ABSTRACT
Safety measurement and its analysis has been a challenging and well-researched topic in transportation. Conventionally, surrogate safety measures have been used as safety indicators in simulation models for safety assessment, in control formulations for driver assistance systems, and in data analysis of naturalistic driving studies. However, surrogate indicators give partial insights on traffic safety i.e., they only indicate a predetermined set of possible pre-crash situations for an interacting vehicle pair. Recently, a safety indicator called the driving safety field based on field theory has been proposed for two-dimensional vehicle interactions. However, the objectivity of its functional form and validity are yet to be tested. This paper provides a qualitative and quantitative comparison of different safety indicators as a risk measure to demarcate their mathematical properties and evaluate their usefulness in quantifying trajectory risk. We compare five relevant safety indicators: inverse time to collision (iTTC), post-encroachment time (PET), potential indicator of collision with urgent deceleration (PICUD), warning index and safety field force. Their formulations are mathematically analyzed to yield qualitative insights and their values over simulated vehicle trajectories are evaluated to yield quantitative insights. Our results acknowledge the limitations and demarcate the functional utilities of the selected safety indicators.

Keywords: Surrogate safety indicators, Risk measure, Safety field force, Automated Driving
INTRODUCTION
Safety is a key performance indicator of any transportation system. Road safety research has received considerable attention owing to the enormous societal losses incurred in road accidents worldwide, with about 1.25 million fatalities, and between 20 and 50 million non-fatal injuries (1). Recent efforts in safety research are primarily focusing on the use of surrogate safety indicators (SSM), as a proactive and cost efficient method to evaluate safety, acknowledging the limitations of using crash records (2) e.g. road safety assessment (3, 4); ex-ante safety evaluation in driver assistance and automation systems (5, 6); and behavior modeling of human drivers in safety critical scenario (7). The advent of intelligent vehicles has brought in uncertainties especially with regard to vehicle interactions. The uncertainties stem from the fact that an intelligent vehicle possesses enhanced communication and control capabilities compared to a human-driven vehicle, but lacks in the spatial and temporal anticipative capabilities. Achieving an agreement on a set of objective safety indicators that are applicable in mixed traffic is a methodological challenge. Hence, the selection of a safety indicator has profound implications on the quality and agreeability of the safety research findings.

Essentially, a safety indicator is a measure of risk associated with a vehicle interaction. Surrogate safety measures (SSM) are the most common risk indicators used in safety studies. The risk delineated by SSM could vary depending on their formulation and parameter consideration. More importantly, they are often discontinuous as their validity is limited to a prescribed set of interacting vehicle configurations. For example, time to collision is not defined in a car following situation with a faster leader. Recently, a safety model has been proposed that is capable of describing risk continuously over the vehicle path. This safety model is based on field theory and defines driving risk as a spatial field (8). However, the objectivity of its functional form and validity are yet to be tested. Therefore, despite the wide range of safety indicators, selection of an appropriate indicator warranting validity and agreeable results is intricate.

Safety indicators are usually selected based on their study scope and methodological suitability, making it difficult to generalize their findings. Even though safety indicators have been extensively reviewed and empirically validated in the past, limited literature exists on the demarcation of their mathematical properties; representation of risk causal factors in their formulation; evaluation of their usefulness in quantifying trajectory risk. To that end, in this paper, we compare relevant safety indicators for their qualitative and quantitative aspects. Their formulations are mathematically analyzed to yield qualitative insights and their values over simulated vehicle trajectories are evaluated to yield quantitative insights. Our results acknowledge the limitations and demarcate the functional utilities of the selected safety indicators.

The remainder of this paper is organized as follows: Following the introductory section, a literature review is presented focused on identifying relevant safety indicators. The theoretical properties of these indicators are then analysed qualitatively and further verified quantitatively using numerical vehicle trajectory simulation.

LITERATURE REVIEW
Crash statistics have been traditionally used for road safety evaluation. Even though relevant, it has drawbacks such as the unavailability of sufficient crash data to derive statistically significant conclusions and inability to be used for ex-ante evaluation. These drawbacks made researchers turn towards a complementary approach that uses surrogate safety measures (SSM). The characteristics of SSM are that they are more frequent than crashes; they are observable in traffic; and they represent crash causality and crash mechanisms (9).
SSM have been critically and extensively reviewed over time (6, 10–13). Generally, SSM define the collision risk of an interacting vehicle pair as a function of their instantaneous kinematic states (acceleration, velocity and position) and depend on their spatial configuration. Hence, these indicators can be categorized into longitudinal and lateral indicators based on the location of the interacting vehicles. Longitudinal indicators have been widely used in forward collision warning systems, safety assessment of highways and human behavioral modelling in rear-end crash scenarios. Common longitudinal-SSM are Time To Collision (TTC), inverse Time To Collision (iTTC), Time Exposed Time to collision (TET), Time Integrated Time to collision (TIT) (14), Deceleration Required To Avoid Collision (DRAC), Potential Indicator of Collision with Urgent Deceleration (PICUD)(12). Lateral-SSM like Post Encroachment Time (PET) have been used as a risk measure in lane change controllers, safety assessment of intersections and lateral vehicle maneuvers.

SSM that are not intrinsically bounded to lateral or longitudinal interactions can be found in the literature. For instance, Crash Potential Index (CPI) and Aggregated Crash Index based on a predetermined set of probable set of evasive maneuvers (6, 15). The functionality of these probabilistic indicators is restricted to certain driving regimes due to the difficulty of exhaustively listing all possible maneuvers. Additionally, predictive risk maps have been proposed to estimate the future risk based on the predicted trajectories of interacting vehicles (16). Even though this approach is efficient for ex-ante safety evaluation in controllers, its performance inherently depends on the prediction modules and does not fall within the scope of this work. Recently, Wang et al proposed an alternative risk assessment methodology for two-dimensional vehicle interactions based on field theory (8). They model risk as a vector field and incorporate road, vehicle and driver characteristics into a unified field formulation. In this study, we focus on five safety indicators: iTTC, PICUD, Warning Index (relevant longitudinal indicators with different parameter considerations), PET (relevant lateral indicator), and safety field force (two-dimensional safety indicator).

**QUALITATIVE ANALYSIS**

Qualitative analysis of the selected indicators was performed with the following objectives: to evaluate the mathematical properties of their functional form in the multi-vehicle scenario; and to benchmark their formulation with expected causal tendencies of major risk contributing variables.

**Desirable Mathematical Properties for a Risk Measure in Multivehicle Scenario**

In this section we present the desirable mathematical properties of safety indicators to verify the applicability of selected safety indicators in multivehicle scenarios. Mathematical measure theory has prescribed criteria for a function to be termed as a measure (17). Being a risk measure of vehicle interaction, it is desirable for safety indicators to adhere to these criteria as follows:

Let $X$ be the set of all interacting vehicles $V$ under consideration, and $\Sigma$ be the collection of possible subsets of $X$. A risk measure $\mu : \Sigma \rightarrow \mathbb{R}$ from $\Sigma$ to the real number line $\mathbb{R}$ is a mathematical risk measure if the following conditions are satisfied:

Non-negativity: The risk measure $\mu$ of any vehicle $V$ with index $k$ in $\Sigma$ is a non-negative value.

$$\mu(V_k) \geq 0$$

This property is desirable considering that a negative risk value is non-intuitive and its use is ambiguous in multi-vehicle scenarios, i.e. it could cancel a positive risk value.
Countable additivity: The risk measure $\mu$ should indicate the union of risk values due to the interacting vehicles $M$ in a multi-vehicle scenario. Wherein, the risk measure of a countable disjoint collection of vehicle units $\{V_i\}_{i=1}^{M}$ is the same as the sum of all risk measures of each vehicle unit as follows:

$$\mu \left( \bigcup_{i=1}^{M} V_i \right) = \sum_{i=1}^{M} \mu(V_i) \quad (2)$$

This property simplifies the individual risk calculations for complex multivehicle interactions; and it allows the addition of individual risk measures to estimate the total societal/collective risk. However, this is not an essential property to indicate the risk associated with vehicle pair interaction like car following.

**Risk Factors and Expected Causal Tendencies**

In this section we detail the major contributing factors of risk and their expected causal tendencies. This expectation is based on reasoning and relationships that are reported in previous empirical and physics-based crash studies. Dynamics and causality of a crash are directly and indirectly influenced by various factors, and it would be farfetched to exhaustively list them. But few of these factors have been reported to have a causal relationship with vehicle collisions. Firstly, the probability of a collision between two road users is expected to increase with their approaching rate and decrease with the inter-vehicle spacing (shorter time for the driver to react; the lesser possibility of risk mitigation or evasive maneuver). Secondly, the collision impact is expected to increase with an increase in velocity (18) and mass (19) of the conflicting vehicles (with higher vehicular velocity, the driver should react more rapidly to avoid a collision; higher vehicular mass results in higher kinetic energy transferred and higher collision severity). Thirdly, the collision impact is reported to increase with delta-V or the change in vehicle velocity as the result of an impact (20). Finally, the roadway characteristic like surface friction (21) and driver characteristic like reaction time (22) are expected to influence the collision risk.

**Benchmarking the Safety Indicators with Expected Risk Tendencies**

In this section, we compare the expected risk tendency of a factor with the risk tendency as described by the partial derivative of the indicator with respect to the factor.

**Time to Collision**

TTC is defined as the time required for two vehicles to collide if they continue in their present velocity along the present path. $TTC = \frac{s_n}{\Delta v_n}$, where, $v_n$ denotes the instantaneous velocity of the vehicle $n$; $\Delta v_n = v_n - v_{n-1}$ and $s_n$ denotes the relative velocity and forward spacing of vehicle $n$ with respect to the front vehicle $n-1$. Inverse time to collision ($iTTC$) is the inverse formulation of TTC and is widely used in controllers like adaptive cruise controller (23) and to assess human driver behavior (24). A higher value represents higher risk and the interaction risk is often captured by the minimum-TTC or maximum-$i$TTC over the interaction period. It is formulated as follows

$$iTTC = \frac{v_n - v_{n-1}}{s_n} \cdot \text{if } v_n > v_{n-1} \quad (3)$$
\[ \frac{\partial \text{ITTC}}{\partial \Delta v} = \frac{\partial \text{ITTC}}{\partial v_n} = \frac{1}{s} > 0, \] indicates that the risk increases with an increase in approaching rate. 
\[ \frac{\partial \text{ITTC}}{\partial \Delta v} = \frac{s}{\partial v_n} = \frac{-\Delta v}{s^2}, \] indicates that the risk decreases with an increase in the spacing of the slower leader. 
As shown in Table 1, both these indications are in agreement with the expected risk tendencies.

**Potential Indicator for Collision with Urgent Deceleration (PICUD)**

PICUD is defined as the forward spacing between two vehicles if both of them brake with a maximum deceleration (12) as follows:

\[ \text{PICUD} = s_n + \frac{v_{n-1}^2 - v_n^2}{2a_{\text{max}}} - t_h v_n \]  \hspace{1cm} (4)

where \( a_{\text{max}} \) denotes the maximum deceleration and \( t_h \) denotes the time delay of human response and smaller PICUD indicate higher risk.

\[ \frac{\partial \text{PICUD}}{\partial \Delta v} = -\frac{v_n + v_{n-1}}{2a_{\text{max}}} < 0, \] \hspace{1cm} \( \frac{\partial \text{PICUD}}{\partial v_n} = -\left( \frac{v_n}{a_{\text{max}}} + t_h \right) < 0 \) and \( \frac{\partial \text{PICUD}}{\partial t_h} = -v_n < 0 \) indicates that the risk increases with an increase in approaching rate, vehicle velocity and human reaction time respectively.

\[ \frac{\partial \text{PICUD}}{\partial \Delta v} = 1 > 0, \] indicates that the risk decreases at a constant rate with an increase in spacing. As shown in Table 1, PICUD is in agreement with the expected risk tendencies. In this paper we use

**Warning Index (w)**

Warning index, \( w \) is a safety indicator used in collision warning algorithms (23). This indicator also includes factors like tire-road friction and system delay. A lower \( w \) represents higher risk and it is formulated as follows

\[ w = \frac{s_n - d_w}{d_w - d_{br}} \]  \hspace{1cm} (5)

\[ d_{br} = \Delta v_n t_s + f(\mu) \left( \frac{v_n^2 - v_{n-1}^2}{2a_{\text{max}}} \right) \]  \hspace{1cm} (6)

\[ d_w = \Delta v_n t_s + f(\mu) \left( \frac{v_n^2 - v_{n-1}^2}{2a_{\text{max}}} \right) + v_n t_h \]  \hspace{1cm} (7)

where, \( d_{br} \) denotes the required braking distance; \( d_w \) denotes the required warning distance; \( f(.) \) denotes the friction scaling function and \( \mu \) is the estimated value of tire-road friction. \( t_s \) is the system delay and \( t_h \) is the delay of human response.

The decreasing \( w \) indicates an increasing risk. Considering this,

\[ \frac{\partial w}{\partial \Delta v} = -\frac{f(\mu)(v_n + v_{n-1}) - t_s}{v_n t_h} < 0 \] and \[ \frac{\partial w}{\partial \Delta v} = -\frac{w}{t_h} < 0 \], indicates that the risk increases with an increase in approaching rate and human reaction time respectively.

\[ \frac{\partial w}{\partial t_h} = \frac{1}{v_n t_h} > 0, \] indicates that the risk decreases with an increase in spacing.

As shown in TABLE 1, \( w \) is in agreement with the expected risk tendencies. However, there are some relations that contradict the expected risk tendencies.

\[ \frac{\partial w}{\partial \Delta v} > 0 \] is subject to the condition \[ \frac{\Delta v_n t_s}{v_n t_h} < \frac{s}{v_n t_h} + \frac{f(\mu)}{2a_{\text{max}} t_h} + \frac{f(\mu) v_{n-1}^2}{2a_{\text{max}} t_h v_n^2}, \]

\[ \frac{\partial w}{\partial \Delta v} > 0 \] if \( f(\mu) \) is an increasing function of \( \mu \). This indicates that the risk increases with an increase in road friction coefficient while approaching a faster leader. In this paper we use
Post Encroachment Time (PET)

PET is used as a risk measure in scenarios involving lateral maneuvers. PET denotes the time lapse between the end of the encroachment of the turning vehicle and the time when the vehicle actually arrives at the potential point of collision (25). The encroachment line $x^e$ in case of a lane changing maneuver is defined as a virtual line perpendicular to the lane dividing marker and crossing the intersection point of the lane dividing marker, and the lane change trajectory. To understand the variation of PET chronologically, we predict the encroachment line and the corresponding PET at every time step, using kinematic prediction with constant velocity assumption. In a situation where two vehicles pass the encroachment line one after the other, the PET definition as per the above assumption is as follows:

$$PET = \frac{x^e - x_j}{v_j} - \frac{x^e - x_i}{v_i}$$  \hspace{1cm} (8)

where, $x_j$ and $v_j$ are the position and velocity of the first vehicle respectively; $x_i$ and $v_i$ are the position and velocity of the second vehicle respectively. $x^e$ is the longitudinal position of the encroachment line. Since this formulation does not directly involve $\Delta v$, we do not further analyze the mathematical properties.

Driving Safety Field

Field theory has been used to model traffic flow (26). In this theory, moving road objects such as vehicles and non-moving road objects such as lane markings are represented as component fields and their union represents the total driving risk. Based on field theory, Wang et. al (8) proposed a Driving Safety Field (DSF). DSF of a road object is a physical field that denotes its influence on driving safety. This influence is determined by the driver behaviour characteristics, road condition, attributes and kinematic state of the road object. The magnitude and direction of this influence are denoted by the field strength vector. A vehicle in the aforementioned field experience a safety field force (SF) which denote its current driving risk. The proposed field strength and field force for two moving vehicles are given as follows:

$$E_{cj} = kR_iM_i\left(1 + DR_i\right)e^{\theta_0}e^{k_1\Delta v_i}$$  \hspace{1cm} (9)

$$F_{cj} = E_{cj}R_iM_i\left(1 + DR_i\right)e^{k_2\theta_0}$$  \hspace{1cm} (10)

where, $E_{cj}$ and $F_{cj}$ denote the safety field strength vector and the safety field force vector, respectively on vehicle $j$ due to a moving vehicle $c$; $r_{cj}$ denotes the radial distance vector from vehicle $c$ to vehicle $j$. $\theta$ (clockwise positive) is the angle between directions $v_c$ and $r_{cj}$; $\emptyset$ is the angle between directions $r_j$ and $r_{cj}$. $k$, $k_1$ and $k_3$ are the calibration coefficients. The driver risk factor $DR_i$ is a dimensionless value between 0 (safe driver) and 1 (risk taking driver). $M_i$ denotes a virtual mass related to a moving or non-moving object, parameterized by its mass, vehicle type and velocity. $R_i$ denotes a road condition influencing factor that is parameterized by factors including road-tyre friction coefficient, curvature, slope and visibility. In this study, we have used the values of parameters as suggested in (8). FIGURE 1 demonstrates the spatial distribution of the safety field strength caused by vehicle $c$. A larger $F_{cj}$ (blue colour) means a higher driving risk for vehicle $j$. 

Mullakkal Babu, Wang, Farah, van Arem, Happee
\[ \frac{\partial F_{c_j}}{\partial \Delta v} = k_1 F_{c_j} > 0, \] indicates that the risk increases with an increase in approaching rate.

\[ \frac{\partial F_{c_j}}{\partial r_{c_j}} = -k_3 F_{c_j} |r_{c_j}|^{k_3-1}, \] indicates that the risk decreases with an increase in spacing.

The original paper (8) does provide a detailed formulation of \( R_i, DR_i \) and \( M_i \). If \( R_i \) is defined as an increasing function of \( f(\mu) \), \( F_{c_j} \) decreases with road friction coefficient. If \( DR_i \) is defined as an increasing function of \( t_h \), \( F_{c_j} \) increases with human reaction time. If \( M_i \) is defined as an increasing function of \( v_n \), \( F_{c_j} \) increases with vehicular velocity. This holds for vehicular mass as well. As shown in TABLE 1, the indications are in agreement with the expected tendencies.

**FIGURE 1** Demonstration of safety field strength due to a moving vehicle. Blue colour indicates higher risk; unit of field strength is Newton.

**Findings of the Qualitative Analysis**

The theoretical verification of the five safety indicators described above reveals the following:

- the selected indicators have limited consideration of risk factors and the SF formulation incorporates the largest number of factors.
- the selected indicators formulations represent the expected risk tendencies. However, a contradiction was found in the case of the \( w \) (See Table 1).
- the selected SSM do not account for vehicle mass in their formulation.

Examination of the mathematical properties of the selected indicators reveals the following:

- none of the selected safety indicators can claim countable additivity property as they are defined for vehicle pairs. Even though the SF on a vehicle is additive, in its present vector
formulation the risk due to the presence of multiple vehicles cannot be added. For example, forces acting in opposite direction tend to cancel out, but the risk measure due to two vehicles cannot cancel out.

- PICUD and PET can have a negative risk value which is undesirable in a multi-vehicle scenario (See Table 1).
- quantitatively, iTTC, w and SF may go to infinity at limiting conditions (See Table 1). Even though theoretically plausible, this property violates the principle of countable additivity and necessitates an upper bound definition. For instance, a risk measure tending to $\infty$ is computationally undesirable for adaptive cruise control systems (23).

TABLE 1 Theoretical Verification of Safety Indicators

<table>
<thead>
<tr>
<th>Aspects</th>
<th>Factors</th>
<th>Expected tendency</th>
<th>iTTC</th>
<th>PICUD</th>
<th>Warning Index</th>
<th>Safety Force</th>
</tr>
</thead>
<tbody>
<tr>
<td>Proximity to point of collision</td>
<td>relative velocity</td>
<td>increase</td>
<td>increase</td>
<td>increase</td>
<td>increase</td>
<td>increase</td>
</tr>
<tr>
<td></td>
<td>spacing</td>
<td>decrease</td>
<td>decrease</td>
<td>decrease</td>
<td>decrease</td>
<td>decrease</td>
</tr>
<tr>
<td>Collision impact</td>
<td>vehicle velocity</td>
<td>increase</td>
<td>increase</td>
<td>increase</td>
<td>subject to the condition</td>
<td>increase</td>
</tr>
<tr>
<td></td>
<td>vehicle mass</td>
<td>increase</td>
<td>NA</td>
<td>NA</td>
<td>NA</td>
<td>increase</td>
</tr>
<tr>
<td>Roadway characteristics</td>
<td>surface friction</td>
<td>decrease</td>
<td>NA</td>
<td>NA</td>
<td>decrease subject to condition</td>
<td>decrease</td>
</tr>
<tr>
<td>Human factors</td>
<td>reaction time</td>
<td>increase</td>
<td>NA</td>
<td>increase</td>
<td>Increase</td>
<td>increase</td>
</tr>
<tr>
<td>Range</td>
<td>NA</td>
<td>(0, $\infty$)</td>
<td>(−$\infty$, $+\infty$)</td>
<td>(0, $\infty$)</td>
<td>(0, $\infty$)</td>
<td></td>
</tr>
</tbody>
</table>

*NA implies not applicable

FIGURE 2 depicts the forward spacing vs relative velocity plot representing the vehicle operational space as suggested by (27). A vehicle trajectory can be visualized on this plot as a continuous line with a plausible direction of motion. The principle concerning the plausibility of the direction of motion is demonstrated using arrows in FIGURE 2a. The risk measures, for a vehicle moving at 10m/s, described by different safety indicators are depicted as color map on this plot. In this paper, the parameters values for $w$ are $a_{max} = 3.3 \text{ m/s}^2$, $t_s = 0.5 \text{s}$, $t_h = 1 \text{s}$, $f(\mu) = 1$; and for PICUD are $a_{max} = 3.3\text{m/s}^2$ and $t_h = 1\text{s}$. We use this plot to visually examine the indicators for their validity and the risk variation along a trajectory. As shown in FIGURE 2a, iTTC is not defined for the lower quadrant, which depicts a faster leader. PICUD and $w$ have smoother transition from “safe-green” to “unsafe-blue” than iTTC and SF. Moreover, the iTTC risk indication abruptly disappears in a transition from upper to lower quadrant of the plot.
FIGURE 2 Visualization of risk measures over the operational space using different safety indicators. Blue colour indicates higher risk.

SIMULATION EXPERIMENTS
Here we extend our study from theoretical findings to simulation-based comparison of risk values associated with two-dimensional trajectories. In particular, we examine peaks of the risk measures, analyze the ability to represent the risk related to vehicle maneuvers and inspect the continuity of the risk measure over typical trajectories.

Towards this, we defined two typical safety critical scenarios on highways (28): emergency braking (Experiment 1) and cut-in (Experiment 2). The two experiments were done as a numerical simulation (simulation time of 20 s and a time step of 0.2 s) of a vehicle pair: a leader with a predesigned trajectory to facilitate the scenario simulation and a follower. Longitudinal follower trajectories were simulated using the Intelligent Driver Model with default parameters as in the original paper (29). Another conservative simulation assumption used is that a vehicle would be identified as the leader only if it is ahead on the same lane. This implies that a vehicle cutting-in will be detected only after it crosses the lane boundary. The safety indicators considered are iTTC (threshold 0.5 s⁻¹) (23), PET (threshold 0.45 s), PICUD (threshold 0 m) and SF. The above thresholds describe the safe ranges (12, 23, 26).

Experiment 1
In this experiment, we simulate a leader applying sudden braking (predefined) and three possible evasive maneuvers of the follower vehicle. Here, the leader vehicle travelling at 5 m/s and a
spacing of 10.5 m ahead of the follower suddenly brakes (-2.5 m/s²) at 5 s and reaches a complete halt at 7 s.

FIGURE 3 Results of experiment 1: (a) when leader brakes and follower brakes to avoid a collision; (b) when leader brakes and follower changes lane via trajectory A (timely lane change); (c) when leader brakes and follower changes lane via trajectory B (late lane change).
FIGURE 3a shows the risk profiles calculated using various safety indicators when the follower brakes to avoid a collision as dictated by the car following model IDM. It can be seen that iTTC is defined only in the time interval when the leader is slower than the follower. All three indicators depict an increasing risk measure as the leader brakes. PICUD and iTTC show the highest risk when the leader reaches a complete halt and thereafter the risk decreases, whereas, SF indicates an increase in risk starting with the braking of the leader and reaches the maximum when the subject vehicle stops.

Trajectory planning systems often compare the risk levels of alternate trajectories to select the safer path (5). To verify if the selected indicators are capable of trajectory comparison, we simulate two evasive lane change trajectories A and B as possible alternative responses to the braking leader on a two-lane highway (one-way). FIGURE 3b shows the risk profiles calculated using various safety indicators when the follower adopts trajectory A. The follower trajectory A begins with deceleration at 5.2 s in response to the lead vehicle braking, followed by a left lane change beginning at 5.4 s and ending at 9.6 s when the vehicle reaches the left lane centre. FIGURE 3c shows the risk profiles calculated using various safety indicators when the follower adopts trajectory B (late lane change). The follower trajectory B begins with deceleration at 5.2 s in response to the lead vehicle braking, followed by a left lane change beginning at 6.4 s and ending center of the left lane at 10.6 s. The lane change in two trajectories follows an “S” shaped path defined by a fifth degree polynomial parameterized by lane change duration of 4.3 s which is the typical value indicated in (30) and the lateral displacement 3.75 meter which is the typical lane width of a highway. The follower begins to accelerate once the lane boundary is crossed and finally passing the leader on the adjacent lane at 9 s (Trajectory A) and 10.2s (Trajectory B). In both cases, PICUD and iTTC show an increasing risk while approaching; however they were discontinued after the lane change. PET shows a decreasing risk starting from the beginning of the lane change via Trajectory A. SF shows continuous risk variation throughout the evasive maneuver and indicates the highest risk corresponding to a passing maneuver (See FIGURE 3b and 3c). The SF indicates a lower risk peak for evasive lane change trajectory A compared to evasive braking (See FIGURE 3a). The total risk measure using SF (area under the plot) associated with trajectory B (late lane change) is higher compared to trajectory A (timely lane change). Note that the other indicators cannot be used for comparison as they are discontinuous over the simulated trajectory.

Experiment 2
In this experiment, we simulate a three-lane highway (one-way) with two vehicles (on right and middle lanes) moving with a forward spacing of 10.5m. The vehicle travelling ahead on the right lane starts to cut-in towards the middle lane at 2s and reaches its center at 6.2s. We simulate the two possible evasive maneuvers of the vehicle initially travelling behind on the middle lane. Risk profiles (using selected indicators) when the follower brakes are shown in FIGURE 4a. PICUD and iTTC indicate the highest risk when cut-in is detected and decreases thereafter. PET indicates the highest risk earlier at the beginning of the cut-in and thereafter decreases. SF indicates risk from the beginning of cut-in; however the highest risk is indicated at a later point when the cut-in vehicle reaches the center of the middle lane and thereafter decreases.

Risk profiles (in terms of selected indicators) when the follower performs evasive left lane change is shown in FIGURE 4b. The follower begins to change lane at 6.4 s and reaches the left lane center at 10.6 s. PICUD and iTTC indicate the highest risk as the leader cut-in is detected. However, they are not defined during the evasive lane change as there is no leader in the left lane.
PET indicates increasing (yet below threshold) risk with leader cut-in. SF indicates the highest risk for passing maneuver (at 13.6 s) and reducing risk thereafter.

![Graph showing safety measures](image)

**FIGURE 4** Results of experiment 2 (a) the leader cuts-in and follower brakes to avoid collision (b) the follower changes the lane to avoid collision.

**Findings of the Simulation Analysis**

- The point in time corresponding to the highest risk for a maneuver differs with the risk measures. For example, PICUD and TTC indicate the highest risk corresponding to the point of cut-in whereas the SF indicates the point of passing to be the riskiest (See FIGURE 4).
- SSM are defined for a collision course and hence have limited ability to capture the precautionary risk measure; on the contrary SF is able to indicate risk in the absence of a collision course, for example, a passing maneuver (See FIGURE 3 and 4).
- SSM are defined for the prescribed set of vehicle configurations and therefore indicate a sudden drop or rise in risk profile during a change in this vehicle configuration. For example, the risk measures by TTC and PICUD increase suddenly when a vehicle is detected ahead.
(See FIGURE 4b at 4.6 s). Additionally, from a control perspective, this is a false negative risk indication, i.e. the indicator wrongly suggests zero risk for a vehicle cutting-in ahead before being identified as a leader. We note that SF is free of this drawback as it is independent of vehicle configuration assumptions.

- Even though PET and SF describe the risk profile of lateral maneuvers, the risk measures have limitations. The PET fluctuates throughout the maneuver (See FIGURE 4b) and do not discriminate near miss events where vehicle passes at a low lateral distance. On the contrary, in the present formulation of SF, inter-vehicle spacing hold a high weightage and therefore it consistently indicates the highest risk for lateral vehicle passing even if both vehicles follow their lane center.

**DISCUSSION**

As suggested in previous studies, the usefulness/validity of a safety indicator does not (only) depend on the extent to which expected accident numbers can be correctly estimated, but also on whether safety problems can be detected or not, and/or road safety counter-measures/treatments can be compared or evaluated (31). In this study, we did not explore the empirical validity. We reviewed the indicators on the basis of their ability to theoretically represent the expected risk tendencies and to evaluate safety problems along simulated trajectories in critical highway situations.

From the perspective of vehicle control systems accounting for safety utility, it is of interest to have smooth and objective risk measures (33). Simulation analysis showed that all the selected indicators are capable of delineating risk continuously in a one-dimensional interaction like car following. However, SSM like iTTC, PET, w and PICUD often display fluctuating or/and discontinuous values. For example, iTTC (and TTC) have an abrupt change when crossing the line of $\Delta v = 0$ (See FIGURE 2). The measured relative speed may oscillate from positive to negative due to sensing errors, and this will result in fluctuating risk measures and in turn the control signals based on them. Moreover, as shown in the simulation analysis, discontinuous risk measures cannot be used in trajectory planners to compare alternate trajectories. Secondly, these indicators do not possess mathematical properties that are desirable in a multivehicle scenario. Thirdly, benchmarking the safety level based on an indicator threshold value is difficult due to limited number of parameters considered by these indicators (16, 26). This is because the threshold may vary with road characteristics, interacting vehicle type and driver reaction time. For example, a TTC that is considered safe on a high friction road could be deemed unsafe on a low friction or icy road. Moreover, most of the SSM do not account for the conflict severity. Hence, the decision-making modules of intelligent vehicles using these indicators cannot identify the trajectory of lesser crash severity in an unavoidable collision situation. Finally, as shown in simulation analysis, indicators defined for a prescribed set of vehicle interaction configurations often lead to false negative risk measures.

Our findings also have implications in regard to the use of safety indicators for traffic safety assessment. The one-dimensional safety indicators yield partial insights in safety as they are only valid for a predefined set of vehicle interactions and do not account for collision severity. Additionally, these indicators cannot be used to estimate collective risk as they do not possess the property of countable additivity. As reported in our simulation results, SSM differs based on the threshold definition and underlying kinematic assumptions. This makes it difficult to reach an objective consensus on the safety impact. Finally the limitations of the lateral indicators like fluctuation (PET) and sensitivity to the spacing (SF) questions the objectivity of safety assessments for multilane highways. SF can potentially be used to model precautionary measures taken by human drivers as it can describe risk despite a collision course.
All the studied safety indicators are based on underlying deterministic assumptions on the future kinematic state of the interacting vehicles, rather than acknowledging the uncertainty in vehicle movement. For example, consider a scenario where a vehicle follows a leader at a spacing of 1 m. This scenario would be deemed safe by a TTC indicator that is based on constant velocity assumption. However, this scenario cannot be regarded as safe if we consider the probability that the leader may brake. Even though the safety field approach does not depend on kinematic assumptions, it also does not explicitly account for uncertainties. Moreover automated vehicles further contribute to the necessity of accounting vehicle uncertainty in the risk measures. Automated vehicles may attain more precise control as compared to manual driving, and may thereby safely pass at low distances; however the precision of automated vehicles will be affected by its perception quality. Therefore we note this inconsideration for uncertainties related to vehicle state is a drawback of these indicators.

Our study demonstrates the advantages of the safety field framework in depicting risk of two-dimensional vehicle interactions. Recently, Wang et al demonstrated the use of an SF-based indicator for collision warning applicable in multivehicle scenarios (33). Moreover, if augmented with prediction paradigms, SF can be used for ex-ante safety evaluation in path planners. However, the formulation has to be fine-tuned or/and refined for practical applications. Unlike SSM, SF does not represent the collision causal mechanism, and therefore interpretation of the safety field risk measure could become ambiguous.

CONCLUSION AND FUTURE WORK
In this study we compared safety indicators based on their qualitative and quantitative aspects as a risk measure. Our results showed that all the selected indicators are capable of delineating risk continuously in a one-dimensional interaction like car following. Moreover, the selected safety indicators in general match the expected risk tendencies. However, in agreement with previous research (6, 10–13) our findings acknowledge the mathematical limitations of selected safety indicators like discontinuity over the operational space, omission of uncertainty in vehicle state assumption and the inability to account for crash severity. We also note that all these indicators lack mathematical properties to account for multiple vehicles and the safety field framework is a promising approach that allows risk estimation in two-dimensional vehicle interactions. Our analysis could be further improved by verifying the findings using empirical accident data. Future research should also focus on defining a safety indicator addressing the limitations of existing indicators found in this study.

ACKNOWLEDGEMENT
This work is supported by STW Technology Foundation, the Netherlands, under the project “From Individual Automated Vehicles to Cooperative Traffic Management - Predicting the benefits of automated driving through on-road human behavior assessment and traffic flow models (IAVTRM)”-STW#13712.
REFERENCES