Coulomb drag in intermediate magnetic fields

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We theoretically investigated the Coulomb drag effect in coupled two-dimensional electron gases in a wide interval of magnetic field and temperature $1/\tau \ll \omega_c \ll E_F/h$. $T \ll E_F$, $\tau$ being the intralayer scattering time and $\omega_c$ the cyclotron frequency. We show that quantization of the electron spectrum leads to rich parametric dependences of the drag transresistance on the temperature and magnetic field. This is in contrast to usual resistance. Small energy scales are found to cut typical excitation energies to values lower than temperature. This may lead to a linear temperature dependence of the transresistance even in a relatively weak magnetic field, and can explain some recent experimental data. We present a mechanism of Coulomb drag when the current in the active layer causes a magnetoplasmon wind and the magnetoplasmons are absorbed by the electrons of the passive layer providing a momentum transfer. We derived general relations that describe the drag as a result of resonant tunneling of magnetoplasmons. [S0163-1829(99)07211-2]

I. INTRODUCTION

When two two-dimensional electron systems are placed in close proximity, then even in the absence of electron tunneling between layers the current in one layer (the active layer) will cause the current in the other (the passive layer). This phenomenon is known as a frictional drag, and is due to the interlayer Coulomb interaction which causes a momentum transfer from one layer to the other. If no current is allowed in the passive layer, a potential difference develops there to compensate for the frictional interlayer force. The transresistance is measured as the ratio between the electric field developing between layers the current in one layer $T\sim \tau\sim 0$. The density of states in the electronic subsystem is strongly distorted in comparison with its zero-field value. That is why even in the "classical" regime $E_F \gg T \gg h \omega_c$ and $R_c \gg d(R_c = v_F/\omega_c$ being the cyclotron radius, and $d$ the distance between the layers) the polarization function $\chi(\omega = T, q = 1/d)$, which is responsible for the absorption of energy, differs strongly from its zero-field form. As a function of frequency, it consists of a series of well-resolved peaks at multiples of cyclotron frequency. We will see that this circumstance leads to rich parametric dependences of transresistance on temperature and magnetic field. This is to be contrasted with the usual intralayer resistance that exhibits no anomalies except strongly suppressed Shubnikov–de Haas oscillations, and does not manifest the electron density of states.

In the intermediate magnetic field at temperatures $T \gg h \omega_c$, weakly damped boson excitations become important. Those are magnetoplasmons with energies close to multiples of the cyclotron energy. These excitations provide a mechanism of Coulomb drag in the system. The momentum transfer is provided by magnetoplasmons excited in one layer and absorbed into the other. We have found general relations that allow us to present the magnetoplasmon contribution to the drag resistance as the result of the resonant tunneling of magnetoplasmons.

We have assumed a Coulomb mechanism of the drag. It has also been shown that the phonons may play an important role in mediating the drag. They renormalize the electron-electron interaction so that the latter acquires a characteristic $\omega_c q$ dependence. This dependence explains some experimentally observed features of the drag. We have not explic-
FIG. 1. Seven regions of different analytical behavior of the transresistance in the intermediate magnetic-field regime. Note the log-log scale. The first number in each region corresponds to the temperature exponent, and the second number indicates the magnetic-field exponent. The vertical line corresponds to the condition \((\hbar \omega_c)^2 = E_0 \Delta\).

magnetic field (Shubnikov–de Haas oscillations). The oscillations can be hardly investigated analytically except those in simplest cases. In the present paper we present only analytical results for regions VI and IV, and give estimations of the transresistance in regions V and VII for typical filling factors.

The outline of the paper is as follows. In Sec. II we list the theoretical assumptions we made, and present a method which is essentially the same as in Refs. 2, 3, 6 and 7. Details of the polarization function and the magnetoplasmon spectrum are presented in Sec. III. In Sec. IV we give a detailed description of the magnetoplasmon mechanism of the drag. A phenomenological description of the resonant tunneling of magnetoplasmons is elaborated upon in Sec. V. We list our analytical results for high temperatures in Sec. VI. Section VII is devoted to an evaluation of the transresistance at low temperatures.

II. METHOD

In the present paper, we cover the parameter region \(\Delta \approx \hbar \omega_c \approx E_0 = \hbar v_F / d\). Here \(\Delta\) stands for the width of the Landau level. It determines the maximum one-particle density of states in a magnetic field. The ratio between \(\hbar \omega_c\) and \(T\) can be arbitrary.

We assume here that Landau levels acquire width due to scattering by impurities and, following Ref. 9, treat the effect in the self-consistent Born approximation (SCBA). This approximation is known to lift the difficulties related to the high Landau-level degeneracy. In this approach, \(\Delta^2 = (2\pi \hbar \omega_c / \tau) / T\). This expression for \(\Delta\) is valid for a short-range (\(\delta\)-correlated) random potential.

We also assume the Coulomb mechanism of the drag, so that the dc drag current results from the rectification by the passive layer of the ac fluctuating electric field created by the active one. In diagrammatic language, the transconductance is given by a diagram composed of three-body correlation functions connected by Coulomb interaction lines (photon propagators). In Refs. 3 and 7, it was argued that under very general conditions the three-body correlation functions can be expressed in terms of electron polarization functions \(\chi_{1,2}(\omega, q)\) in each layer.

This yields the following expression for the diagonal element of the transresistivity tensor [Eq. (28) of Ref. 7]:

\[
\rho_{12}^{\omega} = -\frac{\hbar^2}{2e^2} \frac{1}{n_1 n_2 T} \int \frac{d^2 q}{(2\pi)^2} q^2 \int_0^{\infty} d\omega |V_{12}(q)|^2 |\chi_1(\omega, q)|^2 |\chi_2(\omega, q)|^2 \frac{\text{Im} \chi_1(\omega, q) \text{Im} \chi_2(\omega, q)}{\sinh^2(\hbar \omega / 2T)}. \tag{1}
\]

Here \(n_1\) and \(n_2\) are electron concentrations in the layers, \(V_{12}(q)\) is the Fourier component of the interlayer Coulomb interaction, and \(\mathcal{E}(\omega, q)\) describes (dynamical) screening of this interaction. Electrostatics gives \(V_{12} = V(q) \exp(-qd)\) and \(V(q) = 2\pi e^2 / eq\), \(e\) being the bulk dielectric constant, and

\[
\mathcal{E}(\omega, q) = [1 + V(q) \chi_1][1 + V(q) \chi_2] - V_{12}^2(q) \chi_1 \chi_2. \tag{2}
\]

It has been argued that Eq. (1) is valid for an arbitrary magnetic field provided \(q^{-1} \ll 1, R_c\), \(l\) being the mean free path. Our checks confirm that, so we use Eq. (1) in our calculations in the intermediate magnetic-field regime.

As a reference, we here give the expression for transresistance in the absence of a magnetic field for temperature \(T \ll E_0\) (Refs. 2 and 3):
\[ \rho_{12}^{xx} = \frac{\xi(3)}{64} \frac{\hbar}{e^2} \frac{1}{n_{1}n_{2}a_{B}^4} \frac{T^2 v_{F}^2}{h^2 v_{F1} v_{F2}}. \]  

Here \( \xi(3) \approx 1.202 \) is the Riemann \( \zeta \) function, \( v_{F1} \) and \( v_{F2} \) are the corresponding Fermi velocities, and \( a_{B} \) is the Bohr radius.

To make use of Eq. (1), we shall evaluate the polarization function \( \chi(\omega,q;B) \). This we do in Sec. III.

We will assume that the layers are macroscopically identical. We keep indices 1 and 2 that label the layers in the formulas solely for the sake of physical clarity. The exception will be the discussion of Shubnikov–de Haas oscillations in regions IV and VI. We allow there for different filling factors in the layers.

III. POLARIZATION FUNCTION AND MAGNETOPLASMONS

We start with the following expression for the imaginary part of the polarization function (see Appendix A):

\[ \text{Im} \chi(\omega,q) = v_{\nu} \sum_{n,m} J_{n-m}^{2}(qR_{c}) \int_{-\infty}^{+\infty} \frac{d\epsilon}{\pi} \left[ n_{F}(\epsilon) - n_{F}(\epsilon + \omega) \right] \text{Im} G_{n}^{*}(\epsilon) \text{Im} G_{m}(\epsilon + \omega). \]  

Here \( v = m_{0}/\pi \hbar^{2} \) is the two-dimensional thermodynamic density of states in the absence of a magnetic field, \( n_{F} \) is the Fermi distribution function, and \( G_{n}^{*} \) is the retarded Green’s function of the electrons in the \( n \)th Landau level. In the above formula, we have taken into account that under the conditions considered in this paper (\( T, \hbar, \omega_{c} \ll E_{F} \)), only large Landau-level numbers are important. Thus the bare vertex function is reduced to its quasiclassical form, which is the Bessel function of argument \( qR_{c} \). In the limit \( \Delta \rightarrow 0 \) (no disorder) this expression is equivalent to the semiclassical approximations employed in Refs. 10 and 11.

Expression (4) disregards vertex corrections due to disorder. This is safe since we always assume that \( v_{F} q \gg 1/\tau \) and \( qR_{c} \gg 1 \). We will also disregard the rapidly oscillating part of the Bessel function squares at \( qR_{c} \gg 1 \), i.e., we assume that \( J_{n}^{2}(qR_{c}) \approx 1/\pi qR_{c} \). We discuss the relevance of this assumption in Appendix B.

Using the SCBA expression for \( \text{Im} G \) (Ref. 9) in the limit of large \( n \),

\[ \text{Im} G_{n}^{*}(\epsilon) = -\frac{2}{\Delta} \sqrt{1 - \frac{\epsilon - \epsilon_{n}^{*}}{\Delta}} \Theta \left[ 1 - \left( \frac{\epsilon - \epsilon_{n}^{*}}{\Delta} \right)^{2} \right], \]  

where \( \Theta \) is the step function and \( \epsilon_{n} = (n + 1/2)\hbar \omega_{c} \), we obtain from Eq. (4) that

\[ \text{Im} \chi(\omega,q) = v_{\nu} \frac{4 \omega_{c}}{\pi^{2}} \frac{\omega_{c}}{q v_{F}} \]  

\[ \times \sum_{n,m} \left[ n_{F}(\epsilon_{n}) - n_{F}(\epsilon_{n} + \omega) \right] \]  

\[ \times X_{im} \left( \frac{\epsilon_{n} - \epsilon_{m} + \hbar \omega}{2\Delta} \right). \]  

Here we define the dimensionless function \( X_{im}(x) \equiv \frac{1}{2 \Delta} \left[ (1 + x^{2})E(\sqrt{1 - x^{2}}) - 2 x^{2}F(\sqrt{1 - x^{2}}) \right] \) for \( |x| \leq 1 \), and \( X_{im}(x) = 0 \) otherwise. Functions \( F(x) \) and \( E(x) \) are complete elliptic integrals of the first and second kind, respectively. Note that \( \int_{-1}^{1} dx X_{im}(x) = \pi^{2}/8 \). The expression for \( \text{Im} \chi(\omega,q) \) assumes different forms depending on the temperature. In the most interesting case \( T \gg \hbar \omega_{c} \) and \( q v_{F} \gg \omega \) (the latter inequality is equivalent to \( T \ll E_{F} \), because the characteristic \( q \approx 1/d \) and the frequency cannot be larger than the temperature) from Eq. (6) we obtain

\[ \text{Im} \chi(\omega,q) = v_{\nu} \frac{4 \omega_{c}}{\pi^{2}} \frac{\omega_{c}}{q v_{F}} \sum_{n} X_{im} \left( \frac{\omega - j \omega_{c}}{2\Delta} \right). \]  

Since \( \chi(\omega) \) is an analytical function of \( \omega \), we can easily obtain the real part from Eq. (7). In the vicinity of the \( j \)th cyclotron resonance, \( |\omega - j \omega_{c}| \ll \omega_{c} \), this reads

\[ \text{Re} \chi(\omega,q) = v_{\nu} \frac{4 \omega_{c}}{\pi^{2}} \frac{\omega_{c}}{q v_{F}} \sum_{n} X_{re} \left( \frac{\omega - j \omega_{c}}{2\Delta} \right) \]  

\[ X_{re}(x) = \frac{1}{\pi} \text{v.p.} \int_{-1}^{+1} \frac{dy X_{im}(y)}{(y-x)}. \]

The functions \( X_{re}(x) \) and \( X_{im}(x) \) are plotted in Fig. 2.

In the opposite limit of \( \Delta \ll T \ll \hbar \omega_{c} \) and \( \omega \ll 2\Delta \) and \( qR_{c} \gg 1 \) we obtain, from Eq. (6),

\[ \text{Im} \chi(\omega,q) = v_{\nu} \frac{4 \omega_{c}}{\pi^{2}} \frac{\omega_{c}}{q v_{F}} X_{im} \left( \frac{\omega}{2\Delta} \right) \frac{1}{qR_{c}} f_{n}(1 - f_{n}), \]  

where \( f_{n} = \frac{1}{2} \left[ 1 + \exp(\epsilon_{n} - \mu)/T \right] \) is the filling factor of the \( n \)th Landau level in the layer, and \( \mu \) is the chemical potential.

Finally, at \( T \ll \Delta \) we can set \( n = m \) and integrate expression (4) over \( \epsilon \) in the close vicinity of \( \mu \). This gives

\[ \text{Im} \chi = \frac{4 v_{\nu}^{2} \omega_{c}}{\pi^{2} \Delta} \frac{\omega}{q v_{F}} \left( 1 - \frac{(\mu - \epsilon_{n})^{2}}{\Delta^{2}} \right). \]  

Although the magnetoplasmon modes of two-dimensional electron gas have been extensively studied, very little attention has been paid to their properties at high frequencies in the short-wave limit. Since those are of interest for us, we investigated them in some detail. The dispersion curves of the magnetoplasmon modes in the case of weak damping are
where $z$ and their damping, in terms of frequencies that in the interesting region of $q$ are close to cyclotron harmonics $j\omega_c$. Therefore, we concentrate on a close vicinity of $q_0$ (right panel of Fig. 3).

In this vicinity $\chi(\omega,q)$ can be expanded in Taylor series in terms of $w=\omega-j\omega_c\sim 2\Delta$ and $\kappa=q-q_0$, assuming that $w\ll \Delta$ and $\kappa \ll q_0$:

$$\chi/\nu = C_2 \frac{w}{2\Delta} + \frac{\kappa}{q_0} + iC_1 \left(\frac{w}{2\Delta}\right)^2 \Theta(-w).$$

Here $C_{1,2}$ are numerical constants characterizing the behavior of $X_{im}$ and $X_{re}$ near $x=1$, $C_1=6.23$, and $C_2=2.19$. This determines the dispersion law of magnetoplasmons,

$$w_\pm = -\frac{2\Delta}{C_2}(z_\pm + \kappa/q_0),$$

and their damping,

$$\Gamma = \Theta(-w) \frac{C_1 w^2}{C_2 \Delta},$$

where $z$ is taken at $q=q_0$. Symmetric and asymmetric modes are split by

$$\delta\omega = \Delta \frac{q_0 \alpha_B}{C_2 \sinh q_0 \alpha_B}.$$  

We see that $\Gamma \ll w \ll \Delta$ and $\delta\omega \ll \Delta$, and $\delta\omega$ can be comparable to $\Gamma$.

### IV. Magnetoplasmon Contribution

In this section we consider the magnetoplasmon mechanism of the Coulomb drag. In the absence of a magnetic field there are two plasmon modes in the double-layer system, one with the electron densities in the two layers oscillating in phase (the optic mode), and the other where the oscillations are out of phase (the acoustic mode). It was pointed out in Ref. 3 that the drag effect can be greatly enhanced by dynamical “antiscreening” of the interlayer interaction due to coupled plasmon modes. Since the plasmon modes lie beyond the $T=0$ particle-hole continuum, temperatures of the order of the Fermi energy are required for a large plasmon enhancement of the drag effect. Only then do the thermally excited electrons and holes with plasmon velocities provide sufficient damping of the plasmon modes, and thus facilitate plasmon interaction with electrons.

In the case of an intermediate magnetic field, the magnetoplasmons have even better chances to enhance the drag. First there are many modes, and their typical energies are of the order of $\hbar \omega_c$. Therefore, these modes can be excited at temperatures much lower than the Fermi energy. Second, the magnetoplasmons in our model acquire natural damping: due to the finite Landau-level width, they may lie within the particle-hole continuum. The finite temperature without disorder does not lead to magnetoplasmon damping, and to the effect. This is in contrast to the situation without a magnetic field, where $\Im \chi$ at the plasmon frequency was calculated for collisionless plasma.

The magnetoplasmon mechanism of the Coulomb drag, when the current in the active layer causes a magnetoplasmon wind and the magnetoplasmons are absorbed by the electrons of the passive layer leading to transfer of the momentum, must be quite general. In this section, we evaluate the magnetoplasmon contribution using Eq. (1). It turns out that the answer can be expressed through only two quantities for each double plasmon mode: frequency splitting $\delta\omega$ and damping $\Gamma$. Any concrete model would only set specific expressions for $\Gamma$ and $\delta\omega$. This clarifies the physical meaning of Eq. (1).

To prove this, we rederive the result in Sec. V in a phenomenological framework.

Let us expand $\chi(\omega,q)$ around the frequency $\omega(q)$ when $\Re \chi=0$,

$$\chi(w,q) = \chi' w + i\chi'',$$

$w$ being the frequency deviation. Here $\chi' = \Re d\chi/d\omega(\omega(q))$ and $\chi'' = \Im \chi(\omega(q))$. The expression for $\mathcal{E}$ is reduced to the form

$$\mathcal{E} = [V^2(q) - V_{12}^2(q)](w\chi' + i\chi'' + \nu z_+)(w\chi' + i\chi'' + \nu z_-).$$

Consequently, the integrand in Eq. (1) has a sharp maximum near $\omega(q)$ as a function of $\omega$. Hence we can now integrate over $w$ in infinite limits.

Equation (16) determines the magnetoplasmon spectrum, and suggests that mode splitting $\delta\omega = \nu(z_- - z_+) / \chi'$, damp-
ing $\Gamma = 2 \chi''/\chi'$. All factors $V_{12}$ and $V$ can be absorbed into these two quantities. Typical values of $\omega$ that contribute to the integral are of the order of $\max(\Gamma, \delta \omega)$. The approximation is valid if $\chi$ and statistical factor $\sinh^2(\omega 2T)$ do not change much in this frequency window, i.e., $\omega(q) \gg \delta \omega$, $d\Gamma/d\omega \ll 1$, and $\max(\Gamma, \delta \omega) \ll T/\hbar$.

Provided these conditions are fulfilled, we can reduce the expression for the transresistance to the elegant form

$$\rho_{12} = -\frac{\hbar}{16e^2n_1n_2} \int \frac{d^2 q q^2}{(2\pi)^2} \sum_{\text{modes}} \frac{1}{\sinh^2(\hbar \omega(q)/2T)} \frac{(\delta \omega)^2}{(\delta \omega)^2 + \Gamma^2}.$$  \hfill (17)

Summation over modes means summation over all possible roots of $\Re \chi = 0$.

V. RESONANT TUNNELING OF MAGNETOPLASMONS

Here we give another derivation of this formula which clarifies its physical meaning. To describe resonant tunneling of plasmons between the layers, we introduce for each plasmon mode a density matrix $\rho_{ij} = (b_i^\dagger b_j)$. Here $i, j = 1, 2$ label the layers, and $b_i^\dagger$ and $b_j$ are boson creation and annihilation operators. In the absence of dissipation, i.e., plasmon emission and absorption, the density matrix obeys the equation

$$\frac{\partial \rho_{ij}}{\partial t} = i \sum_l (H_{il}\rho_{lj} - \rho_{il}H_{lj}).$$  \hfill (18)

For identical layers, the diagonal elements of the Hamiltonian are equal to each other and dissappear from the equation. The nondiagonal element that is responsible for the plasmon tunneling between the layers can be readily expressed in terms of splitting $\delta \omega$ between the symmetric and asymmetric plasmon states: $H_{12} = H_{21} = \delta \omega/2$.

The dissipation takes place independently in each layer. It contributes to the time derivative of the diagonal density matrix elements in the following way:

$$\left( \frac{\partial \rho_{ii}}{\partial t} \right)_{\text{diss}} = \Gamma_1(n^B_i - \rho_{ii}),$$  \hfill (19)

the two terms corresponding to generation and absorption of the plasmons, respectively. The temperature of the Bose distribution function $n^B_i$ corresponds to the electron temperature of the $i$th layer. The nondiagonal matrix elements acquire a damping equally from both layers

$$\left( \frac{\partial \rho_{ij}}{\partial t} \right)_{\text{diss}} = -\frac{(\Gamma_1 + \Gamma_2)}{2} \rho_{ij}.$$  \hfill (20)

The system of equations that incorporates both dissipation and resonant tunneling reads as follows:

$$\frac{\partial \rho_{11}}{\partial t} = \Gamma_1(n^B_1 - \rho_{11}) + i \frac{\delta \omega}{2} (\rho_{12} - \rho_{21}),$$

$$\frac{\partial \rho_{22}}{\partial t} = \Gamma_2(n^B_2 - \rho_{22}) + i \frac{\delta \omega}{2} (\rho_{21} - \rho_{12}),$$

$$\frac{\partial \rho_{12}}{\partial t} = -\frac{(\Gamma_1 + \Gamma_2)}{2} \rho_{12} + i \frac{\delta \omega}{2} (\rho_{11} - \rho_{22}),$$

where $\rho_{21} = \rho_{12}^\dagger$. The stationary solution takes the form

$$\rho_{11} = n^B_1 - \frac{\Gamma_2}{\Gamma_1 + \Gamma_2} \frac{\delta \omega^2(n^B_1 - n^B_2)}{\delta \omega^2 + \Gamma_1 \Gamma_2}.$$  \hfill (21)

The expression for $\rho_{22}$ is obtained by reverting indices 1 and 2.

We can now evaluate the drag force acting on electrons of each layer by equating it to the momentum flow between the layers. We sum over modes with all possible $q$, and obtain

$$F = -\sum_q \hbar q \left( \frac{\partial \rho_{ij}}{\partial t} \right)_{\text{diss}} = -\sum_q \hbar q \frac{\Gamma_1 \Gamma_2}{\Gamma_1 + \Gamma_2} \frac{\delta \omega^2(n^B_1 - n^B_2)}{\delta \omega^2 + \Gamma_1 \Gamma_2}.$$  \hfill (22)

where $\Gamma$, $n^B_i$, and $\delta \omega$ may be $q$ dependent.

We assume that the current in layers 1 and 2 and the drag force in layer 1 are equilibrated by the electric field. The transresistivity is essentially the ratio of this field to that current. The effect of the current is that the $n^B_2$ is the equilibrium Bose distribution in the reference frame where the electrons of the second layer are in average at rest, rather than in the laboratory reference frame, so that

$$n^B_2[\epsilon(q)] = f_B[\epsilon(q) - \hbar \mathbf{v}_{\text{drift}}(q)]$$

$$= f_B[\epsilon(q)] - \hbar \mathbf{v}_{\text{drift}}(q) \frac{\partial f_B}{\partial \epsilon},$$  \hfill (23)

$\mathbf{v}_{\text{drift}}$ being the drift velocity.

Substituting Eq. (23) into Eq. (22), we obtain

$$F = \hbar^2 \mathbf{v}_{\text{drift}} \int \frac{d^2 q q^2}{8 \pi^2} \frac{\Gamma_1 \Gamma_2}{\Gamma_1 + \Gamma_2} \frac{\delta \omega^2}{\delta \omega^2 + \Gamma_1 \Gamma_2} \frac{\partial f_B[\epsilon(q)]}{\partial \epsilon}.$$  \hfill (24)

The last things to note are that $F = e n_1 E$ and $I = e n_2 \mathbf{v}_{\text{drift}}$. If we use this and set $\Gamma_1 = \Gamma_2 = \Gamma$, we reproduce Eq. (17).
VI. RESULTS: HIGH TEMPERATURES

In this section we consider the drag resistance at temperatures $T \gg \hbar \omega_c$, which are sufficiently high to excite magnetoplasmons and electrons in many Landau levels. As we can see in Fig. 1, at high temperature we encounter at least three distinct regions with different temperature and magnetic-field exponents.

First of all, we shall explain why there are so many regions. If we compare the imaginary part of the polarization function with and without a magnetic field, we see that their values averaged over frequency intervals larger than $\hbar \omega_c$ are the same $\langle \text{Im} \chi \rangle = \langle \text{Im} \chi(H=0) \rangle$. However, $\text{Im} \chi = 0$ beyond narrow adsorption bands (Fig. 3). This means that in the bands $\text{Im} \chi$ is significantly enhanced in comparison with its zero-field value, typically by a factor $\omega_c/\Delta$. Surprisingly enough, the enhancement of $\text{Im} \chi$ can lead both to enhancement and suppression of the drag.

If $\text{Im} \chi < \nu$, $\text{Re} \chi \approx \nu$. The denominator $\mathcal{E}$ in Eq. (1) which is responsible for screening of the interlayer potential is the same as without a magnetic field. We refer to this situation as the normal screening regime. In this regime, the transresistance is enhanced in comparison to its value without a magnetic field, since the effect is proportional to $\langle (\text{Im} \chi)^2 \rangle / \langle (\text{Im} \chi) \rangle^2$.

Upon further increase of $\text{Im} \chi$, $\text{Re} \chi$ develops as well, so that both $|\text{Re} \chi|$ and $\text{Im} \chi$ become larger than $\nu$. The denominator $\mathcal{E}$ strongly increases. This efficiently screens out the interlayer interaction and leads to a drastic decrease of the transresistance. This we will call overscreening.

However, $\mathcal{E}$ can also decrease with increasing $\text{Im} \chi$ and pass zero. Near this line, the interlayer interaction is greatly increased. This is where the magnetoplasmon contribution dominates. The actual value of the drag effect is thus determined by interplay of these three competing tendencies.

Let us first evaluate the magnetoplasmon contribution. Substituting expressions (13) and (14) into Eq. (17), we notice that the integrand has a sharp extremum near $\kappa = 0$ so we can formally integrate over $\kappa$ in infinite limits. This yields the relation which is valid for all regions,

$$
\rho_{12}^{xx} = -0.00221 \frac{\hbar \Delta}{e^2 n_1 n_2 d^4 T} \left( \frac{a_B}{d} \right)^{3/2} \sum_{j=1}^{\infty} \frac{(\hbar \omega_j/T)^4}{\sinh^2(\hbar \omega_j/2T)} \left( \frac{\alpha \hbar \omega_j/T}{\sinh(\alpha \hbar \omega_j/T)} \right)^{3/2},
$$

where $\alpha = 0.19 (T \omega_c / \Delta E_0)$.

This expression can be simplified further. The region I ($\hbar \omega_c \ll E_0 \Delta / \hbar \omega_c$) corresponds to $\alpha \ll 1$. Since $\hbar \omega_c \ll T$, the characteristic values of $j$ in the sum (25) are much larger than unity. Therefore we can convert the sum over $j$ into the integral. Since $\int_0^\infty x^4 \sinh^{-2}(x/2) = 16 \pi^4/15$, we obtain

$$
\rho_{12}^{xx} = -0.0003 \frac{\hbar}{e^2 n_1 n_2 d^4} \left( \frac{\hbar \omega_c}{E_0} \right)^3 T \left( \frac{a_B}{d} \right)^{3/2}. \tag{26}
$$

Region II is defined by inequalities $T \gg E_0 \Delta / \omega_c \gg \omega_c$. Here $\alpha \gg 1$. As a result, the characteristic frequencies here $\omega_j \approx E_0 \Delta / \omega_c \ll \omega_c$. These frequencies, however, are still much larger than the cyclotron frequency: $\omega_j \gg \omega_c$. This enables us again to introduce the continuous variable $x = \alpha \omega_j / T$, and convert the sum into the integral:

$$
\rho_{12}^{xx} = -0.0095 \frac{\hbar}{e^2 n_1 n_2 d^4} \left( \frac{T}{E_0} \right)^{3/2} \left( \frac{a_B}{d} \right). \tag{27}
$$

In region III ($\omega_c \gg \sqrt{E_0 \Delta}$), the $j = 1$ term dominates the sum. The corresponding expression for the plasmon drag resistance is exponentially small:

$$
\rho_{12}^{xx} = -2.7 \times 10^{-6} \frac{\hbar}{e^2 n_1 n_2 d^4} \left( \frac{\hbar \omega_c}{E_0} \right)^{9/4} \left( \frac{a_B}{d} \right)^{3/2} \times \exp[-0.28(\hbar \omega_c)^2/\Delta E_0]. \tag{28}
$$

This is due to the fact that the value of $q$ which is needed to bring the magnetoplasmon pole to the vicinity of the Landau level is large compared to $1/d$. A similar situation occurs in region IV, with the exponential suppression due to low temperature:

$$
\rho_{12}^{xx} = -1.12 \times 10^{-5} \frac{\hbar}{e^2 n_1 n_2 d^4} \left( \frac{\hbar \omega_c}{E_0} \right)^8 \left( \frac{a_B}{d} \right)^{3/2} \times \exp(-\hbar \omega_c / T). \tag{29}
$$

Below we will see that even the exponentially suppressed magnetoplasmon contribution can efficiently compete with the quasiparticle one.

Now we will estimate the quasiparticle contribution in all three regions. Let us first consider region I ($\omega_c \ll E_0 \Delta / \omega_c$). In this parameter interval the characteristic values of $\hbar \omega \approx T$ and $qd \approx 1$. It follows from Eqs. (7) and (8) that $\text{Re} \chi = \nu$ and $\text{Im} \chi \approx \nu$. Thus we are in the normal screening regime. Using the fact that $V(q) \nu \approx 1$ (in other terms, $qa_B \ll 1$) from Eq. (1) we obtain

$$
\rho_{12}^{xx} = -0.00725 \frac{1}{e^2 n_1 n_2 d^4} \frac{T^2 a_B^2}{\hbar^2 v_{f1} v_{f2}} \frac{\omega_c}{\Delta}. \tag{30}
$$

This magnetotransresistance is larger than the zero-field value [Eq. (3)] by a factor of $\omega_c / \Delta \gg 1$. This is due to the discreteness of the electron spectrum in the magnetic field when the density of states within the Landau level increases remarkably.

In region II ($\omega_c \ll E_0 \Delta / \omega_c \ll T$), though the main contribution to the drag is due to the magnetoplasmon mechanism, it is instructive to give an estimation of the quasiparticle...
As in region I, $q = 1/d$. The characteristic frequency, however, is restricted by value $\omega_{cut} \sim E_0 \Delta/\omega_c$, which is much smaller than the temperature (though still much larger than the cyclotron frequency). The reason for this is the overscreening. For estimations, we approximate $\text{Im } \chi$ near the cyclotron resonance by its value from Eq. (7) at $q d = 1$. This gives $\text{Im } \chi = v \omega_0/\omega_{cut}$. A good estimation for $\text{Re } \chi$ near the resonance is $v$ for $v \gg \text{Im } \chi$ and $\text{Im } \chi$ otherwise. Thus the integrand in Eq. (1), $(\text{Im } \chi)^2/\varepsilon^2 \chi^2 (\text{Re } \chi + \text{Im } \chi)^2$, achieves a maximum at $\omega_{cut}$. Since the interval of integration over frequency is effectively smaller than the temperature, the transresistance exhibits a linear temperature dependence: $\rho_{12}^{\text{xx}} \sim (h/e^2)(1/n_1 n_2 d^4) (a_B^2/d^2)(T/E_0)$.

Let us compare this with the $B = 0$ case. Then the linear temperature dependence starts at $T > E_0$. This is because the absorption of energy due to the Landau damping mechanism is possible only for frequencies smaller than $v_F/d$. Thermal frequencies are ineffective, because the corresponding phase velocities are larger than $v_F$. In the presence of magnetic field, the role of a cutoff energy is taken by a much smaller energy $E_0 \Delta/\omega_c$. Thus the linear temperature dependence of the transresistance starts at much lower temperatures.

In region III ($\sqrt{E_0} \Delta \ll h \omega_c \ll T_c$), $\omega_{cut}$ becomes smaller than $\omega_c$. There is overscreening at $q d = 1$ for all cyclotron resonances. The contribution of resonances to integral Eq. (1) is of the order of $\rho_{12}^{\text{xx}} \sim (h/e^2)(1/n_1 n_2 d^4)(R_c^2 a_B^2/d^4)(\Delta^3/\omega_c^3)$.

The main contribution is determined by the frequencies $\omega \sim \Delta \ll \omega_c$, so that the quasiparticles are created within the same Landau level. For these frequencies we always have the normal screening situation. We can set $\text{Re } \chi = v$. $\text{Im } \chi$ is determined by the $j = 0$ term of Eq. (7). We obtain

$$\rho_{12}^{\text{xx}} = -\frac{\hbar}{e^2} \frac{2}{\pi^2} \frac{1}{n_1 n_2} \int \frac{d^2 q}{(2 \pi)^2} \frac{1}{\sin^2(qd)} \frac{T a_B^2}{\Delta^2 R_c^4 c^2} \int_0^{2 \Delta} \frac{d \omega}{2 \pi} \chi_{e 2 m} \left( \frac{\omega}{2 \Delta} \right) \approx -0.0011 \frac{\hbar}{e^2} \frac{1}{n_1 n_2 d^4} \frac{\omega_e^2 T a_B^2}{\Delta v_F v F_2}. \quad (31)$$

It is interesting to note that this magnetic and temperature dependence coincides precisely with the observed one if we assume that $\Delta$ does not depend on the magnetic field. Indeed, the magnetic dependence of $\Delta$ is rather weak. These experiments were performed in a rather strong magnetic field, where only a few Landau levels were occupied so that one should not expect quantitative agreement with our calculations. On the other hand, the linear $T$ dependence is remarkable. We believe that in any case this indicates a reduction of typical excitation energies to values much smaller than $T$, possibly due to overscreening at energies of the order of $T$.

Now we are in a position to compare the quasiparticle and magnetoplasmon contribution, and thus to set the borders of the gray-shaded regions in Fig. 1. The magnetoplasmon contribution dominates throughout region II. In region I we compare expressions (26) and (30). The magnetoplasmon contribution dominates provided $T = E_0 \Delta/\omega_c (a_B/d)^{1/4}$. Since experimentally $d = a_B$, this happens in fact close to the border between regions I and II ($T = E_0 \Delta/\omega_c$). The exponentially small magnetoplasmon contribution given by Eq. (28) competes with Eq. (31) in region III and dominates provided $0.28 a_B^2/(E_0 \Delta) < \ln(\omega_e^2/\Delta^2 d/a_B)^{1/2}$. It also dominates in region IV if $T/\hbar \omega_c > 1/\ln(\omega_e^4 d^2/(E_0^2 a_B^2 d^2))$. The latter condition is obtained by comparing expressions (29) and (32).

### VII. Results for Low Temperatures

In this section we present our results for low temperatures $T \ll \hbar \omega_c$. Owing to energy limitations, only the states of the upper partially filled Landau level are involved in the drag. This makes the transresistance sensitive to the concrete value of the filling factor. In addition to the slow dependence on the magnetic field, the drag effect exhibits an oscillatory dependence on the inverse magnetic field related to the filling factor. The detailed study of these Shubnikov–de Haas oscillations is beyond the limits of the present work, and will be presented elsewhere. Therefore, we provide here analytical results for regions IV and VI only. As to regions V and VII, we present below estimations of the transresistance for typical filling factors rather than detailed analytical results.

In region IV ($\Delta \ll T \ll \hbar \omega_c \ll \sqrt{E_0}$) the situation is most straightforward, since it follows from Eq. (9) that $\text{Im } \chi \ll v$. Thus here we encounter normal screening and may set $\text{Re } \chi = v$. For the transresistance we obtain

$$\rho_{12}^{\text{xx}} = -0.0011 \frac{\hbar}{e^2} \frac{1}{n_1 n_2 d^4} \frac{a_B^2}{v F_1 v F_2} \frac{\omega_e^4}{T \Delta} \times f_{n_1}(1 - f_{n_1}) f_{n_2}(1 - f_{n_2}). \quad (32)$$

$f_{n_1}$ being filling factors in the layers. Even if $T \gg \Delta$, we encounter overscreening in region V ($\sqrt{E_0} \ll \hbar \omega_c \ll E_0$). For estimations, we set $\text{Im } \chi \sim \text{Re } \chi \gg v$ and obtain

$$\rho_{12}^{\text{xx}} \sim (h/e^2)(1/n_1 n_2 d^4) (a_B^2 R_c c_2 d^4)(T^3/\omega_c^2 \Delta). \quad (33)$$

As a consequence of overscreening, the transresistance decreases rapidly with increasing magnetic field.

At low temperature $T \ll \Delta$ the integral in Eq. (1) is contributed by $\omega \approx T$. This gives rise to the featureless $T^2$ temperature dependence of Eq. (3) with the coefficient depending on a magnetic field. Region VI ($T \ll \Delta$, $\omega_c \ll \sqrt{E_0}$) again corresponds to normal screening. We take $\text{Im } \chi$ from Eq. (10) and set $\text{Re } \chi = v$. This yields

$$\rho_{12}^{\text{xx}} \sim (h/e^2)(1/n_1 n_2 d^4) (a_B^2 R_c c_2 d^4)(T^3/\omega_c^2 \Delta). \quad (34)$$
\[ \rho_{12}^{xx} = -0.0031 \frac{\hbar}{e^2 n_1 n_2 d^4} \frac{v_F}{v_F} \frac{1}{T^2 \omega_n^4} \left( 1 - \left( \frac{\mu_1 - \epsilon_n}{\Delta} \right)^2 \right) \times \left( 1 - \left( \frac{\mu_2 - \epsilon_n}{\Delta} \right)^2 \right), \] (34)

where \( \mu_{1,2} \) are the chemical potentials in the layers. They are related to filling factors \( f_{n_1,n_2} \) by means of

\[ f_n = 1/2 + (1/\pi) \left[ \left( \frac{\mu_1 - \epsilon_n}{\Delta} \right)^2 - \left( \frac{\mu_2 - \epsilon_n}{\Delta} \right)^2 \right] + \arcsin \left( \frac{\mu_1 - \epsilon_n}{\Delta} \right). \] (35)

For a typical filling factor, the effect is larger than the zero field transresistance by a factor of \( (\omega_n/\Delta)^2 \).

With increasing magnetic field, we enter the region of overscreening (region VII, \( T \ll \Delta, \omega_c \gg \sqrt{E_F \Delta} \)). Again we set \( \text{Re} \chi \gg \nu \) and obtain

\[
\chi(i \omega_n, q) = -\frac{2T}{S} \sum_{\epsilon_n} \sum_{n,X} G_n(i \epsilon_n) G_m(i \epsilon_n + i \omega_n)(nX|e^{i \vec{q} \cdot \vec{r}}|mX')(mX'|e^{-i \vec{q} \cdot \vec{r}}|nX),
\] (A1)

where \( G_n \) is the dressed Matsubara Green’s function of the electrons in the \( n \)th Landau level, \( S \) is the area of a sample, \( \epsilon_n = \pi T(2n+1) \) are fermionic frequencies, and \( X \) and \( X' \) are the quantum numbers which give the position of the Landau oscillator center. Since Green’s functions do not depend on \( X \), we can reduce the above expression to the following:

\[
\chi(i \omega_n, q) = -\frac{T}{\pi \lambda^2} \sum_{\epsilon_n} \sum_{n,m} G_n(i \epsilon_n) G_m(i \epsilon_n + i \omega_n) |f_{nm}(q)|^2.
\] (A2)

Here \( \lambda \) is the magnetic length, and \( |f_{nm}(q)|^2 = e^{-x(m!/n!)}x^{n-m}|L_{n-m}^m(x)|^2 \) for \( m \leq n \) (the corresponding expression for \( n < m \) can be obtained by interchanging of indices \( n \) and \( m \), \( x = q\lambda^2/2 \) and \( L_{n-m}^m \) are the Laguerre polynomials. Note that for high Landau levels \( n,m \gg 1 \), we obtain, using the asymptotics of Laguerre polynomials, \( |f_{nm}(q)|^2 \approx J_{n-m}^2(q\lambda \sqrt{n+m+1}) = J_{n-m}^2(qR_c) \), where we have used that \( \sqrt{2n\lambda} = v_F/\omega_c \approx R_c \) is the cyclotron radius (\( n \approx E_F/\hbar \omega_c \gg 1 \)).

The polarization function we need can be obtained from Eq. (A2) by the analytic continuation \( i \omega_n \rightarrow \omega + i0^+ \). Writing the sum (A2) over \( \epsilon_n \) as a contour integral, and deforming the contour in the standard manner, we obtain

\[
\chi(\omega, q) = \frac{1}{\pi \lambda^2} \sum_{n,m} J_{n-m}^2(qR_c) \int_{-\infty}^{+\infty} \frac{d \epsilon}{2\pi i} n_F(\epsilon) \left[ \left( G_n'(\epsilon) - G_n''(\epsilon) \right) G_m'(\epsilon + \omega) + G_m''(\epsilon - \omega) \right] \left( G_n''(\epsilon) - G_n'(\epsilon) \right).
\] (A3)

Here \( n_F \) is the Fermi distribution function, and \( G'^\pm \) are retarded and advanced Green’s functions. While deriving Eq. (A3) we have used that the Bessel function square does not change with the changing of the index sign. Taking the imaginary part of Eq. (A3), we finally obtain Eq. (4) of the main text.

**APPENDIX B**

In this work we disregard rapidly oscillating parts of the polarization function given by Eq. (4). This means we approximate the Bessel function squares in Eq. (4) in the limit of \( qR_c \gg 1 \) as \( J_{n-m}^2(qR_c) \approx 1/\pi qR_c \), rather than

\[
J_{n-m}^2(qR_c) \approx \frac{1}{\pi qR_c} \left[ 1 + \cos \left( 2qR_c - \pi m - \frac{\pi}{2} \right) \right],
\] (B1)

which is the mathematically correct expression. Let us explain why.

First let us note that if we take these oscillating parts into account, it would significantly alter our results. In the normal screening regime it would give an extra factor of \( 1/2 \), since the
answer is proportional to $(\text{Im} \chi)^2$. The answer would change even more drastically in the overscreening regime. The point is that the oscillating terms would set the polarization function to (almost) zero for $q$ corresponding to zeros of the Bessel function. No overscreening would occur near these points and their close vicinity would dominate the drag.

All this would lead to a very sophisticated and extremely unstable picture of the drag effect. Fortunately, we are able to present some arguments that allow one to disregard the oscillating terms in the polarization function.

The physical origin of the oscillating terms can be best understood in the language of semiclassical electron trajectories in a magnetic field. A classical trajectory cannot move from the starting point further than $2R_e$. Hence the polarization function in the coordinate representation, $\chi(x,x')$ has a sharp edge at $|x-x'| \approx 2R_e$. This gives rise to Fourier components $\approx \cos(2qR_e)$.

If the edge is not sharp, the oscillating part is exponentially suppressed. The suppression is of the order of $\exp[-q^2(\delta R)^2]$, $\delta R$ being a typical rounding of the edge. By virtue $\delta R$ is the typical uncertainty of the coordinate of the electron which makes half of the Larmor circle. Such an uncertainty can be of quantum-mechanical origin. In this case we estimate $\delta R = \sqrt{R_e/k_F} \ll R_e$. Another cause of uncertainty may be small-angle scattering by smooth potential fluctuations in the heterostructure. For this case we estimate $\delta R = \sqrt{R_e/l_{\text{sa}}}$, where transport mean free path $l_{\text{sa}} \gg R_e$. It is interesting to note that scattering on pointlike defects does not contribute to $\delta R$ provided $\omega, \tau \gg 1$.

Now we note that typical $q$ values contributing to the drag resistance are of the order of $1/d$. We conclude that the oscillating part is exponentially suppressed provided $d < \sqrt{R_e/k_F}$ or $d < R_e/R_c \sqrt{l_{\text{sa}}}$. We assume that at least one of these conditions is fulfilled.


