Mr. Ohtsuki has performed fundamental research on sediment concentration distributions under the combined effect of waves and current in a quiescent, independent way. The research led to promising results.

How do you calculate the near-reversal?
STUDY OF THE DIFFUSION ACTIVITY IN CASE OF

NON-BREAKING WAVES WITH A CURRENT

PART I: TEXT

by

M. OHNISHI

March 1989
Sediment transport causes morphological changes of coastal areas, and it is one of the most important objects in coastal engineering study. Recently, various experiments have been performed to investigate the sediment transport mechanism under a combination of waves and currents and to find the underlying relationships among various parameters which control the sediment transport mechanism is a main topic of research. But still there are many unknown and unexpected facts in this phenomenon. Therefore, further efforts to investigate the basic mechanism of this phenomenon is inevitable. The diffusion coefficient which expresses the intensity of the diffusion activity is one of the most important parameters controlling this phenomenon.

In this report, two different approaches to estimate the diffusion coefficient in case of non-breaking waves with a current are discussed. The first one is based on the concentration distribution, and the second one is based on the velocity distribution. For these approaches, the experimental data measured by E.N.Nap and H.F.A.van Kampen (August 1988 TU Delft) are mainly used and some of the experimental data measured by D.Heijboer (May 1988 TU Delft) are selected as comparisons.

For convenience this report is divided in two parts. Part I contains all texts and illustrative figures, Part II contains all figures of calculation results. (All computer programs, their input data and outputs for this study are available in the Delft University of Technology).

I would like to thank three of my supervisors, Prof. Bijker, Dr. van de Graaff and Ir. van Ingen for their support and pieces of advice for this study.

Masahiro Ohnishi

March 1989
Preface
Contents
Summary

1. The aims of this study
1.1 The aims of this study
1.2 Basic equations of sediment transport mechanism
1.3 Methods to investigate the diffusion coefficient

2. Used experimental data and outlines of the experiments
2.1 The experiments by E.N.Nap and H.F.A.van Kampen
2.2 The experiments by D.Heijboer

3. The first method with measured concentration data
3.1 The calculation method
3.2 The \( \varepsilon_s(z) \) distribution by CONC program
3.3 The \( \varepsilon_f(z) \) distribution by CONCFL program
3.4 The additional calculations for comparisons
3.5 Study of the results

4. The second method with measured velocity data
4.1 The calculation method
4.1.1 Velocity distribution under waves with a current
4.1.2 Curve fit of velocity distribution
4.1.3 Water particle movement under waves with a current
4.2 Bottom shear stress and \( \varepsilon_f(z) \) distribution by the original Bijker approach
4.2.1 Original Bijker approach
4.2.2 The calculation results of \( \varepsilon_f(z) \) distribution by the original Bijker approach
4.3 Bottom shear stress and εf(z) distribution by
the modified Bijker approach

4.3.1 Modified Bijker approach

4.3.2 The calculation results of εf(z) distribution
by the modified Bijker approach

4.4 Additional internal shear stress

4.4.1 Internal shear stress

4.4.2 Additional internal shear stress

4.4.3 J-value and τs for a following current with
waves

4.4.4 J-value and τs for an opposing current with
waves

4.5 The εf(z) results calculated by the modified Bijker
approach with additional shear stress idea

4.5.1 The calculation results of εf(z) distribution

4.5.2 Study of the results

5. Conclusion and recommendations

List of symbols

List of figures

References

Contents of Part II

Figures of the calculation results

The experimental data, computer programs and their output that
were used or developed for this study are available in the Delft
University of Technology.
In order to estimate the diffusion coefficient of the fluid, $\varepsilon_f(z)$, under combination of waves and currents, two different approaches have been performed in this study. The first is the method using time-averaged concentration distribution data $C(z)$, which was mainly developed by Van de Graaff, and the second is the method using time-averaged velocity distribution data $V(z)$, with the modified Bijker approach and the additional shear stress assumption. The data for these approaches were the experimental data measured by E.H.Nap and H.F.van Kampen (August 1988, TU Delft) as the main data and some additional experimental data measured by D.Heijboer (May 1988, TU Delft).

From the $\varepsilon_f(z)$ distributions calculated by these two methods, the followings have been revealed.

1) By the first method using $C(z)$ distribution data;
   - In most of the cases of Nap and Van Kampen's data, $\varepsilon_f(z)$ distribution in case of a current with waves tend to be relatively smaller than that of the mere current case in the middle layers of the water column. These results seem to be contradictory from a physical point of view. But the $\varepsilon_f(z)$ distributions in the cases of Heijboer's data show quite reasonable tendencies.
   - The reasons of such under-estimation could be the armour effects, probably the coarsening of sand particle size distribution in suspension in case of no-artificial sand supplying experiment, and different $\beta$-effect for higher diffusion activity field than the upper limit of $\beta$-predictor.

2) By the second method using $V(z)$ distribution data;
   - The time-averaged velocity distribution, $V(z)$, under waves with a current is different from that of the mere current case, and it is also different from that explained by present existing idea. Most of present existing idea for the condition under waves with a current also assume the velocity distribution as the logarithmic distribution that is an upward shifted logarithmic distribution of the mere current case. But, from the experimental data, faster or slower velocities than the expected ones appear in the middle or
upper layers in the water column under waves with a current.

- The time-averaged internal shear stress distribution, $\tau(z)$, under waves with a current could be different from a triangle distribution explained by present existing idea.

- Assuming the additional internal shear stress that could be caused by the interaction of waves and current, and applying the modified Bijker approach that estimates the bottom shear stress, more or less reasonable $\varepsilon f(z)$ distributions can be calculated.

- In this case, $\tau(z)$ distribution is double triangle shape for a following current with waves, trapezoidal shape for an opposing current with waves.

- The $\varepsilon f(z)$ distribution by the second method resembles that by the first method for the cases of D.Heijboer's data, even though the magnitude of order of $\varepsilon f(z)$ is about 1.5 times bigger by the second method than by the first method.

In conclusion, the following can be stated;

- The first method using C(z) distribution data is a logically correct approach and a quite strong tool for the investigation of diffusion activity under waves and currents. But, before it is applied to measured C(z) data, especially the armour effect which causes some unbalances in input boundary conditions of this method has to be checked. Otherwise, the estimated $\varepsilon f(z)$ distributions tend to be rather small. Preventing such mistake, the experiments of which data are used for the $\varepsilon f(z)$ calculation by this method should be performed with relatively high sorted materials ($D_{90}/D_{10} \approx 1$), or the sorting ratio ($D_{90}/D_{10}$) after armour effect happens should be somehow estimated for the input data of this method. The experiments with artificial sand supply seem to be preferable for the data of this method.

- As to the second method using V(z) distribution data, it can be stated that this method leads to more or less reasonable $\varepsilon f(z)$ distribution under a current with waves. But, it has to be stated that this method uses several assumptions, like that of $p = 1$ at height $z_t$ in the modified Bijker approach or the additional shear stress due to the wave-current interactions. In fact, these assumptions should be confirmed by the extra experiments or by further theoretical approaches considering the energy or momentum
exchanges between waves and a current, and also the bottom conditions.

The author hopes that these two methods will be a help to find the detailed underlying relationships which control sediment transport mechanism, in the near future.
1. The aims of this study

1.1 The aims of this study

Many theoretical and experimental approaches for sediment transport due to wave action or a combination of waves and current have been carried out in recent years. However, many fundamental aspects of this phenomenon are still unknown, because of the complex relationships between waves, current, sediment particles and bottom conditions. Therefore, it is necessary to find and understand the basic mechanism of this complex phenomenon and the fundamental relationships between and underlying the various parameters which control the phenomenon.

One of the most important parameters for this phenomenon is the diffusion coefficient of the fluid, denoted as $\varepsilon_f$. This parameter links the other parameters, such as the concentration distribution $C(z)$, the velocity distribution $V(z)$ and the diffusion coefficient of sediment, denoted as $\varepsilon_s$. So, it is important to estimate $\varepsilon_f$ value under waves and currents for understanding the basic mechanism of the sediment transport due to waves and currents.

In this study, two different approaches are applied to some experimental data in order to estimate $\varepsilon_f$ value under non-breaking waves with a current condition.

The first method is a method based on the concentration distribution data considering the sorting-effect and $\beta$-effect of sediment particles.

The second method is a method based on the velocity distribution data with some assumptions of internal shear stress.

The aims of this study are to calculate $\varepsilon_f$ values by these two methods and discuss the calculated $\varepsilon_f$ values from a physical point of view.
1.2 Basic equations of sediment transport mechanism

In this section, the basic equations concerning sediment transport, concentration distribution, \( \beta \)-effect, velocity distribution and diffusion coefficient of the fluid will be explained respectively.

(1) Sediment transport

A simple method to obtain sediment transport is to multiply the sediment concentration and to integral over the depth with the velocity.

\[
\eta(t)
\]

\[
S(t) = \int_0^h C(z, t) \times V(z, t) \, dz
\]  

(1.1)

where;

- \( S(t) \): Local instantaneous sediment transport rate per unit width to the current direction [kg/ms]
- \( C(z, t) \): Local instantaneous sediment concentration [kg/m³]
- \( V(z, t) \): Local instantaneous current velocity [m/s]
- \( z \): Height above the mean bed level [m]
- \( t \): Time [s]
- \( \eta(t) \): Water surface elevation from the mean bed level [m]

Taking time-(and bed-)average of eq.(1.1), the general expression of time-(and bed-)averaged sediment transport is obtained, as eq.(1.2).

\[
\bar{S} = \frac{\int_0^h C(z, t) \times V(z, t) \, dz}{h} = \frac{\int_0^h \bar{C}(z) \times \bar{V}(z) \, dz + \int_0^h \dot{C}(z, t) \times \dot{V}(z, t) \, dz}{h}
\]  

(1.2)

where;

- \( \bar{S} \): Time-(and bed-)averaged sediment transport rate per unit width to the current direction [kg/ms]
\[ C(z) \]: Time- (and bed) averaged component of the local instantaneous concentration \[ \text{[kg/m}^3\text{]} \]

\[ C(z,t) \]: Fluctuating component of the local instantaneous concentration \[ \text{[kg/m}^3\text{]} \]

\[ V(z) \]: Time- (and bed) averaged component of the local instantaneous current velocity \[ \text{[m/s]} \]

\[ V(z,t) \]: Fluctuating component of the local instantaneous current velocity \[ \text{[m/s]} \]

\[ h \]: Time- (and bed) averaged water depth \[ \text{[m]} \]

\[ \tilde{} \]: Symbol of time- (and bed) averaging

\[ C(z,t) = \overline{C(z)} + C^r(z,t) \] \hspace{1cm} (1.2.a)

\[ V(z,t) = \overline{V(z)} + V^r(z,t) \] \hspace{1cm} (1.2.b)

Eq.(1.2) shows that sediment transport is divided into two parts. The first is determined by time averaged concentration and current velocity. It represents the transport of sediment by time averaged velocity. Therefore this part is called "the current-related sediment transport" in the present report. The second part is determined by fluctuating components mainly caused by the orbital movements of the fluid. Thus, this part is called "the wave-related sediment transport" in the present report.

Eq.(1.2) is a useful approximation of eq.(1.1). In practice, only time-averaging concentrations and velocities are determined. Because it is quite difficult to measure the instantaneous concentrations distribution and the instantaneous velocity distribution.

Generally, there are two types of sediment transport mechanisms near the coast. One is called longshore sediment transport and the other is called cross-shore sediment transport. (See Fig.1.1) The longshore sediment transport mechanism can be described as; Waves, approaching a coast, will reach the coast under a small angle due to refraction. The radiation stress generated by waves
and the bottom friction stress results in a longshore current. A longshore current can be also generated by local conditions, such as tide or winds. The stirred up sediment particles by the waves and current are transported to the longshore direction by a rather steady longshore current. Thus, the longshore sediment transport can be represented by the current-related sediment transport:

\[ S = \int_{0}^{h} C(z) \ast V(z) \, dz \tag{1.3} \]

from eq.(1.2) rather well. Because the current velocity does not depend on time, the longshore current is rather steady. The time-averaged concentration \( C(z) \) can be used and the use of instantaneous concentration \( C(z,t) \) can be avoided.

The cross-shore sediment transport mechanism can be described as; Tide and waves also generate currents perpendicular to the coastline with local conditions. And the stirred up sediment particles are transported to the direction perpendicular to the coastline. In this case, the instantaneous current velocity \( V(z,t) \) and the instantaneous concentration \( C(z,t) \) do strongly depend on time. Thus, for cross-shore sediment transport, the wave-related sediment transport plays a much important role.

In this study, of more basic investigations of the sediment transport due to waves and currents, only time-(and bed ) averaged parameters, such as \( C(z) \) and \( V(z) \), are treated. Then, the basic equation of the sediment transport corresponds to eq.(1.3). The concept of eq.(1.3) is illustrated in Fig.1.2. This means, this study is a basic study relating to the current-related sediment transport and the longshore sediment transport mechanism.
Fig. 1.1 Longshore and Cross-shore sediment transport

Fig. 1.2 Sediment transport = Velocity * Concentration
(2) Concentration distribution
Sediment concentration is defined as the mass of sediment particles per unit water volume and its distribution is expressed with a help of a diffusion type model. (See Fig.1.3)
Assuming a steady state condition without gradients in the x- and y-directions, the vertical distribution of the sediment concentration can be described as eq.(1.4).

\[
W \times C(z) + \varepsilon_s(z) \times \frac{dC(z)}{dz} = 0 \tag{1.4}
\]

where;
\[
\begin{align*}
W & : \text{Fall velocity of particles} \quad [\text{m/s}] \\
C(z) & : \text{Time-(and bed-) averaged sediment concentration} \\
& \text{at height } z \text{ above the bed} \quad [\text{kg/m}^3] \\
\varepsilon_s(z) & : \text{Diffusion coefficient of the particles at height } z \\
& \text{above the bed} \quad [\text{m}^2/\text{s}] \\
\end{align*}
\]
(The time-(and bed-) averaging symbol "---" will be omitted further.)

Eq.(1.4) means that the concentration moving downward with the fall velocity \(W\) is balancing the concentration moving up/downward due to the diffusion activity at height \(z\).

In practice, the sediment particles stirred up by fluid motion consist of particles with different sizes and shapes, and each of them has different fall velocity and feels different diffusion activity. Generally smaller particles can be easily stirred up and reach higher level in the water column in comparison with bigger particles. Then, sediment concentration at each elevation consists of different particle size components. This is called "sorting effect". Unless the bed material is perfectly uniform, it is inevitable to take into account the sorting effect when \(\varepsilon_s\) values are estimated by eq.(1.4).
Fig. 1.3 Principle of diffusion type model
The relationship between the diffusion coefficient of the particles and that of the fluid is generally described as eq.(1.5) with the help of parameter $\beta$.

$$\epsilon_s = \beta \times \epsilon_f$$

where;

- $\beta$ : Factor
- $\epsilon_f$: The diffusion coefficient of the fluid

As to the $\beta$ value, many investigations have been performed and it has become clear that the $\beta$ value depends on the fall velocity of the particle and the diffusion coefficient of the fluid. Van de Graaff(1988) found from experiments an equation to estimate the $\beta$ value for the particle fall velocity and the diffusion coefficient of the fluid as eq.(1.6).

$$\beta = 1 + \left( \frac{0.008455}{\epsilon_f} - 11.01 \right) \times W$$

(holds for app. $0.000300 \text{ m}^2/\text{s} < \epsilon_f < 0.002500 \text{ m}^2/\text{s}$)

This equation means that the sediment particles with different fall velocities feel different diffusion activities from the fluid and their intensities also depend on the diffusion activities of the fluid.

In order to estimate $\epsilon_f$ values by eq.(1.4) and (1.5), the $\beta$-effect should be taken into account as well as the sorting effect.

(4) Velocity distribution

The velocity distribution $V(z)$ is linked to the internal shear stress with a help of the diffusion coefficient of the fluid as eq.(1.7).

$$\tau(z) = \rho \times \epsilon_f(z) \times \frac{dV(z)}{dz}$$
\[ \tau(z) = \rho \cdot \{ \ell_e(z) \} \cdot \left[ \frac{dV(z)}{dz} \right]^2 \]  

(1.8)

So,

\[ \varepsilon f(z) = \{ \ell_e(z) \} \cdot \left| \frac{dV(z)}{dz} \right| \]  

(1.9)

where;

\( \ell_e(z) : \) The mixing length at height \( z \) above the bed  \([\text{m}]\)

By using special assumption that;

\[ \ell_e(z) = \kappa \cdot \frac{Z}{h} \cdot \left[ \frac{h - Z}{h} \right]^{0.5} \]  

(1.10)

Eqs. (1.8), (1.9) and (1.10) lead to the "well-known" logarithmic velocity distribution law;

\[ V(z) = \frac{V_*}{\kappa} \cdot \ln \left( \frac{Z}{Z_0} \right) \]  

(1.11)

\[ V_* = \left( \frac{\tau_0}{\rho} \right)^{1/2} \]  

(1.12)

where;

\( V_* : \) Shear velocity  \([\text{m/s}]\)
\( \kappa \): Von Kármán constant ( = 0.4)

\( Z_0 \): The elevation at which the velocity is zero [m]

\( \tau_0 \): The bottom shear stress [N/m²]

From eqs. (1.9), (1.10) and (1.11), \( \varepsilon_f(z) \) under a mere current condition is derived as parabolic distribution.

\[ \varepsilon_f(z) = \kappa \cdot V_* \cdot Z \cdot (1 - \frac{Z}{h}) \]  

(1.13)

On the other hand, many researchers about the bottom shear stress have reported that \( \tau_0 \) in case of combination of waves and a current becomes bigger than that of the mere current case (Bijker 1967), and the bottom roughness \( (r=33Z_0) \) increases comparing that of the mere current case. Then, when \( \varepsilon_f(z) \) is estimated by eq. (1.7) in case of a current with waves, this change should be also taken into account.

(5) Diffusion coefficient of the fluid under waves and currents

Diffusion coefficient of the fluid under a mere current is expressed as parabolic function as eq. (1.13). On the other hand, the diffusion coefficient of the fluid under waves or a current with waves, it has not been fixed how to express the diffusion coefficient distribution as a function of boundary conditions. But it has been apparent that waves induce a quite strong diffusion activity in the entire water column from various experiments.

At least, it can be stated that \( \varepsilon_f \) values under a current with waves are bigger than those in case of the mere current, since the orbital motions of the water particles and the increased bottom shear stress due to wave action cause stronger diffusion activities in the entire water column than those in case of the mere current.

\[ \varepsilon_{fw} > \varepsilon_c \]  

(1.14)

where:

\( \varepsilon_{fw} \); Diffusion coefficient of the fluid under a current with waves [m²/s]

\( \varepsilon_c \); Diffusion coefficient of the fluid in case of the mere current [m²/s]
According to the research by Van de Graaff (1988), two kinds of equations relating the separate contributions of waves and current to the resultant diffusion coefficient have been proposed and investigated.

\[
\varepsilon_{fv+c} = \varepsilon_{fw} + \varepsilon_{fc} \quad (1.15)
\]

\[
\varepsilon_{fv+c} = \sqrt{[\varepsilon_{fw}]^2 + [\varepsilon_{fc}]^2} \quad (1.16)
\]

where:

\(\varepsilon_{fw}\); Diffusion coefficient of the fluid in case of only waves [m²/s]

However, the relationship among \(\varepsilon_{fw}, \varepsilon_{fc}\) and \(\varepsilon_{fw}\) is not clearly revealed.

1.3 Methods to investigate the diffusion coefficient

In order to investigate the diffusion coefficient of the fluid, there are at least two methods.

One is based on the concentration distribution data. This method uses mainly eqs. (1.4) and (1.5) with consideration of sorting effect and \(\beta\)-effect. Recently a fairly good technique to calculate the diffusion coefficient of the fluid has been developed by Van de Graaff and has been applied to the measured concentration data under waves or waves with a current. This method will be explained in detail in chapter 3.

The other is based on the velocity distribution data. This method uses mainly eq.(1.7) with some assumptions. This method is proposed in the present study for the first time. The details of this method will be explained in chapter 4.

In the present study, these methods are applied to some available measurement data explained in chapter 2.

In Fig.1.4, these two methods are shown illustratively.
Fig. 1.4 Flow-chart of the two methods for $\varepsilon_f(z)$ investigation
2. Used experimental data and outline of the experiments

In this study, the experimental data measured by E.N.Nap and H.F.A.van Kampen are selected for main calculations and for the comparison, some experimental data measured by D.Heijboer are used. In this chapter, these experimental data and outlines of these experiments will be explained.

2.1 The experiments by E.N.Nap and H.F.A.van Kampen

Their experiment has been performed in a flume of the Laboratory of Fluid Mechanics of the Faculty of Civil Engineering of the Delft University of Technology in 1988, in order to investigate the concentration and velocity distribution under waves and current. The flume is sketched in Fig.2.1. The flume, with a total length of about 45 m, width of 0.8 m and depth of 1.0 m, could make it possible to perform experiments with 30 m bed-length, 0.12 m bed-height and mean water depth of 0.5 m. The wave generator created irregular waves; wave action in combination with a following or opposing current could be performed.

The bed material size was selected as $D_{50}$ of approximately 100 $\mu$m and $D_{90}/D_{10}$ (sorting ratio) of approximately 2. The bed condition was a movable bed with about 10 cm thickness.

The wave conditions were selected as non-breaking irregular waves with peak period of approximately 2.5 s and significant wave height of approximately 0.075 m, 0.10 m, 0.15 m and 0.18 m respectively.

The current conditions were selected as following or opposing current with mean water depth of approximately 0.5 m and depth averaged velocity of approximately 0.4 m/s, 0.2 m/s and 0.1 m/s respectively.

Additionally, four only waves cases and two only current cases have been performed.

Totally, 29 tests with different waves or current conditions were selected and performed. For convenience, each test will be called its test name shown in Table2.1. The first figure indicates its significant height, the second figure indicates its depth averaged
velocity and the sign of + or - indicates following current or opposing current respectively.

Just after measuring under waves with current, the velocity distribution under the mere current with same discharge was measured for each case. Main parameters are shown in Table 2.2. The following parameters were measured by the following instruments.

- Discharge by Rehbock weir
- Mean bed level by PROFO (profile follower) and integrator
- Water level by measuring scale
- Wave parameters and spectrum by electric resistance probe
- Time-and bed-averaged sediment concentration and current velocity
- Sediment concentration by concentration sampler instrument on the moving carriage and volume meters
- Current velocity by E.M.S. (electro-magnetic velocity meter) on the moving carriage
- Ripple parameters by PROFO
- Particle diameters of the bed material by sieving method
- Fall velocity by DUST (Settling Tube of the Delft University)
- Water surface slope, only for mere current condition, by static pitot tubes

The flume consists of various sections;

A - Wave-generator section
B - In- and outflow section
C - Test section
D - Section with wave damping slope structure
E - In- and outflow section
Fig. 2.1 Sketch of the Nap and Kampen's experimental flume
Measuring procedure

A list of the actions, step by step, in the measuring procedure will be given here:

(Preparation)
1. Read the static pitot tubes.
2. Calibration of the velocity meter at still water.
3. Generation of the desired discharge and water depth.
4. Generation of the desired significant height.
5. Wait period (about half an hour) for generation of the characteristic ripple pattern.
6. Measure water temperature.
7. Switch off the wave generator.
8. Mark 10 ripple crest positions at flume window (for determination of ripple migration velocity).

(Test measurements)
9. Read the discharge.
10. Make ripple registrations in the three longitudinal sections at the measuring section.
11. At the same time as 7., determine the mean bed level in the measuring section with the integrator.
12. Measure distance still water level to flume bottom.
13. Installation of the concentration sampler and velocity meter, about 2 [cm] above the mean bed level.
14. Start the moving carriage.
15. Check whether the concentration sampler and velocity meter do hit the bed ripples. (If so, return to 10.)
16. Start the wave generator.
17. Start spectrum computer program.
18. Start pumping out water-sediment samples at 10 heights above mean bed level. (Change buckets after about 15 minutes, for each pump.)
19. Measure fluid velocities at 10 heights above mean bed level. (About 5 minutes per measurement)
20. Determine sediment concentrations with the volume meter. (Twice, for two series of buckets.) Put the samples in sample bottles.
21. Read wave spectra, determine wave parameters by running the spectrum analyzer program.
22. Switch off the wave generator.
23. Stop moving carriage.
24. Determine ripple migration velocity at 10 locations. (See point 7, this is done only once)

Procedure No.9 to 23 have been carried out three times for each experiment.
25. Make ripple registrations, determine mean bed level.
26. Read discharge.  
   (Current alone measurement)
27. Measure fluid velocity at the measuring section at 10 heights above the bed, with moving carriage.
28. Read static pitot tubes every 10 minutes, determine the average water surface slope.  
   (At last)
29. Turn off flow.
30. Take three sediment samples from the sand bed, at cross sections 5, 27 and the measuring section.
31. Determine the sediment particle parameters (D_{10}, D_{50} and D_{90}) by sieving the samples after drying.
32. If necessary, resupply and remix the sand bed.
33. Determine the median fall velocity of the concentration samples in the settling tube (DUST). (This has been done in the period after all the experiments were carried out.)

The details of this experiment have been reported in "Sediment transport in case of irregular non-breaking waves with a current" (E.N.Nap and H.F.A.van Kampen, August 1988).
Table 2.1 Test name list of Nap and Van Kampen’s experiments

<table>
<thead>
<tr>
<th>$H_s$ [m]</th>
<th>0</th>
<th>0.075</th>
<th>0.10</th>
<th>0.15</th>
<th>0.18</th>
</tr>
</thead>
<tbody>
<tr>
<td>$V_m$ [m/s]</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>-0.4</td>
<td>T0-40</td>
<td>T7.5-40</td>
<td>T10-40</td>
<td>T15-40</td>
<td>T18-40</td>
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<td>T7.5-20</td>
<td>T10-20</td>
<td>T15-20</td>
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<td>-0.1</td>
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<td>T18-10</td>
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<td></td>
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<td>T7.5+40</td>
<td>T10+40</td>
<td>T15+40</td>
<td>T18+40</td>
</tr>
</tbody>
</table>
Table 2.2 Conditions of Nap and Van Kampen's experiments

<table>
<thead>
<tr>
<th>CASE</th>
<th>h [m]</th>
<th>Vm [m/s]</th>
<th>Hs [m]</th>
<th>Tp [s]</th>
<th>Δr [mm]</th>
<th>λ [mm]</th>
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</thead>
<tbody>
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<td>14.8</td>
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<td>0.0705</td>
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<td>0.1792</td>
<td>2.33</td>
<td>7.0</td>
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<td>78.7</td>
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Symbol; h is mean water depth
Vm is depth averaged velocity
Hs is significant wave height
Tp is peak wave period
Δr is averaged ripple height
λ is averaged ripple length
This experiment also has been performed in a flume of the Laboratory of Fluid Mechanics of the Faculty of Civil Engineering of the Delft University of Technology in 1988, in order to investigate the diffusion activity under waves and currents. The flume is sketched in Fig.2.2. In this experiment, the bed condition was a fixed bed with artificial ripples. Several sand materials with different \( D_{50} \) sizes were applied to the series of tests. The sand materials were supplied at the upstream position of the flume. The wave conditions were regular waves with wave period of 1.7 s and wave height of 0.115 m, or with wave period of 2.3 s and wave height of 0.128 m. The current conditions were following currents with depth averaged velocity of approximately 0.1 m/s, 0.2 m/s and 0.3 m/s. Similar parameters as to the Nap and Van Kampen’s experiment were also measured in this experiment.

The details of this experiment have been reported in "Zandconcentratie- en stroom snelheidsverdelingen onder golven en stroom"(D.Heijboer, May 1988).

In the present study, only two cases of Heijboer’s experimental data were used for the comparisons. The selected data cases were:

- \( D_{50} = 107 \, \mu m \), \( Vm = 0.3 \, m/s \), \( T = 1.7 \, s \), \( H = 0.115 \, m \)
- \( D_{50} = 197 \, \mu m \), \( Vm = 0.2 \, m/s \), \( T = 1.7 \, s \), \( H = 0.115 \, m \)

where:

\( Vm \) : the depth averaged velocity
\( T \) : the wave period
\( H \) : the wave height
3. The first method with measured concentration data

3.1 The calculation method

This method has been developed by Van de Graaff in order to investigate sediment concentration and diffusion activities due to wave action. It is based on the diffusion type model equation and Coleman-type equation for each small vertical calculation step. With the best fitting concentration distribution curve which is selected by special fit procedure and the bed material size parameters, the diffusion coefficient of the particles and the particle size distribution for each elevation are estimated step by step from the bottom of the water column. In this section, the procedure will be explained.

In the present study, the program called "CONC" that was developed by D.Heijboer to calculate the diffusion coefficient of the particles based on Van de Graaff’s method, and the program called "CONCFL" that was an improved "CONC" program by the author to calculate the diffusion coefficient of the fluid with β-effect equation. The list of these programs are shown in Part III of the report.

(1) Basic equations

The distribution of the bed material diameters is assumed to be the perfect log-normal type distribution that is expressed by three parameters ,D_{10},D_{50} and D_{90}.

\[
\phi(t) = \int_{-\infty}^{t} \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{t^2}{2}\right) \, dt \quad (3.1.a)
\]

\[
t(x) = \frac{x - \mu}{\sigma} \quad (3.1.b)
\]

\[
\ln D_x = \ln D_{10} + \frac{t(x) - t(10)}{t(50) - t(10)} \ln \frac{D_{50}}{D_{10}} \quad (3.1.c)
\]
for \( x > 50 \);

\[
\ln D_x = \ln D_{50} + \frac{t(x) - t(50)}{t(90) - t(50)} \times \ln \frac{D_{90}}{D_{50}} \tag{3.1.d}
\]

where;

\( \phi(t) \): Integrated value of standard normal distribution curve from \(-\infty\) to \(t\)

\( t \): Parameter of standard normal distribution

\( x \): Weight finer percentage [\%]

\( \mu \): Mean of \( x \) (=50)

\( \sigma \): Scale factor

\( t(x) \): Normalized value of \( x \)

\( t(10) \): Normalized value of 10 : \( t(10) = -1.21815518 \)

\( t(50) \): Normalized value of 50 : \( t(50) = 0 \)

\( t(90) \): Normalized value of 90 : \( t(90) = 1.21815518 \)

\( D_x \): Particle diameter of \( x \% \) finer by weight [\(\mu\text{m}\)]

\( D_{10}, D_{50}, \text{ and } D_{90} \) are defined in the same way.

It is illustrated in Fig.3.1.

Using the distribution of the bed material diameters, the particle size classes are classified as follows.

\[
D_{ch,i} = D_{[5i - 2.5]} \tag{3.2}
\]

where;

\( D_{ch,i} \): Characteristic diameter of the \( i \)-th class [\(\mu\text{m}\)]

\( i \): Class number \((i = 1,2,\ldots,20)\)

Eq.(3.2) yields \( D_{2.5}, D_{7.5}, D_{12.5}, \ldots \) and \( D_{97.5} \) as characteristic diameters. An example of \( D_{ch,i} \) is illustrated in Fig.3.1.

For the fall velocity \( W \), the relationship as proposed by the Delft Hydraulics (1983) is used.

\[
\frac{1}{W} = 10 \left[ A \ast (\log D)^2 + B \ast (\log D) + C \right] \tag{3.3}
\]

for \( 50 \ \mu\text{m} < D < 400 \ \mu\text{m} \)
where;
A,B,C : Parameters depending on the particular case
A = 0.495, B = 2.410, C = 3.740 for fresh water with
temperature of 18 °C are used in this program.
D : Particle diameter in meter \([\text{m}]\)
W : Fall velocity of particle \([\text{m/s}]\)

So, the fall velocity of each particle diameter can be calculated
by eq.(3.3).

It can be assumed that each of the 20 fractions 'builds up' its
own vertical concentration distribution. The resulting
concentration at an arbitrary elevation \(z\) above the bottom then
equals the sum of the contributions of the 20 sub-divisions.

\[
C(z) = \sum_{i=1}^{n} C(i,z) \quad (3.4)
\]

where;
\(C(z)\) : Concentration at height \(z\) above the bed \([\text{kg/m}^3]\)
\(C(i,z)\): Concentration of the \(i\)-th sub-division class
consisting of \(D_{ch,i}\) size particles at height \(z\)
above the bed \([\text{kg/m}^3]\)
\(n\) : The number of sub-division classes (=20)

For the bottom concentration, it is assumed that all concentration
sub-divisions are equal to each other.

\[
C(1,0) = \frac{1}{n} C(0) \quad (3.5)
\]

In a small vertical step \(\Delta z\), it is assumed that for each
concentration sub-division "Coleman-type model" is valid.

\[
C(i,z) = C(i,z-\Delta z) \cdot \exp \left[ \frac{- W_i \cdot \Delta z}{e_S(i, z-0.5\Delta z)} \right] \quad (3.6)
\]

where;
\(\Delta z\) : Small vertical step for calculation \([\text{m}]\)
Wi : Fall velocity of the i-th sub-division class \[ \text{[m/s]} \]

\( \varepsilon_s(i,z-0.5\Delta z) \) : Diffusion coefficient of the i-th sub-division class at height \((z-0.5\Delta z)\) above the bed \[ \text{[m}^2\text{/s]} \]

The Coleman type model assumes that the diffusion coefficient of the particle \( \varepsilon_s \) is constant in the entire water column. It is illustrated in Fig.3.2. In this present study, the Coleman type model is valid for each concentration sub-division in a small vertical step \( \Delta z \).

For "CONC" program;
Assuming that all particle feel the same \( \varepsilon_s \) in a small vertical step \( \Delta z \), the following equation can be derived from eqs.(3.4) and (3.6).

\[
C(z) = \sum_{i=1}^{n} C(i,z-\Delta z) \times \exp \left[ \frac{-Wi \times \Delta z}{\varepsilon_s(z-0.5\Delta z)} \right] \tag{3.7}
\]

The calculated diffusion coefficient of the particles by eq.(3.7) is called the obvious \( \varepsilon_s \).

Eqs.(3.1),(3.3),(3.5) and (3.7) are basic equations that make it possible to calculate the obvious \( \varepsilon_s(z) \) distribution by iteration method. These equations are used in "CONC" program. The calculation of the obvious \( \varepsilon_s \) values starts from the bottom, \( z = 0 \), toward the water surface with the estimated \( C(z) \) values by the special fit procedure. This special fit procedure will be explained in the next paragraph.

During calculating \( \varepsilon_s \) values at each elevation, \( C(i,z) \) are also estimated simultaneously. Then the sediment particle diameter distribution at an arbitrary height \( z \) above the bed, such as \( D_{10}(z), D_{50}(z) \) and \( D_{90}(z) \), also can be calculated by summing up of concentration sub-divisions.

For "CONCFL" program;
Instead of eq.(3.6), the following \( \beta \)-effect equation is applied to the original method. This improvement makes it possible to calculate the diffusion coefficient of the fluid, \( \varepsilon_f \), at each
level.

\[ \beta_i = 1 + \left[ \frac{0.008455}{\varepsilon_f} - 11.01 \right] \ast W_i \quad (3.8) \Rightarrow (1.6) \]

where;

\[ \beta_i \quad : \beta \text{ value for the } i\text{-th sub-division class} \]

\[ C(z) = \sum_{i=1}^{n} C(i,z-\Delta z) \ast \exp \left[ \frac{-W_i \ast \Delta z}{\beta_i \ast \varepsilon_f(i,z-0.5\Delta z)} \right] \quad (3.9) \]

Eqs. (3.1), (3.3), (3.5), (3.8) and (3.9) are basic equations for "CONCFL" program to calculate \( \varepsilon_f \) values by iteration method. The calculation procedure is the same as "CONC" program.
Fig. 3.1 Illustration of the perfect log-normal distribution for bed material diameters

\[ C(z) = C(0) \cdot \exp \left( -\frac{Wz}{E_s} \right) \]

Fig. 3.2 \( \varepsilon_s(z) \) and \( C(z) \) distribution of Coleman type model
(2) **Fit procedure for measured concentration distribution**

For performing the calculation for finding $ef(z)$ and $D_{50}(z)$ distributions over the depth, a mathematical description of the concentration distribution over the water depth is required. The fit procedure developed by Van de Graaff seems tolerably objective and fairly reliable results are obtained for model and prototype conditions.

Generally, it can be stated that:
- the highest concentrations occur near the bed,
- a gradual decrease in concentration occurs towards the higher levels above the bed.

Defining the special coordinate system with $z_\ast (=z/h)$ and $d\ln(C)/dz_\ast$ axes that is called "standard square" shown in Fig.3.3, the three requirements are assumed for the gradient curve in the standard square. From the general characteristics of sediment concentration distributions, the three requirements are as follows:

1. For $z_\ast = 0$, the gradient is 1 and at its maximum.
2. No negative gradients.
3. No large gradients close to the bed in the standard square.

The curves in Fig.3.3 can be derived by:

$$\frac{d \ln(C)}{d z_\ast} = b + (1-b) \ast \left[\frac{(a-z_\ast)}{a}\right]^p \quad (3.10)$$

where:
- $C$ : Sediment concentration at height $z_\ast$ [kg/m$^3$]
- $z_\ast$ : Relative height ($=z/h$)
- $z$ : Height above the bed ($z = 0$ being the bed) [m]
- $h$ : Water depth [m]
- $b$ : Horizontal ordinate point A ($0.025 \leq b \leq 1$)
- $a$ : Vertical ordinate point A (in the proposal procedure "a" can be $1.0$, $0.9$, $0.8$ or $0.7$; these values are rather subjective)
- $p$ : Exponent of fit parabola for gradient curve ("p" can be $2$, $4$, $6$ or $8$)
Eq. (3.10) describes gradients in the standard square. The actual gradients in a \( \ln(C) \) versus \( z_\ast \) plot become:

\[
\frac{d \ln(C)}{dz_\ast} = q \left[ b + (1-b) \left( \frac{(a-z_\ast)/a}{p} \right)^p \right]
\]

(3.11)

where;
\( q \) : (negative) Scale factor

Integration of eq. (3.11) yields:

\[
\ln(C) = q \left[ b \cdot z_\ast - (1-b)(a-z_\ast)\left( \frac{(a-z_\ast)/a}{p+1} \right)^p + d \right]
\]

(3.12)

where;
\( d \) : Integration constant

The procedure in a computer program starts with \( a = 1 \) and \( p = 8 \). The 'best' values of the parameters \( q \), \( b \) and \( d \) are determined with a least square method. If it is turns out that \( b < 0.025 \) or \( b > 1.0 \), then \( b \) is fixed at 0.025 or 1.0 respectively and the procedure is restarted. The parameters \( q \) and \( d \) are determined now.

In the next step it is checked whether a better result is found with \( a = 1 \) and \( p = 6 \), and so on. The 'best' result is stored. The procedure then continues with \( a = 0.9 \) and again \( p = 8 \). It is checked whether for this \( a = 0.9 \) case a better result than the stored one is obtained. And so on. As stated the least 'a' value is taken as 0.7.
The calculation method

The flow-chart of the first method is shown in Fig. 3.4.

The outlines of "CONC" and "CONCFL" program are as follows.

1. Calculation of the characteristic diameters, \( D_{ch,i} \) (\( i = 1, 2, \ldots, 20 \)), with the data of \( D_{10}, D_{50} \) and \( D_{90} \) of the bottom material by eqs. (3.1) and (3.2).

2. Finding the best fit curve, \( \ln(C) \), with the help of the fit procedure and the measured concentration data.

3. Calculation of the characteristic fall velocity, \( W_i \) (\( i = 1, 2, \ldots, 20 \)), by eq. (3.3).

4. Iterative calculation for \( \epsilon_s \) values by eqs. (3.4), (3.5), (3.6) and (3.7) from the bottom toward the higher elevation by \( \Delta z \) step. (In case of "CONC" program)

4' Iterative calculation for \( \epsilon_f \) values by eqs (3.4), (3.5), (3.8) and (3.9) from the bottom toward the higher elevation by \( \Delta z \) step. (In case of "CONCFL" program)

Fig. 3.3 Standard square for fit procedure
DATA

- $D_w$, $D_a$ and $D_m$ of the bottom material

- The measured concentration data: $C(z)$

**Calculation of $D_i$ and $W_i$**

**Fit procedure to find the best fit curve of $C(z)$**

**start from $Z = 0$**

**Iterative calculation**
- $C_5$ at height $Z + 0.5 \Delta Z$ (for "CONC" program)
- $C_f$ at height $Z + 0.5 \Delta Z$ (for "CONCFL" program)

**$Z = Z + \Delta Z$**

**$Z > h$**

**end**

---

**Fig. 3.4 Flow-chart of the first method**

---

31
3.2 The $\varepsilon_s(z)$ distribution by CONC program

The calculations for the obvious diffusion coefficient of the particles, $\varepsilon_s$, for the Nap and Van Kampen's experiments have been done by "CONC" program with $\Delta z = 0.005$ m. Fig.II-1.1 to II-1.5 show the results. The $\varepsilon_s$ values have been calculated from the bottom to the highest sediment concentration measuring level by this program.

Although these results seem to contain a lot of mess, they show some tendencies. Bigger $\varepsilon_s$ values exist with faster current velocity than those with slower current velocity, bigger $\varepsilon_s$ values exist at higher elevation than those of the bottom in most of cases. These tendencies are seen in both of the following and opposing currents with waves.

In order to find more closer relationships between $\varepsilon_s$ values and the boundary conditions, several attempts have been made. Fig.II-1.6.1 to II-1.6.3 show the relationships among $\varepsilon_s(0)$, the ripple height $\Delta r$, the orbital velocity amplitude of the water particle at the bottom, $U_b$, and that at $z = 0.25$ m, $U$. The orbital velocity amplitude of the water particle was estimated for the root mean square wave height, $H_{rms}$, in this study. The calculation method of the orbital velocity amplitude of the water particle in case of waves with a current will be explained in section 4.1.3.

From these figures, it seems that $\varepsilon_s(0)$ tends to be more or less proportional to $\Delta r$ but does not have clear relation with $U_b$.

Fig.II-1.7 to II-1.12 show the comparison between $\varepsilon_s(z)$ distribution and the diffusion coefficient distribution of the fluid in case of the mere current $\varepsilon_{fc}(z)$. The $\varepsilon_{fc}$ values have been calculated from the parameters of the fittest logarithmic velocity distribution curve for the measurement velocity data under the mere current condition. The method to find these parameters will be explained in section 4.1.2 and these parameters are shown in Table 4.1. Fig.II-1.7 to II-1.12 show that most of $\varepsilon_s$ values, that is $\beta \cdot \varepsilon_f$ value, are smaller than $\varepsilon_{fc}$ value. It implies that the diffusion activities of the particles under waves with a current are smaller than that of the fluid under the mere current in most of cases. Fig.II-1.13 shows the relationship between $[\varepsilon_s - \varepsilon_{fc}]$
versus \( \hat{U} \) at \( z = 0.25 \) m. This figure also indicates that in most of the cases the diffusion activity in case of a current with waves is smaller than that of the mere current in the middle layer in the water column and its difference, \([\varepsilon_f-\varepsilon_{fc}]\), does not show a clear tendency with \( \hat{U} \).

Can it be stated that \( \beta \) is less than one from these results?

The meaning of \( \varepsilon_s(z) \) distribution calculated by "CONC" program is the one kind of representative diffusion coefficient value of the particles at each elevation in the water column. Actually there are a lot of different size particles at each elevation which feel different diffusion activities, then also different \( \beta \)-effects, from the fluid. The \( \varepsilon_s \) values by "CONC" program indicate one kind of averaged diffusion coefficient value of the particles. But it is not clear which particle size is represented by the obvious \( \varepsilon_s \) value. Only if the perfect sorted material \( (D_{90}/D_{10} = 1) \) is applied for the bottom material, then the particle size relating to the obvious \( \varepsilon_s \) value directly can be fixed. In practice, it is impossible to use the perfect sorted material for experiments because of the expensive cost of such material. In fact, the bottom material in the Nap and Van Kampen's experiments was sorted as \( D_{90}/D_{10} \approx 2.0 \). Therefore, it cannot be stated that \( \beta \) is less than one from these results. That is, the \( \varepsilon_s \) values calculated by "CONC" program cannot be compared to \( \varepsilon_{fc} \) values directly, unless some \( \beta \)-effect estimation is performed for each particles at each elevation under the water column.

So, in order to find the diffusion coefficient distribution of the fluid \( \varepsilon_f(z) \) under waves and a current, and to compare \( \varepsilon_f(z) \) and \( \varepsilon_{fc}(z) \), "CONCFL" program has been developed based on "CONC" program with a help of \( \beta \)-effect eq.(1.6).
3.3 The $\varepsilon_f(z)$ distribution by CONCFL program

As explained in section 3.1, this program can be used to calculate the diffusion coefficient distribution of the fluid, $\varepsilon_f(z)$, considering the $\beta$-effect by eq.(1.6). The calculation results of $\varepsilon_f(z)$ for the Nap and Van Kampen's experiments by CONCFL program are shown in Fig.II-2.1 to II-2.13. In these graphs, $\varepsilon_f(z)$ distribution under waves and a current and $\varepsilon_{fc}(z)$ distribution can be compared. In this time, $\varepsilon_f$ values have been calculated from the bottom to the mean water surface level and been plotted up to $z = 0.4$ m above the bottom in these figures.

In order to see relationships among $\varepsilon_f$ values and boundary conditions, such as $\Delta r$ and $\hat{U}$, Fig.II-2.14.1 to II-2.14.3 were plotted.

These figures show that:

- The tendencies seen in these figures are similar to the tendencies that the $\varepsilon_s$ values calculated by "CONC" program show.

- The diffusion coefficient of the fluid $\varepsilon_f$ under waves and a current calculated by "CONCFL" program, $\varepsilon_f$, are smaller than that of the mere current case, $\varepsilon_{fc}$, in the middle (and upper in some cases) part of the water column, in most cases. This tendency becomes dominant with increasing of current velocity. It implies that if $\varepsilon_f$ values calculated by "CONCFL" program for the Nap and Van Kampen's experimental data are true, $\varepsilon_f$ under waves with a current is smaller than $\varepsilon_{fc}$ in the middle (and upper) part of the water column. This seems contradictory from the general physical sense and does not fit the expression of eq.(1.14) that $\varepsilon_{fwc} > \varepsilon_{fc}$. Normally $\varepsilon_f$ value under waves and a current is thought to be bigger than $\varepsilon_f$ under the mere current.

In order to check whether "CONCFL" program well simulates the real phenomenon, the measured $D_{50}$ data at several elevations and those calculated by "CONCFL" program have been plotted in Fig.II-3.1 to II-3.10. (See Fig.II-3.1 to II-3.10). These figure show that the measured $D_{50}$ values are about 5 to 10 % bigger than those calculated by "CONCFL" program, and at the bottom the measured $D_{50}$
is always bigger than that input to "CONCFL" program.
As the boundary condition of bottom material, D_{10}, D_{50} and D_{90} data were input into "CONC" and "CONCFL" programs. These data were measured from the sampled bed material just after each test procedure. (See measuring procedure No.30 and 31).

It can be thought that during the test the finer particles were washed out by the water movements and the bottom surface was covered by mainly coarser particles. Indeed the water in this flume was not circulated and during the period for creating the stable ripple pattern, that was about 30 minutes, the finer particles could be washed out from the bottom surface. On the other hand, the data for D_{10}, D_{50} and D_{90} of the bottom material were estimated from the bed material samples that could include finer particles existing relatively deeper area from the surface of the bottom. Consequently, D_{10}, D_{50} and D_{90} values used for "CONC" and "CONCFL" program could be relatively finer values comparing the actual ones at the bottom surface.

So, it can be stated that one of the reason why $\varepsilon f$ calculated by "CONCFL" program is smaller than $\varepsilon fc$ is due to armour effects during the testing period. In other words, the input D_{10}, D_{50} and D_{90} were too small in comparison with the real and too small D_{10}, D_{50} and D_{90} values yield too small $\varepsilon s$ and $\varepsilon f$ values.

Unfortunately, only D_{50} value at the bottom during the testing period is available, and D_{10} and D_{90} values at the bottom during the testing period are not available, for the Nap and Van Kampen's experimental data. Then, it seems quite difficult to calculate more reasonable $\varepsilon f(z)$ distributions for these data.
3.4 The additional calculations for comparisons

Here, two kinds of additional calculations have been performed for the comparisons.

(1) Comparing calculation 1
Instead of the Nap and Van Kampen's experimental data, some of Heijboer's experimental data have been used. The selected cases are as follows.

- \( D_{50} = 107 \, \mu m, \, V_m = 0.3 \, m/s, \, T = 1.7 \, s, \, H = 0.115 \, m \)
- \( D_{50} = 197 \, \mu m, \, V_m = 0.2 \, m/s, \, T = 1.7 \, s, \, H = 0.115 \, m \)

The calculation results are shown in Fig.II-4.1 and II-4.2. In these cases, \( \varepsilon_f(z) \) distribution under waves and a current show quite reasonable values comparing \( \varepsilon_{fc}(z) \) distribution. These figures show that the diffusion coefficient under waves and a current is bigger than that of the mere current case in the whole water column. These results are in accordance with the general physical sense. In Fig.II-4.3, the measured and the calculated sediment particle size distributions, \( D_{10}(z) \), \( D_{50}(z) \) and \( D_{90}(z) \), are compared for \( V_m = 0.3 \, m/s \) case. From that figure, it is stated that the \( D_{10}(z) \), \( D_{50}(z) \) and \( D_{90}(z) \) calculated by "CONCFL" program are more or less equal to the measured values respectively, and the \( D_{10} \), \( D_{50} \) and \( D_{90} \) values used for "CONCFL" program are exactly the same as the measured values respectively at the bottom. Because, in this calculation these particle sizes, \( D_{10} \), \( D_{50} \) and \( D_{90} \), as the boundary condition were selected from the measured data during the testing period. It means that the selected \( D_{10} \), \( D_{50} \) and \( D_{90} \) at the bottom were representing the actual particle size distribution at the bottom during testing period quite well even if armour effect existed at that moment. Also, in these experiments the sands were supplied continuously from the upstream part of the flume. Therefore, it could be thought that more or less steady concentration distribution consisting of the same range of the particle size distribution as that of the bottom material was kept during the testing period. In other words, the artificial sand supplying could compensate the unbalance due to armour effect to some extent.
From the comparing calculation 1, it can be stated that "CONCFL" program can calculate quite reasonable $\varepsilon f(z)$ distribution as long as the adequate data of the bottom particle size distribution and the concentration distribution are used.

(2) Comparing calculation 2
Just for a trial, using the modified data of the bottom particle size distribution parameters, T15-20 and T15+20 cases have been re-calculated by "CONCFL" program.
The modified $D_{10}$, $D_{50}$ and $D_{90}$ are calculated by the following equations.

\[
\log D_{10} = \log D_{10} \ast \left[ \frac{\log D_{50}}{\log D_{50}} \right] \quad (3.13.a)
\]

\[
\log D_{90} = \log D_{90} \ast \left[ \frac{\log D_{50}}{\log D_{50}} \right] \quad (3.13.b)
\]

where;

$D_{10}$, $D_{50}$, $D_{90}$ : Characteristic diameters of the bed material from the original data \([\mu m]\)

$D_{50}$ : $D_{50}$ value at the bottom measured during testing period \([\mu m]\)

$D_{10}$, $D_{90}$ : Modified $D_{10}$ and $D_{90}$ values at the bottom \([\mu m]\)

$D_{10}$, $D_{50}$ and $D_{90}$ are the bottom particle diameters considering armour effect in a sense. The results of these cases are shown in Fig.II-4.4 and II-4.5. The $\varepsilon f(z)$ calculated with the modified bottom particle sizes are slightly bigger than those of the original data, but it is still smaller than $\varepsilon fc$ in the middle part of the water column. In these cases, the bottom particle size $D_{50}$ as the input data is exactly equal to the measured one.

Probably, this assumption to estimate the armour effect might be too simple to estimate the real armour effect. But, it can be stated that considering the armour effect by such a simple modification, $\varepsilon f(z)$ calculated by "CONCFL" program changes to the more reasonable side.
3.5 Study of the results

From the calculation results by "CONC" and "CONCFL" programs for the Nap and Van Kampen's data and the additional calculations, it becomes clear that;

- The first method based on the measured concentration data calculates smaller $\varepsilon_f$ values in case of waves with a current than $\varepsilon_{fc}$ values in the middle part of the water column in some cases. These results seem to be contradictory from a physical point of view. (Nap and Van Kampen's experimental data)
- This method can calculate quite reasonable $\varepsilon_f(z)$ distributions in some cases. (Heijboer's experiment data)
- One of the reason of the under-estimating $\varepsilon_f$ values could be the armour effect during the testing period. Then, the $D_{10}$, $D_{50}$ and $D_{90}$ values at the bottom as the boundary condition for this method should be selected considering armour effect. It is preferable to select these values from the measured data at the bottom surface during the testing period, not from the data of the total bottom material, for the input data of the first method.
- In the Nap and Van Kampen's experiments, it could be thought that during the first stage for creating stable ripple pattern of the movable bed with waves and a current relatively finer particles suspended in the water column were washed out from the flume and gradually the proportion of the particle size distribution suspended in the water column was changing. Therefore, the measured concentration data could be thought to consist of relatively coarser particles than those of the bottom material.
- In the Heijboer's experiments, the proportion of the particle size distribution suspended in the water column was more or less stable, because of the artificial continuous sand supplying from the sand bucket upstream. The artificial sand supplying could be thought to compensate the armour effect to some extent.
• This difference of the experimental method could also affect the $\varepsilon f(z)$ calculation by the first method.

• $\beta$-effect equation used in "CONCFL" program is valid for app. $3 \times 10^{-4} \text{m}^2/\text{s} < \varepsilon f < \text{app.} 25 \times 10^{-4} \text{m}^2/\text{s}$. If the actual $\varepsilon f$ values under waves and a current are much bigger than the upper limit of that predictor, there could be another possible explanation why $\varepsilon f$ values for the Nap and Van Kampen's experimental data were under-estimated. This will be referred to in chapter 5.
4. The second method with measured velocity data

4.1 The calculation method

As the second method to investigate the diffusion coefficient of the fluid under waves with a current, a quite simple method with measured velocity data can be considered. The concept of this method is as follows.

Eq. (1.7) yields:

\[ \varepsilon f(z) = \frac{\tau(z)}{\rho \ast \frac{dV(z)}{dz}} \quad (4.1) \]

Then, if \( \tau(z) \) and \( \frac{dV(z)}{dz} \) are known from the measured data, \( \varepsilon f(z) \) distribution can be estimated by eq. (4.1).

4.1.1 Velocity distribution under waves with a current

With respect to the velocity distribution under waves with a current, quite interesting phenomena have been reported from the Nap and Van Kampen's experiments. The velocity distribution under a following current with waves is reduced near the bottom and enlarged in the middle layers and again reduced in the surface layer as compared to the velocity distribution in case of the mere current. On the other hand, the velocity distribution under an opposing current with waves is reduced near the bottom and enlarged in the middle and surface layers as compared to that of the mere current. Fig. 4.1 shows the typical velocity profiles for these cases.

These phenomena have been also seen in the Heijboer's experiments that used following currents with regular waves. With respect to the wave-induced changes in velocity distribution near the bottom, it has been reported by several researchers that the extra turbulence of wave movement under a current with waves leads to an increase of the apparent bottom roughness. Consequently, \( V(z) \) distribution near the bottom under a current with waves becomes the shifted up logarithmic velocity distribution of the mere current case, and the velocity near the
bottom becomes smaller than that of the mere current case. However, the wave-induced changes in velocity distribution in the middle and surface layers reported in the Nap and Van Kampen's or the Heijboer's experiments do not simply fit the shifted up logarithmic velocity distribution of the mere current case. In the middle and surface layers V(z) distribution under a current with waves has a different shape from a logarithmic one. This point will be discussed in section 4.4. Therefore, it can be stated that the velocity distribution under waves and a current are different from that of the mere current case and probably the differences are caused by the current and waves interactions.
Fig. 4.1 Sketch of $V(z)$ distribution in cases of following and opposing currents with waves

Fig. 4.2 Sketch of $V_d(z)$ distribution in cases of following and opposing currents with waves
4.1.2 Curve fit of velocity distribution

In order to calculate $\varepsilon f$ values by eq.(4.1), it is necessary to describe the velocity description mathematically. The velocity distribution under waves and a current can be thought to consist of two components, the original velocity distribution part due to the mere current which is a logarithmic distribution and the extra part of velocity distribution caused by waves and current interaction. This is expressed as;

\[ V(z) = V_c(z) + V_d(z) \]  \hspace{1cm} (4.2)

where;
- $V(z)$: Time-(and bed-) averaged velocity distribution under waves and a current \([\text{m/s}]\)
- $V_c(z)$: The original velocity distribution due to the mere current (original current part: logarithmic one)\([\text{m/s}]\)
- $V_d(z)$: The extra part of velocity distribution caused by waves and current interaction (disturbed part)

From now on, $V_c$ will be called original current part $V_d$ will be called disturbed part in the present study, because the interaction of waves and a current seems to disturb the generating logarithmic velocity distribution in the middle and surface layers.

In Fig.4.2, $V_d(z)$ for following and opposing current with waves are sketched. Generally, $V_c(z)$ can be described by a logarithmic velocity distribution as eq.(1.11).

After several trials to express $V_d(z)$ in a mathematical expression, it has been revealed that $V_d(z)$ can be described by a combination of a constant value and positive and negative exponential functions.

So,

\[ V_c(z) = a_{c0} + a_{c1} \ln(z) \]  \hspace{1cm} (4.3.a)

\[ V_d(z) = a_{d0} + a_{d1} \exp(z) + a_{d2} \exp(-z) \]  \hspace{1cm} (4.3.b)

\[ V(z) = a_{c0} + a_{c1} \ln(z) + a_{d0} + a_{d1} \exp(z) + a_{d2} \exp(-z) \]  \hspace{1cm} (4.3.c)

where;
aco, ac1 : Coefficient in original current part
ado, ad1, ad2 : Coefficient in disturbed part
c, d : Suffix indicating current or disturbed part respectively

Probably other functions for Vd(z) distribution might be possible. But in the present study, the expression of eq.(4.3.b) was selected.

In the Nap and Van Kampen’s experiments, the velocity distribution in case of the mere current has been measured just after the measuring in case of waves with a current for every test. Then the bottom condition can be thought to be almost same for both situations.

Thus, Vc(z) can be calculated by the least square method with the measured velocity data in case of the mere current. After that, Vd(z) can also be calculated by least square method with the Vc(z) distribution and the measured velocity data under the current with waves.

Table 4.1 shows the calculated results and Fig.II-5.1 to II-5.8 show some examples of V(z), Vc(z) and Vd(z) distributions. The detail results are shown in the output list of "EFVEL" program in Part III of the report. "EFVEL" is the program name of the second method in the present study.

From these calculation results, dV(z)/dz and dVd(z)/dz can be calculated. In Fig.II-6.1 to II-6.8 some examples of these distributions have been shown.

From these figures, the next statements can be made;

• The fit curves of eq.(4.3.c) can describe the velocity distribution under a current with waves quite well.
• In case of a following current with waves, V(z) has its maximum value in a middle layer of the water column.
• In case of an opposing current with waves, V(z) is simply increasing from the bottom towards the upper layers.
• Considering V(z) distribution under a current with waves, Vd(z) part is not negligible and it plays an important role in the phenomenon.
• In case of a following current with waves, dv/dz has its positive maximum near the bottom and becomes zero in a middle
layer at which $V(z)$ is its maximum, and becomes negative in the upper layers above the position of $dV/dz=0$.

- In case of an opposing current with waves, $dV/dz$ is positive in the entire water column, and its maximum is located near the bottom, and it decreases towards the middle and upper layers.

Table 4.1 Coefficients of fit curves for velocity profiles

<table>
<thead>
<tr>
<th>CASE</th>
<th>$a_{co}[m/s]$</th>
<th>$a_{ci}$</th>
<th>$a_{do}[m/s]$</th>
<th>$a_{d1}$</th>
<th>$a_{d2}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>T0-40</td>
<td>0.5312</td>
<td>0.08757</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>T0+40</td>
<td>0.5384</td>
<td>0.08715</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>T7.5-40</td>
<td>0.5754</td>
<td>0.10457</td>
<td>-1.1187</td>
<td>0.4965</td>
<td>0.5805</td>
</tr>
<tr>
<td>T7.5-20</td>
<td>0.2424</td>
<td>0.03701</td>
<td>0.7821</td>
<td>-0.2499</td>
<td>-0.5739</td>
</tr>
<tr>
<td>T7.5-10</td>
<td>0.1261</td>
<td>0.01885</td>
<td>0.3124</td>
<td>-0.0701</td>
<td>-0.2774</td>
</tr>
<tr>
<td>T7.5+10</td>
<td>0.1495</td>
<td>0.02541</td>
<td>0.7172</td>
<td>-0.2806</td>
<td>-0.4605</td>
</tr>
<tr>
<td>T7.5+20</td>
<td>0.2751</td>
<td>0.04503</td>
<td>0.8532</td>
<td>-0.3201</td>
<td>-0.5677</td>
</tr>
<tr>
<td>T7.5+40</td>
<td>0.5367</td>
<td>0.08715</td>
<td>1.2735</td>
<td>-0.5147</td>
<td>-0.8034</td>
</tr>
<tr>
<td>T10-40</td>
<td>0.5726</td>
<td>0.09843</td>
<td>-0.4239</td>
<td>0.2334</td>
<td>0.1438</td>
</tr>
<tr>
<td>T10-20</td>
<td>0.2537</td>
<td>0.03683</td>
<td>0.4583</td>
<td>-0.0907</td>
<td>-0.4188</td>
</tr>
<tr>
<td>T10-10</td>
<td>0.1309</td>
<td>0.01716</td>
<td>0.7158</td>
<td>-0.2198</td>
<td>-0.5401</td>
</tr>
<tr>
<td>T10+10</td>
<td>0.1339</td>
<td>0.01951</td>
<td>1.4990</td>
<td>-0.5787</td>
<td>-0.9582</td>
</tr>
<tr>
<td>T10+20</td>
<td>0.2873</td>
<td>0.04582</td>
<td>2.5294</td>
<td>-0.1008</td>
<td>-1.5683</td>
</tr>
<tr>
<td>T10+40</td>
<td>0.5118</td>
<td>0.08799</td>
<td>0.8982</td>
<td>-0.3388</td>
<td>-0.6017</td>
</tr>
<tr>
<td>T15-40</td>
<td>0.5366</td>
<td>0.09325</td>
<td>0.2232</td>
<td>0.0035</td>
<td>-0.2818</td>
</tr>
<tr>
<td>T15-20</td>
<td>0.2643</td>
<td>0.04270</td>
<td>0.6640</td>
<td>-0.1567</td>
<td>-0.5597</td>
</tr>
<tr>
<td>T15-10</td>
<td>0.1388</td>
<td>0.01967</td>
<td>0.7107</td>
<td>-0.1921</td>
<td>-0.5628</td>
</tr>
<tr>
<td>T15+10</td>
<td>0.1285</td>
<td>0.01901</td>
<td>0.2527</td>
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<td>-1.5552</td>
</tr>
<tr>
<td>T15+20</td>
<td>0.2804</td>
<td>0.04590</td>
<td>2.5208</td>
<td>-1.0058</td>
<td>-1.5686</td>
</tr>
<tr>
<td>T15+40</td>
<td>0.6093</td>
<td>0.12558</td>
<td>0.2429</td>
<td>-0.1361</td>
<td>-0.0957</td>
</tr>
<tr>
<td>T18-40</td>
<td>0.5750</td>
<td>0.10981</td>
<td>0.1834</td>
<td>0.0371</td>
<td>-0.2805</td>
</tr>
<tr>
<td>T18-20</td>
<td>0.2589</td>
<td>0.03948</td>
<td>0.5550</td>
<td>-0.0925</td>
<td>-0.5205</td>
</tr>
<tr>
<td>T18-10</td>
<td>0.1382</td>
<td>0.01761</td>
<td>1.2588</td>
<td>-0.0394</td>
<td>-0.9185</td>
</tr>
<tr>
<td>T18+10</td>
<td>0.1344</td>
<td>0.01883</td>
<td>2.3688</td>
<td>-0.0976</td>
<td>-1.4330</td>
</tr>
<tr>
<td>T18+20</td>
<td>0.2794</td>
<td>0.04528</td>
<td>2.7783</td>
<td>-1.1309</td>
<td>-1.7032</td>
</tr>
</tbody>
</table>
According to the special position where \(dV/dz\) becomes zero, the elevation of that position above the bed is calculated by Newton-convergence method.

\[
f(z) = \frac{dV(z)}{dz} = \frac{a_{c1}}{z} + a_{d1} \exp(z) - a_{d2} \exp(-z)
\]

(4.4.a)

\[
f'(z) = -\frac{a_{c1}}{z^2} + a_{d1} \exp(z) + a_{d2} \exp(-z)
\]

(4.4.b)

\[
z_{k+1} = z_k + \frac{f(z_k)}{f'(z_k)}
\]

(4.4.c)

where;
- \(f(z)\) : The first deviation of \(V(z)\) with respect to \(z\)
- \(f'(z)\) : The second deviation of \(V(z)\) with respect to \(z\)
- \(k\) : Suffix indicating iteration number

Method: Iterate the calculation of \(f(z_k)\) value until \(f(z_k)\) becomes approximately zero with eqs.(4.4.a),(4.4.b) and (4.4.c). Finally, \(z_k\) value that yields \(f(z_k)=0\) approximately is the answer.

Denoting the special position as \(z_J\), and define the ratio between the mean water depth \(h\) and \(z_J\) as \(J\).

\[
J = \frac{z_J}{h}
\]

(4.5)

where;
- \(J\) : Ratio between \(Z_J\) and \(h\) [-]
- \(Z_J\) : Elevation where \(dV/dz = 0\) [m]
- \(h\) : Mean water depth [m]

The calculation results are shown in Table 4.2. In Fig.II-7.1, \(J\) versus \(V_m\) and \(J\) versus \(H_s\) are shown.

From these results, the following tendencies are seen.

- The \(J\) value in a following current with waves increases with increasing of depth averaged velocity \(V_m\).
- The \(J\) value in a following current with waves decreases with
Increasing of significant wave height $H_s$ in case of $V_m = +0.1$ m/s and $+0.2$ m/s, and increases with increasing of $H_s$ in case of $V_m = +0.4$ m/s.

In most of cases, the $J$ value for a following current with waves is between 0.4 to 1.0. But for the case $T_{15+40}$, the calculated $J$ value is 1.23. It means that there is no special position $z_J$ in the water column in this case. This case can be thought to be a exception that might be caused by the measuring and/or fit curve estimation error.

Table 4.2 $Z_J$ and $J$ in case of a following current with waves

<table>
<thead>
<tr>
<th>CASE</th>
<th>$Z_J$ [m]</th>
<th>$J$</th>
</tr>
</thead>
<tbody>
<tr>
<td>T7.5+10</td>
<td>0.3488</td>
<td>0.6840</td>
</tr>
<tr>
<td>T7.5+20</td>
<td>0.4138</td>
<td>0.8168</td>
</tr>
<tr>
<td>T7.5+10</td>
<td>0.3939</td>
<td>0.7629</td>
</tr>
<tr>
<td>T10+10</td>
<td>0.2983</td>
<td>0.5906</td>
</tr>
<tr>
<td>T10+20</td>
<td>0.2847</td>
<td>0.5814</td>
</tr>
<tr>
<td>T10+40</td>
<td>0.4863</td>
<td>0.9784</td>
</tr>
<tr>
<td>T15+10</td>
<td>0.2445</td>
<td>0.4950</td>
</tr>
<tr>
<td>T15+20</td>
<td>0.2855</td>
<td>0.5665</td>
</tr>
<tr>
<td>T15+40</td>
<td>0.6215</td>
<td>1.2308</td>
</tr>
<tr>
<td>T18+10</td>
<td>0.2269</td>
<td>0.4575</td>
</tr>
<tr>
<td>T18+20</td>
<td>0.2881</td>
<td>0.5210</td>
</tr>
</tbody>
</table>
4.1.3 Water particle movement under waves with a current

When a current is combined with waves, how is the water particle moving and how is its movement described?

If $V(z)$ is assumed to be constant, $V_{\text{con.}}$, in the entire water column, the movement of the water particle can be expressed as a combination of the constant velocity component and the periodic elliptical moving component. This is illustrated in Fig.4.3.

Defining a relative coordinate system ($\text{or-}x\text{r-}z$) that is moving with the same speed as the constant speed $V_{\text{con.}}$ and assuming the linear wave theory for the relative coordinate system and the conservation of wave number, the water particle movement is described as follows, with respect to the fixed reference system ($o-x-z$);

\[
\begin{align*}
U_a & = V_{\text{con.}} + U_r \\
W_a & = W_r \\
C_a & = V_{\text{con.}} + C_r \\
C_a & = \frac{L}{T} \\
C_r & = \frac{L}{T_{\text{r}}} 
\end{align*}
\]

where;

$U_a$ : Absolute horizontal velocity of the water particle [m/s]

$V_{\text{con.}}$ : Constant current velocity [m/s]

$U_r$ : Relative horizontal velocity of the water particle [m/s]

$W_a$ : Absolute vertical velocity of the water particle [m/s]

$W_r$ : Relative vertical velocity of the water particle [m/s]

$C_a$ : Absolute celerity [m/s]

$C_r$ : Relative celerity [m/s]

$L$ : Wave length [m]

$T$ : Wave period [s]

$T_{\text{r}}$ : Relative wave period [s]

$a, r$ : Suffixes indicating the absolute system and relative system respectively
For wave length \( L \) and relative celerity \( C_r \);

\[
C_r = \left[ \frac{g \cdot L}{2 \cdot \pi} \cdot \tanh \left( \frac{2 \cdot \pi \cdot h}{L} \right) \right]^{0.5} \tag{4.7}
\]

From these equations, the following implicit equation can be derived;

\[
L = \frac{g \cdot T^2}{2 \cdot \pi} \left( 1 - \frac{V_{\text{con.}} \cdot T}{L} \right)^{-2} \cdot \tanh \left( \frac{2 \cdot \pi \cdot h}{L} \right) \tag{4.8}
\]

So, the wave length \( L \) can be calculated iteratively. Then, \( C_a, C_r \) and \( T_r \) can be calculated by eqs.(4.6.a) to (4.6.e), and \( U_r \) and \( W_r \) can be estimated by linear wave theory.

For the Nap and Van Kampen's experiments, the depth averaged velocity \( V_m \), the peak period \( T_p \), the root mean square wave height \( H_{rms} \) and the mean water depth \( h \) have been selected as parameters of the water particle movement.

The calculation results are shown in Table 4.3.

The orbital motion parameters at the bottom are calculated by the following equations;

\[
\hat{U}_b = \frac{\pi \cdot H_{rms}}{T_r} \cdot \frac{1}{\sinh \left( \frac{2 \cdot \pi \cdot h}{L} \right)} \tag{4.9}
\]

\[
a_b = \frac{H_{rms}}{2} \cdot \frac{1}{\sinh \left( \frac{2 \cdot \pi \cdot h}{L} \right)} \tag{4.10}
\]

where;

\( \hat{U}_b \) : Horizontal velocity amplitude of orbital motion at the bottom \([\text{m/s}]\)

\( a_b \) : Horizontal displacement amplitude of orbital motion at the bottom \([\text{m/s}]\)

\( H_{rms} \) : Root mean square wave height \( (= H_s/\sqrt{2}) \) \([\text{m}]\)

By linear wave theory, these parameters at an arbitrary elevation can be calculated in the same way. These calculation results will
be used for the calculation of the bottom shear stress in the following sections and for the representative parameter of the water movement under a current with waves.

Fig. 4.3 Sketch of water particle movement under waves and a constant current

\[ u_r = \hat{u}_r \sin \left( \frac{2\pi}{T} t - \frac{2\pi}{L} x_r \right) \]

\[ \omega_r = \hat{\omega}_r \cos \left( \frac{2\pi}{T} t - \frac{2\pi}{L} x_r \right) \]
Table 4.3 Parameters of water particle movement

<table>
<thead>
<tr>
<th>CASE</th>
<th>L[m]</th>
<th>Ca[m/s]</th>
<th>Cr[m/s]</th>
<th>Tr[s]</th>
<th>Ub[m/s]</th>
<th>ab[m]</th>
</tr>
</thead>
<tbody>
<tr>
<td>T7.5-40</td>
<td>3.787</td>
<td>1.604</td>
<td>1.993</td>
<td>1.900</td>
<td>0.0930</td>
<td>0.0281</td>
</tr>
<tr>
<td>T7.5-20</td>
<td>4.111</td>
<td>1.835</td>
<td>2.019</td>
<td>2.036</td>
<td>0.0934</td>
<td>0.0303</td>
</tr>
<tr>
<td>T7.5-10</td>
<td>4.483</td>
<td>1.932</td>
<td>2.026</td>
<td>2.203</td>
<td>0.0981</td>
<td>0.0344</td>
</tr>
<tr>
<td>T7.5, 0</td>
<td>4.785</td>
<td>2.062</td>
<td>2.062</td>
<td>2.320</td>
<td>0.1019</td>
<td>0.0376</td>
</tr>
<tr>
<td>T7.5+10</td>
<td>5.116</td>
<td>2.205</td>
<td>2.105</td>
<td>2.430</td>
<td>0.1000</td>
<td>0.0387</td>
</tr>
<tr>
<td>T7.5+20</td>
<td>5.361</td>
<td>2.301</td>
<td>2.110</td>
<td>2.540</td>
<td>0.1043</td>
<td>0.0422</td>
</tr>
<tr>
<td>T7.5+40</td>
<td>5.844</td>
<td>2.508</td>
<td>2.144</td>
<td>2.726</td>
<td>0.1062</td>
<td>0.0460</td>
</tr>
<tr>
<td>T10-40</td>
<td>4.134</td>
<td>1.634</td>
<td>2.023</td>
<td>2.044</td>
<td>0.1363</td>
<td>0.0443</td>
</tr>
<tr>
<td>T10-20</td>
<td>4.263</td>
<td>1.822</td>
<td>2.020</td>
<td>2.111</td>
<td>0.1417</td>
<td>0.0476</td>
</tr>
<tr>
<td>T10-10</td>
<td>4.530</td>
<td>1.944</td>
<td>2.050</td>
<td>2.210</td>
<td>0.1381</td>
<td>0.0486</td>
</tr>
<tr>
<td>T10, 0</td>
<td>4.860</td>
<td>2.071</td>
<td>2.071</td>
<td>2.320</td>
<td>0.1435</td>
<td>0.0530</td>
</tr>
<tr>
<td>T10+10</td>
<td>5.102</td>
<td>2.190</td>
<td>2.091</td>
<td>2.440</td>
<td>0.1438</td>
<td>0.0559</td>
</tr>
<tr>
<td>T10+20</td>
<td>5.319</td>
<td>2.283</td>
<td>2.080</td>
<td>2.557</td>
<td>0.1435</td>
<td>0.0584</td>
</tr>
<tr>
<td>T10+40</td>
<td>5.717</td>
<td>2.454</td>
<td>2.107</td>
<td>2.714</td>
<td>0.1350</td>
<td>0.0583</td>
</tr>
<tr>
<td>T15-40</td>
<td>4.514</td>
<td>1.684</td>
<td>2.064</td>
<td>2.186</td>
<td>0.1895</td>
<td>0.0659</td>
</tr>
<tr>
<td>T15-20</td>
<td>4.252</td>
<td>1.825</td>
<td>2.033</td>
<td>2.091</td>
<td>0.1973</td>
<td>0.0657</td>
</tr>
<tr>
<td>T15-10</td>
<td>4.471</td>
<td>1.919</td>
<td>2.040</td>
<td>2.192</td>
<td>0.2025</td>
<td>0.0707</td>
</tr>
<tr>
<td>T15, 0</td>
<td>4.719</td>
<td>2.034</td>
<td>2.034</td>
<td>2.320</td>
<td>0.2166</td>
<td>0.0800</td>
</tr>
<tr>
<td>T15+10</td>
<td>5.701</td>
<td>2.193</td>
<td>2.101</td>
<td>2.714</td>
<td>0.2110</td>
<td>0.0911</td>
</tr>
<tr>
<td>T15+20</td>
<td>6.061</td>
<td>2.322</td>
<td>2.129</td>
<td>2.846</td>
<td>0.2131</td>
<td>0.0985</td>
</tr>
<tr>
<td>T15+40</td>
<td>5.851</td>
<td>2.511</td>
<td>2.125</td>
<td>2.754</td>
<td>0.1982</td>
<td>0.0869</td>
</tr>
<tr>
<td>T18-40</td>
<td>4.039</td>
<td>1.635</td>
<td>2.030</td>
<td>1.989</td>
<td>0.2285</td>
<td>0.0723</td>
</tr>
<tr>
<td>T18-20</td>
<td>4.239</td>
<td>1.811</td>
<td>2.020</td>
<td>2.098</td>
<td>0.2429</td>
<td>0.0811</td>
</tr>
<tr>
<td>T18-10</td>
<td>4.462</td>
<td>1.915</td>
<td>2.046</td>
<td>2.181</td>
<td>0.2423</td>
<td>0.0841</td>
</tr>
<tr>
<td>T18, 0</td>
<td>5.383</td>
<td>2.094</td>
<td>2.094</td>
<td>2.574</td>
<td>0.2550</td>
<td>0.1043</td>
</tr>
<tr>
<td>T18+10</td>
<td>5.694</td>
<td>2.190</td>
<td>2.104</td>
<td>2.706</td>
<td>0.2566</td>
<td>0.1105</td>
</tr>
<tr>
<td>T18+20</td>
<td>6.041</td>
<td>2.324</td>
<td>2.141</td>
<td>2.822</td>
<td>0.2547</td>
<td>0.1144</td>
</tr>
</tbody>
</table>
4.2 Bottom shear stress and \( \varepsilon f(z) \) distribution by the original Bijker approach

In order to calculate \( \varepsilon f(z) \) distribution, it is necessary to know the internal shear stress distribution \( \tau(z) \). In mere current case, the internal shear stress distribution \( \tau(z) \) is described as a triangle shape which has a zero value at the surface and its maximum at the bottom. Then, \( \tau(z) \) distribution is determined if the bottom shear stress is known in this case.

For estimating the bottom shear stress under waves and a current, Bijker (1967) proposed a quite straightforward approach. As the first trial in the present study, the original Bijker approach was applied to estimate the bottom shear stress.

4.2.1 Original Bijker approach

According to the original Bijker approach, the wave-induced water particle movement is directly superimposed on the logarithmic velocity distribution of the mere current case. The bottom shear stress is calculated on the plane of the elevation \( Ztc \) which is the elevation for velocity profile tangency in case of the mere current. The orbital motion due to waves at elevation \( Ztc \) is thought to be proportional to the orbital motion at the bed, \( Ub \), due to waves, and it is expressed as the multiplication of \( Ub \) by a factor \( p \). Considering the angle between the current direction and wave direction, time-averaged bottom shear stress in the current direction is calculated by the following equation. The concept of this approach is sketched in Fig.4.4.

\[
\tau_{oc} = \rho \times 2 \times V_{lc}^2
\]

\[
\tau_o = \frac{\rho \times 2^2}{T} \int_{-T/2}^{T/2} \sqrt{V_{lc} + (pUb)^2 + 2pUbV_{lc} \sin \phi} \times
\left[ V_{lc} + pUb \sin \phi \right] dt
\]

\[
Ub = \hat{Ub} \sin \left( \frac{2 \times \pi}{T} \times t \right)
\]

\[
\hat{Ub} = \hat{Ub} \sin \left( \frac{2 \times \pi}{T} \times t \right)
\]
where;

\( \tau_c \) : Bottom shear stress due to mere current \([\text{N/m}^2]\)

\( \tau_o \) : Time-averaged bottom shear stress due to waves and a current in the current direction \([\text{N/m}^2]\)

\( V_{tc} \) : Current velocity at the elevation \( Z_{tc} \) \([\text{m/s}]\)

\( Z_{tc} \) : The elevation for velocity profile tangency in mere current case \([\text{m}]\)

\( p \) : Dimensionless parameter \([-]\)

\( \phi \) : Angle between current direction and wave direction \([\text{deg.}]\)

In this study, \( \phi = 90 \text{ deg.} \) for following current cases and \( \phi = -90 \text{ deg.} \) for opposing current cases. Then, eq.(4.11.a) becomes simpler as follows;

\[
\tau_o = \frac{\rho \times \kappa^2}{T} \int_{-T/2}^{T/2} \left| V_{tc} \pm p Ub \right| \times ( V_{tc} \pm p Ub ) \, dt \quad (4.12)
\]

where;

+ sign for following current cases

- sign for opposing current cases

The dimensionless parameter \( p \) depends on the ratio between the elevation for velocity profile tangency in case of a mere current, \( Z_{tc} \), and the bottom boundary layer thickness due to only waves. This parameter has the relationship with Jonsson's dimensionless friction coefficient \( f_v \);

\[
p = \frac{1}{\kappa} \sqrt{\frac{f_v}{2}} \quad (4.13)
\]

\[
f_v = \exp \left[ -5.977 + 5.213 \times \left( \frac{ab}{r} \right) \right]^{-0.194} \quad (4.14)
\]

where;

\( f_v \) : Dimensionless friction coefficient \((f_v \leq 0.32) \) \([-]\)

\( r \) : Bottom roughness \([\text{m}]\)

In mere current case the bottom roughness \( r \) is equal to \( 33 \times Z_o \).

It is a controversial point how to estimate with an acceptable degree of accuracy the bottom roughness in case of waves with a current. In the present study Swart’s bottom roughness estimation
was used, because it can estimate the bottom roughness considering the ripple dimensions. According to Swart(1976), the bottom roughness \( r \) under waves and currents is described as follows;

\[
\Delta r^2 \geq 25 \frac{\Delta r}{\lambda} \tag{4.15}
\]

where;
- \( \Delta r \) : Ripple height [m]
- \( \lambda \) : Ripple length [m]

\( Z_{tc} \) and \( V_{tc} \) can be calculated by the following equations;

\[
Z_{tc} = e \times Z_{oc} \tag{4.16}
\]

\[
V_{tc} = \frac{V*C}{\kappa} \tag{4.17}
\]

where;
- \( Z_{oc} \) : Elevation at which the logarithmic velocity distribution becomes zero [m]
- \( V*C \) : Shear velocity in the mere current case [m/s]

From the fit curve parameters in Table 4.1, \( Z_{oc} \) and \( V*C \) are calculated as follows;

\[
V*C = \kappa \times a_{c1} \tag{4.18}
\]

\[
Z_{oc} = \exp \left[ - \frac{a_{c0}}{a_{c1}} \right] \tag{4.19}
\]

Using eqs. above and the data of ripple shape, \( V_{tc} \), \( p \), \( U_t \) are calculated. Then, \( \tau_0 \) also can be calculated by eq.(4.12). The calculations of eq.(4.12) have been done numerically in this study and the results are shown in Table 4.4.

Here, it should be noticed that \( Z_{tc} \) has the relationship with the bottom roughness \( r \) that \( Z_{tc} = e \times r / 33 \) in the mere current case. However, the \( Z_{tc} \) values calculated eqs.(4.16) and (4.19) that are shown in Table 4.4 tend to be bigger than \( Z_{tc} \) values calculated from that relationship and Swart's \( r \). It implies that the Swart's \( r \) for the present data are underestimated to some extent as compared to those calculated by the relation of \( r = (33/e) \times Z_{tc} \).
(a) mere current case

(b) combination of waves and current

Fig. 4.4 Concept of original Bijker approach
Table 4.4 Bottom shear stress by the original Bijker approach

<table>
<thead>
<tr>
<th>CASE</th>
<th>V·c</th>
<th>Z tc</th>
<th>V tc</th>
<th>r</th>
<th>fw</th>
<th>p</th>
<th>r o</th>
</tr>
</thead>
<tbody>
<tr>
<td>T7.5-40</td>
<td>4.183</td>
<td>11.078</td>
<td>10.457</td>
<td>4.70</td>
<td>0.3200</td>
<td>1.0000</td>
<td>2.442</td>
</tr>
<tr>
<td>T7.5-20</td>
<td>1.480</td>
<td>3.889</td>
<td>3.701</td>
<td>2.52</td>
<td>0.3200</td>
<td>1.0000</td>
<td>0.723</td>
</tr>
<tr>
<td>T7.5-10</td>
<td>0.754</td>
<td>3.381</td>
<td>1.885</td>
<td>3.59</td>
<td>0.3200</td>
<td>1.0000</td>
<td>0.379</td>
</tr>
<tr>
<td>T7.5+10</td>
<td>1.016</td>
<td>7.570</td>
<td>2.541</td>
<td>3.70</td>
<td>0.3200</td>
<td>1.0000</td>
<td>0.523</td>
</tr>
<tr>
<td>T7.5+20</td>
<td>1.801</td>
<td>6.041</td>
<td>4.503</td>
<td>4.78</td>
<td>0.3200</td>
<td>1.0000</td>
<td>0.987</td>
</tr>
<tr>
<td>T7.5+40</td>
<td>3.486</td>
<td>5.771</td>
<td>8.715</td>
<td>3.52</td>
<td>0.3200</td>
<td>1.0000</td>
<td>2.106</td>
</tr>
<tr>
<td>T10-40</td>
<td>3.937</td>
<td>8.088</td>
<td>9.843</td>
<td>4.38</td>
<td>0.3200</td>
<td>1.0000</td>
<td>2.978</td>
</tr>
<tr>
<td>T10-20</td>
<td>1.473</td>
<td>2.771</td>
<td>3.683</td>
<td>2.85</td>
<td>0.2843</td>
<td>0.9426</td>
<td>1.015</td>
</tr>
<tr>
<td>T10-10</td>
<td>0.686</td>
<td>1.323</td>
<td>1.716</td>
<td>2.69</td>
<td>0.2647</td>
<td>0.9095</td>
<td>0.440</td>
</tr>
<tr>
<td>T10+10</td>
<td>0.780</td>
<td>2.842</td>
<td>1.951</td>
<td>3.40</td>
<td>0.2884</td>
<td>0.9494</td>
<td>0.544</td>
</tr>
<tr>
<td>T10+20</td>
<td>1.833</td>
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<td>4.582</td>
<td>3.45</td>
<td>0.2808</td>
<td>0.9368</td>
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</tr>
<tr>
<td>T10+40</td>
<td>3.520</td>
<td>8.095</td>
<td>8.799</td>
<td>4.68</td>
<td>0.3200</td>
<td>1.0000</td>
<td>2.595</td>
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<tr>
<td>T15-40</td>
<td>3.730</td>
<td>8.614</td>
<td>9.325</td>
<td>5.90</td>
<td>0.3200</td>
<td>1.0000</td>
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<td>1.708</td>
<td>5.574</td>
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<td>0.1284</td>
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<tr>
<td>T15-10</td>
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<td>1.967</td>
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<td>0.1867</td>
<td>0.7218</td>
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<td>0.760</td>
<td>3.152</td>
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<td>0.1631</td>
<td>0.7139</td>
<td>0.585</td>
</tr>
<tr>
<td>T15+20</td>
<td>1.836</td>
<td>6.043</td>
<td>4.590</td>
<td>3.88</td>
<td>0.2002</td>
<td>0.7909</td>
<td>1.595</td>
</tr>
<tr>
<td>T15+40</td>
<td>5.023</td>
<td>21.241</td>
<td>12.558</td>
<td>4.81</td>
<td>0.2847</td>
<td>0.9095</td>
<td>4.995</td>
</tr>
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<td>T18-40</td>
<td>4.392</td>
<td>14.481</td>
<td>10.981</td>
<td>5.20</td>
<td>0.3200</td>
<td>1.0000</td>
<td>5.311</td>
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<td>0.704</td>
<td>1.062</td>
<td>1.761</td>
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<td>0.1216</td>
<td>0.6163</td>
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<tr>
<td>T18+10</td>
<td>0.753</td>
<td>2.160</td>
<td>1.883</td>
<td>2.88</td>
<td>0.1407</td>
<td>0.6631</td>
<td>0.654</td>
</tr>
<tr>
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<td>1.811</td>
<td>5.682</td>
<td>4.528</td>
<td>3.84</td>
<td>0.1722</td>
<td>0.7336</td>
<td>1.741</td>
</tr>
</tbody>
</table>

Comment: V·c by eq.(4.18) in cm/s
Z tc by eq.(4.16) in mm
V tc by eq.(4.17) in cm/s
(Swart bottom roughness)
r by eq.(4.15) in cm
fw by eq.(4.14)
p by eq.(4.13)
\( r o \) in N/m²

56
The velocity profile obtained with the fit curve method and the bottom shear stress obtained with the original Bijker method are known, now. The distribution of $\varepsilon f(z)$ can be estimated by eq.(4.1) if the internal shear stress distribution can be assumed to be of a triangular shape, the same as that of the mere current. However, there is a problem in case of a following current with waves if a triangular shape distribution of $\tau(z)$ is applied to eq.(4.1) then $\varepsilon f$ value becomes infinite at height $Z_J$ where $dV/dz = 0$. It is quite impractical that the diffusion activity is infinite from a physical point of view.

It could be true that the $\tau(z)$ distribution is a linear function of $z$, because the time-averaged internal shear stress balances the time-averaged surface slope. In case of a following current with waves and in order to evade that problem it is inevitable that $\tau$ value at height $Z_J$ equals zero. Then $\tau(z)$ distribution in case of a following current with waves could be a straight line passing the points $(0, \tau_0)$ and $(Z_J, 0)$ shown in Fig.4.5 (a). In the layers above the $Z_J$ level $\tau$ is negative in this case. This $\tau(z)$ distribution is quite different from the triangular shape in case of the mere current and is rather similar to $\tau(z)$ distribution of pipe or culvert flow. In pipe or culvert flow the internal shear stress is positive in the lower half area and negative in the upper half area around its center position, because of the boundary shear stresses of the low side and upside walls. So, in case of a current with waves it could be thought that some boundary shear stress exists near the surface due to waves and current interaction and it changes the $\tau(z)$ distribution. So, in this stage let's take a tentative assumption. Further investigation about $\tau(z)$ distribution will be done in section 4.4.

As a tentative assumption, it is assumed that $\tau(z)$ distribution in case of a following current with waves is a double triangular shape with zero value at height $Z_J$ and $\tau(z)$ distribution in case of an opposing current with waves is a triangular shape with zero value at the mean water surface, for a while. This assumption of $\tau(z)$ distribution is illustrated in Fig.4.5.
For the calculation of $\sigma f(z)$ distribution, $dV(z)/dz$ is calculated by the following equations.

\[
\frac{dV(z)}{dz} = \frac{ac_1}{z} + ac_1 \cdot \exp(z) - ad_2 \cdot \exp(-z) \quad \text{for } z \geq Z_{zc}
\]

\[
\frac{dV(z)}{dz} = ac_c \quad \text{for } z < Z_{zc}
\]

(4.20.a)

(4.20.b)

\[
ac_c = ac_1 \cdot \exp\left(\frac{ac_0}{ac_1} - 1\right)
\]

(4.20.c)

where:

- $ac_c$ : The gradient of tangent line for $Vc(z)$

The program to calculate $dV(z)/dz$ values by this method is called "EFVELOB" in the present study.

---

(a) following current with waves

(b) opposing current with waves

Fig.4.5 Tentative assumptions of $\tau(z)$ distribution
Using the velocity fit curve, the bottom shear stress calculated by the original Bijker approach and the tentative assumption of $\tau(z)$ distribution, the diffusion coefficient distribution of the fluid, $\varepsilon_f(z)$, for Nap and Van Kampen’s experimental data have been calculated by eq.(4.1). These results are shown in Fig.II-8.1 to II-8.4.

Investigating the tendencies of these results, it becomes clear that some contradictory tendencies appear in these figures from a general physical sense, in spite of some reasonable tendencies that $\varepsilon_f > \varepsilon_{fc}$ in most of the cases. These contradictory tendencies are the following.

- In cases of the opposing current with mean velocity $-10$ cm/s and $-20$ cm/s, $\varepsilon_f$ values are more or less same for all Hs cases. In some cases, the $\varepsilon_f$ value of lower Hs case becomes bigger than that of higher Hs case.

- In most of the opposing cases, $\varepsilon_f(z)$ distribution are quasi-parabolic shape that has small values near the bottom and surface. The $\varepsilon_f$ value in upper layer in case of T7.5-40 is smaller than that of the mere current case.

- In cases of the following current with mean velocity $+10$ cm/s and $+20$ cm/s, the largest $\varepsilon_f$ value appears in case of the lowest Hs case (T7.5+10 and T7.5+20).

The main reason of such strange behavior could be attributed to that the bottom shear stress in case of higher Hs is underestimated by this approach.

In the original Bijker approach, the bottom shear stress is calculated at height $Z_{tc}$ that is the elevation for velocity profile tangency in mere current case. The $Z_{tc}$ value is thought to represent one of the measure of the thickness of the bottom boundary layer in mere current case. But, in case of a combination of waves and a current the thickness of the bottom boundary layer could be affected by the wave actions near the bottom and the ripple shapes. It was reported that the bottom roughness under a current with waves is bigger than that of the mere current. It implies that the thickness of the bottom boundary layer under a current with waves is also thicker than that of the mere current. As mentioned before, the velocity distribution near the bottom
under a current with waves is shifted upward, with some disturbed component, from the logarithmic distribution in the mere current. So, the velocity distribution \( V(z) \) under a current with waves differs from that of the mere current and the water particles can be thought to be moving with the time-averaged velocity \( V(z) \) with the orbital motions. Therefore, in order to estimate the bottom shear stress under a current with waves, theses points should be considered. Extending the concept of the original Bijker approach to the measured velocity distribution under a current with waves, the following idea appears.

\( Z_t \) elevation that is the elevation for velocity profile tangency in case of a current with waves could be thought to represent the measure of the thickness of the boundary layer in case of a current with waves. In fact, \( Z_t \) elevation seems to be much suitable position to estimate the bottom shear stress instead of \( Z_{tc} \) elevation, because \( Z_t \) elevation is estimated from the time-averaged velocity distribution with consideration of the wave-induced current component and the water particle is moving on \( V(z) \) distribution with the orbital motion, not on \( Vc(z) \) distribution.
4.3 Bottom shear stress and $\varepsilon f(z)$ distribution by the modified Bijker approach

In order to consider the difference of the time averaged velocity distributions in case of a current with waves and in the mere current case, let's compare $V(z)$ distribution and $V_c(z)$ distribution. In Fig.4.6(= Fig.II-9.1), $V(z)$ and $V_c(z)$ profiles of T15-20 and T15+20 cases are shown.

These figures indicate that:

The time averaged velocity under a current with waves is smaller than that of the mere current near the bottom. The $Z_l$ point which is defined as the elevation for velocity profile tangency in case of a current with waves is located at higher elevation than $Z_{lc}$. So, the measure of the thickness of the bottom boundary layer in case of a current with waves seems to be much thicker than that of the mere current. This implies that the elevation for calculating the bottom shear stress $\tau_0$ in case of a current with waves should be $Z_l$ instead of $Z_{lc}$. This idea seems to fit the original Bijker's idea that $\tau_0 = \rho x^2 V_l^2$ at the elevation for velocity profile tangency.

In this section, the bottom shear stress will be estimated at height $Z_l$ and applied to the calculation of $\varepsilon f(z)$ distribution. This method is named modified Bijker approach in the present study.
Fig. 4.6 $V(z)$ and $V_c(z)$ distributions near the bottom
Using the results of the fit curve coefficients shown in Table 4.1, \( Z_l \) value can be calculated by Newton convergence method, and \( V_l \) also can be calculated with \( Z_l \) and \( V(z) \). The results of these parameters are shown in Table 4.5.1. The comparisons of these parameters in case of a current with waves versus those in case of the mere current are shown in Table 4.6. In Fig.II-9.2 to II-9.7, several comparisons, such as \( Z_l \) and \( V_l \) versus \( V_m, H_s, \Delta r \) and Swart's bottom roughness \( r \), are plotted.

From these comparisons, the following tendencies appear:

- The calculated measure of the bottom boundary layer thickness, \( Z_l \), is thicker than that of the mere current case, \( Z_{lc} \), except only one case. The ratio \( Z_l/Z_{lc} \) tends to increase with increasing of \( H_s \) and decrease with increasing of \( V_m \) in cases of a following and opposing current with waves. The range of \( Z_l/Z_{lc} \) is about 1.2 to 2.0 for the present data.

- From the Fig.II-9.6, it appears that the \( Z_l \) value is bigger than the half of the ripple height \( \Delta r \) for all cases. It means that the elevation \( Z_l \) is higher than the top level of the ripples.

- The time-averaged velocity \( V_l \) at height \( Z_l \) under a current with waves is faster than \( V_{tc} \) at height \( Z_{lc} \) under the mere current. The ratio \( V_l/V_{tc} \) tends to increase with increasing of \( H_s \) and decrease with increasing of \( V_m \) as same as \( Z_l/Z_{lc} \). The range of \( V_l/V_{tc} \) is about 1 to 1.7.

In order to estimate the bottom shear stress \( \tau_0 \) by eq.(4.12), it is necessary to estimate \( p \) value. However, in this modified Bijker approach there is no suitable empirical or theoretical equation to calculate \( p \) value at \( Z_l \) elevation instead of \( Z_{lc} \). But, considering the tendencies as mentioned above, it seems to be reasonable to assume that \( p = 1 \) at \( Z_l \) elevation in the present study cases. The reason is that the \( p \) value means the ratio between \( Z_{lc} \) and the bottom boundary layer thickness in case of waves (See Fig.4.4 (b)), and now the calculated \( Z_l \) values are bigger than \( Z_{lc} \) values, and if using eqs.(4.13) and (4.14) for estimating \( p \) values then the calculated \( p \) values tends to be equal or close to 1. It means that \( Z_l \) elevations for the present cases could be thought to
be thick enough for waves to reach its bottom velocity amplitude \( \bar{U}_b \). Therefore in this study, the assumption that \( p = 1 \) at height \( Z_l \) seems reasonable.

\[
p = 1 \text{ at } z = Z_l \quad (4.21)
\]

So, eqs. (4.12) and (4.21) yield;

\[
\tau_0 = \frac{2 \pi \rho \chi^2}{T} \int_{-T/2}^{T/2} |V_{IC} \pm U_b| \star (V_{IC} \pm U_b) \, dt \quad (4.22)
\]

where;

+ sign for following current cases
- sign for opposing current cases

Using eq. (4.22), the bottom shear stress \( \tau_0 \) can be calculated. The results of \( \tau_0 \) calculation by the modified Bijker approach and comparisons of the parameters of the original and the modified Bijker approaches are shown also in Tables 4.5.1, 4.5.2 and 4.6. In Fig. 4.7, the concept of this approach is sketched comparing the original approach.

From these results, it can be stated that the calculated \( \tau_0 \) by the modified Bijker approach is bigger than that by the original Bijker approach and that the ratio \( \tau_{0,\text{mod.}}/\tau_{0,\text{ori}} \) is in range of about 1 to 2.7 and it increases with increasing of \( H_s \) and decreases with increasing of the absolute value of \( V_m \). In the cases of \( V_m = \pm 40 \text{ cm/s} \) the ratio is almost 1.0.

Here, it might be noticed that in the \( \tau_0 \) calculation by the original Bijker approach the bottom roughness \( r \) obtained by Swart's empirical predictor, eq. (4.15), was used in the present study. However this predictor estimates the bottom roughness with some error because of the scattering of data that were used for finding the predictor. If using other predictor of the bottom roughness \( r \), for example \( r = 33^*Z_{oc} \), the resulted \( \tau_0 \) values would change. Actually, if the relation that \( r = 33^*Z_{oc} \) is used, then the \( p \) values becomes 1.0 in most of the cases and the calculated \( \tau_0 \) values by the original Bijker approach tend to be close to the \( \tau_0 \) values obtained by the modified Bijker approach. Therefore, the
difference between the calculated $\tau_0$ values by these two methods depends on the bottom roughness predictor.
The time-averaged velocity distribution $V(z)$ and $Z_t$ value are in fact calculated from the fit curve of the measurement data, then the reliable range of these values is from the lowest measuring point to the highest measuring point. Therefore, the lower part of $V(z)$ distribution than the lowest measuring point is the extrapolated curve and its reliability is less than that in the measuring range. From Table 4.5.1, it can be seen that about half of the calculated $Z_t$ elevations are located at the higher position than the lowest measuring elevation and the rests of them also located near the lowest measuring point. Therefore, it can be stated that the calculated $Z_t$ values are more or less reliable in the present study cases.

As to the $Z_0$ point in case of a current with waves, that parameter might have some physical relationship with roughness condition, such as $Z_{oc} = r/33$ in case of mere current.

Although the fit curve under the lowest measuring point is the extrapolated curve, it is interesting to see the difference between the zero velocity elevation $Z_0$ in case of a current with waves and $Z_{oc}$ in case of the mere current. Table 4.5.2 shows the $Z_{oc}$ and $Z_0$ values calculated from the parameters in Table 4.1. $Z_0$ values can be calculated by Newton convergence method. Table 4.5.2 shows that the ratio $Z_0/Z_{oc}$ is in range of about 1 to 18 and that it tends to increases with increasing of $H_s$ and decreases with increasing of the absolute value of $V_m$. And the ratio $r/Z_0$ ($r$ is Swart’s bottom roughness) is in range of about 2 to 10 and it tends to increases with increasing the absolute value of $V_m$.

However, in order to discuss the relationship between $Z_0$ and boundary conditions precisely, more detailed velocity measurements than the present data are required quite close to the bed. Then further research is recommended.
Original Bijker approach

\[ T_0 = \frac{\rho k^2}{T} \int_{-T/2}^{T/2} |V_{\infty} + pU_b| (V_{\infty} + pU_b) \, dt \]

Modified Bijker approach

\[ T_0 = \frac{\rho k^2}{T} \int_{-T/2}^{T/2} |V_t + U_b| (V_t + U_b) \, dt \]

Assuming: \( Z_t \) is the elevation of the touching point of the tangent line from the origin to the \( V(x) \) curve

\( V(x) \) is the time and bed-averaged velocity distribution curve

At the elevation of \( Z_t \), \( p = 1 \)

Fig. 4.7 Original Bijker approach and Modified Bijker approach for the bottom shear stress
Table 4.5.1 Bottom shear stress by modified Bijker approach

<table>
<thead>
<tr>
<th>CASE</th>
<th>$Z_t$ [mm]</th>
<th>LMP [mm]</th>
<th>$V_t$ [cm/s]</th>
<th>$\alpha$ [1/s]</th>
<th>$\tau_o$ [N/m²]</th>
</tr>
</thead>
<tbody>
<tr>
<td>T7.5-40</td>
<td>13.07</td>
<td>19.0</td>
<td>10.334</td>
<td>7.9097</td>
<td>2.401</td>
</tr>
<tr>
<td>T7.5-20</td>
<td>11.95</td>
<td>17.7</td>
<td>4.067</td>
<td>3.4108</td>
<td>0.800</td>
</tr>
<tr>
<td>T7.5-10</td>
<td>21.67</td>
<td>11.0</td>
<td>2.317</td>
<td>1.0694</td>
<td>0.487</td>
</tr>
<tr>
<td>T7.5+10</td>
<td>19.30</td>
<td>15.0</td>
<td>2.861</td>
<td>1.4822</td>
<td>0.591</td>
</tr>
<tr>
<td>T7.5+20</td>
<td>13.00</td>
<td>17.5</td>
<td>4.808</td>
<td>3.7006</td>
<td>1.058</td>
</tr>
<tr>
<td>T7.5+40</td>
<td>9.62</td>
<td>18.0</td>
<td>8.974</td>
<td>9.3377</td>
<td>2.183</td>
</tr>
<tr>
<td>T10-40</td>
<td>13.01</td>
<td>17.0</td>
<td>9.966</td>
<td>7.6817</td>
<td>3.021</td>
</tr>
<tr>
<td>T10-20</td>
<td>10.47</td>
<td>14.7</td>
<td>4.015</td>
<td>3.8387</td>
<td>1.174</td>
</tr>
<tr>
<td>T10-10</td>
<td>17.19</td>
<td>11.7</td>
<td>2.234</td>
<td>1.3056</td>
<td>0.631</td>
</tr>
<tr>
<td>T10+10</td>
<td>19.59</td>
<td>14.7</td>
<td>2.636</td>
<td>1.3457</td>
<td>0.777</td>
</tr>
<tr>
<td>T10+20</td>
<td>14.40</td>
<td>20.0</td>
<td>5.329</td>
<td>3.7058</td>
<td>1.594</td>
</tr>
<tr>
<td>T10+40</td>
<td>13.07</td>
<td>23.0</td>
<td>9.122</td>
<td>6.9851</td>
<td>2.705</td>
</tr>
<tr>
<td>T15-40</td>
<td>15.53</td>
<td>25.0</td>
<td>9.761</td>
<td>6.2846</td>
<td>3.937</td>
</tr>
<tr>
<td>T15-20</td>
<td>18.90</td>
<td>17.3</td>
<td>5.006</td>
<td>2.6481</td>
<td>2.034</td>
</tr>
<tr>
<td>T15-10</td>
<td>22.05</td>
<td>16.0</td>
<td>2.748</td>
<td>1.2485</td>
<td>1.137</td>
</tr>
<tr>
<td>T15+10</td>
<td>28.53</td>
<td>17.0</td>
<td>3.235</td>
<td>1.1340</td>
<td>1.396</td>
</tr>
<tr>
<td>T15+20</td>
<td>18.41</td>
<td>18.3</td>
<td>5.535</td>
<td>3.0073</td>
<td>2.430</td>
</tr>
<tr>
<td>T15+40</td>
<td>19.45</td>
<td>23.3</td>
<td>12.469</td>
<td>6.4119</td>
<td>5.374</td>
</tr>
<tr>
<td>T18-40</td>
<td>24.93</td>
<td>24.7</td>
<td>11.753</td>
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<td>5.715</td>
</tr>
<tr>
<td>T18-20</td>
<td>17.55</td>
<td>18.0</td>
<td>4.678</td>
<td>2.6851</td>
<td>2.329</td>
</tr>
<tr>
<td>T18-10</td>
<td>22.32</td>
<td>14.7</td>
<td>2.864</td>
<td>1.2802</td>
<td>1.417</td>
</tr>
<tr>
<td>T18+10</td>
<td>18.10</td>
<td>16.0</td>
<td>2.630</td>
<td>1.4535</td>
<td>1.377</td>
</tr>
<tr>
<td>T18+20</td>
<td>19.26</td>
<td>20.0</td>
<td>5.525</td>
<td>2.8689</td>
<td>2.889</td>
</tr>
</tbody>
</table>

LMP: Elevation of the lowest measuring point
### Table 4.5.2 Comparison of Zoe and Zo values

<table>
<thead>
<tr>
<th>CASE</th>
<th>Zoe [mm]</th>
<th>Zo [mm]</th>
<th>Zo/Zoe</th>
<th>r/Zo</th>
</tr>
</thead>
<tbody>
<tr>
<td>T7.5-40</td>
<td>4.075</td>
<td>4.826</td>
<td>1.184</td>
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</tr>
<tr>
<td>T7.5-20</td>
<td>1.431</td>
<td>4.244</td>
<td>2.968</td>
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</tr>
<tr>
<td>T7.5-10</td>
<td>1.244</td>
<td>7.398</td>
<td>5.940</td>
<td>4.86</td>
</tr>
<tr>
<td>T7.5+10</td>
<td>2.785</td>
<td>6.808</td>
<td>2.445</td>
<td>5.44</td>
</tr>
<tr>
<td>T7.5+20</td>
<td>2.222</td>
<td>4.671</td>
<td>2.102</td>
<td>10.23</td>
</tr>
<tr>
<td>T7.5+40</td>
<td>2.123</td>
<td>3.502</td>
<td>1.650</td>
<td>10.05</td>
</tr>
<tr>
<td>T10-40</td>
<td>2.975</td>
<td>4.763</td>
<td>1.601</td>
<td>9.20</td>
</tr>
<tr>
<td>T10-20</td>
<td>1.020</td>
<td>3.734</td>
<td>3.661</td>
<td>7.63</td>
</tr>
<tr>
<td>T10-10</td>
<td>0.487</td>
<td>5.752</td>
<td>11.811</td>
<td>4.68</td>
</tr>
<tr>
<td>T10+10</td>
<td>1.046</td>
<td>6.462</td>
<td>6.178</td>
<td>5.26</td>
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<tr>
<td>T10+20</td>
<td>1.892</td>
<td>5.022</td>
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<tr>
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<td>2.978</td>
<td>4.746</td>
<td>1.594</td>
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<tr>
<td>T15-40</td>
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<td>0.882</td>
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<td>8.706</td>
<td>7.505</td>
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<td>T15+20</td>
<td>2.223</td>
<td>6.335</td>
<td>2.850</td>
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<tr>
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<td>7.814</td>
<td>7.174</td>
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<tr>
<td>T18-40</td>
<td>5.320</td>
<td>8.948</td>
<td>1.682</td>
<td>5.81</td>
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<tr>
<td>T18-20</td>
<td>1.488</td>
<td>6.063</td>
<td>4.075</td>
<td>2.94</td>
</tr>
<tr>
<td>T18-10</td>
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<tr>
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<td>2.095</td>
<td>6.804</td>
<td>3.152</td>
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Table 4.6 Comparison of original and modified Bijker approach parameters

<table>
<thead>
<tr>
<th>CASE</th>
<th>Zt/Zt_C</th>
<th>Vt/Vt_C</th>
<th>T_{o, \text{mod.}} / T_{o, \text{orig.}}</th>
</tr>
</thead>
<tbody>
<tr>
<td>T7.5-40</td>
<td>1.180</td>
<td>0.988</td>
<td>0.983</td>
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<td>T7.5-20</td>
<td>3.073</td>
<td>1.101</td>
<td>1.107</td>
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<td>6.410</td>
<td>1.229</td>
<td>1.232</td>
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<td>T7.5+10</td>
<td>2.550</td>
<td>1.126</td>
<td>1.130</td>
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<td>T7.5+20</td>
<td>2.152</td>
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<td>1.072</td>
</tr>
<tr>
<td>T7.5+40</td>
<td>1.667</td>
<td>1.030</td>
<td>1.037</td>
</tr>
<tr>
<td>T10-40</td>
<td>1.609</td>
<td>1.012</td>
<td>1.014</td>
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<td>T10-20</td>
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<td>1.090</td>
<td>1.157</td>
</tr>
<tr>
<td>T10-10</td>
<td>12.993</td>
<td>1.302</td>
<td>1.434</td>
</tr>
<tr>
<td>T10+10</td>
<td>6.893</td>
<td>1.351</td>
<td>1.428</td>
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<tr>
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<td>2.800</td>
<td>1.163</td>
<td>1.246</td>
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<tr>
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<td>1.615</td>
<td>1.037</td>
<td>1.042</td>
</tr>
<tr>
<td>T15-40</td>
<td>1.803</td>
<td>1.047</td>
<td>1.051</td>
</tr>
<tr>
<td>T15-20</td>
<td>3.391</td>
<td>1.172</td>
<td>1.836</td>
</tr>
<tr>
<td>T15-10</td>
<td>9.411</td>
<td>1.397</td>
<td>1.937</td>
</tr>
<tr>
<td>T15+10</td>
<td>9.051</td>
<td>1.702</td>
<td>2.386</td>
</tr>
<tr>
<td>T15+20</td>
<td>3.047</td>
<td>1.206</td>
<td>1.524</td>
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<tr>
<td>T15+40</td>
<td>0.916</td>
<td>0.993</td>
<td>1.076</td>
</tr>
<tr>
<td>T18-40</td>
<td>1.723</td>
<td>1.070</td>
<td>1.076</td>
</tr>
<tr>
<td>T18-20</td>
<td>4.339</td>
<td>1.186</td>
<td>1.900</td>
</tr>
<tr>
<td>T18-10</td>
<td>21.084</td>
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<td>2.639</td>
</tr>
<tr>
<td>T18+10</td>
<td>8.380</td>
<td>1.396</td>
<td>2.106</td>
</tr>
<tr>
<td>T18+20</td>
<td>3.390</td>
<td>1.220</td>
<td>1.659</td>
</tr>
</tbody>
</table>

where; $T_{o, \text{orig.}}$ : $T_{o}$ by the original Bijker approach

$T_{o, \text{mod.}}$ : $T_{o}$ by the modified Bijker approach
4.3.2 The calculation results of $\varepsilon f(z)$ distributions by the modified Bijker approach

Using the velocity fit curve $V(z)$, the bottom shear stress calculated by the modified Bijker approach and also the tentative assumption of $\tau(z)$ distribution shown in Fig.4.5, the diffusion coefficient distribution $\varepsilon f(z)$ for the Nap and Van Kampen’s experimental data have been calculated by eq.(4.1). These results are shown in Fig.II-11.1 to II-11.6.

For this calculation, $dV(z)/dz$ is calculated by the following equations.

\[
\frac{dV(z)}{dz} = \alpha \frac{z}{V_t} + a_1 \exp(z) - a_2 \exp(-z) \quad \text{for } z \geq Z_{tc} \\
\frac{dV(z)}{dz} = \alpha \quad \text{for } z < Z_{tc}
\]

\[
\alpha = \frac{V_t}{Z_t}
\]

where;

$\alpha$ : The gradient of tangent line for $V(z)$

The program for this method is called "EFVEL" in the present study.

From Fig.II-11.1 to II-11.6, the following can be stated:

- In cases of an opposing current of the same $V_m$ with waves, $\varepsilon f$ value in case of higher $H_s$ tends to be more or less bigger than that in case of lower $H_s$ in the low and middle layers of the water column. But this tendency is not so clear in the upper layer of the water column.

- In cases of an opposing current with waves of the same $H_s$, $\varepsilon f$ value in case of faster $V_m$ is not always bigger than that in case of slower $V_m$.

- Most of $\varepsilon f(z)$ distributions in cases of an opposing current with waves are quasi-parabolic shape and $\varepsilon f$ values in the lower and upper layers tend to be smaller than $\varepsilon f$ values in the middle layers.
In cases of a following current of the same \( V_m \) with waves, \( \varepsilon_f \) value in case of higher \( H_s \) tends to be bigger than that in case of lower \( H_s \) in more or less whole layers.

In cases of a following current with waves of the same \( H_s \), \( \varepsilon_f \) in case of faster \( V_m \) is always bigger than that in case of slower \( V_m \).

In cases of a following current with waves, \( \varepsilon_f(z) \) distribution tends to increase from the bottom toward the upper layer and its shape is similar to the \( \varepsilon_f(z) \) distribution that has been calculated by "CONCEFL" program with the Heijboer's data.

\( \varepsilon_f \) values are always bigger than \( \varepsilon_{fc} \) values in the following cases, but \( \varepsilon_f \) values are not always bigger than \( \varepsilon_{fc} \) values in the opposing cases.

Comparing the \( \varepsilon_f(z) \) distribution results of the following and opposing cases, the \( \varepsilon_f(z) \) distribution results of the following cases seems to be more reasonable than that of the opposing cases from a physical point of view.

The reasons are that the \( \varepsilon_f \) value in case of faster \( V_m \) should be bigger than that in case of slower \( V_m \) in the series of test with same \( H_s \), and if \( \varepsilon_f(z) \) distribution is a parabolic or a quasi-parabolic shape then the shape of the concentration distribution becomes quite different shape from those which have been measured. In Fig.4.7, a typical measured concentration distribution and a calculated concentration distribution with a parabolic \( \varepsilon_f(z) \) distribution are sketched.

The main reason of such contradictory results in cases of the opposing current with waves can be thought to be the triangular internal shear stress distribution assumed for the cases of opposing current with waves. As long as the triangular internal shear stress is assumed for \( \varepsilon_f(z) \) calculation, the resulted \( \varepsilon_f(z) \) distribution becomes a quasi-parabolic shape inevitably and the \( C(z) \) distribution obtained from the quasi-parabolic \( \varepsilon_f(z) \) distribution becomes too small in the upper layer as compared to the measured \( C(z) \).

It seems necessary to reconsider the tentative assumption of \( \tau(Z) \) distribution.
4.4 Additional internal shear stress

The $\sigma_f(z)$ distribution calculated by eq.(4.1) with the bottom shear stress obtained by the modified Bijker approach and the assumption of the double triangular shaped $\tau(z)$ distribution for cases of following current with waves indicates quite reasonable tendencies. On the other hand, the $\sigma_f(z)$ distribution calculated by eq.(4.1) with the bottom shear stress by the modified Bijker approach and the assumption of the triangle shaped $\tau(z)$ distribution for cases of opposing current with waves indicates some contradictory tendencies. The tentative assumptions of $\tau(z)$ distribution for following and opposing currents with waves seem to cause these differences mainly.

In this section, the internal shear stress in case of a current with waves will be reconsidered and discussed, in order to find a more suitable assumption of $\tau(z)$ distribution and to get more reasonable $\sigma_f(z)$ distribution under a current with waves.

4.4.1 Internal shear stress

Generally in case of a mere current, the internal shear stress distribution $\tau(z)$ is of a single triangular shape described by eq.(4.24), and the internal shear stress balances the gravity force component via the surface slope (See Fig.4.10).

$$\tau_c(z) = \tau_{oc} \times \left[ 1 - \frac{z}{h} \right]$$  \hspace{1cm} (4.24)

$$\tau_{oc} = \rho \times g \times h \times Ic$$  \hspace{1cm} (4.25)

where:

$\tau_c(z)$ : Internal shear stress at height $z$ in a mere current case  \hspace{1cm} [N/m²]

$\tau_{oc}$ : Bottom shear stress in a mere current case  \hspace{1cm} [N/m²]

$Ic$ : Surface slope in a mere current case  \hspace{1cm} [rad.]

On the other hand, in case of a following current with waves, the assumption of the double triangular shape of $\tau(z)$ distribution leads to reasonable $\sigma_f(z)$ distributions. Therefore, this double
triangular distribution of $\tau(z)$ could be closer to the real internal shear stress distribution in case of a following current with waves.

This fact implies the existence of "additional internal shear stress" in case of a current with waves.

Originally this double triangular shape of $\tau(z)$ distribution was assumed in order to prevent $\psi$ value from being infinite at the elevation $Z_J$ where $dV(z)/dz = 0$. Such special elevation $Z_J$ can be seen in other experimental data by D.Heijboer(1988) or by V.d.Kaaij and Nieuwjaar(1986) in cases of a following current with waves. The wave conditions were non-breaking irregular waves for the Nap and Van Kampen's experiments and the V.d.Kaaij and Nieuwjaar's experiments, and non-breaking regular waves for the D.Heijboer's experiments. This fact implies that $Z_J$ point actually exists in the water column in case of a following current with waves and it is not caused by the measuring errors. The additional internal shear stress can be thought to exist at least in case of a following current with waves.

### 4.4.2 Additional internal shear stress

If the additional internal shear stress which causes $Z_J$ point where $dV(z)/dz = 0$ is assumed to be true, the characteristics of the additional internal shear stress can be listed up as follows.

- It happens in case of the combination of a following (probably opposing) current with waves.
- It happens in experiments with non-breaking regular waves as well as non-breaking irregular waves.
- It is not reasonable to consider that the additional internal shear stress is caused by the change of radiation stress due to wave breaking and the change of the wave shape. Since it happens in experiments with non-breaking waves and the change of the wave shape in the flume can be thought to be negligibly small.
- It should balance some driving force. The driving force acting on the water column seems to be only gravitational force in the experiments, then the additional internal shear stress should balance the gravitational force component.
(a) Typical measured $C(z)$

(b) $C(z)$ calculated by parabolic shaped $\varepsilon_f(z)$

Fig.4.8 Typical measured $C(z)$ distribution and $C(z)$ distribution calculated with parabolic shape $\varepsilon_f(z)$ distribution
Fig. 4.9-1 $V_c(z)$ and $\tau_c(z)$ distribution

Fig. 4.9-2 $V(z)$ and $\tau(z)$ distribution in case of following current with waves
From these characteristics and conditions as mentioned before, the additional shear stress seems to be caused by current-wave interaction and balances the gravity force component. Therefore, it could be considered that the additional shear stress causes the change of the mean surface slope consequently.

Denoting the additional shear stress at height $z$ as $\tau_{a}(z)$, the additional shear stress at the mean surface level, $h$, as $\tau_{s}$ and the surface slope due to the additional shear stress as $I_s$, let's consider the force balance acting on water body due to the additional internal shear stress. (See Fig. 4.10)

The force balance of the control volume $(1-1'-2-2')$ is described as eq. (4.26) assuming hydrostatic pressure distribution.

$$\tau_{a}(z) \Delta X = \tau_{s} \Delta X + 0.5 \rho g \frac{(h-z)^2 - (h'-z)^2}{(h-z)^2 - (h'-z)^2}$$

(4.26.a)

$$I_s = \frac{h - h'}{\Delta X}$$

(4.26.b)

where:
- $\tau_{a}(z)$: Additional internal shear stress at height $z$ [N/m²]
- $\tau_{s}$: Additional shear stress at height $h$ [N/m²]
- $\Delta X$: Length of the control volume $(1-1'-2-2')$ [m]
- $I_s$: Surface slope due to the additional shear stress [rad.]

$1,1',2,2'$: Marks indicating the position of the control volume

Neglecting the second order terms of $I_s$, eqs. (4.26.a) and (4.26.b) yield:

$$\tau_{a}(z) = \tau_{s} + \rho g I_s (h-z)$$

(4.27)

Now, it seems reasonable to assume that $\tau_{a}(z)$ becomes relatively small near the bed, since the additional internal shear stress is caused by the current-wave interaction, then, its intensity depends on the current velocity and the orbital motion due to waves, and the current velocity near the bed becomes quite small comparing the upper layers. Therefore, the current-wave
interaction and consequently the additional internal shear stress could be quite small near the bed. Then, it could be assumed that;

$$\tau_a(0) = 0$$  \hspace{1cm} (4.28)

Eqs. (4.27) and (4.28) yield;

$$\tau_a(z) = \tau_s \cdot \frac{z}{h}$$  \hspace{1cm} (4.29)
$$\tau_s = -\rho \cdot g \cdot h \cdot I_s$$  \hspace{1cm} (4.30)

Eq. (4.29) means that $$\tau_a(z)$$ distribution is of a triangular shape with $$\tau_s$$ at height $$h$$ and zero at the bed. So, $$\tau_a(z)$$ distributes as if a special boundary wall exists at height $$h$$ and this surface shear causes the additional internal shear stress, $$\tau_a(z)$$. In this meaning, $$\tau_a(z)$$ could be called the internal shear stress due to the surface shear stress caused by the current-wave interaction.

Actually, the water body feels the internal shear stress due to the bottom shear stress. Then, the total shear stress, $$\tau(z)$$, can be described as sum of these components. Fig. 4.11 shows this concept.

$$\tau(z) = \tau_b(z) + \tau_a(z)$$  \hspace{1cm} (4.31)

where;
$$\tau_b(z) : \text{Internal shear stress due to the bottom shear stress}$$  \hspace{1cm} [N/m²]

$$\tau_b(z)$$ is described in the same manner as eq. (4.24) for a mere current case.

$$\tau_b(z) = \tau_0 \cdot [1 - \frac{z}{h}]$$  \hspace{1cm} (4.32)

where;
$$\tau_0 : \text{Bottom shear stress in case of a current with waves}$$  \hspace{1cm} [N/m²]
As mentioned before, $\tau_o$ is bigger than $\tau_{oc}$ because of wave orbital movement near the bottom and can be calculated by the modified Bijker method.

From eqs. (4.29), (4.31) and (4.32), $\tau(z)$ can be described as follows.

$$\tau(z) = \tau_o \ast \left[ 1 - \frac{z}{h} \right] + \tau_e \ast \frac{z}{h} \quad (4.33)$$

If $\tau_e$ is negative, then $\tau(z)$ distribution becomes of a double triangular shape as shown in Fig. 4.11.1, and it matches the assumption of $\tau(z)$ distribution in case of a following current with waves. If $\tau_e$ is positive, then $\tau(z)$ distribution becomes of a trapezoidal shape as shown in Fig. 4.11.2.

As mentioned before, time-averaged velocity distribution, $V(z)$, in case of an opposing current with waves, there is not a special point $ZJ$ where $dV(z)/dz = 0$ in the water column, but the velocity increases towards the mean surface. And $dV(z)/dz$ value near the mean surface is bigger than that of the mere current case. That means that the water particle near the mean surface in case of an opposing current with waves is moving faster than that of the mere current case. On the other hand, $dV(z)/dz$ value near the mean surface in case of a following current with waves is moving slower than that of the mere current case.

Therefore, it is probably reasonable to presume that an opposing current with waves corresponds to the case that $\tau_e > 0$ and $\tau(z)$ distribution for an opposing current with waves corresponds to a trapezoidal one, and a following current with waves corresponds to the case that $\tau_e < 0$ and $\tau(z)$ distribution for a following current with waves corresponds to a double triangular one.

Further, according to the residual averaged surface slope, the following equations can be derived;

$$I = I_b + I_s \quad (4.34)$$

$$I = \frac{1}{\rho \ast g \ast h} \ast \left( \tau_o - \tau_e \right) \quad (4.35)$$

78
where;

I : Residual averaged surface slope [rad.]

I_b : Averaged surface slope due to bottom shear stress [rad.]

I_s : Averaged surface slope due to surface shear stress
      (additional shear stress by current-wave interaction) [rad.]

Eq.(4.35) implies that the residual averaged surface slope becomes steeper than the averaged surface slope caused by only the bottom shear stress when $\tau_s$ is negative value, and the residual averaged surface slope becomes milder than the averaged surface slope caused by only the bottom shear stress when $\tau_s$ is positive value.

However it should be noticed that the assumption of the internal shear stress is simply based on the experimental facts not on the theoretical approach. Therefore, it is recommended to investigate the additional internal shear stress under a current with waves by further theoretical way considering the exchanges of the energy and momentum between waves and a current.
Fig. 4.10 Control volume (1-1'-2-2') and force balance

Fig. 4.11-1 Internal shear stress: $\tau_b(z)$, $\tau_a(z)$ and $\tau(z)$ in case of $\tau_s < 0$

Fig. 4.11-2 Internal shear stress: $\tau_b(z)$, $\tau_a(z)$ and $\tau(z)$ in case of $\tau_s > 0$
4.4.3 J-value and $\tau_*$ for a following current with waves

Assuming the discussion above to be true, $\tau_0$ and $\tau_*$ can be calculated with the modified Bijker approach and J-value. Eq.(4.22) yields $\tau_0$ by the modified Bijker approach. Eqs.(4.5) and (4.33) yield;

$$\tau(zJ) = \tau_0 \ast (1-J) + \tau_0 \ast j = 0$$

then,

$$\tau_0 = \frac{\tau(zJ)}{1 - \frac{1}{J}}$$

(4.36)

In case of a following current with waves, using eq.(4.33) and J-value shown in Table 4.2, $\tau_*$ has been calculated by eq.(4.36). The results are shown in Table 4.7 and Fig.II-12.1. From these results, the following tendencies can be seen;

- $\tau(z)$ distributions in cases of the following current with waves are of double triangular shape, except T15+40 case.
- $\tau_0$ increases with increasing of Hs and also increasing Vm.
- $\tau_*$ values are negative, except T15+40 case.
- In cases of Vm = +10 cm/s and +20 cm/s, the absolute $\tau_*$ value increases with increasing of Hs and also increasing of Vm. But, in cases of Vm = +40 cm/s, such tendency is not clear.
- The absolute $\tau_*$ values seem to have a different peak value in the series of tests with the same Hs and different Vm, or in the series of tests with the same Vm and different Hs.

Also, considering eq.(4.35) with the results of $\tau_*$ values in case of a following current with waves, it could be thought that the resulting averaged surface slope in case of a following current with waves becomes steeper than that caused only by the same bottom shear stress, since $\tau_*$ is negative.
Table 4.2 $\tau_O$ and $\tau_s$ for a following current with waves

<table>
<thead>
<tr>
<th>CASE</th>
<th>$\tau_O$ [N/m2]</th>
<th>$\tau_s$ [N/m2]</th>
</tr>
</thead>
<tbody>
<tr>
<td>T7.5+10</td>
<td>0.591</td>
<td>-0.273</td>
</tr>
<tr>
<td>T7.5+20</td>
<td>1.058</td>
<td>-0.238</td>
</tr>
<tr>
<td>T7.5+40</td>
<td>2.183</td>
<td>-0.678</td>
</tr>
<tr>
<td>T10+10</td>
<td>0.777</td>
<td>-0.539</td>
</tr>
<tr>
<td>T10+20</td>
<td>1.594</td>
<td>-1.148</td>
</tr>
<tr>
<td>T10+40</td>
<td>2.705</td>
<td>-0.060</td>
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<tr>
<td>T15+10</td>
<td>1.396</td>
<td>-1.424</td>
</tr>
<tr>
<td>T15+20</td>
<td>2.430</td>
<td>-1.859</td>
</tr>
<tr>
<td>T15+40</td>
<td>5.374</td>
<td>1.008</td>
</tr>
<tr>
<td>T18+10</td>
<td>1.377</td>
<td>-1.633</td>
</tr>
<tr>
<td>T18+20</td>
<td>2.889</td>
<td>-2.656</td>
</tr>
</tbody>
</table>
4.4.4 J-value and τe for an opposing current with waves

In cases of an opposing current with waves, the J-values are unknown, since V(z) simply increases from the bottom to the surface. Then, τe can not be calculated in the same manner as mentioned in the previous section. Another condition is necessary to calculate τe in cases of an opposing current with waves.

Comparing the sediment concentration distributions in case of a following current and an opposing current with waves, the C(z) distribution have similar shapes in the both cases, as shown in Fig.1.2 and Fig.4.8(a). This fact implies that the εf(z) distributions in cases of a following and an opposing currents with waves also are similar to each other.

From Fig.II-11.4 to II-11.6 which are the εf(z) distributions obtained by the modified Bijker approach with the double triangular τ(z) distribution for a following current with waves, it can be seen that εf(z) increases from the bottom towards the upper elevations, and its increasing rate, dεf(z)/dz, is big near the bottom and becomes smaller and smaller with increasing of the elevation and becomes nearly zero near the surface. This means that εf(z) in the upper layer seems to be more or less constant.

Therefore, it seems to be reasonable assumption that;

\[ εf(z) \approx \text{Const}_1. \quad \text{for near the surface} \quad (4.37) \]

where;

\[ \text{Const}_1. \text{ Constant value} \]

With eq.(1.7), this assumption yields;

\[ τ(z) = \rho \times \text{Const}_1. \times \frac{dV(z)}{dz} \quad \text{for near the surface} \quad (4.38) \]

Using the mean water depth h as the representative elevation near the surface, eq(4.39) can be derived.
\[ \tau(h) = \rho \cdot \text{Const}_1 \cdot \left. \frac{dV(z)}{dz} \right|_{z=h} \]  (4.39)

On the other hand, \( \tau(z) \) is a linear function of \( z \) and from eq. (4.33) it can be described that;

\[ \tau(z) = \text{Const}_2 \cdot z + \tau_0 \]  (4.40)

where;

\( \text{Const}_2 \) : Constant value \( \left[ = \frac{\tau_s - \tau_0}{h} \right] \)

Calculating the derivative \( d\tau(z)/dz \) from eqs. (4.38) and (4.40), the following equation emerges for the height \( h \).

\[ \rho \cdot \text{Const}_1 \cdot \left. \frac{d^2V(z)}{dz^2} \right|_{z=h} = \text{Const}_2. \]  (4.41)

From eq. (4.3.c), the first and second derivative terms, \( dV(z)/dz \) and \( d^2V(z)/dz^2 \), can be calculated.

\[ \left. \frac{dV(z)}{dz} \right|_{z=h} = \frac{a_{c_1}}{h} + a_{d_1} \cdot \exp(h) - a_{d_2} \cdot \exp(-h) \]  (4.42.a)

\[ \left. \frac{d^2V(z)}{dz^2} \right|_{z=h} = -\frac{a_{c_1}}{h^2} + a_{d_1} \cdot \exp(h) + a_{d_2} \cdot \exp(-h) \]  (4.42.b)

From eqs. (4.39), (4.40) and (4.41);

\[ \text{Const}_2 \cdot h + \tau_0 = \text{Const}_2 \cdot \frac{F_1(h)}{F_z(h)} \]  (4.43)

where;

\( F_1(h) \) : The value of the first derivative of \( V(z) \) with respect to \( z \) at height \( h \) (= eq. (4.42.a))

\( F_z(h) \) : The value of the second derivative of \( V(z) \) with respect to \( z \) at height \( h \) (= eq. (4.42.b))

Then, \( \text{Const}_2 \) can be calculated as;
\[ \text{Const2.} = - \tau_0 \ast \left[ h - \frac{F_1(h)}{F_2(h)} \right]^{-1} \quad (4.44) \]

With eqs.(4.40) and (4.44), \( \tau(z) \) can be described in a different way as follows.

\[ \tau(z) = \tau_0 - \tau_0 \ast \left[ h - \frac{F_1(h)}{F_2(h)} \right]^{-1} \ast z \quad \text{for near the surface} \quad (4.45) \]

Now, the J-value can be calculated by eq.(4.45) with the condition that \( \tau \) is zero at the height \( Z_J \). Then;

\[ Z_J = h - \frac{F_1(h)}{F_2(h)} \quad (4.46) \]

\[ J = \frac{1}{h} \ast \left[ h - \frac{F_1(h)}{F_2(h)} \right] \quad (4.47) \]

In Table 4.3, the \( Z_J \) and J-value for an opposing current with waves calculated by eqs.(4.46) and (4.47) are shown. In Table 4.4 and Fig.II-12.2 the \( \tau_0 \) and \( \tau_3 \) values calculated by eqs.(4.22) and (4.36) are shown.

For case T7.5-40 and T10-40, eq.(4.46) leads to the negative \( Z_J \) value. Since the assumption that \( \varepsilon f \) is constant is valid near the surface, \( Z_J \) should be positive. In this meaning, it can be stated that \( Z_J \) and J-value could not be found in the significant zone in these two cases.
Table 4.3  $Z_j$ and $J$-value for a opposing current with waves

<table>
<thead>
<tr>
<th>CASE</th>
<th>$Z_j$ [m]</th>
<th>$J$ [-]</th>
</tr>
</thead>
<tbody>
<tr>
<td>T7.5-10</td>
<td>0.7464</td>
<td>1.5527</td>
</tr>
<tr>
<td>T7.5-20</td>
<td>0.5112</td>
<td>1.0397</td>
</tr>
<tr>
<td>T7.5-40</td>
<td>N.F.</td>
<td>N.F.</td>
</tr>
<tr>
<td>T10-10</td>
<td>0.4992</td>
<td>1.0107</td>
</tr>
<tr>
<td>T10-20</td>
<td>0.8156</td>
<td>1.6817</td>
</tr>
<tr>
<td>T10-40</td>
<td>N.F.</td>
<td>N.F.</td>
</tr>
<tr>
<td>T15-10</td>
<td>0.5863</td>
<td>1.1973</td>
</tr>
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<td>T15-20</td>
<td>0.7155</td>
<td>1.4475</td>
</tr>
<tr>
<td>T15-40</td>
<td>1.1829</td>
<td>2.3471</td>
</tr>
<tr>
<td>T18-10</td>
<td>0.4538</td>
<td>0.9187</td>
</tr>
<tr>
<td>T18-20</td>
<td>0.8795</td>
<td>1.8096</td>
</tr>
<tr>
<td>T18-40</td>
<td>1.3325</td>
<td>2.6475</td>
</tr>
</tbody>
</table>

N.F.: Mark indicating that the value is not found.

Table 4.4  $\tau_o$ and $\tau_s$ for a opposing current with waves

<table>
<thead>
<tr>
<th>CASE</th>
<th>$\tau_o$ [N/m2]</th>
<th>$\tau_s$ [N/m2]</th>
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<tbody>
<tr>
<td>T7.5-10</td>
<td>0.467</td>
<td>0.166</td>
</tr>
<tr>
<td>T7.5-20</td>
<td>0.800</td>
<td>0.030</td>
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<tr>
<td>T7.5-40</td>
<td>2.401</td>
<td>N.F.</td>
</tr>
<tr>
<td>T10-10</td>
<td>0.631</td>
<td>0.007</td>
</tr>
<tr>
<td>T10-20</td>
<td>1.174</td>
<td>0.476</td>
</tr>
<tr>
<td>T10-40</td>
<td>3.020</td>
<td>N.F.</td>
</tr>
<tr>
<td>T15-10</td>
<td>1.137</td>
<td>0.187</td>
</tr>
<tr>
<td>T15-20</td>
<td>2.034</td>
<td>0.629</td>
</tr>
<tr>
<td>T15-40</td>
<td>3.937</td>
<td>2.260</td>
</tr>
<tr>
<td>T18-10</td>
<td>1.417</td>
<td>-0.125</td>
</tr>
<tr>
<td>T18-20</td>
<td>2.329</td>
<td>1.042</td>
</tr>
<tr>
<td>T18-40</td>
<td>5.715</td>
<td>3.556</td>
</tr>
</tbody>
</table>

N.F.: Mark indicating that the value is not found.
From the Fig.II-12.2, the following tendencies can be seen.

- \( \tau(z) \) distributions in cases of an opposing current with waves are of trapezoidal shape, except T18-10 case.
- \( \tau_o \) increases with increasing of Hs and also Vm.
- \( \tau_s \) values are positive, except T18-10 case.
- In cases of \( V_m = -20 \text{ cm/s} \) and \(-40 \text{ cm/s}\), \( \tau_s \) increases with increasing of Hs, but in cases of \( V_m = -10 \text{ cm/s} \) such a tendency is not clear.
- In cases of Hs = 10 cm, 15 cm and 18 cm, \( \tau_s \) increases with increasing of the absolute Vm, but in cases of Hs = 7.5 cm such a tendency is not clear.
- \( \tau_s \) value seems to have a different peak value in the series of tests with the same Hs and different Vm, or tests with the same Vm and different Hs.

Also, considering eq. (4.35) with the results of \( \tau_s \) value in case of an opposing current with waves, it can be thought that the resulting surface slope becomes milder than that caused by only the same bottom shear stress, since \( \tau_s \) is positive.
4.5 The $\varepsilon_f(z)$ results calculated by the modified Bijker approach with the additional internal shear stress

In previous section, the additional shear stress due to current-wave interaction was introduced in order to explain the velocity profile of a current with waves, and applied to $\tau(z)$ distributions in cases of a current with waves. Now it becomes clear that because of this additional internal shear stress the total internal shear stress distribution is of a double triangular shape in cases of a following current with waves and of a trapezoidal shape in cases of an opposing current with waves. In this section, $\tau(z)$ distribution estimated by this idea will be used to calculate $\varepsilon_f(z)$ distribution under a combination of waves and a current.

4.5.1 The calculation results of $\varepsilon_f(z)$ distribution

For the cases of a following current with waves, $\varepsilon_f(z)$ distribution have already calculated in section 4.3 by the modified Bijker approach and $\tau(z)$ distribution considering the additional internal shear stress. So, $\varepsilon_f(z)$ distributions for a following current with waves are shown in Fig.II-11.4 to II-11.6. For the cases of an opposing current with waves, $\varepsilon_f(z)$ distribution have been recalculated with $\tau(z)$ distribution, which is of a trapezoidal shape, considering the additional internal shear stress. The results are shown in Fig.II-13.1 to II-13.3.

From these figures, it can be stated that:

- The calculated $\varepsilon_f(z)$ distributions for an opposing current with waves have similar shape to that for a following current with waves and also the $\varepsilon_f(z)$ distribution shape that has been calculated by "CONCFL" program for the Heijboer's experiment data.
- The calculated $\varepsilon_f(z)$ distribution in case of a higher significant wave height tends to be bigger than that of a lower significant wave height in the entire water column, except a few cases, in a series of tests with the same mean velocity.
- The calculated $\varepsilon_f(z)$ distribution in case of a faster mean velocity tends to be bigger than that of a slower mean velocity.
velocity, except a few cases, in a series of tests with the same significant height.

- The calculated $\varepsilon f(z)$ distribution is bigger than the $\varepsilon fc(z)$ in the entire water column.
- This time, the calculated $\varepsilon f(z)$ distributions in cases of an opposing currents with waves seems to be quite reasonable, in comparison with the $\varepsilon f(z)$ distributions obtained from the assumption of a single triangular shape $\tau(z)$ distribution.

4.5.2 Study of the results

From Fig. II-11.4 to II-11.6 and II-13.1 to 13.3 which are $\varepsilon f(z)$ distributions for a current with waves obtained by the modified Bijker approach and the additional internal shear stress idea, it can be stated that;

By using the modified Bijker approach to calculate the bottom shear stress, $\tau_0$, and $\tau(z)$ distribution considering the additional internal shear stress, quite reasonable $\varepsilon f(z)$ distributions can be calculated in cases of a following and opposing current with waves. Since, the calculated $\varepsilon f(z)$ distributions have reasonable tendencies as follows, from a physical point of view.

- $\varepsilon f$ at the bottom is non-zero value that corresponds to the stirring-up mechanism of the sediment particles at the bottom.
- $\varepsilon f(z)$ increases from the bottom towards the upper layers, except inside of the bottom boundary layer. The increasing rate, $d\varepsilon f(z)/dz$, is bigger near the bottom than in the upper layers, and it becomes more or less zero near the surface.
- $\varepsilon f(z)$ is always bigger than $\varepsilon fc(z)$. It fits the concept of eq.(1.14) that explain that the existence of waves on a current increases the diffusion activity as compared to that of the mere current case.
- $\varepsilon f(z)$ distribution in case of a current with higher waves tends to be bigger than that of a current with lower waves.
- $\varepsilon f(z)$ distribution in case of a faster current tends to be bigger than that of a slower current with waves of the same wave height.

Though there is a few exceptional cases, these tendencies can be seen clearly in most of cases. And these tendencies are quite
realistic and reasonable from a physical point of view. In Fig.II-14.1 and II-14.2, the $\varepsilon f(z)$ distributions obtained by "CONCFL" program and "EFVEL" program with the additional internal shear stress for the Heijboer's data cases are compared. From these figures, it can be stated that:

- For both cases in the Heijboer's experimental data, the $\varepsilon f(z)$ distribution calculated by "EFVEL" program is about 1.5 times bigger than that by "CONCFL" program, but these shapes of $\varepsilon f(z)$ distributions are quite similar to each other.
- It is not easy to conclude which $\varepsilon f(z)$ distribution is more reliable than the other only by few calculations. If the $\varepsilon f(z)$ distribution obtained by "CONCFL" program is assumed to be closer to the real than that by "EFVEL" program, the main reason would be attributed to the estimation method of the bottom shear stress, $\tau_0$, and probably the assumption that $p = 1$ at $Z_l$. Therefore, before generalizing the second method, it seems to be necessary to check the assumptions used in the second method.
- If the $\varepsilon f(z)$ distribution obtained by "EFVEL" program is assumed to be closer to the real, the assumption used in "CONCFL" program should be checked.

As to the relationships $\varepsilon f$ values and the boundary conditions, some comparisons are plotted in Fig.II-15.1 to II-15.6. From these figures the following things can be seen.

- From Fig.II-15.1 and II-15.2, $\varepsilon f$ values at the bottom tend to be proportional to the orbital velocity amplitude, $\hat{U}_b$, quite well, but do not show any clear relationship with the mean ripple height.
- From Fig.II-15.3 and II-15.4, the value $[\varepsilon f - \varepsilon f_c]$ or $[\varepsilon f^2 - \varepsilon f^2_c]^{0.5}$ at the height 0.25 m increases with increasing of the orbital velocity amplitude, $\hat{U}$, at the same elevation, and they tend to be proportional to $\hat{U}$ when $\hat{U}$ is smaller than 0.2 m/s but when $\hat{U}$ is bigger than 0.2 m/s the scattering ranges of these parameters become big.
- In Fig.II-15.5 and Fig.II-15.6, the value $[\varepsilon f - \varepsilon f_{c, res.}]$ or $[\varepsilon f^2 - \varepsilon f_{c, res.}^2]^{0.5}$ are plotted for the corresponding $\hat{U}$ at the height 0.25 m. The $\varepsilon f_{c, res.}$ means the diffusion coefficient in
case of the mere current considering the increasing bottom shear stress, that is equal to $\tau_0$, not $\tau_{\infty}$. Here, the similar tendencies as mentioned above also can be seen in these figures.

- The reason of such a wide scattering in higher range of the orbital motion seems that the current velocity and the bigger orbital motion create some additional diffusion activity.

In conclusion, the modified Bijker approach and the additional internal shear stress that are proposed in the present study could be thought to be useful tools or idea for investigating the diffusion activity under waves with a current. However, this proposing method starts from the measured time-averaged velocity data and uses the fit curve approximation for $V(z)$ distribution in order to express $V(z)$ mathematically. And also several assumptions, such as the linear wave theory for the relative coordinate system, or that $p = 1$ at height $Z_t$, or the additional internal shear stress due to current-wave interaction, or that $\varepsilon f$ is constant near the surface, are used to calculate $\varepsilon f(z)$ distribution. Therefore, the calculated $\varepsilon f(z)$ distributions are affected by the measuring error and the errors due to the fit curve process and the difference between the real phenomenon and the used assumptions in some extend. Especially, the assumption of the internal shear stress used for this method are based on the experimental facts, not on the theoretical investigation. Therefore, it is necessary to verify these assumptions used in the proposed method by other ways, for example, a special experiments for checking these assumptions more theoretical investigations considering the current-wave interactions.
In this study, two difference methods have been applied to some experimental data in order to calculate the diffusion coefficient distribution of the fluid, $\varepsilon f(z)$, under waves and a current. From the calculated results by these two methods, the following can be concluded.

According to the first method;

- It leads to smaller $\varepsilon f$ values in case of a current with waves than that of the mere current case for the most of Nap and Van Kampen's experimental data. These results seem to be contradictory from a physical point of view.
- It leads quite reasonable $\varepsilon f(z)$ distribution for some of the Heijboer's experimental data.
- The main reason of the underestimation of $\varepsilon f$ values for the Nap and Van Kampen's data could be that;
  1. Armour effect took place at the bottom surface and rather coarse particles covered the bottom, but this effect can not be considered adequately in the calculation.
  2. In the Heijboer's experiments, sands were supplied continuously from the upstream point of the flume. Then the proportion of particle sizes in the suspension load could be kept more or less stable. On the other hand, in the Nap and Van Kampen's experiments, there was not such an artificial sand supply then the proportion of particle sizes in the suspension load might be changing to a coarser one during test period, but, this change of the particle sizes has not been considered in the calculation.
  3. If the $\varepsilon f(z)$ distribution calculated by the second method is assumed to be true, the ranges of $\varepsilon f$ values for the Nap and Van Kampen's experimental data are bigger than the upper limit of $\beta$-effect predictor applied to the first method. Then in order to improve the first method it would be necessary to find a $\beta$-effect predictor for the higher range of $\varepsilon f$.

In conclusion, the first method is a logically correct method and strong tool for investigation of the diffusion activity with the
concentration distribution data. It can calculate reasonable $\varepsilon f(z)$ distributions as far as the input data match the real phenomenon well. If the input data do not match the real phenomenon due to the armour effects or the changes of the particle size proportion in the suspension load, the calculated $\varepsilon f(z)$ values become unreliable. The artificial sand supply seems to compensate such effects in some extent. It is necessary to investigate the $\beta$-effect predictor for the higher diffusion activity area than the upper limit of the present one.

According to the second method;

- In most of cases for the Nap and Van Kampen's experimental data, the second method using the modified Bijker approach and the additional internal shear stress idea leads to quite reasonable $\varepsilon f(z)$ distribution.
- In some cases, unreasonable $\varepsilon f(z)$ distribution appears by this method. The reasons of such unreasonable results seem the measuring errors, the error of the fit curve process, the difference of the bottom shape conditions because of the movable bed and the difference between the assumptions and the real phenomenon.
- This method uses several assumptions, therefore it is necessary to verify these assumptions by other experiments or theoretical investigations.

In conclusion, this method can be thought to be useful and will be more useful for $\varepsilon f(z)$ estimation after verifying and improving the assumptions. The idea of the modified Bijker approach to calculate the bottom shear stress and the additional internal shear stress in case of a current with waves play important roles in this method. However, these idea are quite new, then it is also necessary to verify them by further research.

The author hopes that these two method used in this study will be advanced and improved by the further study in near future and will be a help the further understanding of the sediment transport mechanism.
<table>
<thead>
<tr>
<th>Parameter</th>
<th>Definition</th>
<th>Unit</th>
</tr>
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<tbody>
<tr>
<td>(a)</td>
<td>vertical ordinate point A</td>
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<tr>
<td>(a_b)</td>
<td>displacement amplitude of orbital motion of water particle near the bed</td>
<td>(m)</td>
</tr>
<tr>
<td>(a_{\alpha 0})</td>
<td>current part parameter of velocity curve fit</td>
<td>(m/s)</td>
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<tr>
<td>(a_{\alpha 1})</td>
<td>current part parameter of velocity curve fit</td>
<td>(m/s)</td>
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<td>(a_{\delta 0})</td>
<td>disturbed part parameter of velocity curve fit</td>
<td>(m/s)</td>
</tr>
<tr>
<td>(a_{\delta 1})</td>
<td>disturbed part parameter of velocity curve fit</td>
<td>(m/s)</td>
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<td>(a_{\delta 2})</td>
<td>disturbed part parameter of velocity curve fit</td>
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<td>(A)</td>
<td>coefficient in shear stress equation</td>
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<td>(A)</td>
<td>parameter in fall velocity equation</td>
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<tr>
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<tr>
<td>(B)</td>
<td>parameter in fall velocity equation</td>
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</tr>
<tr>
<td>(C)</td>
<td>concentration</td>
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<tr>
<td>(C)</td>
<td>parameter in fall velocity equation</td>
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<td>(C_r)</td>
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<td>Const1.</td>
<td>constant coefficient</td>
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<tr>
<td>Const2.</td>
<td>constant coefficient</td>
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<td>(d)</td>
<td>integration constant of concentration fit</td>
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<tr>
<td>(D)</td>
<td>grain diameter</td>
<td>(m)</td>
</tr>
<tr>
<td>(D\times)</td>
<td>grain diameter exceeded by (x) %</td>
<td>(m)</td>
</tr>
<tr>
<td>(D_{ch,i})</td>
<td>characteristic diameter of the (i)-th sub-division</td>
<td>(m)</td>
</tr>
<tr>
<td>(e)</td>
<td>base of natural logarithmic function</td>
<td>-</td>
</tr>
<tr>
<td>(exp(x))</td>
<td>(e) to the (x)</td>
<td>-</td>
</tr>
<tr>
<td>(f)</td>
<td>symbol of function</td>
<td>-</td>
</tr>
<tr>
<td>(f')</td>
<td>the first derivative of function (f)</td>
<td>-</td>
</tr>
<tr>
<td>(f_{\omega})</td>
<td>Jonsson’s friction factor</td>
<td>-</td>
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</table>
F1  the first derivative of velocity fit curve
F2  the second derivative of velocity fit curve
\( g \)  gravitational acceleration \( \text{m/s}^2 \)
\( h \)  water depth \( \text{m} \)
\( H \)  wave height \( \text{m} \)
\( H_{\text{rms}} \)  root mean square wave height \( \text{m} \)
\( H_s \)  significant wave height \( \text{m} \)
i  class number
\( I \)  surface slope
\( I_b \)  surface slope due to bottom shear stress
\( I_c \)  surface slope in case of mere current
\( I_s \)  surface slope due to surface shear stress
\( J \)  ratio between the elevation of \( dV/dx=0 \) and the mean water depth
\( l_e \)  mixing length \( \text{m} \)
\( \ln \)  natural logarithm
\( \log \)  \( 10 \log \)
\( L \)  wave length \( \text{m} \)
n  number of classes
\( p \)  parameter in Bijker equation of bottom shear stress
\( p \)  exponent in concentration fit
\( q \)  scale factor in concentration fit
\( r \)  Swart’s bottom roughness \( \text{m} \)
\( \Delta r \)  mean ripple height \( \text{m} \)
\( S \)  sediment transport \( \text{kg/m/s} \)
\( t \)  time \( \text{s} \)
\( t \)  parameter in standard normal distribution
\( T \)  wave period \( \text{s} \)
\( T_p \)  peak wave period \( \text{s} \)
\( T_r \)  relative wave period \( \text{s} \)
u: horizontal velocity component
u_b: horizontal velocity component near the bed

O: maximum orbital velocity
O_b: maximum orbital velocity near the bed

Ua: horizontal velocity for absolute system
Ur: horizontal velocity for relative system

V: time- (and bed-) averaged horizontal velocity
Vcon.: constant current velocity

Vc: time- (and bed-) averaged velocity of current part
Vd: time- (and bed-) averaged velocity of disturbed part

Vm: depth averaged current velocity
Vt: time- (and bed-) averaged velocity at zt
Vtc: Vt in the current case

V*: shear velocity
V*_c: shear velocity in case of the current

W: fall velocity of sediment particle

W_a: vertical velocity for absolute system
W_i: characteristic fall velocity of the i-th class
W_r: vertical velocity for relative system

x: horizontal ordinate
x: parameter of %

Δx: horizontal length of control volume

y: horizontal ordinate
z: vertical ordinate

z_J: elevation of J point where dV/ dz = 0
z_0: elevation of zero velocity
z_0o: elevation of zero velocity in the current case
z_t: elevation for velocity profile tangency
z_t_c: elevation for velocity profile tangency in the current case
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Meaning</th>
<th>Unit</th>
</tr>
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<td>$z_e$</td>
<td>relative height above the bed</td>
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<tr>
<td>$\alpha_e$</td>
<td>gradient of velocity profile tangency</td>
<td>-</td>
</tr>
<tr>
<td>$\alpha_{e,c}$</td>
<td>gradient of velocity profile tangency in mere current case</td>
<td>-</td>
</tr>
<tr>
<td>$\beta$</td>
<td>factor $[\ell_S/\ell_I]$</td>
<td>-</td>
</tr>
<tr>
<td>$\varepsilon_f$</td>
<td>diffusion coefficient of fluid</td>
<td>m$^2$/s</td>
</tr>
<tr>
<td>$\varepsilon_{fc}$</td>
<td>diffusion coefficient of fluid due to current</td>
<td>m$^2$/s</td>
</tr>
<tr>
<td>$\varepsilon_{fc,res}$</td>
<td>diffusion coefficient of fluid due to current considering increased bottom shear stress</td>
<td>m$^2$/s</td>
</tr>
<tr>
<td>$\varepsilon_{fw}$</td>
<td>diffusion coefficient of fluid due to waves</td>
<td>m$^2$/s</td>
</tr>
<tr>
<td>$\varepsilon_{fw,c}$</td>
<td>diffusion coefficient of fluid due to waves and current</td>
<td>m$^2$/s</td>
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<tr>
<td>$\varepsilon_s$</td>
<td>diffusion coefficient of grain particles</td>
<td>m$^2$/s</td>
</tr>
<tr>
<td>$\eta$</td>
<td>surface water level</td>
<td>m</td>
</tr>
<tr>
<td>$\kappa$</td>
<td>Von Karman's constant</td>
<td>-</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>mean ripple length</td>
<td>m</td>
</tr>
<tr>
<td>$\mu$</td>
<td>mean of variables for normal distribution</td>
<td>Var.</td>
</tr>
<tr>
<td>$\rho$</td>
<td>mass density of fluid</td>
<td>kg/m$^3$</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>scale factor in normal distribution equation</td>
<td>Var.</td>
</tr>
<tr>
<td>$\tau$</td>
<td>internal shear stress</td>
<td>N/m$^2$</td>
</tr>
<tr>
<td>$\tau_{a(z)}$</td>
<td>additional internal shear stress</td>
<td>N/m$^2$</td>
</tr>
<tr>
<td>$\tau_{b(z)}$</td>
<td>internal shear stress due to bottom shear stress</td>
<td>N/m$^2$</td>
</tr>
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<td>$\tau_{c(z)}$</td>
<td>internal shear stress in mere current case</td>
<td>N/m$^2$</td>
</tr>
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<td>$\tau_o$</td>
<td>bottom shear stress</td>
<td>N/m$^2$</td>
</tr>
<tr>
<td>$\tau_{oc}$</td>
<td>bottom shear stress in mere current case</td>
<td>N/m$^2$</td>
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<tr>
<td>$\tau_{o,mod.}$</td>
<td>bottom shear stress by modified Bijker approach</td>
<td>N/m$^2$</td>
</tr>
<tr>
<td>$\tau_{o,orig.}$</td>
<td>bottom shear stress by original Bijker approach</td>
<td>N/m$^2$</td>
</tr>
<tr>
<td>$\tau_s$</td>
<td>surface shear stress</td>
<td>N/m$^2$</td>
</tr>
<tr>
<td>$\phi$</td>
<td>angle between wave and current</td>
<td>rad.</td>
</tr>
<tr>
<td>$\phi(t)$</td>
<td>standard normal distribution of t</td>
<td>-</td>
</tr>
</tbody>
</table>
LIST OF FIGURES

Fig.1.1 Longshore and Cross-shore sediment transport
Fig.1.2 Sediment transport = Velocity * Concentration
Fig.1.3 Principle of diffusion type model
Fig.1.4 Flow-chart of the two methods for $\varepsilon_f(z)$
investigation
Fig.2.1 Sketch of the Nap and Kampen's experimental flume
Fig.2.2 Sketch of the Heijboer's experimental flume
Fig.3.1 Illustration of the perfect log-normal distribution
for bed material diameters
Fig.3.2 $\varepsilon(z)$ and C(z) distribution of Coleman type model
Fig.3.3 Standard square for fit procedure
Fig.3.4 Flow-chart of the first method
Fig.4.1 Sketch of V(z) distribution in cases of following
and opposing currents with waves
Fig.4.2 Sketch of Vd(z) distribution in cases of following
and opposing currents with waves
Fig.4.3 Sketch of water particle movement under waves and
a constant current
Fig.4.4 Concept of original Bijker approach
Fig.4.5 Tentative assumptions of $\tau(z)$ distribution
Fig.4.6 V(z) and Vc(z) distributions near the bottom
Fig.4.7 Original Bijker approach and Modified Bijker
approach for the bottom shear stress
Fig.4.8 Typical measured C(z) distribution and
C(z) distribution calculated with parabolic
shape $\varepsilon_f(z)$ distribution
Fig. 4.9-1 \( V_c(z) \) and \( \tau_c(z) \) distribution

Fig. 4.9-2 \( V(z) \) and \( \tau(z) \) distribution in case of following current with waves

Fig. 4.10 Control volume (1-1'-2-2') and force balance

Fig. 4.11-1 Internal shear stress; \( \tau_b(z) \), \( \tau_a(z) \) and \( \tau(z) \)
in case of \( \tau_s < 0 \)

Fig. 4.11-2 Internal shear stress; \( \tau_b(z) \), \( \tau_a(z) \) and \( \tau(z) \)
in case of \( \tau_s > 0 \)
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