A Note on the
Estimation of Longitudinal
and Lateral Aircraft Derivatives using
Semi-Empirical Methods

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Summary

A number of semi-empirical and theoretical techniques are used to estimate the longitudinal and lateral aerodynamic derivatives of a twin turbo prop aircraft, the H.P. Jetstream.

Estimates of the aircraft derivatives are performed for the aircraft in a typical cruise condition. Longitudinal parameters are calculated using classical theory and lateral derivatives are calculated using the semi-empirical methods of ESDU.

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Notation

The notation used is based on that proposed in ARC R&M 3562 (Hopkin, 1970). In particular for aeronormalised derivatives, 'dip' notation will be used, e.g. $\tilde{L}_v$.

Only some of the more important parameters are listed here and for quantities not listed here or defined in the text, the reader should refer to the appropriate ESDU item or other reference given.

- Aspect ratio of wing = $b^2/S$
- Effective aspect ratio of fin
- Tailplane aspect ratio
- Tail lift curve slope
- Span of wing
- Tailplane span
- Lift-curve slope of fin (ESDU Item 70011)
- Fin root chord
- Fin tip chord
- Tailplane tip chord
- Maximum body width
- Maximum body height
- Wing vertical position relative to body (defined in ESDU Item 73006)
- Body section heights at 0.25$b$, 0.75$b$
- Body height at fin root quarter chord section
- Body height at wing root quarter chord section
- Height of fin from fin root chord
- Distance of yaw axis from nose
- Body length
- Fin root quarter chord station aft of moment reference centre
- Maximum cross-sectional area of body
- Fin area
- Tailplane area
- Wing reference area
- Area of side elevation of body
- Fin arm
- Height of fin root chord from fuselage datum
- iv -

$z_F$  Height of centre of pressure position of fin from fin root chord.

$z_T$  Height of intersection of fin mounted tailplane from fin root chord.

$z_W$  Height of wing root quarter chord above local body centre line (positive for low wings).

$\Gamma$  Dihedral of wing

$A_q$  Quarter chord sweep angle

$\lambda$  $c_t/c_r$
1. Introduction

In this note, the aerodynamic derivatives for the H.P. Jestream aircraft are estimated at one flight condition using semi-empirical and theoretical techniques. Sufficient data for the aircraft is given to enable estimates to be made for all the significant longitudinal and lateral derivatives at a typical cruise condition of 150 kts, 6500 ft altitude.

Longitudinal derivatives are estimated using simple theory based on consideration of the lift, drag and pitching moment acting on the aircraft. Ref. 1 contains details of the derivation of formulae used.

Lateral derivatives are estimated using semi-empirical methods based on experimental test data. Theoretical corrections are used as guides to the effects of aircraft geometry, angle of attack, etc. Widely available sources of data are those of ESDU and DATCOM, and in this note, ESDU items will be used for the lateral derivatives. Wind axes (aerodynamic body axes) will be used except where stated but transformations to other axes systems can be carried out as detailed by Hopkin.

This note is only intended to give an introduction to the practical estimation of aerodynamic derivatives and so only major effects are considered. The aim has been to introduce the reader to the techniques used. Once familiar with these techniques, reference works such as ESDU or DATCOM can be consulted and worked from in order to make allowance for effects not covered in this introductory exercise.
2. Data

The subject aeroplane, shown in 3-view in Fig. 1, is the H.P. Jetstream, Series 100. Data and dimensions for the aircraft are given in Appendix C, along with datum flight conditions.

Additional information may be found in Ref. 5. The main parameters used from this reference, appropriate to the estimation exercises are:

\[(C_{La})_{WB} = 5.2/\text{rad}\]

\[a_1 = \frac{\partial C_{LT}}{\partial \alpha} = 4.3/\text{rad}\]

Rate of change of downwash, \[\frac{dc}{da} = 0.312\]

The reader is referred to Ref. 5 pp. 4-10 for geometrical data on the Jetstream collected from original authoritative drawings.
3. Estimation of Derivatives

3.1 General

The Jetstream is a turbo prop aircraft with wing mounted engines. In a detailed analysis of the aerodynamic derivatives, the engine effects would be accounted for. However, these are beyond the scope of the present exercise and so are not considered. The engine nacelles will have a small effect on the lateral derivatives and as a first approximation, the calculations described should provide a reasonable first estimate of the derivatives.

Propeller power effects on the longitudinal aerodynamic characteristics are mentioned in Ref. 5. Van Rooyen describes the influence of propellers on the lateral-directional stability of multi-engined aircraft, including the Jetstream.

Control derivatives are not estimated in this exercise. For details of these, the reader is referred to such works as Ref. 1.

3.2 Longitudinal Derivatives

3.2.1 Preliminary Calculations

Quantities such as \( C_D \), \( \frac{dC_m}{dC_L} \) etc will first be evaluated for the flight condition under consideration (details given in Appendix C).

3.2.1.1 \( C_D \)

From Ref. 5, at a \( C_L \) of 0.72, \( C_D = 0.079 \)

3.2.1.2 \( C_{La} \)

Ref. 5, Table 1.6.1, gives \( C_{La} = 5.85/\text{rad} \) for \( M = 0.2 \).
3.2.1.3 \( V_H \)

Tail volume coefficient, \( V_H = \frac{1}{c_S} \frac{T_S}{c_S} \)

\[ = 1.119 \]

3.2.1.4 \( \frac{dC_m}{dC_L} \)

Ref. 5 gives \( \frac{dC_m}{dC_L} = -0.236 \) including effects of fuselage and nacelles.

3.2.2 Calculations

The following longitudinal derivatives are estimated: \( \ddot{x}_u, \dddot{z}_u, \ddot{z}_w, \dddot{z}_w, \ddot{M}_w, \dddot{z}_q, \dddot{M}_q, \ddot{M}_w \). Derivations of the formulae used in estimating these can be found in Ref. 1.

For practical purposes some of the longituinals can be equated to zero and consequently are not estimated here. These are \( \ddot{x}_q, \dddot{M}_u \) and \( \dddot{z}_w \).

3.2.2.1 \( \ddot{x}_u \)

\[ \ddot{x}_u = -2C_D - V \frac{\partial C_D}{\partial V} \]

\[ \ddot{x}_u = -0.158 \] ignoring the contribution from \( V \frac{\partial C_D}{\partial V} \)

3.2.2.2 \( \ddot{z}_u \)

\[ \ddot{z}_u = -2C_L - V \frac{\partial C_L}{\partial V} \]

\[ \ddot{z}_u = -1.44 \] ignoring the contribution from second term.
3.2.2.3 \( \dot{X}_W \)

\[ \dot{X}_W = C_L - \frac{\partial C_D}{\partial \alpha} \]

\[ \dot{X}_W = 0.72 \quad \text{ignoring contribution from the second term.} \]

3.2.2.4 \( \dot{Z}_W \)

\[ \dot{Z}_W = -C_D - \frac{\partial C_L}{\partial \alpha} \]

\[ = -0.079 \times 5.85 \]

\[ \dot{Z}_W = -5.93 \]

3.2.2.5 \( \dot{M}_W \)

\[ \dot{M}_W = \frac{\partial C_m}{\partial \alpha} \]

\[ = \frac{\partial C_m}{\partial C_L} \cdot \frac{\partial C_L}{\partial \alpha} \]

\[ = -0.236 \times 5.85 \]

\[ \dot{M}_W = -1.381 \]

3.2.2.6 \( \dot{Z}_q \)

\[ \dot{Z}_q = -\frac{S_T}{s.c} \cdot \frac{l_T}{a_1} \cdot a_1 \]

\[ = -V_h a_1 \]

\[ = -1.119 \times 4.3 \]

\[ \dot{Z}_q = -4.8 \]
3.2.2.7 $\ddot{M}_q$

$$\ddot{M}_q = -\frac{S_T}{S} \cdot \frac{l_T^2}{c^2} \cdot a_1$$

$$= - V_H \cdot \frac{l_T}{c} \cdot a_1$$

$\ddot{M}_q = -17.34$

3.2.2.8 $\ddot{M}_w$

$$\ddot{M}_w = V_H \cdot \frac{l_T}{c} \cdot a_1 \cdot \frac{d\epsilon}{da}$$

$$= \ddot{M}_q \cdot \frac{d\epsilon}{da}$$

$\ddot{M}_w = -5.41$

3.2.3 Summary of estimates

$\ddot{x}_u = -0.158$
$\ddot{z}_u = -1.44$
$\ddot{x}_w = 0.72$
$\ddot{z}_w = -5.93$
$\ddot{M}_w = -1.381$
$\ddot{z}_q = -4.8$
$\ddot{M}_q = -17.3$
$\ddot{M}_w = -5.41$
3.3 Lateral Derivatives

3.3.1 Preliminary calculations

Using the data given in Appendix C for the Jetstream, we first of all derive parameters needed in the ESDU methods of estimating lateral derivatives.

The span of the aircraft, b, is used as the characteristic length in forming the aeronormalised derivatives (see Hopkin). 

3.3.1.1 Body

The moment reference centre (m.r.c.) will be taken at a distance \( l = 5.66 \) m from the nose.

Height of wing root quarter chord above local body centre - line, \( z_w = 0.8 \) m.

\( h_{BW} \), height of body at wing root quarter chord section

\[ h_{BW} = 1.981 \] m.

\( S_c \), maximum cross-sectional area of body

\[ S_c = 3.08 \text{ m}^2. \]

\( S_b \), area of side elevation of body

\[ S_b = 19.60 \text{ m}^2. \]

\( h_{1}, h_{2} \), body section heights at 0.25\( l_b \), 0.75\( l_b \)

\[ h_{1} = 1.78 \text{ m}, \ h_{2} = 1.0 \text{ m} \]

3.3.1.2 Wing

\( \lambda = 0.3333 \)

\( \Lambda_{c/4} = 1^\circ \)

For \( M = 0.23 \), \( \beta = \sqrt{1 - M^2} = 0.973 \) which gives the equivalent quarter chord sweep at \( M = 0.23 \) as

\[ \Lambda_E = \tan^{-1}\left(\frac{\tan \Lambda_{c/4}}{\beta}\right) \]
\[ \Lambda_E = 1^\circ \]
\[ (a_{10})_M = 5/\text{rad} \]
\[ \kappa = \frac{\beta (a_{10})_M}{2\pi} = 0.774 \]
\[ \frac{\beta A}{\kappa} = 12.57 \]

3.3.1.3 Tailplane

\[ \Lambda_E = 3.1^\circ \]
\[ \kappa = \frac{\beta (a_{10})_M}{2\pi} = 0.496 \]
\[ \frac{\beta A}{\kappa} = 10.99 \]

3.3.1.4 Fin

3.3.1.4.1 Lift curve slope

ESDU Item 70011 is used to estimate the fin lift curve slope using the following parameters.

\[ \Lambda_{c/4} = 42.7^\circ \]
\[ S_F = 5.23 \text{ m}^2 \]
\[ h_F = 2.51 \text{ m} \]
\[ z_{cr_F} = 0.69 \text{ m} \]
\[ h_{BF} = 1.181 \text{ m} \]
\[ A_F = 2.41 \]
\[ \lambda_F = 0.278 \]

\[ A_F \tan \Lambda_{\frac{4}{1}F} = A_F \tan \Lambda_{\frac{4}{1}F} - \left(\frac{1 - \lambda_F}{1 + \lambda_F}\right) \]
\[ A_F \tan \Lambda_\frac{1}{2}F = 1.659 \]
\[ \beta A_F = 2.345 \]

For \( \lambda = 0.25 \), ESDU Item 70011, Fig. 1c* gives
\[ \frac{1}{\Lambda} \frac{dC_L}{d\alpha} = 1.12/\text{rad} \]

so
\[ \left( \frac{dC_L}{d\alpha} \right)_F = (C_{L\alpha})_F = 2.70/\text{rad} \]

3.3.1.4.2 Centre of pressure position

Using ESDU Item 82010, the centre of pressure (c.p.) position on the fin is estimated using the following parameters

\[ z_T = 0.745 \text{ m} \]
\[ \frac{z_T}{h_F} = 0.297 \]
\[ b_T = 2.632 \]

Fig. 4 of ESDU Item 82010 gives
\[ \frac{z_T}{h_F} = 0.435 \]

Thus
\[ \frac{z_T}{h_F} = 1.092 \text{ m} \]

Empirical correction factors put the c.p. position at \( 0.85 \frac{z_T}{h_F} \) above and \( 0.7z_T \tan \Lambda_{\frac{1}{2}F} \) behind the fin root quarter-chord station.

*Figs. referred to in section 3.3 will be those in the ESDU item under discussion unless otherwise stated.
Moment arms of the c.p. relative to the moment reference centre are given by

\[ x_F = m_F + 0.7 \, z_F \tan \Lambda_4 = 5.98 \, \text{m} \]

\[ z_{cr_F} + 0.85 \, z_F = 1.618 \, \text{m} \]

3.3.2 Calculations

The lateral derivatives estimated are as follows:

Derivatives due to sideslip, \( \dot{Y}_V, \bar{N}_V \) and \( \bar{L}_V \).

Derivatives due to roll rate, \( \bar{L}_P \) and \( \bar{N}_P \). (\( \dot{Y}_P \) is negligible).

Derivatives due to yaw rate, \( \bar{L}_Y \) and \( \bar{N}_Y \). (\( \dot{Y}_Y \) negligible).

3.3.2.1 \( \dot{Y}_V \)

The main contribution to this derivative is from the fin.

3.3.2.1.1 Fin contribution

ESDU Item 82010 provides a method for estimating this and includes an allowance for interference between the body, wing, tailplane and fin.

\[ (\dot{Y}_V)_F = -J_B \left( J T J W (C_{La})_F \right) \frac{S_F}{S} \]

Fig.1 of ESDU 82010 gives \( J_B \), the effect of the body on the basic fin lift curve slope as a function of \( \frac{h_{BF}}{h_{BF} + h_F} \)

With \[ \frac{h_{BF}}{h_{BF} + h_F} = 0.319, \quad J_B = 1.10 \]
J allows for the interference effect of the tailplane on the sideforce derivative and for \( \frac{z_T}{h_f} = 0.297 \),
\[ b_T = 2.632, J_T = 0.98 \]
\[ h_F \]

\( J \) allows for the interference between the wing and the body-fin-tailplane assembly and for \( \frac{z_W}{h_{BW}} = +0.454 \)

it has the value 1.23.

So \( (Y_V)_F = -1.10(0.98)(1.23)(2.70)(5.23)/25.08 \)
\[ = -0.747 \]

3.3.2.1.2 Body contribution

\( (Y_V)_B \) is calculated using ESDU Item 79006. The maximum body width, \( d \), is taken as 1.981 m.

\( (Y_V)_B \) is given by:

\[ (Y_V)_B = - \left[ 0.0714 + 0.674 \frac{h^2}{S_b} + \frac{hbFF}{S_b}(4.95 \frac{|z|}{h} - 0.12) \frac{S_b + 0.006 |r|}{S} \right] \]

which includes allowances for body, wing-body and dihedral effects. Graphs in the above ESDU Item give values for the effect of wing height and body width function, \( F \), as 0.057 and the wing planform factor, \( F_w \), as 0.88.

Using these and the rest of the Jetstream data gives

\[ (Y_V)_B = -0.236 \]
3.3.2.1.3 Total $\ddot{Y}_V$

$$\ddot{Y}_V = (\ddot{Y}_V)_F + (\ddot{Y}_V)_B$$

$$= -0.747 + (-0.236)$$

$$\ddot{Y}_V = -0.983$$

Comments on the calculation of $\ddot{Y}_V$ for a complete aircraft are given in ESDU Item 82011. Difficulties with setting the centre of pressure position for the load on the fin can lead to inaccuracies in the estimate of $\ddot{Y}_V$.

3.3.2.2 $\ddot{N}_V$

This derivative is difficult to estimate as it is a balance between a large destabilising component from the body and a large stabilising contribution from the fin.

Interference effects are accounted for implicitly in the standard methods of estimation.

3.3.2.2.1 Fin contribution

ESDU Item 82010 again provides the estimate for the fin contribution to this derivative and is based on the value of $(\ddot{Y}_V)_F$ calculated in section 3.3.2.1.1.

For small $\alpha$, we can use the approximations $\cos \alpha = 1$ and $\sin \alpha = 0$. The formula for $(\ddot{N}_V)_F$ is then:

$$(\ddot{N}_V)_F = -(\ddot{Y}_V)_F (m_F + 0.7 \bar{z}_F \tan \Lambda_{4F})/b$$

giving

$$(\ddot{N}_V)_F = 0.282.$$
3.3.2.2.2 Body contribution

Preliminary quantities required are

\[
\frac{b^2}{S_b} = 9.09
\]

\[
\frac{h_1}{h_2} = 1.78
\]

Fig. 1 ESDU Item 79006 gives

\[
-\bar{N}_v \text{ mid} \frac{S_b}{S_b l_b} = 0.16
\]

So that, about a yaw axis through the mid-point of the body,

\[
\bar{N}_v \text{ mid} = -0.105
\]

Using the m.r.c. in section 3.3.1.1, at \( l = 5.66 \text{ m} \), we have

\[
(\bar{N}_v)_B = \bar{N}_v \text{ mid} + \frac{(l-l_b/2)}{b} \bar{Y}_v
\]

\[
(\bar{N}_v)_B = -0.090
\]

Effect of the engine nacelles is considered negligible.

3.3.2.2.3 Total \( \bar{N}_v \)

ESDU Item 82011 provides some move information on the calculation of \( \bar{N}_v \) for a complete aircraft. For the Jestream aircraft under consideration, we have

\[
\bar{N}_v = (\bar{N}_v)_F + (\bar{N}_v)_B
\]

\[
= 0.282 - 0.090
\]

\[
\bar{N}_v = 0.192
\]
3.3.2.3 $\dot{L}_V$

This is one of the most important derivatives with the main contribution coming from the wing. The wing geometrical features with the most effect are aspect ratio, dihedral, sweep and twist. Body effects and fin side force due to sideslip also make contributions to $\dot{L}_V$. As with $\dot{Y}_V$, difficulties in setting the c.p. position for the load on the fin can lead to inaccuracies in estimating the fin contribution. ESDU Item 81032 provides further information on the calculation of $\dot{L}_V$ for a complete aircraft.

3.3.2.3.1 Fin contribution

This can only be regarded as an approximation because the fin c.p. position is influenced considerably by the position of the fin/rudder with respect to the wing/body slipstream.

ESDU Item 82010 gives

$$\dot{L}_V)_F = (\ddot{Y}_V)_F \left[ (z_{cF} + 0.85 \bar{z}_F) \cos \alpha - (m_F + 0.7z_F \tan \lambda_{RF}) \sin \alpha \right]/b$$

Assuming, for low $\alpha$, that $\cos \alpha = 1$ and $\sin \alpha = 0$ gives:

$$\dot{L}_V)_F = -0.076$$

3.3.2.3.2 Wing contribution

The contribution of full span dihedral to $\dot{L}_V$ is covered in ESDU Item A.06.01.03. For $\lambda = 0.25$ Fig. 1b of this item gives $\frac{(L_V)_F}{\kappa \Gamma} = 0.0158$ and for $\lambda = 0.5$ Fig. 1c gives:

$$\frac{(L_V)_F}{\kappa \Gamma} = 0.0172$$

Thus for $\Gamma = 7^\circ$ and $\lambda = 0.333$ we have

$$\dot{L}_V) = -0.091$$
Wing planform contribution, \((\bar{L}_V)_p\) is covered in ESDU Item 80033. The method uses \(\Lambda_E\) for the half-chord line but in the case of slightly negative sweep for the half-chord line, as on the Jetstream, \(\Lambda_E = 0\) is used. Figures 1b and 1c of ESDU 80033 give values of \(\frac{(\bar{L}_V)_p}{C_L}\) for \(\lambda = 0.25, 0.50\)

and interpolation gives \(\frac{(\bar{L}_V)_p}{C_L} = -0.023 (\lambda = 0.3333)\)

Thus \((\bar{L}_V)_p = -0.017\)

For the current exercise we will neglect the effect of twist. Effects of flaps on \((\bar{L}_V)_w\) are dealt with in ESDU Item 80034, but for the current exercise we have flaps up.

\[
(\bar{L}_V)_w = (\bar{L}_V)_f + (\bar{L}_V)_p
\]

\[
= -0.091 - 0.017
\]

\[
= -0.108
\]

3.3.2.3.3 Body contribution

This consists of two parts (described in ESDU Item 73006), \((\bar{L}_V)_b\), the isolated body contribution and \((\bar{L}_V)_h\) the interference effect due to the vertical positioning of the wing on the body.

With the height of the equivalent body reference cross-sectional area, \(H\), equal to \(h\) and \(h_o\) (ESDU Item 73006) equal to \(z_w\) then the equivalent wing vertical position relative to the body, \(h'\) (shown as \(h\) in Item 73006), is given by

\[
\frac{h'}{H} = \frac{h_o}{H} - \kappa \Gamma
\]
Using Fig. 2 of Item 73006 to give $\kappa = 0.0098$ deg$^{-1}$ (N.B. not the $\kappa$ calculated in section 3.3.1.2), we obtain

$$\frac{h'}{H} = 0.331$$

From Fig. 1a for $A = 6$, $rac{\tilde{L}_V}{h} = 0.0165$

where $W$ is the width of the body reference cross-section, 1.98 m. From Fig. 1b, the aspect ratio function, $f(A)$, for $A = 10$, is 1.17 so

$$\tilde{L}_V = 0.0165 \times 2 \times 1.17$$

$$\tilde{L}_V = 0.039$$

The isolated body contribution, $(\tilde{L}_V)_b$, is given by

$$\frac{(\tilde{L}_V)_b}{a_b} = -0.014 \frac{1}{b} \frac{S_C}{S} \text{ degree}^{-1}$$

$$= -0.00145.$$ 

Say $a_b = 2^\circ$, then $(\tilde{L}_V)_b = -0.0029$ so the total body contribution $= (\tilde{L}_V)_h + (\tilde{L}_V)_b$ = 0.036

As before we will neglect the effects of turboprop nacelles and propeller arrangements for a conventional aircraft layout.

3.3.2.3.4 Total $\tilde{L}_V$

Summing the components estimated in the preceding sections we have:

$$\tilde{L}_V = (\tilde{L}_V)_F + (\tilde{L}_V)_T + (\tilde{L}_V)_P + (\tilde{L}_V)_b + (\tilde{L}_V)_h$$

$$= -0.076 + (-0.091) - 0.017 - 0.0029 + 0.039$$

$$\tilde{L}_V = -0.148$$
3.3.2.4 \( \tilde{N}_p \)

Contributions to \( \tilde{N}_p \) are estimated from the wing and fin. The fin makes a significant contribution which is difficult to estimate due to inaccuracies in estimating fin c.p. position.

3.3.2.4.1 Wing contribution

ESDU Item 81014 is used to estimate \( (\tilde{N}_p)_w \). We assume the yawing axis is at the wing aerodynamic centre giving \( \frac{x_{ac}}{b} = 0 \).

Wing taper has a minor effect and is accounted for in ESDU Item 81014 by using the sweepback related to the quarter chord line. The non-linear dependence of \( (\tilde{N}_p)_w \) on \( C_L \) at high \( C_L \) is ignored here and the variation of viscous drag coefficient with \( \alpha \), \( \frac{dC_D}{d\alpha} \), is taken to be zero.

Fig. 1 of ESDU Item 81014 presents a carpet plot of the linear contribution to \( (\tilde{N}_p)_w \):

\[
\left( \frac{(\tilde{N}_p)_w}{C_L} \right)_{0, \lambda_\alpha = 0} = \frac{A + 4}{A + 4 \cos \lambda_\alpha} \left[ 1 + 6 \left( 1 + \frac{\cos \lambda_\alpha}{12} \right) \frac{x_{ac}}{b} \tan \lambda_\alpha + \frac{\tan^2 \lambda_\alpha}{12} \right]_{C_L} \left( \frac{(N_p)_w}{C_L} \right)_{0, \lambda_\alpha = 0} - \frac{1}{8A} \left( \tan \lambda_\alpha + \frac{1}{\lambda_\alpha} \right) - \frac{1}{2A} \frac{x_{ac}}{b}
\]

For \( A = 10 \), we have

\[
\left( \frac{(\tilde{N}_p)_w}{C_L} \right)_{0} = -0.038
\]

so that for \( C_L = 0.72 \)

\( (\tilde{N}_p)_w = -0.0274 \)
3.3.2.4.2 Fin contribution

This may be estimated using the relationship

\[(N_p)_F = - \frac{X_F}{b} \cdot \frac{Z_F}{b} \cdot (Y_F)_F\]

\[-6 \cdot \frac{1.09}{15.85} \cdot \frac{(-0.747)}{15.85} = 0.0194\]

3.3.2.4.3 Total \(N_p\)

Summing the fin and wing components for \(N_p\) we have

\[N_p = (N_p)_w + (N_p)_F\]

\[-0.0274 + 0.0194 = -0.008\]

3.3.2.5 \(L_p\)

The main contribution to this derivative comes from the wing but an estimate will be made of the contribution from the tailplane. For this exercise, the factored contribution to \(L_p\) from the fin will be ignored as would be usual for initial estimates. \(L_p\) is important in considering rolling performance as well as dynamic stability.

3.3.2.5.1 Wing contribution

ESDU Item A.06.01.01 gives plots of \(-\frac{\beta L_p}{k}\) as a function of \(\frac{BA}{k}\), \(\lambda\) and \(\Lambda_E\).
For $\lambda = 0.25$, $-\frac{\beta L_p}{K} = 0.28$

and for $\lambda = 0.5$, $-\frac{\beta L_p}{K} = 0.285$

giving $-\frac{\beta L_p}{K} = 0.282$ for $\lambda = 0.3333$.

So with $\beta = 0.973$ and $K = 12.57$ we have:

$(\tilde{L}_p)_w = -0.224$

3.3.2.5.2 Tailplane contribution

This is estimated using ESDU Item A.06.01.01 as the $\tilde{L}_p$ of the isolated tailplane based on its own area. The result is then factored for summing with the wing contribution.

Using the appropriate data for the tailplane, we have

$\kappa = 0.496$ (using $a_1 = 3.2$/rad obtained from flight test in ref. 5).

$\frac{\beta A}{\kappa} = 10.99$

and $\Lambda_E = 3.1^\circ$

For $\lambda = 0.25$, $-\frac{\beta L_p}{K} = 0.26$

For $\lambda = 0.5$, $-\frac{\beta L_p}{K} = 0.28$

so for $\lambda = 0.41$, $-\frac{\beta L_p}{K} = 0.272$

giving $(\tilde{L}_p)_T = -0.139$
This result must be factored to take into account the relative sizes of the tailplane and wing before summing with \( \dot{L}_w \).

The tailplane contribution to \( \ddot{L}_p \) is thus given by

\[
\dot{L}_p = K \frac{S}{S} b \left( \frac{b}{b} \right)^2 \dot{L}_p
\]

where \( K \) is to account for induced velocity effects and will be taken as \( \frac{1}{4} \).

So \( \dot{L}_p = 0.027(\ddot{L}_p) \)

\[
= -0.0037
\]

3.3.2.5.3 Total \( \ddot{L}_p \)

Using the results of the preceding sections we have:

\[
\ddot{L}_p = (\ddot{L}_w) + (\ddot{L}_p)_T
\]

\[
= -0.224 - 0.004
\]

\[
\ddot{L}_p = -0.228
\]

3.3.2.6 \( \ddot{N}_r \)

The main contribution to \( \ddot{N}_r \) is from the fin but the body also makes a significant contribution.

3.3.2.6.1 Fin contribution

The dynamic derivative \( (\ddot{N}_r)_F \) can be expressed in terms of the static derivative \( (\ddot{N}_v)_F \) by assuming that the yaw rate produces a local sideslip velocity at the fin equal to the product of the yaw rate and the yawing moment arm of the fin sideforce. ESDU Item 82017 then gives (assuming for low \( a \), that \( \cos a = 1, \sin a = 0 \)).

\[
(\ddot{N}_r)_F = -(\ddot{N}_v)_F \left( m_F + 0.7 z_F \tan \Lambda_F \right) / b
\]

\[
= -(0.282)(5.977)/15.85
\]

\[
= -0.106
\]
3.3.2.6.2 Body contribution

Information on the contribution of the body to $N_r$ is scarce. Slender body theory can be adapted.

A reasonable estimate, bearing in mind the relatively large fuselage side area for the Jetstream, would be to put it equal to or slightly under the fin contribution, say

$$(N_r)_B = -0.09$$

3.3.2.6.3 Wing contribution

ESDU Item 71017 gives $(N_r)_w$ as the sum of two components, one due to the asymmetric distribution of the profile drag and the other due to the asymmetric distribution of the lift-dependent drag due to the trailing vortex system.

$$(N_r)_w = \frac{\dot{N}_r}{C_{Do}} C_{Do} + \frac{\dot{N}_r}{C_L^2} C_L$$

Fig. 1a gives $\left(\frac{\dot{N}_r}{C_{Do}}\right)_{\lambda = 1} = -0.167$

For $\lambda = 0.3333$, $\left(\frac{\dot{N}_r}{C_{Do}}\right)_{\lambda = 0.333} = 0.75$

$\left(\frac{\dot{N}_r}{C_{Do}}\right)_{\lambda = 1}$

So $\left(\frac{\dot{N}_r}{C_{Do}}\right)_{\lambda = 0.333} = -0.167 \times 0.75$

$= -0.125$

Which, for $C_{Do} = 0.032$ means

$\dot{N}_r = -0.0040$
Fig. 2b, for $\lambda = 0.25$ gives
\[
\frac{\dot{N}_{rv}}{C_L^2} = -0.005
\]
and for $\lambda = 0.5$
\[
\frac{\dot{N}_{rv}}{C_L^2} = -0.006
\]
so for $\lambda = 0.3333$
\[
\frac{\dot{N}_{rv}}{C_L^2} = -0.0053
\]
giving $\dot{N}_{rv} = -0.00275$ for $C_L = 0.72$.

Thus $(\dot{N}_r)_w = \dot{N}_{ro} + \dot{N}_{rv}$
\[
= -0.004 - 0.0028
\]
\[
(\dot{N}_r)_w = -0.0068
\]

3.3.2.6.4 Total $\dot{N}_r$

Summing the fin, body and wing contributions gives:
\[
\dot{N}_r = (\dot{N}_r)_F + (\dot{N}_r)_B + (\dot{N}_r)_w
\]
\[
= -0.106 - 0.09 - 0.007
\]
\[
\dot{N}_r = -0.203
\]

3.3.2.7 $\ddot{L}_r$

The wing provides the main contribution to $\ddot{L}_r$ for which ESDU Item 72021 provides a semi-empirical method of estimation. The fin also provides a significant contribution. The tailplane contribution to $\ddot{L}_r$ could be estimated in a similar way to the wing contribution but the accuracy of the estimation is
questionable due to interference effects from the wing, fin and body.

3.3.2.7.1 Wing contribution

ESDU Item 72021 provides the means of estimating contributions to \( (\bar{L}_w) \) from planform, dihedral, twist and flaps.

For \( \Lambda_h = 1^\circ \) Fig. 1b gives a value of sweep factor, \( g(\Lambda_h) = 1.01 \). From Fig. 1a,

\[
\frac{1}{g(\Lambda_h)} \cdot \frac{(\bar{L}_\text{ro})_p}{C_L} = 0.107
\]

So

\[
\frac{(\bar{L}_\text{ro})_p}{C_L} = 0.108
\]

At \( C_L = 0.72 \), \( (\bar{L}_\text{ro})_p = 0.078 \)

The dihedral contribution is estimated from Fig. 3.

\( \gamma = +7^\circ \)

For \( \Lambda_h = 1^\circ \),

\[
\frac{(\bar{L}_\text{ro})_\gamma}{\gamma} = 0.00005 \text{ deg}^{-1}
\]

so

\[
(\bar{L}_\text{ro})_\gamma = 7 \times 0.00005
\]

\[
= 0.00035
\]

If the dihedral contribution had been significant the sweep factor from Fig. 1b could also have been applied.
Fig. 4 ESDU Item 72021 shows the effects of twist on $(\dot{L}_r)_{\nu}$. Washout, $\epsilon = +2^\circ$ (convention as in ESDU Item 72021).

\[
(\dot{L}_r)_{\nu} = -0.0019 \text{ deg}^{-1}
\]

\[
(\ddot{L}_r)_{\nu} = -0.0038
\]

So $(\dot{L}_r)_{\nu} = (\dot{L}_r)_{\nu} + (\dot{L}_r)_{\nu} + (\ddot{L}_r)_{\nu}

= 0.078 + 0.00035 - 0.0038

$(\dot{L}_r)_{\nu} = 0.075$

3.3.2.7.2 **Fin contribution**

This is expressed in terms of the static derivative $(\dot{L}_v)_F$ and the moment arm to an estimated fin c.p. position by assuming that yaw rate produces a local sideslip at the fin given by yaw rate multiplied by the yawing moment arm of the fin sideforce.

For low $\alpha$, with $\cos \alpha = 1$ and $\sin \alpha = 0$. ESDU Item 82017 gives:

\[
(\dot{L}_r)_F = -(\dot{L}_v)_F (m_F + 0.7 z_F \tan \Lambda_{46})/b
\]

\[
= -(-0.076)(5.98)/15.85
\]

\[
= 0.0288
\]

3.3.2.7.3 **Total $\dot{L}_r$**

Using the results of the preceding sections, we have

\[
\dot{L}_r = (\dot{L}_r)_F + (\dot{L}_r)_{\nu}
\]

\[
= 0.0288 + 0.075
\]

\[
\dot{L}_r = 0.104
\]
3.3.3 Summary of estimates

Table 1 shows a summary of the ESDU Items used to calculate the contributions to the various lateral derivatives. The following is a summary of the lateral derivatives estimated for the Jetstream aircraft at an altitude of 2000 m, speed 77 m/s (150 kts) and $C_L = 0.72$, clean configuration:

\[
\begin{align*}
\hat{\gamma}_V &= -0.983 \quad \left[ -0.346/-0.467 \right] \\
\hat{N}_V &= 0.192 \quad \left[ 0.182/ 0.190 \right] \\
\hat{L}_V &= -0.148 \quad \left[ -0.098/-0.124 \right] \\
\hat{N}_P &= -0.008 \quad \left[ -0.124/-0.069 \right] \\
\hat{L}_P &= -0.228 \quad \left[ -0.245/-0.317 \right] \\
\check{N}_r &= -0.203 \quad \left[ 0.207/-0.135 \right] \\
\check{L}_r &= 0.104 \quad \left[ 0.117/ 0.133 \right]
\end{align*}
\]

Values in the square brackets are some typical values obtained using parameter identification techniques by Van Rooyen\textsuperscript{6} for flaps up and flaps down cases respectively. Inaccuracies in the estimate of $\hat{\gamma}_V$ will lead to inaccuracies in $\hat{N}_P$ (see section 3.3.2.4.2). Apart from these two derivatives, the other predictions are quite close to the typical flight values shown.
References


2. Engineering Sciences Data Unit. Aerodynamics sub-series to the aeronautical series of data items.


<table>
<thead>
<tr>
<th>Derivative</th>
<th>Contribution from:</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Wing</td>
</tr>
<tr>
<td>$\dot{Y}_V$</td>
<td>-</td>
</tr>
<tr>
<td>$\dot{N}_V$</td>
<td>-</td>
</tr>
<tr>
<td>$\dot{L}_V$</td>
<td>A.06.01.03 80033</td>
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<tr>
<td>$\ddot{N}_P$</td>
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</tr>
<tr>
<td>$\dot{L}_P$</td>
<td>A.06.01.01</td>
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<td>$\ddot{N}_r$</td>
<td>71017</td>
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<tr>
<td>$\ddot{L}_r$</td>
<td>72021</td>
</tr>
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Table 1. ESDU Items used in calculating lateral derivatives.
Fig. 1 Jetstream General Arrangement
Appendix A - Definition of Derivatives

(See Hopkin^4)

A.1 **Longitudinal**

\[
\begin{align*}
\ddot{X}_u &= \frac{\dot{X}_u}{\frac{1}{2} \rho V S} \\
\ddot{X}_n &= \frac{\dot{X}_n}{\frac{1}{2} \rho V^2 S} \\
\ddot{Z}_u &= \frac{\dot{Z}_u}{\frac{1}{2} \rho V S} \\
\ddot{Z}_n &= \frac{\dot{Z}_n}{\frac{1}{2} \rho V^2 S} \\
\ddot{X}_w &= \frac{\dot{X}_w}{\frac{1}{2} \rho V S} \\
\ddot{M}_n &= \frac{\dot{M}_n}{\frac{1}{2} \rho V^2 S c} \\
\ddot{Z}_w &= \frac{\dot{Z}_w}{\frac{1}{2} \rho V S} \\
\ddot{M}_w &= \frac{\dot{M}_w}{\frac{1}{2} \rho V S c} \\
\ddot{q}_w &= \frac{\dot{q}_w}{\frac{1}{2} \rho V S c} \\
\ddot{M}_q &= \frac{\dot{M}_q}{\frac{1}{2} \rho V S c} \\
\ddot{M}_w &= \frac{\dot{M}_w}{\frac{1}{2} \rho S c^2}
\end{align*}
\]
A.2 Lateral

\[ Y_V = \frac{Y_V}{\frac{1}{2} \rho V S} \]

\[ N_V = \frac{N_V}{\frac{1}{2} \rho V S b} \]

\[ N_P = \frac{N_P}{\frac{1}{2} \rho V S b^2} \]

\[ N_r = \frac{N_r}{\frac{1}{2} \rho V S b^2} \]

\[ L_V = \frac{L_V}{\frac{1}{2} \rho V S b} \]

\[ L_P = \frac{L_P}{\frac{1}{2} \rho V S b^2} \]

\[ L_r = \frac{L_r}{\frac{1}{2} \rho V S b^2} \]
Appendix B - Linearised Equations of Motion

B.1 Longitudinal

\[ \dot{u} = \frac{k_p V S}{m} \left( \ddot{x}_u \bar{u} + \ddot{x}_w \bar{w} + \ddot{x}_q \bar{q} + n \ddot{x}_n \right) - \frac{q}{V} \cos \phi \cdot \dot{\theta} \]

\[ \dot{w} = \frac{k_p V S}{m} \left( \ddot{z}_u \bar{u} + \ddot{z}_w \bar{w} + \ddot{z}_q \bar{q} + n \ddot{z}_n \right) + q \cos \alpha \]

\[ \dot{q} = \frac{k_p V^2 S c}{I_y} \left( \ddot{M}_u \bar{u} + \ddot{M}_w \bar{w} + \ddot{M}_q \bar{q} + n \ddot{M}_n \right) \]

\[ \theta = q \]

where \( q_\gamma = \frac{q_c}{V} \)

and \( - \) indicates the quantity is divided by \( V \).

B.2 Lateral

\[ \dot{V} = \frac{k_p V S}{m} \left( \ddot{y}_V \bar{V} + \ddot{y}_\xi \bar{\xi} + \ddot{y}_\zeta \bar{\zeta} \right) + p \sin \alpha - r \cos \alpha + \frac{q}{V} \cos \phi \cdot \cos \phi \cdot \phi \]

\[ \dot{p} = \frac{k_p V^2 S b}{I_x} \left( \ddot{L}_V \bar{V} + \ddot{L}_p \bar{p} + \ddot{L}_r \bar{r} + \ddot{L}_\xi \bar{\xi} + \ddot{L}_\zeta \bar{\zeta} \right) + \frac{I_{xz}}{I_x} \bar{r} \]

\[ \dot{r} = \frac{k_p V^2 S b}{I_z} \left( \ddot{N}_V \bar{V} + \ddot{N}_p \bar{p} + \ddot{N}_r \bar{r} + \ddot{N}_\xi \bar{\xi} + \ddot{N}_\zeta \bar{\zeta} \right) + \frac{I_{xz}}{I_x} \bar{p} \]

\[ \delta = p \]

with \( \dot{Y}_\xi = \frac{Y_\xi}{2 \rho V^2 S} \) (similarly \( \dot{Y}_\zeta \))

and \( \dot{L}_\xi = \frac{L_\xi}{2 \rho V^2 S b} \) (similarly \( \dot{L}_\zeta \), \( \dot{N}_\xi \), \( \dot{N}_\zeta \))
Appendix C - Data and Dimensions for Jetstream

C.1 Datum Flight Condition

Altitude = 2000 m
Mass, m = 5500 kg
c.g. position = 24%c
U_o = 77 m/s
C_L = 0.72
Flaps up, landing gear up.

C.2 Inertias

I_x = 22050 kg m^2
I_y = 40080 kg m^2
I_z = 69491 kg m^2
I_xz = 9655 kg m^2

(See Ref. 6)
### DATA AND DIMENSIONS

**JETSTREAM 100 SERIES**

**G-AXUII AND G-AXUM**

**TURBOMECA ASTAZOU XIV ENGINES**

#### WING

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<tr>
<th>Parameter</th>
<th>Value</th>
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<tbody>
<tr>
<td>Span b</td>
<td>15.850 m (52 ft.)</td>
</tr>
<tr>
<td>Gross Area S</td>
<td>25.084 m² (270 ft.²)</td>
</tr>
<tr>
<td>Mean Aerodynamic Chord c</td>
<td>1.717 m (5.63 ft.)</td>
</tr>
<tr>
<td>Taper Ratio Tip/Centre Line</td>
<td>0.333</td>
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<tr>
<td>Sweep of 30% Chord Line</td>
<td>70°</td>
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<tr>
<td>Dihedral Angle</td>
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</tr>
<tr>
<td>Root Datum Chord Section</td>
<td>N.A.C.A. 63A418 (Modified)</td>
</tr>
<tr>
<td>Tip Chord Section</td>
<td>N.A.C.A. 63A412 (Modified)</td>
</tr>
<tr>
<td>Root Chord Setting to Longitudinal Datum</td>
<td>+ 2°</td>
</tr>
<tr>
<td>Wing Twist about 30% Chord</td>
<td>- 2° (Washout)</td>
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#### FUSELAGE

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<thead>
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<th>Value</th>
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<tbody>
<tr>
<td>Length</td>
<td>13.35 m (43.8 ft.)</td>
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<tr>
<td>Maximum Diameter</td>
<td>1.98 m (6.5 ft.)</td>
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#### TAILPLANE

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<tr>
<td>Span b</td>
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<tr>
<td>Gross Area S_T</td>
<td>7.785 m² (83.8 ft.²)</td>
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<tr>
<td>Mean Aerodynamic Chord</td>
<td>1.25 m (4.105 ft.)</td>
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<tr>
<td>Tail Arm 1_T</td>
<td>6.184 m (20.29 ft.)</td>
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<tr>
<td>Tailplane Setting to Fuselage Datum</td>
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#### CONTROL AREAS AND GEARINGS

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<tr>
<td>Total Aileron Area Aft of Hinge</td>
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<td>Aileron Mean Chord Aft of Hinge</td>
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<td>Aileron Travel</td>
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<td>Aileron Gearing</td>
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<td></td>
<td>(0.686 rads/ft.)</td>
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<td>Total Elevator Area Aft of Hinge</td>
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<td>Elevator Mean Chord Aft of Hinge</td>
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<td>Elevator Travel</td>
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<td>Elevator Gearing</td>
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<td>Rudder Gearing</td>
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