ONE DIMENSIONAL DESIGN OF CENTRIFUGAL
COMPRESSORS TAKING INTO ACCOUNT FLOW
SEPARATION IN THE IMPELLER

P. FRIGNE & R. VAN DEN BRAEMBUSSCHE

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TABLE OF CONTENTS

Summary ........................................ 1
List of Symbols ................................... ii
List of Figures .................................... v
Introduction ...................................... 1

CHAPTER 1 - BRIEF DESCRIPTION OF THE FLOW THROUGH
A CENTRIFUGAL MACHINE .......................... 2

CHAPTER 2 - INLET GUIDE VANES (IGV) ............... 4
2.1 Purpose of IGV ................................. 4
2.2 Isentropic calculation of IGV ..................... 6
  2.2.1 Constant prerotation IGV .................... 7
  2.2.2 Free vortex IGV .............................. 8
  2.2.3 Forced vortex IGV ........................... 9
2.3 Calculation of IGV losses ....................... 9

CHAPTER 3 - THE ROTOR ............................ 11
3.1 General geometry ................................ 11
3.2 The inlet blockage ................................ 12
3.3 Determination of the inducer hub and tip radii .... 14
  3.3.1 Determination of the inducer hub radius ........ 14
  3.3.2 Determination of the inducer tip radius ........ 14
3.4 The inducer flow ................................ 18
  3.4.1 Physical model and diffusion ratio ............ 18
  3.4.2 Geometrical model ............................ 20
  3.4.3 Flow equations ............................... 22
  3.4.4 Calculation of the separation point ............ 26
3.5 The impeller flow ................................ 27
  3.5.1 Real flow effects ............................. 27
  3.5.2 Flow equations for the jet ..................... 28
  3.5.3 Flow equations for the wake .................... 32
  3.5.4 The energetical value of jet and wake flow .... 34
  3.5.5 Influence of the parameter v on
       impeller performance .......................... 36
  3.5.6 Influence of the impeller outlet width b ....... 38
3.6 The impeller outlet tip

3.6.1 The slip factor $\mu$ for a jet and wake configuration
3.6.2 Equations for the impeller outlet flow
3.6.3 The impeller work equation

3.7 Disc friction

CHAPTER 4 - THE MIXING PROCESS

4.1 Introduction
4.2 Theoretical computation of the mixing zone of a jet-wake flow, taking into account the compressibility of the fluid

4.2.1 Assumptions
4.2.2 Forces acting upon the jet and the wake
4.2.3 Equations for the mixing process
4.2.4 Solution of the equations for the mixing process

4.3 Results of the theoretical computations

CHAPTER 5 - VANELESS DIFFUSERS

5.1 Application field
5.2 Advantages

5.1.1 Advantages
5.1.2 Disadvantages

5.2 Computation method

CHAPTER 6 - VANED ISLAND DIFFUSERS

6.1 General geometry
6.2 Computation method

6.2.1 The vaneless space
6.2.2 Semi-vaneless space
6.2.3 The divergent channel

6.3 Dump diffusion

REFERENCES

APPENDIX - SOLUTION OF THE EQUATIONS FOR THE MIXING PROCESS

TABLES

FIGURES
SUMMARY

A one dimensional computation method for the design of a centrifugal compressor is developed, which takes into account various real flow effects, such as flow separation in the impeller, jet-wake mixing in the vaneless space, transonic vaned diffuser performance and diffuser outlet dump diffusion.

Several numerical examples are worked out in detail, to show the influence of different important geometrical and aerodynamical parameters on the compressor performances.

This approach is also the one used for a VKI computer program COMRAD.
## LIST OF SYMBOLS

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>geometric area</td>
</tr>
<tr>
<td>AR</td>
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<tr>
<td>a</td>
<td>sonic speed</td>
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<tr>
<td>a*</td>
<td>critical speed</td>
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<td>b</td>
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<td>static enthalpy</td>
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<td>Mach number</td>
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<tr>
<td>M_w</td>
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<td>\dot{m}</td>
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<td>N_s</td>
<td>specific speed (Baljé)</td>
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<td>gas constant</td>
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<td>rotational speed</td>
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<td>RV</td>
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<td>time</td>
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<td>shear stress</td>
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<td>angular polar coordinate in meridional plane</td>
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<tr>
<td>ω</td>
<td>loss total pressure loss coefficient</td>
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<td>Ω</td>
<td>angular speed</td>
</tr>
</tbody>
</table>

**Subscripts**

- **0**: inlet plenum
- **1**: inducer inlet
- **sep**: separation section
2 impeller discharge
3 diffuser leading edge
4 diffuser throat section
5 diffuser channel outlet
6 dump diffusion
a absolute
h hub
j jet
m meridional
n normal
t tip
u tangential
w wake
w wall
b1 blade
c1 clearance
df disc friction
diff diffuser
fr friction
hom homogeneous
imp impeller
ind inducer
mn mean
pr pressure side
sh shear layer
suc suction side

**Superscripts**

o total
is isentropic
- mass averaged values
<table>
<thead>
<tr>
<th>number</th>
<th>title</th>
<th>page</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Schematic of compressor stage flow regions</td>
<td>69</td>
</tr>
<tr>
<td>2</td>
<td>Baljé-diagram for optimum specific speed</td>
<td>70</td>
</tr>
<tr>
<td>3</td>
<td>Influence of PR on $M_{W_{1t}}$ for different values of $N_S$</td>
<td>70</td>
</tr>
<tr>
<td>4</td>
<td>Inlet velocity triangles</td>
<td>71</td>
</tr>
<tr>
<td>5</td>
<td>Influence of prerotation on $M_{W_{1t}}$ and $M_2$, for different values of PR</td>
<td>71</td>
</tr>
<tr>
<td>6</td>
<td>Inlet guide vane losses</td>
<td>72</td>
</tr>
<tr>
<td>7</td>
<td>Geometry of centrifugal compressor rotor</td>
<td>73</td>
</tr>
<tr>
<td>8</td>
<td>Influence of blade blockage on inlet velocity triangle</td>
<td>73</td>
</tr>
<tr>
<td>9</td>
<td>Influence of RV on $M_{W_{1t}}$</td>
<td>74</td>
</tr>
<tr>
<td>10</td>
<td>Influence of RPM on $M_{W_{1t}}$</td>
<td>74</td>
</tr>
<tr>
<td>11</td>
<td>Influence of $\alpha_1$ on $M_{W_{1t}}$</td>
<td>75</td>
</tr>
<tr>
<td>12</td>
<td>Influence of $R_{1h}$ on $M_{W_{1t}}$</td>
<td>75</td>
</tr>
<tr>
<td>13</td>
<td>Adjustment of RV</td>
<td>76</td>
</tr>
<tr>
<td>14</td>
<td>Adjustment of RPM</td>
<td>76</td>
</tr>
<tr>
<td>15</td>
<td>Influence of wall curvature on boundary layer development</td>
<td>77</td>
</tr>
<tr>
<td>16</td>
<td>Influence of $\Phi R$ on compressor efficiency</td>
<td>77</td>
</tr>
<tr>
<td>17</td>
<td>Comparison of diffusion performances of rotating inducers and stationary diffusers</td>
<td>78</td>
</tr>
<tr>
<td>18</td>
<td>Influence of DR and $M_{W_{1t}}$ on compressor efficiency $\eta_c$</td>
<td>78</td>
</tr>
<tr>
<td>19</td>
<td>Eckardt's impeller - elliptical profile approximation of a real impeller</td>
<td>79</td>
</tr>
<tr>
<td>20</td>
<td>Elliptical profile approximation of a real impeller</td>
<td>80</td>
</tr>
<tr>
<td>21</td>
<td>Geometrical model for separation</td>
<td>81</td>
</tr>
<tr>
<td>22</td>
<td>Impeller blade shape</td>
<td>82</td>
</tr>
<tr>
<td>23</td>
<td>Flow angle variation</td>
<td>82</td>
</tr>
<tr>
<td>24</td>
<td>Meridional velocity profile</td>
<td>83</td>
</tr>
<tr>
<td>25</td>
<td>Separation section geometry</td>
<td>83</td>
</tr>
<tr>
<td>26</td>
<td>Jet-wake model</td>
<td>84</td>
</tr>
<tr>
<td>27</td>
<td>Tangential equilibrium at impeller outlet</td>
<td>85</td>
</tr>
<tr>
<td>28</td>
<td>T,S diagram of impeller flow</td>
<td>86</td>
</tr>
<tr>
<td>29</td>
<td>Influence of $v = W_{2w}/W_{2d}$ on the wake width $\varepsilon_2$ and on the wake mass flow $\lambda$</td>
<td>86</td>
</tr>
<tr>
<td>Page</td>
<td>Description</td>
<td>Page</td>
</tr>
<tr>
<td>------</td>
<td>------------------------------------------------------------------------------</td>
<td>------</td>
</tr>
<tr>
<td>30</td>
<td>Influence of $v = \frac{W_2}{W_2}\ell$ on the impeller efficiency</td>
<td>88</td>
</tr>
<tr>
<td>31</td>
<td>Influence of the impeller width $b_2$ on the losses.</td>
<td>88</td>
</tr>
<tr>
<td>32</td>
<td>Jet-wake velocity triangles</td>
<td>89</td>
</tr>
<tr>
<td>33</td>
<td>Disc friction coefficient $c_M$ versus axial gap $S$</td>
<td>90</td>
</tr>
<tr>
<td>34</td>
<td>$T,S$ diagram of the impeller outlet state</td>
<td>91</td>
</tr>
<tr>
<td>35</td>
<td>Pressure fluctuations in the vaneless space</td>
<td>92</td>
</tr>
<tr>
<td>36</td>
<td>Mixing process</td>
<td>93</td>
</tr>
<tr>
<td>37</td>
<td>Vaneless diffuser boundary layer development</td>
<td>95</td>
</tr>
<tr>
<td>38</td>
<td>Vaned island diffuser geometry</td>
<td>95</td>
</tr>
<tr>
<td>39</td>
<td>Diffuser channel geometry</td>
<td>96</td>
</tr>
<tr>
<td>40</td>
<td>Calculation of diffuser leading edge state</td>
<td>96</td>
</tr>
<tr>
<td>41</td>
<td>Diffuser throat blockage versus actual static pressure recovery from leading edge to throat</td>
<td>97</td>
</tr>
<tr>
<td>42</td>
<td>Channel pressure recovery versus AR and $L/W$</td>
<td>97</td>
</tr>
<tr>
<td>43</td>
<td>Maximum pressure recovery versus aspect ratio $AS$ and throat blockage $B$</td>
<td>98</td>
</tr>
<tr>
<td>44</td>
<td>Dump diffusion process</td>
<td>98</td>
</tr>
</tbody>
</table>
INTRODUCTION

The real flow through a radial compressor is essentially three dimensional, viscous and instationary. Up to now, there is no mathematical model which allows to predict the flow in such a machine without neglecting some important aspects of the problem. In fact, it would be extremely complicated both due to the complexity of the flow and the complexity of the boundaries of the machine.

Nevertheless, there is an urgent need for the designer to dispose of a calculation method which allows to predict the main characteristics of a centrifugal compressor and to investigate the influence of various parameters on the compressor behaviour. Therefore, the calculation method has to be fast enough to allow iterative operation, but at the same time sufficiently elaborated to take into account some important flow phenomena, such as separation, Mach number influence, boundary layer blockage, losses, etc.

The approach described in this report is the one used in a VKI computer program (COMRAD) to calculate the performances and dimensions of a radial compressor, starting from mass flow, required pressure ratio, RPM and some geometrical relations. The method is based on the actual knowledge of real flow in a centrifugal compressor, as described briefly in chapter 1. In the following chapters, the flow in the different parts (IGV, rotor, diffusor, etc.) will be described in more detail and at the same time the equations used in the program are derived.
CHAPTER 1 - BRIEF DESCRIPTION OF THE FLOW THROUGH A CENTRIFUGAL MACHINE

A compressor can be divided into different parts, as shown in figure 1 (Ref. 1). After being deflected by the inlet guide vanes (IGV) the flow enters the inducer where it is decelerated and turned into the radial direction. The presence of a radial velocity component is responsible for coriolis forces, which together with the blade curvature effect, tends to stabilize the boundary layer at the suction side of the inducer (Refs. 2, 3). Due to this stabilization, the boundary layer becomes less turbulent, and will easily separate under influence of an adverse pressure gradient.

Once the flow is separated, we distinguish a high energetic jet, with a high relative Mach number and a low energetic wake, fed by secondary flows. The jet can be considered as an isentropic core with a constant Mach number in the flow direction (Ref. 1).

After leaving the impeller, a strong mixing takes place between jet and wake, due to the difference in angular momentum. This results in an intensive energy exchange and a fast uniformization of the flow.

In case of a vaned diffuser, the flow enters the semi-vaneless space - this is the region between the leading edge and the throat section of the diffuser - and a rapid adjustment rearranges the isobare pattern from parallel to perpendicular to the flow direction. If the Mach number is higher than 1, a shock wave system will decelerate the flow in such a way that the throat section becomes subsonic (design condition).

In the divergent channel, a further decrease of the velocity is realized with a subsequent increase of static pressure. Sometimes, the boundary layers in this channel are so
weak that separation occurs, which limits the static pressure rise. In case of vaneless diffusers, after the jet-wake mixing process, the flow is further decelerated by an increase of flow section corresponding to the radius increase and influenced by friction on the lateral walls. Each part of the compressor can be characterized by one or more typical parameters, p.e., the diffusion ratio $DR$ for the inducer, the mass flow ratio $\lambda$ for the separated impeller flow, the pressure recovery $C_p$ for the diffuser, etc. Their values are based on experimental data and empirical correlations.
CHAPTER 2 - INLET GUIDE VANES (IGV)

2.1 Purpose of IGV

Figure 2 shows the Baljé-diagram (Ref. 4), which gives the variation of adiabatic compressor efficiency $\Delta \eta_{ad}$ versus the specific speed $N_s$:

$$N_s = \frac{RPM \cdot \sqrt{Q}}{\Delta H^{3/4}} \quad (2.1)$$

with $Q$: inlet volume flow  (ft³/sec)

$\Delta H$: manometer height  (ft)

For these units, the optimum specific speed is approximately 120. For an increasing pressure ratio, PR, at constant value of the specific speed and inlet volume flow, the inducer relative tip Mach number $M_{W1t}$ will increase too. This variation has been calculated by Dean (Ref. 1) for different values of $N_s$ (Fig. 3). From this figure, it can be seen that for the speed $N_s = 100$, the critical value $M_{W1t} = 1$ will already be reached for $PR = 4$, while for a 8:1 compressor this relative tip Mach number will be as high as 1.4.

The supersonic Mach number will not only give rise to strong shock losses, but will also induce early flow separation, which results in very high losses. A possible solution for this problem consists in reducing the specific speed $N_s$ by lowering the RPM (Fig. 3). This has of course its influence on the rotor design. The impeller channels will be longer and more narrow, which involves additional shroud leakage, friction losses and secondary flows, with a subsequent efficiency drop. Another possibility is to make use of preswirl vanes. This can be explained by means of velocity triangles (Fig. 4).

A turning of the flow in the direction of rotation results in a noticeable decrease of the relative velocity component and relative flow angle. When looking to the Euler
we remark a decrease of rotor-work, due to the fact that the inlet tangential velocity is no longer zero. Consequently, an increase of the rotor diameter will be necessary to hold the same pressure ratio. However, the impeller outlet Mach number will increase too. This mechanism is shown in figure 5.
We see that for a PR equal to 6, and the inlet flow prerotation varying from 0° to 40°, the inducer tip Mach number will decrease from .9 to .7, while the impeller outlet Mach number increases from 1.1 to 1.2. Roughly, as long as

\[ \Delta M_{W1t} > -\Delta M_2 \]  

IGV are efficient.
It can be seen from figure 5 that this condition will be fulfilled for \( \alpha_1 < 30° \) \( (N_s = 70) \).
For higher values of PR, the increase of \( M_2 \) will be more important than the decrease of \( M_{W1t} \) when prerotation becomes greater than 30°. The range of IGV is thus limited by an optimization of the Mach number dependent losses in wheel and diffuser.

In conclusion it can be said that IGV provide an additional degree of freedom in determining the relative Mach number at the inducer tip.

For an investigation of the influence of compressor inlet adjustable guide vanes for the control of single shaft gas turbines, we refer to reference 5. This study shows that the use of a variable prewhirl control to produce rated power under a wide range of ambient conditions is very promising, helping the designer to meet specifications otherwise impossible to meet.
2.2 Isentropic calculation of IGV

We consider a compressor with a straight inlet channel wherein the flow is uniform. The IGV modifies this velocity distribution by creating a tangential velocity component $V_u$. The radial equilibrium at the outlet of the IGV can be expressed as:

$$\frac{1}{\rho} \frac{dp}{dR} = \frac{V_u^2}{R}$$  \hspace{1cm} (2.4)

The equation of Barré de Saint-Venant gives a relation between $p$, $V$, $p_0$ and $\rho_0$:

$$\frac{V^2}{2} + \frac{1}{K} \frac{p_0}{\rho_0} \frac{P_0}{K-1} \frac{P_0}{K} = \frac{K}{K-1} \frac{P_0}{\rho_0}$$  \hspace{1cm} (2.5)

When this equation is differentiated and substituted in (2.4), together with relations deduced from the velocity triangles, we obtain:

$$\frac{1}{2 \cos^2 \alpha} \frac{dV^2_a}{dR} + \frac{V_a^2 \sin \alpha}{\cos^3 \alpha} \frac{d\alpha}{dR} + \frac{1}{\rho_0} \left( \frac{p_0}{p} \right) \frac{dp}{dR} = 0$$  \hspace{1cm} (2.6)

As the flow between the inlet plenum and the IGV is isentropic, we have:

$$\left( \frac{p_0}{p} \right)^{1/K} = \frac{\rho_0}{\rho}$$  \hspace{1cm} (2.7)

Introducing (2.7) in (2.6) gives:

$$\frac{1}{V_a^2} \frac{dV^2_a}{dR} + 2 \tan \alpha \frac{d\alpha}{dR} + 2 \frac{\sin^2 \alpha}{R} = 0$$  \hspace{1cm} (2.8)
Equation (2.8) allows to calculate the $V_a(R)$-distribution that corresponds to a given $\alpha(R)$-distribution. In what follows, this is done for three different typical prewhirl models, which are available as options for program COMRAD.

2.2.1 Constant prerotation IGV

This type of IGV has already been used and experimental results are available (Refs. 6, 7).

Substitution of $\frac{d\alpha}{dR} = 0$ in eq. (2.8) gives:

$$\frac{1}{V_a^2} \frac{dV_a^2}{dR} + \frac{2\sin^2 \alpha}{R} = 0 \quad (2.9)$$

Equation (2.9) can be integrated:

$$V_a = a_1 R \quad (2.10)$$

The coefficient $a_1$ can be deduced from the volume flow equation

$$Q = \int_{R_{lh}}^{R_{lt}} a_1 R \cdot 2\pi R \, dR \quad (2.11)$$

With $R_{lh}$: hub radius of IGV

$R_{lt}$: tip radius of IGV

Elimination of $a_1$ from (2.10) and (2.11) gives the distribution of the axial velocity component $V_a$ in function of the radius $R$ and the chosen prerotation angle $\alpha$:

$$V_a(\alpha, R) = \frac{Q(1+\cos^2 \alpha)}{2\pi(R_{lt} - R_{lh})} \frac{-\sin^2 \alpha}{1+\cos^2 \alpha} \quad (2.12)$$
With the assumptions of isentropic and uniform flow, the values \( p_1, T_1 \) and \( \rho_1 \) can be calculated as:

\[
T_1 = T_0 - \frac{V_1^2}{2C_p} \quad (2.13)
\]

\[
p_1 = \rho_0 \left( \frac{T_1}{T_0} \right)^{\frac{K}{K-1}} \quad (2.14)
\]

\[
\rho_1 = \rho_0 \left( \frac{T_1}{T_0} \right)^{\frac{1}{K-1}} \quad (2.15)
\]

A similar method of calculation has been proposed by Vavra (Ref. 8).

2.2.2 Free_vortex_IGV

A free vortex flow is defined by

\[
V_u = \frac{K}{R} \quad (2.16)
\]

The absolute flow angle \( \alpha \) follows from

\[
\tan \alpha = \frac{V_u}{V_a} = \frac{K}{RV_a} \quad (2.17)
\]

By substitution of (2.17) in (2.8), it can be shown that

\[
\frac{dV_a}{dR} = 0 \quad (2.18)
\]

The axial flow component is thus constant over the blade height and can be determined by the continuity equation:

\[
Q = V_a \pi (R_{1t}^2 - R_{1h}^2) \quad (2.19)
\]
The values of $p_1$, $T_1$, $\rho_1$ can be calculated with equations (2.13), (2.14), (2.15).

### 2.2.3 Forced vortex IGV

A forced vortex flow is defined by

$$V_u = KR R$$ \hspace{1cm} (2.20)

The absolute flow angle $\alpha$ follows from

$$\tan \alpha = \frac{V_u}{V_a} = \frac{KR}{V_a}$$ \hspace{1cm} (2.21)

Substitution of (2.21) in (2.8) gives

$$\frac{2}{R} \cdot \frac{2}{\tan \alpha} \cdot \frac{\alpha}{dR} + \frac{2\sin^2 \alpha}{R} = 0$$ \hspace{1cm} (2.22)

Integration of (2.22) yields

$$R = \frac{C_1 \sin \alpha}{\sqrt{1+\sin^2 \alpha}}$$ \hspace{1cm} for $\alpha > 0$ \hspace{1cm} (2.23)

The integration factor $C_1$ can be determined by continuity

$$Q = \frac{\pi K}{C_3} \left[ (1-2C_1^2 R_1 h)^{3/2} - (1-2C_1^2 R_1 t)^{3/2} \right]$$ \hspace{1cm} (2.24)

The values of $p_1$, $T_1$, $\rho_1$ can be calculated with equations (2.13), (2.14), (2.15).

### 2.3 Calculation of IGV losses

The losses in the IGV are very small compared with the losses in other parts of the compressor, and can normally be
neglected except in high deflection cases.

Figure 6 (Ref. 6) gives a qualitative sketch of the main parameters determining the IGV losses. It can be seen that:
- The loss coefficient depends strongly upon the span.
  High losses at the tip of IGV (100% span).
  Negligible losses at the hub (0% span).
- Negative preswirl (opposite to the direction of rotation)
  gives rise to a strong increase of the loss coefficient.
For a theoretical and quantitative loss computation, we refer to Stewart (Ref. 9).
CHAPTER 3 - THE ROTOR

3.1 General geometry

Figure 7 gives the general geometry of a centrifugal compressor rotor. In our model, three important flow sections are taken into account:
- The inlet section (1)
- The separation section (SEP)
- The outlet section (2).

At the inlet section, the flow is axisymmetric. However, all flow quantities vary with the radius R: \( p_1(R) \), \( T_1(R) \), \( V_1(R) \), \( \alpha_1(R) \), ...
The relative flow is then decelerated between the rotor blades until separation occurs. Due to coriolis and curvature effects, the separation point will be located at the shroud-suction side intersection. The separation section is defined as the cross section through the separation point.

In this way the rotor has been divided into two parts which are treated separately:
- the inducer flow (1 \( \rightarrow \) SEP),
- the impeller flow (SEP \( \rightarrow \) 2)

The impeller flow can also be divided into two subflows:
- the jet flow (SEP \( \rightarrow \) 2J)
- the wake flow (SEP \( \rightarrow \) 2W).

This division of the rotor flow into subflows is an important feature in our approach, because in this way some real flow phenomena can be included in the computations.

One of the most important parameters determining the rotor losses will be the wake width \( \varepsilon_2 \), which is function of the separation point location. If the inducer is well designed, and much deceleration can be achieved before separation occurs, this point will be located near to the impeller outlet, and the wake will not develop as much. The wake width \( \varepsilon_2 \) will be proportionally smaller and so do the losses.
At the contrary, if we have a bad inducer, and early separation occurs, then the wake will grow to a large part of the impeller flow and the mixing process behind the wheel will involve very high losses with a subsequent drop of the efficiency.

3.2 The inlet blockage

When the flow enters the wheel, the free frontal area is reduced by the presence of the blades. This involves a flow contraction which changes the velocity triangles by accelerating the axial velocity component from $V_a$ to $V'_a$ (Fig. 8). The relative flow angle $\beta_1$ will be turned over an angle $i_k B_1$ to the new value $\beta'_1$. If the influence of the blade curvature is neglected, then the optimum $\beta'_1$ has to be equal to the geometric blade angle $\beta_{1b1}$. From Stanitz (Ref. 10) we get the value of $i_k B_1$:

$$\tan i_k B_1 = \frac{(1-k_{B1})\tan \beta_1}{1+k_{B1}\tan^2 \beta_1} \quad (3.1)$$

$k_{B1}$ represents the procentual free stream section:

$$k_{B1} = 1 - \frac{dz}{2\pi R \cos \beta'_1} \quad (3.2)$$

with $d$: the blade thickness (normal to camber)
$z$: number of rotor blades
$R$: radius.

The blade angle can thus be calculated as:

$$\beta_{1b1} = \beta'_1 = \beta_1 - i_k B_1 \quad (3.3)$$

The new relative velocity $W'_1$ can be calculated as:

$$W'_1 = W_1 \cdot \sin \beta_1 / \sin \beta'_1 \quad (3.4)$$

The new axial velocity component is:
The absolute flow angle $\alpha_1'$ follows from:

$$\alpha_1' = \arctg \left( \frac{\tan 1}{\tan \beta_1} \right)$$

(3.6)

The velocity $V_1'$ is:

$$V_1' = \sqrt{W_1'^2 + W_1'^2 + 2W_1'U_1\sin \beta_1}$$

(3.7)

The absolute flow angle $\alpha_1'$ can also be calculated as:

$$\alpha_1' = \arctg \left( \frac{U_1 + W_1'\sin \beta_1}{W_1' \cos \beta_1} \right)$$

(3.8)

The new static temperature $T_1'$ can be derived from the energy equation, when accepting that no work has been done on the fluid during the contraction.

$$T_1' = T_1 + \frac{V_{1}^2 - V_{0}^2}{2C_p}$$

(3.9)

Supposing an isentropic process, the static pressure $p_1'$ becomes:

$$p_1' = p_1 \left( \frac{T_1'}{T_1} \right)^{\frac{K}{K-1}}$$

(3.10)

The new density $\rho_1'$ follows from the ideal gas equation:

$$\rho_1' = \frac{p_1'}{R_g T_1'}$$

(3.11)

It is important to notice in previous calculations that, since $U_1$ varies with the radius $R$, all quantities $V_1$, $W_1$, $\alpha_1$, $\beta_1$, $p_1$, $T_1$, and $\rho_1$ are function of $R$. 
3.3 Determination of the inducer hub and tip radii

3.3.1 Determination of the inducer hub radius

The minimum hub radius $R_{1h\min}$ is fixed by mechanical considerations. Two criteria hold in this case:
- The minimum hub radius can be determined by strength considerations, which require a minimum sectional area for the axis to transmit the engine torque and to avoid critical velocities.
- The minimum hub radius $R_{1h\min}$ can be limited by the maximum blade blockage $q_{\max}$ at the hub:

$$R_{1h\min} = \frac{dz}{2\pi q_{\max}} \quad (3.12)$$

$R_{1h\min}$ is often limited by the minimum shaft diameter and is therefore used as an input parameter in our program. The corresponding blockage $q_{\max}$ is calculated.

3.3.2 Determination of the inducer tip radius

The determination of $R_{1t}$ is more complicated, because we want to limit the relative inducer tip Mach number $M_{W_{1t}}$. The relation between these two parameters can be demonstrated with a simple example. Suppose a centrifugal compressor for freon with following characteristics:
- mass flow : $\dot{m} = 2.5$ kg/s
- gas constant : $R_G = 75.3$ J/kg°K
- isentropic exponent : $K = 1.136$
- inlet density : $\rho_1 = 4.$ kg/m$^3$
- maximum inlet blockage : $1-k_{B1} = .02$
- inlet temperature : $T_1 = 283 ^{\circ}K$

The rotational speed RPM, the prerotation angle $\alpha_1$ and the inducer hub radius $R_{1h}$ are taken as input parameters. The sonic velocity $a_1$ is:
a_1 = \sqrt{k R_G T_1} = 155.6 \text{ m/s}

For a uniform inlet flow with constant prerotation, following equations are valid:

\[ W_{1t}^2 = V_{1a}^2 + (U_{1t} - V_{1a} \tan \alpha_1)^2 \]

\[ U_{1t} = \frac{2\pi \text{RPM}}{60} R_{1t} \]

\[ V_{1a} = \frac{m}{k B_1 \pi R_{1t}^2 (R_{1h}^2 - R_{1t}^2) \rho_1} \]

\[ M_{W_{1t}} = \frac{W_{1t}}{a_1} \]

In our model, the inducer hub and tip radii are taken together in one parameter \( RV = R_{1h}/R_{1t} \).

The inducer tip relative Mach number \( M_{W_{1t}} \) can also be expressed as a function of the following parameters:

\[
M_{W_{1t}} = f(\text{RPM}, \alpha_1, RV, R_{1h}) \text{ or }
\]

\[
M_{W_{1t}} = \frac{1}{a_1} \sqrt{\left( \frac{m}{k B_1 \pi \rho_1 R_{1h} \left( \frac{1}{RV^2} - 1 \right)} \right)^2 + \left( \frac{2\pi \text{RPM} R_{1h}}{60 \text{ RV}} - \frac{m}{k B_1 \pi \rho_1 R_{1h}^2 \left( \frac{1}{RV^2} - 1 \right)} \right)^2}
\]

In figure 9 this equation is plotted versus \( RV \) for \( \text{RPM} = 16,000 \quad R_{1h} = .04 \text{ m} \quad \alpha_1 = 0^\circ \)

This graph presents a minimum value of \( M_{W_{1t}} = .85 \) for an inducer hub-tip radius ratio \( RV = .6 \). At the left of this point, for smaller values of \( RV \), the Mach number \( M_{W_{1t}} \) will increase, due to
the higher values of the circumferential velocity \( U_{1t} \). At the right of point M, for higher values of RV, the Mach number \( M_{W_{1t}} \) will increase as well, because the inlet flow section is reduced, with a subsequent increase of the axial velocity \( V_{1a} \).

An important conclusion is that, for a given value of RPM, \( \alpha_1 \) and \( R_{1h} \), it will not always be possible to find a solution for RV (say \( R_{1t} \)), if the relative tip Mach number is limited. The only way to overcome this difficulty is to adapt one of the parameters RPM, \( \alpha_1 \) or \( R_{1h} \). Figures 10, 11 and 12 show the influence of each of these parameters respectively. Figures 10 and 11 show that reduction of RPM or use of positive preswirl results in a decrease of the relative inducer tip Mach number. This conclusion has already been drawn when discussing the IGV (cfr Chapter 2).

Figure 12 shows only a slight decrease of the minimum relative Mach number when the hub radius is reduced. Keeping \( \alpha_1 \) and RPM unchanged, it will not always be possible to keep \( M_{1t} \) below a given value by variation of \( R_{1h} \) and RV only.

In the program COMRAD, following procedure has been adopted:

- The prerotation angle \( \alpha_1 \) and \( M_{W_{1t},\text{max}} \) are fixed input data.
- RPM and RV are also input data, but one of them can be adjusted, in order to reduce the inducer tip Mach number below a given maximum value. The adjustment is done automatically and the new value of the parameter is printed out. (see Figs. 13, 14).
- The inducer hub radius \( R_{1h} \) is initially equal to its minimum value \( R_{1h,\text{min}} \), which is an input value. For given values of \( R_{1h}/R_{1t} \) and RPM, the program will increase \( R_{1h} \) with steps of 5% until \( M_{W_{1t}} \) becomes less than \( M_{W_{1t},\text{max}} \). If this is not possible, one of the parameters RV or RPM will be adjusted, according to the key value 1 and the procedure starts again from \( R_{1h} = R_{1h,\text{min}} \).
3.3.2.1 Adjustment of RV

This method is explained in figure 13. Suppose that a large value is chosen for RV (.75). $R_{1h}$ is then increased from $R_{1h,\text{min}}$ (point a) with steps of 5%. As long as $M_{W_{1t}}$ is decreasing, we continue the iterations. At the right of the minimum b of the curve, $M_{W_{1t}}$ will increase again, without having reached the value of $M_{W_{1t,\text{max}}}$. The program will then automatically decrease the value of the hub-tip radius ratio and start again from $R_{1h} = R_{1h,\text{min}}$ (point c). The new minimum value is less and a new iteration cycle can start (c → d). Adjustment of RV will occur as long as the minimum of the curves is higher than $M_{W_{1t,\text{max}}}$. The computation stops at point h.

It is important to notice that the reduction of the minimum of the curves for decreasing RV is rather weak, and limited by that value of RV that gives a minimum for $M_{W_{1t}}$ at $R_{1h} = R_{1h,\text{min}}$.

3.3.2.2 Adjustment of RPM

This method is very similar to the previous one (cfr Fig. 14). The shift of the minimum of the curves when RPM is decreasing is more important and not limited, because the minimum is shifting to the right of the plane (away from $R_{1h,\text{min}}$). Nevertheless, the RPM of a machine is very often determined by mechanical considerations, and has a direct influence on specific speed and thus also on maximum efficiency.

REMARK: No iteration will be carried out when $M_{W_{1t}}$ is less than $M_{W_{1t,\text{max}}}$ at the beginning of the computation. $R_{1h}$ is then set equal to $R_{1h,\text{min}}$. 
3.4 The inducer flow

3.4.1 Physical model and diffusion ratio

As shown in § 3.1, the inducer is defined as that part of the rotor where the boundary layers are attached. The purpose of the inducer is threefold:
- deflection of the flow in axial direction;
- deflection of the flow in radial direction;
- diffusion of the relative flow to increase static pressure.
This results in a heavy load for the boundary layer, especially at the shroud-suction side intersection. Next considerations explain briefly why.

A. Adverse pressure gradient: diffusion means a deceleration or conversion of dynamic energy into static pressure. Consequently, there will exist an adverse pressure gradient. Separation of the boundary layer will occur when the kinetic energy available in the boundary layer and the one added by entrainment is dissipated.

B. Coriolis effects: because of the radial velocity component, important coriolis accelerations can exist in radial impellers. It is shown by Rothe & Johnston (Ref. 2), that due to the coriolis forces, the boundary layer on the pressure side is destabilized and becomes more turbulent. On the suction side, the boundary layer is stabilized and has less turbulent mixing. Turbulent mixing increases the entrainment and thus also postpones the separation point. We thus can conclude that in a radial impeller separation occurs, or is likely to take place at the suction side of the blades and is very unlikely on the pressure side of the blades. However, the loading distribution along the blade and the existence of a relative flow vortex \(-2\omega\) can decelerate the flow on the pressure side to negative values, so that return flow and a potential separation bubble can occur on the pressure side. The last possibility is not considered in our method because this should be avoided by a good blade to blade design.
C. Wall curvature effects: wall curvature is also creating a transverse pressure gradient. This one is related to centripetal accelerations (Fig. 15). Turbulence is influenced by centripetal accelerations in a very similar way as by coriolis accelerations. The boundary layer is stabilized on the convex surface and destabilized on the concave surface. In a radial compressor rotor the shroud is an annular convex surface and the relative flow will be less resistant against separation. In combination with the coriolis forces on the suction side, we can expect that flow separation starts at the suction side-shroud intersection line. Similar as for axial compressors, the relative velocity at separation can be related to inlet velocity by a diffusion ratio

\[ DR = \frac{W_{1t}}{W_{SEP}} \]

The higher the diffusion ratio, the further downstream will be the separation point and the smaller will be the separated region. Consequently, the efficiency will increase.

According to Dean (Ref. 1), the compressor efficiency relates to DR and \( C_{pd} \) (diffusor pressure recovery) as shown in figure 16. For constant \( C_{pd} \), the efficiency increases with DR. For values of DR higher than 1.4, the efficiency curves become very flat, and it seems to be more interesting to pay attention for the diffuser rather than for the impeller diffusion process.

In our one dimensional design program, the expected DR has to be defined and the method is valid only if the predicted DR can also be achieved. The DR is mainly a function of rotor geometry which means that this one dimensional design method must be completed by a detailed three dimensional design. Reasonable values of DR are shown in figure 17 (Ref. 1) as a function of inducer tip relative Mach number \( MW_{1t} \). On the same figure the maximum diffusion ratio for radial diffusers (DR vs \( M_3 \)) is also shown to indicate how badly inducers are affected.
by coriolis forces and curvature.

An increase of $MW_{1t}$ at constant $DR$ will result in a higher separation velocity $W_{sep}$ and $W_{2j}$ (relative velocity of jet flow at outlet of impeller) will increase proportionally. This results in higher "mixing losses" at the impeller outlet (Fig. 18) and shock losses when the flow becomes transonic.

3.4.2 Geometrical model

3.4.2.1 Hub- and shroud contour

For the computation of the inducer flow, the shroud and hub contour have been represented in the meridional plane by elliptical profiles. Comparison with data from literature (Ref. 11) shows that good agreement is obtained (Fig. 19). This method has the important advantage that both hub- and shroud contours can be determined completely with only 5 parameters: $R_{1h}$, $R_{1t}$, $R_2$, $b_2$, $D_{AX}$. The impeller considered at figure 20 is a small radial compressor with an inducer, similar to those commercially available for turbochargers (Ref. 11bis). In this case, the substitution of hub- and shroud contours by elliptical profiles is not so good due to the linear hub shape at the impeller outlet.

The inducer is further divided into 5 equidistant annular stream surfaces (1 to 5 on Fig. 21) and a hub (6) and shroud (7) stream surface. They will be used to calculate approximated values of local parameters, and also facilitate the calculation of mass averaged values at different sections.

3.4.2.2 Blade angle variation

The blade angle variation used in our model is an approximated one and is based on an investigation of blade profiles of impellers with elliptical inducer blade shape. E. Schnell (Ref. 12) shows that an optimal blade loading is obtained using this kind of inducer shape. However, our defi-
nition makes an extension for the impellers with backward bended blades (Fig. 22):

Definition:

\[ \beta_{b1} = \frac{\beta_{2b1}}{2} + \frac{1}{2} \left( \beta_{1b1} - \frac{\beta_{2b1}}{2} \right) \left[ 1 + \sin \left( \frac{\pi}{2} + 3\phi \right) \right] \]

\[ 0 < \phi < \frac{\pi}{3} \]

\[ \beta_{b1} = \frac{\beta_{2b1}}{2} \left[ 2 + \sin \left( \frac{\pi}{2} + 3\phi \right) \right] \]

\[ \frac{\pi}{3} < \phi < \frac{\pi}{2} \quad (3.14) \]

3.4.2.3 Flow angle variation

At the outlet of the wheel, the flow angle \( \beta_2 \) differs from the blade angle \( \beta_{2b1} \) due to the effect of the slip factor \( \mu \) (cfr 3.6). When \( \mu \) is known (from experimental correlation), the flow angle \( \beta_2 \) can be calculated. In our model, we agree that the slip effect starts at \( \phi = 60^\circ \). From the inducer inlet to \( \phi = 60^\circ \), the flow is supposed to be tangent to the blades. From \( \phi = 60^\circ \) to the outlet, a gradual increase of the slip is taken into account.

Definition:

\[ \beta = \beta_{b1} \]

\[ 0 < \phi < \frac{\pi}{3} \]

\[ \beta = \beta_{b1} + \frac{\phi - \frac{\pi}{6}}{\frac{\pi}{3}} \left( \beta_{2} - \beta_{2b1} \right) \quad \frac{\pi}{3} < \phi < \frac{\pi}{2} \quad (3.15) \]
3.4.3 Flow equations

3.4.3.1 Velocity profile in the meridional plane

According to Vavra (Ref. 13), following relation exists for an axisymmetric stream surface:

\[ \frac{\partial W_m}{\partial n} + W_m k_m = 0 \]  \hspace{1cm} (3.16)

with \( W_m \): the relative meridional velocity

\( k_m \): the curvature

\( n \): the normal direction.

For an annular stream tube (Fig. 24), this expression can be integrated:

\[ W_m = W_{m_i} e^{ \left( k_{m_i} (n - \frac{n^2}{2b}) + k_{m_j} \frac{n^2}{2b} \right) } \]  \hspace{1cm} (3.17)

with \( k_m > 0 \)

\( b \) small compared to \( R_i, R_j \).

The relation between \( W_{m_i} \) and \( W_{m_j} \) is:

\[ W_{m_i} = W_{m_j} e^{ -b(\frac{k_{m_i} + k_{m_j}}{2}) } \]  \hspace{1cm} (3.18)
With this equation, the relative meridional velocity $W_m$ can be determined along each intersection line with the elliptical stream surfaces (Fig. 21).

The calculation starts at the shroud, where

$$W_m^{sep} = \frac{W_{1t} \cos(\beta)_{sep}}{DR}$$  \hspace{1cm} (3.19)

When the meridional velocity profile is determined, the relative velocity follows from:

$$W_{sep}^i = \frac{W_m^{sep}}{\cos(\beta)_{sep}}$$  \hspace{1cm} (3.20)

$i$ refers to a stream surface

Remark: The curvature $k_m$ in a point $(x_0, y_0)$ on the ellipse $\frac{x^2}{A^2} + \frac{y^2}{B^2} = 1$ can be calculated with

$$k_m = A B \left( \frac{A^2 y_0^2 + B^2 x_0^2}{B^2 + A^2} \right)^{-3/2}$$  \hspace{1cm} (3.21)

3.4.3.2 Constant rothalpy

This equation is derived from the energy law and is also valid for non-isentropic flow:

$$C_p(T_{sep,i} - T_{1i}) = \frac{W_{i1}^2 - W_{sep,i}^2}{2} + \frac{U_{sep,i}^2 - U_{1i}^2}{2}$$  \hspace{1cm} (3.22)

It shows that the increase of enthalpy $C_p(T_{sep,i} - T_{1i})$ between inlet and separation is composed of:
\[ \frac{W_{1i}^2 - W_{sep,i}^2}{2} : \text{kinetic energy drop according to the variation of velocity. This term is strongly dependent on the inducer design.} \]

\[ \frac{U_{sep,i}^2 - U_{1i}^2}{2} : \text{increase of "centrifugal" energy. This term cannot be improved, because it is not dependent on the flow.} \]

3.4.3.3 Static pressure

For an isentropic flow, the variation of static pressure is given by:

\[ P_{sep,i} = \rho_{1i} \left( \frac{T_{sep,i}}{T_{1i}} \right) \]

(3.23)

A loss coefficient \( \omega_{ind} \) allows to correct for real flow friction losses:

\[ P_{sep,i} = P_{sep,i} - \frac{1}{2} \rho_{1i} \hat{W}_{1i}^2 \omega_{ind} \]

(3.24)

where \( \rho_{1i} \) and \( \hat{W}_{1i} \) are mass averaged values of \( \rho_{1i} \) and \( W_{1i} \).

3.4.3.4 Gas equation

\[ \rho_{sep,i} = \frac{P_{sep,i}}{R \ T_{sep,i}} \]

(3.25)

3.4.3.5 Boundary layer calculation

The loss coefficient \( \omega_{ind} \) and the boundary layer blockage \( \delta^* \) are calculated by an approximated boundary layer calculation as explained by A. Sarmento in reference 14. The momentum integral equation
\[ \theta_{x+\Delta x} = \theta_{x} + \left[ \frac{1}{2} C_f - \theta \left( \frac{1}{\rho} \frac{\partial p}{\partial x} + \frac{(H+2)}{W} \frac{\partial W}{\partial x} \right) \right] \Delta x \]  

(3.26)

can be integrated along the surfaces, assuming a linear variation of \( p \) and \( W \) between inlet and separation point, and by introducing following empirical correlations from reference 15:

\[ C_f = \frac{0.265}{e} - 1.561 \]  

(3.27)

\[ H = 1.63 - 0.0775 \log_{10}(Re) \]  

(3.28)

\[ \theta_0 = \frac{2}{R_l t} \left( \frac{v}{W_l} \right)^2 \]  

(3.29)

Once we know \( \theta \) and \( H \) (function of the change in relative velocity) we can calculate the boundary layer blockage \( \delta^* \), that will be used in the continuity equation.

The total friction force \( F_r \) in the inducer can be calculated as:

\[ F_r = \int_{S_{ind}} \tau W \, dS = \int_{S_{ind}} \frac{1}{2} \rho \, c_f \, W^2 \, dS \]  

(3.30)

with \( S_{ind} \) : the total wall surface of the inducer.

The loss coefficient \( \omega_{ind} \) is then:

\[ \omega_{ind} = \frac{\Delta p_{ind}}{\frac{1}{2} \rho_{1} \, W_{1}^2} = \frac{\frac{1}{2} \rho_{1} \, W_{1}^2 \, S_{sep}}{F_r} \]  

(3.31)

with \( \Delta p_{ind} \) : the total pressure loss in the inducer

\( S_{sep} \) : the area of the separation section.
3.4.3.6 Continuity

The mass flow through an annular flow section \( i \) (Fig. 21) of the separation section is equal to (Fig. 25):

\[
\dot{m}_{sep,i} = \rho_{sep,i} W_{sep,i} \left( 2\pi R_{sep,i} b_{sep,i} \cos \beta_{sep,i} - z \cdot b_{sep,i} (d + \delta^*) \right)
\]

with
- \( d \) : blade thickness
- \( \delta^* \) : boundary layer blockage
- \( z \) : number of blades.

The total mass flow is then:

\[
\dot{m}_{sep} = \sum_i \dot{m}_{sep,i} \tag{3.33}
\]

3.4.4 Calculation of the separation point

To perform calculations following basic assumption has been made: "when the deceleration of the relative flow at the shroud streamline reaches the value \( \frac{W_{lt}}{DR} \), separation occurs. This point on the shroud contour is called separation point, defined by the radius \( R_{sep} \). The flow section through this point, normal to each of the elliptical stream surfaces (Fig. 21), is defined as the separation section. The calculation starts with an arbitrary first guess of \( R_{sep} \). The meridional velocity profile, function only of the impeller geometry, can then be calculated assuming also that \( W_{sep} = \frac{W_{lt}}{DR} \). The other flow conditions are then given by the equations 3.16 - 3.31.

The value of the mass flow \( \dot{m}_{sep} \) is then calculated with eq. (3.32) and will generally be different from the input data \( \dot{m} \), due to the misestimation of \( R_{sep} \). By checking continuity we will change the value of \( R_{sep} \) in an iterative procedure. The value of \( R_{sep} \) in two consecutive iterations is given by:
\[ R_{\text{sep}} = R_{\text{sep}} - \frac{R_{\text{sep}} - R_{\text{sep}}}{m_{\text{sep}} - m_{\text{sep}}} (\tilde{m}_{\text{sep}} - \hat{m}) \]  \hspace{1cm} (3.34)

with \( n \) : the iteration number.
The iteration stops when the computed value of \( m_{\text{sep}} \) is equal to \( m \) within a precision of 1%.

3.5 The impeller flow

3.5.1 Real flow effects

Figure 26 shows schematically the jet-wake model as used in the program. To find a relation between flow conditions at the separation point and the impeller exit, it is necessary to make the following important approximation, as suggested by Dean (Ref. 1). He asserted that the jet flow is little subjected to shear stresses in the rotor and can be considered as an isentropic core. All flow outside this core can be considered as belonging to the wake. This is even more true when there is a tangential pressure gradient over the jet, due to the rotor work creating secondary flow that moves all low energy fluid to the suction side. For a compressible flow, Dean shows that the mean relative Mach number remains constant along the jet from separation to rotor outlet:

\[ M_{Wj} = \text{cst} \]  \hspace{1cm} (3.35)

or

\[ M_{W2j} = M_{W_{\text{sep}}} \]  \hspace{1cm} (3.36)

The mass flow in the wake is not zero and has been characterized as a percentage of the total mass flow:

\[ \lambda = \frac{\dot{m}_w}{\dot{m}} \]  \hspace{1cm} (3.37)

\( \dot{m}_w \) is developed by secondary flows and by "tip leakage".
The wake can be looked at as a pool, wherein all low energetic fluid is flowing together. All friction- and leakage losses are accumulated in the wake and the corresponding losses will be added to the main flow during the mixing process downstream of the impeller.

In our model, the parameter determining the jet-wake velocity profile in the wake-jet relative velocity ratio $\nu$:

$$\nu = \frac{W_{2w}}{W_{2j}}$$

(3.38)

By fixing $\nu$, instead of $\lambda$, the mass flow in the wake will depend also on the extent of the wake. Obviously, the choice of $\nu$ is not free. It should be correlated to the influencing parameters, secondary flow, clearance, wake width $\varepsilon_2$, etc. However, at the actual state of research, this correlation is unknown. The designer can handle it as a supplementary degree of freedom. Our experience has learnt us to choose the parameter $\nu$ not too large. Based on the results of Eckardt (Ref. 16) and Fowler (Ref. 17), this value can be fixed at $\nu = .2$.

Another important feature of our model is that the same relative outlet angle $\beta_2$ is used for the jet and the wake:

$$\beta_{2j} = \beta_{2w} = \beta_2$$

(3.39)

3.5.2 Flow equations for the jet

Energy equation:

$$c_p(T_{2j} - \hat{T}_{sep}) = \frac{\hat{W}_{sep} - W_{2j}}{2} + \frac{U_2 - \hat{U}_{sep}}{2}$$

(3.40)

$\hat{T}_{sep}$, $\hat{W}_{sep}$, $\hat{U}_{sep}$ are the mass mean values of $T_{sep,i}$, $W_{sep,i}$, $U_{sep,i}$.
Model equation: Constant relative Mach number along the jet flow:

$$M_{W_{2j}} = M_{W_{sep}}$$  \hspace{1cm} (3.41)

Gas equation:

$$\rho_{2j} = \frac{p_{2j}}{R G T_{2j}}$$  \hspace{1cm} (3.42)

Continuity equation: From the definition of the wake to total mass flow ratio $\lambda$, we can deduce that:

$$\dot{m}_j = (1-\lambda)\dot{m} = 2\pi R b_2 k B_2 \cos \beta_{2j} \rho_{2j} (1-\epsilon_2) W_{2j}$$  \hspace{1cm} (3.43)

where $\epsilon_2$ is the procentual passage at the outlet, occupied by the wake. $\lambda$ is not an input data, but will be calculated in function of $v$.

Static pressure $p_{2j}$: For an isentropic jet flow we have:

$$p_{2j} = p_{sep} \left( \frac{T_{2j}}{T_{sep}} \right)^{\frac{k-1}{k}}$$  \hspace{1cm} (3.44)

The static pressure $p_{j}$ is calculated by correction of its isentropic value $p_{2j}^{IS}$ by means of a loss coefficient $\omega_{jet}$:

$$p_{2j} = p_{2j}^{IS} - \frac{1}{2} \omega_{jet} \hat{\rho}_{sep} W_{sep}^2$$  \hspace{1cm} (3.45)

In Dean's theory, the jet flow is assumed isentropic, and all losses are connected with the wake flow. Consequently, $\omega_{jet}$ is zero. However, this excludes the possibility to analyze wall friction losses and clearance losses as a function of rotor geometry.
Therefore, in our model, the jet loss coefficient is a function of the wall friction losses and clearance losses:

\[ \omega_{\text{jet}} = \omega_{\text{fr}} + \omega_{\text{cl}} \]  

(3.46)

The wake flow takes only into account the separation losses.

(a) The friction loss coefficient \( \omega_{\text{fr}} \)

\( \omega_{\text{fr}} \) is estimated by means of hydraulic diameters of and length between the separation section and the rotor outlet:

\[
D_{h_{\text{sep}}} = 4 \frac{\text{Area}}{\text{Contour}_{\text{sep}}} = 4 \frac{\pi (R_{\text{sep},7}^2 - R_{\text{sep},6}^2)}{z \cos \phi}
\]

(3.47)

\[
D_{h_{2j}} = 4 \frac{\text{Area}}{\text{Contour}_{2j}} = 4 \frac{2\pi R_2 b_2}{z (1-\varepsilon_2)}
\]

(3.48)

\[
L_{h_{\text{sep}},2j} \approx \frac{1}{2} \left( D_{\text{ax}} - \frac{b_2}{2} \right) + \frac{R_2 - R_{\text{sep},3}}{\cos \phi} \left( \frac{\pi}{2} - \phi \right) \cos \left( \frac{\beta_{\text{sep},3} + \beta_2 b_1}{2} \right)
\]

(3.49)

\[
\omega_{\text{fr}} = 4c_f L_{h_{\text{sep}},2j} \left[ \frac{1}{2} \left( \frac{1}{D_{h_{\text{sep}}}} + \frac{1}{D_{h_{2j}}} \right) \right]
\]

(3.50)

The Reynolds number in the impeller \( \text{Re}_{\text{imp}} \) is calculated at the separation section:
The wall friction coefficient $c_f$ is a function of the Reynolds number and of the relative wall roughness $k / D_{h_{sep}}$:

$$c_f = c_f\left(Re_{imp}, \frac{k}{D_{h_{sep}}} \right)$$  \hspace{1cm} (3.52)

This relationship is calculated using implicit formula of Colebrook and White for the head loss coefficient $\lambda$:

$$\frac{1}{\sqrt{\lambda}} = -2 \log \left( \frac{k}{3.7D_h Re} \right) - \frac{2.51}{Re}$$  \hspace{1cm} (3.53)

The wall friction coefficient is then

$$c_f = \frac{\lambda}{4}$$  \hspace{1cm} (3.54)

Expression (3.53) can also be written in an explicit form:

$$c_f = \frac{.0625}{\left( \log \left( \frac{k}{3.7D_h Re} \right) - \frac{5}{3.7D_h Re} \log \left( \frac{k}{3.7D_h Re} \right) \right)^2}$$  \hspace{1cm} (3.55)

(b) The clearance loss coefficient $\omega_{cl}$

According to Jansen (Ref. 18), the clearance loss coefficient can be expressed as:

$$\omega_{cl} = 2.43 \frac{\delta_{cl}}{b_2} (1 - \frac{R_1 t}{R_2})^2 \frac{U_2^2}{W_{sep}^2}$$  \hspace{1cm} (3.56)

with $\delta_{cl}$: the impeller blade clearance (m).
3.5.3 Flow_equation_for_the_wake

Energy equation:

\[ c_p(T_{2w} - T_{sep}) = \frac{\hat{W}_{sep}^2 - W_{2w}^2}{2} + \frac{U_{2}^2 - U_{sep}^2}{2} \]  \hspace{1cm} (3.57)

Model equation: The relative wake velocity is determined by the parameter \( \nu \), called the wake)jet relative velocity ratio:

\[ W_{2w} = \nu \cdot W_{2j} \]  \hspace{1cm} (3.58)

Gas equation:

\[ \rho_{2w} = \frac{p_{2w}}{R_G T_{2w}} \]  \hspace{1cm} (3.59)

Continuity equation: The wake mass flow is determined by the parameter \( \lambda \), which has to be calculated in function of \( \nu \):

\[ \dot{m}_w = \lambda \cdot \dot{m} = 2\pi R_2 b_2 k_2 B_2 \cos \beta_2 \rho_{2w} \cdot e_2 W_{2w} \]  \hspace{1cm} (3.60)

Tangential equilibrium: The wake static pressure \( p_{2w} \) cannot be determined by the isentropic equation plus loss coefficient, because this flow is essentially irreversible. It accumulates nearly all the rotor losses. Therefore we deduce the tangential pressure gradient at the rotor outlet from equilibrium equations and derive the mean static wake pressure \( p_{2w} \) from the static pressure in the jet, \( p_{2j} \).

Figure 27 represents the equilibrium of inertial forces for the jet. \( R_C \) is the radius of curvature of the streamlines at the rotor outlet, and is supposed to be constant over the whole section. Starting from

\( p_{2,pr} \) : the static pressure at the pressure side of the blade
\( p_{2,suc} \) : the static pressure at the suction side of the blade
\( p_{2,sh} \) : the static pressure at the shear layer between jet and wake
we define

\[ \frac{p_{2j}}{2} = \frac{p_{2,pr} + p_{2,sh}}{2} \quad (3.61) \]

\[ p_{2w} = \frac{p_{2,sh} + p_{2,suc}}{2} \quad (3.62) \]

The inertial forces in the jet, which are acting upon a flow particle with mass \( dm \), are:

a) the coriolis forces:

\[ F_{\text{cor,}j} = 2dm \Omega W_{2j} \quad (3.63) \]

b) the centrifugal forces:

\[ F_{\text{centr,}j} = dm \Omega^2 R_j \quad (3.64) \]

c) the normal component of the relative flow inertial force:

\[ F_{\text{rel,n,}j} = dm \frac{W_{2j}^2}{R_{c2}} \quad (3.65) \]

d) the tangential component of the relative flow inertial force:

\[ F_{\text{rel,t,}j} = dm \frac{\partial W_{2j}}{\partial \tau} \approx 0 \quad (\tau = \text{time}) \quad (3.66) \]

All these forces are balanced by a force \( F \), corresponding to a tangential pressure gradient \( \partial p_j / \partial u \). By expressing the tangential equilibrium of forces

\[ F + F_{\text{cor,}j,u} + F_{\text{rel,n,}j,u} + F_{\text{centr,}j,u} + F_{\text{rel,t,}j,u} = 0 \quad (3.67) \]

we obtain, after some calculations:
\[
\frac{\partial p}{\partial u}_{2j} = p_{2j} \cos \beta_{2j} \left(2\pi W_{2j} - \frac{W_{2j}}{R_{C2}}\right)
\]  
(3.68)

\(R_{C2}\) is positive when opposite to the direction of rotation.

For the wake, we have similarly:

\[
\frac{\partial p}{\partial u}_{2w} = p_{2w} \cos \beta_{2w} \left(2\pi W_{2w} - \frac{W_{2w}}{R_{C2}}\right)
\]  
(3.69)

If \(p_{2j}\) is known, we can deduce \(p_{2w}, p_{2pr}, p_{2sh}, p_{2suc}\):

\[
p_{2w} = p_{2j} - \frac{1}{2} \frac{2\pi R_{2}(1-\varepsilon_{2})}{z} \left(\frac{\partial p}{\partial u}\right)_{j} - \frac{1}{2} \frac{2\pi R_{2}\varepsilon_{2}}{z} \left(\frac{\partial p}{\partial u}\right)_{w}
\]  
(3.70)

\[
p_{2pr} = p_{2j} + \frac{1}{2} \frac{2\pi R_{2}(1-\varepsilon_{2})}{z} \left(\frac{\partial p}{\partial u}\right)_{j}
\]  
(3.71)

\[
p_{2sh} = p_{2j} - \frac{1}{2} \frac{2\pi R_{2}(1-\varepsilon_{2})}{z} \left(\frac{\partial p}{\partial u}\right)_{j} = p_{2w} + \frac{1}{2} \frac{2\pi R_{2}\varepsilon_{2}}{z} \left(\frac{\partial p}{\partial u}\right)_{w}
\]  
(3.72)

\[
p_{2suc} = p_{2w} - \frac{1}{2} \frac{2\pi R_{2}\varepsilon_{2}}{z} \left(\frac{\partial p}{\partial u}\right)_{w}
\]  
(3.73)

The problem with this method is the radius of curvature \(R_{C2}\), which is a function of slip factor and blade geometry at the rotor outlet. As this is beyond the scope of this program, we introduce the value of \(R_{C2}\) as an input data.

### 3.5.4 The energetical value of jet and wake flow

The jet flow is not subject to any noticeable diffusion. The increase of static pressure is almost completely due to the effect of centrifugal forces. This is also true for the
static temperature increase. Equation (3.40) can be written as

\[ T_{2j} = \hat{T}_{\text{sep}} + \frac{\dot{W}_{\text{sep}}^2 - W_{2j}^2}{2c_p} + \frac{U_2 - U_{\text{sep}}}{2c_p} \approx \hat{T}_{\text{sep}} + \frac{U_2 - U_{\text{sep}}}{2c_p} \]  

(3.74)

In the wake, on the contrary, the static temperature increase is considerable, because it is not only dependent on the effect of centrifugal forces, \( \frac{U_2 - U_{\text{sep}}}{2c_p} \), but also on the term \( \frac{W_{\text{sep}}^2 - W_w^2}{2c_p} \) (cfr eq. 3.57). By making the approximation:

\[ W_{2w} = \nu W_{2j} \approx \nu W_{\text{sep}} \]  

(3.75)

equation 3.57 can be written as

\[ T_{2w} - T_{2j} \approx (1 - \nu^2) \frac{\dot{W}_{\text{sep}}^2}{2c_p} \]  

(3.76)

It shows that there will always be a positive temperature difference between jet and wake.

On the other hand, the static pressure in the wake will be lower than in the jet. By recombining eqs. 3.43, 3.60, 3.61, 3.69, 3.70, one obtains, for a sufficiently high radius of curvature \( R_{C2} \):

\[ p_{2j} - p_{2w} \approx \frac{\Omega \dot{m}}{zb_2} \]  

(3.77)

with \( \Omega \) : the angular velocity (rad/s).

Using the parameter \( \lambda \), we can define the mean static values at the impeller outlet (subscript 2mn)

\[ T_{2mn} = (1 - \lambda) T_{2j} + \lambda T_{2w} \]  

(3.78)
\[ \rho_{2\text{mn}} = (1-\lambda) \rho_{2j} + \lambda \rho_{2W} \approx \rho_{2j} \quad (3.79) \]

\[ p_{2\text{mn}} = R_G T_{2\text{mn}} \rho_{2\text{mn}} \approx (1-\lambda) p_{2j} + \lambda p_{2W} \quad (3.80) \]

Assuming an isentropic jet flow, we can use eqs. 3.76, 3.77, 3.78, 3.80 to draw a T,S diagram of the impeller flow (Fig. 28).

From this figure we can see that the wake flow suffers an important entropy increase, which results in heavy losses for the mean flow.

3.5.5 Influence of the parameter \( \nu \) on impeller performance

From equation (3.76) it is evident that the impeller separation losses are strongly dependent on the parameter \( \nu \), which determines the relative velocity profile at the impeller outlet. However, \( \nu \) is also influencing the wake width \( \varepsilon_2 \) and the wake mass flow ratio \( \lambda \). In order to investigate the influence of \( \nu \) on impeller performances, we first look how \( \varepsilon_2 \) and \( \lambda \) are varying with \( \nu \). From the continuity equations (3.43) and (3.60) we can derive that

\[ \varepsilon_2 \approx \frac{1-C}{1-\nu} \quad (3.81) \]

with

\[ C = \frac{\dot{m}}{2\pi R_2 b_2 \cos \beta_2 \rho_{2j} W_{2j}} \approx \frac{W_{2\text{hom}}}{W_{\text{sep}}}; \quad (3.82) \]

\( W_{2\text{hom}} \) corresponds to a homogeneous impeller outlet velocity; \( C \) is a constant which is inversely proportional to the amount of separated flow:

- \( 0 < C < 1 \) for separated flow
- \( C \rightarrow 1 \) for unseparated flow.
From a physical point of view, we know that the wake width $\varepsilon_2$ must be lower than 1, so that eq. 3.81 limits the choice for $v$ to be lower than the limiting value $C$:

$$v \leq C$$

(3.83)

From eq. 3.43 we find an expression for $\lambda$:

$$\lambda = \frac{v}{1-v} \frac{1-C}{C}$$

(3.84)

Equations (3.81) and (3.84) are represented in figure 29.

We now derive expressions for the static to static efficiency of the jet, the wake and the mean flow (Fig. 28).

a) Jet flow:

$$\eta_{jet} \approx 1$$

(neglecting friction and clearance losses)

b) Wake flow

$$\eta_{wake} = \frac{s,s}{s,s} \frac{T_{2W} - T_{sep}}{T_{2W} - T_{sep}}$$

$$= T_{sep} \frac{\frac{K-1}{K}}{\frac{p_{2w}}{p_{sep}}} - 1$$

$$= T_{sep} \frac{W_{sep}^2}{(1-v^2) \frac{c_p}{2c_p} + T_{2j} - T_{sep}}$$

$$\Rightarrow \eta_{wake} \approx \frac{A}{B+(1-v^2)}$$

$A,B$ : independent on $v$

(3.85)

c) Mean flow:

$$\eta_{mean} = \frac{s,s}{s,s} \frac{T_{2mn} - T_{sep}}{T_{2mn} - T_{sep}} = \frac{T_{sep}}{T_{2mn} - T_{sep}} \left[ \frac{K-1}{K} \right]$$

$$- 1$$
By introducing (3.84) into (3.86) we obtain:

\[
\eta_{\text{mean}} \sim \frac{A'}{B' + \lambda(1 - \nu^2)} \quad A', B' : \text{independent of } \nu \quad (3.86)
\]

\[
\eta_{\text{mean}} \sim \frac{A''}{B'' + \nu(1 + \nu)} \quad A'', B'' : \text{independent of } \nu \quad (3.87)
\]

Figure 30 represents equations (3.85), (3.86), (3.87) for a particular case with \(C = .7\). It is worth mentioning that these efficiencies are valid for the rotor only and do not include the mixing losses.

From figure 30 we see that when \(\nu\) is increasing the wake flow efficiency is increasing. This is due to the fact that the temperature difference between jet and wake becomes smaller (cfr eq. 3.76). In spite of this, the mean flow efficiency \(\eta_{\text{mean}}^{SS}\) is decreasing. The reason for this is that the mass flow through the wake \(\lambda = \dot{m}_w/\dot{m}\) is strongly increasing with \(\nu\) (cfr Fig. 29) which means that the wake flow becomes more important compared with the jet flow. We see that in eq. (3.86) \(\lambda\) appears in a positive term of the denominator. This explains the shape of the \(\eta_{\text{mean}}^{SS}\) curve. The mean flow efficiency is thus decreasing because the wake mass flow is increasing faster with \(\nu\) than the wake efficiency. It is clear from figure 30 that the parameter \(\nu\) has an important influence on the compressor characteristics. The choice is judicious, but we suggest to start calculations with small values for \(\nu\) (f.i., \(\nu = .2\)), according to the measurements of Eckardt (Ref. 16).

3.5.6 Influence_of_the_impeller_outlet_width_b

When the impeller outlet width \(b\) is decreased for constant value of \(\nu\), the wake width will become smaller and smaller until the outlet section is completely occupied by the jet flow, and consequently there are no wake flow losses.
On the other hand, the throughflow section of the jet becomes more and more rectangular, and so the hydraulic diameter \( D_{h_2} \) will decrease. We see from eq 3.50 that \( \omega_{fr} \) will increase considerably due to wall friction.

Equation (3.56) shows that also the clearance losses will decrease when the impeller width is increased. Friction losses, separation losses and clearance losses are shown on figure 31 in a qualitative way. Figure 31 makes clear that there exists an optimum value for \( b_2/R_2 \) where the total losses are minimum.

Remark: to take into account normal boundary layer blockage when the flow is unseparated, a minimum wake width \( \varepsilon_2_{\text{min}} \) can be imposed. When \( \varepsilon_2 \) becomes less than \( \varepsilon_2_{\text{min}} \), the model equation for the wake (3.58) cannot be sustained anymore and must be replaced by:

\[
\varepsilon_2 = \varepsilon_2_{\text{min}} \tag{3.88}
\]

3.6 The impeller outlet tip

3.6.1 The slip factor \( \mu \) for a jet and wake configuration

Due to the limited number of blades, the relative flow at rotor outlet will not be tangent to the blade profile. This has no direct influence on efficiency, but on the rotor work output:

\[
\Delta H = U_2 V_{u_2} - U_1 V_{u_1} \tag{3.89}
\]

The ratio

\[
\mu = \frac{V_{u_2}}{V_{u_2}} \tag{3.90}
\]

is called slip factor.
\( V_{u_2} \) = real tangential velocity at rotor exit

\( V_{u_2}^\infty \) = tangential velocity at rotor exit for infinite blade number

A broad review on the values of the slip factor \( \mu \) has been performed by Wiesner (Ref. 19).

For one dimensional calculations we propose the formula of Eck:

\[
\mu = \frac{1}{1 + \frac{\pi}{2} \cos \beta_2 b_1 \frac{R_2}{R_2 + R_1}} \left( 1 - \sqrt{\frac{R_2^2}{2 R_1^2}} \right) (3.91)
\]

The formula of Buseman is better suited for a first guess, when the rotor outlet radius is not yet known:

\[
\mu = \frac{1}{z \cdot \sqrt{\beta_2 b_1}} (3.92)
\]

To determine the slip factor of the jet \( \mu_j \) and of the wake \( \mu_w \) separately, we consider the slip factor \( \mu \) for a uniform flow as being the mass mean value of the jet wake flow:

\[
\mu = \lambda \mu_w + (1-\lambda) \mu_j (3.93)
\]

3.6.2 Equations for the impeller outlet flow

The calculation of velocity triangles at the impeller outlet is based on the assumption that the jet and wake flow have the same flow direction in the relative motion as stated in eq. (3.39):

\( \beta_2 j = \beta_2 w = \beta_2 \)

Based on the velocity triangles (Fig. 32), the absolute velocities in jet and wake are given by:
The relation between the relative flow angle $\beta$ and the blade angle $\beta_{bl}$ is deduced from figure 32, and is given as a function of the slip factor $\mu$ by the following equations, for jet and wake respectively:

$$
\beta_{2j} = \beta_{2bl} + \arctg \left( \frac{\tan \alpha_{2j} \cos^2 \beta_{2bl}}{\tan \alpha_{2j}} \frac{1}{\mu_j} \right)
$$

(3.96)

$$
\beta_{2w} = \beta_{2bl} + \arctg \left( \frac{\tan \alpha_{2w} \cos^2 \beta_{2bl}}{\tan \alpha_{2w} \tan \alpha_{2j}} \frac{1}{\mu_w} \right)
$$

(3.97)

A relation between the slip factor $\mu_j$ and $\mu_w$ can be found in function of the absolute flow angles, by combining (3.96), (3.97) and (3.39):

$$
\tan \alpha_{2j} \left( \frac{1}{\mu_j} - 1 \right) = \tan \alpha_{2w} \left( \frac{1}{\mu_w} - 1 \right)
$$

(3.98)

or

$$
\mu_w = \frac{\tan \alpha_{2w}}{\tan \alpha_{2j} \left( \frac{1}{\mu_j} + (\tan \alpha_{2w} - \tan \alpha_{2j}) \frac{1}{\mu_j} \right)}
$$

(3.99)

The absolute flow angles of jet and wake are related to the relative flow angles by (Fig. 32)

$$
\tan \alpha_{2w} = \frac{U_{2j} + W_{2w} \sin \beta_w}{W_{2w} \cos \beta_w}
$$

(3.100)
\[ \tan \alpha_2 = \frac{U_2 + W_2 \sin \beta_2}{W_2 \cos \beta_2} \]  
\[ (3.101) \]

or after combination of (3.100) and (3.101):

\[ \tan \alpha_2 \frac{W_2}{W_{2W}} (\tan \alpha_2 - \tan \beta_2) + \tan \beta_2 \]  
\[ (3.102) \]

Replacing \( U_2 \) by \( 2 \pi R_2 \text{RPM}/60 \) in eq. (3.101), we obtain:

\[ R_2 = \frac{W_2}{2 \pi \text{RPM}} (\tan \alpha_2 \cos \beta_2 - \sin \beta_2) \]  
\[ (3.103) \]

All equations (3.93) - (3.102) can be used directly or by iteration to determine the flow characteristics at the impeller outlet: \( \mu, \mu_j, \mu_w, V_2 j, V_2 w, \beta_2, \alpha_2 j, \alpha_2 w \).

Equation (3.103) is used to evaluate the impeller radius \( R_2 \).

3.6.3 The impeller work equation

In order to calculate the total enthalpy rise, the Euler equation has to be applied to jet and wake:

\[ \Delta H_0 = c_p \Delta T_0 = (1-\lambda) U_2 \mu_j V_{2j} + \lambda U_2 \mu_w V_{2w} - \frac{1}{\dot{m}} \int_{R_1 h}^{R_1 t} \dot{m}(R) U_1(R) V_1(R) dR \]

The integral term, however, drops out in case of an axial flow at the rotor inlet.

After some transformations, this equation can be written as:

\[ \frac{\Delta T_0}{T_0} = (K-1) \frac{U_2}{a_0} \left[ (1-\lambda) \mu_j \left( \frac{U_2}{a_0} + \frac{W_2 j}{a_0} \sin \beta_2 b_1 \right) + \lambda \mu_w \left( \frac{U_2}{a_0} + \frac{W_2 w}{a_0} \sin \beta_2 b_1 \right) \right] \]

\[ - \frac{K-1}{\dot{m}} \int_{R_1 h}^{R_1 t} \dot{m}(R) \left( \frac{U_1(R)}{a_0} \frac{V_1(R)}{a_0} \sin \alpha_1(R) \right) dR \]
\[ (3.104) \]
which is the general expression of the impeller work equation.

### 3.7 Disk friction

At the back side of the rotor disc there is a stationary wall very close to it. The fluid between these two walls is rotating with the rotor disc at one side, but stationary at the fixed wall side. This fluid rotation produces intensive whirl and energy dissipation. The extra torque due to this friction is given by:

\[
M = 2 \int_{0}^{R_2} 2\pi R^2 \tau_w dR \quad \tau_w = \text{wall shear stress} \quad (3.105)
\]

We define the disc friction coefficient as:

\[
c_m = \frac{2M}{\rho \Omega^2 R_2^5} \quad (3.106)
\]

The rate of dissipated energy is:

\[
E = M \cdot \Omega = \frac{1}{2} \rho \Omega^3 R_2^5 c_m \quad (3.107)
\]

The value of \( c_m \) is dependent on the type of flow between the two walls. Four flow regimes can be discerned:

- I laminar with separated boundary layers
- II laminar with interfering boundary layers
- III turbulent with separated boundary layers
- IV turbulent with interfering boundary layers

The extent of those regimes is varying with:
- the axial gap \( s \) between rotor and wall
- the Reynolds number based on the disc radius and the circumferential velocity

\[
Re = \frac{\Omega R_2^2}{v} \quad (3.108)
\]
The value of $c_m$ is given in table 1 (Refs. 20, 21) as a function of axial gap ($s/R_2$) and the limiting Reynolds numbers for the four flow regimes.

The energy dissipation is going to heat the flow in the impeller, changing the outlet flow temperature and then affecting the rotor performances.

The flow temperature at the rotor discharge will be:

$$\Delta T_{df} = \frac{E}{c_p m}$$

(3.109)

The optimal value of $s/R_2$ can be chosen from figure 33. We see that $s/R_2 = .05$ is a good value, because it results in a small $c_m$, even for $Re = 10^5$. 
CHAPTER 4 - THE MIXING PROCESS

4.1 Introduction

In section 3 we explained how the flow between the blades of a radial impeller gives rise to a jet wake configuration. At the discharge of the impeller, an intensive energy exchange takes place between the two subflows with different angular momentum (Fig. 32), resulting in a transfer of the impeller separation losses from the wake to the nearly isentropic jet. The mixing process we describe here applies only to separated flows in rotating systems. The behaviour of stationary separated boundary layers, as generated in stationary cascades, is completely different. It is also worth to notice that the denomination "mixing process" does not refer to a "mass exchange" between jet and wake, which is nearly negligible, but rather to the "energy exchange" together with the uniformization of the flow.

4.2 Theoretical computation of the mixing zone of a jet-wake flow, taking into account the compressibility of the fluid

This theory is an extension of the theory of Dean and Senoo for incompressible fluids (Ref. 22) and of the study by Bex (Ref. 23).

4.2.1 Assumptions

a. Identical relative flow angle \( \beta \) for jet and wake;
b. The relative velocity distribution at the impeller outlet is rectangular as shown in figure 26;
c. The development of boundary layer blockage along the walls is neglected;
d. The blade blockage at the impeller discharge is not taken into consideration;
e. The static quantities are identical for jet and wake.

Their initial value is equal to the mass mean value at the impeller outlet (Fig. 34). Consequently, the mixing process
does only refer to dynamic quantities. This assumption results in a sudden total pressure increase for the wake and decrease for the jet, while the total temperature is decreasing for the wake and increasing for the jet.

4.2.2 Forces acting upon the jet and the wake

4.2.2.1 Wall friction

The friction force of the jet against the wall is proportional to the square of the absolute velocity

\[ \tau_j = \frac{1}{2} c_f \rho V_j^2 \quad \text{with} \quad c_f = c_f(Re, \frac{k}{D_h}) \quad (4.1) \]

for the wake:

\[ \tau_w = \frac{1}{2} c_f \rho V_w^2 \quad \text{with} \quad c_f = c_f(Re, \frac{k}{D_h}) \quad (4.2) \]

4.2.2.2 Shear stresses

They are acting upon the shear layer between jet and wake and are parallel to the relative velocity direction and proportional to the square of the difference between the relative velocities of jet and wake:

\[ \tau_M = \frac{1}{2} c_M \rho (W_j - W_w)^2 \quad \text{with} \quad c_M : .094 \quad (4.3) \]

4.2.2.3 Pressure forces

They are also acting upon the jet wake shear layer, but in a direction perpendicular to it. They are generated by variation of the angular momentum of jet and wake. This can be considered as an energy transfer between jet and wake.

In fact, this energy exchange is performed by a pressure force between jet and wake, which is rotating with the rotor and gives rise to a fluctuating pressure in the mixing zone. Due
to this, the wake flow will disappear very soon, in contrast with stationary separated boundary layers. Figure 35 is representing this pressure fluctuation $q$.

4.2.3 Equations for the mixing process

4.2.3.1 Continuity equation for the jet

$$2\pi R b(1-\varepsilon)W_j \cos\beta = 2\pi R_2 b_2 \rho_2 (1-\varepsilon_2)W_2 \cos\beta_2$$

(4.4)

This equation can be differentiated with respect to the radius $R$ to obtain:

$$\frac{1}{R} + \frac{1}{R} \frac{\partial b}{\partial R} + \frac{1}{\rho} \frac{\partial \rho}{\partial R} - \frac{1}{1-\varepsilon} \frac{\partial \varepsilon}{\partial R} + \frac{1}{W_j} \frac{\partial W_j}{\partial R} - \tan \beta \frac{\partial \beta}{\partial R} = 0$$

(4.5)

4.2.3.2 Continuity equation for the wake

$$2\pi R b \varepsilon W_w \cos\beta = 2\pi R_2 b_2 \rho_2 \varepsilon_2 W_2 \cos\beta_2$$

(4.6)

or, after differentiation:

$$\frac{1}{R} + \frac{1}{R} \frac{\partial b}{\partial R} + \frac{1}{\rho} \frac{\partial \rho}{\partial R} + \frac{1}{\varepsilon} \frac{\partial \varepsilon}{\partial R} + \frac{1}{W_w} \frac{\partial W_w}{\partial R} - \tan \beta \frac{\partial \beta}{\partial R} = 0$$

(4.7)

Remark: the term $\frac{\partial b}{\partial R}$ takes into account a variation of the width $b$ of the vaneless space at the impeller discharge.

4.2.3.3 Angular momentum

$$2\pi R b(1-\varepsilon)W_j \cos\beta \frac{d}{dR} (R^2 \Omega + RW_j \sin\beta) = -c_f 2\pi R^2 (1-\varepsilon)V_j (R\Omega + W_j \sin\beta)$$

$$-c_M z b R (W_j - W_w)^2 \tan \beta - z b R \frac{q}{\rho}$$

(4.8)

The left hand term of this equation represents the variation of angular momentum of the jet between the radii $R$ and $R+\partial R$. The first right hand side term gives the influence of the wall friction, the second of the shear stresses between jet and wake, and the third one of the tangential components of the pressure forces.
For the wake we can write:

\[ 2\pi R \varepsilon W_w \cos \beta \frac{d}{dR} (R^2 \Omega + RW_w \sin \beta) = -c_f 2\pi R^2 \varepsilon V_w (R\Omega + W_w \sin \beta) \]

\[ + c_M z b R (W_j - W_w)^2 \tan \beta + z b R q \rho \]

(4.9)

4.2.3.4 Forces in radial direction

For the jet we have:

\[ 2\pi R (1-\varepsilon) W_j \cos \beta \frac{d}{dR} (W_j \cos \beta) + 2\pi R (1-\varepsilon) \frac{1}{\rho} \frac{dp}{dR} - 2\pi (1-\varepsilon) (R\Omega + W_j \sin \beta)^2 \]

\[ + 2\pi R (1-\varepsilon) \frac{c_f}{b} V_j W_j \cos \beta - z q \frac{\tan \beta}{\rho} + c_M z (W_j - W_w)^2 = 0 \]

(4.10)

The first term represents the increase of radial momentum between \( R \) and \( R + dR \). The second one represents the radial pressure gradient, the third one the centrifugal and coriolis forces and the fourth one the wall friction. The last two terms represent respectively the influence of the tangential pressure forces and the shear stresses between jet and wake.

For the wake this equation becomes:

\[ 2\pi R \varepsilon W_w \cos \beta \frac{d}{dR} (W_w \cos \beta) + 2\pi R \varepsilon \frac{1}{\rho} \frac{dp}{dR} - 2\pi \varepsilon (R\Omega + W_w \sin \beta)^2 \]

\[ + 2\pi \varepsilon c_f V_w W_w \frac{\cos \beta}{b} + z q \frac{\tan \beta}{\rho} - c_M z (W_j - W_w)^2 = 0 \]

(4.11)

4.2.3.5 Equation of state

\[ p = R G \rho T \]

becomes, after differentiation with respect to \( R \):

\[ \frac{1}{\rho} \frac{dp}{dR} = \frac{1}{p} \frac{dp}{dR} - \frac{1}{T} \frac{dT}{dR} \]

(4.12)
4.2.3.6 Energy equation

The total enthalpy of the jet-wake flow remains constant:

\[(1-\lambda)(c_p T_j + \frac{V_j^2}{2}) + \lambda (c_p T_w + \frac{V_w^2}{2}) = \text{cst.}\]

Defining a mean flow temperature \(T\) as

\[T = (1-\lambda) T_j + \lambda T_w\]

the energy equation becomes:

\[T + (1-\lambda) \frac{V_j^2}{2c_p} + \lambda \frac{V_w^2}{2c_p} = \text{cst}\]

By differentiation with respect to \(R\), we obtain:

\[\frac{dT}{dR} + (1-\lambda) \frac{dV_j}{c_p} + \lambda \frac{dV_w}{c_p} = 0\] \hspace{1cm} (4.13)

4.2.3.7 Geometrical relations

From the velocity triangles we know that

\[V_j^2 = W_j^2 + U^2 + 2W_j U \sin \beta.\]

By differentiation we find after some calculations for the jet:

\[V_j \frac{dV_j}{dR} = (W_j + \Omega R \sin \beta) \frac{dW_j}{dR} + \Omega RW_j \cos \beta \frac{d\beta}{dR} + \Omega (\Omega R + W_j \sin \beta)\] \hspace{1cm} (4.14)

and for the wake:

\[V_w \frac{dV_w}{dR} = (W_w + \Omega R \sin \beta) \frac{dW_w}{dR} + \Omega RW_w \cos \beta \frac{d\beta}{dR} + \Omega (\Omega R + W_w \sin \beta)\] \hspace{1cm} (4.15)
4.2.4 Solution of the equations for the mixing process

A solution method for the equations governing the mixing process is presented in Appendix A.

4.3 Results of the theoretical computations

Figures 36a,b,c,d show a result of a mixing process calculation. The impeller discharge wake width is $\varepsilon_2 = 0.71$ for a wake mass flow of 17.5%. From figure 36d we see that in the first part of the mixing process $1 < \frac{R}{R_2} < \left(\frac{R}{R_2}\right)_M$ the total temperature of the wake $T_w^t$ is increasing at the cost of the total temperature of the jet $T_j^t$. We also see from figure 36a,b that the absolute velocity $V_w^t$ and the absolute flow angle $\alpha_w^t$ of the wake are increasing and so does the angular momentum. This means that for values of $R/R_2$ less than $(R/R_2)_M$, there is an energy transfer from the jet to the wake. During this period, the wake width $\varepsilon$ remains nearly constant. The relative velocity of the jet $W_j$ is slightly decreasing due to shear friction with the wake.

At $R/R_2 = (R/R_2)_M$ a sudden breakdown of the wake flow takes place, reducing $\varepsilon$ very fastly to its final value $\lambda$. The total temperature $T_w^t$ is decreasing and $T_j^t$ increasing. This means that the direction of the energy transfer is reversed: the jet flow is at this time energized by the wake. We see from figures 36a,b that the angular momentum of the jet is increasing at the cost of the angular momentum of the wake. Figure 36c shows that the total pressure of the jet is not increasing. This is due to the fact that the energy, which is transferred from the wake to the jet, is almost completely balanced by an entropy increase due to wall friction.

At $R/R_2 = (R/R_2)_M$, the wake width is reduced to 10% of its final value. The mixing process will progress now very slowly to a complete uniformization. The computation is stopped when the difference of relative velocities becomes less than 5%.
The non-uniformity of the flow at the impeller outlet gives rise to supplementary losses, which usually are called "mixing losses". This denomination suggests that these losses are due to shear stresses between jet and wake - the shear friction coefficient $c_M$ is about 20 times larger than the wall friction coefficient $c_f$. Nevertheless, calculations have shown that the influence of shear friction losses on the mixing losses is almost negligible compared with the wall friction losses. The reason is that the surface, where wall friction applies to, is much larger as well as the velocity difference:

$$\frac{V_{j,w}^2}{2} c_f R \Delta R > c_M (W_j - W_w)^2 b \Delta R$$

(4.16)

with $c_f \approx 0.005$

$$c_M \approx 0.094.$$  

The real origin of the "mixing losses" is connected with the increased friction losses with the wall, due to the non-uniformity of the flow.

Van den Braembussche (Ref. 24) suggests the following approximation:

$$\Delta p_{\text{jet-wake}}^0 = \Delta p_{\text{uniform}}^0 + \frac{1}{4} \rho \left[ (1-\lambda)V_{j}^2 + \lambda V_{w}^2 - \dot{V}_2^2 \right]$$

$\dot{V}_2$ is introduced to take into account the progressive uniformization of jet and wake.
CHAPTER 5 - VANELESS DIFFUSERS

5.1 Application field

Vaneless diffusers are normally used in low pressure ratio centrifugal compressors with subsonic impeller outlet flow. A brief review of the different advantages and lacks of this kind of diffusers makes this clear.

5.1.1 Advantages

Vaneless diffusers are suited for off-design operation, because they are compatible with a wide range of absolute in-flow angles $\alpha_2$. In the case of vaned diffusers, positive incidence gives rise to "rotating stall" and "surge", while negative incidence leads to "choke". Vaneless diffusers can only choke in the very improbable case that the radial component of the Mach number exceeds unity (cfr Vavra, Ref. 13).

The jet wake mixing process can be achieved without any blade perturbation. For vaned diffusers, a vaneless space has to be inserted between impeller outlet and diffuser leading edge, to allow sufficient flow uniformization.

5.1.2 Disadvantages

"Rotating stall" can also occur in vaneless diffusers. (Refs. 25, 26, 27). Due to the influence of the adverse pressure gradient, the wall boundary layers are deflected in a more tangential direction than the main flow. When the mass flow is sufficiently reduced, it can happen that the radial velocity component in the boundary layer becomes negative at some place along the diffuser wall (Fig. 37). According to Senoo et al. (Ref. 30), this incident is the trigger for the development of a circumferentially periodic stall pattern.
In vaneless diffusers, the streamlines are nearly tangential to the isobars (concentric circles), while in vaned diffusers the isobars are rearranged to become perpendicular to the streamlines. In case of a vaneless diffuser, the whole pressure gradient is applied to the radial velocity component, which has already suffered a diffusion in the impeller, and has a small kinetic energy. This limits the allowable pressure recovery.

In practice, this balance results in a choice between a high pressure ratio device with a reduced flow range, or a low pressure ratio one with a wide operating range.

5.2 Computation method

Several one dimensional computation methods are available in the literature: Stanitz (Ref. 28), Bex (Ref. 23), Bex & Sternotte (Ref. 29). These methods are non-isentropic, but do not take into account the real boundary layer blockage development along the walls.

In our program, vaneless diffusers are calculated by means of an extended mixing process (cfr 4), until the mean flow reaches a given Mach number. This method applies also to diffusers with non parallel walls.
CHAPTER 6 - VANED ISLAND DIFFUSERS

6. General geometry

Figure 38 gives the general geometry of a vaned diffuser. Three different flow regions can be recognized:
- the vaneless space: between impeller outlet and diffuser leading edge;
- the semi-vaneless space: between diffuser leading edge and throat section;
- the divergent channel: from throat section to outlet.

Usually, the shape of the suction side is a logarithmic spiral, which is tangent to the streamlines. A detailed investigation of the complicated flow problems in this region, such as shock wave boundary layer interaction, and the optimal geometry to adopt has been performed by G. Verdonk (Ref. 31) at VKI.

The divergent channel can be represented schematically as shown in figure 39. The channel sections are rectangular, and the side-walls are parallel. Due to the 'blunt trailing edge' aspect of this diffuser type, we see that the diffusion top angle $\theta_0$ and the number of vanes can be chosen independently. Usually between 6 and 20 vanes are used, and the length to throat width ratio is between 7 and 18.

The diffuser width $b$ has an important influence on the diffuser flow range. According to Stiefel (Ref. 32), narrow diffusers have a larger flow range at higher RPM than nominal, while wide diffusers behave better at lower RPM.

6.2 Computation method

This computation method is completely based on reference 33, which is reproduced here in a synoptical version. The calculation is plotted up into three distinct flow regions, as mentioned in § 6.1.
In an older concept, the leading edge of the vanes had to be removed radially, until the flow reaches subsonic conditions. This hypothesis cannot be sustained for high pressure ratio compressors, due to the increase of the engine geometrical proportions. At present, the usual location of the leading edge is at a radial distance of 5 to 10% of the impeller outlet tip. As can be seen from figure 36, the natural mixing process is not necessarily achieved there, and an abrupt mixing takes place due to diffuser leading edge interference with the jet wake flow, inducing supplementary losses.

It would be complicated to translate this interference phenomenon into a suitable mathematical model. In our program, following method has been adopted (Fig. 40):

a) the mixing process is sustained until the flow is nearly uniform. This happens at a radius \( R_{MO} \) (mixed out), which usually is higher than the leading edge radius \( R_{LE} \) (input datum). This mixing process involves two kinds of losses:

- mixing losses
- wall friction losses.

The wall friction losses are too high, due to the imaginary diffusion process between \( R_{LE} \) and \( R_{MO} \).

b) therefore, an inverse computation is performed, to subtract the supplementary wall friction losses and to recalculate the uniform flow conditions at the diffuser leading edge. This is realized using the one dimensional vaneless diffuser equations of Stanitz (Ref. 28).

6.2.2 Semi-vaneless space

In the semi vaneless space, two different flow processes take place:

a) a rapid adjustment of the isobars from nearly parallel to perpendicular to the flow direction. Dean (Ref. 34) suggests that this happens by means of a system of weak oblique shock waves.
b) a normal shock wave, just ahead of the throat, which creates subsonic throat conditions. This is a design requirement to assure a sufficient mass flow range.

Rundstadler (Ref. 35) and Kenny (Ref. 36) have pointed out from their experimental investigations that the boundary layer blockage in the throat section B is the most important parameter for determining the diffuser channel behavior. Therefore it is primordial to find a good approximation of this blockage. The shape of the suction side, the number of vanes and the incidence are of second order importance. In our program, we use the experimental correlation of Kenny (Fig. 41) to calculate the throat obstruction B as a function of only one parameter: the static pressure recovery between leading edge and throat section: \( \Delta p/q_{LE} \)

Note: if the leading edge conditions are subsonic, it is obvious that the rapid adjustment is achieved without shock waves. In our model, we state that the throat conditions are equal to those at the leading edge for subsonic flow.

6.2.3 The Divergent Channel

6.2.3.1 Pressure Recovery

For theoretical prediction of straight channel diffuser performances, we refer to Vavra (Ref. 37), Traupel (Ref. 38) or Huo (Ref. 39). In our model, we use the experimental correlations of Dean & Rundstadler (Ref. 35) which are based on a large amount of data for different values of \( L/W \), \( 2\theta \), AS, \( M_{throat} \), B. Figure 42 gives an example of data reduction.

Further data reduction in function of AS, reveal that for AS = 1, the pressure recovery \( c_p \) will be optimal (Fig. 43). For this reason, in our program AS is set equal to 1.

For design purposes, figure 42 has been recalculated to obtain \( c_p \) in function of B, \( M_t \) and AR (Table 2). For given values of \( M_t \) and AR, and a from figure 41 derived value of B, this table is suitable to calculate the static pressure recovery \( c_p \) of the diffuser. The static pressure \( p_s \) can be calculated as:
However, to find the optimal geometry \((2\theta, L/W)\) which performs that pressure recovery, the user of the program has to go back to the original graphs of reference 35.

### 6.2.3.2 Channel losses

The Reynolds number in the diffuser channel is defined as:

\[
Re_u = \frac{V_u D_{h_4}}{v_u}
\]  

The hydraulic diameter of the throat \(D_{h_4}\) can be taken equal to the diffuser width \(b_2\), since the aspect ratio \(AS\) has been chosen equal to 1.

The wall friction coefficient \(c_f\) is a function of the Reynolds number and the relative wall roughness \(k/D_{h_4}\):

\[
c_f = c_f(Re_u, k/D_{h_4})
\]  

This relation is calculated in the same way as for the impeller wall friction (cfr § 3.5.2), using an explicit form of the implicit formula of Colebrook & White (eq. 3.55).

The calculation of the channel efficiency is based on Traupel (Ref. 40):

\[
n_{\text{diff}} = \frac{1}{1+1.2c_f \frac{AR^2-1}{2\tan \theta}}
\]

The static enthalpy increase is then:

\[
\Delta h = c_p(T_5-T_u) = \frac{c_p T_u}{n_{\text{diff}}} \left( \frac{p_5}{p_u} \right)^\frac{K-1}{K} - 1
\]
The outlet Mach number $M_5$ is:

$$M_5 = \sqrt{\frac{2}{k-1} \left( \frac{T_5^0}{T_5} - 1 \right)} \quad (6.6)$$

Since the diffuser is adiabatic, the total temperature $T^0$ remains constant from the impeller outlet:

$$T_5^0 = T_2^0 = T_0 + \Delta H^0 \quad (6.7)$$

The total pressure $p_5^0$ is then

$$p_5^0 = p_5 \left( \frac{T_5^0}{T_5} \right)^{\frac{k}{k-1}} \quad (6.8)$$

### 6.3 Dump diffusion

To take into account
- the blunt trailing edge shape of "vaned island" diffuser blades,
- the sudden increase of axial width at the scroll inlet,

a dump diffusion calculation is performed at the vaned diffuser outlet.

The equations are deduced from the investigations of Hermann on blunt trailing edge cascades (Ref. 41). Figure 44 gives a schematic representation of the dump diffusion process.

At both states 5 and 6, the Laval number can be defined as:

$$L = \frac{V}{a^*} = \frac{V}{\sqrt{\frac{2k}{k+1} R_g T_0}} \quad (6.9)$$

For this calculation, the Laval number $L_6$ after dump is fixed. The area ratio $A_6/A_5$ required to perform this diffusion can be calculated from:
\[ \frac{A_6}{A_5} = 1 - \sigma + \frac{\alpha_2 + \alpha_3}{\alpha_1} \quad (6.10) \]

with

\[ \sigma = \frac{1 + \frac{2k}{k+1} L_5^2}{1 - \frac{k-1}{k+1} L_5^2} \]
\[ \alpha_1 = \frac{1 - \frac{k-1}{k+1} L_5^2}{L_5} \]
\[ \alpha_2 = \frac{1 - \frac{k-1}{k+1} L_6^2}{L_6} \]
\[ \alpha_3 = \frac{2k}{k+1} L_6 \]

The corresponding total pressure drop is given by:

\[ \frac{p_6^0}{p_5^0} = \left( \frac{1 - \frac{k-1}{k+1} L_5^2}{1 - \frac{k-1}{k+1} L_6^2} \right) \frac{L_5}{L_6} \frac{A_5}{A_6} \quad (6.11) \]

The dump diffusion is adiabatic, thus:

\[ \frac{T_6^0}{T_5^0} = \frac{T_6^0}{T_5^0} \quad (6.12) \]

The static temperature \( T_6 \) is calculated as
The static pressure $p_6$ follows from:

\[
T_6 = T_6^0 \left(1 - \frac{k-1}{k+1} \frac{L_6^2}{L_6^0} \right)^{-1}
\]  \hspace{1cm} (6.13)

\[
p_6 = p_6^0 \left(\frac{T_6}{T_6^0}\right)^{\frac{k}{k-1}}
\]  \hspace{1cm} (6.14)
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41. HERMANN, P.: Further investigations of a blunt trailing edge cascade in the S-3 supersonic wind tunnel. VKI IN 9, June 1964.
APPENDIX - SOLUTION OF THE EQUATIONS
FOR THE MIXING PROCESS

Using equations (4.8) and (4.9), we can eliminate the tangential pressure difference \( q \) from (4.10) and (4.11), which results is

\[
\frac{1}{\rho} \frac{d\rho}{dR} = R\Omega^2 - W_j \frac{dW_j}{dR} - \frac{c_f V_j (R\Omega \sin \beta + W_j)}{bcos \beta} - \frac{zc_M (W_j - W_w)^2}{2\pi R (1-\varepsilon) \cos^2 \beta} \tag{A1}
\]

and

\[
\frac{1}{\rho} \frac{d\rho}{dR} = R\Omega^2 - W_w \frac{dW_w}{dR} - \frac{c_f V_w (R\Omega \sin \beta + W_w)}{bcos \beta} + \frac{zc_M (W_j - W_w)^2}{2\pi R \varepsilon \cos^2 \beta} \tag{A2}
\]

By eliminating the radial pressure gradient \( \frac{d\rho}{dR} \) from (A1) and (A2) we obtain:

\[
W_j \frac{dW_j}{dR} - W_w \frac{dW_w}{dR} + \frac{zc_M (W_j - W_w)^2}{2\varepsilon (1-\varepsilon) \pi R \cos^2 \beta}
\]

\[
+ \frac{c_f}{bcos \beta} \left[ (W_j + R\Omega \sin \beta)V_j - (W_w + R\Omega \sin \beta)V_w \right] = 0 \tag{A3}
\]

By adding (4.8) and (4.9), and differentiating with respect to \( R \), we find:

\[
W_j (1-\varepsilon) \Omega \sin \beta \frac{dW_j}{dR} + W_w \varepsilon \Omega \sin \beta \frac{dW_w}{dR} + R \cos \beta \left[ (1-\varepsilon) \frac{W_j^2}{W_j + \varepsilon W_w} \right] \frac{d\beta}{dR} =
\]

\[-\sin \beta \left[ (1-\varepsilon) W_j^2 + \varepsilon W_w^2 \right] - 2\Omega \left[ (1-\varepsilon) W_j + \varepsilon W_w \right] - \frac{c_f R}{bcos \beta} \left[ (1-\varepsilon) (R\Omega + W_j \sin \beta) V_j \right.
\]

\[+ \varepsilon (R\Omega + W_w \sin \beta) V_w \right] \tag{A4}
\]

By substitution of (4.25) and (4.14) in (4.13) we obtain:
\[
\frac{dT}{dR} + \frac{(1-\lambda)}{c_p} (W_j + \Omega R \sin \beta) \frac{dW_j}{dR} + \frac{\lambda}{c_p} (W_w + \Omega R \sin \beta) \frac{dW_w}{dR}
\]

\[
+ \frac{\Omega R \cos \beta}{c_p} \left( (1-\lambda) W_j + \lambda W_w \right) \frac{d\beta}{dR}
\]

\[
+ \frac{\Omega}{c_p} \left[ (1-\lambda) (\Omega R + W_j \sin \beta) + \lambda (\Omega R + W_w \sin \beta) \right] = 0 \quad (A5)
\]

Substitution of (A5) in (4.12) gives:

\[
\frac{1}{\rho} \frac{d}{dR} = \frac{1}{p} \frac{dp}{dR} + \frac{(1-\lambda)}{c_p T} (W_j + \Omega R \sin \beta) \frac{dW_j}{dR} + \frac{\lambda}{c_p T} (W_w + \Omega R \sin \beta) \frac{dW_w}{dR}
\]

\[
+ \frac{\Omega R \cos \beta}{c_p T} \left( (1-\lambda) W_j + \lambda W_w \right) \frac{d\beta}{dR} + \frac{\Omega}{c_p T} \left[ (1-\lambda) (\Omega R + W_j \sin \beta) + \lambda (\Omega R + W_w \sin \beta) \right]
\]

\[
+ \lambda (\Omega R + W_w \sin \beta) = 0 \quad (A6)
\]

Equations 4.5, 4.7, A1, A3, A4, A6 constitute a set of 6 independent equations with 6 unknowns \( \frac{dW_j}{dR}, \frac{dW_w}{dR}, \frac{d\beta}{dR}, \frac{dp}{dR}, \frac{d\rho}{dR} \).

This set of equations can be integrated step by step with the Euler method to determine in this way the evolution of the characteristic values.
Remark: \( \frac{k}{R} \) is defined as the relative roughness ratio of the disc surface.
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TABLE 2
FIG. 1 - SCHEMATIC OF COMPRESSOR STAGE FLOW REGIONS
FIG 2 - BALJE - DIAGRAM FOR OPTIMUM SPECIFIC SPEED

FIG. 3 - INFLUENCE OF PR ON MW_{1t} FOR DIFFERENT VALUES OF NS
FIG. 4 - INLET VELOCITY TRIANGLES

FIG. 5 - INFLUENCE OF PREROTATION ON $M_{W1t}$ AND $M_2$, FOR DIFFERENT VALUES OF PR.
FIG. 6 - INLET GUIDE VANE LOSSES
FIG. 7 - GEOMETRY OF CENTRIFUGAL COMPRESSOR ROTOR.

FIG. 8 - INFLUENCE OF BLADE BLOCKAGE ON INLET VELOCITY TRIANGLE.
FIG. 9 - INFLUENCE OF RV ON MW₁t

FIG. 10 - INFLUENCE OF RPM ON MW₁t
FIG. 11 - INFLUENCE OF $\alpha_1$ ON $M_{W1t}$

FIG. 12 - INFLUENCE OF $R_{1h}$ ON $M_{W1t}$
FIG. 13 - ADJUSTMENT OF RV

FIG. 14 - ADJUSTMENT OF RPM
\[ \alpha_{\text{CENTR}} = \frac{W^2}{R_C} \]

\( R_C \): Radius of Curvature

**FIG. 15 - INFLUENCE OF WALL CURVATURE ON BOUNDARY LAYER DEVELOPMENT.**

**FIG. 16 - INFLUENCE OF DR ON COMPRESSOR EFFICIENCY.**
FIG. 17 - COMPARISON OF DIFFUSION PERFORMANCES OF ROTATING INDUCERS AND STATIONARY DIFFUSERS.

FIG. 18 - INFLUENCE OF DR AND MW1t ON COMPRESSOR EFFICIENCY ηc
FIG. 19- ECKARDT'S IMPELLER.
ELLiptical profile approximation of
A REAL IMPELLER.

\[
\begin{align*}
\frac{R_{1t}}{R_2} &= 0.706 \\
\frac{R_{1h}}{R_{1t}} &= 0.32 \\
b/R_2 &= 0.125 \\
\frac{D_{AX}}{R_2 - R_{1h}} &= 0.83
\end{align*}
\]

--- REAL PROFILE (LINEAR + CIRCULAR) ---
--- ELLIPTICAL PROFILE APPROXIMATION ---
FIG. 20 - ELLIPTICAL PROFILE APPROXIMATION OF A REAL IMPELLER.
FIG. 21 - GEOMETRICAL MODEL FOR SEPARATION
FIG. 22 - IMPELLER BLADE SHAPE

FIG. 23 - FLOW ANGLE VARIATION.
FIG. 24 - MERIDIONAL VELOCITY PROFILE.

FIG. 25 - SEPARATION SECTION GEOMETRY
FIG. 26 - JET-WAKE MODEL.
FIG. 27 - TANGENTIAL EQUILIBRIUM AT IMPELLER OUTLET.
FIG. 28 - T, S DIAGRAM OF IMPELLER FLOW.
FIG. 29 - INFLUENCE OF $v = \frac{w_2 w}{W_2}$ ON THE WAKE WIDTH $\varepsilon_2$ AND ON THE WAKE MASS FLOW $\lambda$. 
FIG. 30 - INFLUENCE OF $V = \frac{W_2 w}{W_2 j}$ ON THE IMPELLER EFFICIENCY.

FIG. 31 - INFLUENCE OF THE IMPELLER WIDTH $b_2$ ON THE LOSSES.
FIG. 32 - JET-WAKE VELOCITY TRIANGLES.
FIG. 33 - DISC FRICTION COEFFICIENT $c_m$ VERSUS AXIAL GAP $S$
FIG. 34 - T.S DIAGRAM OF THE IMPELLER OUTLET STATE.
FIG. 35 - PRESSURE FLUCTUATIONS IN THE VANELESS SPACE
FIG. 37 - VANELESS DIFFUSER BOUNDARY LAYER DEVELOPMENT

FIG. 38 - VANED ISLAND DIFFUSER GEOMETRY
AR = \frac{\text{OUTLET AREA}}{\text{INLET AREA}}

AS = \frac{b}{W}

FIG. 39 - DIFFUSOR CHANNEL GEOMETRY

FIG. 40 - CALCULATION OF DIFFUSER LEADING EDGE STATE.
FIG. 41 - DIFFUSER THROAT BLOCKAGE VERSUS ACTUAL STATIC PRESSURE RECOVERY FROM LEADING EDGE TO THROAT.

FIG. 42 - CHANNEL PRESSURE RECOVERY VERSUS A.R. AND L/W
FIG. 43 - MAXIMUM PRESSURE RECOVERY VERSUS ASPECT RATIO A:S. AND THROAT BLOCKAGE B

FIG. 44 - DUMP DIFFUSION PROCESS.