DESIGN, CONSTRUCTION AND CALIBRATION
OF THE UTIAS PLANETARY BOUNDARY LAYER SIMULATION TUNNEL

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L. D. Reid, G. D. Schuyler, H. W. Teunissen

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Summary

The UTIAS subsonic wind tunnel has been modified into a multiple jet wind tunnel capable of producing simulated planetary boundary layer flows up to 36" thick. Maximum velocities of 100 fps can be obtained in a test section which is about 12 ft. long and 66 in. wide. Adjustment of jet velocities allows a range of velocity profiles to be produced. The tunnel can also be operated in a low-turbulence mode such that uniform flows up to about 100 fps can be achieved in the test section with a turbulence intensity of 2-3%.
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NOTATION

\( A_j \)  
\text{total jet area}

\( A_2 \)  
\text{open area at jet grid (}\( A_3 - A_j \)\text{)}

\( A_3 \)  
\text{total cross-sectional area of tunnel}

\( f \)  
\text{frequency (Hz)}

\( H \)  
\text{tunnel height}

\( i,j \)  
\( \sqrt{-1} \)

\( k \)  
\text{reduced frequency (}\( f/\bar{U} \)\text{)}

\( L \)  
\text{scale length (see Appendix A)}

\( n \)  
\text{profile power law exponent}

\( P \)  
\text{static pressure}

\( q \)  
\text{dynamic pressure (}\( \frac{1}{2} \rho \bar{U}^2 \))

\( r \)  
\text{velocity ratio } \frac{U_2}{U_j}

\( t \)  
\text{time}

\( T \)  
\text{averaging time period, and temperature (}^\circ \text{F)}

\( u \)  
\text{turbulence velocity in x direction (with zero mean)}

\( u' \)  
\text{rms value of } u (\sqrt{u'^2})

\( \bar{U} \)  
\text{mean velocity in x direction}

\( \bar{U}_G \)  
\text{gradient velocity}

\( U_j \)  
\text{jet velocity}

\( U_2 \)  
\text{secondary flow velocity}

\( U_3 \)  
\text{mixed flow velocity}

\( v \)  
\text{velocity in y direction}

\( w \)  
\text{velocity in z direction}

\( W \)  
\text{tunnel width}

\( x \)  
\text{tunnel coordinate axes}

\( y \)  
\text{tunnel coordinate axes}

\( z \)  
\text{tunnel coordinate axes}
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Z_r$</td>
<td>roughness height</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>area ratio $A_2/A_j$</td>
</tr>
<tr>
<td>$\rho$</td>
<td>air density</td>
</tr>
<tr>
<td>$\tau$</td>
<td>time delay</td>
</tr>
<tr>
<td>$\delta$</td>
<td>boundary layer thickness</td>
</tr>
<tr>
<td>$\Phi_{ii}$</td>
<td>power spectral density (see Appendix A)</td>
</tr>
<tr>
<td>$&lt;&gt;$</td>
<td>ensemble average</td>
</tr>
<tr>
<td>$X^*$</td>
<td>complex conjugate of $X$</td>
</tr>
<tr>
<td>$X * Y$</td>
<td>convolution between $X$ and $Y$</td>
</tr>
<tr>
<td>$\text{FT} { x }$</td>
<td>Fourier transform of $x(t)$</td>
</tr>
<tr>
<td>$\text{Im} [x]$</td>
<td>imaginary part of $x$</td>
</tr>
<tr>
<td>$\text{Re} [x]$</td>
<td>real part of $x$</td>
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I. INTRODUCTION

In recent years there has arisen an increasing need for laboratory facilities in which the atmospheric or planetary boundary layer can be properly simulated. This region of the atmosphere, which usually extends up to heights of about 1600 ft. above the earth's surface, depending on its roughness, is of fundamental importance in many varied disciplines. Low-level flight, pollutant dispersal, industrial aerodynamics and environmental comfort in urban areas are but a few of the more important areas where the interaction of the planetary flow with structures and vehicles is of prime concern. Because of the inherent complexity of problems of this nature, theoretical solutions are in general oversimplified if they can be obtained at all. In addition, full scale experimental measurements are quite expensive and can be extremely difficult to obtain without costly delays while waiting for appropriate conditions. Consequently, the availability of controllable laboratory simulations of the planetary flow is extremely important.

There are at present two basic methods in use for simulation of the planetary boundary layer in a wind tunnel. One of these involves the so-called boundary layer wind tunnel in which a long, rough surface is used to develop a 'natural' boundary layer of usable thickness. The other involves the use of various devices, both passive and active, to create an artificially thick boundary layer without the necessity of the long fetch length required in the boundary layer tunnel. A program has recently been completed at UTIAS to determine the capability of a tunnel using a multiple array of controllable jets for simulating the planetary flow by the second method (Ref. 1). As a result of the success of this program, the existing UTIAS subsonic wind tunnel has been redesigned and modified into the multiple jet configuration. In the present report, the details of the design considerations are outlined (Sec. II), a detailed description of the modified facility is given (Sec. III and IV) and calibration results presented (Sec. V).

II. BASIC DESIGN CONSIDERATIONS

The basic design for modifying the original UTIAS subsonic wind tunnel is a scaled-up, closed-circuit version of the small, 8" square prototype tunnel used and described in Ref. 1. This tunnel (Fig. 1) is an open circuit model which is driven on the ejector principle by an array of 64 jets located across its cross-section. The jets are circular and form an 8 x 8 grid (Fig. 2) with a separation of 1" between adjacent jets (i.e., a 'meshlength' of 1"). Primary air is supplied to the jets through the sides of the tunnel via individual lines from a centrifugal blower. Airfoil sections are used to cover these lines inside the tunnel and thus minimize the losses that they add to the system. Each supply line contains a gate valve and a variable-area rotameter such that the velocity of each jet can be independently and accurately controlled. This capability, together with the use of appropriate roughness and barrier plates, allows simulation of the planetary boundary layer to be achieved as described in detail in Ref. 1.

The basic parameters of the prototype tunnel that could be varied in a new facility are the number of jets, their shape, their geometrical arrangement, and the total jet area as defined by the ratio $\alpha = \frac{A_c}{A_j}$. This ratio is the only parameter whose effect can be quantitatively predicted in detail in Ref. 2. Basically, a simple ejector theory involving the momentum and continuity equations...
along with an assumption of uniform velocity profiles throughout the flow is used to obtain the following equations:

\[ \frac{P_3 - P_j}{q_j} = \frac{2\alpha}{(1 + \alpha)^2} (1-r)^2 \]  

(2-1)

and

\[ \frac{U_3}{U_j} = \frac{1 + r\alpha}{1 + \alpha} \]  

(2-2)

where the symbols are defined in Fig. 3. For an open-circuit tunnel, we have

\[ \frac{P_3 - P_j}{q_j} = r^2 \]

and the above equations yield \( U_j \), \( U_2 \) and the necessary jet horsepower (when tunnel size is also specified) in terms of the desired mixed-flow velocity \( U \) with \( \alpha \) as parameter. For the prototype tunnel a value of 20 was chosen for \( 3\alpha \) and this will again be used in the new facility. Using this value in Eqs. (1) and (2) yields

\[ \frac{U_2}{U_j} = 0.23, \quad \frac{U_3}{U_j} = 0.27 \]

If friction losses are included in the equations, then the above predicted ratios reduce to 0.17 and 0.21, respectively. The experimental results of Ref. 1 were found to agree quite well with these reduced values and consequently they are used for predicting design velocities in the new tunnel.

As for the number of arrangement of jets in the new tunnel, the basic pattern used in the prototype was retained. The eight rows of jets in the small tunnel allowed a good degree of control over the velocity profile and still permitted the production of reasonable turbulence scale values when barriers were used. Equal lateral and vertical jet spacing provided reasonable lateral homogeneity of the flow and was thus also retained. The shape of the individual jets, however, was changed from round to square in the new tunnel in order to simplify their construction. The results of Ref. 1 suggest that such a change should have no significant effect on the flows produced in the mixed-flow region of the tunnel.

III. DETAILS OF NEW WIND TUNNEL

3.1 Major Dimensions

The original closed-circuit UTIAS subsonic wind tunnel is shown in Fig. 4 in schematic form. The most convenient and economical way to modify this facility to the multiple-jet configuration was to replace the contraction cone, test section and diffuser between A and B in the drawing with a new contraction, a jet grid section, and a boundary-layer-growth and test section. Thus a total length of 449" is available for the new section, and it is this dimension that
determined its height. That is, we wish to produce boundary layer flows having the maximum possible thickness in the available space. From Ref. 1, it was found that for the multiple-jet type of tunnel, a usable test section is available between 5-1/2 and 8-1/2 tunnel heights from the jet exit plane. Thus allowing roughly 6-1/2 feet for the contraction cone and jet grid sections, a total length of about 371" is available in which to place about 8-1/2 tunnel heights. This yields the design tunnel height of 44" and should allow boundary layer flows up to 36" thick to be obtained. As for the tunnel width, the results of Ref. 1 showed that in some cases fairly large boundary layers could be encountered on the tunnel walls when a barrier was used to produce turbulence in the test section. For this reason, and because increased tunnel width is advantageous for large terrain models, a width of 66" was selected. A layout of the modified tunnel is given in Fig. 5 and a general view is seen in Fig. 6.

3.2 Jet Grid Section

With a tunnel cross-section 44" x 66" and an area ratio \( \alpha = 20 \), the total jet area is fixed at 138 sq. in. In addition, the use of eight jet rows results in a jet spacing of 5.5" vertically and consequently twelve columns of jets for equal vertical and lateral spacing. The complete array therefore consists of 96 jets each having an area of 1.44 in.\(^2\). Square jets were designed (1.2" x 1.2") with each row being covered by an airfoil to reduce friction losses just as in the prototype. An upstream view of the entire grid is given in Fig. 7 together with the notation system chosen for jet identification. Fig. 8 is a photo of the completed grid.

As seen in Fig. 7, the columns of jets have been identified in groups of three. The reason for this is that each group of three in any row is controlled by a single external butterfly valve as far as the jet velocity is concerned. That is, the velocity of each of these jets cannot be individually controlled and all three in any group must be altered together. This was done to reduce the number of valves and valve control devices from 96 to 32 and thus minimize the system cost. One design feature that is necessitated by this approach, however, is the use of so-called 'trimming valves', one in each jet supply line downstream of the butterfly valve which controls the velocities. These trimming valves are necessary to equalize the friction losses in each of the jets of any group so that all the velocities in a group are the same for any control valve setting. A schematic layout of one-half of a typical row of jets is show in Fig. 9 and the location of the trimming valves is clearly seen, as well as that of the velocity-controlling butterfly valves. The jets in the other half of the row are of course supplied from the opposite side of the tunnel.

3.3 Blower Requirements

It was initially desired to obtain a maximum velocity of about 55 fps in the test section of the modified wind tunnel. This value is sufficiently high to give reasonably large pressure values on typical models while still being not so large as to require unreasonably large input power. Based on the results discussed in Sec. II (i.e., \( U_v/U_\infty = 0.21 \)) we therefore require a jet velocity of \( U_v \sim 265 \) fps. Since the total jet area is \( A_j = 138 \) sq. in., the total input volume flow required of the blower supplying the jets is 15,240 cfm. The static pressure rise that must be supplied by this blower is determined from estimated friction losses in the delivery lines and knowledge of the maximum jet velocity. That is, a velocity of 265 fps represents a dynamic head of 16" \( \text{H}_2\text{O} \) or in general, about 14" more than that at the exit of a typical blower. The static losses due
to friction in the jet supply system (see Fig. 10 for schematic layout) have been estimated at about 4" H₂O, and the flow-straightening screen at the blower exit adds a loss of about one dynamic load or about 2" H₂O. Consequently a total static pressure rise of about 20" H₂O must be supplied by the blower. This figure and the above volume flow requirement were used to select a Canadian Blower and Forge Model 55 MW industrial exhauster with a 75 HP General Electric motor. No speed control was required for this blower since a reduction in its delivery will be required only infrequently and can be achieved by throttling its inlet.

3.4 Tunnel Return Section

We see in Fig. 4 that the modified wind tunnel is still a closed circuit tunnel, in the return section of which an axial fan and drive motor (60 HP) are located. There are two basic consequences of this feature of the facility. First, we must provide exhaust ports somewhere in the circuit so that the primary air supplied by the jet supply blower is allowed to leave the tunnel. Three doors have been provided for this purpose in the region downstream of the test section, as shown in Fig. 5. The area of each of these doors is about 4-1/2 ft.² so that the velocity of the exhausting air is less than 20 fps at maximum operating conditions. The second consequence involves the axial fan itself. It will of course 'windmill' if left off during operation of the tunnel in the ejector-driven mode. In this case, it can be represented in the tunnel performance equations by a small pressure loss term to predict its effect. If, however, it is turned on during a run, the sign of this term can be reversed and the fan can supply a pressure boost to the ejector system. This permits a higher test section velocity to be achieved for a fixed jet velocity or, conversely, it reduces the jet velocity required for a desired test section velocity (i.e., \( U_3/U_1 \) would be increased). Since the degree and nature of the improvement to the ejector system that might be obtained from this fan was not exactly known, it was decided to ignore its possible benefits in determining the tunnel blower requirements and assume only that it could reduce its own contribution to friction losses to zero.

One additional feature of the presence of the axial fan in the tunnel return section is the capability for operation of the tunnel in a reduced-turbulence mode. That is, with the jet supply blower off, operation of the axial fan permits flows of up to 100 fps with 2-3% turbulence intensity to be obtained in the tunnel test section.

3.5 Tunnel Test Section

The new tunnel 'growth' and test sections are shown in some detail in Fig. 11. The growth section consists of the region between the jet exit plane and the test section entrance and contains any barriers used in producing turbulence as well as roughness on the floor. The test section itself is about 12 ft. in length and has an access door in the side wall as well as a roof which may be opened along the last 8 ft. of its length. In addition, the entire test section roof is hinged at the test section entrance so as to permit some control of the static pressure gradients in the test section. The degree of control was chosen so as to allow easy removal of the gradients resulting from boundary layer growth on all walls through the test section.
3.6 Valves and Servo-Control System

As mentioned in Sec. II, the velocity of the jets in the small prototype tunnel is controlled by a simple gate valve and a rotameter in each supply line is used to measure its value. In the new facility this velocity, when required, is measured using a Pitot probe at the jet exit and the static pressure on the tunnel walls at the jet exit plane. The jet velocities are controlled, as mentioned in Sec. 3.2, in groups of three by simple butterfly valves. These valves (Figs. 6 and 9) consist of a rotating flat plate located in a short length of 4" diameter pipe. An 8:1 gear reduction is used to increase position sensitivity and to reduce torques. Sixteen valves are located on each side of the jet grid section of the tunnel.

In order to obtain a particular velocity profile in the tunnel test section, an iterative procedure is used in the new tunnel just as it was in the prototype. That is, a probe is placed at some location in the desired plane of the test section and the flow leaving the appropriate jets in the grid is adjusted until the desired velocity is achieved at the probe location. This procedure is repeated iteratively until the desired flow is obtained throughout the plane. To simplify this procedure, it was decided to mechanize the valve adjustment system by using a servo-motor-and-feedback system for each valve. The complete system of 32 channels allows setting of the position of any valve from a central control panel by the simple adjustment of a command potentiometer. Valve position is indicated by a potentiometer connected to the valve itself and its output is fed back to the control circuit ('manual' mode). The complete control circuit for one channel (i.e., one valve) is shown in Fig. 12 and includes automatic stops and warning lights at the two extreme valve positions (open and closed). A photograph of the control panel is shown in Fig. 13. Note that provision has been made for the future addition of a hot wire anemometer circuit for each channel which could provide the system feedback instead of the valve position potentiometer ('auto' mode). Placement of the hot wire sensors at the appropriate location in the tunnel test section would then automatically maintain any particular velocity profile after its initial setup using the command and potentiometers, regardless of any disturbances that might be caused by models, etc.

IV. INSTRUMENTATION

4.1 Tunnel Temperature

The running temperature of each fan motor and the tunnel test section can be monitored by the operator at the tunnel control panel. This is achieved through the use of thermistors which can be linked through a switch to a simple bridge measurement circuit on the panel.

4.2 Traversing Gear

A system for traversing the tunnel has been constructed utilizing a standard lead screw drive. It is shown in Fig. 14. The probe holder may be driven both horizontally and vertically. The apparatus has been made easily removable and may be moved to different measurement planes manually with a minimum of effort.

In order to position the probe accurately, electronic switches, consisting of a photodiode, a light source, and a blade to interrupt the light,
have been placed on the lead screws. Pulses from these switches are counted by two electronic counters. This allows positioning to within one revolution of the lead screw. This, combined with the probe holder play, allows setting accuracy of approximately 0.1 in. With proper gating, the counters may also be used in conjunction with a pulse generator as a frequency counter. Inputs must be in the form of pulses.

4.3 Data Acquisition System

A schematic of the data acquisition system is shown in Fig. 15. It consists of four channels of DISA 55D01 anemometer accompanied by four DISA 55D10 linearizers. These instruments provide flow measurement capabilities of approximately 3-300 fps with a flat frequency response up to 100 KHz. Outputs from these units are conditioned by a PACE TR48 analogue computer. This is a solid state 10 volt machine with 40 amplifiers, 10 integrators, and 2 multipliers presently available.

Signal filtering can be provided both by the TR48 and a Multimetrix Model AF420 active filter. The latter is a two-channel system which provides 2 channels of high or low pass filter or a single channel band pass filter. It features digital cutoff frequency selection and a 24 db/octave roll-off.

Cross- and auto-correlation may be obtained with the PAR model 100 correlator. This instrument provides 100 analogue estimates of correlation with values of \( \tau \) (time delay) ranging from \( \tau_m/100 \) to \( \tau_m \) where \( \tau_m \) is the maximum value which may be selected as 1, .5, .2, .1, .05, .02, .01, .005, .002, .001, .0005, .0002, or .0001 sec.

Each estimate of the correlation is an average of the correlation over the \( \Delta \tau = \tau_m/100 \) interval.

RMS voltages are measured on the Bruel and Kjaer model 2417 random noise meter. This meter is accurate to within 2% for sinusoidal or Gaussian signals.

Data can be fed via the Hewlett Packard Model 5610A A/D converter into the HP 2100A digital computer. The A/D converter is capable of 100,000 samples per sec. The computer has 24K core storage, supplemented by a magnetic tape system. The computer cycling time is 980 nsec. Algol, Basic, Fortran, and Assembler are the languages used on this system. Other equipment in this system includes a digital plotter, high speed paper tape reader, teletype and tape punch, and D/A converter.

The use of this system for the spectral analysis performed during the present calibration has reduced our data acquisition and computation times by a factor of two.

V. TUNNEL CALIBRATION

5.1 Planetary Boundary Layer Characteristics

The purpose of the new boundary layer tunnel is to simulate the earth's planetary boundary layer. The simulation is based on the following information taken from Ref. 3.

(a) Velocity Profile

The height of the boundary layer as well as its velocity profile
varies with the roughness of the terrain over which it travels. This velocity profile is approximately a power law profile governed by the relation

$$\frac{U}{U_G} = \left(\frac{z}{z_G}\right)^n$$

(5-1)

where $U_G =$ gradient velocity or the mean velocity at the top of the boundary layer

$U =$ mean velocity at height $Z$

$z_G =$ gradient height or the height of the top of the boundary layer

$n =$ power law exponent.

This profile is illustrated in Fig. 16. The variation of this profile with terrain is shown in the following table.

<table>
<thead>
<tr>
<th>SURFACE TYPE</th>
<th>$n$</th>
<th>$Z_G$ (ft)</th>
<th>$Z_r$(ft)</th>
</tr>
</thead>
<tbody>
<tr>
<td>flat</td>
<td>.16</td>
<td>900</td>
<td>3</td>
</tr>
<tr>
<td>forest</td>
<td>.28</td>
<td>1300</td>
<td>30</td>
</tr>
<tr>
<td>urban</td>
<td>.35</td>
<td>1600</td>
<td>100</td>
</tr>
</tbody>
</table>

$Z_r =$ height of roughness elements. The roughness elements are the trees, grass, buildings, etc., that extend into the flow.

(b) Atmospheric Turbulence

The power spectral densities of the turbulent wind velocities of the earth's planetary boundary layer have been found to be well represented by the von Karman model for isotropic, homogeneous turbulence (Ref. 3). This model is illustrated in Fig. 29. This allows the use of the spectral peak method in calculating turbulence scale lengths. This method is outlined in Appendix A. The two integral scale length relations which were used as atmospheric models for comparison with the scale lengths found in the tunnel are

$$L_u^x \sim 20 \sqrt{z}$$

(5-2)

and

$$L_w^x \sim .4z$$

(5-3)

These are plotted in Fig. 17.

The preceding information concerning turbulence in the earth's planetary boundary layer applies to a neutrally stable atmosphere only. That is to say, an atmosphere with no temperature variation (zero lapse rate) with height up to the gradient height.
5.2 Test Section Velocity Profile

The tunnel layout is illustrated in Fig. 18. All measurements were made at the upstream and downstream ends of the test section. These are at \( x = 5.5H \) and \( x = 8.5H \), respectively.

A velocity profile at the \( x = 5.5H \) plane is shown in Fig. 19. The profile has been nondimensionalized by the maximum profile velocity \( U_{max} = 100 \text{ fps} \) and the tunnel width \( (W) \) and tunnel height \( (H) \). This is the profile when the clean tunnel is powered by the upper fan only. A similar profile is obtained when the jets power the tunnel, and the jet exit speeds are equal. The wall boundary layers are fairly small in this configuration as would be expected.

The dished shape of the profile is caused by the abrupt contraction cone. The main consideration in the design of the contraction cone was length. Since the tunnel was designed for high turbulence and fairly low velocities, it was thought that little benefit would be gained from a classic contraction cone design with its excessive length requirements. Consequently, a fairly simple design was used. The abruptness in the contraction causes higher local velocities in the outer flow, resulting in the dish-shaped profile. For further discussion of this effect, the reader is referred to Ref. 4.

This profile is important because if the upper fan is to be used with the jets, then the superposition of this flow on that of the jets must be compensated for by the jets. At very high speed settings of the upper fan, this could limit the degree of profile adjustment available from the jets.

5.3 Velocity Range

The jet drive system was designed for a 55 fps uniform profile with the upper fan overcoming only its own losses. It was found that this 55 fps velocity could be attained without the aid of the upper fan. Utilizing the upper fan plus the jet drive system, a maximum velocity of 120 fps can be reached. With the tunnel powered by the upper fan only, a velocity of 100 fps is attained. In this configuration, however, the profile has the dished shape, mentioned earlier, which would allow use of only the central portion of the test section.

It was mentioned that the tunnel could be operated in a low turbulence mode. This is achieved by utilizing the jets to fill out the profile produced by the upper fan. It was found that in this configuration, turbulence intensity levels \( (u'/\bar{U})_n \) were in the range from .02 to .05.

The wall and floor boundary layers were limited to approximately 4 in. at the upstream end of the test section. This in no way approaches laminar flow, but could be used for such things as calibration of cup anemometers or pitot-static tubes used for industrial purposes. It could also be used for studies of wind loading on structures where aeroelastic effects are not being studied.

5.4 Temperature

A time history of the tunnel temperature is shown in Fig. 20. From this it can be concluded that a half hour warm-up time eliminates much of the temperature drift. The tunnel, being a modified closed circuit type, runs well above the ambient temperature. However, with the jets turned on there is an air exchange of approximately 15,000 cfm. This makes the tunnel more sensitive to
ambient temperature than a closed return tunnel, but also limits the operating temperature to approximately 25°F above ambient. For this reason, the tunnel never reaches a temperature where it must be shut down to cool. This provides a virtually unlimited operation cycle.

5.5 Turbulence Producing Devices

(a) Floor Roughness

Roughness on the floor is provided by a vinyl carpet. It is very durable and clean. Dust particles that accumulate in the mat are easily picked up with a vacuum cleaner. The bristles protrude about one-half inch into the airstream and are formed in clumps of 8 in. a regular array. Figure 21 shows this material.

With the carpet in place, turbulence intensity measurements were again taken. It was found that the turbulence intensity increased only slightly and only in the bottom 6 in. of the tunnel. The floor boundary layer with uniform flow increased somewhat, but wall boundary layers were negligibly affected.

(b) Barrier

Based on the results from the prototype tunnel (Ref. 1), a 6-inch barrier was placed at a point 1-1/2 tunnel heights downstream of the jets (see Fig. 18). Turbulence intensity measurements taken with this barrier are compared with atmospheric and other wind tunnel facility data in Fig. 22. Since these results were of a reasonable shape, but rather high, it was decided to try a lower barrier. The barrier was cut to 5 in. with the results shown in Fig. 22.

The wall boundary layers were increased considerably to about 14 in. at the beginning of the test section by the barrier. The barrier also alters the effects of the first two rows of jets.

5.6 Profile Setting

At present, the only profile to be set, other than the uniform profile, is the power law profile discussed in Sec. 5.1 with \( n = .16 \). This was done at \( x = 5.5H \) using the grid of measurement positions shown in Fig. 23.

As mentioned earlier, the jet grid is controlled in sets of 3 jets horizontally. This allows three jets to be set by one valve. In order to set the profile, the (b) measurement position for each set of three jets was used. After setting the profile, velocity measurements were taken at each of the entire set of measurement positions. Some preliminary results of this profile measurement are shown in Fig. 24. The non-uniformity of the lateral profile is caused by two things. The first factor is the dished profile discussed earlier. This causes the centre of the tunnel to be slow. The second reason for the non-uniformity is the presence of the wall boundary layers.

With the roughness and barrier in place, the wall boundary layers have grown to include the first column of measurement points (see Fig. 25). This causes the velocity measurements at those points to be lower than the free stream velocity by about 10%. This, combined with the velocity defect at the tunnel's centre produces the 10% variation in the lateral velocity profile. This variation cannot be removed with the trimming valves which were designed strictly
to equalize jet exit velocities. They cannot overcome large influences such as wall boundary layers.

In order to achieve better uniformity, the 1c and 4a positions (see Fig. 13) are used for measuring at the edges. This allows the boundary conditions of the flow to be set to a known value. The flow velocities between these measurement positions and the near walls are of no interest. This change in measurement position results in the lateral profile varying less than $\pm 3\%$ between columns 1c and 4a as shown in Fig. 26.

Since each valve controls a horizontal group of 3 jets, and only one measurement position is used for each valve, the measurement positions are further apart in the horizontal than in the vertical direction. This causes the setting of one valve to affect the flow controlled by the valves immediately above and below it more than those to either side. The influence region of each jet is shown in Fig. 27. Since the velocity at each measurement point is greatly affected by the valves controlling the velocity at points immediately above and below it, a simple iterative technique in setting the profile takes a large number of iterations.

The flow velocity is measured for an entire column of measurement positions. Using these, the profile is set at that column by considering the interaction of the jets and adjusting each accordingly. When this process has been completed, the remaining columns are set in the same manner. The entire setting procedure is repeated three times on the average. Using the motorized traversing gear described earlier, about 1-1/2 days are needed to set a new velocity profile.

The two lower rows of jets cannot be used to control the velocity profile near the tunnel floor due to the barrier (see Fig. 18). This, however, is not a serious problem since the natural velocity defect of the floor causes the velocity to fall off in accordance to the power law profile that has been set. These two lower rows of jets are set at near maximum settings. This tends to increase the turbulence in the tunnel by hitting the barrier with high speed air.

The velocity profile was originally set with $\bar{U}_{G} = 55$ fps. It was found, however, that the u-component scale lengths were much too low. In order to rectify the situation an 8 in. barrier was installed. This increased the scales somewhat, but caused severe distortion of the $k\Phi_{uu}(k)$ vs. kz plots. This invalidated any scale measurements since the spectral shape varied a great deal from the von Kármán model.

The original 5-in. barrier was then replaced, and the gradient velocity was increased. This increased the scale lengths without distorting the spectral shape. A study of the variation of scale length with velocity was then performed. It was concluded from this study that a gradient velocity of 90 fps was needed. For this reason the tunnel profile was set with $\bar{U}_{G} = 90$ fps. Figure 28 shows the results of this study.

Since the tunnel exchanges air with the atmosphere at a rate of 15000 cfm, the gradient velocity tends to vary slightly with atmospheric conditions. This is caused by changes in loading of the blower motor due to changes in air density, as well as changes in loading due to temperature changes in the belt drive system. These variations cause the gradient velocity to
fluctuate from day to day with the maximum fluctuation being approximately \(\pm5\%\). This, however, does not affect the non-dimensionalized profile which stays within \(3\%\) of the desired profile.

5.7 Spectral Data

Appendix B describes the techniques of power spectral density analysis used for this calibration. Figures 29 to 32 show plots of \(k\Phi_{uu}(k)\) vs. \(kz\) at different points in the test section. The von Kármán spectrum is also shown for comparison. From these plots it may be seen that the measurement of scale lengths using the spectral peak method, described in Appendix A, is reasonable below the \(0.65\) point of the boundary layer. A typical plot of \(\Phi_{uu}(f)\) vs. \(f\) is shown in Fig. 33.

Figures 34 to 36 show some \(k\Phi_{ww}(k)\) plots. The same restriction on scale measurement applies in this case. Figure 37 shows a plot of \(\Phi_{ww}(f)\) vs. \(f\). Generally speaking the viscous cut-off effects noted in Ref. 1 with the 8" tunnel are not present in these data.

Figure 38 shows the results of scale measurement at various points in the test section using the spectral peak method. Figure 17 compares these with the values calculated from Eqs. (5-2) and (5-3).

As shown in Fig. 38, the variation of scale length with height deviates considerably from Eq. (5-2) near the edges of the test section. If, however, we restrict our measurements to the central part of the tunnel (between columns 2a and 3c), then the scale lengths may be considered satisfactory.

As mentioned earlier, the \(L_u^x\) scale was found to be much too small with the original \(U_g\). \(L_u^x/6\) in that case remained practically constant with height at approximately \(0.25\). In scaling the prototype tunnel, it was intuitively felt that scale length would be proportional to barrier height. The scale length was assumed to be governed by the size of the detached boundary layer region immediately behind the barrier. The size of this region with respect to the size of the barrier seems to vary a great deal with experiment (Ref. 5). This would lead one to suspect that local conditions at the edge of the barrier, and upstream, must have a large effect on the size of the detached region. This makes it very unlikely that the scale would increase exactly linearly with barrier height. In the present case, an increase of speed was found to increase the \(L_u^x\) (see Fig. 28).

VI. CONCLUSIONS

The UTIAS wind tunnel has been calibrated with the following specifications.

- Boundary layer height \((5) = 3\text{ ft.}\)
- Velocity profile power law exponent \(n = 0.16\)
- Turbulence intensity profile (see Fig. 22)
- Variation of \(L_u^x\) and \(L_X^x\) with height (see Fig. 17)
- Profile uniformity \(\pm3\%\)
- Usable test section: \(h = 1.8 - 3\text{ ft.}\)
  \(w = 3\text{ ft.}\)
  \(L = 12\text{ ft.}\)
REFERENCES


APPENDIX A

TURBULENCE DEFINITIONS

A detailed discussion of turbulence theory and definitions is given in Ref. 6. The brief descriptions and definitions given below are taken from Ref. 3.

(a) Velocity

The tunnel flow is represented in Fig. 18. The tunnel axis coincides with the \( \bar{U} \) velocity vector. The velocities are represented by \( u, v, w \), or \( u_1, u_2, u_3 \). Both representations are used extensively depending upon notation convenience.

(b) Correlations

A one-dimensional correlation of two functions of three variables is defined as

\[
R_{x_1y_1z_1} = \lim_{B \to \infty} \frac{1}{2B} \int_{-B}^{B} \int_{-B}^{B} f_1(x_1, y_1, z_1) f_2(x_1, y_1, z_1 + z) dz
\]

where \( x_1, y_1, z_1 \) are constants which separate the arguments of the two functions, and \( z \) is the variable with respect to which the product is being averaged. The operation

\[
\lim_{B \to \infty} \frac{1}{2B} \int_{-B}^{B} dz
\]

is the averaging operation. For this calibration, the averaging process will be done with respect to time only. Therefore the superscripts on the correlation may be dropped as it will be understood that these are one-dimensional correlations with respect to time. For convenience we can define

\[
\int_{-T}^{T} f_1(t) f_2(t) dt
\]

In this calibration, the functions of interest are the velocities \( u, v, w \). These are functions of the four variables, \( x, y, z, t \).

The argument of the correlation function defines the separation in space and time of the two velocity functions as shown in Fig. 39. Therefore

\[
R_{uv}(x_1, y_1, z_1, \tau_1) = u(0, 0, 0, t) v(x_1, y_1, z_1, t + \tau_1)
\]
\[ R_{uv}(x_1, y_1, z_1, \tau) = v(0,0,0,t)u(x_1,y_1,z_1,t+\tau) \]

\( x_1, y_1, z_1, \tau \) are constants.

For convenience, any zeroes in the correlation argument are dropped thus:

\[ R_{uv}(\tau) = R_{uv}(0,0,0,\tau) \]

\[ \text{etc.} \]

The correlation \( R_{uv}(\tau) \) is a single value while \( R_{uv}(\tau) \) is a function of the variable \( \tau \). Likewise, \( R_{uv}(x) \) is a function of the variable \( x \) where \( x \) is the separation in the \( x \)-direction.

One property of the correlation function that is useful is

\[ R_{uv}(0) = u(t) v(t) \]

or

\[ R_{uv}(0) = u(t) v(t) = u(t)^2 = \text{the mean square value.} \]

The correlations \( R_{ii}(\tau) \) are known as autocorrelations. The correlations \( R_{ij}(\tau) \) are known as cross-correlations.

The correlation function can be non-dimensionalized by the mean product

\[ \hat{R}_{uv}(\tau) = \frac{R_{uv}(\tau)}{R_{uv}(0)} = \text{the correlation coefficient. This function approaches 1 as } \tau \to 0. \]

(c) Scales

\[ L_i^x = \int_{-\infty}^{\infty} R_{ii}(x) \, dx \]

\[ L_i^y = \int_{-\infty}^{\infty} R_{ii}(y) \, dy \]

\[ L_i^z = \int_{-\infty}^{\infty} R_{ii}(z) \, dz \]

\[ T_i = \int_{0}^{\infty} \hat{R}_{ii}(\tau) \, d\tau \]
The first three scales have the dimensions of length. These scales are the area under the appropriate correlation functions. If there is a high correlation at large separations, then the scale will be large, indicating that the size of the turbulent eddies in the flow must be fairly large since there is very slight correlation between eddies. In this manner the length scales are a measure of the size of the turbulence.

The last scale has the dimension of time and represents a characteristic time inherent in the flow.

(d) Power Spectral Densities

Another use of the correlation function is for obtaining spectral information. The one-dimensional power spectral density is defined as the Fourier transform of the one-dimensional correlation function. Multidimensional spectral densities are Fourier transforms of multidimensional correlation functions. The one-dimensional spectral density is the only one of interest here.

\[
\Phi_{ij}(k) = 2 \int_{-\infty}^{\infty} R_{ij}(x) e^{-j2\pi k_x x} \, dx
\]

\[
\Phi_{ij}(k_y) = 2 \int_{-\infty}^{\infty} R_{ij}(y) e^{-j2\pi k_y y} \, dy
\]

\[
\Phi_{ij}(k_z) = 2 \int_{-\infty}^{\infty} R_{ij}(z) e^{-j2\pi k_z z} \, dz
\]

\[
\Phi_{ij}(f) = 2 \int_{0}^{\infty} R_{ij}(\tau) e^{-j2\pi f \tau} \, d\tau
\]

\[
\Phi_{ij}(k) = 2 \int_{0}^{\infty} R_{ij}(\tau) e^{-j2\pi k \tau} \, d\tau
\]

where \( k \) is the reduced frequency \( f/\bar{U} \) and \( k_x, k_y, k_z \) are wave numbers corresponding to the \( x, y, z \) directions.

(e) Taylor's Hypothesis

This is the assumption that the flow's velocity structure is constant while the changes in velocity at a fixed measurement point are caused by the movement of this "frozen" flow past the measurement point at the mean velocity. This assumption is a good one if \( \bar{U} \gg u, v, w \). It allows us to substitute time separation in correlations for spatial separation in the \( x \) direction. Thus we have
\[ R_{ij}(x_1) = R_{ij}(\tau_1) \]

where
\[ x_1 = \bar{u} \tau_1 \]

and
\[ L_{ix} = \bar{u} \int_{0}^{\infty} R_{i1}(\tau) \, d\tau \]

and
\[ \phi_{ii}(k_x) = 2 \int_{-\infty}^{\infty} R_{i1}(\tau)e^{-j2\pi k_x \tau} \, d\tau \]

and from the equation for \( \phi_{ii}(k) \).

\[ \phi_{ii}(k_x) = \bar{u} \phi_{ii}(k) \text{ with } k_x = k \]

(f) Spectral Peak Method of Estimating \( L \)

As mentioned earlier, the spectral peak method of measuring scales was used. If we assume that the von Kármán model for power spectral density holds then

\[
\frac{k_x \phi_{uu}(k_x)}{u^2} = \frac{4 \frac{L_x}{L_u} k_x}{[1 + 70.7(L_u x k_x)^2]^{5/6}}
\]

\[
\frac{k_x \phi_{ww}(k_x)}{u^2} = \frac{4 \frac{L_x}{L_w} x [1 + 188.4(2L_x x k_x)^2]}{[1 + 70.7(2L_x x k_x)^2]^{11/16}}
\]

By differentiating the right hand side with respect to \( k_x \) and equating it to zero to find the peak value we get

\[
k_{xpu} = \frac{0.146}{L_u} \text{ for } u
\]

and

\[
k_{xpw} = \frac{0.106}{L_w} \text{ for } w
\]

This gives \( L_u x = \frac{0.146}{k_{xpu}} \) and \( L_w x = \frac{0.106}{k_{xpw}} \)
Since frozen flow is an underlying assumption here then $k_x = k$ and $\Phi_{ii}(k_x) = \bar{v} \Phi_{ii}(k)$ (see item (e) above). Thus the scale can be determined from the peak position of the $k\Phi_{ii}(k)/u^2$ vs. $kz$ plot.
INTRODUCTION

The power spectral density measurements utilized in evaluating the flow in the present tunnel were found by Fourier transforming correlation estimates produced by a PAR Model 100 correlator. The following material outlines the properties of this system in order to relate the measured spectra to the true underlying spectra.

CORRELATION ESTIMATE

The cross-correlation between two time signals \( x(t) \) and \( y(t) \), which are non-zero for \( t > 0 \) only, can be defined as

\[
R_{xy}(\tau) = \lim_{T \to \infty} \frac{1}{T} \frac{1}{T} \int_{0}^{T} x(t) y(t+\tau) \, dt
\]

For ergodic processes, such as are assumed to exist in the wind tunnel, the cross-correlation can also be represented by the ensemble average for any value of \( \tau \)

\[
R_{xy}(\tau) = \langle x(t) y(t+\tau) \rangle
\]

The PAR correlator is a hybrid device that produces estimates of \( R_{xy}(\tau) \) by sampling the time signal connected to channel A, \( a(t) \), at equally spaced intervals of \( \Delta \tau \) sec. and quantizing the results to produce \( a(m\Delta \tau) \). The output of the quantizer is fed into a shift register which as a result contains values of \( a(m\Delta \tau - n\Delta \tau) \) for \( n \) going from 0 to 99. The contents of each register is continuously multiplied by the time signal connected to channel B, \( b(t) \), and the product is fed into one of one hundred first order low pass filters. The output of the low pass filter associated with the \( n \)th register (containing \( a(m\Delta \tau - n\Delta \tau) \)) is the estimate of \( R_{ab} ([n + \frac{1}{2}] \Delta \tau) \).

The contents of the \( n \)th register can be represented by (assuming that quantization level effects can be ignored) a sum of boxcar functions (see Fig. 40)

\[
r_n(t) = \sum_{q=0}^{M-1} a(q\Delta \tau) h_1(t-q\Delta \tau)
\]

where

\[
M\Delta \tau = T
\]

\[
h_1(\alpha) = 1(\alpha) - 1(\alpha-\Delta \tau)
\]

and \( 1(\alpha) \) is the unit step function. The contents of the \( n \)th register are given by
\[ r_n(t) = r_0(t-n\Delta\tau) \]

and \( r_n(t) = 0 \) for \( t < n\Delta\tau \).

The output from the PAR correlator can be represented by

\[ PAR_{AB}(n\Delta\tau) = \int_{n\Delta\tau}^{T+n\Delta\tau} b(v)r_n(v)h_2(T+n\Delta\tau-v)dv \]

where \( T \) is called the running time and \( h_2(t) = e^{-bt} \) is the impulse response function of the first order filters. Substituting \( \alpha + n\Delta\tau = v \) and noting that \( r_n(\alpha + n\Delta\tau) = r_0(\alpha) \) obtain

\[ PAR_{AB}(n\Delta\tau) = \int_0^T b(\alpha+n\Delta\tau) \int_0^{M-1} a(q\Delta\tau)h_1(\alpha-q\Delta\tau)h_2(T-\alpha)d\alpha \]

If \( x(t) \) is fed into channel A and \( y(t) \) into channel B then

\[ PAR_{xy}(n\Delta\tau) = \int_0^T y(\alpha+n\Delta\tau) \int_0^{M-1} x(q\Delta\tau)h_1(\alpha-q\Delta\tau)h_2(T-\alpha)d\alpha \]

and the expected value of \( PAR_{xy}(n\Delta\tau) \) is given by

\[ < PAR_{xy}(n\Delta\tau) > = \int_0^{M-1} \int_0^{(q+1)\Delta\tau} < x(\alpha\Delta\tau)y(\alpha+n\Delta\tau) > h_2(T-\alpha)d\alpha \]

\[ = \sum_{q=0}^{M-1} \int_{q\Delta\tau}^{(q+1)\Delta\tau} R_{xy}(\alpha+n\Delta\tau-q\Delta\tau)h_2(T-\alpha)d\alpha \]
let \( \alpha - q \Delta \tau = \gamma \)

\[
< \text{PAR}_{xy}(n\Delta \tau) > = \sum_{q=0}^{M-1} \int_{0}^{\Delta \tau} R_{xy}(\gamma + n\Delta \tau) h_2(T - \gamma - q\Delta \tau) \, d\gamma
\]

Now the first order filter represented by \( h_2 \) has been used to approximate the performance of an averaging circuit over the time period \( T \). If a pure averaging circuit were to replace this first order filter \( h_2 \) would be replaced by \( 1/T \) and we would have

\[
< \text{PAR}_{xy, T}(n\Delta \tau) > = \frac{1}{T} \sum_{q=0}^{M-1} \int_{0}^{\Delta \tau} R_{xy}(\gamma + n\Delta \tau) \, d\gamma
\]

\[
= \frac{1}{T} \int_{0}^{\Delta \tau} R_{xy}(\gamma + n\Delta \tau) \, d\gamma
\]

\[
= \frac{1}{\Delta \tau} \int_{0}^{\Delta \tau} R_{xy}(\gamma + n\Delta \tau) \, d\gamma \quad \text{since } T = M\Delta \tau
\]

This is the value of \( R_{xy}(\tau) \) averaged over the range \( n\Delta \tau - (n+1) \Delta \tau \) and we would call this our estimate of \( R_{xy}([n+\frac{1}{2}]\Delta \tau) \). Thus to the extent that the first order filter approximates an averaging circuit we will call \( \text{PAR}_{xy}(n\Delta \tau) \) our estimate of \( R_{xy}([n+\frac{1}{2}]\Delta \tau) \). Similarly, \( \text{PAR}_{yx}(n\Delta \tau) \) will be taken as our estimate of \( R_{xy}([-n+\frac{1}{2}]\Delta \tau) \).

**POWER SPECTRAL DENSITY ESTIMATE**

The cross-power spectral density between two time signals \( x(t) \) and \( y(t) \) can be defined as the Fourier transform of their cross-correlation function

\[
\Phi_{xy}(j\omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{-j\omega \tau} R_{xy}(\tau) \, d\tau
\]

Note that this leads to two-sided power spectral density functions.

In the present situation an estimate of the power spectral density is found by performing a digital Fourier transformation on the output from the PAR correlator.
\[
\Phi_{\text{EST}}(j\omega) = \frac{1}{2\pi} \sum_{n=0}^{99} e^{-j\omega(n+\frac{1}{2})\Delta\tau} \text{PAR}_{xy}(n\Delta\tau)\Delta\tau
\]

\[
+ \frac{1}{2\pi} \sum_{n=0}^{99} e^{j\omega(n+\frac{1}{2})\Delta\tau} \text{PAR}_{yx}(n\Delta\tau)\Delta\tau
\]

And

\[
\langle \Phi_{\text{EST}}(j\omega) \rangle = \frac{1}{2\pi} \sum_{n=0}^{99} e^{-j\omega(n+\frac{1}{2})\Delta\tau} \langle \text{PAR}_{xy}(n\Delta\tau) \rangle \Delta\tau
\]

\[
+ \frac{1}{2\pi} \sum_{n=0}^{99} e^{j\omega(n+\frac{1}{2})\Delta\tau} \langle \text{PAR}_{yx}(n\Delta\tau) \rangle \Delta\tau
\]

\[
= \frac{1}{2\pi} \sum_{n=0}^{99} e^{-j\omega(n+\frac{1}{2})\Delta\tau} \sum_{q=0}^{M-1} \int_{0}^{\Delta\tau} R_{xy}(\gamma+n\Delta\tau)h_{2}(T-\gamma-q\Delta\tau) d\gamma \Delta\tau
\]

\[
+ \frac{1}{2\pi} \sum_{n=0}^{99} e^{j\omega(n+\frac{1}{2})\Delta\tau} \sum_{q=0}^{M-1} \int_{0}^{\Delta\tau} R_{yx}(\gamma+n\Delta\tau)h_{2}(T-\gamma-q\Delta\tau) d\gamma \Delta\tau
\]

\[
= \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{-j\omega\tau} \bar{V}_{100}^{+}(\tau) \sum_{q=0}^{M-1} \int_{0}^{\Delta\tau} R_{xy}(\gamma+\tau-\frac{\Delta\tau}{2})h_{2}(T-\gamma-q\Delta\tau) d\gamma d\tau
\]

\[
+ \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{j\omega\tau} \bar{V}_{100}^{-}(\tau) \sum_{q=0}^{M-1} \int_{0}^{\Delta\tau} R_{yx}(\gamma-\tau-\frac{\Delta\tau}{2})h_{2}(T-\gamma-q\Delta\tau) d\gamma d\tau
\]

Where

\[
\bar{V}_{100}^{+}(\tau) = \Delta\tau \sum_{n=0}^{99} \delta(\tau-[n+\frac{1}{2}]\Delta\tau)
\]

\[
\bar{V}_{100}^{-}(\tau) = \Delta\tau \sum_{n=0}^{99} \delta(\tau+[n+\frac{1}{2}]\Delta\tau)
\]

and \(\delta(t)\) is the Dirac delta function.
From Appendix C it is seen that if

\[ x_3(t) = x_1(t) \cdot x_2(t) \]

then \( FT(x_3) = FT(x_1) \ast FT(x_2) \) where \( \ast \) indicates convolution and \( FT\{x\} \) is the Fourier transform of \( x(t) \). This theorem will be applied to equation (1). In Appendix D it is shown that

\[
FT \left\{ \frac{t}{100} \right\} = \frac{\Delta t}{2\pi} \frac{\sin(100\omega \Delta t/2)}{\sin(\omega \Delta t/2)} e^{\frac{\pm j100\omega \Delta t}{2}} \]

\[
= \frac{\Delta t}{4\pi} \frac{\sin(100\omega \Delta t)}{\sin(\omega \Delta t/2)} \pm \frac{j\Delta t}{4\pi} \frac{(1-\cos(100\omega \Delta t))}{\sin(\omega \Delta t/2)}
\]

Define

\[ D_1(j\omega) = FT\{V^+_{100}(\tau)\} \]

\[ D_2(j\omega) = FT\{V^-_{100}(\tau)\} \]

Note that

\[ D_1(j\omega) = D_2(j\omega) \]

Also

\[
FT\left\{ \sum_{q=0}^{M-1} \int_0^{\Delta T} R_{xy}(\gamma + \tau - \frac{\Delta T}{2}) h_2(T-\gamma - q\Delta T) d\gamma \right\}
\]

\[
= FT \left\{ \sum_{q=0}^{M-1} \int_0^{\infty} R_{xy}(\gamma + \tau - \frac{\Delta T}{2}) h_1(\gamma) h_2(T-\gamma - q\Delta T) d\gamma \right\}
\]

Substituting \( u = \gamma + \tau - \frac{\Delta T}{2} \) obtain

\[
= FT \left\{ \sum_{q=0}^{M-1} \int_0^{\infty} R_{xy}(u) h_1(\frac{\Delta T}{2} - [\tau-u]) h_2(T-[q+\frac{1}{2}]\Delta T + [\tau-u]) du \right\} \tag{2}
\]
In Appendix E it is shown that if

\[ x_3(\tau) = \int_{-\infty}^{\infty} x_1(u)x_2(\tau-u)du \]

then

\[ \text{FT}\{x_3\} = 2\pi \text{FT}\{x_1\} \cdot \text{FT}\{x_2\} \]

Thus (2) becomes

\[
2\pi \text{FT}\left\{ R_{xy}(u) \right\} \sum_{q=0}^{M-1} \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{-j\omega v} \frac{h_1(\Delta \tau/2 - v)}{h_2(T-[q+\frac{1}{2}])} dv
\]

\[
= \Phi_{xy}(j\omega) \sum_{q=0}^{M-1} \int_{-\infty}^{\infty} e^{-j\omega v} \frac{e^{-b(T-[q+\frac{1}{2}] \Delta \tau + \nu)}}{e^{b(T-q\Delta \tau + \nu)}} dv
\]

\[
= \Phi_{xy}(j\omega) \left( e^{-bT} \sum_{q=0}^{M-1} e^{(q+\frac{1}{2})b\Delta \tau} \right) \left( \frac{e^{(j\omega+b)\frac{\Delta \tau}{2}} - e^{-(j\omega+b)\frac{\Delta \tau}{2}}}{(j\omega+b)} \right)
\]

\[
= \Phi_{xy}(j\omega) \mathcal{G}_1(j\omega)
\]

Similarly

\[
\text{FT}\left\{ \sum_{q=0}^{M-1} \int_0^{\Delta \tau} R_{xy}(\gamma-\tau+\frac{\Delta \tau}{2})h_2(T-\gamma-q\Delta \tau) d\gamma \right\}
\]

\[
= \Phi_{xy}(j\omega) \mathcal{G}_1(j\omega)
\]

since \( R_{xy}(\tau) = R_{yx}(-\tau) \).

And (3) becomes

\[
= \text{FT}\left\{ \sum_{q=0}^{M-1} \int_{-\infty}^{\infty} R_{xy}(-\tau+\gamma+\frac{\Delta \tau}{2})h_1(\gamma)h_2(T-\gamma-q\Delta \tau) d\gamma \right\}
\]

substituting \( v = \tau + \frac{\Delta \tau}{2} \) obtain from (4)

\[
e^{-\frac{j\omega \Delta \tau}{2}} \text{FT}\left\{ \sum_{q=0}^{M-1} \int_{-\infty}^{\infty} R_{xy}(v-\gamma)h_1(\gamma)h_2(T-q\Delta \tau - \gamma) d\gamma \right\}
\]
applying the theorem of Appendix E obtain

\[ 2\pi e^{jw\frac{\Delta \tau}{2} \sum_{q=0}^{M-1} \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{-jw\gamma} h_1(\gamma) h_2(T-q\Delta \tau-\gamma) d\gamma } = e^{jw\frac{\Delta \tau}{2} \sum_{q=0}^{M-1} \Delta \tau - jw\gamma - b(T-q\Delta \tau-\gamma) } \]

\[ = \Phi_{xy}(jw) e^{jw\frac{\Delta \tau}{2} \sum_{q=0}^{M-1} e^{qb\Delta \tau} \left( \frac{1-e^{-(jw-b)\Delta \tau}}{(jw-b)} \right) } \]

\[ = \Phi_{xy}(jw) \left( -bT \sum_{q=0}^{M-1} e^{(q+\frac{1}{2})b\Delta \tau} \right) \left( \frac{(jw-b)\Delta \tau}{2} - \frac{e^{-(jw-b)\Delta \tau}}{(jw-b)} \right) \]

\[ = \Phi_{xy}(jw) G_2(jw) \]

Note that \( G_1(jw) = G_2(jw) \). Thus when the theorem of Appendix C is applied to equation (1) one obtains

\[ <\Phi_{xy}(jw)> = D_1(jw) \left\{ \Phi_{xy}(jw) G_1(jw) \right\} + D_2(jw) \left\{ \Phi_{xy}(jw) G_2(jw) \right\} \]

\[ = 2 \text{Re}[D_1(jw)] \left\{ \Phi_{xy}(jw) \text{Re}[G_1(jw)] \right\} \]

\[ - 2 \text{Im}[D_1(jw)] \left\{ \Phi_{xy}(jw) \text{Im}[G_1(jw)] \right\} \]

\( D_1(jw) \) is called a spectral window and because it is periodic in \( \omega \) it can lead to aliasing (i.e., the leaking of power in one frequency band into the power estimates at other frequencies, see Ref. 8). The real part of the window \( D'(jw) \) behaves much like the QOA window of Ref. 8 except that it has alternate positive and negative peaks. The imaginary part is antisymmetric. The side lobes which exist can be suppressed by introducing Hanning into the computational process and this was done as follows.

Replace \( \text{PAR}_{xy}(n\Delta \tau) \) by

\[ \frac{1}{2} \left\{ 1 + \cos \left( \frac{\pi[n+\frac{1}{2}]\Delta \tau}{T_m} \right) \right\} \text{PAR}_{xy}(n\Delta \tau) \]
where \( \tau_m = 100 \Delta \tau \) and replace \( \text{PAR}_{yx}(n \Delta \tau) \) by

\[
\frac{1}{\tau_m} \left\{ 1 + \cos\left( \frac{\pi(n+\frac{1}{2}) \Delta \tau}{\tau_m} \right) \right\} \cdot \text{PAR}_{yx}(n \Delta \tau)
\]

Thus with this Hanning factor present the expected value of the power spectral density estimate becomes (after applying the theorem of Appendix C)

\[
<\Phi_{xy/EST}(jw)>_H = \text{FT} \left\{ \frac{1}{\tau_m} (1+\cos\left( \frac{\pi \Delta \tau}{\tau_m} \right)) \right\} * <\Phi_{xy/EST}(jw)>
\]

where

\[
\text{FT} \left\{ \frac{1}{\tau_m} (1+\cos\left( \frac{\pi \Delta \tau}{\tau_m} \right)) \right\} = \delta(w-\frac{\pi}{\tau_m}) + \frac{1}{\tau_m} \delta(w) + \frac{1}{\tau_m} \delta(w+\frac{\pi}{\tau_m})
\]

\[
= H(w)
\]

The resulting spectral window \((H(w)*D_1(jw))\) has a central lobe pattern for the real part similar to the \(Q_{2A}\) window in Ref. 8.

When \(G_1(jw)\) is computed for the conditions applicable to the present experiment \(b = 0.05 \text{ sec}^{-1}, T = 100 \text{ sec}, 2 \mu s < \Delta \tau < 10 \mu s\) it turns out that \(\text{Im}\left[G_1(jw)\right]\) is effectively zero and

\[
\text{Re}\left[G_1(jw)\right] = \frac{\sin(\omega \Delta \tau/2)}{\omega \Delta \tau/2}
\]

Thus the expected value of the estimate of power spectral density used in this study is taken to be represented by

\[
<\Phi_{xy/EST}(jw)>_H = 2 \frac{\omega \Delta \tau/2}{\sin \left( \frac{\omega \Delta \tau}{2} \right)} \cdot \text{Re}\left[D_1(jw)\right] * H(\omega) * \left\{ \Phi_{xy}(jw) \frac{\sin(\omega \Delta \tau/2)}{\omega \Delta \tau/2} \right\}
\]

The factor \(\frac{\omega \Delta \tau}{2} \sin \left( \frac{\omega \Delta \tau}{2} \right)\) is used as a simple means of partially removing the effect of \(\text{Re}\left[G_1(jw)\right]\). The removal will be exact as

\[
2 \text{Re}\left[D_1(jw)\right] * H(\omega) \text{ approaches } \delta(\omega).
\]

Qualitatively, the aliased spectral window \(D_1(jw) * H(\omega)\) has the characteristics shown in Fig. 41. Since only the real part is of importance to the
present power spectral density estimates, the following comments will be restricted
to this.

In order to avoid aliasing problems the distance between adjacent lobes

\((2\pi/\Delta \tau)\)

must be greater than the bandwidth of significant power in \(\Phi_{xy}(j\omega)\) (i.e.,

\(2\omega_0\)). (Note that both positive and negative frequencies must be considered).
Thus, when the central positive lobe is placed at the selected measurement fre­

quency \(\omega\), the adjacent window lobes must lie at frequencies containing insignifi­

cant amounts of power. In the case of auto power spectral densities the extent

of this problem can be seen from Fig. 42. Here the window is convoluted with an
even function in frequency. When the central lobe is placed at \(\pi/\Delta \tau\) the adjacent

negative lobe lies at \(-\pi/\Delta \tau\) and the power estimate formed by the convolution is

identically zero. Thus measurement frequencies should be kept well below \(\pi/\Delta \tau\).

At measurement frequencies below this critical value \(\Delta \tau\) must be selected to keep
the adjacent negative lobe to the left of \(-\omega_B\). To ensure this, one must make
certain at the highest measurement frequency, \(\omega_H\), that \(\omega_H + \omega_B < 2\pi/\Delta \tau\). In our
work we selected \(\omega_H = \pi/2\Delta \tau\) and \(3\pi/2\Delta \tau > \omega_B\). In some cases it may be useful to
reduce \(\omega_3\) by filtering the raw data before processing it. It should be remembered
that \(\Delta \tau\) also determines the bandwidth of the spectral window lobes (i.e., the reso­
lution of the power spectral density measurements) which may be approximated by

\(2\pi/100\Delta \tau\) (see Fig. 41). Thus as resolution is improved by increasing \(\Delta \tau\), \(\omega_H\) is
reduced.

The spectral data presented in this report were obtained by averaging
5 estimates found using \(T = 100\) sec. The mean and standard deviations of these
estimates are plotted on the graphs of power spectral density. These data are
plotted as one sided spectra i.e., twice the above \(\Phi_{xy}(j\omega)\) but plotted for \(\omega\)
positive only.
APPENDIX C

Let the Fourier transform of $x(t)$ be written as $\text{FT}\{x(t)\} = X(\omega)$. Then the transform and inverse transform pair are

$$X(\omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} x(t) e^{-j\omega t} \, dt$$

$$x(t) = \int_{-\infty}^{\infty} X(\omega) e^{j\omega t} \, d\omega$$

Thus, given $x_3(t) = x_1(t) \cdot x_2(t)$ then

$$X_3(\omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} x_1(t)x_2(t)e^{-j\omega t} \, dt$$

$$= \frac{1}{2\pi} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} x_1(t) X_2(\nu)e^{j\nu t} e^{-j\omega t} \, d\nu dt$$

Changing the order of integration

$$X_3(\omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} x_1(t)e^{-j(w-v)t} X_2(\nu) \, d\nu dt$$

$$= \int_{-\infty}^{\infty} x_1(w-v)X_2(\nu) \, d\nu$$

Thus

$$\text{FT} \left\{ x_3(t) \right\} = \text{FT}\left\{ x_1(t) \right\} \ast \text{FT}\left\{ x_2(t) \right\}$$
\[ D_1(j\omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{-j\omega \tau} \nabla^+_{100}(\tau) d\tau \]

\[ = \frac{\Delta \tau}{2\pi} \sum_{n=0}^{\infty} e^{-j\omega \frac{\Delta \tau}{2}} e^{-jn\omega \Delta \tau} \]

The sum is that of a geometric progression and is equal to

\[ \frac{a(r^n - 1)}{r - 1} \]

where \( a = 1 \)

\( r = e^{-j\omega \Delta \tau} \)

\( m = 100 \)

Thus

\[ D_1(j\omega) = \frac{\Delta \tau}{2\pi} e^{-j\omega \frac{\Delta \tau}{2}} \frac{-j100\omega \Delta \tau}{(e^{\omega \Delta \tau} - 1)} \]

\[ = \frac{\Delta \tau}{2\pi} e^{-j\omega \frac{\Delta \tau}{2}} \frac{-j100\omega \frac{\Delta \tau}{2}}{e^{\omega \frac{\Delta \tau}{2}} - e^{-\omega \frac{\Delta \tau}{2}}} \frac{-j100\omega \frac{\Delta \tau}{2}}{e^{\omega \frac{\Delta \tau}{2}} - e^{-\omega \frac{\Delta \tau}{2}}} \]

\[ = \frac{\Delta \tau}{2\pi} e^{-j100\omega \frac{\Delta \tau}{2}} \frac{\sin(100\omega \frac{\Delta \tau}{2})}{\sin(\omega \frac{\Delta \tau}{2})} \]

and expanding

\[ D_1(j\omega) = \frac{\Delta \tau \sin(100\omega \Delta \tau)}{4\pi \sin(\omega \frac{\Delta \tau}{2})} - j \frac{\Delta \tau \left(1 - \cos(100\omega \Delta \tau)\right)}{4\pi \sin(\omega \frac{\Delta \tau}{2})} \]
Similarly

\[ D_2(j\omega) = \frac{\Delta T}{2\pi} e^{\frac{j100\omega}{2}} \frac{\Delta T}{\sin(100\omega \frac{\Delta T}{2})} \frac{\Delta T}{\sin(\omega \frac{\Delta T}{2})} \]

This is part of a larger calculus expression and it is used in engineering contexts.
APPENDIX E

Given

\[ x_3(t) = \int_{-\infty}^{\infty} x_1(u)x_2(t-u) du \]

then

\[
\text{FT} \left\{ x_3(t) \right\} = \frac{1}{2\pi} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} e^{-j\omega t} x_1(u)x_2(t-u) dudt
\]

Proof

change the order of integration and substitute \( t - u = v \). Thus

\[
\text{FT} \left\{ x_3(t) \right\} = \frac{1}{2\pi} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} e^{-j\omega(v+u)} x_1(u)x_2(v) dvdu
\]

\[
= \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{-j\omega v} x_2(v) dv \int_{-\infty}^{\infty} e^{-j\omega u} x_1(u) du
\]

\[
= 2\pi \text{FT} \left\{ x_1(t) \right\} \text{FT} \left\{ x_2(t) \right\} .
\]
FIG. 2 PROTOTYPE TUNNEL JET SECTION
MIXING REGION

FIG. 3
FIG. 4 ' ORIGINAL WIND TUNNEL AERODYAMIC OUTLINE
FIG. 5 WIND TUBELE AERODYAMIC OUTLINE
FIG. 6 GENERAL VIEWS OF NEW TUNNEL
Columns

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<tr>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
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<tbody>
<tr>
<td>a</td>
<td>b</td>
<td>c</td>
<td>a</td>
</tr>
</tbody>
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55" (TYP)

2·3" (TYP)

5·5" (TYP)

1·2" (TYP)

66"

Rows

A
B
C
D
E
F
G
H

44"

FIG. 7 LAYOUT OF JET GRID SECTION
FIG. 8 UPSTREAM VIEW OF JET GRID
Airfoil Shape: NACA 0012, 18" Chord

FIG. 9 LAYOUT OF A JET ROW AND SUPPLY LINES
FIG. 10 LAYOUT OF JET SUPPLY SYSTEM
FIG. 11 TUNNEL GROWTH AND TEST SECTIONS
FIG. 12 BUTTERFLY-VALVE CONTROL CIRCUIT
FIG. 13 BUTTERFLY VALVE CONTROL PANEL
TRAVERSING GEAR
SCHEMATIC OF DATA HANDLING SYSTEM

FIG. 15
n = .16 VELOCITY PROFILE

FIG. 16
Data Taken On Tunnel Centreline

$U_G = 90\text{fps.}$
$n = .16$
Barrier Ht. = 5"
$s = 36"$

INTEGRAL SCALES

FIG. 17
Jet Exit Plane
Jet Centrelines

TUNNEL LAYOUT

FIG. 18

TUNNEL VELOCITY PROFILE

FIG. 19
TUNNEL TEMPERATURE

FIG. 20

FLOOR ROUGHNESS

FIG. 21
TURBULENCE INTENSITY PROFILES

FIG. 22

Centreline
$U_G = 90$ fps.
n = .16
$\delta = 36''$
$X = 5.5H$
- Barrier Ht. = 6''
- Barrier Ht. = 5''
- Barrier Ht. = 5''

Sale, Australia, Ref.(4)
Wind Tunnel, Ref.(12)
Rugby, U.K., Ref.(4)
Note: Positions Coincide
With Jet Exit Locations
At \( X = 0 \)

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<tr>
<th>( Y/W )</th>
<th>.042</th>
<th>.123</th>
<th>.208</th>
<th>.292</th>
<th>.375</th>
<th>.458</th>
<th>.542</th>
<th>.625</th>
<th>( 3'08 )</th>
<th>.792</th>
<th>.875</th>
<th>.960</th>
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<td>+</td>
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<td>+</td>
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</tr>
<tr>
<td>( .077 )</td>
<td>.063</td>
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<td>+</td>
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<td>+</td>
<td>+</td>
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</tr>
</tbody>
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MEASUREMENT POSITIONS LOOKING UPSTREAM

FIG. 23

\[ U/U_p \]

\[ U_p = 90 \text{ fps} \]
\[ U = \text{Desired Velocity} \]
\[ n = .16 \]
Barrier HL = 5"
B = 36"
X = 5.5H

LATERAL VELOCITY PROFILE

FIG. 24
WALL BOUNDARY LAYER

FIG. 25

IMPROVED LATERAL VELOCITY PROFILE

FIG. 26
JET INFLUENCE REGIONS

FIG. 27

SCALE VS. VELOCITY

FIG. 28
Level C
$U_G = 90 \text{ fps.}$
$n = 0.16$
Barrier Ht. = 5"

- $X = 5.5\,H$
- $X = 8.5\,H$

FIG. 29
C, Level E

$u_G = 90$ fps.

$n = .16$

Barrier Ht. = 5"

$X = 5.5$ H

$8 = 36"$

FIG. 33
\[ \frac{k \Phi_{ww}(k)}{u^2} \]

**FIG. 34**

- \( \bar{U}_G = 90 \text{fps} \)
- \( n = 0.16 \)
- \( \delta = 36^\circ \)
- Barrier \( H_t = 5'' \)
- \( \text{Level C} \)
- \( \triangle X = 8.5 \, H \, (\text{rear}) \)
- \( \bullet X = 5.5 \, H \, (\text{front}) \)
$U_g = 90$ fps.

$n = .16$

Barrier Ht. = 5"

$x = 85$ H

$\xi$ Level D

$s = 36"$

FIG. 37
Note: For Column Location
See Fig. 10

Scale vs Height

Fig. 38
CORRELATION ARGUMENT

FIG. 39

\[ r_n(t) \]

\[ h_{\tau}(t - q\Delta\tau) \]

\[ r_n(t) \text{ and } h_{\tau}(t - q\Delta\tau) \]

FIG. 40
SPECTRAL WINDOW $D_i^{\times H}$

FIG. 41
ESTIMATE OF $\phi_{xx}$

FIG. 42
The UTIAS subsonic wind tunnel has been modified into a multiple jet wind tunnel capable of producing simulated planetary boundary layer flows up to 36" thick. Maximum velocities of 100 fps can be obtained in a test section which is about 12 ft. long and 66 in. wide. Adjustment of jet velocities allows a range of velocity profiles to be produced. The tunnel can also be operated in a low-turbulence mode such that uniform flows up to about 100 fps can be achieved in the test section with a turbulence intensity of 2-3%.