Contrasting behavior between dispersive seismic velocity and attenuation: Advantages in subsoil characterization

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Abstract: A careful look into the pertinent models of poroelasticity reveals that in water-saturated sediments or soils, the seismic (P and S wave) velocity dispersion and attenuation in the low field-seismic frequency band (20–200 Hz) have a contrasting behavior in the porosity-permeability domain. Taking advantage of this nearly orthogonal behavior, a new approach has been proposed, which leads to unique estimates of both porosity and permeability simultaneously. Through realistic numerical tests, the effect of maximum frequency content in data and the integration of P and S waves on the accuracy and robustness of the estimates are demonstrated.

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1. Introduction

Attenuation and velocity dispersion of seismic waves in water-saturated sediments have been studied in detail in the past and various poroelasticity models have been suggested to explain the observation (e.g., Biot, 1962; Stoll and Bryan, 1970; Stoll, 1977; Buckingham, 2000; Chotiros and Isakson, 2004). The relative importance of various material parameters involved in propagation of compressional (P) and shear (S) waves in marine sediments has been investigated (e.g., Badiey et al., 1998; Buchanan, 2005, 2006). On a compiled database of field-observed seismic wave dispersion in water-saturated land sediments (soils), we have also examined the influence of the unknown model parameters. All these studies suggest that Darcy permeability (k) is the parameter which is usually unknown but has the strongest influence on the dispersion of seismic waves. The other important unknown is porosity (n). These two parameters are generally interrelated (e.g., Carman, 1956); however, an empirical estimation of one from the other on field data usually results in large uncertainties. Although the influence of n and k on the dispersive velocity and attenuation is well investigated, the individual behavior of velocity and attenuation in the n-k domain has remained unexplored. In this context, attenuation and velocity dispersion of seismic waves are interrelated; however, they can be derived from seismic data mutually independently.

Previous attempts to use seismic wave dispersion for estimating n or k have generally considered hard materials or rocks and frequencies in kHz to tens of kHz range (e.g., Burns, 1990; Yamamoto, 2003; Lin et al., 2009; Baron and Holliger, 2011). Near-surface seismic data acquired in soft sediments or soils do not have such high frequencies. Earlier observations of S-wave intrinsic dispersion in soft soils (e.g., Kudo and Shima, 1970; Michaels, 1980) have not led to a quantitative estimation of both n and k from the low-frequency field seismic data. Foti et al. (2002) have discussed the possibility of using both P- and S-wave velocities measured in field to estimate n from the theory of linear poroelasticity in the low-frequency limit, thus assuming the fluid-saturated porous medium as a closed (undrained) system with pore fluid moving in phase with the soil skeleton. This approach does not address in situ k.
This letter presents the results of our research which concentrates on $P$ and $S$ wave dispersion in water-saturated soils and at field seismic frequency range for soft soils (approximately 20–200 Hz). A study on the behavior of dispersive seismic velocity and attenuation in the $n$-$k$ domain reveals a significant difference. This contrasting behavior has been utilized in a cost function integration scheme, leading to stable and unique estimate of both $n$ and $k$. The effects of using only $S$ wave and both $P$ and $S$ waves and the maximum frequency content in the data have been examined for different soil types (gravel, medium sand, fine sand, sandy clay) through numerical tests using realistic material parameters.

2. Dispersive seismic velocity and attenuation in water-saturated soils

For water-saturated unconsolidated sediments, Stoll’s adaptations of the Biot’s theory (Stoll and Bryan, 1970; Stoll, 1977) have earlier been tested on field seismic data (Holland and Brunson, 1988). These models have been found to be capable of predicting the field-observed seismic dispersion for a variety of soil types. In addition to these models, we have looked into the BICSQS model [Biot model with grain contact squirt flow and shear drag by Chotiros and Isakson (2004)] and the Rayleigh–Plesset–Biot model (Smeulders and van Dongen, 1997; Vogelaar, 2009), and have examined them on a compiled database of seismic dispersion observed in soft soil sites. In agreement with Holland and Brunson (1988), we find that the Stoll and Bryan (1970) model can indeed explain well the observed seismic dispersion, except for coarse-grained soils for which BICSQS appears to be more suitable. In this letter, we shall illustrate our findings on the behavior of the dispersive seismic velocity and attenuation in the $n$-$k$ domain, using the Stoll and Bryan (1970) model. The above-mentioned behavior will be little affected if a different model is used. In practice, one may use the model that fits the observed dispersion data best.

Figures 1(a) and 1(c) illustrate, respectively, the dispersive velocity and attenuation (inverse quality factor) for $P$ wave in water-saturated soft soil, estimated using the Stoll and Bryan (1970) model. The dispersion curves for four different porosity and permeability values ($n = 0.35$, $k = 1.09 \times 10^{-9}$ m$^2$; $n = 0.4$, $k = 1.05 \times 10^{-10}$ m$^2$; $n = 0.45$, $k = 1.42 \times 10^{-11}$ m$^2$; and $n = 0.55$, $k = 3.31 \times 10^{-12}$ m$^2$) are shown. These values generally correspond to gravel, medium sand, fine sand, and sandy clay, respectively. As in

![Fig. 1. $P$ and $S$ wave dispersive velocity and attenuation (inverse quality factor) estimated using the Stoll and Bryan (1970) model for four different permeability ($k$) and porosity ($n$) values, representing four different soil types. The dashed box represents the typical field seismic frequency band in soft soil.](image-url)
Holland and Brunson (1988), we have used the Kozeny–Carman relationship to obtain realistic values of $k$ from $n$:

$$k = \frac{1}{KS_0^2(1-n)^2},$$  \hspace{1cm} (1)

where $K$ is an empirical constant and equal to 5 for spherical grains. $S_0$ can be defined analytically for a sphere as $S_0 = 6/d$, where $d$ is the grain diameter. Following Berryman (1981) and Hovem and Ingram (1979), respectively, the tortuosity ($\gamma$) and the pore size parameter ($a$) are obtained as

$$\gamma = 1 - r\left(1 - \frac{1}{n}\right),$$  \hspace{1cm} (2)

$$a = \frac{nd}{3(1-n)},$$  \hspace{1cm} (3)

where $r = 0.5$ for spheres, and lies between 0 and 1 for other family of ellipsoidal surfaces.

Figures 1(b) and 1(d) show the dispersive velocity and attenuation for $S$ wave for these four sets of $n$ and $k$ values. The $S$-wave velocity ($V_S$) range of 120–150 m/s and $P$ wave velocity ($V_P$) range of 1500–1800 m/s represent soft, water-saturated soils. The dashed box in Fig. 1 indicates the typical field seismic frequency band for soft soils. Note that for gravel and medium sand ($k \sim 10^{-9} \text{ m}^2$ and $k \sim 10^{-10} \text{ m}^2$), the velocity dispersion is discernible even in the low field-seismic frequency band. The attenuation is, however, sensitive to even finer grain (lower permeability) soils, as we recognize in the field-frequency band changes in the inverse quality factor ($Q^{-1}$) even for $k \sim 10^{-11} \text{ m}^2$. In other words, at field seismic frequencies, seismic attenuation is sensitive to permeability variation over a wider range than seismic velocity. This effect of the maximum frequency content in data on the permeability sensitivity will have an important bearing on the results to be discussed next.

3. Results and discussion

We calculate the changes—separately for velocity and attenuation over the entire frequency ($f$) range of interest—when $n$ and $k$ both vary, and all other parameters in the model are assigned fixed realistic values. The individual cost functions for velocity and attenuation ($C^V_i$ and $C^\alpha_i$, respectively) are defined as follows:

$$C^V_i = \left( \frac{\sum_j |\Delta^V_{ij}|^\beta}{\left( \sum_j |\Delta^V_{ij} a^V_i(f)|^\beta \right)_{\max}} \right)^{1/\beta},$$  \hspace{1cm} (4)

$$C^\alpha_i = \left( \frac{\sum_j |\Delta^\alpha_{ij}|^\beta}{\left( \sum_j |\Delta^\alpha_{ij} a^\alpha_i(f)|^\beta \right)_{\max}} \right)^{1/\beta},$$  \hspace{1cm} (5)

where $\Delta^V_{ij} = V_{ij}(f, n, k) - \tilde{V}_i(f)$, $V_{ij}(f, n, k)$ being the velocity estimated from the Stoll and Bryan (1970) model and $\tilde{V}_i(f)$ representing the field observed velocity dispersion,
with i indicating the different wavetypes (P or S). The notations are similar for attenuation a, shown in Eq. (5). The attenuation coefficient $a_i$ is related to $Q_i$ as $a_i = 8.686(\pi f/Q_i)\text{dB/m}$. $\sigma^i_v(f)$ and $\sigma^i_a(f)$ denote the standard deviation normalized to the mean value. The denominators in Eqs. (4) and (5) compensate for the differences in sensitivity and noise or fluctuation in velocity and attenuation data. We have used $\beta = 2$ ($L_2$-norm) for noise-free data. Figures 2(a) and 2(b) show, respectively, the cost functions for P-wave velocity and attenuation for a whole range of n and k. Figures 2(c) and 2(d) illustrate the same for S-wave. For this numerical study, field seismic frequency band (20–200 Hz) is considered. Here, the dispersion curves representing the observation, obtained also from Stoll and Bryan (1970), correspond to $n = 0.4$ and $k = 5 \times 10^{-11} \text{m}^2$, which are realistic values for fine-grained alluvial sand. The deep blue indicates the cost function minimum.

Remarkably, one can notice in Figs. 2(a) and 2(b) that the orientation of the cost function minima line is sharply different between dispersive velocity and attenuation for P waves. The same also holds for S waves [Fig. 2(c) and 2(d)]. While for velocity, the cost function minima line is nearly parallel to the k-axis, this line is generally parallel to the n axis for attenuation. This shows that while dispersive velocity of seismic waves is more sensitive to n and excepting at very high k (generally corresponding to coarse-grained soils) is nearly insensitive to k, frequency-dependent seismic attenuation is primarily sensitive to k, and n-sensitivity (especially for S waves) is relatively insignificant. The cost functions ($C^v_p$ and $C^a_p$) for P wave shows a sharper minimum in the n-k domain than the cost functions for S wave. [Compare between Figs. 2(a) and 2(c) and Figs. 2(b) and 2(d)].

Clearly if only seismic velocity or attenuation is used, then one gets a whole set of local minima aligned in the n-k domain and no well-defined global minimum [Figs. 2(a)–2(d)]. Therefore, it is impossible to obtain unique and robust estimates of either n or k in water-saturated soils from field-observed dispersive velocity or attenuation alone.

The difference between velocity and attenuation in the n-k domain is driven by the underlying physics of seismic wave propagation in water-saturated sediments or soils. We have verified that if a different model of poroelasticity is used [than Stoll and Bryan (1970)] then this observed difference between velocity and attenuation remains unaffected.

Because the behaviors of dispersive velocity and attenuation are nearly perpendicular to each other in the n-k domain, their integration is expected to result in a sharp convergence into a global minimum. We have defined an integrated cost function as follows:

$$C^v_p + C^a_p + C^v_s + C^a_s,$$

Fig. 2. Cost functions in the n-k domain [see Eqs. (4)–(6)] for (a) $V_p$, (b) $a_p$, (c) $V_s$, (d) $a_s$, (e) integrated $V_p + a_p + V_s + a_s$, and (f) integrated $V_s + a_s$. The frequency band used in 20–200 Hz. The deep blue color indicates the cost function minimum. The difference in behavior between $V_p$ and $a_p$ and that between $V_s$ and $a_s$ in the n-k domain are driven by the underlying physics of poroelasticity pertinent to such media.
where $C_{P,S}^{V\alpha}$ is the integrated cost function for $P$ and $S$ wave velocity and attenuation. $N$ is the number of attributes (i.e., $V_P$, $a_P$, $V_S$, $a_S$) to be integrated. Figure 2(e) shows the cost function in the $n$-$k$ domain when $V_P$, $a_P$, $V_S$, and $a_S$ are all integrated. Because of the much higher velocity of $P$ wave compared to $S$ wave in water-saturated soil, measuring $P$-wave dispersion in the field is more challenging than $S$-wave dispersion. We have, therefore, investigated also the cost function in the $n$-$k$ domain when only $V_S$ and $a_S$ are integrated. The result is shown in Fig. 2(f). Note that in both Figs. 2(e) and 2(f) the local minima lines have shrunk into sharp global minima points. The global minimum is sharper when both $P$- and $S$-wave data are used. However, with only $V_S$ and $a_S$, the convergence is still very sharp. Unique and correct estimates of both $n$ and $k$ ($n = 0.4, k = 5 \times 10^{-11} \text{m}^2$) are obtained.

In the previous section and in Fig. 1 we have discussed that the maximum frequency content in field data determines the lowest value of $k$ that is sensed by seismic attenuation. Lower $k$ generally corresponds to finer grain soil. The typical $k$ for gravel, fine sand, and sandy clay are, respectively, $10^{-9} \text{m}^2$, $10^{-11} \text{m}^2$, and $10^{-12} \text{m}^2$ (Berry and Reid, 1987). $k$ typically ranges between $10^{-10} \text{m}^2$ and $10^{-12} \text{m}^2$ for very coarse to very

$$C_{P,S}^{V\alpha} = \left( \frac{\sum_f |\Delta V_P|^\beta}{N\left( \sum_f |\Delta V_P(f)|^\beta \right)_{\text{max}}} + \frac{\sum_f |\Delta a_P|^\beta}{N\left( \sum_f |\Delta a_P(f)|^\beta \right)_{\text{max}}} \right)^{1/\beta}, \tag{6}$$

The cost function $C_{P,S}^{V\alpha}$ is shown in Fig. 3. The three columns represent three different values of the maximum frequency in the data. The four rows represent four different values of $n$ and $k$ (taken as true values); from top to bottom: $n = 0.35$, $k = 1.09 \times 10^{-9} \text{m}^2$; $n = 0.40$, $k = 1.05 \times 10^{-10} \text{m}^2$; $n = 0.45$, $k = 1.42 \times 10^{-11} \text{m}^2$; and $n = 0.55$, $k = 3.31 \times 10^{-12} \text{m}^2$. The typical soil types representing these permeabilities are marked.

![Figure 3](image-url)
fine grained sands. We have looked into the effect of the maximum frequency content on the nature of the integrated cost function minima in the \( n-k \) domain. The result is illustrated in Fig. 3.

The four rows in Fig. 3 corresponds to four sets of values for \( n \) and \( k \). Figure 3(a) shows the velocity and attenuation integrated cost function [Eq. (6)] for \( S \) wave [similar to Fig. 2(f)]. Figure 3(b) shows the same when both \( P \) and \( S \) waves are used [similar to Fig. 2(e)]. The three columns in Fig. 3(a) represent three maximum frequency values in \( S \)-wave dispersion data: 80, 140, and 200 Hz. The three columns in Fig. 3(b) correspond to three maximum frequencies in \( P \)-wave data: 100, 200, and 300 Hz, while \( S \)-wave maximum frequency is constant at 80 Hz. The minimum frequency is 20 Hz for all. These frequency limits are realistic for field data acquired in soft soil with conventional seismic sources.

It is clear from these numerical tests that \( n \) can be estimated accurately in all cases. However, estimating correctly the value of \( k \) depends strongly on the maximum frequency available in the data, because the \( k \) variation is sensed by the higher frequencies, especially attenuation (Fig. 1). For high \( k \) \( (\sim 10^{-9} \text{ m}^2) \), typical in coarse-grained sands and gravels, the integration of dispersive velocity and attenuation of \( S \) wave with only 80 Hz maximum frequency provides a clear global minimum [Fig. 3(a1)]. As the maximum frequency limit for \( S \) wave increases to 140 and 200 Hz [Figs. 3(a2) and 3(a3)] or when \( P \)-wave is supplemented to \( S \)-wave [Fig. 3(b1)–3(b3)], the convergence becomes much sharper. However, when \( k \) is low \( (\sim 10^{-11} \text{ m}^2) \), the 20–80 Hz frequency band of \( S \)-wave is no longer sufficient to provide a sharp global minimum, and hence one cannot estimate \( k \) accurately [Fig. 3(a7)]. However, for this low \( k \), if the \( S \)-wave frequency band is extended slightly to 20–140 Hz, then the global minimum becomes quite sharp. The global minimum is unambiguous in the results of inversion. Note that even for a very low value for \( k \) \( (\sim 10^{-12} \text{ m}^2) \), typical for sandy clay and very fine sand) a maximum frequency of 200 Hz in the \( S \)-wave data or supplementing \( P \) data with low-frequency \( S \) data [see Fig. 3(a12) and 3(b12)] leads to a sharp global minimum.

We find that this integrated cost function minimization approach, taking advantage of the contrasting behavior between dispersive velocity and attenuation, is robust. In the illustrations presented in Figs. 2 and 3 we have considered noise-free data [hence, \( \sigma_i'(f) \) and \( \sigma_i^2(f) \) equal to 1 in Eqs. (4)–(6)]. However, we have verified that the presence of \( \pm 10\% \) random noise in the velocity dispersion data and \( \pm 100\% \) random noise in attenuation data leads to 2\% error in \( n \) and 20\% error in \( k \). The use of standard deviation normalized to the mean value in the data \( [\sigma_i'(f) \text{ and } \sigma_i^2(f)] \) in Eqs. (4)–(6) helps significantly in minimizing the effect of random noise in the observed dispersion curves. The error due to incorrect model has also been investigated. We have generated the dispersion curves representing the observation using Stoll and Bryan (1970) while for estimating \( n \) and \( k \) we have used the BICSQS model (Chotiros and Isakson, 2004). These two models are considerably different. We notice that the error in the estimated \( n \) and \( k \) are, respectively, 1.5 and 25\%, in case only \( S \) wave data with a frequency band of 20–80 Hz is used. The error will be less if more higher frequencies are available in the data and/or \( P \) wave is supplemented to \( S \) wave. Because the difference between the two models is more conspicuous for seismic attenuation than for seismic velocity, and attenuation is primarily sensitive to \( k \), the error due to an inaccurate model is larger for \( k \) than for \( n \). All parameters in the model other than \( n \) and \( k \) have been assigned realistic constant values; if reasonable uncertainties are allowed to those parameters then the errors in the estimated \( n \) and \( k \) are still small and they do not exceed the above-mentioned effect of model uncertainty.

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References and links


