Hydroplaning

Lubrication theory
Research question

Can we model hydroplaning fast and accurate using lubrication theory?
Can we model hydroplaning fast and accurate using lubrication theory?
Research question

Can we model hydroplaning fast and accurate using lubrication theory?
Fast?
Fast?
Fast?
Content
Content

1. Hydroplaning
Content

I. Hydroplaning

II. Fluid Mechanics: Lubrication Theory
Content

I. Hydroplaning

II. Fluid Mechanics: Lubrication Theory

III. Solid Mechanics: Tire modelling
Content

I. Hydroplaning

II. Fluid Mechanics: Lubrication Theory

III. Solid Mechanics: Tire modelling

IV. Fluid Structure interaction

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I. Hydroplaning
II. Fluid Mechanics: Lubrication Theory
III. Solid Mechanics: Tire modelling
IV. Fluid Structure interaction
V. Results
Content

I. Hydroplaning

II. Fluid Mechanics: Lubrication Theory

III. Solid Mechanics: Tire modelling

IV. Fluid Structure interaction

V. Results
Hydroplaning
Hydroplaning
Observations
Observations

- Footprint
Observations

- Footprint
Observations

- Footprint
- Bow wave
Observations

- Footprint
- Bow wave
- Spin down
Observations

• Footprint
• Bow wave
• Spin down
• Loss of:
Observations

- Footprint
- Bow wave
- Spin down
- Loss of:
  - Traction
Observations

- Footprint
- Bow wave
- Spin down
- Loss of:
  - Traction
  - Directional control
Parameters
Parameters

- Fluid:
Parameters

- Fluid:
  - Viscosity
Parameters

- Fluid:
  - Viscosity
  - Inertia
Parameters

- Fluid:
  - Viscosity
  - Inertia

- Tire:
Parameters

- Fluid:
  - Viscosity
  - Inertia
- Tire:
  - Tread design
Parameters

- Fluid:
  - Viscosity
  - Inertia
- Tire:
  - Tread design
  - Width
Parameters

- Fluid:
  - Viscosity
  - Inertia
- Tire:
  - Tread design
  - Width
Parameters

- Fluid:
  - Viscosity
  - Inertia

- Tire:
  - Tread design
  - Width

- Surface:
Parameters

- Fluid:
  - Viscosity
  - Inertia
- Tire:
  - Tread design
  - Width
- Surface:
  - Texture
Parameters

- Fluid:
  - Viscosity
  - Inertia
- Tire:
  - Tread design
  - Width
- Surface:
  - Texture
Parameters

- Fluid:
  - Viscosity
  - Inertia
- Tire:
  - Tread design
  - Width
- Surface:
  - Texture
  - Pavement crown
Parameters

- Fluid:
  - Viscosity
  - Inertia
- Tire:
  - Tread design
  - Width
- Surface:
  - Texture
  - Pavement crown
Parameters

- Fluid:
  - Viscosity
  - Inertia
- Tire:
  - Tread design
  - Width
- Surface:
  - Texture
  - Pavement crown
- Vehicle:
Parameters

- Fluid:
  - Viscosity
  - Inertia
- Tire:
  - Tread design
  - Width
- Surface:
  - Texture
  - Pavement crown
- Vehicle:
  - Weight
Parameters

- Fluid:
  - Viscosity
  - Inertia
- Tire:
  - Tread design
  - Width
- Surface:
  - Texture
  - Pavement crown
- Vehicle:
  - Weight
Operating parameters
Operating parameters

- Inflation pressure
Operating parameters

- Inflation pressure
- Vehicle velocity
Hydroplaning formula

\[ v = 6.36 \sqrt{p} \]

Vehicle velocity: \( v \) in \( \frac{km}{hour} \)

Tire pressure: \( p \) in \( kPa \)
Dominant fluid effects

- Viscosity
- Inertia
Dominant fluid effects

- Viscosity
- Inertia
Modelling
Modelling

- Analytical

\[ F = \frac{\rho b V^2 R \lambda}{3} \]
Modelling

- Analytical
- FEM & FVM
Modelling

- Analytical
- FEM & FVM
- CFD
Modelling

• Analytical
• FEM & FVM
• CFD
• Lubrication theory
Lubrication theory
Lubrication theory
Why?

Full 3D simulation

Pressure

Velocity: x-, y-, z-direction
Why?

Full 3D simulation

Pressure
Velocity: x-, y-, z-direction
Why?

Full 3D simulation
Why?

- Full 3D simulation
- Simplified Reynolds model

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Pressure

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Why?

Full 3D simulation

Simplified Reynolds model

4

1
II: Lubrication theory
II: Lubrication theory

- Reynolds equation
II: Lubrication theory

- Reynolds equation
II: Lubrication theory

- Reynolds equation
- Inertia correction
II: Lubrication theory

- Reynolds equation
- Inertia correction
II: Lubrication theory

- Reynolds equation
- Inertia correction
II: Lubrication theory

- Reynolds equation
- Inertia correction
- Inlet condition
II: Lubrication theory

- Reynolds equation
- Inertia correction
- Inlet condition
Navier-Stokes equations

Incompressible Newtonian fluid

\[
\rho \left( \frac{\partial \mathbf{v}}{\partial t} + \mathbf{v} \cdot \nabla \mathbf{v} \right) = -\nabla p + \mu \nabla^2 \mathbf{v}
\]

- **Inertia (per volume)**
- **Unsteady acceleration**
- **Convective acceleration**
- **Divergence of stress**
- **Pressure gradient**
- **Viscosity**

Navier & Stokes (1822)
Assume: thin film

\[
\rho \left( \frac{\partial \mathbf{v}}{\partial t} + \mathbf{v} \cdot \nabla \mathbf{v} \right) = -\nabla p + \mu \nabla^2 \mathbf{v}
\]

- Inertia (per volume)
- Unsteady acceleration
- Convective acceleration
- Divergence of stress
- Pressure gradient
- Viscosity
Assume: thin film

\[ 0 = \left\{ \begin{array}{c}
\text{Divergence of stress} \\
\text{Pressure gradient} \\
\text{Viscosity}
\end{array} \right\} = -\nabla p + \mu \nabla^2 v \]
Assume: no slip
Assume: no slip

Poiseuille
Assume: no slip

Poiseuille
Assume: no slip

Poiseuille + Couette

$U_1$ $U_2$
Assume: no slip

Poiseuille + Couette

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Assume: no slip

Poiseuille

\[ \frac{-h^3}{12\mu} \nabla p \]

Couette

\[ \rightarrow U_2 \quad \rightarrow U_1 \quad \rightarrow U_1 \]
Assume: no slip

Poiseuille

\[ -\frac{h^3}{12\mu} \nabla p + \]

Couette

\[ \rightarrow U_2 \]

\[ \rightarrow U_1 \]
Assume: no slip

Poiseuille

\[-\frac{h^3}{12\mu} \nabla p\]  

+  

\left(\frac{U_1 + U_2}{2}\right) h

Couette

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Assume: no slip

Poiseuille

\(-\frac{h^3}{12\mu} \nabla p\) + \left(\frac{U_1 + U_2}{2}\right) h

Couette
Assume: no slip

Poiseuille

\[-\frac{h^3}{12\mu} \nabla p + \left( \frac{U_1 + U_2}{2} \right) h\]

Couette
Assume: no slip

Poiseuille

\[ \nabla \cdot \left( \frac{-h^3}{12\mu} \nabla p \right) + \left( \frac{U_1 + U_2}{2} \right) h \]

Couette
Assume: no slip

Poiseuille

\[ \nabla \cdot \left( \frac{-h^3}{12\mu} \nabla p \right) + \left( \frac{U_1 + U_2}{2} \right) h = \]

Couette
Assume: no slip

Poiseuille

\[ \nabla \cdot \left( \frac{-h^3}{12\mu} \nabla p \right) + \left( \frac{U_1 + U_2}{2} \right) h = \]

Couette

Lubrication

Tires

FSI

Results

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Assume: no slip

\[ \nabla \cdot \left( \frac{-h^3}{12\mu} \nabla p \right) + \left( \frac{U_1 + U_2}{2} \right) h = \frac{\partial h}{\partial t} \]
Assume: no slip

Poiseuille

Couette

\[ \nabla \cdot \left( \frac{-h^3}{12\mu} \nabla p \right) + \left( \frac{U_1 + U_2}{2} \right) h = \frac{\partial h}{\partial t} \]
Inertia correction
Inertia correction: 1D
Inertia correction: 1D

Reynolds model (no inertia)
Inertia correction: 1D

Reynolds model (no inertia) → Pressure
Inertia correction: 1D

Reynolds model (no inertia) → Pressure

Velocity
Inertia correction: 1D

Reynolds model (no inertia) → Pressure

Inertia

\[ \rho \frac{Dv}{Dt} \]

Velocity

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Inertia correction: 1D

Navier-Stokes equations → Reynolds model (no inertia) → Inertia $\rho \frac{Dv}{Dt}$ → Pressure

Velocity
Inertia correction: 1D

Velocity

Reynolds model
(no inertia)

Pressure

Navier-Stokes equations

Inertia
\( \rho \frac{Dv}{Dt} \)

Velocity

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Inertia correction: 1D

1. Navier-Stokes equations
2. Reynolds model (no inertia)
3. Inertia $\rho \frac{Dv}{Dt}$
4. Pressure
5. Velocity
Inertia correction: 1D

- Navier-Stokes equations
- Reynolds model
- Inertia \( \rho \frac{Dv}{Dt} \)
- Pressure
- Velocity

\[ U_1 \quad U_2 \]
Squeeze

- Circular disc
Squeeze

Hydrodynamic load

Load [N] vs. sinkage [m]

- FlowVision
- Viscous
- Viscous + inertia

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Inertia correction: 2D

Reynolds model (no inertia) → Pressure

Inertia $\frac{Dv}{Dt}$ → Velocity
Inertia correction: 2D

Reynolds model (no inertia) → Inertia $F_{\text{inertia}}(z)$ → Pressure

Velocity
Inertia correction: 2D

Average
\[ \int_0^h \left( \rho \frac{Dv}{Dt} \right) dz \]

Reynolds model (no inertia)

Inertia
\[ F_{\text{inertia}} (z) \]

Pressure

Velocity

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Inertia correction: 2D

\[ F_{\text{inertia}}(z) = \int_0^h \left( \rho \frac{Dv}{Dt} \right) dz \]

- **Velocity**
  - Reynolds model (no inertia)
  - Pressure
  - Inertia
  - Average
  - Poiseuille
  - Couette
  - Velocity

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Inertia correction: 2D

\[ F_{\text{inertia}}(z) \]

\[ \rho \frac{Dv}{Dt} \int_0^h dz \]

Velocity

Reynolds model

Pressure

Average

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Inlet condition
Inlet condition

- Stagnation pressure

Bernoulli (1738)
Inlet condition

- Stagnation pressure

Beroulli (1738)
Inlet condition

- Stagnation pressure
Inlet condition

- Stagnation pressure

\[ p = \frac{1}{2} \rho v^2 \]
Summary

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Summary
Summary
Summary
Tire modelling
Tire modelling
Tire construction

- Tread
- Nylon belt
- Steel belt
- Plies
- Liner
- Filler
- Sidewall
- Bead
- Chafer
Linear elastic tire
Linear elastic tire
Linear elastic tire

\[ F \]
Linear elastic tire

$F$
Linear elastic tire

\[ Ku = F \]
Linear elastic tire

\[ Ku = F \]
\[ u = K^{-1}F \]
The tire model contains some additional compounds to what is described before:

- **Minibase**: A thin rubber layer between tread and reinforcement package with intermediate stiffness softer than breaker package but stiffer than tread.
- **Begs**: The Breaker Ending Gum Strip, sometimes called GumStrip or Breaker Wedge, prevents crack growth in breaker ending region.

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**Figure 4.17**: BMW Coarse

**Figure 4.18**: BMW Coarse
‘real’ tire
Rolling tire model
Rolling tire model

Oscillations

h [mm] vs. time [s]
Rolling tire model

Oscillations → Problem for Reynolds

h [mm]

0.2 0.4 0.6 0.8 1.0 1.2 1.4 1.6 1.8 2.0

0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1.0

time [s]

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Solutions?
Solutions?

• Contact penalty
Solutions?

• Contact penalty
• Time step reduction
Solutions?

• Contact penalty
• Time step reduction
• Mesh refinement
Solutions?

• Contact penalty
• Time step reduction
• Mesh refinement
• Move road (in FSI)
Solutions?

- Contact penalty
- Time step reduction
- Mesh refinement
- Move road (in FSI)
Fluid Structure Interaction
Fluid Structure Interaction
Fluid structure interaction

- Monolithic vs. Partitioned
Fluid structure interaction

- Monolithic vs. Partitioned
Monolithic vs. Partitioned
Monolithic vs. Partitioned
Monolithic vs. Partitioned
Monolithic vs. Partitioned
Monolithic vs. Partitioned
Classical staggering

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Classical staggering

Elastic tire

Reynolds model
Classical staggering

Elastic tire

$h, U$

Reynolds model
Classical staggering

Elastic tire

$h, U \quad p$

Reynolds model
Classical staggering

\[ h, U \quad \rightarrow \quad p \quad \rightarrow \quad h, U \]

Elastic tire \quad \rightarrow \quad 
Reynolds model

Elastic tire \quad \rightarrow \quad 
Reynolds model

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Classical staggering

\[ h, U \rightarrow p \rightarrow h, U \rightarrow p \rightarrow h, U \rightarrow p \]

- Elastic tire
- Reynolds model

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Interface Quasi Newton

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Interface Quasi Newton

- Elastic tire
- Elastic tire
- Reynolds model
- Reynolds model
Interface Quasi Newton

\[ h^k \]

Elastic tire

Reynolds model

Elastic tire

Reynolds model
Interface Quasi Newton

\[ h^k \]

Elastic tire

Reynolds model

Reynolds model

Elastic tire
Interface Quasi Newton

Elastic tire

\[ h^k \]

\[ p^k \]

Reynolds model

Elastic tire

Reynolds model

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Interface Quasi Newton

\[ \begin{align*}
& h^k \\
\end{align*} \]

Reynolds model

\[ p^k \]

Elastic tire

\[ h^{k+1} \]

Elastic tire

Reynolds model

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Interface Quasi Newton

Elastic tire

Reynolds model

Reynolds model

\( h^k \)

\( p^k \)

\( h^{k+1} \)
Interface Quasi Newton

\[ h^k \rightarrow p^k \rightarrow \text{Reynolds model} \]

\[ h^{k+1} \rightarrow p^{k+1} \rightarrow \text{Elastic tire} \]

\[ h^k \rightarrow \text{Reynolds model} \]

\[ h^{k+1} \rightarrow \text{Elastic tire} \]
Interface Quasi Newton

\[
h^k \quad h^{k+1}
\]

\[
p^k \quad p^{k+1}
\]

Elastic tire

Reynolds model

Reynolds model
Interface Quasi Newton

\[ h^k \quad \rightarrow \quad \Delta d \]
\[ p^k \quad \rightarrow \quad \Delta p \]

Elastic tire

Reynolds model

Reynolds model

I Hydroplaning - II Lubrication - III Tires - IV FSI - V Results
Interface Quasi Newton

\[ h^k \quad \cdots \quad h^{k+1} \]

\[ p^k \quad \cdots \quad p^{k+1} \]

Elastic tire

Elastic tire

Reynolds model

Reynolds model
Interface Quasi Newton

\[ h^k \quad \cdots \quad h^{k+1} \quad \cdots \quad h^{k+2} \]

\[ p^k \quad \downarrow \quad p^{k+1} \]

Elastic tire

Reynolds model

Reynolds model

Reynolds model

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Abaqus - Matlab

Figure 4.16: BMW Coarse revolved

The tire has a diameter of 155 mm and an inflation pressure of 0.0 bar. The model is built up from 4 different materials:

- Compound Type E
  - ν: KG
  - Breaker: Hyper elastic
  - Thickness: 9.8876

- Apex Hyper elastic
  - Thickness: 9.586

- Ply Hyper elastic
  - Thickness: 9.886

- Liner Hyper elastic
  - Thickness: 9.886

- Gsbg Hyper elastic
  - Thickness: 9.886

- Chafer Hyper elastic
  - Thickness: 9.886

- Sidewall Hyper elastic
  - Thickness: 9.886

- Miniskirt Hyper elastic
  - Thickness: 9.886

- Tread Hyper elastic
  - Thickness: 9.886

- Beads Hyper elastic
  - Thickness: 9.886

- Overlay Hyper elastic
  - Thickness: 9.886

- Minibase Hyper elastic
  - Thickness: 9.886
Abaqus - Matlab

The tire has a diameter of 55 mm and an inflation pressure of 0 bar. The model is built up from 4 different materials:

- Compound Type E
  - Breaker - Hyper elastic
  - Apex - Hyper elastic
  - Ply - Hyper elastic
  - Liner - Hyper elastic
  - Casing - Hyper elastic
  - Tread - Hyper elastic
  - Beads - Elastic
  - Beads - Hyper elastic
  - Overlay - Hyper elastic
  - Minibase - Hyper elastic

Subroutine VUFIELD

Subroutine DSLOAD
Figure 4.16: BMW Coarse revolved

The tire has a diameter of 155 mm and an inflation pressure of 0.0 bar.
The model is built up from 4 different materials:

- Compound Type E: $\nu_{KG} = 0.586$
- Breaker: $\nu = 0.8876$
- Apex: $\nu = 0.8876$
- Ply: $\nu = 0.586$
- Liner: $\nu = 0.586$
- Tread: $\nu = 0.86$
- Beads: $\nu = 0.86$
- Chafers: $\nu = 0.86$
- Liner: $\nu = 0.586$
- Side Wall: $\nu = 0.586$
- Miniskirt: $\nu = 0.586$
- Tread: $\nu = 0.86$
- Beads: $\nu = 0.86$
- Chafers: $\nu = 0.86$
- Liner: $\nu = 0.586$
- Tread: $\nu = 0.86$
- Beads: $\nu = 0.86$
- Chafers: $\nu = 0.86$
- Liner: $\nu = 0.586$
- Tread: $\nu = 0.86$
- Beads: $\nu = 0.86$
- Chafers: $\nu = 0.86$
- Liner: $\nu = 0.586$
- Tread: $\nu = 0.86$
- Beads: $\nu = 0.86$
- Chafers: $\nu = 0.86$
- Liner: $\nu = 0.586$
- Tread: $\nu = 0.86$
- Beads: $\nu = 0.86$
- Chafers: $\nu = 0.86$
- Liner: $\nu = 0.586$
- Tread: $\nu = 0.86$
- Beads: $\nu = 0.86$
- Chafers: $\nu = 0.86$
- Liner: $\nu = 0.586$
- Tread: $\nu = 0.86$
- Beads: $\nu = 0.86$
- Chafers: $\nu = 0.86$
- Liner: $\nu = 0.586$
- Tread: $\nu = 0.86$
- Beads: $\nu = 0.86$
- Chafers: $\nu = 0.86$
- Liner: $\nu = 0.586$
- Tread: $\nu = 0.86$
- Beads: $\nu = 0.86$
- Chafers: $\nu = 0.86$
- Liner: $\nu = 0.586$
- Tread: $\nu = 0.86$
- Beads: $\nu = 0.86$
- Chafers: $\nu = 0.86$
- Liner: $\nu = 0.586$
- Tread: $\nu = 0.86$
- Beads: $\nu = 0.86$
- Chafers: $\nu = 0.86$
- Liner: $\nu = 0.586$
- Tread: $\nu = 0.86$
- Beads: $\nu = 0.86$
- Chafers: $\nu = 0.86$
- Liner: $\nu = 0.586$
- Tread: $\nu = 0.86$
- Beads: $\nu = 0.86$
- Chafers: $\nu = 0.86$
- Liner: $\nu = 0.586$
- Tread: $\nu = 0.86$
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- Tread: $\nu = 0.86$
- Beads: $\nu = 0.86$
- Chafers: $\nu = 0.86$
- Liner: $\nu = 0.586$
- Tread: $\nu = 0.86$
- Beads: $\nu = 0.86$
- Chafers: $\nu = 0.86$
- Liner: $\nu = 0.586$
- Tread: $\nu = 0.86$
- Beads: $\nu = 0.86$
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- Liner: $\nu = 0.586$
- Tread: $\nu = 0.86$
- Beads: $\nu = 0.86$
- Chafers: $\nu = 0.86$
- Liner: $\nu = 0.586$
- Tread: $\nu = 0.86$
- Beads: $\nu = 0.86$
- Chafers: $\nu = 0.86$
- Liner: $\nu = 0.586$
- Tread: $\nu = 0.86$
- Beads: $\nu = 0.86$
- Chafers: $\nu = 0.86$
- Liner: $\nu = 0.586$
- Tread: $\nu = 0.86$
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- Chafers: $\nu = 0.86$
- Liner: $\nu = 0.586$
- Tread: $\nu = 0.86$
- Beads: $\nu = 0.86$
- Chafers: $\nu = 0.86$
- Liner: $\nu = 0.586$
- Tread: $\nu = 0.86$
- Beads: $\nu = 0.86$
- Chafers: $\nu = 0.86$
- Liner: $\nu = 0.586$
- Tread: $\nu = 0.86$
- Beads: $\nu = 0.86$
- Chafers: $\nu = 0.86$
- Liner: $\nu = 0.586$
- Tread: $\nu = 0.86$
- Beads: $\nu = 0.86$
- Chafers: $\nu = 0.86$
- Liner: $\nu = 0.586$
- Tread: $\nu = 0.86$
- Beads: $\nu = 0.86$
- Chafers: $\nu = 0.86$
- Liner: $\nu = 0.586$
- Tread: $\nu = 0.86$
- Beads: $\nu = 0.86$
- Chafers: $\nu = 0.86$
- Liner: $\nu = 0.586$
- Tread: $\nu = 0.86$
- Beads: $\nu = 0.86$
- Chafers: $\nu = 0.86$
- Liner: $\nu = 0.586$
Figure 4.16: BMW Coarse revolved

The tire has a diameter of 5.5 mm and an inflation pressure of 0 bar. The model is build up from 4 different materials:

- Compound Type E: Hyper elastic
- Breaker: Hyper elastic
- Apex: Hyper elastic
- Ply: Hyper elastic
- Liner: Hyper elastic
- Gast: Hyper elastic
- Chafer: Hyper elastic
- Sidewall: Hyper elastic
- Miniskirt: Hyper elastic
- Tread: Hyper elastic
- Beads: Hyper elastic
- Overlay: Hyper elastic
- Minibase: Hyper elastic

Subroutine

VUFIELD

DSLOAD

\( p \)
Abaqus - Matlab

The tire has a diameter of &.5 mm and an inflation pressure of -0- bar0. The model is build up from 4 different materials:

- Compound Type E
  - Hyper elastic (ν)
  - KG: 9
- Breaker
  - Hyper elastic (ν)
  - KG: 9
- Apex
  - Hyper elastic (ν)
  - KG: 9
- Ply
  - Hyper elastic (ν)
  - KG: 9
- Liner
  - Hyper elastic (ν)
  - KG: 9
- Gostg
  - Hyper elastic (ν)
  - KG: 9
- Chafer
  - Hyper elastic (ν)
  - KG: 9
- Sidewall
  - Hyper elastic (ν)
  - KG: 9
- Miniskirt
  - Hyper elastic (ν)
  - KG: 9
- Tread
  - Hyper elastic (ν)
  - KG: 9
- Beads
  - Hyper elastic (ν)
  - KG: 9
- Begs
  - Hyper elastic (ν)
  - KG: 9
- Overlay
  - Hyper elastic (ν)
  - KG: 9
- Minibase
  - Hyper elastic (ν)
  - KG: 9

Subroutine VUFIELD

Subroutine DSLOAD

\( p \)
The tire has a diameter of 155 mm and an inflation pressure of 0.5 bar. The model is built up from 4 different materials:

- Compound Type E: Hyper elastic with a $\nu$ value of 0.586
- Breaker: Hyper elastic with a $\nu$ value of 0.586
- Apex: Hyper elastic with a $\nu$ value of 0.586
- Ply: Hyper elastic with a $\nu$ value of 0.586
- Liner: Hyper elastic with a $\nu$ value of 0.586
- Gstg: Hyper elastic with a $\nu$ value of 0.586
- Chafer: Hyper elastic with a $\nu$ value of 0.586
- Sidewall: Hyper elastic with a $\nu$ value of 0.86
- Bead: Elastic
- Begs: Hyper elastic with a $\nu$ value of 0.586
- Overlay: Hyper elastic with a $\nu$ value of 0.5848
- Minibase: Hyper elastic with a $\nu$ value of 0.88

Subroutines used:
- VUFIELD
- DSLOAD

I Hydroplaning - II Lubrication - III Tires - IV FSI - V Results
Results
<table>
<thead>
<tr>
<th></th>
<th>Wert</th>
<th>Einheit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Breite</td>
<td>168.00</td>
<td>mm</td>
</tr>
<tr>
<td>Bruttofläche</td>
<td>108.43</td>
<td>cm²</td>
</tr>
<tr>
<td>Nettofläche</td>
<td>91.78</td>
<td>cm²</td>
</tr>
<tr>
<td>SSI</td>
<td>-25.77</td>
<td>%</td>
</tr>
<tr>
<td>CL/SH</td>
<td>0.92</td>
<td></td>
</tr>
<tr>
<td>N/G</td>
<td>0.85</td>
<td></td>
</tr>
</tbody>
</table>

**NB of Nodes**: 356400  
**NB of Elements**: 332640  
**Degree of Freedom**: 2

---

**Results**

1. Hydroplaning  
2. Lubrication  
3. Tires  
4. FSI  
5. Results
Results: Benchmark
Results: Linear elastic tire

- Reynolds
- Benchmark
- Reynolds + Bernoulli

![Chart showing lift force vs speed for different tire models](chart.png)

- Lift force
- Speed [km/h]

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Results: ‘real’ tire

- Lift force vs. Speed [km/h]

- Benchmark and Reynolds + Bernoulli comparison

- Speed range: 0 to 2500 km/h

- Lift force range: 0 to 2000 N
Results: ‘real’ tire

Lift force

Benchmark
Reynolds + Bernoulli

Speed [km/h]

0 500 1000 1500 2000 2500

I Hydroplaning - II Lubrication - III Tires - IV FSI - V Results
Results: ‘real’ tire

- Benchmark
- Reynolds + Bernoulli

<table>
<thead>
<tr>
<th>Speed [km/h]</th>
<th>Lift force</th>
</tr>
</thead>
<tbody>
<tr>
<td>90</td>
<td>2000</td>
</tr>
</tbody>
</table>

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Footprint
Reynolds + Bernoulli Benchmark
Footprint

Reynolds + Bernoulli Benchmark
Footprint
Reynolds + Bernoulli Benchmark
Fast?
Fast?

- Benchmark: 24 - 48 hours / 16 CPU’s
Fast?

- Benchmark: 24 - 48 hours / 16 CPU’s
- Interface method promising
Research question

Can we model hydroplaning fast and accurate using lubrication theory?
Research question

Can we model hydroplaning fast and accurate using lubrication theory?
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Research question

Can we model hydroplaning fast and accurate using lubrication theory?
Research question

Can we model hydroplaning fast and accurate using lubrication theory?
Questions?
Hydroplaning

Lubrication theory
Problem description
Problem description
Newton’s 2nd Law

\[ F = m \cdot a \]
Moving control volume

\[ \rho \frac{d}{dt} (v(x,y,z,t)) = b \]

Infinitesimally small fluid element of fixed mass moving with the flow
Moving control volume

\[ \rho \frac{d}{dt} (\mathbf{v}(x, y, z, t)) = \mathbf{b} \]

\[ \rho \left( \frac{\partial \mathbf{v}}{\partial t} + \frac{\partial \mathbf{v}}{\partial x} \frac{dx}{dt} + \frac{\partial \mathbf{v}}{\partial y} \frac{dy}{dt} + \frac{\partial \mathbf{v}}{\partial z} \frac{dz}{dt} \right) = \mathbf{b} \]

Infinitesimally small fluid element of fixed mass moving with the flow.
Moving control volume

\[
\rho \frac{d}{dt} (v(x, y, z, t)) = b
\]

\[
\rho \left( \frac{\partial v}{\partial t} + \frac{\partial v}{\partial x} \frac{dx}{dt} + \frac{\partial v}{\partial y} \frac{dy}{dt} + \frac{\partial v}{\partial z} \frac{dz}{dt} \right) = b
\]

\[
\rho \left( \frac{\partial v}{\partial t} + v \cdot \nabla v \right) = b
\]

Infinitesimally small fluid element of fixed mass moving with the flow.
Moving control volume

\[ \rho \frac{d}{dt}(v(x, y, z, t)) = b \]

\[ \rho \left( \frac{\partial v}{\partial t} + \frac{\partial v}{\partial x} \frac{dx}{dt} + \frac{\partial v}{\partial y} \frac{dy}{dt} + \frac{\partial v}{\partial z} \frac{dz}{dt} \right) = b \]

\[ \rho \left( \frac{\partial v}{\partial t} + v \cdot \nabla v \right) = b \]
Body forces

The net force on the control volume is due to the gradient of these stresses, see figure 3.3 where this has been demonstrated for forces in the x-direction:
Body forces

$$\rho \frac{Dv}{Dt} = \nabla \cdot \sigma + f$$
Body forces

\[ \rho \frac{D\mathbf{v}}{Dt} = \nabla \cdot \sigma + f \]

\[ \sigma_{ij} = \begin{pmatrix} \sigma_{xx} & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \sigma_{yy} & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \sigma_{zz} \end{pmatrix} = - \begin{pmatrix} p & 0 & 0 \\ 0 & p & 0 \\ 0 & 0 & p \end{pmatrix} + \begin{pmatrix} \sigma_{xx} + p & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \sigma_{yy} + p & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \sigma_{zz} + p \end{pmatrix} = -pI + \mathbb{T} \]
Body forces

\[ \rho \frac{Dv}{Dt} = \nabla \cdot \sigma + f \]

\[ \sigma_{ij} = \begin{pmatrix} \sigma_{xx} & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \sigma_{yy} & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \sigma_{zz} \end{pmatrix} = - \begin{pmatrix} p & 0 & 0 \\ 0 & p & 0 \\ 0 & 0 & p \end{pmatrix} + \begin{pmatrix} \sigma_{xx} + p & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \sigma_{yy} + p & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \sigma_{zz} + p \end{pmatrix} = -pI + \mathbb{T} \]

\[ \tau_{ij} = \mu \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) \]
Bored?

- Existence
- Smoothness
Bored?

- Existence
- Smoothness
Assume: no body force

\[ 0 = \left\{ \begin{array}{c}
-\nabla p \\
\text{Pressure gradient}
\end{array} \right\} + \left\{ \begin{array}{c}
\mu \nabla^2 v \\
\text{Viscosity}
\end{array} \right\} + \left\{ \begin{array}{c}
f \\
\text{Other body forces}
\end{array} \right\} \]
Assume: no body force

\[ 0 = \left( \nabla p \right) + \mu \nabla^2 v. \]
Assume: $\mu$ constant

Conservation of momentum:

\[
\frac{\partial p}{\partial x} = \mu \frac{\partial^2 u}{\partial^2 z} \\
\frac{\partial p}{\partial y} = \mu \frac{\partial^2 v}{\partial^2 z} \\
\frac{\partial p}{\partial z} = 0
\]
Assume: $\mu$ constant

Conservation of momentum:

$$\frac{\partial p}{\partial x} = \mu \frac{\partial^2 u}{\partial^2 z}$$
$$\frac{\partial p}{\partial y} = \mu \frac{\partial^2 v}{\partial^2 z}$$
$$\frac{\partial p}{\partial z} = 0$$

Conservation of mass:

$$0 = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}$$
Continuity

\[ u(x, z) = \frac{1}{2\mu} \frac{\partial p}{\partial x} (z^2 - hz) + \frac{U_2 - U_1}{h} z + U_1 \]

\[ v(x, z) = \frac{1}{2\mu} \frac{\partial p}{\partial y} (z^2 - hz) + \frac{V_2 - V_1}{h} z + V_1 \]

\[ 0 = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} \]
Reynolds equation

\[ \nabla \cdot \left( \frac{h^3}{12 \mu} \nabla p \right) = \nabla \cdot (\bar{U}h) + \frac{\partial h}{\partial t} \]

Reynolds (1886)
Squeeze

- Iterative scheme:
  - Solve the pressure
  - Determine velocity
  - Re-solve Reynolds, including inertia
Sliding: iterative 1D

\[ 0 = -\frac{\partial p}{\partial x} + \mu \frac{\partial^2 u_c}{\partial z^2} \]
\[
p \rho \left( u_v \frac{\partial u_v}{\partial x} + w_v \frac{\partial u_v}{\partial z} \right) = -\frac{\partial p}{\partial x} + \mu \frac{\partial^2 u_c}{\partial z^2}
\]
Sliding: Average 1D

\[
\rho \left( \frac{1}{h} \int_0^h \left( u \frac{\partial u}{\partial x} + w \frac{\partial u}{\partial z} \right) dz \right) = -\frac{\partial p}{\partial x} + \mu \frac{\partial^2 u}{\partial z^2}
\]
Sliding: Average 1D

\[
\rho \left( \frac{1}{h} \int_0^h \left( u \frac{\partial u}{\partial x} + w \frac{\partial u}{\partial z} \right) dz \right) = - \frac{\partial p}{\partial x} + \mu \frac{\partial^2 u}{\partial z^2}
\]

Independent of \( z \)
Sliding: Iterative & Average 2D

\[ 0 = -\frac{\partial p}{\partial x} + \mu \frac{\partial^2 u}{\partial z^2} \]

\[ 0 = -\frac{\partial p}{\partial y} + \mu \frac{\partial^2 v}{\partial z^2} \]

\[ \frac{\partial p}{\partial z} = 0 \]
Sliding: Iterative & Average 2D

\[ \rho \left( \frac{1}{h} \int_0^h \frac{Du}{Dt} \, dz \right) = -\frac{\partial p}{\partial x} + \mu \frac{\partial^2 u}{\partial z^2} \]

\[ \rho \left( \frac{1}{h} \int_0^h \frac{Dv}{Dt} \, dz \right) = -\frac{\partial p}{\partial y} + \mu \frac{\partial^2 v}{\partial z^2} \]

\[ \frac{\partial p}{\partial z} = 0 \]
Inlet condition

• Stagnation pressure

\[ p = \frac{1}{2} \rho v^2 \]

• Energy & Momentum correction

• Converges to zero
Fill rate
Fill rate

\[ \nabla \cdot \left( \frac{-h^3}{12\mu} f \nabla p + \bar{U} h f \right) = 0 \]

\[ p = \xi \text{ for } \xi \geq 0 \]
\[ f = 1 \text{ for } \xi \geq 0 \]
\[ f = 1 + c_f \xi \text{ for } \xi < 0 \]
Models

- Elastic half space
- Abaqus/Explicit model
- ‘real’ tire
Elastic half space

Influence matrix

The exact expression for the influence matrix in the half-space area are determined by Boussinesq [24] and Cerruti. The deformation is then defined by:

\[ w(x, y) = \frac{2\pi E'}{\pi E} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} p(x', y') \sqrt{(x - x')^2 + (y - y')^2} \, dx' \, dy' \]  

(4.1)

Discrete influence matrix

The discrete elastic deformation is then given by:

\[ w(x_i, y_j) = w_{i,j} \approx \frac{2\pi E'}{\pi E} \sum_{k=1}^{n_x} \sum_{l=1}^{n_y} D_{ijkl} p_{kl} \]  

(4.2)

Where the influence coefficients are found from:

\[ D_{ijkl} = \int \int \sqrt{(x - x')^2 + (y - y')^2} \, dx' \, dy' \]  

(4.3)

An analytical solution for this integral is given by Love [25]:

\[ \varepsilon \approx \varepsilon_{\text{half-space}} \]

\[ \varepsilon \ll 1 \]

Elastic body
Elastic half space

- Influence matrix:

\[ w(x_i, y_j) = w_{i,j} \approx \frac{2}{\pi E'} \sum_{k=1}^{n_x} \sum_{l=1}^{n_y} D_{ijkl} p_{kl} \]

\[ D_{ijkl} = \int \int \frac{1}{\sqrt{(x - x')^2 + (y - y')^2}} \, dx' \, dy' \]
Elastic half space
The wheel is modelled with a finite element model in Abaqus Explicit – version 6.8 and newer – see figure:

Figure 4.3: Grosch wheel

The model is built up using 3D quadrilateral elements – consisting of 8 nodes. Two material models have been used – both neo-hookean – with the following parameters:

- **Model 1:**
  - Bulk modulus: 200 MPa
  - Shear modulus: 2 MPa

- **Model 2:**
  - Bulk modulus: 200 MPa
  - Shear modulus: 1 MPa

A Grosch wheel model with a cavity of 10 mm wide and 10 mm high was also tested as well as a Grosch wheel without a groove.
Problem: oscillations
Eigenmodes?

Scale Factor: +1.00
Problem: Energy
Monolithic vs. Partitioned

Continuous solution at interface:
- velocity
- location of interface
- pressure (stresses)

At the interface boundary differences can occur in:
- velocity
- location of interface
- pressure (stresses)

External boundary

Data exchange
Monolithic vs. Partitioned
Monolithic vs. Partitioned
Interface Quasi Newton

Half-space

Elastic half-space

$\varepsilon \approx \varepsilon_{\text{half-space}}$

$\varepsilon \ll 1$

Elastic body

Reynolds model

$R^k$

$R^0$

$R^1$

$R^2$

$\Delta R^0 = R^0 - R^1$

$\Rightarrow d_s^2$

Figure 4.2: Half-space approximation

Influence matrix

Exact expression for the influence matrix in the half-space area are determined by Boussinesq [24] and Cerruti. The deformation is then defined by:

$$w(x, y) = \frac{2\pi E'}{\infty} \int \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} p(x', y') \sqrt{(x-x')^2 + (y-y')^2} \, dx' \, dy'$$

(4.1)

Discrete influence matrix

The discrete elastic deformation is then given by:

$$w(x_i, y_j) = w_{i,j} \approx \frac{2\pi E'}{nx \sum_{k=1}^{n} ny \sum_{l=1}^{D_{ijkl}} p_{kl}}$$

(4.2)

Where the influence coefficients are found from:

$$D_{ijkl} = \int \int \sqrt{(x-x')^2 + (y-y')^2} \, dx' \, dy'$$

(4.3)

An analytical solution for this integral is given by Love [25]:

$$R_k R_0 = R_0 - R_1$$

$$R_0 R_2 = R_1 - R_2$$

$$R_2 R_3 = R_1 - R_2$$

$$\Rightarrow d_s^2$$
Reynolds model

Influence matrix

Exact expression for the influence matrix in the half-space area are determined by Boussinesq [24] and Cerruti. The deformation is then defined by:

\[ w(x, y) = \frac{2\pi E'}{\pi} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} p(x', y') \sqrt{(x-x')^2 + (y-y')^2} \, dx' \, dy' \]  

(4.1)

Discrete influence matrix

The discrete elastic deformation is then given by:

\[ w(x_i, y_j) = w_{i,j} \approx \frac{2\pi E'}{\pi} \sum_{k=1}^{n_x} \sum_{l=1}^{n_y} D_{ijkl} p_{kl} \]  

(4.2)

Where the influence coefficients are found from:

\[ D_{ijkl} = \int \int \sqrt{(x-x')^2 + (y-y')^2} \, dx' \, dy' \]  

(4.3)

An analytical solution for this integral is given by Love [25]:

\[ R_k R_0 R_1 = R_0 R_1 R_2 \]

\[ R_0 R_1 R_3 = R_0 R_1 R_2 \]

\[ \Delta R^0 = R^0 - R^2 \]

\[ \Delta R^1 = R^1 - R^2 \]

\[ \Rightarrow d_s^3 \]
Interface Quasi Newton

Half-space

Elastic half-space

$\varepsilon \approx \varepsilon_{\text{half-space}}$

$\varepsilon \ll 1$

Elastic body

Reynolds model

$R^k$

$R^0$ $R^1$ $R^2$ $R^3$

$\Delta R^0 = R^0 - R^2$

$\Delta R^1 = R^1 - R^2$

$\Rightarrow d^3_s$
Abaqus - Matlab
Abaqus - Matlab
Abaqus - Matlab

- Mesh mapping
Abaqus - Matlab

- Mesh mapping
Abaqus - Matlab

• Mesh mapping
Abaqus - Matlab

- Mesh mapping
Abaqus - Matlab

Domain 1

- DSLOAD
- Matlab engine 1

Domain 2: contact

- VUFIELD
- Matlab engine 2

Domain x

- DSLOAD
- Matlab engine x

Abaqus model

.mat file
Results

• Elastic half space:

<table>
<thead>
<tr>
<th>Speed [km/h]</th>
<th>Reynolds</th>
<th>+ Bernoulli inlet</th>
<th>+ Energy correction</th>
<th>+ Momentum correction</th>
<th>Fill rate</th>
<th>CEL</th>
<th>FV</th>
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<tbody>
<tr>
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<td>46.66</td>
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</table>

• Grosch wheel:

<table>
<thead>
<tr>
<th>Speed [km/h]</th>
<th>Normal load [N]</th>
<th>Water layer [mm]</th>
<th>Reynolds + Bernoulli inlet</th>
</tr>
</thead>
<tbody>
<tr>
<td>50</td>
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<td>5</td>
<td>30.80</td>
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<tr>
<td>15</td>
<td>100</td>
<td>3</td>
<td>3.50</td>
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</table>

• ‘real’ tire:

<table>
<thead>
<tr>
<th>Speed [km/h]</th>
<th>Normal load [N]</th>
<th>Water layer [mm]</th>
<th>Reynolds + Bernoulli inlet</th>
<th>FV</th>
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<tbody>
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<td>3924</td>
<td>3</td>
<td>2200</td>
<td>2000</td>
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</tbody>
</table>
Results: Grosch wheel
Results: Grosch wheel

| Speed [km/h] | 50 |

Thursday, January 21, 2010
# Results: Grosch wheel

<table>
<thead>
<tr>
<th>Speed [km/h]</th>
<th>Reynolds + Bernoulli</th>
</tr>
</thead>
<tbody>
<tr>
<td>50</td>
<td>188%</td>
</tr>
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</table>
Recommendations

• Reynolds equation
• Inlet condition
• Fill rate
• Tire model
• Contact algorithm
• Fluid structure interaction
• Interface quasi Newton