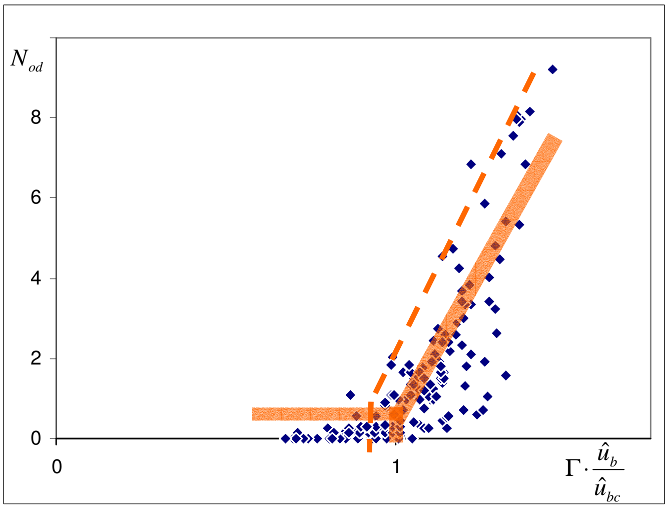
Toe structures for rubble mound breakwaters

Analysis of toe bund design tools and a method for toe rock stability description



MSc thesis
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June 2008

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A dissertation submitted in partial fulfilment of the requirements for the degree of Master of Science at Delft University of Technology in Civil Engineering, track Hydraulic Engineering, specialisation Coastal Structures.

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Abstract

This thesis investigates toe bunds for rubble mound breakwaters and more specifically the stability of rocks in toe bunds under wave attack. Recent physical model tests for design projects have suggested that stability of toe rocks may deviate from desk study calculations. This implies that present design tools lack a reasonable degree of accuracy to assess required element dimensions. Accurate predictions are required in design practice to ensure stability of toe bund rocks on one hand and to prevent over-dimensioning on the other.

The goal of this research is to improve the insight in the physical process related to stability of toe bund elements under wave load. This should lead to more accurate predictions for the required rock size in toe bunds. An analysis is made of the presently recommended design methods. A new hypothesis with a partly theoretical basis is developed for toe rock stability. This is verified with already available data sets.

Presently the methods of Gerding (1993) and Van der Meer (1998) are recommended in design manuals for determination of the required toe rock size. The methods are based on the same date set (tests by Gerding) and are different in geometric description only. Analysis led to the following conclusions:

a. The formula by Van der Meer has the same field of applicability as the formula of Gerding. Since Gerding's formula has better resemblance to the data set, Van der Meer's formula is no improvement.
b. In both methods, the stability number is related to the amount of damage with a power curve. This power curve is inappropriate because it has no physical foundations and it introduces unnecessary uncertainty.
c. Many of Gerding's tests resulted in higher damage than the model of Van der Meer prescribes. Therefore too small rock sizes may be recommended in design for the intended acceptable damage level.

A new hypothesis is formulated to describe toe rock stability. Important propositions are:

a. There is a critical value for the load on toe rocks (threshold of movement), instead of a power relation between damage and stability.
b. The stability problem should be regarded for local conditions at the top surface of the toe bund.

Two invalidated assumptions are made:

a. The combination of down rush and porous outflow of water is normative for toe rock stability. This assumption follows from the theoretical view on the physical process and from suggestions in literature.
b. Flow, turbulence and accelerations can be represented by one characteristic parameter, namely the velocity amplitude of local oscillatory flow.

The concept for this study's model is based on two steps:

Step 1: Assessment of the amplitude of local water velocity at the toe bund. This is calculated by summation of the contributions of the incoming wave and down rush, taking a phase difference into account.

Step 2: Description of the critical velocity for a toe rock. The Rance/Warren stability criterion is used with a theoretical adaptation, which accounts for the effect of porous outflow.

Coupling these two steps implies that a rock will move if the occurring velocity exceeds the critical velocity.

An evaluation of the hypothesis is performed with multiple options of characteristic parameters for irregular waves. The option with significant wave height and period has the best resemblance to the available data. This study's model is eventually empirically fitted to the data set of Gerding. Data from tests with non-representative toe bunds have been discarded. The accuracy of stability assessment is increased and this study's model resembles the data set better than the previous models. The model can be used to predict the required toe rock size. This is verified for the applicability range of the data set of Gerding.

The result of the evaluation is different for the data set of Docters van Leeuwen (1996). This difference with Gerding's data set is not explained by this study's model. Different rock properties may be a cause but information is not at hand. Damage was not counted properly in Docters van Leeuwen's tests, but this is probably not the only cause of the difference.

In addition, a classification is given for damage to toe bunds. It is based on the approach with critical load. This implies that toe rocks are considered generally stable under the critical load and generally unstable above it. The classification is:

Insignificant damage	$N_{od} < 0.4$
Transition	$0.4 < N_{od} < 0.8$
Significant damage	$N_{od} > 0.8$

This classification does not describe severity of the damage level for the breakwater structure. Dimensions of a toe bund have to be designed in such a way that the structure can cope with the expected damage.

Regarding the dimensions of a toe bund itself, it is found that present design guidance is limited. Recommendations for toe bund width vary and have no argumentation. A toe bund should stabilize the armour layer, but some breakwaters may also be stable without a toe bund. More common functions of a toe bund are to aid in construction of the lower part of the armour layer and to shorten its downward extent. Required dimensions for a toe bund should be dependent on functions that are assigned to the bund for the breakwater system.

Further research is recommended to test the applicability of this study's model outside the verified range, to investigate the empirical factors and to verify the calculation of water velocities.

Acknowledgements

This thesis is the final report of a research project undertaken to obtain the degree of Master of Science at Delft University of Technology. I would not have been able to complete this study by myself and therefore I want to thank all people who cooperated in my graduation project.

First of all I wish to thank my parents. You provided the opportunity for me to become an engineer. Thank you for supporting me during my entire study time. My sister has been an example for me in conducting thorough academic research.

I am grateful to Boskalis and Hydronamic for offering me a practical internship in breakwater construction and for facilitating my graduation research. I thank all colleagues for the pleasant working atmosphere and especially my fellow graduation students Merijn and Gerben down the hall.

Many thanks to my rowing mates, union board pals and all other friends for making the past years as a student a wonderful time. I especially want to mention Thomas, with whom I have not only shared my entire study and rowing time, but finally even our office room at Boskalis. I enjoyed discussing ideas about both our projects, which certainly contributed to my research. Renske, thank you for your support and in particular for all due diversion. Roel, thanks for the effort you've done for this report.

I want to thank my graduation committee for their advice and supervision of my research process. From Delft UT: Marcel Stive, Wim Uijttewaal and Henk Jan Verhagen. From Boskalis I want to thank Jeroen van den Bos. Your enthusiasm for my research and for the speciality of coastal structures has been an incentive for me. I also want to thank Jelle Olthof. Your input and experience have kept me on my desired track of combining the scientific and practical sides of this research.

I am very grateful to the specialists of Deltares (former Delft Hydraulics) Marcel van Gent and Bas Hofland for investing their precious time in my project.

Stephan Baart

Papendrecht, June 2008

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Notation

a_h	orbital stroke	[m]
B_t	toe width	[m]
C_{DL}	coefficient for drag and lift	[-]
C_{PF}	coefficient for porous flow	[-]
C_W	coefficient for weight	[-]
D	particle size or typical dimension of concrete armour unit	[m]
D_{50}	diameter of rock that exceeds the 50% value of sieve curve	[m]
D_n	nominal block diameter or equivalent cube size	[m]
D_{n50}	median nominal diameter	[m]
E_{kin}	kinetic energy	[J]
E_{pot}	potential energy	[J]
F	force	[N]
F_{DL}	combined force due to drag and lift	[N]
F_F	force between neighbouring rocks	[N]
F_{PF}	force due to porous outflow	[N]
F_{pi}	vertical pressure force on a grain, buoyancy force	[N]
f	friction factor for W (in rock balance)	[-]
f	coefficient for bottom friction in long wave momentum balance	[-]
g	gravitational acceleration	[m/s^2]
H	wave height, from through to crest	[m]
$H_{1/3}$	significant wave height based on time domain analysis, average of highest 1/3 of all wave heights	[m]
$H_{2\%}$	wave height exceeded by 2% of waves	[m]
H_D	design wave height in a regular wave field	[m]
H_s	significant wave height, H_s=$H_{1/3}$	[m]
h	water depth	[m]
h_m	local water depth at structure	[m]
h_t	toe depth, water depth above the toe relative to still water level (SWL)	[m]
i	hydraulic head gradient of piezometric level	[-]
K	Darcy permeability coefficient, hydraulic conductivity	[m/s]
K_r	wave reflection coefficient, ratio of reflected and incoming wave	[-]
k	wave number, $k=2\pi/L$	[m^{-1}]
L	length of breakwater section along structure axis (in definition of N_{od})	[m]
L	local wave length	[m]
L_0	deep water wave length, $L_0=gT^2/2\pi$	[m]
L_{TA}	horizontal distance between the middle of the toe bund top surface and	[m]

	intersection of front slope and SWL (toe-armour distance)		
m	mass		[kg]
m	foreshore slope (gradient)		[-]
N	number of displaced rocks		[-]
Nh	deviation from hydrostatic conditions (by Francalanci et al.)	$Nh = -\frac{1}{\rho g}\frac{\partial p}{\partial z} = 1 + \frac{u_s}{K} = 1 + i$	[-]
N_{od}	damage parameter, average number of displaced toe elements per D_{n50}-unit of breakwater length		[-]
N_{odB}	damage parameter, average number of displaced toe elements per D_{n50}^2-unit of top surface of toe bund		[-]
N_s	stability number in USACE notation, $N_s=H_s/\Delta D_{n50}$		[-]
P	notional permeability factor		[-]
R^2	coefficient of determination		[-]
R_d	run-down level, relative to SWL		[m]
$R_{d2\%}$	2% run-down, run-down level below which only 2% pass		[m]
R_u	run-up level, relative to SWL		[m]
$R_{u2\%}$	2% run-up, run-up level exceeded by only 2% of run-up tongues		[m]
s	wave steepness, s=H/L		[-]
s_{om}	fictitious wave steepness for mean period wave, $s_{om}=2\pi H_s/(gT_m^2)$		[-]
T	wave period		[s]
$T_{1/3}$	mean period of the highest one-third of waves		[s]
T_m	mean wave period		[s]
$T_{m-1,0}$	wave period based on zeroth and first negative spectral moment, mean energy wave period		[s]
T_p	spectral peak wave period		[s]
T_s	significant wave period, $T_s=T_{1/3}$		[s]
u_b	water velocity at the top surface of the toe bund		[m/s]
$\hat{u}_b$	amplitude of water velocity at the toe bund		[m/s]
$\hat{u}_{bdr}$	amplitude of the component of velocity at the toe bund due to down rush		[m/s]
$u_{b,rms}$	horizontal root mean square velocity at the toe bund		[m/s]
$\hat{u}_{bi}$	amplitude of the component of velocity at the toe bund due to the incoming wave		[m/s]
$\hat{u}_h$	amplitude of horizontal water velocity		[m/s]
u_s	seepage velocity		[m/s]
W	weight of a rock		[N]
W_{50}	median weight of a rock grading		[N]
z_t	toe height		[m]

α	angle of front slope of breakwater	[rad] or [°]
Γ	empirical fit parameter for relative load, ratio of γ_b and γ_{bc}	[-]
γ	breaker index (H/h)	[-]
γ_b	empirical fit parameter for calculation of amplitude of velocity at the toe bund ($\hat{u}_b$)	[-]
γ_{bc}	empirical fit parameter for critical velocity at the toe bund ($\hat{u}_{bc}$)	[-]
γ_{dr}	down rush coefficient, reduction coefficient for slope roughness in calculation of $\hat{u}_{bdr}$	[-]
Δ	relative density of rock in water	[-]
ε	theoretical expansion by Battjes of Munk and Wimbush breaking criterion, $\varepsilon=4\pi^2H/gT^2\sin^2\alpha$	[-]
θ	reflection phase angle (by Hughes)	[-]
ξ	surf similarity parameter, Iribarren number $\xi = \tan\alpha\sqrt{s}$	[-]
ξ_0	surf similarity parameter based on calculation with $L_0=gT^2/2\pi$	[-]
ξ_h	inverted Iribarren-type parameter based on depth (by Hughes et al.)	[-]
ξ_m	surf similarity parameter for mean wave period T_m	[-]
ρ_s	density of rock material	[kg/m^3]
ρ_w	density of water	[kg/m^3]
τ^*	dimensionless shear stress, Shields number	[-]
τ_b	boundary shear stress, bed shear stress	[N/m^2]
φ	phase difference between contribution to the velocity by the incoming wave and the resulting velocity	[rad] or [°]
φ_{TA}	phase difference between incoming wave and down rush	[rad] or [°]
ψ	Shields number	[-]
ω	angular frequency of waves, $\omega = 2\pi/T$	[s^{-1}]

Chapter 1

Introduction and problem description

This thesis studies toe bunds for rubble mound breakwaters. A general introduction to the subject is given in section 1.1. From then on the problem is described in more detail. The position of this thesis in the study field of coastal engineering is discussed. Definitions of the problem and goal are given and research questions are formulated. The approach for this research is explained. The final section of this chapter describes the structure of this report.

1.1 General introduction

1.1.1 Rubble mound breakwaters and toe structures

All over the world people have constructed breakwaters. A breakwater is a structure protecting a shore area, harbour, anchorage or basin from waves. Their most common function is to provide shelter against wave action for ships entering a port or for moored ships. Breakwaters can also be constructed to protect coastlines from wave attack or to redirect currents. Some are attached to the shore and others are detached. (SCHIERECK 2004)

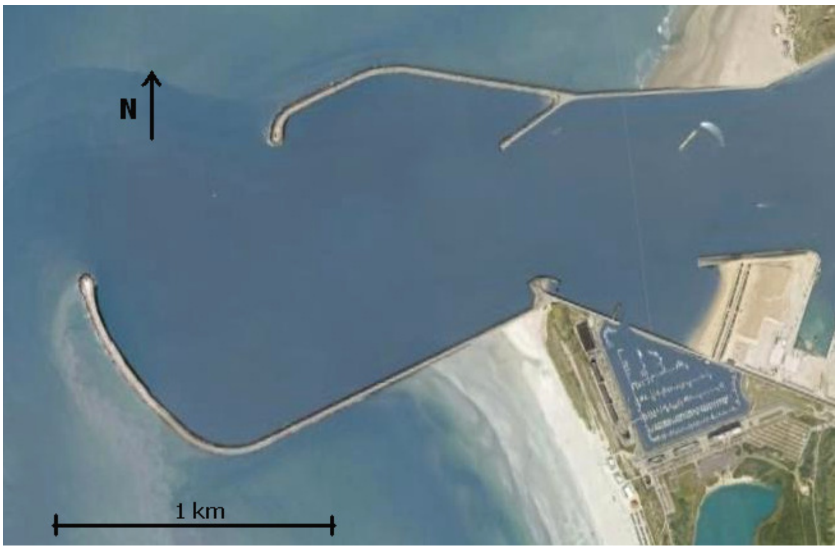

Figure 1: Breakwaters at port entrance of IJmuiden, The Netherlands
(Image from: Google Earth)

Two main types of breakwaters exist, namely rubble mound breakwaters, built up of large heaps of loose elements, and monolithic breakwaters, which perform as a solid block placed on a prepared foundation. Furthermore there are combinations of these types and there are special types, for example floating and pile breakwaters. This research concerns toes for the rubble mound breakwater type.

The hydraulic function of a breakwater is to reduce wave heights in the area behind it. Breakwaters are designed to remain intact under the attack of a predefined sea state. This state is defined as a combination of a water level and a wave climate with a certain return period.

A rubble mound breakwater itself is protected against wave impact by the outer layer on the seaside. This layer consists of so-called armour elements. These can be rocks, concrete cubes or concrete units with special shapes for increased mutual interlocking and porosity.

The area on the seaside where the front slope intersects with the bed level is called the toe. This is considered to be one of the four most important parts of breakwater trunks regarding stability issues, as will be described in this report (section 1.3).

The armour layer may extend to the bed, but commonly a toe structure is applied to provide support to the armour layer or to protect the toe region against scour.

The deeper the toe is, the smaller the water forces are on the elements. Toe elements can therefore often be smaller than armour units. This can reduce costs and improve constructability. The question for designers of breakwaters is how small the toe elements can be, without loosing their stability. In other words, what is the minimal required toe element size for a specific breakwater design case?

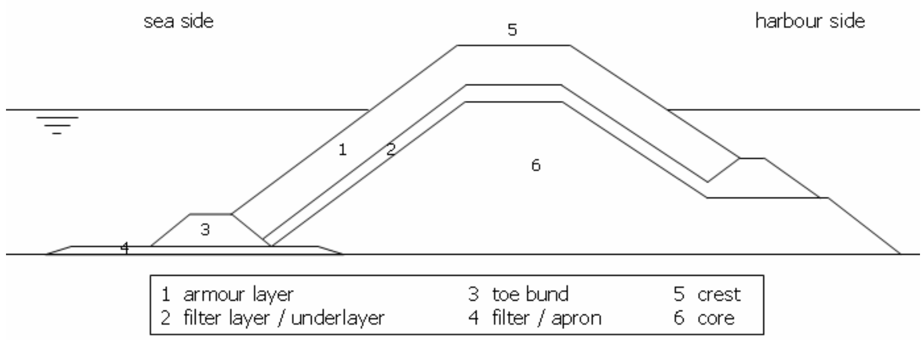

Figure 2: Example of a typical trunk cross section of a conventional rubble mound breakwater

In this thesis toe structures are considered that have the shape of a bund. These toe bunds serve no hydraulic function such as wave energy reduction. They are merely a part of the total structure. Some breakwaters have large (toe) berms which decrease the wave load on the upper slope. This category is not part of the scope of this thesis. Toe bunds are relatively small heaps of rock, compared to for example armour and core volumes (see Figure 2).

1.1.2 Toe bund construction

A common way of toe bund construction is from a pontoon with a back hoe or grab. This floating equipment has a large excavator for placing material individually. It can stabilize itself by jacking on spuds.

Figure 3 a and b: Grab and back hoe pontoons (pictures courtesy Boskalis)

Depending on factors such as volumes, element weight, accessibility and tolerance requirements, contractors can also choose to place toe material by land based equipment such as excavators and cranes or by marine based side stone dumping vessels. After placement, the profile is surveyed and repaired if necessary.

Figure 4 a and b: Toe bunds can be placed with land based or marine based equipment.
On these pictures the equipment is working on the underlayer (left) and armour.
(pictures courtesy Boskalis)

1.2 Breakwater design practice

When engineers design a rubble mound breakwater, they consider multiple options of dimensions and materials. Working with predefined boundary conditions, the goal is to design a structure that performs to hydraulic demands, is safe and stable, and is economically optimized. In a design process an iterative loop is practiced in which the designer gets closer to the eventual structure by adjusting its mutually dependent characteristics. For each step in the design loop, an engineer draws up dimensions for the different parts of the breakwater. Then design tools are used to determine required sizes of specific available materials.

Design tools are recommendations formulated by researchers. Breakwaters manuals provide design tools for elements or hydraulic performance under wave attack. Overall stability of layers and parts of the breakwater is treated as well, but less extensively.

Design tools can be presented as equations. These are used to make estimations for the performance or stability. Many of these equations are empirical fits, although nowadays the trend is to make more use of theoretic approaches.

The Spanish scientist Iribarren theoretically derived stability relations for rock elements on a slope in 1938. These derivations developed into a stability parameter in which the wave height is proportional to the diameter and relative density of the element. This parameter is presently in use in most design tools for armour and toe elements. This stability parameter was also empirically found by Hudson in the 1950's, only written in a different order. It is nowadays known as a Hudson-type stability parameter. Generally it has the form

$$\frac{H}{\Delta D} \tag{1}$$

in which Δ is the relative density. Subscripts for H and D provide information on which exact measurement for wave height and rock diameter is used (D'ANGREMOND *et al.* 2001).

Design manuals should provide reasons to apply toe structures. This is however not always explicitly the case. Designers have to make choices in dimensions of each part of the structure. Most focus in manuals is however on determination of the required rock size. For a designer who reads a manual, it might come across as obvious to apply a toe structure, while this may not always be necessary.

The present design tools for toe rock size are empirical curve fits of physical test results. As input the governing conditions for the design case are used, such as wave height, water depth and geometry. The primary interest is in the output of the formula, but much depends on the spread of the answer as well. The certainty to expect for stability directly follows from the accuracy of the formula. This aspect is important in design.

In this thesis formulae for toe element stability are regarded as the description of a parameter model that represents the real world. If the model is sufficiently accurate, it can be used in design practice to determine a required rock size.

Better models, increasing computer power and more knowledge have led to better descriptions of the physics of element stability. It should however be kept in mind that the process in reality deals with capricious natural factors such as rock shape and irregular wave climate. This is a limitation for the accuracy of a deterministic prediction.

A designer should keep focus on the target breakwater system in its environment rather than on paperwork with precise lines. This also holds for scale modelling. BRUUN (1985) warns us for example for erosion effects:

> "When breakwater designs are confirmed by model tests, modellers tend to forget erosion effects because the model is built on a solid base, which can lead to toe failures in reality."

1.3 Origin and relevance of this research

Experience from design projects in the last few years has suggested that stability of toe bunds in physical model tests may deviate from what was expected from desk study calculations. This implies that presently recommended formulae may not correctly represent the physical process that determines movement of toe rocks.

This study has been set up to further investigate the stability of rocks in toe bunds of rubble mound breakwaters. Apart from scientific curiosity, the study also has practical value. There is a demand from design practice for better prediction tools. Time and money can be saved if the required dimensions can be assessed adequately.

The armour layer is the most important part of a breakwater regarding stability and costs. In the design loop initially less attention is paid to the toe. The toe has namely less influence on the final design than the armour layer, because the elements and total volume are smaller. This is also the reason that research initially focussed on armour elements. The knowledge level of toes is lagging behind on armour. Nowadays much is known about armour unit stability and attention has shifted towards other topics as well, such as the toe, crest and inner slope.

BRUUN (1985) mentions four main overall stability topics. In summary they are:

a. Slidings of the armour layer as a whole or mass slides (including slip circles) penetrating deeper in the mound.
b. Mass departures of individual blocks from the armour layer, exposing the underlayer.
c. Toe failures, expanding upwards and finally causing a mass failure of the armour. Such incident may also start as a failure of the mattress below the toe.
d. Mass breakdowns by heavy overwashes of the crest, peeling off layer after layer, washing most of the material down in the inside of the mound.

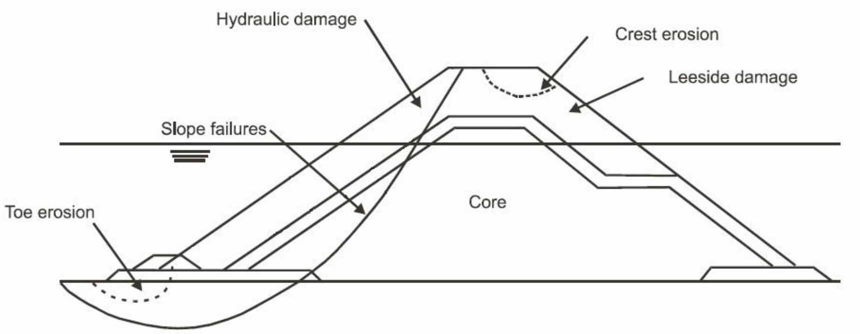

Figure 5: Failure modes of rubble mound breakwater trunks
(Figure from: CIRIA *et al.*, 2007; simplified)

Toe failure is thus one of the major failure mechanisms of breakwaters. It is an important mechanism to overcome in advance, as Bruun remarks:

> "Toe failures are often hard to observe because severe toe damage usually expands upward in the mound leaving no direct evidence of its occurrence. The reason for the failure is either scour by currents or by deep down rushes or by both."

After a breakwater failure occurred, it is not easy to determine whether the cause of the failure was in fact toe instability. Commonly a design for a large breakwater is tested as a scale model. Therefore most inadequate rock size calculations are corrected. Failures of breakwaters still occur though. Two examples of incidents are described here. For these cases it is likely that failure of the toe was the cause of failure of the entire structure.

Example 1: Sines, Portugal

From: SMITH *et al.* (1983)

The armour layer of the breakwater at Sines consists of a face with Dolos units, which collapsed during construction in 1978. In the paper, Smith and Gordon describe the possibility of failure of the toe structure as a cause, together with the individual failure of many Dolos units themselves. The toe is perched, situated half way up the slope. The slope under the toe is steep (3:4), leaving only a small capacity to withstand hydraulic forces. In the deep water high waves are able to reach the breakwater. The toe is situated only 1.4 times H_s beneath the still water level. The waves may have caused large gradients in pore pressure, pushing out toe rock. In the paper this is regarded as fluidization of the toe material.

The book of BRUUN (1985) also records this failure. Bruun explains that waves were higher than expected and the toe was too small. This leads to instability of the armour layer, through which the breakwater slowly but surely collapsed. Unfortunately the book does not mention whether the elements or the toe bund dimensions were too small, and for which functions.

Example 2: Arzew al Djedid, Algeria

From: BRUUN (1985)

The breakwater at Arzew was built in 1978 after desk top design and model tests. Its failure in 1980 was most likely due to a combination of underestimation of wave conditions, low friction between armour layer and underlayer and poor toe construction. The dimensions of the toe structure were too small and the toe was made of too small elements. There is doubt about whether the toe had been constructed at all at some places, as no toe material was recovered after the failure. This cannot be verified because other material covered the toe completely after the collapse. Failure of the toe led to sliding of the armour layer and destruction of the breakwater. The pictures below make clear that toe damage or failure is not immediately identifiable as such after a collapse.

Figure 6 Damage to the breakwater of Arzew el Djedid, Algeria
(From: BRUUN 1985)

These examples illustrate the importance of adequate toe design. This thesis is relevant for engineering practice if it can increase toe design reliability. This can either be by providing an improved design tool or by clarifying how to handle present tools.

From a scientific point of view this thesis is relevant because the problem is approached from a new angle. Unfounded assertions are brought up for discussion.

1.4 Problem definition

In the previous sections the importance of adequate toe design was illustrated. It is found that design manuals are not complete yet, as will be described in more detail in chapter 2 (Literature). Incompleteness of guidance is indicated by:

a. Various sources do not fully agree on when to use which type of toe structure and what the dimensions should be.
b. It is unclear how much displacement of toe elements is acceptable.
c. There is much scatter around the equations that are provided as design tools for determining the toe rock size. The formulae lack theoretical background and the applicability is limited.

Of these issues, the description of stability of rock elements in a toe bund is the main subject of this thesis. The problem of this thesis research is therefore defined as follows:

Design tools for toe bunds of rubble mound breakwaters that are presently used in engineering practice lack a reasonable degree of accuracy to be able to assess required element dimensions.

1.5 Research objective

The main objective of the research is defined as follows:

To improve the insight in the physical process related to stability of toe bund elements under wave load. This should eventually lead to more adequate guidance for breakwater design practice.

1.6 Research questions

In this chapter research questions have arisen. These are summed up here.

a. Under which circumstances are the present methods for determination of the required rock size applicable and reliable?

b. The presently recommended design tools include a power curve between damage number and stability number. Is this appropriate or is it better to use a threshold of movement for elements in the toe?

c. Is it possible to obtain a better model of the physical process that determines stability of rocks in a toe bund?

d. Is this model, meant in the question above, a more adequate tool for design practice than the presently recommended tools to determine the required toe rock size?

e. Is it possible to assign a classification for the amount of damage to a toe bund?

f. What amount of damage should be considered as acceptable or severe?

1.7 Scope of the research

The scope of the research is given by the following constraints:

a. The subject is toe structures of rubble mound breakwaters in general, but more specifically stability of toe bunds elements.

b. The focus is on sizes of toe elements in relation with wave load and breakwater geometry. In the literature chapter, attention is also paid to dimensions and functions of toe bunds for sake of completeness of the overview on toe knowledge.

c. More insight is sought in the physics of toe element behaviour. This involves the influences of yet untested parameters, which have to be tested in further research.
d. Stability is only considered for a 2D situation, i.e. for physical tests performed on breakwater trunk sections in wave flumes. Oblique wave incidence and flow is not considered.
e. The focus of this thesis is on analysis of present tools and a conceptual and theoretical approach for assessment of stability of toe rocks. The hypothesis of this research is tested and calibrated with already available data sets and no new flume tests are performed.

1.8 Research approach and structure of the report

The approach for this research is based on sequencing steps. The structure of the report is analogous to these steps. Views on quantitative analysis are used from BAARDA *et al.* (2006). The steps taken in this research are:

a. Making an inventory of present knowledge (Chapter 2)
b. Analyzing the problem and its causes (Chapter 3)
c. Setting up a hypothesis (Chapter 4)
d. Verification and calibration with data sets (Chapter 5)
e. Drawing conclusions (Chapter 6)

The emphasis of this study is on analysis of the problem and on conceptual modelling. The explicit result of the verification is however important for the relevance for design practice. Therefore the problem is always viewed in perspective of what is important to know for a breakwater design.

Chapter 2 summarizes knowledge on toes that is found in literature. This includes views on toe bund dimensions and functions of the toe as well. After this chapter the scope of the thesis narrows to stability of the rocks in a toe bund.

An analysis is performed of the presently recommended methods for toe rock stability in Chapter 3. It is investigated what the cause may be of the difference between the actual damage in flume tests and expectations based on the present methods.

In Chapter 4 the hypothesis for this study's model is described. The solution to the problem is sought in a basis of physics for a parameter model. Not all the parameters that may be of influence have been varied in previously performed tests. Therefore unvaried parameters cannot be a source of scatter in the present methods. The influences of parameters that are presently included in the equations are apparently not fully understood. Therefore first a better parameter model is required, before it is useful to physically test the influences of additional parameters and expand the model.

In presently used methods, boundary condition parameters (like wave height and water depth) are directly coupled to the resulting damage. This study should describe the physical mechanisms with theoretical background. An important proposition for the hypothesis is that the local water motion at the toe bund governs the forces on toe elements and thus also their displacements. This requires a different stability number.

The local water motion at the toe is governed by the geometry of the breakwater and the sea state. Therefore the description of physics is approached in two steps:

Step 1: In this step we describe the influence of the boundary condition parameters on the water motion at the toe. A characteristic parameter is sought for the local conditions at the bund.

Step 2: On the level of the local process, a relation is sought between the characteristic parameter of step 1 and the stability of toe rocks.

The approach of this study introduces more theoretical background but remains partially empirical.

Another proposition for the hypothesis is based on the principle that toe elements have a capacity to remain stable, by gravity and interlocking. The equilibrium holds as long as the capacity for stability is not exceeded. Above this level of load, elements are unstable. The final section of Chapter 4 lists the parameters that are in- and excluded in this study's model.

The hypothesis is verified in Chapter 5. The final model is calibrated to presently available test data. It is described how this study's model can be used in design practice. A calculation example is provided in Appendix G.

Chapter 6 summarizes the conclusions of this study. Recommendations are given for further research.

Chapter 2

Literature

This chapter reviews literature on toe design. This is divided into two parts. Content of books and publications is summarized objectively per source of literature in part 2.1. Comment, comparisons of authors and personal views are given per topic in part 2.2. This distinction has been made for reasons of clarity, to enable the reader to easily discern which considerations were in the original sources and what is new insight in this report.

Part 2.1 consists of two parts as well. Section 2.1.1 deals with toe types and guidelines for dimensioning of toe bunds. The second part, section 2.1.2, concerns stability of toe rocks. This is described in chronological order. Additionally, Appendix A contains content on grain stability in flow, run-up, down rush and reflection.

Part 2.2 evaluates the literature and compares content of various sources per topic. This starts on breakwater scale, down to description of stability of elements in the toe.

The structure of this literature chapter is schematized in the following figure.

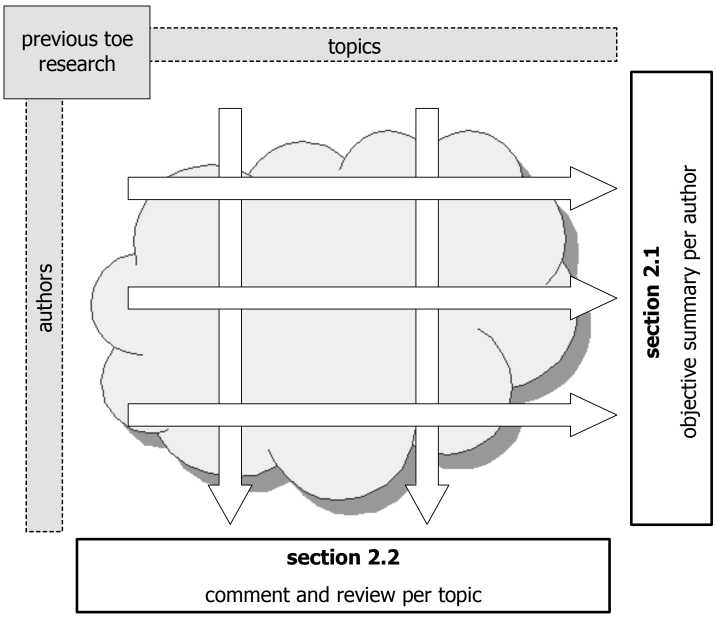

Figure 7: Schematic representation of the structure of this chapter.

2.1 Summary of previous research

2.1.1 Toe types, dimensions and purposes

The minimum required toe rock diameter is the main topic of breakwater literature. Less attention is paid to general dimensions, types and shapes of toe structures, and perhaps rather contradictory, when and why to apply a toe structure. This section summarizes content with relevance for this thesis.

As a starting point we regard ECKERT (1983). Here is described that a toe of a rubble mound breakwater can consist of an apron, a bund or combinations. The first purpose of a toe apron is to prevent scouring of bottom sediments from under the primary armour. The second purpose is to prevent a scour hole adjacent to the structure, deep enough to destabilize the embankment structure by the oversteepening and lengthening of the ocean side slope. For vertical breakwaters, the purpose of the toe is different, namely to supply passive earth support to the structure. Neither a reason for supporting the armour layer is mentioned by Eckert, nor a specific toe bund requirement.

Scour in front of a rubble mound structure may occur anywhere within one fourth of a wavelength of the incident wave, but the area to be protected is generally not that wide. The geotechnical concerns of slope stability and foundation bearing strength will already be fulfilled if the bottom sediments are protected within a width of the face such as shown in the figure below.

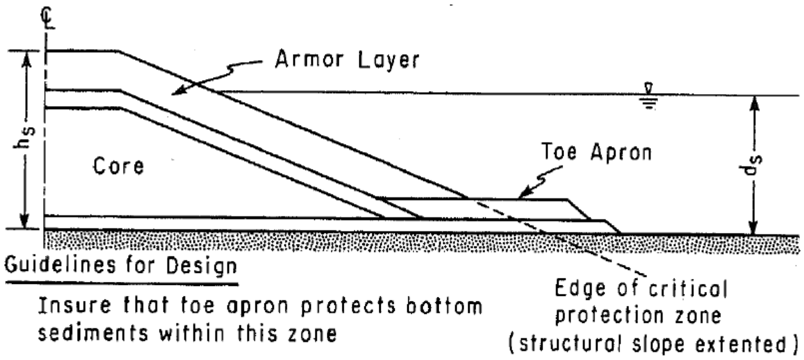

Figure 8: Guideline for minimum toe width B_t for structural stability (ECKERT 1983)

With respect to geotechnical stability, BRUUN (1985) distinguishes four different soil mechanics problems for rock mounds:

a. Stability of the foundation including static stability against overall slides by spiral or by slip-circle passing through subsoil and/or core fill.
b. Dynamic-geotechnical stability, which includes porous flow, wave attack and internal rock failures.
c. Stability against sliding of the armour layer along the first sublayer and of sublayers on sublayers or on the core.
d. Stability against toe damage and erosion at the toe.

There is much emphasis in BRUUN (1985) on stability of the toe area, since it is one of the four main geotechnical issues. Bruun concludes that the main cause of damage to the toe is the occurrence of deep wave troughs in front of the breakwater. Destabilizing forces for toe elements are down rushes, but also pore pressures and porous flow push out toe rocks.

Bruun mentions the toe as a buttressing layer for the main armour: “The toe shall support the armour layer against sliding and against undermining by scour. Material may have the block size of the armour layer. If the top elevation of the toe is close to the elevation of the deepest down rush, it is best to extend the armour layer across the toe.”

In 1986 the US Army Corps of Engineers conducted a field survey of toe structures throughout the United States (MARKLE 1986). They discern toe buttressing stone and toe berms. The purpose of the buttressing stone is to stabilize the onslope armour by preventing downslope slippage. The primary function of a toe berm is to protect a breakwater on an erodible bottom from being undermined by scour, causing, again, downslope slippage.

The front part of a breakwater that reaches the bottom is called the toe. British Standard 6349 (BSI 1991) states that this does not imply that a special structure is always needed at the toe. An apron against scour can be applied if necessary. A toe structure for armour support, other than an apron, is called a toe bund. A toe bund is considered necessary when the water depth is less than twice the significant wave height and the slope is steeper than 1:3. No reason is mentioned. This British Standard gives five structure types for toes, including the circumstances in which they should be applied:

a. A toe bund of underlayer material (for $h_t>2H_s$)
b. A toe bund of armour material (for $h_m<2H_s$)
c. Lower armour units resting in a trench (for $h_m<1,5H_s$ and rock bottom)
d. A toe bund on top of bed protection and replacement material (for an original bed of soft material)
e. A toe bund on an extended anti-scour apron.

In these applicability recommendations h_t is the water depth above the toe (toe depth) and h_m is the water depth right in front of the breakwater, see also Figure 13.

In 1991 CIRIA and CUR bundled breakwater knowledge in a publication which became known as the Rock Manual (CIRIA 1991). The first version of the Rock Manual recommends that the toe berm thickness should be at least a double layer, just as with armour layers, in order to protect inner layers on all locations. The top level of the toe h_t is generally selected as 1-1,5 times design H_s below low water level. The exact level depends on weight of the stones. For toes in shallow water, a trench can be dredged for the entire breakwater.

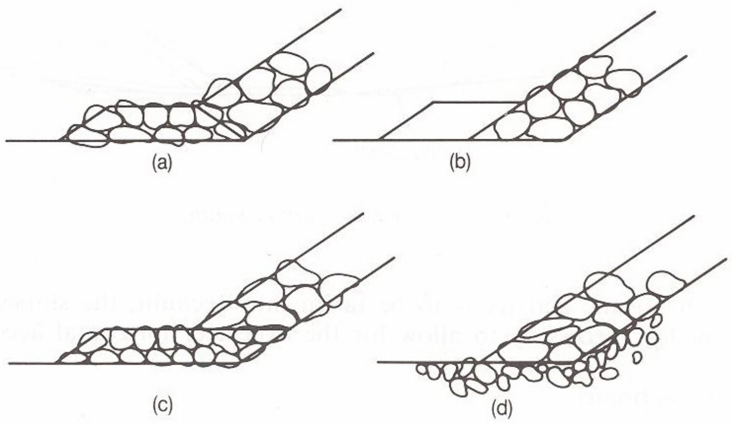

Figure 9: Alternative arrangements of a toe berm in rubble-mound breakwater (CIRIA 1991).

Four alternative arrangements of toe structures examples are given: a] is conventional, b] is an alternative for this with a different construction sequence. Option c] will give poor support for the lower armour stones. In case of careless construction a] will become like c] if underlayer rocks tumble down the slope. Case d] can be used if the bed consists of coarse stones. There is no information in the Rock Manual '91 about when a toe berm is required or not.

The latest version of the Rock Manual (CIRIA 2007) gives four examples of options for toe structures with corresponding application conditions (Figure 10).

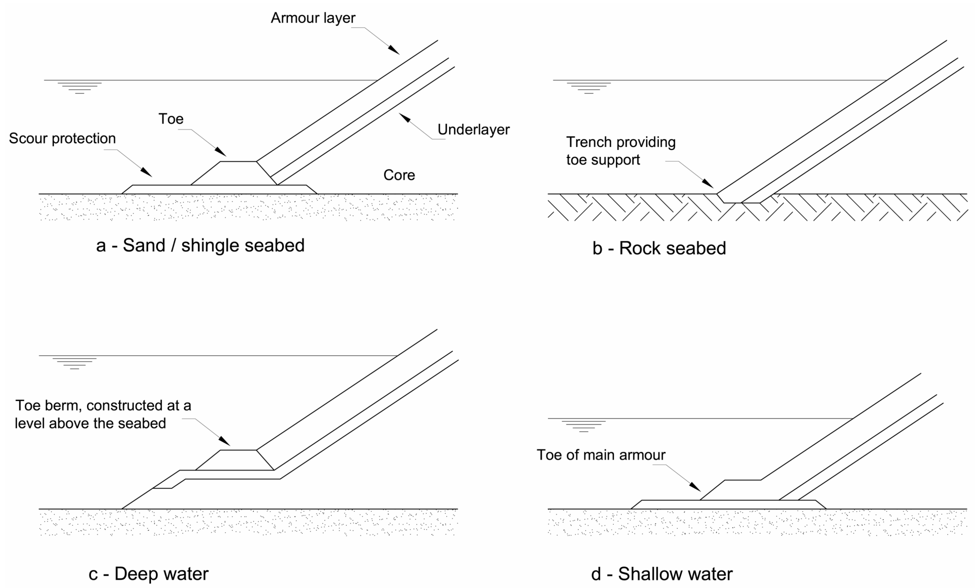

Figure 10: Schematic examples of toe details for rubble mound breakwaters (CIRIA 2007).

For breakwaters with special concrete armour units is suggested that the product developer should provide guidance on the toe details. Especially for interlocking units, the edges of the armour layer are less stable and therefore great attention should be paid to the toe. In cases where even the slightest settlement is unacceptable, concrete piles or dredged trenches may be required.

The Coastal Engineering Manual (USACE 2006) gives the same functions for a toe berm as Eckert and Bruun. It can support the armour layer and prevent damage resulting from scour. Toe stability is increased if armour units are displaced and come to rest on the toe berm. Toe rocks can be smaller than the armour, but in very shallow water the toe should be built of armour stone. In deep water the toe can be constructed halfway on the slope (see Figure 11).

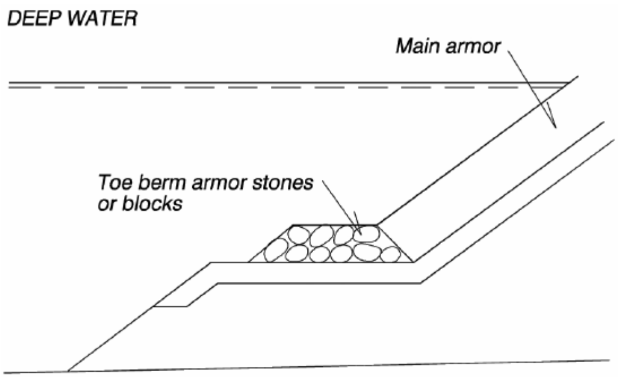

Figure 11: Toe structure for deep water conditions (figure from CEM (USACE 2006))

For sloping rock beds it is possible that the lowest rocks on the bottom slip seaward. This risk can be reduced by digging a trench or anchoring the lowest armour stones to the bed.

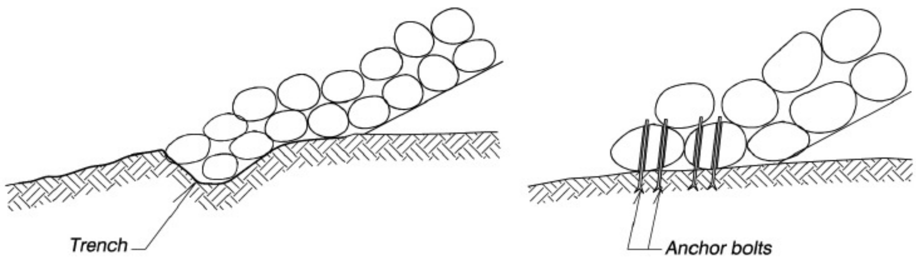

Figure 12: Trench and anchors for toes on rocky beds (figure from CEM (USACE 2006))

The various sources that were mentioned in this section provide diverse guidelines on the width B_t of the top surface of the toe structure. The recommendations are summarized in the table below.

Table 1: Toe width recommendations

Publication	*Width recommendation*
Eckert (1983)	Prevent scour within structural slope
SPM (1984)	At least 2 stones
USACE (1987)	At least 3 D_{n50}
CIRIA (1991, 2007)	At least 3 stones
BSI (1991)	At least 4 stones

2.1.2 Stability of elements in the toe

This section treats relevant publications on stability of toe elements. The information in this section is presented chronologically per publication. All sections are a summary of each publication unless otherwise indicated with an additional reference.

Eckert 1983

Toe scour by waves reveals a strong correlation to the wave reflection coefficient. Studies by Sawaragi (1966) showed that for a reflection of 0.25 or less, there will be little scour of the bottom sediments. The occurrence of scour may be expected in shallow water when the depth is less than twice the incident wave height and the reflection coefficient exceeds 0.25, as it will for almost any rubble mound structure. The scour depth is greater where the down rush of the waves extends to the toe structure.

CERC 1984

In many papers on toe stability is referred to the Shore Protection Manual (SPM) from 1984. The weight of toe material weight is related to the weight of armour material with a ratio of 1:10. This rule-of-thumb implies to use a toe rock diameter of about half the armour diameter.

The SPM quotes research conducted by Brebner and Donnelly in 1962. With regular waves they tested toe material for vertical composite breakwaters. To describe stability a dimensionless parameter is introduced for the relative water depth above the toe, namely h_t/h_m (varied in the range 0.3 - 0.8). This parameter indicates how deep the top surface of the rubble foundation is located compared to the water depth in front of the structure.

For rubble mound breakwaters with $h_t/h_m > 0.5$, the SPM recommends to use a value of N_s of 6 to 7. Here N_s is the stability parameter, which is defined by a wave height, weight and nominal diameter of the stone, and densities of stone and water. This is similar to the Hudson-type stability parameter (equation 1). Rewritten it reads the same as in European literature:[1]

$$N_s \equiv \frac{H_D}{\Delta D_{n50}} \tag{2}$$

In 1977, Gravesen and Sorensen related N_s for toes to h_t/H_D. They concluded that higher wave steepness created more damage to toe structures. They based this conclusion on a few data points only. Therefore this conclusion has not been generally accepted.

[1] The tests were performed with monochromatic waves and thus the analysis used a single design wave height H_D in the stability number. More recent publications use the significant wave height or the 2% wave height. This is indicated by different subscripts.

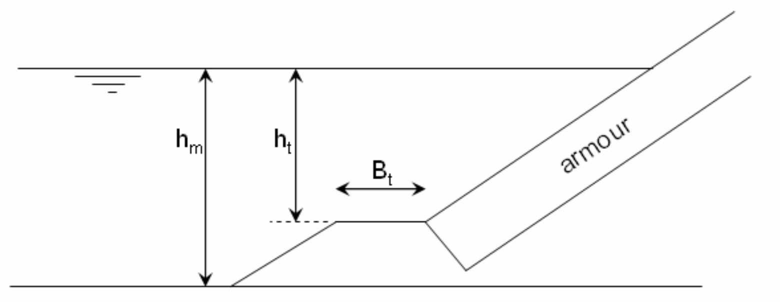

Figure 13: Traditional toe parameter definition sketch

USACE 1987

Guidance for toe rock size was found to be inadequate in 1987 because field experience showed that there were damaged structures. Recommendations had been rules of thumb (such as 1/10th of the armour weight). Therefore new tests were performed by USACE.

The final report for all toe tests, including data sets, was published in 1989 (USACE 1989). The SPM (1984) gave the stability number as $N_s = H_D/\Delta D_{n50}$. Therefore the relation between weight and the stability number is $W_{50} \sim N_s^{-3}$. The cubed stability number was plotted against h_t/h_m, just as Brebner and Donnelly had done for vertical breakwaters. Only tests were used in the analysis that had resulted in 'acceptable levels of damage'.

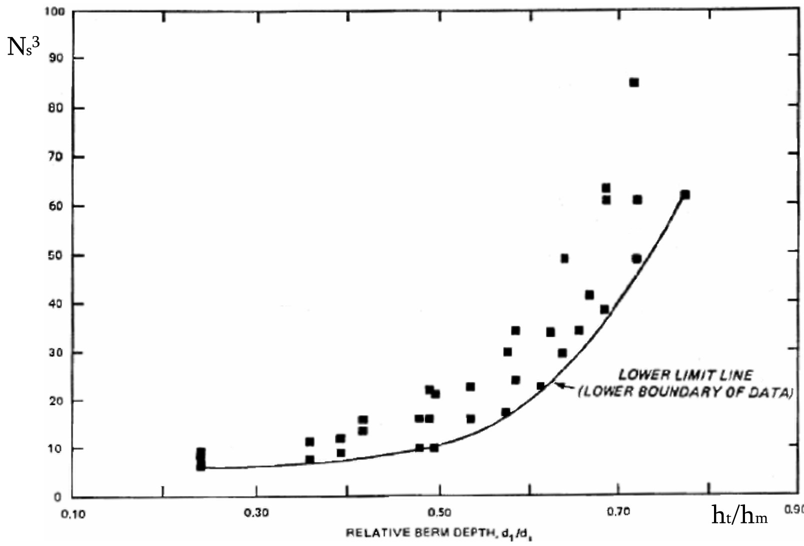

Figure 14: Toe design recommendation for depth limited waves (USACE 1987)

According to this paper, the vertical spread in the dataset seems to be a function of water depth and wave period (for example in parameters h/L), but this could not be described adequately. A lower limit line is given as design recommendation. This recommendation should be used for depth limited breaking waves and otherwise the rule of thumb of 1/10th of the armour stone weight from the SPM should be used.

The report of 1989 clarifies that a vertical displacement monochromatic wave generator was used. Unfortunately no description is given of the 'acceptable' damage level.

British Standard no. 6349, 1991

This British Standard can be interpreted to give underlayer material as the minimum diameter size for toe elements although this is not explicitly mentioned. For $h_t<2H_s$ the guidance of ECKERT (1983) is recommended, with the armour material as a maximum size. Support to the armour is mentioned as a function of the toe. For toe apron material a diagram for the threshold of movement is given.

CIRIA et al. 1991

A consequent research line can be indicated for stability of toe elements of rubble mound breakwaters. It starts with the Rock Manual by CIRIA *et al.* (1991) and develops via GERDING (1993) and DOCTERS VAN LEEUWEN (1996) to VAN DER MEER (1998).

The first Rock Manual states that the toe will be stable if the rock has the same size as the armour. In most cases reduction of the stone size in the toe is desired. A smaller rock grading reduces costs and the quarry can be used more efficiently.

For the Rock Manual, tests on toe element stability were carried out at Delft Hydraulics. The results were analyzed by Van der Meer. Wave boundary conditions were established for the following criteria:

a. 0-3 % no movement of stones, or only a few
b. 3-10% toe flattened out a little, but the function of the toe is intact and the damage is acceptable
c. >20-30% failure, toe has lost its function and damage is not acceptable.

A design curve is fitted to the dataset for low and acceptable damage (0-10%). The wave conditions were depth limited, so H_s/h_m was close to 0.5. The tested range was from h_t/h_m = 0.5 - 0.8, with stability numbers N_s 3.3 - 6.3 respectively. The relation that was fitted to the data points is:

$$\frac{H_s}{\Delta D_{n50}} = 8.5\left(\frac{h_t}{h_m}\right)^{1.4} \tag{3}$$

The stability is in this case also related to h_t/h_m (like in USACE 1987). According to the Rock Manual it is acceptable to relate h_t to h_m in a depth limited situation. The results of the analysis are thus applicable for depth-limited situations.

For the foundation of vertical breakwaters a lower stability number should be used because of the amplification of water velocities due to reflection. In practice stability numbers will be close to 2 for vertical breakwaters.

Gerding 1993

In the thesis of Gerding (GERDING 1993) is written that existing knowledge on toe structure stability is limited. Therefore he decides to perform new scale model tests. The research was later also presented as a paper by VAN DER MEER *et al.* (1995).

First of all a new damage parameter N_{od} is introduced. Damage was previously indicated as a percentage. According to Gerding "the disadvantage is that if the same number of stones is displaced from different toe structures, the percentage changes but the amount of damage is actually the same."

N_{od} is defined as the number of stones removed from the toe structure in a strip with a width of 1 D_{n50}. In various sources after the original thesis, different definitions are given for N_{od}. They aim to describe the same number. In the present report N_{od} is presented in the following formula to prevent misunderstandings:

$$N_{od} \equiv \frac{N}{\left(L/D_{n50}\right)} \qquad (4)$$

N_{od} is the damage parameter. N is the number of displaced stones and L is the length of the breakwater section (along the axis) in which those displaced rocks are counted, for example the width of a wave flume.

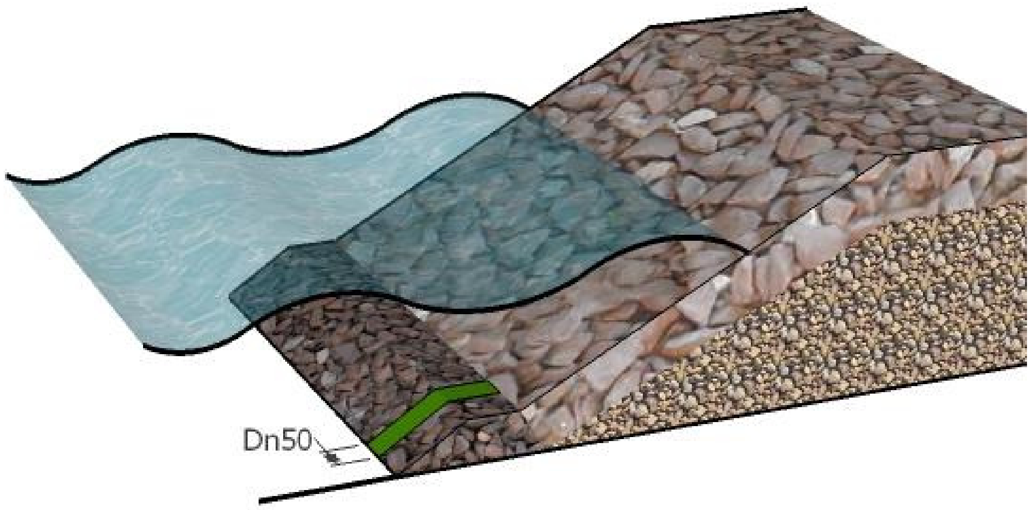

Figure 15: N_{od} interpretation - the average number of displaced stones in a strip with a width of 1 D_{n50}.

In the report is remarked: "The advantage of using the damage number N_{od} is that the damage is not related to height or width of the toe structure and the same amount of moved stones give the same damage area for all toe shapes. In this way the amount of damage is independent of the shape of the toe structure. It should be noted however that the effect of a certain damage level on several toe structures is different with the shape of the toe structure."

In the evaluation, N_{od} values of 0.5, 2 and 4 are used as constant damage levels for further analysis, which are called fixed damage levels. Test results are rounded to these N_{od} levels. This was done to be able to group data for analysis. For values of $N_{od} \geq 4$ is said that the toe has lost its function. The report does not describe movement of the armour layer. So whether the toe indeed lost its function is not reported.

With respect to the influence of the toe width B_t is written: “The average trend is that a constant wave height can be found for varying toe width if a constant damage level is reviewed. (…) The conclusion must therefore be that the width of the toe structure has no influence on the stability of the toe structure. (…) It can however be suggested that for a wider toe structure more damage is acceptable if the damage is defined by the damage number N_{od}. This could lead to the conclusion that for wider toes a damage percentage is more suitable.”

The tests were performed in a depth limited situation, especially in the cases of low relative water depths. Parameter analysis starts with a relation between N_{od} and H_s. According to Gerding, the significant wave height will be used in every relation as the parameter to describe toe stability. It is suggested that a power-curve fits best through the points in the relation between H_s and N_{od}.

The first attempt to describe all data output at once is based on the traditional parameter of h_t/h_m. This compares well to Van der Meer’s equation (CIRIA *et al.* 1991) but the equations deviate more for small toes. Gerding writes: “Because the relation between shallow water significant wave height H_s and the stone diameter D_{n50} is linear but not through the origin, a bias is introduced. To compensate for this bias one of the parameters in the stability number must be used again in a relation with the stability number.” The reason is mentioned that the toe is not directly attacked by the waves, as is the case for armour layer elements.

This led to the final formula of his research:

$$\frac{H_s}{\Delta D_{n50}} = \left(0.24\left(\frac{h_t}{D_{n50}}\right) + 1.6\right) \cdot N_{od}^{0.15} \qquad (5)$$

The relation is based on a smoothed data set with fixed damage levels. This relation is less conventional, but is said to fit the data set better than the relation with h_t/h_m. N_{od}=2 is suggested as a design criterion.

Wave height measurements show decreasing $H_{2\%}$ (in relation to H_s) from deep water to the toe, which is proof of depth limited conditions. Gerding obtained no increase in accuracy by analysis with $H_{2\%}$ instead of H_s. This resulted in the same relation, only multiplied by a factor 1.4. This factor is in accordance with the Rayleigh distributions for deep water wave heights. This is strange since there clearly were depth limited conditions.

To verify the results, the new relation (equation 5) is compared to MAST project test results, which are collected commercial tests by Delft Hydraulics (GERDING 1992). Both datasets show the same trend, but Gerding’s relation seems to be conservative for the MAST dataset. The difference could be in parameters as a steeper foreshore slope or the damage determination. It is recommended to investigate the influence of the foreshore slope and the slope and size of the breakwater armour.

Docters van Leeuwen 1996

Docters van Leeuwen used her Master's Thesis in 1996 to extend the knowledge on toe stability. The focus was on relative rock density Δ in equation 5.

Box 1: Surf similarity and depth limited waves

The reflection coefficient, run-up, overtopping and more parameters are usually expressed as functions of the surf similarity parameter. This is also known as the Iribarren number ξ which is defined as

$$\xi = \frac{\tan\alpha}{\sqrt{s}}$$

in which α is the angle of the slope and s is the wave steepness H/L. Wave length L is however commonly not used, but replaced by the deep water expression for wave length that depends on the wave period T. The deep water wave length L_0 is calculated by

$$L_0 = \frac{gT^2}{2\pi}$$

and therefore

$$\xi = \tan\alpha \Big/ \sqrt{\frac{2\pi \cdot H}{gT^2}}$$

In an irregular wave field for H and T characteristic values are used. For T these are for example the mean period T_m or the period of the peak of the energy density spectrum T_p. This results in ξ_m or ξ_p.

Waves break in different shapes and breaker types, which are classified as surging, collapsing, plunging or spilling. Per classification the surf similarity parameter has a fixed range of values.

Waves in shallow water may break because the height of the wave is limited by the water depth. For regular waves or an individual wave in an irregular wave field, breaking will occur if the water depth has decreased such that H/h ≈ 0.78. Because all waves are different in an irregular field, they will not start to break at the same point. Instead a criterion for the significant wave height is used, which is in the order of H_s/h = 0.5 - 0.6, depending on the foreshore slope. This ratio is called the breaker index γ, defined by

$$\gamma = \frac{H}{h}$$

(BATTJES 2001)

Gerding had found that the wave steepness s_{op} (=H/L_0) had no influence on toe stability in his tests. A possible reason for this is given by Docters van Leeuwen. For the performed tests applies that ξ=3.3 and higher (see Box 1). In the reflection equation for rough permeable slopes by Postma 1989, the reflection coefficient K_r supposedly does not change much for the tested range of ξ. Therefore K_r does not vary much with differences in wave steepness. Hence there will be similar stability.

Four parameters involved with damage are distinguished: H_s, $\hat{u}_{orb}$, h_t/h_m and $H_s/\Delta D_{n50}$. Parameter $\hat{u}_{orb}$ is a measurement for the peak velocities in the oscillatory water motion directly above the toe. H_s and $\hat{u}_{orb}$ are coupled parameters for load (also including reflection and transmission), h_t/h_m is a geometric parameter and ΔD_{n50} is a parameter for strength.

In the wave flume two tests are conducted at once by separating the flume in half with a board. The toe width B_t and wave steepness s_{op} are not varied because Gerding concluded that these do not influence the stability.

In this research N_{od} is calculated by dividing the number of displaced stones by the number of stones that fit next to each other in the wave flume width (0.39 m). This differs from Gerding's N_{od} value with a factor of about 1.1, because Gerding uses D_{n50}-units for the width. The reason is not clear, but might be based on a misinterpretation of Gerding's definition of the damage number. This suspicion arises because Docters van Leeuwen states a different definition of the damage number in her report.

It is concluded that the Δ-value is used correctly in the Gerding relation, because all data points with varying rock density appear mixed in the same 'cloud' of data points.

When h_m/D_{n50} is plotted against the stability number, for both tested toe heights clearly two separate lines can be distinguished. The choice to use h_t/D_{n50} is approved by Docters van Leeuwen, because it can make a better prediction for practical use. It seems that h_m does have an influence however. This is shown once more by plotting the two tested values of h_m for Gerding's axes h_t/D_{n50} and $H_s/\Delta D_{n50}$. Two separate trends can be seen for the water depths, so there is some influence which is not regarded in the formula. The lines are close though, so the influence is not large. The influence of h_m is equal for Gerding's data.

It is concluded that the shape of the stones is of influence to their stability. The development of damage proceeds slower with sharper edges.

Regarding the toe stability formula (equation 5 in this report), the same trend of the formula is found. Docters van Leeuwen concludes however that the damage in the new tests is about half of Gerding's damage. Reasons for differences are suggested to be found in only counting seaward displaced rocks, the number of waves in a test and a less steep foreshore slope. Also packing or permeability could be different.

Additionally, an analysis is performed based on results from research for threshold of movement for grains (see Appendix A.1). The rock size from calculations is compared to the rock size in tests. In the report is said: “At a certain critical velocity the stones will start to move; in this study this is indicated as damage level N_{od}=0.5.” N_{od} = 0.5 is used because this was indicated by Gerding as ‘start of damage’. For calculations Docters van Leeuwen used an elaboration of the criterion of Rance and Warren that can also be found in SCHIERECK (2004):

$$D_{n50} = \frac{2.15 \cdot \hat{u}_h^{2.5}}{T_p^{0.5} \cdot (\Delta g)^{1.5}} \tag{6}$$

Here $\hat{u}_h$ is calculated by using orbital velocity from regular linear wave theory, but increasing the wave height by adding the reflection coefficient K_r.

$$\hat{u}_h = \omega(1 + K_r)\frac{H_s}{2} \cdot \frac{\cosh k(h_m - h_t)}{\sinh kh_m} \tag{7}$$

K_r was measured during the tests.

A selection of test results was made which correspond with N_{od} = 0.5. Since all other parameters are also known for the selected data, the same parameters can be used to calculate a theoretically required diameter with the formulae above.

Now the test results for N_{od} = 0.5 and the results from the calculation are plotted on the dimensionless parameter space of the Gerding-formula. The computations match the measurements quite well. Only a scalar factor seemed to be the difference. In the paper by VAN DER MEULEN *et al.* (1997), which was written as a result of the thesis, the tuning factor was found to be 1.7. Since the highest waves are responsible for the damage, this is reviewed as the difference between H_s and the highest waves. In a Rayleigh distribution for wave heights this would mean that 1.7 * H_s = $H_{0.5\%}$.

In the reprinted figures below the results for computed diameters and the diameters from the tests are compared.

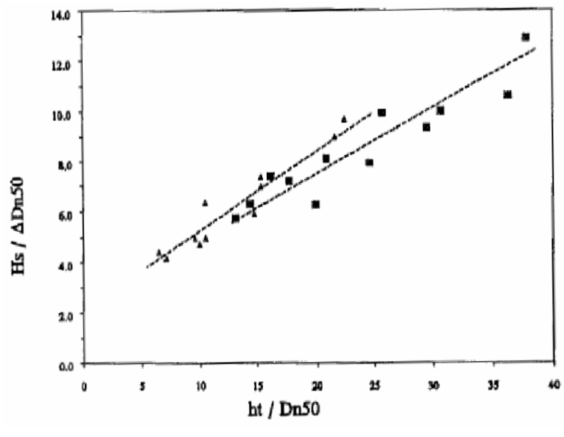

Figure 16: Stability of the tests sorted out to water depth h_m (VAN DER MEULEN 1997)

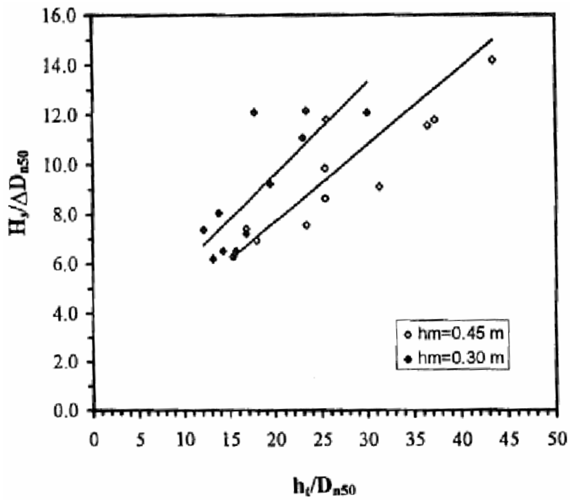

Figure 17: Results of computations based on Rance and Warren (VAN DER MEULEN 1997)

Van der Meer 1998

According to this publication, N_{od}=0.5 is a safe figure for design. D_{n50} is present in h_t/D_{n50} and in $H_s/\Delta D_{n50}$. Therefore for low structures unrealistic and even negative diameters can be calculated (this will be explained in section 2.2.5). Gerding's data set was re-analysed. A simple formula that is rather similar to the formula from CIRIA *et al.* (1991) is given, but now it is based on Gerding's data set. It reads:

$$\frac{H_s}{\Delta D_{n50}} = \left(6.2 \left(\frac{h_t}{h_m} \right)^{2.7} + 2 \right) \cdot N_{od}^{0.15} \tag{8}$$

The formula does not start in the origin. For high toe levels, stability comes close to the stability of armour elements. These have stability numbers close to 2, which is consistent with this formula. The formula applies for a so-called 'standard toe size' (about 3-5 stones wide and 2-3 stones high).

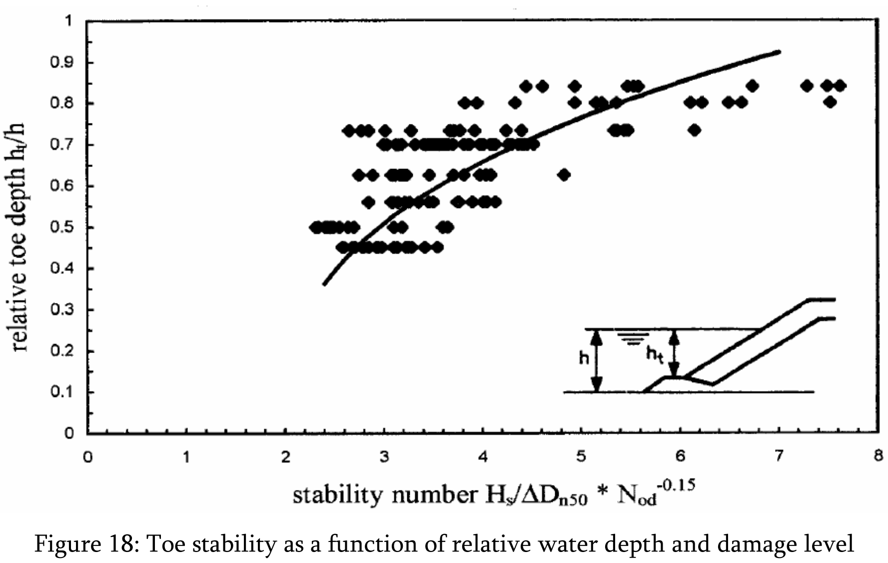

Figure 18: Toe stability as a function of relative water depth and damage level (VAN DER MEER 1998).

This formula is nowadays commonly used, since it appears in the new Rock Manual (CIRIA *et al.* 2007). The Coastal Engineering Manual by USACE (USACE 2006) still recommends the formula of Gerding (equation 5).

Sayao 2007

Particularly for shallow waters and steeper nearshore slopes, design guidance is limited, according to SAYAO (2007). For some geometries, the Van der Meer method of 1998 (equation 8) does not give better predictions than the Gerding formula of 1993 (equation 5). Various authors have shown influence of the foreshore slope on stability of breakwater armour. Some parameters were not reviewed yet for toes, such as nearshore slope m, Iribarren number ξ, wave period and breaker index γ_b (see Box 1). Dimensionless parameter analysis was performed by Sayao.

Analysis for the foreshore slope m only shows a trend of increasing stability for a steeper foreshore slope.

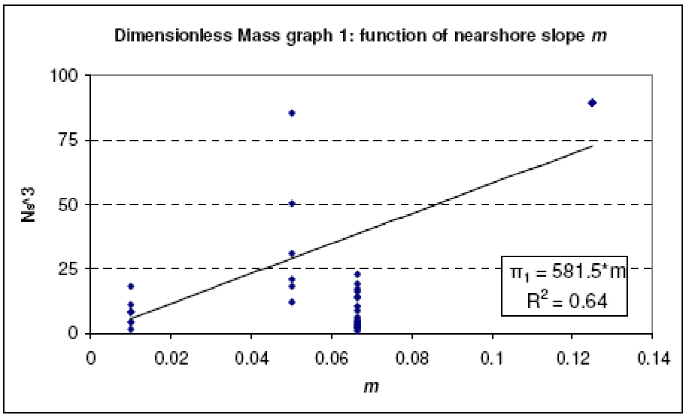

Figure 19: Dimensional analysis by Sayao for m only (figure from SAYAO, 2007)

The influence of the Iribarren number is also presented, with large scatter. The relative depth ratio h_t/h_m does not seem to give a clear correlation. Therefore the relation from USACE (1987) is used. It is concluded that although ξ and m were not yet included in toe element stability formulae, these parameters play an important role. The result is a design chart which includes the influence of the dimensionless parameters described before, in relation to the cubed stability number:

$$N_s^3 = 4.5 \cdot m^{-2/3} \cdot e^{\left(5.67 h_t/h_s - 0.63\xi_s\right)} \quad (9)$$

Recommendations for a new design method are based on this relation. Furthermore, the water depth in front of the toe (h_m) should be defined based on the breaker travel distance.

2.2 Evaluation of literature

In this second part of the literature chapter the content of section 2.1 is discussed. First terminology concerning toes is treated. Secondly is elaborated on the functions of a toe and the recommendations for toe dimensioning. Thereafter damage to toe bunds is considered, followed by views on toe element stability. Finally some remarks are made about specific topics that recur further in the report.

2.2.1 Definitions

Comparing sources of literature has led to some confusion, because the word 'toe' is used for several somewhat different parts of a breakwater.

In this section, the rubble mound breakwater is first decomposed (DE RIDDER 2006). This means that on the top scale level, the entire breakwater structure is seen from the roundhead to the shore connection as one system, including bed protection, slip circle areas, crown, inner slope etcetera. The lay-out of a breakwater structure changes dynamically during the planning, design, construction and service phases. This system consists of subsystems, sub-subsystems and so on, until the lowest scale level of loose elements, for example individual rocks.

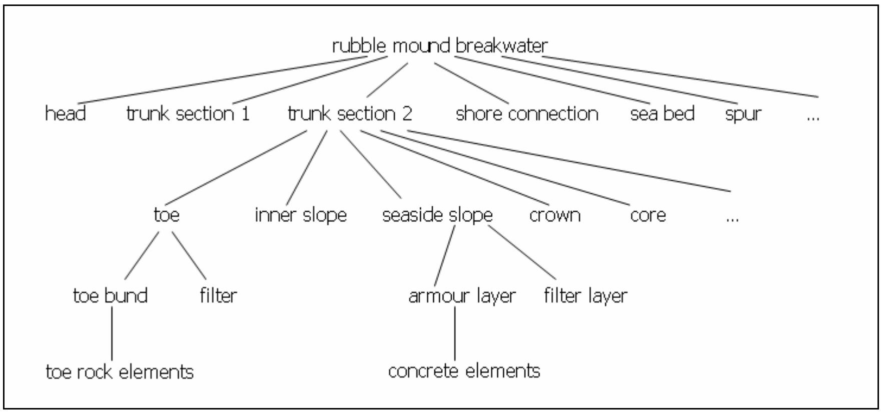

Figure 20: Example of decomposition of a rubble mound breakwater

From this perspective, a number of expressions is defined here, in order that their meaning is clear for further use in this report.

a. The *toe* is the area – with rough boundaries – where the seaside slope intersects the bed level (see Figure 21).
b. In this toe area a *toe structure* may or may not be applied. The toe structure can for example be a toe apron, a toe bund, piles, a trench or a combination.
c. A *toe bund* is a type of toe structure, generally a trapezoidal or dike-shaped heap of rocks, situated at the downward extent of the armour layer. It supports the armour layer or protects the vulnerable toe region.

d. A *toe apron* is a layer of rock or gravel that protects the bed in the toe region from scour. It generally has a larger width than a toe bund.
e. With *toe material* the elements are meant with which a toe structure is built, which is usually rock. In this report the term toe material is used for toe bund rocks unless otherwise indicated.

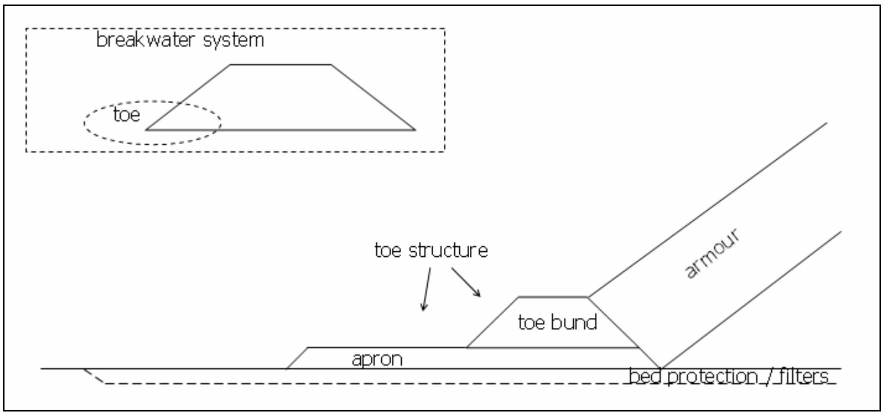

Figure 21: Combination of an apron and a bund as toe structure

2.2.2 Functions of a toe structure

In the literature study a number of functions for a toe bund have been found. They are listed and their validity is discussed.

a. Supporting the armour layer

From the recommendation by ECKERT (1983) (see Figure 8) we may conclude that a toe bund is not always required to give support to an armour layer. If a trapezoidal toe bund is included in the design, automatically the function of armour layer support is introduced (in USACE literature the term 'support berm' or 'toe buttressing rock' is used). The toe then has to perform this function throughout the entire lifetime of the breakwater, since it lies underneath the armour layer. This is illustrated in Figure 22.

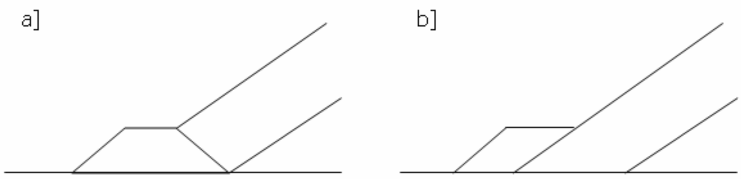

Figure 22: The lay-out of a toe bund can create the support function

Case a] is a toe bund with a supporting function (whether or not this was the reason to add it to the design). This bund is an essential subsystem to the system's stability, because if the subsystem fails, the system fails as well. Toe bund b] supplies extra safety. The system may not be in direct danger when the toe structure is damaged. Notice the difference in construction sequence. To what degree safety is added or if support is needed at all, depends on slope angles, material properties, friction between the layers mutually and on subsoil stiffness. This topic has not been treated yet in literature for toes.

b. Prevent bed material loss from undermining the cover layer armour units

If bed material washes out from under the lowest elements of the armour layer, the layer can deform. According to Eckert, there is no problem if scour only occurs outside of the critical zone confined by the fictitious extended slope line (see Figure 8). Scour within the area under this line means that the armour elements will have to find a new equilibrium. For small amounts of scour and settlements, this is not necessarily a problem. But if the entire armour layer slides down, the underlayer may become exposed in the next storm. For single layer special concrete units even small settlement is often a problem, because interlocking is decreased quickly. Solving this problem does not particularly require a toe bund, but an apron or filter is also an option. For this purpose a bund can be favourable over other toe structures if it can combine multiple functions. The bund is then used as an extra grading layer between bed protection and armour rock.

c. Prevent a scour hole adjacent to the structure

ECKERT (1983) mentions the function to prevent a scour hole adjacent to the structure. By 'adjacent to the structure' is probably meant outside the toe zone as in Figure 21. With scour is probably pointed at a hole that starts well outside the critical zone, but may grow into it. For this function a toe bund is less appropriate than an apron. This report will focus on toe bunds only and not on toe aprons. This function is in fact a bit similar to function b. since it eventually prevents movement of particles from within the critical area. The problem is different in the place where scour starts. Furthermore it may trigger a different failure mechanism. At function b. namely, the armour layer may slide if the toe erodes, but if a scour hole in front of the slope forms, slip circle instability may become normative due to loss of counterweight.

d. Construction aid or basis for armour elements

This function can be desirable for example in case of special concrete armour units. The toe bund is useful as an aid to construction if it is constructed before the armour layer, as in most cases. The armour elements (either concrete or rock) in the bottom rows can be placed nicely in the so-called 'neck' between the toe and underlayer (see Figure 23). The toe bund is used as a passive-support heap that is more stable than the sea bed.
During construction the pressures in the soil are changing. A bund provides counterweight. Relaxation takes place before construction is completed.

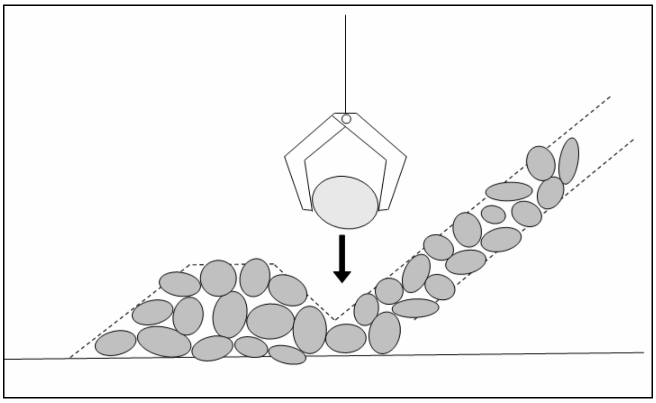

Figure 23: Toe bund with a construction aid function

e. Shortening the downward extent of the armour layer

In deep water (relative to the wave heights), it is not necessary to extend the armour layer to the sea bed. The toe bund may be situated higher on the slope (perhaps made of underlayer material) so that the required amount of armour material is reduced.

f. Provide extra safety

A toe bund can provide extra safety in general, as extra material gives more resistance to the loads and can have a positive effect on the slip circle balance and balance against sliding of complete layer parts. A toe bund also relocates the weak spot of the toe region from the bottom of the armour layer on the bed to the upper seaward edge of the toe. Initial damage to the breakwater is less severe here. When the dimensions of the toe get significant, in the sense that they influence the hydraulic functioning of the breakwater, special attention has to be paid to those hydrodynamic effects. A problem may in fact be created by applying the toe structure, for example when it makes waves break exactly on the slope, thus increasing impact on the armour elements. This can only happen when toe bunds are large. These effects are not a subject of this report.

About functions for a toe bund in general the following is concluded. It is questionable whether the armour layer support function (function a.) is the main reason to apply a toe bund in a breakwater design. There are however enough other functions for a toe bund in many design cases. It can even be useful solely for construction, but it may not always be necessary. If a toe structure is applied, it is trivial that the minimum required rock size has to be known.

2.2.3 Toe dimensioning and toe structure stability

There are not many guidelines for when to use which type of toe structure. The reason is of course that each specific situation has different demands and may require a different structure type. A bit more guidance would however be very useful for designers, in particular for when a toe structure is required at all.

Very different guidelines were found regarding toe width (see Table 1). For B_t the recommendations vary from 2 times D_{n50} (of the toe material) to 4 times D_{n50}.

None of the sources provide a reason for their recommendation. The recommendations imply that if a smaller toe rock diameter will suffice, the dimensions of the bund decrease as well. This does not seem logical.

So if we look at the toe bund and why it is applied in the design, it would make more sense to relate toe dimensions to the functions the toe should perform. For example for support and construction the required dimensions should be related to properties of the armour layer. Other factors than stability can be normative for B_t such as available equipment, construction sequence and the tolerances that are obtainable and desired.

It is commonly accepted that a toe bund has a minimum height of two layers of rock, because of the same reasons that are valid for armour and filter layers. The toe should fully cover the rocks in the layer below, even when some damage has developed.

2.2.4 Damage

In this review three subjects concerning damage are distinguished, namely:

a. The expression of the quantity of damage
b. The development of damage, in time or load increase
c. The amount of damage that is acceptable for a structure

a. The quantity of damage

There are various ways to measure damage in physical models of breakwaters. The measuring method for armour slopes is usually profiling, but according to VIDAL *et al.* (2003) digital image processing can give good results as well. For toes counting the displaced rocks is common. The quantity of damage is initially either determined by the area from which rocks are displaced or by the amount of displaced rocks. For toes, the latter is usually denoted as percentage of the total amount of toe rocks or as N_{od}, defined by Gerding (see section 2.2.1, Gerding).

Both N_{od} and percentage have advantages and disadvantages. Gerding uses N_{od} because it is independent of the toe height and width. That a percentage depends on the size of the toe is more useful than N_{od} for the severity to the stability of the system. A percentage is on the other hand not very useful to describe the process of stability of rocks on the exposed surfaces of the toe. Whether this disadvantage is taken away by Gerding by introducing N_{od} is again questionable, since more exposed elements lead to higher probabilities for displacements. So the length and width are in fact of influence to the damage quantity. These properties might be introducing scatter in the analysis of toe element stability.

Gerding based his classification of N_{od} on CIRIA *et al.* (1991), where a classification is given of damage percentages. So in spite of Gerding's effort to develop a new independent parameter for toe damage, the old and the new parameter are coupled. The parameters correspond in the following way (based on GERDING 1993 and CIRIA 1991):

Table 2: Coupling of classifications for damage parameter by Gerding

% CIRIA (1991)	*description*	N_{od} Gerding (1993)	*description*
0-3 %	no movement of stones (or only a few) in the toe	< 0.5	hardly any damage
3-10 %	toe flattened out a little bit but function is intact and the damage is acceptable	0.5 – 2.0	acceptable damage, design criteria
> 20-30 %	failure; the toe has lost its function and this damage level is not acceptable	> 4	unacceptable damage, toe structure has lost its function

The coupling implies that Gerding should use about 16 to 20 stones per D_{n50}-strip of toe bund. For broad variation of toe dimensions and toe element diameters, this classification of N_{od} cannot be used universally. This seems to be misinterpreted in design practice.

Gerding recommends to use $N_{od} = 2$. Van der Meer recommends $N_{od} = 0.5$ for a safe design a few years later. Unfortunately they give no reasons for these values. The reason will probably be that in their personal opinion these levels of damage are acceptable. For the N_{od}-classification above, DOCTERS VAN LEEUWEN (1996) presents the following figure:

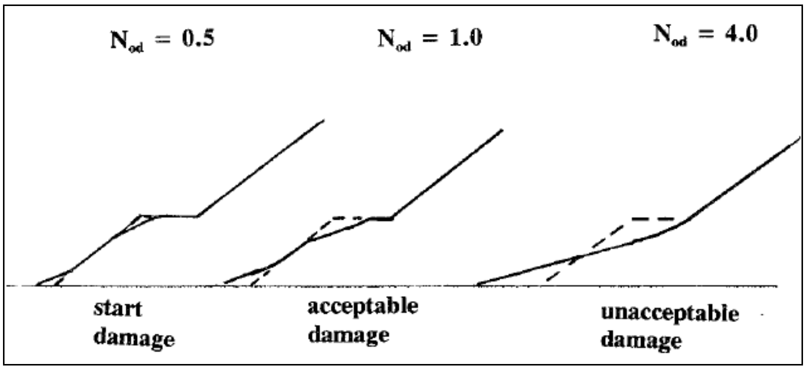

Figure 24: Indication of classification of N_{od} (DOCTERS VAN LEEUWEN 1996)

Some of the previously mentioned disadvantages for the present damage parameters are illustrated by the simple sketch of toe cross sections in Figure 25. The first two toes are standard toes as described in D'ANGREMOND (2001): "(…) a height of 2-3 D and a width of 3-5 D." The third toe is a sketch of a test performed by Docters van Leeuwen in her thesis.

Mind the differences in value of the damage parameters (N_{od} and percentage). Their meaning is different for stability analysis and for damage level significance. Imagine for example a damage level N_{od}=4 as if 4 rocks have displaced from the drawn cross sections, which is a big difference in severity per drawn toe bund. The difference between the test of Docters van Leeuwen (below) and her interpretation of N_{od} levels in Figure 24 is also remarkable.

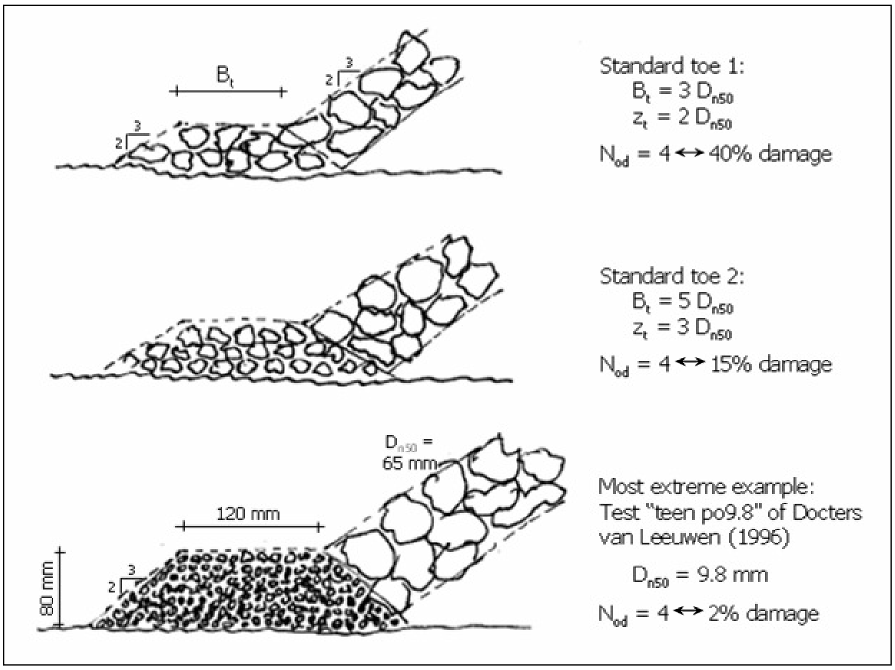

Figure 25: Sketch indicating differences between damage parameters N_{od} and percentage

b. Development of damage

Development of damage can be tracked in time (number of waves) or by increasing the load. Unfortunately no source has been found that accurately describes where on the toe bund damage starts. Docters van Leeuwen suggests in the report that damage starts at the seaside upper edge of the bund (Figure 24). Recent commercial tests have shown that initial damage may occur in the top layer of the bund across the entire toe width.

USACE (1987) does not describe damage at all. The reports of Gerding and Docters van Leeuwen are the only two that describe the development of damage, but only in load increase. Gerding concludes that N_{od} is related to wave height by the power of 0.15. Docters van Leeuwen suggests that this power is dependent on the shape of the rocks. The wave height here is the only load increase considered.

Docters van Leeuwen suggests that her damage should be about 1.4 times as high as Gerding's. She used 2000 waves in her test where Gerding used 1000 and according to DOCTERS VAN LEEUWEN (1996) "the damage grows with the root of the multiplication of the wave number". This can be regarded as development of damage over time. Docters van Leeuwen writes however that this growth factor of 1.4 cannot be observed from the measurements. The reason for this might be the following. Chances of wave exceedence in deep water, where the Rayleigh distribution is valid, increase with the square root of time. But since most of the tests were in depth limited conditions, the height of the largest wave is limited. Therefore damage does not need to develop with the square root of time (or number of waves). It is likely on the other hand that the highest wave that is possible at the toe occurs more frequently.

In armour slope stability formulae, such as the common Van der Meer formulae and the formula in MELBY (1998), damage does actually increase with the root of the number of waves. This could be because waves attack the slope elements differently. More waves lead to more damage. For toes the destabilizing mechanism is different. For toe elements an approach with critical load will be used in this research.

c. Acceptable damage

USACE (1987) does not describe the amount of damage they call acceptable. This is probably because at first hand engineers in design practice are not interested in the development of damage, but they want a recommendation with which they can calculate the required rock size. USACE aimed to provide this. Considering that USACE used monochromatic waves, it may be the case that they experienced a threshold of movement for the toe elements. They disregarded tests without damage and used the data points corresponding with the start of significant movements of elements. RANCE *et al.* (1968) prove that for monochromatic oscillatory flow tests determination of the threshold is possible with visual observations.

If a threshold exists, the increase in damage beneath and above the threshold is very different. Above the threshold it may be highly unpredictable.

Gerding did not mention arguments for his design recommendation of N_{od} = 2. Based on CIRIA (1991) he stated that above N_{od} = 4 (or 20%) the toe has lost its function (see Table 2). First of all, which function is that? Secondly, if it is to support the armour layer, no records have been found that for high N_{od} values the armour layer indeed collapsed. Wouldn't that have been reported if it had occurred? The armour layer was still supported and the toe did not loose that function.

In present design practice little damage is accepted for toe structures. There are three reasons for this.

a. For toe bunds, small amounts of damage may be relatively large compared to slope armour damage criteria.
b. As Bruun denoted, the toe bund is one of the four most important parts of the breakwater for overall stability, so a high safety level is desired. Increasing toe element size rapidly increases safety, but the increase in costs is relatively small as well.
c. It is an economic decision to accept little damage in design, because maintenance for toes is expensive. It is a precision job under water. After all, it is also hard to discover toe damage in reality because the toe is always under water.

In this report is suggested that the amount of damage that is acceptable depends on the functions of the toe. As long as the functions stay intact, any amount of damage is acceptable for the structure. When the damage becomes a threat to the structure, it becomes severe.

The severity of toe damage states to what extent damage is acceptable. Severity is determined by the threat of an amount of damage to the functions of a toe structure. Different toe structures can have different functions. Therefore it is principally impossible to give one amount of damage as an acceptable amount for all cases. This is summarized in the following diagram.

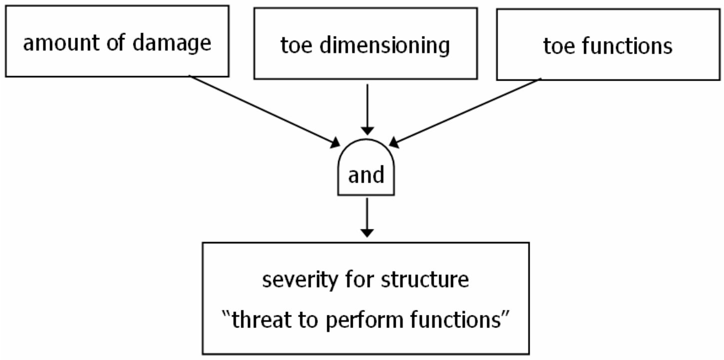

Figure 26: Severity of toe damage related to three themes

2.2.5 Choice of toe element size

The maximum diameter is generally considered to be slope armour size. No explicit recommendations for a minimum diameter have been found during the literature study, but BSI (1991) suggests underlayer material.

Formulae are used to determine the minimum required diameter. Arguments for choosing parameters in such stability formulae are often not mentioned. Therefore the conclusion is drawn that these choices do not originate in sound theoretical background.

For the first Rock Manual (CIRIA 1991), Van der Meer analyzed tests by Delft Hydraulics. Because these were not sufficient, Gerding performed more tests in 1993. Soon after the work of Gerding was published, a problem was found, namely that the recommendation for the required diameter can become irrationally small and even negative. How this is possible becomes clear if the formula of Gerding (equation 5) is rearranged:

$$\frac{H_s}{\Delta D_{n50}} \cdot N_{od}^{-0.15} = 0.24 \frac{h_t}{D_{n50}} + 1.6 \tag{10}$$

$$D_{n50} = \frac{0.625 H_s}{\Delta N_{od}^{0.15}} - 0.15 h_t \tag{11}$$

So $D_{n50} < 0$ if $0.15 h_t > \frac{0.625 H_s}{\Delta N_{od}^{0.15}}$

If figures for Δ and N_{od} are used which are common in design practice, for example $\Delta = 1.6$ and $N_{od} = 0.5$, negative diameters are obtained if

$$H_s < 0.35 \cdot h_t \tag{12}$$

This is not a rare case. This means that a different criterion is decisive for the toe diameter in these cases and that Gerding's formula is not applicable outside its data range. The latter was already written in the original report. So if the problem of negative diameters occurred, it does not necessarily mean that the formula is wrong; it is more likely that it was applied in a wrong way.

Three years later successive research was done by Docters van Leeuwen. She concluded that the relative density was included in the Gerding formula correctly. An influence of h_m for Gerding's data was found in the same way as for the new data, while this parameter was not in the formula.

Docters van Leeuwen initiated a theoretical approach to the problem. The data points corresponding with damages of approximately N_{od} = 0.5 were used as a threshold of movement. For this threshold in oscillatory conditions, the approach of RANCE *et al.* (1968) was used. Attention should be paid to the determination of the threshold and the applicability of the criterion. N_{od} = 0.5 was used because Gerding called this 'start of damage'. The question is if this is correct and whether it is comparable to the threshold criterion of Rance and Warren?

In VAN DER MEULEN *et al.* (1997) is written that the correction factor for the theoretical calculations and the test results is 1.7. In a Rayleigh distribution, they write, this would mean that $H_{0.5\%}$ is responsible for the damage. First of all, the Rayleigh distribution is not valid here, in particular for such small exceedence probabilities. Secondly it suggests that only the 10 highest (deep water) waves out of 2000 are responsible for the damage. If there is resemblance between the calculations and test results, the correction factor also has other causes.

Because of the problem with the formula of Gerding, the data is reanalyzed in VAN DER MEER (1998). With an example is illustrated that the scatter around the new relation is too large for use as an adequate design tool (see Figure 27). If the relative depth of the toe is 0.7, the recommendation for the required stability number N_s=H/ΔD is about 4.4. The scatter in the figure shows that the actual required N_s is probably between 2.6 and 6.1. For a certain design wave, this makes a difference of a factor 2.3 in required diameter, which is almost a factor 13 in required weight class. One of these data cloud boundaries may even exceed the range that is possible in practice.

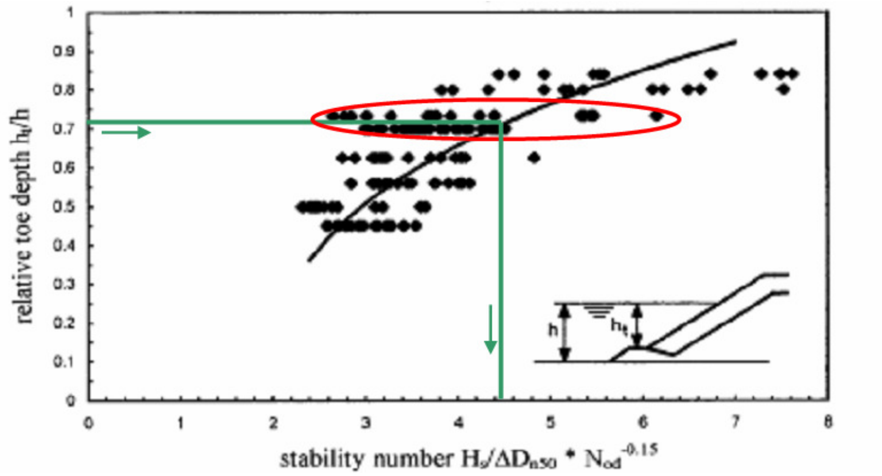

Figure 27: Toe stability according to VAN DER MEER (1998), with a scatter example

Based on this example, h_t/h_m does not seem to be the correct parameter to describe toe stability. But the problem for this relation should not be sought in geometric parameters alone. The scatter could also be due to inappropriate description of damage and its development or an inappropriate stability parameter.

Throughout the development of description of toe element stability, relative depth is a popular parameter. There is however no sound theoretical base for the dependence of stability on this exact parameter. The parameter is practical because it is dimensionless. For rubble mound breakwater toes it was first used in 1987 by USACE. Thereafter Van der Meer uses it in 1991 (CIRIA, 1991). Gerding proposes to use h_t/D_{n50} which seems to decrease scatter, but proves to have its own problems. In 1998 Van der Meer uses the relative depth once more. Sayao continues with it in 2007 in spite of lack of correlation with stability (see Figure 28).

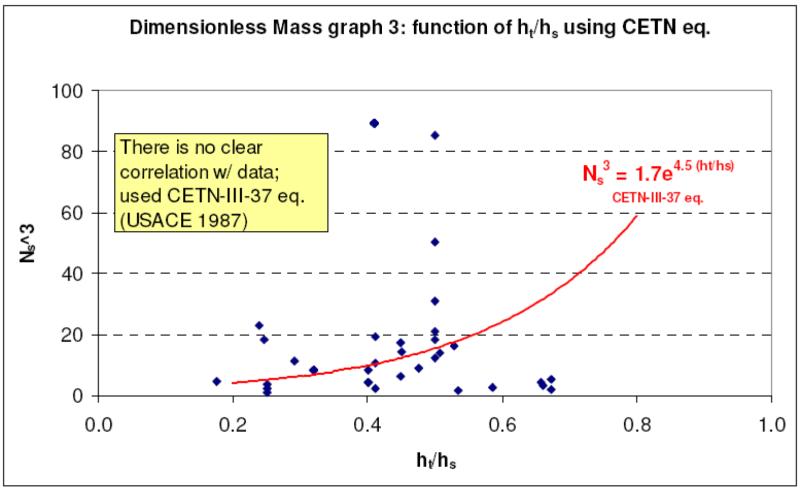

Figure 28: No clear correlation between h_t/h_m and stability (SAYAO 2007).
Depicted curve is from USACE (1987), equation fitted by Sayao.

The following example illustrates that relative depth does not seem appropriate. In Figure 29 two cases are depicted of breakwaters with toe bunds. Case a] has the same h_t/h_m ratio as b], namely 0.75, and the same wave height. If the formula of VAN DER MEER (1998) is used (equation 8), both situations result in the same required toe element diameter. This is no problem at forehand, if h_t/h_m is indeed the correct geometric parameter to describe stability. But experienced experts, looking at situations a] and b], would feel this is incorrect.

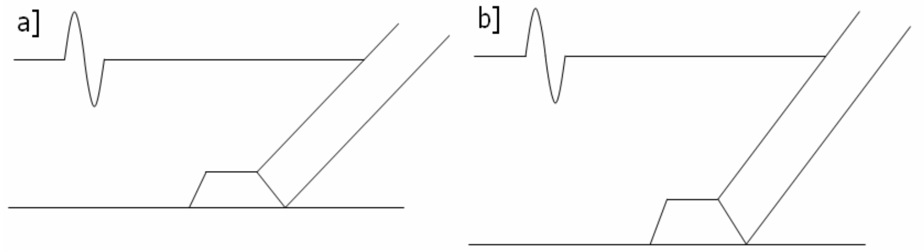

Figure 29: Example of two different cases with equal h_t/h_m parameter

In DOCTERS VAN LEEUWEN (1996) is concluded that h_t is the governing geometric parameter of the two (see Figure 16). Also h_m plays a role, but its influence is smaller than that of h_t. From this can be concluded that h_t/h_m is not an ideal parameter for the description of stability of toe elements because with this parameter the influence of h_t and h_m is always equal.

For cases a] and b] wave conditions are also different, regarded as wave height relative to the water depth. Thus the wave height is a parameter that indicates in which sort of condition the toe stability issue is situated, so H_s belongs to the same category of parameters as h_t and h_m (for comparing situations). It may therefore be undeservedly present in the stability parameter for toes.

The relative depth parameter is probably popular because:

a. it is practical in analysis by nature (dimensionless)
b. it was first adapted from research on vertical breakwaters
c. no better alternative has been found yet.

The maximum value of h_t/h_m should practically be about 0.9 (or anyway not 1), because a toe bund as meant here has a minimum thickness. The highest value tested by Gerding is 0.84.

2.2.6 Other topics

a. Stability parameter

In the Hudson-type stability parameter the ratio of the wave height and the rock diameter is fixed (H~D). Since it was widely used in armour stability descriptions, it is logical that this parameter is also used for toes. The parameter has a theoretical basis for rocks on slopes (D' ANGREMOND 2001). GRAVESEN *et al.* (1977) also used $H/\Delta D$ for toe elements, but in relation to h_t/H. This changes the ratio of H to D since H appears on both sides of the equation.

Similarly we can say the same for equation 3 (first Rock Manual, CIRIA 1991). It is stated that the h_t/h_m parameter is only appropriate for depth limited conditions. In that case the water depth is a good indicator to show how deep the toe level relatively is. But for these conditions, h_m and H have a fixed ratio (namely γ, see Box 1). Therefore h_t/h_m can be replaced by h_t/H, with a linear factor. This also influences the power of the relation between H and D. This holds for equation 8 as well.

An armour layer is always situated in the fluctuating water level and the toe is always submerged. The displacing mechanism is different and therefore it is possible that different stability parameters should be used for these different parts of a breakwater. For near bed structures, the stability parameter $\hat{u}^2/g\Delta D$ is used, where û is a figure representing the amplitude of velocity near the bed.

b. Toe guidance for patented armour units

In the Rock Manual (CIRIA 2007) is said that developers of special concrete units should provide guidance on toe details. In practice this is not always the case. It is also not necessary. Developers should give demands, such as maximum subsidence, orientation etcetera, for the bottom rows of their armour units so that the units can perform their functions. From these demands, designers can derive functions that a toe should have and then select their favourable toe structure for the job. Thereafter material properties of the toe structure need to be calculated.

c. Rayleigh distribution in breaking waves

In deep water wave heights have a Rayleigh distribution in which H_s and $H_{2\%}$ have a ratio of 1.4. Gerding shows that this applies in his test at deep water and that this ratio is different near the structure, in the depth limited zone. But when stability is analyzed with $H_{2\%}$ instead of H_s, Gerding finds a relation that has a factor 1.4 difference with the significant wave height fit. This is rather unexpected.

d. Wave steepness

Docters van Leeuwen gives an explanation for absence of influence of wave steepness in the stability function of Gerding. A basic assumption is that above ξ=3.3 the value of the reflection coefficient does not increase very fast. In fact, the curve is not that different for values below ξ=3.3.

This explanation is also not in accordance with the theoretical approach that Docters van Leeuwen develops further in the report. This approach is based on the threshold criterion of Rance and Warren (1968), where the wave period clearly plays a role in orbital velocities. For equal wave heights, this would imply shorter periods (higher steepness) to represent more load and result in more damage. GRAVESEN *et al.* (1977) report however that longer periods require lower berm elevations. So does the wave period have influence?

Chapter 3

Analysis of existing methods for toe rock stability description

This chapter describes the analysis of presently recommended design tools for the determination of toe rock size. The treated methods are those of GERDING (1993) and VAN DER MEER (1998). As was found in Chapter 2, Gerding's result is recommended by USACE (USACE, 2006), and Van der Meer's result is recommended by CIRIA *et al.* (2007), but the latter also reports Gerding's results. The Gerding method and Van der Meer method are quite similar; the difference is only in influence of geometric parameters. Since the Van der Meer method is the most recent one, this method is chosen as the main subject of this chapter to explain the train of thought. Most analysis is similarly applicable for the method of Gerding as well.

This chapter commences with philosophy of science about models. This is done to clarify the terminology concerning models further on in this report and to position the use of models in design methods. Thereafter the equation by Van der Meer is compared to the data set on which it was based. Then Van der Meer's model is compared to Gerding's original model. Thereafter the findings are compared to the data set of Docters van Leeuwen and the data set of USACE. Finally the conclusions are summarized.

3.1 Philosophy of models in science

This section concerns remarks from philosophy of models in science with a specific view on toe element stability. Considerations and views are based on FRIGG *et al.* (2006).

Models in science can be assigned to the following semantic classifications (i.e. what is the representational function of the model?):

a. Models that represent a selected part of the real world, which is called the target system. There are models of phenomena and models of data.
b. Models that represent a theory, in the sense that the models are what the theory describes with a set of sentences.

These classifications are not mutually exclusive. This means that scientific models can be a combination of these classifications.

With respect to a method such as that by GERDING (1993) and VAN DER MEER (1998), two model types are discernable that both belong to semantic classification a. Gerding performed physical tests in a flume, which are models of phenomena. These are hereby called material models. They represent actual breakwaters as their target systems. The second type is the model of data, which the researchers describe with a mathematic equation. For this latter model type the target system is the data set of the physical tests.

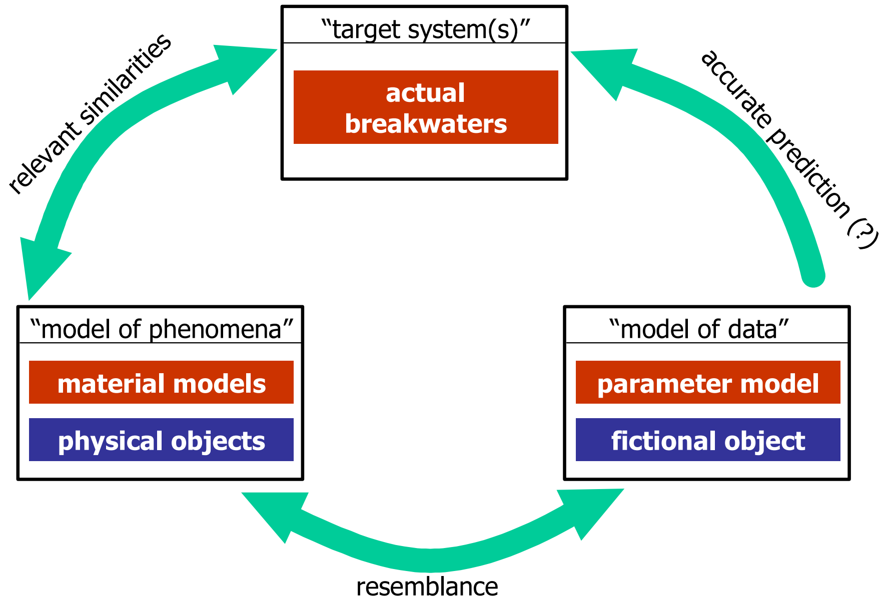

Figure 30: Breakwaters and two ways of representation by models.

Ontologically speaking, models can be divided into two groups (i.e. what kind of thing is the model?), namely fictional objects and physical objects. A scale structure in the flume is a physical object. This material model and the target system should have relevant similarities. This implies that properties of interests of the target are simulated by properties of the model. The results of the tests are data output. For commercial tests a breakwater design is tested, so there is an actual target to be built when the design is complete. For scientific tests such as Gerding's, no actual target existed yet. Therefore such a material model cannot be called a scale model, because it is no resized copy of a target system. There is however a range of possible future targets of which a few were selected for material modelling.

The model of data represents the data output of the physical tests. From hereon this model of data is called the parameter model. The set of all data points is a target system for the *parameter model*. Ontologically speaking, the parameter model is a fictional object. It represents relations between parameters and might therefore be addressed as a mathematical model. By empirical curve fitting resemblance was sought between the parameter model and the actual data output. The parameter model is accurate if the approximation is close to the data set in a relevant sense. The parameter model has relevant resemblance with the target system if

a. the equation approximates the data set with an absolute value
b. the mutual influences of parameters are alike for the equation and the data set

An equation is not the same as a model of data. An equation is a syntactic item. It can be written or arranged in multiple ways, for example with other parameters or annotations, while the model remains the same. The equation is a mathematical description of the model of data. Van der Meer displayed the data set and the parameter model (as a curve) in a 2D space, see Figure 18. In the figure, each data point (target) is the actual output of one physical test. The value calculated by filling in Van der Meer's equation, is its model.

The parameter model can predict properties of the actual breakwater if the breakwater and material model have relevant similarity and if the parameter model resembles the results from the physical tests. This is called epistemology (how do we learn from models?).

3.2 The parameter model of Van der Meer

Data points are results from tests with physical objects. Scientific methodology prescribes that theory should determine the shape of the curve, although the data themselves can statistically underwrite an inference concerning the curve's shape (FRIGG *et al.*, 2006). Empirical equations, such as Van der Meer's toe stability equation, are sometimes mistaken for theory itself. But this equation does not describe a theory, because a hypothesis has not been stated before the data analysis. In any case no hypothesis can be found in VAN DER MEER (1998). Regarding the persistence of searching in the same direction, it is likely that Van der Meer suspects stability to depend on relative toe depth.

The equation that describes the parameter model consists of three dimensionless parts. These are:

a. *Stability*, expressed in $H_s/\Delta D_{n50}$, indicating the combination of rock properties and wave height for any case.
b. *Geometry*, expressed in h_t/h_m, indicating which configurations of breakwaters have comparable properties.
c. *Damage* (or perhaps mobility), expressed in N_{od}, indicating the amount of displaced elements.

Each test has a specific geometry and a combination of rock and wave size. This resulted in a level of damage to the toe bund, measured in N_{od}. The model should be able to predict which nominal diameter is required for an accepted level of damage. In other words: we want to be able to predict the level of damage with reasonable accuracy for a test with certain waves, rocks and geometry.

Box 2: The stability number and stable conditions

Merely the value of the stability number does not indicate if elements in the toe are stable or how stable they are. For example, tests can be set up with any ratio of wave height and rock class that a researcher desires without being able to conclude in advance whether the situation will have stable toe elements. One might think the stability number is actually an instability number, because a higher value suggests a higher ratio of load and resistance.

For a certain case, there is a critical stability number, i.e. a critical ratio of H_s and ΔD_{n50}. If this critical ratio is exceeded for that case, more rocks will move than indicated by the damage number. If for a case with different geometry a higher value of the critical stability number will suffice, this means that the conditions for toe elements are in that case more stable. Therefore a higher critical stability number indicates more stable conditions, and the designation 'stability number' is correct in this sense.

Van der Meer related the three dimensionless parts of the model. The relation describes which ratio of rocks and waves is required for a certain configuration to obtain the desired damage level.

A part of the model describes the relation between damage and stability number. This part of the equation was copied from GERDING (1993). Only the geometric part is different for Van der Meer. Both relations describe for example the decrease in damage for a certain increase in relative toe depth. Or the increase in damage for a certain decrease in rock size.

Since the Van der Meer model consists of three parts, a 3D space is set up. Let each of the three parts form an axis. So we have a stability, a geometry and a damage axis. The equation of Van der Meer is depicted below in 3D space set-up by the axes. This forms a double curved plane (Figure 31).

If the parameter model has accurate resemblance, the data points from the tests should be on or near the depicted double curved plane. In this way we can check how well the parameter model fits the data set. In simple words, the 3D space is like a shoe box, filled with floating data points, which should be near the double curved plane.

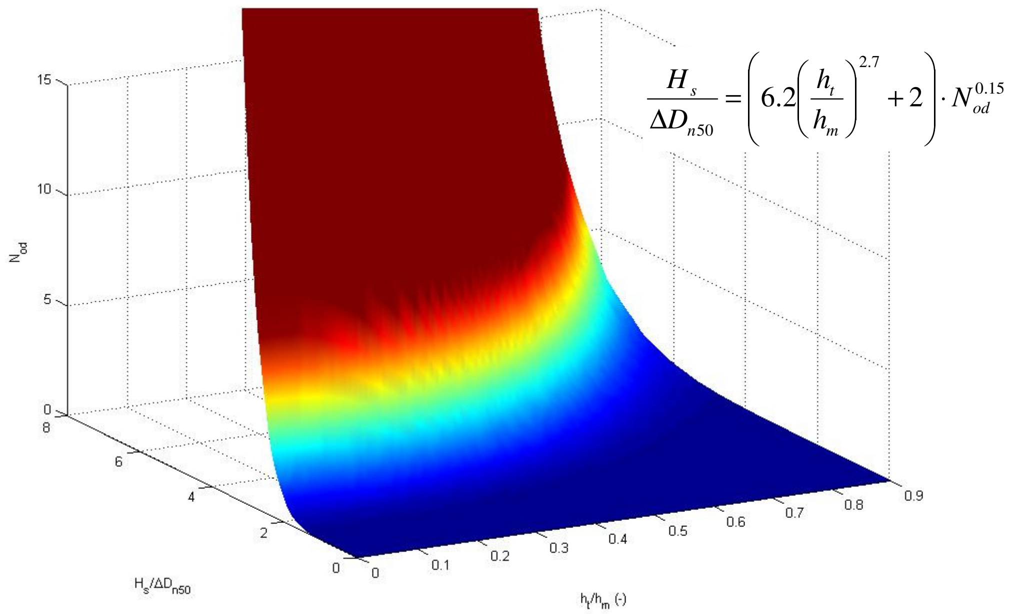

Figure 31: 3D visual interpretation of the parameter model of Van der Meer for toe element stability

A part of the model is the power curve relation between stability and damage. This relation was made by Gerding. The argumentation to use this relation is short and rather poorly stated:

> "Through the points in the figures the best fit seems to be a power curve. For some series of points a straight line seems a better fit (…) but the total tendency is that a power curve is the best fit through the points." (GERDING, 1993)

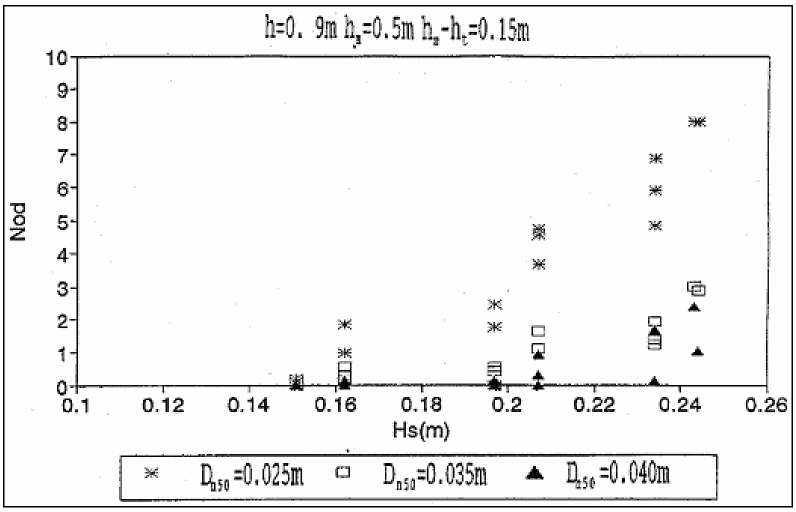

Figure 32: Example of Gerding's figures as motivation for a power curve relation between wave height and damage. Non-varied toe height and water depth. (From: GERDING, 1993)

Figure 32 is an example of the basis of assuming a power relation between wave height and damage (notice that the origin of the wave height axis is not in the figure). The figures on their own are not very convincing evidence that there is a power relation. If we view the data points empirically like Gerding did, a straight line that does not go through the origin of the figure, would seem a better fit. We will come back to this in section 3.3.

Curve fitting is mathematically possible for a space with many axes. But it is not easy to visually grasp what happens. Therefore Van der Meer depicted his model in a 2D space, which was probably also the basis for the curve fitting procedure (see Figure 18). But this presentation of the model and data does not make it easy for a designer to see for himself how accurate the model predicts a damage level for a certain breakwater configuration.

If we want to assess the accuracy of damage prediction visually, we have to compare the model and data in another way. The model consists of 3 parts. Van der Meer and Gerding combined the stability and damage parts to obtain a 2D figure. In this report the geometry and stability axes are combined for a 2D figure. Through this transformation of the axes we can see how much damage can be expected for a certain combination of geometry and stability number. We can also see how accurate the model predicts this. This is helpful for adequate toe design.

Step by step is explained in section 3.3 how the axis transformation is made. But first we want to investigate how combining two axes affects the presentation of the model. This is done in this section. Gerding and Van der Meer have combined the stability and damage axis. We will apply a similar method to obtain a 2D figure with the combined axis of stability and geometry (this eventually results in Figure 39).

First we look at the 3D visualization (Figure 31) from above (top view of the shoe box). This results for the parameter model only in the following figure:

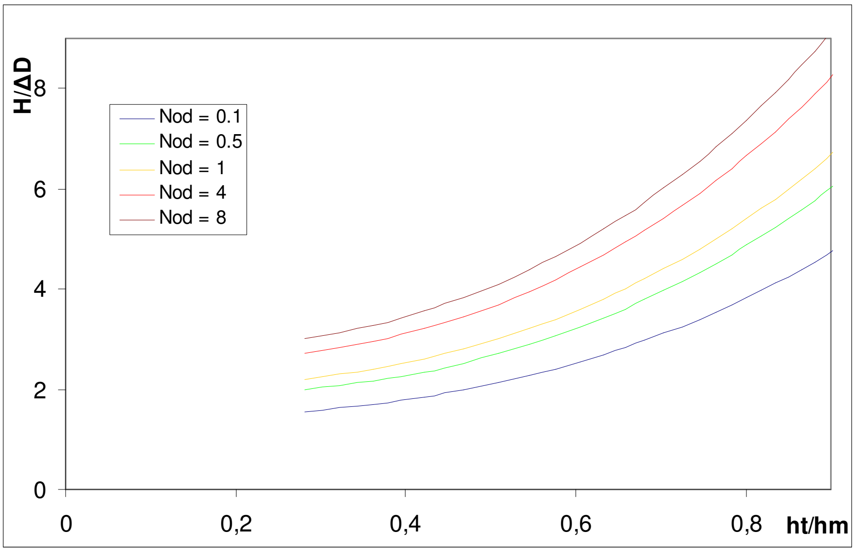

Figure 33: Top view of 3D image of Van der Meer's parameter model

A safe design recommendation by VAN DER MEER (1998) for required toe rock diameter is N_{od} = 0.5, so in Figure 33 this is depicted as the green line.

In the paper is found that for $h_t/h_m = 0$ (where the equation intersects the axis), the stability number should be about 2. This is because 2 is a regular stability number for armour elements and a toe is supposed to resemble an armour layer for $h_t/h_m = 0$. At this geometric value, the stability number for the recommended design line is $2 * 0.5^{0.15} = 1.8$. Actually the stability number has the value 2 for $N_{od} = 1$ and not for $N_{od} = 0.5$.

Now we view the data set from above.

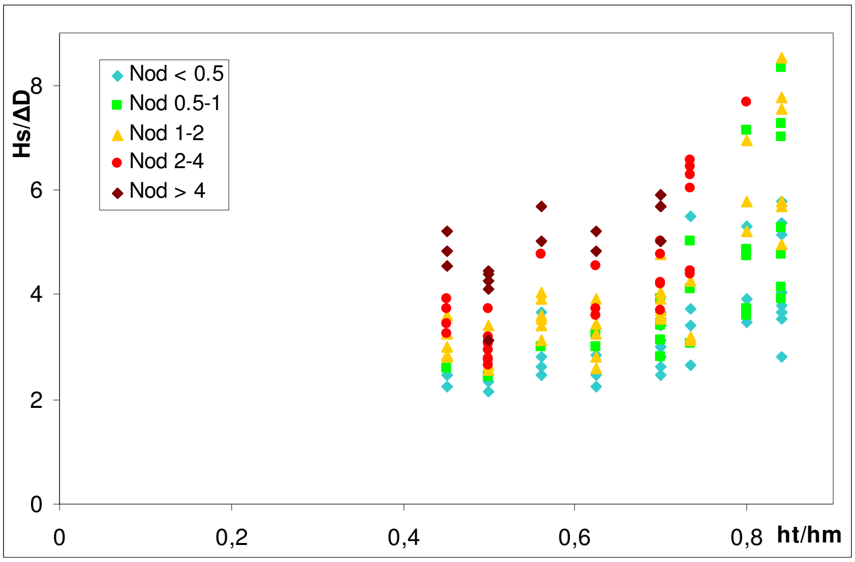

Figure 34: Top view of Gerding's data set in Van der Meer's model space

From comparing Figure 33 to Figure 34 it is clear that the model is less accurate for smaller toes. This is unfortunate, because this is the interesting part for design. The deeper the toe namely, the more likely it is that rock sizes can be chosen smaller than the armour layer sizes.

USACE (1989) based their recommendation on data with the test results "acceptable damage", because a relation for higher amounts of damage is irrelevant to designers. This approach would result in a curve, fitted as a lower limit for the green data points in Figure 34.

This was however not the approach for the method of Van der Meer. Analogous to the analysis of Gerding, it was chosen to use all test data in the analysis. By using the (supposed) relation between damage and stability, all test output can be used in the comparison.

Now the model and data are viewed again in the original presentation with two axes. One of these is the combination of damage and stability. This is equal to Figure 18, only the axes are transposed for convenience.

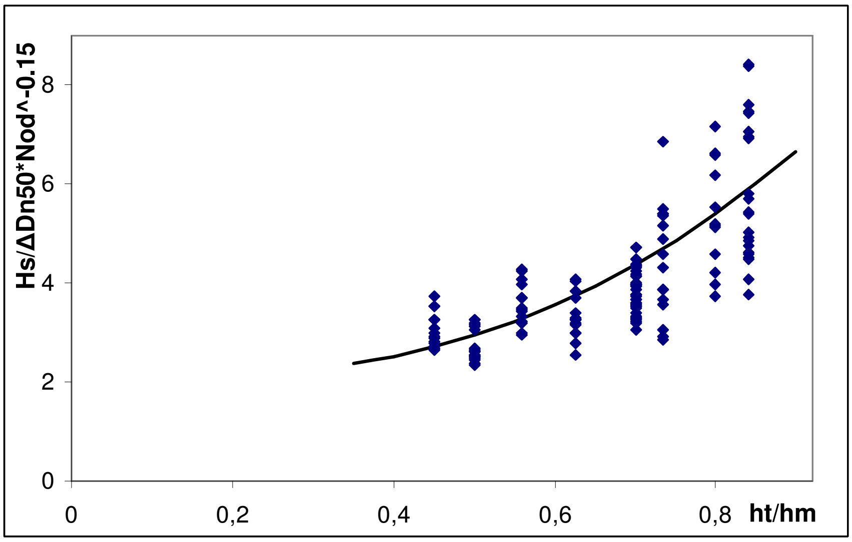

Figure 35: Data set from GERDING (1993) and the parameter model of VAN DER MEER (1998)

The axis transformation from 3D to 2D implies that the figure above is not the same as the top view of the 3D space. The model prescribes that we know that the relation between damage and stability is a function through the origin with power 0.15. The result of a material test is a damage level for a certain stability number. According to the model, we know what the stability number should be to obtain another damage level, by applying the 0.15 power relation. This is the basis to obtain Figure 35.

For the model goes that all the lines in Figure 33 coincide in one single line in Figure 35. If we want to read out the required stability number (and thus D_{n50}) on the vertical axis directly, we fill in the desired level of damage (N_{od}) and obtain a design line as in Figure 33. This means that for different values of N_{od} the recommendation line is shifted up or down in the graph. This also applies for the data set. How this works exactly for a single geometric test configuration is explained with a visual interpretation in the next section.

3.3 Comparing damage and stability for similar geometries

We will consider a vertical cross section of the 3D framework (Figure 31), at a certain geometric value. The parameter model implies that the relation between stability and damage is equal for all cases with the same relative depth, which thus makes those cases geometrically comparable. As an example the cross section of the geometry at $h_t/h_m = 0.63$ is considered.

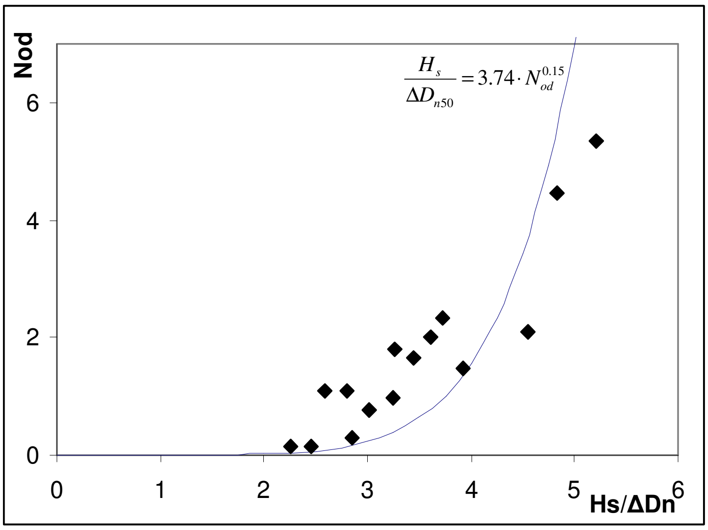

Figure 36: Test data (Gerding) and its parameter model (Van der Meer) for geometric cross section at h_t/h_m=0.63

The line in Figure 36 depicts the parameter model at this cross section. If we enter $h_t/h_m = 0.63$ in the Van der Meer equation (equation 8), we obtain:

$$\frac{H_s}{\Delta D_{n50}} = 3.74 \cdot N_{od}^{0.15} \tag{13}$$

In Figure 35 equation 13 is represented by a single point on the curve, namely (0.63;3.74). To interpret how the 3D depiction relates to the 2D depiction, we can imagine that the multiplication of stability with $N_{od}^{-0.15}$ implies that all model points are scaled (in the sense of damage) to where the model point would lay if their damage value would have been N_{od}=1. The data points can be 'scaled' to compare them in a 2D plane.[2]

Once again, this applies not only for the model but also for the data set. By the operation of multiplying the stability number with its test result N_{od} to the power -0.15, all data points are scaled and are compare in one plane. This is shown in the following figure.

[2] The combined stability-damage number can also be scaled to any other damage value. This could be for example N_{od} = 0.5 (Van der Meer's safe design recommendation), resulting in the green line in Figure 33, which would be useful for design practice as it immediately would return the required value of the stability number on the vertical axis, see Appendix G.2.

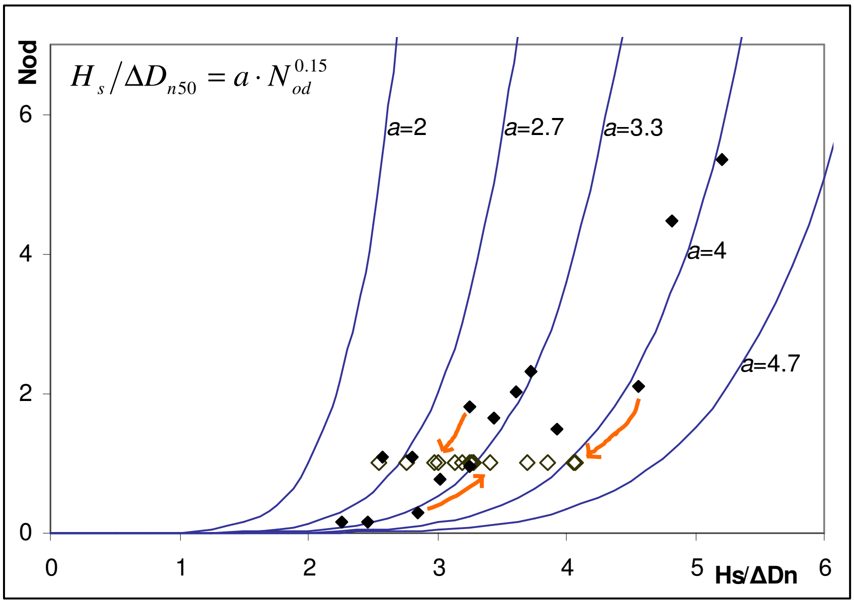

Figure 37: Cross section of model in 3D frame for h_t/h_m=0.63 with actual data points and their images on the level of N_{od} = 1

Since the dots are not exactly on the line $H_s/\Delta D_{n50} = 3.74 \cdot N_{od}^{0.15}$, the data points do not form a single point in Figure 35 (like the model does). The data points are projected on the horizontal plane of N_{od}=1 along a line where

$$\frac{H_s}{\Delta D_{n50}} = a \cdot N_{od}^{0.15} \tag{14}$$

in which *a* is the value such that this line goes through the data point.

As a result, when we look at Figure 35, we see the axis $H_s/\Delta D_{n50}*N_{od}^{-0.15}$. This can be interpreted as looking at a horizontal cross section of the 3D space at N_{od}=1. All data points are projected on this plane as shown in Figure 37. Every test result is depicted with a stability number that it would have had, according to the model, if the damage value were 1. The curve in Figure 35 represents the model.

Above we have seen what it means to use an axis that combines stability and damage. Now a figure will be created with an axis that combines stability and geometry. Thereby the damage level, which is the test result, can be seen independently. This means we now use a vertical cross section of the 3D space instead of a horizontal cross section.

The same example is used: h_t/h_m = 0.63. Every data point is plotted with what the stability number would have been if the geometry was h_t/h_m = 0.63. We know how to calculate this imaginary stability number because the relation between stability and geometry is prescribed by the model. This means shifting each data points along a line in as in Figure 38 until it reaches geometry h_t/h_m = 0.63. The lines match the properties of the model. The damage number N_{od} remains unchanged. The lines cross the vertical axis at $H_s/\Delta D_{n50}$ at value 2, because this is a basis for the model.

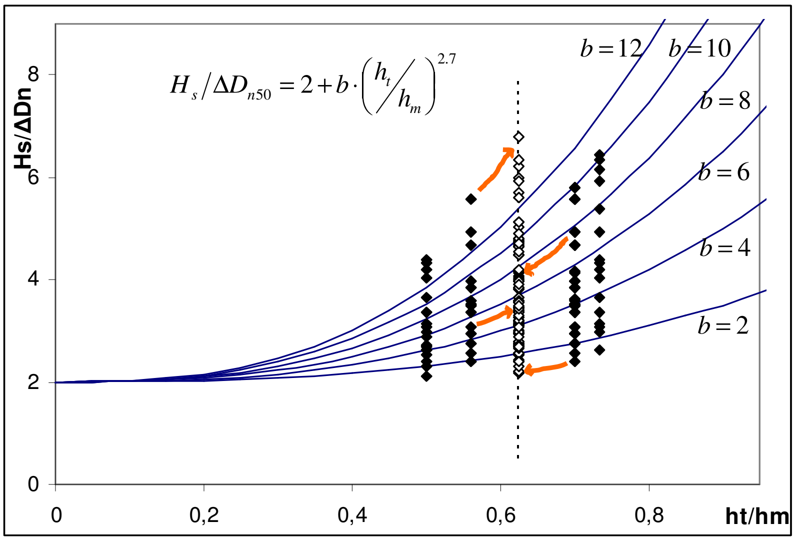

Figure 38: Top view of 3D space with data points and images on cross section at h_t/h_m = 0.63. (The value of *b* for the model is 6.2 for N_{od} = 1)

This analysis is done only with the test results close to the selected cross section (h_t/h_m=0.5-0.73). Otherwise it may become significant how the set-up of the imaging lines is chosen exactly. The result of this axis transformation is the cross section for h_t/h_m=0.63 that is now shown below.

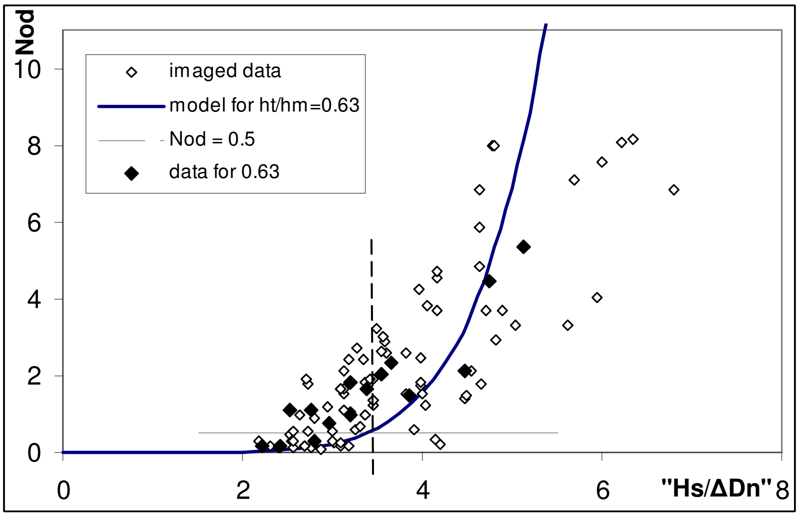

Figure 39: Cross section of 3D space at h_t/h_m=0.63 with scaled data points of other cross sections[3]

[3] The actual horizontal axis of this figure is not the stability number, but the combined axis for geometry and stability. The expression for this axis is $\left((H_s/\Delta D_{n50} - 2) \cdot \left(h_t/h_m \cdot 0.63^{-1} \right)^{-2.7} \right) + 2$. This is analogous to the original 2D space of the curve fit, where $H_s/\Delta D_{n50} \cdot N_{od}^{-0.15}$ forms an axis and not the stability number alone.

The data points that originally already were in this cross section have not moved of course. This is indicated here by the fully coloured markers. The fully coloured markers thus have the exact same position as in Figure 36. The images from other geometric cross sections are markers which are blank in the middle.

Figure 39 shows that at the line where N_{od} = 0.5 (which is the area of interest for design), most data points are at the left hand side of the model. Here the model prescribes at N_{od}=0.5 to use a value of 3.4 for $H_s/\Delta D_{n50}$. From the figure is derived here on the other hand that test results with damage levels up to N_{od}= 3 are also possible for this combination of parameters. This is not conservative for design. If for a design only N_{od} < 0.5 would be acceptable, Figure 39 shows that a more appropriate value for the stability is about 2.4 (for h_t/h_m=0.63). This implies a 40% larger required diameter (more than 2.5 higher weight class)!

The problem for a lower limit is that there is also test output with low damage for higher stability numbers. So there are also cases where smaller toe element diameters would suffice. In such cases, over-dimensioning leads to unnecessary high costs.

Regardless of which design level of N_{od} is desired, 65% of the actual tests have a higher damage value than their representing value in the parameter model. This is visible in Figure 35 where 99 of 152 data points lie lower than the curve. In other words: 65% of the test results lie above the double curved model plane in the 3D space (as in Figure 31). This is significantly important for actual design tests. The probability is reasonably high that more damage will occur than is predicted by filling in the Van der Meer equation.

The power relation between damage and stability

The power relation is investigated in more detail based on Figure 40. The tests with geometry h_t/h_m=0.63 are used again.

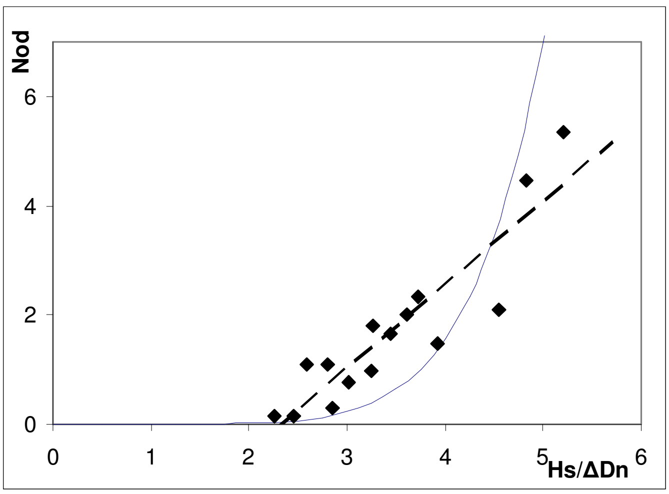

Figure 40: Gerding data for h_t/h_m = 0.63. The Van der Meer model is represented by the blue curve (R^2= 0.13). The straight regression line has R^2= 0.83.

A statistical measurement for correlation between the model and data is the coefficient of determination, expressed as R^2. The value in Figure 40 for R^2 is 0.13 for the curve and for 0.83 for the straight regression line. The straight line thus fits this data set better. No argument of physical theory was given for the power curve either, so the straight line is better.[4]

A significant reduction for the R^2 value for the curve is produced by points with higher stability number and damage result. This is because the model is far above the data points. Since this is not the area of interest, it is fair to exclude the highest data point in the comparison. Even then the R^2 value is 0.24 for the curve (0.74 for the line). The value of R is therefore smaller than 0.5 and the correlation therefore classifies as weak while correlation of the line classifies as strong.

It is certainly not claimed here that the relation should be a straight line. No theoretical background is present at this time for such a statement. That the straight line fits this data set better than the power curve is merely used as an argument; the power curve is in any case not a good description. Use of this relation produces much scatter for the Van der Meer formula. Moreover, to present the model in this way decreases the insight in certainty about the predicted damage. In Figure 41 this effect is cleared up, because two other axes are combined than in the figure presented by VAN DER MEER (1998).

In conclusion is stated that the Van der Meer design tool is not very accurate. For the model applies:

a. The power relation between damage and stability is not correctly assumed.
b. Is Nod the best option to describe damage and why?
c. The choice for Hs/ΔDn50 was rather arbitrary, because it was simply copied from armour slope analysis.
d. The geometric parameters require improvement. (see also section 2.2.5)

Stability as well as geometry and damage are investigated in the new hypothesis in Chapter 4.

3.4 The parameter model of Gerding

The parameter model of GERDING (1993) has several similarities to the model by Van der Meer. This is logical, since Van der Meer only adjusted the geometric part. Therefore the analysis can be similar as well.

The original figure with Gerding's test results and the parameter model in his 2D space is given here:

[4] For the R^2-value various definitions exist for curves. Here a common definition is used: $R^2 \equiv 1 - \frac{\sum (y_i - f_i)^2}{\sum (y_i - \bar{y})^2}$ (DEKKING *et al.* 2005)

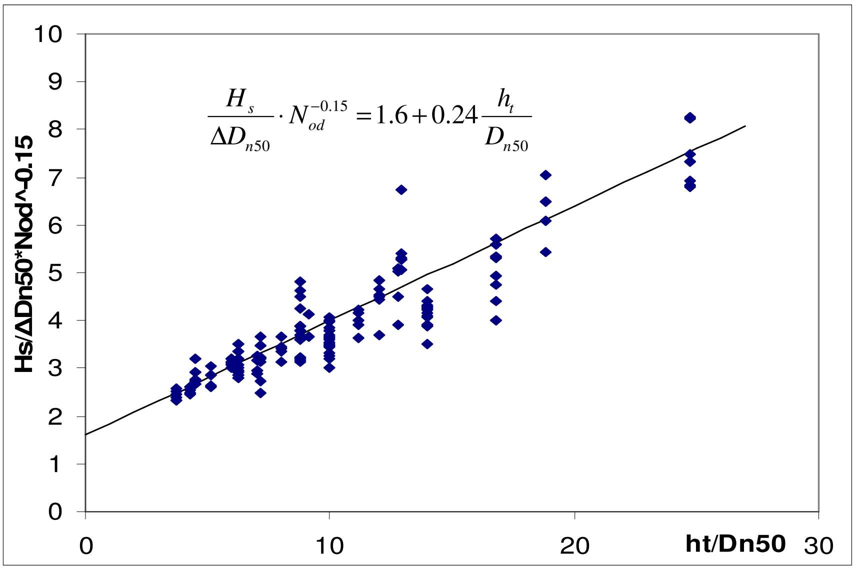

Figure 41: Data set and parameter model of GERDING (1993)

This parameter model is a better fit to the test results than Van der Meer's model. The correlation is higher, measured again as the coefficient of determination R^2. This value is 0.82 for Gerding's fit and 0.52 for Van der Meer's fit[5].

The problem that Gerding's equation can return negative recommendations for diameters is actually not solved by the method of Van der Meer. Van der Meer's model is based on Gerding's data set as well. The equation of Van der Meer has therefore the same field of applicability. Since the correlation of the Gerding model is better, this is a better design tool. Presently no recommendation is available for non-depth-limited situations.

According to the Gerding model, material tests with equal h_t/D ratios are comparable. As an example $h_t/D_{n50} = 8.8$ is given in the figure below.

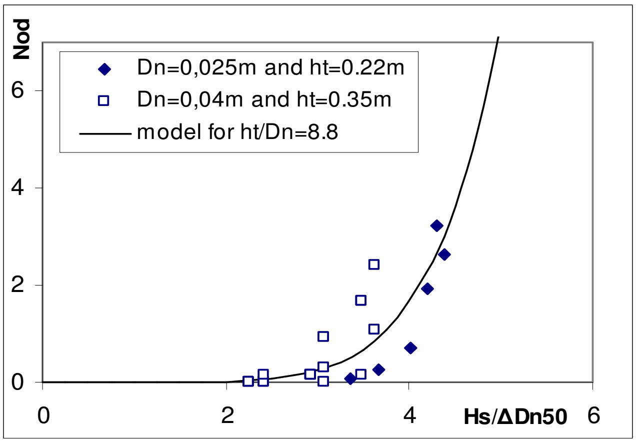

Figure 42: Test results and Gerding model for h_t/D_{n50}=8.8

[5] Measured for the original presented forms, as in Figure 41 and Figure 35.

For this geometric ratio, 2 combinations of h_t and D_{n50} are available in the data set. Absolute values of h_m are different, namely 0.3 m and 0.5 m. The figure show that there is a difference between the two tests groups, but the model does resemble the test results to a certain extent. We could conclude that h_t/D_{n50} alone is not enough to compare different geometries because h_m also has influence. But these conclusions are only true for validity of these propositions:

a. that N_{od} is a correct way to measure damage
b. that $H_s/\Delta D_{n50}$ is a good stability parameter.

In Figure 43 test results for h_t/D_{n50} =10 are depicted.

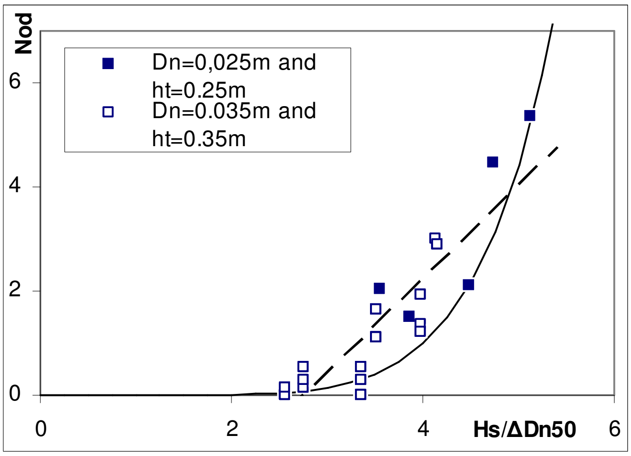

Figure 43: Test results and Gerding model for h_t/D_{n50}=10.0

Here the test results are mixed in one 'cloud' of data points. The difference in diameters is smaller than in Figure 42, as well as the difference in h_m (namely 0.4 m and 0.5 m).

The fit of the model's power curve and a straight regression line are compared for Gerding's model as well. Here the curve has an R^2 value of 0.67 and the line has an R^2 value of 0.80. It is repeated that it is unknown which shape the relation should have. But since even a simple straight line fits the data set better, a power curve is not appropriate. No theoretical background is given for a power curve.

In DOCTERS VAN LEEUWEN (1996) the influence of h_m is evident. Even though this parameter is absent in the model of Gerding, the linear relation in Figure 41 seems apparent. The influence of h_m could be that it influences the H_s for depth limited conditions. It is concluded that h_t is the governing of both depth parameters.

3.5 Comparison with the data set of Docters van Leeuwen

Van der Meer adjusted Gerding's parameter model in 1998. By this time the data set of Docters van Leeuwen (1996) was also available, but it was not used. Docters van Leeuwen concluded in her thesis that using the parameter combination ΔD_{n50} is applicable. The test results appear with similar distribution in one cloud of data points. In the report was also concluded that the new tests resulted in smaller damage levels than Gerding's tests.

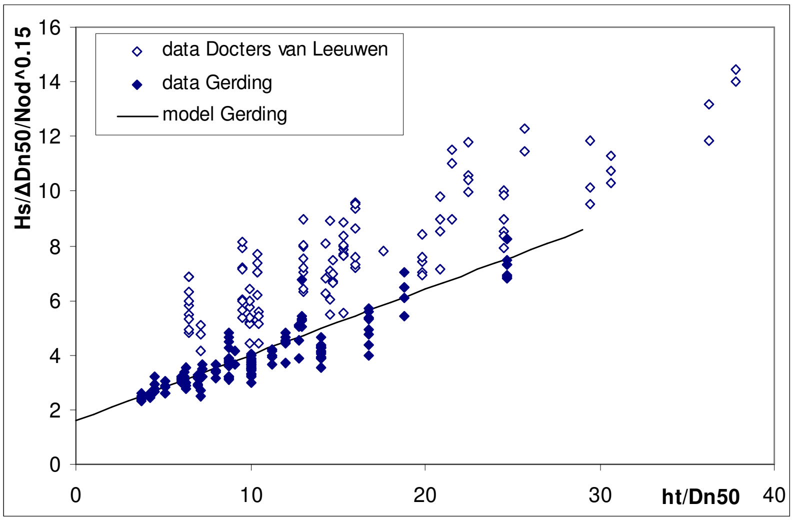

Figure 44: Comparison of data sets of GERDING (1993) and DOCTERS VAN LEEUWEN (1996)[6]

Docters van Leeuwen gives suggestions for the supposed differences in damage:

a. Only seaward rock displacements were counted.

b. Difference in foreshore slope

c. There may have been a difference in the packing of the stones.

These reasons are not verified by Docters van Leeuwen. Since 2000 waves per test were used where Gerding used only 1000, damage could actually also have been larger. A difference for the data sets is also visible in Van der Meer's model:

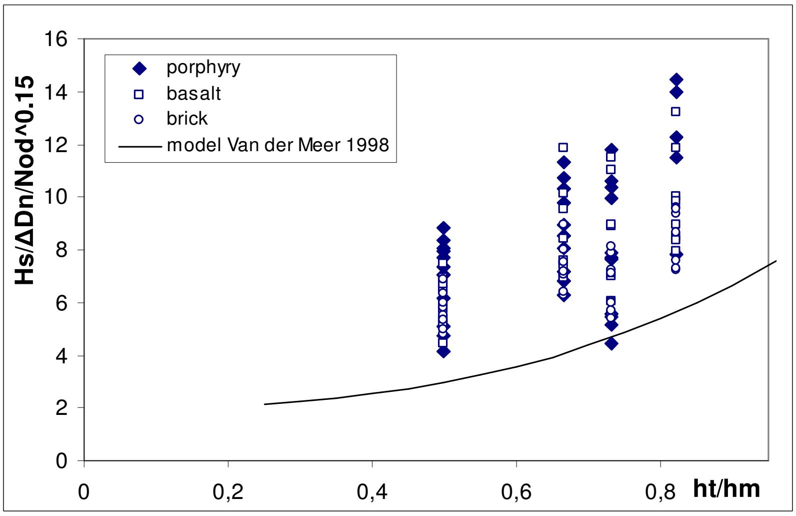

Figure 45: Data set of DOCTERS VAN LEEUWEN (1996) and the model of VAN DER MEER (1998)

[6] Damage levels in terms of N_{od} for the data set of DOCTERS VAN LEEUWEN (1996) have been recalculated in accordance to the definition of GERDING (1993) (use of D_n instead of D). Because of this, the difference with the values in the original report is about 10%.

Between the parameter model and this data set there is no resemblance in absolute sense. This is not applicable for design. This is also visible when a single geometric cross section is regarded, for example h_t/h_m=0.73. Docters van Leeuwen used only 4 separate geometric configurations, namely 2 toe heights and 2 local water depths. Therefore there are 4 combinations (see Figure 45). All test data for equal h_t/h_m-ratio thus also have the same absolute h_m and h_t.

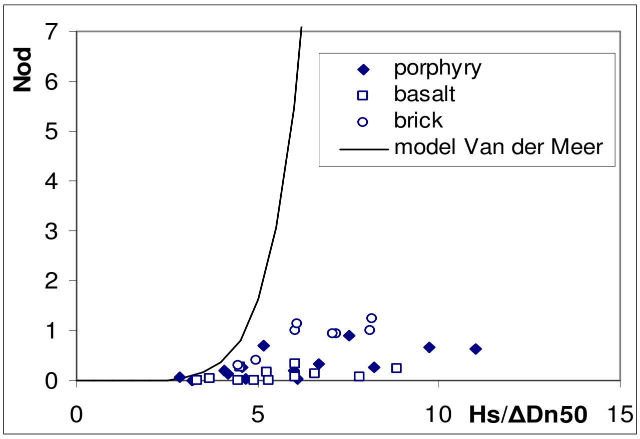

Figure 46: Test data of Docters van Leeuwen for h_t=0.22m and h_m=0.3m

The model does not resemble the data of Docters van Leeuwen, where for Gerding's data the model was at least close. Are the test results really that different from Gerding's tests? Are other parameters such as foreshore slope, which are not present in the parameter model, of that level of influence to the damage result? And if we also consider that the number of waves was doubled?

Box 3: Differences between test series set-up of Gerding and Docters van Leeuwen.

The foreshore slope of Gerding is 1:20 and that of Docters van Leeuwen 1:50. HOVESTAD (2005) concluded that foreshore slope steepness has influence on stability. He used 1:8 for a steep slope and 1:30 for a mild slope. We may expect that the difference in the foreshore slopes between Gerding and Docters van Leeuwen has influence. But since they both classify as a relatively mild slope, this influence will not be very large.

Concerning that Docters van Leeuwen only counted seaward displacement, the following was already mentioned in her report: "from communication with Gerding it seemed that landward damage was never more than 30% of the total damage". Therefore the deviation cannot be larger than 30%.

The difference in number of waves simulates a twice as long duration of a storm for the tests of Docters van Leeuwen. She remarks that this should actually result in about 40% more damage than for the tests of Gerding.

We conclude that the amount of damage in the tests of Docters van Leeuwen and Gerding should be more or less comparable, or in any case not deviate to the extent that is visible by comparing Figure 44 to Figure 46. Otherwise an influential factor was different that is not regarded here.

There can be a difference in the test results for Gerding and Docters van Leeuwen. But as follows from Box 3, the difference is probably not very large. It is more likely that the dimensionless parameters are chosen incorrectly. Of course scatter exists due to irregularities, but per data set, scatter like in Figure 35 and Figure 45 is extraordinary. Fitting a curve to data points is no problem, but the real question is which (dimensionless) parameters should be chosen on the axes before the relations between these parameters are regarded.

Comparison preferably begins simple and we should keep as much parameters equal as possible. Fortunately, there is one test setup identical in the series of Docters van Leeuwen and Gerding. For this set-up h_m=0.30 m and h_t=0.22 m. There is a combination of ΔD_{n50} in both series with the same value, namely 0.029 m. For this configuration, this is the smallest rock class of Gerding's series and the largest of Docters van Leeuwen's. Now for these tests the only variable parameter is the wave height. This results in the following figure:

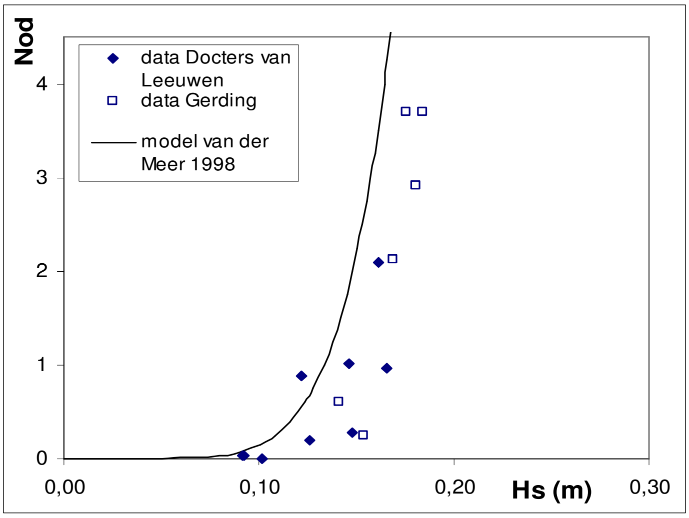

Figure 47: Test data by Docters van Leeuwen (1996) and Gerding (1993) for h_m=0.30m, h_t=0.22m and ΔD_{n50}=0.029 m

In this figure is visible that the results for these test series are not that different. The results are not only close to each other, but for this particular case they are also relatively close to the parameter model. So when using only the wave height as a variable parameter, the test results are similar here. But when we compare data for other varying $H_s/\Delta D_{n50}$ values, the results are not similar anymore.

The conclusion seems to be that we found a new clue that the Hudson-type stability parameter is not usable for toe element stability analysis. Only one test configuration is however regarded. Figure 44 and Figure 45 give the impression that there are large differences between the test results from Gerding and Docters van Leeuwen. It was suggested by Docters van Leeuwen that the differences fully originate in circumstances for the tests series. It is now likely that at least part of the difference is caused implicitly by the choice of dimensionless parameters. The

Van der Meer model is not usable for the function it should perform, namely to make reasonably accurate predictions for the required toe rock diameter.

As a remark is said that in general Docters van Leeuwen tested smaller diameters than Gerding did (for equal absolute toe bund dimensions). This explains why in general larger stability numbers are found (regardless of the damage result, which is limited by the available number of stones in the toe), see Figure 44. To interpret this visually, we regard Figure 48. Figure a] indicates where most test results should be, if the test set-up had been equal except for wave height. The circles in figure b] indicate where generally test results of Docters van Leeuwen and Gerding are for the present model.

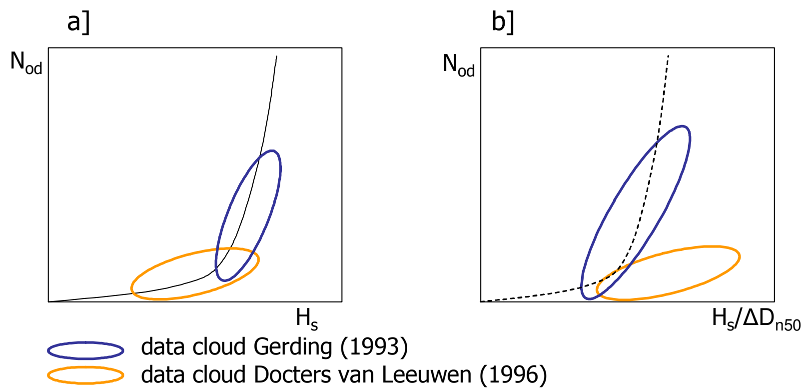

Figure 48: General comparison between test set-ups for two data sets

As comment on the test series of Docters van Leeuwen can be said that some tests were performed with very small toe elements. The ratio of h_t/D_{n50} of up to almost 40 is not realistic in practice. Furthermore, the toe width was 12 times D_{n50} and the toe heights were 8 and 15 times D_{n50}, which are dimensions that are far from standard for bunds investigated in this report (see Figure 25). Material models with these properties may not correctly represent possible target systems. Besides that, not that many tests have resulted in high damage, so it is difficult to study the relation between damage and stability based on the data set by Docters van Leeuwen.

3.6 Comparison of Van der Meer and USACE

Because the parameter models of VAN DER MEER (1998) and USACE (1987 and 1989) make use of the same geometric parameter, namely relative depth, they can be compared. There are however restrictions. These are:

a. The model of Van der Meer was based on the data set of Gerding. He used irregular waves. USACE used monochromatic waves. Their stability number Ns is calculated with the regular wave height. We have to compare monochromatic waves with irregular waves.

b. The parameter model of Van der Meer uses data points with all levels of damage (except 0). USACE uses only test output with ‘acceptable’ levels of damage. This level is unknown.

c. USACE have drawn a lower limit design line. SAYAO (2007) presents fitted equation for this line (see Figure 28). Van der Meer fitted his line as a mean for all data points (slightly on the upper side actually).

Since Van der Meer recommends to use N_{od}=0.5 for a safe design, we fill in this value in the equation. This parameter model now also forms a curve in a 2D space (equal to the green line in Figure 33). This results in the following figure:

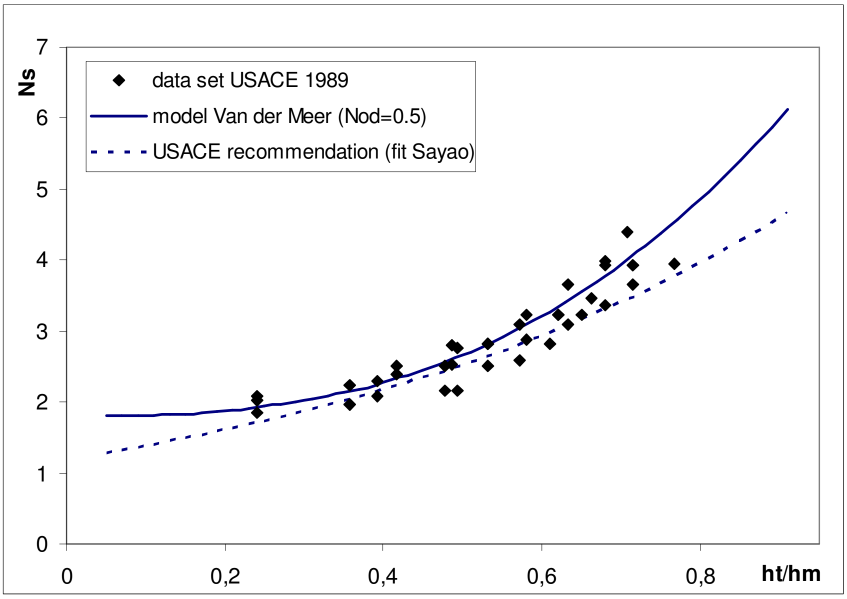

Figure 49: Comparison parameter model Van der Meer and data set USACE

Notice that the vertical axis uses regular wave height for the USACE data and USACE model. The vertical axis includes H_s for the Van der Meer model.

The resemblance of the parameter model to the data set is very good. This is unexpected, because above limitations of the comparison are mentioned. The shape of Van der Meer’s curve fits the USACE data set even better than the fit by Sayao. This is remarkable because the relation of VAN DER MEER (1998) was fit to the data set of GERDING (1993). Two questions remain:

a. Which criterion did USACE use in 1987 as ‘acceptable damage’?

b. Is H_s in Van der Meer applicable here for comparison to regular H in the USACE tests?

These influences might compensate each other. The resemblance in absolute sense might be coincidental.

A restraint is that the present parameter model is not conservative. It is better to recommend a lower limit design line. This results in larger recommended toe element diameters and has the disadvantage of possible over-dimensioning. Nonetheless, the comparison of Figure 49 seems to imply that results from monochromatic tests are comparable to test results with irregular waves. This conclusion has to be handled with great care though.

Despite the unknown criterion by USACE for "acceptable damage" we compare the data sets of USACE and Gerding in the next figure. We use the following compromises for comparison:

a. On the vertical axis Hs for irregular waves and H for regular waves, similar to Figure 49.
b. 'Acceptable damage' for USACE and N_{od} = 0.25-0.75 for Gerding. The latter classification is close to Van der Meer's design recommendation.

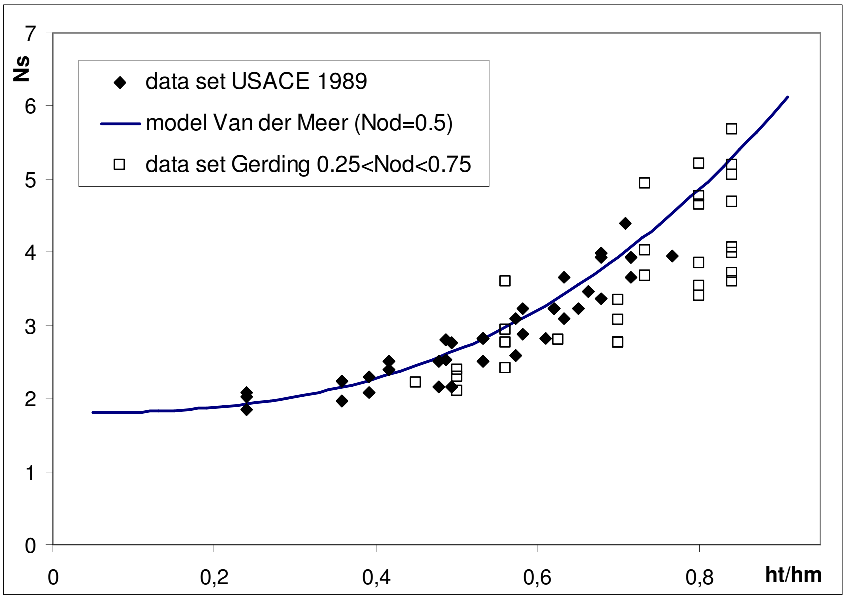

Figure 50: Comparison data of USACE (1989) and data of GERDING (1993)

The data sets seem to have similar properties with regards to geometric influences. The clouds of dots compare well. The transition zone from "low damage" to "unacceptable damage" is more stretched for irregular waves than for monochromatic waves, especially for large relative depth. As a remark may be given that this comparison certainly does not provide similar results for the data set of Docters van Leeuwen.

3.7 Conclusions

The parameter model of Van der Meer is less accurate for deeper toes, thus for larger relative depths. It is not possible to say that the model deviates more from tests with higher or lower damage.

The power curve relation between damage number and stability parameter does not seem to fit the general shape of the cloud of data points. This follows from viewing the parameter model in a different 2D figure, namely Figure 39. In this figure not stability and damage are combined, but stability and geometry. Using the power relation produces scatter for the models. Statistically is proven that even a simple straight line fits the data set better (see Figure 40 and Figure 43).

At design recommendation level N_{od} = 0.5 most data points have significantly larger damage than the model predicts (Figure 39). The model recommends a too small diameter for many cases. This is certainly not a conservative design method. Increasing or decreasing the value of N_{od} for a design recommendation does not increase the accuracy of the model due to use of the power relation between damage and stability. Probabilities are high that future design tests do not result in the predicted amount of damage. It would be safer to work with a lower limit curve, but that can lead to strong over-dimensioning.

For adequate design, the scatter around the model needs to be reduced. This applies for both the GERDING (1993) model and the VAN DER MEER (1998) model. The relations need improvement, but the most profit is to be gained in the description of damage, stability and geometry individually.

The model of Gerding fits the data set better than the model of Van der Meer and this is therefore a better design tool. Since the empirical parameter models have been fitted to the same data set, they have the same field of applicability. The Van der Meer formula is therefore not an improvement. Toe depth h_t has more influence than h_m.

For both models applies that they do not resemble the data set of Docters van Leeuwen. This is for a part of the data set caused by the chosen diameters and wave heights, because:

a. Most tests are not representative material models for possible target systems.
b. Most material models are not tested to result in considerable amounts of damage.

For other parts of the data set this is probably due to the set up of the parameter models. The data sets of Gerding and Docters van Leeuwen may different, but not as much as Figure 44 and Figure 45 suggest.

Comparison of the data set of GERDING (1993) and USACE (1987/1989) leads to a cautious conclusion that test results with regular and irregular waves have useful similarities.

Chapter 4

Hypothesis for description of toe element stability

4.1 Approach

In the previous chapters is described that the equations that are presently used for determining the required toe element diameter are not very accurate. The formula from 1998 by Van der Meer was no improvement of the formula of 1993 by Gerding. This is because the formulas have the same field of applicability and herein Gerding's formula has better resemblance with the test results.

The equations have certain similarities with test results from material models, but in general they fail to accurately describe the physical process that determines stability. The attempt of SAYAO (2007) with parameter analysis of multiple datasets did not immediately result in obvious improvements.

A more accurate model has to be made for the parameters that were varied in the previous test series. Thereafter new flume tests are useful to determine the influences of other parameters, such as slope roughness, wave period, foreshore slope angle and front slope angle. Since the existing methods do not display a promising path to improvement, the search for a better description is approached by means of a new concept, explained in section 4.4.

It is assumed in this research that progress will be made fastest if the physics are understood. Therefore the theoretic approach is chosen. First predictions are made with a hypothesis and afterwards is checked if the hypothesis resembles test data. A short discussion can be found in Appendix D about some disadvantages for the empirical approach for this research subject.[7]

In this chapter a hypothesis is developed. This will be compared to test data in Chapter 5. First two propositions for the hypothesis will be treated individually:
a. how to express the result of a test
b. applicability of a threshold of movement

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[7] A theoretical approach with testing a hypothesis is not necessarily better than an empirical approach (respectively called verifying research and explorative research in BAARDA *et al.* 2006). The practical value is equal as long as a usable tool for design with small scatter is obtained.

These are investigated separately first, to see whether they provide a sound basis to continue with the rest of the hypothesis. Therefore the structure of this chapter is as follows:

First is considered in section 4.2 how damage can be expressed. In section 4.3 the approach with a threshold of movement for toe rocks is described and evaluated. Section 4.4 is the start of assessing the influence of all parameters for the problem. Here the concept for the hypothesis is explained. Separate parts of the hypothesis are worked out in detail thereafter in the sections 4.5 and 4.6. In section 4.7 is explained how the hypothesis can be tested with the available data. Finally, section 4.8 summarizes the influences of all parameters that are included or excluded in the calculation method that is developed in this chapter.

4.2 Damage

For analysis of description of stability of toe elements, a parameter is required that indexes how much damage was done by the waves to the toe bund during the test. This parameter cannot be an indicator for the severity of this amount of damage to the system or subsystem at the same time. A certain situation may cope with more damage than another, which depends on the functions the toe structure has to perform (see section 2.2.4).

Here the interest lies in description of the process and at the moment not in severity of damage. Therefore we solely look at the rock displacements in a context of process analysis. The damage parameter needs to meet the requirement that it has the same absolute value for situations with comparable damage. This means that if we consider that two situations have the same level of damage, the value of the damage parameter needs to be equal for these situations. In Chapters 2 and 3 advantages and disadvantages were mentioned of the two previously used damage parameters, namely percentage and N_{od}.

In this section is explained that two damage parameters could be correct for analysis, depending on how and where damage starts and develops. The percentage is not one of them, but N_{od} and an adaptation thereof, N_{odB}, are.

First option: N_{od}

If, in a cross sectional plane of the breakwater, the rock on the upper most seaward edge of the bund is generally the first rock to displace, then the parameter N_{od} is a good parameter for comparing different situations. In these situations the normative forces apparently act on these particular rocks ‘on the edge’ of the bund. Damage will grow towards the armour slope, because the neighbouring rock in the cross section is the new ‘rock on the edge’. How much damage is done to the toe bund by the water motion, can in this case be interpreted as follows:

> The damage number N_{od} is the amount of elements that have actually displaced from the toe bund edge, with respect to the amount of elements that were lying on the toe bund edge before the test.

This is very well expressed by the original N_{od} parameter by Gerding[8]. For completeness, the definition in a formula for N_{od} is repeated here:

$$N_{od} \equiv \frac{N}{\left(L / D_{n50} \right)} \tag{15}$$

L is the length of the breakwater section in which the displaced rocks are counted. That N_{od} is a good parameter for comparing damage on the toe bund edge, is illustrated in the following schematic examples (Figure 51).

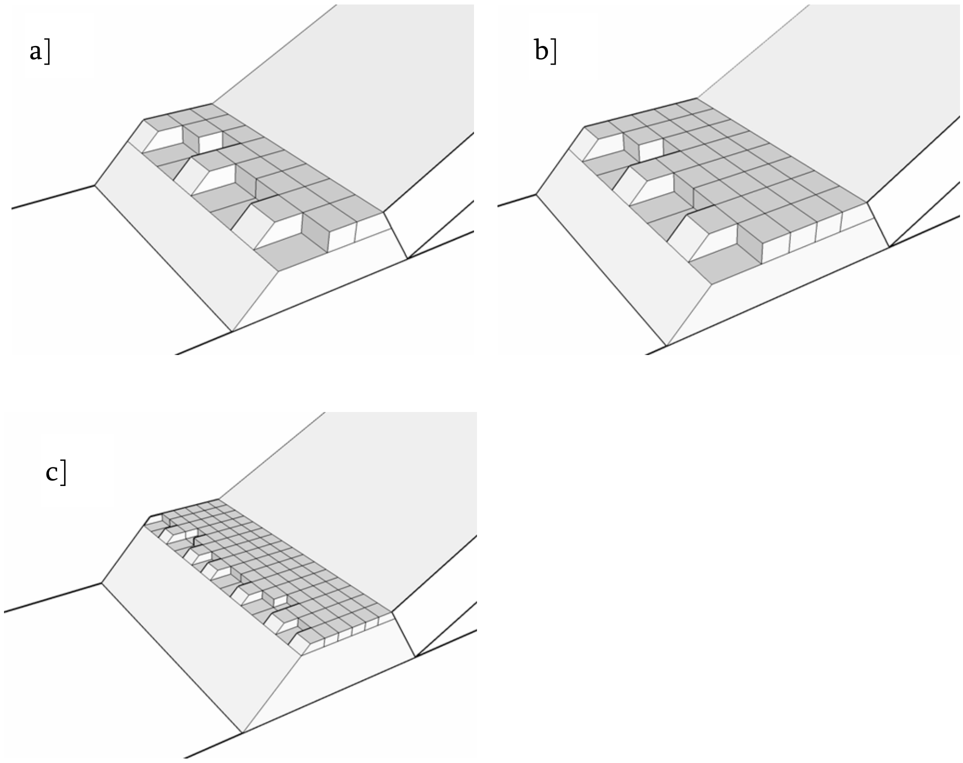


Figure 51: Comparison for damage on the edge of the bund. All cases have equal relative amounts of damage in terms of N_{od}. Here N_{od} is a correct parameter because it has the same value for similar relative damage amounts.

Case a] is a reference case. Case b] has a larger toe width B_t, but for process analysis the damage is equal. Case c] has twice the amount of displaced rocks

[8] The only difference with this interpretation is that in N_{od} the amount of displaced rocks is divided by the number of times D_{n50} fit on the toe bund edge, instead of the number of elements that fit there (see Figure 15). But this difference is only a scalar factor (of about 0.84), which has the dimension rocks-per-D_{n50} [m^{-1}]. This is the error in damage calculation of Docters van Leeuwen, who used D instead of D_{n50} for N_{od}.

compared to a], but the damage can be considered equal. The damage number is an indication of what percentage of rocks on the edge has been displaced.[9]

Second option: N_{odB}.

The process could also be different, namely that not only the rocks on the edge of the bund can displace first, but that damage can start at all rocks in the top layer of the toe bund. Then the parameter to describe the amount of damage also has to be different to account for toe width.

The amount of displaced rocks has to be described relative to the amount of rocks that are exposed to critical forces (just as in the previous case). The damage by wave action to the toe bund is interpreted as follows for this case:

> The damage number N_{odB} is the amount of elements that have actually displaced from top surface layer of the toe bund with respect to the amount of elements that were lying in this layer before the test.

In a formula, this is expressed as:

$$N_{odB} \equiv \frac{N}{\left(L/D_{n50} \cdot B_t/D_{n50} \right)} \tag{16}$$

Here N_{odB} is the damage parameter for damage on the entire toe bund top layer instead of just on the seaside edge. It is the number of displacement that accounts for toe bund width. The meaning of N_{odB} is illustrated in the following figure (Figure 52).

In the figure case a] and b] have the same absolute amount of displaced rocks, so the value of N_{od} would be the same, but it is clearly visible that the relative amount of damage is less in case b]. In case c] damage is equal to a], while the absolute number of displaced rocks is twice as high as in case c] of Figure 51.

[9] This interpretation of N_{od} does not imply that the interpretation proposed in Chapter 3 is wrong, where N_{od} was regarded as the number of rocks that displace from a cross section strip with a width of 1 D_{n50}. Both interpretations can be used simultaneously.

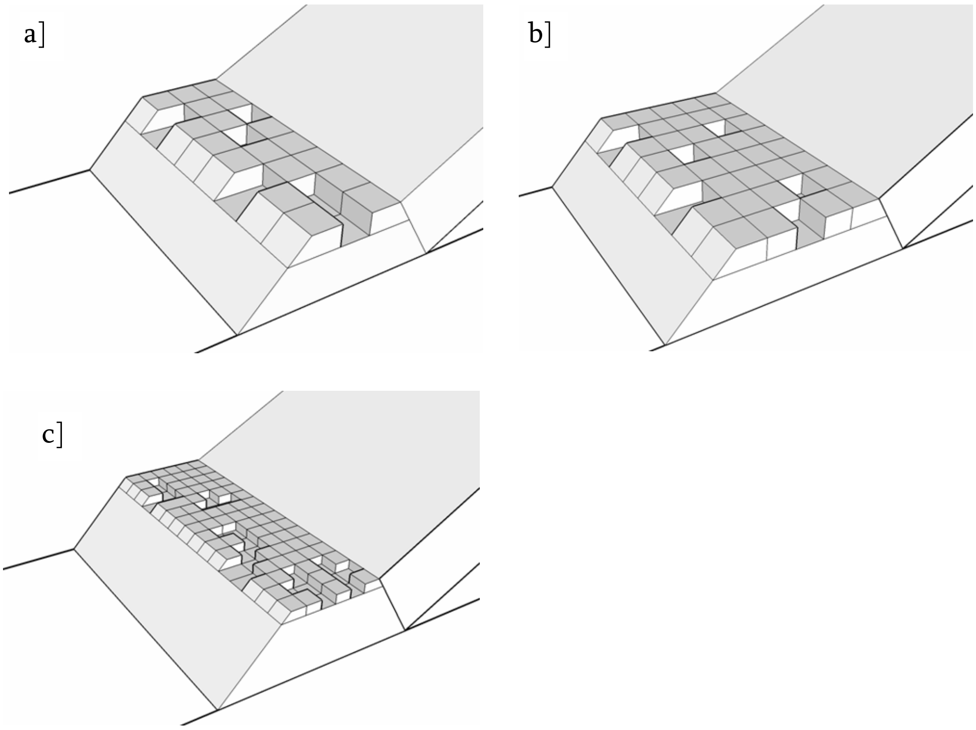


Figure 52: Comparison for damage to the top layer of the bund. Here case a] and b] do not have the same relative amount of damage. Case a] and c] do. Here N_{odB} is a correct damage number, because of the place of displacements.

The denominators of expressions in equation 16 and 15 are an indication for the amount of rocks that are in a position to displace. Thus for example N_{odB}=0.15 would mean that 15% of the rocks on the top layer of the toe bund have indeed displaced. N_{od}=0.60 would mean that 60% of the rocks on the edge of the toe bund have displaced.[10]

Which of these two damage parameters should be used, is determined by where significant damage starts, namely on the seaside edge or on the entire top surface of the toe bund. This has not been reported yet and cannot be derived presently from the data sets of Gerding and Docters van Leeuwen. Docters van Leeuwen suggests in her report that damage starts on the bund edge, but recent tests have shown that it may also be on the entire bund surface. This would be very interesting to record for further testing. Both options will be used in the evaluation.

4.3 Threshold of movement

The methods of Gerding and Van der Meer have attempted to describe the amount of damage in relation to the load conditions with a power function. The present research investigates the possibility to work with a critical value for the

[10] From this point of view, both damage parameters are unitless, since the operation is (number of rocks)/(number of rocks).

load. Above this critical value, considerable movement of elements is discernable and under it elements are generally stable. This is a different approach to the stability problem. Toe element stability has been regarded with an approach similar to armour stability, but now it is viewed with principles adapted from bed stability. First this proposition for the hypothesis is elaborated in a conceptual sense. Then its applicability is checked with the data sets.

4.3.1 Hypothesis on the threshold of movement

The threshold part of the hypothesis will be visually compared to the model of Van der Meer. The latter model consists of a stability, damage and geometry part. In the next figure the threshold-model is compared to the Van der Meer model as in Figure 31. The relation between stability and damage is very different in principle for the present hypothesis. It not increasing along a curve, but per geometry the model can be divided in a stable and an unstable part.

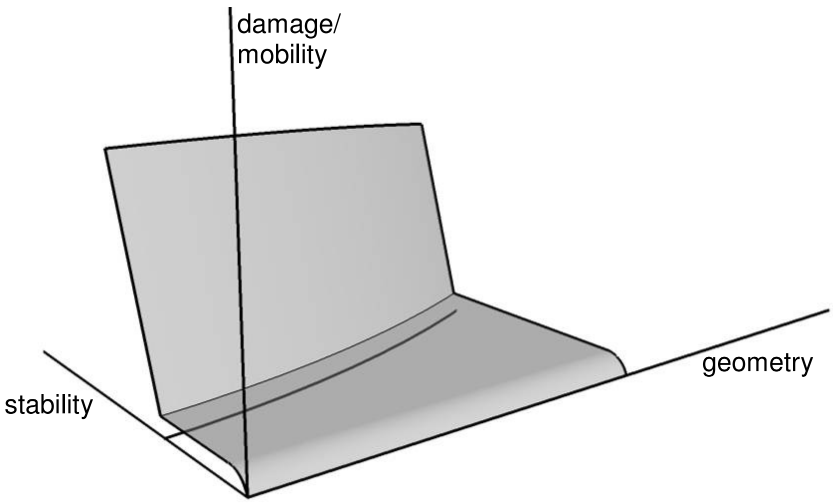


Figure 53: Conceptual adaptation to the parameter model for the relation between damage and stability. Here a threshold of movement is present per geometry, which is not the case for the models of Gerding and Van der Meer.

At this time we are not sure of the mutual influences of geometric parameters. Therefore threshold analysis cannot be performed for the entire data set at once yet. In section 3.3 and 3.4 was described that the power curve relation for stability and damage is inappropriate.

We are interested in the applicability of a *threshold of movement*[11] for this approach. First is described conceptually what to expect and thereafter this is compared to the data set.

[11] The terminology used here is 'threshold of movement' since this was also used in the research of RANCE *et al.* (1968). European literature prefers 'threshold of motion' for bed particles, but this might suggest that elements will remain in motion for some time, while

Influences of geometric parameters are yet unknown and the stability parameter with ratio of wave height and rock diameter is questionable. Therefore the starting point will be, like in section 3.3, to evaluate situations with equal geometries, which can be interpreted as a vertical cross section in Figure 53. Influences of geometric variations are ruled out. The basic figure for critical load is as follows:

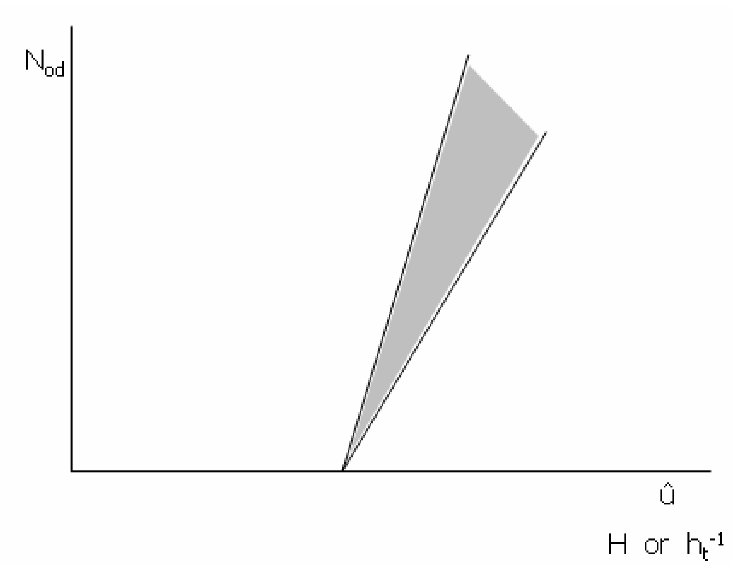


Figure 54: Basic figure for the idea of threshold of movement

For increasing the load the following should be noticeable: First the rocks are fully able to absorb the load. Some settlement of the elements may take place but this is not regarded as damage. Above a critical load, elements will move.

The measurement for load is the amplitude of local velocity û and damage is measured in terms of N_{od}. The grey plume indicates where test output should be, as a cloud of data points. The way in which damage is measured does not matter here, since we are not comparing only one geometric set-up. Therefore qualitatively the shape of the figure (whether that is a curve or line or plume of data points) will be exactly alike for using percentage, N_{od}, N_{odB} or the absolute number of displaced stones.

The measurement for load is amplitude of local velocity û, since the hypothesis is that local water motions determine the stability of the elements. This amplitude of local velocity was however not measured. Therefore we have to use another parameter for load that has been recorded, which has influence on û. An obvious choice is wave height (N_{od}- vs - H analysis), but the important thing is to use test results where all parameters are equal except one load increasing parameter.

toe elements only move, as in 'displace', over a short distance. Therefore in this report 'threshold of movement' is preferred.

We also do not know the relation between load and element size. Therefore we also have to use cases with equal toe rock diameters for now. If every parameter is equal, it is therefore also possible to use toe height z_t (or h_t^{-1}) on the horizontal axis, since we know that this increases load on the toe elements.

The conceptual figure is changed into the figure below.

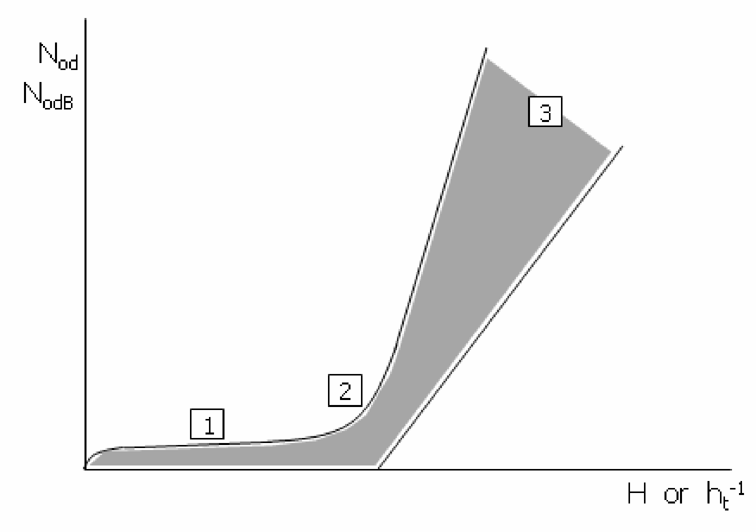


Figure 55: Modified basic threshold figure, adapted to irregularities and load influence parameter

Figure 55 is different from Figure 54 in description of the load axis and in shape of the 'plume'. The plume is the envelope of the data points that represent test results. For cases with equal geometry and equal D_{n50} it is not important which parameter is used for the damage axis[12]. The practical shape differs in three properties from the basic figure (Figure 54), namely:

1. Beneath the threshold value small amounts of damage are possible due to factors such as unstably positioned rocks, very small rocks in the grading, a single high wave in the distribution, etcetera.
2. The transition from stable to unstable is not that sharp, due to wide grading of the rock, irregular shapes of the rocks, and most importantly the irregularities of the incoming waves, of which some may be load above the threshold and some below it.
3. The plume or 'cloud of data points' is wider, again because of wave and rock randomness.

[12] This is only specifically not true for analysis with h_t as load influence and damage in 'percentage of all toe rocks'. The different geometry (higher toe) would appear to give smaller damage.

A power curve can also be fitted to the depicted plume in Figure 55. Therefore the description of Gerding is not really strange. The problem with the power curve is however:

a. Displacements above the threshold are rather unpredictable and to fit a curve here is difficult
b. Tests with high damage have a high influence on the curve fit while the interest for design is in the values at the start of damage
c. The uncertainty of the start of damage is increased by the scatter of high damage tests (see section 3.3).

The aim is to see if better resemblance is obtained with the threshold concept than with the power curve concept.

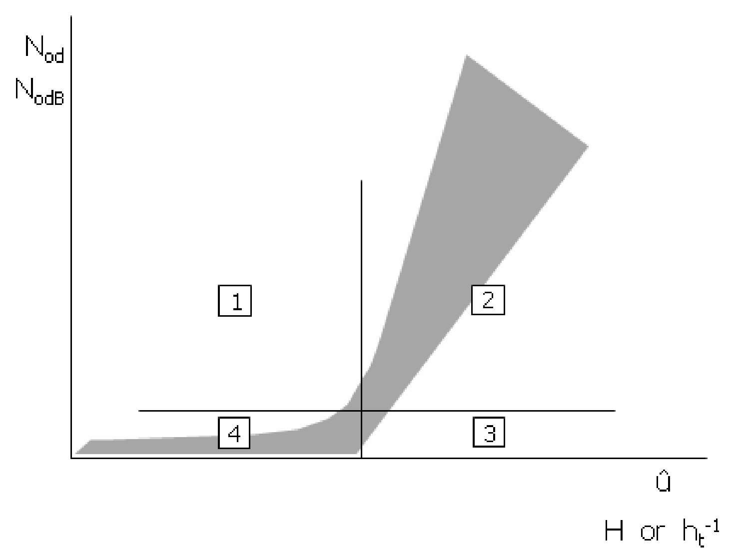


Figure 56: Threshold concept with critical load value and minimum damage.

In Figure 56 the same shape of the data cloud is depicted. The horizontal line indicates the minimum amount of damage that may be present beneath the threshold value. Significant movement of elements lies above this line. The vertical line indicates the critical value of the load, or the threshold of movement. It is more important to find the position of the horizontal and vertical lines in Figure 56 than to fit a curve through the plume.

For correct analysis, it is important that the test results lay in quadrant 2 and 4 (stable and unstable). For design practice, it is important that there are no test results in quadrant 1 and 3. Not in 1, because this would mean that higher damage is possible than one would count on in design. Not in 3, because this would mean that stable situations are possible above the described threshold. The latter would result in a too conservative design for that case with probably unnecessary high costs.

Since only one parameter may be varied at a time in this analysis, there are many different figures and only a few data points per figure instead of a full plume. A figure would appear as follows on the next page.

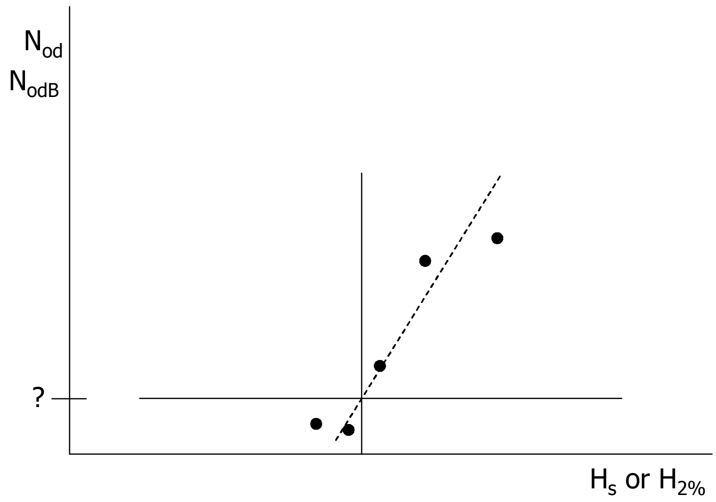


Figure 57: Fictional example indicating the uncertainty of a threshold value.

The data points in this figure could be extrapolated well down to the axis of $N_{od}=0$, similar to the approach of Shields for bed material (see Appendix A.1). The real threshold is a little higher, as can be seen in Figure 56. The problem is that because no damage values for low loads are available, for this figure it is impossible to say how many points are below the threshold and thus what is the minimum amount of damage. In other words, the shape of the plume cannot be determined very well with this amount of data points.

The extrapolation to zero can be done for the combination of load and damage, but in Figure 57 the wave height is used instead of the real load on the elements. The relation between load (û) and wave height may not be linear, and in that case the regression may not have to be linear as well. Furthermore, different wave heights are used. A different wave height may not only imply higher load but also a different water motion. And if the wave height is increased, but the steepness remains unchanged (as in Gerding's tests), the wave period also changes. So are the situations still comparable and are we actually changing only one parameter at the time then in this analysis?

For these reasons the best way to define a threshold of movement would be to keep all boundary conditions for this problem the same, to be certain that the load is exactly equal for each case. In this way we are not searching for the critical value of the load, but for the critical value of the resistance (or: 'strength') for one certain set of load conditions. Thus we may vary the element size, and obtain the following type of figure. For the principles of the approach and concept in the present research, this is the best way of qualitative threshold analysis, because we are sure there is only one influencing factor.

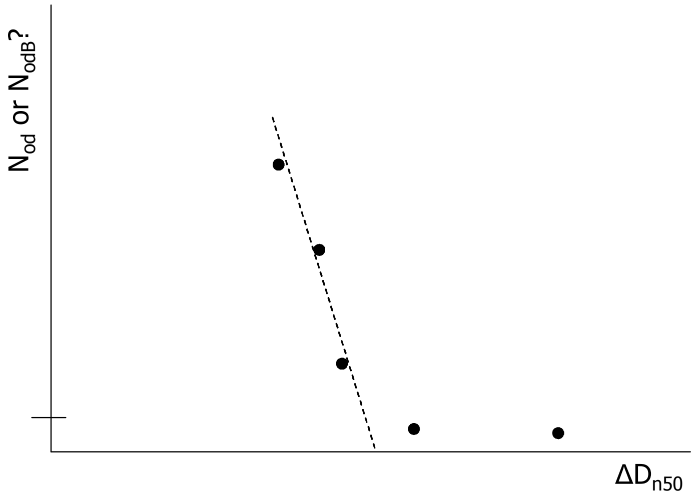


Figure 58: Threshold determination by critical diameter concept (fictional example).

In this figure different diameter sizes are present. Therefore it is also qualitatively important which damage parameter is used. For regression to zero damage, it is important to not include test results in the regression calculation that are under the threshold of movement (the two points with highest ΔD_{n50} values in Figure 58). Better yet, the threshold is best determined by manually or visually fitting a regression line. There are namely so little data points per figure that the analyst must decide for himself which points to in- or exclude. Which test results seem most valuable for the regression is hard to indicate statistically because of the small amount of data points.

4.3.2 Qualitative analysis of the threshold proposition with data sets

Now that it is known what behaviour is expected for the threshold, the data set is used to verify whether this behaviour can be retrieved in the test results. The previous section gave three ways of investigating the existence of a threshold of movement, namely:

a. Damage number versus wave height
b. Damage number versus toe depth (inversed)
c. Damage number versus element nominal diameter and density

Here a. and b. are comparisons of damage versus load increase and in c. damage versus resistance decrease.

a. Damage number versus wave height

We commence with the analysis of damage number versus wave height. One of the original figures from the report of GERDING (1993), the same one which was also reprinted in Chapter 3, namely Figure 32, is repeated once more.

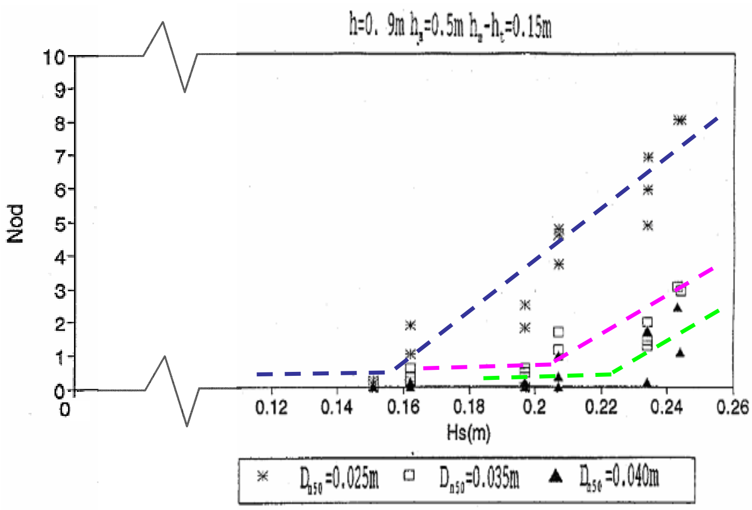


Figure 59: Example of Gerding's figures, now used as a basis for threshold determination instead of a power curve. Different lines for different diameters.

For this figure, a linear regression line can be drawn per rock diameter. The threshold of movement is where this regression line intersects with the line of insignificant damage (kink in the line), or it may be extrapolated down to zero damage (which is only a minor difference).

Hereafter two examples are given from Gerding's data set, with on the left side damage measured in N_{od} and on the right side in N_{odB}.

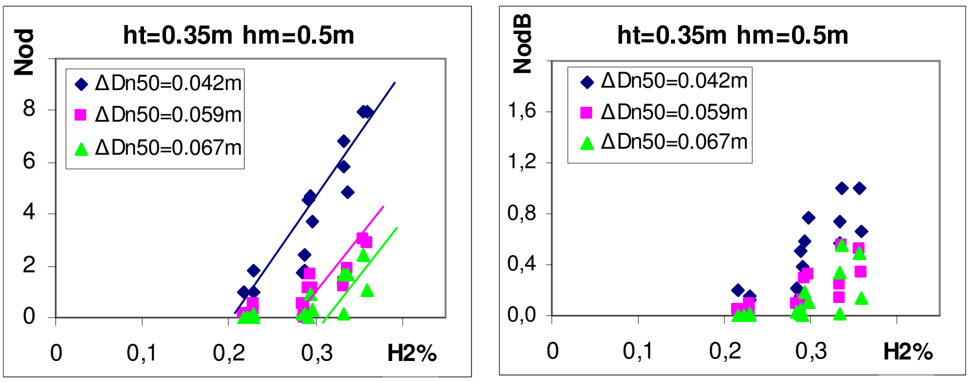


Figure 60: Examples from Gerding's test results, for similar geometries, three different rock diameters. This example concerns the same tests (and results) as in Figure 59. Damage is measured in N_{od} on the left and in N_{odB} on the right for comparison.

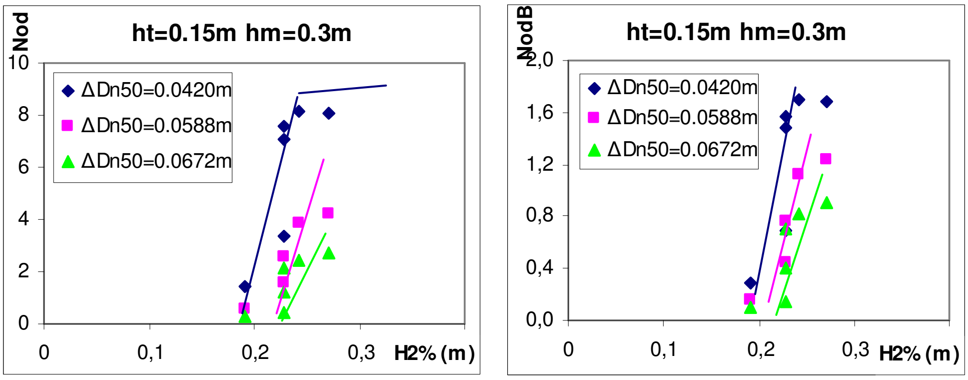


Figure 61: Example from Gerding's data set for similar geometries, only varying wave height. Damage is measured in N_{od} on the left and in N_{odB} on the right.

In these figures it is easy to draw a regression line through the data points down to zero damage. To be able to compare multiple cases at once, three different rock types are plotted together in one figure. Because of this, the influence of using a different damage parameter becomes clear. Using a different damage number does not change the shape of data cloud of one rock type. But using N_{odB} instead of N_{od} mutually shifts the data clouds per rock type vertically with respect to each other.

At a certain point a maximum value for N_{od} is reached, depending on the geometric case, because there simply is a limited number of rocks in a cross section of a toe (see Figure 61 left). It seems generally easier to fit a regression line through the comparison with N_{od} than with N_{odB}.

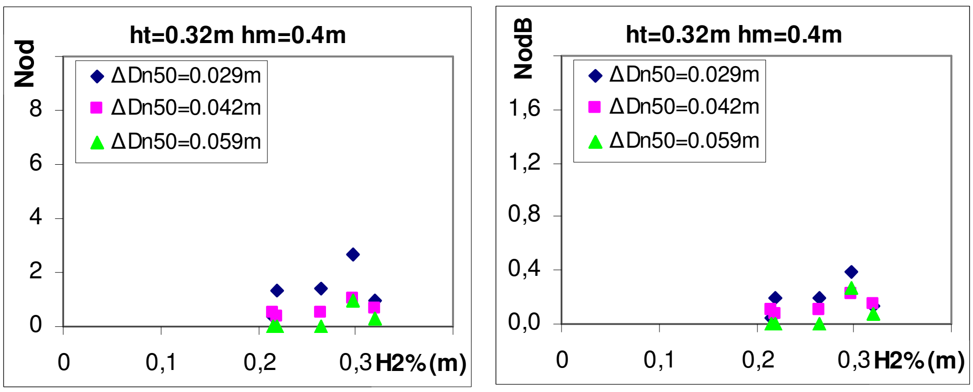


Figure 62: Example from Gerding's test results, with similar geometries and three rock sizes, no regression line to be fitted. Damage measured in N_{od} on the left and in N_{odB} on the right.

In the figure above another example from the data set of Gerding is depicted with again three different rock types for this geometric case. Here a regression line cannot be drawn. Most of the data points may be under the threshold of movement or near the transition from stable to unstable (part [1] and [2] respectively from Figure 55).

Two examples are given from the data set of DOCTERS VAN LEEUWEN (1996).

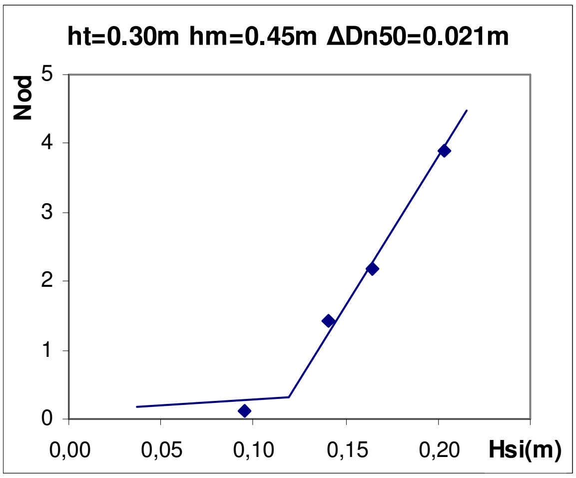


Figure 63: Example of test data of Docters van Leeuwen with similar geometries. Threshold regression well possible.

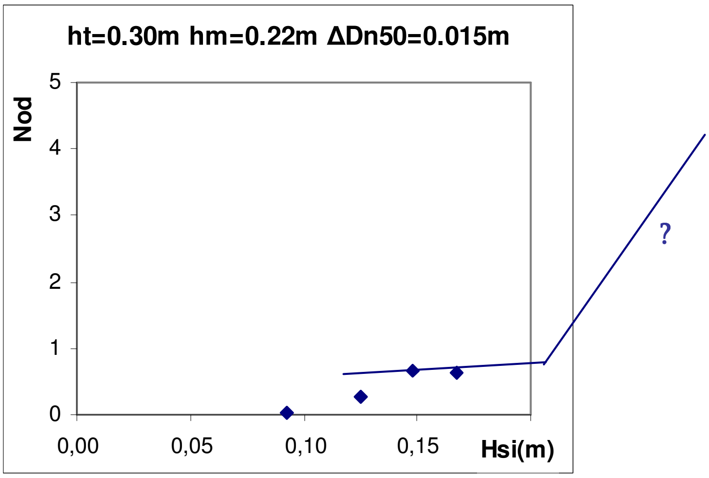


Figure 64: Example of test data of Docters van Leeuwen with similar geometries. Threshold regression not possible.

The first of the two figures above confirms the threshold hypothesis quite clearly. Most test results by Docters van Leeuwen are however like the latter of the figures, by which a threshold value cannot be determined.

The test series from Gerding and Docters van Leeuwen are set up for general stability analysis of toe bund rock. The MAST data set (as found in GERDING 1992) is a collection of commercial tests that were set up as a material model for one specific case. The following figure is an example from series 1 of the MAST data set.

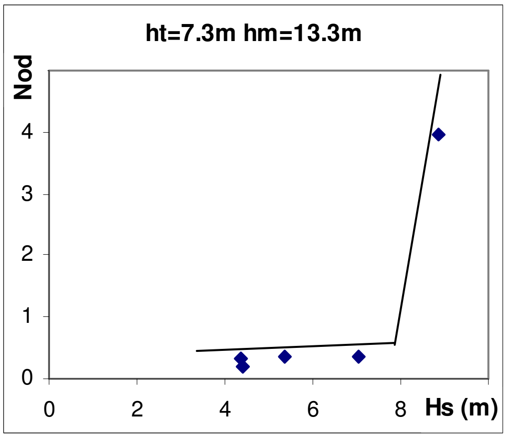


Figure 65: Example of test data from MAST data set (GERDING 1992) series 1.

The threshold hypothesis is confirmed again, showing small amounts of damage beneath the threshold of movement, and suddenly a major increase in damage. The disadvantage for this set on the other hand is that it is not possible to draw a regression line, because there is only one test result with high damage. So we know that the critical significant wave height for this geometric case is probably somewhere between 7 m and 9 m, but we cannot determine where exactly[13].

In Appendix B more figures can be found for the analysis of damage number versus wave height for the data sets of Gerding, Docters van Leeuwen and MAST.

b. Damage number versus toe depth (inversed)

Now the analysis is continued with another load increasing factor. All conditions are kept equal, including water depth, the 2% wave height at the toe and the rock diameter. The level of the top surface of the toe bund varies however, which makes the load on the bund elements different. Gerding's test series were not intended to perform this analysis and therefore there are not that many data points per comparison.

Because h_t has an inversely proportional influence on the load, the figure is 'flipped horizontally' in comparison to the analysis with varying wave height. This is shown in the following figure, where for the load influencing axis on the left h_t is used, and on the right h_t^{-1}.

[13] For this commercial test this is not important if the significant wave height that is given as a boundary condition for extreme conditions is smaller than 7m.

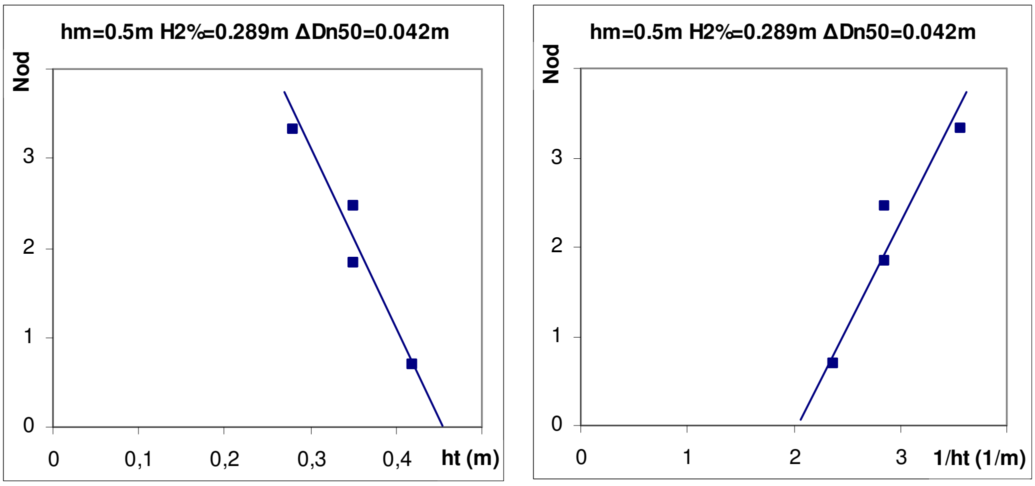


Figure 66: Example from Gerding's data set. Only the toe height varied.
Horizontal axes: left figure h_t (m) and right figure $1/h_t$ (m^{-1})

This analysis with toe depth also seems to confirm the threshold proposition. For this analysis type, the following remark is made. The threshold point in Figure 66, where the regression line crosses the horizontal axis, has a toe depth of about 0.45 m. Since the water depth is 0.50 m and the rock diameter is about 3 cm, such a toe bund would be almost impossible. This implies that with this rock size always an unstable situation is obtained. The ratio of representative wave height (here $H_{2\%}$) and water depth is $\gamma = 0.6$. In literature was found that for such conditions armour size elements should be applied for the toe, see for example section 2.1 of this report referring to British Standard (BSI 1991).

In appendix B more examples of Gerding's data set can be found.

c. Damage number versus element nominal diameter and density

In this analysis, the load conditions are identical, and the rock properties are varied. For each comparison, geometry and wave height are equal. First examples from Gerding's test series are shown.

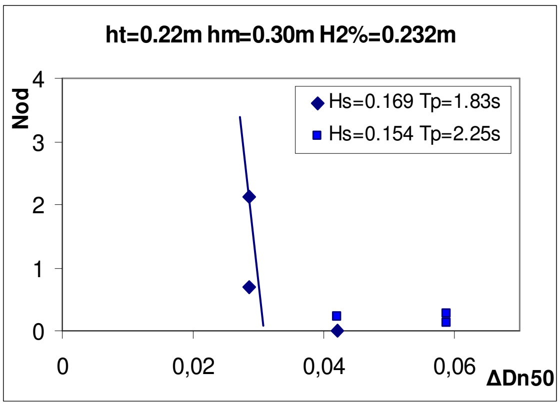
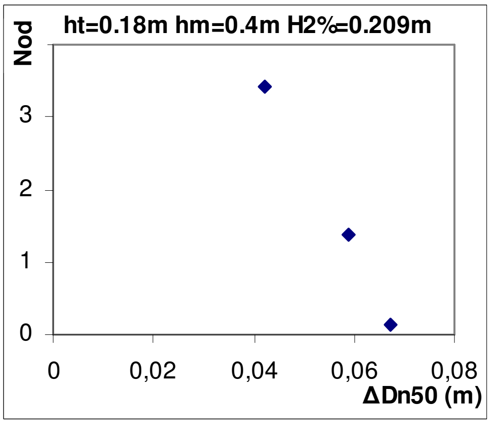


Figure 67: Example from Gerding's test results, similar load conditions, varied rock size.

Since for each configuration Gerding tested only three different rock types, test results are only possible at three values of ΔD_{n50}. This complicates finding the threshold value. Hereafter a better example is shown, comparing the use of N_{od} on the left and N_{odB} on the right again.



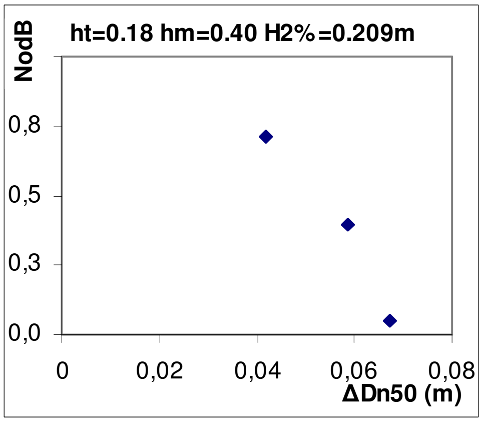
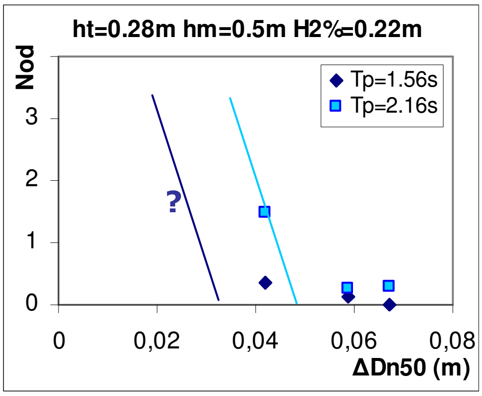


Figure 68: Example from Gerding's test results, similar load conditions, varied rock size. Damage measured in N_{od} on the left and in N_{odB} on the right.

Here it is possible to fit a threshold regression line in both cases. The use of N_{od} fits better to a straight line, but in a statistical sense drawing this conclusion based on three data points would be nonsense. Furthermore, it is unknown whether the increase in damage should be on a straight line. High damage results are required to be able to determine the combination of parameters at a threshold, but how damage develops above this value exactly, is not in the interest of designers. It cannot be concluded with the presently available information, which is the best damage parameter.



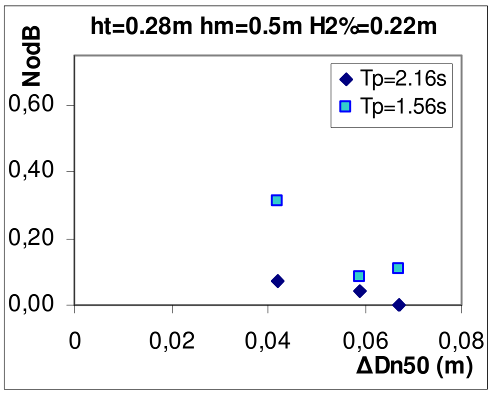


Figure 69: Example from Gerding's test results, similar load conditions, varied rock size. Damage measured in N_{od} on the left and in N_{odB} on the right. Threshold regression not possible.

Figure 69 is another example from Gerding's data set that illustrates the complication of determining the threshold. There is a data point above the threshold, but the inclination of the regression line is unknown. Besides that it is unknown if the regression line should go exactly through this data point or if this particular test result is slightly higher or lower than average. Determining the exact threshold may be difficult for many of these cases, but most test results can be categorised as stable or unstable.

The comparison of Figure 69 may confirm that the wave period has influence (difference between dark and light blue data points). Although statistically this conclusion may not be drawn firmly, it is another indication that that wave period (or steepness) has influence. Since this parameter is not present in the relations of Gerding and Van der Meer, this is also a source of scatter for those models.

Hereafter examples from the data set of Docters van Leeuwen are provided for this analysis type with damage versus rock size.

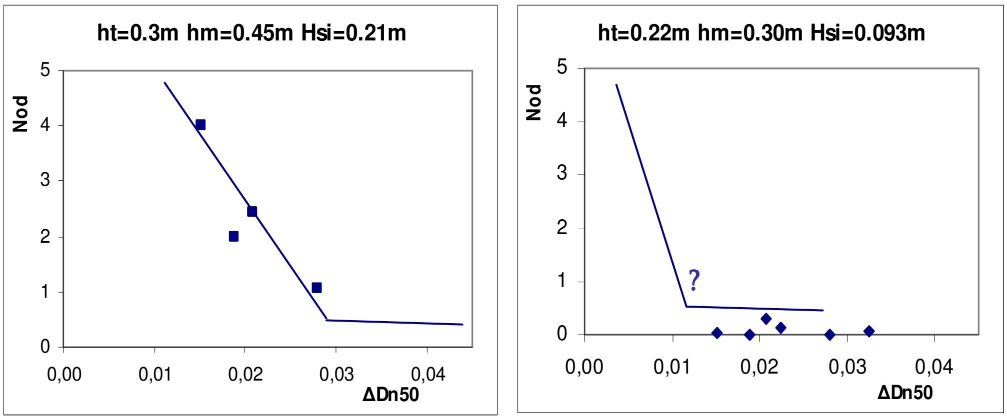


Figure 70: Examples of test results of Docters van Leeuwen for equal load conditions and varied rock properties. Threshold regression possible on the left and not possible on the right.

Again the hypothesis is plausible, but analysis is only possible for part of the data set. The advantage of the data set of Docters van Leeuwen for this analysis is that there are more variations of ΔD_{n50} compared to Gerding's series set-up. The figure on the right is still useful, despite of lack of threshold regression. More figures such as these show that the maximum values for 'low' or insignificant damage is for most cases somewhere between N_{od} =0.4 and N_{od} =0.8, or between N_{odB} = 0.1 and N_{odB} = 0.2, depending on toe width for the latter damage parameter.

For the commercial tests in the MAST data set, analysis with damage versus rock properties is not possible, because only one diameter is used in these tests.

4.3.3 Evaluation of the threshold analysis

Three ways of threshold analysis have been applied. All show to fit within the expectations of the proposition that there is a threshold of movement. This means that it is a concept that is applicable for further analysis.

The interest of a designer is solely in the value and certainty of stable elements. But we may conclude that for determination of the threshold it is important to have test results with high and low damage results. The shape of the plume above the threshold is not that important, but high damage values are necessary to find a regression line to zero damage. Many data points in the low-damage region are necessary to determine the level of insignificant (or unavoidable) damage. During testing the set-up and scheme for the series may have to be adjusted to obtain these results. The data sets of Gerding and Docters van Leeuwen are not ideal for the present research.

Analysis with damage expressed in N_{od} seems to work slightly better than with N_{odB}. But this can not be concluded firmly. Future tests have to show which parameter is better. Observations must determine where significant damage to the toe bund starts. It may be the case that damage starts on the edge first and that thereafter rocks from the entire top layer of the bund displace as the toe has flattened a bit. Altogether, with this small number of data points per analysis, statistical back-up for affirming the hypothesis is difficult.

The transition from small amounts of damage beneath the threshold of movement to considerable amounts above it, seems to lay between N_{od} = 0.4 and 0.8. This is more or less between N_{odB} = 0.1 and 0.2. Thus damage results with N_{od} < 0.4 or even up to < 0.8 in some cases can be called insignificant. Therefore it is chosen indicate damage for process analysis in the following classification:

Stable:	$N_{od} < 0.4$
Transition:	$0.4 < N_{od} < 0.8$
Unstable:	$N_{od} > 0.8$

The choice for this classification will be proven more evidently in Chapter 5, where the hypothesis is verified with data sets. Firstly the complete hypothesis is described in the following sections. If influences of the geometric and load parameters are predicted, all test data can be analyzed in one figure.

4.4 Concept for assessing the physical process of toe element stability

4.4.1 A concept with two steps

Now that has been found that the concept with threshold of movement is applicable, the interest is in determining this threshold. In previous research (see Chapter 2) external boundary conditions, such as wave height and water depth, have directly been curve-fitted to the damage results of the material model tests. One of the propositions in this research is that stability of toe elements is determined by the forces that directly act on them. Therefore the stability issue is regarded with the local motions of the water surrounding an element.

The accuracy of the description of stability can be improved by assessing the local physical process at the toe bund. Since this has not been measured in the tests of Gerding and Docters van Leeuwen, the local conditions will be calculated theoretically. This concept adds a basis of a set of theoretic descriptions to the solution of the problem. This has the advantage that this part of the hypothesis may be extrapolated outside the data set, but it must be carefully noticed that this is only true for the applicability range of the theoretical derivations used here. In the figure below, the implementation of this approach is schematically represented.

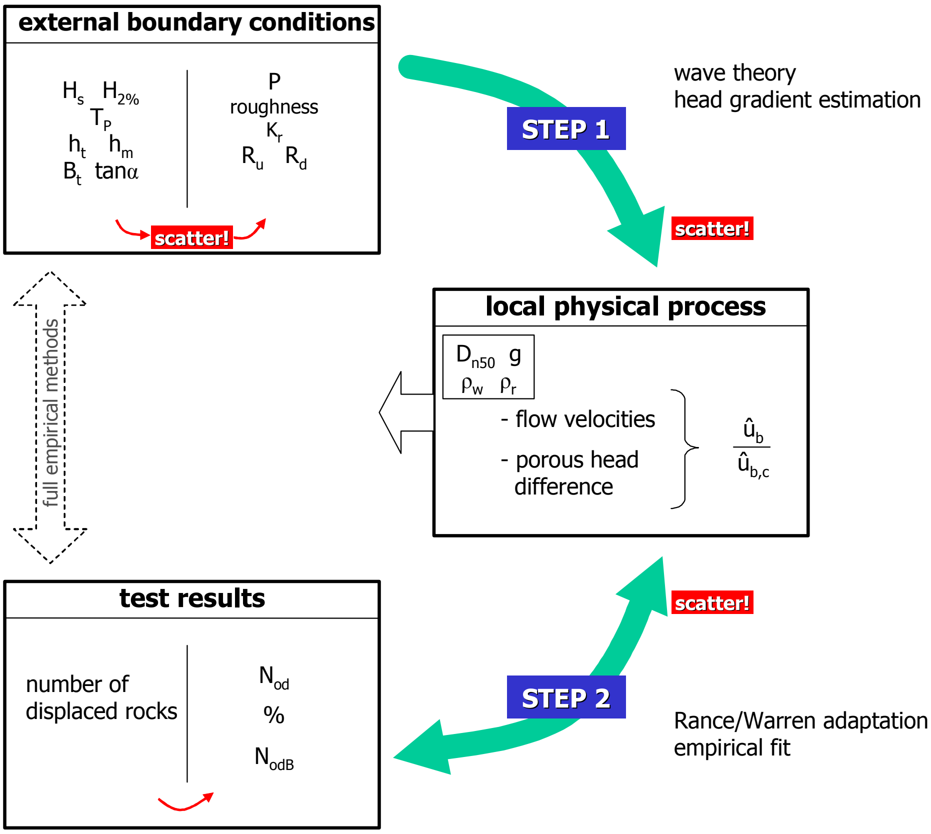


Figure 71: Schematic representation of the concept for assessment of stability of toe bund elements.

The external boundary conditions can be divided into two groups. The first group concerns parameters that are manually adjustable in advance, such as geometry and wave properties, forming the so-called set-up parameters. The second group is measurable or not adjustable exactly, such as run-up and reflection, since these are dependent on the adjustable set-up conditions. The entire group of boundary condition parameters determines the local process that takes place at the top surface of the toe bund. Making estimations for these conditions is step 1. How this is done is described in the next sections.

Under the influence of these local conditions, toe rocks will be stable or unstable. The stability problem is now regarded locally and not for the wave height or water depth directly. The capacity of the rocks to withstand the forces that are exerted by the water is determined by rock properties, expressed in D_{n50} and Δ.

When the local physical process has been assessed, this has to be coupled to whether toe elements are stable or unstable, which is step 2. The threshold of movement for grains in oscillatory water motion has been assessed before in previous research, such as that of Rance and Warren (RANCE *et al.* 1968) and Izbash (as described in SCHIERECK 2004), see also Appendix A.1. The conditions for this problem are not exactly like those researches. Therefore it is expected that the principles are similar and usable, but an adaptation has to be made and the value for the threshold has to be fitted empirically.

In previous methods the external boundary conditions were coupled to the test results in on stroke. The local conditions have not been measured in the tests. Both steps in the new method are parameter models that introduce scatter. They have to be calibrated in one comparison, because of the lack of data on the local process near the bund. It will presently not be possible to see in the evaluation of the hypothesis which of the two steps introduces most scatter or to what extent. This is a disadvantage because now we are still comparing damage to the external conditions. This method leaves much room on the other hand for further research per step.

Further advantages of this concept are as follows. The concept can in principle be applied to depth limited as well as non-depth limited conditions, as long as the local conditions at the toe bund are assessed correctly. The separate parts (step 1 and 2) can in the long run be investigated independently from each other. This makes it possible to separately check if both propositions are usable. On one hand this is coupling external boundary conditions to local conditions such as $\hat{u}_b$. On the other hand this is coupling local velocities to damage results.

Another advantage over previous methods is that tests with the result 'zero damage' (that is $N_{od}=0$) are included in the evaluation. These test results are very valuable because they indicate which situations are absolutely stable for toe elements. In the methods of Gerding and Van der Meer these test results are not usable. They fall out of the data cloud figures (Figure 35 and Figure 41) due to the division by $N_{od}^{0.15}$, which is division by zero for these test results. Valuable test output is lost, which is not the case for the method of this research.

4.4.2 Sources of scatter in the parameter model

The concept described in the previous section is a parameter model. With external boundary conditions predictions can be made whether significant damage will evolve or not. The model will not describe the test results exactly. The test results have a certain spread around their model value, the scatter. This has a number of causes, namely that the model does not describe reality into full detail and that stochastic variables cause the amount of damage to be a little different for each particular case. To categorize this, we regard the stability process for a single element in the toe bund.

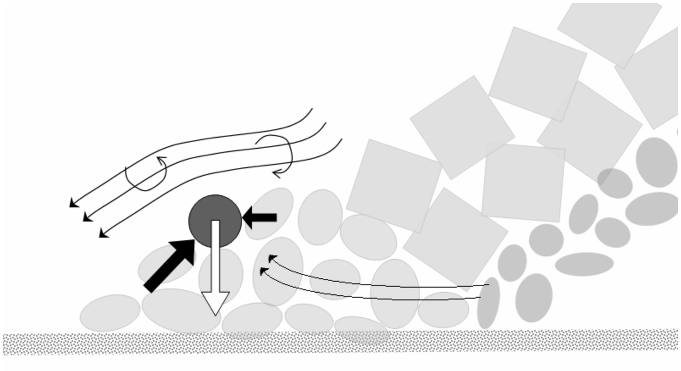


Figure 72: Schematization of forces on an element in the toe bund during down rush.

In Figure 72 the cross section of a breakwater toe with a bund is schematized. The stabilizing forces are gravity and interlocking. The destabilizing water motions can be divided into two parts, namely porous flow exiting the structure and flow over the toe bund. The latter consists of forces by the flow itself as a 'mean' velocity, by turbulence as the fluctuations around this mean velocity and by accelerations. In the approaches of Izbash and Rance/Warren these are all represented by one representative value of the velocity, namely the amplitude û.

For this concept the sources of scatter are also indicated by the red scatter patches in Figure 71. In this report they are classified in the following categories:

Category A: natural or stochastic fluctuations

a. rocks (shape, grading on site, placement)
b. waves (irregularities, waves coinciding exactly at the structure)

Category B: incompleteness of the parameter model to resemble behaviour of the target

a. step 1: description of the local process by the external boundary conditions
   - calculation of velocity of flow over the bund
   - calculation of porous outflow
   - the ratio of influence of both flow types
b. step 2: relation between rock movement and the local process
   - using the correct damage parameter
   - deciding what to call significant or insignificant movement
   - assuming that the criterion of Rance/Warren is applicable

Category A is an amount of scatter that is present even if the parameter model resembles small details well. If this scatter type was not present, it would be easier to make a better parameter model. This makes it easier to reduce scatter in category B. This is the goal of this research, namely to understand the physics.

The type of scatter of category A is also visible in Figure 55. A data plume more similar to Figure 54 is obtainable by pre-testing the structure with lower load to 'pre-displace' unstable elements in advance. Using regular waves also has this result. Under the validity of this hypothesis, a clearer threshold should be demonstrable.

A method to obtain a good design tool could be the following: first find out about the physical processes with regular waves and well sorted grains. Thereafter assess what adaptations are required for adequate design in irregular circumstances.

4.5 Step 1: Influence of boundary conditions on local water motions

4.5.1 Local conditions: contributions of destabilizing forces

The determination of local conditions at the toe bund is approached for regular waves. First the motion of the free water level is regarded. In the following figure, the phase difference between the motion on the structure slope and the motion above the toe bund is approximately ¼ T, or in other words about a quarter of a wave length. This phase shift is used here to explain the concept, but it is different for each case. It is also fluctuating for each wave in an irregular wave field.

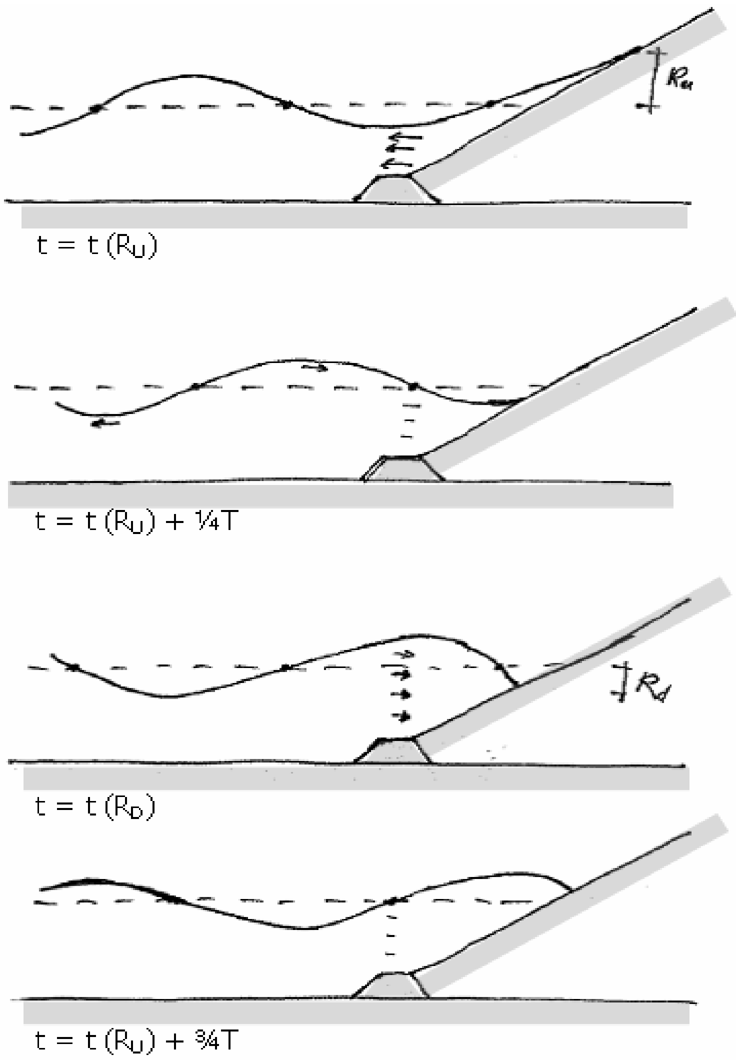


Figure 73: Sketch of water level motion on a slope and above the toe for every ¼ of a period with regular waves

In Appendix C screen shots from an arbitrary flume test movie are shown to correspond with the sketches in Figure 73.

The figure shows incoming waves and the effect of the reflected waves from the slope. The type of wave is 'surging', as it is for most breakwater cases in design conditions. At the moment the run-up is maximum, the through of the incoming wave is above the toe bund here. Then the down rush begins, until the moment of run-down, when the wave crest is above the toe.

Now the moment is sought that toe elements are least stable. To see when this is, we regard the motions of the water, divided in separate contributions. The following three contributions are distinguished:

a. Flow over the toe bund top surface due to the incoming wave
b. Flow over the toe bund top surface due to down rush or the reflected wave
c. Flow through the pores of the breakwater due to head differences

The total seaward flow over the toe bund surface during the down rush period is a combination of a. and b. For convenience the contributions to the flow are modelled as a velocity component with a sinusoidal shape. The amplitudes of the sinuses and the phase lag between them determine the amplitude of the resulting flow. They are thus important for the total load on the toe elements.

The phase difference is determined by how far (horizontally) the toe is situated from the intersection of water level and outer slope of the breakwater. This is a function of the local wave length and the angle of the slope. The following figure depicts the summation of contributions a. and b. to the velocity at the bund.

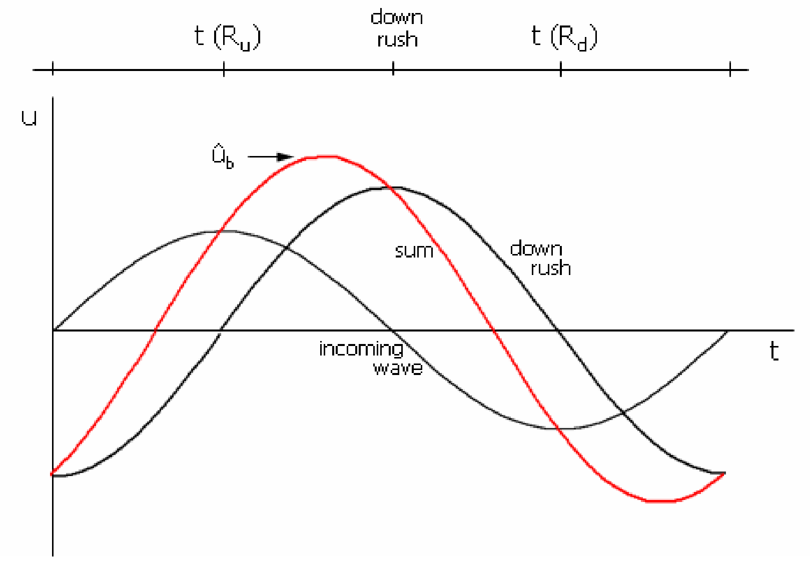


Figure 74: Summation (red line) of the contributions of incoming and wave and down rush to the velocity over the toe bund top surface.

The contribution of the incoming wave comes first, drawn here with the smaller amplitude. In the example, the toe is ¼L from the intersection of slope and water level (see also Figure 73). At the moment of run-up ($t(R_u)$), the through of the incoming wave creates seaward flow. This is the moment that down rush begins.

A quarter of a period later, the contribution of the incoming wave has passed because the water level is at mean level, but the down rush is maximum. Therefore moment of the maximum combined velocity at the toe bund is between run-up and down rush.

The value of this maximum is to be assessed by estimating the amplitudes of both contributions. The incoming wave is not symmetrical and breaking in most cases of the tests of Gerding. For the reflected wave, the up-rush is probably much smaller than the down-rush. For reasons of simplicity these effects are not included in the figure, also because this is not the interesting part of the graph. The interesting part is namely what happens between the moments of run-up and down rush, as is shown in the next figure.

For the rocks in the top layer of the bund, it does not matter whether the flow is towards the breakwater or seaward. So the resulting force due to flow over the toe bund (the red line in Figure 73) is depicted in absolute form because it is always destabilizing.

The third contribution to the forces of the water on the toe elements will now be added, which is porous flow. In Figure 72 can be seen that at the moment of run-up there is a head difference between the level on the slope and above the toe bund. In the porous structure this creates a head gradient and porous flow through the structure. This exits through the armour slope, but also through the toe. The toe rocks are therefore subjected to a force from below that lifts them up. At the moment of run-down, this gradient is reversed as water flows into the structure, pushing the elements tighter in their place. This force is depicted in Figure 75 as the purple line. The black line is the summation of forces due to porous flow and flow over the bund surface.

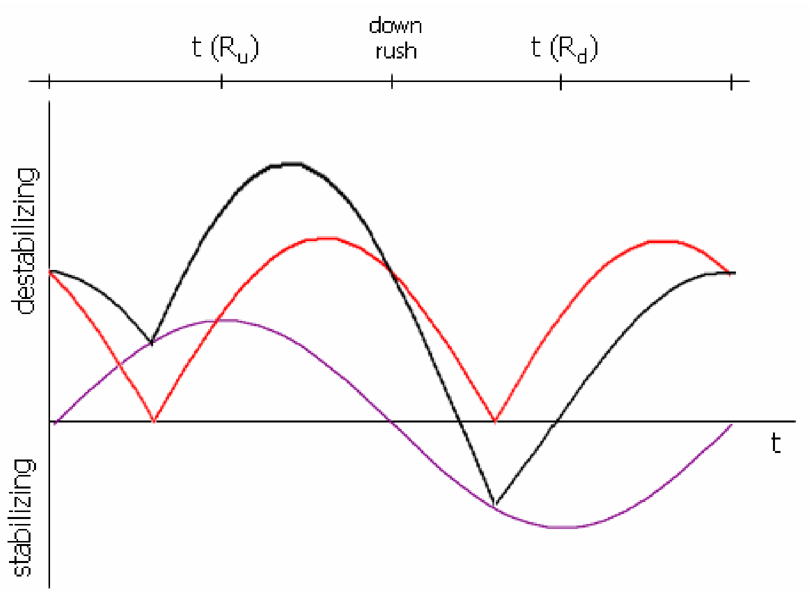


Figure 75: Resulting force (black line) on toe bund elements as summation of contributions of porous flow (purple line) and flow over the bund surface (red line).

This leads to the proposition that toe bund elements are least stable somewhere between the moment of run-up and maximum down-rush. Both the peak of the flow velocity and the peak head difference are between the moment of run-up and the moment of down rush. The toe elements are least stable in quick succession to the moment of run-up. These influences determine the local conditions and thus from hereon the focus is on estimating the amplitudes and phase lags of the basic contributions to the destabilizing forces.

There are a few restrictions.

a. The threshold hypothesis implies that rocks have a certain capacity to withstand forces exerted by the water. If the black line in Figure 75 exceeds this capacity, the rock will be unstable and start to move. This proposition is not exactly correct, because in practice stones will be rocking first and start moving thereafter. This is a transition range again.

b. This model-of-thought is valid for the conventional rubble mound breakwater type. For large-berm breakwaters, the stability of toe elements (if a bund is present at all) is very different. An important mechanism is the water rushing down the slope after the run-up moment. This is not present for breakwaters with berms close to still water level. The upper part of the wave breaks completely over the structure and sinks into it, because of which down rush may barely be present. Perhaps an adaptation could be made with reduction of the down rush contribution.

c. In ECKERT (1983) and BRUUN (1985) additional clues were found that down rush is normative for toe element displacements. On the other hand, movies of research on an Icelandic breakwater have shown displacement of elements from the lower slope during up rush, which subsequently roll down during down rush.

4.5.2 Local conditions: determining the flow velocity contributions

For wind waves in coastal and oceanic waters, the velocity field beneath a wave can be calculated quite accurately with short wave theory. Wind waves are short waves, because their length is not large with respect to the water depth. For short wave hydrodynamics it is important that the pressure under the waves cannot be modelled as hydrostatic because the flow lines are significantly curved in the vertical plane (BATTJES 2001). The vertical velocity field of short waves is thus non-uniform.

In long wave theory ($h<<L$) the water motion is modelled with a uniform distribution of vertical velocity. Numerical calculations have proven that wave motion on a breakwater slope can be modelled with reasonable resemblance using long wave theory (VAN GENT 1994). The validity of long and short wave theory is shown in the following figure.

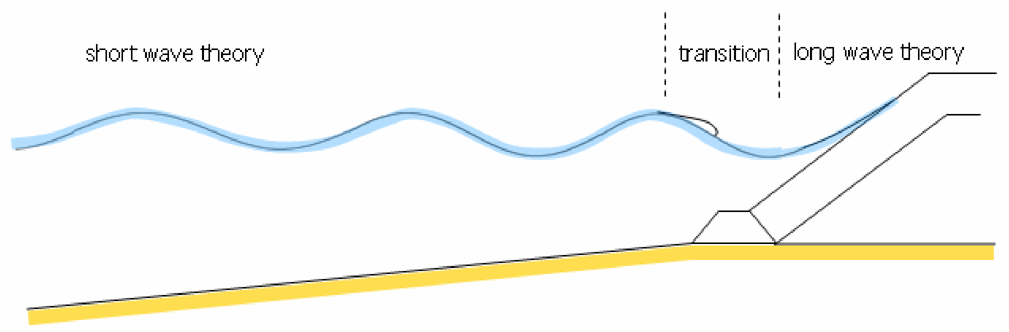


Figure 76: Validity area's for short and long wave theory for velocity calculations

The toe bund is situated in the transition zone between short and long wave behaviour. Near the breakwater structure, the short waves that are generated off shore resemble properties of long waves already quite well. The velocity distribution over the vertical for depth limited waves is quite straight.

The approach is to estimate the contribution of the incoming wave with a short wave equation and the contribution of the down rush with long wave properties. The incoming wave still generates oscillatory motion of water particles that is rotation free. The down rush is on the other hand a highly turbulent water motion with parallel streamlines.

Calculation of the velocity of the water flow over the toe bund

Two methods are regarded for the calculation of the representative amplitude of velocity over the toe bund:

a. First the method of the previous section is elaborated. Here the contributions of the incoming wave and the down rush are separated.
b. Secondly the method of HUGHES *et al.* (1995) is regarded. The reflected wave is added to the incoming wave, accounting for a phase difference.

Method 1: down rush energy

This first method is a more detailed elaboration of separately adding the contributions of the incoming wave and the down rush. The amplitude of the velocity contribution by the incoming wave (through) is calculated by

$$\hat{u}_{bi} = \omega \frac{H}{2} \frac{\cosh(k(h_m - h_t))}{\sinh(kh_m)} \tag{17}$$

By using this formula, the horizontal velocity is calculated at a height of the top surface of the toe bund in a water depth equal to h_m. There are however other possibilities. Firstly we could calculate the velocity as if the wave motion is already reduced to a water depth that is h_t instead of h_m. Secondly we could use the representative velocity near the bed in front of the structure where $z=-h_m$ instead of using $z=-h_t$.

The latter alternative would not make a significant difference. Due to the relatively long wave in relatively shallow water, the horizontal velocity distribution is nearly vertical, especially near the bed. The first alternative is not used, because in the boundaries of the research was stated that only relatively small toes are regarded, in the sense that they do not exert influence to the general hydraulic properties (no significant hydrodynamic influence). Then we would be regarding berm-like structures. Furthermore, because of the large permeability of the toe bund, a significant portion of wave is not influenced by the toe structure because the wave 'travels further' into the structure. Because of these reasons the velocity is calculated at a height of h_t in a water depth of h_m.

For the second contribution to the seaward velocity, namely down rush, a logical choice for a calculation method is not directly at hand. It would appear at first that an obvious choice might be a long wave momentum equation, adapted for use on a slope, like is used as well in VAN GENT (1994) for example for numerical calculations:

$$\frac{\partial hu}{\partial t} + \frac{\partial hu^2}{\partial x} = -gh\frac{\partial h}{\partial x} - gh\tan\alpha - \frac{1}{2}fu|u| \tag{18}$$

This implies however the use of a momentum balance, with of course accompanying boundaries and imposed conditions. This cannot be used to assess the velocity at the toe analytically. This implies the use of one balance area, with too significant simplifications. Furthermore, a mass (or volume) balance is required. In this method no statements are made about the exchange of water through the permeable interface of sea and structure front slope. In reality a water discharge is present there (in- and outflow). Approximating this analytically is quite complex.

Instead we will use a calculation method similar to VAN DER MEER *et al.* (1990), which is based on an energy balance. For a free falling particle without friction, the velocity can be calculated by

$$E_{kin} = \tfrac{1}{2}mu^2 = mgh = E_{pot} \tag{19}$$

In the approximation in this study we use the average drop in height of all water particles near the structure to calculate the contribution to the velocity by down rush. Since a highly turbulent water motion is assumed, a uniform distribution of the velocity over the vertical is applicable above the toe.

As can be seen in Figure 73 at $t=t(R_u)$, the shape of the wave on the slope is in approximation triangular, so the average height of all particles above still water level (SWL) is 1/3 R_u. The maximum drop of the free water surface on the slope is R_d. If we model the down rush as a jet-like flow that runs more or less horizontally over the bund, the average height beneath SWL of the water particles is $R_d/2$.

Therefore the approximation for the average height drop of the water particles during down rush is $R_u/3 + R_d/2$. The run-up and run-down levels are calculated as found in The Rock Manual (CIRIA *et al.* 2007), summarized in Appendix A.2.

Substituted in the energy balance, this results for the depth averaged velocity over the toe bund in

$$\hat{u}_{bdr} = \gamma_{dr}\sqrt{2g\left(R_u/3 + R_d/2\right)} \tag{20}$$

Here γ_{dr} is the down rush roughness coefficient. For a frictionless slope, this coefficient would be 1. Because of the large roughness of the armour slope, the velocity is much lower than the maximum velocity based on the energy balance. Previously namely we assumed a highly turbulent water motion. For other hydraulic properties on rough permeable slopes, such as run-up and overtopping, the reduction factor due to roughness is about 0.4. The down rush roughness reduction coefficient γ_{dr} is therefore assumed here to be in the same order. The first predictions will thus use:

$$\gamma_{dr} = 0.4 \tag{21}$$

The down rush motion is certainly not sinusoidal in time considering the up rush is not the mirrored mechanism. But in this calculation we will assess the down rush velocity as if it can be approximated by a sine function for the time period between $t(R_u)$ and maximum down rush, see Figure 74.

The resulting (or: occurring) velocity is calculated by adding the two sinusoidal contributions for incoming wave and down rush in the following way:

$$u_b = \hat{u}_b \sin(\omega t + \varphi) = \hat{u}_{bi} \sin(\omega t + \varphi_{TA}) + \hat{u}_{bdr} \sin(\omega t) \tag{22}$$

Herein φ_{TA} is the phase lag of the contribution of the down rush with respect to the contribution of the incoming wave. The φ in the resulting sine function is the phase difference between the maximum resulting velocity at the bund and the moment of run-up $t(R_u)$, see Figure 74 ($0<\varphi<\varphi_{TA}$). The calculation method for $\hat{u}_b$ and φ is quite simple, if the amplitudes and phase difference are known, see Appendix E.[14]

We are for this research interested in the amplitude $\hat{u}_b$ and not particularly in the value of the phase difference φ. To confirm the hypothesis, it is interesting to investigate whether this calculation method applies. This means that it is interesting to visually confirm in further research whether rocks generally indeed start to move between $t(R_u)$ and $t(R_u)$ + ¼T. There is however also a phase difference between the moment of maximum destabilizing forces and the moment of maximum velocity. This is the result of porous flow, see Figure 75.

The phase difference φ_{TA} between the two contributions is dependent on the horizontal distance between the toe and the point where the still water level

[14] This method applies for $\varphi_{TA} < \pi$, which is $L_{TA} < ½L$, so only quite close to the structure. This boundary prevails anyhow, because further from the structure the down rush model is not applicable.

intersects with the slope of the breakwater. This horizontal distance is indicated with L_{TA}. The subscript TA stands for length between toe and armour.

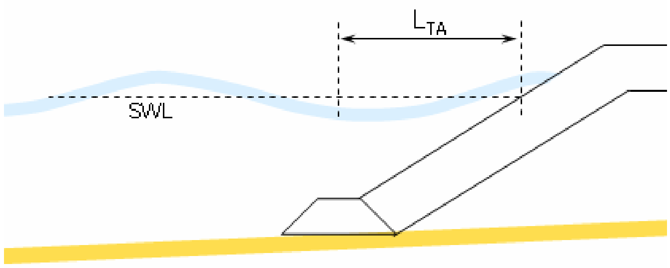


Figure 77: Definition of the horizontal distance between the middle of the toe bund and the intersection of the still water level with the slope of the structure.

The phase difference φ_{TA} can now be calculated by

$$\varphi_{TA} = kx = 2\pi \cdot \frac{L_{TA}}{L} \tag{23}$$

in which L is the local wave length (still using regular wave approach here) which is iteratively calculated by

$$L = L_0 \cdot \tanh\left(\frac{2\pi \cdot h_m}{L}\right) \tag{24}$$

This local wave length was also used to calculate k and ω in equation 17. L_{TA} can be assessed easily for each separate design project, but for quick calculations on the data set of Gerding (which is described in the next chapter) a parametric formula comes in handy. Therefore L_{TA} is calculated by

$$L_{TA} = {}^{B_t}\!/_{2} + \frac{h_t}{\tan\alpha} \tag{25}$$

Now all parameters are known in order to be able to calculate $\hat{u}_b$ in equation 22. This concludes the first method.

Method 2: reflection (Hughes)

The second method for calculating water velocities is that of HUGHES *et al.* (1995). This method also uses the linear wave theory approximation. An effect for the reflected wave is added, based on the reflection coefficient K_r. The reflection coefficient was implicitly measured in the tests of Gerding, because Gerding used a reflection compensating wave generator. The reflection coefficients have however not been recorded in the report. The reflection coefficient must therefore be calculated. This can be done with the formula given in HUGHES *et al.* (1995) itself or with more recent formulae like in VAN DER MEER *et al.* (2006) or DEKKER *et al.* (2007), see Appendix A.3.

For this Hughes-reflection method the velocity at the toe bund will be calculated for a water depth h_m and at a height h_t. This is the same as for the down rush energy method. This Hughes-reflection method has the advantage that it is based on adding wave components in an irregular wave field. For irregular waves, the root-mean-squared velocity is calculated by adding all components of the individual waves. The following form is used for a single wave component:

$$u_{b,rms}^2 = \left(\left(\frac{gk}{\omega} \right)^2 \frac{\left(H/2 \right)^2}{2} \frac{\cosh^2 k(h_m - h_t)}{\cosh^2 kh_m} \right) \cdot \left(1 - 2K_r \cos(2kx + \theta) + K_r^2 \right) \tag{26}$$

in which x is the distance relative to the toe. In the tests of Hughes, the armour slope simply ends at the bed because there is no toe bund. Therefore "the toe" may be regarded as the intersection of the bed with the armour front slope line (section 2.2.1). For the analysis in this research, x is thus 0. The calculated squared velocity $u_{b,rms}^2$ can be regarded as the variance of the horizontal velocity time series.

In the formula, θ is the reflection phase angle. The implementation of the part of the formula that includes θ implies that there are nodes and antinodes for the summation of incoming and reflected wave. The location of the toe bund, relative to the breakwater at SWL and the length of the wave, has therefore influence on the magnitude of the load on the toe elements. This is rather similar to the use of φ_{TA} in the down rush energy method. The reflection phase angle θ is calculated in HUGHES *et al.* (1995) by the empirical formula

$$\theta = 4\pi(1 - 2.11\xi_h) \tag{27}$$

in which θ is given in radians. The parameter ξ_h is of the same form of an inverted Iribarren number except water depth replaces wave height and the factor 2π is not present. This parameter is given by

$$\xi_h = \frac{\sqrt{\dfrac{h_m}{gT^2}}}{\tan\alpha} \tag{28}$$

In this second method the phase difference of incoming and reflected wave apparently depends on the front slope angle and the water depth, just like it does for the first method.

The paper by HUGHES *et al.* (1995) provides a formula to calculate K_r. This formula is:

$$K_r = \frac{0.1415}{0.1415 + \xi_h^{0.804}} \tag{29}$$

All parameters are now known to be able to calculate $u_{b,rms}$. In the paper of HUGHES (1992) is described that for assessing the maximum velocities, H_{m0} (≈ $H_{1/3}$ ≈ H_s) and T_p are used to calculate u_{rms} for a single wave component. Then the amplitude of the velocity $\hat{u}_b$ can be calculated by

$$\hat{u}_b = 2 \cdot u_{b,rms} \tag{30}$$

for the amplitude of velocity corresponding to an H_s wave.

Calculation of porous flow

Now that the velocity of flow at the toe bund has been evaluated, we proceed with the flow through the pores of the toe. As was said, a destabilizing force on the toe rocks exists due to a head gradient, see Figure 73 at $t=t(R_u)$. This head gradient is a force that creates a flow through the structure. The flow exerts force on the rocks in the structure. Rocks that are inside the structure, are held in place by the surrounding rocks, except for the rocks on an edge.

If there was no outflow through the armour layer, the piezometric head gradient would be more or less constant over the slope and through the toe. Then, in this example where $\varphi_{TA} \approx \pi/2$ (see Figure 78), the average gradient i would be

$$i = \frac{\Delta h}{\Delta x} = \frac{H/2 + R_u}{L_{TA} + R_u/\tan\alpha} \tag{31}$$

in which here L_{TA} is thus approximately L/4. R_u is calculated according to the formula in the Rock Manual (CIRIA 2007), see Appendix A.2. Notice that here $R_{u2\%}/H_s$ is used, and that this may have to be adapted for calculations if stability of toe rocks in regular waves is tested.

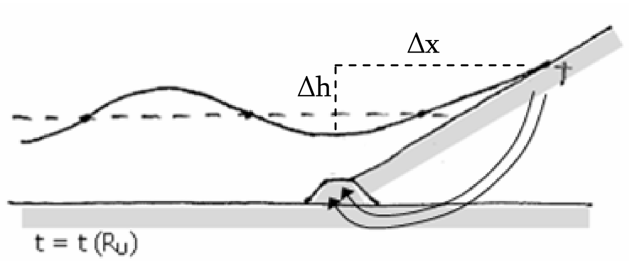


Figure 78: A head gradient between the water in the structure and in the toe induces porous flow.

It is more realistic that the gradient decreases towards the toe bund, because of outflow through the armour layer. From this point of view immediately becomes clear that the deeper the toe is, the smaller the up-lifting force due to porous flow.

If the through of the incoming wave is not above the toe at $t(R_u)$, the gradient is smaller. In an irregular wave field this is a stochastic variable. Therefore this calculation method for the head gradient is regarded as an upper bound from hereon in this report, so that i in equation 31 is i_{max}.

At the moment of run-down ($t(R_d)$) in Figure 73) the inflow into the structure stabilizes the toe rocks. This is a different mechanism than outflow. It is evident that the formula of the porous flow force is not quite sinusoidal, opposed to its presentation in Figure 75 (purple line). But this is no problem since the value of inflow is not of interest for stability of the toe elements in this study's model.

4.6 Step 2: Influence of local water motions on stability of rocks

In section 4.5 the influences of external boundary condition parameters on the local physical process have been determined (step 1). The second step is to find a relation between those local physics and the stability of rocks in the toe bund.

Previously was stated that the approach of this research is to see if existing relations for rocks in oscillatory water motion are applicable for this problem. Among available relations for rocks are those of Izbash (as found in SCHIERECK 2004) and Rance/Warren (RANCE *et al.* 1968).

The criterion of Rance/Warren is based on tests with monochromatic oscillatory flow over a horizontal bed. The concept of this research is that flow due to the waves over the toe bund is liable to similar effects as in the oscillatory flow over a bed. As was mentioned previously in section 4.4.2, the oscillatory water motion has multiple properties, namely the flow itself, as an instantaneous 'mean' velocity, turbulence as the fluctuations around this mean velocity, and accelerations. In the relation of Rance/Warren these are all represented by the amplitude of the periodic velocity û, see Appendix A.1. The relation of Izbash uses the local velocity near an element.

The flow forces on a rock in the toe are different from the forces on a rock in a bed. Porous outflow also has to be taken into account. Description of the forces due to flow over the bund is similar to the approach found in SCHIERECK 2004. A theoretic basis is created for a stability parameter by using three phenomena's:

a. Drag
b. Lift
c. Shear

The extra destabilizing force for toe rocks is:

d. Porous outflow

If the element is in equilibrium, counteracting forces stabilize the element. These forces are:

a. Weight (due to gravity)
b. Interaction forces with other elements

The forces drag, lift and shear are all in the form:

$$F = C\rho_w u^2 A \tag{32}$$

in which C is a coefficient, with C_D for drag, C_L for lift and C_S for shear. A is the surface on which the force acts, which is proportional to $(D_{n50})^2$.

Since the flow is highly turbulent, the shear component will be very small. Shear is neglected from hereon, although this does not matter for the elaboration in this section. The drag and lift forces are thus proportional to:

$$F = C\rho_w u^2 D_{n50}^2 \tag{33}$$

A significant difference with bed stability is that a porous flow exits the toe bund. On the boundary of the porous layer, the flow exits perpendicularly to the interface with 'free' water. Therefore the porous flow force to the toe elements in the top layer of the bund is modelled as vertical. The porous flow force on a grain per unit of volume is described in SCHIERECK (2004) and given by

$$F_{PF} = \rho_w g i \tag{34}$$

The subscript PF indicates that this concerns the force due to porous flow.[15] The local gradient in piezometric head is given by i. For the force on a rock, we use the volume of a median rock, which is D_{n50}^3 by definition. This results in

$$F_{PF} = C_{PF} \rho_w g i D_{n50}^3 \tag{35}$$

Herein C_{PF} is a coefficient that accounts for a number of factors:

a. The shape and the orientation of the rock
b. The flow force has to act on the water that surrounds the rock as well before the rock will move
c. In the previous section was found that the calculation of the gradient i was an upper bound. Empirically fitting coefficient C_{PF} will include this effect as well.

In summary, the forces acting on a single rock in the top layer of the toe bund are depicted below.

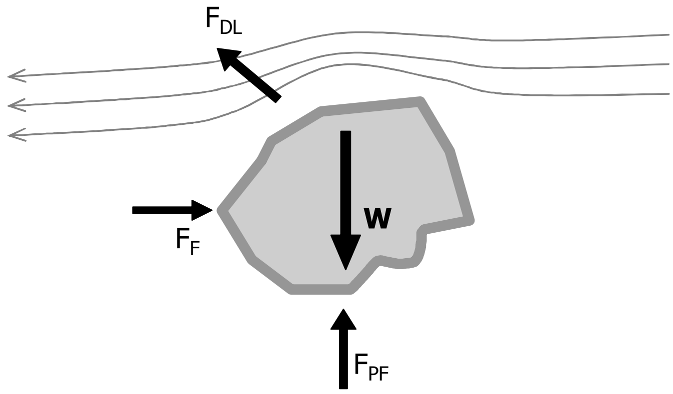


Figure 79: Forces on a rock in the top layer of a toe bund

In this figure, F_{DL} is the vector summation of the forces by drag and lift. F_F is the reaction force of the neighbouring rock, which is depends on the (submerged!) weight by a friction factor as $F_F=f \cdot W$.

The equilibrium of forces will now be regarded in the direction of initial motion (as it exists just before the motion of a rock). This means that for each force vector

[15] The derivation of the force on a grain in a sand layer was based on the linear relation of Darcy. For rocks the estimation of the force might be improved by using a Forchheimer-type relation. This is level of detail is not incorporated in this report because of the uncertainties surrounding the estimation of the (temporary) head gradient.

a sine or cosine contribution is included. This sine or cosine factor is from hereon regarded included in the coefficients. The equilibrium then reads:[16]

$$C_W \cdot W = C_{DL} F_{DL} + C_{PF} F_{PF}$$

$$C_W \left(\rho_s - \rho_w\right) g D_{n50}^3 - C_{PF} \rho_w g i D_{n50}^3 = C_{DL} \rho_w \hat{u}_b^2 D_{n50}^2$$

$$\frac{\hat{u}_b^2}{\left(\Delta - C_{PF} \cdot i\right) g D_{n50}} = C \tag{36}$$

From this is concluded that the principle of forces works rather similar to the Izbash-type stability parameter ($u^2/\Delta gd$, SCHIERECK 2004). For this problem, the amplitude of velocity is used, indicated by the circumflex accent. Therefore equation 36 is addressed in this report as the “adapted Izbash criterion”. The extra porous flow term ($C_{PF}i$) can be regarded as a reduction on the relative density of the rock. In the calculations with this adapted criterion the same value for constant C is used as in Izbash, namely 1.7, see Appendix A.1

The derivation that is described above is lend force by FRANCALANCI *et al.* (2008). Here an adaptation to the Shields number is described for uniform flow over a bed of sand with vertical seepage. The Shields number is based on shear stress on the bed due to uniform flow ($\psi=\tau/(\rho_s-\rho_w)gd$, SCHIERECK 2004). In FRANCALANCI *et al.* (2008) a dimensionless parameter Nh=1+i is derived for the non-hydrostatic conditions caused by the outflow (seepage). An adaptation is made for the Shields number.

The derivation and practical implementation are quite similar to the derivation for equation 36, see Appendix A.4. The parameter Nh characterizes the reduction of buoyancy by the head gradient. In the research by Francalanci is concluded that the generalization of the Shields number is able to account for non-hydrostatic (seepage) conditions. There is resemblance between experiments and numerical calculations in which the new parameter is used.

A remark in the paper is given that the derivation is valid for grains within the bed and not exactly on the bed surface. Extra remarks are given in this report that the Francalanci paper concerns sand and uniform flow and this report concerns rocks and oscillatory flow. Furthermore, in the experiments of Francalanci, the head gradient is known and in this report the head gradient is approximated by an upper bound. Nevertheless, we may conclude from the results by Francalanci that porous in- and outflow (i.e. seepage) have indeed effect on the stability of grains and that the effect is dependent on the head gradient.

The same adaptation of equation 36 is now applied to the criterion of Rance/Warren (see Appendix A.1). This is a reduction to the relative density. For

[16] From other fields of rock equilibrium study, the porous flow term can be recognised similar to a Morrison-type term for inertia.

the sake of completeness, the subscript c is added to the orbital velocity. This indicates that this is a critical value for the velocity.

$$(\hat{u}_{bc})^{2.5} = 0.46\sqrt{T} \cdot ((\Delta - C_{PF} \cdot i)g)^{1.5} \cdot D_{n50} \tag{37}$$

Izbash or Rance/Warren

The research of DOCTERS VAN LEEUWEN (1996) has shown that there is a large difference in results for calculating with the criterion of Rance/Warren or the criterion of Sleath. For the calculations further on in this report, use of the adapted criterion of Rance/Warren is favoured over the adapted criterion of Izbash. The reasons for this are the following:

a. Docters van Leeuwen presented a rough impetus to a theoretical approach. The best results were obtained with the Rance/Warren criterion.
b. It is unclear how the diameter should be defined exactly in the original Izbash criterion and this criterion is not meant for use in oscillatory conditions.
c. The criterion for displacement in the analysis of Rance and Warren is more similar to the criterion used in the present analysis, namely the movement of only a few rocks over a short distance.
d. The tests by Rance and Warren are specifically set-up for coastal conditions. The tests simulate large grains, periods in the range T= 5-15 s and a semi orbit range of 0-4.5 m.

The expectation is that better results will be obtained with the (adapted) criterion of Rance/Warren than with the adapted criterion of Izbash.

Even though the influence of porous flow is included, there still is a different situation for the toe bund than in the tests of Rance and Warren. They studied movement of elements in a bed, which is different from elements on the edge or in top surface of a toe bund. An extra adaptation may be required, to compensate for the small number of rocks in a toe bund that even could become in motion. In fact, the proposition that a threshold approach is applicable for toe elements also makes or breaks the applicability of equation 37.

4.7 Predictions and how to evaluate the hypothesis

In section 4.5 a method is described to calculate the amplitude of velocities at the toe bund. In section 4.6 a method is described to calculate the critical velocity for movement of the toe elements. These are step 1 and step 2 respectively from Figure 71.

If for a certain test the occurring velocity ($\hat{u}_b$ in equation 22) exceeds the critical velocity ($\hat{u}_{bc}$ in equation 37), we should have a test with the result 'movement'. Based on the threshold-proposition, the test results are divided in two groups, namely 'movement' and 'no movement'. This means that a low damage level is chosen. Above this level, damage is called significant. Under this level, damage is called insignificant.

The analysis of above-mentioned predictions is presented in the next chapter of this report.

4.8 Influence of the parameters

In this hypothesis is accounted for the influence of most of the governing parameters. There is a number of parameters that may also have influence. In section 4.8.1 the parameters are summarized that are included. Thereafter in section 4.8.2 potential factors of influence are summed up that the new model does not take into account.

4.8.1 Included parameters

Wave height – It is evident that a higher wave height increases load on the toe elements. The wave height is present in the equations for the contribution to the velocity by the incoming wave. Furthermore the wave height determines the run-up level. The run-up in its turn determines the down rush and the porous outflow in the toe. Most calculations are based on a regular wave height, whereas in the tests of Gerding (and in reality!) the waves are irregular. The wave height parameter is by this model categorized in the same class as water depth, toe height, slope angle etcetera. These parameters all play a role in the local conditions at the toe bund. The wave height is therefore not present anymore in the stability parameter.

Water depth – The water depth in front of the breakwater is present in the velocity contribution formula for the incoming wave, but not in that for the down rush. This implies that the water depth in front of the breakwater does not influence the effect of the returning wave on the toe elements (the toe depth does). The water depth however determines the wave height for depth limited conditions, which conditions are present in most of Gerding's tests. The effect of limiting water depth is implicitly included in the local wave height at the structure.

Wave period – The wave period was not present in previous methods. In the model of this research, the wave period influences many parameters that contribute to the water velocities at the toe bund. One for example is expressed in the angular velocity. Furthermore, the wave period influences the steepness and thus the Iribarren number. This is important for the motion of the wave on the slope of the structure, and implicitly the run-up. Together with the water depth, the period determines the wave length and thus (by the phase lag) the addition of the influence of the reflected wave on the velocity at the toe bund.

Toe depth – This parameter is included in the formula for $\hat{u}_{bi}$ because the velocity is calculated at the toe bund top level (at $z=-h_t$) in the water depth h_m. The toe depth also is of influence to the phase lag of the reflected wave. The head gradient that induces the porous outflow through the toe is strongly dependent on the toe depth. Furthermore toe depth also influences the down rush velocity, since the energy by down rush flow is divided over a larger cross section if the toe is deeper. A discrepancy is that in the formula for the velocity contribution by down rush, h_t is not present. The analysis in section 4.3.2 clearly shows that a higher toe level is equivalent to a higher load on the toe elements. This may thus very well be a source of scatter in the model. It may be solved in further research by including the estimating the deepness of the toe (=h_t) relative to the extent of the down rush, perhaps indicated in relation to the run down level R_d.

Toe width – The width of the toe is included in the calculation of the phase lag of the reflected wave. This is however a minor influence. In the present model, the width of the toe does not influence the stability, in the sense that for example a wider toe can cope with more load. This is analogous to the findings of Gerding on this matter. The width of the toe can however be important for the description of damage, if future tests show that toe damage may develop on the entire top surface of the toe. Then B_t is of influence in comparing damage and determining the threshold of movement or indicating a significant level of damage, see section 4.2.

Toe rocks – The properties of the toe rocks, that is to say nominal diameter and density, mainly determine how much load the rocks can handle. This is represented in the adapted Rance/Warren criterion.

Structure slope angle – The angle of the front slope of the breakwater is included in the calculation of the run-up, the phase lag and the head gradient that influences the porous flow. Therefore the slope angle has a significant influence in the present model, where it was not included at all in the models of Van der Meer and Gerding. The influence of the slope angle can however not be verified, because it was not varied in any test series. Comparing different test series only for the influence of the slope angle may provide large scatter. This happens for example in SAYAO (2007) where α is included in ξ. Per test series there are also many other parametrical differences. Verification of the influence of the slope angle therefore has to take place in future testing.

4.8.2 Excluded parameters

In this section variables are mentioned that are not included in the present model. These may however have influence, of which the extent is not assessed.

Breaker index – The breaker index γ (H/h) is not explicitly included in the equations. This was not necessary, since both H and h_m are separately measurable and separately included in the equations. This parameter may still have influence, since a large value of γ indicates that the largest waves in the incoming wave field are breaking (usually the spilling-type), which influences the water motion and limits the applicability of linear wave theory. This influence is probably not large, since these waves with larger periods in relatively shallow water have a quasi-uniform profile (vertically) for the horizontal water velocities.

Permeability – The permeability of the armour layer and the toe bund as well have a large influence. It plays a role in for example the run-up. This part is included in the model, because a formula is used that is specifically adapted for permeable structures. The permeability also influences the noticeable effect of the head difference in the toe bund, because of outflow through the armour influences C_{PF}. In the same category roughness and thickness of the layers can be mentioned, which influence down rush velocity and hydraulic conductivity.

Other rock properties – The shape of the rocks and the method of construction influence the interlocking of the elements and thus the stability. The grading may also be of influence, since a wider grading has more small rocks and thus damage may begin earlier. This increases the maximum level of insignificant damage beneath the threshold, smoothens the transition from stable to unstable and widens the cloud of data points, see Figure 55. It is of influence to where one might define the threshold point. In DOCTERS VAN LEEUWEN (1996) is concluded that the shape of the rocks, measurable in friction angle, influences the rate of increase in toe damage.

Flow and incidence – In the model, (quasi-)stationary flow, for example due to the tide, has not been included. The model is specifically set-up for the 2D flume situation. No 3D effects are included. Oblique wave incidence is not regarded as well.

Wave spectrum – The wave field properties are represented by characteristic parameters such as H_s, $H_{2\%}$, T_P and T_m. Other shapes of the wave spectrum are not included. The shape of the spectrum changes as the wave field approaches the shore or structure. Other, not reported parameters may be more characteristic for the wave spectrum, such as $T_{m-1,0}$ (see next chapter).

Breaker types – The model is made for surging conditions. It is expected that plunging conditions provide a very different water motion, and thus also a different load on the toe elements. Therefore the model has to be handled carefully and only in applicable cases.

Foreshore slope – Steep foreshores may increase load (HOVESTAD 2005). This is not included in the model. It appears that waves transform differently on steeper slopes which may produce different acceleration forces on the rocks. This is a property that is not visible in the wave spectrum.

Interaction of armour layer and toe bund – An effect that is particularly interesting for breakwaters in reality, is that toe bunds and armour layers can have a stabilizing effect on each other. If an armour elements rolls from its position and lands on the toe berm, this smoothens the armour slope a bit, increasing the capacity to withstand load. This element also armours the toe bund, leading to less damage to the toe. Preliminary results on this subject are described in LAMBERTI (1994). For this effect the toe width B_t is an important factor, since a wider bund is more likely to stop the loose armour element from rolling.

Chapter 5

Verification of the hypothesis

In Chapter 4 a hypothesis was described for stability of elements in a toe bund under wave attack. In this chapter the hypothesis is tested and calibrated, mainly with the data set of Gerding. Other data sets are used to verify the findings.

In section 5.1 is explained that resemblance is sought between relevant properties of the model and the data set. In section 5.2 a systematic comparison of model and test data is executed by multiple steps. Section 5.3 is an evaluation of the comparison and the best method is chosen from the tested alternatives. In the final section of this chapter the relevance for toe design is treated.

5.1 Resemblance

As was stated in this report before, resemblance between the test output and a parameter model is sought in two senses:

Resemblance type A: The parameter model approximations should be close to each material model test result in absolute value.

Resemblance type B: The mutual relations between parameters should be equal for the model and the data sets; i.e. resemblance should exist amongst relevant properties of the target and the model.

In this research, resemblance type B is more important, because this means that at least parts of the theoretic approach apply to describe physical processes. The model is empirically tuned to obtain resemblance type A as best as possible as well. In this way the model is a combination of the semantic classifications 'model of theory' and 'model of phenomena', see section 3.1.

The shape of a cloud of data points is determined by how the horizontal and vertical axes are defined. If the cloud of data points proves to have a shape that it was expected to have, we can state that resemblance of type B is clearly present. Then, after all, the parameter model can predict how the test results relate to each other mutually. Resemblance of type A is obtained if the cloud of data points is also in the absolute position it was expected to be (that is for example not shifted along the horizontal or vertical axis).

5.2 Systematic comparison

The test results of the different available data sets are compared to the model in a systematic way that consists of multiple steps.

Step A: In this step the applicability is compared of the adapted stability criterions of Izbash and Rance/Warren to see whether these are useful for continuation in the next steps. For this step the method of down rush energy (see section 4.5.2).

Step B: The data sets of Gerding and Docters van Leeuwen are compared to the model. A representative data set is chosen for further comparisons. The presence of rock density in the criterion of rock movement is investigated.

Step C: In Chapter 4 two methods are described for calculation of water velocities at the toe, namely the ‘down rush energy’-method and the ‘Hughes-reflection’-method. These alternatives are compared. For the first method, comparisons are also made for different characteristic parameters for irregular waves. In the theoretic descriptions regular waves were assumed. Some factors of the calculations can be adjusted to obtain better resemblance.

Step D: In section 4.2 two parameters for comparing damage were described, namely N_{od} and N_{odB}. These are compared in this step.

Step E: The model is compared to two extra data sets, namely that of MAST, series 1 (as found in GERDING 1992), and that of USACE (1987).

In section 5.2.6 the steps are evaluated and the final parameter model is chosen.

5.2.1 Step A: Criterions for the threshold of movement

In section 4.6 two adapted criterions are described for assessing the threshold of movement for rocks in oscillatory flow, namely that of Sleath and that of Rance/Warren. It was expected that the criterion of Rance/Warren would give better results, mainly because it was developed for larger grains and coastal conditions. Both criterions were adjusted to account for the additional effect of porous outflow through the toe.

The data set of Gerding is used to analyse the threshold criterions here. The data set is divided into three parts, namely stable ($N_{od}<0.4$), transition ($0.4<N_{od}<0.8$) and unstable ($N_{od}>0.8$), analogous the results of section 4.3.3. The method of down rush energy (see section 4.5.2) is used to calculate the amplitude of occurring velocity at the toe bund.

In the derivation of this method monochromatic waves are assumed. Therefore characteristic values from the irregular wave field are used in the formulas. For H we use H_s. For T it is logical to use T_s. T_s is defined in HOLTHUIJSEN 2007 as the mean period of the highest one-third of waves. This is sometimes written as $T_{1/3}$ to distinguish the measured T_s from the visually estimated T_s.

For the local conditions only T_p is reported by Gerding. Therefore in this JONSWAP wave field T_s is calculated from T_p multiplied by a constant factor. From HOLTHUIJSEN 2007 follows that $T_s \approx T_p$ for deep water swell waves and $T_s \approx 0.95\ T_p$ for deep water 'wind sea' (locally generated young wind waves). In BATTJES 2001 is found that for spectrums of wind drives waves ('sea') $T_s \approx 0.9\ T_p$. In the calculations for Gerding's near shore conditions $T_s \approx 0.9\ T_p$ is used here.[17] This is probably not directly applicable for the near shore conditions in (partly) depth limited conditions, but it is the best available option at hand for this data set.

For both criterions applies: If the for a particular test the occurring velocity exceeded the critical velocity, damage should have occurred in that test. The results are shown below.

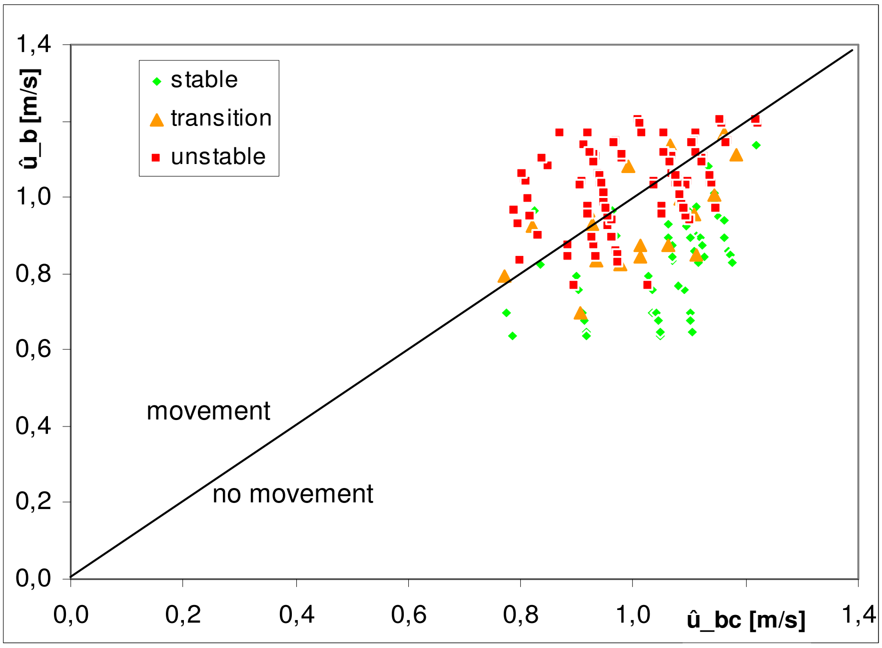


Figure 80: Data of Gerding divided in three categories, adapted Rance/Warren criterion for $\hat{u}_{bc}$, down rush energy method with H_s for $\hat{u}_b$.

For this first comparison with the Rance/Warren criterion, the test results are quite well approximated by the model (division by the straight line). The tests with considerable damage are to the upper left and the stable tests are to the lower right of the figure. Thus resemblance type B is discernable. The position of the line, where occurring and critical velocity are equal, is a bit off, but not that far. Therefore for this first comparison resemblance type A is fairly reasonable.

[17] This implies that here $T_s \approx T_{m-1,0}$, because $T_s = 0.9 T_p$ is used and for a JONSWAP spectrum at deep water applies that $T_{m-1,0} = T_p/1.1$ (VAN GENT 2001 and VAN DER MEER 2006). $T_{m-1,0}$ may be a better characteristic parameter for spectrums with irregular shapes.

Below the adapted criterion of Izbash is evaluated.

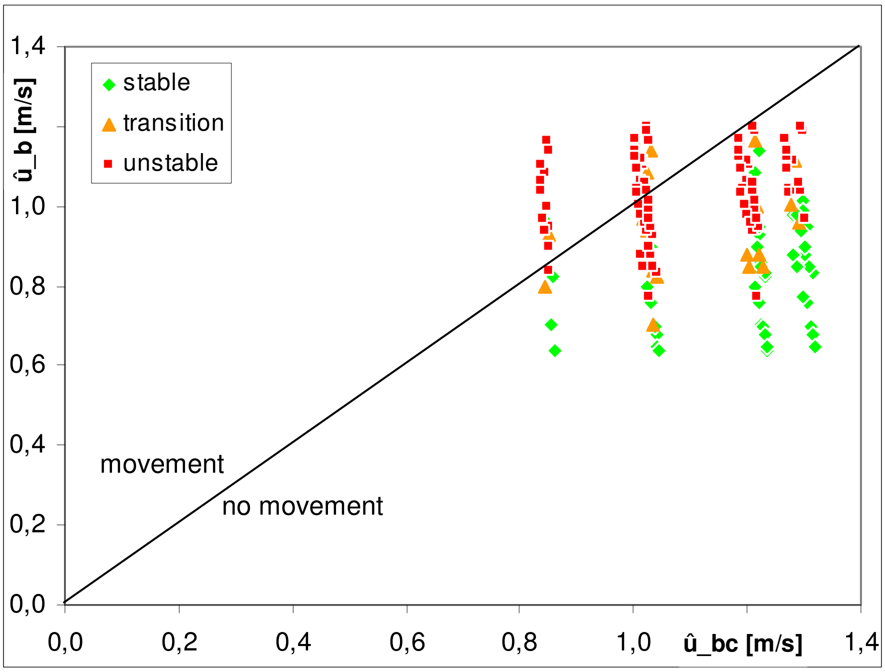


Figure 81: Stability analysis with Gerding data, adapted Izbash criterion for $\hat{u}_{bc}$, down rush energy method with H_s for $\hat{u}_b$.

The figure makes clear that the division in stable and unstable by the model is not close to the data set. The shape of the data cloud is recognisable, but the transition from stable to unstable should be parallel to the model line. In the data cloud it is more horizontal. Therefore this adapted criterion does not represent mutual influences of the parameters. The resemblance is worse than with the Rance/Warren criterion.

Of the two criterions, the Rance/Warren criterion is apparently the best suitable for this problem. This agrees with the expectations (see section 4.6). Therefore from hereon only this criterion is used.

5.2.2 Step B: Selection of a representative data set

We will now compare the data set and the model in another way. The calculated occurring velocity is divided by the calculated critical velocity, to obtain the ratio $\hat{u}_b/\hat{u}_{bc}$ for each data point. In this way the occurring load is viewed in relation to the critical load. If this ratio has the value 1, the test was performed at the threshold of movement. If the ratio is larger than 1, the occurring load was larger than the critical load. In that case the result should be “significant damage”. The result of this comparison should resemble the theoretical shape, as in Figure 55 and Figure 56 in section 4.2.

Firstly, we regard the exact same calculation method as in Figure 80. This results in the following figure:

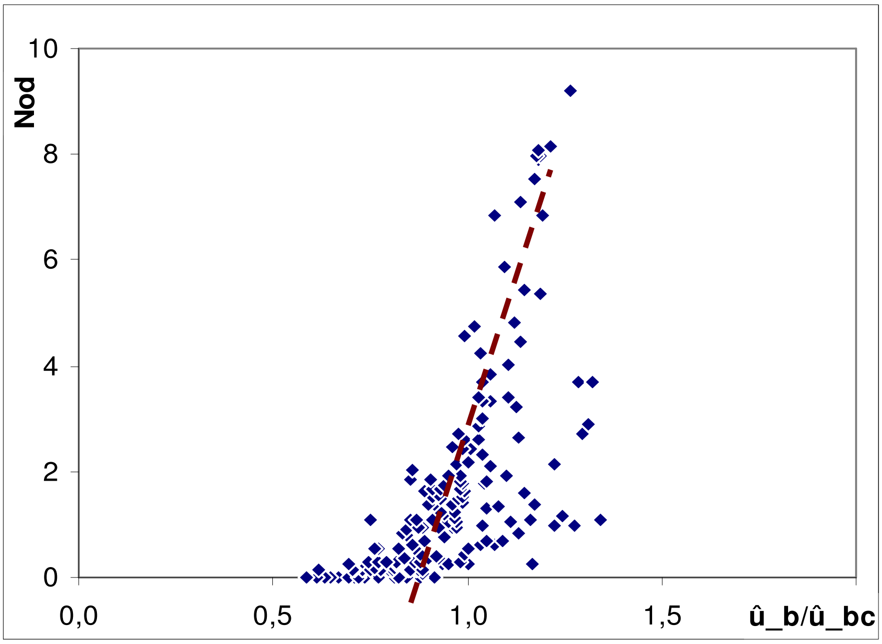


Figure 82: Gerding data set with damage versus relative load, calculated by adapted Rance/Warren criterion and down rush energy method with H_s.

The shape of this data cloud is fairly close to the model, that is to the expected shape. Results with low and zero damage lie to the left and results with mediate and high damage lie to the right. The ratio of relative load is about 10% lower than predicted, as was also concluded from Figure 80. This is therefore a reasonable result.

Now the data set of Docters van Leeuwen is added.

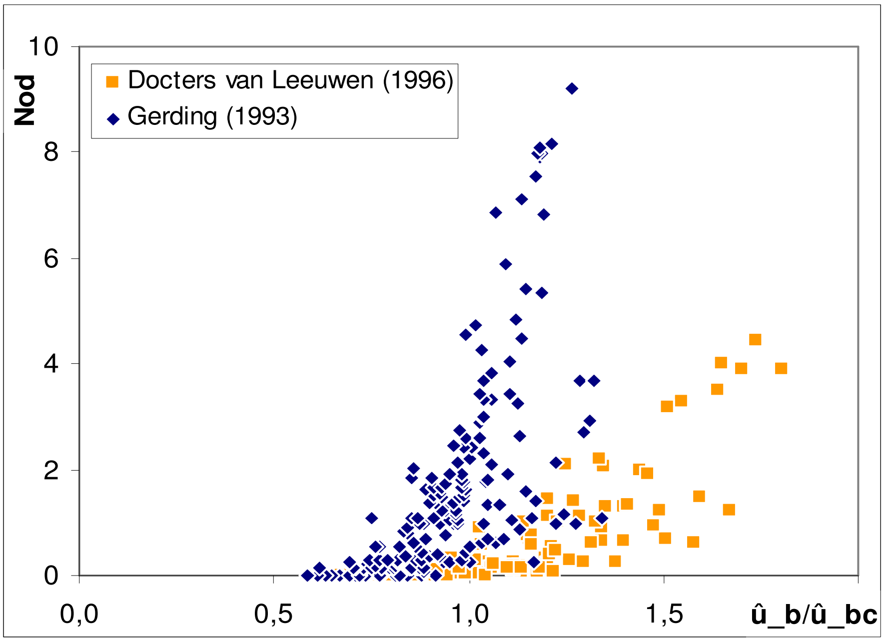


Figure 83: Same comparison as in Figure 82, but now both data sets of Gerding and Docters van Leeuwen are included.

It was expected that with this model the difference between the two test series would be smaller than with the data set of Gerding. This was described in section 3.5. In other words, the new model should describe both data sets equally. There still is a difference between these data sets. The damage level in the results for Docters van Leeuwen should be 30% higher or lower at maximum. Even then, this data set would not fully coincide with the data set of Gerding.

Some tests by Gerding and much tests by Docters van Leeuwen have a small rock size. These tests are not in accordance with available design guidance, see Table 1 in section 2.1.1. A figure is now presented that excludes the smallest tested diameter by Gerding. All tests of Docters van Leeuwen are filtered for the same rock size. For the discarded tests applies that B_t/D_{n50} > 7. This is not a representative value. The resulting figure is as follows:

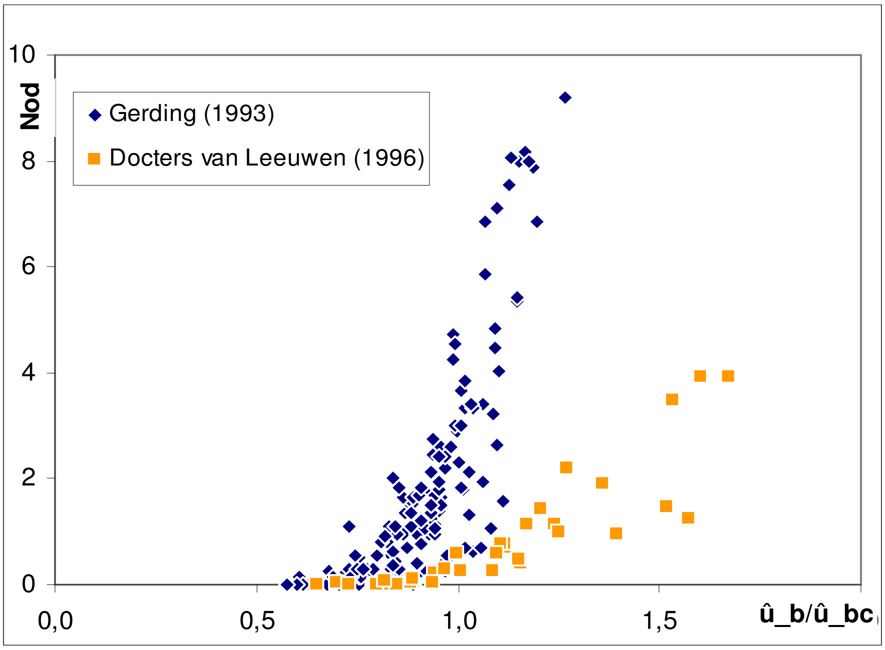


Figure 84: Data sets of Gerding and Docters van Leeuwen without non-representative tests. Here tests are discarded with rocks smaller than D_{n50} = 0.017m, thus B_t/D_{n50} > 7 for the normal toe widths (0.12m).

In the remaining data set there still is a difference for the tests by Gerding and Docters van Leeuwen, see Figure 84 above. The origin of this difference is not explained by the model of this research. The shape of the data cloud of Gerding has improved clearly, since the scatter around the area (1.2;2) in the figure has been discarded. Absolute toe width does not seem to be a source of scatter.

Further comparisons will be made with the representative part of the data sets (thus without the smallest rocks). The analysis will be based on the remaining data set of Gerding. Thereafter is shown what the result is for the data set of Docters van Leeuwen.

A remark is made that for the discarded rocks, both data sets appear to be similar, see Appendix F.1. A reason for different behaviour of the smaller rock grading may have to be sought in scaling effects. Another reason may be that the hydraulic conductivity of a toe with more rocks of a smaller size is lower. Therefore the destabilizing force by the head gradient is smaller. Regardless of this, the tested configurations are not representative structures for possible future target systems. Therefore discarding them is justified.

The discarded part of the data set of Docters van Leeuwen is not useless for this research. In the original Gerding formula D was proportional to Δ, but in the new adapted criterion D is proportional to $(\Delta\text{-}C_{PF}i)^{-1.5}$. To test the applicability the different densities are plotted together in one figure.

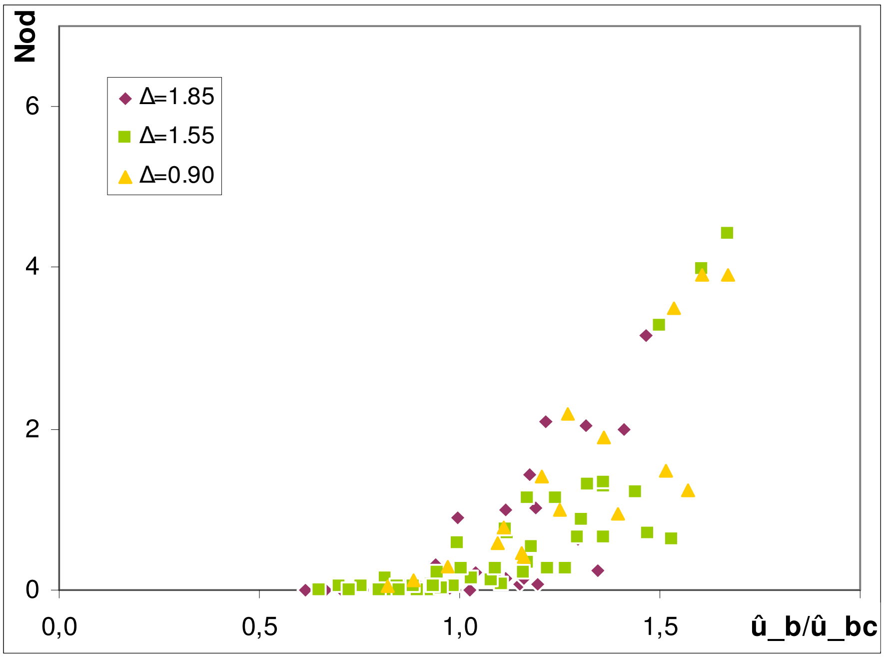


Figure 85: Data set of Docters van Leeuwen for the adapted Rance/Warren criterion and down rush energy method. In this figure C_{PF} = 0.5.

From the figure it can be concluded that the relative density is included appropriately, since the data points are mixed very well. When C_{PF} was set to 1 however, three separate groups could be identified, thus with smaller overlapping. This may indicate that the occurring head gradient is indeed smaller than the maximum gradient.

5.2.3 Step C: Comparison of different velocity calculation methods

In the previous step a representative data set has been composed and the criterion for critical amplitude of velocity is selected. The next step is to compare different calculation methods for the occurring amplitude of velocity. Three alternatives are reviewed:

Alternative 1A: Down rush energy method with H_s

Alternative 1B: Down rush energy method with $H_{2\%}$

Alternative 2: Reflection method by Hughes (with H_s and T_p)

The down rush energy method (see section 4.5.2) was based on an approach with regular waves and the tests were preformed with irregular waves.[18] Therefore characteristic parameters of the irregular field are used in the formulas. For the down rush energy method we evaluate alternative 1A with H_s and T_s and alternative 1B with $H_{2\%}$ and T_p. For the latter would then apply that the highest waves in the wave field are responsible for damage.

For alternative 2, which is the reflection method, H_s and T_p are used. This Hughes method is based on contributions of all waves in the field, but for calculations is recommended to evaluate only one wave component. In the paper by HUGHES (1992) H_s and T_p are used and therefore in the present research as well.

For these alternatives, empirical fit factors can be tuned manually to see whether this leads to improvements for the resemblance. The factors to be tuned for the alternatives are:

Table 3: Empirical factors that have to be tuned in the comparisons of the alternatives

Alternative 1A and 1B	C_{PF}	equation 35 and 37
	γ_{dr} (thus $\hat{u}_{bdr}$)	equation 20
Alternative 2	C_{PF}	equation 35 and 37

For each method the un-tuned calculation and calculations with tuned factors are compared.

A scalar multiplication factor is applied for some parameters to test the sensitivity of the model. These factors are:

Table 4: Factors that can be tuned in the comparisons to test the sensitivity

Alternative 1A and 1B	factor for $\hat{u}_{bi}$	equation 17
	factor for φ_{TA}	equation 23
Alternative 2	factor for T_p in calculation of ξ_h	equation28
	factor for θ	equation 27
	factor for K_r	equation 29

Adapting the factors of Table 3 and Table 4 changes the relative and absolute resemblance. Therefore an adaptation to a sensitivity factor can also improve the resemblance.

Alternative 1A: down rush energy method with H_s

In Appendix F.2 is explained how exactly each parameter from the irregular waves is used in the regular method. The un-tuned figure for this alternative is equal to

[18] Gerding used a JONSWAP spectrum in deep water with reflection compensator.

Figure 82, however without the smallest rocks. Therefore this is the same as the blue data points in Figure 84. The un-tuned figure is included in Appendix F.2.

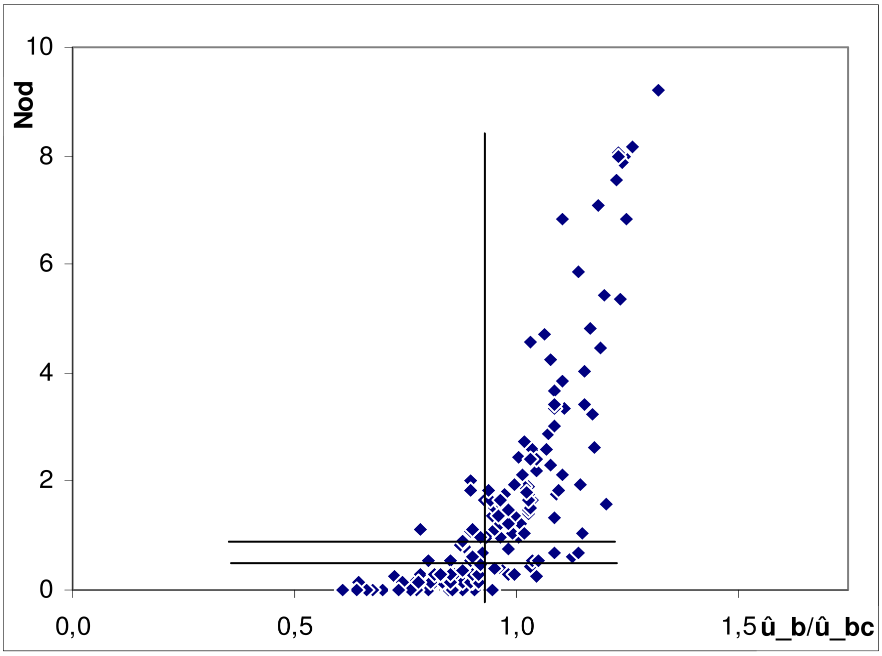


Figure 86: Down rush energy method with Hs and tuned factors for representative Gerding data set.

In attempts to adjust some factors in the calculations (see Table 3), not much improvements are obtained with respect to the un-tuned result. During the adjusting, the main focus was on the shape of the data cloud, thus resemblance type B (see section 5.1). Therefore it is important, that analogously to Figure 56 in section 4.3.1, there are no data points in quadrants 1 and 3 and as much as possible lay in quadrants 2 and 4. Improvement by tuning is therefore not easily measurable by statistics but is assessed visually. The interest is thus mainly in the region near the cross of lines that indicates the threshold and the transition from insignificant to significant damage. The tuning procedure was to make small adjustments and to see if this made the plume shape better or worse. The tuned factors for Figure 86 are:

factor for $\hat{u}_{bi}$ =1 (un-tuned 1) γ_{dr} = 0.45 (first attempt 0.4)
factor for φ_{TA} = 1.1 (un-tuned 1) C_{PF} = 0.4 (first attempt 0.5)

An overall factor eventually has to be applied to the horizontal axis in order to get the line of threshold of movement at the value $\hat{u}_b/\hat{u}_{bc}$ = 1.

It was found that adjusting C_{PF} does not change the shape of the data cloud very much, but mainly influences the absolute value. This means that the value of C_{PF} is important for the required diameter, but that it is not equal for all cases. It may be dependent on how much of the head gradient is still noticeable in the toe bund. Therefore it depends on how much water flows out of the armour layer, and thus on how deep the toe is situated (h_t). A better description of C_{PF} or the actually occurring head gradient in the toe will decrease some scatter.

Figure 86 shows that there are only a few data points in quadrants 1 and 3 and that these are close to the boundaries. The position of the data points is not important for design as long as they are in quadrants 2 and 4. This figure may therefore become an accurate design tool.

Alternative 1B: down rush energy method with $H_{2\%}$

For this alternative the un-tuned figure is presented in appendix F.2. Below the tuned figure is presented.

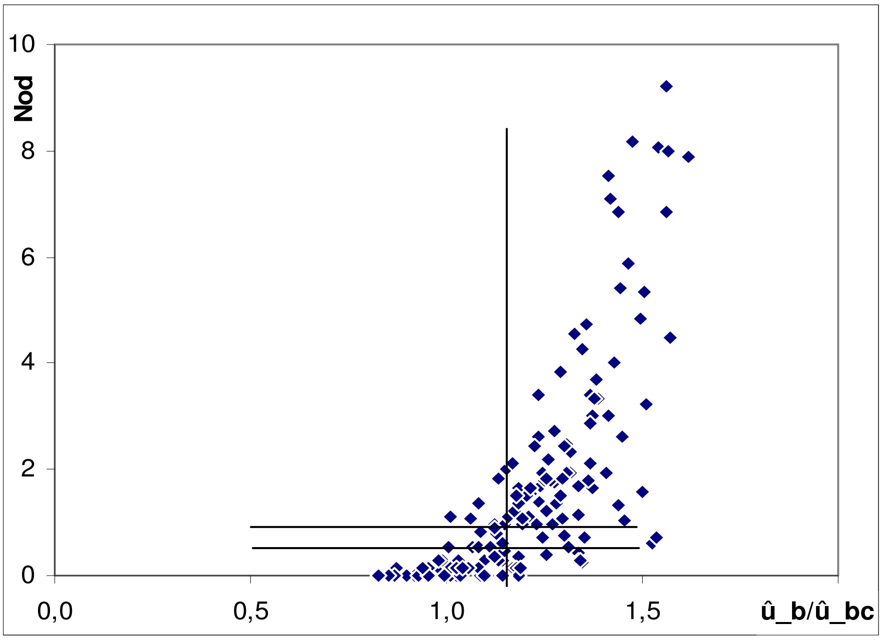


Figure 87: Tuned down rush energy method with (measured) $H_{2\%}$.

The tuned factors are:

factor for $\hat{u}_{bi}$ =1 (un-tuned 1) γ_{dr} = 0.4 (first attempt 0.4)
factor for φ_{TA} = 0.9 (un-tuned 1) C_{PF} = 0.3 (first attempt 0.5)

In this figure slightly more data points lay in the wrong quadrants, if we compare it to Alternative 1A. This alternative of the model shows less resemblance (type B) to the data set. Furthermore, if we take a look at the tuned factors, we derive that most factors are lowered, simulating a situation with smaller waves, which is thus more similar to the calculation with H_s.

Alternative 2: Hughes-reflection method

Now that the down rush energy method has been evaluated, we continue with the Hughes-reflection method. The figure below shows the results for the un-tuned calculations. In this method the reflection coefficient is calculated according to the latest publication, namely DEKKER *et al.* (2007).

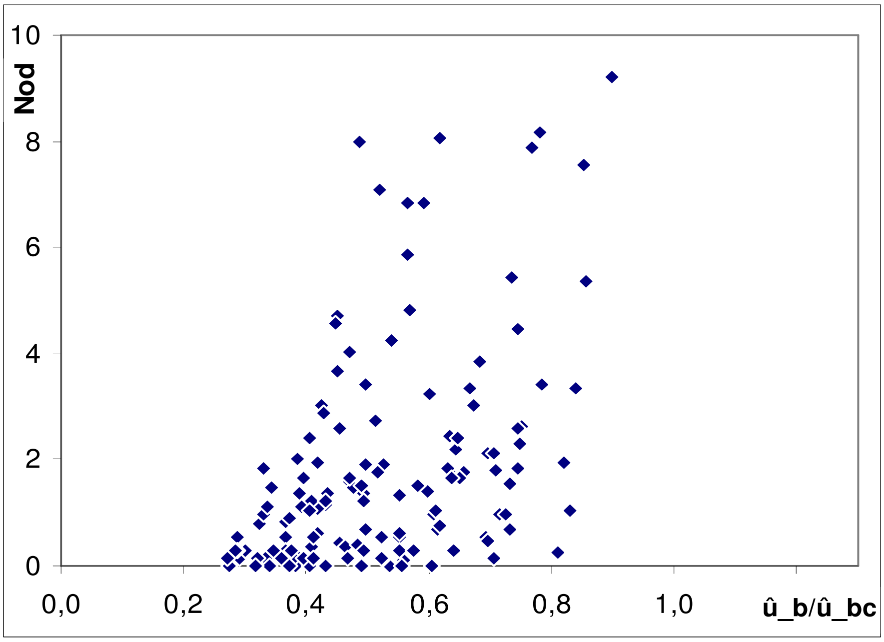


Figure 88: Un-tuned Hughes-reflection method with K_r by DEKKER *et al.* (2007)

The figure shows very little resemblance of this model to the data set. With strong adjustments to the tuning factors, a better figure is obtainable, see Appendix A.2. This is however not very appropriate for design. There is a number of available options for calculation of the reflection coefficient (see Appendix F.2), but this is not the main source of scatter.

Adjustment to the sensitivity factors seems to have a large influence, such as for the phase angle θ (see Table 4). Since this method is very sensitive to small differences it is not a good design tool.

It seems that the way in which θ is calculated (according to HUGHES 1995) may not be applicable for the experiments by Gerding. When the maximum possible velocity in front of the structure is calculated (by entering π in the formula for all θ), an improvement is obtained (see Appendix F.2). This is however still no accurate method.

It may be the case that actual velocities at the toe bund can be assessed better with this reflection method. After all, the method by Hughes was verified for a number of cases and the down rush energy method was not. This would imply that actual velocities are lower than calculated with the down rush velocity method. With the Hughes-reflection method namely, the calculated velocities are generally lower. The latter method is however not a better design tool for toe element stability. The resemblance for relevant properties is better for the down rush energy method.

Evaluation of alternatives

The best result is obtained with Alternative 1A, which is the down-rush energy method with using H_s. Resemblance type B (see section 5.1) is quite well present, and resemblance type A can be obtained by applying an overall factor to the relative load.

The Hughes-reflection method is not adequate because of the high sensitivity. A small change produces a very different prediction. Furthermore the resemblance is lower.

In further steps of this analysis the tuned Alternative 1A is used.

5.2.4 Step D: Comparison of N_{od} and N_{odB}

In this section the two ways of damage description are compared. In section 4.2 was described what the difference is between N_{od} and N_{odB}.

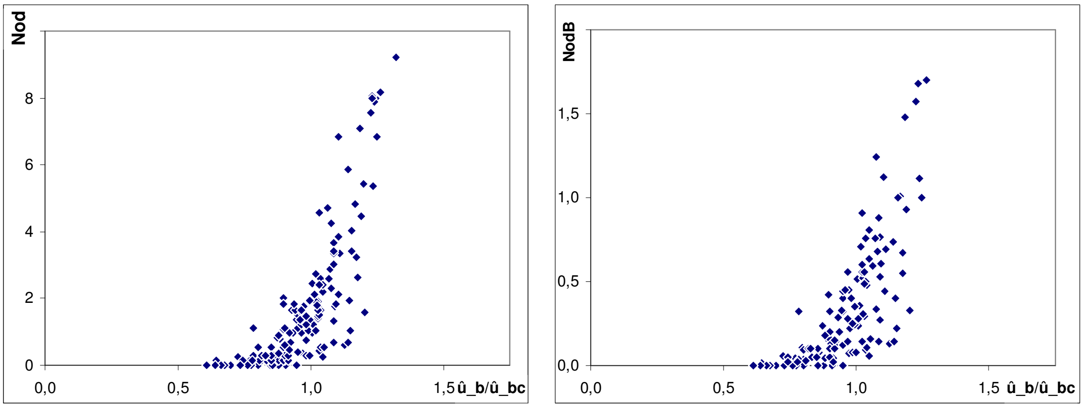


Figure 89: Comparison of using N_{od} (left) or N_{odB} (right) to measure damage.

From the figures it is concluded that using N_{odB} is no improvement. The small difference could be explained by the small variation in diameters in the remaining part of Gerding's data set. Wider toe bunds still remain in the data set though and these seem to fit in properly. Therefore the conclusion is that N_{od} is the better damage parameter for this data set.

5.2.5 Step E: Comparison to MAST data set and USACE data set

In this step the model is compared to other data sources. First series 1 of the MAST data set is used (as found in GERDING (1993) or GERDING (1992). For convenience the representative Gerding data set is depicted as well.

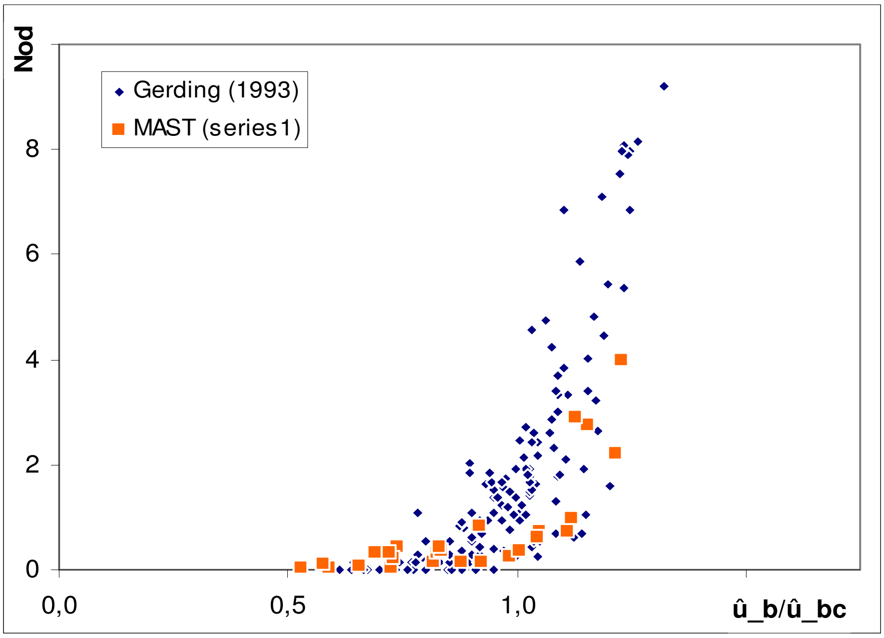


Figure 90: Comparison of alternative 1A to the MAST data set (series 1).

Resemblance is quite similar for both data sets. It appears that this MAST test case could cope with a slightly higher load than the Gerding tests. This means that this study's model would result in a conservative diameter of a few extra percent.

In the following figure, the model is compared to the data set by USACE (1987). This test series used monochromatic waves. Furthermore, the amount of damage is unknown. Unfortunately only the test results with 'acceptable damage' are included in the report. In this figure, we will interpret 'acceptable damage' as an amount of damage that is just a bit higher than the threshold.

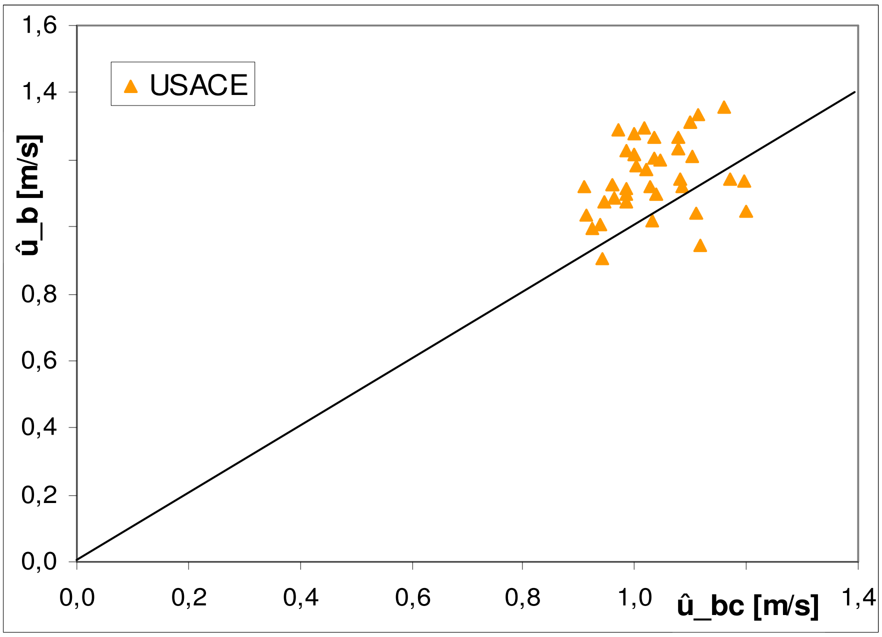


Figure 91: Comparison of the model to the USACE (1989) data set.

This result is quite satisfying. For most data points the characteristic occurring velocity is slightly higher than the characteristic critical velocity. The data cloud is partly on the lower right of the line. This is similar to Figure 80, so the same adjustment (with an overall factor for relative load) would lead to a good result.

In the comparisons of step E the down rush energy method with H_s proved very reasonably applicable.

5.2.6 Evaluation of the analysis

From the analysis in this section is concluded that:

Step A: The adapted criterion of Rance/Warren works better than Izbash for this problem.

Step B: The resemblance improves when only representative rocks sizes are used. There still is a difference between the data sets of Gerding and Docters van Leeuwen.

Step C: Of the proposed alternatives, the down rush energy method with H_s is the best method for description of stability of toe bund elements.

Step D: Damage parameter N_{od} provides better results and N_{odB}.

Step E: Comparison to the MAST and USACE data sets also confirms the hypothesis. Results are close to Gerding's data set.

This confirms that the hypothesis describes the physical process to a reasonable extent.

5.3 Final parameter model

5.3.1. Fitting the model to the data set of Gerding

The final step is to empirically adjust the model. Thereby we obtain the best resemblance of type A as well, which is in absolute sense. In section 5.2.3 was found that use of γ_{dr} = 0.45 and C_{PF} = 0.4 results in slight improvements for the shape of the data cloud. Also adjusting the sensitivity factor for φ_{TA} has influence. This was however originally not a factor that had to be filled in empirically (see Table 4). Therefore this factor is kept unadjusted. Now the ratio of occurring amplitude of velocity and critical amplitude of velocity is empirically tuned (see section 4.4.1) in such a way that the threshold of movement lies at 1. Written in a formula, this implies that

$$1 = \Gamma \cdot \frac{\hat{u}_b}{\hat{u}_{bc}} \tag{38}$$

In which Γ is the fit parameter. In the following figure is shown what the result is for Γ = 1.05 for the remaining data set of Gerding.

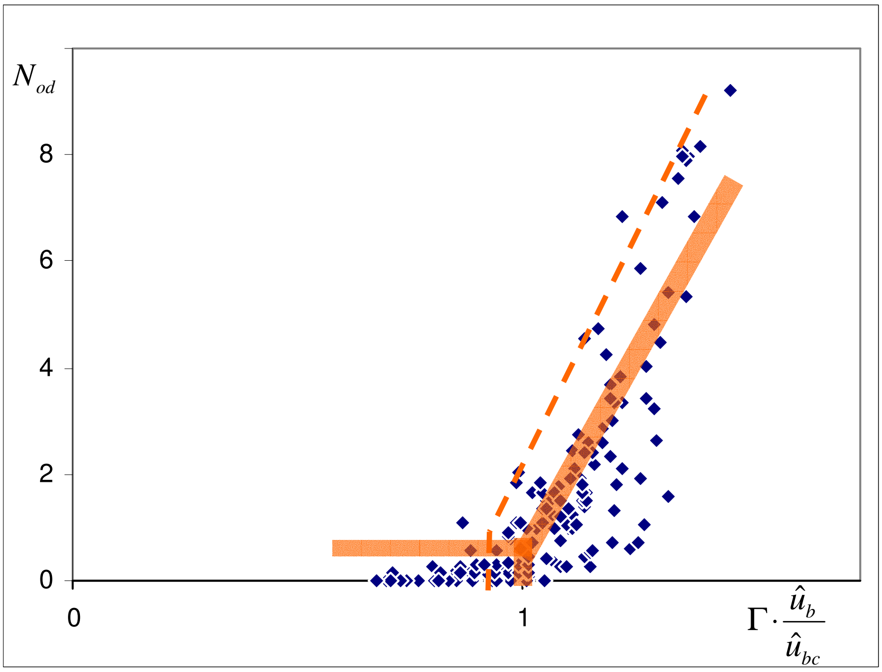


Figure 92: Final model with fit parameter Γ = 1.05, for γ_{dr} = 0.45 and C_{PF}=0.4

As a conservative design method a line to the left of the threshold can be used (see the dashed line in Figure 92). This means that we do not use $\Gamma \cdot \hat{u}_b / \hat{u}_{bc} = 1$ but for example $\Gamma \cdot \hat{u}_b / \hat{u}_{bc} = 0.94$ (see also in the calculation example in Appendix G). This yields using an increased relative load of 6.6%. A designer may choose his safety level between an increase in relative load of 5% to 10%. This means for a safe design to use:

$$\Gamma \cdot \frac{\hat{u}_b}{\hat{u}_{bc}} = 0.91 - 0.95 \tag{39}$$

Considering the assessment of the physical process, the following remarks are stated:

a. Since irregular waves are used, there is no single amplitude of occurring velocity.
b. The same applies for the critical velocity amplitude, since the rocks are irregular as well.
c. Turbulence and accelerations are included in the amplitudes of velocities, and therefore the value of the calculated parameter is rather a characteristic parameter that includes these effects, than an actually occurring velocity.

In the concept of this hypothesis, see Figure 71, assessing the local stability process is approached from two sides. These are:
Step1: estimation of the local conditions due to the waves
Step2: estimation of the critical conditions for the stability rocks

Both these steps have to be tuned empirically in further research. We know that the overall empirical fit factor Γ is about 1.05 for Gerding's data set, but we don't know whether it is the critical or occurring velocity that has to be adjusted. Most likely they are both a bit off and the good resemblance of type A (only a factor 1.05) is coincidental. Written in a formula this yields:

$$1 = \Gamma \cdot \frac{\hat{u}_b}{\hat{u}_{bc}} = \frac{\gamma_b \cdot \hat{u}_b}{\gamma_{bc} \cdot \hat{u}_{bc}} \tag{40}$$

so that Γ is the ratio of two separate fit parameters for steps 1 and 2, according to

$$\Gamma = \frac{\gamma_b}{\gamma_{bc}} \tag{41}$$

in which γ_b is the fit parameter for calculation of the occurring amplitude of velocity at the toe bund (hence the subscript b of "bund") and in which γ_{bc} is the fit parameter for the calculation of the critical amplitude of velocity at the toe bund (hence the added subscript c for "critical").

It is likely that both these fit parameters are smaller than 1. The threshold criterion of Rance/Warren was based on tests with rock movements on a bed. For a toe a small number of moving rocks is already considered as movement. It is likely that in the tests of RANCE *et al.* (1968) more movement was considered as the threshold. Furthermore, rocks on the edge of the toe bund have smaller stability because stabilizing interlocking is missing on the seaside of the rock. For these reasons γ_{bc} is probably smaller than 1.

The reflection method of Hughes resulted in smaller velocities. This method is based on theory and verified by tests in which velocities were measured. This is regarded here as a clue that actual occurring velocities are lower than calculated with the down rush velocity method, and that therefore $\gamma_b < 1$ as well.

Because velocities were not measured in the tests of Gerding, fitting these parameters is unfortunately not possible. Of course the ratio Γ can still remain 1.05 in the case that both γ_b and γ_{bc} are smaller than 1.

5.3.2 Results for data set of Docters van Leeuwen

In step B of the analysis (section 5.2.2) a large part of the data set of Docters van Leeuwen was discarded. The evaluation of the model is also done for the the remaining part of Docters van Leeuwen's data set. Adjustments to the empirical fit factors (see Table 3) affect the horizontal position (absolute resemblance) of the data cloud. But the effect of the adjustments for the shape of the data cloud is hard to discern, because there are not many data points left. The same empirical fit factors are therefore used as for Gerding. This implies $\gamma_{dr} = 0.45$ and $C_{PF} = 0.4$. For Gerding was found that $\Gamma = 1.05$. The following figure shows the result for the remaining data set of Docters van Leeuwen with $\Gamma = 0.84$ (20% difference).

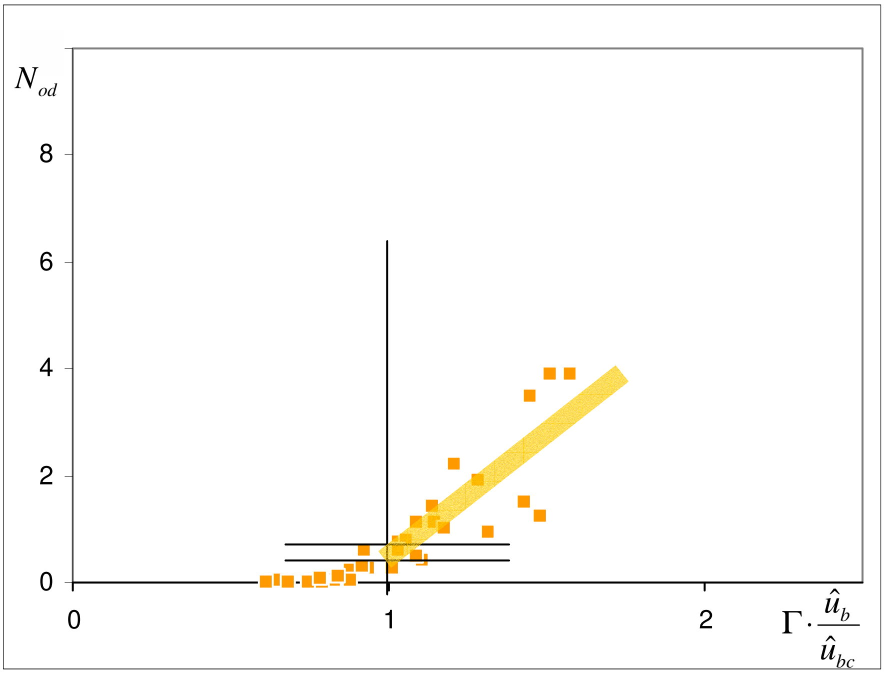


Figure 93: Fit of the model to the remaining data set of Docters van Leeuwen for Γ=0.84.

The value for Γ illustrates that there still is a difference for the data set of Gerding and Docters van Leeuwen. This difference is not explained by this study's model. Although Docters van Leeuwen only counted seaward damage, the test results cannot be ignored. That Docters van Leeuwen's calculation of N_{od} was not in accordance with Gerding's definition has been corrected in this research. This can therefore no longer be a source of scatter.

Differences between the data sets of Gerding and Docters van Leeuwen may be:

a. The steeper foreshore slope of Gerding could be a cause of higher load (see box 3 in section 4.5).
b. It is possible that the rocks of Docters van Leeuwen were different from Gerding's in shape and placement. This could imply a change in the application of the adapted Rance/Warren criterion. Perhaps the critical velocity was therefore higher for Docters van Leeuwen.
c. If there were differences in rock properties, this could be also a cause of the difference in development of damage above the threshold (inclination of the thick lines in Figure 92 and Figure 93). The friction angle of the material plays a role in the development of damage, as is described in DOCTERS VAN LEEUWEN (1996). Unfortunately it is at present not possible anymore to check whether there was a difference in internal friction angle between the toe material of Gerding and that of Docters van Leeuwen.
d. Another difference is the breaker index (see Box 1), which indicates whether the situations were depth-limited. For Gerding the average value of γ is 0.44 and for Docters van Leeuwen 0.39. This shows that Gerding had more tests with depth-limited waves.

As a design tool it is recommended to use the fit of this study's model to the data set of Gerding, because:

a. The fit to Gerding is based on more data points
b. Using the fit to Gerding is conservative for the results of Docters van Leeuwen.
c. Actual numbers of displacement in the tests of Docters van Leeuwen were different since she did not count all displacements.

5.3.3 Ways to obtain better resemblance

The concept of this hypothesis has a number of advantages, as was described in section 4.4.1. Further improvements to the parameter model can be obtained by a number of investigations:

a. Further research on the fit parameters γ_b and γ_{bc}.
b. $T_{m-1,0}$ is a more characteristic parameter for non-standard wave spectra than T_p, according to VAN GENT (2001). In the calculations a scaled version of measured T_p was used, namely T_s=0.9T_p, because that is the only reported period parameter by Gerding. Therefore measuring (and reporting!) $T_{m-1,0}$ can lead to improvements.
c. Use of a representative wave height between $H_{2\%}$ and H_s.
d. Theoretical elaborations on the relation between regular wave theory and irregular wave tests.
e. Using regular wave tests (see also section 4.4.2). Some of the scatter in Figure 92 could simply be stochastic band width.
f. Use of better run-up and especially better run-down formulas. For calculation of the run-down level, only a formula for $R_{d2\%}$ was available in The Rock Manual, see Appendix A.2.
g. The formula for down rush contribution to $\hat{u}_b$ does not include h_t, while this parameter should have influence (see also section 4.8.1). This leaves also scope for improvement.
h. The head gradient i in the toe that is responsible for porous outflow is approximated rather roughly (as i_{max}). The fit factor C_{PF} for the head gradient was fitted simultaneously with γ_{dr}.
i. Some rocks may displace at the first waves, because they have been placed in an unstable orientation. This introduces scatter, but this may of course also happen in reality. For understanding, assessing and describing the physical process on the other hand, it can be helpful to pre-test the material model with small waves first.
j. It is possible that the smaller rocks in the grading of a certain test are generally spoken the first rocks to move. Accounting for this effect could lead to improvements. Because of this, displaced rocks need to be weighed to see if this assumption is correct.

5.3.4 Comparison to the previous model

The new parameter model is compared to the model by VAN DER MEER (1998). The results for the present method are represented by Figure 92. The method by Van der Meer is represented in the following figure. This figure is almost the same as Figure 39 and thus also includes tests with the result N_{od}=0. The figure is shown as well without the smallest fraction of the rocks, for fair comparison. For the sake of completeness is remarked that discarding the tests with the smallest rocks did not change the figure much, actually.

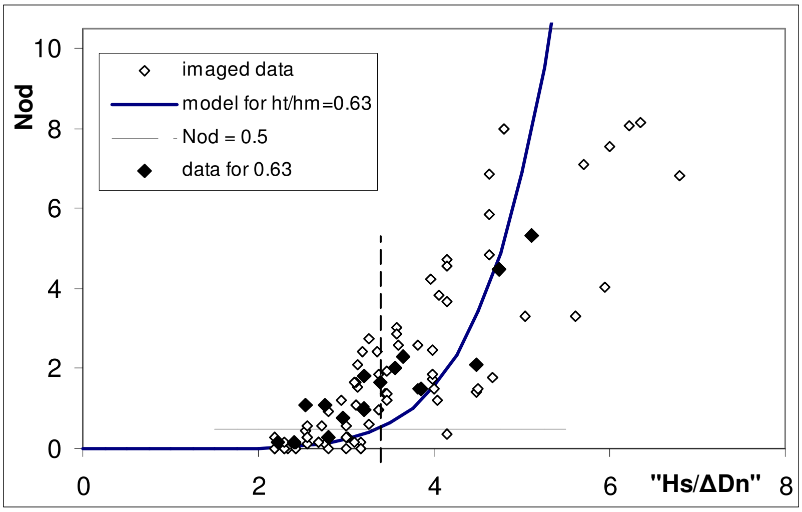


Figure 94: Van der Meer model and Gerding data set (here without smallest rock grading), at h_t/h_m=0.63 and images of tests with other relative toe depths (0.5<h_t/h_m<0.73) .

The new model has indeed better resemblance with the theoretically predicted shape. This was defined in the hypothesis (Chapter 4) before the analysis. The comparison between the new method and the method by Van der Meer should be regarded especially for the area that is of interest for toe design, namely near the threshold. But we may conclude as well that more careful use in practice of the Van der Meer and Gerding method can also improve expectations (see Chapter 3).

5.3.5 Applicability range

In section 5.3.2 it is recommended to use the fit of this study's model to the data set of Gerding. Since this final fit is empirical, application of the model is only allowed within the boundaries of the data set. Application outside these boundaries has to be done with great care. This study's model may be verified or calibrated for other fields of applicability in further research.

D'ANGREMOND *et al.* (2001) provides a validity range for the formula of Gerding (1993) in terms of h_t/h_m and h_t/D_{n50}. In Chapter 2 and Chapter 3 has come forward that these are inappropriate parameters for comparing different geometric configurations. The range of applicability has to be given with other parameters.

This study's model is based on assessment of the local water motions at the toe bund (see Figure 71). This is founded on the summation of contributions of the incoming wave and the water mass that rushes down the slope after the moment of run-up (see section 4.5.1). The limitations to the applicability of this study's model are given by the applicability of this theoretical foundation.

For the contribution of the incoming wave applies that equation 17 is quite uniformly applicable. Therefore no restrictions are given in terms of h_t/h_m. One should bear in min that the equation is based on regular linear wave theory.

For the contribution of down rush energy applies that the model-of-thought for equation 20 is not widely in use yet. The approach should not be extrapolated beyond the data set of Gerding. In section 4.8.1 is explained that toe depth is of influence to the down rush contribution, but the parameter is not present in equation 20. An increasing ratio of R_d/h_t decreases stability, since a higher toe is more affected by down rush. If the toe is too deep, this model does not represent the occurring mechanism. H_s and R_d are proportional (for equal wave steepness anyway, see Appendix A.2). Therefore H_s/h_t may be an appropriate indicator of the applicability of equation 20. Based on Figure 95 it is proposed to use this study's model only for $H_s/h_t > 0.5$.

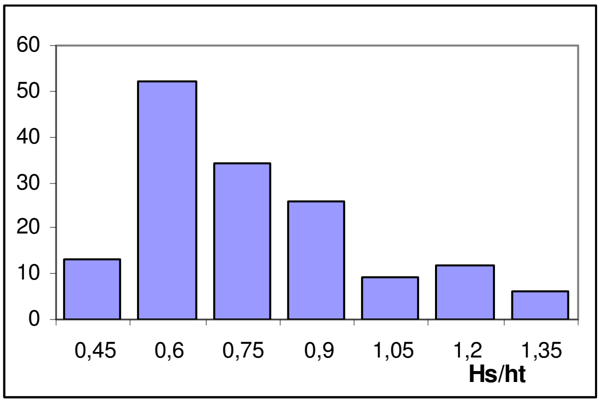


Figure 95: Number of tests in data set of Gerding, sorted to H_s/h_t

Most tests were performed in depth-limited waves. The water motion at the toe bund may be different for non-depth-limited waves. Therefore the limitation is suggested: $H_s/h_m > 0.35$, see the figure below.

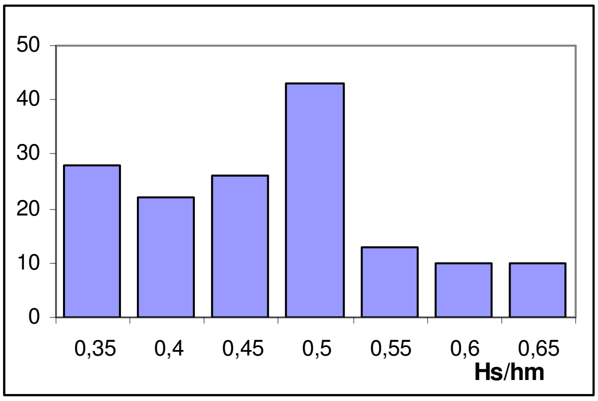


Figure 96: Number of tests in data set of Gerding, sorted to breaker index

The calculation of run-up has an upper limit (see Appendix A.2). The calculations of Chapter 5 make clear that in cases of the tests of Gerding, the run-up acquired the values of this upper limit or were close to it. For use of this study's model in design practice, it is important that the motion of water on the slope is similar to Gerding's tests. Therefore it is suggested that for application in design cases, the run-up level should be close to maximum.

In summary the validity range of this study's model is given by:

a. Standard size toe bunds, as defined in section 2.2.1
b. Conventional permeable rubble mound breakwaters
c. Rough front slope with $\tan\alpha \approx 0.67$
d. Run-up level should be close to upper limit
e. $H_s/h_t > 0.5$
f. $H_s/h_m > 0.35$

5.4 Toe bund design

5.4.1 Required nominal diameter for toe rock

In this thesis a method is developed to describe the stability of toe bund elements. Eventually the result is Figure 92. This method can be applied in toe bund design for determination of the required rock diameter, with reasonable accuracy. This is certainly true in the long term if more tests are done to verify step 1 and step 2 (see Figure 71).

The occurring velocity amplitude at the toe bund $\hat{u}_b$ can be calculated by equations of the down rush method in section 4.5.2. An appropriate value for equation 39 can be chosen on the desired level of safety. Then the required amplitude of critical velocity $\hat{u}_{bc}$ can be derived. With the adapted Rance/Warren criterion the required toe bund diameter is calculated. In reality and in material tests, partly independent of the applied safety, a damage level of $N_{od} = 0.4$ or up to $N_{od} = 0.8$ has to be expected.

What does using this description of the physical process mean for the required toe rock diameter? From equation 37 follows that the required D_{n50} is proportional to $\hat{u}_{bc}^{2.5}$. This is the only formula that includes D_{n50} and therefore also applies:

$$D_{n50} \propto \left(\Gamma \cdot \frac{\hat{u}_b}{\hat{u}_{bc}} \right)^{-2.5} \tag{42}$$

If we use the right hand side of this expression on the horizontal axis the following figure is obtained:

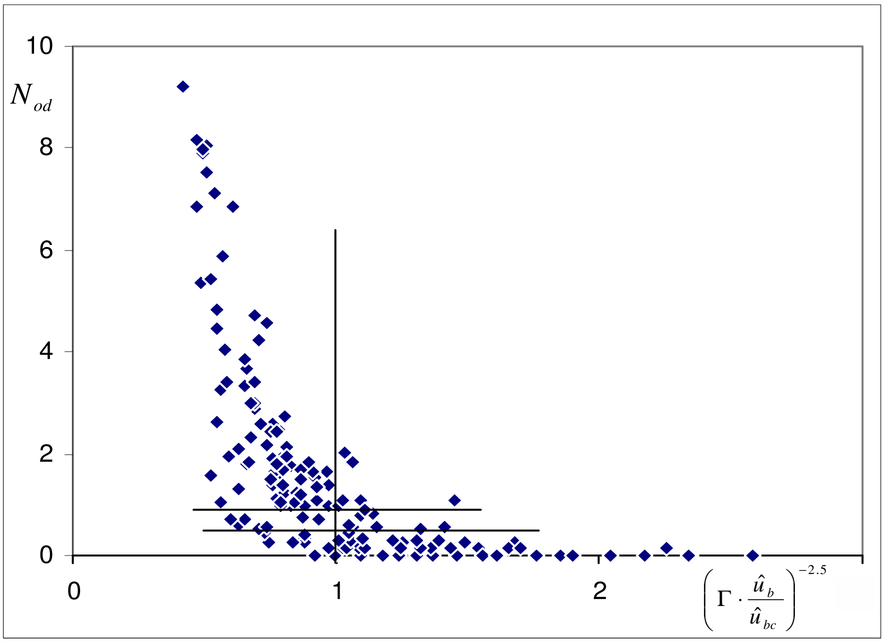


Figure 97: This study's parameter model compared to data set of Gerding with horizontal axis proportional to D_{n50}.

In this figure the horizontal axis is proportional to D_{n50}. One may interpret this as the tested diameter relative to the critical diameter (at value 1). The figure confirms that with this method a reasonable prediction can be made for the required rock size. For smaller rocks than the critical rock size the outcome of a test is highly unpredictable. For example for 0.7 times the critical diameter (thus 0.7 = value on the horizontal axis of Figure 97) the test result can be somewhere between N_{od} = 0.6 and N_{od} = 5. For larger rocks than the critical diameter we are fairly certain of the damage result that we should expect.

5.4.2 Toe bund design procedure

Ideally the procedure to design a toe would be as follows:

1. Define the functions that a toe structure should perform for the breakwater system and determine whether a toe structure is indeed desired.
2. Select the preferred toe structure for those functions.

If the desired toe structure is a toe bund, continue with:

3. Determine the required toe bund dimensions to fulfil the predefined functions. (This is not further described in this research)
4. Select the material and required size (mind availability).
5. Add the expected damage level (perhaps up to N_{od} = 0.8) to the required dimensions so that the structure remains intact after some damage has developed.

A fictional design case example is provided in Appendix G. Here the method of this report is used and compared to the methods of Gerding and Van der Meer.

Unfortunately presently there are no recommendations available on required bund dimensions for toe functions as meant in step 3 of the procedure above. Therefore this design procedure is not directly applicable yet and further research is required. For now the procedure must remain to design a toe bund with standard dimensions as found in common literature (see Table 1). Then an argumentation has to be provided why a damage level between N_{od} = 0.4 and N_{od} = 0.8 is not harmful to the stability of the structure. This can for example be done with the recommendation by Eckert (see Figure 8) and additional safety.

A final remark is made. We have seen that wave load just under the critical load can also result in small damage. Therefore a few storms with lower frequency of occurrence than the design storm can also result in small damage, for each separate storm. Therefore a bit more material may have to be added. This is however a probabilistic design issue that should be incorporated in the design procedure of determining the design conditions for a breakwater. It is not a part of the tests by Gerding and not a part of this research, since only the physical interconnection of wave load and toe damage is investigated in this thesis.

Chapter 6

Conclusions and recommendations

In this chapter the conclusions of this study are presented. In section 6.1 the research questions are answered. In section 6.2 conclusions are drawn based on the problem and goal of this research. Thereafter in section 6.3 additional conclusions are discussed. The final section provides recommendations for further research.

6.1 Answers to the research questions

In section 1.6 research questions have been formulated. The main conclusions of this thesis are presented here as answers to those questions.

a. *Under which circumstances are the present methods for determination of the required rock size applicable and reliable?*

The methods of GERDING (1993) and VAN DER MEER (1998) are both empirical methods and are based on the same data set. Therefore the methods have the same field of applicability. Since Gerding's equation has more resemblance with the test data it is the better one. Accuracy of the methods decreases when toe depth increases. The scatter of data is considerable, certainly for the Van der Meer method. A lower limit curve fit may result in a very conservative design. If a design value of N_{od} = 0.5 is used in the Van der Meer equation, probabilities are considerable that a flume test will result in higher damage than the expected N_{od} = 0.5. The Van der Meer model predicts lower damage than actually occurred for 65% of Gerding's tests.

b. *The presently recommended design tools include a power curve between damage number and stability number. Is this appropriate or is it better to use a threshold of movement for elements in the toe?*

This study's parameter model includes a critical value for the load. This implies use of a threshold principle. For design practice it is important to create a situation with smaller load than the critical load. This leads to low (insignificant) and well predictable damage levels. It is presently not possible to describe the development of damage accurately above the critical load. It is not recommended to design above the critical load.

Gerding and Van der Meer used a relation with a power curve for damage and stability. This is inappropriate and results in much uncertainty for the damage to expect in a test and thus in reality as well. Working with the threshold approach is more reliable for design. It is simply not attempted to predict the level of damage above the threshold. Tests with high damage are required though to be able to assess the threshold. Tests with high damage may be helpful for commercial tests as well to assess the threshold (see Figure 90).

The Van der Meer and Gerding methods are presented with different axes than in the original graphs. This improves reliability of these methods as well (Figure 39/Figure 94). Tests with the result N_{od}=0, which are valuable, are now included in the analyses and the spread of damage around the model is visible independently from other parameters.

c. *Is it possible to obtain a better model of the physical process that determines stability of rocks in a toe bund?*

It certainly seems so. This study's model has more resemblance to the data set than the previous methods and introduces more theoretical background. Besides the increase in resemblance, there are more advantages. This study's model provides more leads (points of departure) for further research and further improvements. The model also accounts for the influence of more parameters and no distinction is made between depth-limited and non-depth limited waves. Although it is only validated for Gerding's data set, the model is adaptable for both situations in further research.

d. *Is this model, meant in the question above, a more adequate tool for design practice than the presently recommended tools to determine the required toe rock size?*

This study's model has more resemblance than the presently recommended models with the same data set. This particularly applies for the range that is relevant for design practice, namely where significant damage begins to develop. Therefore it seems indeed the case that a basis for a better design tool has been found. There are parameters in the equations that were not varied in the tests, such as $\tan\alpha$. Some influences have been derived theoretically, but some parameters have been fitted empirically to the Gerding data set, such as C_{PF} and γ_{dr}. These could be of significant influence to Γ and thus also to the required rock size. Therefore great care has to be applied with use outside of Gerding's data range. The hypothesis gains strength from other data sets (MAST and USACE). Analysis for the data set of Docters van Leeuwen also confirms the hypothesis, but a deviation of 20% for the overall fit parameter Γ is found compared to Gerding. Use of the fit to Gerding is recommended because it is conservative for Docters van Leeuwen and based on more tests.

e. *Is it possible to assign a classification for the amount of damage to a toe bund?*

In literature a classification of toe damage was found (Table 2). This is a classification for the amount of damage that simultaneously indicates the severity of these amounts of damage for the structure. This cannot be combined, since a certain damage level may be more severe to one structure than to another. This study classifies the amount of damage for Gerding's data set as follows:

Insignificant damage (generally stable)	$N_{od} < 0.4$
Transition	$0.4 < N_{od} < 0.8$
Significant damage (generally unstable)	$N_{od} > 0.8$

The transition is partly due to irregularities of rocks and waves (stochastic band width) and partly due to incompleteness of the model. The transition can be interpreted as that for some toe bunds the generally stable cases have damage up to N_{od}=0.8 and that for other bunds significant damage may start at N_{od}=0.4. Figure 97 shows that damage levels up to N_{od}=0.4 occur for gradings with larger rocks than critical. Eventually damage levels decrease to 0 for increasing rock size.

f. What amount of damage should be considered as acceptable or severe?

Gerding classified N_{od}=4 as loss of function (of the structure). But for some breakwaters this damage level may not be severe at all. In this research it is recommended to regard this issue from the opposite side, see section 5.4. To obtain a reliable design, the minimum required dimensions for a toe bund should be determined. Then damage is not severe, as long as rocks are not removed from within the minimum dimensions. The bund dimensions in the final design should be so much larger than minimum, that the structure can cope with the expected damage. The minimum dimensions can be different for the construction stage and during the lifetime. Implementation of this approach in present design practice is difficult, because present manuals do not provide minimum toe bund dimensions for specific toe functions.

6.2 Conclusions regarding the problem and objective

In this section feedback is given on the problem and objective. These have been defined at the start of the thesis in sections 1.4 and 1.5.

The problem for this thesis research was defined as follows:

Design tools for toe bunds of rubble mound breakwaters that are presently used in engineering practice lack a reasonable degree of accuracy to be able to assess required element dimensions.

In this thesis a new approach to the problem is chosen. The scatter in previous models could not have come from parameters like slope angle and foreshore slope, because these have not been varied in Gerding's tests. A more accurate model was required before expansion with additional parameters was useful. No new material model tests have been performed for this research. For this study a theoretical approach is chosen where possible, instead of the empirical approach. This study's model is at least as good as the previous models and additionally it provides new leads. The result of this research is not merely the final fit of the parameter model (Figure 92). The strength of the applied concept is just as valuable. It is essential that the local conditions at the toe bund are assessed. The stability of elements is evaluated for those local conditions, and no longer with a stability parameter that includes the wave height directly. The local stability problem is grasped from two sides, namely load (water motion due to wave action) and stability (critical value of the load), which are step 1 and step 2 in Figure 71. Unfortunately the characteristic parameter where these steps are coupled, which is $\hat{u}_b$, is not verified yet. Some relevant parameters in Gerding's tests were not measured or reported. An empirical total fit parameter Γ is applied for both step 1 and step 2 at once.

The objective of the research was defined as follows:

> *To improve the insight in the physical process related to stability of toe bund elements under wave load. This should eventually lead to more adequate guidance for breakwater design practice.*

There is resemblance of this study's model to the data set in absolute and relative sense (types A and B, see section 5.1). Because of this, the hypothesis is confirmed to a reasonable extent. The model, expressed in the presented equations, describes the governing physics. This is applicable as a more accurate design tool for the range of Gerding's data set. The model may become applicable outside that range as well if parts of the hypothesis are verified, which are included in the model but have not been varied in Gerding's tests.

6.3 Discussion and additional conclusions

In this section some additional conclusions, other than treated in section 6.1 and 6.2, are mentioned.

Data set parameters

The available data sets proved to be incomplete for this study because some parts of the hypothesis could not be verified. These are assessment of water velocities, threshold analysis and the use of wave periods. Some parameters were implicitly measured in Gerding's tests, but they have not been reported, like $T_{m-1,0}$ and the reflection.

Data set of Docters van Leeuwen

In Chapter 3 is put up for discussion whether the test results of Docters van Leeuwen really are that different from Gerding's. Docters van Leeuwen gives possible causes for a difference in test results, but it is not likely that the difference is as large as the presentation in Gerding's model space suggests (Figure 44). Both data sets live up to the hypothesis of this study, but there still remains a difference in the overall fit factor of 20%. Causes of the difference may be:

a. Foreshore slope gradient m is different for the two data sets. This may cause a small difference in stability.
b. The angle of internal friction of toe material plays a role in the development of damage, as was also concluded in DOCTERS VAN LEEUWEN (1996). No information is available on the internal friction angle in Gerding's tests.

c. The average breaker index is different for Gerding and Docters van Leeuwen, which shows that the conditions in the tests of Docters van Leeuwen were not always depth-limited.

The data set of Docters van Leeuwen has a difference with Gerding's data set, but the data sets of USACE and MAST series 1 seems to match Gerding's data set better. A large part of the data set of Docters van Leeuwen and a small part of the data set of Gerding are discarded because these tests do not represent possible target systems.

Damage as research topic

Two parameters are evaluated to describe damage, namely Gerding's original parameter N_{od} and the new parameter N_{odB}, which accounts for the toe width B_t. Eventually the description of damage has proven not to be that important altogether. In any way N_{odB} is not better than N_{od} (for Gerding's data set). The variety in absolute diameters in the tests is however limited. On the other hand, B_t was in fact varied in Gerding's tests.

Range of applicability

There are restrictions to the applicability of this study's model. These restrictions follow from the research scope and data set (Gerding) to which the model was eventually fitted. It is recommended to use this study's model for determination of the required toe rock diameter only for:

a. Standard toe bunds for conventional permeable rubble mound breakwaters with a rough front slope with $\tan\alpha \approx 0.67$
b. $H_s/h_t > 0.5$
c. $H_s/h_m > 0.35$

Toe design guidance

Design manuals provide varying recommendations on toe bund dimensions. Terminology in various sources can be a bit confusing as well. From one source "a berm" can be interpreted as a large and wide heap of armour material for a breakwater just below SWL. In another source "a berm" can point to a small bund near the toe. Manuals give armour layer support as a function for the toe bund. This may not always be required. More common functions are to aid in construction and to avoid extending the armour layer down to the bed.

Alternative calculation methods

Two methods are evaluated in this thesis for calculation of amplitude of wave induced velocity of water. These are the down rush energy method, which was developed in this study, and the previously available reflection method by Hughes. For the toe element stability problem applies that the down rush energy method provides better applicable results. For an element's capacity to withstand load, the criterion of Rance/Warren provides better results than the criterion of Izbash. Both criterions were adapted to account for the influence of porous outflow through the toe bund.

6.4 Recommendations

This section is divided into three parts, namely recommendations on modelling, testing and toe bund design. There is overlap in these categories, because of these fields of study are interwoven.

Modelling and parameter influences

a. The final model of this thesis was calibrated by applying one single fit factor Γ for step 1 (hydrodynamics) and step 2 (critical load for toe rock). In view of theory, this factor is the ratio of γ_b and γ_{bc}. These parameters could not be assessed because of lack of data. This has to be conducted in further research.

b. A few parameters, other than Γ, were fit for this particular data set of Gerding. First of all how the head gradient i (or rather: i_{max}) is calculated, which is the force behind porous outflow. Furthermore the parameters C_{PF} and γ_{dr} are fit as a combination for this data. These parameters determine not only the shape of the data cloud, but adjustments to these parameters also shift the data cloud horizontally in Figure 92. This thus influences the appropriate Γ and safety factor for a specific design, which has to be regarded carefully.

c. Two important assumptions have been made during the derivation of the model:
   1. That down rush is normative for movement of toe elements.
   2. That flow, turbulence and accelerations can all be described by a single characteristic parameter û.

d. In the calculation of the critical velocity $\hat{u}_{bc}$ two adapted criterions were used, namely adapted Rance/Warren and adapted Izbash, see section 4.6 and 5.2.1. It is recommended for further research to test a criterion based on shear stress as well, such as the method of Sleath (see Appendix A.1). The Sleath criterion may be in accordance with the Rance/Warren criterion, although it is rather set up for smaller grains (sand).

e. Better results and more practical relevance could be obtained by investigating how to apply regular wave theory on irregular wave test results. Moreover the problem was now regarded for the 2D flume situation only. Influences of 3D matters, such as tide-induced flows and obliquely incident waves, have not been included.

f. In available data sets, damage was measured by counting rocks. Therefore damage parameters are now based on the number of displaced rocks. Perhaps it could be interesting to investigate the use of other damage parameters if tests are for example based on profile measurements or photographic measurements of damage areas.

g. The model is based on situations with surging breakers on the slope, as they are for most rubble mound breakwaters. Plunging breakers probably produce a different water motion at the toe bund. Smaller waves than the design wave may even be normative for toe elements, if plunging breakers prove to have more effect on toe element stability.

h. A numerical model may also be used to assess local water velocities at the bund. A proper option may for example be the numerical model ODIFLOCS (VAN GENT 1994). Perhaps a method can be found to implement rock stability in a numerical model.

Testing

i. For understanding the physical process, some adjustments to the test method can be applied to reduce scatter. Thereafter, if the physics are understood well, the difference with stability in more natural and capricious conditions can be assessed. Testing with regular waves makes local velocity assessment easier. This also leads to a better discernable threshold of movement for the rocks.

j. Pre-testing the breakwater structure with smaller waves can decrease the scatter that is due to movements of unstably placed rocks. These movements happen in reality as well of course, but it is not a part of the mechanism we want to investigate at first hand, scientifically spoken. This increases insight in the minimum damage level beneath the critical load. Furthermore it is questionable whether these unstable rocks add to the stability of the system.

k. In the applied parameter model of this thesis no distinction is made between a depth-limited and non-depth-limited situation. The model is compared with the tests of Gerding which are mostly depth-limited. Further testing should verify whether the model also applies outside Gerding's range or an adjustment is required.

l. After a study is finished, more parameters may prove to be useful than were important for the aimed research. For example, Gerding did not report all measurable parameters, which is unfortunate for this research. Concerning damage specifically, it is interesting to denote where displacements actually occur, namely on the edge or on the top surface of the bund. This tells whether N_{od} or N_{odB} should be used. Insignificant damage might start anywhere and that significant damage might start on the edge of the bund. If the toe has flattened a bit, higher significant damage may thereafter develop anywhere on the bund, like on the top surface, edge and front slope (if still discernable). If displaced rocks had been weighed after each test, we could conclude whether the smaller rocks from the grading are the first to displace.

m. For investigations on value of C_{PF} it can be useful to study regular wave tests and vary the rock diameter and density rather than the wave height and period. In this way the load on the rocks can be kept equal. Description of local conditions in terms of boundary parameters is then less important. Analysis as in section 4.3.2 part c. can be applied for parameters that influence the value of C_{PF}.

Toe bund design

n. In Chapter 2 was mentioned that the required dimensions for the toe bund itself should be dependent on the predefined functions that are subscribed to the toe structure. This topic has not recurred further in the research, because it was not within the research boundaries. To make a design manual complete, this has to be investigated. It raises questions like: What are the functions that toe structures may have to perform (briefly inventoried in section 2.2.2)? and: What are the minimum required dimensions to perform those functions? The envelope of the minimum dimensions for the bund can be different for the construction phase and the service phase.

o. Besides the armour-support function there are many other possible functions for a toe bund. It is questionable whether a toe bund is indeed required to ensure the stability of the armour layer. Other reasons may be more important to apply a toe bund. Sometimes a toe bund may not be required at all.
With the numerical soil simulation software Plaxis, some swift preliminary investigations have been performed during this study. These concerned the influence of a toe bund on the overall stability of a breakwater and the influence of the toe bund on the stability of the armour layer (armour support function). It seems that these influences of a toe bund, with dimensions as meant in this report, are limited. The overall stability of an armour layer may only become the critical mechanism when the subsoil is very stiff and there is very small friction between the armour and the rocks of the layer underneath it. Even when this mechanism is critical, the design may stay within the safety limits. There are however many limitations to the performed simulations. Mainly these are caused by large differences in stiffness of the separate layers (including the subsoil and the model boundaries), which are difficult to model numerically. The calculations sometimes led to (numerical?) failure of the breakwater by micro-instability before larger failure mechanisms of interest could really develop.

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Appendix A

Additional literature

A.1 Stability of grains water motion

Uniform flow

(summary of SCHIERECK 2004)

Stability of non-cohesive loose sand and grains is described in SCHIERECK (2004). The basis is the equilibrium of forces on a grain in a bed. By the uniform flow, three forces are exerted, namely

a. Drag force
b. Shear force
c. Lift force

These are proportional to

$$F = C\rho_w u^2 A$$

in which C is a coefficient, with C_D for drag, C_L for lift and C_S for shear. A is the surface on which the force acts, which is proportional to D^2. The stabilizing force is submerged weight, which is expressed in

$$W = (\rho_s - \rho_w) g D^3$$

This results in relation between load and strength in the form

$$u_c^2 = C\Delta g d \qquad \text{with} \qquad \Delta = \frac{\rho_s - \rho_w}{\rho_w}$$

Herein u_c is the critical velocity at which elements start to move. The constant C has to be found empirically.

Izbash expressed this relation for local water motion (no depth influence) in the dimensionless form

$$\frac{u}{\sqrt{\Delta g d}} = C \qquad \text{and fitted} \qquad \frac{u}{\sqrt{\Delta g d}} = 1.7$$

A well-known formula for stability of elements in uniform flow is the one by Shields. Shields gives a relation based on shear stress on the bed due to the flow:

$$\frac{\tau}{(\rho_s - \rho_w)gd} = \psi$$

Herein ψ is the stability parameter. In each of Shields' experiments the transport was measured. A measurement for a threshold of motion was found by extrapolating the transport down to zero, see the figure below.

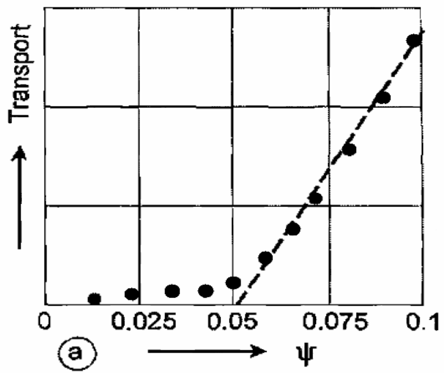


Figure 98: Determination of threshold of movement for comparable situations (in terms of Reynolds number) by Shields in 1936 (figure from SCHIERECK 2004)

In this way, a critical value ψ_c for the stability number was found, where the subscript c stands for the critical value.

Oscillatory flow

With the criterion of Shields thresholds could be determined for uniform flow. RANCE *et al.* (1968) presented experimental data from which it is possible to predict the threshold of movement of shingle for oscillatory water motion. Before conducting the tests, they tried to define initiation of movement in a precise way, similar to the method of Shields. They constructed probability curves for the distance moved by a particle and plotted that against velocity amplitude. The point of intersection with the axis was taken as the threshold condition.

Observation showed three phases in bed movement, namely firstly rocking, secondly movement over a short distance by some stones, and finally, with a small increase in velocity, many grains in motion. The second phase was found to coincide with the threshold, and was therefore further used for analysis. By this method the observer was 'calibrated' and could visually determine the threshold of movement. The tests were set up to simulate a period range of 5-15 s and an orbit range of 0-4.5 m, therefore very suitable for coastal waves. The oscillatory motion seems to have been tested for regular wave conditions. From plotting the acceleration number against the Froude number, it is concluded that neither drag force nor acceleration force can be considered insignificant.

The result of the research by Rance and Warren was:

$$\frac{a_h}{T^2 \Delta g} = 0.025 \left(\frac{a_h}{D_{50}} \right)^{-2/3}$$

in which a_h is the orbital stroke, defined as $a_h=\hat{u}_h/\omega$. Using this and $D_{n50}=0.84D_{50}$ results in (with left and right hand side of the equation in units [m]):

$$D_{n50} = \frac{2.15 \cdot \hat{u}_h^{2.5}}{T_p^{0.5} \cdot (\Delta g)^{1.5}}$$

In 1978 Sleath summarized various measurements (SCHIERECK 2004), among which those of Rance and Warren, in oscillating flow, which resulted in an adaptation of the Shields diagram for waves in stead of uniform flow. For a large diameter (with a turbulent boundary layer), Sleath found a constant value of $\psi_c = \hat{u}_{c*}^2/\Delta gd$ to be 0.055. Sleath's result is summarized in the following diagram, together with the original result of Shields for uniform flow.

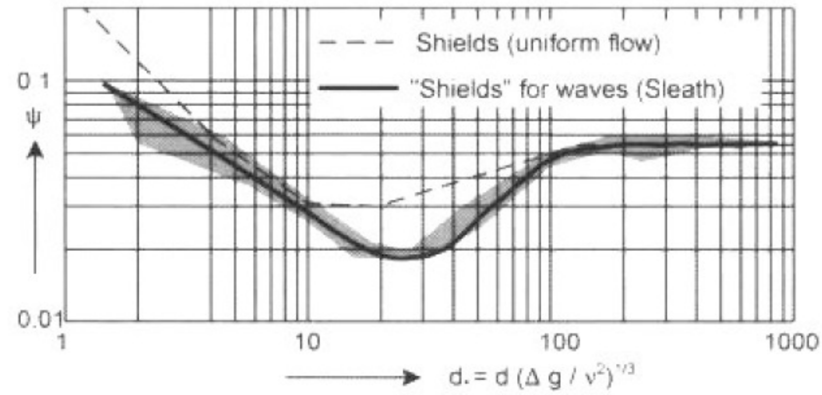


Figure 99: Sleath curve for grains in oscillatory motion (from: SCHIERECK 2004)

A.2 Down rush, run-down and run-up

In this report by down rush is meant the downward water movement on the slope of the structure between the moment of the highest extreme water level (run-up R_u) and the moment of the lowest extreme water level (run-down R_d) reached by a wave.

ECKERT (1983) describes that down rush is responsible for scour of bottom sediments in front of rubble mound breakwaters. This thus seems to be a heavier load on toe elements than the water motion running up the slope.

According to BRUUN (1985), decrease in primary armour weight should only be made below an elevation where individual wave run-down R_d may penetrate. The run-down can be calculated with research results of Günbak (1979):
$R_d/H=-0.27 \cdot \xi$ for $\xi < 3.7$ and
$R_d/H=-1.0$ for $\xi > 3.7$
Here for H the maximum wave height should be used, where $H_{max}/H_s=1.8$. For large ξ-values (steep slopes) no decrease in armour weight is allowed until below 2 H_s.

So ξ is an important parameter for run-down and down rush, although the way to calculate ξ is not specified. The relation is bent at ξ = 3.7, as depicted in Figure 100. An explanation could be that around this value the transition was in Günbak's tests between surging and plunging. These are different water motions over the toe, which represent therefore a different load.

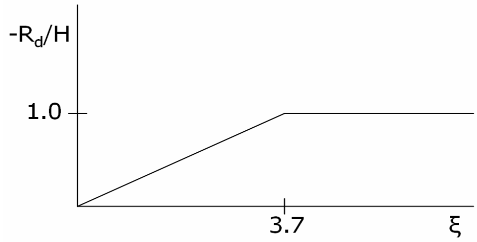


Figure 100: Run-down dependent on Iribarren number according to Günbak

The Rock Manual (CIRIA 2007) provides a design tool to calculate run-down and run-up. These levels are defined vertically relative to the still water level (SWL). R_d has a positive value if it is below SWL. For run-down on rough slopes of porous, rubble structures, the following formula is recommended:

$$\frac{R_{d2\%}}{H_s} = 2.1\sqrt{\tan\alpha} - 1.2P^{0.15} + 1.5e^{-60s_{om}}$$

R_u on the other hand has a positive value if it is above SWL. The basic approach is that run-up (relative to the incident significant wave height) is a linear function of the surf similarity parameter ξ. For simple configurations a Rayleigh distribution may be assumed for run-up levels, because of which only one characteristic parameter is required to know the entire distribution. Usually the 2% exceedence level is used. For rough permeable slopes with a permeable core the relative run-up has a maximum of 1.97. The formula to be used in the case of tests of Gerding and Docters van Leeuwen is:

$$\frac{R_{u2\%}}{H_s} = 1.17\xi_m^{0.46} \quad \text{for} \quad \xi_m > 1.5 \quad \text{with a maximum of 1.97}$$

For exceedence levels with higher frequency, formula's are also provided. The highest percentage given is 10%. The formula in this case is:

$$\frac{R_{u10\%}}{H_s} = 0.94\xi_m^{0.42} \quad \text{for} \quad \xi_m > 1.5 \quad \text{with a maximum of 1.45}$$

A.3 Reflection

In this section reflection is described firstly according to VAN DER MEER *et al.* (2006) and thereafter according to DEKKER *et al.* (2007).

Van der Meer et al. (2006)

In physical model testing reflection must be compensated to be able to generate a predefined incoming wave field. This means that for all stability tests etcetera reflection data are available, if properly denoted. Recent European projects DELOS and CLASH have produced much reflection data for all kinds of structures. Analysis of previous data and formulae has been performed, including Postma (1989) and Davidson (1996) for rock slopes and Allsop (1989) for armour units. In the analysis the surf similarity parameter ξ_0 is used (see box 1 in section 3.1.2). For the wave period the spectral period at the toe $T_{m-1,0}$ is used which is $T_p/1.1$ for single peak spectra (such as the JONSWAP-spectrum). With the existing datasets Van der Meer developed a new formula, which represents physical bounds and is compatible for all slope types. It is given by:

$$K_r = \tanh(a \cdot \xi_0^{\ b})$$

in which ξ_0 is calculated with $T_{m-1,0}=T_p/1.1$. The scalars a and b differ with varying slope types and are given by the following table:

Table 5: Coefficients a and b to be included in the reflection formula.

	a	b	γ_f
Rock permeable	0.12	0.87	0.40
Armour units	0.12	0.87	various
Rock impermeable	0.14	0.90	0.55
Smooth	0.16	1.43	1.00

In this table γ_f is the roughness factor that is used in the overtopping discharge formula in the same paper.

The following figure displays the data fitted with the new formula for rock permeable and armour unit slopes.

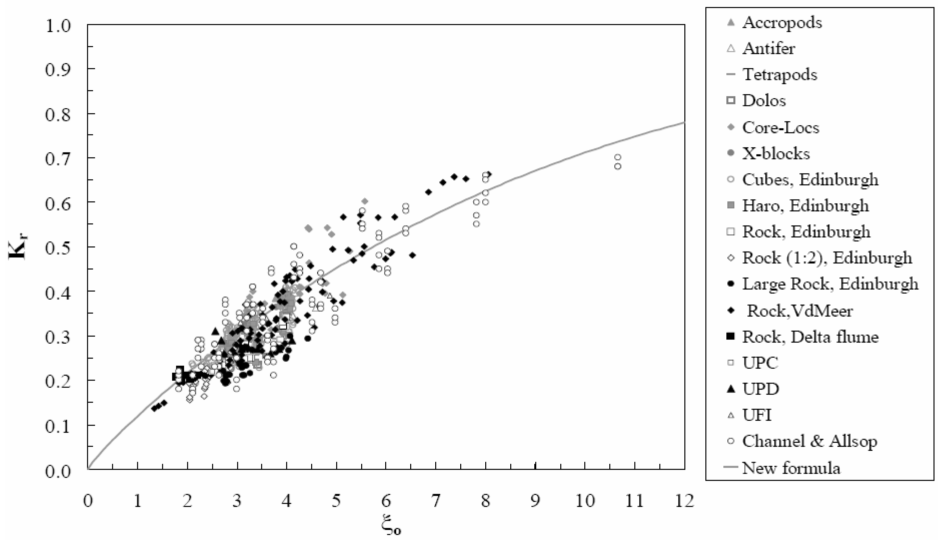


Figure 101: Relation between K_r and ξ_0 for rock permeable and armour unit slopes.

Dekker et al. (2007)

Since the research of Battjes from 1974 (as also found in BATTJES 2001), wave reflection from coastal structures is usually expressed as a function of the Iribarren number (see box 1 in section 3.1.2). These formulas have commonly used T_p as a characteristic parameter for the wave period. Similar to the work of VAN GENT (2001) a wide range of local incident wave spectra have been analysed. In the choice for a characteristic period, $T_{m-1,0}$ proved to be more appropriate. Furthermore the new formula for the reflection coefficient is not based on the Iribarren parameter. In stead it includes breaker parameter ε as theoretically derived by Battjes (see also BATTJES 2001). This parameter is calculated by:

$$\varepsilon = \frac{4\pi^2}{g}\frac{H/T^2}{\sin^2\alpha}$$

For irregular waves Dekker recommendes to use $T_{m-1,0}$ as characteristic parameter for T and H_s for H. This leads to an empirically fitted equation for the calculation of the reflection coefficient K_r on rough permeable slopes:

$$K_r = 0.79 \cdot \exp\left(-2.0(\varepsilon \tan\alpha)^{0.85}\right)$$

(in the publication of Dekker et al. for the K_r the notation C_{rB} is used)

A.4 Adaptation of the Shields number for a bed with vertical seepage

In section 4.6 an adaptation to the criterion of Shields and of Rance/Warren was made. This section describes the similarity of that adaptation with the adaptation of the Shields number in FRANCALANCI *et al.* (2008).

In section 4.6 the balance of forces is described with the submerged weight of a rock. In the paper by Francalanci, this is split in the gravitational weight and the buoyancy force. The buoyancy force exists because of the (vertical) pressure gradient over the rock. The buoyancy force is the integral of pressure differences surrounding the rock, namely

$$\iiint \frac{\partial p}{\partial x_i} dV$$

In hydrostatic conditions the pressure gradient only act vertically, and thus the buoyancy force is

$$F_{pi} = \rho_w g \frac{4}{3}\pi\left(\frac{D}{2}\right)^3$$

A dimensionless number Nh, characterizing deviation from hydrostatic conditions, is defined as

$$Nh = \frac{1}{\rho_w g}\frac{\partial p}{\partial x_3}$$

Regarded for vertical flow and with the implementation of Darcy's law this provides

$$Nh = \frac{1}{\rho_w g}\frac{\partial p}{\partial z} = 1 + \frac{u_s}{K} = 1 + i$$

in which u_s is the seepage velocity, K is the hydraulic conductivity and i is the head gradient in the bed. The buoyancy force on an immersed particle is now described by

$$F_{pi} = \rho_w g \frac{4}{3}\pi\left(\frac{D}{2}\right)^3 \cdot Nh$$

where Nh is evaluated at the sediment bed. This results in an adaptation of the Shields number (see Appendix A.1) as

$$\tau_* = \frac{\tau_b}{(\rho_s - \rho_w Nh)gD} \text{ in which } \tau_b \propto u^2 \rho_w$$

From this point, the similarity with the derivation in section 4.6 of this report is shown by the following elaboration of the formula above.

$$\tau_* \propto \frac{u^2\rho_w}{(\rho_s - \rho_w Nh)gD} = \frac{u^2\rho_w}{(\rho_s - \rho_w(1+i))gD} = \frac{u^2\rho_w}{(\rho_s - \rho_w - \rho_w i)gD}$$

Dividing the numerator and the denominator by ρ_w results in

$$\tau_* \propto \frac{u^2}{\left(\frac{\rho_s - \rho_w}{\rho_w} - i\right)gD} = \frac{u^2}{(\Delta - i)gD}$$

The difference of this expression with the derivation in section 4.6 is the factor C_{PF}. This factor concerns:

a. the effect of decrease of head gradient due to outflow through the armour layer
b. the effect of rock shape irregularities
c. the effect of added mass by water surrounding the rock.

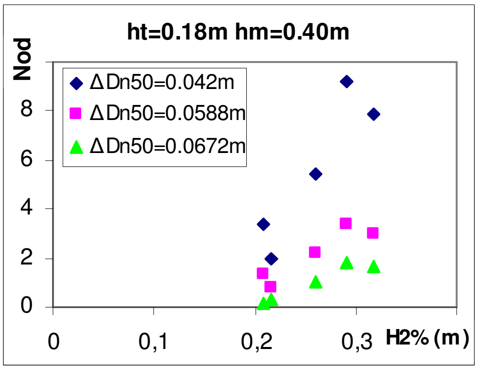
Other differences are that the nominal diameter and amplitude of velocity are used.

Appendix B

More threshold analysis figures

In this appendix more figures are presented that are used in the threshold analysis as described in section 4.3.2.

a. Damage number versus wave height



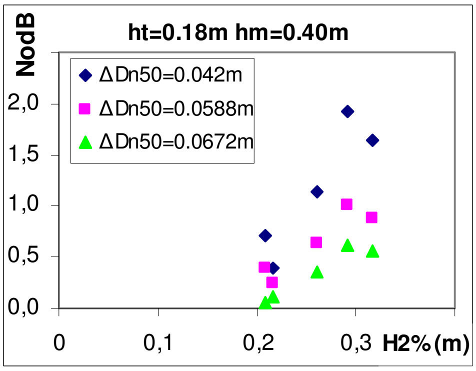


Figure 102: Data set of Gerding, damage number versus wave height analysis.

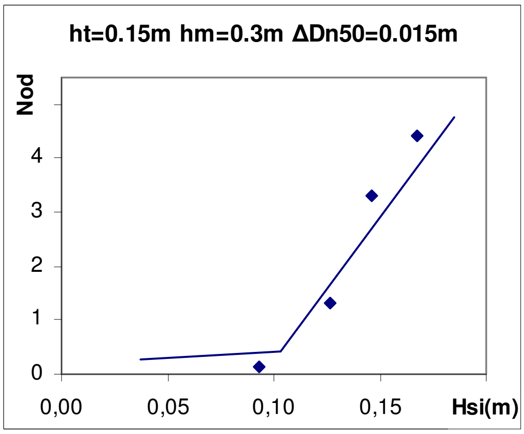


Figure 103: Data set of Docters van Leeuwen, damage number versus wave height analysis.

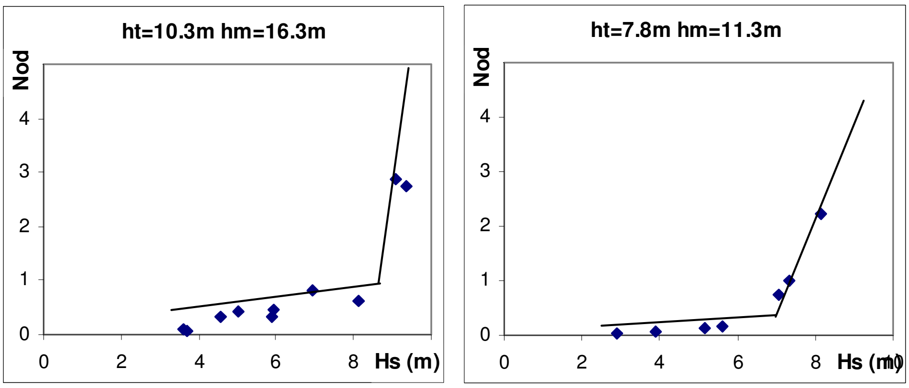


Figure 104: MAST data set, series 1. Damage number versus wave height analysis, two different geometric configurations. (D_{n50}=1.15 m)

b. Damage number versus toe depth

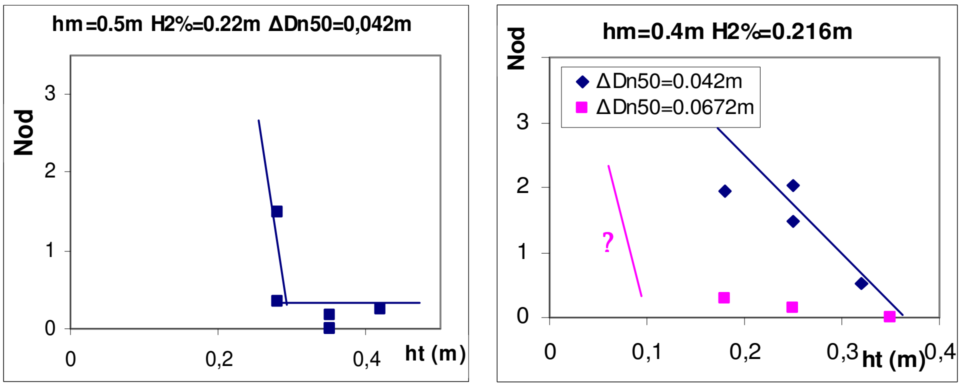
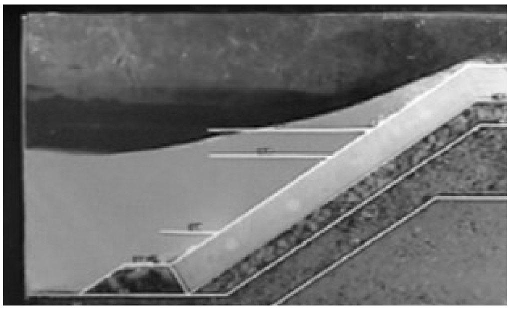


Figure 105: Gerding data set, damage number versus toe depth analysis.

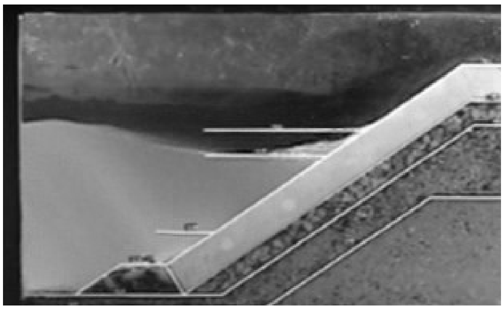
Appendix C

Screen shots

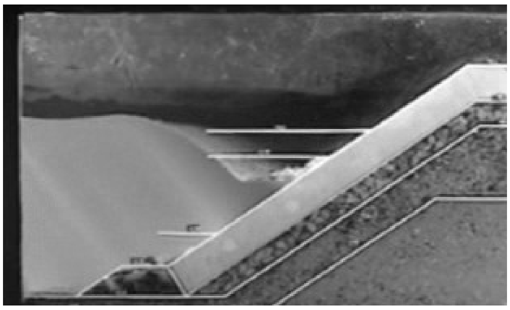
In this appendix screen shots are depicted from wave flume test movies. The screen shots in the figure below correspond with the sketches in section 5.5.



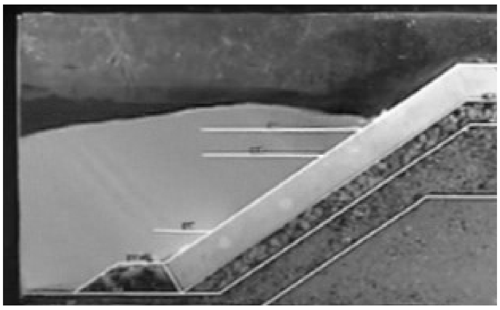
t = t (R_U)



t = t (R_U) + ¼T



t = t (R_D)



t = t (R_U) + ¾T

Figure 106: Screen shots from flume test movie for every ¼ wave period.

Appendix D

Empirical approach as an alternative

For this research it was chosen not to use an empirical approach to the problem, but to see whether a theoretical approach leads to more progress. The reason is the following. The shape of a cloud of data points and its slimness are governed by which parameters are chosen for the axes. This determines how well a curve can be fitted. It was expected that finding appropriate (dimensionless) combined parameters for the axes would be easier with theoretical concept than with choosing dimensionless parameters for an empirical approach.

In analyses such as Gerding's and Van der Meer's dimensionless combined parameters are used for a relation, which describes the influence that the parameters in the formula have on each other. If cases have the same value of a dimensionless parameter, the influences of that combined parameter are similar for the situations. If for example the relation from VAN DER MEER (1998) is used, two cases with the same value for h_t/h_m would require the same ratio of H_s and ΔD. Thus two situations with different absolute values for h_t and h_m are then comparable situations in this sense.

The relation of Gerding does not include h_m. In Van der Meer's relation h_t and h_m are equally important. Docters van Leeuwen showed that both these parameters have influence, but it is not equal. How can these parameters be separated to test their individual influence? For comparison they need to be dimensionless, because otherwise they would lay far apart (in a graph) from the target values. It would be possible to make both depth parameters dimensionless with dividing by another length-scale parameter, such as deep water wave length. Then some function would result in the form

$$\frac{H_s}{\Delta D_{n50}} = \left(\frac{h_t}{L_0}\right)^{\alpha} \cdot \left(\frac{L_0}{h_m}\right)^{\beta} \cdot \ldots$$

After an iterative process some values for α and β might be found with which a reduction of the scatter is obtained, compared to the original formula. But now a resulting influence of the wave length is introduced. And what should we do when this influence does not resemble what happens in reality?

Furthermore, the wave length has implicit influence on the stability parameter. One case namely with a certain value for a dimensionless parameter including L cannot have a fixed ratio of H_s and ΔD because there is a resulting influence of the wave length. So if H_s is increased (which would imply to simply use an equally larger D) this would change the wave steepness if L must be kept the same, or otherwise the ratio of H_s and ΔD would be different. Thus this proposed method with L will always include scatter, because of its basis.

Other methods with different dimensionless parameters might give a very usable result in design practice after some effort. An empirical method is not less valuable if it works. The idea for this research was however that a theoretical basis would lead to a good result in the quickest way, that it is better expandable in the future with other influences and that it would be better to try a different approach than the empirical methods so far, which seem to lead to a path where improvement is difficult.

The previous methods have attempted to directly couple the governing parameters like wave height and water depth to the results of the tests (in terms of damage). In the present method the damage is related to what physically happens near the elements of the toe themselves. Better insight in conditions for the elements should improve description of a parameter model for stability of toe elements. While theoretical background is included for understanding the stability behaviour of toe elements, complicated scaling of parameters to assess the influence of a single parameter like h_m is simply avoided.

Appendix E

Adding sine functions

The contribution to the seaward horizontal velocity over top surface the toe bund due to the incoming wave is calculated with a sine function. The contribution of the down rush is for that part also modelled as a sine function. Adding these two sine functions gives the resulting horizontal velocity. These two sine functions have the same period and therefore the result of the summation is also a sine function with the same period. We are primarily interested in the amplitude of this resulting velocity. The summation is depicted in the figure below:

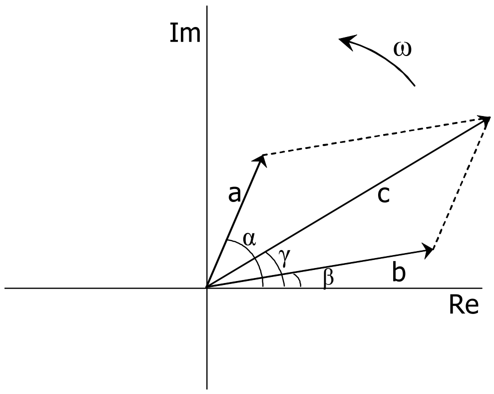


Figure 107: Two sine functions and their summation in the complex plane.

For calculation of the amplitude and phase a vector summation is applied.

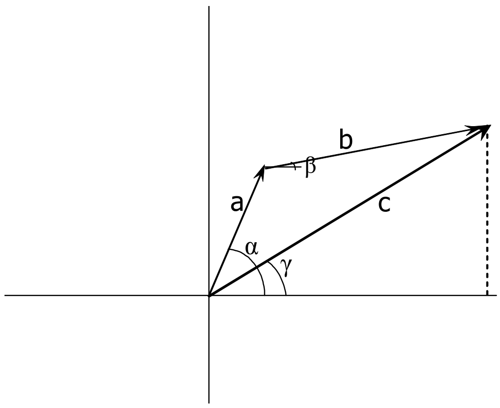


Figure 108: Summation of two vectors.

Summation of the two sine functions yields:

$$\hat{a}\sin(\omega t+\alpha)+\hat{b}\sin(\omega t+\beta)=\hat{c}\sin(\omega t+\gamma)$$

Now the amplitude and phase of the resulting sinus function are calculated. The amplitude is equal to the length of the resulting vector, which is calculated by:

$$\hat{c}=\sqrt{\left(\hat{a}\sin\alpha+\hat{b}\sin\beta\right)^2+\left(\hat{a}\cos\alpha+\hat{b}\cos\beta\right)^2}$$

For adding the velocity contributions in Chapter 4, we are only interested in the amplitude, so we fill in t=0 for convenience. Then $\alpha=\varphi_{TA}$ and $\beta=0$. Any other t will result in the same $\hat{u}_b$.

The phase is the angle between the vector and the x-axis, which is calculated by:

$$\gamma=\arctan\frac{\hat{a}\sin\alpha+\hat{b}\sin\beta}{\hat{a}\cos\alpha+\hat{b}\cos\beta}$$

Appendix F

Calculation sequence and comparison of model and data

In this appendix additional figures are shown that are used in the comparison of the model to the data sets as described in section 6.2. Furthermore section F.2 contains the description of how each parameter is exactly calculated.

F.1 Step B

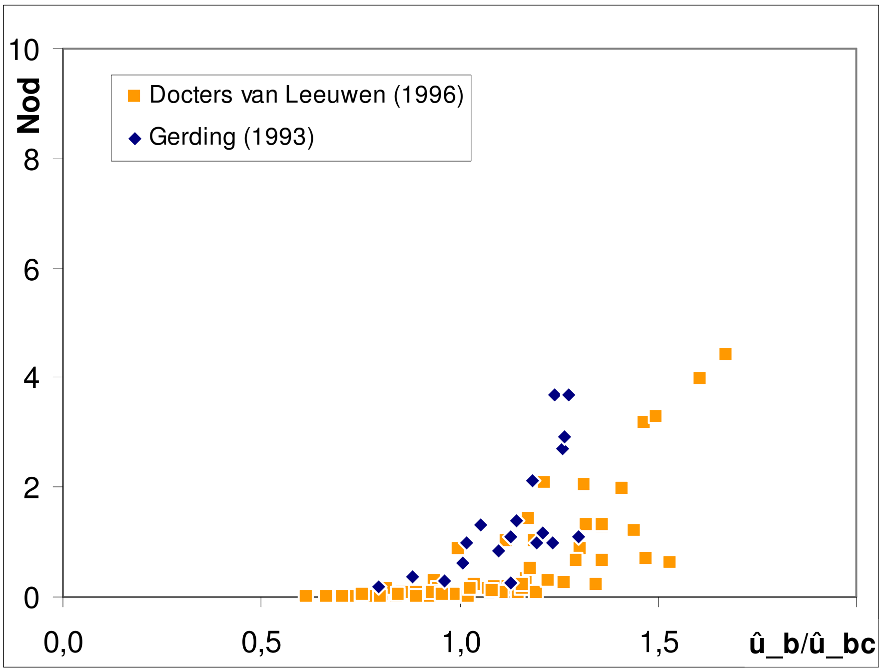


Figure 109: Discarded tests of Gerding and Docters van Leeuwen on the criterion $D_{n50} < 0.017$m ($B_t/D_{n50} > 7$).

F.2 Step C: calculation sequence

In this section is explained which parameters are exactly used in each step of the calculation methods for comparing different methods in section 6.2.3. Step by step is shown how the calculations are exactly done for Alternative 1A, which is the down rush energy method, calculated with H_s and T_s. In the table blocks, the header of each column in the calculation sheet is copied. The calculation is done per test in the rows under the column headers. The numbers of the equations correspond with the equation number in the report text.

First of all, the known parameters are given as the input.

Geometric configuration of the toe

h_m	h_t	B_t	tan(α)
[m]	[m]	[m]	[-]

Properties of the rock

D_n50	Δ
[m]	[-]

Wave characteristics

T_s	H_s
[s]	[m]

Herein T_s is calculated as $0.9T_p$, because T_p is the only available local period parameter. Preferably $T_{m-1,0}$ would have been used in the analysis, but this was not reported by Gerding. For alternative 1B, T_p is used for the period character and $H_{2\%}$ for the wave height.

Test results

N_od	N_odB
[-]	[-]

The number of displaced rocks was not given, so N_{odB} is calculated by $N_{od}/(B_t/D_{n50})$, see section 5.2.

Up to this point we have included the given parameters. Now we have to calculate the amplitude of occurring velocity and the critical velocity. First the amplitude of the contribution of the incoming wave is calculated with

$$\hat{u}_{bi} = \omega \frac{H}{2} \frac{\cosh(k(h_m - h_t))}{\sinh(kh_m)} \qquad (17)$$

For the local angular velocity ωs is used and for the local wave number k_s. These values are based on the local T_s and local wave length (thus for the H_s wave). Therefore L_s is calculated, by applying

$$L = L_0 \cdot \tanh\left(\frac{2\pi \cdot h_m}{L}\right) \qquad (24)$$

Now we calculate the wave number k_s with $2\pi/L_s$. Thus in the calculation sheet we have added for the contribution of the incoming wave:

L_s	k_s	ω_s	û_bi
[m]	[1/m]	[1/s]	[m/s]

Now we have to calculate the contribution of down rush and thereafter the phase difference between the two contributions. For the down rush we need the run-up and the run-down level. These parameters are calculated with the formulae from the Rock Manual (CIRIA *et al.* 2007) as is described in Appendix A.2. Unfortunately, formulae for significant run-up and run-down levels are not given, so we use the best available alternatives. For the run-up this is the $R_{u10\%}$ level for rough permeable slopes, which is

$$\frac{R_{u10\%}}{H_s} = 0.94\xi_m^{0.42} \quad \text{for} \quad \xi_m > 1.5 \quad \text{with a maximum of 1.45}$$

For the run-down on rough permeable slopes, only one formula is available. This best available option is:

$$\frac{R_{d2\%}}{H_s} = 2.1\sqrt{\tan\alpha} - 1.2P^{0.15} + 1.5e^{-60s_{om}}$$

For these calculations is used: notional permeability factor P=0.4 and wave steepness $s_{om}=H_s/(gT_m^2/2\pi)$. Surf similarity parameter ξ_m is also required in the calculations of the run-up and run-down. The value is obtained by

$$\xi_m = \tan\alpha \Big/ \sqrt{\frac{2\pi \cdot H_s}{gT_m^2}}$$

Since T_m was not recorded, the ratio $T_m=0.8T_p$ is used[19], which is for this particular calculation equal to $T_m=(0.8/0.9)T_s$. The amplitude of the down rush contribution can now be calculated by

$$\hat{u}_{bdr} = \gamma_{dr}\sqrt{2g\left(R_u/3 + R_d/2\right)} \tag{20}$$

in which for γ_{dr} eventually the factor value is used:

γ_dr
0,45

These calculations provide enough information to include the following steps in the calculation sheet:

ξ_m	R_u10%/H_s	R_u10%	R_d2%/H_s	R_d2%	û_bdr
[-]	[-]	[m]	[-]	[m]	[m/s]

Now the contributions by incoming wave and down rush need to be added up. Therefore the phase difference φ_{TA} is calculated by

$$\varphi_{TA} = kx = \frac{2\pi}{L} \cdot L_{TA} \tag{23}$$

[19] This ratio applies for the single peak JONSWAP spectrum (BATTJES 2001 and HOLTHUIJSEN 2007) that was imposed in the tests at deep water. This relation is probably not exactly valid anymore for shallow water and (partly) depth limited conditions, but it is the best available option.

in which for k and L the previously calculated k_s and L_s are used again. L_{TA} is calculated by

$$L_{TA} = {B_t}/{2} + \frac{h_t}{\tan\alpha} \tag{25}$$

The occurring velocity is the combination of the incoming contribution and down rush contribution. The amplitude of the occurring velocity is defined in

$$u_b = \hat{u}_b \sin(\omega t + \varphi) = \hat{u}_{bi} \sin(\omega t + \varphi_{TA}) + \hat{u}_{bdr} \sin(\omega t) \tag{22}$$

This $\hat{u}_b$ is calculated according to

$$\hat{c} = \sqrt{\left(\hat{a}\sin\alpha + \hat{b}\sin\beta\right)^2 + \left(\hat{a}\cos\alpha + \hat{b}\cos\beta\right)^2}$$

from Appendix E in which the difference between α and β is equal to φ_{TA}. For convenience is chosen to use α = 0 and β = φ_{TA}, but this choice for the point in time is of course irrelevant for the value of amplitude $\hat{u}_b$ of the resulting sine function. We are for now not particularly interested in the value of the phase difference φ between the maximum velocity at the bund and the moment of run-up. To confirm the hypothesis, it is interesting to investigate whether this calculation method applies. This means that it is interesting to visually confirm in further research whether rocks generally indeed start to move between $t(R_u)$ and $t(R_u)$ + ¼T, see Figure 74 and Figure 75.

The steps mentioned above result in the following parameters in the calculation sheet:

L_TA	φ_TA	û_b
[m]	[rad]	[m/s]

With these steps we have obtained the characteristic value for the amplitude of occurring velocities.

Next we proceed with the calculation of the critical velocity. This means we need to insert the adapted criterion of Rance/Warren, for which the upper bound of the head gradient i is calculated by

$$i = \frac{\Delta h}{\Delta x} = \frac{{H}/{2} + R_u}{L_{TA} + {R_u}/{\tan\alpha}} \tag{31}$$

All parameters in this formula have been calculated in preceding steps. The following equation is used for the critical velocity:

$$\hat{u}_{bc} = \left(0.46\sqrt{T} \cdot ((\Delta - C_{PF} \cdot i)g)^{1.5} \cdot D_{n50}\right)^{\left({1}/{2.5}\right)} \tag{37}$$

In the calculation sheet the following columns are thus added:

i_max	û_bc
[-]	[m/s]

and we include the factor C_{PF} for the noticeable head gradient in the toe

C_PF
0,4

Now we also have obtained the critical velocity. If the occurring velocity exceeds the critical velocity, rocks will displace.

When the results from the tests are grouped to the three classifications stable, transition and unstable, Figure 80 is obtained by using $\hat{u}_b$ and $\hat{u}_{bc}$ on the vertical and horizontal axes.

Figure 86 and similar figures are obtained by using N_{od} (or N_{odB}) on the vertical axis and the ratio $\hat{u}_b/\hat{u}_{bc}$ on the horizontal axis.

û_b/û_bc
[-]

Eventually the ratio of relative load is multiplied with the overall fit factor Γ:

Γ
1,05

by which Figure 92 is obtained.

F.3 Step C: Additional figures

In this section additional figures are shown that belong to step C in section 6.2.3.

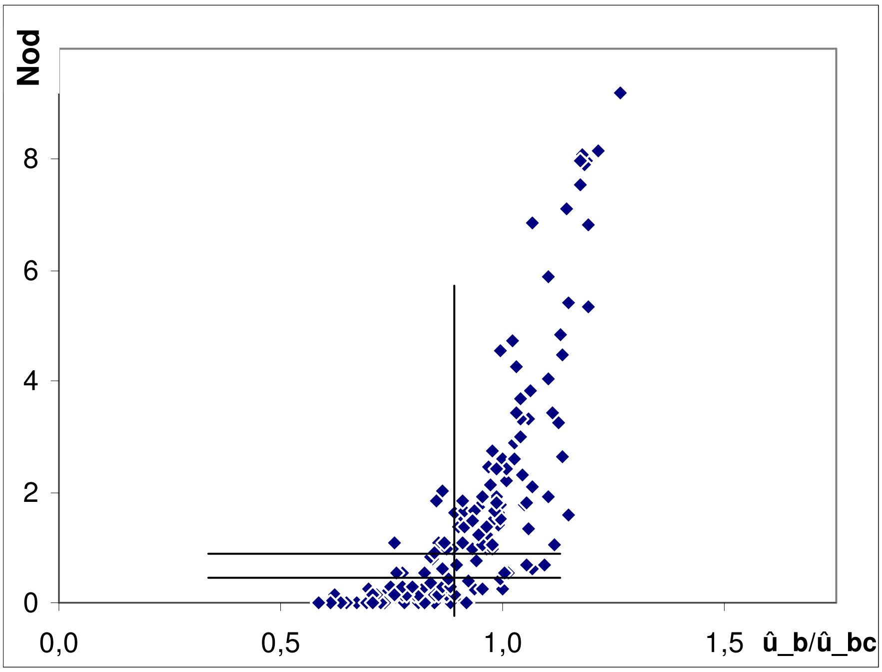


Figure 110: Alternative 1A: Un-tuned down rush energy method with H_s.

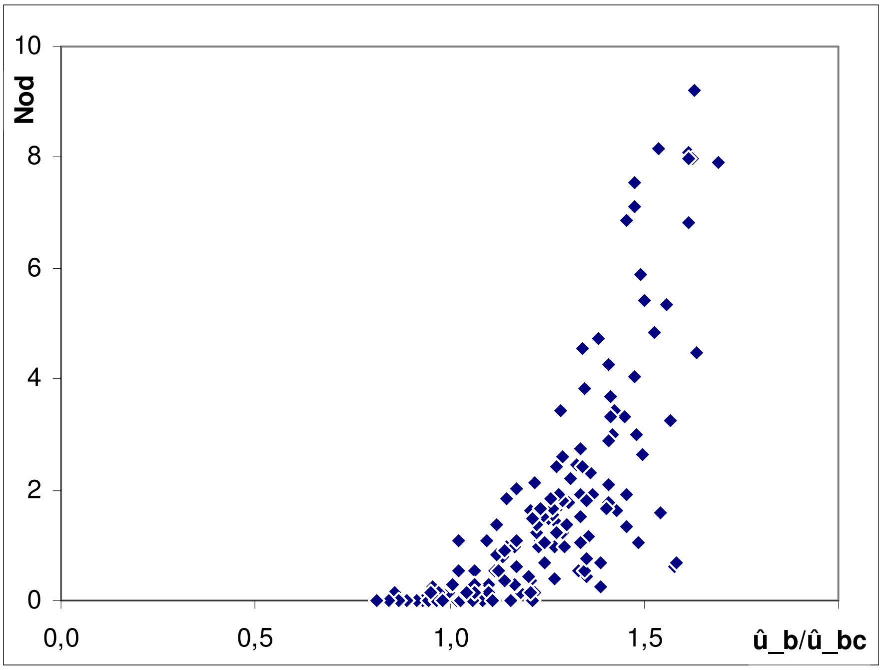


Figure 111: Alternative 1B: Un-tuned down rush energy method with $H_{2\%}$.

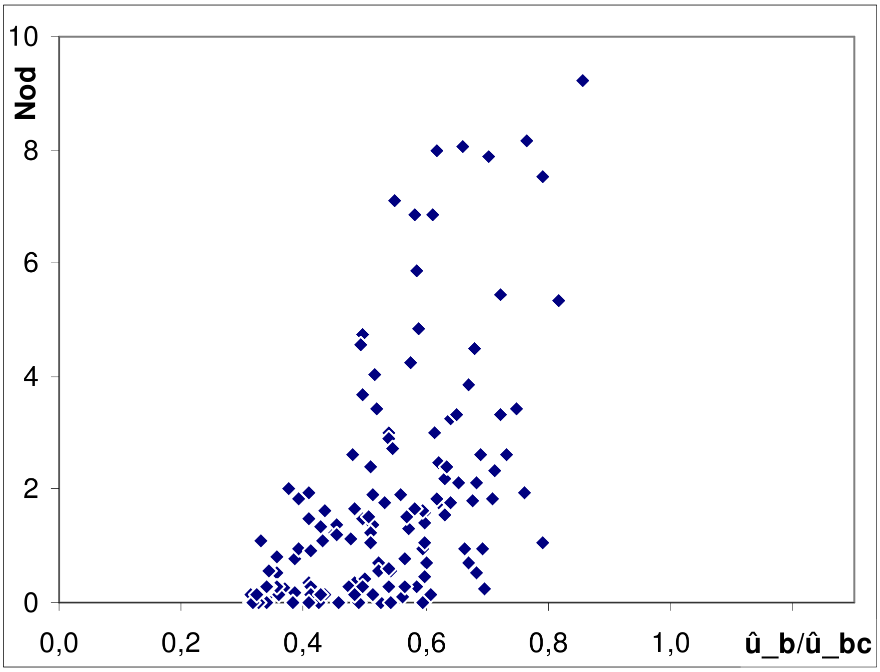


Figure 112: Alternative 2: Hughes-reflection method untuned, K_r by HUGHES (1995)

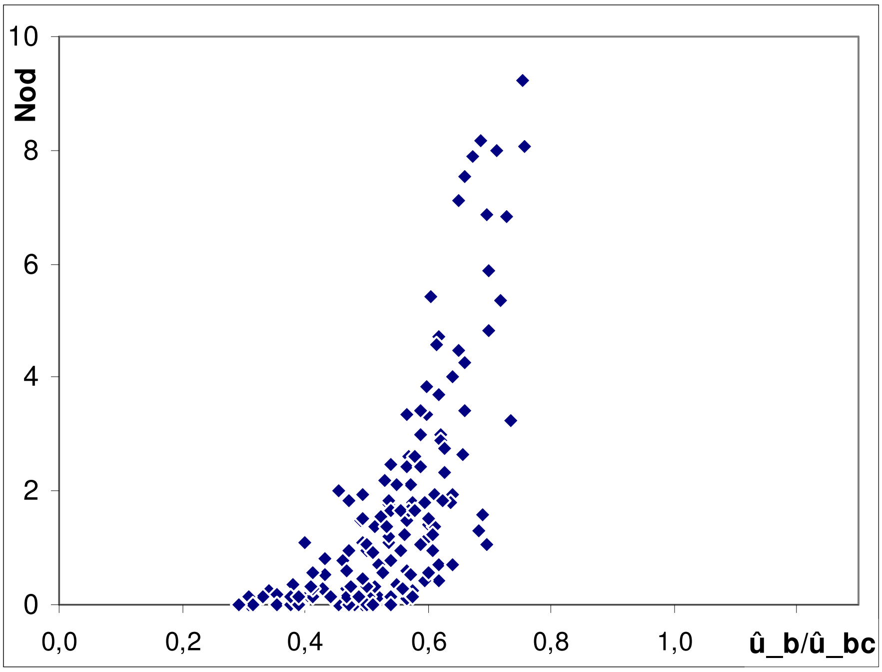


Figure 113: Alternative 2: Hughes-reflection method tuned, K_r by HUGHES (1995)

In the figure above the following tuned factors are used:

factor for T_p in ξ_h = 0.9 (un-tuned 1)   factor for θ =0.8 (un-tuned 1)
factor for K_r =0.4 (un-tuned 1) C_{PF} = 0.5 (first attempt 0.5)

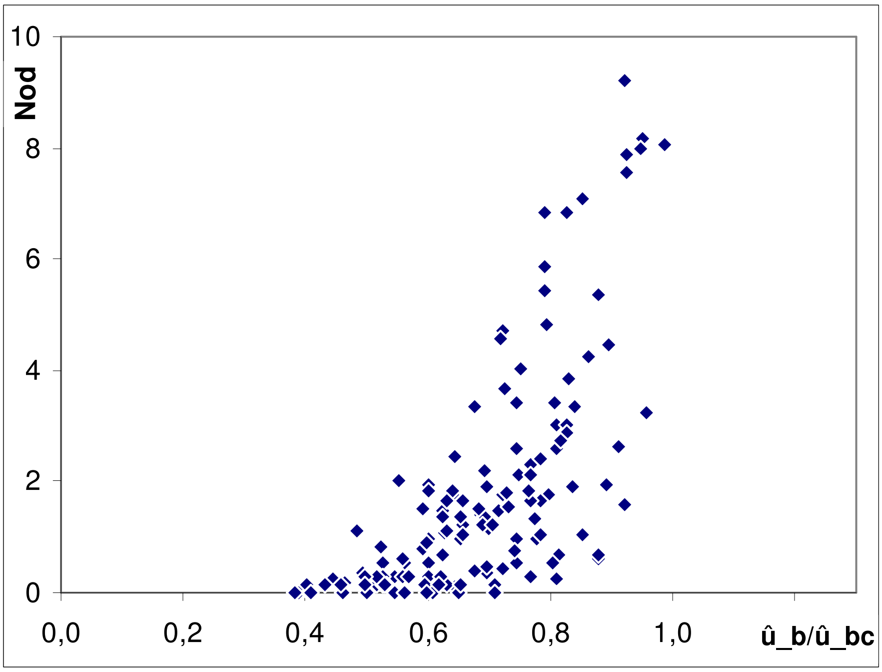


Figure 114: Alternative 2: Hughes-reflection method, K_r by Dekker (2007) and θ=π

In the figure above the factor for T_p in ξ_h = 0.9 (un-tuned 1) is used.

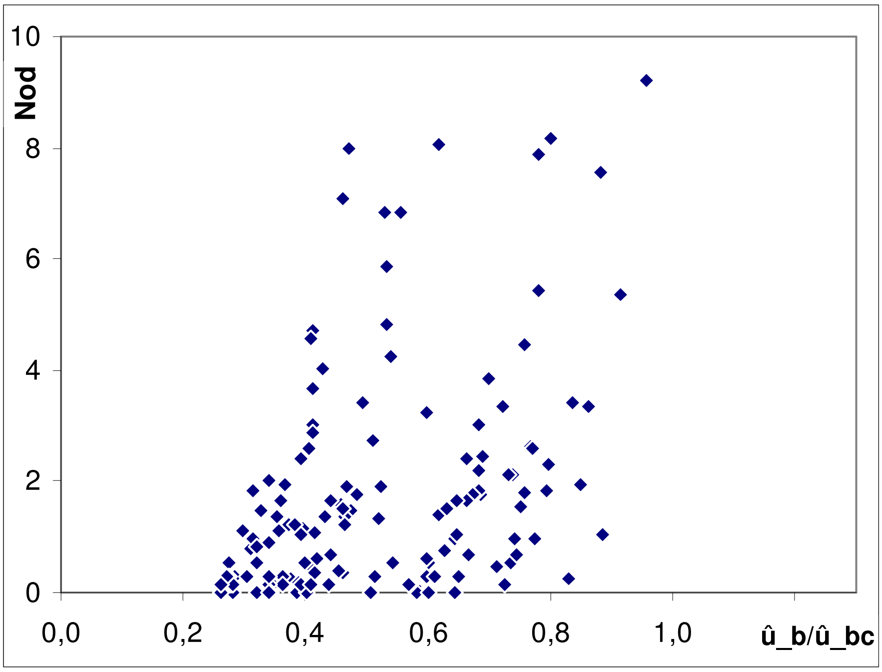


Figure 115: Alternative 2: Hughes-reflection method un-tuned, K_r by Van der Meer (2006)

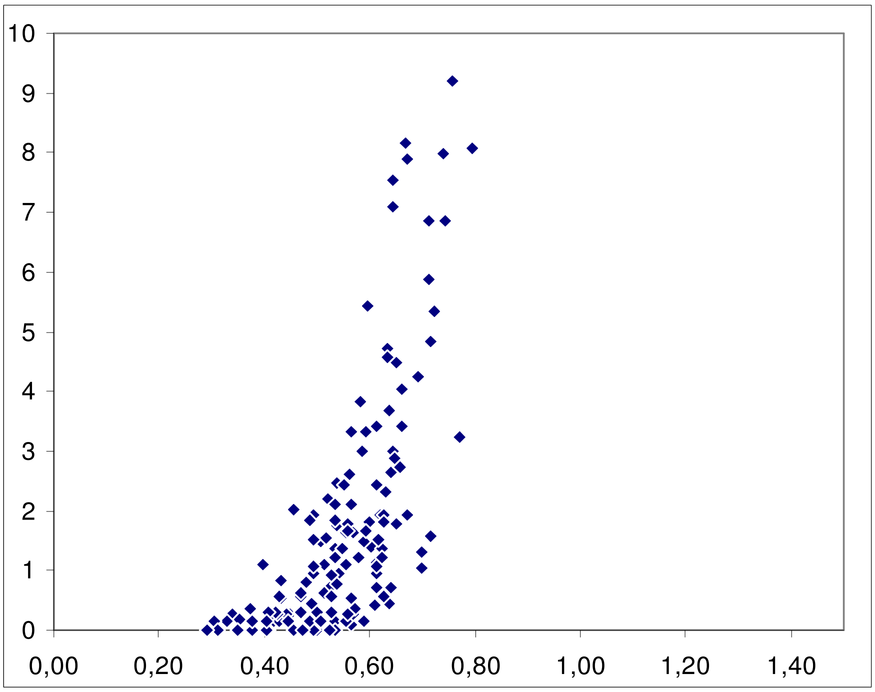


Figure 116: Alternative 2: Hughes-reflection method tuned, K_r by Van der Meer (2006)

In the figure above the following tuned factors are used:

factor for T_p in ξ_h = 0.9 (un-tuned 1)   factor for θ =0.8 (un-tuned 1)
factor for K_r =0.3 (un-tuned 1) C_{PF} = 0.5 (first attempt 0.5)

Appendix G

Design example

In this appendix a fictional example is worked out. The goal is to determine the required toe rock nominal diameter. In section G.1 the method of the present research is used. In section G.2 the obtained values are compared to the recommended values of the previous methods by GERDING (1993) and VAN DER MEER (1998).

G.1 Design example

Boundary conditions

In this example, a breakwater is designed to protect a new port entrance. With a numerical model simulations have been made to calculate the wave field transformations in the area where the breakwater is to be built, based on deep water conditions. The trunk of the breakwater is divided in separate sections. For each section boundary conditions have been determined. From the numerical model characteristic values of wave field properties are determined for the breakwater sections. It is decided to design a rubble mound breakwater with a permeable core and concrete cubes in the armour layer. For the trunk section in this example the boundary conditions are specified as follows:

$h_m = 8.5$ m H_s=4.7m

T_p=8.7 s T_m=6.9 s $T_{m-1,0} \approx T_s = 7.8$ s

There is a sandy bed, which is improved with a number of filter layers on the bed. On top of this filter the breakwater will be built. The seaside slope of the breakwater is 1:1.5. The armour layer consists of concrete cubes with $D_n = 1.45$ m and a layer thickness t = 3.2 m.

This is the first loop of the design cycle, which results in the preliminary design. After this cycle the design may be optimized and all trunk sections are geared to one another. But now the toe region is regarded and it is determined what general solution is to be applied. According to section 5.4 the following steps are taken:

Step 1: Functions

For this rubble mound breakwater trunk section the following functions are assigned to the toe:

A. The toe should act as an aid in construction (construction phase function). A so-called 'neck' is required for the placement of the lowest armour blocks (see Figure 23).

B. Support for the armour layer (service life function). This function follows from function A. Because the toe is underneath the armour, the toe has to stay intact at least in accordance with the criterion of Eckert (see Figure 8), plus a safety margin.

Step 2: Choice of structure type

It is decided that a toe structure is required to perform the functions for the toe. A toe bund is chosen as the structure type.

Step 3: Bund dimensions

The required bund dimensions follow from the assigned functions of step 1.

Function A

The dimension requirements are determined as follows: Seaside slope of the toe bund 1:1.5. Slope of the toe bund at the structure 1:1. The slope of the armour layer and therefore the underlayer as well is 1:1.5. But the opposite slope of the toe bund is not designed at 1.5:1, as the inverse of 1:1.5 (this would result in a squared angle). This is very hard to construct and not desired, since the armour layer consists of two layer of blocks. The second layer of cubes is now staggered with respect to the first layer. The minimum toe height $z_t = 3.2/\sqrt{2} = 2.3$ m. Therefore $h_t = 6.2$ m. For this function a toe width of 2 times the D_n of the armour layer is chosen, thus $B_t = 2.9$ m.

Function B

Toe bund dimensions should be in accordance with the criterion of Eckert (see Figure 8 in section 3.1.1). A safety margin is added. Here for example could be chosen that $B_t > 1D_{n(armour)}$.

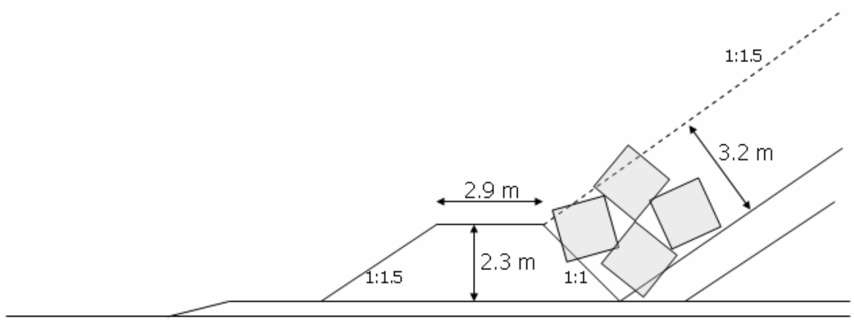


Figure 117: Chosen toe dimensions for this example case summarized in a schematic figure.

NB: Arguments for the kind of choices in steps 1, 2 and 3 have not been treated in more detail than discussed in Chapter 2. Therefore step 1, 2 and 3 are not elaborated in this example but only summarized.

Step 4: Material and size

The following material is available from a nearby quarry: local rock with Δ=1.68. Since this is the first loop of the design cycle, now the minimum required toe material size is determined. Thereafter in the second cycle will be evaluated how the quarry can be used efficiently (regarding material for the underlayer, core, harbour side slope, other trunk sections etcetera).

The required toe element nominal diameter D_{n50} will be calculated. A high safety is required for this project, since damage will not be visible and maintenance is expensive.

The material model tests of Gerding have relevant similarities with this design case (see Figure 30 in section 4.1). Therefore the method of this report is applicable, in which some fit factors were determined based on Gerding's data. It is thus chosen to use Γ = 1.05 and a safety factor in equation 39 of 0.94.

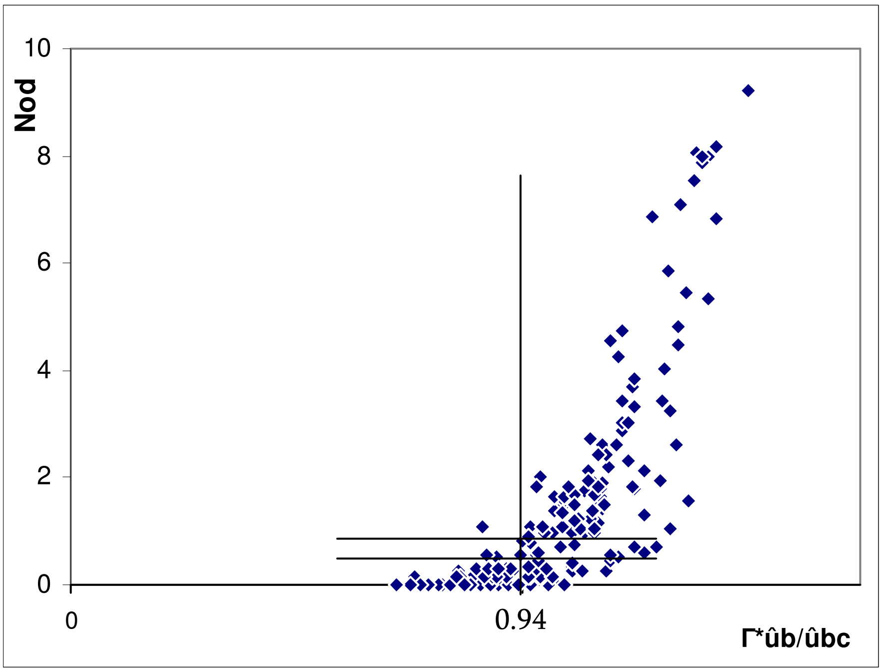


Figure 118: Design chart with Gerding data set for Γ = 1.05 and safety factor 0.94 for desired relative load (safe value).

We can see that there is only one test result (out of 152 in the selected data set) in quadrant 1 (see Figure 55). Therefore this diagram with safety value 0.94 is a safe design.

The calculation commences. In Appendix F.2 is shown how to calculate $\hat{u}_b$. Some values of intermediate calculation steps are given:

L(s)	k(s)	ω(s)	û_bi
[m]	[1/m]	[1/s]	[m/s]
64,5	0,10	0,81	2,10

ξ_m	Ru10%/Hs	R_u10%	Rd2%/Hs	R_d2%	γ_dr	û_bdr
[-]	[-]	[m]	[-]	[m]		[m/s]
2,67	1,45	6,82	0,67	3,15	0,45	3,91

L_TA	φ_TA	û_b
[m]	[rad]	[m/s]
10,9	1,06	5,26

Thus $\hat{u}_b$ proves to be 5.26 m/s. To calculate the required D_{n50}, the value of $\hat{u}_{bc}$ we have to use is

$$\hat{u}_{bc} = \frac{\Gamma}{0.94} \cdot \hat{u}_b \tag{39}$$

Thus the required $\hat{u}_{bc} = 1.12 \cdot 5.26 = 5.87$ m/s.

The median nominal diameter now follows from

$$\hat{u}_{bc} = \left(0.46\sqrt{T} \cdot ((\Delta - C_{PF} \cdot i)g)^{1.5} \cdot D_{n50}\right)^{\left(1/2.5\right)} \tag{37}$$

as was shown in appendix F.2. Rearranging this formula leads to

$$D_{n50} = \frac{2.15 \cdot \hat{u}_h^{2.5}}{\sqrt{T_s} \cdot ((\Delta - C_{PF} \cdot i)g)^{1.5}}$$

Hereby for the required toe element nominal diameter is obtained: D_{n50} = 1.13 m.

Step 5: Extra material

It is assumed that during the construction phase no toe material will displace. Therefore extra material is already present for the service phase, which is more than the damage that can be expected for the design storm (N_{od} = 0.4 - 0.8). This means that no extra material has to be added. The structure can cope with the expected damage in the service phase. The toe structure will be safe and will remain able to perform its functions after a wave attack with design storm magnitude.

G.2 Comparison of the results to the methods of Gerding and Van der Meer

Present method

In the example calculation a conservative safety factor for equation 39, namely 0.94 was chosen. If not a conservative approach was used, but the model that describes the average of the test results, this yields a safety factor of 1. This is shown in the following figure, which results in a required D_{n50} of 0.98m.

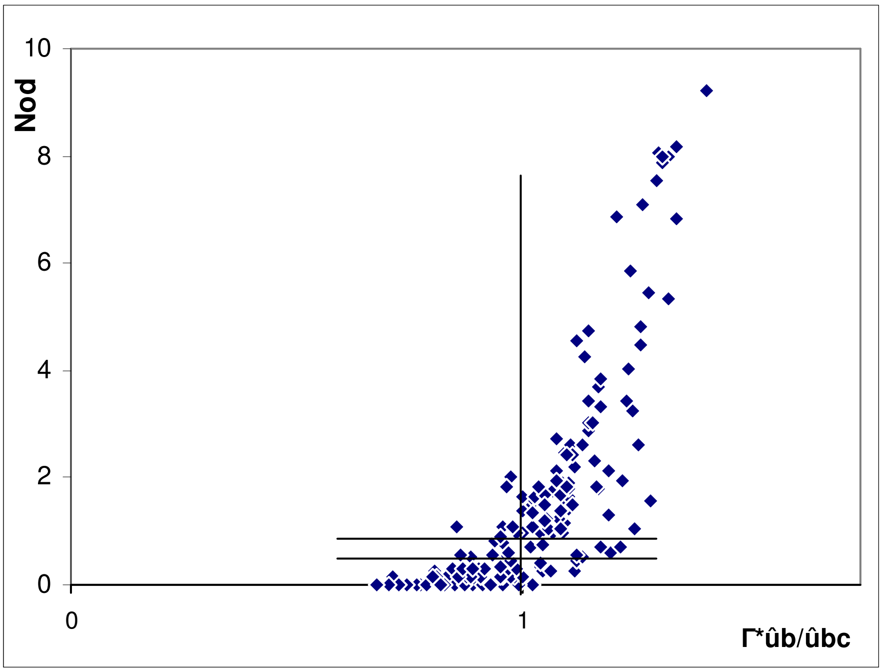


Figure 119: Best description of the physics with the present parameter model, for Γ=1.05.

If the demand would not be a safe structure, but the certainty that the toe structure is not over-dimensioned, we would have used a safety factor of 1.07. This results in the next figure.

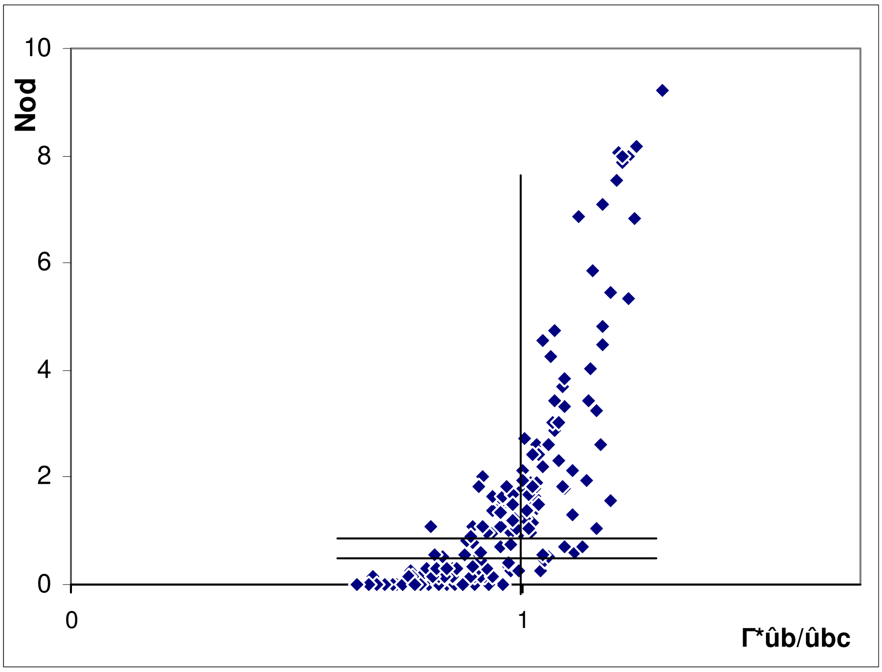


Figure 120: With Γ=1.05 and a safety factor of 1.07 (unsafe!)

For this value of the safety factor, there is only one test result (of the selected 152) in quadrant 3 (see Figure 55).

Gerding method

The design recommendation of Gerding is $N_{od} = 2$. The design formula is rewritten in a different order to obtain

$$D_{n50} = \frac{0.625 H_s}{\Delta N_{od}^{0.15}} - 0.15 h_t \tag{11}$$

For this design case this yields:

$$D_{n50} = \frac{0.625 \cdot 4.7}{1.68 \cdot 2^{0.15}} - 0.15 \cdot 6.2 = 0.65 m$$

Now the certainty of the damage level after a test (or storm in reality) is analyzed. In the graph below, the value in the domain (horizontal axis) is h_t/D_{n50}=9.5.

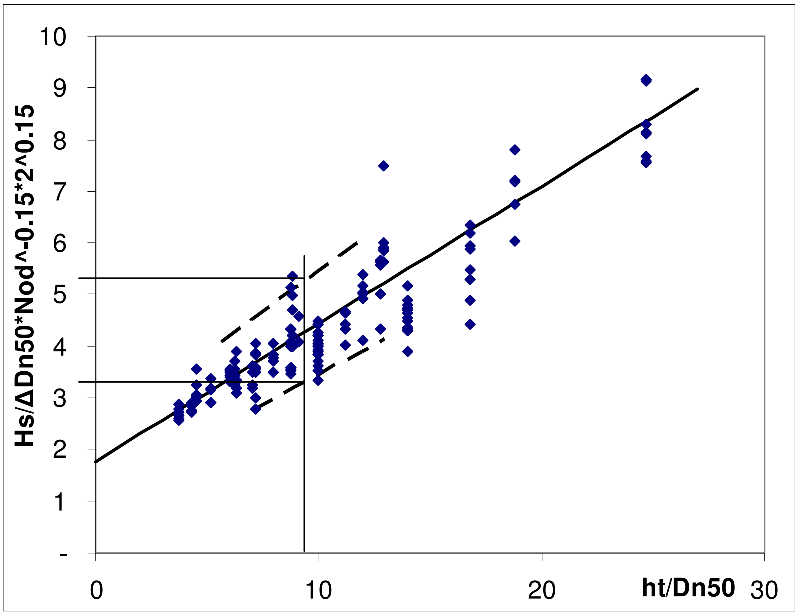


Figure 121: Method of Gerding (1993) with accompanying data set. Figure drawn for design recommendation N_{od}=2 (adjusted vertical axis for direct read out of required $H_s/\Delta D_{n50}$ value for this design recommendation)

In this figure the formula of Gerding is depicted together with his test results. The figure is the same as Figure 40 in section 4.4. Now the vertical axis is however multiplied with $2^{0.15}$ because then the read out value on the axis directly is the required value of the stability number. For the domain value h_t/D_{n50} = 9.5 the range (vertical axis) is between the boundaries 5.3 and 3.9 for the required stability parameter. Therefore the recommended value of D_{n50} is 0.65 m and the accompanying range is 0.53 m to 0.72 m.

If a design value of N_{od} = 0.5 had been used, the diameter sizes would have been a factor $(2/0.5)^{0.15}$ = 1.23 higher. This gives the recommendation D_{n50} = 0.80 m in the range 0.65 m – 0.89 m.

Another philosophy can also be applied for finding the upper and lower boundaries of the recommendation according to Gerding's model. Since D_{n50} also appears in the horizontal axis for this model, the boundaries also result in a different value on the horizontal axis. For the boundary lines of Gerding's data set in Figure 121 and Figure 122 the following equations are used:

upper line: $$\frac{H_s}{\Delta D_{n50}} = \left(0.28\left(\frac{h_t}{D_{n50}}\right) + 2.1\right) \cdot 2^{0.15}$$

which results in D_{n50} = 0.37 m with h_t/D_{n50} = 16.6

lower line: $$\frac{H_s}{\Delta D_{n50}} = \left(0.21\left(\frac{h_t}{D_{n50}}\right) + 1.0\right) \cdot 2^{0.15}$$

which results in D_{n50} = 1.22 m with h_t/D_{n50} = 5.1

This is depicted in the following figure:

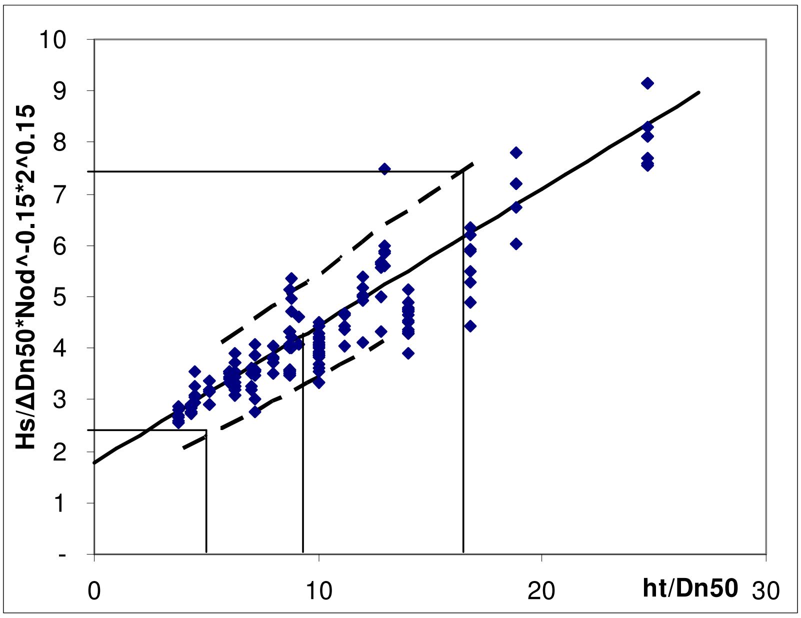


Figure 122: Recommended range (vertical axis) for this design case if the equations of the upper and lower boundary line are used.

We can see that the range (vertical axis) is enormous now. This illustrates that the Gerding formula is very sensitive to the exact values that are used in his fit.

Van der Meer method

In the paper that presents this method is stated that N_{od}=0.5 is a safe figure for design. Therefore opposed to the Gerding method we now use N_{od}=0.5. The formula of Van der Meer is:

$$\frac{H_s}{\Delta D_{n50}} = \left(6.2\left(\frac{h_t}{h_m}\right)^{2.7} + 2\right) \cdot N_{od}^{0.15} \qquad (8)$$

Rewritten for the required diameter this is:

$$D_{n50} = \frac{H_s}{\Delta}\left(6.2\left(\frac{h_t}{h_m}\right)^{2.7} + 2\right)^{-1} \cdot N_{od}^{-0.15}$$

This results for this case in D_{n50} = 0.67 m. For this example case, the relative toe depth h_t/h_m = 0.73. In the following figure Van der Meer's parameter model is presented together with Gerding's data set. The domain is the relative toe depth and the range is the required stability number.

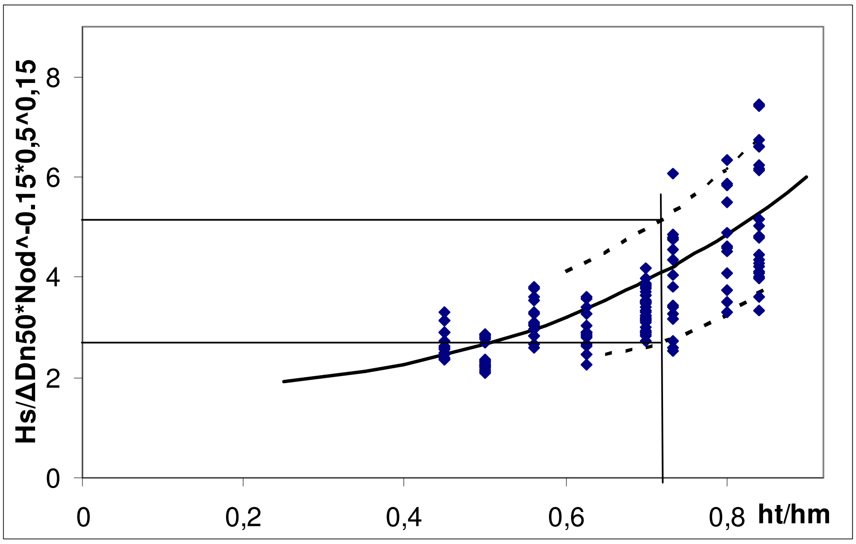


Figure 123: Method of Van der Meer (1998) with data set of Gerding (1993). Figure drawn for design recommendation N_{od}=0.5 (adjusted vertical axis for direct read out of required $H_s/\Delta D_{n50}$ value for this design recommendation)

The figure is almost equal to Figure 35 in section 3.2 and Figure 18 in section 2.1.2. The present figure is however slightly different because the vertical axis is multiplied with $0.5^{0.15}$. Now the absolute value that is read out on the vertical axis is the required value of the stability number. For this design case, we can see that for the domain value 0.73 (horizontal axis), the range is between the boundaries 5.2 and 2.7 (vertical axis). Therefore the recommended value of D_{n50} is 0.67 m and the range is from 0.53 m to 1.04 m.

Comparison

In conclusion can be said for the different methods:

Gerding 1993:

Aimed (and thus expected) damage level for material test or target system: N_{od} = 2

This damage level is to be obtained with a rock grading with D_{n50} = 0.65 m (for N_{od} = 0.5 this is D_{n50} = 0.80 m). From the way in which the model determines the shape the data cloud, we derive that this median D_n can vary between 0.53 m and 0.72 m and still result in the same damage level (0.65 m - 0.89 m for N_{od} = 0.5). This follows from the way in which the parameter model is presented. A crucial part of the model namely prescribes that the relation between wave height and damage number is a power function. If the range for D_{n50} is calculated with equations for the upper and lower boundary lines of the data set, this results in 0.37 m – 1.22 m.

Van der Meer 1998:

Aimed and expected damage level for material test or target system: N_{od} = 0.5

This damage level is to be obtained with a rock grading with D_{n50} = 0.67 m. The required median D_n of the grading can however lie between 0.53 m and 1.04 m and still result in the same amount of damage. The same statement applies for this method as for Gerding, since the model is based on the same power relation between damage and wave height. Figure 39 shows that for D_{n50} = 0.67 actually a damage result in the range $0.5 < N_{od} < 3$ should be expected.

Method of the present report:

Maximum damage to expect is between N_{od}=0.4 and N_{od} = 0.8.

A high certainty of this expected maximum result is obtained with using D_{n50} = 1.11 m. According to the description of stability of this method, damage will increase if a grading is used with D_{n50} < 0.98 m (threshold of movement). High certainty that no over-dimensioning took place is obtained by using a grading with D_{n50} = 0.82 m. For this diameter the probability is however very low that we have stable toe elements.

The following remarks are made:

a. Under the validity of the present research, in this example case the Gerding and Van der Meer method result in a serious underestimation of the required rock size. It is absolutely not claimed though, that this would be the case for all designs based on those methods.

b. Given the defined functions, the required bund dimensions are different for the construction and service phase for this example case. A damage level of N_{od}=2 would be acceptable here. The present method however claims that the damage level is unpredictable above the threshold, or that anyway the spread around the mean is very large. This can for example be seen in Figure 119 at the relative load value 1.1 or 1.2 on the horizontal axis (10 % or 20 % higher load than the threshold).

c. When the method of Gerding or Van der Meer is applied, using a smaller value of N_{od} in the formula of course leads to a safer design, because then a larger D_{n50} is recommended. But the certainty about the actual damage result (for material tests or for a target system) does not increase by using a lower damage level. If we fill in N_{od}=0.5 in the formula, we expect that the result of a test will be N_{od}=0.5. And if we fill in N_{od}=2, we expect that that will be the test result. The probability that this happens is equal for every damage level that is used as input for the formula. This applies because the parameter models prescribe a power relation between the stability number and the damage number. Therefore is claimed that the decrease in damage can be predicted for a certain increase in rock diameter. This furthermore implies that for any chosen damage level all data points in Gerding's set can supposedly be used to predict which D_{n50} is required. In other words, we can use the model for any damage level, but the data set as well. Therefore according to the model it is justified in Figure 121 and Figure 123 to multiply the vertical axis with a scalar and move not only the model, but the data set as well.

d. The range that is found from the graph for the Van der Meer method should be interpreted as follows: It is aimed to obtain a future test result or damage level to a target system of N_{od}=0.5. There is a distribution for the required D_{n50}. The mean of the distribution is given by the equation. If we use the lowest stability number, the probability is high that the actual damage value will not exceed the aimed 0.5. This results in the highest value for D_{n50}. If we use the highest stability number (thus lowest D_{n50}), we are not over-dimensioning the structure.

e. The same applies for the Gerding method. The boundaries of the Gerding method can however also be interpreted differently. If the boundaries themselves are used to calculate the required diameters, the domain itself (h_t/D_{n50}) changes as well. This results in an extraordinary large range for the required D_{n50}.

f. For the Gerding method a different design recommendation was used, with respect to the accepted level of damage, than for the Van der Meer method.

g. The parameter model of this report does not prescribe how much damage to expect above the threshold. The model describes however with more accuracy what we actually want to know as designers, namely what rock diameter should be used for a breakwater's toe bund. The present model focuses on describing the physical process at the start of distinct movement of rocks. This has expressed itself by attempts to:
- shape the data cloud in such a way that no test results lay in quadrants 1 and 3 (see Figure 55)
- determine the position on the horizontal axis of $\Gamma \cdot \hat{u}_b / \hat{u}_{bc} = 1$ (equation 38)

h. The present method is different from the previous methods in the sense that:
- Here it is not attempted to fit all data points to a curve (this is not particularly of practical interest).
- The damage level recommendation follows from the data cloud, thus from the test results. The level of damage that is called acceptable therefore follows from the damage that we may expect below the threshold, based on the tests.

Appendix H

Data sets

In this section the data sets are given that are used in this report.

H.1 Gerding 1993

test	deep T_m [s]	deep H_s [m]	deep $H_{2\%}$ [m]	deep h [m]	local T_p [s]	local H_s [m]	local $H_{2\%}$ [m]	local h_m [m]	h_t [m]	B_t [m]	tan(α) [-]	D_{n50} [m]	Δ [-]	N_{od} [-]	N_{odB} [-]
t1	1,36	0,161	0,234	0,90	1,56	0,151	0,220	0,50	0,42	0,12	0,67	0,017	1,68	0,18	0,03
t1	1,36	0,161	0,234	0,90	1,56	0,151	0,220	0,50	0,42	0,12	0,67	0,025	1,68	0,26	0,05
t1	1,36	0,161	0,234	0,90	1,56	0,151	0,220	0,50	0,42	0,12	0,67	0,035	1,68	0,00	0,00
t2	1,83	0,160	0,241	0,90	2,16	0,162	0,230	0,50	0,42	0,12	0,67	0,017	1,68	0,30	0,04
t2	1,83	0,160	0,241	0,90	2,16	0,162	0,230	0,50	0,42	0,12	0,67	0,025	1,68	0,79	0,16
t2	1,83	0,160	0,241	0,90	2,16	0,162	0,230	0,50	0,42	0,12	0,67	0,035	1,68	0,14	0,04
t3	1,59	0,207	0,304	0,90	1,80	0,197	0,290	0,50	0,42	0,12	0,67	0,017	1,68	0,97	0,14
t3	1,59	0,207	0,304	0,90	1,80	0,197	0,290	0,50	0,42	0,12	0,67	0,025	1,68	0,70	0,15
t3	1,59	0,207	0,304	0,90	1,80	0,197	0,290	0,50	0,42	0,12	0,67	0,035	1,68	0,00	0,00
t4	1,78	0,206	0,312	0,90	2,05	0,204	0,302	0,50	0,42	0,12	0,67	0,017	1,68	0,85	0,12
t4	1,78	0,206	0,312	0,90	2,05	0,204	0,302	0,50	0,42	0,12	0,67	0,025	1,68	1,14	0,24
t4	1,78	0,206	0,312	0,90	2,05	0,204	0,302	0,50	0,42	0,12	0,67	0,035	1,68	0,14	0,04
t5	1,78	0,205	0,321	0,90	2,05	0,212	0,300	0,50	0,42	0,12	0,67	0,017	1,68	1,09	0,15
t5	1,78	0,205	0,321	0,90	2,05	0,212	0,300	0,50	0,42	0,12	0,67	0,025	1,68	0,44	0,09
t5	1,78	0,205	0,321	0,90	2,05	0,212	0,300	0,50	0,42	0,12	0,67	0,035	1,68	0,14	0,04
t6	2,83	0,192	0,315	0,90	3,58	0,218	0,368	0,50	0,42	0,12	0,67	0,017	1,68	1,15	0,16
t6	2,83	0,192	0,315	0,90	3,58	0,218	0,368	0,50	0,42	0,12	0,67	0,025	1,68	0,61	0,13
t6	2,83	0,192	0,315	0,90	3,58	0,218	0,368	0,50	0,42	0,12	0,67	0,035	1,68	0,27	0,08
t7	1,77	0,248	0,371	0,90	2,01	0,234	0,340	0,50	0,42	0,12	0,67	0,017	1,68	0,97	0,14
t7	1,77	0,248	0,371	0,90	2,01	0,234	0,340	0,50	0,42	0,12	0,67	0,025	1,68	1,32	0,28
t7	1,77	0,248	0,371	0,90	2,01	0,234	0,340	0,50	0,42	0,12	0,67	0,035	1,68	0,41	0,12
t8	2,48	0,238	0,372	0,90	2,86	0,239	0,340	0,50	0,42	0,12	0,67	0,017	1,68	1,09	0,15
t8	2,48	0,238	0,372	0,90	2,86	0,239	0,340	0,50	0,42	0,12	0,67	0,025	1,68	1,58	0,33
t8	2,48	0,238	0,372	0,90	2,86	0,239	0,340	0,50	0,42	0,12	0,67	0,035	1,68	0,55	0,16
t9	1,39	0,155	0,225	0,70	1,56	0,141	0,193	0,30	0,22	0,12	0,67	0,017	1,68	0,61	0,09
t9	1,39	0,155	0,225	0,70	1,56	0,141	0,193	0,30	0,22	0,12	0,67	0,025	1,68	0,09	0,02
t9	1,39	0,155	0,225	0,70	1,56	0,141	0,193	0,30	0,22	0,12	0,67	0,035	1,68	0,00	0,00
t10	1,83	0,155	0,235	0,70	2,25	0,154	0,232	0,30	0,22	0,12	0,67	0,017	1,68	0,24	0,03
t10	1,83	0,155	0,235	0,70	2,25	0,154	0,232	0,30	0,22	0,12	0,67	0,025	1,68	0,26	0,05
t10	1,83	0,155	0,235	0,70	2,25	0,154	0,232	0,30	0,22	0,12	0,67	0,035	1,68	0,14	0,04
t11	1,60	0,199	0,287	0,70	1,83	0,169	0,232	0,30	0,22	0,12	0,67	0,017	1,68	2,12	0,30

t11	1,60	0,199	0,287	0,70	1,83	0,169	0,232	0,30	0,22	0,12	0,67	0,025	1,68	0,70	0,15
t11	1,60	0,199	0,287	0,70	1,83	0,169	0,232	0,30	0,22	0,12	0,67	0,035	1,68	0,00	0,00
t12	1,77	0,200	0,300	0,70	2,12	0,176	0,243	0,30	0,22	0,12	0,67	0,017	1,68	3,70	0,52
t12	1,77	0,200	0,300	0,70	2,12	0,176	0,243	0,30	0,22	0,12	0,67	0,025	1,68	1,93	0,40
t12	1,77	0,200	0,300	0,70	2,12	0,176	0,243	0,30	0,22	0,12	0,67	0,035	1,68	0,96	0,28
t13	2,03	0,198	0,312	0,70	2,56	0,181	0,275	0,30	0,22	0,12	0,67	0,017	1,68	2,91	0,41
t13	2,03	0,198	0,312	0,70	2,56	0,181	0,275	0,30	0,22	0,12	0,67	0,025	1,68	3,24	0,68
t13	2,03	0,198	0,312	0,70	2,56	0,181	0,275	0,30	0,22	0,12	0,67	0,035	1,68	1,92	0,56
t15	1,75	0,230	0,330	0,70	2,04	0,184	0,252	0,30	0,22	0,12	0,67	0,017	1,68	3,70	0,52
t15	1,75	0,230	0,330	0,70	2,04	0,184	0,252	0,30	0,22	0,12	0,67	0,025	1,68	2,63	0,55
t15	1,75	0,230	0,330	0,70	2,04	0,184	0,252	0,30	0,22	0,12	0,67	0,035	1,68	1,78	0,52
t17	1,37	0,158	0,229	0,80	1,52	0,149	0,216	0,40	0,32	0,12	0,67	0,017	1,68	0,36	0,05
t17	1,37	0,158	0,229	0,80	1,52	0,149	0,216	0,40	0,32	0,12	0,67	0,025	1,68	0,53	0,11
t17	1,37	0,158	0,229	0,80	1,52	0,149	0,216	0,40	0,32	0,12	0,67	0,035	1,68	0,00	0,00
t18	1,83	0,158	0,241	0,80	2,21	0,162	0,219	0,40	0,32	0,12	0,67	0,017	1,68	1,33	0,19
t18	1,83	0,158	0,241	0,80	2,21	0,162	0,219	0,40	0,32	0,12	0,67	0,025	1,68	0,35	0,07
t18	1,83	0,158	0,241	0,80	2,21	0,162	0,219	0,40	0,32	0,12	0,67	0,035	1,68	0,00	0,00
t19	1,60	0,216	0,302	0,80	1,83	0,195	0,265	0,40	0,32	0,12	0,67	0,017	1,68	1,39	0,20
t19	1,60	0,216	0,302	0,80	1,83	0,195	0,265	0,40	0,32	0,12	0,67	0,025	1,68	0,53	0,11
t19	1,60	0,216	0,302	0,80	1,83	0,195	0,265	0,40	0,32	0,12	0,67	0,035	1,68	0,00	0,00
t20	2,13	0,201	0,316	0,80	2,57	0,200	0,320	0,40	0,32	0,12	0,67	0,017	1,68	0,97	0,14
t20	2,13	0,201	0,316	0,80	2,57	0,200	0,320	0,40	0,32	0,12	0,67	0,025	1,68	0,70	0,15
t20	2,13	0,201	0,316	0,80	2,57	0,200	0,320	0,40	0,32	0,12	0,67	0,035	1,68	0,27	0,08
t21	1,78	0,244	0,342	0,80	2,03	0,215	0,297	0,40	0,32	0,12	0,67	0,017	1,68	2,70	0,38
t21	1,78	0,244	0,342	0,80	2,03	0,215	0,297	0,40	0,32	0,12	0,67	0,025	1,68	1,05	0,22
t21	1,78	0,244	0,342	0,80	2,03	0,215	0,297	0,40	0,32	0,12	0,67	0,035	1,68	0,96	0,28
t23	1,36	0,163	0,232	0,90	1,56	0,151	0,222	0,50	0,35	0,12	0,67	0,025	1,68	0,18	0,04
t23	1,36	0,163	0,232	0,90	1,56	0,151	0,222	0,50	0,35	0,12	0,67	0,035	1,68	0,00	0,00
t23	1,36	0,163	0,232	0,90	1,56	0,151	0,222	0,50	0,35	0,12	0,67	0,040	1,68	0,00	0,00
t24	1,83	0,161	0,237	0,90	2,16	0,162	0,216	0,50	0,35	0,12	0,67	0,025	1,68	0,96	0,20
t24	1,83	0,161	0,237	0,90	2,16	0,162	0,216	0,50	0,35	0,12	0,67	0,035	1,68	0,14	0,04
t24	1,83	0,161	0,237	0,90	2,16	0,162	0,216	0,50	0,35	0,12	0,67	0,040	1,68	0,00	0,00
t25	1,59	0,208	0,299	0,90	1,80	0,197	0,288	0,50	0,35	0,12	0,67	0,025	1,68	2,46	0,51
t25	1,59	0,208	0,299	0,90	1,80	0,197	0,288	0,50	0,35	0,12	0,67	0,035	1,68	0,00	0,00
t25	1,59	0,208	0,299	0,90	1,80	0,197	0,288	0,50	0,35	0,12	0,67	0,040	1,68	0,15	0,05
t26	1,78	0,205	0,319	0,90	2,05	0,207	0,297	0,50	0,35	0,12	0,67	0,025	1,68	3,68	0,77
t26	1,78	0,205	0,319	0,90	2,05	0,207	0,297	0,50	0,35	0,12	0,67	0,035	1,68	1,10	0,32
t26	1,78	0,205	0,319	0,90	2,05	0,207	0,297	0,50	0,35	0,12	0,67	0,040	1,68	0,30	0,10
t27	1,77	0,251	0,368	0,90	2,01	0,234	0,337	0,50	0,35	0,12	0,67	0,025	1,68	4,83	1,01
t27	1,77	0,251	0,368	0,90	2,01	0,234	0,337	0,50	0,35	0,12	0,67	0,035	1,68	1,92	0,56
t27	1,77	0,251	0,368	0,90	2,01	0,234	0,337	0,50	0,35	0,12	0,67	0,040	1,68	1,67	0,56
t29	1,39	0,153	0,220	0,70	1,56	0,141	0,191	0,30	0,15	0,12	0,67	0,025	1,68	1,40	0,29
t29	1,39	0,153	0,220	0,70	1,56	0,141	0,191	0,30	0,15	0,12	0,67	0,035	1,68	0,55	0,16
t29	1,39	0,153	0,220	0,70	1,56	0,141	0,191	0,30	0,15	0,12	0,67	0,040	1,68	0,30	0,10
t30	1,83	0,154	0,230	0,70	2,25	0,154	0,229	0,30	0,15	0,12	0,67	0,025	1,68	3,33	0,69
t30	1,83	0,154	0,230	0,70	2,25	0,154	0,229	0,30	0,15	0,12	0,67	0,035	1,68	1,55	0,45
t30	1,83	0,154	0,230	0,70	2,25	0,154	0,229	0,30	0,15	0,12	0,67	0,040	1,68	0,45	0,15
t31	1,60	0,198	0,281	0,70	1,83	0,169	0,229	0,30	0,15	0,12	0,67	0,025	1,68	7,10	1,48
t31	1,60	0,198	0,281	0,70	1,83	0,169	0,229	0,30	0,15	0,12	0,67	0,035	1,68	2,60	0,76
t31	1,60	0,198	0,281	0,70	1,83	0,169	0,229	0,30	0,15	0,12	0,67	0,040	1,68	1,21	0,40
t31a	1,77	0,201	0,291	0,70	2,12	0,176	0,229	0,30	0,15	0,12	0,67	0,025	1,68	7,54	1,57

t31a	1,77	0,201	0,291	0,70	2,12	0,176	0,229	0,30	0,15	0,12	0,67	0,035	1,68	2,60	0,76
t31a	1,77	0,201	0,291	0,70	2,12	0,176	0,229	0,30	0,15	0,12	0,67	0,040	1,68	2,12	0,71
t32	2,03	0,196	0,305	0,70	2,56	0,181	0,271	0,30	0,15	0,12	0,67	0,025	1,68	8,07	1,68
t32	2,03	0,196	0,305	0,70	2,56	0,181	0,271	0,30	0,15	0,12	0,67	0,035	1,68	4,25	1,24
t32	2,03	0,196	0,305	0,70	2,56	0,181	0,271	0,30	0,15	0,12	0,67	0,040	1,68	2,73	0,91
t33	1,75	0,231	0,324	0,70	2,04	0,184	0,242	0,30	0,15	0,12	0,67	0,025	1,68	8,16	1,70
t33	1,75	0,231	0,324	0,70	2,04	0,184	0,242	0,30	0,15	0,12	0,67	0,035	1,68	3,84	1,12
t33	1,75	0,231	0,324	0,70	2,04	0,184	0,242	0,30	0,15	0,12	0,67	0,040	1,68	2,42	0,81
t35	1,37	0,160	0,226	0,80	1,52	0,149	0,216	0,40	0,25	0,12	0,67	0,025	1,68	2,02	0,42
t35	1,37	0,160	0,226	0,80	1,52	0,149	0,216	0,40	0,25	0,12	0,67	0,035	1,68	1,10	0,32
t35	1,37	0,160	0,226	0,80	1,52	0,149	0,216	0,40	0,25	0,12	0,67	0,040	1,68	0,15	0,05
t36	1,83	0,158	0,237	0,80	2,21	0,162	0,216	0,40	0,25	0,12	0,67	0,025	1,68	1,49	0,31
t36	1,83	0,158	0,237	0,80	2,21	0,162	0,216	0,40	0,25	0,12	0,67	0,035	1,68	1,09	0,32
t36	1,83	0,158	0,237	0,80	2,21	0,162	0,216	0,40	0,25	0,12	0,67	0,040	1,68	0,15	0,05
t37	1,60	0,204	0,296	0,80	1,83	0,188	0,261	0,40	0,25	0,12	0,67	0,025	1,68	2,11	0,44
t37	1,60	0,204	0,296	0,80	1,83	0,188	0,261	0,40	0,25	0,12	0,67	0,035	1,68	0,96	0,28
t37	1,60	0,204	0,296	0,80	1,83	0,188	0,261	0,40	0,25	0,12	0,67	0,040	1,68	0,30	0,10
t38	2,13	0,201	0,316	0,80	2,57	0,199	0,317	0,40	0,25	0,12	0,67	0,025	1,68	4,47	0,93
t38	2,13	0,201	0,316	0,80	2,57	0,199	0,317	0,40	0,25	0,12	0,67	0,035	1,68	1,64	0,48
t38	2,13	0,201	0,316	0,80	2,57	0,199	0,317	0,40	0,25	0,12	0,67	0,040	1,68	0,76	0,25
t39	1,78	0,244	0,346	0,80	2,03	0,215	0,294	0,40	0,25	0,12	0,67	0,025	1,68	5,35	1,11
t39	1,78	0,244	0,346	0,80	2,03	0,215	0,294	0,40	0,25	0,12	0,67	0,035	1,68	2,32	0,68
t39	1,78	0,244	0,346	0,80	2,03	0,215	0,294	0,40	0,25	0,12	0,67	0,040	1,68	1,81	0,60
t41	1,36	0,162	0,232	0,90	1,56	0,151	0,222	0,50	0,28	0,12	0,67	0,025	1,68	0,35	0,07
t41	1,36	0,162	0,232	0,90	1,56	0,151	0,222	0,50	0,28	0,12	0,67	0,035	1,68	0,14	0,04
t41	1,36	0,162	0,232	0,90	1,56	0,151	0,222	0,50	0,28	0,12	0,67	0,040	1,68	0,00	0,00
t42	1,83	0,159	0,236	0,90	2,16	0,162	0,222	0,50	0,28	0,12	0,67	0,025	1,68	1,49	0,31
t42	1,83	0,159	0,236	0,90	2,16	0,162	0,222	0,50	0,28	0,12	0,67	0,035	1,68	0,27	0,08
t42	1,83	0,159	0,236	0,90	2,16	0,162	0,222	0,50	0,28	0,12	0,67	0,040	1,68	0,30	0,10
t43	1,59	0,207	0,297	0,90	1,80	0,197	0,288	0,50	0,28	0,12	0,67	0,025	1,68	3,33	0,69
t43	1,59	0,207	0,297	0,90	1,80	0,197	0,288	0,50	0,28	0,12	0,67	0,035	1,68	1,51	0,44
t43	1,59	0,207	0,297	0,90	1,80	0,197	0,288	0,50	0,28	0,12	0,67	0,040	1,68	0,61	0,20
t44	1,78	0,204	0,316	0,90	2,05	0,207	0,294	0,50	0,28	0,12	0,67	0,025	1,68	4,03	0,84
t44	1,78	0,204	0,316	0,90	2,05	0,207	0,294	0,50	0,28	0,12	0,67	0,035	1,68	1,23	0,36
t44	1,78	0,204	0,316	0,90	2,05	0,207	0,294	0,50	0,28	0,12	0,67	0,040	1,68	1,36	0,45
t45	1,77	0,250	0,369	0,90	2,01	0,234	0,337	0,50	0,28	0,12	0,67	0,025	1,68	6,84	1,43
t45	1,77	0,250	0,369	0,90	2,01	0,234	0,337	0,50	0,28	0,12	0,67	0,035	1,68	1,78	0,52
t45	1,77	0,250	0,369	0,90	2,01	0,234	0,337	0,50	0,28	0,12	0,67	0,040	1,68	1,51	0,50
t47	1,37	0,159	0,227	0,80	1,52	0,149	0,216	0,40	0,18	0,12	0,67	0,025	1,68	1,93	0,40
t47	1,37	0,159	0,227	0,80	1,52	0,149	0,216	0,40	0,18	0,12	0,67	0,035	1,68	0,82	0,24
t47	1,37	0,159	0,227	0,80	1,52	0,149	0,216	0,40	0,18	0,12	0,67	0,040	1,68	0,30	0,10
t48	1,83	0,158	0,234	0,80	2,21	0,162	0,209	0,40	0,18	0,12	0,67	0,025	1,68	3,42	0,71
t48	1,83	0,158	0,234	0,80	2,21	0,162	0,209	0,40	0,18	0,12	0,67	0,035	1,68	1,37	0,40
t48	1,83	0,158	0,234	0,80	2,21	0,162	0,209	0,40	0,18	0,12	0,67	0,040	1,68	0,15	0,05
t49	1,60	0,204	0,290	0,80	1,83	0,188	0,261	0,40	0,18	0,12	0,67	0,025	1,68	5,43	1,13
t49	1,60	0,204	0,290	0,80	1,83	0,188	0,261	0,40	0,18	0,12	0,67	0,035	1,68	2,19	0,64
t49	1,60	0,204	0,290	0,80	1,83	0,188	0,261	0,40	0,18	0,12	0,67	0,040	1,68	1,06	0,35
t50	2,13	0,200	0,313	0,80	2,57	0,199	0,317	0,40	0,18	0,12	0,67	0,025	1,68	7,89	1,64
t50	2,13	0,200	0,313	0,80	2,57	0,199	0,317	0,40	0,18	0,12	0,67	0,035	1,68	3,01	0,88
t50	2,13	0,200	0,313	0,80	2,57	0,199	0,317	0,40	0,18	0,12	0,67	0,040	1,68	1,67	0,56
t51	1,78	0,244	0,346	0,80	2,03	0,215	0,291	0,40	0,18	0,12	0,67	0,025	1,68	9,21	1,92

t51	1,78	0,244	0,346	0,80	2,03	0,215	0,291	0,40	0,18	0,12	0,67	0,035	1,68	3,42	1,00
t51	1,78	0,244	0,346	0,80	2,03	0,215	0,291	0,40	0,18	0,12	0,67	0,040	1,68	1,82	0,61
t53	1,36	0,162	0,232	0,90	1,56	0,151	0,222	0,50	0,35	0,20	0,67	0,025	1,68	0,00	0,00
t53	1,36	0,162	0,232	0,90	1,56	0,151	0,222	0,50	0,35	0,20	0,67	0,035	1,68	0,00	0,00
t53	1,36	0,162	0,232	0,90	1,56	0,151	0,222	0,50	0,35	0,20	0,67	0,040	1,68	0,00	0,00
t54	1,83	0,160	0,232	0,90	2,16	0,162	0,229	0,50	0,35	0,20	0,67	0,025	1,68	0,96	0,12
t54	1,83	0,160	0,232	0,90	2,16	0,162	0,229	0,50	0,35	0,20	0,67	0,035	1,68	0,55	0,10
t54	1,83	0,160	0,232	0,90	2,16	0,162	0,229	0,50	0,35	0,20	0,67	0,040	1,68	0,00	0,00
t55	1,59	0,207	0,301	0,90	1,80	0,197	0,284	0,50	0,35	0,20	0,67	0,025	1,68	1,75	0,22
t55	1,59	0,207	0,301	0,90	1,80	0,197	0,284	0,50	0,35	0,20	0,67	0,035	1,68	0,55	0,10
t55	1,59	0,207	0,301	0,90	1,80	0,197	0,284	0,50	0,35	0,20	0,67	0,040	1,68	0,15	0,03
t56	1,78	0,204	0,315	0,90	2,05	0,207	0,294	0,50	0,35	0,20	0,67	0,025	1,68	4,73	0,59
t56	1,78	0,204	0,315	0,90	2,05	0,207	0,294	0,50	0,35	0,20	0,67	0,035	1,68	1,64	0,29
t56	1,78	0,204	0,315	0,90	2,05	0,207	0,294	0,50	0,35	0,20	0,67	0,040	1,68	0,91	0,18
t57	1,77	0,250	0,364	0,90	2,01	0,234	0,333	0,50	0,35	0,20	0,67	0,025	1,68	5,87	0,73
t57	1,77	0,250	0,364	0,90	2,01	0,234	0,333	0,50	0,35	0,20	0,67	0,035	1,68	1,37	0,24
t57	1,77	0,250	0,364	0,90	2,01	0,234	0,333	0,50	0,35	0,20	0,67	0,040	1,68	1,66	0,33
t58	2,35	0,241	0,382	0,90	2,86	0,243	0,356	0,50	0,35	0,20	0,67	0,025	1,68	7,98	1,00
t58	2,35	0,241	0,382	0,90	2,86	0,243	0,356	0,50	0,35	0,20	0,67	0,035	1,68	3,01	0,53
t58	2,35	0,241	0,382	0,90	2,86	0,243	0,356	0,50	0,35	0,20	0,67	0,040	1,68	2,42	0,48
t59	1,36	0,162	0,232	0,90	1,56	0,151	0,222	0,50	0,35	0,30	0,67	0,025	1,68	0,00	0,00
t59	1,36	0,162	0,232	0,90	1,56	0,151	0,222	0,50	0,35	0,30	0,67	0,035	1,68	0,14	0,02
t59	1,36	0,162	0,232	0,90	1,56	0,151	0,222	0,50	0,35	0,30	0,67	0,040	1,68	0,00	0,00
t60	1,83	0,161	0,236	0,90	2,16	0,162	0,229	0,50	0,35	0,30	0,67	0,025	1,68	1,84	0,15
t60	1,83	0,161	0,236	0,90	2,16	0,162	0,229	0,50	0,35	0,30	0,67	0,035	1,68	0,28	0,03
t60	1,83	0,161	0,236	0,90	2,16	0,162	0,229	0,50	0,35	0,30	0,67	0,040	1,68	0,15	0,02
t61	1,59	0,207	0,298	0,90	1,80	0,197	0,288	0,50	0,35	0,30	0,67	0,025	1,68	1,84	0,15
t61	1,59	0,207	0,298	0,90	1,80	0,197	0,288	0,50	0,35	0,30	0,67	0,035	1,68	0,28	0,03
t61	1,59	0,207	0,298	0,90	1,80	0,197	0,288	0,50	0,35	0,30	0,67	0,040	1,68	0,15	0,02
t62	1,78	0,204	0,311	0,90	2,05	0,207	0,291	0,50	0,35	0,30	0,67	0,025	1,68	4,56	0,38
t62	1,78	0,204	0,311	0,90	2,05	0,207	0,291	0,50	0,35	0,30	0,67	0,035	1,68	1,10	0,13
t62	1,78	0,204	0,311	0,90	2,05	0,207	0,291	0,50	0,35	0,30	0,67	0,040	1,68	0,00	0,00
t63	1,77	0,251	0,364	0,90	2,01	0,234	0,333	0,50	0,35	0,30	0,67	0,025	1,68	6,85	0,57
t63	1,77	0,250	0,364	0,90	2,01	0,234	0,333	0,50	0,35	0,30	0,67	0,035	1,68	1,23	0,14
t63	1,77	0,250	0,364	0,90	2,01	0,234	0,333	0,50	0,35	0,30	0,67	0,040	1,68	0,15	0,02
t64	2,35	0,244	0,389	0,90	2,86	0,244	0,359	0,50	0,35	0,30	0,67	0,025	1,68	7,98	0,67
t64	2,35	0,244	0,389	0,90	2,86	0,244	0,359	0,50	0,35	0,30	0,67	0,035	1,68	2,88	0,34
t64	2,35	0,244	0,389	0,90	2,86	0,244	0,359	0,50	0,35	0,30	0,67	0,040	1,68	1,06	0,14

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The value of N_{od} has been recalculated according to equation 4. The original report provides the number of displaced rocks.

local	local	local								
T_p	H_{si}	h_m	h_t	B_t	tan(α)	D_{n50}	Δ	N_{od}	N_{odB}	K_r
[s]	[m]	[m]	[m]	[m]	[-]	[m]	[-]	[-]	[-]	[-]
1,34	0,093	0,30	0,22	0,12	0,67	0,0210	1,55	0,05	0,01	0,195

1,27	0,104	0,30	0,22	0,12	0,67	0,0210	1,55	0,00	0,00	0,193
1,59	0,134	0,30	0,22	0,12	0,67	0,0210	1,55	0,22	0,04	0,228
1,54	0,149	0,30	0,22	0,12	0,67	0,0210	1,55	0,27	0,05	0,227
1,69	0,168	0,30	0,22	0,12	0,67	0,0210	1,55	0,70	0,12	0,291
1,59	0,142	0,45	0,37	0,12	0,67	0,0210	1,55	0,00	0,00	0,254
1,49	0,167	0,45	0,37	0,12	0,67	0,0210	1,55	0,00	0,00	0,247
1,82	0,209	0,45	0,37	0,12	0,67	0,0210	1,55	0,27	0,05	0,297
1,37	0,097	0,45	0,30	0,12	0,67	0,0210	1,55	0,00	0,00	0,229
1,64	0,143	0,45	0,30	0,12	0,67	0,0210	1,55	0,05	0,01	0,249
1,49	0,170	0,45	0,30	0,12	0,67	0,0210	1,55	0,05	0,01	0,244
1,82	0,208	0,45	0,30	0,12	0,67	0,0210	1,55	1,13	0,20	0,297
1,37	0,094	0,30	0,15	0,12	0,67	0,0210	1,55	0,00	0,00	0,191
1,27	0,105	0,30	0,15	0,12	0,67	0,0210	1,55	0,00	0,00	0,193
1,59	0,125	0,30	0,15	0,12	0,67	0,0210	1,55	0,59	0,10	0,207
1,54	0,148	0,30	0,15	0,12	0,67	0,0210	1,55	0,75	0,13	0,224
1,69	0,170	0,30	0,15	0,12	0,67	0,0210	1,55	1,13	0,20	0,282
1,34	0,093	0,30	0,22	0,12	0,67	0,0144	1,55	0,15	0,02	0,195
1,27	0,104	0,30	0,22	0,12	0,67	0,0144	1,55	0,04	0,00	0,193
1,59	0,134	0,30	0,22	0,12	0,67	0,0144	1,55	0,18	0,02	0,228
1,54	0,149	0,30	0,22	0,12	0,67	0,0144	1,55	0,33	0,04	0,227
1,69	0,168	0,30	0,22	0,12	0,67	0,0144	1,55	0,89	0,11	0,291
1,59	0,142	0,45	0,37	0,12	0,67	0,0144	1,55	0,00	0,00	0,254
1,49	0,167	0,45	0,37	0,12	0,67	0,0144	1,55	0,04	0,00	0,247
1,82	0,209	0,45	0,37	0,12	0,67	0,0144	1,55	0,26	0,03	0,297
1,37	0,097	0,45	0,30	0,12	0,67	0,0144	1,55	0,04	0,00	0,229
1,64	0,143	0,45	0,30	0,12	0,67	0,0144	1,55	0,15	0,02	0,249
1,49	0,170	0,45	0,30	0,12	0,67	0,0144	1,55	0,18	0,02	0,244
1,82	0,208	0,45	0,30	0,12	0,67	0,0144	1,55	1,29	0,16	0,297
1,37	0,094	0,30	0,15	0,12	0,67	0,0144	1,55	0,07	0,01	0,191
1,27	0,105	0,30	0,15	0,12	0,67	0,0144	1,55	0,04	0,00	0,193
1,59	0,125	0,30	0,15	0,12	0,67	0,0144	1,55	0,22	0,03	0,207
1,54	0,148	0,30	0,15	0,12	0,67	0,0144	1,55	0,66	0,08	0,224
1,69	0,170	0,30	0,15	0,12	0,67	0,0144	1,55	1,22	0,15	0,282
1,37	0,099	0,45	0,37	0,12	0,67	0,0098	1,55	0,00	0,00	0,209
1,64	0,144	0,45	0,37	0,12	0,67	0,0098	1,55	0,08	0,01	0,239
1,82	0,208	0,45	0,37	0,12	0,67	0,0098	1,55	0,70	0,06	0,302
1,37	0,093	0,30	0,22	0,12	0,67	0,0098	1,55	0,03	0,00	0,188
1,59	0,125	0,30	0,22	0,12	0,67	0,0098	1,55	0,28	0,02	0,236
1,55	0,148	0,30	0,22	0,12	0,67	0,0098	1,55	0,65	0,05	0,251
1,75	0,167	0,30	0,22	0,12	0,67	0,0098	1,55	0,63	0,05	0,315
1,41	0,098	0,45	0,30	0,12	0,67	0,0098	1,55	0,00	0,00	0,232
1,59	0,142	0,45	0,30	0,12	0,67	0,0098	1,55	0,53	0,04	0,254
1,59	0,170	0,45	0,30	0,12	0,67	0,0098	1,55	1,31	0,11	0,245
1,82	0,212	0,45	0,30	0,12	0,67	0,0098	1,55	4,00	0,33	0,297
1,37	0,093	0,30	0,15	0,12	0,67	0,0098	1,55	0,13	0,01	0,185
1,59	0,126	0,30	0,15	0,12	0,67	0,0098	1,55	1,33	0,11	0,212
1,54	0,146	0,30	0,15	0,12	0,67	0,0098	1,55	3,29	0,27	0,239
1,76	0,168	0,30	0,15	0,12	0,67	0,0098	1,55	4,42	0,36	0,312
1,41	0,098	0,45	0,37	0,12	0,67	0,0151	1,85	0,00	0,00	0,202
1,59	0,146	0,45	0,37	0,12	0,67	0,0151	1,85	0,04	0,00	0,236
1,49	0,171	0,45	0,37	0,12	0,67	0,0151	1,85	0,08	0,01	0,228

1,82	0,212	0,45	0,37	0,12	0,67	0,0151	1,85	0,15	0,02	0,297
1,37	0,093	0,30	0,22	0,12	0,67	0,0151	1,85	0,00	0,00	0,192
1,59	0,126	0,30	0,22	0,12	0,67	0,0151	1,85	0,00	0,00	0,210
1,54	0,149	0,30	0,22	0,12	0,67	0,0151	1,85	0,00	0,00	0,239
1,69	0,170	0,30	0,22	0,12	0,67	0,0151	1,85	0,08	0,01	0,310
1,41	0,095	0,45	0,30	0,12	0,67	0,0151	1,85	0,00	0,00	0,232
1,64	0,141	0,45	0,30	0,12	0,67	0,0151	1,85	0,08	0,01	0,253
1,49	0,165	0,45	0,30	0,12	0,67	0,0151	1,85	0,31	0,04	0,246
1,82	0,204	0,45	0,30	0,12	0,67	0,0151	1,85	1,43	0,18	0,298
1,37	0,092	0,30	0,15	0,12	0,67	0,0151	1,85	0,04	0,00	0,195
1,27	0,102	0,30	0,15	0,12	0,67	0,0151	1,85	0,00	0,00	0,199
1,59	0,122	0,30	0,15	0,12	0,67	0,0151	1,85	0,89	0,11	0,204
1,49	0,146	0,30	0,15	0,12	0,67	0,0151	1,85	1,01	0,13	0,218
1,69	0,161	0,30	0,15	0,12	0,67	0,0151	1,85	2,09	0,26	0,287
1,37	0,099	0,45	0,37	0,12	0,67	0,0102	1,85	0,00	0,00	0,209
1,64	0,144	0,45	0,37	0,12	0,67	0,0102	1,85	0,03	0,00	0,239
1,82	0,208	0,45	0,37	0,12	0,67	0,0102	1,85	0,63	0,05	0,302
1,37	0,093	0,30	0,22	0,12	0,67	0,0102	1,85	0,00	0,00	0,188
1,59	0,125	0,30	0,22	0,12	0,67	0,0102	1,85	0,13	0,01	0,236
1,55	0,148	0,30	0,22	0,12	0,67	0,0102	1,85	0,08	0,01	0,251
1,75	0,167	0,30	0,22	0,12	0,67	0,0102	1,85	0,24	0,02	0,315
1,41	0,098	0,45	0,30	0,12	0,67	0,0102	1,85	0,00	0,00	0,232
1,59	0,142	0,45	0,30	0,12	0,67	0,0102	1,85	0,21	0,02	0,254
1,59	0,170	0,45	0,30	0,12	0,67	0,0102	1,85	0,16	0,01	0,245
1,82	0,212	0,45	0,30	0,12	0,67	0,0102	1,85	1,99	0,17	0,297
1,37	0,093	0,30	0,15	0,12	0,67	0,0102	1,85	0,13	0,01	0,185
1,59	0,126	0,30	0,15	0,12	0,67	0,0102	1,85	1,02	0,09	0,212
1,54	0,146	0,30	0,15	0,12	0,67	0,0102	1,85	2,04	0,17	0,239
1,76	0,168	0,30	0,15	0,12	0,67	0,0102	1,85	3,16	0,27	0,312
1,41	0,098	0,45	0,37	0,12	0,67	0,0231	0,90	0,06	0,01	0,202
1,59	0,146	0,45	0,37	0,12	0,67	0,0231	0,90	0,77	0,15	0,236
1,49	0,171	0,45	0,37	0,12	0,67	0,0231	0,90	0,41	0,08	0,228
1,82	0,212	0,45	0,37	0,12	0,67	0,0231	0,90	1,48	0,29	0,297
1,37	0,093	0,30	0,22	0,12	0,67	0,0231	0,90	0,30	0,06	0,192
1,59	0,126	0,30	0,22	0,12	0,67	0,0231	0,90	1,01	0,19	0,210
1,54	0,149	0,30	0,22	0,12	0,67	0,0231	0,90	0,95	0,18	0,239
1,69	0,170	0,30	0,22	0,12	0,67	0,0231	0,90	1,24	0,24	0,310
1,41	0,095	0,45	0,30	0,12	0,67	0,0231	0,90	0,12	0,02	0,232
1,64	0,141	0,45	0,30	0,12	0,67	0,0231	0,90	1,42	0,27	0,253
1,49	0,165	0,45	0,30	0,12	0,67	0,0231	0,90	2,19	0,42	0,246
1,82	0,204	0,45	0,30	0,12	0,67	0,0231	0,90	3,91	0,75	0,298
1,37	0,092	0,30	0,15	0,12	0,67	0,0231	0,90	0,59	0,11	0,195
1,27	0,102	0,30	0,15	0,12	0,67	0,0231	0,90	0,47	0,09	0,199
1,59	0,122	0,30	0,15	0,12	0,67	0,0231	0,90	1,90	0,36	0,204
1,49	0,146	0,30	0,15	0,12	0,67	0,0231	0,90	3,49	0,67	0,218
1,69	0,161	0,30	0,15	0,12	0,67	0,0231	0,90	3,91	0,75	0,287

H.3 USACE 1987

Values for h_m, h_t, L_1, L_s, B_t, Δ and D_{n50} are calculated from the original report (USACE 1989), where parameters are given in dimensionless form or in non-SI units.

test	plan	T [s]	H_D [ft]	H_D [m]	L1 [m]	Ls [m]	ds [ft]	hm [m]	d1=ht [m]	tan(α) [-]	Bt [m]	Δ [-]	Dn50 [m]
1	1	1,78	0,75	0,229	2,299	2,754	0,90	0,274	0,182	0,67	0,122	1,642	0,0403
1	2	1,78	0,65	0,198	2,363	2,752	0,90	0,274	0,196	0,67	0,099	1,642	0,0330
2	2	1,52	0,70	0,213	1,981	2,294	0,90	0,274	0,196	0,67	0,099	1,642	0,0330
4	1	1,67	0,70	0,213	1,963	2,425	0,80	0,244	0,151	0,67	0,122	1,642	0,0403
4	2	1,67	0,60	0,183	2,044	2,438	0,80	0,244	0,166	0,67	0,098	1,642	0,0330
5	2	1,43	0,71	0,216	1,725	2,042	0,80	0,244	0,166	0,67	0,098	1,642	0,0330
7	2	1,92	0,55	0,168	2,144	2,661	0,70	0,213	0,135	0,67	0,099	1,642	0,0330
9	2	1,57	0,65	0,198	1,731	2,130	0,70	0,213	0,135	0,67	0,099	1,642	0,0330
10	1	2,82	0,45	0,137	1,540	3,048	0,40	0,122	0,029	0,67	0,123	1,642	0,0403
10	2	2,82	0,35	0,107	1,819	3,048	0,40	0,122	0,044	0,67	0,098	1,642	0,0330
11	1	1,90	0,44	0,134	1,009	2,032	0,40	0,122	0,029	0,67	0,120	1,642	0,0403
11	2	1,90	0,40	0,122	1,247	2,032	0,40	0,122	0,044	0,67	0,100	1,642	0,0330
11	3	1,90	0,38	0,116	1,335	2,032	0,40	0,122	0,051	0,67	0,089	1,642	0,0295
11	4	1,90	0,35	0,107	1,418	2,013	0,40	0,122	0,058	0,67	0,078	1,642	0,0258
12	1	2,32	0,47	0,143	2,147	3,048	0,60	0,183	0,090	0,67	0,120	1,642	0,0403
12	2	2,32	0,46	0,140	2,325	3,048	0,60	0,183	0,105	0,67	0,100	1,642	0,0330
12	3	2,32	0,45	0,137	2,374	3,048	0,60	0,183	0,112	0,67	0,088	1,642	0,0295
12	4	2,32	0,45	0,137	2,480	3,048	0,60	0,183	0,119	0,67	0,077	1,642	0,0258
13	1	1,62	0,50	0,152	1,219	1,905	0,50	0,152	0,060	0,67	0,121	1,642	0,0403
13	2	1,62	0,50	0,152	1,347	1,905	0,50	0,152	0,074	0,67	0,098	1,642	0,0330
13	3	1,62	0,45	0,137	1,422	1,905	0,50	0,152	0,081	0,67	0,090	1,642	0,0295
13	4	1,62	0,40	0,122	1,476	1,905	0,50	0,152	0,089	0,67	0,077	1,642	0,0258
14	6	1,78	0,70	0,213	2,363	2,735	0,90	0,274	0,196	0,67	0,099	1,642	0,0330
14	8	1,78	0,55	0,168	2,447	2,748	0,90	0,274	0,210	0,67	0,078	1,642	0,0258
15	5	1,67	0,70	0,213	1,963	2,425	0,80	0,244	0,151	0,67	0,122	1,642	0,0403
15	6	1,67	0,70	0,213	2,044	2,425	0,80	0,244	0,166	0,67	0,098	1,642	0,0330
15	7	1,67	0,70	0,213	2,080	2,425	0,80	0,244	0,173	0,67	0,089	1,642	0,0295
16	6	1,90	0,35	0,107	1,247	2,013	0,40	0,122	0,044	0,67	0,100	1,642	0,0330
16	7	1,90	0,40	0,122	1,335	2,032	0,40	0,122	0,051	0,67	0,089	1,642	0,0295
16	8	1,90	0,30	0,091	1,418	2,032	0,40	0,122	0,058	0,67	0,078	1,642	0,0258
17	5	2,32	0,60	0,183	2,147	3,048	0,60	0,183	0,090	0,67	0,120	1,642	0,0403
17	6	2,32	0,55	0,168	2,325	3,048	0,60	0,183	0,105	0,67	0,100	1,642	0,0330
18	5	1,62	0,45	0,137	1,219	1,905	0,50	0,152	0,060	0,67	0,121	1,642	0,0403
18	6	1,62	0,50	0,152	1,347	1,905	0,50	0,152	0,074	0,67	0,098	1,642	0,0330
18	7	1,62	0,45	0,137	1,422	1,905	0,50	0,152	0,081	0,67	0,090	1,642	0,0295
18	8	1,62	0,45	0,137	1,476	1,905	0,50	0,152	0,089	0,67	0,077	1,642	0,0258
19	5	2,82	0,40	0,122	1,540	3,048	0,40	0,122	0,029	0,67	0,123	1,642	0,0403
19	6	2,82	0,40	0,122	1,819	3,048	0,40	0,122	0,044	0,67	0,098	1,642	0,0330
21	6	1,32	0,45	0,137	1,089	1,524	0,50	0,152	0,074	0,67	0,099	1,642	0,0330
21	7	1,32	0,40	0,122	1,142	1,524	0,50	0,152	0,081	0,67	0,089	1,642	0,0295