at the vehicle layer.

The vehicle layer deals with the manoeuvres of one vehicle and the controller updates in the order of 1 second. The control task consists of maintaining a desired speed, following with a desired distance and changing lanes. The vehicle controller receives microscopic information from the vehicle component, via the internal (wired) communication channel such as the Controller Area Network (CAN) bus, and sends back the reference acceleration and steering angle to the vehicle components via the internal communication channel.

This thesis focuses on the longitudinal control of IVs to improve traffic operations and sustainability. To this end, four control concepts in literature at the platoon and vehicle layer are most relevant for the thesis, being autonomous following, multi-anticipation, cooperative manoeuvring and eco-driving support.

Autonomous following refers to the ADAS function that automates the following behaviour in a non-cooperative way, i.e. determining the following speed and distance using only on-board sensors. This class of ADAS is referred to as Adaptive Cruise Control (ACC) systems (VanderWerf et al., 2001; Van Arem et al., 2006; Shladover et al., 2012), or Intelligent Cruise Control (ICC) system (Minderhoud, 1999), or Autonomous Intelligent Cruise Control (AICC) (Rao & Varaiya, 1993).

The multi-anticipation concept refers to utilising V2V to acquire the downstream information beyond the direct preceding vehicle (Kovacs et al., 2006) and choosing control decisions accordingly. This concept entails that downstream IVs share information to upstream vehicles, and has been applied to develop Cooperative ACC (CACC) (VanderWerf et al., 2001; Van Arem et al., 2006, 2007; Shladover et al., 2012).

While the multi-anticipation concept focuses on sharing information to peer IVs, the cooperative manoeuvring concept entails peer cooperative IVs to negotiate for consensus in the decision-making process and to coordinate their manoeuvres under a common goal by exchanging information via V2V communications. The information flows bidirectionally between peer IVs. Kovacs et al. (2006) proposed a coordinated braking application where IVs brake together to avoid collisions. A similar concept was proposed by Nakayama et al. (2002).

Ecological driving (eco-driving) strategies encourage drivers to drive at the energy-efficient speed, to anticipate traffic flow conditions to avoid sharp deceleration and acceleration, and to shift to higher gear early, etc. (ECOWILL, 2010; Bakenbus, 2010). Eco-driving support systems entail ADAS to support drivers in controlling IVs under eco-driving strategies, which are referred to as Ecological Driver Assistance Systems (EcoDAS) (Kamal et al., 2010). Examples of EcoDAS include Eco-cruise
Table 2.1: Comparison of different control layers

<table>
<thead>
<tr>
<th>Layer</th>
<th>Control task</th>
<th>Typical control measures</th>
<th>Control cycle</th>
<th>Spatial range</th>
<th>Message from lower layer</th>
<th>Message to lower layer</th>
<th>Communication</th>
</tr>
</thead>
<tbody>
<tr>
<td>Network</td>
<td>Route and flow distribution</td>
<td>Route guidance, perimeter control</td>
<td>≥ 1 hour</td>
<td>Road network in an area</td>
<td>Macroscopic flow, speed, density</td>
<td>Reference route, link speed, link density</td>
<td>I2I, V2I</td>
</tr>
<tr>
<td>Link</td>
<td>Flow and speed control</td>
<td>Speed control, ramp metering, lane advice</td>
<td>A few minutes - 1 hour</td>
<td>Single road stretch</td>
<td>Meso-/Microscopic speed, density/gap</td>
<td>Reference section speed, platoon size and trajectory</td>
<td>I2I, V2I</td>
</tr>
<tr>
<td>(Platoon) Trajectory control</td>
<td>-</td>
<td>A few seconds - 1 minute</td>
<td>A few vehicles</td>
<td>Microscopic traffic state</td>
<td>Reference speed, gap, microscopic traffic state</td>
<td>V2I, V2V</td>
<td></td>
</tr>
<tr>
<td>Vehicle</td>
<td>Longitudinal and lateral driving task</td>
<td>Lane choice, speed and following distance control</td>
<td>≤ 1 second</td>
<td>Single vehicle</td>
<td>Microscopic traffic state</td>
<td>Reference acceleration, braking, and steering</td>
<td>V2I, V2V, CAN bus</td>
</tr>
</tbody>
</table>

control (Park et al., 2012) in free driving conditions and Eco-Adaptive Cruise Control (EcoACC) (Kamal et al., 2010) in car-following conditions.

### 2.2.3 Summary

It is important to build the ADAS concepts and identify functional needs before developing operational algorithms. Different control concepts have been proposed in literature under a hierarchical architecture. The road-based layers are generic in a sense that they can function in traditional traffic systems without IVs. The platoon layer, however, requires the upper-level link layer to compute the reference platoon size and trajectory and lower-level vehicle layer to execute the control signals (Varaiya & Shladover, 1991; Halle, 2005; Baskar, 2009). To the best of our knowledge, no algorithms are available on how to compute the reference platoon size and path at the manoeuvre level, i.e. a reference path in continuous \((x, y)\) positions for the vehicle layer to follow. Not all reviewed architectures include the platoon layer in the hierarchy, i.e. PROMETHEUS (Cremer, 1992) and CVIS (Kovacs et al., 2006). The vehicle layer can function with and without the presence of platoon layer, and even function without the presence of roadside layers, which constitutes a distributed or fully decentralised system with intelligent vehicles spatially distributed on the road. The in-vehicle actuation of traffic control signals concept forces IVs to cooperate with the traffic management systems, setting a good example of interaction between vehicle-based layers and road-based layers.

No cooperation prevails in the autonomous following and eco-driving support concepts. The multi-anticipation concept requires the IVs to share information to other IVs, but the control decision is still non-cooperative, i.e. there is no negotiation or
consensus during the decision-making process. The cooperative manoeuvring concept entails the peer IVs to negotiate with each other and coordinate their behaviour under a common goal.

In the next section, we review the control algorithms that operationalise the aforementioned control concepts at the platoon and vehicle layer.

### 2.3 Control algorithms for ADAS at platoon and vehicle levels

This section reviews existing control algorithms for ADAS at platoon and vehicle levels. Figure 2.2 shows an abstract representation of an ADAS controller. For a generic ADAS controller, the on-board sensors measure the local driving environment and the measurements serve as inputs for the controller. The controller may receive traffic control signals from the network or link controller via V2I communication, e.g. reference speeds, or information from the other vehicles in the neighbourhood via V2V communication, e.g. positions and speeds of other vehicles. With all information available, the vehicle controller determines the control signal, i.e. the acceleration of the IV, and the vehicle actuators execute the control signal automatically. The ADAS controller may communicate its local measurements and its acceleration to other vehicles via V2V or to roadside controllers via V2I communications. As the ADAS vehicle moves, the vehicle-driving environment changes, and the controller updates the control signal in the next control cycle. The grey rectangle in the figure is the focus of this section.

ACC algorithms are benchmarks for ADAS controller design at platoon and vehicle level, since many controllers and algorithms of more advanced control concepts are developed upon ACC algorithms. In the remaining of this section, we first revisit algorithms for ACC systems, and then the Ecological ACC (EcoACC) and Cooperative ACC controllers (CACC) are reviewed. The characteristics of existing control methods and algorithms are summarised and discussed at the end of the section.

#### 2.3.1 ACC algorithms

Under the autonomous following concept, an ACC system uses local measurements from its own on-board sensors to estimate the system state. It usually operates in two modes, being cruising mode (or speed control mode) and following mode (or gap control mode). The two modes are distinguished by a distance gap threshold $s_f$. Without loss of generality, we consider ACC vehicle $i$, as shown in Figure 2.3(a). We use $a_i$, $v_i$, $\Delta v_i$, $s_i$, $x_i$ and $l_i$ to denote the acceleration, speed, relative speed, gap, (rear bumper) position and length of ACC vehicle $i$ respectively. The preceding vehicle is indexed with $i - 1$. Notice that in our definition $s_i = x_{i-1} - x_i - l_i$ and $\dot{s}_i = v_{i-1} - v_i = \Delta v_i$. 
Figure 2.2: Abstract representation of an ADAS controller.

Figure 2.3: Schematic illustration of (a) ACC, (b) Multi-anticipative CACC and (c) Looking-backward ACC controllers.
In cruising mode, where \( s_i > s_f \), the ACC system regulates the vehicle speed towards a free speed \( v_0 \), and the speed controller is universally a linear proportional (P) controller (Godbole et al., 1999; Hogema, 1999; VanderWerf et al., 2001; Van Arem et al., 2006; Shladover et al., 2012) where acceleration is calculated with:

\[
a_i^{\text{cruising}} = K_v(v_0 - v_i)
\]  

(2.1)

where \( K_v \) is the constant feedback gain.

In following mode, where \( s_i \leq s_f \), the ACC system regulates the gap towards a desired gap \( s_d \) and at the same time tries to match the speed of the preceding vehicle. Generally speaking, the state-feedback ACC controllers output the reference acceleration \( a_i \) of vehicle \( i \) as an explicit function of the system state \( x = (s_i, \Delta v_i, v_i)^T \), i.e. Eq. (2.2), while the optimisation-based ACC controllers or ACC controllers using artificial intelligence (AI) techniques generate the reference acceleration using an implicit function.

\[
a_i = f(x) = f(s_i, \Delta v_i, v_i)
\]  

(2.2)

One design choice for a complete ACC controller is determining the gap threshold \( s_f \) to distinguish the two operating modes. In (VanderWerf et al., 2001), a fixed distance of 100 metres is used for \( s_f \), while in Godbole et al. (1999), the looking-forward sensor range is used as \( s_f \). Alternatively, the operating mode is implicitly determined by the acceleration value of two modes, not on a fixed distance (Hogema, 1999; Van Arem et al., 2006, 2007). At each time instant, the controller compares Eq. (2.6) and Eq. (2.1), and the lower acceleration will be used as the final reference signal \( a_i^{\text{final}} \):

\[
a_i^{\text{final}} = \min(a_i, a_i^{\text{cruising}})
\]  

(2.3)

with \( a_i \) calculated with (2.6) and \( a_i^{\text{cruising}} \) evaluated with (2.1).

ACC algorithms in following mode are most interesting from traffic operation perspective, since congestion occurs in following mode. The following mode algorithms will be discussed in detail in the ensuing.

**Constant spacing policy**

The constant spacing policy is probably the most simple gap control law for ACC vehicles (Chandler et al., 1958; Chu, 1974; Hedrick et al., 1994; Swaroop, 1994) in following mode. This control policy aims at maintaining a fixed desired gap \( L \) and matching the predecessor speed, and the algorithm follows:

\[
a_i = K_s(s_i - L) + K_{\Delta v}\dot{s}_i
\]  

(2.4)

where \( K_s \) and \( K_{\Delta v} \) are constant feedback gains for the gap error and speed error.

The constant spacing policy is a linear state-feedback controller, i.e. a proportional derivative (PD) controller. No prediction of the leader-follower dynamics is included
in the control law. The control algorithm does not guarantee collision-free behaviour and the desired gap is not related to vehicle speed, which is contrary to plausible car-following behaviour (Treiber & Kesting, 2011). Thus the user acceptance of the ACC system employing this control policy is very low.

### Constant time headway/gap policy

The most widely-studied gap control algorithm is the constant time headway (CTH) policy (Chu, 1974; Ioannou & Chien, 1993; Hedrick et al., 1994; Swaroop, 1994; Minderhoud, 1999; Godbole et al., 1999; Rajamani & Shladover, 2001; VanderWerf et al., 2001; Marsden et al., 2001; Hogema, 1999; Davis, 2004; Van Arem et al., 2006, 2007; Jiang et al., 2009; Shladover et al., 2012). Instead of keeping a fixed desired gap $L$, the CTH policy keeps a desired gap $s_d$ depending on the vehicle speed:

$$s_d = v_i t_d + s_0$$

with $s_0$ denoting the minimum gap at standstill conditions, and $t_d$ denotes the preferred time gap. The control algorithm reads:

$$a_i = K_s (s_i - s_d) + K_{\Delta v} \dot{s}_i = k_s (s_i - t_d \cdot v_i - s_0) + K_{\Delta v} \dot{s}_i$$

(2.6)

where $K_s$ and $K_{\Delta v}$ are constant feedback gains for the gap error and speed error. In fact, the term Constant Time Headway (CTH) is not correct, since $t_d$ is the time gap, while the time headway $t_h = t_d + \frac{l}{v_i}$ is determined by the vehicle length $l$ and vehicle speed $v_i$. The time headway $t_h$ is not constant through different traffic conditions. In the remainder of the thesis, we refer this type of controller as Constant Time Gap (CTG) controller.

The CTG policy is a linear state-feedback algorithm without prediction of the future state, which is essentially a Helly-type car-following model without reaction time (Helly, 1959). The ACC system employing the CTG policy does not guarantee collision-free at safety-critical conditions, e.g. approaching a standstill vehicle with high speeds (Godbole et al., 1999). In practice, the ACC system is switched off at safety-critical conditions as well as in dense traffic conditions (Viti et al., 2008; Klunder et al., 2009), since the system does not satisfy human desires in these conditions (Marsden et al., 2001; Bifulco et al., 2013).

### Variable time gap policy

Several authors proposed control algorithms where the desired gap of ACC vehicle $i$ is variable and not linear proportional to the vehicle speeds (Hoogendoorn & Minderhoud, 2002; Xu et al., 2002; Wang & Rajamani, 2004; Zhou & Peng, 2005). As a

\[2\] We drop the vehicle index $i$ in $s_d$ in the ensuing for notation simplicity. The same holds for $t_d$ and $t_h$ in this chapter.
consequence, the desired time gap $t_d = s_d/v_i$ is variable rather than constant. Thus this class of algorithms as variable time gap (VTG) policy (Wang & Rajamani, 2004; Zhou & Peng, 2005). Hoogendoorn & Minderhoud (2002) proposed a quadratic distance gap controller for ACC vehicles, where the desired gap is a quadratic function of vehicle speed:

$$s_d = s_0 + z_1 v_i + z_2 v_i^2$$  \hspace{1cm} (2.7)

where $z_1$ and $z_2$ are positive control parameters. A similar quadratic policy has been reported by Zhou & Peng (2005).


The VTG policy is a class of non-linear state-feedback algorithms without predicting the predecessor behaviour. The VTG policy has potentials to improve traffic flow stability compared to the CTG policy (Wang & Rajamani, 2004).

**Safe distance control algorithm**

Broqua et al. (1991) proposed a safe distance controller for ACC systems, which is similar to the safe distance car-following model (Gipps, 1981). This controller ensures that the following gap $s_i$ is not smaller than a safe gap $s_{safe}$ for vehicle $i$. The assumption is that if the predecessor suddenly brakes to a full stop, the stopping distance of the ACC vehicle should not be smaller than a minimum gap $s_0$. The safe gap $s_{safe}$ can be calculated by the kinematic equation:

$$s_{safe} = s_0 + v_i \tau + \frac{v_i^2}{2b} - \frac{v_{i-1}^2}{2b'}$$  \hspace{1cm} (2.8)

where $\tau$ is the time lag and $b$ and $b'$ are the braking capabilities of the ACC vehicle and the predecessor.

Equation (2.8) gives a constraint on the vehicle speed and gap, but does not output a reference acceleration. How the controller regulates the vehicle actuators to satisfy the constraint is a design choice and entails introducing extra control parameters.

Minderhoud (1999) proposed a two-regime algorithm for following mode under the safe distance concept. If the actual gap is smaller than the safe distance $s_i < s_{safe}$, the acceleration is calculated with:

$$a_i = -2 \frac{s_{safe} - s_i}{T_{u,prev}^2}$$  \hspace{1cm} (2.9)

where $T_{u,prev}$ is a controller parameter. Otherwise, the acceleration is determined by the CTG policy of Eq. (2.6).

Eq. (2.9) is a non-linear state-feedback algorithm without prediction of the future state. No test is performed on the user acceptance of this control algorithm.
Car-following models as ACC algorithms

Since acceleration is the control signal of ACC controllers, two car-following models have been used as gap control algorithms in literature. The Intelligent Driver Model (IDM) is used to design ACC controllers to resemble human car-following behaviour (Kesting et al., 2008). IDM calculates the controlled acceleration with the following equation:

\[ a_i = a_{\text{max}} \left( 1 - \frac{v_i}{v_0} \right)^{\delta} - \left( \frac{s_d(v_i, \Delta v_i)}{s_i} \right)^2 \] (2.10)

with

\[ s_d(v_i, \Delta v_i) = s_0 + \max \left( 0, v_i t_d + \frac{v_i(-\Delta v_i)}{2a_{\text{max}} b_{\text{conf}}} \right) \] (2.11)

where \( a_{\text{max}} \) is the maximum acceleration, \( b_{\text{conf}} \) is the comfortable deceleration and \( \delta \) is a model parameter. Eq. 2.10 shows that the acceleration \( a_i \) is a balance between the tendency to accelerate to cruising speed \( v_0 \) and the tendency to decelerate due to interaction with the predecessor.

The optimal velocity model (OVM) and variations are also used to calculate reference acceleration for controlled vehicles (Nakayama et al., 2002; Hasebe et al., 2003; Ge et al., 2006). The OVM with speed difference is used to calculate acceleration:

\[ a_i = \alpha (v_{\text{opt}}(s_i) - v_i) \] (2.12)

where \( \alpha \) is the sensitivity factor in the original OVM, or the feedback gain on the deviation from the optimal velocity \( v_{\text{opt}} \) as a function of gap (Bando et al., 1995).

Both IDM and OVM are non-linear state-feedback control algorithms, giving an acceleration value under each initial condition. No prediction of system state is included. The intelligent braking strategy of the IDM model prevents the ACC colliding with the preceding vehicle. However, the OVM cannot guarantee collision-free behaviour (Treiber et al., 2000), and thus the OVM-based ACC system has to be switched off in some conditions. The IDM algorithm has been put into ACC equipped vehicles, resulting in high user acceptability (Kesting et al., 2010). The user acceptability is not expected to be high since the OVM of Eq. (2.12) does not generate plausible car-following behaviour.

Optimisation-based ACC algorithms

Chu (1974) proposed an optimisation-based controller for automated vehicles in a platoon, assuming linear system dynamics and choosing the objective function as quadratic, i.e. linear quadratic regulator (LQR). The optimal control problem is formalised as:

\[ a^*(t) = \arg\min_a J(x, a|x_0) \] (2.13)
with cost function, also termed as cost functional, objective function, or performance index $J$ defined as:

$$J = \frac{1}{2} \int_0^\infty p(s_i(t) - L)^2 + q\Delta v_i(t)^2 + a_i(t)^2 \, dt$$

(2.14)

where $p$ and $q$ are constant weight factors, and $L$ is a constant spacing between vehicles in a platoon. The optimal control problem (2.13) is subject to the linear vehicle dynamics equations.

In this special ACC controller as a LQR, the reference acceleration can be calculated with a linear feedback control law of the state $x$ (Pontryagin et al., 1962), which is the same as the constant spacing policy, i.e. Eq. (2.4). A similar formulation has been taken by Mårtensson et al. (2012) by replacing the constant spacing $L$ with the speed dependent gap $s_d$ as in Eq. (2.5). Although the link between linear state feedback algorithms and optimal control algorithms holds for linear systems under specific formulation (cf. Section 3.3), in more general cases the reference accelerations of optimal controllers cannot be explicitly computed and numerical solutions are necessary to find the optimal accelerations. Same as the constant spacing and CTG policy, the LQR controller cannot guarantee collision-free, and thus the system cannot function in all conditions.

Recently, Li et al. (2011) reported a model predictive ACC controller in which the reference acceleration is assessed in a rolling horizon way. The ACC controller aims at minimising deviation from desired gap, deviation from predecessor speed, accelerations and jerk, and the deviation from a human desired acceleration calculated with Helly model. The cost function is of quadratic form and the acceleration is constrained to a range of $[-1.5, 0.6] \, \text{m/s}^2$, which restricts the usage of the system from highly dynamic traffic conditions. Although the safety is used as constraint of the system, the limited acceleration range cannot guarantee collision-free at safety-critical conditions.

**Artificial intelligence based ACC algorithms**

Michon (1993) proposed a rule-based controller for generic intelligent driver support systems, which lies in the category of artificial intelligence (AI) techniques. The vast number of the rules and scenarios involved makes the controller highly non-linear and it is not straightforward to mathematically describe the controller. There is no prediction in this control algorithm. The safety aspect is addressed by incorporating a collision-free, rule but the user acceptability of the system is not clear. Other AI technologies are also reported in ACC controller design, such as fuzzy logic or self-learning systems (Müller & Nöcker, 1992; Shaout & Jarrah, 1997).
Table 2.2: Comparison of ACC algorithms in following mode

<table>
<thead>
<tr>
<th>Algorithms</th>
<th>Control methods</th>
<th>Linearity</th>
<th>Control objectives</th>
<th>State prediction</th>
<th>Collision-free</th>
<th>User acceptability</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant spacing</td>
<td>State-feedback</td>
<td>yes</td>
<td>Maintain constant gap; match predecessor speed;</td>
<td>no</td>
<td>no</td>
<td>low</td>
</tr>
<tr>
<td>CTG</td>
<td>State-feedback</td>
<td>yes</td>
<td>Maintain constant time gap; match predecessor speed;</td>
<td>no</td>
<td>no</td>
<td>fair</td>
</tr>
<tr>
<td>VTG</td>
<td>State-feedback</td>
<td>no</td>
<td>Maintain desired gap (variable time gap); match predecessor speed;</td>
<td>no</td>
<td>n.r.</td>
<td>high</td>
</tr>
<tr>
<td>Safety-distance</td>
<td>State-feedback</td>
<td>no</td>
<td>Keep a safe gap;</td>
<td>no</td>
<td>yes</td>
<td>n.r.</td>
</tr>
<tr>
<td>model</td>
<td>feedback</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>IDM</td>
<td>State-feedback</td>
<td>no</td>
<td>Resemble human behaviour</td>
<td>no</td>
<td>yes</td>
<td>high</td>
</tr>
<tr>
<td>OVM</td>
<td>State-feedback</td>
<td>no</td>
<td>Maintain a gap-dependant speed</td>
<td>no</td>
<td>no</td>
<td>low</td>
</tr>
<tr>
<td>LQR ACC</td>
<td>Optimal</td>
<td>yes</td>
<td>Minimise deviation from desired gap; minimise deviation from predecessor speed;</td>
<td>yes</td>
<td>no</td>
<td>low/fair</td>
</tr>
<tr>
<td></td>
<td>control</td>
<td></td>
<td>minimise acceleration/deceleration</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Model predictive</td>
<td>MPC</td>
<td>yes</td>
<td>Minimise deviation from desired gap; minimise deviation from predecessor speed;</td>
<td>yes</td>
<td>no</td>
<td>fair</td>
</tr>
<tr>
<td>ACC</td>
<td></td>
<td></td>
<td>minimise accelerations and jerk; minimise deviation from a human desired acceleration</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Rule-based ACC</td>
<td>AI</td>
<td>no</td>
<td>n.r.</td>
<td>no</td>
<td>yes</td>
<td>n.r.</td>
</tr>
</tbody>
</table>

AI: artificial intelligence; n.r.: not reported

Summary of ACC algorithms

Table 2.2 gives an overview of the characteristics of ACC algorithms in following mode in literature. The widely-studied linear CTG policy has been implemented in modern cars (Mårtensson et al., 2012; Bifulco et al., 2013) and extended to multi-anticipative following control algorithms. However, the linearity of the controller cannot handle highly dynamic traffic conditions, resulting in deactivation of the systems in these conditions. Using non-linear car-following models as ACC control algorithms has the potentials to increase the application of ACC systems and enhance user acceptance.

Different from the feedback ACC controllers, optimisation-based control algorithms predict the system state in the future horizon and output a time-series trajectory of reference acceleration $a^*(t)$ \(^3\). The flexibility of the control approach allows one to design controllers under multiple criteria.

2.3.2 Cooperative ACC algorithms

With V2V communication, the vehicle controller can access not only local measurements from its own sensors, but also information from other vehicles, as shown in

---

\(^3\)Although linear feedback controllers are equivalent to optimal controllers under specific controller formulations, we distinguish the two methods for more general cases where they are not equivalent.
The vehicle controller may benefit with V2V communication from two ways.

The first way to improve the controller performance is to use the more accurate information transmitted via V2V communication instead of the information from own on-board sensors, e.g. using the speed transmitted by V2V communication to replace the relative speed measurements from on-board radar (Rajamani & Shladover, 2001; Shladover et al., 2012). This is a rudimentary form of cooperation. The control algorithm remains the same as the CTG policy (2.6). There is no improvement in the control algorithm itself and thus we do not discuss this type of controller in more detail.

Another way of using V2V communication is to provide more information than the local measurements from on-board sensors, enabling the controller to look further ahead beyond the direct preceding vehicle, or even look backwards to its followers, as depicted in Figure 2.3(b) and 2.3(c). In this case, the reference acceleration of the controlled vehicle $i$ in a cooperative driving system can be generalised as follows:

$$a_i = f(s_i, v_i, s_{i-1}, v_{i-1}, ..., s_{i-j}, v_{i-j})$$

where $k = 1, 2, ..., j = 1, 2, ...$ denoting the $k$th vehicle downstream of vehicle $i$ and $j$th vehicle upstream of vehicle $i$. We classify the CACC controllers reported in literature into two groups based on the control concepts, being multi-anticipative ACC controllers under the multi-anticipation concept and backwards-looking ACC controllers under the cooperative manoeuvring concept.

### Multi-anticipative ACC algorithms under multi-anticipation concept

Without loss of generality, we consider ACC vehicle $i$ as shown in Figure 2.3(b). Van Arem et al. (2007) extended the CTG policy to cooperative ACC (CACC) controllers under the multi-anticipation concept. This CACC system, which we will termed as MACC system, entails downstream vehicle $i-1$ transmitting information (i.e. position, gap and speed) to the CACC vehicle $i$. The controller can be generalised as a multi-anticipative Helly model (Hoogendoorn et al., 2006) without reaction time, which expresses the acceleration as:

$$a_i = K_{ctg}(s_i - v_it_d - s_0) + K_{Δs}Δ_i + \sum_{k=i-n+1}^{i-1} [α_k(s_k - v_kt_d - s_0) + β_k\dot{s}_k]$$

where $n \geq 2$ denotes the number of vehicles that the CACC vehicle $i$ looks downstream and $α_k$ and $β_k$ denote the control gains of the gap deviation and relative speed of vehicle $k$ respectively (Note that $n = 2$ in Figure 2.3(b)). The first two terms in the right-hand side of Eq. (2.16) represents the CTG policy for ACC systems. The third term consists of the multi-anticipation terms, implying that the CACC vehicles tend to accelerate when downstream CACC vehicles are driving with a gap larger than their desired gap or with a lower speed than their predecessor, and tend to decelerate when vice versa.
VanderWerf et al. (2001) and Van Arem et al. (2006) extended the CTG policy to include the acceleration of the predecessor. Different from the work of Van Arem et al. (2007), V2V communication is used to transmit the acceleration and braking capabilities of the predecessor rather than gap and relative speed of the predecessor. The controller is still a feedback controller with the following form:

\[
a_i = K_s(s_i - v_it_d - s_0) + K_a \dot{s}_i + Ka_{i-1}
\]

(2.17)

where \(K_a\), \(K_{\Delta v}\) and \(K_s\) are positive feedback gains. The first two terms in the right-hand side of the CACC controller in Eq. (2.17) is the CTG policy. The third term in Eq. (2.17) distinguishes the CACC controller from the ACC controller, which implies that a CACC vehicle tends to accelerate when the predecessor is increasing speed and tends to decelerate when vice versa. The CACC controller tends to copy the acceleration of the direct predecessor. Transmitting the predecessor acceleration to facilitate the controller design of CACC vehicles is also reported in Ploeg et al. (2014) with a vehicle kinematic model.

In the scenario as shown in Figure 2.3(b), the CACC vehicle \(i-1\) is employing a CTG policy since it cannot anticipate further if the leader is not a CACC vehicle. Then its acceleration can be calculated with:

\[
a_{i-1} = K_s(s_{i-1} - v_{i-1}t_d - s_0) + K_{\Delta v} \dot{s}_{i-1}
\]

(2.18)

One can re-write Eq.(2.17) as:

\[
a_i = K_s(s_i - v_it_d - s_0) + K_{\Delta v} \dot{s}_i + K_a K_s(s_i - v_i t_d - s_0) + K_a K_{\Delta v} \dot{s}_{i-1}
\]

(2.19)

Equation (2.17) is equivalent to Eq. (2.16) with \(n = 2\), \(\alpha_{i-1} = K_a K_s\), and \(\beta_{i-1} = K_a K_{\Delta v}\). Hence the feedback controller with anticipation of the predecessor dynamics, i.e. Eq. (2.17), is essentially a multi-anticipative CACC controller, i.e. Eq. (2.16).

Some authors also propose a similar concept of regulating the acceleration difference in addition to the CTG feedback law (Halle, 2005). The reference acceleration is calculated as:

\[
a_i = K'_s(s_i - v_it_d - s_0) + K'_{\Delta v} \dot{s}_i + K_a(a_{i-1} - a_i)
\]

(2.20)

with \(K'_{\Delta v}\), \(K'_s\) and \(K_a\) being constant control gains. One can show that Eq. (2.20) is equivalent to Eq. (2.17) by removing \(a_i\) to the left hand-side and letting \(K_{\Delta v} = K'_{\Delta v} / (1 + K_a)\), \(K_s = K'_s / (1 + K_{\Delta a})\), and \(K_a = K_{\Delta a} / (1 + K_{\Delta a})\). Hence the controller that regulates the relative acceleration in addition to the CTG policy, i.e. Eq. (2.20), is also a multi-anticipative CACC controller.

Anticipating downstream vehicle acceleration is also reported for controlled vehicles in a platoon based on constant spacing policy (Shladover et al., 1991; Hedrick et al., 1994; Swaroop, 1994). Difference is that the followers do not anticipate the predecessor acceleration, but the acceleration of the platoon leader. This entails platoon leader communicating its position, speed and acceleration to all the vehicles in the platoon.
Table 2.3: Comparison of control design methods

<table>
<thead>
<tr>
<th>Control methods</th>
<th>Computational complexity</th>
<th>Constraints</th>
<th>Predictive/Reactive</th>
<th>Flexibility</th>
</tr>
</thead>
<tbody>
<tr>
<td>State-feedback</td>
<td>low</td>
<td>difficult</td>
<td>reactive</td>
<td>low</td>
</tr>
<tr>
<td>Optimisation-based control</td>
<td>high</td>
<td>yes</td>
<td>predictive</td>
<td>high</td>
</tr>
<tr>
<td>AI techniques</td>
<td>medium</td>
<td>difficult</td>
<td>reactive</td>
<td>low</td>
</tr>
</tbody>
</table>

Backwards-looking ACC algorithms

Nakayama et al. (2002) proposed a cooperative driving system in which the controlled vehicle not only looks ahead to downstream vehicles, but also look backwards to its followers. The reference acceleration of vehicle $i$ is determined by:

$$a_i = \alpha (v_{opt}(s_i) + v_B(s_{i+1}) - v_i)$$

(2.21)

where $\alpha$ and $v_{opt}(s_i)$ are the same as in Eq. (2.12). $v_B(s_{i+1})$ is the optimal velocity determined by the gap behind vehicle $i$, and is a decreasing function of $s_{i+1}$, i.e. when the gap behind vehicle $i$ becomes smaller, the vehicle $i$ tends to increase its speed to create more space for the follower. Similar formulation has been proposed in Hasebe et al. (2003) and Ge et al. (2006).

2.3.3 EcoACC algorithms

Few work has been devoted to operationalise eco-driving support concept. Park et al. (2012) proposed an eco-cruise control system using model predictive control approach, aiming at minimising fuel consumption taking into account the road gradients. However, the systems is only valid for cruising mode, not for following mode. On the contrary, Kamal et al. (2010) proposed an ecological adaptive cruise control (EcoACC) system that regulates the car-following behaviour of the controlled vehicles. The controller aims at minimising fuel consumptions, minimising the deviation from the desired gap and minimising accelerations/decelerations. No cruising mode is included in the controller. The two controllers reported are non-linear due to the inclusion of fuel consumption model in the cost function and can only be solved numerically. Generally speaking, adding fuel consumption into the objective function has the tendency to control vehicle speeds towards the fuel-efficient speed, which is between 70 and 80 km/h for passenger cars (Barth & Boriboonsomsin, 2008).

2.3.4 Discussion

As the end of this section, we summarise the ADAS control algorithms at the platoon and vehicle level and discuss ADAS design considerations, control methods and the
main differences in algorithms under different control concepts.

**General design issues for ADAS**

User acceptance is of crucial importance to achieve rapid market penetration, and is of primary concern for vehicle manufacturers. Among other aspects, the safety and user acceptability are key issues that should always be considered and tested when designing ADAS controllers. Although several authors claimed that the drivers should take the responsibility of monitoring the ADAS systems operations and overrule the systems when necessary (VanderWerf et al., 2001; Kamal et al., 2013). We argue that a good ADAS design should always take safety and driver desires into consideration and try to reduce the deactivation of the system as much as possible.

**Changes from traditional ACC to cooperative and ecological ACC systems**

Traditional non-cooperative ACC systems pertain to the autonomous following concept. The ACC algorithms serve as benchmark for other algorithms at the platoon and vehicle level, particularly the constant time gap policy.

Multi-anticipative CACC systems under the multi-anticipation concept use information shared by downstream CACC vehicles through V2V communications and react to multiple predecessors. This has potentials to react earlier to downstream disturbances compared to the non-cooperative ACC system. We have mathematically proved that copying or matching the acceleration of the direct predecessor has the same effects as anticipating the behaviours of multiple predecessors.

Backwards-looking ACC systems using V2V communications under the cooperative manoeuvring concept entails the IV considering the situation of its follower when making decisions. This implies that the controlled vehicles may have to compromise its own benefits in some situations.

The EcoACC algorithms under the eco-driving support concept add fuel consumption into the control objectives of ACC vehicles. This entails a multiple objective approach to design EcoACC algorithms.

**Control methods for ADAS**

The main objective of the PhD research is to develop a generic control framework for ADAS, which entails choosing a control method. In literature, three different control approaches have been found for ADAS controllers, being state-feedback control, optimisation-based control and artificial intelligence (AI) techniques. The majority of the controllers is of a state-feedback form, with few controllers reported using optimisation-based control approach and AI techniques. All the three control approaches have been used to design ACC controllers. State-feedback control approach
has been used to design multi-anticipative CACC and backwards-looking ACC controllers, and optimisation-based approach has been used to design EcoACC controllers.

Table 2.3 compares the three control methods in different dimensions. From computational demand perspective, state-feedback methods have the lowest computational complexity, which is quite appropriate for on-line applications. The optimal control and Model Predictive Control (MPC) methods requires higher computation power, but the computational efficiency has been improved a lot for the past decade. The computational load of AI techniques is between the two.

It is difficult to use static state-feedback methods to deal with constraints, particularly hard constraints. For longitudinal ADAS controllers, not colliding with the preceding vehicle is a hard constraint. The safety constraint can be taken into account directly into the optimisation-based approach.

Optimisation-based methods are predictive, which entail predicting the local system state, i.e. the leader-follower dynamics, and making optimal control decisions. The state-feedback and AI techniques do not make predictions, and thus are reactive and myopic in this sense. Prediction of the future evolution has potentials in improving controller performance under disturbances (Hoogendoorn et al., 2012) and compensating the negative impacts of delay.

Optimisation-based methods are flexible in that the system state vector, control input vector as well as the objective function can be changed easily depending on the applications. This is a unique property compared to the state-feedback method or AI techniques. Particularly the flexibility in the objective function is promising for including vehicle-vehicle cooperation and sustainability in the controller.

The choices of ADAS control methods and parameter settings affect individual system characteristics and the collective vehicular performance. From vehicle manufacturers and users perspective, it is important to know how individual system behaves in real-world scenarios. From the road operators perspective, the primary concern is what are the effects of ADAS on traffic flow operations and sustainability? To answer that, assessment of the controller performance has to be conducted. In the remaining of this chapter, we will discuss the approaches in literature to assess the ADAS impacts and the results of the assessment.

2.4 Methods for ADAS impact assessment

As discussed in the previous section, assessment of the individual and collective ACC controller impacts are of fundamental importance to system users, vehicle manufacturers as well as road operators. This section reviews the methods that enable one to do such an evaluation. We first summarise the performance indicators for ADAS impacts. Then the impacts models for traffic operations and environmental sustainability
are reviewed. The advantage and disadvantages of different models are discussed in the end.

2.4.1 Performance measures and indicators

ADAS take over part of or all the driving tasks from human drivers, which may induce changes in individual vehicular performance at microscopic level and the collective traffic performance at macroscopic level. At microscopic level, we are primarily interested in how ACC systems behave under typical driving scenarios, i.e. free driving, cutting-in, cutting-down, approaching a constant predecessor, following a decelerating predecessor, following an accelerating predecessor, emergency braking. (Godbole et al., 1999; Bareket et al., 2003), and whether the ACC systems can guarantee local stability (Treiber & Kesting, 2011; Wilson, 2008). Indicators reflecting the average and amplitude of responses of ACC vehicles under these scenarios can be used to assess their performance, such as maximum acceleration/deceleration, average speed and speed variance, average gap and gap variance, etc.

Numerous indicators can be used to evaluate macroscopic traffic operations. Generally speaking, the performance indicators for traffic operations can be categorised into three groups, being efficiency, predictability and smoothness, and safety (Minderhoud, 1999; Neudorff et al., 2003; Shaw et al., 2003). The most important indicator for efficiency is capacity. Total time spent (TTS) in traffic and delay are also common indicators for efficiency. Predictability and smoothness at macroscopic level can be assessed by string and flow stability, speed and travel time variations. Safety can be evaluated by the risks of collisions, e.g. time-to-collision.

Broad definition of sustainability entails the three pillars of economic, environmental, and social sustainability, which are often mutually reinforcing (World Bank, 1996; United Nations, 1987), while narrow definition of sustainability particularly refers to environmental sustainability. In this thesis, we focus on environmental sustainability, which is typically assessed by the fuel consumption and emissions rates for individual vehicles at microscopic level and for all traffic at macroscopic level (Litman & Burwell, 2006; Jeon & Ameikudzi, 2005; Litman, 2007).

In practice, it is difficult and expensive to measure the indicators the empirically, particularly at the design stage for new technologies such as ADAS. Therefore, traffic flow models and fuel consumption and emission models provide a cost-effective way for qualifying and quantifying the impacts of ADAS systems. In remaining of this section, we review the models that has been used for calculating the indicators for evaluating traffic operations and environmental sustainability.
Table 2.4: Overview of models for traffic operations impacts of ADAS

<table>
<thead>
<tr>
<th></th>
<th>$u, v, \Delta v, s$ plots</th>
<th>Local stability</th>
<th>Capacity</th>
<th>String/flow stability</th>
<th>TTS, delay, etc.</th>
<th>Requirement on ACC algorithms</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Microscopic</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>SL CF models</td>
<td>+</td>
<td>+</td>
<td>+</td>
<td>+</td>
<td>+</td>
<td>no</td>
</tr>
<tr>
<td>SL micro. models with discontinuity</td>
<td>+</td>
<td>+</td>
<td>+</td>
<td>+</td>
<td>+</td>
<td>no</td>
</tr>
<tr>
<td>ML micro. models</td>
<td>+</td>
<td>+</td>
<td>+</td>
<td>+</td>
<td>+</td>
<td>no</td>
</tr>
<tr>
<td><strong>Mesoscopic</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Gas-kinetic models</td>
<td>+</td>
<td>+</td>
<td>+</td>
<td></td>
<td></td>
<td>yes</td>
</tr>
<tr>
<td>Headway distr. models</td>
<td>+</td>
<td>+</td>
<td>+</td>
<td></td>
<td></td>
<td>yes</td>
</tr>
<tr>
<td><strong>Macroscopic</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Payne-type models</td>
<td>+</td>
<td>+</td>
<td>+</td>
<td></td>
<td></td>
<td>yes</td>
</tr>
<tr>
<td><strong>Hybrid models</strong></td>
<td>+</td>
<td>+</td>
<td>+</td>
<td>+</td>
<td></td>
<td>no</td>
</tr>
</tbody>
</table>

2.4.2 Models for ADAS impact assessment on traffic operations

In literature, there are in general three classes of models used for assessing the impact of ADAS on traffic flow, being microscopic, mesoscopic and macroscopic models (Hoogendoorn & Bovy, 2001; Van Wageningen-Kessels, 2013). Microscopic traffic models describe the space-time behaviour of individual driver-vehicle units as well as their interactions at high levels of detail. Macroscopic traffic models describe traffic at high level of aggregation as a flow without distinguishing its constituent units. Mesoscopic traffic models describe traffic flow at a medium detail level, describing the collective behaviour of driver-vehicle units while remaining some individual behaviour characteristics (Hoogendoorn & Bovy, 2001). An overview of the models and their capability in assessing the ACC controller performances is shown in Table 2.4.

**Microscopic traffic models**

Since ACC algorithms generate reference accelerations, it is straightforward to assess the impacts of ACC on traffic flow operations using microscopic traffic models using ACC algorithms as car-following models.

Based on the model complexity, we found three types of microscopic models that have been used to assess ADAS impacts, being single-lane (SL) car-following (CF) models, single-lane models with discontinuity, and multi-lane microscopic models. Thanks to the detailed representation of vehicle behaviour, microscopic performance (time-series plots of microscopic traffic variables and local stability) can be easily assessed using all microscopic models. Example CF models used together with ACC algorithms include Pipes model (Bose & Ioannou, 2003), OVM (Zhang & Ioannou, 2006), cellular automata (CA) model (Jiang et al., 2009).

Single-lane car-following models examine the ACC controller performance in a platoon of vehicles where a human car-following model is (possibly) used to represent
human driver behaviour. This model type can also assess the capacity and string sta-

bility, but it is difficult to use this model class to assess TTS and delay, due to the 

simplification of road infrastructure. This is improved by single-lane models with 

discontinuities. The discontinuity entails a lateral behaviour model for the merging process, allowing one to examine the ACC controller under more realistic scenarios. This type of models are used for ACC assessment in VanderWerf et al. (2001, 2002).

Multi-lane microscopic models with both car-following and lane-changing compo-

nents can represent the driving scenarios more realistically. Therefore, they are often used to study the impacts of ACC controller on traffic operations. Examples include the microscopic model under three-phase theory (Kerner et al., 2008), and simulation packages SPEACS (Broqua et al., 1991), SmartAHS/SmartPath (Rao & Varaiya, 1993; Rao et al., 1993; Godbole et al., 1999), MIXIC (Hogema, 1999; Van Arem et al., 2006), SIMONE (Minderhoud, 1999), PELOPS (Ludmann et al., 1999), FLOWSIM (Marsden et al., 2001), AIMSUN (Kamal et al., 2010; Shladover et al., 2012), VISSIM (Benz et al., 2003), PARAMICS (Baskar, 2009), ITS modeller (Wilmink et al., 2007; Klunder et al., 2009), MOVSIM (Kesting et al., 2008), MOTUS (Schakel & van Arem, 2014), and the models reported in Davis (2004).

In general, any microscopic traffic model can be used to assess the impact of ACC controllers on traffic flow. Most microscopic models describe the free speeds (also called desired speeds) of human-vehicle units, they can model ACC operations in both cruising and following modes without putting requirements on the ACC algorithms. Although the validity of the human driver models involved in the models, particularly the car-following models, chosen for the testing may influence the assessment results, it is nevertheless beyond the scope of this review to discuss the validity of human driver models. For more detailed review and discussion on human car-following models, we refer to Brackstone & McDonald (1999) and Ossen (2008).

Mesoscopic traffic models

Mesoscopic models describe traffic flow at a medium detail level in the sense that ve-

hicular characteristics are not distinguished nor traced individually, but are rather de-

scribed in more aggregate terms using probability distribution functions (Hoogendoorn & Bovy, 2001). However, the dynamics of these distributions are generally governed by various processes describing individual vehicle’s behaviour, e.g. acceleration, interaction between vehicles, lane changing (Hoogendoorn & Bovy, 2001).

In literature, the so-called gas-kinetic models have been used the model ADAS (Tampère, 2004; Ngoduy et al., 2009; Ngoduy, 2012), in which the dynamics of vehicle speeds distributions are described. These models are developed from a multiclass gap-kinetic theory (Hoogendoorn & Bovy, 2000), treating human-driven vehicles and intelligent vehicles as different user classes and model them in the same modelling framework (Tampère, 2004).
To analyse ADAS impacts on traffic flow in gas-kinetic models, one can either specify changes due to ADAS at the individual driver behavioural level to obtain a new kinetic model formulation and integrate to the macroscopic level, or start from the macroscopic properties (Tampère, 2004). In either way, macroscopic flow properties such as capacity, flow stability and TTS and delay can be evaluated. It is also possible to derive the evolution of average values of microscopic indicators, but the accuracy and the descriptive power of such average values of microscopic indicators are questionable. They cannot be used to assess local stability. In addition, it is difficult to include the ADAS algorithms where the reference acceleration cannot be described by an explicit function of system state in the model framework, i.e. model predictive ACC controllers and AI based ACC controllers.

Headway distribution models are also used to study the impacts of AICC vehicles on traffic flow. Rao & Varaiya (1993) used time headway distribution model to derive the probability distribution of platoon size and the theoretical upper bound of capacity in AHS. However, headway distribution models neglect the role of traffic dynamics. They cannot describe dynamic properties of macroscopic flow, such as string and flow stability, TTS and delay. Headway distribution models cannot output the temporal evolution of microscopic traffic variables nor assess the local stability of ACC vehicles. Similar to the gas-kinetic models, it is difficult to include model predictive ACC controllers and AI based ACC controllers in the model class.

**Macroscopic traffic models**

Macroscopic models describe traffic at a high level of aggregation as a flow without distinguishing its constituent parts (Hoogendoorn & Bovy, 2001). Payne-type traffic flow models have been used to model effects of ADAS on traffic characteristics. The general form of Payne model can be expressed as follows (Payne, 1971; Hoogendoorn & Bovy, 2001):

\[
\frac{\partial V}{\partial t} + V \frac{\partial V}{\partial x} = \left( V^e(\rho) - V \right) / T - \left( \frac{1}{\rho} \right) \frac{\partial P}{\partial x}
\]

(2.22)

where \( V(x,t) \) denotes the vehicle speed at location \( x \) and time \( t \). \( V^e \) is the equilibrium speed as a function of the density \( \rho \). \( T \) is the reaction time and \( D \) is the distance headway with respect to the preceding vehicle. \( P \) is the traffic pressure. The convection term describes changes in the mean speed due to inflowing and outflowing vehicles, and the relaxation term represents the tendency of drivers to adjust their speeds to an equilibrium speed-density relationship. The anticipation term describes the driver’s anticipation of traffic conditions downstream. The equilibrium speed-density relationship is described by the following equation:

\[
V^e(\rho) = V_f \left[ 1 - \left( \frac{\rho}{\rho_{\text{max}}} \right)^m \right]^{m}
\]

(2.23)
where $\rho_{\text{max}}$ is the maximum density and $V_f$ is the free flow speed. $l$ and $m$ are model parameters.

Darbha & Rajagopal (1999) proposed a modified Payne-type model to describe the macroscopic flow behaviour of traffic consisting of ACC vehicles. This model is formulated as:

$$\partial_t V + \left(V + \rho \dot{V}^e (\rho)\right) \partial_x V = \frac{(V^e (\rho) - V)}{T}$$

(2.24)

Compared to the original form of Eq. (2.22), they include a new component $\rho \dot{V}^e (\rho) \partial_x V$ in the convection term on the left hand-side. Note that for plausible traffic models, the equilibrium speed is monotonically decreasing with density, thus $\rho \dot{V}^e (\rho) < 0$, which means the influence of speed change due to the inflowing vehicles is less in ACC vehicular flows compared to the manual vehicular flow. The anticipation term is missing in Eq. (2.24), due to the fact that actions of an ACC vehicle depends solely on the motion of its preceding vehicle. The proposed model is based on homogeneous traffic compositions of ACC vehicles, and thus the effects of different penetration levels could not be assessed.

Karaaslan et al. (1990) reported another modification to model the macroscopic flow consisting of vehicle platoons with manual and automatic vehicles. They replaced the anticipation term by a control term, including a target speed in it. The target speed can be reached using individual vehicle controllers at microscopic level, e.g. ACC, or achieved using roadside infrastructure, e.g. Variable Message Signs.

Due to the inclusion of dynamics of velocity $V$ and close relation to car-following behaviour, it is possible for the Payne-type models to derive the average traffic flow accelerations and speeds dynamics. Similar to the mesoscopic models, such an average description does not have satisfactory accuracy and nor can they be used to assess local stability of ACC systems. Macroscopic performance can be assessed straightforwardly, given the preconditions that the effects of ACC controllers can be qualitatively expected, e.g. no anticipation driving of ACC vehicles (Darbha & Rajagopal, 1999). This might be a daunting task for more complex controllers such as model predictive controllers or AI based controllers. In addition, the Payne-type models have been challenged for their validity due to unrealistic gas-like behaviour in some traffic conditions (Daganzo, 1995). Assessments using this type of models must be designed under scrutinisation. Improvements and generalisation of the second order models have been reported in the past decade (Aw & Rascle, 2000; Zhang, 2002; Li & Zhang, 2013). The improved higher order models have not been found in literature on evaluating the traffic flow impact of ADAS systems.

Hybrid models

Apart from the three model classes of microscopic, mesoscopic and macroscopic levels, there are also hybrid approaches to use models at different levels to assess the
performance of ADAS systems. To this end, microscopic performance can be assessed using microscopic models, while macroscopic performance is to be evaluated by mesoscopic or macroscopic models.

Demir (2003) used the general form of the Payne model as in Eq. (2.22), but modified the equilibrium speed-density relationship by adding two new parameters $\mu_1$ and $\mu_2$:

$$V^e(\rho) = V_f \left[ 1 - \left( \frac{\mu_1 \rho}{\rho_{\text{max}}} \right)^{\mu_2} \right]^m$$

(2.25)

where $\mu_1$ represents the influence of including ACC systems on traffic flow, which results in a fact that the equilibrium density with ACC vehicles employing smaller time headways is increased and vice versa. $\mu_2$ represents the non-linear influence of the ACC controller on the correlation between the traffic density, maximum traffic density and free flow speed.

The two new parameters for different ACC vehicle penetration rate and time headway settings are estimated using results from a microscopic simulation model. In other words, Demir (2003) calibrated the parameters of the equilibrium equation using a microscopic simulation model. The hybrid modelling approach preserves the capability of both microscopic and macroscopic models. However, consistency between the models at different levels should be guaranteed using this approach, which is not a trivial task.

### 2.4.3 Models for ADAS impacts on sustainability

Environmental sustainability can be assessed by evaluating the resource use such as fuel consumption, greenhouse effects and air pollution, noise pollutions. Here we only review fuel consumption and emission models, where spatial or temporal fuel consumption and emission rates are estimated based on road, vehicle, driving behaviour and ambient environmental conditions (Cloke et al., 1998; Esteves-Booth et al., 2002). The models are categorised into aggregated emission factors, average speed models, traffic situation models, traffic variable models, cycle variable models, simple modal models, statistical modal models and physical modal models. An overview is shown in Table 2.5. Since fuel consumption models and emission models bear the same (mathematical) structure, hereinafter we use the term *emission models* to represent both model types.

**Aggregated emission factors**

Aggregated emission factors belong to the simplest emission model type that calculates the total fuel consumption and emissions in the considered area based on the total distance travelled on different types of road (e.g. urban, rural and motorway) and vehicle classes (e.g. truck and car) (Cloke et al., 1998; Esteves-Booth et al., 2002). The input traffic data are traffic volume or total distance travelled (usually estimated from
traffic volume), which can be modelled by traffic demand models without the need of assignment. The model class fails to capture the variation of traffic dynamics, and is only suitable for large-area emission inventory estimations. NAEI model is an example of this model family (Boulter et al., 2007).

**Average speed models**

Compared to the emission factors models, the average speed models take into account the average trip speed, which captures the vehicle operations at different traffic conditions to some extent. The input traffic data are traffic volume and average trip-speed, which can be provided by traffic demand and supply models with route assignment. Average speed models distinguish vehicle classes, and as output they produce a local emission factor in mass per unit distance \( (g/km) \) per vehicle on average. Road type is used as model input. The resolution of the model is course due to the incapability of capturing variations in speeds during a trip and road facilities. They are often used for estimating regional emission inventories. Examples of this model family include COPERT (Ntziachristos et al., 2009), MOBILE 5 and 6 (EPA, 2013a), DMBR (HA, 2007), EMFAC (CARB, 2013).

**Traffic situation models**

Traffic situation models translate the traffic volume and speed into a finite set of traffic situations or driving patterns, e.g. free traffic, congested traffic or stop-an-go traffic. The traffic situations are defined in terms of level of service (LOS). Road type and the corresponding characteristics such as speed limits are needed as model inputs. The model class captures traffic dynamics to some extent, but within a traffic situation, no differences are distinguished among vehicle activities, e.g. acceleration, stop, etc. The model class can be used to estimate regional inventories and to assess network level control schemes, coupled with traffic demand and supply models with route assignment and traffic flow models. Examples of this emission model class include HBEFA (INFRAS, 2013).

**Traffic variable models**

Contrary to the traffic situation models, the traffic variable models take into account quantifiable traffic variables describing traffic conditions instead of a finite set of traffic situations. The traffic variables include volume, density, queue length, and speed. As output, the model delivers emission rates per unit distance of single vehicle. This model class is suitable for assessing link level control schemes. To this end, macroscopic, mesoscopic or microscopic traffic flow models representing the congestion dynamics are needed to link this model family. Examples of this model class include Traffic Emissions and Energetics (TEE) (Negrenti, 1998) and the queue-based Matzoros Model (Matzoros & Vliet, 1992).

**Cycle variable models**

Cycle variable models output the emission rate in per unit distance per vehicle as a function of various driving cycle variables, including idle time, average speed, positive kinetic energy, number of stops etc., at high temporal resolutions (seconds). This
model class is derived based on regression of measured data from driving cycles. It requires detailed movements of each vehicle and road characteristics such as gradient. Thus only microscopic traffic models with additional road characteristics information are linked to this model. They are suitable for evaluating link, platoon and control level controllers, such as ADAS (Boulter et al., 2007; Smit et al., 2008; Wismans, 2012). Examples of this model class include MEASURE and VERSIT+ (Gense et al., 2001).

Simple modal models

In modal models, emission rates are related to specific modes of vehicle operation during the trip. The simplest form of modal model categorises vehicle operations into idle, acceleration, deceleration and cruise modes. More detailed ones aim to provide a more accurate representation of vehicle emissions in a look-up table or map. For each operation mode, the emission rates are fixed and the total emission during a trip is a weighted with the time spent in each mode. Modal emission models require detailed vehicle operation data, and thus only microscopic traffic models are linked to this model class. This approach can be applied to evaluate the impacts of link, platoon and vehicle level controllers, such as ADAS. Examples of this model class include the UROPOL (Taylor & Herbert, 1993) and MOVES (EPA, 2013b).

Statistical modal models

Statistical modal models calculate instantaneous emission rate per unit time based on the instantaneous speed and acceleration values. This model class is usually derived by statistical regression. Thus the model parameters have no physical meanings. Road characteristics are not necessary input for the model. Each vehicle type pertains to a set of parameters generated at certain test conditions. Microscopic traffic flow models are appropriate to connect this model class. This approach can be applied to evaluate the impacts of link, platoon and vehicle level controllers, such as ADAS. Example of this model class is VT-Micro (Rakha et al., 2004).

Physical modal models

Physical modal models are the most sophisticated emission models, which include a physical model calculating the required engine load instantaneously. Engine load or power demand is the most influencing factor of instantaneous fuel consumption and emissions. This model requires detailed vehicle characteristics, road gradient as well as microscopic traffic data. Thus microscopic traffic models are appropriate to connect this model class in terms of levels of modelling details. Nevertheless, one should notice the difference between synthetic trajectories generated from microscopic traffic models and those from real world tests (Vieira da Rocha et al., 2013). In general, this model family can be applied to evaluate the impacts of link level traffic control measures or ADAS systems on fuel consumption and emissions. Typical examples of this model family are CMEM (Barth et al., 1996), PHEM (Boulter & McCrae, 2013), and ARRB model (Akcelik, 1989).
### Table 2.5: Overview of fuel consumption and emission models

<table>
<thead>
<tr>
<th>Model class</th>
<th>Traffic input data</th>
<th>Road input data</th>
<th>Vehicle input data</th>
<th>Application range</th>
<th>Link to traffic models</th>
<th>Examples</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aggregated emission factors</td>
<td>Traffic volume</td>
<td>Road type</td>
<td>Vehicle class</td>
<td>National or regional inventory estimation</td>
<td>Traffic demand models (without route assignment)</td>
<td>NAEI</td>
</tr>
<tr>
<td>Average speed models</td>
<td>Traffic volume</td>
<td>Road type</td>
<td>Vehicle class</td>
<td>Regional inventory estimation</td>
<td>Traffic demand and supply models</td>
<td>COPERT, MOBILE, DMBR, EMFAC</td>
</tr>
<tr>
<td>Traffic situation models</td>
<td>Traffic volume LOS</td>
<td>Road type</td>
<td>Vehicle class</td>
<td>Regional inventories and network level controllers assessment</td>
<td>Traffic demand and supply models + Traffic flow models (Macro-, meso- or micro-scopic)</td>
<td>HBEFA</td>
</tr>
<tr>
<td>Traffic variable models</td>
<td>Traffic volume</td>
<td>Road type</td>
<td>Vehicle class</td>
<td>Link level controllers assessment</td>
<td>Traffic flow models (Macro-, meso- or micro-scopic)</td>
<td>TEE, Matzoros Model</td>
</tr>
<tr>
<td>Cycle variable models</td>
<td>Speed, acceleration</td>
<td>Road gradient</td>
<td>Vehicle type</td>
<td>Link, platoon and vehicle level controllers schemes</td>
<td>Microscopic models flow VERSIT+</td>
<td></td>
</tr>
<tr>
<td>Simple modal models</td>
<td>Speed, acceleration</td>
<td>Road gradient</td>
<td>Vehicle type</td>
<td>Link, platoon and vehicle level controllers assessment</td>
<td>Microscopic models flow UROPOL, MOVES</td>
<td></td>
</tr>
<tr>
<td>Statistical modal models</td>
<td>Speed, acceleration</td>
<td>-</td>
<td>Vehicle type</td>
<td>Link, platoon and vehicle level controllers assessment</td>
<td>Microscopic models flow VT-Micro</td>
<td></td>
</tr>
<tr>
<td>Physical modal models</td>
<td>Speed, acceleration</td>
<td>Road gradient</td>
<td>Vehicle type</td>
<td>Link, platoon and vehicle level controllers assessment</td>
<td>Microscopic models flow CMEM, PHEM, ARRB model</td>
<td></td>
</tr>
</tbody>
</table>
2.4.4 Summary on impact models

In this section, we summarise the reviews on impacts models.

Traffic flow impact models

Since the ACC algorithms can be treated as car-following models, microscopic traffic simulation are common approach used for assessing ACC systems performance at both microscopic and macroscopic level. This approach does not put requirements on the ACC algorithms to be evaluated, i.e. all types of ACC algorithms can be evaluated using microscopic simulations. Assessment of different penetration rate of IVs can be conducted quite straightforwardly.

Mesoscopic and macroscopic models describe the collective vehicular behaviour. While the macroscopic performance can be assessed using these models, they require a prior knowledge on the collective behaviour of the ACC algorithms to be evaluated, which becomes difficult for the optimisation-based algorithms and AI based algorithms. In addition, it is not possible to assess microscopic performance of ACC controllers with these models. To assess the impacts of different penetration rate of IVs in traffic, the chosen mesoscopic or macroscopic models have to be able to model multiple user classes.

The hybrid modelling approaches preserves the capabilities of models at different levels (Leclercq, 2007), and thus is capable of evaluating both the microscopic and macroscopic traffic impacts of ACC systems. Consistency between models at different levels are required.

Note that for certain type of ACC algorithms which is equivalent to time-continuous car-following models, the local stability, string stability and capacity can be assessed analytically using the ACC algorithm as a car-following model (Herman et al., 1959; Holland, 1998; Treiber & Kesting, 2011; Wilson, 2008). Stability properties of macroscopic models can also be assess analytically (Darbha & Rajagopal, 1999; Wang & Rajamani, 2004; Tampère, 2004; Treiber & Kesting, 2011).

Fuel consumption and emission models

It is evident that different fuel consumption model have different application range and descriptive power. Aggregated emission factors do not consider traffic dynamics, and average speed models only includes average speeds. Demand plays an dominant role in the two model classes. Traffic situation and traffic variable models takes traffic dynamics into account to some extent, thus can be used to assess road-based traffic control schemes. The cycle variable models, and modal models operates at microscopic level and include much more influencing factors into the model. They can be used for evaluating detailed changes in driving behaviour, e.g. induced by ADAS.
Note that more detailed emission models do not necessarily lead to more accurate estimation of fuel consumption and emissions. Synthetic trajectories from microscopic traffic models are usually smoother compared to real world vehicle trajectories. While this might not be an issue to assess the collective impact of IVs on traffic flow, there could be discrepancies when evaluating fuel consumption and emissions using simulated and real world trajectories (Vieira da Rocha et al., 2013). Hence, evaluation experiments using microscopic traffic flow models and model emission models should be designed carefully to reduce estimation errors (Vieira da Rocha et al., 2013) and to make a fair comparison. In addition, more detailed emission models entails more input data such as engine and gear changing schemes of vehicles, and consequently more calibration work. This aspect should be taken into account in making choices.

2.5 Impact studies of ADAS

Many efforts have been reported in examining ADAS impacts on collective traffic flow operations, mainly on capacity and stability and scarcely on environmental sustainability. As stated in Chapter 1, there are different conclusions on the impacts of ADAS systems on traffic operations. This section unveils the roots for the differences by reviewing current impacts studies of ADAS systems and identifies the gaps in impacts of cooperative systems and eco-driving support systems. An overview of the impacts study examples is shown in Table 2.6.

This section starts with the impacts studies of benchmark ACC systems, followed by the impacts of Cooperative ACC systems with V2V communications and EcoACC systems. Summary of results and some open issues are discussed in the end.

2.5.1 Impact of ACC systems

Below we review the impacts of non-cooperative ACC systems on traffic capacity, stability and environmental sustainability.

On capacity

For ACC controllers employing constant spacing policy, the resulting capacity is determined by the desired gap between vehicles (Swaroop, 1994). Rao & Varaiya (1993) and Rao et al. (1993) analytically derived a maximum capacity of 6900 veh/h on a dedicated ACC lane, but Swaroop (1994) questioned the practical feasibility of such an improvement with instability and safety concerns.
<table>
<thead>
<tr>
<th>Work</th>
<th>Controllers</th>
<th>Capacity</th>
<th>Stability</th>
<th>Environment</th>
<th>Approach</th>
<th>Network</th>
<th>Traffic composition</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>ACC systems</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Rao &amp; Varaiya (1993);</td>
<td>Constant spacing</td>
<td>++</td>
<td>-</td>
<td></td>
<td>Analytical approach with headway distribution model, microscopic simulation</td>
<td>2-lane (one dedicated ACC lane) with on-ramp</td>
<td>ACC or manual vehicles</td>
</tr>
<tr>
<td>Rao et al. (1993)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Darbha &amp; Rajagopal (1999)</td>
<td>CTG</td>
<td>-</td>
<td></td>
<td></td>
<td>Analytical approach with macroscopic traffic model, microscopic simulation</td>
<td>Single-lane road with discontinuities</td>
<td>ACC</td>
</tr>
<tr>
<td>Li &amp; Shrivastava (2002)</td>
<td>CTG</td>
<td>+</td>
<td></td>
<td></td>
<td>Analytical approach and macroscopic traffic model</td>
<td>Single-lane homogeneous ring road</td>
<td>ACC</td>
</tr>
<tr>
<td>Minderhoud &amp; Bovy (1999)</td>
<td>CTG</td>
<td>+/-/-0/-</td>
<td>0/-</td>
<td></td>
<td>Microscopic simulation</td>
<td>2-lane with on-ramp</td>
<td>Mixed</td>
</tr>
<tr>
<td>(VanderWerf et al., 2002)</td>
<td></td>
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<td></td>
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<tr>
<td>Hoogendoorn &amp; Minderhoud (2002)</td>
<td>VHG</td>
<td>0</td>
<td></td>
<td></td>
<td>Microscopic simulation</td>
<td>Multi-lane road with discontinuities</td>
<td>Mixed</td>
</tr>
<tr>
<td>Kesting et al. (2008)</td>
<td>Human-like ACC</td>
<td>+</td>
<td>+</td>
<td></td>
<td>Microscopic simulation</td>
<td>3-lane with discontinuities</td>
<td>Mixed</td>
</tr>
<tr>
<td>Ngoduy (2012)</td>
<td>CTG</td>
<td>+</td>
<td></td>
<td></td>
<td>Analytical approach with mesoscopic traffic model</td>
<td>Circular road</td>
<td>Mixed</td>
</tr>
<tr>
<td><strong>CACC systems</strong></td>
<td></td>
<td></td>
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<td></td>
<td></td>
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</tr>
<tr>
<td>Van Arssen et al. (2006)</td>
<td>Multi-anticipative ACC</td>
<td>+</td>
<td>+</td>
<td></td>
<td>Microscopic simulation</td>
<td>Four-lane road with lane drop</td>
<td>Mixed</td>
</tr>
<tr>
<td>Hasebe et al. (2003)</td>
<td>Backward-looking ACC</td>
<td>+</td>
<td></td>
<td></td>
<td>Microscopic simulation</td>
<td></td>
<td>Mixed</td>
</tr>
<tr>
<td><strong>EcoACC systems</strong></td>
<td></td>
<td></td>
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<td></td>
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<td></td>
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</tr>
<tr>
<td>Kamal et al. (2010, 2013)</td>
<td>MPC</td>
<td>+</td>
<td></td>
<td></td>
<td>Microscopic simulation and a modal emission model</td>
<td>Urban road</td>
<td>Mixed</td>
</tr>
</tbody>
</table>

n.a.: not applicable; ++: substantial increase (more than 50%); +: modest positive effects (within 50%); -: negative effects; 0: no effects;
For ACC controllers with a CTG policy, some studies show improvement in the achievable capacity \citep{vanderwerf2001impact} and \citep{vanarem2006impact} using ACC systems, while others are more conservative on the impacts \citep{minderhoud1999traffic, marsden2001traffic, hoogendoorn2002traffic}. The desired time gap is the most influential factor on capacity. \citet{ludmann1997analysis} showed that ACC systems with a time gap of 1.2 seconds result in the same capacity as manual traffic without ACC systems. Capacity gains can be achieved by setting a smaller time gap, usually smaller than 1.0 seconds, and in this case, the resulting capacity increases with the increase of ACC penetration rate in traffic. However, due to comfort reasons, drivers prefer to choose a time gap similar to their driving style, which usually lead to similar capacity of ACC vehicular traffic as normal traffic. Although the exact desired time gap varies through studies, this argument is supported by many studies with CTG policy \citep{cremer1998traffic, minderhoud1999traffic, vanderwerf2001impact, vanderwerf2002impact, vanarem2006impact, shladover2012impact} and with different algorithms like ACC systems with VTG policy \citep{hoogendoorn2002traffic}, and this is even more apparent with ACC systems designed to resemble human car-following behaviour \citep{keesting2008traffic}.

No systematic study has been reported on capacity impacts of ACC systems using optimisation-based or AI based algorithms.

Note that while the acceptable time gap setting leads to no substantial improvement in capacity, dynamically changing the time gap settings may achieve an efficient outflow at downstream of bottlenecks and thus leads to gains in the queue discharge capacity \citep{keesting2008traffic}.

**On stability**

For the constant spacing policy, analytical study showed that string stability cannot be guaranteed by non-cooperative ACC systems \citep{swaroop1994traffic}.

For the widely-used CTG policy, analytical studies give contradictory conclusions. \citet{darbha1999traffic} used a linear stability analysis approach to examine the traffic flow stability with ACC vehicles employing CTG policy on a single-lane stretch with discontinuities. Their analysis on a macroscopic flow model showed that ACC vehicular traffic is unstable, i.e. the density disturbance propagates upstream and unattenuated. On the contrary, analytical study by \citet{li2002traffic} showed that on a circular highway showed the traffic flow induced by the CTG policy is exponentially stable. \citet{wang2004traffic} showed that the contradiction are due to the different network and boundary conditions assumed in the models and he proposed a VTG policy that improves traffic flow stability compared to the CTG policy.

Simulation studies of ACC systems also give inconsistent conclusions. \citet{vanarem2006impact} and \citet{jiang2009traffic} reported the stabilisation effects of ACC systems, while \citet{minderhoud1999traffic} showed that ACC systems destabilise traffic flow when decreasing time headway setting to $0.8s$ \citep{minderhoud1999traffic}. Note that one important parameter of the CTG policy influencing traffic stability is the desired time
Increasing the desired time gap of the ACC vehicle to a larger value implies maintaining a larger gap at the same speed, which tends to suppress disturbances and stabilise traffic. However, one should be aware of the potential loss in road capacity of choosing a larger time headway compared to human drivers.

For human-like ACC (Hasebe et al., 2003; Ge et al., 2006; Kesting et al., 2008), the resulting traffic flow stability property will not change if the car-following models parameters remain the same. No systematic study has been reported on capacity impacts of ACC systems using optimisation-based or AI based algorithms.

**On environmental sustainability**

Compared to the influence on capacity and stability, much less studies have been conducted to examine the sustainability impacts of ACC systems. Simulation on a single-lane road showed environmental benefits of the ACC system as a secondary effects of a platoon of mixed ACC and manual vehicles compared to reference case of a platoon of manual vehicles (Bose & Ioannou, 2003). The environmental indicators are calculated using a physical modal emission model (Barth et al., 1996). A similar assessment method was used in Zhang & Ioannou (2006). The reduction is up to 28.5% for fuels and between 1.5% and 60.6% for pollutants in rapid accelerating transitions. But this environmental benefits depends highly on reference human driver model used and the sequential order of the ACC vehicles in the platoon.

### 2.5.2 Impact of CACC systems

Significant increase in capacity (up to 4550 veh/h/lane) can be achieved due to multi-anticipative behaviour of the CACC vehicles enabled by V2V communications (Van Arem et al., 2006; VanderWerf et al., 2001, 2002). The improvement in capacity is mainly due to a smaller time gap setting (up to 0.5 seconds). Following with small time gaps requires CACC vehicles following each other, and thus the benefits are only pronounced with considerable penetration rate (VanderWerf et al., 2001; Shladover et al., 2012). No study on the capacity changes of backward-looking ACC systems is reported.

Monteil et al. (2012) analytically showed that multi-anticipative ACC vehicles stabilise traffic flow compared to non-cooperative ACC vehicles. Same findings are reported in Hasebe et al. (2003); Ge et al. (2006); Ploeg et al. (2014) using analytical approach and in Van Arem et al. (2006); Schakel et al. (2010) using microscopic simulation. From vehicular behaviour perspective, anticipating multiple predecessor behaviour enable a cooperative vehicle to react earlier to the downstream disturbance, and thus the multi-anticipative ACC system tends to improve stability.

For the backward-looking IVs, it was shown with analytical results and simulations that the backward-looking ACC systems stabilise traffic flow (Nakayama et al., 2002; Hasebe et al., 2003; Ge et al., 2006). However, the results all used the specific formulation of Eq. (2.21). The generality of the conclusions are questionable.
2.5.3 Impact of EcoACC systems

EcoACC systems aim at reducing fuel consumption or emissions of the controlled vehicles. In the only work we found of EcoACC controller with car-following behaviour, Kamal et al. (2010) reported an optimisation-based controller that includes a penalty of deviating from the desired gap. They evaluate the controller performance in a low density urban road scenario with inflow of 405 veh/h. The environmental benefits mainly lie in transient conditions (acceleration and deceleration), not significant in steady-state conditions. No systematic studies have been reported on the impacts of EcoACC systems on traffic operations.

2.5.4 Summary and discussion

Non-cooperative ACC systems autonomously control the longitudinal movements of IVs and reduce the reaction delay in the control loop of the human drivers. Due to the technological limits of sensors, ACC systems can only react to the direct preceding vehicle and are thus myopic compared to human drivers. As a result, the effects of ACC systems on traffic capacity and stability depend on the control algorithms, the control parameter setting, the reference driver models, and the assessment scenario, such as penetration rate of IVs, initial and boundary conditions of the traffic network. The contradictory statements on the impacts of ACC systems can be largely explained by these aspects.

Regarding cooperative systems, it is generally accepted that the multiple ACC systems improve traffic flow stability. Backward-looking ACC systems are also reported to stabilise traffic flow, but the generality of results remains questionable due to the specific formulation of the control algorithm. Collective impacts of eco-driving support systems on traffic and environment have not received much attention so far.

Existing studies also show some trade-off between capacity, stability and sustainability effects of ADAS systems. For instance, for designing ACC systems to increase capacity, one may reduce the desired time gap. However, this may destabilise traffic flow and increase the potential risks for colliding with the predecessor. When taking sustainability into consideration, the design of ADAS controllers becomes even more complex. Designing ACC controller may lead to smoother following behaviour (Kamal et al., 2010). However, this may lead to retarded accelerating behaviours of ACC vehicles when moving out of congestion, which deteriorates the outflow of congested area and exaggerates the capacity drop phenomena (Zhang, 1999). Hence the design and assessment of ADAS systems for efficiency and sustainability should be conducted in a synthesised and multi-criteria way.
2.6 Conclusions

In this chapter, we conducted literature study to identify the state-of-the-art and knowledge gaps on ADAS controller design and their impacts on traffic operations and sustainability. We first reviewed the control concepts at different layers of the hierarchical functional architecture. Autonomous following, multi-anticipation, cooperative manoeuvring, eco-driving support are control concepts at vehicle and platoon level, and in-vehicle actuation of traffic control signals are relevant concepts that may have far-reaching impacts on traffic flow operations and sustainability.

Review on the existing ADAS control methods reveals that it is difficult to use the widely-used state-feedback approach to address multi-objectives and hard constraints, while the flexibility in formulating system state and objective function and in dealing with constraints suggests the optimisation-based approach a good candidate for generic ADAS control design. The user acceptability and safety issues of the existing ADAS algorithms suggest needs for algorithms refinement. Algorithms to operationalise the cooperative manoeuvring and eco-driving support concept are lacking in general, despite few work reported in literature.

To evaluate the collective impacts of ADAS on traffic flow and sustainability, traffic and emission models are helpful or even dispensable. Since the longitudinal ADAS control algorithms can be treated as car-following models, microscopic traffic simulation is a generic approach to assess the impacts of ADAS on traffic flow quality, which can be used to evaluate any ACC algorithm. For certain type of ACC algorithms, traffic operation impacts can be assessed analytically. The fuel consumption and emission models can be linked to traffic flow models and be used to assess the impacts of traffic control systems and ADAS on environmental sustainability.

Regarding the impacts of ADAS on traffic operations, both positive and negative effects of ADAS on road capacity and traffic flow stability have been found in literature. The contradictory conclusions on ADAS impacts can be explained by the control methods and algorithms, parameter settings, reference case, assessment methods and experiment set-up in different studies. Knowledge gaps exist in sustainability impacts of ADAS systems.

Based on these findings, the following chapter (Chapter 3) proposes an optimisation-based control framework and impacts assessment framework that are generic for different ADAS controllers, while the subsequent chapters (Chapter 4 - 7) derive, test, and improve non-cooperative, cooperative and ecological ADAS algorithms under control concepts identified in this chapter.
Chapter 3

Model predictive control framework for ADAS

In the previous chapter, we reviewed ADAS control concepts, algorithms and impact studies of ADAS. We found that the existing ADAS controllers need to be refined and that it is difficult to use the state-feedback controllers to operationalise new control concepts for cooperative systems and eco-driving support systems. It is also concluded that optimal control methods are flexible in dealing with multiple control objectives and constraints and hence have potentials in addressing the difficulties encountered with state-feedback control methods. In this chapter, we propose a generic model predictive control framework for ADAS. Under this framework, we formulate the supported driving task into a receding horizon optimal control problem. The control framework is generic in the way that widely-studied state-feedback algorithm for ACC systems can be derived under the framework. The control problem can be solved by multiple approaches, of which the appropriateness is discussed. To verify the ADAS performance and to assess their collective impact, we also propose a performance assessment framework for ADAS. The control framework, solution approaches and the performance assessment framework will be used in the following chapters to design and evaluate ADAS controllers.

The chapter is structured as follows. Section 3.1 presents the core design assumptions and control objectives for ADAS, followed by the mathematical formulation of the model prediction control framework in Section 3.2. Different solutions approaches to optimal control problems are categorised and compared in Section 3.3. Section 3.4 describes the performance assessment framework for verifying the ADAS performance at individual vehicle level and assessing the impact of ADAS at macroscopic level. The main conclusions are summarised in Section 3.5.
3.1 Core design assumptions and control objectives

Before presenting the mathematical formulation of the control framework, we introduce the assumptions regarding the ADAS design and operations, upon which the control framework is built. The core design assumptions under the generic control framework include the following:

- ADAS predict the evolution of the system state of their vicinity based on the current system state, which involves the expected behaviour of other vehicles, and make control decisions to optimise the predicted cost.
- ADAS make longitudinal control decisions, i.e. accelerations, to fulfil control objectives and the vehicle actuators execute the control signals automatically.
- ADAS replace drivers in observing positions and speeds of other vehicles on the same lane through on-board sensors, e.g. radar or lidar. This sensing strategy is also referred to as autonomous sensing.
- In case of cooperative systems, position and speeds of other cooperative vehicles on the same lane can be acquired through V2V communications. This sensing strategy is referred to as cooperative sensing.
- ADAS acquire their own vehicle position, speed and acceleration from on-board systems.
- Data from on-board sensors and control decisions are updated at regular time intervals.
- Drivers monitor the system operations and are responsible for lateral control of ADAS vehicles, e.g. changing lanes.
- Drivers want to use the system as much as possible. Only in crucial conditions where drivers feel unsafe, systems may be overruled by drivers and hard braking are performed by drivers.

Note that the aforementioned aspects are core design assumptions, and additional assumptions may be necessary for specific ADAS controllers in the following chapters. Note also that although we focus on longitudinal control in this thesis, the control framework can be extended to include lateral control decisions, e.g. lane choices (Hoogendoorn & Bovy, 2009; Wang et al., 2014c).

ADAS controllers are designed to fulfil some control objectives, which can be any combination of the following objectives:

- to maximise travel efficiency;
- to maximise safety and to avoid collision;
- to minimise fuel consumption and emissions;
- to maximise smoothness and comfort.

The importance of each of these objectives may vary according to preferences of drivers, traffic conditions, or vehicles types, e.g. some systems may give priority to safe
driving, while others prefer travel efficiency, accepting smaller headways and higher risk if other influencing factors (speed and relative speed) are the same.

3.2 Control framework formulation

This section presents the general control framework and the mathematical formulation for ADAS controllers, with a controller design example showing the applicability of the control framework.

3.2.1 Supported driving as an optimal control cycle

Figure 3.1 shows an abstract representation of a model predictive ADAS controller in the grey rectangle, extending the generic controller shown in Figure 2.2. Let us consider the system from the perspective of an ADAS controlled vehicle $i$, of which the state $\mathbf{x}$ can be described by the relative positions and relative speeds of the controlled vehicle and surrounding vehicles. At time instant $t_0$, the controller receives the positions and speeds of surrounding vehicles from observations either made by its on-board
sensors (i.e. non-cooperative systems) or transmitted from other sensors through V2V and/or V2I communications (i.e. cooperative systems). Based on this information, the controller estimates the current state of the system $\hat{x}(t_0)$ and uses a system dynamics model to predict its future state in a time horizon $T_p$, with the current system state as the initial conditions. The reference control signal/input/variable $u$, which is acceleration in the vehicle following control case, is determined to minimise some criterion or cost reflecting undesirable situations. The control decisions of ADAS vehicles may be influenced by control commands from link- or network-level controllers if present (cf. Section 2.2). In a rolling horizon implementation, the control signal is often discretised and only the first sample from $t_0$ to the next time instant $t_1$ is executed by the on-board actuators. As the vehicle manoeuvres, the system state changes, and the optimal control signal $u$ is recalculated with the newest information regarding the system state at regular time interval $\Delta t$, i.e. when $t_1 = t_0 + \Delta t$. The rolling horizon implementation of the optimal control signal is depicted in Figure 3.2.

### 3.2.2 Formulation of the control problem

Here we formulate the optimal control problem of ADAS within one cycle as a mathematical programme. To impart some intuition of the considered problem, we illustrate the formulation with examples throughout this section, while acknowledging that the specification of the formulation may vary according to different ADAS applications.

Let us consider the longitudinal control of vehicles as a time-continuous dynamic system, of which the dynamics can be described by the following ordinary differential equation (ODE):

$$
\dot{x} = f(x(t), u(t), t)
$$

(3.1)

\footnote{In this thesis, we focus on continuous-time systems and exclude the formulation and discussion for discrete-time systems.}
with $x(t) \in \mathbb{R}^m$ denotes the state vector, which for instance can be the gap and relative speed of the controlled vehicle with respect to the preceding vehicle. $u(t) \in \mathbb{R}^n$ denotes the control input vector, which for instance can be the acceleration of the controlled vehicle. Equation (3.1) describes the kinematic motion of the controlled vehicle. Notice that the system dynamics model does not nee to be deterministic. The stochastic optimal control theory allows us to include uncertainty in the system dynamics model (Fleming & Soner, 1993).

The state and control variables are subject to constraints:

$$x(t) \in X, \text{ and } u(t) \in U, \forall t \geq 0$$

(3.2)

where $X \in \mathbb{R}^m$ denotes the state constraints. For instance, the gap between two vehicles should be larger than zero and the relative speed should not exceed the maximum speed a vehicle can travel. $U \in \mathbb{R}^n$ denotes the admissible control (Pontryagin et al., 1962), i.e. the controlled accelerations are bounded between the minimum value $u_{\text{min}}$ and the maximum value $u_{\text{max}}$ due to physical characteristics of vehicles and driver/passenger comfort.

Suppose that at time $t_0$, we have an estimate of the system state $\hat{x}_0$. The optimal control problem for ADAS at time $t_0$ is to seek some optimal control $u^*$ over a prediction horizon $T_p$ that drives the system along a trajectory such that the predicted cost is minimised. This can be formulated as the following mathematical programme:

$$u^*_{[t_0, t_0+T_p]} = \arg\min_u J(x, u, t|\hat{x}(t_0))$$

(3.3)

with the predicted cost

$$J(x, u, t|\hat{x}(t_0)) = \int_{t_0}^{t_0+T_p} \mathcal{L}(x(t), u(t), t)dt + \mathcal{G}(x(t_0+T_p), t_0+T_p)$$

(3.4)

The optimisation is subject to system dynamic equation (3.1), state and control constraints (3.2) and initial condition of

$$x(t_0) = \hat{x}_0$$

(3.5)

In Eq. (3.4), $J$ denotes the cost functional, or simply the cost function (also termed as objective function or performance index) to be minimised. It describes the predicted/expected cost accumulated from current time $t_0$ over a time horizon $T_p$, measuring the deviation of the predicted state $x$ driven by the control input $u$ from the desired state (Chen & Allgöwer, 1998). The desired state for ADAS can be, for instance, maintaining a desired speed when in free driving conditions or a desired gap to the predecessor in constrained driving conditions. The cost function (3.4), including the desired state and parameters, is selected or designed by engineers while the system dynamic equation (3.1) is fixed by the physics of the system (Lewis, 2012).

$\mathcal{L}$ denotes the so-called running cost, describing the costs incurred during an infinitesimal period $[\tau, \tau + \delta\tau]$, which is additive over time. The function $\mathcal{G}$ denotes the so-called terminal cost. It describes the cost remaining at the end of the prediction horizon.
The cost function formulation (including specification of $L$, $G$, and the choice of $T_p$) and the system dynamic equation influence the controller performance as well as the choice of solution approaches to the optimal control problem. Under specific cost formulation for linear system dynamic equation, one may get a closed-form solution to the considered problem. In the sequel of this section, we show such an example that requires specific choices of $L$, $G$, $T_p$.

### 3.2.3 Worked example: linear ACC algorithm

To show the general applicability of the control framework, we apply the framework to derive the widely-studied linear ACC algorithm with Constant Time Gap (CTG) policy (cf. Section 2.3) by choosing a specific controller formulation.

Let us define the system state $x$ from the perspective of an ACC vehicle as the deviation from the desired gap and the relative speed with respect to the preceding vehicle:

$$x = (s - s_d, \Delta v)^T$$

where $s_d$ and $\Delta v$ denote the desired gap and relative speed of the ACC vehicle respectively, and the superscript $T$ denotes the transpose of a vector. The desired gap is calculated as:

$$s_d = vt_d + s_0$$

where $t_d$ is the desired time gap and $s_0$ is the minimum gap between two vehicles at standstill conditions. Assuming the predecessor is travelling with constant speed within the prediction horizon, the state dynamics follow the linear ODE:

$$\dot{x} = Ax + Bu$$

with

$$A = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}, B = \begin{pmatrix} -t_d \\ -1 \end{pmatrix}$$

$u = u$ is the acceleration of the ACC vehicle.

The optimal control problem is formulated as:

$$\min_u J(x, u|x(t_0)) = \min_u \int_{t_0}^{t_0+T_p} \mathcal{L}(x(\tau), u(\tau)) \, d\tau$$

where running cost function is defined as:

$$\mathcal{L} = \frac{1}{2} x^T Q x + \frac{1}{2} u^T R u$$

and with

$$Q = \begin{pmatrix} \beta_1 & 0 \\ 0 & \beta_2 \end{pmatrix}, R = 1$$
where $\beta_1 > 0$ and $\beta_2 > 0$ are constant cost weights. Eq. (3.10) shows that the controller aims to regulate speed towards the state $x = (0, 0)^T$. The considered ACC controller with quadratic cost function as in Eq. (3.10) and linear system dynamics (3.8) is a classical Linear Quadratic Regulator (LQR) and the optimal control law is a feedback control with the form:

$$u^* = -R^{-1}B^TP(t)x$$

(3.11)

where $P(t)$ is the solution of the following Algebraic Riccati Equation (ARE):

$$-\dot{P}(t) = P(t)A + A^TP(t) + Q - P(t)BR^{-1}B^TP(t)$$

(3.12)

Under stabilizability condition (Lewis, 2012), when $T_p \to \infty$, the considered time invariant system will settle down to the steady state $x = (0, 0)^T$. Thus, Eq. (3.12) relaxes to the infinite horizon Continuous Algebraic Riccati Equation (CARE) of:

$$PA + A^TP + Q - PBR^{-1}B^TP = 0$$

(3.13)

with positive definitive solution $P$. Let $K = -R^{-1}B^TP = (K_s, K_v)$ denote the constant control gain, the optimal control law is expressed as:

$$u^* = Kx = K_s(s - vt_d - s_0) + K_v\Delta v = K_s(s - vt_d - s_0) + K_v\dot{s}$$

(3.14)

Equation (3.14) is exactly the linear state-feedback ACC algorithm with Constant Time Gap policy as in Eq. (2.6). This implies that the algorithm is actually optimising an objective function, i.e. Eqs. (3.9, 3.10).

While the control framework is generic, the example solution approach we showed in this section is specific. In Section 3.3, we explore more general solution approaches to optimal control problems.

### 3.3 Solution approaches to optimal control problems

The model predictive control framework for ADAS entails solving an optimal control problem at each time step. To this end, solution approaches are required to find the optimal control signals. This section presents several widely-used solution approaches in the solution family of optimal control problems and discusses appropriateness of these approaches. We are not aiming at constructing an exhaustive list of all possible solution methods, but revisit and compare the typical methods under different paradigms. Particularly, we distinguish four main streams in the solution family as shown in Figure 3.3, which are dynamic programming approach, indirect numerical methods by solving necessary conditions for optimality (Pontryagin’s Principle), direct numerical methods by evaluating the cost function and analytical solutions under specific control problem formulations.
3.3.1 Dynamic programming solutions

The dynamic programming approach entails solving the so-called *value function* $W(x,t)$, which is the optimum value of the predicted cost $J(x,u,t)$ as a function of the system state $x$ and time $t$ (Bertsekas, 2005; Lewis, 2012; Fleming & Soner, 1993). If we can find the value function satisfying a partial differential equation (PDE) for all possible states, we can determine the optimal trajectory from all initial states, which is *sufficient condition for optimum* Bertsekas (2005). We define the value function as:

$$W(x,t) = \min_u J(x,u,t)$$ (3.15)

where $J$ is defined similarly as Eq. (3.4) but is not restricted to the current state of the system $x_0$. The PDE which $W(x,t)$ satisfies is known as the following *Hamilton-Jacobi-Bellman (HJB)* equation:

$$-\frac{\partial W}{\partial t} = \min_u \mathcal{H}(x, \frac{\partial W}{\partial x}, u, t)$$ (3.16)

with $\mathcal{H}$ denoting the Hamiltonian expressed as:

$$\mathcal{H}(x, \frac{\partial W}{\partial x}, u, t) = \mathcal{L}(x, u, t) + \left(\frac{\partial W}{\partial x}\right)^T f(x, u, t)$$ (3.17)

The HJB equation can be derived from the *Bellman’s principle of optimality* and Taylor expansion. For details regarding the derivation, we refer to Lewis (2012).

Assuming smoothness of the Hamiltonian, the optimum of the right-hand side of Eq. (3.16) can be found by the so-called *stationarity condition*:

$$\frac{\partial \mathcal{H}}{\partial u} = 0$$ (3.18)
In most cases, the stationarity condition allows us to express the optimal control input \( u^* \) as a function of the state \( x \), the derivatives of the value function \( \frac{\partial W}{\partial x} \), removing one unknown in the HJB equation. The key issue remained is to solve the HJB equation subject to the boundary condition governed by the terminal cost function \( G(x_0 + T_p) \), so that we can obtain an optimal control policy \( u^* \) that minimises the cost function \( J \). This implies that the HJB equation is determined \textit{backward in time}, which is the feature of the dynamic programming approach.

One distinctive merit of the dynamic programming approach is that if the HJB equation can be solved, the optimal control is approximated as a state feedback in \textit{graphical or tabular form} (Lewis, 2012). However, as we can see from Eqs. (3.16, 3.17), the HJB equation is an \( n + 1 \) dimensional PDE, with \( n \) equalling the dimension of the state vector \( x \). Solving the HJB equation requires evaluating partial derivatives of the value function \( \frac{\partial W}{\partial x} \) for all possible state, which can be done using numerical approximation schemes such as \textit{finite difference} or \textit{finite volume} methods. Due to the \textit{curse of dimensionality} (Bryson, 1996), it is in general computationally expensive and is only applicable to systems with low dimensions, i.e. dimensions less than three (Lewis, 2012; Bryson, 1996).

The HJB equation (3.16) is a first-order PDE. Thus it requires at least first-order differentiability (\( C^1 \)) of the value function everywhere. The simple bound constraints on control variables can be included by the so-called \textit{viscosity solutions} (Fleming & Soner, 1993), but it is not easy to deal with inequality constraints on state variables.

### 3.3.2 Numerical methods based on Pontryagin’s Principle

Indirect numerical approaches relate closely to the dynamic programming approach, but are much more efficient in solving the optimal control problem. This solution family is based on the Pontryagin’s Minimum Principle (PMP) (Pontryagin et al., 1962), which is also known as the \textit{necessary conditions} for optimality. In this section we present the main ideas of the methods based on PMP.

The PMP based solution class streamlines the computation by solving a set of coupled ordinary differential equations (ODEs) instead of the HJB PDE. While the dynamic programming approach evaluates \( \frac{\partial W}{\partial x} \) everywhere in the state-space, it is not necessary to know \( \frac{\partial W}{\partial x} \) for all possible states. Only the derivatives along the optimal trajectory \( x^* \) suffice. In most cases, the derivatives of the value function along the optimal trajectory, which is known as the \textit{costate} or the adjoint (Pontryagin et al., 1962; Bertsekas, 2005), can be calculated more efficiently compared to the HJB equation. Particularly, the costate satisfies a certain set of ODEs, called the \textit{costate dynamic equations} or the \textit{adjoint equations}.

The PMP can be derived by the method of \textit{Lagrangian multipliers} (Bryson & Ho, 1975; Lewis, 2012). It uses costate \( \lambda = \frac{\partial W}{\partial x^*} \), which reflects the marginal change in the
optimal cost due to a small change in the state, to adjoin the state dynamic equation (3.1) to the running cost function $L$, which results in the Hamiltonian as:

$$H(x, \lambda, u, t) = L(x, \lambda, u, t) + \lambda^T f((x, u, t)) \quad (3.19)$$

Introducing the auxiliary costate variable $\lambda$ gives us extra degree of freedom and the general control problem of Eqs. (3.3, 3.4) is recast into the following mathematical programme (Pontryagin et al., 1962; Lewis, 2012):

$$u^* = \arg\min_u H(x, \lambda, u, t) \quad (3.20)$$

subject to state dynamic equation

$$\dot{x} = \frac{\partial H}{\partial \lambda} = f(x, u, t) \quad (3.21)$$

with initial condition

$$x^*(t_0) = \hat{x}_0 \quad (3.22)$$

and costate dynamic equation

$$\dot{\lambda} = -\frac{\partial H}{\partial x} \quad (3.23)$$

with terminal condition

$$\lambda(t_0 + T_p) = \frac{\partial G(x)}{\partial x} \quad (3.24)$$

Equations (3.21) and (3.23) establish a set of conjugated ordinary differential equations (ODEs). Furthermore, the stationarity condition $\frac{\partial H}{\partial u} = 0$ is generalised by the following condition, which allows constraints on control variables to be included easily:

$$H(x, \lambda, u^*, t) \leq H(x, \lambda, u, t) \quad \text{for } \forall u \in U \quad (3.25)$$

where $U$ denotes the admissible control set.

Equation (3.25) allows us to express the optimal control input $u^*$ as a function of the state $x$ and the costate $\lambda$, reducing one unknown in the considered problem. The remaining issue is to solve the coupled ODEs (3.21, 3.23), with initial conditions for state dynamic equation and terminal conditions for costate dynamic equation, i.e. two-point boundary value problem (BVP). Numerical methods such as single/multiple shooting methods and gradient methods are exemplary solution methods to the BVP problem (Binder et al., 2001; Falb & de Jong, 1969).

**Single/multiple shooting methods**

The core idea of shooting methods is to change the difficult BVP into an initial value problem (IVP), which is much easier to solve. *Single shooting methods* guess the missing initial values of the costate $\hat{\lambda}(t_0)$ and only integrate the system and costate dynamic equations (3.21, 3.23) forward in time to get an estimate of the costate at terminal
time. The difference between the estimated value of costate at terminal time and the known costate at terminal time from Eq. (3.24), is evaluated to find the direction to improve the initial guess, using for instance a Newton method or bisection scheme (Breakwell, 1959; Falb & de Jong, 1969; Cervantes & Biegler, 2009). This procedure is repeated until the difference between the estimated terminal costate and the given terminal costate reaches a satisfactory level. Figure 3.4 illustrates the main idea of single shooting methods.

![Figure 3.4: Illustration of indirect single shooting methods.](image)

Shooting methods usually guarantee very accurate solutions provided that good initial guesses for the costate are available (Pytlak, 1999). The main disadvantage of single shooting methods is that the solution is very sensitive to the initial guess, particularly for non-linear or unstable systems. This drawback is improved substantially by multiple shooting methods. Multiple shooting methods bear the same idea as single shooting methods, but the integration horizon is divided into smaller subintervals. The missing values are not only guessed at initial time, but also at several points at the subintervals (Bulirsch et al., 1991; Cervantes & Biegler, 2009). At each subinterval, the state and costate dynamic equations are solved, and continuity at the subintervals must be guaranteed. The main difficulty for the multiple shooting methods is the proper guess for the state/costate at the connection points.

**Gradient methods**

Gradient methods entail guessing initial values of the control input $u(t)$ from initial time to terminal time. Then, the system dynamic equations are integrated *forward in time*...
time} to obtain the state trajectory \( x(t) \), using the initial guesses of the control input \( u(t) \). The state trajectory is then used to solve costate dynamic equations \textit{backward in time}. After that, state and costate trajectories are used together to calculate the gradients for the Hamiltonian \( H \) with respect to the control (Bryson & Ho, 1975; Pytlak, 1999). Adjusting the guess of control input in the opposite direction of the gradients \( \frac{\partial H}{\partial u} \) so that each iteration results in a lower value of \( H \). The procedure is repeated until the gradients \( \frac{\partial H}{\partial u} \) are close to zero (Bryson & Ho, 1975).

Gradient methods usually show substantial improvements in the first few iterations but have poor convergence characteristics as the optimal solution is approached. The convergence of the gradient methods can be accelerated by exploring the second order gradients (Bryson & Ho, 1975).

### 3.3.3 Direct numerical solutions to optimisation problems

Instead of indirectly determining the optimal control input by solving the necessary conditions as in the indirect methods, direct methods involves the generation of a sequence of control functions with the property that each successive control functions \textit{directly} results in a lower value of the cost function \( J \) (3.4) (Binder et al., 2001). Direct numerical solutions regard the control process purely as an optimisation problem, of which the basic idea is to transcribe the infinite dimensional problem into a finite dimensional Nonlinear Programming (NLP) problem (Binder et al., 2001; Sargent, 2000). To this end, an NLP solver, typically Successive Quadratic Programming (SQP) \(^5\), in addition to an ODE solver is employed to seek a time-series control inputs that minimises the cost function subject to the system dynamic equations and other constraints on state and control variables.

Based on the discretisation of the control and state variables and the feasibility of the intermediate solutions in the iterative solution process, direct methods can be generally classified into two categories: being sequential approach and simultaneous approach.

#### Sequential approach

The sequential approach only discretises control variables in time. This strategy entails parametrising the control input variable as a function of time, integrating system dynamic equation with the parametrised control function and searching the optimal parameter settings to minimise the cost function (3.4). Hence the methods are also referred to as \textit{control (vector) parametrisation methods}. Mathematically, the control parametrisation can be represented by the following equation:

\[
\begin{align*}
  u(t, q), & \quad t \in [t_0, t_0 + T_p) \\
\end{align*}
\]

\(^5\text{We omit the equivalent term }\text{Sequential Quadratic Programming} \text{ to avoid confusion with the sequential approach to optimal control problems}\)
where \( \mathbf{q} = (q_0, q_1, ..., q_{N-1})^T \) is the vector of \( N \) control parametrisation parameters. Typical parametrised functions forms used are polynomial or exponential, i.e. the control input \( \mathbf{u} \) is parametrised as a polynomial or exponential function of time. The simplest form of the control parametrisation methods is to discretise and parametrise the control input as a \textit{piece-wise constant} function of time, as illustrated in Figure 3.5.

![Figure 3.5: Piecewise constant parametrisation of a control function \( \mathbf{u}(t, \mathbf{q}) \) and the integrated state \( \mathbf{x}(t, \mathbf{q}) \) with \( N = 5 \).](image)

In this case, the control function reads:

\[
\mathbf{u}(t; \mathbf{q}) := q_i, \; t \in [t_i, t_{i+1})
\]  

(3.27)

where \( i = 0, 1, ..., N-1 \) and \( t_0 < t_1 < ... < t_N = t_0 + T_p \).

Inserting Eq. (3.26) into Eq. (3.4) transcribes the original the optimal control problem into:

\[
\min_{\mathbf{q}} J(\mathbf{u}(t; \mathbf{q}) | \mathbf{x}(t_0), t_0, t_0 + T_p)
\]  

(3.28)

subject to the transcribed state dynamic equation of:

\[
\dot{\mathbf{x}}(t) = \mathbf{f}(\mathbf{x}(t), \mathbf{u}(t, \mathbf{q}), t)
\]  

(3.29)

with the same initial condition of Eq. (3.5).

The sequential approach is of \textit{feasible path} type (Cervantes & Biegler, 2009). At each optimisation iteration, the system dynamic equation (3.29) is first solved by a numerical integration method for the current guess of control parameters, which produces the value of the cost function (3.28). Then the cost function and its \textit{gradients with respect to the control parameters} are evaluated by an NLP solver to find better estimates of the control parameters. The predicted evolution of the system (by an ODE solver) and the
optimisation of the systems (by an NLP solver) are performed sequentially, one after the other (Sargent, 2000; Binder et al., 2001).

Direct single shooting methods are a typical sequential solution approach (Binder et al., 2001; Allgöwer et al., 2004; Diehl et al., 2009). Notice that the major differences of the direct shooting methods compared to the shooting methods under indirect approaches include the objective function to be minimised and initial values to be shoot. The objective function to be minimised in the direction shooting methods is the transcribed cost function (3.28) and the direction methods shoot the initial values of the control parametrisation parameters \( q \) instead of minimising the Hamiltonian function (3.19) and guessing the missing values of the costate variables \( \lambda \) in the indirect shooting methods.

While the direct sequential methods are computationally efficient compared to the indirect methods, the strategy may change the structure of the problem due to the specific parametrisation function used, thus leading to sub-optimal solutions (Cervantes & Biegler, 2009; Binder et al., 2001). Furthermore, due to the use of NLP solver, sequential methods introduce optimisation error caused by finite number of iterations in addition to the numerical error in the integration of the system dynamic equations. Hence the methods are not as accurate as the dynamic programming approach and the indirect methods.

**Simultaneous approach**

As opposed to the sequential strategy, the simultaneous approach discretises both control and state variables. The solution to the differential equation and the optimisation of the cost function are obtained simultaneously. The differential equation of the system dynamics is discretised over the whole prediction horizon and enter the optimisation problem as additional nonlinear constraints, which can be violated during the optimisation procedure. However, at the solution, the constraints have to be satisfied (Binder et al., 2001; Allgöwer et al., 2004). Typical simultaneous approach are direct multiple shooting and direct collocation methods (Cervantes & Biegler, 2009; Binder et al., 2001; Allgöwer et al., 2004; Diehl et al., 2009).

Direct multiple shooting bears the same idea as direct single shooting, i.e. the horizon \([t_0, t_0 + T_p]\) is divided into \( N \) subintervals, \( t_0 < t_1 < ... < t_N = t_0 + T_p \) and the control function is parametrised with parameter \( q = (q_0, q_1, ..., q_{N-1})^T \) at each interval as in Eq. (3.26). In an important second step, \( N + 1 \) additional parameters \( p = (p_0, p_1, ..., p_N)^T \) of the same dimension of the state variable \( x \) are introduced, which are referred to as node values (Binder et al., 2001). The node values need to be guessed at the initialisation of the solution methods and serve as initial values for the \( N \) independent initial value problems (IVPs) over the prediction horizon.
At each subinterval, the transcribed cost is a function of $p$ and $q$ only:

$$J_i(p_i, q_i) := \int_{t_i}^{t_{i+1}} L(x(t, p_i, q_i), u(t, q_i)) dt$$  \hspace{1cm} (3.30)

The *decoupled* optimisation problems at each subinterval are solved independently by an NLP solver but are connected to ensure continuity of the state variables at nodes with:

$$p_{i+1} = x(t_{i+1}, p_i, q_i), \quad i = 0, 1, \ldots, N - 1$$  \hspace{1cm} (3.31)

and

$$p_0 = x(t_0)$$  \hspace{1cm} (3.32)

Hence the original optimal control problem is transcribed as:

$$\min_{p, q} \bar{J}(p, q) = \min_{p, q} \sum_{i=0}^{N-1} J_i(p_i, q_i) + G(p_N)$$  \hspace{1cm} (3.33)

Note that the arguments we seek to minimise the transcribed cost function is not only the control parameter $q$ as in the sequential methods, but also the state parameter $p$. Although this increase the size of the NLP problem, the *decoupled* feature of the sub-problems at subintervals leads to sparse Jacobian and Hessian matrices, which can be exploited to accelerate the iteration process.

Figure 3.6 illustrates the idea of direct multiple shooting methods. Notice again that during the optimization process, the continuity constraint (3.31) can be violated until the solution is reached.

![Figure 3.6: Illustration of the direct multiple shooting method with piecewise constant representation of control function $u(t, q)$ and integrated state $x(t, p, q)$.](image-url)
Direct collocation methods bear much similarity with direct multiple shooting methods Binder et al. (2001), of which the idea is shown in Figure 3.7. The time horizon \([t_0, t_0 + T_p]\) is still divided into \(N\) subintervals, and the control function is parametrised with parameter \(q\) at each interval with \(N + 1\) node state values \(p\). While the state trajectory is obtained by integrating the system dynamics equation (3.1) at each subinterval \([t_i, t_{i+1}]\), direct collocation further discretises the subinterval into \(M\) finite elements \(t_i = t_i < t_{i,1} < ... < t_{i,M-1} = t_{i+1}\), resulting in \(N \times M + 1\) collocation points. The state trajectory in each subinterval is approximated with polynomials up to a certain degree, using parameters \(p_i\) and \(p_{i+1}\) at two nodes of the subintervals and additional parameters \(c\) relates to the polynomials. The error of the polynomial representation of the state trajectory at each finite element, which can be evaluated by the difference between the derivatives of the state calculated by system dynamic equation (3.1) and the gradients of the polynomial-approximated state function at collocation points. This error should equal zero, which serves as additional equality constraints into the transcribed NLP problem. The collocation methods results in large-size NLP problems due to the additional arguments \(c\) in the optimisation related to polynomials as well as the additional equality constraints. Nevertheless, the sparsity of the Jacobian and Hessian matrices is preserved and can be exploited to accelerate the iterative procedure.

In general, simultaneous approaches are more stable compared to sequential approaches and both equality and inequality constraints on control and state variables can be dealt with easily. However, the initialisation of the optimisation procedure entails initial
guesses of the control and state over the whole horizon, and results in large scale NLP problems. Special techniques much be sought to efficiently solve the resultant large scale NLP problems. The same as sequential methods, simultaneous approaches introduce extra optimisation error into the original optimal control problem and may yield infeasible path due to the relaxation of the system dynamic constraints during the iterative process, which leads to sub-optimal solutions (Binder et al., 2001).

### 3.3.4 Closed-form solutions for specific problems

While the numerical solutions are relatively computationally expensive, practical engineers seek simplifications of the optimal control problems that allows one to get a closed-form state-feedback control law. One important feature for the state-feedback control algorithms is that they can be applied in a real-time setting. The unique solution in the linear quadratic regulator is a typical example of the specific formulation, which requires the running cost function to be quadratic of the state and control vectors and system dynamics to be linear, which restricts the design choices to some extent. Here we present an alternative formulation that relaxes this restriction but still allows one to find analytical solution to the problem.

The optimal control problem described here introduces a time discounted factor in the cost function, and specify the running cost and terminal cost as:

\[
\mathcal{L}(x,u,t) = e^{-\eta t} \tilde{L}(x,u), \quad \mathcal{G} = e^{-\eta T_p} \tilde{G}(x)
\]

(3.34)

The corresponding cost function becomes:

\[
J(x,u,t|x(0)) = \int_{t_0}^{t_0+T_p} e^{-\eta \tau} \tilde{L}(x,u) d\tau + e^{-\eta(t_0+T_p)} \tilde{G}(x(t_0 + T_p))
\]

(3.35)

Equation (3.35) implies that the controller weights the cost in the near future more than the cost in the further future.

Notice that Eq. (3.35) is a specific formulation of the general controller formulation (3.4).

The parameter \( \eta \) (\( \eta > 0 \)) denotes the so-called discount factor (Fleming & Soner, 1993), which reflects a trade-off between the cost incurred in the near term and the far future cost. A larger \( \eta \) implies a more myopic controller which increase the emphases on situations in the near future than those in the far future. The prediction horizon \( T_p \), reflects a trade-off between the integral in Eq. (3.35) and the discounted terminal cost \( e^{-\eta(t_0+T_p)} \tilde{G} \), i.e. the longer \( T_p \) is, the less important the discounted terminal cost is.

When the prediction horizon becomes very long, i.e. \( T_p = \infty \), the terminal cost can be disregarded (which equals zero). The optimal control problem relaxes to:

\[
\min_u J(x,u|x(0)) = \min_u \int_{t_0}^{\infty} e^{-\eta \tau} \tilde{L}(x(\tau),u(\tau)) d\tau
\]

(3.36)
subject to system dynamics of Eq. (3.1).

Applying the Bellman’s principle of optimality yields the HJB equation as in Eq. (3.16). The property of discounted cost term allows express the value function as:

$$W(t, x) = e^{-\eta t} \tilde{W}(x)$$  \hspace{1cm} (3.37)

where \( \tilde{W} \) only depends on \( x \). The Hamiltonian in this specific formulation becomes:

$$\mathcal{H}(x, u, \frac{\partial W(x)}{\partial x}) = e^{-\eta t} \tilde{L} + \lambda^T f$$  \hspace{1cm} (3.38)

The costate \( \lambda \), reflecting relative change of \( W(t, x) \) due to small change in \( x \), is defined as:

$$\lambda = \frac{\partial W(t, x)}{\partial x} = e^{-\eta t} \frac{\partial \tilde{W}(x)}{\partial x}$$  \hspace{1cm} (3.39)

Let \( \tilde{\lambda} = \frac{\partial \tilde{W}(x)}{\partial x} \) denote the marginal cost in \( \tilde{W}(x) \) with respect to \( x \). Inserting Eq. (3.38,3.39) to the costate dynamics equation (3.23) arrives at the following relation (Fleming & Soner, 1993; Hoogendoorn & Bovy, 2003):

$$\tilde{\lambda}(x) = \frac{1}{\eta} \frac{\partial L}{\partial x} + \frac{1}{\eta} \frac{\partial (\lambda^T f)}{\partial x}$$  \hspace{1cm} (3.40)

Equation (3.40) results in an explicit expression of the marginal cost as a function of the state \( x \). Using the stationarity condition of \( \frac{\partial H}{\partial u} = 0 \), one can find the optimal control \( u^* \) as an explicit function of the state \( x \) and the marginal cost of \( \tilde{\lambda}(x) \), which is a state-feedback control law.

### 3.3.5 Discussions

In practice, the solution approaches to the considered optimal control problem are determined by the characteristics of the problem and the solution approach, i.e. the linearity of the system dynamics, specification of the cost function, presence of equality/inequality constraints on state and control variables, and the computational requirements. Table 3.1 lists the main characteristics of the main solution methods.

In principle, most methods can handle nonlinear system dynamics except the analytical CARE solution of LQR. However, nonlinear system dynamics adds more complexity to the considered problem. In practice, linearisation techniques are often sought to guarantee solvability of the problem or to solve the problem more efficiently.

Dynamic programming method entails a numerical solver to the PDE of \( n + 1 \) dimension, i.e. the HJB equation, with \( n \) equals the dimension of the state vector \( x \), while indirect methods require a numerical solver to solve a \( 2 \times n \) ODEs. Direct methods requires ODE solver to solve the \( n \) dimensional state dynamic equation and an NLP solver for the optimisation. The specific formulation with discounted cost and LQR problems finds analytical solution to the problems and hence no numerical solver is needed.
Table 3.1: Comparison of solution approaches of optimal control problems

<table>
<thead>
<tr>
<th>Characteristics</th>
<th>Dynamic programming</th>
<th>Indirect methods</th>
<th>Direct sequential methods</th>
<th>Direct simultaneous methods</th>
<th>Analytical solution for discounted cost problem</th>
<th>CARE solution for LQR</th>
</tr>
</thead>
<tbody>
<tr>
<td>Handling nonlinear system dynamics</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>No</td>
</tr>
<tr>
<td>Numerical solver needed</td>
<td>PDE</td>
<td>ODE</td>
<td>ODE, NLP</td>
<td>(ODE,) NLP</td>
<td>n.a.</td>
<td>n.a.</td>
</tr>
<tr>
<td>Smoothness of cost function</td>
<td>$C^1$</td>
<td>$C^1$</td>
<td>$C^2$</td>
<td>$C^2$</td>
<td>$C^1$</td>
<td>$C^2$</td>
</tr>
<tr>
<td>Other requirements on cost function formulation</td>
<td>Explicit expression of $u^*$</td>
<td>Explicit expression of $u^*$</td>
<td>Quadratic cost function of $x$ and $u$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Resultant objective function for optimisation</td>
<td>Hamiltonian</td>
<td>Hamiltonian</td>
<td>Transcribed cost function</td>
<td>Transcribed cost function</td>
<td>Hamiltonian</td>
<td>Cost function</td>
</tr>
<tr>
<td>Resultant arguments for optimisation</td>
<td>$u$</td>
<td>$u$</td>
<td>$q$</td>
<td>$q, p$</td>
<td>$u$</td>
<td>$u$</td>
</tr>
<tr>
<td>Handling constraints on control</td>
<td>Easy</td>
<td>Easy</td>
<td>Easy</td>
<td>Easy</td>
<td>Difficult</td>
<td>Difficult</td>
</tr>
<tr>
<td>Handling constraints on state</td>
<td>Possible</td>
<td>Possible</td>
<td>Possible</td>
<td>Easy</td>
<td>Difficult</td>
<td>Difficult</td>
</tr>
<tr>
<td>Numerical error</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>No</td>
<td>No</td>
</tr>
<tr>
<td>Optimisation error</td>
<td>No</td>
<td>No</td>
<td>Yes</td>
<td>Yes</td>
<td>No</td>
<td>No</td>
</tr>
<tr>
<td>Computational efficiency</td>
<td>Low</td>
<td>Medium</td>
<td>Medium</td>
<td>High</td>
<td>Very high</td>
<td>Very high</td>
</tr>
</tbody>
</table>

$C^i$: Continuous and differentiable to $i$th order. n.a.: not applicable.
The cost function needs to be at least first-order differentiable \((C^1)\) for the dynamic programming, indirect methods and the analytical solution approach with discounted cost, since these approaches require the evaluation of *costates*, which are first-order derivatives of the value function. For direct methods, NLP solvers usually evaluate the Jacobian (first-order derivatives) and Hessian (second-order derivatives) matrices of the cost function with respect to the arguments, and hence second-order differentiability of the cost function is preferable. The CARE solution method entails second-order differentiability of the cost function.

The dynamic programming approach, indirect methods and the analytical solution method with discounted cost formulation necessitate the cost function to be specified in such a way that explicit expression of the optimal control law \(u^*\) is yielded from the *stationarity condition*. Although this puts forward some requirement on the selection of performance index, it gives insights into the solution direction where the cost function decreases fastest. While the requirement on selection of cost function is relaxed for the direct methods, transcribing the original optimal control problem into NLP problems may lose insights into the control problem. The CARE solution for LQR also gives insights into the optimal control law which minimises the cost function to specific problem formulations, but the the cost function must be a quadratic form of state and control variables which restricts the application of the solution methods to some extent.

The dynamic programming, indirect methods and the analytical solution minimise the Hamiltonian with continuous control \(u\) as arguments. Direct methods minimise the transcribed cost function with control function parameters \(p\) (for sequential approaches) or with control and state function parameters \(p\) and \(q\) (for simultaneous approaches) as arguments, which results in large size NLP problems of finite size. The CARE solution method minimises the original cost function with continuous control \(u\).

In practice, constraints commonly exist on control and state variables. Inequality constraints on control input be dealt with easily in most numerical methods, but are difficult to address with the analytical solution approaches. Inequality constraints on state variables can be deal with straightforwardly with direct simultaneous approaches. Special techniques, such as introducing penalties in the cost function when the constraints are violated or discretising the inequality constraint equations (Binder et al., 2001), are needed to address inequality constraints on state variables in dynamic programming, indirect methods and direct sequential methods. It is difficult to address state constraints using the analytical solution approaches.

Dynamic programming and indirect methods are more accurate than direct methods, since only numerical errors exist in the methods due to the approximation of the differential equation solution. Direct methods use both ODE and NLP solvers, and hence extra optimisation errors are introduced (due to finite iterations) apart from the numerical errors, which result in sub-optimal solutions. Analytical solutions are most accurate, since no numerical errors nor optimisation errors prevail in these methods.
Dynamic programming approach entails solving PDEs, which is computationally expensive. Thus the method is restricted to small scale systems, i.e. systems with dimensions $n \leq 3$ (Bryson & Ho, 1975). The indirect methods reduce the computational load by solving the coupled ODEs instead of PDEs as in the dynamic programming approach, and can treat large scale systems with high dimensions. Direct methods can make use of efficient NLP solvers and are computationally efficient. Analytical solutions generate state-feedback control laws, which have the lowest computational load. Direct methods and analytical solutions are also favourable in dealing with large scale systems from computation perspective.

A trade-off characterises the selection of an appropriate solution approach to the considered optimal control problem. One may use a complex nonlinear system dynamics model to achieve good prediction of system state. But this may result in a heavy burden in computing the optimal control input, thus prohibiting the real-time application of the solution method. Similarly, one may choose a simple cost function to enable fast solution methods, such as a quadratic function of the state and the control variables, this nevertheless may not achieve the desired controller performance under complex situations. It is important to know the consequences of design choices in the controller design on the solution approaches that can be sought to, and there are usually multiple solution methods to one specific problem rather than a unique one.

In subsequent chapters of the thesis, we use different solution approaches to the ADAS control problem under the proposed model predictive control framework in Section 3.2, based on the characteristics of the problem at hand and the solution approaches discussed in this section. We do not aim at seeking one generic solution approach that is appropriate throughout the variant ADAS controllers.

Note that in this thesis, we skip the discussion on the detailed NLP solvers, for instance the SQP methods and interior point methods (Diehl et al., 2009), as well as global optimisation methods such as random search and genetic algorithm (Resende & Pardalos, 2002). While acknowledging the increasing interests and considerable advances of the NLP methods, detailed discussions on the many available NLP algorithms are beyond the scope of this thesis. For an overview of the NLP algorithms, we refer interested readers to Betts (2010) and Resende & Pardalos (2002).

### 3.4 ADAS performance assessment framework

Controller design is an iterative process and it is indispensable to assess the ADAS performance, cf. Section 2.1. In Sections 2.4 and 2.5, we reviewed the assessment methods, with a focus on the merits and drawbacks of traffic and sustainability impact models for assessing ADAS controller performance. In this section, we present an assessment framework for ADAS controller, including the indicators and criteria, simulation and analytical approaches for performance assessment at both microscopic and macroscopic. The assessment framework will be used for the subsequent chapters.
3.4.1 Performance indicators and criteria

At microscopic level, the performance assessment is to verify whether the ADAS controller generate desired behaviour under real-world scenarios, while at macroscopic level the assessment focuses on qualifying and quantifying the impact of ADAS controller on macroscopic flow operations. Microscopic performance is important to individual users and manufacturers while macroscopic impact is important to road operators (Minderhoud, 1999).

Microscopic performance

At microscopic level, we are primarily interested in the following questions regarding the performance of ADAS controllers:

- How fast does the ADAS vehicle achieve its desired speed or desired gap under representative scenarios?
- Can the ADAS controller prevent rear-end collisions in safety-critical situations where emergency braking is needed?
- How stable is the response of the ADAS vehicle under disturbances of representative scenarios?
- How many fuels the ADAS vehicle consumes and how many emissions it produces per unit time and unit distance?

The first question relates to the agility and alertness of the ADAS vehicle, and indicators such as maximum acceleration $a_{\text{max}}$, maximum braking $b_{\text{max}}$, and the time needed to accelerate/decelerate to desired speed or desired gap can be used to answer this question.

The second question is about the safety of the ADAS controller. This can be evaluated by checking the criterion that whether the gap of the ADAS vehicle remains above zero in safety-critical situations.

The third question relates to the local stability of the ADAS controller. This can be examined by the criterion that whether the disturbance in the ADAS vehicle speed profile will be damped out or remain unattenuated with the course of time.

The last question is about the sustainability impact. The indicators of spatial and instantaneous fuel consumption and emission rate can be used to answer this question.

Macroscopic impact

At macroscopic level, we are primarily interested in the collective impacts of ADAS on traffic flow quality, particularly on capacity, throughput, string/flow stability and
 environmental sustainability. Indicators at the macroscopic level include capacity, total time spent and delay, speed variance, total fuel consumption and/or emissions.

Analytical methods can be sought to for ADAS evaluation when the algorithms satisfy certain requirements, cf. Chapter 5. In this thesis, both simulation and analytical studies are used to assess the proposed ADAS controller performance.

### 3.4.2 Simulation study under representative scenarios

As pointed out in Section 2.4, microscopic simulation is very useful for assessing ADAS performance due to the descriptive power. In this thesis simulation studies will be used to assess the proposed ADAS controllers, particularly for algorithms derived with numerical solution approaches (cf. Section 3.3).

The aforementioned microscopic and macroscopic performance indicators related to traffic operations are calculated directly from trajectory data in microscopic simulator. The synthetic trajectories are also used as inputs for macroscopic or microscopic fuel consumption and emission models to calculate the microscopic and macroscopic indicators related to sustainability. Although different layouts of the network are used in different chapters, the following representative scenarios are generic and hence are used in setting up simulation experiments throughout the thesis:

- **Accelerating from standstill to free speed.**
  
  This scenario represents accelerating without constrained by preceding vehicles. The ADAS vehicle is expected to increase its speed gradually until the free speed.

- **Following a vehicle decelerating from equilibrium conditions.**
  
  This scenario represents decelerating disturbance, e.g. approaching the congestion tail (upstream congestion front). The ADAS vehicle is expected to decelerate as a reaction to the insufficient gap and the braking behaviour of the leading vehicle. It should maintain the same speed of the leader and travels at its desired gap after sufficient time.

- **Following a vehicle accelerating from equilibrium conditions.**
  
  This scenario represents accelerating disturbance, e.g. moving out of the congestion head (downstream congestion front). The expected behaviour is to accelerate as a reaction to the increase in gap and maintain the leader speed at its own desired gap after sufficient time.

- **Approaching slow vehicles with high speeds.**
  
  This represents scenarios where safety-critical situations where hard braking is necessary to avoid collisions. A collision-free controller should be able to brake in sufficient distance and sufficient amplitude such that rear-end collision with the preceding vehicle is avoided.

The representative scenarios are selected based on a comprehensive study (Bareket et al., 2003).
### 3.4.3 Analytical approach for state-feedback algorithms

As pointed out in Section 2.4, analytical approaches are available for state-feedback ADAS algorithms. In longitudinal ADAS applications, the state-feedback algorithms entail $u$ as an explicit function of the following gap $s$, relative speed $\Delta v$ and vehicle speed $v$: $u = f(s, \Delta v, v)$. For a homogeneous traffic composition with ADAS vehicles, the equilibrium speed-gap and flow-density relationship can be analytically derived by setting $u = 0$ and $\Delta v = 0$, i.e. all vehicles in traffic travel at same speeds and at their desired gaps:

$$u = f(s_e, 0, v_e)$$  \hspace{1cm} (3.41)

Equation (3.41) usually results in expressing the equilibrium gap as a function of equilibrium speed, $s_e(v_e)$ or vice versa, $v_e(s_e)$. Using the relationship of density $\rho_e$ (in veh/km) and gap $s_e$ (in m) of $\rho_e = 1000/(s_e + l)$ and the macroscopic flow-density-speed relation of $q_e = \rho_e v_e$, one can get the analytical expression of the flow-density relationship, where the maximum flow (capacity) can be derived (Treiber & Kesting, 2011).

Since state-feedback optimal algorithms are equivalent to time-continuous car-following models, Fourier transform can be used to analytically examine the local stability and string stability criteria for ADAS systems (Treiber & Kesting, 2011). The analytical approach will be presented in detail in Chapter 5.

### 3.5 Conclusions

In this chapter, we presented the model predictive control framework, solution approaches and performance assessment framework for ADAS systems, which builds on the findings of the previous chapter. Under the model predictive control framework, the longitudinal motion of IVs is recast into a rolling horizon optimisation problem, seeking an optimal acceleration that minimises a cost function. Thus the research question 4 in Chapter 1 is answered. We have illustrated the generic nature of the control framework by deriving the widely-studies ACC controller employing constant time gap policy.

We categorised and discussed several solution approaches, and compared the merits and drawbacks of them, which answers the research question 5 in Chapter 1. Dynamic programming methods entail solving PDE and generate solutions to the entire state space, but they are limited to small scale systems, i.e. with dimensions $n \leq 3$ due to the computational burden. The PMP based approaches reduce the computational load by solving the coupled ODEs instead of PDE as in the dynamic programming approach and can be applied to large scale systems. Direct numerical solutions discretise the original optimal control problem and transcribe it into NLP problems, which can be solved more efficiently but with less accuracy.
The classical CARE solution to the linear quadratic control problem gives a linear state-feedback control law. However, the solution approach requires the format of the cost function to be quadratic. As a result, the linear feedback controller cannot handle highly non-stationary conditions such as preventing collision when approaching a standstill leader with high speeds (Godbole et al., 1999), nor it can be applied to Eco-driving support systems which entails incorporating non-quadratic fuel consumption and emission models into the cost function. The analytical solution approach to the problem with discounted cost does not require the running cost function to be quadratic, and it is possible to generate non-linear optimal control laws using this solution approach.

A performance assessment framework for evaluating the performance of ADAS controllers at both microscopic and macroscopic level is also proposed. Both simulation study and analytical methods can be sought to for evaluating ADAS controllers depending on the solution approaches used.

The control framework, solution approaches will be used for deriving specific control algorithms in the following chapters and the assessment methods will be used to assess the ADAS performance.
Chapter 4

ACC and EcoACC controllers with dynamic programming solution

In the previous chapter, we introduced the generic model predictive control framework for Advanced Driver Assistance Systems (ADAS) and discussed different solution approaches to the optimal control problem. To show the workings of the framework and to test its flexibility in the cost function formulation, this chapter applies the framework to derive and to assess operational control algorithms of Adaptive Cruise Control (ACC) and Ecological Adaptive Cruise Control (EcoACC) systems under the autonomous following and ecodriving support concepts (cf. Section 2.2). The ACC system aims at maximising driving safety, efficiency and comfort, while the EcoACC system minimises CO\textsubscript{2} emissions in addition to the objectives of the ACC system. Both systems pertain to non-cooperative ADAS identified in Section 2.2, i.e. the controller relies solely on its own sensors and only optimises its own situation.

The algorithms for ACC and EcoACC systems are derived using the dynamic programming approach by solving the sufficiency conditions in the form of a partial differential equation (PDE), i.e. the so-called Hamilton-Jacobi-Bellman (HJB) equation. To this end, a finite difference numerical scheme is employed. To enable real-time implementation, the algorithms are accelerated using an off-line computing strategy. This strategy entails discretisation of the state-space and off-line calculation and storage of the optimal accelerations under each possible state. In on-line applications, the optimal accelerations corresponding to the most recent system state will be called by the controller. This avoids solving PDEs on-line for all vehicles at each time step. The microscopic performances of the ACC and EcoACC controllers and the collective impacts on traffic operations and sustainability are assessed by simulation of large platoons on a single-lane ring road.

This chapter is structured as follows. Section 4.1 presents control objectives and design assumptions of ACC/EcoACC systems, followed by the controller formulation in Section 4.2. The numerical solution approach to find the optimal accelerations is presented in Section 4.3. Section 4.4 introduces the simulation experimental set-up,
followed by the analyses of microscopic behaviour and collective behaviour of ACC and EcoACC platoons in Section 4.5 and 4.6 respectively. The conclusions are summarised in Section 4.7.


### 4.1 Control objectives and design assumptions

This section describes the control objectives and assumptions on the operations of the ACC and EcoACC systems addressed in this chapter. ACC systems operationalise the autonomous following concept (cf. Section 2.2), aiming at maximising efficiency, safety and driving comfort. EcoACC systems operationalise the eco-driving support concept (cf. Section 2.2), which minimises fuel consumption or $CO_2$ emissions in addition to the objectives of ACC systems.

The ACC/EcoACC controller follows the general assumptions for ADAS described in Section 3.1. The controller receives estimates of the system state from on-board sensors, i.e. autonomous sensing, and uses a model to predict the system evolution in a future horizon. The controller outputs optimal accelerations based on the minimisation of an objective function and the on-board actuators automatically execute the reference accelerations. We assume that the prediction of the system state is not perfect and there is a fixed delay of vehicle response to the change of system state, due to the lag in vehicle dynamics.

Regarding the role of drivers, it is assumed that drivers want to use the systems as much as possible. However, due to the limited detection range of on-board sensors and limited decelerating capabilities, an ACC/EcoACC vehicle may face safety-critical situations, i.e. when approaching a slower predecessor at high cruising speeds. Hence drivers need to take over the control of vehicles in such conditions. The interactions between human drivers and ADAS are illustrated in Figure 4.1. In normal driving conditions, ADAS take over all the tasks of sensing, computing and executing accelerations. Drivers are assumed to continuously monitor the operations of the system and check whether the vehicles are driving too close to the preceding vehicle. In critical conditions where drivers feel unsafe, drivers overrule the system and brake hard to gain control. It is assumed that drivers will return the control of vehicles to ADAS as soon as the conditions are perceived as non-critical.

A general procedure to design an ADAS controller under the generic control framework includes determining a system dynamics model, specifying the cost functions, deriving the solutions to the optimal control problem and testing the controller performance. The remaining of the chapter will describe these elements.
4.2 ACC and EcoACC controller formulation

Under the generic model predictive control framework, accelerations of the controlled vehicle are determined to optimise a cost function, taking into account the predicted behaviour of other vehicles. The control decisions are re-assessed at regular time intervals, using the newest information regarding the system state. This section presents the mathematical formulation of the ACC and EcoACC controller under the model predictive control framework. The system dynamics model which is used by the controller for prediction is firstly determined, then the specifications of the cost function are presented.

4.2.1 Stochastic system dynamics

The system state $\mathbf{x}$ from the perspective of an ACC/EcoACC vehicle $n$ is described by the bumper-to-bumper gap (or range) $s_n$ and the relative speed (or range rate) $\Delta v_n$ with respect to its predecessor $n - 1$, as shown in Figure 4.2.

Thus:

$$\mathbf{x} = (x_1, x_2)^T = (s_n, \Delta v_n)^T = (p_{n-1} - p_n - l_{n-1}, v_{n-1} - v_n)^T$$  \hspace{1cm} (4.1)

where $p$ and $l$ denote the front bumper position and the vehicle length, with subscripts denoting vehicle indices. The subscripts of the system state components denote the corresponding dimension of the state space.
We assume that the ACC/EcoACC system is not able to predict the system state perfectly due to errors in observations. Hence, the system dynamics can be predicted by the following kinematic equations:

\[ \dot{x}_1 = \Delta v_n + \xi_1 \]  
\[ \dot{x}_2 = u_{n-1} - u_n + \xi_2 \]

where \( u_{n-1} \) and \( u_n \) denote the accelerations of the preceding vehicle and the controlled vehicle respectively. \( \xi_1 \) and \( \xi_2 \) in Eq. (4.2, 4.3) denote Gaussian noise terms with a standard deviation of \( \sigma_1 \) and \( \sigma_2 \) respectively. The noise terms reflect the errors of on-board sensors, and the Gaussian noise is chosen due to the thermal nature of typical on-board sensors, e.g. radars (Skolnik, 2001). We further assume that the noise terms are uncorrelated for the sake of solution simplicity. It is possible to include correlation of the noise terms at the expense of adding complexity to the numerical solutions.

For ACC/EcoACC systems based on autonomous sensing technologies without Vehicle-to-Vehicle (V2V) communications, the estimation of the predecessor acceleration \( u_{n-1} \) entails differentiating the gap measurements to the second order, which reduces the signal-to-noise ratio substantially. Therefore, we assume other vehicles are driving at constant speed in the finite prediction horizon, i.e. constant speed heuristics (Treiber et al., 2006a). Thus, the system dynamics equation (4.3) relaxes to

\[ \dot{x}_2 = -u_n + \xi_2 \]

Since the control vector only has one dimension in this case, i.e. \( \mathbf{u} = u_n \), we will drop the index \( n \) in the remaining formulation for the sake of notation simplicity.

The mismatch between the prediction model and the real-world system dynamics stemming from the constant speed heuristics and uncorrelated noise terms implies that the prediction of the system state is not perfect. Nevertheless, the feedback nature of the rolling horizon control framework corrects the prediction permanently and guarantees the controller performance as we show in the ensuing.

In normal conditions without driver intervention, the control signal \( u_n \) is determined by the algorithm described in the following section. In case of safety-critical conditions, drivers will override the system and brake hard to gain in safety, which is similar to the
design principles of ACC systems in (Godbole et al., 1999). If the following conditions are met, an emergency situation is judged to be present, and the drivers will override the system and brake with $u = -4 \, \text{m/s}^2$:

\[ t_{TTC} = \frac{s}{\Delta v} \leq \epsilon, \text{ if } \Delta v < 0 \]  

(4.5)

$t_{TTC}$ denotes the so-called time-to-collision, which is an indicator for safety (Minderhoud, 1999), and $\epsilon$ is a threshold for time-to-collision. It is assumed that the drivers return control to the ACC/EcoACC systems once the $t_{TTC}$ is larger than the threshold value.

### 4.2.2 Cost specification for ACC and EcoACC systems

Since the system state prediction model is stochastic and time-invariant, the expected cost $J$ of the controlled vehicle $n$ can be formulated by slightly changing the general cost function of Eq. (3.4) in the deterministic case into:

\[ J(x, u, t) = E \left\{ \int_{t_0}^{t_0+T_p} L(x, u, t) \, dt + G(x(t_0+T_p), t_0+T_p) \right\} \]  

(4.6)

where $E$ denotes the expected value for the stochastic cost. $t_0$ and $T_p$ denote the current time and the prediction horizon respectively. $L$ and $G$ denote the so-called running cost and terminal cost respectively (cf. Section 3.2).

ACC and EcoACC systems operate in two modes, being cruising mode and following mode. In cruising mode, the system regulates the vehicle speed irrespective of the behaviour of the preceding vehicle, while in following mode the system regulates the following distance taking into account the interaction with the predecessor. Therefore, a two-regime running cost function for ACC and EcoACC systems is proposed. The running costs for the ACC system are composed of safety cost, efficiency cost and comfort cost, reflecting control objectives of the ACC system, cf. Section 4.1. The EcoACC system consists of an ecological cost in addition to the cost compositions of the ACC system. To this end, the ACC system can be regarded as a special case of the EcoACC system. Hence, we specify the cost for the more generic EcoACC system as follows:

\[ L = \begin{cases} 
  c_1 \Delta v^2 + c_2 (s_d(v) - s)^2 + \frac{1}{2} u^2 + c_4 e^{-d/s} (v - v_{eco})^2 & \text{if } s \leq s_f \\
  c_3 (v_0 - v)^2 + \frac{1}{2} u^2 + c_4 e^{-d/s} (v - v_{eco})^2 & \text{if } s > s_f 
\end{cases} \]  

(4.7)

Eq. (4.7) shows that the two operating modes of EcoACC systems are distinguished by a gap threshold $s_f$, i.e. following mode if $s \leq s_f$ and cruising mode if $s > s_f$. The gap threshold is determined by:

\[ s_f = v_0 t_d + s_0 \]  

(4.8)
where \( v_0 \) is the free speed, \( t_d \) is the desired time gap and \( s_0 \) is the minimum distance the controlled vehicle keeps at standstill, i.e. \( v = 0 \text{m/s} \). The gap threshold can be interpreted as the spatial anticipation range for controlled vehicles, since beyond this range, no cost stems from the interaction with the preceding vehicle. Thus, controlled vehicles do not react to the behaviour of the preceding vehicle in cruising mode.

The safety cost only exists in the following mode and stems from deviations from the preceding vehicle’s speed. \( c_1 \) is a weight factor related to the safety cost.

The efficiency cost in following mode arises from deviations from a desired gap, which is a function of vehicle speed \( v \):

\[
s_d = v t_d + s_0
\]

The efficiency cost in the cruising mode stems from deviations from the free speed \( v_0 \). \( c_2 \) and \( c_3 \) are weight factors corresponding to the efficiency cost in following and cruising mode respectively.

The comfort cost stems from accelerating or decelerating manoeuvres of the controlled vehicle and is the same for cruising and following modes.

To represent the control objective of minimising fuel consumption or \( CO_2 \) emissions, a penalty is included when vehicles travel at a speed deviating from the fuel efficient or environment-friendly speed \( v_{eco} \), herein referred to as Eco-cruising speed. The Eco-cruising speed \( v_{eco} \) is usually lower than the free speed \( v_0 \). Here, we choose the \( CO_2 \) emission model developed by Barth & Boriboonsomsin (2008) to determine the Eco-cruising speed. The reason for choosing this model is that it sets up a direct relationship between the \( CO_2 \) emissions and vehicle speed, and it can predict reasonable \( CO_2 \) emission rates under real world conditions. The Eco-cruising speed is acquired through minimizing the spatial emission rate of the \( CO_2 \) emission model, being around 72 km/h. The exponential term in Eq. (4.7) shows that the Eco cost decreases exponentially when the gap is smaller than some distance \( d \), since the vehicle is operating in constrained conditions and cannot drive towards the Eco-cruising speed with small gaps compared to situations where the gap is large. \( c_4 \) is the weight factor of the ecological cost. Notice that when \( c_4 = 0 \), the controller relaxes to the ACC controller.

It should be pointed out that the general concept of Eco-driving is broader than the EcoACC controller we formulated here. General Eco-driving concepts include anticipating traffic flow, maintain a steady speed at low RPM, shift to higher gear early etc. (ECOWILL, 2010). It is possible to include Eco-driving strategies such as anticipating traffic flow under the same control framework (Wang et al., 2014b).

The terminal cost \( G \) is set to zero in this study. We emphasise that the formulation of the running cost and terminal cost is not unique. The focus of the cost function formulation here is to show the flexibility of the framework in cost function formulation, e.g. including non-quadratic and non-linear cost functions, and to compare the performance differences of ACC and EcoACC controllers. The control framework allows other specifications of running costs and non-zero terminal cost.
4.3 Optimal acceleration for ACC and EcoACC systems

This section describes the solution algorithm which gives the optimal acceleration minimising the cost function (4.6). Since the system state has only two dimensions, a dynamic programming approach which gives the optimal solution in the entire state-space is chosen to solve the optimal control problem. To this end, a finite difference method is used to solve the so-called Hamilton-Jacobi-Bellman (HJB) equation, and the solution process is accelerated by an off-line computing strategy. The accelerations are bounded within a comfortable range, and an explicit delay is included in the algorithm.

In the sequel of this section, the solution approach based on dynamic programming is firstly presented, followed by the numerical scheme to the HJB equation. The off-line computing strategy is described afterwards, followed by the treatment of acceleration bounds and explicit delay.

4.3.1 Dynamic programming approach

We define the so-called value function $W(x,t)$ as the cost of a controlled vehicle when optimal control $u^*$ is applied starting from time $t$:

$$W(x,t) = \min_u J(x,u,t) \quad (4.10)$$

To find the optimal acceleration $u^*$ that minimises the expected cost $J$, the dynamic programming approach is used, yielding the so-called Hamilton-Jacobi-Bellman (HJB) equation with a diffusion term due to the stochastic system dynamics (Fleming & Soner, 1993; Hoogendoorn & Hoogendoorn, 2011):

$$\frac{\partial W}{\partial t} = \mathcal{H}^*(s, \Delta v, u^*, \frac{\partial W}{\partial s}, \frac{\partial W}{\partial \Delta v}, \frac{\partial^2 W}{\partial s^2}, \frac{\partial^2 W}{\partial \Delta v^2}) \quad (4.11)$$

where $\mathcal{H}$ denotes the Hamiltonian function as follows:

$$\mathcal{H}^* = \min_u \left\{ L + \Delta v \frac{\partial W}{\partial s} + (-u) \frac{\partial W}{\partial \Delta v} + \sigma_1 \frac{\partial^2 W}{\partial s^2} + \sigma_2 \frac{\partial^2 W}{\partial \Delta v^2} \right\} \quad (4.12)$$

Assuming no bounds on control inputs and taking the necessary condition of $\frac{\partial \mathcal{H}^*}{\partial a} = 0$, we arrive at the following optimal control strategy:

$$u^* = \frac{\partial W}{\partial \Delta v} \quad (4.13)$$

Eq. (4.13) is the applied optimal control strategy that decreases the costs most rapidly. It shows that the optimal acceleration equals the marginal cost of the relative speed, which describes the relative change in the optimal cost due to a small change in the relative speed $\Delta v$. 
4.3.2 Numerical solution based on finite difference method

Although Eq. (4.13) gives some insights into the applied optimal control strategy, to obtain the exact acceleration value, we still have to solve the HJB equation. In many cases, it is not possible to solve the partial differential equation (PDE) analytically. Hence, numerical schemes are often employed. In this study, we use the following finite difference method to approximate the first and second order partial derivatives (Fleming & Soner, 1993; Hoogendoorn & Hoogendoorn, 2011): 

\[
\frac{\partial W}{\partial s} \approx \nabla_s W = \frac{W(s + \delta s, \Delta v) - W(s, \Delta v)}{\delta s}
\]  

(4.14)

\[
\frac{\partial W}{\partial \Delta v} \approx \nabla_{\Delta v} W = \frac{W(s, \Delta v + \delta v) - W(s, \Delta v)}{\delta v}
\]  

(4.15)

\[
\frac{\partial^2 W}{\partial s^2} \approx \nabla^2_s W = \frac{W(s + \delta s, \Delta v) - 2W(s, \Delta v) + W(s - \delta s, \Delta v)}{\delta s^2}
\]  

(4.16)

\[
\frac{\partial^2 W}{\partial \Delta v^2} \approx \nabla^2_{\Delta v} W = \frac{W(s, \Delta v + \delta v) - 2W(s, \Delta v) + W(s, \Delta v - \delta v)}{\delta v^2}
\]  

(4.17)

where \(\delta s\) and \(\delta v\) denote the mesh size of the system state space. Using the so-called upwind scheme (Fleming & Soner, 1993), Eq. (4.12) becomes:

\[
\mathcal{H}^* = \min_u \{ L + \{ \Delta v \}^+ \nabla_{\Delta v}^+ W - \{ \Delta v \}^- \nabla_{\Delta v}^- W + \{-u\}^+ \nabla_{\Delta v}^+ W - \{-u\}^- \nabla_{\Delta v}^- W + \sigma_1 \nabla^2_s W + \sigma_2 \nabla^2_{\Delta v} W \}
\]  

(4.18)

where the operator \(\{ \}\)^± means \(\{x\}^+ := \max(x, 0)\) and \(\{x\}^- := \min(x, 0)\), for \(\forall x\). For details of this numerical approach, we refer to Fleming & Soner (1993).

4.3.3 Computing the optimal strategy off-line

Solving PDEs simultaneously for a large number of vehicles is computationally expensive and difficult to be implemented in a real-time setting. In the simulation, we simplify the process by discretising the possible gap, speed and predecessor speed into finite mesh grids, since the running cost (4.6) is calculated based these three variables. For each predecessor speed we compute the optimal acceleration for all plausible gaps and relative speeds off-line. The discretisation step of gap is \(\delta s = 1m\) and the step of speeds is \(\delta v = 1km/h\). The discretised grids should satisfy the so-called Courant-Friedrichs-Lewy (CFL) condition to ensure numerical stability (Fleming & Soner, 1993). During the simulation, the optimal accelerations calculated off-line are called each time step by the ADAS controller, which relieves us from solving a PDE on-line.
4.3.4 Bounded acceleration and explicit delay

The numerical approximation of the optimal acceleration based on dynamic programming may result in inadmissible accelerations to decrease the predicted cost. To find feasible solutions, accelerations are bounded within the range of \([-3, 2.5]\) m/s\(^2\). The accelerations bounds are truncated to the optimal accelerations after the dynamic programming process, which guarantees feasibility but may result in sub-optimal solutions.

A fixed system delay \(\tau\) of 0.5 seconds is chosen to represent the lag of the actuating the optimal acceleration by the on-board actuators. Thus, the actual acceleration \(a\) of EcoACC vehicle is expressed as:

\[
a(t) = u^*(t - \tau)
\]

where \(\tau\) is the system delay. Equation (4.19) shows that the actual acceleration at the current time \(t\) is the optimal acceleration of a previous time \(t - \tau\).

In case of safety-critical conditions, a deceleration of \(u = -4\) m/s\(^2\) will be applied by the driver, cf. Section 4.2.1.

4.4 Simulation experimental design

To test the control framework and algorithms for ACC and EcoACC systems and to compare the differences of the two systems on individual and collective vehicular behaviour, we simulate platoons of homogeneous ACC or EcoACC vehicles on a single-lane ring road of 1 km.

To examine the controller performance, we first simulate two densities with different numbers of vehicles on the ring road to represent free traffic and congested traffic conditions. In the first experiment, 20 vehicles are placed in the ring road to represent free traffic conditions, while 40 vehicles in the second experiment create moderately congested conditions. The platoons are homogeneous ACC or EcoACC vehicle platoons. Each simulation run lasts 5 minutes, which is sufficient for the platoons to settle to stationary traffic conditions.

At initial conditions, all vehicles are evenly distributed on the first half of the ring road, with an initial speed of 20 km/h. When the simulation starts, the 1st vehicle of the platoon should accelerate because it is operating in the cruising mode and it is driving with a very low speed. After a while when the 1st vehicle approaches the last vehicle of the platoon, the system is switched to the following mode and it should decelerate. Other vehicles follow the 1st vehicle in the platoon consequently. The set-up of initial conditions leads to disturbances in the platoon, which allows us to analyse the vehicular behaviour under representative scenarios described in Section 3.4, including accelerating in free driving conditions, following the leader with accelerating and decelerating disturbance and approaching slow vehicles with high speeds.
To test the differences in macroscopic flow-density relations of the two driving strategies, we run simulations with different numbers of vehicles on the ring road until stationary conditions, and calculate flow and density from vehicle trajectories based on Edie’s definition (Edie, 1965).

We examine the controller performance by visualising the evolution of acceleration, speed and CO\textsubscript{2} emission rate of representative vehicles in the platoon at the microscopic level. At the macroscopic level, we calculate the average flow, speed, time headway, vehicle kilometres travelled (VKMT), CO\textsubscript{2} emissions of the whole platoon and the stationary flow-density relations to examine the differences in the resultant flow of the two controllers. To gain more insights into the differences of ACC and EcoACC systems on the total CO\textsubscript{2} emissions, we use the flow-density-speed relationship of $V = Q/K$, where $Q$, $K$, $V$ are equilibrium flow, density and speed to calculate total CO\textsubscript{2} emissions of any $Q$ and $K$ and plot the emission contours in the flow-density plane. The aforementioned CO\textsubscript{2} emission model (Barth & Boriboonsomsin, 2008) to calculate Eco cost is used to calculate CO\textsubscript{2} emissions.

Since the focus of the research is to test the workings of the framework and the differences of the two driving strategies, same parameter settings are used for both systems, with $t_d = 1.5$ s, $v_0 = 30$ m/s (108 km/h), $T_p = 2$ s, $c_1 = 0.5$ s\textsuperscript{-2}, $c_2 = 0.01$ s\textsuperscript{-4}, $c_3 = 0.01$ s\textsuperscript{-2}, $s_0 = 2$ m and $l = 5$ m. The standard deviations of the noise terms are set at $\sigma_1 = 2.5$ m and $\sigma_2 = 2.5$ m/s Hoogendoorn & Hoogendoorn (2011). Only difference is the Eco cost weight of $c_4$ which is set to 0.1s\textsuperscript{-2} for EcoACC systems and 0s\textsuperscript{-2} (no ecological cost) for ACC systems.

\section{4.5 Results and discussion}

The simulation results are analysed and discussed in this section. We first compare the behaviours of representative vehicles in the platoon with different controllers, and followed by the collective impact on traffic flow operations and environment. An extended discussion is presented at the end of this section.

\subsection{4.5.1 Microscopic performance of ACC and EcoACC controllers}

Temporal evolutions of speed, acceleration and spatial CO\textsubscript{2} emission rate of the 1st and 5th vehicle in the platoon are plotted in Figures 4.3 and 4.4. Clearly, we can see from the speed and acceleration profile that at the start of the simulation, vehicles are operating at non-stationary conditions, with strong fluctuations in acceleration. However, after sufficient time, the vehicles settle down to equilibrium conditions with small variations in acceleration profiles. The speed and acceleration profiles show the closed-loop stability of the proposed ACC and EcoACC controllers.
The fluctuation in the speed profile of the 1st vehicle in the ACC/EcoACC platoon is caused by the ring road simulation set-up. At the start of the simulation when the gap in front of the 1st ACC vehicle is large, it travels at cruising mode. The behaviour of the 1st vehicle in the ACC platoon is dominated by the cost stemming from not driving at its free speed $v_0$. In the EcoACC platoon, the behaviour of the 1st vehicle in the free cruising mode is dominated by the cost due to not driving at the free speed and not driving at the eco-cruising speed $v_{eco}$. The incurred cost demand both platoon leaders to accelerate at the start of the simulation and the other vehicles in the platoon follow the vehicle to accelerate subsequently. When approaching the end of the platoon, the on-board sensor of the 1st vehicle detects a slower vehicle ahead, and the system will switch to following mode. The resultant acceleration of the 1st vehicle is now a trade-off among the cost of driving at a non-zero relative speed, not driving at the desired gap and not driving at the eco-cruising speed (for EcoACC only). The switching of operation mode causes a change in the cost of the 1st vehicle. As a result it has to decelerate due to the presence of safety cost, although its predecessor, which is the last vehicle in the platoon is accelerating. After this transition, the predecessor of the 1st vehicle is still increasing its speed, and the 1st vehicle is convicted to accelerate again.

For both systems, the fluctuation lasts longer in time at moderately congested conditions compared to free traffic conditions. In moderately congested conditions, the fluctuation in the EcoACC platoon is damped out faster compared to that in the ACC platoon. In the ACC platoon, both the 1st and 5th vehicles exhibit oscillatory behaviours for about 220 seconds at moderately congested conditions, while at the same condition, the fluctuation disappears in the speed profiles of the 1st and 5th vehicles in the EcoACC platoon after 90 seconds. This is explained by the extra Eco cost term in the EcoACC controller, which results in more agile accelerating behaviour of the 1st vehicle in the platoon and demands the EcoACC vehicles to match the fuel-efficiency speed after the initial fluctuation. The more agile accelerating behaviour of EcoACC vehicles damps out the perturbation faster compared to ACC vehicles.

At moderately congested conditions the fluctuation in the speed profile travels downstream from the 1st vehicle to the 5th vehicle in the same platoon. In the EcoACC platoon, the amplitude of the fluctuation is smaller when reaching the 5th vehicle, as we can see from the peaks in the acceleration profiles in Figure 4.4. However, the amplitude of the disturbance does not decrease when travelling downstream to the 5th vehicle in the ACC platoon. The behaviours of the EcoACC vehicles are smoother compared to the ACC vehicles in congested conditions.

The stationary speed in free traffic conditions of the ACC vehicles is around 100 km/h, and is higher than the equilibrium speed of the EcoACC vehicles, which is closer the Eco-cruising speed. However, the EcoACC strategy results in a higher equilibrium speed at moderately congested conditions compared to the ACC strategy.

In free traffic conditions, both the 1st and the 5th vehicle of the EcoACC platoon generate fewer spatial $CO_2$ emission rates than the 1st vehicle of the ACC platoon. The same holds for congested conditions. The benefits in $CO_2$ emission rate in moderately
Figure 4.3: Comparison of speed, acceleration, and spatial CO$_2$ emission rate of the 1st and 5th vehicle in EcoACC platoon (blue line) and ACC platoon (red line) in free traffic conditions.

Congested conditions are more significant than those in free traffic conditions, due to the oscillatory behaviours in the ACC platoon.

In moderately congested conditions, we observe driver intervened situations of 1st vehicles in both platoons with deceleration of -4 m/s$^2$, as shown in the acceleration profiles in Figure 4.4(c).

4.5.2 Collective behaviour of ACC and EcoACC vehicles

To investigate the macroscopic impacts on traffic, we calculate the stationary average speed, average time headway, flow, vehicle kilometres travelled, spatial CO$_2$ emission rate and total CO$_2$ emission for the ACC and EcoACC platoons with different densities. The results are shown in Table 4.1.

At free traffic conditions, the EcoACC system steers vehicles towards the eco-cruising speed, which reduces the level of service in free traffic conditions, resulting in a lower flow at the same density (20 veh/km) compared to the ACC system. On the contrary, at moderately congested conditions, the EcoACC system leads to a higher traffic speed and flow compared to the ACC system. This can be explained by the difference in cost.
Figure 4.4: Comparison of speed, acceleration, and spatial CO$_2$ emission rate of the 1st and 5th vehicle in EcoACC platoon (blue line) and ACC platoon (red line) in congested traffic conditions.

Table 4.1: Overall impact on traffic flow and CO$_2$ emissions

<table>
<thead>
<tr>
<th>Average density</th>
<th>Indicators</th>
<th>EcoACC</th>
<th>ACC</th>
<th>Relative change</th>
</tr>
</thead>
<tbody>
<tr>
<td>20 veh/km</td>
<td>Speed (km/h)</td>
<td>84.4</td>
<td>101.8</td>
<td>-17.0%</td>
</tr>
<tr>
<td></td>
<td>Time headway (s)</td>
<td>2.13</td>
<td>1.77</td>
<td>20.3%</td>
</tr>
<tr>
<td></td>
<td>Flow (veh/h)</td>
<td>1689</td>
<td>2035</td>
<td>-17.0%</td>
</tr>
<tr>
<td></td>
<td>VKMT (veh·km)</td>
<td>140.73</td>
<td>169.4</td>
<td>-17.0%</td>
</tr>
<tr>
<td></td>
<td>CO$_2$ rate (g/km/veh)</td>
<td>202.61</td>
<td>209.0</td>
<td>-3.1%</td>
</tr>
<tr>
<td></td>
<td>Total CO$_2$ (kg)</td>
<td>28.5</td>
<td>35.4</td>
<td>-19.5%</td>
</tr>
<tr>
<td>40 veh/km</td>
<td>Speed (km/h)</td>
<td>55.0</td>
<td>41.9</td>
<td>31.3%</td>
</tr>
<tr>
<td></td>
<td>Time headway (s)</td>
<td>1.63</td>
<td>2.15</td>
<td>-24.2%</td>
</tr>
<tr>
<td></td>
<td>Flow (veh/h)</td>
<td>2199</td>
<td>1675</td>
<td>31.3%</td>
</tr>
<tr>
<td></td>
<td>VKMT (veh·km)</td>
<td>183.3</td>
<td>139.6</td>
<td>31.3%</td>
</tr>
<tr>
<td></td>
<td>CO$_2$ rate (g/km/veh)</td>
<td>205.8</td>
<td>226.2</td>
<td>-9.0%</td>
</tr>
<tr>
<td></td>
<td>Total CO$_2$ (kg)</td>
<td>37.7</td>
<td>31.5</td>
<td>19.7%</td>
</tr>
</tbody>
</table>
specifications of the two systems. At stationary conditions where the speed variations among vehicles in the same platoon are small, the optimal accelerations of EcoACC vehicles are determined by the cost due to deviating from the desired gap and deviating from the eco-cruising speed, while the optimal accelerations of the ACC vehicles are dominated by the cost due to deviating from the desired gap. The differences in cost compositions result in a much larger time headway of the Eco-driving platoon at free traffic conditions and a smaller time headway at moderately congested conditions.

In EcoACC platoons, lower average spatial emission rates are observed compared to the ACC system, with 3.1% less in free traffic conditions and 9.1% less at moderately congested conditions. At free traffic conditions, the benefits of the Eco-driving strategy on total CO\(_2\) emissions are amplified by the lower flows of the Eco-driving platoon, resulting in 19.5% less of total CO\(_2\) emissions compared to the ACC system. However, at moderately congested conditions, benefits on total CO\(_2\) emissions are outweighed by the higher flows of the Eco-driving platoon, resulting in a 19.7% increase of total CO\(_2\) emissions. Results imply that benefits of ADAS or dynamic traffic management measures on total emissions are related to both demand and traffic flow dynamics. Similar findings have also been reported in Wang et al. (2011). It should be noticed that in the ring road simulation, where the density on the road is fixed, the demand is proportional to the traffic speed. If the demand is not elastic, which is true in reality in short terms, the changes in the total CO\(_2\) emissions are the same as the changes in spatial emission rates. In other words, the EcoACC system will reduce CO\(_2\) emissions in both free traffic and congested conditions.

To examine the differences in the flow-density relations of the two controllers, we run simulations with different densities. The resulting flow-density relationship are depicted in Figure 4.5, from 2 veh/km to 80\(^6\) veh/km. In general, we can see two branches in both ACC and EcoACC platoons. At low density, flow increases with the density, while at high density, flow decreases with the increase of density. Results bear much resemblance with flow-density relationship in human-driven vehicular traffic (Richards, 1956; Treiber & Kesting, 2013), where the flow-density relationship often displays a fundamental diagram consisting of a free traffic branch and a congested branch. In free traffic branch, the flow increases with the increase of density, while in the congested branch, the flow decreases with the increase of density. The density where the traffic changes from free flow branch to congested branch is the so-called critical density. It is clear from Figure 4.5 that the critical density of EcoACC platoon is around 40 veh/km, which is higher than that of the ACC platoon. Furthermore, the resulting capacity of the EcoACC traffic is around 2200 veh/h, which is also higher than the ACC platoon, being around 2000 veh/h.

At loose traffic conditions where the density is lower than 30 veh/km, the EcoACC system results in lower flow and lower speed compared to the ACC system at the same density, while at high density (higher than 30 veh/km), the EcoACC system leads to higher flow and higher speed at the same density.\(^6\)

---

\(^6\)Collision occurs with densities larger than 80 veh/km for both ACC and EcoACC systems.
The contour plot of total $CO_2$ emissions in the fundamental diagram are drawn in Figure 4.6, which shows the difference on total $CO_2$ emissions of the two controllers. When the density is higher than 12 veh/km but lower than 25 veh/km, the total $CO_2$ emission of EcoACC vehicles is lower than ACC vehicles. When the density is higher than 25 veh/km, the EcoACC system leads to higher flow and the resultant total emission is higher. However, higher flow and capacity imply less congestion. In reality where the demand is not elastic in short terms, the changes in the total $CO_2$ emissions are the same as the changes in spatial emission rates.
4.5.3 Discussion on Eco-driving strategies

The Eco-cruising speed minimising the spatial emission rate of the $CO_2$ emission model is 72 km/h (Barth & Boriboonsomsin, 2008). Travelling with speeds below 72 km/h generates higher spatial $CO_2$ emission rates, since vehicles spend more time on the road, which results in higher $CO_2$ emissions for a fixed travel distance. Travelling with speeds over the fuel optimum speed can also produce a higher spatial $CO_2$ emission rate, since travelling with those high speeds requires more vehicle power (thus more fuel consumption) per time unit. In the end, travelling with speeds higher than the fuel optimum speed causes a negative effect on $CO_2$ emissions for a fixed travel distance.

In free traffic conditions, the EcoACC system in our simulation reduces speed towards fuel efficiency speed, but it increases travel time and decreases the level of service of the ring road. There is a trade-off between travel efficiency and average spatial $CO_2$ emissions rate in this situation. However, in congested conditions, where the average speed is low, reducing congestion will increase average speed, and thus reduce the spatial $CO_2$ emission rate as well. Thus in congested conditions, increasing travel efficiency and reducing average spatial $CO_2$ emission rate lie in the same line.

In real world, stop-and-go waves with many accelerating and decelerating behaviours produce substantial fuel consumption and emissions (Rakha et al., 2004). If the EcoACC systems can stabilise traffic flow and damp out stop-and-go waves, this will lead to an additional reduction of fuel consumption and emissions.

Notice that the quantitative impacts of ADAS on traffic and environment depend on the choice of controller formulation and parameters. In our case study of designing the EcoACC algorithm, the running costs are to some extent arbitrary, and the proposed controller needs to be refined to improve the operational range, e.g. avoiding collision at high density conditions. Other definitions of costs and parameters settings for EcoACC systems are also possible, i.e. including road topology information (Park et al., 2012).

4.6 Conclusions

In this chapter, we applied the generic model predictive control framework for ADAS to controller design of non-cooperative ACC and EcoACC systems. Operational algorithms for ACC and EcoACC controllers are derived and are implemented in a single-lane ring road scenario to examine the controller performance at microscopic level and the resultant flow characteristics at collective level. It is found that the accelerating and decelerating behaviours of the EcoACC vehicles are smoother compared to the ACC vehicles. At macroscopic level, two ADAS result in different fundamental diagrams. The EcoACC system leads to lower speed (and thus lower flow in the ring road with fixed density) compared to ACC system at free traffic conditions, but higher speed
(and thus higher flow in the ring road with fixed density) at moderately congested conditions.

From sustainability perspectives, the EcoACC system results in a lower spatial $CO_2$ emission rate than ACC system, both in free traffic and in moderately congested conditions. In reality where the demand is not elastic in short terms, the changes in the total $CO_2$ emissions are the same as the changes in spatial emission rates.

Although we designed a human driver intervention mechanism in the ACC and CACC controller, simulation experiments show that the ACC and EcoACC systems are not collision-free at dense traffic conditions, i.e. when travelling with small gaps. Hence, refinements on the controller formulation are necessary to avoid collision at dense traffic conditions. This will be addressed in the following chapter.

Regarding the efficiency of the solution approach, the off-line computation strategy for dynamic programming approach is feasible for the non-cooperative ADAS with low dimensionality in this chapter. Information and communication technologies have enabled V2V communications to extend the spatial anticipation range of ADAS, which entails expansion of the system state-space and adds complexity in computing the optimal accelerations. It becomes a daunting task to compute and to store the optimal control strategy off-line for cooperative ADAS with high dimensions. This poses challenges on how to design controllers to capture the V2V cooperation and how to solve the optimal control problem with more efficient solution approaches. These issues will also be dealt with in the following chapter.
Chapter 5

Refined ACC and C-ACC controllers with analytical solution

In the previous chapter, we applied the generic control framework to design non-cooperative ACC and EcoACC controllers, with an off-line computing strategy to accelerate the dynamic programming solution approach. Simulation experiments showed that the ACC and EcoACC controllers proposed in the previous chapter cannot guarantee collision-free behaviour under dense traffic conditions and it is difficult to extend the solution approach to cooperative ADAS with many state dimensions, cf. Section 4.6. The objective of this chapter is threefold. The first objective is to refine the non-cooperative ACC controller to avoid collisions at smaller gaps, which is achieved by giving a high penalty when an ACC vehicle approaches the leader at small gaps. The second objective is to apply the generic control framework to controller design of cooperative ACC (C-ACC) systems under the cooperative manoeuvring concept identified in Chapter 2, where a C-ACC controller chooses optimal accelerations to optimise a joint cost function representing its own situation as well as the situation of its follower. The third objective is to analyse the controller properties and the resulting flow characteristics, with a focus on capacity and stability.

To derive the optimal algorithms for the refined ACC and C-ACC controllers, an analytical solution approach as sketched in Chapter 3 is used. The solution approach gives analytical solutions to the considered optimal control problem in a state-feedback form, which is very efficient for computation and hence is a desirable feature for real-time implementation. To assess the controller characteristics, an analytical approach for deriving local and string stability criteria and equilibrium flow-density solutions is proposed.

This chapter is structured as follows. Section 5.1 introduces the additional assumptions and control objectives for ACC and C-ACC systems based on the general description in Section 3.1. Section 5.2 presents the ACC and C-ACC controller formulation and the analytical solutions. An analytical framework for analysing the controller performance and characteristics is presented in Section 5.3. Section 5.4 and 5.5 present the
controller performance verification results and the traffic flow characteristics of ACC and C-ACC vehicles. The conclusions are summarised in Section 5.6.


5.1 Controller design assumptions and control objectives

We make the following assumptions for the ACC and C-ACC controllers and systems in addition to the core assumptions in Section 3.1:

- The state information available for the ACC/C-ACC controller is perfect. Therefore we do not consider noise in measurements and state prediction.
- ACC vehicles get the state information solely from their own on-board sensors. There is no delay in on-board sensors and actuators.
- When two C-ACC vehicles form a leader-follower pair, they exchange information regarding their positions and speeds via V2V communications.
- V2V communications take place at a frequency much higher than the sampling frequency of control decisions and thus the communication delay can be neglected. At the start of each control cycle, the newest information regarding the system state will be used as the current state of the system.
- The ACC/C-ACC controller predicts the behaviours of surrounding vehicles and controls the accelerations to optimise a cost function, based on the current state and the prediction of the future state of the system.
- The ACC and C-ACC controllers assume that the surrounding vehicles are travelling at equilibrium conditions with zero accelerations during a prediction horizon, i.e. constant speed heuristics.
- An ACC vehicle only optimises its own situation, while a C-ACC vehicle optimises the situations of both itself and its direct follower.

ADAS controllers are designed to fulfil some control objectives, which can be any combination of the following objectives:

- To maximise travel efficiency;
- To maximise safety;
To minimise fuel consumption and emissions;
• To maximise smoothness and comfort.

The importance of each of these objectives may vary according to preferences of drivers, traffic conditions, or vehicles types, e.g. some systems may give priority to safe driving, while others prefer travel efficiency, accepting smaller headways and higher risk if other influencing factors (speed and relative speed) are the same.

The control objectives for ACC systems are the same as in the previous chapter, i.e. maximising efficiency, safety and driving comfort. The control objectives for the C-ACC systems are to maximise the overall efficiency, safety and driving comforts of the C-ACC vehicle and its direct follower.

5.2 ACC and C-ACC controller formulation

In this section, we present the mathematical formulation of the ACC and C-ACC controllers and derive the control algorithms for both controllers. We first describe the system state specification and state dynamics models. Then, the cost functions for ACC and C-ACC systems are specified, formulating the control of vehicles into an infinite horizon problem with discounted cost. After that, an analytical solution approach based on Pontryagin’s Minimum Principle (PMP) for the infinite horizon problem is presented and used to derive the ACC and C-ACC algorithms.

5.2.1 System dynamics model

The system state \( x \) from the perspective of ACC vehicle \( n \) is fully described by the gap (net distance headway) \( s_n \) and its own speed \( v \). Thus \( x = (x_1, x_2)^T = (s_n, v_n)^T \). Since we do not consider noise in the sensors (cf. Section 5.1), the system dynamics can be described by the following deterministic equations:

\[
\frac{d}{dt} x = \frac{d}{dt} \begin{pmatrix} s_n \\ v_n \end{pmatrix} = \begin{pmatrix} \Delta v_n \\ u_n \end{pmatrix} = f(x, u) \tag{5.1}
\]

where \( \Delta v_n = v_{n-1} - v_n \) denotes the relative speed of vehicle \( n \) with respect to its predecessor \( n - 1 \). \( u_n \) is the controlled acceleration. The considered system is a time invariant system, i.e. the system dynamics model \( f \) does not depend explicitly on time \( t \). Note that the system dynamics includes an exogenous variable, which is the speed of the predecessor \( v_{n-1} \).

For Cooperative ACC (C-ACC) controllers, the system state for vehicle \( n \) is extended to include the situation of its follower \( n + 1 \), \( x = (s_n, v_n, s_{n+1}, v_{n+1})^T \), where \( s_{n+1} \) and
\( v_{n+1} \) denote the gap and speed of the vehicle \( n+1 \) respectively. The system dynamics now follows the following differential equation:

\[
\frac{d}{dt} \mathbf{x} = \frac{d}{dt} \begin{pmatrix} s \\ v_n \\ s_{n+1} \\ v_{n+1} \end{pmatrix} = \begin{pmatrix} \Delta v_n \\ u_n \\ \Delta v_{n+1} \\ u_{n+1} \end{pmatrix} = f(\mathbf{x}, \mathbf{u}) \tag{5.2}
\]

with \( u_{n+1} \) denoting the acceleration of the follower. Note that it becomes very difficult to use the dynamic programming approach and the off-line computing strategy for the C-ACC system with four dimensions, cf. Section 4.6.

For both ACC and C-ACC systems, the preceding and following vehicles are assumed to be travelling at constant speeds within the prediction horizon, i.e. driving as a non-cooperative differential game (Hoogendoorn & Bovy, 2003). Thus \( u_{n-1} = u_{n+1} = 0 \) for the state prediction model.

Notice that when applying the controller, the preceding and following vehicles may not travel at constant speed, which implies a mismatch between the prediction model and the actual system due to the constant-speed heuristics. The feedback nature of the receding horizon process, which entails reassessing the control input at regular time intervals \( \Delta t \) with the newest information of other vehicles, is permanently corrected, and thus robust to the mismatch. One advantage of this system definition is that the C-ACC controller can be applied in a generic context irrespective of vehicle type of its follower, i.e. the system prediction model holds no matter the follower \( n+1 \) is a cooperative vehicle or not.

### 5.2.2 Cost function specification

In the present work, we formulate the ACC and C-ACC controllers as infinite horizon optimal control problems with discounted cost (cf. Section 3.2 ). As we will show in the ensuing, this formulation allows one to get an analytical optimal acceleration in the state-feedback form, which is a desired feature from computational perspectives.

For infinite horizon problems with discounted cost, the optimal control problem is described by the following mathematical programme:

\[
\mathbf{u}^*_{(t_k, \infty)} = \arg \min_j J(t_k, \mathbf{x}|\mathbf{u}) = \arg \min_j \int_{t_k}^{\infty} e^{-\eta \tau} \mathcal{L}(\mathbf{x}, \mathbf{u}) d\tau \tag{5.3}
\]

subject to:

\[
\frac{d}{dt} \mathbf{x} = f(\mathbf{x}, \mathbf{u}) \tag{5.4}
\]

The cost function \( J(t_k, \mathbf{x}|\mathbf{u}) \) describes the predicted cost when the system is driven by the control input \( \mathbf{u} \) from the current state of the system \( \mathbf{x}(t_k) \) to the future. \( \mathcal{L} \) denotes the so-called running cost, describing the cost incurred during an infinitesimal period \([\tau, \tau + d\tau]\), which is additive over time.
The parameter $\eta > 0$ with a unit of $s^{-1}$ denotes the so-called discount factor (Fleming & Soner, 1993), which reflects some trade-off between cost incurred in the near term and future cost. Cost incurred in the future will be weighed less importantly by the controller. $\eta >> 0$ results in a short-sighted driving behaviour where the controller optimises the immediate situation and does not care too much about the future. Particularly, the cost after a future horizon $\frac{1}{\eta}$ decreases exponentially.

In a rolling horizon implementation where the state information is received at a discrete time instant $t_k$, the control input $u$ is usually discretised and the first sample $u^*(t_k)$ is implemented to update the system state. The control input will be re-assessed at regular time intervals $\Delta t = t_{k+1} - t_k$, using the most current estimates of the system state (at time $t_{k+1}$).

### 5.2.3 Analytical solution approach

This section discusses an analytical solution approach to the considered problem of Eqs. (5.3, 5.4) based on Pontryagin’s Minimum Principle (Fleming & Soner, 1993; Hoogendoorn & Bovy, 2003). Let us denote $W(t_k, x)$ as the so-called value function, which is the optimal cost function under optimal control $u^*$:

$$W(t_k, x) = J(t_k, x|u^*)$$  \hspace{1cm} (5.5)

Applying Bellman’s Principle of Optimality yields the Hamilton-Jacobi-Bellman (HJB) equation with discount factor (Fleming & Soner, 1993):

$$\eta W(x) = \mathcal{H} \left( x, u^*, \frac{\partial W(x)}{\partial x} \right)$$  \hspace{1cm} (5.6)

where $\mathcal{H}$ is the so-called Hamilton equation (Hamiltonian), which satisfies:

$$\mathcal{H} \left( x, u^*, \frac{\partial W(x)}{\partial x} \right) = L + \frac{\partial W(x)}{\partial x} \cdot f$$  \hspace{1cm} (5.7)

Let $\lambda = \frac{\partial W(x)}{\partial x}$ denote the so-called co-state or marginal cost of the state $x$, reflecting the relative extra cost of $W$ due to making a small change $\delta x$ on the state $x$. Pontryagin’s Principle (Fleming & Soner, 1993; Hoogendoorn & Bovy, 2003) requires that the co-state along the optimal trajectory satisfies the following equation:

$$\lambda = \frac{1}{\eta} \frac{\partial \mathcal{H}}{\partial x} = \frac{1}{\eta} \frac{\partial L}{\partial x} + \frac{1}{\eta} \frac{\partial (\lambda \cdot f)}{\partial x}$$  \hspace{1cm} (5.8)

Using the Hamiltonian of Eq. (5.7), we can derive the following necessary condition for the optimal control $u^*$:

$$\mathcal{H}(x, u^*, \lambda) \leq \mathcal{H}(x, u, \lambda), \ \forall u$$  \hspace{1cm} (5.9)
In nearly all cases, this requirement will enable expressing the optimal control \( u^* \) as a function of the state \( x \) and the co-state \( \lambda \).

For problems without constraints on control, the *stationarity condition* of \( \frac{\partial \mathcal{J}}{\partial u} = 0 \) gives the following optimal control law for ACC vehicle \( n \):

\[
\mathbf{u}^* = -\lambda_2 = -\frac{1}{\eta} \left( \frac{\partial \mathcal{L}}{\partial v_n} - \lambda_1 \right) \tag{5.10}
\]

where \( \lambda_2 \) denotes the co-state of the speed and \( \lambda_1 \) denotes the co-state of the gap, and is given by:

\[
\lambda_1 = \frac{1}{\eta} \frac{\partial \mathcal{L}}{\partial s_n} \tag{5.11}
\]

Eq. (5.10) states that the optimal acceleration of an ACC vehicle is negative to the *marginal cost of its speed*, determined by the gradient of the running cost function with respect to its speed and the *marginal cost of its gap*.

For the C-ACC controller, the change in the system state and dynamics results in the following optimal control law when applying the same solution approach:

\[
\mathbf{u}^* = -\lambda_2 = -\frac{1}{\eta} \left( \frac{\partial \mathcal{L}}{\partial v_n} - \lambda_1 + \lambda_3 \right) \tag{5.12}
\]

with \( \lambda_1 \) given in Eq. (5.11) and

\[
\lambda_3 = \frac{1}{\eta} \frac{\partial \mathcal{L}}{\partial s_{n+1}} \tag{5.13}
\]

Equation (5.12) shows that the optimal acceleration for a C-ACC vehicle is also negative to the marginal costs of its speed, determined by the gradient of the running cost function with respect to its speed, the *marginal cost of its gap* as well as the *marginal cost of its follower’s gap*. Clearly, the inclusion of the marginal cost of the follower’s gap in the optimal control law captures the *cooperative nature* of the C-ACC controller.

### 5.2.4 Derivation of ACC algorithm

In this part, we apply the aforementioned analytical solution approach to derive the algorithm for the ACC system.

**Cost specification and optimal acceleration**

As the ACC controller proposed in the previous chapter cannot guarantee collision-free at small gaps, cf. Section 4.6, a slightly different formulation of running cost function is chosen to remedy this. The weight factor on safety cost is formulated in such a way that the safety costs increase substantially when approaching the preceding vehicle at small gaps.
Chapter 5. Refined ACC and C-ACC controllers with analytical solution

Similar to the ACC controller in the previous chapter, the refined ACC controller distinguishes cruising (free driving) mode and following mode for the proposed ACC system. In cruising mode, ACC vehicles try to travel at a user defined free speed $v_0$. In following mode, ACC vehicles try to maintain a desired gap $s_d$ while at the same time avoiding driving too close to the predecessor. For the sake of notation simplicity, we drop the vehicle index $n$ in the ACC controller formulation.

Mathematically, the two-regime running cost function can be formulated as:

$$L_{\text{ACC}} = \begin{cases} 
  c_1 e^{\frac{s_0}{\Delta v}} \cdot \Theta(\Delta v) + c_2 (s_d(v) - s)^2 + \frac{1}{2}u^2 & \text{if } s \leq s_f \\
  c_3 (v_0 - v)^2 + \frac{1}{2}u^2 & \text{if } s > s_f 
\end{cases}$$

(5.14)

where $s_f$ is the gap threshold to distinguish cruising mode ($s > s_f$) from following mode ($s \leq s_f$) and is calculated with $s_f = v_0 \cdot t_d + s_0$, where $v_0$ is the free speed and $s_0$ is the distance between two vehicles at completely congested (standstill) conditions. $t_d$ denotes the user-defined desired time gap. $s_d$ is the so-called desired gap in following mode and is determined by:

$$s_d(v) = vt_d + s_0$$

(5.15)

$\Theta$ is a Heaviside function which follows the form:

$$\Theta(\Delta v) = \begin{cases} 
1 & \text{if } \Delta v \leq 0 \\
0 & \text{if } \Delta v > 0 
\end{cases}$$

(5.16)

Equation (5.14) implies that the controller makes some trade-off among the safety cost, efficiency cost and comfort cost when following a preceding vehicle:

- The safety cost only incurs when approaching the preceding vehicle, i.e. $\Delta v < 0$; $c_1 > 0$ is a constant weight factor. The exponential term $e^{s_0/\Delta v}$ of the safety cost ensures a large penalty when driving too close to the predecessor, i.e. $s \leq s_0$. The safety cost is a monotonic decreasing function of gap $s$, reflecting the fact that the sensitivity to the relative speed tends to decrease with the increase of following distance. There is no safety cost in cruising mode.

- The efficiency cost term in following mode incurs deviating from the desired gap; $c_2 > 0$ is a constant weight factor. The user-set desired time gap $t_d$ reflects driver preference and driving style, i.e. a smaller $t_d$ tends to an aggressive driving style, while a larger one implies more timid driving behaviour. This cost also stems from the interaction with the predecessor, and thus vanishes in the cruising mode.

- The travel efficiency cost in cruising mode stems from not driving at free speed $v_0$, with a constant weight $c_3 > 0$. 


• The comfort cost is represented by penalising accelerating or decelerating behaviour.

Notice that in following mode, Eq. (5.14) is continuous and differential to second order in \( s \) and \( v \), and differential to first-order in \( \Delta v \). Thus, the first-order gradients of \( L \) are well defined.

**Optimal acceleration of ACC vehicles**

Employing the solution of Eq. (5.10) arrives at the following optimal control law:

\[
u^*_{\text{ACC}} = \begin{cases} 
\frac{2c_1 e^{s_0}}{\eta} \left( \Delta v \cdot \Theta(\Delta v) - \frac{sv_0 \Delta^2 \Theta(\Delta v)}{2\eta v^2} \right) + \frac{2c_2 (1+\eta d)}{\eta} (s-s_d(v)) & \text{if } s \leq s_f \\
\frac{2c_3}{\eta} (v_0 - v) & \text{if } s > s_f
\end{cases}
\]  

Equation (5.17) shows that the optimal acceleration is a non-linear function of the state \( x = (s,v)^T \) and the relative speed \( \Delta v \). The first term in following mode (when \( s \leq s_f \)) describes the tendency to decelerate when approaching the predecessor, while the second term describes the tendency to accelerate when the current gap is larger than the desired gap and the tendency to decelerate when vice versa. In cruising mode ACC vehicles adjust their speed towards the free speed \( v_0 \) to minimise the efficiency cost, with an acceleration proportional to the speed difference with respect to the free speed. For details of the derivation, we refer the reader to Appendix A.

In reality, accelerations of vehicles are usually limited by the power train, i.e. \( u \leq 2.5 m/s^2 \). For the optimal acceleration function (5.17), it achieves its maximum \( u_{\text{max,f}} \) in following mode when \( s = s_f, v = 0 km/h, \) and \( \Delta v \geq 0 km/h \) and achieves its maximum \( u_{\text{max,c}} \) in cruising mode when \( v = 0 km/h \) for \( s > s_f \) and all \( \Delta v \):

\[
u_{\text{max,f}} = u^*_{\text{ACC}}(s_f, \Delta v, 0) = \frac{2c_2 v_0 t_d (1+\eta d)}{\eta^2}, \text{ for } \Delta v \geq 0
\]

and

\[
u_{\text{max,c}} = u^*_{\text{ACC}}(s, \Delta v, 0) = \frac{2c_3 v_0}{\eta}, \text{ for } s > s_f
\]

To smooth the transition from following mode to cruising mode, we let \( u_{\text{max,f}} = u_{\text{max,c}} \), which leads to the following relationship between the two cost weights of \( c_2 \) and \( c_3 \):

\[
c_3 = \frac{c_2 t_d (1+\eta d)}{\eta}
\]

In doing so, the total number of parameters in the model has been reduced. The default parameters of the model are shown in Table 5.1. The default parameters are chosen to guarantee local stability and produce similar string stability/instability types as human drivers, as we will show in the ensuing.
Table 5.1: ACC/C-ACC controller parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Physical meaning</th>
<th>Default value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>$v_0$</td>
<td>free speed</td>
<td>33.3 (120)</td>
<td>m/s (km/h)</td>
</tr>
<tr>
<td>$c_1$</td>
<td>weight on safety cost</td>
<td>0.2</td>
<td>s$^{-2}$</td>
</tr>
<tr>
<td>$c_2$</td>
<td>weight on efficiency cost</td>
<td>0.002</td>
<td>s$^{-4}$</td>
</tr>
<tr>
<td>$\eta$</td>
<td>discount factor</td>
<td>0.25</td>
<td>s$^{-1}$</td>
</tr>
<tr>
<td>$t_d$</td>
<td>desired time gap</td>
<td>1.0</td>
<td>s</td>
</tr>
<tr>
<td>$s_0$</td>
<td>desired gap at standstill</td>
<td>1</td>
<td>m</td>
</tr>
<tr>
<td>$l$</td>
<td>vehicle length</td>
<td>5</td>
<td>m</td>
</tr>
</tbody>
</table>

5.2.5 Derivation of C-ACC algorithm

As a second example, we apply the control framework to design Cooperative-ACC (C-ACC) systems where the controlled vehicle does not only consider its own situation but also the situation of its follower when making control decisions. The cooperation mechanism is applied when a C-ACC vehicle is followed by another C-ACC vehicle. The two vehicles exchange their gaps and speeds via V2V communications and the considered vehicle, the leader of the two C-ACC vehicles, determines its accelerations to minimise a joint cost function, reflecting the situations of both C-ACC vehicles.

Joint running cost function for C-ACC

The cooperative behaviour entails minimising a joint cost. Since there is no interaction in cruising mode, we assume that the cooperative behaviour only occurs when both the controlled vehicle and its follower are operating in following mode. Thus, we only change the running cost at following mode, which becomes:

$$L_{\text{C-ACC}} = \underbrace{c_1 e^{\frac{s_0}{2}} \Delta v_n^2 \cdot \Theta(\Delta v_n) + c_2 (s_n - s_d(v_n))^2 + \frac{1}{2} u_n^2 + \text{forward cost} (L_{\text{ACC}})}_{\text{forward cost}} + \underbrace{c_4 e^{\frac{s_0}{2}} \Delta v_{n+1}^2 \cdot \Theta(\Delta v_{n+1}) + c_5 (s_{n+1} - s_d(v_{n+1}))^2 + c_6 u_{n+1}^2 + \text{backward cost}}_{\text{backward cost}}$$

(5.21)

The running cost function (5.21) shows that in following mode, the cooperative controller aims to optimise both the situation in front (forward cost) and the situation behind (backward cost). The joint costs stem from safety costs due to approaching their direct predecessors, efficiency costs due to not driving at desired gaps and comfort costs of the C-ACC vehicle and its follower.

c_4$, $c_5$, and $c_6$ are weight factors related to the backward situation, which are selected to reflect the anisotropic nature of the C-ACC controller, i.e. the C-ACC controller
weeds the forward situation more than the backward situation. Thus $0 \leq c_4 \leq c_1$, $0 \leq c_5 \leq c_2$, and $0 \leq c_6 \leq \frac{1}{2}$.

**Optimal control of C-ACC vehicles**

Applying the analytical solution approach yields the following optimal acceleration control algorithm of the C-ACC vehicle:

$$u_{\text{C-ACC}}^* = \frac{2c_1 e^\frac{\Delta v_n}{\eta}}{\eta} \left( \Delta v_n \Theta(\Delta v_n) - \frac{s_0 \Delta v_n^2 \Theta(\Delta v_n)}{2 \eta s_n^2} \right) + \frac{2c_2 (1 + \eta t_d)}{\eta} (s_n - s_d(v_n))$$

$$- \frac{2c_4 e^\frac{\Delta v_{n+1}}{\eta}}{\eta} \left( \Delta v_{n+1} \Theta(\Delta v_{n+1}) - \frac{s_0 \Delta v_{n+1}^2 \Theta(\Delta v_{n+1})}{2 \eta s_{n+1}^2} \right)$$

$$- \frac{2c_5 (1 - \eta t_d)}{\eta^2} (s_{n+1} - s_d(v_n))$$

(5.22)

Notice that $\Delta v_{n+1} = v_n - v_{n+1}$, Eq. (5.22) shows that the optimal acceleration of a C-ACC vehicle $n$ is a *nonlinear* function of the relative speed with respect to its predecessor $\Delta v_n$, and the state vector $x = (s_n, v_n, s_{n+1}, v_{n+1})^T$, including the gaps and speeds of both itself and its follower.

The first two terms in Eq. (5.22) correspond to the ACC algorithm in Eq. (5.17). By the virtue of the Heaviside function $\Theta$, the third term is always non-negative. It implies that the C-ACC vehicle tends to accelerate when its follower is approaching itself, which can be interpreted as the *cooperative braking* behaviour. The fourth term implies that the C-ACC vehicle changes its acceleration to adjust the gap towards the desired gap. Particularly, when $1 - \eta t_d > 0$, the C-ACC vehicle tends to accelerate to make room for its follower when the follower’s gap is below the desired gap, and to decelerate to close the gap behind it when its follower’s gap is larger than the desired value. In doing so, the joint cost function (5.21) is optimised. The backward-looking behaviour in the third and fourth term shows how the *backward situation* affects the optimal control.

**5.3 Analytical approach for assessing controller characteristics**

The control algorithms of Eqs. (5.17, 5.22) for the proposed ACC and C-ACC systems are *non-linear state-feedback* algorithms, which are explicit functions of the system
state. Mathematically, the algorithms are equivalent to car-following models without explicit delay. This property allows one to perform analytical analyses to the controller characteristics without extensive simulation work, cf. Section 2.4. In this section, we present an analytical framework for assessing controller characteristics, with a focus on equilibrium flow-density relations and (local and string) stability. The analytical framework turns out to be very useful in testing how the proposed controllers perform under disturbances and evaluating their impact on road capacity and traffic flow stability.

The analytical framework is based on recent advances in traffic flow theory, particularly on linear stability analysis of car-following models (Holland, 1998; Treiber & Kesting, 2011; Wilson & Ward, 2011). We extend the linear stability analysis approach to include the cooperative behaviour of the C-ACC algorithm. To this end, we consider a generalised expression for the acceleration function, with gap, relative speed, and speed of vehicle $n$ and the gap and speed of its follower, vehicle $n + 1$, as independent variables. Thus the model or algorithm is represented by the following ordinary differential equation without explicit delay:

$$\frac{dv_n}{dt} = u_n(s_n, \Delta v_n, v_n, s_{n+1}, v_{n+1}). \quad (5.23)$$

The analytical approach is proposed to analyse the stability properties of homogeneous traffic where the traffic is composed of homogeneous vehicles on a homogeneous single-lane road. Discontinuity in road infrastructure and lane-changing behaviour are not taken into account explicitly. Rather, we consider an arbitrary initial disturbance (cf. Section 5.3.3) in equilibrium traffic, which may be caused by road inhomogeneity or lane-changing behaviour and examine how the disturbance evolves in the homogeneous vehicular flow. Although heterogeneities prevail in real-world traffic, the stability analysis on homogeneous traffic is important to characterise and to understand the influencing factors of controller stability and the resultant traffic flow stability (Treiber & Kesting, 2013), particularly the effects of the control parameters.

The remainder of this section is organised as follows: first, we introduce several stability concepts which are essential for the analytical framework. Then, the method for deriving equilibrium flow-density relations is presented, followed by the linear stability analysis approach.

### 5.3.1 Relevant definitions on stability

Before presenting the analytical method for stability analysis, we need to clarify some relevant concepts related to stability property of car-following models, including linear and non-linear stability, local and string stability, and convective and absolute instability (Treiber & Kesting, 2011; Wilson & Ward, 2011).
Linear and non-linear stability

Stability or instability of a dynamical system is always analysed around equilibrium conditions. If a deviation (initial disturbance) from the equilibria is damped out or bounded with the course of time, the system is stable at the equilibria. Otherwise, if the disturbance is amplified (unstable) for the studied system, the system is unstable.

Based on the amplitude of the initial disturbance, linear stability/instability is distinguished to non-linear stability/instability. If a (traffic) system is linearly unstable, any arbitrarily small disturbance increases with the course of time (Treiber & Kesting, 2013). Even if a (traffic) system is linearly stable, it can still display non-linear instability, i.e. small perturbations decay but large ones grow to persistent traffic waves with the course of time. Mathematically, dynamics of small perturbations can be approximately characterised by linear differential equations around equilibria using Taylor series for the linear stability analysis, as we will show in the ensuing, while this is not the case for large perturbations in non-linear stability analysis. For a more rigorous and detailed definition of linear and non-linear stability, we refer the readers to Treiber & Kesting (2013).

Linear stability analysis of car-following models is well developed and more analytically tractable compared to non-linear stability analysis. Analytical studies on linear stability of car-following models have been reported for more than five decades (Herman et al., 1959; Holland, 1998; Treiber & Kesting, 2011; Wilson & Ward, 2011), with new advances providing theoretical and empirical evidence of linear instability of traffic flow (Treiber & Kesting, 2011). Analytical methods for non-linear stability of traffic flow have been scarcely studied (Helbing & Moussaid, 2009), but strong and unrealistic assumptions are made to ensure analytical tractability. A common way to study the non-linear stability property is using simulation (Treiber & Kesting, 2013).

Note that the widely-used linear and non-linear stability analysis of car-following models in traffic flow theory are different from stability of linear systems and non-linear systems. The linear and nonlinear stability is distinguished by the amplitude of the perturbation and small perturbations allows the linearisation of the system around equilibrium points (Herman et al., 1959; Holland, 1998; Treiber & Kesting, 2011; Wilson & Ward, 2011). The linearity nature of car-following models describing the system dynamics is defined by the mathematical property of the car-following equations. Alternative approaches to treat the stability criterion of car-following models or the leader-follower systems can be found in mathematical control theory, such as Lyapunov’s first stability approach (Sontag, 1998).

Local and string stability

Local stability refers to the stability property of a single vehicle or the follower in a leader-follower pair, where an initial perturbation is typically introduced by a temporary drop in the leader speed profile (Treiber & Kesting, 2013). The model is locally
stable if the follower settles down at the reduced speed with the course of time under the decelerating disturbance caused by the leader, as depicted in Figure 5.1(a). Otherwise, the system is locally unstable, as shown in Figure 5.1(b). Local stability is a microscopic property of traffic flow models. For plausible car-following models, local stability is a necessary feature (Treiber & Kesting, 2011; Wilson & Ward, 2011).

Figure 5.1: Illustration of local and string stability and instability (reproduced after Pueboobpaphan & van Arem (2010)).

String stability involves more vehicles compared to local stability. The traffic system is string stable if the initial disturbance decays for all the vehicles in the platoon after sufficient time, as shown in Figure 5.1(c). String stability is a more restrictive concept compared to local stability, i.e. even if every vehicle exhibits locally stable behaviour, the collective traffic system can still display string instability, as shown in Figure 5.1(d). The string stability property pertains to both microscopic and macroscopic level.

Notice that in literature, some authors distinguished flow stability to string stability (Darbha & Rajagopal, 1999; Tampère, 2004), of which different interpretations exist and a rigorous definition is lacking (Pueboobpaphan & van Arem, 2010; Treiber & Kesting, 2013). Tampère (2004) explained that to generate flow instability, traffic flow must contain enough long platoons combined with short inter-platoon gaps, so that instability within one platoon is transferred to the next platoon, with the perturbation growing in amplitude. Hence, string instability is a necessary condition for flow instability. In this study, we are interested in the necessary conditions of flow instability, i.e. under which condition/criterion, the string instability occurs.

It is worth mentioning that there are different definitions of string stability in literature (Ploeg et al., 2014). While acknowledging that our definition of string stability is
based on the controller/model performance, the string stability/instability concept we introduced here has been proved to be quite relevant for macroscopic traffic flow phenomena such as stop-and-go waves Wilson (2008); Treiber & Kesting (2011). Hence, we use this definition in the remainder of the thesis.

**Convective and absolute string instability**

If traffic flow is string unstable, the string instabilities can be further classified into convective upstream or downstream instability and absolute instability. If the initial perturbation propagates both in the vehicle travelling direction and opposite the travelling direction with the course of time, the instability is of *absolute string instability*, as illustrated in Figure 5.2(b). If the initial perturbation propagates only in one direction, the traffic flow is of convective string instability. Based on the propagation direction, we distinguish between *convective upstream instability* (as illustrated in 5.2(a)) where the perturbation propagates only opposite the travelling direction and *convective downstream instability* where the perturbation propagates only in the vehicle travelling direction. Empirical study shows that the majority of instabilities of human-driven vehicular flow is of convective upstream type (Treiber & Kesting, 2011).

![Figure 5.2: Illustration of (a) convective upstream instability and (b) absolute instability in the spatio-temporal (x-t) plane, with vehicles travelling in the direction of increasing x. (Reproduced after Treiber & Kesting (2011)).](image)

Since stability is always evaluated at equilibrium conditions, we first present the method to derive the equilibrium conditions of traffic flow in the ensuing.

**5.3.2 Equilibrium flow-density relation**

At equilibria in homogeneous traffic, all vehicles travel at the same speed with the same gap and zero acceleration. Hence, the acceleration function (5.23) relaxes to the following equation:

\[
    u_p(s_e, 0, v_e, s_e, v_e) = 0
\]  

(5.24)
The solution to Eq. (5.24), if it can be solved, gives a unique equilibrium speed as a function of gap $v_e(s_e)$, or an equilibrium gap as a function of speed $s_e(v_e)$. Thanks to the homogeneous traffic composition, the equilibrium density $\rho_e$ can be calculated with

$$\rho_e = \frac{1}{s_e + l}$$

where $l$ denotes the vehicle length. Using the flow density relation of $q_e = \rho_e \cdot v_e(s_e) = \rho_e \cdot v_e(\rho_e)$, one gets the equilibrium flow-density relation. The capacity can be derived as:

$$q_c = \max q_e(\rho_e)$$

and the critical density corresponding to the capacity given by the following argument:

$$\rho_c = \arg\max \rho_e q_e(\rho_e)$$

### 5.3.3 Linear stability analysis

Let us assume a small deviation $h_n$ and $g_n$ of the $n$th vehicle in the homogeneous platoon from the steady-state gap $s_e$ and speed $v_e$ respectively, then the gap and speed of vehicle $n$ can be written as:

$$s_n = s_e + h_n, \quad v_n = v_e + g_n$$

The first and second order derivatives of $h_n$ are given by:

$$\dot{h}_n = \Delta v_n = g_{n-1} - g_n, \quad \ddot{h}_n = u_n - u_n$$

Approximating $u_{n-1}$ and $u_n$ in Eq. (5.28) around equilibria using Taylor series to the first order arrives at:

$$\ddot{h}_n = u_s(h_{n-1} - h_n) + u_{\Delta v}(h_{n-1} - h_n) + u_v h_n + u_{s_b}(h_n - h_{n+1}) + u_{v b} h_{n+1}$$

with the coefficients (gradients of acceleration) evaluated at equilibria:

$$u_s = \frac{\partial u_n}{\partial s_n} |_{e}, \quad u_{\Delta v} = \frac{\partial u_n}{\partial \Delta v_n} |_{e}, \quad u_v = \frac{\partial u_n}{\partial v_n} |_{e}, \quad u_{s_b} = \frac{\partial u_n}{\partial s_{n+1}} |_{e}, \quad u_{v b} = \frac{\partial u_n}{\partial v_{n+1}} |_{e}$$

The equilibrium solutions $v_e(s_e)$ restrict the coefficients from being independent from each other. The acceleration and relative speed along the equilibrium solutions should always be zero. This property leads to the following relationship by approximating acceleration around equilibria with Taylor expansion to the first order:

$$(u_s + u_{s_b}) = -v'_e(s_e) \cdot (u_v + u_{v b})$$

where $v'_e(s_e) = \frac{dv_e}{ds_e}$ is the gradient of the equilibrium speed function, reflecting the sensitivity of equilibrium speed to the change of gap.
Local stability criterion

For local stability property, we are primarily interested in the response of the following vehicle, while assuming the leader is driving constantly after the initial disturbance. In this case, Eq. (5.29) will relax to:

\[
\ddot{h}_n + (u_{\Delta v} - u_v)\dot{h}_n + u_s h_n = 0 \quad (5.31)
\]

Equation (5.31) is a harmonic damped oscillator which can be solved using the following ansatz (Treiber & Kesting, 2011):

\[
h = h_0 e^{\gamma t} \quad (5.32)
\]

where \(\gamma = \sigma + i\omega\) \((i = \sqrt{-1})\) is the complex growth rate and \(h_0\) reflects the amplitude of the initial disturbance. We can reformulate the damped oscillator as:

\[
\gamma^2 + (u_{\Delta v} - u_v)\gamma + u_s = 0 \quad (5.33)
\]

with two solutions of the following form:

\[
\gamma_{\pm} = \frac{-(u_{\Delta v} - u_v) \pm \sqrt{(u_{\Delta v} - u_v)^2 - 4u_s}}{2} \quad (5.34)
\]

Local stability requires that the real parts of both solutions of Eq. (5.33), \(\gamma_+\) and \(\gamma_-\), to be negative (Wilson, 2008; Treiber & Kesting, 2011), which is satisfied by the following criterion:

\[
u_{\Delta v} - u_v > 0 \quad (5.35)
\]

String stability criterion

For string stability, we are interested in how a small disturbance propagates through the increasing index of vehicles. We state the following proposition for string stability of generalised car-following models in the form of (5.23).

**Proposition 1** If \(u_v + u_{v_b} < 0\), string stability is guaranteed by the inequality:

\[
v'_{e}(s_e)^2 \leq v'_{e}(s_e)(u_{\Delta v} - u_{v_b}) + \frac{u_s - u_{s_b}}{2} \quad (5.36)
\]

**Proof** The generalised disturbance dynamic equation of (5.29) can be solved using Fourier analysis with the following ansatz:

\[
h_n = h_0 e^{\gamma t + i\omega k} \quad , \quad g_n = g_0 e^{\gamma t + i\omega k} \quad (5.37)
\]

where \(\gamma = \sigma + i\omega\) \((i = \sqrt{-1})\) is the complex growth rate. The real part \(\sigma\) denotes the growth rate of the oscillation amplitude while the imaginary part \(\omega\) is the angular frequency from the perspective of the vehicle. The dimensionless wave number \(k \in (-\pi, \pi)\) indicates the phase shift of the traffic waves from one vehicle to the next at a
given time instant, and the corresponding physical wavelength is \(2\pi(s_e + l)/k\) (Treiber & Kesting, 2011).

To find the limit for string instability, we insert Eq. (5.37) into Eq. (5.29), which yields the following quadratic equation of the eigenvalue \(\gamma\):

\[
\gamma^2 + p(k)\gamma + q(k) = 0
\]  

(5.38)

The solutions of Eq. (5.38) are given by:

\[
\gamma_{\pm}(k) = -\frac{p(k)}{2} \pm \frac{\sqrt{p^2(k) - 4q(k)}}{2}
\]  

(5.39)

with coefficients:

\[
p(k) = u_{\Delta v}(1 - e^{-ik}) - u_v - u_{vb}e^{ik}, \quad q(k) = u_s(1 - e^{-ik}) + u_{sb}(e^{ik} - 1)
\]  

(5.40)

For a given wave number \(k\), only two complex growth rates \(\gamma_+\) and \(\gamma_-\) are possible and \(\text{Re}(\gamma_+) \geq \text{Re}(\gamma_-)\). The model is string stable if \(\text{Re}(\gamma) < 0\) for both solutions and for all wave numbers (relative phase shifts) in the range \(k \in [-\pi, \pi]\).

It can be proven that the first instability of time-continuous car-following models without explicit delay always occurs for wave number \(k \to 0\) (Wilson, 2008). Thus we can expand coefficients of the \(p(k)\) and \(q(k)\) with Taylor series around \(k = 0\):

\[
p(k) = p_0 + p_1 k + O(k^2), \quad q(k) = q_1 k + q_2 k^2 + O(k^3)
\]  

(5.41)

with coefficients:

\[
p_0 = p(0) = -u_v - u_{vb}, \quad p_1 = p'(0) = i(u_{\Delta v} - u_{vb})
\]

\[
q_1 = q'(0) = i(u_s + u_{sb}) = iv'(s_e)p_0, \quad q_2 = \frac{q''(0)}{2} = \frac{u_s - u_{sb}}{2}
\]  

(5.42)

Expanding root \(\gamma_+\) around \(k = 0\) to second order of \(k\) and using the Taylor series of square root of \(\sqrt{1 - \varepsilon} = 1 - \varepsilon/2 - \varepsilon^2/8 + O(\varepsilon^3)\) gives:

\[
\gamma_+ = -\frac{q_1}{p_0} k + \left(\frac{q_1 p_1}{p_0^2} - \frac{q_2}{p_0} - \frac{q_1^2}{p_0^3}\right) k^2 + O(k^3)
\]  

(5.43)

Notice that the first term in Eq. (5.43) is purely imaginary and the second term is a real number. String stability is governed by the sign of the second term. For string stability, it is required that:

\[
\frac{q_1 p_1}{p_0^2} - \frac{q_2}{p_0} - \frac{q_1^2}{p_0^3} \geq 0
\]  

(5.44)

If \(u_v + u_{vb} < 0\), which implies \(p_0 > 0\), moving the last term in the inequality to the right side and multiply \(p_0\) will give:

\[
\frac{q_1^2}{p_0^2} \leq \frac{q_1 p_1}{p_0} - q_2
\]  

(5.45)
Replacing the coefficients with Eqs. (5.42) in the inequality (5.44) and divide by $p_0^2$ will give:

$$v'_e(s_e)^2 \leq v'_e(s_e)(u_{\Delta v} - u_{vb}) + \frac{u_s - u_{sb}}{2}$$

(5.46)

which is the string stability criterion.

Q.E.D.

For ACC systems or car-following models that only react to the direct predecessor, the string stability criterion relaxes to:

$$v'_e(s_e)^2 \leq v'_e(s_e)u_{\Delta v} + \frac{u_s}{2}$$

(5.47)

When comparing Eq. (5.36) with Eq. (5.47), we can obtain the following analytical criterion for stabilisation effects of cooperative systems. If a cooperative system keeps the equilibrium speed-gap relationship and the gradients of acceleration $u_s$, $u_{\Delta v}$ and $u_v$ the same as a non-cooperative system, the stabilisation effect of the cooperative behaviour compared to the non-cooperative model, is determined with:

$$-u_{vb}v'_e(s_e) - \frac{u_{sb}}{2} > 0, \text{ cooperative system is more stable};$$

$$-u_{vb}v'_e(s_e) - \frac{u_{sb}}{2} = 0, \text{ string stability property remains unchanged};$$

$$-u_{vb}v'_e(s_e) - \frac{u_{sb}}{2} < 0, \text{ cooperative system is more unstable}. \quad (5.48)$$

**Convective and absolute instability limits**

Several studies have revealed that the instability in traffic flow is of a convective type (Wilson & Ward, 2011; Treiber & Kesting, 2011). To find the limits for convective and absolute instability, we need to state a more rigorous definition of the perturbation $Z$.

Let $Z_e(x,0)$ denote the initial perturbation to an equilibrium traffic flow at the local level around $x = 0$:

$$Z_e(x,0) = \begin{cases} 
\epsilon \text{ if } |x| < \frac{1}{2\rho_e} \\
0 \text{ otherwise}
\end{cases}$$

(5.49)

where $\epsilon > 0$ is the arbitrarily small perturbation in front of a vehicle at the local region, i.e. the space of one vehicle occupies at equilibrium conditions $|x| < \frac{1}{2\rho_e}$.

Let $Z(x,t)$ denote the spatio-temporal evolution of an initial perturbation $Z_e(x,0)$. Traffic flow is convectively unstable if it is linearly unstable and if

$$\lim_{t \to +\infty} Z(0,t) = 0$$

(5.50)

Intuitively, Eq. (5.50) means that the perturbation will eventually convect out of the system after a sufficient time (Wilson & Ward, 2011; Treiber & Kesting, 2011). Otherwise, if traffic flow is linearly unstable but does not satisfy Eq. (5.50), it is absolutely unstable.
To investigate the limits of convective instability, the Fourier transform of a linear response function is used to determine the spatio-temporal evolution of the perturbation $Z(x,t)$ (Treiber & Kesting, 2013). This approach involves finding the wave number corresponding to the maximum growth rate and expanding the complex growth rate around the wave number. For details of the mathematical procedure, we refer to the book of Treiber & Kesting (2013). Employing this approach allows one to obtain the spatio-temporal evolution of the perturbation as:

$$Z(x,t) = \text{Re} \frac{Z_0}{\sqrt{-2\pi \gamma''(k_0^p)t}} \exp \left[i(k_0^p x - \omega_0^p t)\right] \exp \left[\left(\sigma_0 + \frac{(c_g - \frac{s}{k} i)^2}{2(i\omega_{kk}^p - \sigma_{kk}^p)}\right)t\right]$$ (5.51)

where $k_0^p$ denotes the physical wave number with the maximum growth rate, and is determined by the dimensionless wave number $k_0$:

$$k_0^p = \frac{k_0}{s + l}, \quad k_0 = \text{arg max}_k (\text{Re} \gamma(k))$$ (5.52)

and

$$\sigma_0 = \text{Re} \gamma(k_0), \quad \omega_0 = \frac{v_e k_0}{s + l} + \text{Im} \gamma(k_0), \quad \sigma_{kk}^p = (s + l)^2 \text{Re} \gamma''(k_0), \quad \omega_{kk}^p = (s + l)^2 \text{Im} \gamma''(k_0)$$

$$c_g = v_e + (s + l) \text{Im} \gamma'(k_0), \quad c_p = \frac{\omega_0}{k_0^p} = v_e + (s + l) \frac{\text{Im} \gamma'(k_0)}{k_0}$$ (5.53)

In Eq. (5.53), $c_p$ denotes the phase velocity, which is defined by the movement of points of constant phase. It represents the propagation velocity of a single wave. For human-driven vehicular traffic, the phase velocity $c_p$ is of the order of $-15 \text{km/h}$ in congested traffic (Treiber & Kesting, 2013). $c_g$ is the group velocity, with which the overall shape of the wave amplitudes propagates through space (Lighthill, 1965). More intuitively, the middle of a wave group (or perturbation) propagates with group velocity (Treiber & Kesting, 2013). The group velocity can be influenced by several waves.

While group velocity represents the propagation of the centre of a wave group, signal velocity $c_s$ is more representative in describing the spatio-temporal dynamics of disturbance in dissipative media like vehicular traffic flow (Ward & Wilson, 2011). The signal velocity represents the propagation of waves that neither grow nor decay. It can be calculated using Eq. (5.51), by considering the growth rate of $Z(x,t)$ along the trajectory of $x = c_s t$ and setting it to be zero, which gives:

$$\sigma_0 = \text{Re} \gamma \left(\frac{(c_g - c_s)^2}{2 \gamma''}\right) = \sigma_0 - \frac{(c_g - c_s)^2}{2D_2}$$ (5.54)

where $D_2 = -\sigma_{kk}^p \left(1 + \frac{(\omega_{kk}^p)^2}{(\sigma_{kk}^p)^2}\right)$. If there is any string instability, we have two signal velocities:

$$c_s^\pm = v_g \pm \sqrt{2D_2 \sigma_0}$$ (5.55)

Equation (5.55) shows that the perturbed region grows spatially at the constant rate of $2\sqrt{2D_2 \sigma_0}$. Convective instability types can be classified as:
if $c_s^- < 0 < c_s^+$, traffic flow is absolutely string unstable.

- if $c_s^+ < 0$, traffic flow is upstream convectively unstable.

- if $c_s^- > 0$, traffic flow is downstream convectively unstable.

This completes the linear stability analysis framework for deriving equilibrium flow-density relations, local and string stability criteria, and convective and absolute instability criteria. In the ensuing, we will show the results of the assessment of controller performance at microscopic level and the collective traffic flow characteristics using the proposed analytical approach.

Notice that the existence of $\Delta v\Theta(v)$ in Eqs. (5.17, 5.22) implies a discontinuity in the first order derivative with respect to the speeds, while the analytical approach for stability analysis requires the acceleration function to be differential to the first order. Thus a smoothed approximation of the $\Delta v\Theta(\Delta v)$ is used. Details of the approximation are described in Appendix B.

5.4 Microscopic performance of the ACC controller

One nice property of the state-feedback optimal ACC control algorithm is that it allows us to examine the controller properties analytically. To verify whether the proposed ACC controller generates desired behaviour, we check the mathematical property of the acceleration function (5.17), which is equivalent to a car-following model without explicit delay.

Several authors have provided basic requirements for plausible car-following models (Treiber & Kesting, 2011; Wilson & Ward, 2011). Let $u(s, \Delta v, v)$ denote a general class of car-following models where the acceleration is a function of gap $s$, relative speed $\Delta v$ and speed $v$. The basic requirements for car-following models can be summarised with:

1. The acceleration is an increasing function of the gap to the predecessor and is not influenced by the gap when the predecessor is far in front:

$$\frac{\partial u(s, \Delta v, v)}{\partial s} \geq 0, \lim_{s \to \infty} \frac{\partial u(s, \Delta v, v)}{\partial s} = 0 \quad (5.56)$$

2. The acceleration is an increasing function of relative speed with respect to the preceding vehicle, and is not influenced by the relative speed at very large gaps:

$$\frac{\partial u(s, \Delta v, v)}{\partial \Delta v} \geq 0, \lim_{s \to \infty} \frac{\partial u(s, \Delta v, v)}{\partial \Delta v} = 0 \quad (5.57)$$
3. The acceleration is a strictly decreasing function of speed, and equals zero when vehicles travel with free speed at very large gaps

$$\frac{\partial u(s, \Delta v, v)}{\partial v} < 0, \lim_{s \to \infty} u(s, \Delta v, v_0) = 0$$

(5.58)

It can be shown that the proposed optimal ACC control law of Eq. (5.17) satisfies the three basic requirements. Thus, the ACC controller generates plausible human car-following behaviour.

Figure 5.3(a) shows the contour plot of the optimal acceleration for different gaps and relative speeds when following a predecessor driving constantly with a speed of 54 km/h using default parameters. Clearly we can see the two regimes of following mode and cruising mode distinguished at the gap of around 35 m. At cruising mode, the acceleration is above zero, because all the possible speeds (between 36 km/h and 72 km/h) in the contour plot are below the free speed of 120 km/h. In following mode, the acceleration increases with the increase of headway and relative speed, and consequently decreases with the increase of vehicle speed. The thick line between the green and yellow area shows the neutral line where the accelerations equal zero. Most of the left plane in following mode show a negative acceleration, as a result of the safety cost. This asymmetric property of the optimal acceleration prevents vehicles from driving too close to the leader. The two-regime formulation of ACC acceleration algorithms is a common practice in literature (VanderWerf et al., 2001; Van Arem et al., 2006; Shladover et al., 2012), and field test did not show user inconvenience on the mode transition (Alkim et al., 2007).

Figure 5.3(b) shows how the system evolves from a high cost area to a low cost area of an ACC vehicle following a predecessor driving constantly with a speed of 54 km/h. The initial state is $s = 15 m$ and $\Delta v = -14 km/h$ ($v = 68 km/h$), denoted with ‘O’ in the figure, using the default parameters. The contour lines show the cost, while the dark star line shows the trajectory of the vehicle, with the optimal acceleration evaluated every 0.25 s. At the start, the ACC controller has safety cost due to approaching the leader and travel efficiency cost due to driving higher than the desired speed of around 47 km/h. The vehicle starts to decelerate until the relative speed is 0 km/h. Then it continues to decelerate because driving at 54 km/h is still higher than the desired speed, which has changed to around 36 km/h (at the gap of 12 m). As a result, the vehicle will travel with a lower speed and the gap to the predecessor will increase, leading to an increase of the desired speed. The vehicle starts to accelerate when the desired speed is higher than the vehicle speed. The trade-off between the travel efficiency and safety cost will finally lead to the behaviour as shown in the figure, ending with ‘D’ with zero cost in the figure after a simulation period of 50 s.

The local stability property is examined using the analytical approach described in the previous section. It can be shown with Eq. (5.17) that in following mode $u^*_{\Delta v} > 0$ and $u^*_v < 0$, thus the local stability condition (5.35) is always satisfied. In cruising mode, the ACC controller does not react to the disturbance in the leader speed profile,
thus local stability is always guaranteed in cruising mode. This signifies that the ACC controller with algorithm of Eq. (5.17) is \textit{unconditionally locally stable}. Figure 5.4 shows the two roots of linear growth rate $\gamma_1$ and $\gamma_2$ calculated with solution (5.34). We can clearly see from the figure that the real parts of the two roots are below zero.

5.5 Macroscopic flow characteristics of ACC and C-ACC vehicles

Macroscopic flow characteristics of the ACC and C-ACC controllers are discussed in this section, with a focus on the equilibrium flow-density relations (fundamental diagram) and string stability. The analytical framework described in the previous section is used to examine the characteristics of the ACC and C-ACC algorithms.
Chapter 5. Refined ACC and C-ACC controllers with analytical solution

5.5.1 Fundamental diagram

For the ACC model of Eq. (5.17), following the equilibrium solutions in the previous section (when $\Delta v = 0$ and $a^* = 0$) gives a unique relationship between equilibrium speed and gap:

$$v_e = \begin{cases} \frac{s_e - s_0}{t_d} & \text{if } s_e \leq s_f \\ v_0 & \text{if } s_e > s_f \end{cases}$$

(5.59)

Assuming constant vehicle length $l$ and using the relationship between gap and local density $\rho$: $\frac{1000}{\rho} = s + l$ in veh/km, we will get the classic triangular fundamental diagram of the steady-state flow-density relationship as:

$$q = \begin{cases} 3.6v_0\rho & \text{if } \rho \leq \frac{1000}{v_0t_d + s_0 + l} \\ \frac{1000}{v_0t_d + s_0 + l} & \text{if } \rho > \frac{1000}{v_0t_d + s_0 + l} \end{cases}$$

(5.60)

with $q$ denoting traffic flow in the unit of veh/h and $\rho$ in the unit of veh/km.

Figure 5.5(a) shows the steady-state speed-gap relationship and Figure 5.5(b) depicts the equilibrium flow-density relation for two different desired time gaps. The two branches in each of the fundamental diagrams are distinguished by the operating mode of the ACC controller. On the left branch, ACC vehicles operate in cruising mode, while at the right branch ACC vehicles operate in following mode. With the default parameter $t_d = 1.0$ s, the resulting flow reaches the capacity of 3050 veh/h at a critical density of around 25 veh/km, while a desired time gap of 1.5 s leads to a capacity of 2142 veh/h at a critical density of around 18 veh/km. The critical density is determined by the gap threshold $s_f$. The figures show that the desired time gap has a strong influence on the capacity.

The equilibrium solutions of the C-ACC model with the same gap settings are the same as of the ACC model, and both of them display the fundamental diagram as Eq. (5.60) and Figure 5.5. With V2V communications, it is possible to increase the capacity.
by choosing smaller gaps for C-ACC systems, which has been discussed in literature (VanderWerf et al., 2001; Van Arem et al., 2006; Shladover et al., 2012).

### 5.5.2 String stability of the ACC controller

Since there is no interaction with other vehicles in the optimal acceleration at cruising mode, both local stability and string stability are guaranteed in cruising mode for both the ACC and the C-ACC controller. The string stability analysis in the ensuing focuses on following mode.

#### String stability threshold

To find the string stability threshold, we evaluate the gradients of $u^*$ in Eq. (5.17) at equilibria and the derivative of equilibrium speed in Eq. (5.59) as:

$$u^*_s = \frac{2c_2(1 + \eta t_d)}{\eta^2}, \quad u^*_{\Delta v} = \frac{c_1 e^{\frac{\eta_0}{\eta}}}{{\eta}}, \quad u^*_v = -\frac{2c_2(1 + \eta t_d)t_d}{\eta^2}, \quad v'(s_e) = \frac{1}{t_d}$$  

(5.61)

The stability condition (5.47) gives the following criteria to guarantee string stability:

$$\frac{c_1 t_d}{\eta} + \frac{c_2}{\eta^2} \left( \frac{1}{\eta^2} + \frac{t_d}{\eta} \right) \geq 1$$  

(5.62)

Equation (5.62) gives the following properties of model parameters on the string stability:

- Increasing safety cost weight $c_1$ will stabilise homogeneous flows. Microscopically, a larger $c_1$ leads to a higher sensitivity to the relative speed and thus a more anticipative driving style, since relative speed reflects future gaps, which is a simple form of anticipation (Treiber & Kesting, 2013).

- Increasing efficiency cost weight $c_2$ will stabilise homogeneous flows. A larger $c_2$ means that the controller has a higher sensitivity to the deviation from the desired speed. Notice that the maximum acceleration is proportional to $c_2$ in Eq. (5.18), a larger $c_2$ means a more responsive agile driving style, which tends to suppress string instabilities (Treiber & Kesting, 2013). However, physical constraints of vehicles limit the choice of too large $c_2$, i.e. increasing $c_1$ from default value from $0.002 s^{-2}$ to $0.004 s^{-2}$ with other default parameters already changes the maximum acceleration from $2.5 m/s^2$ to $5 m/s^2$.

- Increasing the discount factor $\eta$ will destabilise traffic. Notice that a larger $\eta$ implies a shorter anticipation horizon $\frac{1}{\eta^2}$ or in other words a more short-sighted driving style. A controller only optimising its immediate situation favours string instability.
Chapter 5. Refined ACC and C-ACC controllers with analytical solution

- Increasing the desired time gap $t_d$ will increase the left hand side of the inequality (5.62), which implies more stable flow. A larger $t_d$ tends to suppress string instability by following with a larger distance at equilibria. Hence the vehicles need less strong accelerations to adapt to the new equilibrium condition.

Figure 5.6 shows thresholds of stability and instability with different parameters in a two-dimensional parameter plane. The area above the line is string-stable under those parameter settings, while the area below the lines is string-unstable. The stabilisation effects of the parameters are clearly seen.

**Convective instability**

With Eq. (5.40), the coefficients of the quadratic equation for the complex growth rate $\gamma$ of the ACC model are specified:

$$p(k) = \frac{2c_1}{\eta} e^{\frac{s_0}{\eta}} (1 - e^{-ik}) + \frac{2c_2(1 + \eta t_d) t_d}{\eta^2} \quad q(k) = \frac{2c_2(1 + \eta t_d)}{\eta^2} e^{-ik} \quad (5.63)$$

The first and second order derivatives of $p(k)$ and $q(k)$ can be obtained straightforwardly.

The linear stability analysis framework enables one to plot the linear growth rate and the propagation velocities of a disturbance for the ACC model as a function of wave number under an equilibrium speed of $54 \text{ km/h}$, as depicted in Figure 5.7. Numerically, we can find the dimensionless wave number $k_0$ corresponding to the maximum growth rate with the argument in Eq. (5.52), which is 0.082 in this case. The physical wavelength is $\frac{(s_e + l)2\pi}{k_0} \approx 1.5km$ and the number of vehicles per wave is around $\frac{2\pi}{k_0} \approx 77$ vehicles. The maximum growth rate is $0.0028s^{-1}$ (the red point in Figure 5.7(a)), which is a slow growth implying that it may take some time for a small disturbance growing to traffic breakdown (Treiber & Kesting, 2013). The phase and group
velocity corresponding to this maximum growth rate are $-16\, km/h$ and $-11\, km/h$ respectively, with the negative sign indicating the propagation direction is against vehicle travelling direction, as depicted in Figure 5.7(b).

Figure 5.8(a) and 5.8(b) show the phase, group and signal velocities as a function of equilibrium speed and density respectively. Since traffic is always string stable in cruising mode, traffic flow is always stable below the critical density of $\rho_{c1} = 1000/(s_f + l) \approx 25\, veh/km$. As long as the density is higher than the critical density $\rho_{c1}$, traffic becomes absolutely unstable when $c_{s+} > 0$ and $c_{s-} < 0$, with disturbances travelling both upstream and downstream. When the density increases to another critical density $\rho_{c2} \approx 42\, veh/km$, the traffic becomes convectively upstream unstable, with disturbances travelling upstream only. When the density increases further to above another critical density $\rho_{c3} \approx 96\, veh/km$, the traffic becomes stable again, which is the so-called restabilisation effect (Treiber & Kesting, 2013). With the default parameters, the ACC model displays absolute and convective upstream instability, which is different from human drivers (Treiber & Kesting, 2013; Wilson & Ward, 2011).

Figure 5.8(c) and 5.8(d) show the spatio-temporal evolution of the system using the analytical disturbance function of Eq. (5.51) with different equilibrium speeds of $48\, km/h$ (density of $52\, veh/km$) and $72\, km/h$ (density of $38\, veh/km$). We can clearly see from the figure that:

- at the equilibrium speed of $48\, km/h$, the initial disturbance travels upstream, while at the equilibrium speed of $72\, km/h$, the disturbance travels both upstream and downstream.

- absolute instability grows faster in amplitude, which can be see from the ranges of the speed contour plots of Figures 5.8(c) and 5.8(d).

- the centre of the disturbance travels with group velocity and each signal wave travels with phase velocity.

- two signal velocities limit the region of disturbance in the spatio-temporal plane.

When choosing different parameters, one can get different stability characteristics of the model. Figure 5.9(a) shows the one dimensional parameter safety cost weight $c_1$ and the resulting stability at different equilibrium speeds at following mode with other default parameters. If we increase $c_1$ to a slightly higher value than the default one, traffic will become convectively upstream stable and stable in following mode, which is similar to human-driven vehicular traffic. When choosing $c_1$ higher than $0.22s^{-2}$, the traffic is always stable, while $c_1$ lower than $0.12s^{-2}$ leads to co-existence of convective downstream, absolute and convective upstream instability in the congested branch of the fundamental diagram. One can easily show the influence of other parameters using the same approach.
Chapter 5. Refined ACC and C-ACC controllers with analytical solution

Figure 5.7: (a) Growth rate of the more unstable branch $\gamma_+$ as a function of wave number under $v_e = 54 \text{ km/h}$; (b) phase and group velocity as a function of wave number under $v_e = 54 \text{ km/h}$ of the ACC algorithm with default parameters.

Figure 5.8: (a) Phase, group, signal velocities as a function of equilibrium speed and (b) phase, group, signal velocities as a function of equilibrium density and spatio-temporal evolution of initial disturbance at the equilibrium speeds of (c) 48 km/h and (d) 72 km/h of ACC model with default parameters. Driving direction in (c) and (d) is from top to bottom.
5.5.3 Stabilisation/destabilisation effect of cooperative systems

The local stability is no longer of interest for the C-ACC controller, since we will consider at least three vehicles for a C-ACC controller. For the optimal acceleration of the C-ACC controller in Eq. (5.22), the gradients are given:

\[
\begin{align*}
    u^*_v &= \frac{2c_2(1 + \eta t_d)}{\eta^2}, \quad u^*_{\Delta v} = \frac{c_1 e^{\frac{\gamma_0}{\eta}}}{\eta}, \quad u^*_v = -\frac{2t_d(c_2(1 + \eta t_d) - c_5(1 - \eta t_d))}{\eta^2} - \frac{c_4 e^{\frac{\gamma_0}{\eta}}}{\eta} \\
    u^*_{s_b} &= -\frac{2c_5(1 - \eta t_d)}{\eta^2}, \quad u^*_{v_b} = \frac{c_4 e^{\frac{\gamma_0}{\eta}}}{\eta}, \quad v'_e(s_e) = \frac{1}{t_d}
\end{align*}
\] (5.64)

Condition (5.36) gives the following criteria for string stability of C-ACC controllers:

\[
\frac{(c_1 - c_4)t_d e^{\frac{\gamma_0}{\eta}}}{\eta} + \frac{(c_2(1 + \eta t_d) + c_5(1 - \eta t_d))t_d^2}{\eta^2} \geq 1
\] (5.65)

The stabilisation effect of the C-ACC controller with reference to the ACC controller is governed by the condition (5.48), which reads:

\[
-v'_e(s_e)u^*_v - \frac{u^*_{s_b}}{2} = -\frac{c_4 e^{\frac{\gamma_0}{\eta}}}{\eta t_d} + \frac{c_5(1 - \eta t_d)}{\eta^2}
\] (5.66)

Condition (5.36) implies that increasing the value of Eq. (5.66) tends to stabilise traffic while decreasing the value destabilises traffic. With the virtue of the gradients in Eq. (5.64) and the analytical criteria for the stabilisation effect of cooperative systems (5.66), we find that:

- \(u^*_v > 0\), which destabilises traffic.
- If \(1 - \eta t_d > 0\), \(u^*_s < 0\), which stabilises traffic; otherwise, \(u^*_s\) destabilises traffic.
- Increasing the cost weight \(c_4\) destabilises traffic.
- Increasing the cost weight \(c_5\) stabilises traffic.

Equations (5.65, 5.66) provide guidance for parametrising cooperative ACC controllers to improve traffic stability.

To classify the convective instability, we need to specify the coefficients of the quadratic equation (5.38) as:

\[
p(k) = u^*_{\Delta v}(1 - e^{-ik}) - u^*_v - u^*_{v_b} e^{ik}, \quad q(k) = u^*_v(1 - e^{-ik}) + u^*_s b(e^{ik} - 1)
\] (5.67)

The first and second order derivatives of \(p(k)\) and \(q(k)\) can be obtained straightforwardly.

The linear stability analysis framework enables us to calculate signal velocity at different equilibrium speeds and different parameter settings. Figure 5.9(b) and 5.9(c) show
Chapter 5. Refined ACC and C-ACC controllers with analytical solution

Figure 5.9: Stability plot with safety cost weight $c_1$ and equilibrium speed of (a) ACC model with default parameters of $c_2 = 0.0019 \text{ s}^{-4}$, $v_0 = 33.3 \text{ m/s}$, $t_d = 1.0 \text{ s}$ and $s_0 = 1 \text{ m}$; (b) C-ACC model with $c_4 = 0$, $c_5 = 0.9 c_2$; (c) C-ACC model with $c_4 = c_1/2$, $c_5 = c_2/2$. S: Stable region; U: region with convective Upstream instability; A: region with Absolute instability; D: region with convective Downstream instability.

the resulting stability/instability types of one dimensional parameters with different settings of $c_4$ and $c_5$. By careful tuning, it is possible to design a C-ACC controller to achieve better string stability compared to the ACC controller, as shown in Figure 5.9(b). However, this should be done under a rigorous guidance. Otherwise, under certain parameter settings, the C-ACC controller may destabilise traffic, as shown in Figure 5.9(c).

5.6 Conclusions

We have applied the optimal control framework to controller design for both non-cooperative and cooperative systems. The supported driving process has been recast into a infinite horizon problem with discounted cost. One unique feature of the solution approach in this chapter is that closed-form control algorithms can be derived analytically, which relieves us from solving partial differential equations as in the dynamic
programming approach in the previous chapter.

The ACC controller is refined based on the result from the previous chapter. An explicit safety mechanism to prevent rear-end collisions has been formulated explicitly into the running cost function. The proposed ACC controller is shown to be unconditionally local-stable and generates plausible car-following behaviour. A cooperative ACC (C-ACC) controller is designed based on the optimisation of a joint cost consisting the situations of the cooperative vehicle and its direct follower.

Insights into the impact of ADAS and cooperative systems on flow operations are gained via linear stability analysis. Equilibrium solutions and string stability criteria are derived for ACC and C-ACC controllers. It is found that the ACC and C-ACC controllers lead to the classic triangular fundamental diagram, with capacity largely determined by the desired time gap. With small time gap settings, ACC and C-ACC systems can achieve higher capacity compared to human drivers. With careful choice of parameters, the ACC model only displays convective upstream instability at following mode, which is similar to human car-following models. Increasing safety cost weight, efficiency cost weight and desired time gap will stabilise traffic, while increasing the discount factor (decreasing the anticipation horizon) will destabilise traffic. The stabilisation and destabilisation effect of the C-ACC controller with reference to the ACC controller is analytically determined. By careful tuning, the C-ACC controller achieves better string stability compared to the ACC controller. The control framework and analytical results provide guidance in developing controllers for driver assistance systems to improve flow operations.

Although the refined ACC controller includes high penalties when approaching the preceding vehicle in small gaps, this mechanism may lead to very strong decelerations due to the limited spatial anticipation range of the constant time gap policy included in the efficiency cost. This will be addressed in the next chapter. Besides, although the proposed C-ACC control optimises a joint cost function consisting of the situation of both itself and its direct follower, the follower is assumed to be driving at equilibrium conditions and not influenced by the control decisions of the C-ACC vehicle. More intensive cooperation entails cooperative vehicles take into account the expected response of followers to the control decisions, which will be captured in the C-ACC controller in the next chapter.
Chapter 6

Flexible ACC and C-ACC controllers with fast numerical solution

In the previous chapter, we proposed an ACC controller that gets large penalties in safety-critical car-following conditions and a cooperative ACC (C-ACC) controller optimising a joint cost function consisting of the situations of both the C-ACC vehicle and its direct follower (cf. Section 5.2). While the ACC controller prevents rear-end collisions at small gaps, it may generate extremely hard braking when approaching slow preceding vehicles with high speeds. This problem is solved in this chapter by refining the controller formulation to increase the spatial anticipation range of the ACC controller, which results in a more generic formulation of ACC controller with variable time gaps. In addition, the cooperative control strategy proposed in the previous chapter is extended to take into account the expected responses of followers to the decisions of the controlled vehicle and to include the cooperation between intelligent vehicles and human-driven vehicles, resulting in two novel C-ACC controllers.

Optimal accelerations for the improved ACC and C-ACC controllers are determined by a new indirect solution approach, which can efficiently solve large scale systems with multiple dimensions (as opposed to the dynamic programming approach used in Chapter 4) and can deal with constraints on state and control variables (in contrast to the analytical solution approach used in Chapter 5).

The proposed controllers for cooperative systems are implemented to a platoon of 10 controlled vehicles following an uncontrolled leader with predefined behaviour, resulting in a system state space with cardinality of 20. Performance of the individual vehicles and the whole platoon under representative scenarios are assessed using microscopic simulation on a single lane road stretch under the ADAS performance assessment framework proposed in Chapter 3. Evaluation of the platoon performance provides insights into the potential impact of cooperative systems on traffic operations, particularly on capacity and stability.

This chapter is structured as follows. Section 6.1 presents the design assumptions and control objectives of ACC and C-ACC systems. Section 6.2 formulates the controllers
under different control strategies, followed by a description of the numerical solution algorithm based on Pontryagin’s Principle in Section 6.3. Section 6.4 shows the experimental design for performance assessment of the proposed controllers at individual and platoon levels. Section 6.5 describes the key results of the ACC controller performance at representative scenarios, while Section 6.6 assesses the performance of the C-ACC controllers in a large platoon in comparison to ACC controllers. The conclusions are summarised in Section 6.7.

Acknowledgement. Parts of the contents of this chapter are based on the following papers:


6.1 Design assumptions for ACC and C-ACC systems

In this section, we present the control concepts and controller design assumptions of ADAS using schematics and pictures. The assumptions on ACC systems are the same as in the previous chapter, where measurement errors of on-board sensors are not considered. An ACC vehicle detects the gap and speed difference with respect to the direct preceding vehicle solely based on its own on-board sensors, e.g. forward-looking radar, and does not communicate with other vehicles. The ACC vehicle predicts the evolution of the gap and relative speed with respect to its predecessor based on the current state of the system, assuming the predecessor is driving at equilibrium conditions, i.e. constant speed heuristics. The accelerations of the ACC vehicle are controlled to optimise the forward situation with respect to its predecessor. The control objectives of ACC systems include maximising safety, efficiency and driving comfort. Figures 6.1(a) and 6.1(b) show the non-cooperative ACC systems in two platoons with different vehicle compositions.

When cooperative vehicles form a platoon, cooperative ACC (C-ACC) vehicles exchange their gaps and speeds via V2V communications and make joint control decisions under a common objective, i.e. the cooperative control strategy, as shown in Figure 6.1(c) The common objective for the two neighbouring equipped vehicles is represented by a joint cost function, consisting of the safety, efficiency and comfort of both followers. The V2V communication is assumed to be perfect and communication delay is negligible compared to the sampling rate of C-ACC controllers. To avoid
abuse of terms, we refer to the C-ACC controller for cooperation of Homogeneous equipped vehicles with Perfect knowledge of each other as C-ACC-HP controller.

The cooperative control strategy is not restricted to cooperation between intelligent vehicles. It can be extended to the cooperation between equipped vehicles and human-driven vehicles, as illustrated in Figure 6.1(d). In car-following situations, human drivers determine their vehicle accelerations based on the behaviour of their predecessors. Hence, it is possible to indirectly control a human-driven vehicle when it follows an equipped vehicle. To achieve that, an additional backward-looking sensor is needed for the equipped vehicle to detect the gap and speed difference with respect to its human follower, i.e. *backward situation*. The equipped vehicle uses a model to predict the behaviour of the human follower, and thus can make decisions to optimise a joint cost function reflecting the situations of both. We further assume that the prediction of the human follower behaviour is not perfect. The C-ACC controller in a Mixed platoon with Imperfect knowledge of the human follower is referred to as C-ACC-MI controller hereinafter.

As opposed to the *constant speed heuristics* used by the C-ACC controller for predicting behaviour of surrounding vehicles in the previous chapter, the C-ACC-HP and C-ACC-MI controllers in this chapter do not assume other vehicles to travel at constant speeds. The expected behavioural changes of other vehicles as a reaction to the control decisions of the C-ACC-HP and C-ACC-MI controllers are taken into account in the state prediction and hence the accelerations of other vehicles are not necessarily zero, nor constant.

![Diagram of controllers](image)

Figure 6.1: Illustration of controllers: (a) two ACC follower with an exogenous leader; (b) ACC follower and a human-driven vehicle with an exogenous leader; (c) cooperative controller for cooperation with equipped vehicles with perfect knowledge of follower behaviour (C-ACC-HP) and (d) cooperative controller for cooperation of equipped vehicle and human-driven vehicle with imperfect knowledge of follower behaviour (C-ACC-MI).

Note that the multi-anticipative ACC (MACC) controller widely reported in literature (Van Arem et al., 2006; Shladover et al., 2012) can be formulated and implemented
using the framework and solution approach described in this chapter (Wang et al., 2014b). We exclude the formulation and discussion of the MACC controller to avoid lengthy chapters. Interested readers are referred to the work in Wang et al. (2014b).

6.2 Controller design formulation

In this section, we formulate the improved ACC, C-ACC-HP and C-ACC-MI controllers under the generic model predictive control framework proposed in Section 3.2.

6.2.1 Control problem formulation

The class of longitudinal driving control problems can be described by the ordinary differential equation:

$$\dot{x}(t) = f(x(t), u(t))$$

(6.1)

where \( x(t) \in \mathbb{R}^m \) and \( u(t) \in \mathbb{R}^n \) denote the vector of state and control input respectively (Pontryagin et al., 1962). The state and input variables are subject to constraints

$$x(t) \in X \text{ and } u(t) \in U, \forall t \geq 0$$

(6.2)

where \( X \in \mathbb{R}^m \) denotes the closed state constraint set and \( U \in \mathbb{R}^n \) denoting the closed admissible control region. For \( X \), it is clear that the gap \( s \) with respect to the preceding vehicle should be larger than zero and the vehicle speeds should be larger than zero and smaller than the maximum vehicle speed \( v_{\text{max}} \) limited by the vehicle type or road speed limits. The physical meaning of \( U \) is also clear, i.e. the accelerations of vehicles are bounded by the maximum deceleration \( u_{\text{min}} \) and the maximum acceleration \( u_{\text{max}} \) respectively, which are set according to vehicle characteristics and driving comfort.

The finite horizon optimal control problem at time \( t = 0 \) with initial state \( x(0) = x_0 \) can be formulated as:

$$\min_u J(x, u, t|x(0)) = \min_u \int_0^{T_p} L(x(\tau), u(\tau), \tau)d\tau + G(x(T_p), T_p)$$

(6.3)

subject to Eq. (6.1), constraints (6.2) and initial condition:

$$x(0) = x_0$$

(6.4)

In Eq. (6.3), \( J \) denotes the cost functional to be minimised. \( L \) denotes the so-called running costs, describing the costs incurred during an infinitesimal period \( [\tau, \tau + d\tau] \), which are assumed additive over time. The function \( G \) denotes the so-called terminal costs. It describes the costs remaining at the end of the prediction horizon \( T_p \).

Among other design choices, the terminal cost function influences the controller stability and performance, and is not trivial to specify (Chen & Allgöwer, 1998). In general,
the terminal cost function should be specified to ensure closed-loop stability when no terminal constraints on state variable prevail (Mayne et al., 2000). It has also been proved that when the prediction horizon is large enough, the closed-loop stability can be guaranteed without the need of a terminal cost (Rawlings & Muske, 1993). However, large prediction horizons lead to heavier computational load and may render the controller infeasible for online implementation. Our previous study has verified that when increasing the prediction horizon to 5 seconds or larger, the influence of the zero terminal cost on the ADAS performance is negligible in typical car-following situations (Wang et al., 2012a). Hence in this chapter, we set $G = 0$ and $T_p = 5 \text{ s}$ as a trade-off between closed-loop controller performance and computational load (Wang et al., 2012a).

In the ensuing, we will formulate the specific controllers under the generic control problem formulation.

### 6.2.2 ACC controller with variable time gap

As concluded in the previous chapter (cf. Section 5.6), the ACC controller with an explicit safety mechanism may generate very uncomfortable brakings under highly non-stationary conditions, e.g. approaching a standstill vehicle with free speed. This is due to the limited spatial anticipation range determined by the constant time gap, see Eqs. (4.9, 5.15). This section shows the refined formulation with a variable time gap policy which allows the ACC vehicle to avoid rear-end collisions with a constrained braking capacity.

#### State dynamics model

The system state from the perspective of an ACC vehicle $n$ is described by the gap (or distance) $s_n$ and relative speed $\Delta v_n = v_{n-1} - v_n$ to the preceding vehicle, $x = (s_n, \Delta v_n)^T$, where $v_n$ and $v_{n-1}$ denote the ACC vehicle speed and the preceding vehicle speed respectively. The controller assumes that the preceding vehicle is travelling at constant speed within the prediction horizon, i.e. its acceleration equals zero. Hence the system dynamics equation follows:

$$\frac{d}{dt}x = \begin{pmatrix} \Delta v_1 \\ -u_1 \end{pmatrix} = f(x, u)$$  \hspace{1cm} (6.5)

with $u = u_n$ denoting the control input, which is in this case the acceleration of the ACC vehicle $n$. 
Running cost specification

The running cost function is similar as in the previous chapter, and is specified as:

\[ L = \begin{cases} 
\frac{c_1}{s} \Delta v^2 \cdot \Theta(\Delta v) + c_2 (s_d - s)^2 + \frac{1}{2} u^2 & \text{if } s \leq s_f \\
1 & \text{if } s > s_f 
\end{cases} \]  \hspace{1cm} (6.6)

where \( s_f = v_0 t_{d,m} + s_0 \) is the gap threshold distinguishing cruising mode (\( s > s_f \)) from following mode (\( s \leq s_f \)). The gap threshold is determined by a user-defined maximum desired time gap \( t_{d,m} \) at following mode, the free speed \( v_0 \), and the minimum gap between vehicles at standstill conditions \( s_0 \).

Here \( s_d \) is the so-called desired gap, which is determined by the current speed and a desired time gap:

\[ s_d = v(t) \cdot t_d(s) + s_0 \]  \hspace{1cm} (6.7)

where \( t_d = t_d(s) \) denotes a gap-dependent desired time gap and is calculated with:

\[ t_d = t_{d,0} + \frac{s}{s_f} (t_{d,m} - t_{d,0}) \]  \hspace{1cm} (6.8)

with \( t_{d,0} \) denoting the minimum desired time gap at following mode. Equations (6.7, 6.8) show that the new ACC controller employs a variable time gap policy in following mode, i.e. it aims to keep larger time gaps with respect to the preceding vehicle at loose traffic conditions (or larger gaps) and smaller time gaps when vice versa. The variable time gap policy of ACC controller is more generic and flexible in addressing the safety and stability issues of ACC controllers (Wang & Rajamani, 2004; Zhang & Kim, 2005).

Note that when \( t_{d,0} = t_{d,m} \), our variable time gap formulation relaxes to the constant time gap policy.

Equation (6.6) implies that the controller makes a trade-off among the safety cost, efficiency cost and comfort cost when following a preceding vehicle:

- The safety cost only incurs when approaching the preceding vehicle in following mode, i.e. when \( \Delta v < 0 \) and \( s \leq s_f \), and vanishes in cruising mode. The safety cost is a monotonic decreasing function of gap \( s \), reflecting the fact that the sensitivity to the relative speed decreases with the increase of following distance. The term \( \frac{c_1}{s} \) ensures that the ACC vehicle will get a large penalty when driving too close to the predecessor, i.e. \( s \ll 1m \).

Note that \( \Theta \) is a Heaviside step function which follows the form:

\[ \Theta(x) = \begin{cases} 
1 & \text{if } x \leq 0 \\
1 & \text{if } x > 0 
\end{cases} \]  \hspace{1cm} (6.9)
Chapter 6. Flexible ACC and C-ACC controllers with fast numerical solution

where \( w \in [0, 1] \) is a dimensionless asymmetric acceleration factor for \( \forall x \in \mathcal{R} \), reflecting the asymmetric nature of the accelerating and decelerating behaviours of vehicles. When the ACC vehicle is approaching the predecessor, the safety cost weight is \( c_1 \), while the weight is reduced to \( c_1w \) when the predecessor is driving away.

- The efficiency cost in following mode incurs when deviating from the desired gap. The user-set desired time gaps \( t_{d,0} \) and \( t_{d,m} \) reflect driver preference and driving style in following mode, i.e. smaller desired gaps represent an aggressive driving style, while larger ones result in more timid driving behaviour. Notice that \( t_{d,m} \) influences the gap threshold \( s_f \). A larger \( t_{d,m} \) implies a longer spatial anticipation range, within which the controlled vehicle predicts and reacts to the gap and relative speed with respect to the predecessor. As we will show in the ensuing, \( t_{d,m} \) should be set in such a way that the controlled vehicle has enough time to avoid collision in the most safety-critical situation.

The efficiency cost in cruising mode stems from deviating from the free speed \( v_0 \).

- The comfort cost is represented by penalising large accelerations/decelerations, thus favouring smoother driving behaviour.

- \( c_1 > 0, c_2 > 0 \) and \( c_3 > 0 \) are weight factors related to safety cost, efficiency cost in following mode and efficiency in cruising mode respectively.

**Braking capability and spatial anticipation range**

The most safety-critical situation for an ACC vehicle is approaching a standstill vehicle at free speed, since the ACC vehicle has a limited look-ahead distance due to the capability of on-board sensors. While our model has an explicit safety cost term in the cost function, the braking capability (the minimum acceleration allowed) of the controlled vehicle must also be taken into account to prevent collision at the most safety-critical situation. Since ACC vehicles have a limited spatial anticipation range, which equals the gap threshold between following mode and cruising mode, \( s_f = v_0 t_{d,m} + s_0 \), hard braking behaviour may be necessary in this situation.

The basic requirement to prevent collision at the most safety-critical condition is that the minimum distance travelled by the ACC vehicle from free speed to full stop, the distance travelled when the ACC vehicle applies the minimum admissible acceleration (maximum deceleration) \( u_{min} < 0 \) during the whole decelerating process, should be smaller than the gap \( s_f - s_0 \). Hence:

\[
\frac{v_0^2}{-2u_{min}} \leq s_f - s_0 \Rightarrow \frac{v_0^2}{-2u_{min}} \leq v_0 t_{d,m}
\]
Table 6.1: The new ACC controller parameters

<table>
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<tr>
<th>Parameter</th>
<th>Physical meaning</th>
<th>Default value</th>
<th>Unit</th>
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</thead>
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<td>m/s^2</td>
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<td>s^{-2}</td>
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<td>m</td>
</tr>
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<td>$l$</td>
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<td>m</td>
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<td>-</td>
</tr>
</tbody>
</table>

This gives the following relationship between $t_{d,m}$ and $u_{min}$:

$$t_{d,m} \geq \frac{v_0}{-2u_{min}} \quad (6.10)$$

Equation (6.10) relates the controller parameters $u_{min}$ and $t_{d,m}$. For example, a maximum deceleration of $-5m/s^2$ with a free speed of $33.3m/s$ ($120km/h$) requires $t_{d,m}$ to be no less than $3.33s$ (corresponding to a gap threshold of $s_f \approx 112m$) to avoid collision in the safety-critical situation, while $u_{min} = -8m/s^2$ results in a minimum $t_{d,m}$ of $2.1s$ (corresponding to a gap threshold of $s_f \approx 71m$). Nevertheless, the limited detection range of on-board sensors posts upper bounds for the $t_{d,m}$, since the gap threshold $s_f$ should be shorter than the detection range of state-of-the-art on-board sensors, which is around 150 to 200m. In the default parameter setting as shown in Table 6.1, we add some safety margin and set $t_d = 2.5s$ (corresponding to a gap threshold of $s_f \approx 85m$) assuming an acceptable deceleration of $u_{min} = -8m/s^2$.

The default parameter setting for ACC controller is shown in Table 6.1. The choices of the parameters are based on the verification of the controller performance at the microscopic level (cf. Section 6.5) and the resulting flow property (Wang et al., 2014a). The derivation of the optimal acceleration for ACC controllers will be shown in Section 6.3 and the sensitivity of the parameters on controller performance will be tested in Section 6.5.

### 6.2.3 C-ACC-HP controller formulation

Here, we formulate the C-ACC-HP controller with two cooperative vehicles as shown in Figure 6.1(c). Extension of the controller formulation to more than two vehicles
is straightforward. The two cooperative vehicles in a platoon exchange their gaps and relative speeds and determine their optimal accelerations to optimise the joint objective of both vehicles. To this end, the system state for the two cooperative vehicles is defined as: \( \mathbf{x} = (x_1, x_2, x_3, x_4)^T = (s_1, \Delta v_1, s_2, \Delta v_2)^T \), with \( s_n \) and \( \Delta v_n \) denoting the gap and relative speed of the \( n \)th \((n = 1, 2)\) follower respectively. The system dynamics equation is as follows:

\[
\frac{d}{dt} \mathbf{x} = \begin{pmatrix}
\Delta v_1 \\
u_1 - u_1 \\
\Delta v_2 \\
u_1 - u_2
\end{pmatrix} = f(\mathbf{x}, \mathbf{u}) \tag{6.11}
\]

where \( \mathbf{u} = (u_1, u_2)^T \) is the control vector, with \( u_1 \) and \( u_2 \) denoting the acceleration of the first and second follower respectively. \( u_l \) denotes the acceleration of the leader. If the leader is an uncontrolled vehicle, it is assumed to be zero, i.e. constant speed heuristics. If the leader is a controlled vehicle, it will be included in the control vector without loss of generality.

To represent the situations of the two neighbouring equipped vehicles, a joint cost function needs to be specified. The most straightforward formulation of the joint cost function is simply summing up the cost of the two vehicles, which gives the running cost function as:

\[
L = \sum_{n=1}^{2} \left\{ c_1 \frac{\Delta v_n^2}{s_n} \cdot \Theta(\Delta v_n) + c_2 (s_{dn} - s_n)^2 + \frac{1}{2} u_n^2 \right\} \tag{6.12}
\]

Equation (6.12) shows that the cooperative controller, which is referred to as C-ACC-HP controller hereafter, aims to minimize the joint safety, efficiency and comfort cost for both followers. Notice that the cooperation only occurs in following mode. In cruising mode, the C-ACC-HP controller functions as the ACC controller, i.e. the controlled vehicles aims at minimising the deviation from free speed and accelerations as in Eq. (6.6).

Although we only show the example with two sequential equipped vehicles, the system state and cost function can be easily extended to include more equipped vehicles forming a platoon. In the simulation experiment in Section 6.6, we test a large platoon with 10 equipped vehicles, which amounts to a system state with 20 dimensions.

The derivation of the optimal acceleration for C-ACC-HP controllers will be shown in Section 6.3.

### 6.2.4 C-ACC-MI controller formulation

As we have explained in Section 6.1, the cooperative control concept is not restricted to the cooperation of equipped vehicles. It can capture the cooperation between intelligent vehicles and human-driven vehicles, which is formulated in the ensuing with the platoon example as shown in Figure 6.1(d).
Consider a platoon of heterogeneous followers as Figure 6.1(d), where the first follower is a cooperative vehicle and the second follower is a human-driven vehicle. The on-board system of the first follower can detect not only the situation in front (its own gap and relative speed), but the situation behind (the gap and relative speed of second follower) as well. The equipped vehicle predicts the behaviour of the human follower using a dynamic model. Note that in this case, V2V communication is not required nor feasible.

We assume that the cooperative vehicle cannot predict its follower’s behaviour perfectly. Hence the cooperative vehicle predicts the human follower behaviour with one model, while the human follower moves according to a different one. In the case studies of this chapter, the C-ACC-MI predicts the human follower behaviour using the non-cooperative ACC controller, while the human-driven vehicle actually moves with the Helly car-following model (Helly, 1959). The Helly model reads:

\[ u_2 = K_v \Delta v_2 + K_s (s_2 - s^*) \]  

(6.13)

where \( K_s > 0, K_v > 0 \) are constant gains in desired gap and relative speed. The desired gap \( s^* \) for human follower is determined by:

\[ s^* = vt^* + s_0 \]  

(6.14)

where \( t^* \) is the desired time gap. In this case study, we use the parameter settings as in (Schakel et al., 2010), with \( K_s = 0.3m/s^2, K_v = 1.0m/s, t^* = 1.3s. \)

The imperfect knowledge of human follower behaviour implies a mismatch between the system dynamics model and the actual system and leads to inconsistencies of the predicted system state \( \hat{x} \) in the open-loop optimal control problem solved at each time step and the actual system state \( x \) in the closed-loop process. As we will show in the simulation experiment, the feedback nature of the receding horizon process can compensate this mismatch. Hence, the framework is robust to the mismatch.

The system state from the perspective of the cooperative equipped vehicle in Figure 6.1(d) is the same as the C-ACC-HP controller, i.e. \( x = (s_1, \Delta v_1, s_2, \Delta v_2)^T \). The state prediction model is the same as Eq. (6.11) and the running cost function is specified with the same as in Eq. (6.12).

This completes the specification of the controller. In the sequel, we will show how to solve the control problem to obtain the optimal acceleration of the proposed controllers.

### 6.3 Derivation of optimal accelerations

In this section, we present an indirect solution approach based on Pontryagin’s Minimum Principle (PMP) to optimal control problems and apply it to derive the optimal
accelerations for the ACC, C-ACC-HP and C-ACC-MI controllers. The PMP based solution approach is chosen because it can deal with admissible control constraints and the low differentiability of the cost functions (6.6, 6.6) and particularly its scalability to large scale systems with high dimensionality, cf. Section 3.3.

### 6.3.1 A numerical solution algorithm based on Pontryagin’s Minimum Principle

The solution approach based on Pontryagin’s Minimum Principle (PMP) (Pontryagin et al., 1962) has been discussed in Section 3.3. Without going too much into details, the solution approach entails defining the Hamiltonian

\[ H(x, u, \lambda) = L(x, u) + \lambda^T \cdot f(x, u) \]  

(6.15)

where \( \lambda \) denotes the so-called co-state or marginal cost of the state \( x \), which reflects the relative extra cost of \( J \) incurred due to making a small change \( \delta x \) on the state \( x \).

Using the Hamiltonian, we can derive the following necessary condition for the optimal control \( u^* \):

\[ u^* = \min_{u} H(x, u, \lambda) \text{, s.t. } u \in U, \ x \in X \]  

(6.16)

Equation (6.16) shows that the optimal control \( u^* \) minimises the Hamiltonian in the admissible range \( U \) and in the bounded set \( X \). In nearly all cases, this requirement will enable expressing the optimal control input \( u^* \) as a function of the state \( x \) and the co-state \( \lambda \).

The co-state has to satisfy the following dynamic equation:

\[ -\frac{d}{dt} \lambda = \frac{\partial H}{\partial x} = \frac{\partial L}{\partial x} + \lambda \cdot \frac{\partial f}{\partial x} \]  

(6.17)

subject to the terminal conditions at the end of the prediction horizon \( t = T_p \):

\[ \lambda(T_p) = \frac{\partial G}{\partial x}(x(T_p)) \]  

(6.18)

The solution to the optimal control problem now is relaxed to solving the set of ordinary differential equations of (6.1) and (6.17). While for the state dynamic equation the initial conditions \( x(0) \) are known, we only know the terminal conditions \( \lambda(T_p) \) for the co-state dynamic equation. This makes the resulting set of differential equations (known as two-point boundary value problem) difficult to solve.

To solve the aforementioned problem, we put forward an algorithm which iteratively solves the state dynamic equation (6.1) forward in time and subsequently the co-state equation (6.17) backward in time (Hoogendoorn et al., 2012; Wang et al., 2012b). The algorithm will be referred to as iPMP (iterative solution based on Pontryagin’s Minimum Principle) hereinafter. The algorithm is summarised by the following procedure:
Table 6.2: The iPMP solution algorithm

1. Choose a weight factor $\alpha$ ($0 < \alpha < 1$) for smoothly updating the co-state, set the iteration number $n = 1$, and set the error threshold $\varepsilon_{\text{max}}$.
2. Set the initial co-state $\Lambda^{(0)}(t) = 0$ for $0 \leq t \leq T_p$.
3. Solve the state dynamic equation:
   \[ \frac{d}{dt}x^{(n)} = f(x^{(n)}, u^*(x^{(n)}, \Lambda^{(n-1)})) \]
   subject to $x^{(n)}(0) = x_0$ forward in time.
4. Solve the co-state dynamic equation:
   \[ -\frac{d}{dt}\lambda^{(n)} = \frac{\partial H}{\partial x}(x^{(n)}, u^*(x^{(n)}, \Lambda^{(n-1)})) \]
   subject to $\lambda^{(n)}(T_p) = \frac{\partial G}{\partial x}(x^{(n)}(T_p))$ backward in time.
5. Update the co-state $\Lambda^{(n)}$ with the weight factor $\alpha$:
   \[ \Lambda^{(n)} = (1 - \alpha)\Lambda^{(n-1)} + \alpha\lambda^{(n)}. \]
6. If $\varepsilon = ||\Lambda^{(n)} - \lambda^{(n)}||^2 < \varepsilon_{\text{max}}$ then stop, otherwise set $n := n + 1$ and go to step 3.

The key is in choosing $\alpha$ such that the scheme will converge, and that it will converge as fast as possible.

We emphasises that the iPMP solution approach is flexible such that it can be applied to both autonomous and cooperative vehicle systems (Wang et al., 2014a,b) and to Ecodriving support systems with non-regular cost functions (Wang et al., 2012a).

6.3.2 Optimal accelerations for ACC controllers

For the non-cooperative ACC vehicles as in Figure 6.1(a) and (b), applying the PMP solution and taking the necessary condition of Eq. (6.16) yields the following optimal law:

\[ u^* = \lambda_2 \] (6.19)

where $\lambda_2$ denotes the co-state corresponding to state $x_2 = \Delta v$. In other words, the optimal acceleration, minimising the cost function $J$, equals the marginal cost of the relative speed with respect to the predecessor. It is the direction in which the cost $J$ decreases fastest.

The co-state dynamics of the ACC controller are as follows:

\[ -\frac{d}{dt}\lambda_1 = \frac{\partial H}{\partial x_1} = -c_1s \Delta v^2 \cdot \Theta(\Delta v) + 2c_2(s_d - s) \left( \frac{v(t_{d,m} - t_{d,0})}{s_f} - 1 \right) \] (6.20)

\[ -\frac{d}{dt}\lambda_2 = \frac{\partial H}{\partial x_2} = \frac{2c_1}{s} \Delta v \cdot \Theta(\Delta v) + \lambda_1 \] (6.21)
6.3.3 Optimal accelerations for C-ACC-HP controllers

For the cooperative vehicles in the exemplar platoon in Figure 6.1(c), we can define the Hamiltonian $H$ and derive the optimal control and the co-state dynamics. Using the necessary condition of (6.16), we find the optimal control law as:

$$u_1^* = \lambda_2 - \lambda_4$$  \hspace{1cm} (6.22)

$$u_2^* = \lambda_4$$  \hspace{1cm} (6.23)

where $\lambda_2$ and $\lambda_4$ denote the co-state (or the marginal cost) corresponding to the relative speed of the first follower ($\Delta v_1$) and the co-state (or the marginal cost) corresponding to the relative speed of the second follower ($\Delta v_2$).

Equation (6.22) shows that the first follower determines control actions based on its own marginal cost as well as on the marginal cost of the second follower, i.e. the first follower tends to decrease its speed when the marginal cost of the second follower is negative and it tends to accelerate when vice versa. Clearly, the presence of $\lambda_4$ in the optimal control law of the first follower captures the cooperative nature of the C-ACC-HP controller. Since there is no vehicle behind the second follower in the two follower platoon and the second follower is not able to improve the situation of the first follower, the second follower only considers his own marginal cost when making decisions, as shown in Eq. (6.23).

Derivation of the co-state dynamics is straightforward and yields the following:

$$-\frac{d}{dt} \lambda_1 = \frac{\partial H}{\partial x_1} = -\frac{c_1}{s_1^2} \Delta v_1^2 \cdot \Theta(\Delta v_1) + 2c_2 (s_{d_1} - s_1) \left( \frac{v_1(t_{d,m} - t_{d,0})}{s_f} - 1 \right)$$  \hspace{1cm} (6.24)

$$-\frac{d}{dt} \lambda_2 = \frac{\partial H}{\partial x_2} = 2 \frac{c_1}{s_1} \Delta v_1 \cdot \Theta(\Delta v_1) + \lambda_1$$  \hspace{1cm} (6.25)

$$-\frac{d}{dt} \lambda_3 = \frac{\partial H}{\partial x_3} = -\frac{c_1}{s_2^2} \Delta v_2^2 \cdot \Theta(\Delta v_2) + 2c_2 (s_{d_2} - s_2) \left( \frac{v_2(t_{d,m} - t_{d,0})}{s_f} - 1 \right)$$  \hspace{1cm} (6.26)

$$-\frac{d}{dt} \lambda_4 = \frac{\partial H}{\partial x_4} = 2 \frac{c_1}{s_2} \Delta v_2 \cdot \Theta(\Delta v_2) + \lambda_3$$  \hspace{1cm} (6.27)

Scaling the optimal acceleration laws and the state dynamics to large platoons is straightforward. When more than two cooperative vehicles form a platoon, each cooperative vehicle determines its control actions based on its own marginal cost as well as the marginal cost of its direct follower except the last follower of the platoon. The only changes in the optimal control laws and the costate dynamic equations are the vehicle indices.
6.3.4 Optimal accelerations for C-ACC-MI controllers

For the cooperative controller in a mixed platoon as in Figure 6.1(d), the necessary condition yields the same form of the optimal control law as in Eq. (6.22). Since in the open-loop prediction, the behaviour human-driven vehicle is assumed to be an ACC controller, the dynamics equations of the co-state vector are the same as in the C-ACC-HP controller, which entails that the cooperative equipped vehicle determines its optimal acceleration based on both its own marginal cost and the predicted marginal cost of its human follower.

Using the proposed numerical solution algorithm finds the optimal acceleration of the equipped vehicle, which minimises both its own situation and its follower’s situation.

6.4 Experimental design for ACC and C-ACC performance assessment

In this section, we give an overview of the experimental setup for assessing the proposed controllers at individual and platoon levels under the ADAS assessment framework discussed in Section 3.4. The key research questions are first elaborated, followed by the simulation setup for ACC and C-ACC controllers. Particularly, we distinguish individual controller performance where an autonomously controlled vehicle is examined and platoon performance where several controlled and/or uncontrolled vehicles following each other are examined.

6.4.1 Key research questions

Several key research questions are identified regarding the performance of the proposed controllers and the proposed solution algorithm:

1. How does an ACC vehicle behave under representative scenarios (cf. Section 3.4), including free driving, emergency braking, and following the leader with decelerating and accelerating disturbances?
2. How do the control parameters, particularly the weight factors, influence the ACC controller performance?
3. Can the C-ACC-HP and C-ACC-MI controller achieve better platoon performance compared to the ACC controller under decelerating and accelerating disturbances?
4. How does the computational complexity change with the increase of platoon size?

The first two questions will be answered by simulation experiments with a leader-follower pair where the leader behaviour is predefined and the follower is controlled,
while the last two questions will be answered by examining a platoon where several confined followers prevail after the leader with predefined behaviour.

### 6.4.2 Experiments for assessing individual ACC performance

To verify whether the proposed ACC controller generates desired performance, we simulate a pair of vehicles of which the leader behaviour is exogenous and the follower is an ACC vehicle. The leader behaviour is predefined to represent real world scenarios under the assessment framework in Section 3.4. Sensitivity of the controller performance in relation to control parameters is also examined.

The representative scenarios for testing the ACC performance include situations of free driving, emergency-braking and following under decelerating and accelerating disturbance, cf. Section 3.4. The free driving situation involves accelerating from standstill to free speed and maintaining free speed in cruising mode. The emergency-braking situation refers to the most safety-critical conditions where ACC vehicles approach a stopped vehicle with free speed. The normal following situation is represented by an ACC vehicle following a predecessor with decelerating disturbance and accelerating disturbance.

In the free driving and emergency braking simulation scenario, the controlled vehicle starts at standstill with a 2 km gap to a stopped leader. The expected behaviour under this scenario is that the controlled vehicle accelerates from standstill to free speed, maintains the free speed for some time and finally decelerates to full stop. The controller parameters to be tested in the first scenario include the efficiency cost weight $c_3$ in cruising mode, the safety cost weight $c_1$, the maximum desired time gap $t_{d,m}$ and the braking capability $u_{\text{min}}$, since they are relevant for free driving and emergency-braking behaviour. The experiments and parameter settings for the first scenario are shown in Table 6.3 with the experiment number 1 to 3.

In the normal following under decelerating and accelerating disturbances scenario, we add accelerating and decelerating manoeuvres to the leader behaviour to represent a disturbance in normal following situations in real traffic. In a simulation of 100 seconds, the leader starts driving with a constant speed of $20\,\text{m/s}$ for the first three seconds, then decelerates with $-3\,\text{m/s}^2$ for two seconds, and travels constantly afterwards until 53rd seconds. From the 53rd to 56th second, the leader accelerates with $2\,\text{m/s}^2$ back to the initial speed, then maintains this speed until the end of the simulation. The ACC follower starts at equilibrium conditions, i.e. travelling with the same speed as the leader and with its desired gap, and adjusts its speed to minimise the predicted costs. The parameters to be tested are the safety cost weight $c_1$ and the efficiency cost weight $c_2$, since they reflect the trade-off between different cost terms in following mode. The experiments and parameter settings for the second scenario are shown in Table 6.3 with the experiment number 4 to 5.
Table 6.3: Overview of experimental setup

<table>
<thead>
<tr>
<th>Experiment no.</th>
<th>Scenarios</th>
<th>Controller type</th>
<th>Parameters</th>
<th>Indicators and/or plots</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Free accelerating and emergency braking</td>
<td>ACC</td>
<td>$c_3 = 0.005, c_3 = 0.01, c_3 = 0.02$</td>
<td>s, v, u plots</td>
</tr>
<tr>
<td>2</td>
<td>Free accelerating and emergency braking</td>
<td>ACC</td>
<td>$c_1 = 1, c_1 = 2, c_1 = 5$</td>
<td>s, v, u plots</td>
</tr>
<tr>
<td>3</td>
<td>Free accelerating and emergency braking</td>
<td>ACC</td>
<td>$t_d = 3.6, u_{min} = -5,$</td>
<td>s, v, u plots</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>$t_d = 2.5, u_{min} = -8,$</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>$t_d = 1.8, u_{min} = -10$</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>Normal following with decelerating and accelerating disturbances</td>
<td>ACC</td>
<td>$c_2 = 0.01, c_2 = 0.05, c_2 = 0.1$</td>
<td>Cost components, s, $\Delta v$, v, u plots</td>
</tr>
<tr>
<td>5</td>
<td>Normal following with decelerating and accelerating disturbances</td>
<td>ACC</td>
<td>$c_1 = 1, c_1 = 2, c_1 = 5$</td>
<td>Cost components, s, v, $\Delta v$, u plots</td>
</tr>
<tr>
<td>6</td>
<td>Decelerating transition Platoon (a) v.s. Platoon (c) with 10 followers</td>
<td>w = 0, w = 1</td>
<td>Cost, v, u plots, $\Delta v_{max}$, speed variance, mean gap and speed, $t_{g}^{in}, t_{g}^{out}$, $t_{h}^{in}, t_{h}^{out}$</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>Accelerating transition Platoon (a) v.s. Platoon (c) with 10 followers</td>
<td>w = 0, w = 1</td>
<td>Cost, v, u plots, $\Delta v_{max}$, speed variance, mean gap and speed, $t_{g}^{in}, t_{g}^{out}$, $t_{h}^{in}, t_{h}^{out}$</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>Decelerating transition Platoon (b) v.s. Platoon (d) with 10 followers</td>
<td>w = 1</td>
<td>Cost, v, u plots, $\Delta v_{max}$, speed variance, mean gap and speed, $t_{g}^{in}, t_{g}^{out}$, $t_{h}^{in}, t_{h}^{out}$</td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>Accelerating transition Platoon (b) v.s. Platoon (d) with 10 followers</td>
<td>w = 1</td>
<td>Cost, v, u plots, $\Delta v_{max}$, speed variance, mean gap and speed, $t_{g}^{in}, t_{g}^{out}$, $t_{h}^{in}, t_{h}^{out}$</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>Decelerating transition Platoon (a) v.s. Platoon (c) with 1-20 followers</td>
<td>w = 1</td>
<td>Computation time</td>
<td></td>
</tr>
<tr>
<td>11</td>
<td>Accelerating transition Platoon (a) v.s. Platoon (c) with 1-20 followers</td>
<td>w = 1</td>
<td>Computation time</td>
<td></td>
</tr>
</tbody>
</table>

[1] Platoon type refers to Figure 6.1.
6.4.3 Experiments for assessing C-ACC platoon performance

To test the platoon performance of different control strategies (research question 5), we simulate two platoons of vehicles using a standard leader. When all followers in the platoon are homogeneous cooperative vehicles, such as the platoon (c) in Figure 6.1, we compare with platoon (a) in Figure 6.1, where all followers are non-cooperative vehicles. When the followers are a mixture of cooperative vehicles in cooperative systems and human-driven vehicles, such as platoon (d) in Figure 6.1, we compare with platoon (b) in Figure 6.1, where the equipped vehicles are non-cooperative vehicles.

To avoid trivial results from small-sized platoons, the number of followers of all platoons in Figure 6.1 is extended to 10 in the simulation by duplicating the pair of followers four times. For instance, the platoon (c) will have ten CACC followers, while platoon (d) also have ten followers, of which the odd numbered followers are cooperative equipped vehicles and the even numbered followers are human-driven vehicles.

The standard leader behaviour is predefined to represent disturbances in real traffic situations, which is the same as the normal following scenario in the previous section. We distinguish two scenarios for each platoon:

- **Decelerating transition scenario**: the leader starts with an initial speed of 20\text{m/s}, decelerates with \(-3\text{m/s}^2\) from 3 to 5 seconds to 14\text{m/s} and maintains this speed until the end of the simulation period.

- **Accelerating transition scenario**: the leader starts with an initial speed of 14\text{m/s}, accelerates with 2\text{m/s}^2 from 3 to 6 seconds back to 20\text{m/s} and maintains this speed until the end of the simulation period.

Each scenario lasts 100 seconds. The platoon starts from equilibrium conditions, i.e. all followers travel at the same speed of the platoon leader and at their desired gaps. The simulation time is chosen to ensure that the platoon settles down to equilibrium conditions at the end of the simulation. The experiment setup for the two scenarios with C-ACC-HP and C-ACC-MI controllers are shown in Table 6.3 with the experiment numbers 6 to 9.

Temporal evolution of cost, speed and acceleration of representative vehicles (the first, second, fifth and tenth followers) in each platoon are plotted and maximum speed difference during the decelerating and accelerating phases are compared. Total platoon cost and speed variance of the whole platoon during the simulation are also calculated to assess the platoon performance.

To gain some insights into the changes in the collective flow operations, we plot the evolution of average gap and average speed of all followers in the two platoons at each simulation time step in the speed-gap \((v(t),s(t))\) plane, which gives an indication of the dynamic platoon performance in the state transitions. Furthermore, we calculate the mean time gap \(t_g\) and time headway \(t_h\) of the followers during the deceleration and
acceleration transition, which gives an indication of the inflow upstream and outflow downstream of the low speed state. For each vehicle we identify the time when it starts to decelerate/accelerate, and the time it settles down to the new state with low/high equilibrium speed. To avoid numerical error, we put a bound of 5% of speed change to identify the equilibrium state, which is \((20 - 14) \cdot 5\% = 0.3 \text{ m/s} \). More specifically, in the deceleration phase, when the follower’s speed drops below \(19.7 \text{ m/s} \), we assume it starts to decelerate and include it in calculation of mean upstream time gap \(t_{\text{up}}^{\text{g}}\) and time headway \(t_{\text{up}}^{\text{h}}\), and when its speed stabilises between the range of \([13.7 \text{ m/s}, 14.3 \text{ m/s}]\), we assume the follower settles down to low speed equilibrium state and exclude it from the calculation. In the acceleration phase, when the follower’s speed reaches \(14.3 \text{ m/s} \), we assume the follower starts to accelerate and include it in the calculation of mean downstream time gap \(t_{\text{down}}^{\text{g}}\) and time headway \(t_{\text{down}}^{\text{h}}\), and when its speed stabilises in the range of \([19.7 \text{ m/s}, 20.3 \text{ m/s}]\), we assume the follower settles down to the high speed equilibrium state and exclude it from calculation (Jelali, 2006).

To examine the computational complexity and scalability of the solution algorithm to large scale systems, we examine the influence of the number of controlled vehicles or equivalently dimensionality of the state space on the computation time (research question 6). Note that for the C-ACC-HP controller, the system state space expands with the increases number of homogeneous followers, while the C-ACC-MI controller considers a system state with cardinality of 4. Hence, we only compare the ACC controller with the C-ACC-HP controller for this test.

### 6.5 Individual ACC performance under representative scenarios

In this section, we present the simulation results on the performance of the proposed ACC controller in representative scenarios.

#### 6.5.1 Free driving and emergency-braking behaviour

In the first simulation scenario, we examine how the ACC vehicle accelerates to free speed in cruising mode (free driving conditions) and how it behaves when emergency-braking is required to avoid collision in the most safety-critical conditions.

**Accelerating to free speed**

Figure 6.2 (a) and (b) show the spatial evolution of speed and the applied acceleration of the ACC follower respectively, using different efficiency cost weights \(c_3\). In cruising mode, the acceleration is determined by the efficiency cost and the comfort cost.
A larger $c_3$ leads to higher efficiency cost, and thus results in sharper accelerating behaviour. This is verified by the speed and acceleration profiles in Figure 6.2, i.e. ACC vehicles with higher $c_3$ perform larger accelerations before reaching free speed. The braking procedure of the ACC vehicles with different $c_3$ are exactly the same, since the parameters of the ACC vehicles in following mode are the same.

As we can see from the red lines in Figure 6.2, the ACC vehicle with $c_3 = 0.02s^{-2}$ travels with the maximum acceleration of $2m/s^2$ from the starting position of $s = 2000m$ for around 100 meters to the position of $s \approx 1900m$. When the comfort cost stemming from applying large accelerations outweighs the efficiency cost stemming from not driving at the free speed, the ACC vehicle gradually reduces its acceleration till it reaches its free speed. After that, the ACC vehicle maintains the free speed until it reaches the gap threshold ($s = s_f \approx 122m$), where the ACC system switches to following mode. After this position, the safety cost starts to dominate the following behaviour, and the ACC vehicle starts to decelerate hard to avoid collision with the standstill leader until it reaches a full stop.

As shown with blue lines in Figure 6.2, the ACC vehicle with the lowest $c_3 = 0.005s^{-2}$ starts with a maximum acceleration of around $1.5m/s^2$, and reduces its acceleration as long as it starts to move, due to the (relatively) large comfort cost. As a consequence, it travels with much lower speed compared to the vehicles with higher $c_3$ before approaching the standstill leader. The emergency-braking profiles of the three ACC vehicles with different $c_3$ are the same, since $c_3$ does not influence the vehicle behaviour in following mode, as shown in Figure 6.2. All three vehicles stop before colliding with the standstill leader. It takes 67.6 seconds to reach the position where the ACC vehicle with $c_3 = 0.02s^{-2}$ switches to following mode, but a significantly longer time of 78.4 seconds for the ACC vehicle with $c_3 = 0.005s^{-2}$ to arrive at the same position.

**Emergency braking**

In the first scenario, when the ACC vehicles approach the standstill leader with free speeds, the safety cost prevents the ACC vehicles from colliding to the leader. Different values of the safety cost weight $c_1$, the maximum desired time gap $t_{d,m}$ and the minimum admissible acceleration (maximum deceleration) $u_{min}$ are evaluated to examine the effects on the emergency-braking behaviour.

Figure 6.3(a) and (b) show the spatial evolution of the speed and acceleration of ACC vehicles with different $c_1$. To get a closer look at the braking manoeuvre, only the last 150 metres of the journey are depicted, since there is no difference in the free driving part before the ACC vehicles switch to following mode ($c_3$ is the same in this test). As we can see from Figure 6.3, after the ACC vehicle reaches the gap $s = s_f = 84.5m$, the ACC vehicle switches to following mode and the maximum braking is performed to avoid collision. The higher the safety cost weight $c_1$ is, the later the ACC vehicle relaxes from the maximum braking. As a result, the ACC vehicle with the highest
Figure 6.2: Experiment 1: spatial evolution of (a) speed and (b) acceleration in the free driving and emergency braking scenario with different $c_3$ of 0.005, 0.01 and 0.02. Other parameters are set as default values.

$c_1 = 5m/s^2$ stops with the largest gap among the ACC vehicles. The stopping distance of the ACC vehicle with the highest $c_1 = 6m/s^2$ is $4.5m$, which is $2.3m$ longer than the stopping distance of the ACC vehicle with $c_1 = 1m/s^2$. The simulation results imply that a higher $c_1$ reflects a more conservative driving style.

Figure 6.4 shows the spatial evolution of speed and acceleration with different combinations of $t_{d,m}$ and $u_{min}$ for the last 150m of the simulated journey. It is evident that increasing the maximum desired time leads to earlier braking manoeuvres of ACC vehicles as a result of the increased spatial anticipation range. We can also see that the default parameters of $t_{d,m} = 2.5s$ and $u_{min} = -8m/s^2$ succeed in preventing colliding with the standstill leader. When we constrained the maximum braking with $u_{min} = -5m/s^2$, larger $t_{d,m}$ is necessary to avoid collisions. When decreasing the time gap to $t_{d,m} = 2.0s$, larger braking capabilities are needed to avoid collision. All the three parameter settings in experiment 3 prevent collision with the standstill leader. However, if we reduce the $t_{d,m}$ to 2.0s without changing the braking capability, the ACC vehicle will collide with the standstill leader, since the parameter setting does not satisfy condition (6.10). For comfort reasons, the maximum allowed braking capability should be limited.

### 6.5.2 Follow-the-leader under speed disturbance

In the second scenario, we test how the ACC vehicle performs when following a leader with decelerating and accelerating manoeuvres. Figure 6.5 shows the temporal evolution of instantaneous cost, deviation from the desired gap $(s - s_d)$ and relative speed, and the acceleration of the leader and the ACC follower with default model parameters.
Figure 6.3: Experiment 2: emergency-braking behaviour with different safety cost weight $c_1$. Other parameters are set as default values.

Figure 6.4: Experiment 3: emergency-braking behaviour with different $t_{d,m}$ and $u_{min}$. Other parameters are set as default values.
The ACC vehicular behaviour in the decelerating phase is dominated by the safety cost and comfort cost. The ACC vehicle first travels with constant speed for a few seconds, since there is no cost incurred at equilibrium conditions, i.e. the vehicle travels at its desired gap and at the same speed as its leader. After the leader starts to brake, the ACC vehicle incurs safety cost which demands the ACC vehicle to brake. As the vehicle starts to decelerate, comfort cost starts to play a role which restricts the magnitude of the applied acceleration. The trade-off between the safety cost and the comfort cost results in the increase of braking magnitude from 3 to 5 seconds, and the relaxation of the deceleration from 5 to 12 seconds. After 12 seconds, the ACC vehicle settles down to the equilibrium condition again.

The ACC behaviour in the accelerating phase is dominated by the efficiency cost and the comfort cost, since there is no safety cost with the default parameter of $w = 0$. As the leader increases its speeds from 53 to 56 seconds, the ACC follower incurs efficiency cost which demand the ACC vehicle to accelerate. As it increases its speed, comfort cost emerges. The trade-off between the efficiency cost and the comfort cost leads to the increase of acceleration to the peak at around 56 seconds and relaxation of the acceleration afterwards.

![Figure 6.5](image)

Figure 6.5: Experiment 4: evolution of (a) efficiency and safety cost; (b) gap deviation from desired gap and relative speed; and (c) optimal acceleration with default parameters in the normal following scenario.

To test the sensitivity of the efficiency cost weight $c_2$ on the controller performance, we choose different values of $c_2$ and simulate the ACC vehicle behaviour under the same initial conditions. Figure 6.6 (a)-(d) show the evolution of gap deviation from the desired gap ($s_d - s$), relative speed, speed and acceleration of the ACC followers with different $c_2$. Clearly, we can see from Figure 6.6:
• Larger $c_2$ results in similar decelerations in the decelerating phase but a larger accelerations in the accelerating phase.

• Larger $c_2$ results in smaller deviation between the actual gap and the desired gap, since the controller puts more emphasis on the efficiency cost.

• Larger $c_2$ leads to smaller speed difference both in the decelerating phase and the accelerating phase, implying a more agile car-following behaviour of the ACC vehicle.

In general, $c_2$ reflects the agility of the controlled vehicle. The higher $c_2$ is, the more agile the behaviour of the controlled vehicle is. When $c_2$ is very small, the controlled vehicle considers more comfort cost, at the expenses of deviating from the desired gap for a longer time and a longer time to settle down to the equilibrium after the disturbance, as seen in the blue lines in Figure 6.6. This implies that increasing $c_2$ stabilises the controller, which is in line with the analytical results in the previous chapter.

![Figure 6.6](image_url)

Figure 6.6: Experiment 4: evolution of (a) gap deviation from desired gap; (b) relative speed; (c) speed; (d) acceleration of ACC vehicles in the normal following scenario with different $c_2$. Other parameters are set at default values.

To test the sensitivity of the controller performance to the safety cost weight $c_1$, a pair of vehicles is simulated for the normal following scenario with different $c_1$ values. Figure 6.7 shows the evolution of gap, relative speed, gap deviation from the desired gap $(s_d - s)$ and the optimal acceleration of the ACC followers with different $c_1$ values. The effects of the safety cost weight $c_1$ are summarised as follows:
• Larger $c_1$ results in larger deceleration in the decelerating phase and similar acceleration in the accelerating phase.

• Larger $c_1$ results in less oscillations in the gap and speed profiles in both decelerating and accelerating phases.

• Larger $c_1$ results in a smaller absolute relative speed in the decelerating phase.

The results show that the disturbance is attenuated faster with larger $c_1$. Hence increasing $c_1$ stabilises the controller, which is in line with the analytical results in the previous chapter.

Figure 6.7: Experiment 5: evolution of (a) gap; (b) relative speed; (c) speed and (d) acceleration of the ACC vehicle in the normal following scenario with different $c_1$. Other parameters are set at default values.

This completes the verification of the ACC controller performance and the sensitivity analysis of the controller parameters. In the following section, we will investigate the cooperative controller performance in a large platoon, with the non-cooperative ACC controller as reference.

### 6.6 Platoon performance of cooperative controllers

In this section, we present the key results regarding platoon performance with difference controllers and computational complexity of the proposed solution algorithm. The controllers for cooperative systems are compared to the ACC controller for non-cooperative systems.
Table 6.4: Indicators for different test platoons during two scenarios (experiments 6-9)

<table>
<thead>
<tr>
<th></th>
<th>ACC (w = 0)</th>
<th>C-ACC-HP (w = 0)</th>
<th>C-ACC-HP (w = 1)</th>
<th>ACC + Human</th>
<th>C-ACC-MI + Human</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Deceleration scenario</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Platoon cost</td>
<td>32.29</td>
<td>20.94</td>
<td>21.08</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>max $\Delta v$ of 1st follower (m/s)</td>
<td>0.06</td>
<td>0.09</td>
<td>0.07</td>
<td>0.11</td>
<td>0.25</td>
</tr>
<tr>
<td>max $\Delta v$ of 10th follower (m/s)</td>
<td>0.20</td>
<td>0.02</td>
<td>0.01</td>
<td>0.08</td>
<td>0.02</td>
</tr>
<tr>
<td>Platoon speed variance ($m^2/s^2$)</td>
<td>1.48</td>
<td>0.81</td>
<td>0.80</td>
<td>1.06</td>
<td>0.91</td>
</tr>
<tr>
<td>$t_h^{up}$ (s)</td>
<td>1.65</td>
<td>1.73</td>
<td>1.73</td>
<td>1.70</td>
<td>1.72</td>
</tr>
<tr>
<td>$t_g^{up}$ (s)</td>
<td>1.35</td>
<td>1.41</td>
<td>1.41</td>
<td>1.40</td>
<td>1.42</td>
</tr>
<tr>
<td><strong>Acceleration scenario</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Platoon cost</td>
<td>21.48</td>
<td>15.85</td>
<td>12.33</td>
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<td>-</td>
</tr>
<tr>
<td>min $\Delta v$ of 1st follower (m/s)</td>
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<td>-0.14</td>
<td>-0.08</td>
<td>-0.45</td>
<td>-0.42</td>
</tr>
<tr>
<td>min $\Delta v$ of 10th follower (m/s)</td>
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<td>-0.04</td>
<td>-0.01</td>
<td>-0.11</td>
<td>-0.05</td>
</tr>
<tr>
<td>Platoon speed variance ($m^2/s^2$)</td>
<td>1.92</td>
<td>1.05</td>
<td>0.60</td>
<td>1.69</td>
<td>0.95</td>
</tr>
<tr>
<td>$t_h^{down}$ (s)</td>
<td>1.81</td>
<td>1.77</td>
<td>1.66</td>
<td>1.77</td>
<td>1.71</td>
</tr>
<tr>
<td>$t_g^{down}$ (s)</td>
<td>1.55</td>
<td>1.51</td>
<td>1.39</td>
<td>1.50</td>
<td>1.44</td>
</tr>
</tbody>
</table>

6.6.1 C-ACC-HP performance

The simulation results regarding the C-ACC-HP platoon (Figure 6.1(c)) performance are analysed here, with comparison to the reference ACC platoon (Figure 6.1(a)).

Decelerating characteristics

Figure 6.8(b)(d)(f) depicts the temporal evolution of cost, speed and acceleration of representative followers (the 1st, 2nd, 5th and 10th followers) of the C-ACC-HP platoon during the decelerating transition scenario, while the Figure 6.8(a)(c)(e) shows their ACC counterparts. The platoon cost, maximum/minimum $\Delta v$ of the first and the last follower, platoon speed variance, and the average inflow/outflow time headways and time gaps are calculated in Table 6.4.

As shown in Figure 6.8, all vehicles in both platoons generate plausible decelerating...
behaviour. As we can see from the figures, all followers in the C-ACC-HP platoon and the ACC platoon first decrease their speeds from the equilibrium state of 20m/s as a reaction to the decelerating disturbance, and then recover the equilibrium state of 14m/s.

![Figures 6.8: Experiment 6: evolution of (a) incurred cost J, (c) speed and (e) acceleration of representative vehicles in the platoon with homogeneous ACC followers and (b) incurred cost, (d) speed and (f) acceleration of representative vehicles in the platoon with homogeneous C-ACC-HP followers in the decelerating scenario.](image)

The C-ACC-HP vehicles exhibit cooperative behaviour. As we can see from Figure 6.8(a) and (b), the cost of the first C-ACC-HP follower is higher than the first ACC follower. The C-ACC-HP follower considers his own marginal cost of its own relative speed as well as the marginal of its follower’s relative speed, cf. Eq. (6.22). This considerate behaviour demands the first C-ACC-HP follower to increase its own cost in order to minimise the overall cost. As a result, the 2nd, 5th and 10th C-ACC-HP followers incur a lower cost than their non-cooperative counterparts. The total cost of the C-ACC-HP platoon in the decelerating phase is 20.94, much lower than the total cost of the ACC platoon of 32.29, as shown in Table 6.4.

The C-ACC-HP vehicles generate smoother decelerating behaviour compared to the ACC vehicles. As we can see from Figure 6.8(c)(d), there is clear overshooting phenomena observed in the ACC platoon, i.e. the ACC followers decelerate more than necessary, and they have to accelerate again to reach equilibrium state. The maximum
speed difference with respect to the equilibrium state of $14m/s$ is $0.06m/s$ for the 1st ACC follower, but increases to $0.20m/s$ for the 10th ACC follower. The overshooting phenomenon is much less in the C-ACC-HP platoon, with the maximum speed difference of $0.09m/s$ for the 1st C-ACC-HP follower, but is attenuated to a negligible $0.02m/s$ for the 10th C-ACC-HP follower. The speed variance of the C-ACC-HP platoon is smaller than that of the ACC platoon.

The C-ACC-HP vehicles react earlier to the decelerating disturbance, which is evidenced by the acceleration profiles of representative vehicles shown in Figure 6.8(e)(f). The 2nd, 5th and 10th C-ACC-HP vehicles start to decelerate earlier than their ACC counterparts. This is caused by the difference in the prediction model of the two controllers. The ACC controller assumes other vehicles driving with zero accelerations, and can only correct this imperfect prediction with feedbacks from its own sensors. On the contrary, C-ACC-HP vehicles coordinate to determine their optimal accelerations together via V2V communications, and hence are capable of making a more accurate prediction with non-zero accelerations in dynamic conditions. The predictive behaviour of the C-ACC-HP controller leads to the earlier response to the disturbance compared to the reactive behaviour of the ACC controller.

**Accelerating characteristics**

Figure 6.9(b)(d)(f) shows the evolution of cost, speed and acceleration of representative followers of the C-ACC-HP platoon during the first hundred seconds in the accelerating phase, while Figure 6.9(a)(c)(e) depicts the profiles of the ACC followers. In the accelerating phase, all followers in the two platoons first accelerate from the low equilibrium speed of $14m/s$ to the high equilibrium speed of $20m/s$, and settle down to the new equilibrium conditions after sufficient time. The indicators are calculated in Table 6.4.

In the accelerating scenario, the C-ACC-HP preserves behavioural characteristics shown in the decelerating phase.

- The C-ACC-HP vehicles show cooperative behaviour. The followers in front of the C-ACC-HP platoon (the 1st and 2nd followers) increase their cost in order to optimise the platoon performance, as shown in Figure 6.9(a)(b). As a result the subsequent followers (the 5th and the 10th followers) incur smaller cost compared to their ACC counterparts.

- The C-ACC-HP vehicles generate smoother behaviour than the ACC vehicles, as shown in Figure 6.9(c)(d) and the minimum speed difference and variance in Table 6.4. The overshooting phenomenon in the speed profiles of the C-ACC-HP platoon is much smaller than in the ACC platoon.

- The C-ACC-HP vehicles react earlier than the ACC vehicles, particularly for the downstream vehicles (5th and 10th follower) in the platoon, as shown in the acceleration profiles of Figure 6.9(e)(f).
Figure 6.9: Experiment 7: evolution of (a) incurred cost $J$, (c) speed and (e) acceleration of representative vehicles in the platoon with homogeneous ACC followers and (d) incurred cost, (d) speed and (f) acceleration of representative vehicles in the platoon with homogeneous C-ACC-HP followers in the accelerating scenario.

**Hysteresis loops of C-ACC-HP controller**

To gain insights into the resultant traffic flow dynamics, we plot the evolution of average gap and average speed of all followers in the two platoons at each simulation time step in the speed-gap $(v(t), s(t))$ plane in Figure 6.10(a), with arrows indicating the time sequence.

For the ACC platoon, the trajectory in the speed-gap plane evolves clockwise, with a lower gap in the deceleration phase and a larger gap in the acceleration phase, as shown with the red line and arrows in Figure 6.10(a). This is caused by the asymmetric decelerating and accelerating behaviour. This controller characteristic is a result of our design choices, particularly the cost function formulation. For the ACC controller, the asymmetric factor $w = 0$, which implies the ACC follower reacts only on the deviation from desired gap. Since gap is the second order integral of vehicle accelerations, it has to wait some time for the gap deviation to grow sufficiently large to trigger the accelerating manoeuvre, which causes the slow-to-start or retarded behaviour in the accelerating phase. As a consequence, the mean downstream time gap $t_{down}^{g}$ and time
The mean downstream time gap $t_{down}^d$ and time headway $t_{down}^h$ of the ACC platoon are larger than the mean upstream time gap $t_{up}^u$ and time headway $t_{up}^h$, as shown in Table 6.4.

The *hysteresis loop* and the retarded behaviour in the accelerating phase are also common features observed in human-driven vehicular traffic (Treiterer & Myers, 1974; Zhang, 1999; Hoogendoorn & Bovy, 2001). The retarded behaviour in the accelerating phase results in a larger following gap for human-driven vehicles, particularly at the downstream front of congestion, which leads to the so-called *capacity drop* (Treiterer & Myers, 1974; Zhang & Kim, 2005; Treiber et al., 2006b). The overshooting behaviour at the upstream front of congestion often leads to the propagation of traffic waves (Treiterer & Myers, 1974; Zhang & Kim, 2005; Treiber et al., 2006b). From traffic operations perspective, if we can control vehicles to react early to decelerating disturbances and to reduce the overshooting phenomena and the *slow-to-start* effect under acceleration disturbances, we can stabilise traffic flow upstream of jams and mitigate the *capacity drop* downstream of jams.

The C-ACC-HP controller results in a different hysteresis loop and has potentials to improve traffic flow operations compared the ACC controller. The cooperative and predictive behaviour of the C-ACC-HP controller results in a higher gap in the decelerating transition and a lower gap in the accelerating transition compared to the ACC controller, as shown with the blue line and arrows in Figure 6.10(a). Comparing the mean time headway and time gap suggests the benefits of the C-ACC-HP in terms of traffic operations. The mean downstream time gap $t_{down}^d$ and time headway $t_{down}^h$ of the C-ACC-HP platoon are smaller than that of the ACC platoon, while the mean upstream time gap $t_{up}^u$ and time headway $t_{up}^h$ are larger than that of the ACC controller, as shown in Table 6.4. The larger time headway at the upstream section of jams has the potential to reduce inflow into the congestion and stabilise traffic while the smaller time headway at downstream sections of jams has the potential to increase the outflow of congestion.

Notice that the hysteresis loop of the C-ACC-HP platoon in Figure 6.10(a) still evolves clockwise. As a consequence, the mean downstream time gap $t_{down}^d$ and time headway $t_{down}^h$ of the C-ACC-HP platoon are still larger than the mean upstream time gap $t_{up}^u$ and time headway $t_{up}^h$, as shown in Table 6.4. This is still caused by the *slow-to-start* behaviour with the asymmetric factor $w = 0$. Increasing this factor can mitigate the *slow-to-start* behaviour and hence increase the resultant outflow at downstream congestion front.

The decelerating behaviour of the C-ACC-HP with $w = 1$ is nearly the same as the C-ACC-HP controller with $w = 0$, since in the decelerating phase, the asymmetric factor plays a negligible role. The indicators of the two C-ACC-HP controllers are nearly the same, as shown in Table 6.4.

The difference of the C-ACC-HP platoon performance becomes apparent in the accelerating phase. Figure 6.11(b)(d)(f) shows the evolution of cost, speed and acceleration of the C-ACC-HP followers with $w = 1$ compared to the reference the ACC followers
Figure 6.10: Experiments 6 and 7: hysteresis loops in the average speed-gap $(v(t), s(t))$ plane for (a) ACC followers and C-ACC-HP followers with $w = 0$; (b) ACC followers and C-ACC-HP followers with $w = 1$. Arrows indicate the evolution direction of the loops.

in Figure 6.11(b)(d)(f). As we can see, the 1st C-ACC-HP follower with $w = 1$ still increases its cost, but the incurred cost for the 2nd follower is already lower than the ACC counterpart. The overshooting phenomenon is nearly negligible for the C-ACC-HP platoon with $w = 1$, with smaller speed difference, speed variance and platoon cost in the accelerating scenario. Notice that to make a fair comparison, all the platoon costs are calculated with Eq. (6.6) using $w = 0$.

The hysteresis loop of the C-ACC-HP platoon with $w = 1$ is quite different from the ACC platoon and the C-ACC-HP platoon with $w = 0$. It mainly evolves anti-clockwise rather than clockwise in the ACC platoon and the C-ACC-HP platoon with $w = 0$. The mean downstream time gap $t_{g}^{down}$ and time headway $t_{h}^{down}$ of the C-ACC-HP platoon are smaller than that of the ACC platoon and the C-ACC-HP platoon with $w = 0$, while the mean upstream time gap $t_{g}^{up}$ and time headway $t_{h}^{up}$ are larger than the ACC platoon and the C-ACC-HP platoon with $w = 0$, as shown in Table 6.4.

### 6.6.2 C-ACC-MI performance

In the last experiment, we test the performance of the mixed platoon with non-cooperative ACC followers and human-driven vehicles (platoon (c) in Figure 6.1) and the mixed platoon with C-ACC-MI followers and human-driven vehicles (platoon (d) in Figure 6.1) in the decelerating and accelerating scenarios. The parameter setting of the C-ACC-MI controller is the same as the C-ACC-HP controller with $w = 1$. The resultant speed and acceleration trajectories of representative vehicles in both scenarios are shown in Figure 6.12, and the indicators are shown in Table 6.4. Notice that due to the inclusion of human-driven vehicles, whose behaviour is not determined by cost optimisation, we omit the calculation and comparison of overall cost in this experiment.

The ACC vehicle only reacts to its predecessor and uses constant speed heuristics to
Figure 6.11: Experiment 7: (a) incurred cost \( J \), (c) speed and (e) acceleration of representative vehicles in the platoon with homogeneous ACC followers and (d) incurred cost, (d) speed and (f) acceleration of representative vehicles in the platoon with homogeneous C-ACC-HP followers with \( w = 1 \) in the accelerating scenario.
predict the future behaviour of the predecessor. The C-ACC-MI vehicle uses *constant speed heuristics* to predict the behaviour of its predecessor, but uses dynamic speed heuristics to predict its follower behaviour. It assumes the human follower behaves as an ACC controller and reacts on the acceleration of the C-ACC-MI vehicle, while in reality the human follower moves according to the Helly model.

It is interesting to see that the C-ACC-MI controller still results in better platoon performance compared to ACC controller despite the imperfect knowledge of the human follower behavioural rule. As shown in Figure 6.12(a)(b)(e)(f), the overshooting phenomena in the decelerating and accelerating phases are less in platoon (d) than those in platoon (c). The amplitude of the speed differences in platoon (d) are attenuated faster than in platoon (c). In decelerating phase, the speed difference is 0.11m/s for the first follower and 0.08m/s for the last follower in platoon (c), while the first follower in platoon (d) has a larger speed difference of 0.25m/s, the last follower in platoon (d) only has a speed difference of 0.02m/s. In other words, the cooperative vehicles in platoon (d) sacrifice their own situation to benefit the sequel followers in the platoon. The same behavioural characteristics of the C-ACC-MI behaviour are observed in the accelerating phase.

The prediction of the human follower behaviour results in earlier reaction of the C-ACC-MI vehicles to the disturbances. As we can see from the acceleration profiles in Figure 6.12(c)(d)(g)(h), the 5th and 10th followers in platoon (d) start to decelerate/accelerate earlier than their counterparts in platoon (c) in the deceleration/acceleration phase. This has potentials in damping out the disturbances.

The predictive and considerate behaviour of the C-ACC-MI vehicles in platoon (d) results in smoother following behaviour under decelerating and accelerating disturbances. The speed variance of platoon (d) is smaller than that of platoon (c) in both decelerating and accelerating scenarios. The smoothing effect is more evident in the accelerating phase due to the increased sensitivity of the C-ACC-MI controller to the relative speed with \( w = 1 \).

Plotting the hysteresis loops of the mixed platoons gives more insights into the changes in the phase transitions. Figure 6.13 shows the evolution of platoon trajectory in the average gap-speed plane. As shown in the figure, both loops evolve clockwise. Platoon (d) maintains a slightly larger gap during the deceleration transition and significantly smaller gap during the acceleration transition compared to the reference platoon (c). The mean time headway and time gap at upstream section are 1.72s and 1.42s in platoon (d), larger than 1.70s and 1.40s in platoon (c). The mean time headway and time gap of platoon (d) at downstream section are 1.77s and 1.44s, smaller than 1.77s and 1.40s of platoon (c).

The comparison of the platoon performance in the mixed platoons reassures the benefits of the cooperative controller proposed in this chapter in terms of traffic flow operations. The cooperative controller stabilises traffic and reduces inflow at upstream congestion front and increases outflow at downstream congestion front compared to
Figure 6.12: Experiments 8 and 9: (a) speed and (c) acceleration in the decelerating phase of mixed platoon with ACC vehicles; (b) speed and (d) acceleration in the decelerating phase of mixed platoon with C-ACC-MI vehicles; (e) speed and (g) acceleration in the accelerating phase of mixed platoon with ACC vehicles; (f) speed and (h) acceleration in the accelerating phase of mixed platoon with C-ACC-MI vehicles.
the non-cooperative controller.

Figure 6.13: Experiments 10 and 11: hysteresis loops in the average speed-gap \((v(t), s(t))\) plane for ACC + human followers (red line) and C-ACC-MI + human followers (blue line) with \(w = 1\). Arrows indicate the evolution direction of the loops.

6.6.3 Computational experiments

Figure 6.14 shows the CPU time needed for simulation of 100 seconds on a desktop computer (Intel Core(TM)2 i5-2400, 3.10 GHz) for the deceleration transition and acceleration transitions of the ACC and C-ACC-HP platoons with different platoon sizes. When the number of controlled vehicles increases, the dimensionality of the state space of the ACC and C-ACC-HP platoon increases. As a result, the computation time for ACC and C-ACC-HP algorithms increases as well. The computation time for the C-ACC-HP platoon is larger than their ACC counterpart, since each C-ACC-HP vehicle calculates not only its own marginal cost but the marginal cost of its follower. It is interesting to see that the increase in computation time is linearly for both ACC and C-ACC-HP controllers and even when 20 followers are controlled simultaneously, equivalent to a large system state with cardinality of 40, the simulation can still be performed in a real-time setting with the test computer. This shows the efficiency and the scalability of the iPMP algorithm.

It should be noted that the computer used in these experiments is much more powerful than on-board computers in modern cars. However, the finding that the computational load increases linearly with the increase of cardinality of the system state holds regardless of the computer used in the test. In addition, the algorithms are implemented in Matlab and are based on centralised optimisation for all vehicles. There are potentials in speeding up the algorithms such as implementing the algorithm in more efficient languages and in a distributed fashion.
Figure 6.14: Experiments 10 and 11: CPU time (Computation time) as a function of the number of followers using the iPMP algorithm for the ACC and C-ACC-HP platoons, simulation period of 100 seconds with $\alpha = 0.01$ and $\epsilon_{\text{max}} = 0.1$.

### 6.7 Conclusions

In this chapter, we propose and test an ACC controllers with variable time gaps and two cooperative ACC controllers (C-ACC-HP and C-ACC-MI) that capture vehicle-vehicle cooperation under the generic control framework. A flexible numerical solution based on Pontryagin’s Minimum Principle (iPMP) is proposed and applied to control design of both autonomous and cooperative vehicle systems. Key research questions regarding the performance of the proposed controllers and the solution algorithm are identified and answered through a series of simulation experiments.

To verify the ACC performance at representative scenarios, we simulated a leader-follower pair where the leader behaviour is predefined. Simulation results shows that the proposed ACC controller generates desired performance under free driving conditions and prevents rear-end collision with bounded deceleration when approaching a standstill leader with free speed. The ACC controller also exhibits plausible follow-the-leader behaviour in normal driving situations with decelerating and accelerating disturbances.

To identify the influence of control parameters on controller performance, heuristic sensitivity analyses of the control parameters were conducted. It is found that the weight of efficiency cost in cruising mode determines how fast an ACC accelerate to the free speed. Increasing weight of the safety cost $c_1$ implies more anticipative driving style and increasing weight of the efficiency cost in following mode implies more agile
driving style. The weight factors influence the stability of the ACC controller. The desired time gap and maximum braking of the controlled vehicle should be designed together to avoid rear-end collision at safety-critical conditions.

To show the potential benefits of the cooperative control strategy, performance of a platoon with different controllers under decelerating and accelerating transitions were examined. Compared to the non-cooperative ACC controller, the predictive and cooperative nature of the C-ACC-HP and C-ACC-MI vehicles results in earlier reaction to disturbances smoother following behaviour under decelerating and accelerating scenarios. This leads to a larger time headway upstream of the low speed state in the decelerating transition and a smaller time headway downstream of the low speed state. Simulation results support the benefits of the proposed cooperative systems in stabilising traffic flow at upstream front of congestion, and increasing queue discharge rate at downstream front of congestion.

To verify the computational complexity of the solution algorithm, computation times for different number of controlled followers are recorded and compared. The results show that the solution algorithm can deal with large scale systems and the increase of the computational complexity is proportional to the increase of system state cardinality.

Our work demonstrates that the control framework and the solution algorithm are flexible and can be easily extended to cooperative systems. With proper formulation, the ACC controller generate plausible behaviour in all representative scenarios and the cooperative ACC controllers improve the platoon performance and have potentials to improve macroscopic flow operations. The changes in the platoon performance involving cooperative vehicles are caused by the fact that the downstream cooperative vehicles compromise their own situations to benefit the upstream vehicles, which is the nature of cooperation. It is desirable to take the follower’s behaviour into consideration, since the cooperative behaviour leads to better performance of the platoon, even though the prediction of the follower’s behaviour is not perfect.

Note that in this chapter we focus on performance and characteristics of the proposed ADAS controllers at microscopic and platoon levels. The sequence of the controlled vehicles and human-driven vehicles in the platoons of the experiments is predefined and the simulation is conducted on a single-lane scenario. Although the functionality of the cooperative controllers is not dependent on the order of the followers and the network layout, the quantitative effects may change due to the distribution of controlled vehicles in traffic and simplification of the road network. In the following chapter, we investigate the macroscopic impact of ADAS on traffic operations and sustainability in more realistic scenarios with controlled vehicles randomly distributed on a multi-lane motorway stretch.
Chapter 7

Impacts of ADAS systems on traffic operations and sustainability

In the previous chapter, we designed ACC and C-ACC controllers and tested the controller performance in a large platoon consisting of controlled and uncontrolled vehicles in a predefined order. While the simulation results show the characteristics of the proposed controllers at microscopic and platoon levels, the experimental set-up does not capture scenarios with lane-changing manoeuvres and the variability of platoon composition due to random distribution of intelligent vehicles (IVs) in networks. Hence the question of what are the impacts of ACC and C-ACC systems on traffic flow operations and sustainability has only been partially answered so far. In this chapter, we simulate and analyse traffic dynamics at a bottleneck with ACC/C-ACC vehicles randomly distributed on a two-lane motorway. The focus is on the formation and propagation of a jam type widely observed in real world, the stop-and-go wave, which is highly related to the flow stability, capacity and fuel consumption. The simulation of the 14 km motorway stretch forms a traffic system with more than 500 vehicles running in the network. To realise simultaneous control of many ACC and C-ACC vehicles in the large scale system, decentralised and distributed algorithms are implemented for ACC and C-ACC controllers respectively.

This chapter is structured as follows. Section 7.1 presents the assumptions and operational algorithms of ACC and C-ACC controllers in microscopic traffic simulations. Section 7.2 describes the simulation experimental set-up. The simulation results regarding the impacts on dynamic traffic operations are presented in Section 7.3. Section 7.4 concludes the findings.
7.1 Assumptions and algorithms for ACC and C-ACC controllers

To study the impacts of ACC and C-ACC systems on traffic operations and sustainability, we assume that a certain proportion of vehicles in the network are equipped with ACC or C-ACC systems developed in the previous chapter. We only consider scenarios where ACC systems and C-ACC systems are not present in the network at the same time in order to separate the impacts of the two systems. In the remainder of this section, we describe the operations of ACC and C-ACC systems and present the corresponding assumptions and implementable algorithms in microscopic simulations.

7.1.1 ACC controller and decentralised algorithm

An ACC vehicle detects the gap and speed difference with respect to the predecessor solely based on its own on-board sensors, e.g. forward-looking radar, and do not communicate with other vehicles. The ACC vehicle predicts the behaviour of its predecessor based on the current gap and vehicle speeds with constant speed heuristics and determines its optimal accelerations to minimise its own cost.

The ACC system operates in two modes, being following mode and cruising mode. In cruising mode, the ACC system aims to maximise driving efficiency and comfort, while in following mode the system aims to maximise safety in addition to efficiency and comfort. The two modes are distinguished by a critical gap \( s_f \) determined by the controller parameters of free/desired (cruising) speed \( v_0 \) and the maximum desired gap \( t_{d,m} \). The ACC controller formulation has been detailed in Section 6.2. It is noteworthy that both variable time gap and constant time gap policies can be operationalised under this formulation.

When simulating a large scale system with many ACC vehicles as subsystems, the subsystems interact with each other via state vectors, i.e. each ACC vehicle determines its accelerations based on the relative position and speed with respect to the preceding vehicle. This feature allows straightforward decentralised implementation of the ACC control algorithm in microscopic simulation models, i.e. at each time instant \( t_k \), each subsystem/vehicle solves its local non-cooperative optimal control problem of Eqs. (6.3, 6.6) subject to system dynamics equation (6.5), state and control constraints (6.2), and initial conditions (6.4), using the constant speed heuristics to predict the trajectory of its predecessor. The iterative numerical solution algorithm based on Pontryagin’s Minimum Principle (iPMP) (cf. Section 6.3) is used to compute the optimal control/acceleration trajectory, and only the first portion of the acceleration trajectory is implemented to updated the system state. The optimal accelerations are re-calculated after each control cycle in a receding horizon manner, using newest information regarding the system state available from on-board sensors. Details of the ACC controller formulation and the solution algorithm have been described in Section 6.2 and
6.3 respectively.

It is assumed that the lane-changing decisions of ACC vehicles are made and executed by human drivers. Hence in microscopic simulations, the longitudinal motions of ACC vehicles are updated using the ACC algorithm rather than car-following models, while the lane-changing logics are the same as human-driven vehicles.

7.1.2 C-ACC controller and distributed algorithm

Two C-ACC controllers have been designed in Chapter 6, being the C-ACC-HP (Cooperative ACC in Homogeneous platoon with Perfect knowledge of follower behaviour) controller as depicted in Figure 6.1(c) and the C-ACC-MI (Cooperative ACC in Mixed platoon with Imperfect knowledge of follower behaviour) controller as depicted in Figure 6.1(d). The C-ACC controller considered in this chapter combines the ACC controller, C-ACC-HP controller and C-ACC-MI controller. Each C-ACC vehicle is assumed to be equipped with both a forward-looking sensor detecting the gap and relative speed with respect to its predecessor and a backward-looking sensor detecting the gap and relative speed of its follower. Furthermore, each C-ACC vehicle is equipped with a V2V communication unit to transmit and receive state and control information. The V2V communication is assumed to be perfect and the communication delay is negligible compared to the control cycle of C-ACC controllers. To summarise, the C-ACC controller in this chapter functions as:

(i) Non-cooperative ACC controller if there is the gap of its follower is larger than the gap threshold \( s_f \), i.e. no interaction or cooperation between the leader-follower pair is considered in cruising mode.
(ii) C-ACC-HP controller if conditions (i) is not satisfied and the follower is also a C-ACC vehicle.
(iii) C-ACC-MI controller if conditions (i) is not satisfied and the follower is an uncontrolled vehicle driven by a human driver. Hence no V2V communication prevails between the C-ACC vehicle and its follower.

When controlling a large scale system with many C-ACC vehicles\(^7\), it is \textit{practically infeasible} for the \textit{centralised} implementation of the cooperative algorithm minimising the performance of the whole system due to the computation and communication requirements (Dunbar & Murray, 2006; Scattolini, 2009). Therefore, the C-ACC algorithm is implemented in a \textit{distributed} fashion, where each vehicle as a subsystem solves a local cooperative optimal control problem simultaneously, taking into account the predicted dynamics of its direct predecessor and follower (Dunbar & Murray, 2006; Scattolini, 2009).

\(^7\)In the scenario with 100% C-ACC vehicles, we consider simultaneous control of more than 500 controlled vehicles on a two-lane motorway of 14-km length.
When operating in condition (i), the implementation of the C-ACC algorithm is the same as the aforementioned ACC algorithm. Therefore we only detail the algorithms in conditions (ii) and (iii) below.

At each time instant $t_k$, a C-ACC vehicle $n$ solves its local cooperative optimal control problem of Eqs. (6.3, 6.12) subject to system dynamics equation (6.11), state and control constraints (6.2), and initial conditions (6.4) using the iPMP solution algorithm (cf. Sections 6.2, 6.3). Solving the local optimal control problem entails predicting the dynamics of its predecessor, vehicle $n-1$. If the predecessor is an uncontrolled vehicle, its dynamics is predicted with the constant speed heuristics. If the predecessor is a cooperative vehicle, the predecessor transmits its most recent state ($s_{n-1}(t_k)$, relative speed $\Delta v_{n-1}(t_k)$) and its assumed control/acceleration trajectory $\hat{u}_{n-1}[t_k, t_k+T_P]$ in the prediction horizon $T_P$ obtained from the last time instant $t_{k-1}$ to the considered vehicle $n$ via V2V communication before the local iPMP solution procedure starts. Vehicle $n$ also receives current gap and speed of its following vehicle $n+1$ via V2V communications if vehicle $n+1$ is a cooperative vehicle, i.e. condition (ii), or from on-board sensors if vehicle $n+1$ is a human-driven vehicle, i.e. condition (iii). With all the information available, vehicle $n$ predicts the predecessor dynamics with $\hat{u}_{n-1}[t_k, t_k+T_P]$ and determines its own optimal trajectory $u_n[t_k, t_k+T_P]$ by the iPMP algorithm. Likewise, when operating in condition (iii), vehicle $n$ also transmits its current gap $s_n(t_k)$ and relative speed $\Delta v_n(t_k)$ and its assumed control trajectory $\hat{u}_n[t_k, t_k+T_P]$ obtained from the last time instant $t_{k-1}$ to its cooperative follower before each optimization to facilitate the decision-making of vehicle $n+1$.

For each receding horizon update, every distributed optimal control problem is solved synchronously, i.e. at the same time instant. At the next time instant $t_{k+1}$, the aforementioned procedure repeats, whereas the cooperative vehicles update and transmit the most recent assumed control trajectory with $\hat{u}_{n-1}[t_{k+1}, t_{k+1}+T_P] = u_{n-1}[t_{k+1}, t_{k+1}+T_P]$ and $\hat{u}_n[t_{k+1}, t_{k+1}+T_P] = u_n[t_{k+1}, t_{k+1}+T_P]$. Note that in this way, the optimisation and communication are limited to vehicles directly following each other (Dunbar & Murray, 2006; Scattolini, 2009; Muller et al., 2012).

The same as the ACC vehicles, lane change decisions of C-ACC vehicles are determined and executed by human drivers through steering wheels. For cooperative vehicles in microscopic simulations, the accelerations are computed by the iPMP algorithm in a receding horizon way and a communication channel needs to be modelled through which the current gap, relative speed and the predicted acceleration trajectory are transmitted to their cooperative peers.

Here we complete the description on implementation of the algorithms for ACC and C-ACC controllers. In the sequel, we will present the simulation experimental set-up for assessing impacts of ACC and C-ACC controllers on traffic flow characteristics near a bottleneck, followed by discussions on simulation results.
Chapter 7. Impacts of ADAS systems on traffic operations and sustainability

7.2 Experimental set-up

This section describes the simulation model and the experimental set-up for assessing traffic dynamics under a bottleneck, with a focus the formulation and propagation of moving jams.

7.2.1 Bottleneck and necessary modelling aspects

In real traffic, jams are caused by bottlenecks (Treiber & Kesting, 2013). A bottleneck is defined as a local reduction of the road capacity (Treiber & Kesting, 2013), which can be permanent or temporary. Permanent bottlenecks are usually caused by inhomogeneity in road infrastructure, such as on-ramps and off-ramps, waving areas, curves, and uphill and downhill gradients. Temporary bottlenecks are usually caused by accidents, roadworks or temporary change in traffic regulations such as speed limits. We choose the second type of bottleneck induced by temporal change of speed limits in this study, since the focus of the thesis is on longitudinal driving control (cf. Chapter 1) and the car-following manoeuvres determines the traffic flow dynamics at this type of bottleneck to a great deal.

When a bottleneck is activated, traffic breaks down at the bottleneck and congested traffic forms at upstream of the bottleneck. Different congested states or jam patterns at motorway bottlenecks have been reported and defined in literature (Treiber et al., 2000), which can be in general categorised into two types: one with an upstream moving downstream front (jam head) and upstream front (jam tail), which is often called shock wave or stop-and-go wave or moving jam, and one with fixed downstream front at the bottleneck location (Treiber & Kesting, 2013). The first type of jam is the focus of this study, since it is highly related to the flow stability and capacity of traffic where the proposed controllers have potentials to influence.

The jam head of a shock wave propagates against the driving direction with a characteristic velocity in the order of -10 to -20 km/h\(^8\) and the propagation velocity of the jam tail depends on the inflow and outflow of the bottleneck (Lighthill & Whitham, 1955; Cassidy & Bertini, 1999; Leclercq et al., 2011; Treiber & Kesting, 2011, 2013). One important feature of traffic flow operations at bottlenecks is the so-called capacity drop or two capacities phenomenon. The capacity drop refers to the phenomenon that the maximum outflow observed downstream of a jam (referred to as queue discharge flow) is usually smaller than the maximum flow observed before traffic breaks down to congestion (Cassidy & Bertini, 1999; Treiber et al., 2000; Tampère, 2004; Treiber & Kesting, 2013). The discrepancy is reported to be around 10 - 30\% (Cassidy & Bertini, 1999; Tampère, 2004). Although the discussions on the capacity drop phenomenon have last for decades (Hall et al., 1986; Daganzo et al., 1999; Leclercq et al., 2011; 8The minus sign indicates that the shock wave is propagating in the upstream direction, i.e. against the vehicle travelling direction.
Laval & Daganzo, 2006; Treiber et al., 2006b; Coifman & Kim, 2011), one plausible microscopic explanation for the capacity drop is that drivers tend to keep a larger gap in the transition from an equilibrium state with low speeds to an equilibrium state with high speeds and keep a smaller gap when vice versa, i.e. the microscopic hysteresis (Treiterer & Myers, 1974; Chen et al., 2012).

The aforementioned traffic flow properties, in particular the backward propagating shock wave and capacity drop phenomena, should be resembled by the traffic simulation model. Furthermore, multiple driver-vehicle classes, e.g. human drivers, ACC and C-ACC systems, should be distinguished in the model. In the following, we describe the chosen simulation model that is able to address these modelling issues.

7.2.2 Simulation model and network settings

We choose a simulation model called MOTUS for the impact study. MOTUS is an open-source microscopic traffic simulation package that is developed in Java (Schakel et al., 2010, 2012). Among many other features, the MOTUS model uses an improved Intelligent Driver Model (IDM) as the car-following module (Schakel et al., 2010), hereinafter referred to as IDM+, and a lane change model with relaxation and synchronisation (Schakel et al., 2012), hereinafter referred to as LMRS. IDM+ generates more realistic shock wave patterns and macroscopic capacity compared to the original IDM (Treiber et al., 2000). LMRS integrates the IDM+ for a complete microscopic traffic model and resembles better multilane traffic at a macroscopic level regarding the amount of traffic volume per lane, the traffic speeds across lanes and the onset of congestion at bottlenecks. For details of the models, we refer to Schakel et al. (2010, 2012).

Simulation is set up in analogy to a long motorway stretch where shock wave is the major type of jams. The road network is a two-lane motorway of 14 km, with a demand of 1900 veh/h on both lanes. Loop detectors are placed every 250 metres on each lane along the motorway, collecting flow and time mean speed every 30 seconds.

A bottleneck is created by posting low speed limits on VSL gantries on parts of the motorway. The speed limits are activated for 2 minutes, with speed values of 80 km/h, 60 km/h and 40 km/h displayed at the location of 11 km, 11.5 km and 12 km respectively.

Parameters for IDM+ in MOTUS is chosen based on the face validation on the resultant capacity drop and shock wave propagation characteristics, as we will show in Section 7.3.1.

7.2.3 Experimental scenarios

The variables to be tested for the impact study is the controller type (ACC or C-ACC), the penetration rate of controlled vehicles in traffic (5%, 10%, 50%, 100%). Together
Table 7.1: Experimental scenarios for impact study of ACC/C-ACC systems

<table>
<thead>
<tr>
<th>Scenarios</th>
<th>Traffic composition</th>
<th>Desired time gap (s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>100% human drivers</td>
<td>1.3</td>
</tr>
<tr>
<td>2</td>
<td>5% ACC + 95% human drivers</td>
<td>1.3</td>
</tr>
<tr>
<td>3</td>
<td>10% ACC + 90% human drivers</td>
<td>1.3</td>
</tr>
<tr>
<td>4</td>
<td>50% ACC + 50% human drivers</td>
<td>1.3</td>
</tr>
<tr>
<td>5</td>
<td>100% ACC</td>
<td>1.3</td>
</tr>
<tr>
<td>6</td>
<td>5% CACC + 95% human drivers</td>
<td>1.3</td>
</tr>
<tr>
<td>7</td>
<td>10% C-ACC + 90% human drivers</td>
<td>1.3</td>
</tr>
<tr>
<td>8</td>
<td>50% C-ACC + 50% human drivers</td>
<td>1.3</td>
</tr>
<tr>
<td>9</td>
<td>100% C-ACC</td>
<td>1.3</td>
</tr>
</tbody>
</table>

with the reference scenario with 100% human drivers, this amounts to 9 simulation scenarios as shown in Table 7.1.

As we have discussed earlier Section 2.5, the desired time gap settings for ACC and C-ACC systems impact the road capacity and traffic flow stability. For ACC and C-ACC systems developed in the previous chapter, there are two desired time gap parameters, being the minimum desired time gap \( t_{d,0} \) at highest density and the maximum desired time gap \( t_{d,m} \) at the density corresponding to the gap threshold between cruising mode and following mode. In this study, we choose the same desired time setting as human drivers in MOTUS with \( t_{d,0} = t_{d,m} = 1.3s \). This leads to the same equilibrium flow-density relation and theoretical capacity at the macroscopic level. Hence, the potential changes in dynamic traffic operations depend predominantly on the accelerating and decelerating characteristics of ACC/C-ACC vehicles from one equilibrium state to another. This allows us to investigate the potentials of advanced model predictive control strategies of ACC/C-ACC vehicles in improving traffic operations without setting smaller time gaps.

### 7.2.4 Assessment indicators

Impact on traffic flow will be assessed by the following indicators:

- Total time spent (TTS) in network. TTS is calculated from the vehicle trajectories of the simulation as:

  \[
  TTS = \sum_{n=1}^{N_{veh}} ts_n
  \]

  where \( ts_n \) denotes the time spent in the network for vehicle \( n \) and \( N_{veh} \) denotes the total number of vehicles generated in the network during the simulation period.
- **Average outflow:** $\bar{Q}_{\text{out}}$. This is measured by the most downstream detector on the motorway stretch and averaged in the simulation period. For homogeneous motorway stretch, this gives an indication on the effective capacity at the bottleneck.

- **Jam area:** $A_{\text{jam}}$. Jam area is calculated with

  \[ A_{\text{jam}} = \sum_{i=1}^{K_{\text{sim}}} L_{\text{jam},i} \cdot dT \]  

  where $dT$ is the detector aggregation time, which is 30s in this case and $K_{\text{sim}}$ is the number of aggregation time intervals for the whole simulation period. $L_{\text{jam},i}$ is the spatial length of the jam at aggregation interval $i$, and the location of a jam is present is detected by:

  \[ V_{i,j} \leq V_{\text{max}} \]  

  where $V_{i,j}$ is the detector speed at time interval $i$ and location $j \cdot dX$ and $dX$ denotes the distance between loop detectors, which is 250 metres in this case. $V_{\text{max}}$ is the speed threshold to distinguish free flow and congested traffic, which is 50 km/h. Note that when there is no jam in the network, $A_{\text{jam}} = 0$.

- **Flow** $Q_{\text{jam}}$ and speed $V_{\text{jam}}$ of jam area. These are the average flow and speed of the whole jam area determined by the speed threshold $V_{\text{max}}$, which indicate the traffic state in the jam.

- **Downstream jam front velocity** $C_{\text{head}}$. This is measured with the position of jam head determined by $V_{\text{max}}$, which reflects how fast the resultant travels. To avoid large estimation errors due to small jam size, we only calculate $C_{\text{head}}$ for jams that last longer than 5 minutes.

- **Mean absolute speed difference across lanes** $DV_{\text{lane}}$ for the network. This gives an indication on the inhomogeneity of traffic states across lanes.

For sustainability indicators, we focus on average spatial fuel consumption rate per vehicle (AFC). To this end, a modal fuel consumption model is employed, because it captures the operational characteristics of vehicle engines and uses instantaneous vehicle speed and acceleration to calculate (temporal) fuel consumption rate (Akcelik, 1989). All the model parameters are available in (Akcelik, 1989). Assuming the specific engine type in (Akcelik, 1989), this model estimates instantaneous fuel consumption rate of vehicle $n$, $F_{t\text{n}}$, as a function of vehicle speed $v_{n}$ and acceleration $a_{n}$:

\[
F_{t\text{n}} = \begin{cases} 
\sum_{j=0}^{3} b_{j} v_{n}^{j} + c_{1} v_{n} a_{n} + c_{2} v_{n}^{2} a_{n}^{2} & \text{if } a_{n} \geq 0 \\
\sum_{j=0}^{3} b_{j} v_{n}^{j} + c_{1} v_{n} a_{n} & \text{if } a_{n} < 0 
\end{cases} 
\]  

where $b_{j}$ and $c_{j}$ are model parameters. For details of the fuel consumption model, we refer to Akcelik (1989). The AFC is calculated by dividing total fuel consumption with the total distance travelled by all vehicles in the network.
Apart from the indicators, we also visualise the simulation results in different scenarios, including the spatial-temporal contour plots of flow $Q$ and speed $V$, flow-density scatter plots. In the flow-density plots, we differentiate the whole motorway into upstream, downstream and jam regions when a jam prevails, which gives insights into the differences in the inflow and outflow of a jam area. Furthermore, to gain insights into the changes at microscopic level, we plot the average gap and speed relationships in different scenarios. Particularly, we distinguish the trajectory data samples into the acceleration state, deceleration state and equilibrium state. The different states for each vehicle $n$ at time $t$ are differentiated with an acceleration threshold of $0.01 \, m/s^2$ in acceleration:

- if $a_n > 0.01$, the vehicle is in acceleration state;
- if $a_n < -0.01$, the vehicle is in deceleration state;
- if $a_n \in [-0.01, 0.01]$, the vehicle is in equilibrium state.

The trajectory data points are aggregated into the same spatial and temporal length of the detectors with different states.

7.3 Simulation results on dynamic traffic operations

This section describes the simulation results, with a focus on impacts of ACC and C-ACC systems on formation and propagation of shock wave at the bottleneck. First, the traffic states and jam patterns in the reference case is described, showing the face validity of the simulation model. Then the flow characteristics with ACC and C-ACC systems are discussed subsequently.

7.3.1 Verification of the reference scenario

In this study, a bottleneck is activated by temporarily changing speed limits on part of the motorway stretch, resulting in a lower road capacity than the demand. Figure 7.1(a)(b) show the spatio-temporal evolution of flow and time mean speed per lane on the simulated motorway collected from loop detectors. As we can see from the figures, the traffic speed drops from $120 \, km/h$ to about $40 \, km/h$ at around $12 \, km$ from 15 minutes, due to the change in speed limits. The speed limits cause vehicles in the bottleneck area to slow down, increasing the density in the bottleneck area and limiting the outflow from the bottleneck. After the vehicles move out of the bottleneck, they start to accelerate back to the free speed. This leads to a wave propagating downstream with low flow and high speed in the flow and speed contour plots. The bottleneck is only active for two minutes. Directly after the release of the bottleneck, the desired speeds of vehicles in the bottleneck switch back to $120 \, km/h$. The vehicles in the
Figure 7.1: (a) Flow contour and (b) time mean speed contour plots, and (c) flow-density plots and (d) gap-speed plots in Scenario 1 with 100% human drivers.

bottleneck with high density start to accelerate consequently, leading to a high flow and high speed state propagate downstream, as shown in Figure 7.1(a)(b). Traffic flow theorists have shown that the high density state with high flow is not stable (Treiber & Kesting, 2013; Wilson, 2008). After a while, traffic breaks down with speeds degrading gradually, which leads to a persistent shock wave propagating backwards against the driving direction (Treiber & Kesting, 2013; Wilson, 2008).

The average traffic speed and flow in the jam (shock wave) is 11.7 km/h and 402 veh/h respectively, with the jam head, or the downstream jam front, propagating with a characteristics velocity of -11.8 km/h, as shown in Table 7.2. The traffic state downstream of the jam area is quite homogeneous, i.e. there are hardly any variations in the flow and speed, as shown in Figure 7.1(a) and (b).

The capacity drop phenomenon is clearly visible in the flow contour plot of Figure 7.1(a) and the flow-density scatter plot of Figure 7.1(c), i.e. outflow from the jam maintain an average value of 1647 veh/h, which is significantly lower than the average inflow of 1900 veh/h. The discrepancy between outflow and the capacity is in accordance with the reported values of around 10 - 30% (Cassidy & Bertini, 1999; Tampère, 2004; Treiber & Kesting, 2013). Due to this discrepancy, the size of the jam increases with the course of time, with the upstream jam front (jam tail) travelling...
upstream with a faster speed than that of the downstream jam front (jam head). This leads to a total jam size of $41.7 \text{ veh} \cdot \text{h}$.

It is commonly accepted that the microscopic explanation of the capacity drop phenomenon is that drivers keep a larger gap in accelerating phase compared to decelerating phase, i.e. the microscopic hysteresis phenomenon (Treiterer & Myers, 1974; Chen et al., 2012). This common phenomenon is reproduced in our simulation, as depicted by the average gap and speed scatter plots in the decelerating and accelerating phase in 7.1(d).

### 7.3.2 Impacts of ACC systems on flow characteristics

Compared to the reference case, scenarios 2-5 with different penetration rate of ACC systems bear some resemblance regarding flow characteristics at the bottleneck. The activation of the bottleneck creates a traffic wave of low flow and high speed traveling downstream and followed by a wave with high flow and high speed propagating downstream immediately after the release of the bottleneck. The bottleneck leads to backward propagating shock waves in all ACC scenarios, as shown with one representative simulation run in Figure 7.2.

There are several differences that need special attention regarding the traffic dynamics in scenarios with ACC systems compared to the reference case. We discuss the qualitative differences with Figure 7.2, Figure 7.3, and the quantitative differences with indicators of Table 7.2. We structure the results into traffic efficiency, stability and jam propagation, and sustainability.

#### Traffic efficiency

ACC systems increase traffic efficiency and mitigate the capacity drop phenomenon. The average total time spent (TTS) in the network and jam sizes in scenarios 2-5 are much smaller than the reference scenario. The TTS and jam size decrease to 477.4 $\text{veh} \cdot \text{h}$ and 24.9 $\text{km} \cdot \text{min}$ from 562.8 $\text{veh} \cdot \text{h}$ and 41.7 $\text{km} \cdot \text{min}$ respectively in scenario 2 and decrease further to similar values in scenarios 3 and 4. The TTS and jam size are reduced to only 443.8 $\text{veh} \cdot \text{h}$ and 5.3 $\text{km} \cdot \text{min}$ when all vehicles are controlled by the ACC system.

The outflow from jams in scenarios 2 is higher than that of the reference scenario, but still lower than the inflow. Hence the capacity drop and microscopic hysteresis phenomenon remains in scenario 2, as we can see from Figure 7.3(a)(b). As a result, the spatial size of the jam increases with the course of time in scenario 2, as shown in Figure 7.2(a)(b). When the penetration rate increases to 10% and higher, the outflow increases more or less to the value of the inflow, resulting in shock waves with more or less constant sizes, as we can see from Figure 7.2(d)(f)(h). Hence capacity drop is not pronounced in scenarios 3, 4 and 5, which is also evidenced by the flow-density plots.
of Figure 7.3(c)(e)(g) and gap-speed plots of Figure 7.3(d)(f)(h). The reason for this is that the formulation and parameter settings for ACC controllers result in a more agile driving style, i.e. the ACC vehicles accelerate towards the high speed state faster than human drivers under same conditions of gaps, relative speeds and speeds, which leads to smaller headway and hence higher flow in the acceleration transition. Furthermore, the human-driven vehicles following ACC vehicles are convicted to the follow-the-leader rule in mixed traffic scenarios, and hence are dragged by the ACC vehicles to keep smaller headways than their do in the reference scenario 1, which consequently increases the outflow from the jam area.

Stability and jam propagation

As we can see from Figure 7.2 and Table 7.2, the traffic speed and flow in the jam of scenario 2 are higher than those in scenario 1. This implies that the ACC vehicles improves the traffic flow stability of the jam area, i.e. the traffic does not break downs to the speed as low as the reference case. When the penetrate rate increases to 10% or higher, the stabilisation effects are more pronounced, with average speed in the jam area stays around bottleneck speed of 40 km/h. The stability effects can be explained by the controller design of ACC systems. As we have explained in the previous chapter, in decelerating transitions, the safety cost due to approaching the preceding vehicle dominates the ACC vehicular behaviour (cf. Section 6.5). Hence ACC vehicles are more sensitive to the relative speed in the decelerating phase, exhibiting a more anticipative driving style which stabilises traffic flow approaching the jam tail.

Regarding the propagation of the jam head, although the size and period of the jams are different, they all propagate in the upstream direction after the start of the bottleneck. However, unlike the reference case where the jam head propagates upstream with a constant characteristic velocity, the jam propagation velocity differs with different penetration rate of equipped vehicles across scenarios, as shown in Figure 7.2 and Table 7.2. In scenario 2, the jam head propagates faster than that of scenario 1 while in scenario 3, 4 and 5, the jam heads propagate slower than in scenario 1. In scenario 5 as shown in Figure 7.3(h), the shock wave first propagates upstream from 17 minutes to about 25 minutes, then it gradually changes its velocity and stays at around 9.75 km for 10 minutes and finally propagates in the reversed direction. After a few minutes the jam dissolves when propagating in the downstream direction.

It is noteworthy that several waves propagating in the downstream direction from the jam head are observed in speed contour plots in ACC scenarios, e.g. from around 40 minutes in Figure 7.2(b), from around 42 minutes onwards in Figure 7.2(d), and from around 34 minutes in Figure 7.2(f). This is quite different compared to the homogeneous traffic speeds downstream of the jam area in the reference scenario 1, as shown in Figure 7.1(b). These waves are not sustained and vanishes in the free flow region after propagating downstream for a few minutes. Although these disturbances does not
result in persistent waves, it does raise some concerns on the stability property at downstream area of the jam. Since the ACC vehicles accelerate faster and keep a smaller gap from the jam area to the downstream free flow area compared to human-driven vehicles, the human-driven vehicles in Scenario 2 that follow ACC vehicles also accelerate faster and maintain a smaller gap compared to the normal situations in the reference case. Although this increases the outflow from the jam area, it may destabilise traffic flow downstream of the jam area.

The existence of ACC vehicles in the mixed traffic scenarios of 2, 3 and 4 increase the inhomogeneity of traffic states across lanes. As we can see from Table 7.2. The average speed differences across lanes in the network are higher in scenarios 2, 3 and 4 than those in scenarios 1 and 5. The inhomogeneity of traffic states across lanes are caused by the intrinsic differences in the car-following rules between human drivers and ACC vehicles.

**Sustainability**

From sustainability perspectives, the reduction of the stop-and-go waves has clear benefits in reducing fuel consumptions, since the accelerating and decelerating manoeuvres and their durations are substantially reduced with the decreasing size of jams. As we can see from Table 7.2, the average spatial fuel consumption rates are reduced when ACC vehicles present in the network, and the benefits increase with the increase of penetration rate in general.
(a) Flow of scenario 2 with 5% ACC

(b) Speed of scenario 2 with 5% ACC

(c) Flow of scenario 3 with 10% ACC

(d) Speed of scenario 3 with 10% ACC

(e) Flow of scenario 4 with 50% ACC

(f) Speed of scenario 4 with 50% ACC

(g) Flow of scenario 5 with 100% ACC

(h) Speed of scenario 5 with 100% ACC

Figure 7.2: Spatio-temporal evolution of flow and speed of ACC with different penetration rate (Scenarios 2 - 5) in one simulation run.
Chapter 7. Impacts of ADAS systems on traffic operations and sustainability

Figure 7.3: Flow-density plots for ACC impact study with different penetration rate (Scenarios 2 - 5) in one simulation run.
Table 7.2: Indicators for different scenarios averaged over ten simulation runs for each scenario

<table>
<thead>
<tr>
<th>Scenarios</th>
<th>TTS (veh·h)</th>
<th>$Q_{out}$ (veh/h)</th>
<th>$A_{jam}$ (km·min)</th>
<th>$Q_{jam}$ (veh/h)</th>
<th>$V_{jam}$ (km/h)</th>
<th>$C_{head}$ (km/h)</th>
<th>$D_{lane}$ (km/h)</th>
<th>AFC (l/100/km)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 (100% Human)</td>
<td>562.8</td>
<td>1647</td>
<td>41.7</td>
<td>402</td>
<td>11.7</td>
<td>-11.8</td>
<td>1.7</td>
<td>4.07</td>
</tr>
<tr>
<td>2 (5% ACC)</td>
<td>477.4</td>
<td>1818</td>
<td>24.9</td>
<td>1082</td>
<td>28.5</td>
<td>-13.2</td>
<td>3.0</td>
<td>3.91</td>
</tr>
<tr>
<td>3 (10% ACC)</td>
<td>453.8</td>
<td>1885</td>
<td>11.5</td>
<td>1586</td>
<td>38.5</td>
<td>-9.3</td>
<td>3.6</td>
<td>3.81</td>
</tr>
<tr>
<td>4 (50% ACC)</td>
<td>454.4</td>
<td>1881</td>
<td>10.9</td>
<td>1629</td>
<td>41.4</td>
<td>-7.7</td>
<td>3.8</td>
<td>3.84</td>
</tr>
<tr>
<td>5 (100% ACC)</td>
<td>443.8</td>
<td>1897</td>
<td>5.3</td>
<td>1600</td>
<td>37.1</td>
<td>-7.8</td>
<td>2.3</td>
<td>3.69</td>
</tr>
<tr>
<td>6 (5% C-ACC)</td>
<td>476.5</td>
<td>1824</td>
<td>26.5</td>
<td>1150</td>
<td>29.9</td>
<td>-14.3</td>
<td>3.4</td>
<td>3.92</td>
</tr>
<tr>
<td>7 (10% C-ACC)</td>
<td>442.3</td>
<td>1896</td>
<td>3.6</td>
<td>1554</td>
<td>41.1</td>
<td>-15.3</td>
<td>3.4</td>
<td>3.67</td>
</tr>
<tr>
<td>8 (50% C-ACC)</td>
<td>435.1</td>
<td>1897</td>
<td>2.8</td>
<td>1420</td>
<td>46.4</td>
<td>-31.44*</td>
<td>1.9</td>
<td>3.55</td>
</tr>
<tr>
<td>9 (100% C-ACC)</td>
<td>433.8</td>
<td>1898</td>
<td>1.9</td>
<td>1456</td>
<td>45.4</td>
<td>n.a.**</td>
<td>1.5</td>
<td>3.49</td>
</tr>
</tbody>
</table>

*: only two jams longer than 5 minutes are observed out of ten simulation runs; **: no jam longer than 5 minutes is observed.
7.3.3 Impacts of C-ACC systems on flow characteristics

Compared to human drivers and ACC systems, C-ACC systems have clear benefits in improving traffic flow operations and sustainability at the bottleneck type in this study.

Traffic efficiency

In scenario 6 with 5% C-ACC vehicles, the average outflow from the jam is 1824 veh/h, which is much higher than 1647 veh/h in the reference case but still lower than the inflow of 1900 veh/h. Hence the capacity drop phenomenon prevails and spatial jam size increases with the course of time, as shown in Figure 7.4(a)(b). The jam size is in the same order as scenario 2.

When the penetration rate of C-ACC systems increases to 10% and 50%, the differences between the C-ACC scenarios and their ACC counterparts become apparent. The average outflow are much higher and the TTS and jam size in scenario 7 are much smaller than those in scenarios 6 and 3. The indicators in scenario 7 with 10% C-ACC systems are even better compared scenario 5 with 100% ACC vehicles. As we have shown in the previous chapter, C-ACC systems generate more anticipative and responsive accelerating behaviour compared to ACC systems, which account for the substantial improvement in traffic efficiency.

In scenarios 8 and 9, after the release of bottleneck, the outflow downstream fo the jam area remains at very high values, and the resultant TTS and jam size decrease with the increase of penetration rate of C-ACC vehicles.

The improvement in traffic efficiency with ACC systems are clearly seen in the flow-density plots. As we can see in Figure 7.5, the data points in jam area decreases with the increase of C-ACC penetration rate, and the data points are less scattered with the increase of C-ACC penetrate rate.

Stability and propagation of jams

The cooperative control strategy of C-ACC systems leads to smoother decelerating behaviour and improves stability of flow approaching the jam. As we can see from Table 7.2, the traffic speed and flow in the jam areas are higher than those in the reference scenario 1 and those with their ACC counterparts. In scenarios 7, 8 and 9, average traffic speeds in the jam are even higher than the bottleneck speed of 40 km/h. This is explained by the controller characteristics, i.e. the smoother behaviour of C-ACC controllers decreases the overshooting effects in the decelerating transition (cf. Section 6.6) and vehicles in scenarios 7, 8 and 9 do not decelerate further to speeds lower than 40 km/h.

Although the bottleneck results in a persistent stop-and-to wave in scenario 6, with only 10% of C-ACC vehicles in the network in scenario 7, the stop-and-go wave dissolves
itself after propagating upstream a few minutes, as we can see from Figure 7.4(c)(d). The shock waves also dissolve themselves in scenarios 8 and 9, as shown in Figure 7.4(e)(f)(g)(h).

Regarding the propagation characteristics of the jam head, although the jam head speed still varies across different scenarios with different C-ACC vehicle penetration rates, it is clear that for all the scenarios where jams longer than 5 minutes are observed, the jam head in C-ACC scenarios propagates with a faster speed compared to the reference case and to their ACC counterparts. The propagation speed also increases with the penetration rate of C-ACC systems. This can be explained by the distributed cooperative algorithm that V2V communication enables, i.e. the current state and the predicted acceleration trajectory of a C-ACC vehicles are transmitted to their C-ACC follower when applicable and are taken into account by the C-ACC follower in determining cooperative optimal accelerations. This allows the C-ACC vehicles react earlier to the downstream disturbances compared to uncontrolled vehicles and ACC vehicles.

Similar to scenarios with ACC systems, we also observe several forward propagating waves originating from the jam head characterised with relative high speeds (between 60 and 80 km/h) and very high flow (around 2000 veh/h), e.g. at about 18 minutes, 24 minutes and 37 minutes in Figure 7.4(c)(d).

Similar to the ACC scenarios, the mixtures of C-ACC and human-driven vehicles in the network increase the inhomogeneity in the traffic states across lanes compared to scenario 1. The average speed difference across lanes are much higher in scenarios 6 and 7, compared to reference scenario 1 and scenarios 8, 9.

**Sustainability**

The improvement in sustainability with C-ACC systems are obvious compared to human drivers, as shown in Table 7.2. In all scenarios with C-ACC systems, the average spatial fuel consumption rates are lower compared to the reference scenario 1. Even compared to ACC system, the benefits are still clear. While the average fuel consumption rate in scenario 6 remains at a similar level compared to scenario 2, the average spatial fuel consumption rates with C-ACC systems in scenarios 7, 8 and 9 are considerably lower than their ACC counterparts in scenarios 3, 4 and 5, as a result of the reduced jam size. This suggests improvement in sustainability of when C-ACC vehicles travel in networks.

**7.3.4 Discussion on changed flow characteristics**

The impact study show some new insights into traffic flow characteristics with ACC and C-ACC systems. In this subsection, we summarise and discuss the changes in flow dynamics at the bottleneck.
Figure 7.4: Spatio-temporal evolution of flow and speed of CACC with different penetration rate (Scenarios 6 - 9) in one simulation run.
(a) Scenario 6 with 5% ACC  
(b) Scenario 7 with 10% ACC  
(c) Scenario 8 with 50% ACC  
(d) Scenario 9 with 100% ACC  

Figure 7.5: Flow-density plots for CACC impact study with different penetration rate (Scenarios 6 - 8) in one simulation run.
Traffic downstream of jam area

Downstream of jam area, vehicles are in acceleration transition from low speed state to high speed state. Our simulation shows that under the designed parameter setting, ACC systems lead to more efficient flow moving out of jam and reduce the capacity drop. The microscopic explanation of this change is the responsive behaviour of ACC systems, i.e. ACC vehicles recovers the high speed faster than human drivers, and thus they follow with a smaller gap in the accelerating phase. However, this is achieved at the expenses of compromising stability, since the decentralised ACC vehicles has limited knowledge of the predecessor behaviour and make simple assumptions in the state prediction, i.e. constant speed heuristics for predecessor. This has a potential risk for triggering new jams at downstream area of the jam with ACC systems.

When employing cooperative controls strategy, C-ACC vehicles have better knowledge of the predecessor behaviour when they are following their C-ACC peer, and hence can predict the dynamics of the predecessor more accurately. As a result, they are able to react earlier to the accelerating stimuli while at the same time preventing the overshooting in the transition from low speed state to high speed state, cf. Section 6.6. This leads to smaller headways in the accelerating phase, but also improves the stability of downstream area of a jam.

Traffic upstream of jam area

Stability/instability of flow approaching the jam tail is important property in the formation of jams. The stability is determined by two counteracting processes: the retarded adaptation to the low speed and the ability to anticipate downstream traffic (Tampère, 2004). For the human-driven vehicular flow, the retarded adaptation outweighs the anticipation, and hence the temporary speed drop leads to formation of jam.

The stability of traffic flow approaching the jam tail is improved with ACC vehicles in the network. This is explained by the controller formulation, when ACC vehicles predicts costs under decelerating disturbances, the weight on safety cost increases with the decreasing gap in Eq. (6.6). Hence the ACC vehicles reacts more to the relative speed with respect to the preceding vehicle. As the relative speed is a simple form of anticipation for the future gap (Treiber & Kesting, 2013; Wang et al., 2013), this implies that ACC vehicles exhibits anticipative driving style in the decelerating phase. Compared to ACC systems, C-ACC systems based on the cooperative control strategy lead to more anticipative and smoother decelerating behaviour by manoeuvring together as a platoon (cf. 6.6), and hence further improve the stability of traffic approaching the jam tail compared to ACC systems.
Propagation of jam

For stop-and-go waves in human-driven vehicular flow, the jam head has a characteristic velocity (Treiber & Kesting, 2013). The jam head velocity changes in scenarios with different penetration rate of ACC systems in traffic. At low penetration rate of 5%, the jam heads travel faster, while at scenarios with 10% or higher ACC vehicles in the network, the jam heads travel slower compared to the reference case. According to kinematic wave theory (Lighthill & Whitham, 1955), the jam head velocity is determined by the traffic state in the jam and traffic state downstream of the jam, both influenced by the specific compositions of traffic (penetration rates and locations of ACC vehicles) in the respective areas. This makes the jam head velocity less characteristic with ACC/C-ACC systems.

Although the jam head velocity also varies with different penetration rate of C-ACC systems, it is quite clear that the jam head travels much faster when C-ACC systems exist in the network due to V2V communication.

Implications for dynamic traffic management

The changes in flow characteristics have implications for dynamic traffic management. Under the same strength of a bottleneck, the ACC and C-ACC system may stabilise the upstream traffic approaching the bottleneck that reduces the probability of traffic breakdown, which consequently lowers the necessity for the traffic controller to intervene. Even if the jam prevails, the more efficient outflow due to the presence of ACC/C-ACC systems reduces the size of the jam, which is also favourable for traffic controllers since this implies less control efforts.

However, possible difficulties are also expected when controlling traffic flow with ACC/C-ACC systems. Firstly, the resultant jam state is difficult to prediction due to the inhomogeneous distribution of controlled vehicles in the network. Furthermore, ACC systems may destabilise traffic flow in the accelerating transition, and hence increase the risk of triggering new jam downstream of the considered jam.

7.4 Conclusions

In this chapter, we tested the ACC and C-ACC algorithms in multilane traffic scenarios and examined the impacts of ACC and C-ACC systems on traffic flow operations. Decentralised algorithms and distributed algorithms are proposed and implemented for ACC and C-ACC controllers in the microscopic simulator respectively. The ACC and C-ACC algorithms are successfully implemented on a two-lane motorway stretch of 14 km, with simultaneous control of more than 500 vehicles. The proposed algorithms work well under discontinuities in state variables (i.e. gap and relative speed.
with respect to the preceding vehicle) caused by lane-changing manoeuvres and parameters (free/desired speeds) due to sudden drop and increase in motorway speed limits. In principle, the decentralised ACC and distributed C-ACC algorithms can be implemented in any microscopic simulation model.

Simulation results provide insights into the impacts of ACC systems on traffic operations. ACC systems mitigate the capacity drop phenomenon and improve the stability of traffic flow upstream of the jam area. The jam propagation speed changes with different penetrate rate of ACC vehicles in the network, and the jam propagation direction is even reversed in some cases, resulting in a new phenomenon which is not observed in human-driven vehicular flow. ACC systems may destablise traffic downstream of the jam area due to the closer following distance in the transition from low speed state to high speed state. Fuel consumption is reduced with ACC systems in traffic compared to reference scenario with all human-driven vehicles.

The C-ACC systems employing the cooperative control strategy is more predictive and anticipative, since the predicted acceleration trajectory of the cooperative predecessor is taken into account in the state prediction of the cooperative follower. These characteristics improve the stability at both jam tail and jam head, while at the same time increase the outflow in the accelerating phase. Under the bottleneck setting in this study, the disturbance caused by reduced speed limit is damped out and hence does not evolve into persistent waves with only 10% C-ACC vehicles in traffic. The fact that C-ACC vehicles predict the future of the predecessor behaviour and taking into account the expected behaviour of the follower have clear benefits in the bottleneck. C-ACC systems stabilise traffic flow and increase the effective capacity at the bottleneck compared to human driver and ACC systems. One noteworthy flow property is that C-ACC systems result in faster stop-and-go waves propagating upstream due to V2V communications.

At very low penetration rate of ACC and C-ACC vehicles in traffic, e.g. less than 5%, human drivers still dominate the traffic flow characteristics. Hence the flow characteristics remain qualitatively the same, i.e. capacity drop and jam propagation, as reference case with 100% human drivers.

Note that different types and strength of bottlenecks may result in different jam types and flow patterns other than the stop-and-go waves discussed in this study. Hence, it remains an interesting question on how the proposed decentralised ACC and distributed C-ACC systems influence the characteristics of other jam types and flow patterns.

Based on the impact study results, it can be concluded that the ACC and C-ACC system change flow characteristics substantially, and roadside controller is likely to be necessary to resolve stop-and-go waves when ACC vehicles and human-driven vehicles determine the flow operations. In the next chapter, a proof-of-concept study will be conducted for integrating traffic controller with ACC systems via Vehicle-to-Infrastructure communications.
Chapter 8

Integrated variable speed limit control system with ACC vehicles

In the previous chapter, we investigated the impacts of ACC and C-ACC systems on traffic operations. It is found that ACC and C-ACC systems change the stability and effective capacity at the bottleneck and the changed flow characteristics show both merits and difficulties for traffic control with ACC and C-ACC systems. In this chapter, we carry out a proof-of-concept study for integrating link-level traffic controller with vehicle-level ACC controllers via Vehicle-to-Infrastructure (V2I) communications, operationalising the control concept of in-vehicle actuation of variable speed limit (VSL) discussed in Chapter 2. The main objective is to investigate the feasibility and potential benefits of the integrated control paradigm in resolving stop-and-go waves. A VSL control algorithm designed to resolve stop-and-go waves, namely SPECIALIST, and the ACC algorithm tested in the previous chapters are chosen for the case study. It is assumed that each ACC vehicle receives VSL commands from the traffic controller and uses it as variable parameter for the local ACC controller. The effectiveness of this integrated control paradigm is tested with simulation experiments.

This chapter is structured as follows. Section 8.1 introduces the integrated traffic control concept. Section 8.2 operationalises the integrated control concept with VSL controller at link-level and ACC controller at vehicle-level. The experimental design is discussed in Section 8.3, followed by the test results and performance analysis of the proposed control paradigm. The conclusions are summarised in Section 8.5.

8.1 Introduction

In Chapter 2, we discussed control concepts that have been proposed for traffic systems with intelligent vehicles (IVs), most of which pertains to a hierarchical structure. Based on the spatial and temporal scope of different control layers, several levels can be distinguished, including network, link, platoon and vehicle levels. In this chapter we
focus on integrating link-level traffic controller with vehicle-level ADAS controllers via V2I communications. This section introduces the concepts of the integrated traffic control with ADAS.

Figure 8.1 shows a schematic representation of the controllers at link and vehicle levels and their interaction based on the generic ADAS controller of Figure 2.2, with arrows indicating the direction of information flow. At the upper level, traffic controller monitors the state of the road network. Road-based sensors such as loop detectors or cameras record the traffic volumes and speeds at a cross-section or over a spatial range, and average them over space and time. The measurements are transmitted to the traffic controller at traffic control centres. The traffic controller, such as a VSL controller, estimates the global traffic state of the road network based on measurements from road-based sensors and on information from IVs in the network via vehicle-to-infrastructure (V2I) communication. Depending on the specific control approach employed by the traffic controller, it may involve a procedure of predicting the global traffic state, evaluating necessity and feasibility of traffic control schemes, such as variable speed limit (VSL). The traffic control signals are transmitted to and executed by road-based actuators, such as VSL gantries. With the availability of V2I communication, the traffic control signal can be transmitted directly to IVs from traffic controllers or from road-based actuators. As the vehicles in the network move based on the local interactions and the traffic control signals, the global traffic system state changes, and the traffic controller enters the next control cycle, which is typically in the order of tens of minutes.

At the lower level, the vehicle controller, such as ACC controller, estimates the local traffic state surrounding the vehicle based on the measurements from on-board sensors. The traffic control commands from the road-based controller are transmitted to the vehicle via V2I communications, which are used by the local ADAS controller for computing the reference vehicle control signals. The reference signal of vehicle control system is executed by the vehicle actuators. The control cycle of vehicle controller is typically in the order of less than one second. The state and control information of the vehicle controller may also be transmitted to the upper-level traffic controller via V2I communication, or to other vehicles via V2V communication, which is not the focus of this case study.

While transmitting individual vehicle information to traffic controller via V2I communication can improve the traffic state estimation (Treiber et al., 2011; Yuan et al., 2012; Netten et al., 2013) and achieve faster detection of traffic jams (Hegyi et al., 2013), this case study focus on test of the concept of in-vehicle actuation of variable speed limits discussed in Section 2.2. In the sequel, we use a VSL controller at link level and ACC controller at vehicle level to operationalise this concept, implement it and test its effectiveness in simulation.
Figure 8.1: Schematic representation of bi-level control problem. Dashed lines are not covered in this study.
8.2 Control design of integrated VSL control with ACC systems

This section describes main control design issues of the integrated traffic control concept and illustrates its workings using a VSL control algorithm designed to resolve shock waves and the model predictive ACC algorithm tested in the previous chapter. First, the assumptions for the integrated traffic control systems is described. Then the VSL control algorithm at link level and ACC algorithm at vehicle level are briefly recalled, followed by the description of the controllers and algorithms implementation with microscopic traffic simulations.

8.2.1 Assumptions of integrated control

The following assumptions are made for the operation of the integrated VSL control with ACC system on a motorway stretch:

- There are loop detectors every 250 metres along the motorway, collecting aggregate flow and speed data every 30 seconds. VSL gantries are positioned every 500 metres along the motorway. Errors from detectors are not considered in this study, but are addressed in another work (Hegyi et al., 2013).

- The VSL controller uses data from loop detectors to estimate and to predict the state of the traffic system on the motorway. The estimation of the traffic state has the same sampling rate as loop detectors.

- The transmission of loop detector data to the traffic control centre and the transmission of VSL control schemes to VSL gantries are via cables. The transmission of VSL signals from VSL gantries to ACC vehicles are via wireless V2I communications (Ioannou et al., 2007). Dedicated short range communication (DSRC) units are assumed to be equipped on both VSL gantries and ACC vehicles. The DSRC communication range is assumed to be 200 metres. Communication delays for both cabled and DSRC communications are negligible compared to the VSL control cycle.

- ACC vehicles are assumed to fully comply with the speed limits, while human drivers do not fully comply. Under the speed limit of 60 km/h, human drivers are assumed to choose their desired speeds at 72 km/h, which is in accordance with field test results (Hegyi & Hoogendoorn, 2010)\(^9\).

\(^9\)Note that the driver compliance with speed limits depends highly on the presence of speed enforcement.
8.2.2 VSL control algorithm: SPECIALIST

At the link level, a VSL control algorithm, namely SPECIALIST (SPEed ControllIng ALgorIthm using Shockwave Theory) (Hegyi et al., 2008), is chosen for this case study. It is a feedforward controller and resolves moving jams based on shock wave theory. The SPECIALIST algorithm is briefly explained in the following, which is necessary for discussing the experiment results in the next section.

Theory revisited

The theory of resolving shock waves by VSL is based on the shock wave theory (Lighthill & Whitham, 1955). One of the basic relationships in shock wave theory is the relationship between the time-space graph of the traffic states (as shown on the left in Figure 8.2) and the density-flow graph (as shown on the right in Figure 8.2). The time-space graph shows the traffic states on a road stretch (along the vertical axis) and their propagation over time (in the horizontal direction). In the figure a short traffic jam is shown that propagates upstream (area 2) and which is surrounded by traffic in free-flow (areas 1). The density-flow diagram shows the corresponding density and flow values for these states. Shock wave theory states that the front (boundary) between two states in the left figure has the same slope as the slope of the line that connects the two states in the right figure. Note that the slopes in both figures have the unit of km/h, and that states in the right figure correspond to areas in the left figure. The orange lines (light grey in black and white) indicate the fundamental diagram (as a reference).

The importance of this relationship is that if the different traffic states on a motorway stretch are known, then their future evolution can be predicted by describing the fronts between them. This basic relationship will be used in the theory for resolving shock waves.

Control approach

The approach to resolve shock waves consists of four phases and starts with a shock wave similar to the example above.

Phase I. A shock wave (as shown in Figure 8.3) is detected on the motorway. We assume that the traffic state upstream (state 6) and downstream (state 1) of the shock wave is in free flow which is generally the case in real traffic. Note that the theory also holds for the cases when state 6 has a higher flow than state 1.

Phase II. As soon as the shock wave is detected, the speed limits upstream of the shock wave are switched on. This leads to a state change in the speed-controlled area from state 6 to state 3 (in Figure 8.3 approximately from 4-8 km), and to the boundary between areas 6 and 3. State 3 has the same density as state 6. However, the flow of

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Figure 8.2: According to the shock wave theory the propagation of the front between two traffic states in the left figure has the same slope as the line connecting the two states in the density-flow diagram in the right figure. The arrow indicates the travel direction. Flow and density values are for two lanes.

state 3 is lower than that of state 6 due to the combination of the same density with a lower speed.

As shown by the density-flow graph, the front between states 2 and 3 will propagate backwards with a lower speed than the front between states 1 and 2, and consequently the two will intersect and the shock wave will be resolved after some time.

At the upstream end of the speed-limited area traffic will flow into this area, with the speed equalling the speed limit and with a density that is in accordance with the speed, typically significantly higher than the density of state 3 (which was the density corresponding to free-flow). This state is called state 4. The front between states 6 and 4 will propagate upstream or downstream depending on the flow of state 4.

Phase III. When the shock wave (area 2) is resolved, there remains an area with the speed limits active (state 4) with a moderate density (higher than in free-flow, but lower than in a shock wave) and a moderate speed. A basic assumption in this theory is that the traffic from such an area can flow out more efficiently than a queue discharging from full congestion as in the shock wave. So, the traffic leaving area 4 will have a higher flow and a higher speed than state 4, represented by state 5. This leads to a backward propagating front between states 4 and 5, which resolves state 4.

Phase IV. What remains is state 5, and state 6 upstream and state 1 downstream of it. The fronts between states 1 and 5, and between states 6 and 5 propagate downstream, which means that eventually the backward propagating shock wave is converted into a forward propagating wave leading to a higher outflow of the link as shown in Figure 8.3.
Figure 8.3: The four phases of the SPECIALIST algorithm. Phase I: The shock wave is detected. Phase II: Speed limits are turned on in areas 2, 3, and 4. The shock wave dissolves. Phase III: The speed-limited area (area 4) resolves and flows out efficiently. Phase IV: The remaining area 5 is a forward propagating high-speed high-flow wave. Flow and density values are for two lanes.

**Operational algorithm**

Based on the theory, an algorithm is developed that is suited for real-world implementation. First, a shock wave is detected by using thresholds $V_{\text{max}}$ (km/h) for the speed measurements, by assuming that in segment $i$, a shock wave is present if $V_i \leq V_{\text{max}}$. After that, traffic states 1-6 are determined using measurements from sensors and control scheme is generated, involving the determination of the various fronts and their intersection points by solving linear equations based on the shock wave theory. After that the control scheme is determined and the speed limit can be activated. For details of the algorithm, we refer to Hegyi et al. (2008, 2013).

Heuristic tuning rules can be given for tuning the algorithm, partially based on offline traffic data and partially based on the online (closed-loop) behaviour of the algorithm. One of the most important tuning parameters is the density associated with state 4. The speed of state 4 is determined by the speed limits, however the choice of the density is a design variable that influences the shape of the control scheme. If the density is chosen higher, the slope between states 4 and 6 will be less steep, which means that the tail of the speed limits will propagate less quickly backwards. This relationship also can be understood the other way around: by letting the tail of the speed limits (the front between states 4 and 6) propagate faster backwards, the density in area 4 can be kept low.

Other tuning parameters are the speed $v_{[5]}$ and flow $q_{[5]}$ of state 5. These parameters determine the slopes 1–5, 4–5 and 5–6. Slope 4–5 determines how the speed limits are...
release at the head of the speed-limited area. By changing the speed of the slope the flow of state 5 can be influenced. The outflow should be not too high to prevent triggering new jams at downstream bottlenecks. Furthermore, \( q_{[5]} \) should be higher than \( q_{[4]} \) and density \( k_{[5]} \) should be lower than \( k_{[4]} \) in order to have a backward propagating front in order to release the speed limits region.

8.2.3 ACC algorithm with variable desired speeds

The ACC algorithm chosen for the case study is the model predictive ACC algorithm designed and tested in the previous chapter. An ACC vehicle detects the gap and speed difference with respect to the predecessor solely based on its own on-board sensors, e.g. forward-looking radar. The ACC vehicle predicts the behaviour of its predecessor and chooses its optimal acceleration to minimise its own cost of (6.6). Lane change decisions of ACC vehicles are determined and are executed by human drivers through steering wheels. We choose the same desired time gap setting of \( t_{d,0} = t_{d,m} = 1.3s \) as in the previous chapter to facilitate cross-comparison of the control effects in different scenarios.

When the VSL signals are transmitted to the ACC controller, the ACC controller changes its parameter of desired speed \( v_0 \) to the speed limit, i.e. full compliance with the speed limits. The optimal acceleration is based on the minimising the original cost function of (6.6) with \textit{variable desired speed} \( v_0 \).

8.2.4 Implementation of integrated control with traffic simulation

The same as in the previous chapter, the \textit{decentralised} ACC algorithm is implemented in a Java-based microscopic traffic simulator, MOTUS (Schakel et al., 2012), to update the longitudinal motion of ACC vehicles. The VSL control algorithm is implemented in Matlab. At each update of the VSL controller, with a sampling rate of 30 seconds, the VSL controller communicates with MOTUS to get the loop detector measurements along the motorway and sends back the VSL control signals to the VSL gantries, resembling the cabled communication between road infrastructure (including loop detectors and VSL gantries) and traffic control centres. The loop detectors and VSL gantries are modelled in MOTUS. The communication between VSL gantries and ACC controller are included in MOTUS, and the ACC vehicle change their desired speeds after entering the DSRC communication range of the VSL gantries, which is 200 metres upstream of each gantry. Human drivers are assumed to react to the VSL gantries 150 metres upstream of each gantry.

Here, we complete the description of the main design issues of the integrated VSL control with ACC systems. In the following, we test the workings of the integrated control paradigm and its effectiveness with different penetration rate of ACC vehicles in the network.
Table 8.1: Experimental scenarios for testing integrated VSL control with ACC systems

<table>
<thead>
<tr>
<th>Scenarios</th>
<th>Traffic composition</th>
<th>VSL control</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>100% human drivers</td>
<td>off</td>
</tr>
<tr>
<td>2</td>
<td>5% ACC + 95% human drivers</td>
<td>off</td>
</tr>
<tr>
<td>3</td>
<td>10% ACC + 90% human drivers</td>
<td>off</td>
</tr>
<tr>
<td>4</td>
<td>50% ACC + 50% human drivers</td>
<td>off</td>
</tr>
<tr>
<td>5</td>
<td>100% ACC</td>
<td>off</td>
</tr>
<tr>
<td>6</td>
<td>100% human drivers</td>
<td>on</td>
</tr>
<tr>
<td>7</td>
<td>5% ACC + 95% human drivers</td>
<td>on</td>
</tr>
<tr>
<td>8</td>
<td>10% ACC + 90% human drivers</td>
<td>on</td>
</tr>
<tr>
<td>9</td>
<td>50% ACC + 50% human drivers</td>
<td>on</td>
</tr>
<tr>
<td>10</td>
<td>100% ACC</td>
<td>on</td>
</tr>
</tbody>
</table>

8.3 Experimental design

To prove the workings and potential benefits of the integrated VSL control system with ACC vehicles, simulation experiments are carried out at a bottleneck which causes shock waves if VSL controller does not intervene. The experimental design is presented in this section.

8.3.1 Bottleneck setting

To make comparison with the scenarios without VSL control, we choose the same bottleneck setting as in the previous chapter. The bottleneck is triggered by imposing temporary low speed limits on parts of a two-lane motorway stretch of 14 km. Speed value of 80 km/h, 60 km/h and 40 km/h is posted on the VSL gantry at 11 km, 11.5 and 12 km of the motorway for two minutes, with a constant demand of 1900 veh/h per lane. The bottleneck setting results in reasonable capacity drop and shock wave propagation characteristics in the reference scenario 1 with 100% human drivers, as we have verified show in Section 7.3.1. The parameters for human drivers and ACC controllers are kept the same as those in the previous chapter.

8.3.2 Deployment scenarios

The variables to be tested is whether to use VSL control and the penetration rate of ACC vehicles in traffic (5%, 10%, 50%, 100%). Together with the reference scenario with 100% human drivers, this amounts to 10 simulation scenarios as shown in Table 8.1.
Scenarios 1 to 5 are exactly the same as those in the previous chapters. We conduct 10 simulation runs for each scenarios.

### 8.3.3 Assessment indicators

Part of the indicators used in the previous chapter is used to assess the effectiveness of the integrated traffic control paradigm, including the Total time spent (TTS) in the network and average fuel consumption (AFC) described in Section 7.2. Extensions of the TTS indicator are included to evaluate the relative changes in traffic efficiency, which are gains in TTS of scenarios 6 - 10 with reference to scenario 1, gains in TTS in scenarios 7 - 10 with reference to scenario 6 of VSL control with 100% human-driven vehicles, and gains in TTS in scenarios 7 - 10 with respect to scenarios 2 - 5 respectively.

The jam related indicators, jam area $A_{\text{jam}}$, jam flow $Q_{\text{jam}}$, jam speed $V_{\text{jam}}$, are not present in this context since the integrated traffic control system resolves the stop-and-go waves. Instead, the VSL area $A_{\text{VSL}}$, speed $V_{[4]}$ and density $K_{[4]}$ of state 4 , and flow $Q_{[5]}$ of state 5 are used to assess the changes in traffic state with different penetration rate of ACC vehicles. The number (#) of detected jams and number of resolved jams are included which give an indication on the effectiveness of the integrated control paradigm in resolving shock waves, and the number (#) and location of new jams triggered by SPECIALIST is used to assess the change in flow stability property due to VSL control.

Numbers of (detected, resolved and new) jams are summed over ten simulation runs, while other indicators are averaged over ten simulation runs in each scenario. VSL area, speed and density of state 4 and flow of state 5 are calculated by extracting different states in the VSL control scheme with loop detector data.

Apart from the indicators, spatio-temporal contour plots of flow and speed from loop detectors are coupled with VSL control scheme in different scenarios to visualise the control effects of the integrated traffic control system.

### 8.4 Simulation results

This section discusses the results of effectiveness of integrated traffic control with ACC vehicles. First, the tuning of SPECIALIST algorithm is discussed. Then the performance of the algorithm is discussed, using the assessment indicators and plots introduced in the previous section.
Table 8.2: SPECIALIST parameter settings for different scenarios

<table>
<thead>
<tr>
<th>Scenario</th>
<th>6 (100% Human)</th>
<th>7 (5% ACC)</th>
<th>8 (10% ACC)</th>
<th>9 (50% ACC)</th>
<th>10 (100% ACC)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$V_{\text{max}}$ (km/h)</td>
<td>50</td>
<td>50</td>
<td>50</td>
<td>50</td>
<td>50</td>
</tr>
<tr>
<td>$V_{\text{front}}$ (km/h)</td>
<td>-12</td>
<td>-13</td>
<td>-8</td>
<td>-8</td>
<td>-8</td>
</tr>
<tr>
<td>$v_{\text{eff}}$ (km/h)</td>
<td>78</td>
<td>75</td>
<td>72</td>
<td>72</td>
<td>72</td>
</tr>
<tr>
<td>$k_{[4]}$ (veh/km)</td>
<td>24</td>
<td>24</td>
<td>23</td>
<td>23</td>
<td>23</td>
</tr>
<tr>
<td>$v_{[5]}$ (km/h)</td>
<td>105</td>
<td>105</td>
<td>115</td>
<td>115</td>
<td>115</td>
</tr>
<tr>
<td>$q_{[5]}$ (veh/h)</td>
<td>1980</td>
<td>1980</td>
<td>1980</td>
<td>2020</td>
<td>2020</td>
</tr>
</tbody>
</table>

8.4.1 Tuned variables

In the previous chapter, we discussed the changes in flow characteristics with ACC systems and their implications for traffic management. One of the changes in flow characteristics is the velocity of the downstream jam front $V_{\text{front}}$, which varies with different penetration rate of ACC vehicles in traffic and even changes when propagating upstream. During the tuning of the algorithm, we use the average downstream jam front velocity for different ACC penetration rates to predict the jam front velocity, as shown in Table 8.2.

The impact study in the previous chapter also suggests that the ACC systems tested may destabilise traffic flow in the acceleration transition. This feature increases the risk of triggering new jam downstream of the speed control region and it is evidenced by the tuning efforts in the experiments. In many parameter settings, new jams emerges downstream of the speed limit region. As a result, the effective speed $v_{\text{eff}}$ and target density $k_{[4]}$ in state 4 and the flow $q_{[5]}$ and speed $v_{[5]}$ of state 5 have to be chosen in such a way that the state 4 and 5 are stable and the number of new jams triggered the VSL control scheme is as few as possible. In general, decreasing the density in state 4 and state 5 reduce the number of new jams triggered by VSL.

The tuning of the SPECIALIST algorithm follows the heuristic method in Hegyi & Hoogendoorn (2010). The tuned parameters for SPECIALIST algorithm are summarised in Table 8.2. Note that we did not optimise the control parameters during the tuning, which involves large number of simulation runs and consequently considerable computation efforts needed due to the receding horizon algorithms of ACC controllers. Nevertheless, as we will show in the ensuing, the chosen parameter settings already prove the feasibility of the integrated control paradigm and show its potential benefits in improving traffic operations and sustainability.
8.4.2 Performance of the integrated control paradigm

Despite the difficulties we envisaged on traffic control with ACC vehicles due to the changed flow characteristics (cf. Section 7.3.4), our simulation experiments show that without changing the SPECIALIST algorithm, the integrated VSL control works well and solves the resultant shock waves efficiently. Table 8.3 shows the number of detected jams and the number of jams solved by the integrated VSL control. In the tested scenarios with ACC systems, all the detected jams are resolved within the spatio-temporal region of the simulation, i.e. around 10 km in space and 40 minutes in time for VSL control. Figure 8.4 and 8.5 show typical contour plots of flow and density in different scenarios where jams are resolved successfully. One out of 10 jams in scenario 1 with 100% human drivers is not resolved eventually, which is shown in Figure 8.6(a)(b).

New jams are triggered by the VSL control scheme in scenario 7, 9 and 10, as shown in Figure 8.6(c)-(h). All the new jams are observed downstream of the speed limit region, which is in accordance with our expectations based on the findings of the impact study of ACC systems in the previous chapter. The ACC vehicles destabilises traffic flow in the accelerating phase after the release of speed limit, which degrade the VSL controller performance to a certain extent.

Resolving shock waves has clear benefits in improving traffic efficiency. Compared to the reference scenario 1 without VSL control, SPECIALIST itself, applying SPECIALIST in the 100% human drivers already brings a saving of $87.7 \text{ veh} \cdot \text{h}$ in the simulation period. With 5% ACC vehicles in traffic, the gain in TTS increases to 119 $\text{veh} \cdot \text{h}$ in scenario 7, and is slightly higher in scenarios 8, 9 and 10.

When comparing to scenario 6 of SPECIALIST with 100% human drives, the benefits of the integrated VSL control are pronounced. The average TTS gain in scenario 7 with 5% ACC vehicles is 31.4 $\text{veh} \cdot \text{h}$, and is higher in scenarios 8, 9 and 10 with more ACC vehicles in traffic. Even comparing to the scenarios with same ACC penetration rate, the benefits of the integrated VSL control in traffic efficiency are still clear. The average TTS gain of in scenario 7 is 33.8 compared to scenario 2. This gain is decreasing with the increasing penetrate rate of ACC vehicles, due to the mitigated capacity drop phenomenon with more ACC vehicles in traffic (cf. 7.3). Nevertheless, a slight gain of 2.1 $\text{veh} \cdot \text{h}$ is still observed of VSL control with 100% ACC compared to the uncontrolled scenario 5, where the jam size is already very small (cf. Table 7.2).

The VSL control size decreases with the increase of ACC penetration rate. This is explained by the reduced jam size with more ACC vehicles in traffic. The reduced jam size implies less control efforts, i.e. less traffic are limited to enter the jam region to resolve the shock wave.

The average speed in state 4 gives an indication of the compliance rate of the traffic under speed limits. At low penetration rate of 5% and 10%, the traffic speeds in state 4 are slightly lower than that scenario 6. When the penetrate rate of ACC vehicles
increases to 50%, the average speed decreases substantially to 67.7 \( km/h \). With 100% ACC vehicles, the average speed in area 4 is 59.9 \( km/h \), due to the fully compliance of ACC vehicles.

The average flow rate in state 5 reflects how efficient the resultant flow is after the release of speed limits. It is clear that flow rates in state 5 in all controlled scenarios are higher than the fixed demand of 1900 \( veh/h \), which is essential to resolve shock waves. It is interesting to see that with low ACC vehicle penetration rate of 5% and 10%, the resultant flow rates of state 5 are much higher than in the 50% and 100% case. This is also explained by the mitigated capacity drop phenomenon due to ACC vehicles. The mechanism that slowing down vehicles to get more efficient flow works still with high penetration rate of ACC vehicles, however, the increase in outflow is less significant compared to scenarios where more human-driven vehicles present in traffic.

From sustainability perspectives, the benefits of integrated traffic control is clear. In all controlled scenarios (6 - 10), the average spatial fuel consumption rates are lower than their uncontrolled counterparts with the same traffic compositions (scenarios 1 - 5). Furthermore, for all controlled scenarios with ACC systems in traffic (scenarios 7 - 10), the average spatial fuel consumption rates are lower than the controlled scenario with 100% human drivers (scenario 6).
Figure 8.4: Spatio-temporal plots of flow for VSL control with different traffic compositions.
Figure 8.5: Spatio-temporal plots of speed for VSL control with different traffic compositions.
Figure 8.6: Spatio-temporal plots of flow and speed in scenarios with the unresolved jam and new jams triggered by VSL control.
Table 8.3: Indicators for different scenarios

<table>
<thead>
<tr>
<th>Scenarios</th>
<th>VSL (AVSL (km·min))</th>
<th># detected jams</th>
<th># resolved jams</th>
<th># and location of new jams</th>
<th>TTS (veh·h)</th>
<th>TTS gain to Scen. 1 (veh·h)</th>
<th>TTS gain to Scen. 6 (veh·h)</th>
<th>TTS gain to Scen. with same penetrate rate (veh·h)</th>
<th>$A_{VSL}$ (km·min)</th>
<th>$V_4$ (km/h)</th>
<th>$Q_5$ (veh/h)</th>
<th>AFC (l/100/km)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 (100% Human)</td>
<td>off</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>562.8</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>4.07</td>
</tr>
<tr>
<td>2 (5% ACC)</td>
<td>off</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>477.4</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>3.91</td>
</tr>
<tr>
<td>3 (10% ACC)</td>
<td>off</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>453.8</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>3.81</td>
</tr>
<tr>
<td>4 (50% ACC)</td>
<td>off</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>454.4</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>3.84</td>
</tr>
<tr>
<td>5 (100% ACC)</td>
<td>off</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>443.8</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>3.69</td>
</tr>
<tr>
<td>6 (100% Human)</td>
<td>on 10</td>
<td>9</td>
<td>0</td>
<td>1,dss**</td>
<td>475.0</td>
<td>87.8</td>
<td>–</td>
<td>–</td>
<td>82.0</td>
<td>75.2</td>
<td>1942</td>
<td>3.91</td>
</tr>
<tr>
<td>7 (5% ACC)</td>
<td>on 11</td>
<td>11</td>
<td>1,dss**</td>
<td>–</td>
<td>443.6</td>
<td>119.2</td>
<td>31.4</td>
<td>33.8</td>
<td>22.4</td>
<td>73.3</td>
<td>2000</td>
<td>3.65</td>
</tr>
<tr>
<td>8 (10% ACC)</td>
<td>on 10</td>
<td>10</td>
<td>0</td>
<td>1,dss**</td>
<td>439.2</td>
<td>123.6</td>
<td>35.8</td>
<td>14.6</td>
<td>10.2</td>
<td>73.5</td>
<td>1982</td>
<td>3.61</td>
</tr>
<tr>
<td>9 (50% ACC)</td>
<td>on 12</td>
<td>12</td>
<td>2, ds</td>
<td>–</td>
<td>440.0</td>
<td>122.8</td>
<td>35.0</td>
<td>14.4</td>
<td>3.7</td>
<td>67.7</td>
<td>1945</td>
<td>3.62</td>
</tr>
<tr>
<td>10 (100% ACC)</td>
<td>on 12</td>
<td>12</td>
<td>2, ds</td>
<td>–</td>
<td>441.7</td>
<td>121.1</td>
<td>33.3</td>
<td>2.1</td>
<td>2.7</td>
<td>59.9</td>
<td>1926</td>
<td>3.65</td>
</tr>
</tbody>
</table>

*: number (#) of detected, resolved and new jams are summed over ten simulation runs in each scenario, while other indicators are averaged over ten simulation runs.

**: ds stands for downstream of the VSL control region.
8.5 Conclusions

In this chapter, we performed a proof-of-concept study for integrated traffic control with intelligent vehicles, where a link-level VSL controller sends traffic control commands to vehicle-level ACC controllers via Vehicle-to-Infrastructure (V2I) communications and the ACC controllers adapt their control parameters according to the link-level control commands. Despite the fact that ACC vehicles change flow characteristics substantially, the integrated control concept works without changing the VSL algorithm (SPECIALIST algorithm) and resolves stop-and-go waves successfully in all the test cases with ACC vehicles in traffic.

Simulation test shows that the integrated VSL control with ACC systems improves traffic efficiency and sustainability, i.e. total time spent in the network and average fuel consumption rate are reduced compared to (uncontrolled and controlled) scenarios with 100% human drivers and to uncontrolled scenarios with the same ACC penetration rates. The improvement is a result of the combination of a smaller capacity drop due to the presence of ACC vehicles and the better compliance of the ACC vehicles to the VSL control commands.

It should be noticed that the fact that the flow characteristics after the release of the speed limits (state 5 in SPECIALIST) are different with ACC vehicles in traffic, i.e. the instability triggered new jams in state 5 for scenarios with ACC vehicles and the flow rates in state 5 are decreasing with the increasing penetrate rate of ACC vehicles in traffic. As SPECIALIST is originally designed for human-driven vehicular flow, re-design of the algorithm taking into account the fundamental changes in traffic flow dynamics at bottlenecks and under variable speed limits may be even more beneficial. This finding suggests that SPECIALIST algorithm may be redesigned to consider these aspects.

Note that apart from the benefits we discussed of in-vehicle actuation of traffic control commands, there are other potentials of connecting IVs with traffic control systems. With V2I communications, IVs can provide more comprehensive, timely and accurate information, i.e. extended floating car data, to the traffic controller to facilitate estimation and prediction of traffic state. This aspect is however, beyond the scope of this study and remains an interesting topic for future research.
Chapter 9

Findings, conclusions, implications and recommendations

The thesis is motivated by the challenges in ADAS control design and performance assessment outlined in Chapter 1. The main objectives are to develop a generic control framework for deriving ADAS control algorithms and to test the performance and impacts of the proposed algorithms on traffic operations and sustainability. Several research questions are stated under the research objectives, which are answered throughout Chapters 2-8. This chapter summarises the answers. First, the main findings are synthesised from the perspectives of ADAS control design and evaluation in Section 9.1. Section 9.2 draws conclusions based on the findings. Section 9.3 discusses the implications of the thesis work for practice, and Section 9.4 recommends future research directions.

9.1 Findings

This thesis deals with the development of a generic control framework for ADAS control design. Application of the framework yields several controller examples under autonomous following, eco-driving support, cooperative manoeuvring, and in-vehicle actuation of traffic control signals concepts, of which the performance and impacts on traffic flow characteristics and sustainability are examined systematically. Here we highlight the main findings of the research from a wider perspective. We structure the main findings around a number of contributions, each of which answers research questions set out in Section 1.3.

Current knowledge on ADAS control design and impacts

The literature study on ADAS control design answers the question of which control concepts and algorithms have been proposed for ADAS to improve traffic operations. It
is found that conceptual control models of autonomous following, eco-driving support, multi-anticipation and cooperative manoeuvring based on Vehicle-to-Vehicle (V2V) communications, and in-vehicle actuation of traffic control signals based on Vehicle-to-Infrastructure (V2I) communications have been proposed at vehicle, platoon and link levels.

The review of the operational algorithms shows that ACC, EcoACC and multi-anticipative ACC algorithms have been developed to operationalise autonomous following, eco-driving support and multi-anticipation concepts respectively. It reveals that the widely used linear ACC algorithms need to be refined to be applicable in safety-critical conditions and EcoACC algorithms need to take car-following behaviour into consideration to be applicable in constrained traffic conditions. Algorithms based on cooperative manoeuvring concept and operationalisation the in-vehicle actuation of traffic control signals concept have not been reported. Comparison of the control methods suggests that optimisation-based control approaches are very flexible in formulating objective function and in dealing with constraints and are hence advantageous for constructing a generic control framework applicable in control design under diversified ADAS concepts.

The synthesis of current knowledge on the impacts of ADAS on traffic operations answers the question of what are the impacts of state-of-the-art ADAS concepts on traffic operations and sustainability. Literature work reveals that both positive and negative effects can be attributed to the presence of ACC systems in road networks. The overall effects depend on ACC algorithms and parameter settings, particularly the desired gap policy and setting compared to human drivers, boundary and network conditions, particularly demand the bottleneck strength etc. Insights into the impacts of EcoACC and ADAS under cooperative manoeuvring concept on traffic flow operations and sustainability are lacking.

**Generic control framework**

A model predictive control framework is proposed to answer the question of how to formulate the control of ADAS vehicles into an optimisation problem. The key feature of the framework is that at each time instant, the ADAS controller solves on-line an optimal control problem, which entails predicting the behaviour of surrounding vehicles based on the current state of the system and determining an optimal acceleration trajectory minimising a cost function. The optimal acceleration trajectory is computed in a receding horizon manner based on the newest information available regarding the system state. The cost function reflects control objectives, such as maximising safety, efficiency, comfort, and sustainability. Illustrative work shows that the widely-studied linear state-feedback ACC controller can be derived under the framework.

Different solution approaches to general continuous-time deterministic optimal control problems are discussed and compared, which answers the question of which solution approaches can be used to solve the optimal control problem. The solution
approaches can be categorised into dynamic programming approaches, indirect solution approaches based on Pontryagin’s Minimum Principle (PMP), direct non-linear programming (NLP) approaches, and analytical solutions under specific formulations. Dynamic programming approaches produce closed-loop solutions to the entire state space, but the computational complexity limits their application to small scale systems with low dimensions. Direct optimisation approaches transcribe the original problem into NLP problems that can be solved more efficiently but with less accuracy, and are flexible in dealing with constraints on state and control variables. The PMP solution family and the analytical solution approaches finds the optimal solution more accurately compared to the NLP approach and more efficiently compared to the dynamic programming approach, and hence are promising in control design for large scale systems. The constraints on state variables should be handled with care for the PMP solution family and analytical solution approaches.

An assessment framework is proposed to assess the performance of the proposed ADAS algorithms at microscopic level and their impacts at macroscopic level, which answers the question of which methods and indicators can be used to assess the performance of ADAS and impacts on traffic and sustainability. The assessment framework consists of microscopic simulation on representative scenarios, analytical approach for capacity and flow stability analysis, and a set of indicators for traffic operations and sustainability. Microscopic traffic simulation models are found to be very useful in assessing the ADAS performance and the impacts on traffic operations, due to their descriptive power in dynamics of individual vehicles. The analytical approach for capacity and stability analysis is feasible for algorithms under analytical solution approaches. Traffic operation indicators can be quantified from microscopic traffic flow models. Fuel consumption and emission models can be used to assess ecological and environmental impacts of ADAS systems, using outputs from microscopic traffic flow models as inputs to calculate sustainability indicators.

**Framework application in designing and testing new ADAS controllers**

Application of the proposed control framework leads to several new ADAS controllers and implementable algorithms, which answers the questions of how to derive ACC, EcoACC and C-ACC controllers and operational algorithms under the control framework. The ACC controller optimising efficiency, safety and comfort is developed and refined throughout the thesis, taking into account collision-free constraints and variable desired time gap policies. The EcoACC controller is designed by incorporating $CO_2$ emissions minimisation in addition to the ACC control objectives.

The C-ACC controller is proposed under the cooperative manoeuvring concept, which entails controlled vehicles cooperating with each other under a joint objective and optimising the situations of a platoon. It captures the nature of cooperation, i.e. compromising own situations to benefit the whole system. This cooperative control strategy
is distinctive from the non-cooperative ACC controller which only optimises the situation of the controlled vehicle/ supported driver. One unique feature of the C-ACC controller is that it is not restricted to cooperation between intelligent vehicles. When a C-ACC vehicle is followed by a human-driven vehicle, it can still exhibit cooperative behaviour by taking into account the expected response of the human-driven vehicle to the control decisions. This is of vital importance for introducing such systems at an early stage, since it does not rely on V2V communication and can function with low penetration rates of intelligent vehicles in traffic. The multi-anticipative ACC controller can also be operationalised under the framework (Wang et al., 2014b).

Solution approaches of dynamic programming, the analytical approach based on infinite horizon problem with discounted cost and a new iterative numerical solution approach based PMP (iPMP) are applied to derive control algorithms for different ADAS controllers. The choice of solution approach depends on the dimensionality of the system state and formulations of the objective function and constraints. It is found that the iPMP solution approach is very generic and leads to efficient algorithms for ADAS controllers. Particularly, the iPMP approach enables successful implementation of decentralised ACC algorithms and distributed C-ACC algorithms for simultaneous control of more than 500 vehicles on a two-lane motorway stretch of 14 km.

Systematic simulation experiments verify the performance of the proposed controllers and thus answers the research question of how the proposed algorithms perform under representative scenarios. It is found that the proposed controllers generate plausible behaviour as expected from the controller formulations, including free driving, emergency braking, following-the-leader behaviour under decelerating and acceleration disturbances. Multi-lane simulation experiments also show that the proposed ACC and C-ACC algorithms work well under discontinuities in state variables (i.e. gap and relative speed with respect to the preceding vehicle) caused by lane-changing manoeuvres and parameters (free/desired speeds) due to sudden drop and increase in motorway speed limits.

**Impacts of proposed ADAS controllers on traffic operations and sustainability**

Systematic investigations into the resulting flow characteristics of proposed controllers are carried out throughout controller design processes, giving insights into the collective controller behaviour and answers the question of what are the impacts of the proposed ADAS controllers on traffic operations and sustainability. It is found that the resulting capacity and stability of traffic with ADAS vehicles are largely determined by the controller parameters, e.g. desired time gaps and weight factors associated to different cost terms in the objective function. Increasing the desired time gap increases the resulting flow stability while sacrificing capacity. Increasing weights on safety cost and efficiency cost improves the resulting flow stability while the equilibrium flow-density relations and capacity remain unchanged.
Particular attention is given to the dynamic flow properties related to stop-and-go waves, including capacity drop, stability and wave propagation speed, in case of a temporary bottleneck with presence of ACC and C-ACC vehicles. Simulation results provide insights into the impacts of ACC systems on flow properties related to stop-and-go waves, including capacity drop, stability and wave propagation speed, in case of a temporary bottleneck. Under the chosen parameter setting, the ACC system mitigates the capacity drop phenomenon and reduces about 21.5% of the total time spent in the network in the simulation with 100% ACC vehicles compared to that with 100% human-driven vehicles. The ACC system improves the stability of traffic flow approaching the jam area, i.e. in deceleration transition from high speed states to low speed states. The ACC system may destabilise traffic downstream of the jam area due to the closer following distance in the transition from low speed state to high speed state. Spatial fuel consumption rate is reduced with ACC systems in traffic compared to reference scenario with 100% human-driven vehicles.

The C-ACC system employing the cooperative control strategy captures the cooperation in the decision-making process and is more predictive and anticipative under disturbances. These characteristics improve the stability for traffic in both deceleration and acceleration transitions, while at the same time increasing the outflow from the jam area. Under the bottleneck setting in this study, the C-ACC system reduces about 23.0% of the total time spent in the network in the simulation with 100% ACC vehicles compared to that with 100% human-driven vehicles. The stop-and-go wave dissolves during propagation with only 10% C-ACC vehicles in traffic. One noteworthy change in flow properties is that C-ACC systems result in faster stop-and-go waves propagating upstream due to V2V communications compared to human-driven vehicles and ACC vehicles. On average, C-ACC vehicles keep a reduced spatial fuel consumption rate compared to human-driven and ACC vehicles.

It should be noted that at very low penetration rate of ACC and C-ACC vehicles in traffic, e.g. less than 5%, human drivers still dominate the traffic flow characteristics. Hence the flow characteristics of capacity drop, stability and jam propagation velocity, remain qualitatively the same in traffic with low penetration rate of IVs as reference case with 100% human-driven vehicles.

The proposed EcoACC system leads to lower traffic speed and lower flow compared to ACC system at free traffic conditions, but higher speed and higher flow at congested traffic conditions. The spatial $CO_2$ emission rate of EcoACC vehicles is lower than that of ACC vehicles. The EcoACC controller leads to collisions in simulation at densities above 80 $veh/km$ and needs to be refined further to be applicable in these conditions.
Findings on integrated traffic control with ACC systems

The proof-of-concept study of integrated variable speed limits (VSL) control with ACC vehicles answers the question of how to integrate traffic control with ADAS and what are the benefits of the integration. The control concept of in-vehicle actuation of traffic control signals is operationalised by integrating a link-level VSL controller and the vehicle-level ACC controller with V2I communication. The link-level controller estimates and predicts the global traffic state and constructs VSL schemes to resolve stop-and-go waves. The VSL control signals are transmitted to the ACC vehicles from VSL gantries via V2I communications. ACC vehicles optimise local situations, using the VSL signals as control commands to adapt their local control parameters of free/desired speeds. Despite the difficulties in predicting traffic states due to the changed flow characteristics with ACC systems, the integrated control concept works without changing the VSL algorithm (SPECIALIST algorithm) and resolves stop-and-go waves successfully in all the test cases with ACC vehicles in traffic. Indicators show that the integrated VSL control with ACC vehicles improves traffic efficiency and sustainability, i.e. total time spent in the network and average spatial fuel consumption rate are reduced compared to scenarios with 100% human drivers and to uncontrolled scenarios with the same ACC penetration rates.

9.2 Conclusions

First, the proposed model predictive control framework for ADAS is generic and can lead to feasible and implementable control algorithms. The classic linear state-feedback ACC controller can be derived under the framework. Both non-cooperative and cooperative ADAS systems can be operationalised with the single mathematical framework. Non-linear ADAS controller with non-quadratic cost formulation and state and control constraints can be dealt with under the framework. The decentralised ACC and distributed C-ACC algorithms with the numerical solution approach based on Pontryagin’s Minimum Principle are implementable for simultaneous control of many controlled vehicles in large scale systems and can be implemented in any microscopic simulation model in principle.

The proposed ACC controller improves the application range of existing ACC controllers and the EcoACC and C-ACC controllers operationalise new ADAS control concepts. The refined ACC controller considers multiple criteria such as safety, efficiency and comfort, and guarantees collision-free in all traffic situations. The EcoACC controller operates in both free driving and car-following conditions, as opposed to the existing eco-driving support controllers which focus only on free driving conditions and neglect the interactions with preceding vehicles. The C-ACC controller entails cooperative vehicles in a platoon manoeuvring together under a common goal and hence captures cooperation in the decision-making process, as opposed to the information-sharing form of cooperation in the multi-anticipative ACC systems in literature. The
distinctive feature of the C-ACC controller is that it can function without V2V communications and can still exhibit cooperative behaviour even if the C-ACC vehicle is followed by a non-cooperative vehicle.

**ACC and C-ACC systems change traffic flow characteristics substantially.** Findings on the impacts of ACC and C-ACC systems imply that the resultant flow characteristics with ACC and C-ACC are substantially different compared to the human-driven vehicular flow. For the bottleneck type where a stop-and-go wave is triggered in the reference case with human drivers, ACC and C-ACC systems can mitigate the capacity drop phenomenon and achieve significant savings in total travel time without reducing the desired time gap compared to human drivers. They change the flow stability and the jam propagation velocity, resulting in new phenomena that have not been observed in traffic with all human-driven vehicles. We emphasise that the impacts of ACC and C-ACC on flow operations depends on controller formulation and parameter settings. Hence, designing ADAS controllers to improve traffic operations must be approached in a rigorous way under profound understanding of the consequences of design choices and parameter settings.

**Cooperative ADAS with vehicle-vehicle cooperation perform better than non-cooperative ADAS and human drivers in terms of throughput, stability and sustainability.** The C-ACC controller takes into account the expected responses of its follower and makes control decisions to optimise the performance of the whole platoon. This feature has clear benefits from traffic operations perspectives. Although the considerate behaviour of the C-ACC controller may entail *compromising its own situation*, it generates benefits in the collective platoon and flow. Systematic investigation and comparison on the flow characteristics of human drivers, ACC and C-ACC systems indicate that cooperative systems have more potentials in improving the flow stability and efficiency compared to human drivers and non-cooperative systems.

**Integrated traffic control based on vehicle-infrastructure cooperation is more effective in reducing congestion than the decentralised control paradigm.** The case study of integrated VSL control with ACC vehicles proves the feasibility and benefits of in-vehicle actuation of traffic control signals concept. Sending link-level control commands to vehicle-level controllers via V2I communication integrates the traffic controller and vehicle controller and demands the controllers at the two levels to cooperate. Hence the traffic control signals are better actuated. The integrated control paradigm with vehicle-infrastructure cooperation is more effective in reducing total time spent in the network compared to reference case with 100% human drivers and to the scenarios with the same penetration rate of the decentralised ACC vehicles.

### 9.3 Implications for practice

The findings and conclusions of this thesis have several important practical implications and contributions, which can be categorised into toolbox development, platoon-
ing strategy for fleet management, provision of assistance and new approach for future traffic management with IVs.

The generic control framework and the solution approach based on Pontryagin’s Principle provide a methodological toolbox for designing new ADAS controllers and deriving operational algorithms. The flexibility of the framework in formulating objective function and in dealing with state and control constraints facilitates ADAS developers in the design process. The assessment framework, including the analytical approach for capacity and stability analysis and simulation methods, can be applied by researchers, industries and policy-makers for testing ADAS performance and the impacts on traffic and sustainability. It is advised to assess the impacts of new ADAS on traffic operations before promoting ADAS to market, since our study have shown that careful design and tuning of ADAS controllers are essential to guarantee positive impacts on collective traffic operations.

The improved platoon performance with vehicle-vehicle cooperation implies that a platoon of cooperative vehicles with V2V communications can better off from the cooperation. This cooperative control strategy can be applied to fleet operations. Fleet owners can equip vehicles with V2V communications and implement cooperative control strategy in fleet vehicles to increase driver comfort, to ensure safety and to improve travel efficiency for the whole platoon.

The insights into the changes in flow characteristics of ADAS can facilitate the researchers and road authorities in reconsidering or redesigning current traffic control measures based on human-driven vehicular flow properties. The changed flow characteristics, including capacity and stability properties, have great implications for design of traffic control measures. The insights into the qualitative and quantitative changes in flow characteristics can facilitate the road authorities in successful implementation of traffic control measures. For instance, in case of stop-and-go waves, the SPECIALIST algorithm may choose a higher speed limit than 60 km/h, since the reduced capacity reduction and jam size due to ADAS vehicles necessitate less control efforts, and integrating the cooperative version of the SPECIALIST algorithm recently developed (Van de Weg et al., 2014) with the model predictive ACC and C-ACC systems proposed in this thesis may bring further benefits in resolving stop-and-go waves since the speed limits can be sent more specifically according to the positions and traffic states of the intelligent vehicles.

Furthermore, the integrated traffic control paradigm based on vehicle-infrastructure cooperation provides a new approach for road authorities to manage future traffic flow with intelligent vehicles. Traffic control signals can be better complied and actuated with V2I communications. It is advised to incorporate the vehicle-infrastructure cooperation with V2I communication at early stages of introducing ADAS in road networks to achieve higher effectiveness of traffic control.
9.4 Recommendations for future research

The virtue of scientific research implies that new questions emerge as the underlying research questions are answered. In this section, we recommend several directions for future research.

The first suggestion for future research is to extend the generic control framework to automation of lateral driving task, i.e. lane-changing. In this thesis, we focus on ADAS functions in the longitudinal dimension, which support drivers in maintaining a free/desired speed or following the leader with a desired distance. The lateral driving task is more complex compared to its longitudinal counterpart, due to the interaction and interdependency of decisions at the two dimensions. Anticipated difficulties in this extension include the discrete nature of lane change decision as control variable and lane choices as state variable, and proper cost specification to yield incentives of lane changes, e.g. gain in travel speed or following keep-right directives. The preliminary work seems producing promising results in the extension (Wang et al., 2014c).

Another future research direction is to test the robustness and effectiveness of the proposed controllers in more practical situations where noises and delays prevail in the control loop. In this thesis, we assume the duality problem of state estimation has been addressed and vehicle lags are negligible. In reality, data from on-board sensors are prone to errors and lags exist in vehicle dynamics. Performance of the proposed controllers under the model predictive control framework with noise and delay are of interest for the robustness and effectiveness of the controllers. The issues may compromise the controller performance and state estimator or improved control strategy may be needed to cope with the inaccuracies and delays in the control loop. For instance, the classic Kalman filter and its extensions can be used to improve the state estimation (Yuan et al., 2012), while the delays can be compensated by prediction to some extent under model predictive control approaches (Diehl et al., 2009). The work of Ploeg et al. (2014) also provides an interesting approach to test the robustness of the proposed controllers in the thesis.

The third suggestion is to test the impacts of ADAS in active bottlenecks with road geometric inhomogeneities, such as on- and off-ramps. In the simulation study on the homogeneous two-lane motorway stretch, we focus on the impacts of ADAS on formation and propagation of stop-and-go waves triggered by temporarily lowering speed limits. In reality, other jam types and traffic states prevail in active bottlenecks such as motorway sections with on-ramps. It is interesting to examine the changes in jam properties and traffic states with ADAS vehicles in active bottlenecks under different bottleneck strength.

For operations of ADAS, human drivers still play an important role. Therefore, human factors issue in ADAS remains an important future topic of research. In this thesis, we assume that human drivers give the longitudinal control of the vehicles to ADAS and only control the steering wheel. Field test has shown deactivation of the existing ACC.
systems by drivers (Viti et al., 2008), which may also be an issue for the proposed controllers and may lead to changes in controller performance. Therefore, it is interesting to consider and test human factors in relation to the proposed ADAS controllers and understand the consequences of the interactions between human drivers and the automated systems on the controller performance.

Last but not least, information and communication technologies are dispensable in implementation of cooperative systems. In this thesis, we make simple assumptions on V2V and V2I communications. It is interesting to consider more technological and implementation issues into the ADAS controller design process, such as connectivity failure, quantisation error, ambient disturbances, etc. (Yan & Bitmead, 2003). This may stimulate new research questions, including how the C-ACC system perform when V2V fails in platooning? which information architecture and information flow employed by cooperative systems can lead to better performance? what are the requirements on the bandwidth, frequency for V2V and V2I communications for certain ADAS applications? Answering these questions entails development of analysis, modelling and simulation tools to assess the feasibility, security and performance related to V2V and V2I communications for ADAS applications (RITA, 2013).
Appendices

Appendix A: Derivation of ACC and C-ACC algorithms

ACC algorithm

Under the cost specification (5.14), we have:

\[ \lambda_1 = \frac{1}{\eta} \frac{\partial L_{\text{ACC}}}{\partial s} = \frac{1}{\eta} \left( -\frac{c_1 s_0}{s^2} e^{s_0 / s} \Delta v^2 \Theta(\Delta v) - 2c_2 (s - s_d(v)) \right) \] (9.1)

\[ \lambda_2 = \frac{1}{\eta} \frac{\partial L_{\text{ACC}}}{\partial v} = \frac{1}{\eta} \left[ -2c_1 e^{s_0 / s} \Delta v \Theta(\Delta v) + 2c_2 t_d(s_d(v) - s) - \lambda_1 \right] \] (9.2)

Inserting Eq. (9.1) into Eq. (9.2) and using \( u^* = -\lambda_2 \) will arrive at the ACC algorithm of Eq. (5.17).

C-ACC algorithm

Under the joint cost specification \( L_{\text{CACC}} \) of Eq. 5.21, we have:

\[ \lambda_3 = \frac{1}{\eta} \frac{\partial L_{\text{CACC}}}{\partial s_{n+1}} = \frac{1}{\eta} \left( -\frac{c_4 s_0}{s^2} e^{s_0 / s} \Delta v_{n+1}^2 \Theta(\Delta v_{n+1}) + 2c_5 (s_{n+1} - s_d(v)) \right) \] (9.3)

\[ \lambda_2 = \frac{1}{\eta} \left( \frac{\partial L_{\text{CACC}}}{\partial v_n} - \frac{1}{\eta} \lambda_1 + \frac{1}{\eta} \lambda_3 \right) \]

\[ = \frac{1}{\eta} \left[ -2c_1 e^{s_0 / s} \Delta v_n \Theta(\Delta v_n) + 2c_2 t_d(s_d(v) - s) + 2c_4 e^{s_0 / s} \Delta v_{n+1} \Theta(\Delta v_{n+1}) + 2c_5 t_d(s_d(v) - s_{n+1}) \right] - \frac{1}{\eta} \lambda_1 + \frac{1}{\eta} \lambda_3 \] (9.4)

with \( \lambda_1 \) evaluated using Eq. (9.1). Inserting Eqs. (9.3, 9.1) into Eq. (9.4) and using \( u^*_n = -\lambda_2 \) will arrive at the C-ACC algorithm of Eq. (5.22).
Appendix B: Approximation of the Heaviside function in the ACC and C-ACC algorithms

The term with the Heaviside function in the Eqs. (5.17, 5.22), being $\Delta v\Theta(\Delta v)$, has the following form:

$$\Delta v\Theta(\Delta v) = \begin{cases} \Delta v & \text{if } \Delta v \leq 0 \\ 0 & \text{if } \Delta v > 0 \end{cases}$$

and the first order derivative given by:

$$\frac{d\Delta v\Theta(\Delta v)}{d\Delta v} = \begin{cases} 1 & \text{if } \Delta v < 0 \\ 0 & \text{if } \Delta v > 0 \end{cases}$$

While Eq. (9.5) is continuous at $\Delta v = 0 m/s$, it is not differentiable at $\Delta v = 0 m/s$. To enable the analytical analysis on the model property, Eq. (9.5) is approximated by the following function

$$\Delta v\Theta(\Delta v) \approx \frac{\Delta v}{2} - \frac{\ln(\cosh(\delta \cdot \Delta v))}{2\delta}$$

(9.5)

where the coefficient $\delta$ determines the range of the transition from $\Delta v\Theta(\Delta v) = \Delta v$ to $\Delta v\Theta(\Delta v) = 0$. The larger $\delta$ is, the sharper the transition is, and the more accurate the approximation is. Thus the first order derivative of Eq. (9.5) gives:

$$\frac{d\Delta v\Theta(\Delta v)}{d\Delta v} \approx 1 - \tanh(\delta \cdot \Delta v)$$

(9.6)

with $\left| \frac{d\Delta v\Theta(\Delta v)}{d\Delta v} \right|_{\Delta v=0} \approx \frac{1}{2}$.

Figure 9.4 depicts the smoothed function compared with the original function using $\delta = 10$, which gives a good approximation of the original function.

![Smoothed function compared with original function](image)

Smoothed function (red line) with $\delta = 10$. Green dashed line depicts the function $f(\Delta v) = 0$ and the blue dashed line depicts the function $f(\Delta v) = \Delta v$.

Notice that the second order term of $\Delta v$ with the Heaviside function in the ACC and C-ACC algorithms, being $\Delta v^2\Theta(\Delta v)$, is continuous and differentiable to the first order.
This becomes evident when writing out the complete expression of the $\Delta v^2 \Theta(\Delta v)$ as:

$$\Delta v^2 \Theta(\Delta v) = \begin{cases} 
\Delta v^2 & \text{if } \Delta v \leq 0 \\
0 & \text{if } \Delta v > 0
\end{cases}$$

Taking the first order derivative of Eq. (9.7) to $\Delta v$ piecewisely and taking the limits around $\Delta v = 0$ gives:

$$\lim_{\Delta v \to 0^-} \frac{d\Delta v^2}{d\Delta v} = \lim_{\Delta v \to 0^+} \frac{d0}{d\Delta v} = 0$$

Thus $\frac{d\Delta v^2 \Theta(\Delta v)}{d\Delta v}|_{\Delta v=0} = 0$. 

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Summary

Model Predictive Control Framework for Advanced Driver Assistance Systems

Advanced Driver Assistance Systems (ADAS) support or even take over drivers in performing driving tasks such as car-following or lane-changing and are seen as a promising approach to improve safety, efficiency and sustainability of transport systems. Numerous conceptual ADAS models and prototype systems have been developed in the past three decades. Despite the significant achievements, a number of scientific challenges prevail in designing and testing ADAS controllers, such as refining existing ADAS, operationalising new ADAS concepts and examining the collective impact of ADAS on traffic flow and environmental sustainability. These challenges motivate the research reported in this thesis.

This thesis deals with the development of a generic control framework for ADAS control design, with a focus on ADAS that automate car-following tasks. Application of the framework yields several new controller examples of Adaptive Cruise Control (ACC), ecological ACC (EcoACC), cooperative ACC (C-ACC) under autonomous following, eco-driving support, and cooperative manoeuvring concepts. Controllers’ performance and their impact on traffic flow characteristics and sustainability are examined systematically. Below we synthesise the research from a wider perspective.

Control framework and application

A model predictive control approach is used to develop the control framework for designing and testing ADAS controllers. Positions and speeds of controlled vehicles and surrounding vehicles constitute the system state while accelerations of controlled vehicles are controlled inputs. The key feature of the approach is that the controller predicts the behaviour of surrounding vehicles based on the current state of the system and determines an optimal acceleration trajectory minimising a cost function. The cost function reflects multiple criteria, such as safety, efficiency, comfort, and sustainability. The optimal acceleration trajectory is recomputed in a receding horizon manner with the newest information regarding the system state.

Different solution approaches to general continuous-time optimal control problems are discussed, compared and applied in the thesis. A new iterative solution approach based
on Pontryagin’s Minimum Principle is proposed. Tests show that the solution approach is flexible in dealing with both autonomous and cooperative ADAS control problems. The solution approach is scalable to large scale systems and yields efficient algorithms that are promising for on-line applications.

Application of the proposed control framework leads to several new ADAS controllers and implementable algorithms. The ACC controller optimising efficiency, safety and comfort is developed and refined throughout the thesis, taking into account collision-free constraints and variable desired time gap policies. The EcoACC controller is designed by incorporating CO₂ emissions minimisation in addition to the ACC control objectives. Unlike the ACC and EcoACC systems based solely on information from on-board sensors, C-ACC systems can share the state and control information via Vehicle-to-Vehicle (V2V) communications and make control decisions together. C-ACC vehicles coordinate their behaviour to optimise the collective situation, which may entail compromising the individual situation to benefit the whole system. One unique feature of the C-ACC controller is that it is not restricted to cooperation between controlled vehicles. When a C-ACC vehicle is followed by a human-driven vehicle, it can still exhibit cooperative behaviour by taking into account the expected response of the human-driven vehicle to the control decisions. This is of vital importance for introducing such systems at an early stage, since the proposed C-ACC controller shows cooperative behaviour even without V2V communication. The controller properties are examined via analytic methods and simulations. Results provide insights into the controller properties, particularly into the influence of controller parameters on controller performance. Examinations on controller performance show that the proposed controllers generate plausible behaviour under representative scenarios as expected from the controller formulations.

New insights on the impact of ADAS

Systematic investigations into the collective flow properties of the proposed controllers are carried out throughout controller design processes, giving insights into the resulting flow characteristics. The resulting capacity and stability of traffic with ADAS vehicles are largely determined by the controller parameters, e.g. desired time gaps and weight factors associated to different cost terms in the objective function. Increasing the desired time gap increases the resulting flow stability, while compromising capacity. Increasing weights on safety cost and efficiency cost improves the resulting flow stability, while the fundamental relation between flow and density remains unchanged.

Particular attention is given to the dynamic flow properties related to stop-and-go waves, including capacity drop, stability and wave propagation speed, in case of a temporary bottleneck with presence of ACC and C-ACC vehicles. The decentralised ACC and distributed C-ACC algorithms are implemented in an open-source microscopic traffic simulator. Simulation experiments on a 2-lane motorway stretch of 14 km with more than 500 vehicles are conducted, with different penetrate rates of ACC and C-ACC vehicles randomly distributed. Results show that ACC systems mitigate the capacity drop phenomenon and improve the stability of the traffic flow upstream of
the jam area. The resulting jam propagation velocity is less characteristic compared to that of the human-driven vehicular flow, i.e. it does not show a constant speed propagating upstream but depends on the compositions of ACC and human driven vehicles in traffic. The C-ACC system improves the stability at both jam tail and jam head and increases the effective capacity at the bottleneck compared to human drivers and ACC systems. One noteworthy flow property is that compared to those from human drivers and ACC systems, C-ACC systems result in stop-and-go waves propagating faster in the upstream direction due to V2V communications.

The proposed EcoACC systems lead to lower traffic speed and lower flow compared to ACC systems at free traffic conditions, but higher speed and higher flow at congested traffic conditions. The spatial CO₂ emission rate (in g/km) of EcoACC vehicles is lower than that of ACC vehicles.

**Connected traffic control and vehicle control**

The control concept of in-vehicle actuation of traffic control signals where intelligent vehicles are used as actuators for traffic control systems is operationalised by integrating a link-level variable speed limit (VSL) controller and the vehicle-level ACC controller with Vehicle-to-Infrastructure (V2I) communication. The link-level controller estimates and predicts the global traffic state and constructs VSL schemes to resolve stop-and-go waves. The VSL control signals are transmitted to the ACC vehicles from VSL gantries via V2I communications. ACC vehicles optimise local situations, using the VSL signals as control commands to adapt their local control parameters of desired speeds. Controlled simulation experiments are conducted to examine the effectiveness of the integrated control paradigm, with different penetration rates of ACC vehicles in traffic. Despite the difficulties in predicting traffic states due to the changed flow characteristics with ACC systems, the integrated control concept works without fundamental changes of the original VSL algorithm and resolves stop-and-go waves successfully in all the test cases with ACC vehicles. The integrated control paradigm with vehicle-infrastructure cooperation is more effective in reducing total time spent in the network compared to the reference case with 100% human drivers and to the scenarios with the same penetration rate of decentralised ACC vehicles.

**Implications and recommendations**

The generic control framework and the solution approach based on Pontryagin’s Principle provide a methodological toolbox for designing new ADAS controllers and deriving operational algorithms. It is advised to assess the impacts of new ADAS on traffic operations before promoting ADAS to market, since our study has shown that careful design and tuning of ADAS controllers are essential to guarantee positive impacts on the collective traffic flow.

The insights into the changes in flow characteristics of ADAS can facilitate researchers and road authorities in reconsidering or redesigning current traffic control measures based on human-driven vehicular flow properties. The integrated traffic control paradigm
based on vehicle-infrastructure cooperation provides a new approach for road authorities to manage future traffic flow with intelligent vehicles. It is advised to incorporate the vehicle-infrastructure cooperation with V2I communication at early stages of introducing ADAS in road networks to achieve higher effectiveness of traffic control.

Several future research directions are recommended, including extension of the generic control framework to lane change decision and manoeuvre control, testing the robustness and effectiveness of the proposed controllers in more practical situations where noise and delay prevail in the control loop, and including the effects of human factors.
Samenvatting

Model Predictive Control Framework for Advanced Driver Assistance Systems


Dit proefschrift beschrijft de ontwikkeling van een generiek regeltechnisch raamwerk voor het ontwerpen van ADAS regelaars, waarbij de focus ligt op ADAS die het afstand houden automatiseren. De toepassing van dit raamwerk omvat voorbeelden van verschillende regelaars van Adaptive Cruise Control (ACC), ecologische ACC (EcoACC), en coöperatieve AAC (C-AAC) bij automatisch afstand houden, ondersteuning voor zuinig rijden, en coöperatieve rijconcepten. Hierbij worden de prestaties van de regelaars en hun invloed op verkeersdoorstroming en duurzaamheid systematisch beoordeeld. Hieronder wordt het onderzoek vanuit een breder perspectief toegelicht.

Regeltechnisch raamwerk en toepassing

Voor het ontwikkelen en testen van ADAS regelaars wordt een zogenaamde ‘model predictive control’ methodiek toegepast. Een essentiële eigenschap van deze methode is dat de regelaar aan de hand van bestaande condities in het transport systeem het gedrag van omringende voertuigen voorspelt. Aan de hand daarvan wordt vervolgens het optimale versnellings- (of vertragings-)profiel bepaald door het minimaliseren van een kostenfunctie. Deze kostenfunctie weerspiegelt meerdere criteria, waaronder veiligheid, efficiëntie, comfort en duurzaamheid. Hiermee wordt het optimale versnellingsprofiel steeds opnieuw berekend aan de hand van de meest recente systeem condities.
In dit proefschrift worden verschillende oplossingsmethoden voor algemene tijd-continue optimale regeltechnisch problemen besproken, vergeleken en getest. Op basis hiervan wordt een nieuwe iterative oplossingsmethode voorgesteld, gebaseerd op Pontryagin’s Minimum Principle. Tests laten zien dat deze methode flexibel omgaat met zowel autonome als coöperatieve ADAS regelproblemen. Bovendien is de methode schaalbaar naar grootschalige systemen en leidt deze tot efficiënte algoritmes die veelbelovend zijn met betrekking tot on-line (i.e., real-time uitvoerbare) toepassingen.

De toepassing van het voorgestelde regeltechnisch raamwerk leidt tot een aantal nieuwe ADAS regelaars en implementeerbare algoritmes. De ACC regelaar voor het optimaliseren van efficiëntie, veiligheid en comfort is in dit proefschrift ontwikkeld en afgesteld met in acht name van variabele volgtijdstrategieën en de restrictie dat de oplossing botsings-vrij is. De EcoACC regelaar is ontworpen door CO\textsubscript{2} emissie minimalisatie toe te voegen aan de ACC regelaar doelstellingen. In tegenstelling tot de ACC en EcoACC systemen die alleen informatie krijgen van sensoren aan boord, delen C-ACC systemen de toestands- en regelinformatie via voertuig-voertuig (V2V) communicatie en worden regelbeslissingen samen gemaakt. C-ACC voertuigen coördineren hun gedrag zodanig dat de prestatie van het totale systeem geoptimaliseerd wordt. Dit houdt in dat compromissen worden gesloten waarbij het individueel belang ondergeschikt is aan dat van het totale systeem. Een uniek kenmerk van de C-ACC regelaar is dat samenwerking niet beperkt blijft tot samenwerking tussen geregelde voertuigen. Als een C-ACC voertuig wordt gevolgd door een voertuig met bestuurder, kan het nog steeds coöperatief gedrag vertonen door de verwachte reactie van de bestuurder van het voertuig mee te nemen in de beslissing van de regelaar. Dit is van essentieel belang bij de invoering van het systeem, omdat dit systeem niet allen afhankelijk is van V2V communicatie. Simulaties met de voorgestelde regelaar laten onder representatieve omstandigheden plausibel gedrag zien, zoals verwacht op basis van de formulering van de regelaar.

**Nieuwe inzichten over de invloed van ADAS**

Gedurende het ontwerpproces van de voorgestelde regelaars is de verkeersstroomsysteematisch in kaart gebracht, waardoor inzicht is verkregen in de resulterende eigenschappen van de verkeersstroom. De capaciteit en stabiliteit van verkeersstromen met ADAS-voertuigen wordt voornamelijk bepaald door de parameters van de regelaar, zoals de gewenste volgtijd en de weegfactoren voor de verschillende elementen in de kostenfunctie. Het vergroten van de volgtijd verbetert de stabiliteit van de verkeersstroom ten koste van de capaciteit. Het verhogen van de weegfactoren voor veiligheid en efficiëntie zorgt ook voor een stabielere verkeersstroom, terwijl de fundamentele relatie tussen verkeersdichtheid en verkeersintensiteit (en dus ook de capaciteit) gelijk blijft.

Er is speciale aandacht gegeven aan dynamische verkeersstroom eigenschappen die zijn gerelateerd aan korte stroomopwaarts bewegende filegolven, waaronder de capaciteitsval, stabiliteit en de snelheid van de filegolf in het geval van een tijdelijk knelpunt bij aanwezigheid van ACC en C-ACC voertuigen. De algoritmes voor gedecentraliseerde ACC systemen en gedistribueerde C-ACC systemen zijn geïmplementeerd in
een open-source microscopisch verkeerssimulatiemodel. Er zijn experimenten uitgevoerd met het simulatiemodel voor een 14 kilometer lange tweestrooms snelweg met verschillende, willekeurig verdeelde, aandelen van ACC en C-ACC voertuigen. ACC systemen verminderen de capaciteitsval en verbeteren de stabiliteit stroomopwaarts van de file. De resulterende filegolf heeft geen constante snelheid stroomopwaarts, maar de snelheid hangt af van de verhouding tussen ACC voertuigen en voertuigen met bestuurders in het verkeer. De C-ACC systemen verbeteren de stabiliteit aan zowel het begin als het einde van een opstopping en verhogen de effectieve capaciteit van het knelpunt in vergelijking met menselijke bestuurders en ACC systemen. Een noemenswaardige eigenschap van C-ACC systemen is dat filegolven zich sneller stroomopwaarts verplaatsen als gevolg van de V2V communicatie.

De voorgestelde EcoACC systemen leiden tot lagere verkeerssnelheden en lagere intensiteiten in vergelijking met ACC systemen in filevrije condities, maar tot hogere snelheid en intensiteiten in staat van congestie. De uitstoot van CO$_2$ per kilometer weg is in geval van EcoACC voertuigen lager dan met ACC voertuigen.

**Verkeersmanagement met ADAS**

Het regelconcept met matrixbord aangestuurde voertuigen, waarbij intelligente voertuigen de actuatoren van het verkeersregelsysteem zijn, is gerealiseerd door een variabele snelheidslimiet (VSL) regelaar op wegvak-niveau te koppelen aan de ACC regelaar op voertuig-niveau door middel van voertuig-infrastructuurbeginsel (V2I) communicatie. De regelaar op wegvak-niveau schat en voorspelt de globale verkeerstoestand en construeert VSL schema’s om de filegolven op te lossen. Deze VSL regelsignalen worden naar de ACC voertuigen doorgestuurd vanaf de VSL-portalen via V2I communicatie. De ACC voertuigen optimaliseren vervolgens de lokale situatie, waarbij de VSL-signalen worden gebruikt als regelcommando om de parameters voor de gewenste snelheid aan te passen. Gecontroleerde simulatieexperimenten zijn uitgevoerd om de effectiviteit te onderzoeken van dit geïntegreerde verkeersregelschema, met verschillende aandelen van ACC voertuigen in het verkeer. Ondanks de moeilijkheden in het voorspellen van de verkeerssituatie door de veranderde eigenschappen van de verkeersstroom bij gebruik van ACC systemen, werkt het regelconcept zonder fundamentele veranderingen van het oorspronkelijke VSL algoritme, en worden de filegolven succesvol opgelost in alle geteste situaties met ACC voertuigen. Het geïntegreerde regelconcept met samenwerking tussen voertuigen en infrastructuur is effectiever in het reduceren van de totale reistijd in het netwerk dan de referentie-situatie met allen menselijke bestuurders en ten opzichte van de scenario’s met een even groot aantal ACC voertuigen dat niet samenwerkt.

**Implicaties en aanbevelingen**

Het generieke regeltechnisch raamwerk en de oplossingsmethode gebaseerd op Pontryagin’s Minimum Principle leveren methodologisch gereedschap voor het ontwerpen van nieuwe ADAS regelaars en het afleiden van operationele algoritmes. De invloed van nieuwe ADAS kan het beste op verkeersdoorstroming worden beoordeeld voordat
ADAS op de markt wordt gebracht, omdat deze studie uitwijst dat het ontwerp en de afstelling van ADAS regelaars essentieel is om een positief effect op de verkeerssituatie te garanderen.

De inzichten in de verandering van de verkeersstroom bij een transitie van menselijke bestuurders naar ADAS kunnen onderzoekers en wegbeheerders ondersteunen bij het heroverwegen en herontwerpen van verkeersregelmechanismen. Het geïntegreerde verkeersregelsysteem met samenwerking tussen voertuigen en infrastructuur geeft wegbeheerders een nieuwe aanpak voor verkeersmanagement met intelligente voertuigen. Er wordt geadviseerd om samenwerking tussen voertuigen en infrastructuur met behulp van V2I communicatie in een vroeg stadium van de ADAS invoering mee te nemen om een hogere effectiviteit te kunnen bereiken.

Er zijn meerdere aanbevelingen voor toekomstig onderzoek, zoals (1) een uitbreiding van het generieke regeltechnische raamwerk met regelaars voor rijstrookwisselingen, (2) het testen van de robuustheid en effectiviteit van de voorgestelde regelaars in de praktijk, waar ruis en vertraging een rol spelen in de regellus, en (3) het meenemen van het effect van het menselijk gedrag.
概述 (Summary in Chinese)

用于先进的辅助驾驶系统的通用模型预测控制框架

先进的辅助驾驶系统（以下简称辅助驾驶系统）能够帮助或替代驾驶员执行类似跟驰或换道的驾驶任务，是一种提高交通运输系统安全、效率和可持续性的有效手段。虽然过去三十年间已经开发出了许多辅助驾驶系统的概念模型和原型系统，取得了显著的成果，但在设计和测试辅助驾驶系统方面仍然存在一系列的技术挑战，包括现有系统的改进、新的概念系统的实施、以及这些系统规模应用后对交通流和环境可持续性影响的检验。这些技术挑战是本论文研究的出发点。

本论文研究开发一种用于辅助驾驶系统控制设计的通用控制框架，重点关注的是跟驰任务的自动化。应用控制框架的应用提出了几种新型的辅助驾驶系统控制器，包括基于自主跟驰概念的自适应巡航系统，基于生态辅助驾驶概念的自动巡航系统和基于合作式操纵概念的合作式自适应巡航系统。论文对提出的控制器的性能和系统规模应用后对交通流和可持续性的影响进行了系统性地检验。下面将从一个更广泛的角度对本论文的研究进行阐述。

控制框架和应用

论文应用模型预测控制方法开发用于设计和测试辅助驾驶系统控制器的控制框架。受控车辆和周围车辆的位置和速度组成了系统的状态，受控车辆的加速度作为控制输入。该方法的主要特征是控制器基于当前系统的状态预测周围车辆在未来一段时间的运动，从而确定一个能够最小化费用泛函（或目标泛函）的最优加速度轨迹。该费用泛函反映了多维指标，如安全、效率、舒适和可持续性等。在下一个时间步系统获得更新后状态信息后，将采用滚动时域控制方式重新计算最优加速度轨迹。

论文对解决连续时间最优控制问题的不同方法进行了对比分析和应用，并且提出了一种新的基于哥特里亚金极值原理的迭代算法。测试表明论文提出的迭代解法在处理自主式和合作式辅助驾驶系统控制的问题上非常灵活。该方法很容易扩展应用到大规模系统的控制设计并且可以导出可用于在线应用的高效控制算法。

本文控制框架的应用形成了一些新型的辅助驾驶系统控制器和可执行的控制算法。自适应巡航控制器的控制目标是最大化出行效率、驾驶安全和驾乘舒适性。论文的不同章节对自适应巡航控制器进行了不断改进，考虑了前后车之间
追尾碰撞的约束条件和可变的期望间距时距策略。生态自适应巡航控制器在自适应巡航控制器的控制目标的基础上还加入了最小化二氧化碳排放量的目标。与完全依赖于车载传感器的自适应巡航控制系统和生态自适应巡航控制系统不同的是，合作式自适应巡航控制系统可以通过车辆间通信交换系统状态和控制信号信息，并且一起做出控制决定。合作式自适应巡航控制车辆协调多个车辆的行为来优化整体的情况，这样可能会导致单个合作车辆对局部利益作出让步而使全局受益。合作式自适应巡航控制器的一个独有的特征是它局限于受控车辆间的合作：当一个合作式受控车辆被普通非受控车辆跟随时，它仍然可以应用人的驾驶行为模型预测普通车辆对受控车辆的控制决定的反应，并根据这些反应来优化整体的性能。这一点在早期引入辅助驾驶系统时至关重要，因为它的工作不再依赖于车车间通信：即使没有车间通信，合作式自适应巡航车辆仍然可以表现出合作的行为。论文采用解析方法和仿真对控制器特性进行了检验，检验结果为控制器特性提供了深刻的认识，特别是控制参数对控制器性能的影响。对控制器性能研究的结果表明提出的控制器在典型的场景下表现出与控制器的数学公式预期相符的合理行为。

辅助驾驶系统对交通流影响的新认识

在设计辅助驾驶系统控制器的过程中，论文对系统规模应用后对宏观交通流特征的影响进行了系统性的研究。研究结果为包含受控车辆的交通流特征提供了新的认识。由受控车辆构成的交通流的通行能力和稳定性在很大程度上取决于控制器参数的设定，例如期望间距时距和费用泛函中的不同费用项的权重。提高间距时距可以改善交通流的稳定性，然而会降低通过能力。提高费用泛函中的安全费用和效率可调项的权重也可以使交通流更加稳定，同时不改变流量-密度的基本关系。

论文特别关注了在临时交通瓶颈下自适应巡航系统和合作式自适应巡航系统对与走-停交通波相关的动态交通流特征的影响，包括通行能力下降，稳定性和交通流的传播速度。我们将分离式自适应巡航控制算法和分布式合作式自适应巡航控制算法在一个开源的微观交通仿真器上进行了实施。在此基础上，建立了包含500多辆车、14公里长的两车道高速公路的仿真模型，并将不同比例的普通车辆、自适应巡航系统车辆和合作式自适应巡航系统车辆随机分布在路网中，开展了大量的仿真实验。仿真结果表明自适应巡航系统可以缓解通行能力下降的现象，并且可以提高拥堵区域上游交通流的稳定性。然而，与人工驾驶车辆组成的交通流相比，拥堵在包含自适应巡航车辆的车流中的传播速度不再具有明显的特征，例如拥堵不再是按固定的速率向上游传播，而是取决于车流中自适应巡航车辆和普通车辆的构成。和普通车流和包含自适应巡航车辆的车流相比，合作式自适应巡航系统可以提高拥堵区域上游和下游交通流的稳定性，并且提高瓶颈区域的有效通行能力。值得一提的是，由于合作式车辆间可以无线通信，合作式自适应巡航系统导致的走-停交通波在车流中传播速度要比普通驾驶员和自适应巡航系统导致的交通波传播速度快很多。

和自适应巡航系统相比，论文所提出的生态自适应巡航系统在自由交通条件下会形成较低的速度和流量，但拥堵交通条件下会形成较高的速度和较高的流量。生态自适应巡航系统车辆的单位里程二氧化碳排放量要低于自适应巡航系统车辆。
交互式交通控制和车辆控制

论文实施了交互式交通控制信号的车载执行概念，将路段层的可变限速控制器和车辆层的自适应巡航控制器通过车辆间无线通信进行集成。路段层的可变限速控制器估计并预测全局交通状态并构建可变限速控制方案以消除车-车交通波。实时的可变限速信号由路侧的可变限速板通过车辆通信传送到车载自适应巡航系统。自适应巡航系统用接收到的可变限速值来调整车辆层的控制参数，即期望车速，从而计算最优加速度来优化车辆周围的状态。论文应用了受控的仿真实验来检验交互式交通控制方式的效果。仿真实验考虑了不同普通车辆和自适应巡航车辆的比例。实验表明，即使自适应巡航车辆的存在使交通流的特征发生了变化而增加了交通状态预测的难度，交互式控制方式在所有的测试实验中都工作良好并且成功消除了车-车交通波。与完全由人工驾驶车辆组成的交通流相比，这种基于车辆间合作的交互式控制方式在减少路段总体行程时间方面更有效。

启示和建议

本文提出的通用的控制框架和基于波特里亚金极值原理的解法为设计新型辅助驾驶系统控制器和开发控制算法提供了一个方法论层面的集成工具包。研究结果显示，对辅助驾驶系统控制器进行谨慎设计和调整可确保其对宏观交通流生产正面影响的必要条件。因此，建议在将新型的辅助驾驶系统控制器向市场推广之前系统性地评估其对交通流运行可能造成的影响。

对辅助驾驶系统对交通流特征影响机理的认识可以帮助帮助研究人员和道路交通管理部门重新考虑或设计目前基于人工驾驶车流特征的交通控制系统。基于车辆合作的交互式交通控制系统为道路交通管理部门管理车道和智能车辆的交通流向提供了新方法。为获得更好的交通控制效果，本研究建议在推广辅助驾驶系统的初期阶段便将基于车辆通信与车辆合作纳入整体系统。

后续研究建议在以下几个方面开展：扩展通用的控制框架以控制换道决定和换道过程，在实际的包含噪声和延迟的控制环路中测试控制器的鲁棒性和性能，以及在控制环路中加入人的因素。
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Curriculum Vitae

Meng Wang was born on November 19, 1980 in Beijing, China. He received his Bachelor of Engineering degree from the Department of Civil Engineering, Tsinghua University in 2003 and his Master of Science degree from Research Institute of Highway (RIOH), Ministry of Transport in 2006, with specialisation in Transport Planning and Management.

From 2006 to 2009, Meng worked as research associate and assistant researcher at National Centre of Intelligent Transport Systems (ITS) Engineering and Technology in RIOH. There he contributed in several projects funded by the Ministry of Science and Technology, Ministry of Transport, and local governments in China, varying from driver behavioural analyses, system architecture design and evaluation for Cooperative (Vehicle Highway) Systems, to ITS policy recommendations. In RIOH, he was also involved in international projects on potential cooperation in ITS between Europe and China, SIMBA and SIMBA II, funded by European Commission under the FP6 and FP7 programmes.

From September 2009, he started working on his PhD project “Sustainability Perspectives of Cooperative Systems” funded by Royal Dutch Shell at the Department of Transport & Planning in TU Delft. He worked on the development of a model predictive control framework for a variety of autonomous and cooperative vehicle systems. During his PhD study, he also worked on two external projects, one funded by NAVTEQ and the other funded by the Dutch Ministry of Infrastructure and Environment. Apart from research, he also assisted in teaching a master course, supervised master students, and served as referees for international journals and conferences.

From May 2014, he started working at the Department of BioMechanical Engineering in TU Delft on the project “Truck Merging Support – a Step towards Autonomous Driving” funded by the Dutch Technology Foundation (STW).

Meng’s main research interests are driver behaviour modelling, control approaches for intelligent vehicle systems and traffic flow theory in relation to intelligent vehicles.
Author’s Publications

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**Book chapter**


**Peer-reviewed conference papers**


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**Technical reports**
