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IMPROVED COMPUTER PROGRAM FOR CALCULATION
OF VISCOUS-INVISCID INTERACTIONS

by Bernard G. GAUTIER and
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RHODE SAINT GENESE BELGIUM
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ABSTRACT

This report describes a theoretical and experimental investigation of the effect of wall cooling on the overall features of shock wave laminar boundary layer interactions in hypersonic two dimensional flow.

The integral method of Lees-Reeves-Klineberg has been modified in order to compute viscous interactions over a continuous range of wall-to-stagnation temperature ratios using a single set of "universal" integral functions.

Comparisons between the theoretical predictions and the experimental results obtained from pressure and heat transfer distribution measurements show satisfactory agreements.

These experiments have been carried out in a moderately hypersonic flow (M = 6) over flat plate-wedge models for different wall cooling rates, corresponding to conditions lying between a highly cooled wall and a quasi-adiabatic surface.

Furthermore, the theory developed herein may be considered to be a significant improvement in the attempt to calculate viscous interactions where the wall temperature distribution is prescribed.
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Comparison between theoretical and measured heat transfer coefficient distributions ($Re_u = 1.5 \times 10^7 / 1 \text{ m}$)

Comparison between theoretical and measured heat transfer coefficient distributions ($Re_u = 1.8 \times 10^7 / 1 \text{ m}$)

Comparison between theoretical and measured heat transfer coefficient distributions ($Re_u = 2.2 \times 10^7 / 1 \text{ m}$)

Normalized experimental pressure and heat transfer distributions
LIST OF SYMBOLS

AR aspect ratio span/chord
a sound velocity, also velocity profile describing parameter
b $S'(O)$ Klineberg's total enthalpy profile describing parameter
B $G'(O)$ normalized total enthalpy profile describing parameter
$\hat{B}$ expression defined at Appendix B
C $\frac{\mu}{\mu_{\infty}}$, $\frac{T}{T_{\infty}}$ Chapman's constant
$\hat{C}$ expression defined at Appendix B
$C_F$ $(\rho u_{\infty}^2/2)$ skin friction coefficient
$C_H$ $q_w/\left(\rho u_{\infty}^2(h_0 - h_w)\right)$ heat transfer coefficient
$C_P$ specific heat at constant pressure
D determinant of the system of eq. (2.31) to (2.34)
$\hat{D}$ expression defined at Appendix B
d skin thickness
$\delta_i$
E $-\frac{1}{\delta_i} \int_0^{SdY} S dy$
$\alpha E$
$\hat{E}$ expression defined at Appendix B
f expression defined at eq. 2.11, also function defined in the Cohen-Reshotko equations
F $\gamma + \frac{1+m}{m_e} (1-E)$
$\hat{F}$ expression defined at Appendix B
$\hat{G}$ expression defined at Appendix B
$\hat{G}$ expression defined at Appendix B
$\hat{H}$ static enthalpy, also expression defined in eq. (2.29)
$h_S$ expression defined in eq. (2.27)
h $C_p T + \frac{u^2}{2}$ total enthalpy
$\gamma_0$ $\delta_i/\delta_i^* \in$ inverse of the shape parameter
I \int_0^\delta p u^2 dy \text{ momentum flux}

J \delta^M_i / \delta^M_i

k \text{ thermal conductivity}

K \int_0^\delta u dy

K_1, K_2 \text{ functions defined in Appendix B}

L \text{ flat plate length}

m \frac{Y-1}{2} M^2, \text{ also exponent of } U_e

\dot{m} \int_0^\delta p u dy \text{ mass flux}

M \frac{u}{a} \text{ Mach number}

N_i \text{ numerators in the system of eq. (2.31) to (2.34)}

P \text{ static pressure}

P \frac{1}{U_e} \left( \frac{3U}{3Y} \right)^{Y=0}, \text{ also function defined in eq. 2.77}

Pr \mu C_p/k \text{ Prandtl number}

q \frac{-k \partial T}{\partial y} \text{ heat flux}

Q \frac{1}{3Y} (\frac{38}{Y})^Y \text{ function defined in eq. 2.28}

\frac{Q}{Q_S} \frac{Q}{Pr_w}

\frac{Q}{\delta_k} \frac{Q}{B/a}

\frac{Q}{\delta_M} \frac{Q}{B/a Pr_w}

R \frac{2 \delta^M_i}{U_e} \left[ \int_0^\delta_0 \left( \frac{3U}{3Y} \right)^2 dY \right]

Re_u \frac{\rho_{\infty} u_{\infty}}{\mu_{\infty}} \times 1 \text{ m. unit Reynolds number of the free stream}

Re_\delta_i \frac{\rho_{\infty} u_{\infty}}{\mu_{\infty}} \delta^M_i

Re_x \frac{\rho_{\infty} u_{\infty}}{\mu_{\infty}} x

S \left( h_0 / h_{0e} - 1 \right) \text{ total enthalpy function}

T(a, b) = - \int_0^{\delta_i} \frac{U}{U_e} S dY, \text{ also static temperature}
\[ T^m = - \frac{1}{\delta^m} \int_0^{\delta^m} \frac{U}{U_e} \, dY \]

\[ \bar{T} = a \bar{T} \]

t  

time (seconds)

\( u, v \)  

velocity components respectively in the direction parallel and normal to the wall

\( U, V \)  

velocity components in the transformed plane

\( x, y \)  

coordinates axis respectively parallel and normal to the wall in the physical plane

\( X, Y \)  

transformed coordinates, also expression defined in eq. 2.77

\[ Z = \frac{1}{\delta^i} \int_0^{\delta^i} \frac{U}{U_e} \, dY \]

\[ a = \frac{1}{\eta} \frac{Y}{\delta^i} \]

\( \alpha \)  

also angle of attack of the model

\( \alpha_w \)  

local wall inclination with respect to the free stream direction

\[ \beta = \frac{2m}{m+1} \]

pressure gradient parameter, also \( \frac{p}{p_e} \), also inclination of an oblique shock wave with respect to the flow direction

\[ \gamma = \frac{C_p}{C_v} \]

ratio of specific heats

\[ \Gamma = \int_0^{\eta} \frac{U}{U_e} \, (1-G) \, d\eta \]

\( \delta \)  

boundary layer thickness

\( \delta^i \)  

transformed boundary layer thickness

\[ \delta = \int_0^\infty (1 - \frac{u}{u_e}) \, dy \]

\[ \delta^i = \int_0^{\delta^i} (1 - \frac{\rho u}{\rho e u_e}) \, dy \]

displacement thickness

\[ \delta^m = \int_0^{\delta^m} (1 - \frac{U}{U_e}) \, dY \]

transformed displacement thickness

\( \varepsilon \)  

perturbation parameter

\( \varepsilon_k \)  

polynomial coefficients defined in Appendix A
non-dimensional coordinate in the Cohen-Reshotko equations (eq. 2.18)

\( \Theta \) wedge angle (or ramp deflection angle)

\( \Theta_e, \Theta_\delta \) local inclination of the outer flow streamlines

\[
\Theta = \int_0^\delta \frac{\rho u}{\rho u_e} (1 - \frac{u}{u_e}) \, dy \quad \text{momentum thickness}
\]

\[
\Theta_i = \int_0^{\delta_i} \frac{U}{U_e} (1 - \frac{U}{U_e}) \, dY \quad \text{transformed momentum thickness}
\]

\[
\Theta^M = \int_0^\delta \frac{\rho u}{\rho u_e} (1 - \frac{u^2}{u_e^2}) \, dy \quad \text{mechanical energy thickness}
\]

\[
\Theta^M_i = \int_0^{\delta_i} \frac{U}{U_e} (1 - \frac{U^2}{U_e^2}) \, dY \quad \text{transformed mechanical energy thickness}
\]

\[
\Theta^M^M = \int_0^\delta \frac{\rho u}{\rho u_e} \left( \frac{h_0}{h_{0e}} - 1 \right) \, dy \quad \text{energy thickness}
\]

\( \mu \) dynamic viscosity of air

\( \nu \) kinematic viscosity of air, also Prandtl-Meyer angle

\( \rho \) density

\( \xi \) flow parameter defined in eq. (2.55)

\[
\sigma = \int_0^{\eta_\delta} S \, d\eta
\]

\[
\Sigma = \int_0^{\eta_\delta} (1 - C) \, d\eta
\]

\( \tau \) \( \frac{\partial u}{\partial y} \) shear stress

\( T_k \) polynomial coefficients defined at Appendix A

\( \phi_k \) polynomial coefficients defined at Appendix A

\[
\bar{x} = \frac{M^3 \sqrt{C}}{\sqrt{Rex}} \quad \text{viscous interaction parameter}
\]

\( \psi \) stream function
Subscripts

B  Blasius solution
C  corner, hinge line
CR critical point
e  outer flow
δ  at the boundary layer edge
i  transformed plane
M  dealing with the surface material
r  reattachment point, also ratios through the critical jump
s  separation point
sh  shock impingement on the boundary layer outer edge
t  stagnation conditions
w  at the wall
o  beginning of the interaction
l  upstream of the jump
2  downstream of the jump
~  free stream flow upstream of the interaction
~+ free stream flow at downstream infinity
1. INTRODUCTION

1.1 Preliminary considerations

The phenomenon of shock wave boundary layer interaction is often encountered in the flight of high speed vehicles. At supersonic speeds the impingement of a shock wave (or a sudden change in the direction of the surface) on the boundary layer may force the flow to separate and to later reattach to the wall.

Flow separation significantly modifies the pressure distribution associated with non separated flows. It has usually a detrimental effect on the expected aerodynamic performance. Furthermore, at hypersonic speeds, the aerodynamic heating problem which is strongly dependent on flow separation becomes a severe design problem and cooling or thermal insulation of the vehicle surface must be considered. It is then of interest to know the location of regions of intense heating as well as the effect of wall cooling upon the flow field itself.

In practical cases, shock wave boundary layer interactions are generally three dimensional (e.g., see ref. 1) and the analytic treatment of such interactions is difficult, therefore one often considers the simpler two dimensional case. Herein, the two dimensional interaction generated a sudden charge in the direction of the surface (e.g., a flat plate equipped with a deflected trailing edge flap) is considered.

We assume that the classical boundary layer equations are valid for the description of any viscous interaction including separated flows. For steady, two dimensional flow these equations are:

Continuity:

\[ \frac{\partial (\rho u)}{\partial x} + \frac{\partial (\rho v)}{\partial y} = 0 \]  

(1)
Momentum:
\[
\rho \frac{\partial u}{\partial x} + \rho v \frac{\partial u}{\partial y} = -\frac{\partial p}{\partial x} + \frac{\partial}{\partial y} \left( \mu \frac{\partial u}{\partial y} \right) \quad (1.2)
\]
\[
\frac{\partial p}{\partial y} = 0 \quad (1.3)
\]

Energy:
\[
\rho \frac{\partial h_0}{\partial x} + \rho v \frac{\partial h_0}{\partial y} = \frac{\partial}{\partial y} \left( \frac{\partial h_0}{\partial y} \right) - \frac{\partial}{\partial y} \left[ \mu \left( \frac{1}{Pr} \right) u \frac{\partial u}{\partial y} \right] \quad (1.4)
\]

In the framework of classical boundary layer theory, the external flow is specified, neglecting its interaction with the viscous layer. A review of these methods is presented in ref. 4. However, there are many problems for which this assumption must be abandoned. For example in shock wave boundary layer interactions with flow separation, the external flow is not known a priori. In such cases the boundary layer equations must be solved simultaneously with the governing equations for the external flow through a coupling equation, a fact which was first pointed out by Crocco-Lees (ref. 2). Two different techniques can be used to solve equations (1.1) to (1.4): A finite difference method which considers the partial differential equations (refs. 26, 27) or an integral method which transforms these equations into ordinary differential equations.

Integrating equations 1.1 to 1.4 across the boundary layer from the wall to an arbitrarily defined outer edge of the boundary layer we get:

Continuity:
\[
\frac{\partial}{\partial x} \left( \rho u e \right) = \frac{\partial \delta}{\partial x} - (\delta - \delta^\infty) \frac{\partial}{\partial x} \left( \log \rho u e \right) \quad (1.5)
\]

Momentum:
\[
\frac{d}{dx} \left( \rho u e \delta^2 \delta \right) + \delta^\infty \rho e u \frac{du}{dx} = (\mu \frac{u}{y}) \quad (1.6)
\]
Energy:

\[ \frac{d}{dx} \left( \rho_e u e^\cdot \hat{\theta} \right) = - \left[ \frac{u}{Pr} \frac{\partial}{\partial y} \left( \frac{h_0}{h_0 e} \right) \right]_v \]  

(1.7)

and

\[ p(y) = p_e = p_v \]  

(1.8)

This set of integro-differential equations reduces to ordinary differential equations provided that the boundary layer integral quantities (δ, δ̇, θ, θ̇, etc) can be related to the flow variables either by semi empirical correlations (refs. 2-5) or by specifying the boundary layer profile.

The boundary layer profiles can be described by simple analytic functions (ref. 6) or, using the local similarity concept first introduced by Thwaites (ref. 10), may be derived from exact solutions of the boundary layer equations, involving similar profiles (refs. 7, 8, 9). All these approaches have been found to be unsuccessful for separated flows. In order to increase the flexibility of integral methods by introducing a larger number of free parameter, Tani (ref. 11) uses moment methods where additional differential equations may be obtained by multiplying the basic differential equations by suitably chosen weighting functions.

Applying the generalized multi moment method to the boundary layer equations, Dorodnitsyn (ref. 12) established the basis of the "integral relations" methods. Further developments (ref. 13) have led to one of the most sophisticated methods (ref. 17).

From the engineer's point of view, we are interested more in the prediction of the overall properties of the flow than in the details of the flow field and the integral methods have the merit of relative simplicity compared to Dorodnitsyn and Nielsen's methods. One of the most promising is the Lees and Reeves method (ref. 3) and its further refinements proposed
1.2 Principle of the Lees-Reeves-Klineberg's method

Considering the set of 3 integro-differential equations (eq. 1.5 to 1.7) an additional moment equation is obtained by multiplying the differential equation of momentum (eq. 1.2) by \( u \) and integrating across the boundary layer:

\[
\frac{d}{dx} \left( \rho e u^3 e^{\delta^i} \right) + 2(\delta^i - \delta u) \rho e u^2 \frac{du}{dx} = 2 \int_0^\delta u \left( \frac{\partial u}{\partial y} \right)^2 dy
\]

The integral quantities appearing in eq. (1.5), (1.6), (1.7) and (1.9) are related to two independent parameters describing respectively velocity and total enthalpy profiles. The family of boundary layer profiles are those obtained from similar solutions but the pressure gradient parameter \( \delta \) is not explicitly related to the streamwise pressure gradient \( \frac{dp}{dx} \). Furthermore, a complete unhooking of velocity and total enthalpy profiles is achieved using two independent descriptive parameters.

The set of governing differential equations reduces to a system of four first order ordinary differential equations which can be written in the following form:

\[
\frac{\delta^i}{M_e} \frac{dM_e}{dx} = K \frac{N_1(M_e \delta^i, a, b)}{D(M_e, a, b)}
\]

\[
\frac{d\delta^i}{dx} = K \frac{N_2}{D}
\]

\[
\frac{\delta^i}{dx} = K \frac{N_3}{D}
\]

\[
\frac{\delta^i}{dx} = K \frac{N_4}{D}
\]

where the flow parameters are respectively; the external flow Mach number \( M_e \), the transformed displacement thickness \( \delta^i \) and the velocity and total enthalpy profile parameters \( a \) and \( b \).
The equations (1.10) to (1.13) are then numerically integrated in the streamwise direction. The initial boundary conditions are not known a priori but must be iterated until a downstream boundary condition is satisfied.

According to the analysis of Lighthill (ref. 18) it has been shown that the boundary layer equations do not contain the initial perturbation term, thus the integration must be initiated by a suitable perturbation of the undisturbed flow. Kubota (ref. 19) has used such a perturbation technique for the adiabatic case. On the other hand, Klineberg (ref. 14) and Holden (ref. 15), considering the case with heat transfer, show that departure conditions depend on whether the boundary layer is subcritical or supercritical. This distinction which arises from the stable or unstable behaviour of the governing integral equations was first pointed out by Crocco (refs. 2, 20) and led to a number of attempts to explain the singular behaviour of the governing equations by physical considerations.

1.3 Subcritical and supercritical boundary layers

Considering the effect of an adverse pressure gradient \( \frac{dp}{dx} > 0 \) on boundary layer growth a distinction between two categories of flows must be done. A thickening boundary layer \( \frac{d\delta}{dp} > 0 \) able to generate its own pressure gradient is termed subcritical, whilst a thinning boundary layer \( \frac{d\delta}{dp} < 0 \) which damps the initial perturbation is termed supercritical. The latter one does not permit upstream propagation of the perturbations and therefore is incompatible with the concept of free interaction derived from experimental observations (ref. 21). This difficulty has been overcome by Klineberg (ref. 14) and Holden (ref. 15) by introducing a jump from a supercritical to a subcritical state at the beginning of the interaction.

On the other hand, it has been shown that continuous transition from a subcritical to a supercritical state occurs with flow acceleration or wall cooling. This transition is re-
lected as a saddle point type singularity in the framework of an integral theory. An analysis of the singular behaviour of the governing equations in multi moment integral theory has been performed by several investigations (refs. 22, 23) and shows that this mathematical feature provides in fact one additional condition which establishes the uniqueness of the solution.

Among the attempts to give a physical interpretation to the critical point singularity, those of Weinbaum and Garvine (ref. 24) point out the similar behaviour of the viscous layer to that of the quasi-one-dimensional inviscid stream tube. A relation similar to that relating pressure and area variations at the throat of the inviscid stream tube can be obtained for the throat station of the two dimensional viscous layer (the throat being defined by \[ \int_0^\delta \frac{M^2-1}{M^2} \, dy + 0 \]).

The question then arises: Is the critical point related to the use of an integral method? Murphy (ref. 25) carried out a comparison between experimental data and the results of several theories dealing with shock wave boundary layer interactions in which both integral and finite difference methods were used. Among them the method of Rheyner-Flügge-Lotz (ref. 26) exhibits none of the numerical problems related to the existence of mathematical singularities, but Weinbaum and Garvine, applying the finite difference method of Baum and Denison (ref. 27) encountered a critical point type behaviour.

The critical boundary, being related to the Mach number profile, is very sensitive to many factors. Holden (ref. 16) shows that the requirement for a supercritical-subcritical jump as posed by Klineberg can be entirely removed by considering the effect of the normal pressure gradient \( \frac{\partial P}{\partial y} \neq 0 \), in the framework of an integral theory. On the other hand, Alziary (ref. 30) develops an integral theory which includes one equation of compatibility (momentum equation at the wall) whilst this condition is not used in Lees and Reeves theory, and then removes
the critical boundary. In conclusion, it must be noted that no evidence of a jump in pressure preceding the free interaction region can be established from experimental results, except perhaps for turbulent boundary layers (Roshko, Thomke, ref. 28 and Todisco, Reeves, ref. 29).

1.4 Conclusions

This survey of different methods dealing with the computation of laminar viscous-inviscid interactions shows that the conventional boundary layer equations are suitable for the description of separated flows provided that a coupling equation linking the viscous layer and the outer flow is included. Furthermore, velocity and total enthalpy profiles derived from similar solutions have been proved valid to provide the set of integral relationships contained within the framework of an integral theory. For the sake of simplicity, Klineberg's method has been selected and modified in order, firstly, to increase its flexibility with respect to an arbitrary choice of wall to stagnation temperature ratio and, secondly, to provide a computational tool able to solve the problem of interactions with prescribed wall temperature.

Section 2 of this report describes the above mentioned extension of the Klineberg's method applied to shock wave boundary layer interactions induced by a compression corner or by an incident shock impinging on a flat plate.

Computations have been carried out for different wall to stagnation temperature ratios. The range covered lies between adiabatic and highly cooled wall conditions. The normalization of the profile dependent integral functions is performed and a numerical evaluation of the accuracy of such an approximate method is presented herein. The third section presents experimental results carried out in hypersonic two dimensional flow (M = 6). These experiments consist of surface pressure and heat transfer measurements over a flat plate wedge model for
several wall temperature conditions. The last section gives a
direct comparison between these experiments and the theoretical
predictions obtained from the generalized theory described in
section 2.
2. THEORETICAL STUDY OF SHOCK WAVE LAMINAR BOUNDARY LAYER INTERACTION

2.1 Introduction

The first part of the analysis follows closely the Lees-Reeves-Klineberg approach. Using classical boundary layer theory the flow field is divided into two regions, a viscous inner part where the boundary layer equations are assumed to be valid and an external inviscid flow field governed by Euler's equations. The two regions are tied together by a coupling equation established by considering the mass flow balance at the boundary layer outer edge. A simplified physical model of the external flow field is based on the assumption of isentropic flow.

The three boundary layer equations of continuity, momentum and energy are treated in the framework of an integral method. One defines a number of integral quantities (mean values of flow variables across the boundary layer), which can be related if the velocity and total enthalpy profiles are specified. These integral relationships can be calculated from the similar solutions. The compatibility equation at the wall is replaced by the first moment of momentum, which in turn un hooks the pressure gradient parameter from the velocity profile. Two independent parameters describe respectively velocity and total enthalpy profiles.

2.2 Analysis

2.2.1 Governing equations

The following system of coordinate axes has been chosen: The abscissa x is taken along the wall from the leading edge and the y coordinate is normal to the wall. The governing partial differential equations for a steady two dimensional compressible laminar boundary layer are:
Continuity:
\[
\frac{3}{3x} (\rho u) + \frac{3}{3y} (\rho v) = 0 \tag{2.1}
\]

Momentum:
\[
\rho u \frac{3u}{3x} + \rho v \frac{3u}{3y} = - \frac{dp}{dx} + \frac{3}{3y} (\mu \frac{3u}{3y}) \tag{2.2}
\]
\[
\frac{dp}{3y} = 0
\]

Energy:
\[
\rho u \frac{3h_0}{3x} + \rho v \frac{3h_0}{3y} = \frac{3}{3y} \left( \frac{\mu}{Pr} \frac{3h_0}{3y} \right) - \frac{3}{3y} \left[ \mu \left( \frac{1-Pr}{Pr} \right) u \frac{3u}{3y} \right] \tag{2.3}
\]
where \( h_0 \) is the total enthalpy \( h_0 = C_{pT} + \frac{u^2}{2} \)

Assuming perfect gas \( \frac{\rho}{u} = RT \)

First moment of momentum:
\[
\rho u^2 \frac{3u}{3x} + \rho uv \frac{3u}{3y} = - u \frac{dp}{dx} + u \frac{3}{3y} (\mu \frac{3u}{3y}) \tag{2.4}
\]

The associated boundary conditions are:
- at the wall \( y = 0 \), \( u = v = 0 \) and \( h_0 = h_0w \)
- at the boundary layer outer edge: \( y = \delta \), \( u = u_e \); \( v = v_e \) and \( h_0 = h_0e \).

The boundary layer thickness is arbitrarily defined as:
\[
\frac{u}{u_e} = 0.99
\]

Following the von Karman approach the basic equations (2.1) to (2.4) are integrated across the boundary layer from \( y = 0 \) to \( y = \delta \). Thus a number of integral quantities are defined \( (\delta, \delta^*, \theta, \theta^*, \text{etc.}) \). The resulting set of integro-differential equations can be simplified to some extent by applying the Ste wartson transformation. Assuming a linear viscosity law:
and isentropic external flow, the transformation is defined by the following equations:

\[
\frac{dX}{dx} = C \frac{\rho_{a}\bar{e}}{\rho_{\infty a_{\infty}}} \quad (2.6)
\]

\[
\frac{dY}{dy} = \frac{\rho_{a}\bar{e}}{\rho_{\infty a_{\infty}}} \quad (2.6)
\]

The integral boundary layer quantities ($\delta^M$, $\theta$, $\theta^*$, etc) are related to their equivalent "incompressible" expression through eq. 2.6 but the x coordinate is not transformed into X.

The resulting set of integro-differential equations can be written as follows:

\[
\frac{d\delta^M}{dx} + \delta^M \frac{d\chi}{dx} + \delta^M (2\chi + 1 + S_{w} \bar{F}) \frac{d\log M_{e}}{dx} = \beta C \frac{M_{\infty}}{M_{e}} e^{-\frac{P}{\delta^M}} \quad (2.8)
\]

\[
\frac{d\delta^M}{dx} + \delta^M \frac{d\chi}{dx} + \delta^M (3\chi + 2S_{w} \bar{T}) \frac{d\log M_{e}}{dx} = \beta C \frac{M_{\infty}}{M_{e}} e^{-\frac{R}{\delta^M}} \quad (2.9)
\]

\[
\frac{d\delta^M}{dx} + \delta^M \frac{d\chi}{dx} + \delta^M (2\chi + 1 + S_{w} \bar{F}) \frac{d\log S_{w}}{dx} = \beta C \frac{M_{\infty}}{M_{e}} e^{-\frac{\chi}{\delta^M}} \quad (2.10)
\]

Note: The term "incompressible" refers in fact to the transformed plane. It has been introduced by analogy with the case of zero heat transfer at the wall.
where 

\[ m_e = \frac{\gamma - 1}{2} M_e^2 \]

\[ \beta = \frac{p_e}{p_{\infty}} \]

\[ \tan \theta_e = \frac{v_e}{u_e} \]

\[ \text{Re} = \frac{\rho_{\infty} a_e}{v_{\infty}} M_{\infty} \delta_i \]

\[ F = \phi + \left( \frac{1 + m_e}{m_e} \right) (1 + S_w F) \]

\[ f = \left[ 2 + \frac{y + 1}{y - 1} \frac{m_e}{1 + m_e} \right] \phi - \left( 3 \frac{y^2 - 1}{y - 1} (1 + S_w F) + \frac{(M_e^2 - 1)}{m_e (1 + m_e)} \right) \]

The transformed boundary layer integral quantities are:

\[ \delta_i = \int_0^\delta_i \frac{dY}{dY} \]  
\[ Z = \frac{1}{\delta_i} \int_0^\delta_i \frac{U}{U_e} dY \]

\[ \delta_i = \int_0^\delta_i \left( 1 - \frac{U}{U_e} \right) dY \]
\[ R = 2 \delta_i \int_0^\delta_i \left( \frac{3}{2} \left( \frac{U}{U_e} \right) \right)^2 dY \]

\[ \theta_i = \int_0^\theta_i \frac{U}{U_e} \left( 1 - \frac{U}{U_e} \right) dY \]
\[ P = \delta_i \left[ \frac{3}{2} \frac{U}{U_e} \right]_{Y=0} \]

\[ \theta_i = \int_0^\theta_i \frac{U}{U_e} \left( 1 - \frac{U^2}{U_e^2} \right) dY \]
\[ Q = -\delta_i \left[ \frac{3}{2} \frac{S}{S_w} \right]_{Y=0} \]

\[ \chi = \frac{\theta_i}{\delta_i} \]
\[ E = \frac{1}{\delta_i} \int_0^\delta_i \left( \frac{S}{S_w} \right) dY \]

\[ J = \frac{\theta_i}{\delta_i} \]
\[ T = \frac{1}{\delta_i} \int_0^\delta_i \left( \frac{U}{U_e} \right) \left( \frac{S}{S_w} \right) dY \]

The integral quantities \( \chi, E, T \) which depend on the total enthalpy profile are slightly different from those defined by Klineberg.
Normalization with respect to $S_w$ will be proved useful for the calculation of a set of polynomial functions representing the integral properties of the flow.

The basic integro-differential equations (2.7 to 2.10) have been established for the most general case of viscous interactions, when an arbitrary wall temperature distribution $T_w(x)$ is given. Considering now the isothermal wall case ($T_w = \text{const}$), the last term in the RHS of equation 2.7 and equation 2.10 vanishes.

### 2.2.2 Coupling between the boundary layer and the outer flow

The local inclination of the outer flow streamline $\theta_e$ appearing in the continuity equation 2.7 must be related to the local external flow Mach number $M_e$.

Assuming a supersonic, irrotational, isentropic external flow $\theta_e$ and $M_e$ can be related through the Prandtl-Meyer relationship

\[
\theta_e = \alpha_w + \theta(M_{\text{ref}}) - \theta(M_e) \tag{2.12a}
\]

where $\alpha_w$ is the local tangent to the surface ($> 0$ for an expansion turn) and

\[
\theta(M) = \frac{\gamma+1}{\gamma-1} \arctg \sqrt{\frac{\gamma-1}{\gamma+1} (M^2-1)} - \arctg \sqrt{M^2-1} \tag{2.12b}
\]

Equations (2.12a) and (2.12b) are valid if the external flow is isentropic.

For the particular case of viscous interaction generated by an incident shock wave impinging on a flat plate, the external flow field is divided into two regions: upstream of the shock impingement point the reference Mach number is the free stream Mach number at upstream infinity ($M_\infty$) (or downstream of the leading edge shock if the model angle of
attack is not zero); downstream of the shock impingement point \(M_{\text{ref}}\) is the free stream Mach number at downstream infinity \(M_\infty\).

2.2.3 Velocity and total enthalpy profiles

In order to transform the set of basic differential equations (2.7 to 2.10) into ordinary differential equations amenable to numerical integration, one must specify both velocity and total enthalpy profiles.

Particular solutions of the boundary layer equations, namely the similar solutions can be used to provide a family of boundary layer profiles. Then we assume that: Any functional relations between the integral quantities established for the similar solutions remain valid for non-similar flows (ref. 3).

Applying the Stewartson transformation (ref. 9) as defined by equations (2.6) to the three compressible flow boundary layer equations (2.1 to 2.3) we get:

Continuity: \(\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0\) (2.13)

Momentum: \(U \frac{\partial U}{\partial X} + V \frac{\partial U}{\partial Y} = U e \frac{dU}{dX} (1+S) + \frac{\partial^2 U}{\partial Y^2}\) (2.14)

Energy: \(U \frac{\partial S}{\partial X} + V \frac{\partial S}{\partial Y} = \alpha \left[ \frac{1}{Pr} \frac{\partial^2 S}{\partial Y^2} - \frac{1-Pr}{Pr} \frac{m e}{(1+m e)} \frac{\partial^2 (U e)^2}{\partial Y^2} \right]\) (2.15)

Taking \(S_w = \text{const}\) and \(Pr = 1\), we assume that the external flow velocity \(U_e\) is given by:

\(U_e = C_1 X^m\) (2.16)

Through the von Mises transformation momentum and energy equations reduce to:
\[ f''' \cdot f'' + \beta (1 + S' \cdot f')^2 = 0 \]  
(2.17a)

\[ S'' + fS' = 0 \]  
(2.17b)

where \( \beta = \frac{2m}{m+1} \)

\[ f' = f'(\eta) = \frac{U}{U_e} \]  
(2.18)

\[ S = \frac{h_0}{h_0e} - 1 \]

and \( \eta = Y \left( \frac{m+1}{2} \right) \frac{U_e}{\delta_e} \)

The boundary conditions are the following:

at the wall \( \eta = 0 \) \( f(0) = f'(0) = 0 \), \( S(0) = S_w \)

in the outer inviscid flow \( \eta \to \infty \) \( f' \to 1 \) \( S \to 0 \)  
(2.19)

Numerical solution to eq. 2.17a and b have been given by Cohen-Reshotko (ref. 8) for fixed values of \( \beta \) and \( S_w \).

Knowing both velocity and total enthalpy profiles in terms of functions \( f(\eta), f'(\eta) \ldots S(\eta), S'(\eta) \), etc. the integral boundary layer quantities as defined by equations (2.12) can be calculated.

In Klineberg's earlier formulation of the problem the total enthalpy profile dependent functions are defined by

\[ E = -\frac{1}{\delta_i} \int_{0}^{\delta_i} S dY \]

\[ Q = \delta_i \left( \frac{3S}{\delta} \right)_{Y=0} \]  
(2.20)

\[ T = -\frac{1}{\delta_i} \int_{0}^{\delta_i} S dY \]
and therefore strongly dependent upon the parameter $S_w$. As a consequence, a new table of integral functions must be established for each value of $S_w$. The main reason for this dependence appears to be the improper choice of the total enthalpy parameter $S$, indeed $S$ appears in the boundary conditions (2.19) of equations 2.17a and b. Herein, we try to remove the direct dependence of integral functions upon $S_w$ by suitably redefine the total enthalpy parameter. Let

$$G = \frac{S - S_w}{-S_w}$$

(2.21)

equations 2.17a and b become:

$$f''' + ff'' + \beta (1 - f'^2 + S_w (1 - G)) = 0$$

(2.22a)

$$G'' + fG' = 0$$

(2.22b)

The associated boundary conditions are:

- at the wall $\eta = 0$ \( f(0) = f'(0) \) and \( G(0) = 0 \)

- in the outer inviscid flow $\eta \rightarrow \infty$, \( \lim f' = 1 \) and

$$\lim_{\eta \rightarrow \infty} G = 1$$

(2.23)

Having standardized the boundary conditions, the effect of $S_w$ enters explicitly only in the equation 2.22a. Since we know that the influence of $S_w$ upon $f$ is rather weak (see ref. 14), it is evident that this influence will be even weaker upon $G$, since the equation determining $G$ (eq. 2.22b) does not contain $S_w$ and is only affected by $S_w$ by the pressure of $f$. Thus, if we write the total enthalpy profile dependent functions in terms of $G$ the major dependence upon $S_w$ can be removed.

When $\beta$ and $S_w$ are fixed, $\eta$ and $Y$ are linearly related for a given value of $X$:

$$\frac{Y}{\eta} = a S_i^w = \text{const.}$$
then we define a scaling parameter $\alpha$:

$$
\alpha = \left[ \int_0^\eta \left( 1 - \frac{U}{U_e} \right) d\eta \right]^{-1}
$$

The resulting set of "universal" integral quantities is:

$$
\begin{align*}
\delta_i &= \alpha \delta_i \eta_i \\
\delta_i^* &= \frac{\delta_i}{\alpha} \\
\theta_i &= \alpha \delta_i \int_0^\eta f'(1-f') d\eta \\
\theta_i^* &= \frac{\theta_i}{\delta_i} \\
J &= \frac{\theta_i}{\delta_i} \\
Z &= \alpha \int_0^\eta f' d\eta \\
R &= \frac{2}{\alpha} \int_0^\eta f'' d\eta \\
P &= \frac{f''(0)}{\alpha} \\
Y &= \frac{G'(0)}{\alpha} \\
E &= \alpha \int_0^\eta (1-G) d\eta \\
F &= \alpha \int_0^\eta f'(1-G) d\eta
\end{align*}
$$

Two independent parameters are chosen in order to describe velocity and total enthalpy profiles. These parameters have to be bounded and single valued over their whole variation range for any given viscous interactions. Therefore the definition of the velocity profile parameter "$\alpha$" is different for attached and separated flows.

Velocity profile parameter "$\alpha$":

$$
\text{attached flow: } a = \left[ \frac{\partial \left( \frac{U}{U_e} \right)}{\partial \eta_i} \right]_{\eta = 0} = \eta_i f''(0) \quad (2.26)
$$

$$
\text{separated flow: } a = \left[ \frac{\eta_i}{\delta_i} \right]_{\frac{U}{U_e} = 0} = \frac{\eta_i f'(0)}{\eta_i} \quad (2.27)
$$
Total enthalpy parameter B:

\[ B = - \frac{S'(0)}{S} \]  

(2.28)

### 2.2.4 Final form of governing equations

Introducing both \( a \) and \( B \) into equations (2.7) to (2.10) we get:

\[
 F \frac{d\delta^*}{dx} + \delta^* \left( \frac{3F}{\delta a dx} + \frac{3F}{\delta B dx} \right) + \frac{f_i}{M_e} \frac{dM}{dx} = \beta C \frac{M_\infty}{M_e} \frac{h_s}{Re} \\
 \chi \frac{d\delta^*}{dx} + \delta^* \frac{d\chi}{dx} + (2\chi + 1 + S_w E) \frac{\delta_i}{M_e} \frac{dM}{dc} = \beta C \frac{M_\infty}{M_e} \frac{P}{Re} \\
 J \frac{d\delta^*}{dx} + \delta_i \frac{dJ}{dx} + (3J + 2S_w T) \frac{\delta_i}{M_e} \frac{dM}{dx} = \beta C \frac{M_\infty}{M_e} \frac{R}{Re} \\
 F \frac{d\delta^*}{dx} + \delta^* \frac{dF}{dx} + (2T + 2S_w F) \frac{\delta_i}{M_e} \frac{dM}{dx} = \beta C \frac{M_\infty}{M_e} \frac{Q_s}{Re} \\
\]

(2.29) (2.30) (2.31) (2.32)

where

\[
 h_s = \frac{M_e}{M_\infty} \left( \frac{1 + m_e}{m_e} \right) \frac{Re}{\epsilon} \delta^* \left[ \tan \theta_e - \frac{\delta^*}{\delta \tilde{S}_w} \frac{dS_w}{dx} \right] \\
 Q_s = \frac{\chi}{Pr_w} - \frac{\delta_i}{\epsilon} \frac{Re}{\delta^*} \frac{M_e}{M_\infty} \frac{T}{\beta C} \frac{d \log S_w}{dx} \\
\]

(2.33) (2.34)

For the case of an isothermal wall \( (S_w = \text{const}) \) these expressions reduce to:

\[
 h_s \equiv h = \frac{M_e}{M_\infty} \left( \frac{1 + m_e}{m_e} \right) \frac{Re}{\epsilon} \frac{\delta_i}{(1 + m_e)} \tan \theta_e \\
 Q_s \equiv \frac{\chi}{Pr_w} = \tilde{Q} \\
\]

(2.35) (2.36)
The equations (2.29) to (2.32) are written in a more convenient form for numerical calculations:

\[
\begin{align*}
\frac{\delta_i^* dM_e}{M_e dx} &= \frac{BC}{Re} \frac{M_\infty}{M_e} N_1 \\
\frac{d\delta_i^*}{dx} &= \frac{BC}{Re} \frac{M_\infty}{M_e} N_2 \\
\frac{\delta_i^* da}{dx} &= \frac{BC}{Re} \frac{M_\infty}{M_e} N_3 \\
\frac{\delta_i^* dB}{dx} &= \frac{BC}{Re} \frac{M_\infty}{M_e} N_4
\end{align*}
\]

(2.37)

where the following relations have been used:

\[
D = B_1 \frac{\partial T}{\partial B} - B_2 \frac{\partial F}{\partial B}
\]

\[
N_1 = B_3 \frac{\partial T}{\partial B} - B_4 \frac{\partial F}{\partial B}
\]

\[
N_2 = B_5 \frac{\partial T}{\partial B} - B_6 \frac{\partial F}{\partial B}
\]

\[
N_3 = B_7 \frac{\partial T}{\partial B} - B_8 \frac{\partial F}{\partial B}
\]

\[
N_4 = \beta B_4 + \beta B_6 + \frac{\partial F}{\partial a} B_8 - h B_2
\]

(2.38)

with

\[
B_1 = A_6 \frac{\partial F}{\partial a} + (A_3 - A_5 F) \frac{\partial \delta_i^*}{\partial a}
\]

\[
B_2 = A_6 \frac{\partial T}{\partial a} + (A_3 - A_5 T) \frac{\partial \delta_i^*}{\partial a}
\]
\[ B_3 = A_2 \frac{3 \dot{F}}{3a} + (A_3 h - A_4 F) \frac{d \%}{d a} \]

\[ B_4 = A_2 \frac{3 \ddot{T}}{3a} + (A_3 \dot{Q} - A_4 \ddot{T}) \frac{d \%}{d a} \]

\[ B_5 = A_7 \frac{3 \dot{F}}{3a} + (A_7 f - A_8 h) \frac{d \%}{d a} \]

\[ B_6 = A_7 \frac{3 \ddot{T}}{3a} + (A_7 \ddot{T} - A_8 \dot{Q}) \frac{d \%}{d a} \]

\[ B_7 = A_6 h - (A_2 f + A_7 F) \]

\[ B_8 = A_6 \dot{Q} - (A_2 + A_7) \ddot{T} \]

and

\[ A_1 = 2 \% + 1 + S_w E \]

\[ A_2 = PJ - \% R \]

\[ A_3 = \% \frac{dJ}{d\%} - J \]

\[ A_4 = P \frac{dJ}{d\%} - R \]

\[ A_5 = 3J + 2S_w \ddot{T} \]

\[ A_6 = A_1 J - A_5 \% \]

\[ A_7 = A_1 R - A_5 P \]

\[ A_8 = A_1 \frac{dJ}{d\%} - A_5 \]

(2.39)

(2.40)
2.2.5 **Numerical calculation of the polynomials representation of profile-dependent integral functions**

Numerical computation of integral functions is performed following the procedure described by Klineberg (ref. 14) for specified values of \( S_w \). These integral functions are transformed into their non-dimensional form simultaneously with the integration of the basic differential equations (2.37).

Similar solutions computations are first performed by numerical integration of both equations (2.17a and b) (ref. 14).

For a given value of \( S_w \), a number of similar profiles are calculated including:
- accelerated flows (favourable pressure gradient \( \beta > 0 \))
- retarded flows (adverse pressure gradient \( \beta < 0 \)) which can be:
  - attached (positive skin friction \( f''(0) > 0 \))
  - separated (negative skin friction \( f''(0) < 0 \)).

For any values of \( \beta \) (pressure gradient parameter) both velocity and total enthalpy profiles are given as functions of \( \eta \) (\( f'(\eta) \) and \( S(\eta) \)). Numerical integration of these functions between \( \eta = 0 \) and \( \eta = \eta_0 \) provides a set of integral quantities as defined by eq. 2.25. When a sufficient number of similar solutions have been calculated (herein = 30 separated and = 50 attached flow profiles) the integral quantities \( J, Z, R, P, \) etc., are tabulated as functions of profiles parameters \( a \) and \( b \) (using Klineberg's notation \( b = -S_w B \)).

However, in order to reduce the number of functions which depend on both velocity and total enthalpy profiles (i.e., \( E, Q, T, \ldots \)), we introduce two scaling functions \( a \) and \( \sigma(b) \) : \( a \) being defined by eq. 2.24,

\[
\sigma(b) = - \int_{0}^{\eta_0} S d\eta \quad (2.41)
\]
Thus integral quantities $Q$, $E$ and $T^*$ (in Klineberg's notations) reduces to:

$$\begin{align*}
Q &= \frac{b}{a(a)} = -S_w \frac{B}{a(a)} = -S_w \tilde{Q} \\
E &= a(a)\sigma(b) = -S_w a(a)\Sigma(B) = -S_w \tilde{E} \\
T^* &= a(a)T(a, b) = -S_w a\Gamma(a, b) = -S_w \tilde{T}
\end{align*}$$

where

$$T(a, b) = \int_0^{\infty} \frac{U}{U_e} Sdn$$

The tabulated values of integral functions are approximated by polynomial functions using a least-square curve-fit. For example:

$$\tilde{\chi}(a) = \sum_{i=0}^{N} C_i a^i$$

The following integral functions which depend upon a single parameter are directly curve-fitted in this way:

$$\tilde{\chi}(a), J(a), Z(a), R(a), P(a), a(a) \text{ and } \sigma(b).$$

The first order derivatives of functions $\tilde{\chi}(a)$, $J(a)$, $a(a)$ and $\sigma(b)$ are approximated through the following simple equation:

$$\frac{d\tilde{\chi}}{da}(a) = \frac{\chi_{i+1} - \chi_i}{a_{i+1} - a_i}$$

where

$$\frac{a}{a} = \frac{a_{i+1} + a_i}{2}$$

The maximum number of polynomial coefficients is 9.

The remaining integral function $T(a, b)$ depends on both parameters $a$ and $b$. A table of data points must be first calculated by multiplying every velocity profile by every enthalpy profile.
point by point and integrating the product across the boundary layer from \( n = 0 \) to \( n = n_0 \) (\( n_0 \) corresponds to the largest profile). From the tabulated values of \( T(a, b) \), a first curve-fit provides a polynomial function in \( a \) for every value of \( b \):

\[ b = b_1, \quad T(b_1, a) = \sum_{k=0}^{M} \tau_k(b_1)a^k \]  

(2.46)

A second curve-fit of coefficients \( \tau_k \) provides the required expression for \( T(a, b) \):

\[ \tau_k(b) = \sum_{l=0}^{N} D_k^l b^l \]  

and

\[ T(a, b) = \sum_{k=0}^{M} \left( \sum_{l=0}^{N} D_k^l b^l \right)a^k \]  

(2.47)

(2.48)

The polynomial functions for the first order partial derivatives \( \frac{\partial T}{\partial a} (a, b) \) and \( \frac{\partial T}{\partial b} (a, b) \) have been obtained from tabulated values of function \( T(a, b) \) derivatives. They can be written as double summations:

\[ \frac{\partial T}{\partial a} (a, b) = \sum_{k=0}^{M} \left( \sum_{l=0}^{N} F_k^l a^l \right)b^k \]  

(2.49)

\[ \frac{\partial T}{\partial b} (a, b) = \sum_{k=0}^{M} \left( \sum_{l=0}^{N} G_k^l b^l \right)a^k \]  

(2.50)

For reasons of computer capability, these polynomial functions are limited to the 5th degree.

Finally, a set of 29 polynomials is obtained for each value of \( S_w \), but due to the double definition of the velocity profile parameter \( a \), two tables dealing respectively with attached and separated flow polynomials are required.

Numerical values of polynomials coefficients are presented in appendix A. Five values of \( S_w \), respectively \( S_w = -0.8, -0.6, -0.4, -0.2 \) and 0 have been considered.
Obviously, the Klineberg's method should gain in flexibility if a single "universal" set of integral functions valid for all values of $S_w$ may be built up. Furthermore, it will become possible to treat the case of viscous interactions where the wall temperature distribution is prescribed.

Comparing the plots of integral functions $\mathcal{L}(a), J(a), Z(a), R(a), \alpha(a)$ for different values of $S_w$ (from ref. 14), it is clear that the dependence upon $S_w$ is very weak, except for $P(a)$ in the separated region.

On the other hand, eliminating $S_w$ from the boundary conditions of eq. 2.22 results that the redefined integral functions $\mathcal{L} = \sigma/(-S_w), \Gamma = T/(-S_w), Q = Q/(-S_w)$ exhibit also a weak dependence upon $S_w$.

Practically the above non-dimensional functions are calculated from the polynomials obtained for a reference value of $S_w$, called $S_{w,\text{ref}}$. For the integral functions where the dependence upon $S_w$ cannot be entirely removed, such as $P(a)$, a linear interpolation between two extreme values of $S_{w,\text{ref}}$ provides a suitable expression.

$$P(a,S_w) = P(a,S_w=0) - \frac{S_w}{0.8} (P(a,S_w=-0.8)-P(a,S_w=0)) \quad (2.51)$$

A plot of $P(a)$ for various values of $S_w$ is presented at fig. 2.2. The effect of the non-dimensionalization of the integral functions $T(a,b)$ with respect to $S_w$ appears clearly by comparison of fig. 2.3 and fig. 2.4 where $\Gamma(a,b)$ has been plotted for both $S_w = -0.8$ and $S_w = 0$. The same result is obtained for the first order partial derivatives $\frac{\partial T}{\partial a}$ and $\frac{\partial T}{\partial b}$ as well as for $L(B)$ and $\frac{dL}{dB}$. These functions are respectively plotted in fig. 2.5, 2.6 and 2.7.
2.3 Application to viscous interactions generated by flat-plate-wedge configurations

2.3.1 Physical model of the flow field

In most of the practical cases, shock wave - boundary layer interaction is generated by a sudden change in the direction at the wall (e.g., a trailing edge flap). However, it is possible to establish an equivalence between the flow field generated by this particular configuration and the case of an incident shock impinging on a flat plate. Both configurations are sketched in fig. 2.1. As shown in fig. 2.1b an externally generated oblique shock wave intersects the boundary layer outer edge at some given point \( x_{SH} \).

The momentum of the viscous layer flow cannot balance the strong streamwise pressure gradient induced by the incident shock wave and is communicated upstream through the viscous sublayer, thus giving rise to flow separation. In order to ensure the continuity of pressure distribution at the impingement point, the incident shock is reflected as an expansion fan which suddenly turns the flow towards the wall, squeezing the boundary layer along the surface until reattachment is completed. Flow deflection in the vicinity of separation and reattachment points gives rise to compression waves which coalesce in the outer flow and generate two shock waves. The intersection of both shock waves originating respectively from separation and reattachment regions forms finally a unique shock which corresponds to the reflection of the incident shock on the solid surface in the inviscid flow model.

Sketc\(h\) 1
The above sketch shows that a rotation of the coordinate axes by an angle $\theta$ equal to the wedge angle makes the two configurations equivalent. Thus assuming:

$$R_{ex}^{SH} = R_{ex}^c$$

and

$$\left( \frac{P^F}{P_i} \right)_I = \left( \frac{P^F}{P_i} \right)_{II}$$

(2.52)

According to sketch 2, both configurations may be identified.

However, due to the fact that the flow passes across two different shock systems, the downstream free stream Mach number $M_\infty$ must be different in each case

$$(M_\infty)_I \neq (M_\infty)_II$$

(2.53)

The validity of this equivalence principle has been verified experimentally at Mach number 2.2 (ref. 37) by carefully measuring the pressure distributions obtained successively with a shock generator and with a flat plate wedge. It has been shown that both pressure distributions can be matched with a good accuracy over the entire interaction region. Nevertheless, care must be taken when extending this principle of equivalence to high hypersonic Mach numbers.
2.3.2 Entropy variation across the incident shock

The static pressure is continuous at shock impingement, assuming in addition that the other flow parameters, $a$, $B$ and $\delta_i$ also remain continuous at this station, the external flow Mach number must be discontinuous due to entropy change through the incident shock.

Let $M_{e1}$ be the Mach number just upstream of shock impingement point and $M_{e2}$ that just downstream of the shock impingement point. Assuming that the external flow streamlines are parallel to the wall up to $x_0$ and are approximately straight lines between the separation point and the shock impingement, we apply the Prandtl-Meyer relationship:

$$\theta_{e_{SH}} - \theta_e = \theta(M_{e0}) - \theta(M_{eS})$$  \hspace{1cm} (2.54)

Using the oblique shock wave relationship $M_{ec}$ and $p_{ec}$, just behind the incident shock, can be calculated as follows:

$$M_{ec} = \left[ \frac{M_{e1}^2 + 5}{7M_{e1}^2 \sin^2(\theta_G + \theta_{SH}) - 1} + \frac{5M_{e1}^2 \cos^2(\beta_G + \theta_{SH})}{5 + M_{e1}^2 \sin^2(\beta_G + \theta_{SH})} \right]^{1/2}$$  \hspace{1cm} (2.55)
\[
\frac{P_e}{P_{e1}} = 7 M_2^2 \sin^2(\theta + \theta_{SH}) - 1
\]  
(2.56)

Thus the Mach number \(M_{e2}\) downstream of the expansion fan centered at \(x_{SH}\) is:

\[
M_{e2} = \left[ 2 \left( \frac{P_e}{P_{e1}} \right) \left( 1 + \frac{y-1}{2} M_c^2 \right) - 1 \right]^{1/2}
\]  
(5.57)

Furthermore, it is assumed that the compression waves originating from the separated and reattachment regions coalesce far from the wall, so that we neglect the entropy changes across these two systems of compression waves. On the other hand, the free stream Mach number \(M_\infty\) at downstream infinity is calculated according to the inviscid flow model. We take into account the entropy change through both the incident and reflected shock waves, so that the final pressure ratio \(p_\infty/p_e\) obtained by streamwise integration of the governing differential equations, is higher than the theoretical inviscid value.

2.3.3 Nature of the solutions

2.3.3.1 Subcritical and supercritical boundary layers

The introduction of a coupling equation linking the viscous layer to the outer flow introduces some elliptic features into the basic differential equations which allow for an upstream propagation of downstream disturbances. In the framework of integral methods this problem is solved as a two point boundary value problem and the solution of the viscous interaction must be initiated by a suitable perturbation of the undisturbed flat plate solution. This derives from the fact that the problem is not well posed as a boundary value problem, indeed the terms which contain the initial perturbation in the full Navier-Stokes equations have been neglected when one
considers the conventional boundary layer equations.

According to the stability of the governing equations describing the viscous interaction problem, a distinction must be drawn between subcritical and supercritical flows. This distinction, first pointed out by Crocco-Lees (ref. 2) is briefly discussed in Chapter 1. One defines a subcritical boundary layer as one which thickens when it is subjected to an adverse pressure gradient and conversely, a boundary layer is termed supercritical when pressure perturbations produce its thinning.

A number of attempts have been carried out in order to give a physical interpretation to the mathematical behaviour of the governing equations. Some of the most interesting are given by Weinbaum and Garvine (ref. 24) and by Holden (ref. 16).

The former consider the analogy between the behaviour of the inviscid quasi one dimensional stream tube on a two dimensional viscous layer subjected to pressure perturbations. The equation relating pressure and cross section area variations of the one dimensional stream tube is:

\[ \frac{1}{\gamma p} \frac{dp}{dx} \frac{M^2-1}{M^2} = - \frac{1}{A} \frac{dA}{dx} \tag{2.58} \]

where \( M \) is the Mach number, \( p \) static pressure, \( A \) the normal cross section area and \( x \) the streamwise coordinate.

The sign of cross section variations \( \frac{dA}{dx} \) depends upon \( M \).

a) If \( M < 1 \) (subsonic flow) a positive pressure gradient \( \frac{dp}{dx} > 0 \) will produce an inverse of cross section area, \( \frac{dA}{dx} > 0 \). By analogy a boundary layer which responds by thickening to an adverse pressure gradient is termed subcritical.

b) If \( M > 1 \) (supersonic flow) a positive pressure gradient \( \frac{dp}{dx} > 0 \) will produce a decrease in cross section area. Thus a boundary layer thinning under a positive pressure gradient is termed supercritical.
Using simplified analytic considerations the following relation for the local inclination of the outer flow streamline may be derived (ref. 14, 24):

\[
\tan \theta = \frac{1}{\gamma p} \int_0^\delta \left( \frac{1-M^2}{M^2} \right) \frac{dy}{\rho} + \ldots
\]

\(S\) being the thickness of the viscous sublayer adjacent to the wall, it is assumed that \(S \ll \delta\).

The sign of the integral on the RHS of eq. 2.59 depends on the Mach number profile, especially on the relative location of the sonic line with respect to the total boundary layer thickness \(\delta\).

Neglecting the momentum transport through the boundary layer outer edge, we assume:

\[
\tan \theta = \frac{d\delta}{dx}
\]

thus,

\[
\frac{d\delta}{dx} = \frac{1}{\gamma p} \int_0^\delta \left( \frac{1-M^2}{M^2} \right) \frac{dy}{\rho} + \ldots
\]

Assuming supersonic external flow \(M_e > 1\) the response of the boundary layer to a positive pressure gradient depends upon the balance between the respective contribution of subsonic and supersonic portion of the Mach number profile.

a) If \(\int_0^\delta \left( \frac{1-M^2}{M^2} \right) \frac{dy}{\rho} > 0\), \(\frac{d\delta}{dx} > 0\) + subcritical behaviour;

b) If \(\int_0^\delta \left( \frac{1-M^2}{M^2} \right) \frac{dy}{\rho} < 0\), \(\frac{d\delta}{dx} < 0\) + supercritical behaviour.

A subcritical boundary layer can generate its own pressure gradient, i.e., an adverse pressure gradient produces a thickening which in turn increases the pressure gradient and
so on. Conversely, a supercritical boundary layer responds by thinning to a positive pressure gradient, therefore damping the initial pressure perturbation.

An important result of the distinction between subcritical and supercritical nature of the boundary layer is that the upstream propagation of disturbances is permissible as long as the flow remains subcritical. Therefore, when a boundary layer initially supercritical is subjected to an adverse pressure gradient, the "upstream influence" must be introduced through a sudden transition from supercritical to subcritical state.

2.3.3.2 Mathematical aspect of the singularities

Within the framework of a multimoment integral theory, the analysis of the singularities of the set of governing differential equations provides a way of selecting physically realistic solutions.

The general form of governing differential equations is:

\[
\frac{d\xi_i}{dx} = \frac{N_i(\xi_1, \ldots, \xi_k)}{D(\xi_1, \ldots, \xi_k)} \quad 1 < i < k
\]  

(2.61)

where \( k \) is the number of unknown flow parameters \( \xi \). The above set of first order ordinary differential equations can be solved numerically provided that \( D \neq 0 \).

Nevertheless, the solution exists if, when \( D(\xi \to 0) \), the simultaneous vanishing of all the numerators \( N_i(\xi) \) can be achieved. A singular point is then located. The existence and the uniqueness of the solution through the critical point depends on the nature of the latter.

Considering now our particular case, \( D \) depends upon \( M_\infty, a \) and \( b \) or \( B \) for a given value of \( S_w \). A typical example
of the behaviour of $D(a, b)$ for fixed values of $M_e$ is presented in fig. 2.8. A unique root of this function is contained within the physically realistic range of variation of parameters $a$ and $b$ (according to shock wave boundary layer interaction problem herein considered). In the framework of an integral theory, the passage through this critical point corresponds to sub-supercritical transition (or vice versa).

2.3.3.3 Transition from subcritical to supercritical boundary layer (and reciprocal)

The locus of critical points for different values of $S_w$ can be derived from a plot of $a_{CR}$ as a function of $b$ and $M_e$ according to the following equation:

$$D(a_{CR}, M_e, b) = 0$$

A plot of $a_{CR}$ versus $M_e$ is presented at fig. 2.9 for $S_w = -0.8$. The plots corresponding to $S_w = -0.6, -0.4$ and $-0.2$ can be found in ref. 50. In the adiabatic case ($S_w = 0$), $a_{CR} > a_{Blasius}$ whatever the value of $M_e$, thus the critical point always lies outside the useful range of the parameter $a$.

A complete analysis of the different types of transition sub-supercritical is reported in ref. 14. Considering only the viscous interaction generated by a compressive disturbance, two cases must be analyzed leading to two different computation procedures. These are related to the boundary layer nature (sub- or supercritical) at the beginning of the interaction $x_0$.

a) Initially subcritical flow: Pressure distributions due to the impinging shock are propagated upstream over a large distance (more than ten boundary layer thicknesses). With viscous dissipation the magnitude of these perturbations decreases exponentially in the upstream direction. The beginning of the interaction $x_0$ is then taken as the point where the magnitude of perturbations is less than an arbitrary value $\varepsilon$. The whole interaction lies in the subcritical domain, and the
integration of governing differential equations proceeds smoothly without singularities.

b) Initially supercritical flows: In the case of interactions on a highly cooled wall, the critical point lies between separation and Blasius solution (e.g., $0 < a_{CR} < a_{Blasius}$) (see ref. 50) even at low Mach numbers. In any shock wave boundary layer interaction where the wall has been sufficiently cooled down, the upstream undisturbed flat plate solution lies in the supercritical domain. Therefore, to allow for the upstream propagation of disturbances, a transition from supercritical to subcritical state must be located between the end of the undisturbed flat plate flow and the separation point. This transition causes a sudden variation of the flow variables $M_e$, $M_i$, $a$ and $b$ which will be assimilated to a discontinuity.

Considering now the flow downstream of the shock impingement point, a second singular point may arise under conditions such that $D \to 0$. This singular point (Crocco-Lees critical point) corresponds to a subcritical-supercritical transition. However, no discontinuity of the flow parameters is required at the critical point location, indeed, the upstream influence is permissible in the subcritical flow regime. It has been shown in ref. 22 and 23 that the Crocco-Lees critical point is a saddle point type singularity. Therefore, the condition that the integral solution must pass through the singular point is sufficient to select the single curve which will enable to satisfy further downstream conditions, however, the critical point location is not known a priori.

Klineberg (ref. 14) shows that smooth subcritical-supercritical transition can be located on a flat surface downstream of a compressible disturbance (for $S_w = -0.8$ and $M_e > 2$). Writing simultaneously $N_1 \to 0$ and $D \to 0$ he deduces the required sign of streamline inclination at the critical point location (throat):

$$a_w + \beta(M_e) - \beta(M_e) < 0 \quad (2.63)$$
In the particular case of viscous interaction generated by a flat plate wedge configuration where \( \alpha_w = \text{cst} \) the above inequality can be satisfied if \( M_e > M_* \), downstream of shock impingement.

The numerical methods of solution applied to shock wave boundary layer interaction calculations derive from the analysis of the mathematical singularities of the governing equations briefly discussed above. According to the subcritical or supercritical nature of the boundary layer approaching the interaction region, two different numerical procedures must be developed.

2.4 Numerical methods of solution

This section describes the methods of solution for the governing equations (eq. 2.37) applied to shock wave boundary layer interaction generated by a compression corner.

2.4.1 The weak interaction region

According to the physical model of the flow described in section 2.3.1 we assume that a self preserved flat plate flow exists "far" upstream and far downstream of the interaction region.

For a sharp-nosed body, the flow in the neighbourhood of the leading edge may be divided into two regions:

- a strong interaction region close to the leading edge where
  \[
  M_0 \frac{\partial \Theta}{\partial x} \gg 1 \quad \text{(hypersonic similarity parameter)}
  \]

- a weak interaction region further downstream where
  \[
  M_0 \frac{\partial \Theta}{\partial x} \ll 1
  \]

Asymptotic expansion of the flow variables can be obtained in both limits. Kubota (ref. 33) performed a second order weak
interaction expansion for the adiabatic case and Klineberg (ref. 14) further extended this result to both strong and weak interaction regimes including heat transfer. Starting from Klineberg's results we include the normalization with respect to $S_w$ as presented in section 2.2.3.

Substituting $\chi$, the viscous interaction parameter for $x$ in equations 2.29 to 2.32:

$$\chi = \frac{M^3 \sqrt{C}}{\sqrt{Re x}} \quad (2.64)$$

and assuming in addition that $\frac{dS}{dx} = 0$ (isothermal surface), we get

$$\frac{F}{\chi} \frac{d\Delta}{d\chi} + \Delta \left[ \frac{3F}{a} \frac{d\alpha}{d\chi} + \frac{3F}{B} \frac{dB}{d\chi} \right] + \frac{F}{\frac{\Delta}{M_e}} \frac{dM_e}{d\chi} =$$

$$= \frac{1}{X} \left[ \frac{\gamma + 1}{2(\gamma - 1)} \frac{1 + m_\infty}{l + m_e} \frac{M^3 + g_e}{m_e X} \right] \quad (2.65)$$

$$\frac{\mu_e}{\chi} \frac{d\Delta}{d\chi} + \Delta \frac{d\mu_e}{d\chi} \frac{da}{d\chi} + (2 \frac{M_\infty}{l + S_w} \frac{1 + \sigma}{\frac{\Delta}{M_e}}) \frac{dM_e}{d\chi} =$$

$$= \frac{1}{X} \left[ \frac{3\gamma - 1}{2(\gamma - 1)} \frac{1 + m_\infty}{l + m_e} \frac{M_\infty}{\frac{P}{M_e}} \right] \quad (2.66)$$
\[
\begin{align*}
J \frac{d\Delta}{d\bar{x}} + \Delta \frac{dt}{d\bar{x}} - \frac{d\bar{d}}{d\bar{x}} - \frac{d\bar{d}}{d\bar{x}} + (3J + 2ST) \frac{-dt}{M_e \frac{d\bar{d}}{d\bar{x}}} = \\
= \frac{1}{X} \left[ J \Delta - 2(\frac{l+m}{l+m_e}) \frac{M_{\infty}}{M_e} \frac{R}{\Delta} \right] \\
= \frac{1}{X} \left[ \frac{3y-1}{2(y-1)} \right]
\end{align*}
\]

\[
\begin{align*}
\frac{T}{T} \frac{d\Delta}{d\bar{x}} + \Delta \left[ \frac{3y-1}{2(y-1)} \right] \\
= \frac{1}{X} \left[ \frac{3y-1}{2(y-1)} \right]
\end{align*}
\] (2.67) (2.68)

where

\[
t_{60} = \int_{M_e}^{M_{\infty}} \frac{\sqrt{M^2 - 1}}{1 + \frac{\gamma - 1}{2} M^2} \frac{dM}{M} 
\] (2.69)

and

\[
\Delta = \frac{Re_{\delta}}{M^3 c} 
\] (2.70)

In the vicinity of the Blasius solution the flow variables are expressed by coordinate expansions in small values of \( \bar{x} \).

\[
M_e = M_\infty(1 + m_1 \bar{x} + m_2 \bar{x}^2 + \ldots)
\]

\[
\Delta = \delta_0(1 + \delta_1 \bar{x} + \delta_2 \bar{x}^2 + \ldots) 
\] (2.71)
Introducing these expressions into equations 2.65 to 2.68 and equating terms of the same degree in \( \overline{X} \) an algebraic set of equations is obtained which provides the numerical values of coefficients \( m_1, m_2, \delta_0, \delta_1, \) etc.

The complete equations for these coefficients and their numerical values for different \( S_w \) are given in Appendix B. A numerical application of the weak interaction theory has been performed. The computer program which has been used is described in Appendix C.

Considering a flat plate flow the free stream conditions are: \( M_e = 6.06, \) \( Re_u = 0.239 \times 10^7 \) per meter, the wall temperature conditions correspond successively to \( S_w = -0.8, -0.6, -0.4, -0.2, \) and 0. Figures 2.10a to 2.10f show the plots of the four flow variables \( M_e, \delta, a, B \) and both skin friction and heat transfer coefficients defined as follow:

\[
C_F = \frac{\mu \frac{3u}{\partial y}}{\rho_e u_e^2} \\
C_H = \frac{k \frac{3T}{\partial y}}{\rho_e u_e (h_0 e - h_0 w)}
\]

versus the Reynolds number based on the streamwise coordinate \( x \).

\[
Re_x = \frac{\rho_e u_e}{u_e^2} x
\]

The numerical application has been first performed using the earlier Klineberg's formulation. A table of integral functions has been calculated for each of the five values of \( S_w \) investigated herein (see Appendix A). But a second step calculation has also been carried out in order to check the reliability of the non dimensional method described in section 2.2.3.
For this purpose the polynomial functions established for 
$S_w = -0.8$ and $S_w = 0$ respectively have been used as "reference" 
values in the calculation of "universal" polynomial functions 
as described in section 2.2.5 and then applied to intermediate 
values of $S_w$.

The comparison with the "exact" computations previously 
done reveals the effect of the "universal" integral functions 
substitution.

Considering figs. 2.10a et 2.10b a perfect agreement 
of both methods is obtained for pressure and transformed displacement thickness distributions, as well as for skin friction 
and heat transfer coefficients (fig. 2.10e and 2.10f). Small discrepancies arise in the distribution of profile parameters 
a(x) and B(x) as indicated in figs. 2.10c and 2.10d.

In conclusion, the non dimensionalization of the 
integral functions with respect to $S_w$ has been proved valid 
for flat plate flow in weak interaction regime.

2.4.2 Departure conditions 
(Upstream boundary conditions)

As previously mentioned the set of governing differential equations (eq. 2.37) is integrated as a two point 
boundary value problem. The initial conditions belong to the 
undisturbed flat plate flow (the weak interaction region) 
whilst the assumption of a self preserved Blasius flow "far" 
downstream of the interaction provides the second boundary 
condition. The departure conditions for streamwise integration 
are obtained by suitably perturbing the weak interaction 
solution at some arbitrary selected point $x_0$, the correct 
location of this point being determined by satisfying the 
downstream boundary condition.

According to the subcritical or supercritical nature 
of the boundary layer at $x_0$ location, two different procedures
must be developed to properly initiate the streamwise integration.

2.4.2.1 Subcritical Boundary Layer

The departure integral solution is generated by perturbing the flat plate solution by a small amount. Kubota (ref. 19) for the adiabatic case and Klineberg (ref. 14) including heat transfer found the appropriate form of the initial perturbation from the linearization of the hypersonic form of the governing differential equations. The departure integral path originating from the weak interaction solution belong to a single parameter family and can be expressed in terms of the initial perturbation $\varepsilon$. Thus:

$$M^e = M^e_0 (1 + P_1 \varepsilon)$$

$$\delta_i^e = \delta_i^e_0 (1 + P_2 \varepsilon)$$

$$a = a_0 (1 + P_3 \varepsilon)$$

$$B = B_0 (1 + P_4 \varepsilon)$$

where

$$P_1 = \frac{dJ}{d\Psi} - J$$

$$P_2 = 3J + 2S_{wT} - (2\Psi + 1 + S_{wE}) \frac{dJ}{d\Psi}$$

$$P_3 = \frac{(2\Psi + 1 + S_{wE})J - (3J + 2S_{wE})\Psi}{\frac{d\Psi}{da}}$$

$$P_4 = -S_{wT}(P_1 + P_2) - P_3 S_{wT} \frac{\partial T}{\partial B}$$
\( \varepsilon < 0 \) generates a departure integral path tending toward a compression and therefore to separation, whilst \( \varepsilon > 0 \) generates an expansion flow. Because of the requirement \( \varepsilon < \ll 1 \) for the linearization of the equations to be valid, a two stage iteration of initial conditions must be performed. \( x_0 \), the location of the beginning of the interaction, must be first iterated for a fixed, small value of \( \varepsilon (\approx 10^{-3}) \) until the complete solution satisfies approximately the downstream boundary condition. Then, \( x_0 \) being fixed, the correct integral path is obtained by completing the iteration on \( \varepsilon \).

### 2.4.2.2 Supercritical boundary layer

According to the discussion of section 2.3.3, a supercritical boundary layer cannot propagate upstream the perturbations felt at some point. Thus, in order to satisfy the physical model of the flow a sudden transition from supercritical to subcritical state is required to properly initiate the integration of the governing equations. In the framework of the integral theory this transition can be simulated by a sudden jump in the flow variables at the beginning of the interaction. Klineberg (ref. 14) writing suitable conservation equations across the discontinuity found a set of initial conditions \( (M, \delta M, a, b, \text{ just downstream of jump location}) \) which initiates the streamwise integration in the subcritical domain. It should be noted that boundary layer assumption must be released at the location of the jump.

[Sketch 4]
According to the schematic representation of the discontinuity sketched above, we write the following conservation equations, assuming that both the effects of skin friction and heat transfer vanish as well as the volume dissipation ($\Delta x_1, \Delta x_2 + 0$).

Mass flux:

$$\dot{m}_2 - \dot{m}_1 = (\rho u_e e_1) (\delta_2 - \delta_1) \quad (2.76)$$

Momentum flux:

$$I_2 - I_1 = (\rho u_e^2 e_1) (\delta_2 - \delta_1) - \delta_2 (p_2 - p_1) \quad (2.77)$$

Total enthalpy:

$$(M_e \delta_{e}^{mN})_2 - (M_e \delta_{e}^{mN})_1 = 0 \quad (2.78)$$

Writing the integral quantities $\dot{m}$, $I$ and $\delta$ in the transformed plane:

$$\delta = \int \rho u dy = \rho_e a_e M_e \delta_{e}^{mZ}$$

$$I = \int \rho u^2 dy = \rho_e a_e u_e M_e \delta_{e}^{m}(Z - \delta)$$

$$\delta = \frac{\rho_e a_e}{\rho_e a_e} \delta_{e}^{m} (m_e F + \delta) \quad (2.79)$$

Although the mechanical energy is not conserv, the integral equation of moment of momentum takes a form similar to the momentum equations as the control volume vanishes ($\Delta x = \Delta x_1 + \Delta x_2 + 0$).

Considering the equation (2.77) and substituting $K = \int u dy$ for $\delta$ Klineberg found the following relation:
\[ G_2 - G_1 = (\rho_e u_e^3) (\delta_2 - \delta_1) - 2\kappa_2 (p_2 - p_1) \]  

(2.80)

where

\[ G = \int_0^\delta \rho u^3 dy = \rho \infty a_\infty u_e^2 M_e \delta_i^\infty (Z-J) \]

and

\[ \kappa = \int_0^\delta u dy = \rho \infty a_\infty M_e \delta_i^\infty \{(1+m_e)(Z-T^\infty) - m_e(Z-J)\} \]

Substituting the expressions for \( m, I, G \) and \( \kappa \) into eq. 2.76, 2.77, 2.78 and 2.80, one gets a set of three algebraic equations relating the flow variables across the jump.

\[ m_e F_2 \left[ \frac{J_1}{m_e F_1} - \frac{J_2}{m_e F_2} \right] + \frac{1}{\gamma M_e^2} \left[ (p_r - 1) \left( m_e F_2 + Z_2 \right) - (1 - p_r u_r^2) (Z_2 - \frac{J_2}{m_e F_1}) \right] \]

\[ + \left( 1 - p_r u_r^2 \right) \left( 1 + \frac{J_1}{m_e F_1} \right) Z_2 = 0 \]

(2.82)

\[ m_e F_2 \left[ \frac{\bar{T}_1}{m_e F_1} - \frac{\bar{T}_2}{m_e F_2} \right] + \left( 1 - p_r u_r^3 \right) (Z_2 - J_2) + \left( 1 - p_r u_r^2 \right) \left( 1 + \frac{J_1}{m_e F_1} \right) Z_2 = 0 \]

(2.83)

\[ m_e F_2 \left[ \frac{\bar{T}_1}{m_e F_1} - \frac{\bar{T}_2}{m_e F_2} \right] + (1 - p_r u_r) \left[ \frac{\bar{T}_2}{m_e F_1} + \frac{\bar{T}_1 Z_2}{m_e F_1} \right] = 0 \]

(2.84)

where subscripts ( )_1 and ( )_2 refer respectively to the flow variable just upstream and just downstream of the discontinuity.

It is assumed that the external flow experiences an oblique shock wave at the jump location, the magnitude of the shock strength being fixed by \( M_{e_2}/M_{e_1} \). The pressure, density and velocity ratios (subscript ( )_r) are obtained from the
oblique shock wave relationship

\[
\frac{u_2}{u_1} = \frac{\rho_2}{\rho_1} = \frac{p_2}{p_1}
\]

The following departure procedure is adopted for the supercritical boundary layer. The jump location \( x_0 \) being arbitrarily taken, the three algebraic jump equations (eq. 2.82 to 2.84) are solved simultaneously by iteration of the three variables \( M_2 \), \( a_2 \) and \( B_2 \). From particular numerical applications it has been found that rapid convergence of the jump equations is obtained at high wall cooling rates \((-0.8 < S_w < -0.5)\) but difficulties have been encountered in the limiting case \( (S_w = -0.4) \) where the magnitude of the discontinuity in flow variables becomes very small. However, the small perturbation departure described in section 2.4.2.1 may apply, the requirement for \( |\epsilon| \ll 1 \) being again justified. On the other hand, Klineberg shows that the jump equations reduce to the governing equations for continuum flow in the limiting case of an infinitesimal jump.

2.4.3 Downstream boundary conditions

In the case of semi infinite ramp length, it is assumed that the flow downstream of the interaction region tends to a Blasius solution at downstream infinity. According to the subcritical or supercritical nature of the boundary layer (i.e., the existence of the Crocco-Lees critical point) a specific boundary condition must be defined for each of the two cases.

2.4.3.1 Downstream subcritical flow

The unstable behaviour of downstream solution inherent to numerical streamwise integration of the governing differential equations provides two families of diverging solutions, the exact anticipated integral path lying between two curves of different types.
Sketch 5 shows qualitatively the two families of pressure distributions obtained from successive iterations on the initial conditions. Curve 1 is typical of the flow going toward an expansion ($\frac{dM_e}{dx} < 0$) which would be located further downstream. Curve 2 is typical of the flow ahead of a second compressive disturbance which would be located further downstream ($\frac{da}{dx} < 0$).

After a number of iterations on both $x_0$ and $\varepsilon$, both integral paths of type 1 and 2 become indistinguishable up to a divergence point (point A on sketch 5) located downstream of the reattachment point. In order to obtain the correct integral path within a reasonable number of iterations on initial conditions, an interpolation procedure has been performed.

When the difference between two adjacent solutions of different type becomes greater than an arbitrary value (herein $\frac{\Delta \xi}{p_e} = 0.5 \times 10^{-3}$), the downstream integration is started again at the mid-point B, and continued until two other diverging solutions are found. This operation is repeated, moving the divergence point A further downstream until the correct integral path is entirely determined.

In principle, this procedure is applied to downstream infinity except when a singular point is encountered.
For the particular case of a compression wedge considered herein, the boundary layer is subcritical in the range $-0.35 < S_w < 0$. On the other hand, it can be seen from plots of the locus of critical points (fig. 2.9) that the critical boundary is quasi independent of Mach number $M_e$ above $M_e = 3$. As a result the nature of the boundary layer (sub- or supercritical) remains the same upstream and downstream of the interaction region as long as the flow is hypersonic and the turning angle ($\Theta_{ramp}$) remains small.

2.4.3.2 Downstream supercritical flow

This situation occurs when the wall has been sufficiently cooled down. Considering fig. 2.9 it can be seen that $0 < a_{CR} < a_{Blasius}$ for $b < b_{Blasius}$ if $M_e > 2$, therefore a transition from subcritical to supercritical boundary layer occurs downstream of the hinge line ($x > x_c$) or the shock impingement point ($x > x_{SH}$). As shown in §2.3.3.3 this transition is associated with a smooth passage through a saddle point type singularity (Crocco-Lees critical point), therefore a single integral path can satisfy this requirement. However, a streamwise numerical integration cannot provide the mathematically exact solution, then the following iteration procedure is applied.

The solution downstream of reattachment is extremely sensitive to initial conditions (i.e., the location of $x_0$) and the number of iterations can be reduced to some extent using the interpolation procedure described in section 2.4.3.1 but the exact location of the critical point cannot be found numerically. The numerical suppression of diverging curves (downstream of reattachment point) forces the solution toward the singularity : $N_1 = D = 0$. When the solutions depart too rapidly to allow for accurate interpolations, this numerical procedure must be stopped.

A graphical extrapolation of the flow variables
2.5 Numerical results

The theory developed herein has been first applied to a parametric study of the wall cooling effect in the overall features of shock wave boundary layer interaction. For this purpose, the original formulation of Klineberg has been adopted. A second application using the non dimensional formulation with respect to the wall cooling rate \( S_w \) has been carried out. The later results have been compared with the previous ones considered as "exact" solutions, thus proving the reliability of the method.

2.5.1 Parametric study of wall cooling effect

Numerical results of this study have been reported in ref. 50, therefore this section is devoted to recalling the main results.

The physical data for theoretical computations are those used in Lewis' experimental study (ref. 35).

Using a flat plate ramp configuration, the hinge line is located 63.5 mm aft of the leading edge and the ramp angle is \( \theta_{\text{ramp}} = 10.25^\circ \). The free stream Mach number and unit Reynolds numbers are respectively \( M = 6.06 \) and \( 2.4 < Re_u < 4.75 \times 10^6 \) per meter.

The wall cooling effect upon pressure, skin friction and heat transfer distributions has been analysed through suc-
cessive calculations carried out at five different wall cooling rates: \( S = -0.8, -0.6, -0.4, -0.2 \) and 0. The results are presented respectively at fig. 2.11 (pressure), fig. 2.12 (skin friction) and 2.13 (heat transfer).

The main conclusions which can be drawn are the following:
- Cooling the surface produces a large decrease of the upstream influence and therefore a decrease of the separated length; consequently, pressure gradients in both separation and reattachment region increase. Figure 2.14 summarizes the effect of surface cooling upon the characteristic lengths of the interaction;
- \( x_0/L \) is the normalized abscissa of the beginning of the interaction;
- \( x_s/L \) is the normalized abscissa of the separation point;
- \( x_r/L \) is the normalized abscissa of the reattachment point. 
L is a reference length; here \( L \equiv x_c \) or \( x_{SH} \).
All these quantities are quasilinear functions of the ratio \( T_w/T_t \). This result is in agreement with the free interaction calculations performed by Curle (ref. 36).
On the other hand, surface cooling produces an increase of skin friction (in the separated region) and gives rise to a sharp heat transfer peak in the reattachment region.

Figures 2.15 and 2.16 show the effect of wall cooling upon the extremes of skin friction and heat transfer distribution. The minimum values \( C_{F\min} \) and \( C_{H\min} \) reached at the hinge line location appear quite independent of the wall cooling rate but a marked increase of the reattachment heat peak must also be noted.

2.5.2 Normalization of the integral functions

According to Klineberg's formulation the total enthalpy profile dependent functions are \( E, Q, T^m \) as well as the profile parameter \( b \). We define:
The integral functions $\bar{E}$, $\bar{Q}$, $\bar{T}$, as well as the velocity profile dependent functions ($\bar{V}$, $J$, $Z$, $E$, etc.) have been found quasi independent on $S_w$ according to discussion and plots reported in section 2.2.5.

On the other hand, it has been shown that the $S_w$ dependence can be reintroduced in a simple way when it cannot be entirely removed (functions $P(a)$, $\frac{dQ}{da}(a)$, $a(a)$ and $\frac{da}{da}(a)$ for example). A unique set of integral functions is then established from one of the five tables already calculated for five different values of $S_w$ (see Appendix A). The selected value of $S_w$ will be called $S_{wref}$.

The normalization of the different functions with respect to $S_w$ is then performed step by step simultaneously with the integration of the basic differential equation. Indeed, for every value of $a$ and $b$, numerical value of the "universal" integral functions are calculated as follows:

$\frac{\sigma(b)}{(-S_{wref})}$, $J$, $Z$, $P$, $\frac{d\bar{Q}}{da}$, $\frac{dJ}{da}$, $a$, $\frac{da}{da}$

and

$\bar{E} = \frac{\sigma(b)}{(-S_{wref})}$, $\bar{T} = \frac{T(a,b)}{(-S_{wref})}$, $\frac{d\bar{E}}{db} = \frac{\sigma}{db}(b)$

$\frac{2\Gamma}{3a} = \frac{2T(a,b)}{(-S_{wref})}$, $\frac{3\Gamma}{3B} = \frac{3T(a,b)}{3b}$

provided that $b = (-S_{wref})B$. 

\begin{equation}
\begin{aligned}
\bar{E} &= \frac{E}{(-S_w)_{\text{ref}}} \\
\bar{Q} &= \frac{Q}{(-S_w)_{\text{ref}}} \\
\bar{T} &= \frac{T}{(-S_w)_{\text{ref}}} \\
B &= \frac{b}{(-S_w)_{\text{ref}}} \\
\end{aligned}
\end{equation} 

(2.86)
The normalized integral functions (eq. 2.86) are then deduced from the above calculations:

\[ E = a \Gamma, \quad T = \alpha \Gamma, \quad Q = \frac{B}{\alpha} \quad (2.88) \]

together with their derivatives:

\[ \frac{3E}{3a} = \alpha \frac{d\alpha}{da} + \frac{3E}{3a} = \alpha \frac{d\beta}{dB}, \quad \frac{3T}{3a} = \Gamma \frac{d\alpha}{da} + \alpha \frac{3\Gamma}{3a}, \quad \frac{3T}{3B} = \alpha \frac{3\Gamma}{3B}, \text{ etc.} \]

In the particular case of an adiabatic wall \( (S_w = 0) \) the above procedure does not apply and the normalized integral functions \( \Gamma, \Gamma, \zeta, \text{ etc.} \) are directly tabulated from similar solutions \(^1\) (see Appendix A).

In order to avoid oscillations associated with the use of a high degree polynomial, the functions dependent on the total enthalpy parameter have been represented by two polynomials, respectively valid for \( B < 0.36 \) and \( B > 0.35 \).

---

\(^1\) Comment: In the adiabatic case, the normalized integral functions defined by eq. (2.86) take a form similar to those used in ref. 38, dealing with the problem of shock wave boundary layer interaction on a spinning body of revolution. 

\( v \) is a transverse velocity component normal to the main flow velocity \( u \). Assuming zero heat transfer, the following equations, similar in form to Falkner-Skan's equations can be derived:

\[ f''' + ff'' - \beta f'^2 + \beta (1+\omega(2g-g^2)) = 0 \quad (2.89) \]

\[ g'' + fg' = 0 \]

where \( f' = \frac{U}{U_e} \) and \( g = \frac{v}{v_0} \).

When the rate of rotation \( \Omega \) and therefore the spin parameter \( \omega \) is zero, the equations (2.89) become similar in form to eq. 2.22a and b, where \( S_w = 0 \), i.e.,

\[ f''' + ff'' + \beta (1-f'^2) = 0 \quad (2.90) \]

\[ G' + fg' = 0 \]

the associated boundary conditions are:

\[ n = 0 \quad f = f' = 0 \quad G = 1 \quad (2.91) \]

\[ n \to \infty \quad f' \to 1 \quad G \to 0 \]
Letting \((1-G) = \frac{V}{V_0}\) the solutions for both systems \((2.89)\) and \((2.90)\) are identical.

Numerical calculations to establish the validity of the non dimensional method have been carried out. The physical data of the viscous interaction considered here are identical to those of § 2.5.1.

Both tables of integral functions established respectively for \(S_w = -0.8\) and \(S_w = 0\) have been successively used as "reference" values for the computation of "universal" integral relationships.

The static pressure distributions dealing with different values of the actual wall cooling rate \(S_w\) are presented in fig. 2.17a. Skin friction and heat transfer distributions (for the two extreme cases \(S_w = -0.8\) and \(S_w = 0\)) are presented at fig. 2.17b and 2.17c respectively.

The main discrepancies between "exact" and approximate calculations occur at the beginning of the interaction, indeed, the magnitude of the jump is very sensitive to the change of integral functions. However, these discrepancies remain less than 15% for the rear part of skin friction and heat transfer distribution.

Finally, a comparison between "exact" and approximate solutions has been carried out for an intermediate value of wall cooling rate \((S_w = -0.4)\), the "universal" integral

---

1 In the adiabatic case the heat flux at the wall vanishes but the heat transfer coefficient \(C_H\) remains finite. The undeterminacy arising from the definition of \(C_H\) has been removed because \(S_v\) cancels in the non dimensional form of the energy equation \((\text{eq. 2.32})\), provided \(dS_v/dx = 0\).

2 The term "exact" refers to the computation performed with \(S_w = S_{w\text{ ref}}\).
functions being again calculated from $S_{\text{ref}} = -0.8$ and $S_{\text{ref}} = 0$. Figures 2.1a to 2.1f show the distributions of pressure, skin friction and heat transfer as well as the variations of the flow parameters $\delta_l$, $a$ and $B$. The error inherent to approximate calculation methods are contained within $5\%$ for static pressure and $10\%$ for skin friction and heat transfer distributions.

Nevertheless, the main interest of this method is the opportunity it offers to treat shock wave boundary layer interactions where the wall temperature is a prescribed function of the streamwise coordinate (i.e., $T_w(x)$).

2.6 Concluding remarks

The shock wave laminar boundary layer interaction problem has been theoretically investigated in the framework of the Lee-Reeves-Klineberg's integral method. The present theory applies to interactions induced either by flat plate ramp configurations or by an incident shock wave impinging on a flat plate. The problem can be entirely solved without any recourse to experimental or empirical informations.

Assuming an isothermal wall, Klineberg's theory has been extended to different wall cooling rates lying between highly cooled and adiabatic wall conditions.

A parametric study of the effects of wall cooling on pressure, skin friction and heat transfer distributions has been carried out.

A suitable normalization of the total enthalpy profile dependent functions partially removes their dependence upon $S_w$. It is then possible to establish a table of "universal" integral functions valid for every value of $S_w$.

It has been shown that this generalized method
applies with good accuracy to a large range of wall cooling rates $- 0.8 < S_w < 0$. Furthermore, the method provides a simple calculation method for flows with prescribed wall temperature distributions.
3. Experimental Investigation of the Effect of Wall Cooling on the Overall Features of Shock Wave Boundary Layer Interaction

3.1 Introduction

The main purpose of these tests is to check the theoretical predictions described in section 2 and, particularly, the trends derived from the parametric study of wall cooling effects.

The viscous interaction is generated by flat plate ramp models placed in a moderately hypersonic airstream at a Mach number of 6. The experiments were carried out in the hypersonic wind tunnel H-3 of the von Karman Institute. This tunnel is equipped with an axisymmetric nozzle giving a uniform supersonic flow at Mach 6. The test chamber has a free jet configuration and the models are placed between the nozzle exit and a downstream conical diffuser. Because of non uniformity in the flow near the axis of the free jet the models are located slightly off the nozzle centreline (22.5 mm below the axis and 3.5 mm downstream of the nozzle exit); the Mach number nonuniformities are then within 1% of the mean value. The nominal free stream Mach number used for all the experiments is $M_\infty = 5.96$. The stagnation pressure can be adjusted between 5 and 30 kg/m$^2$ whilst the stagnation temperature is up to 350°C.

The surface temperature of these models can be kept uniform and constant in the neighbourhood of the three following values: -195°C, +20°C and +120°C corresponding respectively to highly cooled, moderately cooled and quasi adiabatic surface conditions.

Static pressure and heat flux distribution measurements have been carried out. Design and instrumentation of models used for both this purposes are presented in section 3.2.
and section 3.3 respectively. The external geometry of both models has been determined by preliminary tests according to the following requirements: The flow over the whole interaction region must be laminar and two dimensional.

The overall dimensions of the models have to be selected according to the tunnel test chamber requirements, and the two remaining parameters: $\theta$ the ramp angle and $x_c$ the abscissa of the hinge line must be optimized.

3.1.1 Two dimensional flow

According to the conclusions of ref. 35, the flow over a flat plate ramp model can be considered as two dimensional provided that the aspect ratio AR ($\text{span} / x_c$) is greater than 1. For models described herein AR = 2.5. On the other hand, the two dimensional character of the flow field has been confirmed by surface visualization using a sublimation technique (see section 3.3.5).

3.1.2 Transition

$x_{tr}$ the abscissa of the transition point on a flat plate placed in a supersonic flow depends on many factors, particularly on the free stream Reynolds number, or wall to stagnation temperature ratio and on leading edge thickness. It has been shown (refs. 21-40) that the transition located in the reattachment region moves forward when $\theta_{\text{ramp}}$ increases, and is delayed when the wall is cooled down (ref. 41). Thus $\theta_{\text{ramp}}$ has to be selected such that, for adiabatic wall conditions the transition can be located far downstream of the reattachment region.

The ramp angle has been fixed at 7°30' and the length of the flat plate ($x_c$) has been determined in order to obtain a fully laminar interaction for adiabatic wall conditions and the maximum free stream Reynolds number available ($Re_u = 2.5 \times 10^7$ per meter). The laminar nature of the interaction region has
been checked using the criteria established in ref. 44. Finally, a value of \( x_c = 40 \text{ mm} \) for a ramp angle \( \theta = 7^\circ 30' \) has been selected.

### 3.2 Heat transfer measurements

The thin skin transient technique has been used for the heat flux distribution measurements. The model covered with a thin metallic skin is hollow. It is kept prior to the run at a constant and uniform temperature and suddenly injected into the hypersonic stream. The temperature time history of the surface can be recorded at different locations along the model by means of several thermocouples welded to the inner surface of the skin.

With the following assumptions,

1. constant skin thickness,
2. the temperature gradient through the skin can be neglected,
3. no heat is transferred to the interior of the model,
4. no heat is conducted along the model surface,

the local heat flux is:

\[
q = \left( \rho C_p d_M \right) \frac{dT}{dt} \tag{3.1}
\]

where \( \rho_M \) is the specific mass of the skin material,
\( C_{PM} \) is the specific heat of the skin material,
\( d_M \) is the skin thickness.

#### 3.2.1 Model design

Figures 3.1a and 3.1b show the model used. This model is made from stainless steel. The thin skin was fabricated by milling a rectangular groove along the model centerline. This cavity is covered by a sheet of stainless steel (\( 8 \text{ mm} \) thick) equipped with 16 thermocouples welded into small holes (\( .3 \text{ mm} \) \( \phi \)) drilled through the skin. The tubing for coolant circulation was drilled through the remaining solid part of the model. The overall dimensions of the model are: length 150 mm, span 100 mm
flat plate length 40 mm, ramp angle 7°30', Central cavity: length 90 mm, width 50 mm. The thermocouples location are given in fig. 3.1a. These thermocouples are made with two wires of copper and constantan of 0.1 mm diameter.

The model has to be injected quickly into the supersonic flow. For this purpose it is installed on a rocking support moved by a pneumatic system (see fig. 3.2). In order to avoid breakdown of the hypersonic flow during the injection process, the diffusor must be maintained around the model during the test thus a part of this diffusor has been cut out and attached to the support. The whole system closed up around the model after injection. The entire injection movement took less than 1/10 sec.

The heat transfer measurements have been carried out for two different wall cooling rates:

a) the model is initially at room temperature,
b) the model is cooled down prior to run.

In the last case, a low surface temperature can be reached by both internal circulation and spraying the upper surface with a mixture of gaseous and liquid nitrogen. In order to avoid frost formation at the model surface during cooling down, both the model and the spraying tube are enclosed into a plastic bag which traps the dry cold nitrogen around the model; this envelope can be removed quickly before the model injection into the supersonic jet. This system ensures a satisfactory surface temperature uniformity which will be discussed in section 3.2.3.

3.2.2 Instrumentation

The thermocouples outputs are connected to a C.E.C. type 5-124 oscillograph recorder. Twelve channels can be used simultaneously (11 are connected to the model and the 12th measures the stagnation temperature of the free stream). The galvanometer sensitivity lies between 0.015 and 0.045 mV/mm. The time base used for most of the tests is 2 cm/sec and a
useful record is obtained for 5 seconds following injection.

The reference junctions of all thermocouples are immersed in a thermally insulated bath of:
- oil at 20°C for the room temperature tests,
- boiling liquid nitrogen at -195.6°C for highly cooled wall tests.

A direct calibration of every model thermocouple has been done both at ambient and low temperature. The whole model is immersed in an oil bath for the range $20 < T_w < 70°C$. Typical calibration curves are presented in fig. 3.3. At low temperature an isopentane (2 methyl-butane $(CH_3)_2CHCH_2CH_3$) bath was used which made direct calibration possible in the range $-160 < T_w < -110°C$. The reference junctions of the thermocouples were placed into boiling nitrogen. During the tests the model wall temperature remained close to $-185°C$. The calibration curves of the thermocouples must be extrapolated to the range $-185$ to $-160°C$ and a correction using standard tables for copper-constantan junctions have been computed. The corrected slope of the calibration curve was estimated by the following linear expression:

$$a_2 = a_1 \frac{a_1}{a_2}$$

(3.2)

where

$a_2$ ($°C$/mm) is the slope of the extrapolated calibration curve in the range $-185$ to $-160°C$ (no direct measurements),

$a_1$ ($°C$/mm) is the slope of the measured calibration curve in the range $-160$ to $-110°C$ (faired curve from measured values),

$a_1$ (mV/$°C$) is the mean slope of standard curve (for copper-constantan) in the range $-160$ to $-110°C$, and

$a_2$ (mV/$°C$) is the mean slope of standard curve in the range $-185$ to $-160°C$.

Both measured and corrected calibration curves for every model thermocouple are presented in fig. 3.4.
3.2.3 Accuracy of heat transfer measurements

The four basic assumptions of the transient thin skin technique must be checked:

a) Isothermal wall conditions must be achieved prior to the injection of the model into the hypersonic flow and therefore the uniformity of the wall temperature at the instant of injection must be checked.

When the initial wall temperature is close to the room temperature, the maximum difference temperature indicated by the different thermocouples is 1.5°C; however, when the model is cooled down this temperature difference rises to 6°C.

b) The thickness of the instrumented portion of the model skin has been measured: \( d = 0.83 \text{ mm} \pm 0.01 \text{ mm} \).

c) The thermal insulation of the model interior is insured by the low pressure air circulating in the cavity; indeed, this cavity communicates to the rear face of the model, therefore the internal static pressure remains at base pressure during the test.

d) The lateral heat conduction has been minimized using a skin thickness as small as possible and a short exposure time to the supersonic flow.

Nevertheless, there are indications that the first thermocouple close to the leading edge block is affected by heat conduction due to the plate thickness discontinuity in this region. On the other hand, an approximate calculation of the heat flux corrections including the longitudinal (or streamwise) heat conduction shows that a maximum error of 15% (1 sec after injection) of the measured heat flux must be taken into account in the most unfavourable case: namely, for the maximum values of local heat flux measured.

The other main sources of inaccuracies in heat flux measurements are now reviewed.
3.2.3.1 Uniformity of the free stream flow

A fixed model position (as indicated on sketch 6) has been used for all the tests.

The results of ref. 39 show 1% Mach number non-uniformity over the whole length of the model at the location defined in sketch 6. A mean value of $M_\infty = 5.96$ has been used for theoretical calculations. The model angle of attack has been kept to $0° \pm 1/4°$.

3.2.3.2 Effect of leading edge bluntness

The effect of leading edge bluntness may overshadow the strong viscous interaction for

$$\bar{X} \gg \frac{M_\infty^{3/2}}{(Re_d)^{1/2}}$$

where $Re_d = \frac{\rho_\infty u_\infty d}{\mu_\infty}$, $d$ being the leading edge thickness.

In the case considered herein $\bar{X} = 0.6$ ($x = 14$ mm abscissa of the first measurements). The mean value of the leading edge thickness is $0.08$ mm, then $M_\infty^{3/2}/(Re_d)^{1/2} = 0.4$. One concludes that leading edge bluntness has little effect on the measurements and that these measurements lie in the weak interaction regime $\bar{X} < 1$. 
3.2.3.3 Error due to the evaluation of the temperature-time derivative $dT/dt$

A typical example of the records obtained during a test is presented in fig. 3.5 (in the case of the lowest heat flux measured). The temperature-time derivative must be measured at the instant $t = 0$, and this has been done by estimating graphically the slope of the function $T_w(t)$ at time $t = 0$. The linearity of the traces obtained during the first few seconds of the test is sufficient to avoid any drawing errors larger than 10% of a mean value except for measurements made in the neighbourhood of the hinge line.

3.2.3.4 Error due to the calibration of thermocouples

For the case of a highly cooled wall, the calibration curve of thermocouples has been extrapolated as described in section 3.2.2. The actual calibration curve has been replaced by a linear function (slope $a_2$) in the range 77 to 100°C. For these tests the wall temperature lies between 85 and 100°C and the measured temperature-time gradients in the range 0.4 to 15°C/sec.

The temperature gradient $t = 0$ is calculated as follows:

$$\left(\frac{dT}{dt}\right)_{t=0} = \left(\frac{\Delta T}{\Delta y}\right)_{T=T_{\text{winj}}} \times \left(\frac{\Delta y}{\Delta t}\right)_{t=0}$$

where $\left(\frac{\Delta T}{\Delta y}\right)_{T=T_{\text{winj}}} = a_2$

and $\left(\frac{\Delta y}{\Delta t}\right)_{t=0}$ is graphically evaluated on the records.

$\Delta y$ being the measured deflection of the spot recorder during the interval of time $\Delta t$. 
The error in the slope of the calibration has been estimated to be 5% whilst the error in the estimation of \( \frac{\Delta V}{\Delta t} \) may reach 10%.

3.2.3.5 Variation of thermal properties of the skin material

The specific heat of the skin material \( C_p \) enters in the heat flux computation through equation 3.1. Figure 3.6 shows the variations of \( C_p \) for stainless steel 18-8 versus the temperature (data obtained from both manufacturer and ref. 43). The actual function \( C_p(T_w) \) can be locally approximated by a linear function:

\[
C_p = \alpha T_w + \beta
\]

where for \( 77 < T_w < 100^\circ K \)
\[
\alpha = 0.827 \times 10^{-3} \text{ cal/}\text{gr}\text{\textdegree K}
\]
\[
\beta = -25.5 \times 10^{-3} \text{ cal/}\text{gr}\text{\textdegree K}
\]
and \( 200 < T_w < 298^\circ K \)
\[
\alpha = 0.153 \times 10^{-3} \text{ cal/}\text{gr}\text{\textdegree K}
\]
\[
\beta = 68.4 \times 10^{-3} \text{ cal/}\text{gr}\text{\textdegree K}
\]

Introducing eq. (3.3) into (3.1):

\[
q_w = (rd)_m(\alpha T_{\text{inj}} + \beta) \left( \frac{dT_w}{dt} \right)_{t=0}
\]

In practice the estimation of the slope \( \frac{\Delta V}{\Delta t} \) must be done in the following way:

We replace the actual tangent at \( t=0 \) by the chord between \( t=0 \) and \( t=t_1 \). For most of the cases \( t_1 = 1 \text{ sec} \), therefore it is important to analyse the effect of the variations of \( C_p(T_w) \) on the curvature of the function \( T_w(t) \) during the first few seconds of the test. This computation has been done considering the simplified case of the flat plate placed in a uniform flow \( M \) and subjected to heat flux \( q_w \).
Assuming quasi isothermal wall conditions, the heat flux at some given abscissa $x$ is:

$$q_w = -\frac{k}{2} (T_w - T_r) Y_0'(0) \frac{\sqrt{\text{Re}}}{x}$$  \hspace{1cm} (3.5)

Assuming that $C$ (Chapman constant) and $Y_0'(0)$ (function of Prandtl number) remain constant for any small changes in $T_w$, we can equate (3.4) and (3.5), giving:

$$\frac{d(T_w)}{dt} \frac{C_p(T_w)}{T_r - T_w} = \text{const.}$$  \hspace{1cm} (3.6)

Using eq. 3.3 the above equation is integrated whilst a similar integration is performed for $C_p = \text{const}$.

Figure 3.7 shows both calculated functions $T_w(t)$ obtained from (3.6) for $C_p = \text{const}$ and $C_p$ linearized. It must be noted that $\frac{dT_w}{dt}$ calculated as the slope of the chord joining two points of $T_w(t)$ between $t=0$ and $t=t_1$ is reduced when $C_p$ is not taken as a constant.

### 3.2.3.6 Correction due to finite thickness of the skin

It can be shown using the results of ref. 42 that the inner face of the skin reaches approximately the same temperature as the upper face exposed to the flow, respectively 0.15 sec (room temperature tests) and 0.1 sec (cooled wall conditions) after the model injection into the supersonic flow. This time delay may be neglected in the data reduction as well as the corrections to the measured heat flux (0.1%).

In conclusion, the total error in heat flux measurement is estimated to be $\pm 15\%$. 

3.2.4 Results and discussion of heat transfer measurements

The results of heat transfer measurements are presented in a non-dimensional form equivalent to a Stanton number.

\[
C_H = \frac{k \frac{\partial T}{\partial y}}{\rho \cdot u \cdot (h_0 - h_{0_w})} \quad (3.7)
\]

where \( h_0 \), the total enthalpy, is for a steady flow of perfect gas:

\[
h_0 = c_p T + \frac{u^2}{2} \quad (3.8)
\]

The relationship (3.7) can be written explicitly as:

\[
C_H = \frac{-(\rho C_d \frac{dT}{dt})_M}{\gamma \frac{c_p M S_w C_T \sqrt{\frac{T_w}{RT_w}}}} \quad (3.9)
\]

where \( S_w = \frac{h_{0_w}}{h_{0_e}} - 1 \)

For every test the heat transfer distribution has been represented by \( C_H(x) \), where \( x \) is the longitudinal abscissa along the wall taken from the leading edge.

3.2.4.1 Room temperature tests

The equilibrium temperature of the model lies between 24 and 27°C. The wall temperature uniformity was checked prior to the injection of the model into the supersonic flow and was ± 1°C.

Flow visualization on a geometrically similar model using a double pass schlieren system is shown in fig. 3.8a. The separation bubble and its associated shock system can be seen.
Figure 3.9 shows the heat transfer coefficient $C_H(x)$ obtained for different free stream unit Reynolds numbers: $0.5 < Re_u < 2.5 \times 10^7$ per meter. The characteristic phenomenon associated with laminar boundary layer separation is seen; indeed, the heat transfer rate drops rapidly in the separated region, goes through a minimum and then rises in the reattachment region.

An explanation for the unexpected behaviour of the first two points can be found in the previous section 3.2.3. The phenomenon is due to longitudinal heat conduction along the model skin in the neighbourhood of a skin thickness discontinuity.

3.2.4.2 Highly cooled wall tests

The model being cooled down both by internal circulation and surface spraying with liquid nitrogen, an equilibrium temperature close to the boiling point of nitrogen ($\approx 77^\circ K$) can be reached prior to run. The wall temperature uniformity is then $\pm 3^\circ C$. At the instant of model injection its surface temperature lies between 85 and 100$^\circ K$. For a free stream stagnation temperature of $\approx 200^\circ C$ the wall cooling rate is about -0.8. A flow visualization photograph is presented at fig. 3.8b showing a very narrow separation bubble in the vicinity of the hinge line. Figure 3.10 shows the heat transfer coefficient distribution on a highly cooled wall for different free stream Reynolds numbers. The general aspect is similar to one previously analyzed at section 3.2.4.1, but a well defined heat transfer peak appears at the end of the reattachment point.

3.2.4.3 Effect of wall cooling in the measured heat transfer distribution

The experimental results of figures 3.9 and 3.10 are summarized in fig. 3.11. The heat transfer coefficient distribution $C_H(x)$ is shown for two different wall cooling rates.
(respectively $S_w = -0.4$ and $S_w = -0.8$) and approximately the same free stream unit Reynolds number.

The effects of wall cooling appear to be the following:

1. The "cusp like" shape of the heat transfer distribution in the separated region is sharpened by wall cooling, and the separated length is reduced as can be seen in fig. 3.8a and 3.8b.

2. The minimum value of $C_H$ reached in the neighbourhood of the hinge line is rather insensitive to wall cooling.

3. The magnitude of the reattachment heat transfer peak increased and its location moves upstream as the wall is cooled down.

The wall cooling effect on the extremes of $C_{H_{\text{max}}}$ and $C_{H_{\text{min}}}$ of the measured heat transfer distribution have been summarized at fig. 3.12.

3.3 Pressure measurements

3.3.1 Model design

The external geometry of the model used for heat transfer measurements has been kept (i.e., $x_c = 40$ mm, $\theta_{\text{ramp}} = 7^\circ 30'$).

The wall temperature lies between $-195^\circ C$ and $+130^\circ C$, but for each of the wall cooling rate values selected, the surface temperature is kept constant during the whole run.

The model shown in fig. 3.13 is made from copper beryllium alloy. It is hollow and the liquid coolant flows through four internal cells which cover 65% of the model surface (exposed to the flow). The skin thickness is 1.5 mm. The model centerline is equipped with a row of 21 pressure taps (1 mm internal diameter) whose locations are indicated at fig. 3.13, as well as the internal structure of this model. The bottom face of the model is thermally insulated from its support with a layer of "s elastene".
The surface temperature can be controlled by means of 8 thermocouples imbedded 1 mm below the model surface (thermocouples locations are given at fig. 3.13). Both the model injection apparatus and the set up to avoid frost formation, as the model cools down, described in section 3.2.1, have been kept.

Experiments at three different surface temperature values have been carried out:

a) \(-196 < T_w < -173^\circ C\). An internal circulation of liquid nitrogen is established from a pressurized tank (1.5 kg/cm\(^2\) absolute) having a capacity of 130 l. Cooling down the model to 60 K takes \(\approx 3\) minutes.

b) \(15 < T_w < 20^\circ C\). Surface temperature control has been achieved by water circulation.

c) \(110 < T_w < 130^\circ C\). Oil circulation by pump from a thermally insulated reservoir has been used.

3.3.2 Instrumentation

The pressure distribution survey is made simultaneously on two pressure taps by means of two scanivalves (2 \times 12 positions) connected to two pressure transducers which cover the range from 0 to 50 mm Hg. The reference pressure of both transducers is connected to a vacuum tank (\(0.1 < P_{ref} < 0.15\) mm Hg).

A complete survey takes approximately one minute. The pressures are read on a double channel "graphinac" recorder. The 8 model thermocouples have been connected to the multichannels galvanometric recorder (see § 3.2.2) in order to obtain a continuous survey of the wall temperature history during a test. The calibration of these thermocouples has been done in the same way as for those used for heat transfer measurements (see § 3.2.2).
3.3.3 Accuracy of pressure measurements

The model position in the hypersonic jet is identical to that indicated in sketch 6, therefore, the errors in measurement due to non uniformities of the free stream flow and those due to the zero angle of attack adjustment have been discussed in section 3.2.3. The strain gauge pressure transducers have a linear response over their rated pressure range, and an accuracy better than 1%. However, measurements have been done simultaneously using two pressure transducers and their respective response times can be different; thus, in order to prevent a possible scale shift, the last pressure tap is connected to both transducers. During the course of these tests the difference of pressure indicated by both transducers connected to the same pressure tap was $\pm 1\%$.

On the other hand, the main source of errors may be provided by the non uniformities, as well as the unsteadiness of surface temperature during a test. The most critical case arises when the heat flux from the air stream to the model surface is maximum (namely, highly cooled wall and large free stream Reynolds number).

The parts of the model surface which are in direct contact with the cooling cells heat up from 3 to 16°C after a running time of 1 minute whilst the leading edge and rear part of the ramp are subjected respectively to a temperature increase of 100°C and 40°C during the same time.

Considering now the particular case of quasi-adiabatic wall conditions, the temperature of the oil circulating into the model has been adjusted close to the theoretical flat plate recovery temperature $T_r(M_a) + 5°C$. In this case local heating of any parts of the model is provided by non uniform recovery temperature distribution due to pressure gradient effects. Indeed, such a surface temperature distribution cannot be entirely simulated, it has been replaced by a roughly uniform distribution at $T_r(M_a)$. 
An experimental check of the effect of non uniform surface temperature increase on pressure distribution has been carried out in the following conditions:

\[ T_{\text{initial}} = -185^\circ \text{C}, \quad \text{Re}_u = 2.5 \times 10^7 \text{ per meter}. \]

After a running time of 1.5 minutes, the temperature rise is:
- +140°C in the vicinity of the leading edge,
- +40°C at the rear part of the ramp,
- from +2.5°C to +11°C for the other parts of the model.

The pressure variations due to this surface heating measured at a location close to the beginning of the interaction (pressure taps No 7 and 8 on fig. 3.13) have been recorded versus time and found to be negligibly small (less than 1%).

3.3.4 Results and discussion of pressure measurements

The tests conditions were the following:
- the free stream Mach number is \( M_\infty = 5.96 \),
- the model angle of attack is 0°,
- the free stream stagnation temperature has been kept close to 250°C for every test,
- the static pressure distributions have been measured at three different wall temperatures:
  a) highly cooled wall \(-189.5 < T_w < -184^\circ \text{C}, \quad -0.813 < S_w < -0.791\),
  b) moderately cooled wall \(14.6 < T_w < 19.3^\circ \text{C}, \quad -0.35 < S_w < -0.3\),
  c) quasi adiabatic wall \(102 < T_w < 134^\circ \text{C}, \quad S_w = 0\)

The last case corresponds approximately to a thermally insulated surface where \( S_w = 0 \) (if Pr = 1).

A range of unit Reynolds numbers from 0.5 to \(2.5 \times 10^7\) per meter has been covered by five tests.

The results for the three wall cooling rates, cases a, b, c, are presented in figs. 3.14, 3.15, and 3.16 where the ratio \( p/p_\infty \) has been plotted versus \( x \):
- \( p \) is the measured local static pressure at the wall,
- \( p_\infty \) is the theoretical free stream static pressure,
- \( x \) is the streamwise coordinate taken along the wall from the leading edge.
As it can be seen from the above mentioned figures, increasing the free stream Reynolds number results in decreasing the abscissa of the beginning of the interaction \( x_0 \) (\( x_0 \) is defined as the point where the static pressure first departs from the undisturbed flat plate distribution). This result is valid for every wall cooling rate.

An increase in upstream influence as the Reynolds number increases, is a characteristic feature of a purely laminar interaction, it has been confirmed using the criteria defined in ref. 44.

Figure 3.17 shows the variation of the ratio \( P_N/P_e_0 \) versus \( Re_u \) where \( P_N \) is the measured static pressure at some point of abscissa \( x_N \) located in the vicinity of the separation point (the pressure gradient being positive and approximately constant). \( P_e_0 \) is the minimum static pressure at \( x_0 \), the beginning of the interaction. It can be seen from fig. 3.17 that the ratio \( P_N/P_e_0 \) increases as \( Re_u \) increases whatever \( S_w \) results in an increase of upstream influence characteristic of a laminar flow.

The wall cooling effect on pressure distribution is shown in fig. 3.18 and 3.19 respectively, for \( Re_u = 10^7 \) per meter and \( Re_u = 2.5 \times 10^7 \) per meter. The pressure distributions have been measured respectively for \( S_w = -0.8 \), \(-0.32 \) and \( 0 \). From these plots one deduces that surface cooling produces:

1. A boundary layer thinning. Considering the weak interaction region the ratio \( p/p_m \) depends on the boundary layer growth through

\[
\frac{p}{p_m} \frac{d \delta}{dx}
\]

therefore, at a given abscissa \( x < x_0 \) the ration \( p/p_m \) decreases as the surface is cooled down.

2. The length scale (along the \( x \) coordinate) of the whole interaction region is reduced under wall cooling, in particular, the separated length decreases as can be seen from flow visualization photographs (figs. 3.8a and 3.8b).
3. As a result of this sealing effect, the pressure gradient in the separation and reattachment region remains quite unaffected by wall temperature variations. Figure 3.20 shows to an enlarged scale the wall cooling effects on the beginning of the interaction (and the "plateau" region). It can be seen that the "knee" in pressure distribution almost disappears when the wall is cooled down.

3.3.5 Sublimation tests

During the highly cooled wall testing the existence of three dimensional perturbations in a two dimensional boundary layer (in a mean sense) was revealed "accidentally". As has been shown in ref. 45 these perturbations originating from leading edge irregularities in thickness are associated with spatially periodic system of streamwise vortices. Figure 3.21a shows the model surface after a few minutes exposure to the flow. The model was then cooled down (≈85°K) prior to test. The white straight lines indicate a thin film of ice deposited at the surface. This ice film was formed during the test. The spatially periodic pattern is very similar to that observed in fig. 3.21b. This second surface flow visualization has been obtained from the sublimation of an "acenaphtene" layer sprayed uniformly on the model surface prior to test. The sublimation is obtained mainly by skin friction but is also sensitive to heat transfer effects.

3.4 Concluding remarks

An experimental study of wall cooling effect on the overall features of shock wave laminar boundary layer interaction in hypersonic flow (M = 6) has been carried out.

The heat flux distribution for two different wall cooling rates has been measured. The accuracy of such measurements has been estimated to be 15%. It has been found that the main effect of wall cooling is to increase the reattachment
heat transfer peak.

The static pressure distributions for three different surface temperature conditions have been measured; the accuracy has been estimated to be 5%. It has been found that a considerable reduction in separated length occurs under high cooling effects which in turn steepens the pressure gradient over the whole interaction region.

The technical problems encountered at low surface temperature level have been solved satisfactorily and the fully laminar character of the interaction for all thermal conditions has been demonstrated.
4. COMPARISON BETWEEN THE THEORETICAL PREDICTIONS
AND THE EXPERIMENTAL RESULTS

4.1 Limitations of the Lees-Reeves-Klineberg theory

The physical model of the flow field developed in the present theory (Section 2) assumes that the external flow is isentropic over the whole extent of the interaction except at the shock impingement point \( x_{SH} \). Needham (ref. 40) developed a similar model for interactions occurring at high free stream Mach number \( M_\infty > 10 \).

The compression waves originating from the sonic line in the boundary layer coalesce and form two shock waves located respectively in the separation and reattachment region, therefore the external flow cannot be assumed isentropic and the Prandtl-Meyer relationship coupling the local inclination of the external flow streamline \( \theta_e \) and the Mach number \( M_e \) must be abandoned (and replaced by the tangent wedge formula for example).

Furthermore, at high hypersonic Mach numbers, large ramp deflection angles are permissible and the boundary layer thickness in the separated region has the same order of magnitude as the separated length, so that the streamline curvature radii become small at separation and reattachment. This curvature induces centrifugal forces which must be balanced by a normal pressure gradient, thus the assumption \( \frac{3P}{3y} = 0 \) is no longer valid. Holden (ref. 16) developed an integral theory including the momentum equation normal to the wall, however, comparisons with experimental results show not much improvement over the classical theory \( \frac{3P}{3y} = 0 \) for any Mach numbers lower than 10.

Finally, two other assumptions included in the present theory become doubtful for high hypersonic flows:
The equivalence between the interactions generated by either a deflected wall and an incident shock impinging on a flat plate.

The linear viscosity law assumed in the Stewartson transformation is not valid when very high temperature gradients exist inside the boundary layer.

The conclusion naturally arises that the application of the present theory must be restricted to supersonic and low hypersonic flows ($M_{\infty} < 10$).

Another assumption implicitly contained in the theory must be examined carefully: the proposed method of calculation applied to fully laminar interactions.

The location of transition is difficult to detect accurately by experimental techniques. One usually considers as transitional shock wave boundary layer interactions when the transition can be located between separation and reattachment points. However, the transition when it is located in the reattachment region may already influence the development of the interaction; therefore, many of the experimental results found in the literature deal in fact with transitional more than purely laminar flows.

Herein, in order to avoid this ambiguity, we consider the effect of the transition on the whole interaction but no attempt has been made to locate accurately the transition point. The criteria described in ref. 44 and briefly discussed in section 3 is based on the following experimental observation: For any given shock wave boundary layer interaction, as the free stream Reynolds number increases, the location of the beginning of the interaction first moves upstream, reaches a minimum value and then starts to move downstream as the boundary layer becomes transitional. The movement of the early part of the pressure distribution can be detected by measuring the static pressure at a point close to the separation, as indicated on sketch 7.
According to this criteria an interaction may be considered as transitional for \( \Re_u > \Re_u^m \) where \( \Re_u^m \) denotes the maximum of the curve \( \frac{p_N}{p_{e0}} \) versus \( \Re_u \), therefore the theory will apply for \( \Re_u << \Re_u^m \).

### 4.2 Selection of the experimental results

The experimental results available from the literature fall into two classes according to the type of facility which has been used:

- low Mach number \( 2 < M_\infty < 5 \), adiabatic wall conditions, experiments have been performed in continuous supersonic wind tunnels;
- high Mach number, highly cooled wall, experiments carried out in hypersonic blowdown wind tunnels or shock tubes.

The running time being usually a few milliseconds the wall may be considered isothermal.

A few experimental results have been obtained in the intermediate range of wall cooling conditions, such experiments are described in the following references: Lewis (ref. 34) and Gray (ref. 48) for pressure measurements, Johnson (refs 46-47) and Alziary (ref. 30) for both pressure and heat transfer measurements.
The present theory has been applied to the experimental conditions of Lewis (see § 2.5.1). A comparison with the experiments of Needham (ref. 40) and finally with the present experiments repeated in section 3 is presented below.

4.3 Comparison with Needham's experiments

These experiments have been carried out in a shock tube at nominal free stream Mach numbers of 7.5 and 10. Two models have been used: a flat plate model equipped with a deflected trailing edge flap and a flat plate equipped with an external shock generator. The free stream flow is conical and measurements have been corrected to their equivalent two dimensional flow values.

In order to compare these results with the theory, the Mach number at the beginning of the interaction $M_{e0}$ has been matched with the experimental value but the abscissa $x_0$ remains a unknown of the problem and must be determined by iteration and described in section 2.

Figure 4.1 shows the non dimensional pressure and heat transfer distributions on a flat plate, the interaction being generated by an incident shock impinging on a flat plate at $x_{SH} = 152$ mm aft of the leading edge. The overall pressure ratio across the incident shock is $p_*/p_0 = 2.933$. The experimental value of the Mach number at the beginning of the interaction is $M_0 = 7.4$ and the free stream Reynolds number is $Re_{xSH} = 2.2 \times 10^6$. The results have been plotted in a non dimensional form:

- pressure distribution $p/Pf_p$
- heat flux distribution $\dot{q}/\dot{q}_f_p$

where $Pf_p$ and $\dot{q}_f_p$ are respectively the undisturbed flat plate values for pressure and heat flux.

Figure 4.2 shows the non dimensional pressure and heat transfer distributions on a flat plate wedge model.
(θ ramp = 10°). The Mach number at the beginning of the inter-
action is M₀ = 9.7 and the free stream Reynolds number is
Re₅SH = 0.95 x 10⁵. The flat plate length or the abscissa of
the hinge line is also x₅SH = 152 mm.

For both experiments reported in fig. 4.1 and 4.2
it is assumed that the surface temperature is constant and
Sₐ = -0.8 (running time 60 mmiliseconds). From these plots it
can be seen that the theory overestimates the length of sepa-
ration whilst the magnitude of the heat transfer peak is
underpredicted, however, these applications may be considered
to be done in a limiting case according to the requirements
discussed in section 4.1.

4.4 Comparison with the present experiments

The theoretical computation has been performed with
the "non dimensional" theory described in section 2. Three sets
of reference integral functions have been used according to
experimental conditions:
for highly cooled wall : -0.78 < Sₐ < -0.82, Sₐref = -0.8,
for moderately cooled wall : -0.31 < Sₐ < -0.4, Sₐref = -0.4,
for quasi adiabatic wall, the surface temperature of the model
has been matched with the theoretical recovery temperature on
a flat plate:

\[ \frac{T_r}{T_\infty} = 1 + \frac{γ-1}{2} \sqrt{Pr} M_0 \]

and Sₐref = 0.

4.4.1 Comparison between theoretical and
experimental pressure distribution

Figures 4.3a to 4.3e show both experimental and
theoretical pressure distributions where the ratio p/pₐ has
been plotted versus x. pₐ is the theoretical static pressure
of the free stream flow and \( x \) the longitudinal coordinate along
the wall taken from the leading edge.

4.4.1.1 General aspect of pressure distribution

The undisturbed flat plate pressure distribution
\((x < x_0)\) is predicted by the weak interaction theory with
good accuracy for every Reynolds number and wall cooling rate
as it can be seen in fig. 4.4.

Here the characteristic "knee" followed by the
"plateau" pressure reduces to an inflection point (at \( x_{SH} \)) in
the pressure distribution, because the separated length is
very small. Furthermore, in the highly cooled wall case, it
can be seen that the theory exhibits a physically unrealistic
jump, although the measured pressure gradient starts to rise
very rapidly at the beginning of such an interaction.

Downstream of the hinge line, the steep rise of
pressure is well predicted by the theory but all the experi­
mental pressure distributions exhibit a peak pressure before
relaxing to the downstream flat plate value. This singularity
cannot be predicted in the framework of the present theory.

This type of pressure distribution has been found
by many investigators (Needham, Holden, etc.) when the shock
wave boundary layer interaction is generated by a deflected
flap. Looking at the flow visualization (fig. 3.8a and 3.8b)
it can be seen that the shocks formed respectively in the
separation and reattachment region coalesce into a single
shock wave at some location on the ramp downstream of the
reattachment point. The interactions generated by an incident
shock impinging on a flat plate where no such shock interaction
exists exhibit a smooth downstream pressure distribution.

The location of this peak pressure moves forward as
the wall is cooled down. This is due to the shortening of the
separation region under wall cooling which moves the shock intersection point forward.

4.4.1.2 Reynolds number effect

Both the thinning of the boundary layer and the increase of upstream influence as the free stream Reynolds number increases are accurately predicted by the theory. Indeed, fig. 4.4 shows that both the pressure ratio \( \frac{p}{p_\infty} \) in the weak interaction region and \( x_0 \) decrease as the Reynolds number \( R_{eu} \) increases.

Downstream of the hinge line the theoretical pressure distribution overestimates the measured values at low Reynolds number but underestimates at high Reynolds number. The lack of experimental data in the neighbourhood of the hinge line does not permit a correct location of the end of the "plateau" region.

The point where the pressure starts to rise is theoretically located at the given abscissa \( x_{SH} \) but the experiments of Needham (ref. 40) and Holden (ref. 15) show that it must be located further downstream on the ramp. The resulting shift in abscissa of the downstream pressure distribution with respect to the theory is magnified by the very short separated length considered herein.

Generally, the effect of an increase in Reynolds number on upstream influence is magnified by the theory. The appearance of the transition in the reattachment region increases the discrepancy between the theoretical trend and experiments.

4.4.1.3 Wall temperature influence

On each of the figs. 4.3a to 4.3e the pressure distribution has been plotted for three different wall cooling
rates: $S_w = 0.8, -0.4$, and $S_w = 0$.

The contraction of the $x$ scale of the interaction under wall cooling is quite well predicted by the theory except for the case of high cooling rate (in the neighbourhood of the jump location). Excellent agreement is achieved at moderate cooling rate ($S_w = -0.4$) and the weak effect of wall cooling on the "plateau" pressure (fig. 2.14) is correlated by the experimental results.

The separation point location $x_S$ cannot be accurately determined from the experimental pressure distribution, thus only qualitative agreement between the theoretical effect of wall temperature changes on the location of the separation point (fig. 2.13) may be deduced from figs. 4.3a to 4.3e).

4.4.2 Comparison between theoretical and experimental heat transfer coefficient distribution

The fig. 4.5a to 4.5e show both theoretical and experimental distributions of the heat transfer coefficient $C_H$ versus $x$ coordinate. These plots refer to free stream parameters (Mach and Reynolds numbers), model geometry ($x_{SH}$, $\theta_{ramp}$, angle of attack, etc.) similar to those defined in section 4.4.1. Two wall to stagnation temperature levels corresponding to $S_w = -0.8$ and $S_w = -0.4$ have been investigated.

As defined in section 2, the heat transfer coefficient is:

$$C_H = \frac{k(\frac{3T}{\partial y})}{\rho_w u_m (h_{0e} - h_{0w})}$$

where $h_0 = C_p T + \frac{u^2}{2}$
\( C_H \) replaces the usual Stanton number because the present theory assumes \( Pr = 1 \) and applies to an isothermal wall, with a possible extension to the prescribed wall temperature case. It does not therefore allow for a theoretical computation of the recovery temperature distribution to be made. On the other hand, the recovery temperature distribution on a thermally insulated model has not been measured. For this purpose, the development of a steady state technique is not possible with the possible running time of wind tunnel H-3. However, an extrapolation of the temperature time history up to quasi adiabatic conditions \( \frac{dT}{dt} \to 0 \) with the transient technique remains possible but rather inaccurate.

4.4.2.1 General aspect of the heat transfer coefficient distribution

Figures 4.5a to 4.5e show that the present theory effectively predicts a strong heat transfer drop in the separated region and a rise to a large heat transfer peak during the reattachment process.

The large differences existing in the weak interaction region close to the leading edge are due to inaccuracies in the heat flux measurements as indicated in section 3.2.3. The minimum heat transfer appears to be overestimated by 10 to 30\% by the theory but the rear part of the heat transfer distribution downstream of the hinge line is well predicted. A semi empirical correction taking into account the effects of the actual Prandtl number (\( Pr \neq 1 \)) as proposed by Alziary (ref. 30) may improve the general agreement between theory and experiments. Although the actual location of the beginning of the interaction is not a well defined point on the heat transfer distribution it must be noted that the theory underestimates the separated length particularly at high cooling rates. This remark has already been made from the analysis of pressure distribution. It is the result of an inaccurate jump assumption despite the fact that the initial discontinuity
at the beginning of the interaction is very small and does not appear on the plots.

4.4.2.2 Reynolds number effect

The Reynolds number variations are reflected as $x,y$ coordinate scaling effect on the heat transfer coefficient distribution. Indeed, the free stream density $\rho_\infty$ appears explicitly in the denominator of the expression for $C_H$, so that an increase of free stream unit Reynolds number decreases the non dimensional coefficient $C_H$. 

4.4.2.3 Wall temperature influence

$C_H$ versus $x$ has been plotted in fig. 4.3a to 4.3e for two different wall cooling rates and approximately the same free stream Reynolds number. The increase of both the heat transfer gradient and the magnitude of the peak heating downstream of the hinge line, as well as the increase of upstream influence under wall cooling are effectively predicted by the theory.

Nevertheless, the effect of surface cooling on the non dimensional heat transfer coefficient is rather small. A direct comparison between the experimental results and the predicted extremes of the heat transfer distribution is not possible. Indeed, the free stream Reynolds numbers of the tests carried out at the same thermal conditions, are not strictly identical. The scaling effect of the Reynolds number on the heat transfer coefficient cannot be entirely eliminated.

However, the correct qualitative trend may be derived from the comparison of figs. 2.16 and 3.11.

Finally, a check was made on an implicit assumption contained in the framework of the present theory, namely, that the pressure and heat transfer distribution characteristic points can be exactly matched (in particular the beginning of
the interaction $x_0$ which corresponds to both the first increase in pressure and the decrease of heat transfer from the undisturbed flat plate values).

Figure 4.6 shows, for two different wall temperature levels, non dimensional experimental pressure and heat transfer distributions; $q_f^P$ refers to the undisturbed flat plate flow heat flux.

For a moderate cooling rate ($S_w = -0.34$) the beginning of the interaction can be matched on both pressure and heat transfer distributions. At high cooling rate the inaccuracies of the measured heat transfer distributions at the beginning of the interaction rule out any further conclusion. The experimental results of Alziary (ref. 30) suggest that the length scales of pressure and heat transfer distributions are different but other experimental results (Holden, ref. 15, for example) show a perfect matching of both thermal and dynamic phenomena.
5. CONCLUSIONS

A theoretical and experimental study of the effect of wall cooling on the pressure and heat transfer distributions associated with shock wave laminar boundary layer interaction in two dimensional hypersonic flow has been carried out.

The Lees-Reeves-Klineberg theory has been extended to the case of interactions where the wall to stagnation temperature ratio is arbitrarily fixed in the range 0.2 to 1. Using this approximate method it is possible to treat the more general case where the wall temperature distribution is prescribed. The main assumptions of the theory are:

- The conventional boundary layer equations (including $\frac{3p}{3y} = 0$) remain valid for a description of the separated flow.
- A coupling relationship between the inviscid external flow field and the viscous layer can be used leading to the simultaneous solution of both viscous and inviscid flow fields.
- The outer flow is assumed isentropic, therefore the method must be restricted to moderately hypersonic flows ($M_w < 10$).
- The boundary layer profiles are derived from similar solutions. Two independent parameters are used in order to describe respectively the velocity and the total enthalpy profiles, and a complete unhooking of these profiles can be achieved.

A suitable normalization of the total enthalpy profile partially avoids the dependence of the boundary layer integral functions on wall temperature. Thus a set of "universal" integral relations valid over a large range of thermal conditions at the wall ($-0.8 < S_w < 0$) has been established.

The mathematical singularities of the basic set of differential equations (singular points) have been overcome using a simplified model of the flow at the beginning of the interaction (jump assumption) and a graphical procedure has been used in order to obtain the solution through the Crocco-Lees critical point.
A parametric study of the wall cooling effect on the overall features of shock wave boundary layer interaction has been carried out. It has been shown that the characteristic lengths of the interaction (particularly the separated length) vary linearly with the wall to stagnation temperature ratio $T_w/T_t$.

An experimental study has been performed in a hypersonic blowdown wind tunnel at a nominal free stream Mach number of 6. Both pressure and heat transfer measurements have been carried out on flat plate wedge models (the flat plate length and the wedge deflection angle were respectively $x_c = 40$ mm and $\theta_{ramp} = 7^\circ 30'$). The pressure distribution has been measured keeping the model surface at a steady uniform temperature. Three successive wall temperature levels have been investigated corresponding respectively to $S_w = 0.8$, $S_w = -0.4$ and $S_w = 0$.

The surface heat flux distribution has been measured by means of the thin skin transient technique. Two wall temperature levels corresponding to $S_w = -0.8$ and $S_w = -0.4$ have been investigated. Both pressure and heat transfer distributions agreed reasonably well with the theoretical predictions over the whole range of wall temperature ratio. Furthermore, the effects of both the free stream Reynolds number and the wall temperature on the overall features of the interaction particularly on the separated length, have been experimentally correlated with good accuracy.
6. FUTURE WORK

The non-dimensionalization of the integral functions with respect to $S_w$ described in section 2 has been demonstrated to be suitable for the study of shock wave boundary layer interactions over a continuous range of wall cooling rates. Therefore, this procedure leads itself to the analysis of more complicated situations, namely, the interactions where the surface temperature and the parameter $S_w$ are prescribed functions of the streamwise coordinate $x$. For this purpose, the expressions $h_s$ and $Q_s$ (eq. 2.33 and 2.34) containing the streamwise temperature gradient $\frac{dS_w}{dx}$ must be substituted for $h$ and $Q$ which apply to the isothermal wall case in the governing equations (2.29) and (2.32). The analysis of section 2 then holds provided that the terms containing the temperature gradient remain continuous and finite over the whole interaction region.

Additional assumptions are required concerning initial conditions at the beginning of the interaction; indeed, the weak interaction solution developed in section 2.4.1 is only valid for $\frac{dS_w}{dx} = 0$. 
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APPENDIX A

NUMERICAL VALUES OF COEFFICIENTS FOR INTEGRAL FUNCTIONS POLYNOMIALS

The integral functions defined by equations 2-25 are approximated by polynomial functions which fit the data points derived from similar solutions. Five tables containing the numerical values of polynomial coefficients have been established respectively for $S_w = -0.8$, $-0.6$, $-0.4$, $-0.2$ and $0$. The original Klineberg formulation has been retained for the first four values of $S_w$ whilst the normalized notation has been used in the adiabatic case $S_w = 0$.

A.1 Notation used for the single profile dependent integral functions (velocity or total enthalpy profile)

The maximum number of coefficients is 9.

$$F(a) = \sum_{NK=1}^{NK=9} CD(NJ,NK)a_{NK-1}$$

where $F$ is the integral function considered,

$a$ is the velocity profile describing parameter (Summation parameter)

$NJ$ is the definition subscript of function $F$

$NK$ is the summation subscript $1 < NK < 9$

$CD$ are the polynomial coefficients

Example :

$$\Phi(a) = \sum_{NK=1}^{NK=9} CD(6,NK)a_{NK-1}$$
A.2 Notation used for the functions dependent on both profiles (velocity and total enthalpy profiles)

The maximum number of coefficients in the double summation is 6. The integral functions considered are:

\[
T(a,b) = \sum_{k=0}^{5} \tau_k(b) a^k
\]

where

\[
\tau_k(b) = \sum_{NK=1}^{6} CD(NJ,NK)b^{NK-1}
\]

and

\[
\frac{\partial T}{\partial a}(a,b) = \sum_{k=0}^{5} \epsilon_k(b) a^k
\]

where

\[
\epsilon_k(b) = \sum_{NK=1}^{6} CD(NJ,NK)b^{NK-1}
\]

\[
\frac{\partial T}{\partial b}(a,b) = \sum_{k=0}^{5} \phi_k(a) b^k
\]

where

\[
\phi_k(a) = \sum_{NK=1}^{6} CD(NJ,NK)a^{NK-1}
\]

where \(a\) and \(b\) are respectively the describing parameters of velocity and total enthalpy profiles (Summation parameters); \(NJ\) is the subscript of definition of functions \(\tau_k\), \(\epsilon_k\), \(\phi_k\); \(NK\) is the subscript of summation.

Because of the alternative definitions of the velocity profile parameter \(a\), two coefficient tables must be established dealing respectively with separated and attached flow polynomials. On the other hand, functions \(\sigma(b)\) and \(\frac{d\sigma}{db}(b)\) have been represented.
by two polynomials matched at some point $b = b_{\text{div}}$ corresponding to $a = 0$ and dealing with separated and attached flow functions respectively.

### A.3 Notation table

<table>
<thead>
<tr>
<th>Integral function</th>
<th>NJ</th>
<th>Summation parameter</th>
<th>Integral function</th>
<th>NJ</th>
<th>Summation parameter</th>
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<td>$\phi_5$</td>
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A.4 Particular case, adiabatic wall \((s_w = 0)\)

In the normalized formulation described in section 2.2.3, the non-dimensional integral functions \(\Sigma (B), \frac{d\Sigma}{dB} (B), \Gamma (a,B), \frac{3\Gamma}{3a} (a,B), \frac{3\Gamma}{3B} (a,B)\) are substituted for respectively \(\sigma (b), \frac{d\sigma}{db} (b), T(a,b), \frac{2T}{2a} (a,b), \frac{2T}{2b} (a,b)\) so that the describing total enthalpy profile parameter is replaced by \(B = b/(-s_w)\).

The numerical values of the polynomial coefficients are identical to those used in ref. 38. The analogy of the present problem with the case of shock wave boundary layer interaction on a spinning body of revolution in the limiting case of \(\omega \rightarrow 0\) has been explained in section 2.5.2.

On the other hand, any of these integral functions are represented by two polynomials fitting the data points obtained from similar solutions. The matching point is \(B = B_{\mathrm{div}} = 0.35\).

As a result four tables giving the numerical values of polynomial coefficients are required for separated and attached flows and \(B \geq 0.35\).

The notations used are identical to those listed in section A.3 where \(b\) must be replaced by \(B\).

Example:

\[
\Sigma (B) = \sum_{NK=1}^{9} CD(NJ,NK)B^{NK-1}
\]

\[
\Gamma (a,B) = \sum_{k=0}^{5} \tau_k (B)a^k
\]

where

\[
\tau_k (B) = \sum_{NK=1}^{6} CD(NJ,NK)B^{NK-1}
\]
\[ \frac{\partial \Gamma}{\partial a} (a, B) = \sum_{k=0}^{5} \epsilon_k(B) a^k \]

where

\[ \epsilon_k(B) = \sum_{NK=1}^{6} CD(NJ, NK) B^{NK-1} \]

and

\[ \frac{\partial \Gamma}{\partial B} (a, B) = \sum_{k=0}^{5} \phi_k(a) B^k \]

where

\[ \phi_k(a) = \sum_{NK=1}^{6} CD(NJ, NK) a^{NK-1} \]
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### Attenuation Flow

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<th>CN(4, NK)</th>
<th>CN(5, NK)</th>
<th>CN(6, NK)</th>
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### Separation Flow

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### A-7
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### Diagram

- A:9
# Profiles Coefficients

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**A.1.0**
### Profiles Coefficients

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APPENDIX B

WEAK INTERACTION COEFFICIENTS CALCULATIONS

It has been shown in section 2.4.1 that a solution of the basic equations (eq. 2.65 to 2.68) in the vicinity of the Blasius solution may be obtained by suitable linearization of the flow parameters (ref. 33).

Following Klineberg’s approach (ref. 14) we intend to develop the algebraic expressions of the weak interaction series expansions in order to apply the normalization with respect to $S_w$ and thus derive "universal" relationships.

The four flow parameters $M_e$, $\Delta$, $a$ and $B$ are expressed by coordinate expansions in terms of the viscous interaction parameter $\bar{x}$ (limited to the 2nd degree in $\bar{x}$).

$$M_e = M_\infty (1 + m_1 \bar{x} + m_2 \bar{x}^2 + \ldots)$$

$$\Delta = \delta_0 (1 + \delta_1 \bar{x} + \delta_2 \bar{x}^2 + \ldots)$$

$$a = a_0 + a_1 \bar{x} + a_2 \bar{x}^2 + \ldots$$

$$B = B_0 + B_1 \bar{x} + B_2 \bar{x}^2$$

The logarithmic term in the expression for $\Delta$ has been introduced because of the singular nature of the particular solution of equations 2.65 to 2.68. Indeed, the term $\delta_2$ disappears from these equations when series expansions are substituted for the flow variables.

Kubota (ref. 33) has shown that the coefficient $\delta_2$ can be related to the other coefficients according to the following relation:

$$\delta_2 = \frac{1}{2} \delta_1^2 - \epsilon_2 \log \delta_0$$
In the vicinity of the Blasius solution the integral functions can be expanded into Taylor series in terms of both profile parameters $a_B$ and $B_B$.

Example:

$$
\psi (a) = \psi_B + (a-a_0) \left( \frac{d \psi}{da} \right)_B + \frac{(a-a_0)^2}{2!} \left( \frac{d^2 \psi}{da^2} \right)_B + \ldots
$$

Introducing $(B, l)$:

$$
\psi (a) = \psi_B + a_1 \left( \frac{d \psi}{da} \right)_B + a_2 \left( \frac{d^2 \psi}{da^2} \right)_B + \frac{a_1^2}{2} \left( \frac{d^2 \psi}{da^2} \right)_B + \ldots \quad (B, 3)
$$

and

$$
T(a, B) = T_B + \frac{(\frac{\partial T}{\partial a})_B}{B} + B_1 \left( \frac{\partial T}{\partial B} \right)_B \frac{1}{2} + \frac{(\frac{\partial T}{\partial a})_B}{B} + \frac{(\frac{\partial T}{\partial B})_B}{B} + \frac{a_1^2}{2} \left( \frac{\partial^2 T}{\partial a^2} \right)_B + \ldots \quad (B, 4)
$$

The first order derivatives (i.e., $\frac{d \psi}{da}$, $\frac{\partial T}{\partial a}$, $\frac{\partial T}{\partial B}$, ...) have been curve fitted from the similar data points (see Appendix A) whilst the second order derivatives (i.e., $\frac{d^2 \psi}{da^2}$, $\frac{\partial^2 T}{\partial a^2}$, $\frac{\partial^2 T}{\partial B^2}$, ...) are obtained from differentiation of the polynomials for first order derivatives.

The analytic expressions for the coefficients entering into equation $(B, 1)$ are listed below:

0th degree in $\overline{a}$ coefficients

$$(\psi, R)_B = (Pr)_B \quad (B, 5)$$

$$(\psi, R)_B = 2(\frac{P}{\psi})_B \quad (B, 6)$$

$$B_0 = Pr \left( \frac{P}{\psi} \right)_B \frac{a^2}{B} \quad (B, 7)$$
The numerical values for \( a_0 \) and \( B_0 \) derive from the solution of eq. B.5 and B.7. Both these equations can be solved by iteration procedure.

1st degree in \( \chi \) coefficients

\[
m_1 = - \frac{\gamma - 1}{4M_\infty^2 \gamma^2 - 1} \left( m_{11} + \frac{1 + m_\infty}{m_\infty} m_{12} \right)
\]  

(B.8)

where

\[
m_{11} = \frac{\mu_B}{\sqrt{\gamma}} \delta_0
\]

\[
m_{12} = \delta_0 (1 + S_w \bar{E}) B
\]

\[
\delta_1 = (d_{11} - K_1) m_1
\]  

(B.9)

where

\[
d_{11} = \frac{\sqrt{\gamma} \frac{d R}{da}}{\sqrt{\gamma} \frac{d R}{da}} B \left( \frac{\sqrt{\gamma} + 1}{\sqrt{\gamma}} \right) - 2 \left( \frac{\sqrt{\gamma} + 1}{\sqrt{\gamma}} \right) \left( J + S_w \bar{T} \right) - \left( J + S_w \bar{T} \right) - \left( J + S_w \bar{T} \right)
\]

and

\[
K_1 = \left( \frac{3 \gamma - 1}{\gamma - 1} \right) \frac{m_\infty}{1 + m_\infty}
\]

\[
a_1 = a_{11} m_1
\]  

(B.10)

where

\[
a_{11} = \frac{\delta^2_0}{2} \left( J_B \left( 1 + S_w \bar{E} - \bar{E} \right) - 2 S_w \bar{E} \right) - \left( J_B \left( 1 + S_w \bar{E} - \bar{E} \right) - 2 S_w \bar{E} \right)
\]

\[
B_1 = B_{11} m_1
\]  

(B.11)

where

\[
B_{11} = B_0 \left[ \frac{\left( \frac{da}{da} B \right)}{a_B} a_{11} + d_{11} \right]
\]
2nd degree in \( \chi \) coefficients

\[
m_2 = \left[ \frac{\sqrt{M_2-1}}{M_\infty} \right] m_{21} + \left[ \frac{m_\infty}{1+m_\infty} - \frac{1}{2(M_\infty^2 - 1)} \right] m_1
\]

where

\[
m_{21} = \frac{\delta_B}{2} z_B
\]

\[
a_2 = (a_{21} + K_1 a_{22}) m_1 + (a_{11} - a_{22}) m_2
\]

where

\[
a_{21} = \left[ \frac{1}{(\hat{B} - \hat{C})} \right] a_{11} d_{11} + \left[ \frac{\hat{E}}{(\hat{B} - \hat{C})} + \frac{\hat{B} a_{11}}{2(\hat{B} - \hat{C})} \right] a_{11} - \frac{S_{FB} a_{11}}{(B - C)}
\]

\[
a_{22} = a_{11} \frac{(\hat{B} + \hat{C})}{(\hat{B} - \hat{C})}
\]

\[
\hat{B} = J_B \left( \frac{d\chi_B}{da} \right) - \chi_B \left( \frac{dJ_B}{da} \right)
\]

\[
\hat{C} = \frac{2}{\delta_0 B} \left[ \chi_B (\frac{dR}{da}) - J_B (\frac{dP}{da}) \right]
\]

\[
\hat{D} = \left[ \chi_B (\frac{d^2 J_B}{da^2}) - J_B (\frac{d^2 \chi_B}{da^2}) \right] + \frac{2}{\delta_0} \left[ \chi_B (\frac{d^2 R}{da^2}) - J_B (\frac{d^2 P}{da^2}) \right]
\]

\[
\hat{E} = \chi_B \left[ 3 (\frac{d\chi_B}{da}) + 2S (\frac{d^2 \chi_B}{da^2}) \right] - J_B \left[ 2 (\frac{d\chi_B}{da}) + S (\frac{d^2 \chi_B}{da^2}) \right]
\]

\[
\hat{F} = J_B a_B (\frac{d\chi_B}{dB}) - 2 \chi_B (\frac{d^2 \chi_B}{dB^2})
\]
\[ \epsilon_2 = (d_{21} + K_1 d_{22} + K_2) m_1^2 + (K_1 - d_{22}) m_2 \]  

(B.14)

where

\[ K_2 = \frac{(3 \gamma - 1)}{2(\gamma - 1)} \frac{m_\infty}{1 + m_\infty} (1 - \frac{(5 \gamma - 3)}{(\gamma - 1)} \frac{m_\infty}{1 + m_\infty}) \]

\[ d_{22} = \frac{1}{B} \left[ \left( \frac{d \lambda_B}{da} \right)_B + \frac{\lambda_B}{P_B} \left( \frac{d \phi}{da} \right)_B \right] (a_{11} - a_{22}) + 2(2 \frac{\lambda_B}{P_B} + 1 + S \frac{E}{w}) - 1 \]

\[ d_{21} = -\frac{1}{\lambda_B} \left[ \left( \frac{d \lambda_B}{da} \right)_B + \frac{\lambda_B}{P_B} \left( \frac{d \phi}{da} \right)_B \right] a_{21} + \]

\[ \left[ -\frac{1}{\lambda_B} \left( \left( \frac{d \lambda_B}{da} \right)_B a_{11} + 2 \left( \frac{\lambda_B}{P_B} + 1 + S \frac{E}{w} \right) \right) + 1 \right] d_{11} \]

\[ \delta_2 = \frac{1}{2} \frac{d}{d_1^2} \epsilon_2 \log \delta_0 \]  

(B.15)

\[ B_2 = (B_{21} + K_1 B_{22}) m_1^2 + (B_{11} - B_{22}) m_2 \]  

(B.16)

where

\[ B_{22} = \frac{\left( -T_B (d_{22} - d_{11} - 1) - \left( \frac{3 \phi_B}{3 a_B} \right)_B (a_{22} - a_{11}) + \left( \frac{3 \phi_B}{3 a_B} \right)_B B_{11} + T_B \left( \frac{d a}{d a} \right)_B a_{22} \right)}{\left( a_B \left( \frac{3 \phi_B}{3 a_B} \right)_B + \frac{\lambda_B}{P_B \phi_B} \right)} \]
\[ B_{21} = \frac{T_B}{G} \left( a_B(d_{21}+d_{11}) - \left( \frac{da}{da} \right)_B(a_{11}d_{11}+a_{21}) - \left( \frac{d^2a}{da^2} \right)_B \right) \]

\[ + \frac{a_B}{G} \left[ \left( \frac{3T}{3a} \right)_B a_{21} + \left( \frac{3T}{3a} \right)_B a_{11} + \left( \frac{3T}{3B} \right)_B B_{11} \right] \left( 1 + d_{11} \right) + \left( \frac{3^2T}{3a^2} \right)_B \frac{a_{11}}{2} \]

\[ + \left( \frac{3^2T}{3B^2} \right)_B \frac{B_{11}}{2} + \left( \frac{3^2T}{3a3B} \right)_B a_{11} B_{11} \]

where

\[ \hat{G} = - \left( a_B \left( \frac{3T}{3B} \right)_B + \frac{\nu B}{P_B Pr_w} \right) \]

The numerical values of these coefficients depend on the wall cooling rate \( S_w \). In the case of an arbitrary choice of the parameter \( S_w \), the numerical computation of the weak interaction coefficients proceeds as follows. One chooses a reference value of \( S_w \), say \( S_{w,\text{ref}} \), for which the integral function polynomial coefficients have been tabulated (herein \( S_{w,\text{ref}} \) must be \(-0.8, -0.6, -0.4, -0.2 \) or \( 0 \)). Equations B.5 and B.7 must be solved by an iteration process thus providing the numerical values of \( a_0 \) and \( B_0 \). Knowing both \( a_0 \) and \( B_0 \) all the integral functions and their successive derivatives can be computed. The numerical values of the weak interactions coefficients contained in eq. B.1 are computed from eq. B.6 to B.16 for the actual value of \( S_w \).

1 From the theoretical point of view the leading terms \( \delta_0, a_0 \) and \( B_0 \) must be independent from the wall cooling rate \( S_w \), nevertheless some weak dependence must be present in the numerical computations mainly due to inaccuracies of the curve fits used to represent the integral functions.
These numerical values have been tabulated for an "exact" computation (i.e., $S_w = S_{w_{ref}}$) respectively $S_w = -0.8$, $-0.6$, $-0.4$, $-0.2$ and 0.

Klineberg's notation being retained for the first four values, the series expansion for the total enthalpy parameter $b$ is:

$$b = b_0 + b_1x + b_2x^2 + \cdots \quad (B.17)$$

In the adiabatic case ($S_w = 0$) the normalized formulation has been adopted.

**Notation used in the tables of weak interaction coefficients**

<table>
<thead>
<tr>
<th>$CC(MJ,LJ)$</th>
<th>$CC(MJ,1)$</th>
<th>$CC(MJ,2)$</th>
<th>$CC(MJ,3)$</th>
</tr>
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<tbody>
<tr>
<td>$CC(1,LJ)$</td>
<td>$\delta_0$</td>
<td>$a_0$</td>
<td>$b_0$ or $B_0(S_w=0)$</td>
</tr>
<tr>
<td>$CC(2,LJ)$</td>
<td>$m_{11}$</td>
<td>$m_{12}$</td>
<td>$m_{21}$</td>
</tr>
<tr>
<td>$CC(3,LJ)$</td>
<td>$d_{11}$</td>
<td>$d_{21}$</td>
<td>$d_{22}$</td>
</tr>
<tr>
<td>$CC(4,LJ)$</td>
<td>$a_{11}$</td>
<td>$a_{21}$</td>
<td>$a_{22}$</td>
</tr>
<tr>
<td>$CC(5,LJ)$</td>
<td>$b_{11}$ or $B_{11}(S_w=0)$</td>
<td>$b_{21}$ or $B_{21}(S_w=0)$</td>
<td>$b_{22}$ or $B_{22}(S_w=0)$</td>
</tr>
</tbody>
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### Weak Interaction Coefficients $S^*=0.8$

<table>
<thead>
<tr>
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<tbody>
<tr>
<td>$CC(1, LJ)$</td>
<td>$0.17303819E 01$</td>
<td>$0.16351673E 01$</td>
<td>$0.3756153E 00$</td>
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<td>$CC(2, LJ)$</td>
<td>$0.65980899E 00$</td>
<td>$0.34655851E 00$</td>
<td>$0.16130764E 01$</td>
</tr>
<tr>
<td>$CC(3, LJ)$</td>
<td>$0.10192151E 01$</td>
<td>$0.31657905E 01$</td>
<td>$0.24312701E 01$</td>
</tr>
<tr>
<td>$CC(4, LJ)$</td>
<td>$0.19320478E 01$</td>
<td>$0.38467279E 01$</td>
<td>$0.16281033E 01$</td>
</tr>
<tr>
<td>$CC(5, LJ)$</td>
<td>$0.11565837E 00$</td>
<td>$-0.41320077E 01$</td>
<td>$0.21047707E 01$</td>
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### Weak Interaction Coefficients $S^*=0.6$

<table>
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<tr>
<th></th>
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<tbody>
<tr>
<td>$CC(1, LJ)$</td>
<td>$0.17241549E 01$</td>
<td>$0.16336767E 01$</td>
<td>$0.28211760E 00$</td>
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<tr>
<td>$CC(2, LJ)$</td>
<td>$0.66133934E 00$</td>
<td>$0.58851825E 00$</td>
<td>$0.16060264E 01$</td>
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<tr>
<td>$CC(3, LJ)$</td>
<td>$0.12964274E 01$</td>
<td>$-0.38444666E 01$</td>
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<td>$CC(4, LJ)$</td>
<td>$0.19024386E 01$</td>
<td>$-0.56515159E 01$</td>
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<tr>
<td>$CC(5, LJ)$</td>
<td>$0.11233703E 00$</td>
<td>$-0.40482835E 01$</td>
<td>$0.20676102E 01$</td>
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### Weak Interaction Coefficients $S^*=0.4$

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</tr>
</thead>
<tbody>
<tr>
<td>$CC(1, LJ)$</td>
<td>$0.17241990E 01$</td>
<td>$0.16341772E 01$</td>
<td>$0.18793846E 00$</td>
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<td>$CC(2, LJ)$</td>
<td>$0.66106078E 00$</td>
<td>$0.10341641E 01$</td>
<td>$0.16264856E 01$</td>
</tr>
<tr>
<td>$CC(3, LJ)$</td>
<td>$0.15666942E 01$</td>
<td>$-0.50549929E 01$</td>
<td>$0.30966854E 01$</td>
</tr>
<tr>
<td>$CC(4, LJ)$</td>
<td>$0.24876709E 01$</td>
<td>$-0.79538545E 01$</td>
<td>$0.28607391E 01$</td>
</tr>
<tr>
<td>$CC(5, LJ)$</td>
<td>$0.80140364E-01$</td>
<td>$-0.33026561E 01$</td>
<td>$0.16812727E 01$</td>
</tr>
</tbody>
</table>

### Weak Interaction Coefficients $S^*=0.2$

<table>
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<th>$CC(MJ, 3)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$CC(1, LJ)$</td>
<td>$0.17240469E 01$</td>
<td>$0.16340014E 01$</td>
<td>$0.93572139E-01$</td>
</tr>
<tr>
<td>$CC(2, LJ)$</td>
<td>$0.68186177E 00$</td>
<td>$-0.13781654E 01$</td>
<td>$0.16055371E 01$</td>
</tr>
<tr>
<td>$CC(3, LJ)$</td>
<td>$0.18419537E 01$</td>
<td>$-0.43850341E 01$</td>
<td>$0.34234057E 01$</td>
</tr>
<tr>
<td>$CC(4, LJ)$</td>
<td>$0.30817132E 01$</td>
<td>$-0.10742639E 02$</td>
<td>$0.35315498E 01$</td>
</tr>
<tr>
<td>$CC(5, LJ)$</td>
<td>$0.50650164E-01$</td>
<td>$-0.25904259E 01$</td>
<td>$0.13036112E 01$</td>
</tr>
</tbody>
</table>

### Weak Interaction Coefficients $S^*=0.0$

<table>
<thead>
<tr>
<th></th>
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</tr>
</thead>
<tbody>
<tr>
<td>$CC(1, LJ)$</td>
<td>$0.17238450E 01$</td>
<td>$0.16345266E 01$</td>
<td>$0.47048074E 00$</td>
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<tr>
<td>$CC(2, LJ)$</td>
<td>$0.66245412E 00$</td>
<td>$0.17238450E 01$</td>
<td>$0.16142501E 01$</td>
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<td>$CC(3, LJ)$</td>
<td>$0.21196576E 01$</td>
<td>$-0.52645311E 01$</td>
<td>$0.37380135E 01$</td>
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<tr>
<td>$CC(4, LJ)$</td>
<td>$0.36297047E 01$</td>
<td>$-0.18092578E 02$</td>
<td>$0.41543216E 01$</td>
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<tr>
<td>$CC(5, LJ)$</td>
<td>$0.30231481E 00$</td>
<td>$-0.11339723E 02$</td>
<td>$0.56879683E 01$</td>
</tr>
</tbody>
</table>
APPENDIX C

COMPUTER PROGRAMS

C.1 Introduction

The programs described herein have been designed for the computation of a shock wave boundary layer interaction generated either by a flat plate wedge configuration or an incident shock wave impinging on a flat plate. The basic assumptions are:
- The flow is steady, two dimensional and laminar over the whole interaction region.
- The free stream flow is supersonic or moderately hypersonic ($M_\infty < 10$).
- The interaction takes place between two regions of self-preserved flat plate flow respectively located upstream and far downstream of the interaction.
- The wall temperature must be uniform and constant, but arbitrarily taken in the range corresponding to $-0.8 < S_w < 0$.

In order to perform such a computation three sets of programs must be available:
1. A main line program design for the integration of the governing equations of the shock wave boundary layer interaction according to the procedure described in section 2.
2. A program designed for the calculation of similar solutions and the associated integral quantities.
3. A set of programs to curve fit the boundary layer integral quantities and to establish the tables of polynomial coefficients.
4. An additional program dealing with weak interaction calculations has also been written.

Listings and explanation concerning the programs referred to in 1 and 4 are presented in this Appendix C.
The program used for similar solution computations can be found in ref. 14 and the results of curve fits are presented in the form of tables of polynomial coefficients in Appendix A.

C.2 Program organization
(Integration of governing differential equations)

The main line program (called VSWTI) has been designed for use with an IBM 1130 digital computer. The limited storage capacity of the central memory unit (8K) lead to an extensive use of "local" mode therefore this program has been divided into 10 subroutines called successively by a main program. In addition, numerical data have been stored as "files" on a magnetic disk supply. Files Nr 1, 2, 3, and 4 store the intermediate results required in the interpolation procedure.

Prior to starting the computation with the main line program VSWTI one must store the tables of polynomial coefficients according to the value of $S_{ref}$ which has been selected; files Nr 18 to 22 are used for this purpose. 3 tables are required if $S_{ref} = -0.8, -0.6, -0.4$ or $-0.2$.

File Nr 18 Attached flow polynomials
File Nr 19 Separated flow polynomials
File Nr 20 Weak interaction coefficients.

For the adiabatic case 5 tables are required, respectively,
File Nr 18 Attached flow polynomials $B > 0.35$
File Nr 19 Separated flow polynomials $B > 0.35$
File Nr 20 Weak interaction coefficients
File Nr 21 Attached flow polynomials $B < 0.35$
File Nr 22 Separated flow polynomials $B < 0.35$

These tables are presented at Appendix A and R.
### Program Name | Title | Operators
--- | --- | ---
Program VSWTI | Main line program | Call the following subroutines
Subroutine LKHT1 | Inviscid flow calculations | Read input data cards
 |  | Inviscid flow field calculations (model with or without angle of attack)
 |  | Calculation of the equivalent incident shock
Subroutine VSWT2 | Iteration on the initial conditions | Iteration on both $x_0$ and $\epsilon$
 |  | Weak interaction calculations
 |  | Calculation of the perturbed initial values (subcritical boundary layer)
Subroutine VSWT8* | Jump calculations | Iteration procedure of the jump equations (supercritical boundary layer)
Subroutine VSWT9 | Jump calculations | Calculations of the iteration steps of the 3 simultaneous jump equations by Newton's method
Subroutine VSWT3 | Runge-Kutta integration routine | Integration of the four governing differential equations
Subroutine VSWT4 | Tests on solution behaviour | Correction of the integral path in the vicinity of separation, shock impingement and reattachment points
 |  | Tests on the behaviour of the solution
Subroutine LEEK4 | Entropy change calculations | Calculation of the entropy variation across the incident shock wave
Subroutine VSWT5 | Head title | Read input data and print out
Subroutine VSWT6 | Print out | Print out of titles
Subroutine VSWT7 | Interpolation routine | Computation of the downstream solution by interpolation between adjacent solutions (numerical suppression of successive diverging solutions)

* An alternative version of the jump calculations routines called VS88V can be substituted for subroutines VSWT8 and VSWT9. Subroutine VS88V contains a rough iteration method for the three simultaneous jump equations using a halving procedure of fixed initial iteration steps.
C.2.2 List of subroutines used for flat plate calculations (VSWIE)

A separate program has been developed for flat plate flow calculations (VSWIE). This program must also be used for weak interaction coefficient calculations when the actual value of $S_w$ is different from the five reference values tabulated in Appendix B.

<table>
<thead>
<tr>
<th>Program name</th>
<th>Title</th>
<th>Operations</th>
</tr>
</thead>
<tbody>
<tr>
<td>VSWIE</td>
<td>Main line program</td>
<td>Computation of the weak interaction solution</td>
</tr>
<tr>
<td>Subroutine VSWWI</td>
<td>Iteration part</td>
<td>Calculation of the leading terms of series expansions of flow variables $a_0$ and $B_0$</td>
</tr>
<tr>
<td>Subroutine VSWI2</td>
<td>Coefficient calculation</td>
<td>Calculation of the weak interaction coefficients (see Appendix B)</td>
</tr>
<tr>
<td>VSWCL</td>
<td>Coefficient loading</td>
<td>Storage of weak interaction coefficients on file Nr 20</td>
</tr>
</tbody>
</table>

C.2.3 Block diagram of computer program VSWTI

The computation of the undisturbed flat plate flow being relatively straightforward, the program and subroutine organization can be easily deduced from the listings presented at the end of Appendix C. A simplified flow chart of the main line program VSWTI is presented below.
Call VSW75
read $C_c$, $C_{ref}$
declare iteration procedure

Call LEKT2
read physical data
inviscid flow field calculation
equivalent incident shock calculation

Assume $x_0$

Call VSW75
initial values computation
(weak interaction)

Subcritical boundary layer
on stability

Assume $x_0$

Perturbed initial values computation

Call VSW76
title print out

Call VSW73
Runge-Kutta routine
Integration of the $n$ governing equations

Call VSW73
Pseudo-Iutta routine
Integration of the 3 jump equations

Yes

Jump eqns. satisfied

Call VSW75
Iteration steps computation

Downstream solution determination

Call VSW74
Integral path corrections
at separation, shock
impingement and reattachment

Test on the solution behaviour

2 branches defined

Yes

End of downstream solution

Critical point approached

Call VSW7
Interpolation routine

End of interpolations

Graphical extrapolation
read initial conditions for
downstream solution

Complete solution obtained - end

Yes

Downstream solution determined

End

No
C.2.4 Operating procedure

The complete calculation of a shock wave boundary layer interaction (for a given fixed value of $S_w$) must be performed step by step as follows:

1. Selection of the reference value $S_{w_ref}$ and storage of the associated tables of polynomials coefficients (In practice $S_{w_ref}$ must be chosen among the five values tabulated in Appendix A).

2. Computation and storage of the weak interaction coefficients (Execution of program VSWCL).

3. Computation of the entire interaction (Execution of the main lines program VSWTI) — two cases may occur:
   a. Entirely subcritical interaction, the computation proceeds smoothly toward the Blasius solution at downstream infinity;
   b. The Crocco-Lees critical point is encountered downstream of reattachment. The automatic computation must be stopped in the vicinity of the singularity. Then a graphical extrapolation of the flow variable trajectories through the critical point provides new starting conditions for the downstream integration. The input data is entered through the computer typewriter.

4. The above calculation may be completed by executing the program VSWIE (weak interaction region).

The entire calculation of one case of interaction as described above takes approximately 7 hours on the IBM 1130 computer in the most time consuming case (moderately cooled wall when the graphical extrapolation must be performed) and 3 hours for the adiabatic case.
This program has been designed as a research tool, therefore a number of possible outputs are available at the cost of an increase in complexity and time consuming executions. This chapter is an attempt to clarify the numerical difficulties encountered during the numerical applications of the theory and which do not appear in the discussions of section 2.

1. A single set of "universal" integral functions has not been tabulated and curve fitted, but these functions are computed step by step during the streamwise integration of the governing equations from the tables of polynomial coefficients obtained for 5 different values of \( s_w \). Therefore, two values of \( s_w \) are simultaneously used in the computer program:

- \( s_w \) (Fortran symbol SW) is the actual given value derived from physical considerations \( (T_w/T_t) \);
- \( s_w^{\text{ref}} \) (Fortran symbol SWREF) is an arbitrarily chosen reference value for which integral function polynomial coefficients have been tabulated (here in \( s_w^{\text{ref}} \) must be \(-0.8, -0.6, -0.4, -0.2\) or 0).

2. When the reference value \( s_w^{\text{ref}} = 0 \) is used, the equations derived in the general case remain valid provided one sets SWREF = \(-1\) in the input data.

3. Difficulties have been encountered during the streamwise integration with respect to \( x \) in the vicinity of separation and reattachment points, due to the large value of the streamwise gradient \( \frac{\partial a}{\partial x} \). Therefore, the integration of the governing equations in the separated region is performed with respect to \( a \), the velocity profile parameter. Then linear corrections of the integral paths have been applied in order to locate accurately separation, shock impingement and reattachment points.
### C.3 Input data of computer programs

The data cards listed below contain both physical data of the problem and the initial values of iterative parameters. Six data cards are required for the execution of the main program VSWTI, a 7th optional data card may be added. Only one data card is required to execute the program VSWIE (flat plate computations).

#### C.3.1 Program VSWTI

<table>
<thead>
<tr>
<th>1st Card</th>
<th>Read in subroutine VSWT5</th>
<th>Wall temperature conditions</th>
</tr>
</thead>
<tbody>
<tr>
<td>Symbols</td>
<td>SWREF, SW, KT10</td>
<td></td>
</tr>
<tr>
<td>Format</td>
<td>F6.3, F6.3, I1</td>
<td></td>
</tr>
<tr>
<td>Comments</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

- Symbols SWREF, SW, KT10 represent the wall temperature conditions.
- Format F6.3, F6.3, I1 corresponds to the specific data format required.
- Comments explain the significance of the values.

#### 2nd Card

<table>
<thead>
<tr>
<th>2nd Card</th>
<th>Read in subroutine LKHT1</th>
<th>Physical data of the interaction</th>
</tr>
</thead>
<tbody>
<tr>
<td>Symbols</td>
<td>ZMA, TT, REU, XSH, TO, ALP</td>
<td></td>
</tr>
<tr>
<td>Format</td>
<td>F5.4, F4.2, E15.8, F6.3, F5.1, F5.2</td>
<td></td>
</tr>
<tr>
<td>Comments</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

- Symbols ZMA, TT, REU, XSH, TO, ALP represent the physical data of the interaction.
- Format F5.4, F4.2, E15.8, F6.3, F5.1, F5.2 correspond to the specific data format required.
- Comments explain the significance of the values.
<table>
<thead>
<tr>
<th>Comment</th>
<th>Format</th>
<th>Symbol(s)</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial value of the assumed location of the beginning of the interaction (mm if data is given in inches)</td>
<td>$X_0$</td>
<td>X0</td>
<td>Read in subroutine LWRITE</td>
</tr>
<tr>
<td>Iteration step on Xo</td>
<td>$X_0$</td>
<td>X0</td>
<td>Read in subroutine LWRITE</td>
</tr>
<tr>
<td>Accuracy of the interpolation on pressure distribution</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Iteration step on E</td>
<td>$E_{1}$, $E_{2}$</td>
<td>$E, E_1, E_2$</td>
<td>Initial iteration step on E</td>
</tr>
<tr>
<td>Large streamwise integration</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Small streamwise integration</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Print out spacing on pressure distribution</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Prandtl number at the wall</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Integration and iteration steps
### Note:
The superscript H denotes the 4 input data required when operating with subroutines VSWT8 and VSWT9. When the simplified version, subroutine VSS80V is used, the 9 input data listed above are required.

<table>
<thead>
<tr>
<th>6th Card</th>
<th>Read in subroutine VSWT8*&lt;sup&gt;™&lt;/sup&gt; or VSS80V</th>
<th>Initial data for iteration of the jump equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Symbols</td>
<td>A12&lt;sup&gt;™&lt;/sup&gt;</td>
<td>CBI2&lt;sup&gt;™&lt;/sup&gt;</td>
</tr>
<tr>
<td>Format</td>
<td>E6.2</td>
<td>E6.2</td>
</tr>
</tbody>
</table>

Note: The superscript H denotes the 4 input data required when operating with subroutines VSWT8 and VSWT9. When the simplified version, subroutine VSS80V is used, the 9 input data listed above are required.

### 7th Card (optional)
<table>
<thead>
<tr>
<th>Read in subroutine LKHT1</th>
<th>Stagnation pressure</th>
</tr>
</thead>
<tbody>
<tr>
<td>Symbols</td>
<td>PT</td>
</tr>
<tr>
<td>Format</td>
<td>F7.2</td>
</tr>
<tr>
<td>Comments</td>
<td>Stagnation pressure of the free stream (mm Hg)</td>
</tr>
</tbody>
</table>

**This card is read only when Re <sub>2</sub> = 0 in the 2nd data card.**
<table>
<thead>
<tr>
<th>Symbols</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>Polynomial coefficients have been tabulated for $S = S_{\text{ref}}$</td>
<td>P6.3</td>
</tr>
<tr>
<td>Actual value of $S_w$</td>
<td>P6.3</td>
</tr>
<tr>
<td>Prandtl number at the wall</td>
<td>P6.3</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Symbols</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>Polynomial coefficients have been tabulated for $S_w = S_{\text{ref}}$</td>
<td>P6.3</td>
</tr>
<tr>
<td>Free stream Mach number</td>
<td>P5.4</td>
</tr>
<tr>
<td>Free stream unit Reynolds number (per meter)</td>
<td>P5.4</td>
</tr>
<tr>
<td>Stagnation temperature (°K)</td>
<td>P5.4</td>
</tr>
<tr>
<td>Actual value of $S_w$</td>
<td>P6.3</td>
</tr>
<tr>
<td>Prandtl number at the wall</td>
<td>P6.3</td>
</tr>
<tr>
<td>Abscissa of the beginning of the weak interaction computation (mm)</td>
<td>E6.3</td>
</tr>
<tr>
<td>Streamwise step on $x$ for print out (mm)</td>
<td>E7.2</td>
</tr>
<tr>
<td>Total numbers of print outs</td>
<td>I2</td>
</tr>
</tbody>
</table>
### C.3.3 List of data switches used in the program VSWI

<table>
<thead>
<tr>
<th>Switch No</th>
<th>Index</th>
<th>( \phi N )</th>
<th>( \phi F F )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>JJ</td>
<td>Small streamwise integration step ( \Delta X_1 )</td>
<td>Large streamwise integration step ( \Delta X_2 )</td>
</tr>
<tr>
<td>1</td>
<td>KK</td>
<td>No print out except at 1rst and last streamwise integration step</td>
<td>Print out at fixed intervals (WW) on pressure distribution</td>
</tr>
<tr>
<td>2</td>
<td>JL</td>
<td>If switch 12 is ( \phi N ) read input data on the typewriter for downstr.solut.,</td>
<td>No print out</td>
</tr>
<tr>
<td>3*</td>
<td>LI</td>
<td>Print out of ( N_1, N_2, N_3, N_4 ) and DE if switch 4 is ( \phi N )</td>
<td>Output of the jump equation iteration</td>
</tr>
<tr>
<td></td>
<td></td>
<td>#When VSWT8 is processed print out of every jump equation iteration step</td>
<td>#Not to be used during the integration process</td>
</tr>
<tr>
<td>4</td>
<td>J</td>
<td>Print out of every streamwise integration step</td>
<td></td>
</tr>
<tr>
<td>5*</td>
<td>IJ</td>
<td>When VSWT8 is processed read new initial data for the iteration of jump equations (if switch 11 is ( \phi N ))</td>
<td>Print out of REX if switch 13 is ( \phi F F )</td>
</tr>
<tr>
<td>6</td>
<td>IJ</td>
<td>Print out of ( x/L ) if switch 13 is ( \phi F F )</td>
<td>Print out of CF if switch 13 is ( \phi N )</td>
</tr>
<tr>
<td>7</td>
<td>IJ</td>
<td>Print out of ( c_f ) if switch 13 is ( \phi N )</td>
<td>Print out of ( p/p_0 )</td>
</tr>
<tr>
<td>8</td>
<td>IJ</td>
<td>Print out of ( p/p_\infty )</td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>IK</td>
<td>Input of initial data ( x_0, \varepsilon, T ) on typewriter</td>
<td>Print out of the abscissa ( x ) in (mm) (Input data XSH and XN must be in inches)</td>
</tr>
<tr>
<td>10</td>
<td>K</td>
<td>Read new data cards if switch 4 is ( \phi N )</td>
<td></td>
</tr>
<tr>
<td>11</td>
<td>IW</td>
<td>Call data switch 5</td>
<td></td>
</tr>
<tr>
<td>12</td>
<td>NN</td>
<td>Bypass all the tests on solution behaviour, call data switch 2</td>
<td></td>
</tr>
<tr>
<td>13</td>
<td>IØ</td>
<td>Print out of ( C_F ) and ( C_H )</td>
<td>Print out of ( R_{ex} ) and ( \frac{dp}{dx} )</td>
</tr>
<tr>
<td>14</td>
<td>NS</td>
<td>Use small step ( \Delta X_1 ) until ( X_{N} ), print out of each step in the neighbourhood of ( X_{N} ), No interpolations and read new initial data</td>
<td></td>
</tr>
<tr>
<td>15*</td>
<td>NT</td>
<td>Read new initial data prior to iterate on ( x_0 ) #Also arithmetic and transfer traces</td>
<td></td>
</tr>
</tbody>
</table>
### DATA

<table>
<thead>
<tr>
<th>Fortran symbols</th>
<th>M</th>
<th>TETA</th>
<th>REU</th>
<th>XSH</th>
<th>XN</th>
<th>DXO</th>
<th>DX1</th>
<th>DX2</th>
<th>DAS</th>
<th>DP</th>
<th>PRX</th>
<th>PRXO</th>
<th>DIPS</th>
<th>WW</th>
<th>PRW</th>
<th>BDIV</th>
</tr>
</thead>
<tbody>
<tr>
<td>Algebraic symbols</td>
<td>M_0</td>
<td>θ</td>
<td>Re_u</td>
<td>X_SH</td>
<td>X_n</td>
<td>Δx_0</td>
<td>Δx_1</td>
<td>Δx_2</td>
<td>Δa</td>
<td>Δp_x</td>
<td>Pr_x</td>
<td>Δε</td>
<td>WW</td>
<td>Pr_w</td>
<td>b_div</td>
<td></td>
</tr>
<tr>
<td>Comments</td>
<td>See comments of the data cards</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

### EQUIVALENT INCIDENT SHOCK

<table>
<thead>
<tr>
<th>Fortran symbols</th>
<th>MF</th>
<th>MFA</th>
<th>TETAG</th>
<th>FTA</th>
<th>PTB</th>
<th>BTC</th>
<th>PFPI</th>
</tr>
</thead>
<tbody>
<tr>
<td>Algebraic symbols</td>
<td>M_0</td>
<td>-</td>
<td>g</td>
<td>g_A</td>
<td>g_B</td>
<td>g_C</td>
<td>P_e/P_m</td>
</tr>
<tr>
<td>Comments</td>
<td>Mach number at downstream side of the equivalent shock behind the shock wave attached to the hinge line of the equivalent shock generator</td>
<td>Mach number behind the shock</td>
<td>Angle of the equivalent shock generator</td>
<td>Shock angle (Mach angle)</td>
<td>Incident shock angle</td>
<td>Reflected shock angle</td>
<td>Inviscid flow overall pressure ratio</td>
</tr>
</tbody>
</table>

### WEAK INTERACTION

<table>
<thead>
<tr>
<th>Fortran symbols</th>
<th>XO</th>
<th>MO</th>
<th>DTEO</th>
<th>POSPI</th>
<th>DXO</th>
<th>AO</th>
<th>GHO</th>
<th>CI</th>
<th>EPS</th>
<th>TO</th>
<th>CFO</th>
<th>CEO</th>
</tr>
</thead>
<tbody>
<tr>
<td>Algebraic symbols</td>
<td>x_0</td>
<td>M_0</td>
<td>δ_10</td>
<td>po/P_m</td>
<td>Δx_1</td>
<td>a_0</td>
<td>B_g</td>
<td>X</td>
<td>ε</td>
<td>T_t</td>
<td>C_f_0</td>
<td>C_R_0</td>
</tr>
<tr>
<td>Comments</td>
<td>See list of symbols</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Boundary layer nature: initially subcritical or initially supercritical boundary layer according to the sign of determinant D (DE=-D)

Supercritical Subcritical Jump

* This print out is suppressed for initially subcritical boundary layer
### JUMP INITIAL DATA

<table>
<thead>
<tr>
<th>Fortran symbols</th>
<th>AI2</th>
<th>GB2</th>
<th>PRCI</th>
<th>EPSM</th>
</tr>
</thead>
<tbody>
<tr>
<td>Algebraic symbols</td>
<td>$a_2$</td>
<td>$B_2$</td>
<td>$P_{ci}$</td>
<td>$M_2/M_1$</td>
</tr>
<tr>
<td>Comments</td>
<td>See comments of the data cards</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

### JUMP EQUATION ITERATION

<table>
<thead>
<tr>
<th>Fortran symbols</th>
<th>DEL 88</th>
<th>DEL 89</th>
<th>DEL 90</th>
<th>MF1</th>
<th>ME2</th>
<th>DT1</th>
<th>DT2</th>
<th>A1</th>
<th>A2</th>
<th>GB1</th>
<th>GB2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Algebraic symbols</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>$M_1$</td>
<td>$M_2$</td>
<td>$\delta_1$</td>
<td>$\delta_2$</td>
<td>$a_1$</td>
<td>$a_2$</td>
<td>$B_1$</td>
<td>$B_2$</td>
</tr>
<tr>
<td>Comments</td>
<td>Residue of 1st eq.</td>
<td>Residue of 2nd eq.</td>
<td>Residue of 1st eq.</td>
<td>Residue of 2nd eq.</td>
<td>see list of symbols</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

### INTEGRATION

<table>
<thead>
<tr>
<th>Fortran symbols</th>
<th>PE/PI</th>
<th>X</th>
<th>CF</th>
<th>ME</th>
<th>DTE</th>
<th>A</th>
<th>GB</th>
<th>CH</th>
</tr>
</thead>
<tbody>
<tr>
<td>Algebraic symbols</td>
<td>$p/p_{-}$</td>
<td>$x$</td>
<td>$C_F$</td>
<td>$Me$</td>
<td>$\delta_i$</td>
<td>$a$</td>
<td>$B$</td>
<td>$C_H$</td>
</tr>
<tr>
<td>Comments</td>
<td>see list of symbols</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Optional print out: REX or $X/L$  
$Re_X$ or $X/x_{SH}$  
$DPDX$  
$dp/dx$

ETA1 and ETA2 define respectively the type of the previous and of the present downstream solution, according to sketch 5.

ETA2 = 1 corresponds to solution going toward an expansion 1  
ETA2 = 2 corresponds to solution going toward a new separation 2.

Typical example of print out during the execution of program VSWIE

**Head title writing**
**Print out of initial data**

<table>
<thead>
<tr>
<th>Fortran symbols</th>
<th>SWREF</th>
<th>ZMI</th>
<th>REU</th>
<th>TO</th>
<th>SW</th>
<th>PRW</th>
<th>XINIT</th>
<th>DX</th>
<th>MX</th>
</tr>
</thead>
<tbody>
<tr>
<td>Algebraic symbols</td>
<td>$S_{wref}$</td>
<td>$M_w$</td>
<td>$P_{eu}$</td>
<td>$T_t$</td>
<td>$S_w$</td>
<td>$P_{riv}$</td>
<td>$x_{init}$</td>
<td>$\Delta x$</td>
<td>$MX$</td>
</tr>
<tr>
<td>Comments</td>
<td>see comments of the data cards</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
## WEAK INTERACTION COEFFICIENTS

### 1st Part - Iteration on $a_0$ and $B_0$

<table>
<thead>
<tr>
<th>Fortran symbols</th>
<th>$\mu_G$</th>
<th>$J_B$</th>
<th>$P_B$</th>
<th>$R_B$</th>
<th>$AO$</th>
<th>$a_0$</th>
<th>$B_0$</th>
<th>$\Gamma_B$</th>
<th>$\Delta a_0$</th>
<th>$\Delta B_0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Algebraic symbols</td>
<td>$\frac{H}{J}$</td>
<td>$P$</td>
<td>$R$</td>
<td>$AO$</td>
<td>$\alpha$</td>
<td>$a$</td>
<td>$\Gamma$</td>
<td>$B_0$</td>
<td>$\Delta a_0$</td>
<td>$\Delta B_0$</td>
</tr>
<tr>
<td>Comments</td>
<td>Integral function at sinus point</td>
<td>Integral function at sinus point</td>
<td>Integral function at sinus point</td>
<td>Leading term of series expansion at sinus point</td>
<td>Leading term of series expansion at sinus point</td>
<td>Leading term of series expansion at sinus point</td>
<td>Leading term of series expansion at sinus point</td>
<td>Integral function at sinus point</td>
<td>Integral function at sinus point</td>
<td>Integral function at sinus point</td>
</tr>
</tbody>
</table>

### 2nd Part - Print out of the numerical values of weak interaction coefficients

<table>
<thead>
<tr>
<th>Fortran symbols</th>
<th>DO AO GBO R11 R12 R21 D11 D21 D22 All A21 A22 GE11 GB21 GB22</th>
</tr>
</thead>
<tbody>
<tr>
<td>Algebraic symbols</td>
<td>$\delta_0$ $a_0$ $B_0$ $m_1$ $m_2$ $d_1$ $d_2$ $a_{21}$ $a_{22}$ $b_{11}$ $b_{21}$ $b_{22}$</td>
</tr>
<tr>
<td>Comments</td>
<td>see Appendix B</td>
</tr>
</tbody>
</table>

### Print out of the flow parameter streamwise distribution

<table>
<thead>
<tr>
<th>Fortran symbols</th>
<th>X Q1 ME DTE A GB PSPI POSPI CF CH</th>
</tr>
</thead>
<tbody>
<tr>
<td>Algebraic symbols</td>
<td>$x$ $X$ $Me$ $\delta$ $\alpha$ $a$ $B$ $p/p_m$ $p/p_m$ $C_F$ $C_H$</td>
</tr>
<tr>
<td>Comments</td>
<td>see list of symbols</td>
</tr>
</tbody>
</table>

---

*See list of symbols*
### C.5 List of key integers

In order to ease the understanding of Fortran listings of the computer program VSWTI, a list of the key integers in the program logic trace is presented below.

<table>
<thead>
<tr>
<th>Key integer name</th>
<th>Numerical value</th>
<th>Meaning</th>
</tr>
</thead>
<tbody>
<tr>
<td>K1</td>
<td>1</td>
<td>Attached flow upstream of separation point</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>Separated flow inbetween separation and reattachment point</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>Attached flow downstream of reattachment point</td>
</tr>
<tr>
<td>K2</td>
<td>1</td>
<td>Deal with calculations upstream of the hinge line</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>Deal with calculations downstream the hinge line</td>
</tr>
<tr>
<td>K3</td>
<td>1</td>
<td>At the beginning of the integration go into subroutine VSWT7</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>Output of subroutine LKHT4 go through LEEK4 for entropy change calculations and then come back into LKHT4</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>Output LKHT4 and go to the beginning of the integration</td>
</tr>
<tr>
<td>K4</td>
<td>1</td>
<td>If K5=2 1rst recording of some solution, whatever it is</td>
</tr>
<tr>
<td></td>
<td>2</td>
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<td>Recording of the 2nd solution of the 2nd file (whatever it is)</td>
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<td>If the two previous solutions belong to different type recording on the 3rd file</td>
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<td>Return to iteration on the initial values ((x_0 \text{ or } e)) after testing the solution behaviour</td>
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<td>During the interpolation process return to integration into subroutine VSWT3</td>
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<td>If the number of data points stored on a file is too small return to new data input</td>
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<td>1...n</td>
<td>Number of halving operations on the interpolation accuracy for a single record</td>
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<td>2 successive solutions of different type</td>
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<td>In the jump calculations routine (VS80V or VSWT8) Integral functions computations upstream jump location ((\ell_1))</td>
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<td>In the jump calculations routine (VS80V or VSWT8) Integral functions computation downstream of jump location ((\ell_2))</td>
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<td>In the alternative jump calculations routine (VS80V) - iteration of the 1rst jump eq. 2-82</td>
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<td>In the alternative jump calculations routine (VS80V) - iteration of the 2nd jump eq. 2-83</td>
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<td>During the iteration of the jump equations goes through VSWT9 subroutine</td>
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<td>End of iteration of the jump bypass the subroutine VSWT9</td>
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<td>No print out if the type of solution has already been found</td>
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<td>Further iterations on ( x_0 ), ( \Delta x_0 ) are fixed by input data</td>
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<td>Iteration on ( \varepsilon )</td>
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<td>Deal with the definition of the files number ( (S_w = 0 ) only) ( B &gt; 0.35 )</td>
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</table>

Note also that \( J_\Phi \) and \( J_S \) are used for the definition of the file numbers when the tables containing the polynomial coefficients have to be used whilst \( NR \) deals with the file numbers used for intermediate solution storage during the interpolation process.
LIST OF FORTRAN SYMBOLS USED IN
COMPUTER PROGRAMS VSWTI AND VSWIE

Note: *
See list of symbols ahead of this report
**
See list of notation included into Appendix A and B
***
Notation defined in section 2.2.4.

<table>
<thead>
<tr>
<th>Fortran symbols</th>
<th>Algebraic notation</th>
<th>Program name</th>
<th>Definition</th>
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Note: The notation DPDX is also used for CH according to data switch 13.

Integral function derivatives (2nd order)

Jump equation partial derivatives
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<td>EP2</td>
<td>$\epsilon_2$</td>
<td>VSWT2</td>
<td></td>
</tr>
<tr>
<td>F</td>
<td>F</td>
<td>common</td>
<td></td>
</tr>
<tr>
<td>FF</td>
<td>$f$</td>
<td>common</td>
<td></td>
</tr>
<tr>
<td>FNU</td>
<td>$\Gamma(a,B)$</td>
<td>common</td>
<td></td>
</tr>
<tr>
<td>GAM</td>
<td>$\Gamma(a,B)$</td>
<td>common</td>
<td></td>
</tr>
<tr>
<td>GB</td>
<td>B</td>
<td>VSWT8</td>
<td></td>
</tr>
<tr>
<td>GBI</td>
<td>$B_{init}$</td>
<td>VSWT2</td>
<td></td>
</tr>
<tr>
<td>GB2</td>
<td>$B_2$</td>
<td>VSWT8,VSWT9</td>
<td></td>
</tr>
<tr>
<td>GBI2</td>
<td>$B_{2init}$</td>
<td>VSWT8,VSWT9</td>
<td></td>
</tr>
<tr>
<td>GBZ</td>
<td>B</td>
<td>common</td>
<td></td>
</tr>
<tr>
<td>GBO</td>
<td>$B_0$</td>
<td>VSWT2</td>
<td></td>
</tr>
<tr>
<td>GB1</td>
<td>$B_1$</td>
<td>VSWT2</td>
<td></td>
</tr>
<tr>
<td>GB2</td>
<td>$B_2$</td>
<td>VSWT8,VSWIE</td>
<td></td>
</tr>
<tr>
<td>GB11</td>
<td>$B_{11}$</td>
<td>VSWT1</td>
<td></td>
</tr>
<tr>
<td>GB21</td>
<td>$B_{211}$</td>
<td>VSWT2</td>
<td></td>
</tr>
<tr>
<td>GB22</td>
<td>$B_{22}$</td>
<td>VSWT2</td>
<td></td>
</tr>
<tr>
<td>GJ</td>
<td>J</td>
<td>common</td>
<td></td>
</tr>
<tr>
<td>GP</td>
<td>P</td>
<td>common</td>
<td></td>
</tr>
<tr>
<td>GPO</td>
<td>$P(S_w=0)$</td>
<td>common</td>
<td>Integral function $P(a)$ defined from polynomial coeff. table established for $S_w=0$</td>
</tr>
<tr>
<td>Fortran symbols</td>
<td>Algebraic notation</td>
<td>Program name</td>
<td>Definition</td>
</tr>
<tr>
<td>----------------</td>
<td>--------------------</td>
<td>--------------</td>
<td>------------</td>
</tr>
<tr>
<td>GP08</td>
<td>( P(S_w = -0.8) )</td>
<td>common</td>
<td>Integral function ( P(a) ) defined from polynomial coefficient table established for ( S_w = -0.8 )</td>
</tr>
<tr>
<td>GSIG</td>
<td>( E(B) )</td>
<td>common</td>
<td></td>
</tr>
<tr>
<td>GTAB</td>
<td>( T(a, b) )</td>
<td>common</td>
<td></td>
</tr>
<tr>
<td>H</td>
<td>( h )</td>
<td>common</td>
<td></td>
</tr>
<tr>
<td>HR</td>
<td>( H )</td>
<td>common</td>
<td></td>
</tr>
<tr>
<td>PFPI</td>
<td>( P_{\infty}/P_0 )</td>
<td>common</td>
<td>Overall inviscid pressure ratio</td>
</tr>
<tr>
<td>POSPI</td>
<td>( P_{e_0}/P_0 )</td>
<td>common</td>
<td></td>
</tr>
<tr>
<td>PMF</td>
<td>( m )</td>
<td>common</td>
<td></td>
</tr>
<tr>
<td>PMI</td>
<td>( \frac{m}{m_e} )</td>
<td>common</td>
<td></td>
</tr>
<tr>
<td>PMO</td>
<td>( \frac{m}{m_e_0} )</td>
<td>common</td>
<td></td>
</tr>
<tr>
<td>P2P1</td>
<td>( \frac{P_2}{P_1} )</td>
<td>LKHT1</td>
<td>Static pressure ratio across the leading edge shock (model with angle of attack)</td>
</tr>
<tr>
<td>P2SPI</td>
<td>( \frac{P_2}{P_1} )</td>
<td>VSWT8</td>
<td>Static pressure ratio across the jump</td>
</tr>
<tr>
<td>PRCI</td>
<td>VSWT8</td>
<td>VSWT8</td>
<td>Accuracy of the iteration on jump equations</td>
</tr>
<tr>
<td>PRCA</td>
<td>VS8( \theta V )</td>
<td>VS8( \theta V )</td>
<td></td>
</tr>
<tr>
<td>PRCM</td>
<td>PT</td>
<td>common</td>
<td>Stagnation pressure (free str</td>
</tr>
<tr>
<td>PRW</td>
<td>( Pr_w )</td>
<td>common</td>
<td></td>
</tr>
<tr>
<td>PRX</td>
<td>( \epsilon )</td>
<td>common</td>
<td>Accuracy of the iteration on ( \epsilon )</td>
</tr>
<tr>
<td>PRXO</td>
<td>( \chi )</td>
<td>common</td>
<td>Accuracy of the iteration on ( \chi_0 )</td>
</tr>
<tr>
<td>Q1</td>
<td>( K_1 )</td>
<td>VSWT2, VSWT8, VSWT9</td>
<td></td>
</tr>
<tr>
<td>Q2</td>
<td>( K_2 )</td>
<td>VSWT2, VSWT8, VSWT9</td>
<td></td>
</tr>
<tr>
<td>QC</td>
<td>( \epsilon Q )</td>
<td>common</td>
<td></td>
</tr>
<tr>
<td>QI</td>
<td>( \frac{1}{\chi} )</td>
<td>common</td>
<td></td>
</tr>
<tr>
<td>REDT</td>
<td>( Re_{\delta_i} )</td>
<td>common</td>
<td></td>
</tr>
<tr>
<td>REU</td>
<td>( Re_u )</td>
<td>common</td>
<td></td>
</tr>
<tr>
<td>REX</td>
<td>( Re_x )</td>
<td>common</td>
<td>Note—The notation REX is also used for ( C_F ) according to data switch 13</td>
</tr>
<tr>
<td>R( \rho R )</td>
<td>( \rho_r )</td>
<td>VSWT8</td>
<td></td>
</tr>
<tr>
<td>Fortran symbols</td>
<td>Algebraic notation</td>
<td>Program name</td>
<td>Definition</td>
</tr>
<tr>
<td>-----------------</td>
<td>---------------------</td>
<td>--------------</td>
<td>------------</td>
</tr>
<tr>
<td>R1</td>
<td>m₁</td>
<td>VSWT2</td>
<td></td>
</tr>
<tr>
<td>R2</td>
<td>m₂</td>
<td>VSW₁E</td>
<td></td>
</tr>
<tr>
<td>R1₁</td>
<td>m₁₁</td>
<td>VSWWI</td>
<td></td>
</tr>
<tr>
<td>R1₂</td>
<td>m₁₂</td>
<td>VSWI2</td>
<td></td>
</tr>
<tr>
<td>R2₁</td>
<td>m₂₁</td>
<td>common</td>
<td></td>
</tr>
<tr>
<td>SIG</td>
<td>σ</td>
<td>common</td>
<td></td>
</tr>
<tr>
<td>SSM</td>
<td>sin²μ</td>
<td>LKHT₁</td>
<td>μ is the Mach angle</td>
</tr>
<tr>
<td>SSMB</td>
<td>sin²βₘ</td>
<td>LKHT₁</td>
<td>Maximum shock angle with respect to upstream streamline (att. sh.)</td>
</tr>
<tr>
<td>S2B</td>
<td>sin²β</td>
<td>VSWT8-9</td>
<td>Shock angle associated with the shocklike jump</td>
</tr>
<tr>
<td>SW</td>
<td>S_w</td>
<td>SCommon</td>
<td></td>
</tr>
<tr>
<td>SWREF</td>
<td>S_wref</td>
<td>common</td>
<td></td>
</tr>
<tr>
<td>TI</td>
<td>T₀</td>
<td>common</td>
<td></td>
</tr>
<tr>
<td>TO</td>
<td>Tᵣ</td>
<td>common</td>
<td></td>
</tr>
<tr>
<td>TT</td>
<td>θ</td>
<td>LKHT₁</td>
<td>Ramp angle</td>
</tr>
<tr>
<td>TTE</td>
<td>θｏ</td>
<td>LKHT₁</td>
<td>Angle of attack of the equivalent shock generator</td>
</tr>
<tr>
<td>TW</td>
<td>T_w</td>
<td>VSWT8-9</td>
<td>Static temperature ratio across the shocklike jump</td>
</tr>
<tr>
<td>T2ST1</td>
<td>T₂/T₁</td>
<td>VSWT8-9</td>
<td></td>
</tr>
<tr>
<td>UR</td>
<td>uᵣ</td>
<td>u₂/u₁</td>
<td>Velocity ratio across the shocklike jump</td>
</tr>
<tr>
<td>VD</td>
<td>μ</td>
<td>LKHT₁</td>
<td>Dynamic viscosity of air</td>
</tr>
<tr>
<td>WW</td>
<td>-</td>
<td>common</td>
<td>Step spacing of printout on pressure distribution</td>
</tr>
<tr>
<td>X</td>
<td>x</td>
<td>common</td>
<td></td>
</tr>
<tr>
<td>XX</td>
<td>-</td>
<td>VSWT3-4</td>
<td>Current point of variable x</td>
</tr>
<tr>
<td>XN</td>
<td>xᴺ</td>
<td>common</td>
<td></td>
</tr>
<tr>
<td>K₀</td>
<td></td>
<td>common</td>
<td></td>
</tr>
<tr>
<td>XSH</td>
<td>Z</td>
<td>common</td>
<td></td>
</tr>
<tr>
<td>Z</td>
<td>Z</td>
<td>common</td>
<td></td>
</tr>
<tr>
<td>ZM</td>
<td>Me</td>
<td>common</td>
<td>Current point of variable Me</td>
</tr>
<tr>
<td>ZMA</td>
<td>Meₐ</td>
<td>LKHT₁</td>
<td>Mach number behind the shock attached to the hinge line</td>
</tr>
<tr>
<td>ZMB</td>
<td>Meᵦ</td>
<td>LKHT₁</td>
<td>Mach number behind the incident shock</td>
</tr>
<tr>
<td>Fortran symbols</td>
<td>Algebraic notation</td>
<td>Program name</td>
<td>Definition</td>
</tr>
<tr>
<td>-----------------</td>
<td>--------------------</td>
<td>--------------</td>
<td>------------</td>
</tr>
<tr>
<td>ZMC</td>
<td>$M_e$</td>
<td>LKHT1</td>
<td>Mach number behind the reflected shock</td>
</tr>
<tr>
<td>ZME</td>
<td>Me</td>
<td>common</td>
<td>#</td>
</tr>
<tr>
<td>ZMF</td>
<td>$M_{+e}$</td>
<td>common</td>
<td>#</td>
</tr>
<tr>
<td>ZMI</td>
<td>$M_{-e}$</td>
<td>common</td>
<td>#</td>
</tr>
<tr>
<td>ZMO</td>
<td>$M_{e0}$</td>
<td>common</td>
<td>#</td>
</tr>
<tr>
<td>ZM2</td>
<td>$M_{2}$</td>
<td>VSWT8-9</td>
<td>Mach number just downstream of jump location</td>
</tr>
<tr>
<td>ZN1</td>
<td>$N_{1}$</td>
<td>VSWT3-4</td>
<td></td>
</tr>
<tr>
<td>ZN2</td>
<td>$N_{2}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>ZN3</td>
<td>$N_{3}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>ZN4</td>
<td>$N_{4}$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
DEFINE FILE1(1500,16,U,NR)
DEFINE FILE2(1500,16,U,NR)
DEFINE FILE3(1500,16,U,NR)
DEFINE FILE4(600,4,U,N)
DEFINE FILE8(50,16,U,NR)
DEFINE FILE9(50,16,U,NR)
DEFINE FILE10(50,16,U,NR)
DEFINE FILE12(50,16,U,NR)
DEFINE FILE20(5,6,U,NR)
DIMENSION ??(29,9)
COMMON PIN1,Z11,RE1,X11,DI1,DIS,ZMP,XP,X0,PSPI,ZN0,ETEO,PM0,A1,ETA1
ETAT1,XTX1,KTX1,KTX2,THX1,EHC,EM1,??
1003 FORMAT(1X'OUTPUT TYPEWRITEX FORMAT E11.4 ZME FORMAT F8.5
1 DTE FORMAT E11.4 A FORMAT F8.5 GNZ FORMAT F8.5')
1004 FORMAT(E11.4,F8.5,E11.4,A28.5)
CALL VSWH(SUREF,SH,K10)

INPUT DATA AND EQUIVALENT NAME OF SHOCK CALCULATION
1002 CALL VSWH(CT8,PMF,E1,EF,RPX,K6,DP,DEPS,P2PI,IN,PRI,PRXO,NAS,
18DIV)
K1=1
KTH=1
JT=1
J2=1
J3=1
K1=1
K2=1
40 CALL DATSIH(15,HT)
GO TO(1002,1920),NT
1920 GO TO(53,52),KTH
52 GO TO(54,53,55,53),K5
54 CALL MTSWH(14,NS)
GO TO(4,33,NS)
4 IF(000-PRX)1002,53,53
3 IF(000-PRX)1,1,53
K5=2

WEAK INTERACTION CALCULATION
53 CALL VSWH(KT6,CS,DEPS,P2PI,K10,A,39Z,31E,31E,X,PRH,SH,PRXO,
13SUREF,JO,K10)
GO TO(5,61,K10)
5 CALL VSWH(Z11,2,A1,GB2,TAUO,TAUI,TAU2,TAU3,TAU4,ALF,31O,HR1,
31O2,31I,31J,TT,TT2,2,F1,2,F2,CC5A,CC5B,CC5M,CCSM,S28,TST1,PSPI,ROU,
2UR,5A,5B,52,AB,AL,F4,CS,KTX1,KTX2,K12,AL2,OB2,PRC1,ET14,EM1,EPN1,
S,SH,J0,KKB)
GO TO(4000.6),K12
4000 CALL VSWH(Z12,2,A1,GB2,TAUO,TAUI,TAU2,TAU3,TAU4,ALF,31O,HR1,
31O2,31I,31J,TT,TT2,2,F1,2,F2,CC5A,CC5B,CC5M,CCSM,S28,TST1,PSPI,ROU,
2UR,5A,5B,52,AB,AL,F4,CS,KTX1,KTX2,K12,AL2,OB2,PRC1,ET14,EM1,EPN1,
S,SH,J0,KKB)
GO TO(5,61,K10)
6 CALL VSWH(A,ZME,DTE,X,J1,HWP,1J,IK,GB2)
K1=1
K2=1
K3=1
K4=1
K5=1
K6=1
K7=1
K8=1
PES=1
IQ=1
NO=1
NAME=NAS

INTEGRATION AND TESTS
43 GO TO(45,50),KTH
50 CALL DATSIH(2,1J)
GO TO(5,61,UL)
49 CALL DATSIH(12,NN)
GO TO(47,48),NN
47 K5=1
K6=2
KTH=1
UNITE(1,1003)
REAL(6,1004)X,ZME,DTE,AK,GB2
48 CALL VSWH(A,ZME,DTE,X,J1,HWP,1J,IK,CS,K1,K2,K3,K4,K5,K7,PES,
3E1,EF,KNI,KKN2,KN3,PSPO,XM,REX,DPMX,Z11,Z12,Z2,N3,NE,HR,PR,EES,XS,KS,
ZXX,J1,J2,J3,NF,NR1,NR2,NR3,NH,GB2,ZN0,TAU,TAUM,RA,1A,1B,PRHR,SH,10,914,
3SUREF,JO,K10)
44 CALL VSWH(X,A,ZME,DTE,X,J1,HWP,1J,IK,CS,K1,K2,K3,K5,37EES,
3E1,EF,KNI,KKN2,KN3,PSPO,XM,REX,DPMX,Z11,Z12,Z2,N3,NE,HR,PR,EES,XS,KS,
ZXX,J1,J2,J3,NF,NR1,NR2,NR3,NH,GB2,ZN0,TAU,TAUM,RA,1A,1B,PRHR,SH,10,914,
3SUREF,JO,K10)
GO TO(2,43),K3

ENTROPY CHANGE CALCULATION
42 CALL LEAK(XM,E1,BT8,PMF,PES,PSPO,PSPI,ZM0,EES)
GO TO 44

INTERPOLATIONS
2 CALL VSWH(A,ZME,DTE,X,ETAL2,KN,K5,K6,NR,IK,J1,J2,J3,OP,PL,
INF,IN,HR3,HR2,HR1,HR,GB2,PSPI,111)
GO TO(192,3,1002),K6
END
SUBROUTINE LKHT1

COMMON P1, ZM1, REU, DXX, P1, ZMF, P1, ZMA, P2 Pl, WM, PRX0, PS, BDI

COMMON P1, ZM1, REU, DXX, ZMF, X0, POSPI, ZM1, DXX, OMEGA, P1, ETA1, ETA2, XI, XH, KT, KT5, ZMA, EP, EPS

1 FORMAT(F5.4, F4.2, E15.8, F6.3, F5.1, F5.2)
3 FORMAT(5X’DATA’/5X(1H-)/)
4 FORMAT(3X’DATA’/5X(1H-)/)
5 FORMAT(5X’DATA’/5X(1H-)/)

READ(2,1) ZHA, TT, REU, XSH, TO, ALP
I(F(ZMA),4,4,2)
READ(2,2) X0, X1, DXX, DAS, EPS, DP, PRX0, NEPS, WM, PRX, BNI
READ(2,25) X0, XN
ITRA=-1, LOE=0-1
ITBU=1, LOE=0-1
PR=1, LOE=0-5
P2=1...
PO=760...
AD=131,24032
T2=175...
RO=1,28284
PHA=0,2*ZMA*ZMA
C=1/(1.-PM)
T3=-C
TA=PS/SQ(T0/TZ)
AS=PS/SQ(T0/TZ)
VN=1,4E5*SQRT(TS)/(1.+102.5/TS)

READ(2,38) PT

PS=PT*C**3.5
RS=RO*PS*TS/(PO*TS)
REU=AS*ZMA*RS/VO

REU, PT

WRITE(1,3)

WRITE(1,1) ZMA, TT, REU, XSH, XN, DXX, DAS, EPS, DP, PRX0, NEPS, WM, PR1, BNI

CALL DATSH(5, LM)
QD TO(50, 51, LM)
50 READ(2,100)
51 I(1, ALP, 28, 29, 28)

MODEL WITH ANGLE OF ATTACK

ZMA, TO, ALP
PS=PS/TZ/(PO/TS)
REU=PS/ZMA*PS/V0
.HRITE(1,39) PT

WRITE(1,5)

WRITE(1,1) ZMA, XN, DXX, DAS, EPS, DP, PRX0, NEPS, WM, PR1, BNI

CALL DATSH(5, LM)
QD TO(50, 51, LM)
50 READ(2,100)
51 I(1, ALP, 28, 29, 28)

MODEL WITH ANGLE OF ATTACK

ZMA, TO, ALP
PS=PS/TZ/(PO/TS)
REU=PS/ZMA*PS/V0
.HRITE(1,39) PT

WRITE(1,5)

WRITE(1,1) ZMA, XN, DXX, DAS, EPS, DP, PRX0, NEPS, WM, PR1, BNI

CALL DATSH(5, LM)
QD TO(50, 51, LM)
50 READ(2,100)
51 I(1, ALP, 28, 29, 28)

MODEL WITH ANGLE OF ATTACK

ZMA, TO, ALP
PS=PS/TZ/(PO/TS)
REU=PS/ZMA*PS/V0
.HRITE(1,39) PT

WRITE(1,5)

WRITE(1,1) ZMA, XN, DXX, DAS, EPS, DP, PRX0, NEPS, WM, PR1, BNI

CALL DATSH(5, LM)
QD TO(50, 51, LM)
50 READ(2,100)
51 I(1, ALP, 28, 29, 28)
SUBROUTINE WRITE1, S

K1=0
K2=1

IF(TT+.017453292

6 IF(K1=K1+1

R1=R1
ZHI=ZHI
K=0

8 K=K+1

SSB1=(ZHI-ZHI)

SSSN= (S.*ZHI*ZHI-5.*SQRT(3.*S.*ZHI*ZHI*ZHI*ZHI+2C.))/7.*ZHI

F= (ZHI-ZHI+2.)/(ZHI+ZHI)-1.45*ZHI+ZHI+ZHI+ZHI+ZHI

IF(AIIS(SSB1=SSB1)-.10.9,9

IF(AIIS(SSB2=SSB2)-.12.11,10 SSQ=SSB1

GO TO 13

10 SSB=SSB1

GO TO 12

11 SSB=SSB2

GO TO 13

12 SSB=SSS

RT=ATAN(SQRT((1.-SSB))

ZHI=ZHI+5.)/(7.*ZHI+ZHI+SSN-1.)*ZHI+ZHI+ZHI+ZHI+ZHI

SSN=SSN

ZHI=SQRT(ZHI/2)

GO TO(15,10),K1

15 ZHI=ZHI

TP=BT

PPPI=(3.*(ZMA-SIN(BTA)+SIN(BTA)-1.))/.G.

TR=TR/2.

GO TO 6

16 GO TO (17,18),K

17 ZHI=ZHI

ZHI=ZHI

BT=BT

GO TO 3

18 ZHI=ZHI

TC=BT

PPBP=7.*(ZMA-SIN(BTA)+SIN(BTA)-1.)/G.

PFBP=7.*(ZMA-SIN(BTA)+SIN(BTA)-1.)/G.

PPBP=PPBP+PPBP

PPBP=PPBP+PPBP

AP=ABS(R2)

AP=R2/2.

IF(AP-AP)<20,19,19

19 GO TO(20,21),K2

20 IF(D1=127,27,22,22

27 K=2

21 TRM=TR/2.

22 GO TO APR+TRM.

GO TO 7

20 ZHI=ZHI

TF= (1.-.017453292

PPPI=PPPI+P2P1

ZMA=ZMA

WRITE(1.33)*Z1F, ZMA1F, T1E, BTA, 9TA, 9TC, PPPI

PM=2.*Z1F*Z1F

PM=2.*ZHI*ZHI

E+FNU(ZMA)

EF=FNU(2.Z1F)

K7=1

K7=1

RETURN

END
SUBROUTINE VSI'IT2

SUBROUTINE VSI'IT2 (KT6, CS, DEPS, P2P1, KT10, A, GBZ, ZME, NTE, X, P10, SH, PRX0
1, SHREF, JO, KNO)

*DIMENSION CC(5,3), DO(29,9)
COMMON PMI, ZHI, REU, DX0, DX1, DX2, ZMF, XO, POSPI, ZHI0, NTE0, P100, AI, ETA1,
ETA2, T1, XSH, XN, KT4, KT5, ZWI, TC, EPS, OMI, D9

6 FORMAT(/1X,3X, 10F14.3, 5X, 10F14.3, 5X, 10F14.3, 5X, 10F14.3, 5X, 10F14.3, 5X, 10F14.3,
4 FORMAT(/1X,3X, 8F14.3, 5X, 8F14.3, 5X, 8F14.3, 5X, 8F14.3, 5X, 8F14.3, 5X, 8F14.3,

WRITE(6,8)

10 FORMAT(/1X, 'E15.8, 3X, 'INITIALLY SUBCRITICAL N, L',//)
11 FORMAT(/1X, 'E15.8, 3X, 'INITIALLY SUPERCRITICAL N, L',//)
12 FORMAT(/1X, 'DATA TYPEWRITER XO FORMAT F6.3 EPS FORMAT E7.3 TO GFORMAT F5.1'//)
8 FORMAT(/1X, 'SUBROUTINE /5X/WEAK INTERACTION'/5X16(1H-))

ITERATION PART

JO=18
KTB=2
PRX=PRX0
NO TO(13,20), KT4
20 GO TO(16,12), KT6
10 IF(DXO-PRX15,13,13
19 GO TO(11,21), KT10
21 KT5=1
ETAI=ETA2
X0=EPS
12 XF=X0
X0=EPS
KT6=2
11 CALL NTSI(8, KI)
1 WRITE(16,1)
15 TE0, 6)
1EAn(6, 3)XO,EPS,TC
KTS -1
10 TO 11
10 NO TO(98,99), KT5
98 IF(ETA1=ETA2),191, 191
190 KT5=2
99 XO-X0/2,
191 X0=XO+ETA2*X0
NO TO(11,15), KT6
15 EPS=X0
X0=XF
14 KT4=2

WEAK INTERACTION CALCULATION

'NO2A XI=1.29
24 REAP(0014U)(90(HJ,KJ),KJ=1,9)
DO 38 X0 HJ=0,5
38 REAP(2014U)(CC(HJ,LJ),LJ=1,3)
CC=CC(1,1)
1G=CC(1,2)
GB0=CC(1,3)
R11=CC(2,1)
R12=CC(2,2)
R21=CC(2,3)
D11=CC(3,1)
D12=CC(3,2)
D21=CC(3,3)
A11=CC(4,1)
A12=CC(4,2)
A21=CC(4,3)
O11=CC(5,1)
O12=CC(5,2)
O21=CC(5,3)
ETAI=ETA2

REX=REU=X0/1000.
1X=-4*(1.*PM1+*(R11+1.+PM1)+*(R12/PM1))/(4.*ZM1*SQRT(ZH1*ZI1+1.))
2X=SQRT(ZH1*ZHI1+1.)*RZ/ZHI1+5*R1+*(PM1/(1.+PM1)-1./2,*(ZH1*ZI1
1-1.))/R1+R1
1Q=8.*(PM1/1.+PM1)
2Q=QX*(1.-(1.-(PM1/(1.+PM1)))/2,
1X=(QX+Q1)R1
2P=QX*(1.-2./2.)*R1+Q1*(QX-Q2)2)*R2
2(X=Q1/2.)*EP=ALOG(D0)
Al=Al1+R1
A2=2*(A21+Q1+Q2)*R1+R1+(A21-2*A22)*R2
GB1=GB11+R1
GB2=GB21+Q1+GB22+R1+R1+(GB11-2*GB21)*R2
T1=TO/(1.+PM1)
T0=TO+(1.+SH)
C1=SQRT((T1/T0)/(1.+102.5/T1))/(1.+102.5/T)
Q1=SQRT(CS/REX)+ZHI1+3
ZM1=ZHI1+1.+Q1+Q2+Q1*Q1
NTE0=X0/1000.*SQRT(CS/REX)*50*(1.+71.*Q1+EP2+Q1*Q1=ALON(Q1)+N2*Q1*
101*Q1)
AI=AO+AI1+Q2+Q1+Q1
GB1=GB0+GB11+Q1+GB21+Q1
PM1=2*ZM1+2*ZM1

C.29 -
SUBROUTINE VSI2

PAGE 02

POSPI=((1.*PH1)/(1.*PML))+5.
POSP=POSPI+P2P1
WRITE(1,40)X,Z,ML,M3L,DTEDL,POSPI,DX,AL,Q1,EC,T0

KLIEBERG DEPARTURE CONDITIONS

AA=A1
S1MP=sref4M1
S1N=sin(2,1)
S1MPB=s1mp(3,1)
HRR=sin(6,1)
GJ=sin(7,1)
Z=sin(8,1)

THEP=RE

THETA=theta(11,1)
THETA1=theta(12,1)
ALF=theta(15,1)
THETA2=theta(14,1)

43 DATA=DATA(14,1)+AA+

TAU=tau(1,1)
TAU1=tau(15,1)
TAU2=tau(16,1)
TAU3=tau(17,1)
TAU4=tau(18,1)
TAU5=tau(19,1)
EPS0=eps(4,1)
EPS1=eps(20,1)
EPS2=eps(21,1)
EPS3=eps(22,1)
EPS4=eps(23,1)
EPS5=eps(24,1)
PH1=phi(15,1)
PH2=phi(21,1)
PH3=phi(27,1)
PH4=phi(28,1)
PH5=phi(29,1)

04 1Z=1.5
TAU=tau(1,1)+AA+(TAU2+AA*(TAU3+AA*(TAU4+AA*TAU5)))+

E=ALF+S10G(-S1REF)
TT=ALF+OTAB(-S1REF)
DEBA=ALF+SMF
TDTA=ALF+TMF
TOTA=ALF+T2
Z1=ML

PO7=PO7L

FF=(1.*PH1)/(1.*P1)

CF=CF+CH

WRITE(1,102)CF,CH

IF (OE110, 40, 41
SUBROUTINE VSH2

40 WRITE(1,100)DE
41 WRITE(1,101)DE
42 GO TO(22,23),KT10
22 ZHD=ZHL
 DTED=DTEDL
 RETURN

23 CONTINUE
 PE1=HR+DJH-GJ
 PE2=1.*GJ+2.*SH+TT-(2.*HR+1.*SI*ER)*DJH
 PE3={(2.*HR+1.*SI*ER)*DJ-(3.*HR+2.*SH+TT)*HR)/TTH
 PE4={-(SH+TT*(PE1+PE2)-PE3+SH+TT)/TTH/THH
 ZHD=ZHL*(1.+PE1*EPS)
 DTED=DTEDL*(1.+PE2*EPS)
 AI=AI*(1.+PE3*EPS)
 GRI=GRI*(1.+PE4*EPS)
 PMO=2*ZHD+ZMD
 POSPI=POSPI*(1.+PMOL)/(1.+PMO)**3.5
 RETURN
 END
SUBROUTINE VSI 'In
TAU3=0(17, 1)
TAU4=0(18, 1)
TAU5=0(19, 1)
EPS0=0(4, 1)
EPS1=0(20, 1)
EPS2=0(21, 1)
EPS3=0(22, 1)
EPS4=0(23, 1)
EPS5=0(24, 1)
PI+0=0(5, 1)
PHI1=0(25, 1)
PHI2=0(26, 1)
PHI3=0(27, 1)
PHI4=0(28, 1)
DO 65 IZ=1, 5
TAUO=TAUO+(1, IZ=1)*BB**IZ
TAU1=TAU1+(15, IZ=1)*BB**IZ
TAU2=TAU2+(16, IZ=1)*BB**IZ
TAU3=TAU3+(17, IZ=1)*BB**IZ
TAU4=TAU4+(18, IZ=1)*BB**IZ
TAUS=TAUS+(19, IZ=1)*BB**IZ
EPS0=EPS0+(4, IZ=1)*BB**IZ
EPS1=EPS1+(20, IZ=1)*BB**IZ
EPS2=EPS2+(21, IZ=1)*BB**IZ
EPS3=EPS3+(22, IZ=1)*BB**IZ
EPS4=EPS4+(23, IZ=1)*BB**IZ
EPS5=EPS5+(24, IZ=1)*BB**IZ
PI+0=PI+0+(5, IZ=1)*BB**IZ
PHI1=PHI1+(25, IZ=1)*BB**IZ
PHI2=PHI2+(26, IZ=1)*BB**IZ
PHI3=PHI3+(27, IZ=1)*BB**IZ
PHI4=PHI4+(28, IZ=1)*BB**IZ
GO TO 65
65 CONTINUE
Differential Equations

PM=Z*ZH
RED'=RED*UT
EE=FU/U(U)
EI1=(K+K)1/(K+K)-EF
H=ZH+M+P1*RE*SS(1-EE)/(ZH+P1*P1-P1*SS((1-EE))
B=+(M+P1)/(1+P1)+4

X=ALF*SIG1/SURF
Y=-H+(1+P1)+(1+P1)+/PME
Z=+(2, 1+P1)+(1+P1)+(1+P1)+(1+P1)+(1+P1)+/PME
TT=ALF+GTAB/(-SURF)

Differential Equations

tt=PM+2*Z*ZH
RED=RED*UT
EE+FU/U(U)
EI1=(K+K)1/(K+K)-EF
H=ZH+M+P1*RE*SS(1-EE)/(ZH+P1*P1-P1*SS ((1-EE))
B=+(M+P1)/(1+P1)+4

X=ALF*SIG1/SURF
Y=-H+(1+P1)+(1+P1)+/PME
Z=+(2, 1+P1)+(1+P1)+(1+P1)+(1+P1)+(1+P1)+/PME
TT=ALF*GTAB/(-SURF)

Differential Equations

PM=Z*ZH
RED=RED*UT
EE+FU/U(U)
EI1=(K+K)1/(K+K)-EF
H=ZH+M+P1*RE*SS(1-EE)/(ZH+P1*P1-P1*SS((1-EE))
B=+(M+P1)/(1+P1)+4

X=ALF*SIG1/SURF
Y=-H+(1+P1)+(1+P1)+/PME
Z=+(2, 1+P1)+(1+P1)+(1+P1)+(1+P1)+(1+P1)+/PME
TT=ALF*GTAB/(-SURF)

Differential Equations

PM=Z*ZH
RED=RED*UT
EE+FU/U(U)
EI1=(K+K)1/(K+K)-EF
H=ZH+M+P1*RE*SS(1-EE)/(ZH+P1*P1-P1*SS ((1-EE))
B=+(M+P1)/(1+P1)+4

X=ALF*SIG1/SURF
Y=-H+(1+P1)+(1+P1)+/PME
Z=+(2, 1+P1)+(1+P1)+(1+P1)+(1+P1)+(1+P1)+/PME
TT=ALF*GTAB/(-SURF)

Differential Equations

PM=Z*ZH
RED=RED*UT
EE+FU/U(U)
EI1=(K+K)1/(K+K)-EF
H=ZH+M+P1*RE*SS(1-EE)/(ZH+P1*P1-P1*SS((1-EE))
B=+(M+P1)/(1+P1)+4

X=ALF*SIG1/SURF
Y=-H+(1+P1)+(1+P1)+/PME
Z=+(2, 1+P1)+(1+P1)+(1+P1)+(1+P1)+(1+P1)+/PME
TT=ALF*GTAB/(-SURF)

Differential Equations

PM=Z*ZH
RED=RED*UT
EE+FU/U(U)
EI1=(K+K)1/(K+K)-EF
H=ZH+M+P1*RE*SS(1-EE)/(ZH+P1*P1-P1*SS ((1-EE))
B=+(M+P1)/(1+P1)+4

X=ALF*SIG1/SURF
Y=-H+(1+P1)+(1+P1)+/PME
Z=+(2, 1+P1)+(1+P1)+(1+P1)+(1+P1)+(1+P1)+/PME
TT=ALF*GTAB/(-SURF)

Differential Equations

PM=Z*ZH
RED=RED*UT
EE+FU/U(U)
EI1=(K+K)1/(K+K)-EF
H=ZH+M+P1*RE*SS(1-EE)/(ZH+P1*P1-P1*SS((1-EE))
B=+(M+P1)/(1+P1)+4

X=ALF*SIG1/SURF
Y=-H+(1+P1)+(1+P1)+/PME
Z=+(2, 1+P1)+(1+P1)+(1+P1)+(1+P1)+(1+P1)+/PME
TT=ALF*GTAB/(-SURF)

Differential Equations

PM=Z*ZH
RED=RED*UT
EE+FU/U(U)
EI1=(K+K)1/(K+K)-EF
H=ZH+M+P1*RE*SS(1-EE)/(ZH+P1*P1-P1*SS ((1-EE))
B=+(M+P1)/(1+P1)+4

X=ALF*SIG1/SURF
Y=-H+(1+P1)+(1+P1)+/PME
Z=+(2, 1+P1)+(1+P1)+(1+P1)+(1+P1)+(1+P1)+/PME
TT=ALF*GTAB/(-SURF)

Differential Equations

PM=Z*ZH
RED=RED*UT
EE+FU/U(U)
EI1=(K+K)1/(K+K)-EF
H=ZH+M+P1*RE*SS(1-EE)/(ZH+P1*P1-P1*SS((1-EE))
B=+(M+P1)/(1+P1)+4

X=ALF*SIG1/SURF
Y=-H+(1+P1)+(1+P1)+/PME
Z=+(2, 1+P1)+(1+P1)+(1+P1)+(1+P1)+(1+P1)+/PME
TT=ALF*GTAB/(-SURF)

Differential Equations

PM=Z*ZH
RED=RED*UT
EE+FU/U(U)
EI1=(K+K)1/(K+K)-EF
H=ZH+M+P1*RE*SS(1-EE)/(ZH+P1*P1-P1*SS ((1-EE))
B=+(M+P1)/(1+P1)+4

X=ALF*SIG1/SURF
Y=-H+(1+P1)+(1+P1)+/PME
Z=+(2, 1+P1)+(1+P1)+(1+P1)+(1+P1)+(1+P1)+/PME
TT=ALF*GTAB/(-SURF)

Differential Equations

PM=Z*ZH
RED=RED*UT
EE+FU/U(U)
EI1=(K+K)1/(K+K)-EF
H=ZH+M+P1*RE*SS(1-EE)/(ZH+P1*P1-P1*SS((1-EE))
B=+(M+P1)/(1+P1)+4

X=ALF*SIG1/SURF
Y=-H+(1+P1)+(1+P1)+/PME
Z=+(2, 1+P1)+(1+P1)+(1+P1)+(1+P1)+(1+P1)+/PME
TT=ALF*GTAB/(-SURF)

Differential Equations

PM=Z*ZH
RED=RED*UT
EE+FU/U(U)
EI1=(K+K)1/(K+K)-EF
H=ZH+M+P1*RE*SS(1-EE)/(ZH+P1*P1-P1*SS ((1-EE))
B=+(M+P1)/(1+P1)+4

X=ALF*SIG1/SURF
Y=-H+(1+P1)+(1+P1)+/PME
Z=+(2, 1+P1)+(1+P1)+(1+P1)+(1+P1)+(1+P1)+/PME
TT=ALF*GTAB/(-SURF)

Differential Equations

PM=Z*ZH
RED=RED*UT
EE+FU/U(U)
EI1=(K+K)1/(K+K)-EF
H=ZH+M+P1*RE*SS(1-EE)/(ZH+P1*P1-P1*SS((1-EE))
B=+(M+P1)/(1+P1)+4

X=ALF*SIG1/SURF
Y=-H+(1+P1)+(1+P1)+/PME
Z=+(2, 1+P1)+(1+P1)+(1+P1)+(1+P1)+(1+P1)+/PME
TT=ALF*GTAB/(-SURF)
SUBROUTINE VSN3

C3

10 X=X+X
20 GO TO(8,9),KEAT
30 X=X+X+X
40 GO TO10

1) TE=DE+(V(2)+2.*V(3)+2.*V(4)+V(5))/6.
2) MBZ=MBZ+(S(2)+2.*S(3)+2.*S(4)+S(5))/6.
3) GO TO(8,9),KEAT
5) X=X+X
6) GO TO10
7) X=X+(U(2)+2.*U(3)+2.*U(4)+W(5))/6.
8) A=A+DA
9) K1=10000.*K1+0.00005
PME=1.*ZH*ZME
9P0=-1./4.*(1.+PMO)**3.5*ZME=B+CS=ZH=ZNI/(RENT*DE=DE(1.,PME)
1) K1+PES
PES=(E1+PMO)/(1.+PME)**3.5*PES
2) GO TO(20),165.
9) PES=PSP0
PES=PSP0
30 GO TO(51,60),1K
31 X=X+X/25.4
50 RE=SRE+X
CALL DAT(SI(13,10)
60 GO TO(60,61),10
61 GO TO(64,63,64),K1
93 PPO=AP(-1.1+1.55*SA(-7.19)+(-7.1253*A*(20,8563+100.2792*A*
1510,2393*A*(263,587)))
93 PPO=AP(-6.7296+1.8713*A*(-18.2656+13.7356+13,-0.597)))
R=PO+(-1I)+PO-PD0)/R.
94 A(A+n1(10,4)+A*(n1(10,5)+A*(n110,6)
1) A(1010,7)=A*(n110,8)+A*(n110,9))))
62 ALF=ALF(13,12)+A(13,4)+A*(n1(13,5)+A*(13,5)+A*(
13,7)+A*(13,8)+A*(13,9))))
2) X=RE+S+CS=PEIRX=(1.+PMI)/(REU+DE=DE(1.,PME)
3) PHEL=1X/2,9/2.*2,ZE/5.*PEIR
TETU=ESE
C3

SUBROUTINE LEK4

PAGE 01

CALCULATION OF FUTURISTIC CHANGE THROUGH THE SHOCK

CALL DAT(SI(7,4)
1) TO(1,2),4
2) PES=PSP0
3) RETURN
4) .S+PS=PS
5) CONTINUE
6) EN=EN+1(ZET)
7) T=EP+EP
8) TAP=(ZI+ZIF=5.)/(7.*ZIF*ZIF*SIH(NT)*SIH(NT)-1.)
9) TAP2=2.*ZIF*ZIF*CIG(NT)*CIG(NT)/(5.*ZIF*ZIF*CIG(NT)*CIG(NT))
10) TAP=TAP2+TAP1
11) IF2=2.*ZIF*ZIF
12) P=(7.+ZIF*ZIF*SIIH(NT)*SHI(NT)-1.)/6.
14) ZME=SRT(5.+P)
15) ZME=ZME
16) RETURN
17) ENN
SUMROUTINE VSUT4 PACE 03

WRITE(1,1221) A, PS00, X1, REX, ZME, YTE, A, PHZ, NMX
WRITE(1,1111) ZTV, ZMV, ZHS, ZNM, NE
85 I+1
KTV=2
DATA SWITCHES TESTS

62 CALL MATSH(4, J)
ON TO(15, 16), J
15 WRITE(1,1221) A, PS00, X1, REX, ZME, YTE, A, PHZ, NMX
CALL MATSH(13, K)
ON TV(32, 17), K
17 CALL MATSH(3, L)
ON TO(18, 16), L
18 WRITE(1,1111) ZTV, ZMV, ZHS, ZNM, NE
16 CALL MATSH(14, HS)
ON TO(19, 20), HS
19 AXW=ANS(XN-X1)
ON TO(15,5)
FAX=W, 521, 21, 22
21 WRITE(1,1221) A, PS00, X1, REX, ZME, YTE, A, PHZ, NMX
22 FEX=W-K, 523, 23, 26
20 CALL MATSH(7, JJ)
ON TO(23, 24), JJ
23 NEX=WK1
ON TO(5)
24 NEX=WK2
25 FEX=W-K, 525, 26, 26
26 CALL MATSH(1, KK)
ON TO(71, 72, 73, 73), KS
73 ON TO(27,28), KK
28 WRITE(1,1221) A, PS00, X1, REX, ZME, YTE, A, PHZ, NMX
27 IF=WARPK-IN
ON TO(5)
192 KX=1
1H=W-I
ON TO(91, 88, 89, 90), KS
88 M=8-4
ON TO 71
89 M=W-1
ON TO 91
90 M=W-1
91 RETURN
5 KX=3
RETURN
END

SUMROUTINE VSUT5(SURF, SI, KT10)

SUMROUTINE VSUT5(SURF, SI, KT10)

HEAT TITLE WRITING

101 FORMAT('HALL COOLING RATIO SI 'F6.3', AUDIATURE #:IHE KT11 'I1
1ND PROCEDURE SI REFERENCE 'F6.3/)
102 FORMAT('HALL COOLING RATIO ON 'F6.3', AUDIATURE #:IHE KT11 'I1
1ND PROCEDURE SI REFERENCE 'F6.3/)
103 FORMAT('F6.3,11)
115 FORMAT('HALL COOLING RATIO: ANGLE-SHOCK HAVE INTERACTION'/21X37(IF1)=)3
1HLINEPROGS'S 'RETURN'/39X18(114=)/26X'ARBITRARY HALL TEMPERATURE'
/26X26(IF1=+)
21X37(IF1=+)
8F6M2,SURF, SI, KT10
WRITE(1,105)
ON TO(1,2), KT10
1 WRITE(1,101), SI, KT10, SURF
ON TO(3)
2 WRITE(1,102), SI, KT10, SURF
3 IF(SURF> 5, 5, 4
5 SURF=-1,
4 RETURN
END
SURROUTINE VSUT7 PAGE 01

SURROUTINE VSUT7(A, Z4F, TF, X, ETA1, ETA2, K4, K5, K6, N1, N1, K1, J1, J2, J3, "P, P01, N, J1, N1, N2, NP1, PP1, 111, 1)

100 FORMAT(3X, F8.4, 3X, F8.4, 2X, E11.4, 2X, F8.4, 2X, E11.4, 2X, F8.4, 2X, E11.4, 3X)
101 FORMAT(13X, "INTERPOLATIONS")
102 FORMAT(66, /)
103 FORMAT("/ETA1 =0.3,2*ETA2 =0,0")
105 FORMAT(2E11.4, 4X, 12)

94 TF1(1,0) ETA1, ETA2
K6=2
K7=0
TO (1,2,2,3), K5
1 K6=1
TF1(1)
2 TO (4,5), K4
4 KB=2
K5=3
K6=1
TF1(1)

"ADJACENT SOLUTIONS TEST"

5 IF(ETA1+ETA2),/., 6
6 KB=2
JN=J2
J1=J2
J4=J2
N1=NP1
N2=NP2
K8=2
TO (12)
7 KB=3
K5=4
K8=1
J1=J2
J2=J3
N1=NP1
N2=NP2
TO (12)

"TYPE OF SOLUTION" TEST

3 IF(ETA1+ETA2),/., 8
8 J1=J1
J2=J2
N1=NP1
N2=NP2
K8=3

"REA1 FILES"

12 I=0
J=N
"REA"(J1) PO PO, XI, RE, Z1, NT, AA, N1, N2
"REA"(J2) PC, X2, RE, Z1, NT, AA, N1, N2
IF(ANS(XX-X1), "MPC") 21, 21, 2
20 IF(XX-X2),/., 21, 22
21 I=1+1
IF(-XX),/., 45, 45, 11
45 "REA"(J1) PO PO, XX, RE, Z1, NT, AA, N1, N2
IF(ANS(XX-X2), "MPC") 25, 25, 2
22 J=J+1
IF(J-HN2),/., 46, 46, 11
46 "REA"(J2) PO PO, XX, RE, Z1, NT, AA, N1, N2
IF(ANS(XX-X1), "MPC") 24, 24, 22
24 J=J-1
21 I=I-1
22 J=J-1
23 J=J+1
IF(-XX),/., 47, 47, 58
47 "REA"(J1) PO PO, XI, RE, Z1, NT, AA, N1, N2
IF(-XX),/., 48, 48, 58
48 "REA"(J2) PO PO, XI, RE, Z1, NT, AA, N1, N2
AP=AP(2) =0
TO (14, 16, 16), K8
14 IF(A1=0.0),/., 15, 15
15 IF(A2=0.0),/., 23, 23
16 IF(A3=0.0),/., 23, 23
23 TO (26, 27, 28), K8
26 "REA"(1, 101)
11=0
PP1=0.
JN=0
CALL NATSJ(S, L1)
TO (51, 10), L1

"REA1 FILE AND PUNCH CARDS"
SUBROUTINE VSIT7

60 JN=JN+1
61 YEP(JN,Y4),Y4,PS,PS
62 JN=JN(J2,105),Y1,PS,PS,JN
63 IF(JN=JN)51,10,10
64 JN=JN
65 J3=J3
66 J2=J2
67 NR4=NR4
68 NR3=NR3
69 NR2=NR2
70 NT=NT 42
71 JN=JN
72 J3=J3
73 J2=J2
74 NR4=NR4
75 NR3=NR3
76 NR2=NR2
77 NT=NT 42

INTERRATIONS

10 AA=(A1+A2)/2.
11 Z1=(Z11+Z12)/2.
12 T1=(T1+T2)/2.
13 XP=XP
14 (P1+P2)/2.
15 T1=(T1+T2)/2.
16 X=100,X=,0,05
17 X=XP
18 NT=NT 25.4
19 NT=NT 25.4,32,32
20 NT=NT
21 NT=NT
22 NT=NT
23 NT=NT
24 NT=NT
25 NT=NT
26 NT=NT
27 NT=NT
28 NT=NT
29 NT=NT
30 NT=NT
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107 NT=NT
108 NT=NT
109 NT=NT
110 NT=NT
111 NT=NT
INITIALIZATIONS

FUNCTIONS
24  \[ \begin{align*} 
Z_{\text{E}}(8,1) &= AA + \gamma(8,2) + AA + \gamma(8,3) + AA + \gamma(8,4) + AA + \gamma(8,5) + AA + \gamma(8,6) + AA + \gamma(8,7) + AA + \gamma(8,8) + AA + \gamma(8,9) 
\end{align*} \]
c.44

FUNCTION

RETURN

FUNCTION

RETURN

FUNCTION

RETURN
INTEGRAL FUNCTIONS CALCULATIONS

\[
\sin^n(x) = \sum_{k=0}^{n} \binom{n}{k} (-1)^k \sin^{n-2k}(x) \cos^{2k}(x)
\]

\[
\cos^n(x) = \sum_{k=0}^{n} \binom{n}{k} (-1)^k \sin^{2k}(x) \cos^{n-2k}(x)
\]

NEW DERIVATIVES CALCULATIONS
A: Compression wedge

B: Incident shock

FIG. 2-1 SCHEMATIC REPRESENTATION OF SHOCK WAVE-LAMINAR BOUNDARY LAYER INTERACTION
FIG:2-2 INTEGRAL FUNCTION $P(a)$
FIG: 2-3 FUNCTION $T(a, b)$ DISTRIBUTION FOR VARIOUS WALL COOLING RATIOS

- $b = 0.1$, $Sw = -0.8$
- $b = 0.1$, $Sw = -0.6$
- $b = 0.1$, $Sw = -0.4$
- $b = 0.1$, $Sw = -0.2$
The diagram shows the integral function \( \Gamma(a, B) \) for attached flow and separated flow for different values of \( B \) and \( Sw \). The values of \( B \) are 0.1, 0.2, and 0.3. The separated flow lines are for \( Sw = -0.8 \) and \( Sw = 0 \). The x-axis represents the parameter \( a \) ranging from 3.2 to 1.0, and the y-axis represents the integral function values.
FIG: 2-5 INTEGRAL FUNCTION $\frac{\partial f}{\partial a}(a, B)$

**Attached Flow**
- $S_w = -0.8$
- $S_w = 0$

**Separated Flow**
- $B = 0.1$
- $B = 0.3$
Attached flow

$S_w = -0.8$

$S_w = 0$

Separated flow

$\frac{\partial^2 \Gamma}{\partial B^2}(a, B)$

$B = 0.3$

$B = 0.1$

FIG: 2-6 INTEGRAL FUNCTION $\frac{\partial^2 \Gamma}{\partial B^2}(a, B)$
FIG:2-7 INTEGRAL FUNCTIONS $\xi(B)$ and $\frac{d\xi}{dB}(B)$
FIG: 2 - TYPICAL TRAJECTORIES OF FUNCTION $D(Me, a, b)$

Me = 5
Sw = -0.8

subcritical

supercritical

$b = 0.20$
$b = 0.25$
$b = 0.30$
Fig. 2-9 Locus of Critical Points (Sw = -0.8)
$M_\infty = 6.06$  
$Re = 0.239 \times 10^7 / 1m.$

**Figure 2-10a**  WEAK INTERACTION (Pressure distribution)
$M_\infty = 6.06$

$Re = 0.239 \times 10^7 / m$

WEAK INTERACTION (Transformed displacement thickness distribution)
FIG: 2-10c  WEAK INTERACTION (Velocity profile parameter distribution)
FIG: 2-10d WEAK INTERACTION (Normalized total enthalpy profile parameter distribution)
FIG: 2 - WEAK INTERACTION (Skin friction coefficient distribution)
G2-10f WEAK INTERACTION (Heat transfer coefficient distribution)
FIG: 2-11 EFFECT OF SURFACE COOLING ON PRESSURE DISTRIBUTION

- $M_\phi = 6.06$
- $Re_x = 0.152 \times 10^6$
- $\frac{P_{\phi}}{P_0} = 3.812$
- J: Jump
- S: Separation
- R: Reattachment

Legend:
- $Sw = 0.8$
- $Sw = 0.6$
- $Sw = 0.4$
- $Sw = 0.2$
- $Sw = 0$
FIG 2-13 EFFECT OF SURFACE COOLING ON HEAT TRANSFER DISTRIBUTION

$M_{\infty} = 6.06$
$Re_x = 0.152 \times 10^6$
$P_\infty = 3.812$
$P_\theta$

$Sw = -0.8$
$Sw = -0.6$
$Sw = -0.4$
$Sw = -0.2$
$M_\infty = 6.06$  
$\Theta_\Gamma = 10.25^\circ$

open symbols $Re_x = 0.152 \times 10^6$

solid symbols $Re_x = 0.302 \times 10^6$

FIG. 24 EFFECT OF WALL-TO-STAGNATION TEMPERATURE RATIO ON CHARACTERISTIC LENGTHS OF LAMINAR INTERACTION
FIG 2.5: EFFECT OF WALL-TO-STAGNATION TEMPERATURE RATIO ON MINIMUM SKIN-FRICTION

FIG 2.6: EFFECT OF WALL-TO-STAGNATION TEMPERATURE RATIO ON PEAK AND MINIMUM HEAT TRANSFER

$M_\infty = 6.06$

$\Theta_f = 10.25^\circ$

open symbols: $\text{Re}_{X_c} = 0.152 \times 10^6$

solid symbols: $\text{Re}_{X_c} = 0.302 \times 10^6$
Fig: 2-17a APPROXIMATE PRESSURE DISTRIBUTION

\[ M_\infty = 6.06 \]
\[ \text{Re}_x = 0.15 \times 10^6 \]

- \( S_{w_{\text{ref.}}} = -0.8 \)
- \( S_{w_{\text{ref.}}} = 0 \)
FIG: 2-17c APPROXIMATE HEAT TRANSFER COEFFICIENT DISTRIBUTION
FIG. 2-18a COMPARISON BETWEEN APPROXIMATE AND EXACT CALCULATIONS

(Pressure distribution S\textsubscript{w} = 0.4)
FIG: 2-18d (cont'd) (Total enthalpy profile parameter distribution)
**FIG: 3-4a** Heat transfer measurements model (upper surface)

**FIG: 3-4b** Model interior (open cavity)
THERMOCOUPLES CALIBRATION CURVES (Room temperature)
FIG: 3-4 THERMOCOUPLES CALIBRATION CURVES (Low temperature)
FIG:3-5 EXAMPLE OF TEMPERATURE-TIME HISTORY RECORDING (Lower heat flux)
SPECIFIC HEAT OF STAINLESS STEEL VERSUS TEMPERATURE

$C_p$ (Cal/g°C)

---

- Curve used
- Data from manufacturer

$T$(°K)

0 100 200 300
FIG: 3-7 EFFECT OF THE VARIATION OF CP (Stainless steel) ON $T_w(t)$
FLOW VISUALIZATION (Room temperature $S_w = -0.32$)

FLOW VISUALIZATION (Low temperature $S_w = -0.8$)
EXPERIMENTAL HEAT TRANSFER COEFFICIENT DISTRIBUTION
(Room temperature)
FIG. 3-10 EXPERIMENTAL HEAT TRANSFER COEFFICIENT DISTRIBUTION
(Highly cooled wall)
Highly cooled wall | Wall at ambient temperature

<table>
<thead>
<tr>
<th>$S_w$</th>
<th>$R_e \times 10^{-7}$</th>
<th>$S_w$</th>
<th>$R_e \times 10^{-7}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.782</td>
<td>0.721</td>
<td>0.336</td>
<td>0.566</td>
</tr>
<tr>
<td>0.784</td>
<td>1.015</td>
<td>0.364</td>
<td>0.976</td>
</tr>
<tr>
<td>0.819</td>
<td>2.24</td>
<td>0.396</td>
<td>2.125</td>
</tr>
</tbody>
</table>

FIG 3-11 WALL COOLING EFFECT ON THE MEASURED HEAT TRANSFER COEFFICIENT DISTRIBUTION
FIG. 3-12 WALL COOLING EFFECT ON THE EXTREMEUM OF HEAT TRANSFER COEFFICIENT DISTRIBUTION
FIG:3-13 PRESSURE MEASUREMENTS MODEL

<table>
<thead>
<tr>
<th>Pressure tap Nr</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>11</th>
<th>12</th>
<th>13</th>
<th>14</th>
<th>15</th>
<th>16</th>
<th>17</th>
<th>18</th>
<th>19</th>
<th>20</th>
<th>21</th>
</tr>
</thead>
<tbody>
<tr>
<td>X(mm)</td>
<td>14</td>
<td>18</td>
<td>22</td>
<td>25</td>
<td>28</td>
<td>31</td>
<td>34</td>
<td>37</td>
<td>43</td>
<td>46</td>
<td>49</td>
<td>52</td>
<td>55</td>
<td>59</td>
<td>63</td>
<td>67</td>
<td>71</td>
<td>81</td>
<td>91</td>
<td>101</td>
<td>111</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>x thermocouple N°</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>1G</th>
<th>2G</th>
<th>1D</th>
<th>2D</th>
</tr>
</thead>
<tbody>
<tr>
<td>Y(mm)</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>-22</td>
<td>-45</td>
<td>22</td>
<td>45</td>
</tr>
<tr>
<td>X(mm)</td>
<td>11</td>
<td>40</td>
<td>74</td>
<td>114</td>
<td>44</td>
<td>76</td>
<td>44</td>
<td>76</td>
</tr>
</tbody>
</table>
FIG: 3-1. EFFECT OF FREE STREAM REYNOLDS NUMBER ON PRESSURE DISTRIBUTION

<table>
<thead>
<tr>
<th>Experiments</th>
<th>Fitted Curve</th>
<th>Re_x10^-7</th>
</tr>
</thead>
<tbody>
<tr>
<td>□</td>
<td>--</td>
<td>0.745</td>
</tr>
<tr>
<td>○</td>
<td>---</td>
<td>1.072</td>
</tr>
<tr>
<td>△</td>
<td>---</td>
<td>1.528</td>
</tr>
<tr>
<td>+</td>
<td>---</td>
<td>1.89</td>
</tr>
<tr>
<td>◆</td>
<td>---</td>
<td>2.57</td>
</tr>
</tbody>
</table>

X (mm) 0 10 20 30 40 50 60 70 80 90 100

Pressure Distribution
FIG. 3: EFFECT OF FREE STREAM REYNOLDS NUMBER ON PRESSURE DISTRIBUTION
(Ambient wall temperature)
FIG. 36: EFFECT OF FREE STREAM REYNOLDS NUMBER ON PRESSURE DISTRIBUTION

(Quasi adiabatic wall)
Figure 31: Laminarity criterion

<table>
<thead>
<tr>
<th>$S_w$</th>
<th>n</th>
<th>X (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>7</td>
<td>34</td>
</tr>
<tr>
<td>-0.32</td>
<td>7</td>
<td>34</td>
</tr>
<tr>
<td>-0.8</td>
<td>8</td>
<td>37</td>
</tr>
</tbody>
</table>
FIG: 3-2 WALL COOLING EFFECT ON MEASURED PRESSURE DISTRIBUTION

<table>
<thead>
<tr>
<th>Experiments</th>
<th>Fitted Curve</th>
<th>Re (_u \times 10^- 7)</th>
<th>Sw</th>
</tr>
</thead>
<tbody>
<tr>
<td>o</td>
<td>--</td>
<td>1.07</td>
<td>-0.805</td>
</tr>
<tr>
<td>(\triangle)</td>
<td>---</td>
<td>1.13</td>
<td>-0.325</td>
</tr>
<tr>
<td>(\square)</td>
<td>---</td>
<td>1.08</td>
<td>(\approx) 0</td>
</tr>
</tbody>
</table>
FIG: 3-20 WALL COOLING EFFECT ON THE LOCATION OF THE BEGINNING OF THE INTERACTION
FIG: 3-21a  SURFACE VISUALIZATION OF STREAMWISE VORTICES
(Cooled wall)

FIG: 3-21b  (Sublimation technique)
$M_o = 7.4$

$Re_{sh} = 2.2 \times 10^6$

$\frac{P_{\infty}}{P_{\infty}^{\infty}} = 2.933$

$Sw = -0.8$

Shock generator

Experimental data from NEEDHAM

Theoretical

Pressure

Heat transfer

FIG: 4-1 EXPERIMENTAL and THEORETICAL DISTRIBUTIONS OF PRESSURE and HEAT TRANSFER
$M_0 = 9.7$
$Re_{sh} = 0.95 \times 10^5$
$Sw = -0.8$
$\theta_{ramp} = 10^\circ$

Experimental Data from NEEDHAM

Theoretical

Pressure

Heat transfer
COMPARISON BETWEEN THEORETICAL AND MEASURED PRESSURE DISTRIBUTIONS

<table>
<thead>
<tr>
<th>Experiments</th>
<th>Theory</th>
<th>Re_u x10^7</th>
<th>Sw</th>
</tr>
</thead>
<tbody>
<tr>
<td>o</td>
<td></td>
<td>0.83</td>
<td>-0.79</td>
</tr>
<tr>
<td>△</td>
<td></td>
<td>0.825</td>
<td>-0.31</td>
</tr>
<tr>
<td>o</td>
<td></td>
<td>0.683</td>
<td>0</td>
</tr>
</tbody>
</table>
Experiments

Theory

<table>
<thead>
<tr>
<th>$Re_x \times 10^{-7}$</th>
<th>$Sw$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.57</td>
<td>-0.805</td>
</tr>
<tr>
<td>2.24</td>
<td>-0.309</td>
</tr>
<tr>
<td>2.40</td>
<td>0</td>
</tr>
</tbody>
</table>
FIG: 4-4 COMPARISON BETWEEN THEORETICAL AND MEASURED PRESSURE DISTRIBUTION IN THE FREE INTERACTION REGION
G-4-5a COMPARISON BETWEEN THEORETICAL AND MEASURED HEAT TRANSFER COEFFICIENT DISTRIBUTIONS
FIG:4-5b (cont'd)
FIG. 6 NORMALIZED EXPERIMENTAL PRESSURE AND HEAT TRANSFER DISTRIBUTIONS
This report describes a theoretical and experimental investigation of the effect of wall cooling on the overall features of shock wave laminar boundary layer interactions in hypersonic two dimensional flow. The integral method of Lees-Reeves-Klineberg has been modified in order to compute viscous interactions over a continuous range of wall-to-stagnation temperature ratios using a single set of "universal" integral functions. Comparisons between the theoretical predictions and the experimental results obtained from pressure and heat transfer distribution measurements show satisfactory agreements. These experiments have been carried out in a moderately hypersonic flow (M=6) over flat plate-wedge models for different wall cooling rates, corresponding to conditions lying between a highly cooled wall and a quasi-adiabatic surface. Furthermore, the theory developed herein may be considered to be a significant improvement in the attempt to calculate viscous interactions where the wall temperature distribution is prescribed.
Laminar boundary layer
Separation
Integral method
Calculation procedure
Viscous-inviscid interaction
Computer program