THE INFLUENCE OF FRESNEL NUMBER AND SECONDARY GEOMETRIC PARAMETERS
ON THE DIFRACTED FIELD OF PLANAR LAMINA

by

R. L. M. Wong and Zhangwei Hu

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Abstract

The approximate diffraction formalism of Keller and others is re-examined and cast into three explicit terms, each of which is given a physical interpretation. The new approximate forms are valid for Fresnel numbers as small as .05. In this regime the often neglected secondary parameters make significant contributions. The scheme is applied to several test cases and performs very well.
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1. INTRODUCTION

The theory of the diffraction of a spherical sound wave by a semi-plane is basically necessary for the purpose of predicting (and controlling) the sound level behind barriers.

A rigorous solution was obtained by Carslaw [1]. It is an integral representation with no restriction on source location and can be solved only by using a computer. For this reason it is not suitable for many practical applications. On the other hand, some approximate and empirical solutions are available. Among them, Maekawa's empirical curve [2] is widely used. Similar to it, Kurze et al [3] gave a formula for predicting attenuation,

\[ A = 5 \, \text{dB} + 20 \log \frac{(2\pi N)^{1/2}}{\tanh(2\pi N)^{1/2}} \]

Here \( N \) is the Fresnel number, \( N = (R_1 - R)/(\lambda/2) \) [see Eq. (3) and Fig. 1]. Both of these formulas are very simple, but they just present the attenuation by the barrier as a function of \( N \). Actually it also depends on three other parameters, as will be seen later (Section 2). The discrepancy of the measured data (when plotted as a function of \( N \)), as shown in Fig. 2.17 of Ref. 2 by Maekawa, is mainly due to the effect of implicit secondary parameters. Moreover, if the source and observer are not far away from the barrier, the error would be greater.

The fact, as has been pointed out above, implies that secondary parameters should be taken into account for calculating the sound attenuation by the barrier. We use the numerical results of Carslaw's rigorous solution to seek a better understanding of the effect of secondary parameters and try to express this effect by accounting for all four parameters explicitly.

Several approximate solutions which take secondary parameters into account have already been obtained [3-6]. Among them, Keller's formula deduced from
his geometrical theory of diffraction gives an explicit relation between the attenuation and both Fresnel number and secondary parameters. Moreover it is very simple and easy to use. After closely analysing the numerical results we found Keller's formula to be a good approximation to the rigorous solution for \( N \geq 1 \). In this paper, Keller's formula was modified for \( N \leq 1 \). The modified formula agrees very well with the rigorous solution for all \( N \).

In the last section the above formulae are applied to solve certain practical problems which involve the interference of the sound waves.

2. RIGOROUS SOLUTION

Consider a pure tone spherical wave which is emitted from the source point \( p_o(r_o, \theta_o, z_o) \) (Fig. 1). Let \( \phi \) be the total velocity potential at the observation point \( p(r, \theta, z) \) in the diffracted field by a semi-plane, which must satisfy the Helmholtz equation and the rigid boundary condition \( \partial \phi / \partial n = 0 \).

The rigorous solution given by Carslaw [1] is an integral representation in terms of a cylindrical coordinate (Fig. 1):

\[
\phi = ik \int_{-m}^{m} \frac{H_1^{(1)}(\mu^2 + kR)}{\sqrt{\mu^2 + 2kR}} \, d\mu + ik \int_{-m'}^{m'} \frac{H_1^{(1)}(\mu^2 + kR')}{\sqrt{\mu^2 + 2kR'}} \, d\mu
\]  

(1)

where

\[
m = \text{sgn}(\pi - \theta + \theta_o) \sqrt{k(R_l - R)}
\]

\[
m' = \text{sgn}(\pi - \theta - \theta_o) \sqrt{k(R_l - R')}
\]

\( R \) = direct distance from the source to the observer

\[
= [r^2 + r_o^2 + (z - z_o)^2 - 2rr_o \cos(\theta - \theta_o)]^{1/2}
\]

\( R' \) = distance from image source to the observer

\[
= [r^2 + r_o^2 + (z - z_o)^2 - 2rr_o \cos(\theta + \theta_o)]^{1/2}
\]

\( R_l \) = shortest distance from the source to the observer

\[
= [(r + r_o)^2 + (z - z_o)^2]^{1/2} = [R^2 + 4rr_o \cos^2(\theta - \theta_o/2)]^{1/2}
\]
sgn(x) = \begin{cases} 
+1 & \text{for } x > 0 \\
-1 & \text{for } x \leq 0 
\end{cases}

Here, \( H_1^{(1)} \) is the first order Hankel function of the first kind and \( k \) is the wave number. The first integral in Eq. (1) is the velocity potential due to the real source, while the second integral is due to the image source.

Let \( \phi_0 \) be the unshielded velocity potential, \( \phi_0 = e^{ikR/R} \), then the attenuation \( A \) by the barrier is defined as

\[
A = -20 \log \left| \frac{\phi}{\phi_0} \right| \quad (\text{dB})
\]

and

\[
A = -20 \log \left| kR \left\{ \int_{-m}^{\infty} \frac{H_1^{(1)}(\mu^2 + kR)}{\sqrt{\mu^2 + 2kR}} \, d\mu + \int_{-m'}^{\infty} \frac{H_1^{(1)}(\mu^2 + kR')}{\sqrt{\mu^2 + 2kR'}} \, d\mu \right\} \right| \quad (\text{dB})
\]

(2)

From this equation it is found that the attenuation by the barrier depends on nondimensional variables \( kR, kR_1, kR' \) and angles \( \theta, \theta_0 \). But here \( kR' \) is a dependent variable and can be deduced from the other four parameters, namely

\[
kR' = \left\{ (kR)^2 + \frac{(kR_1)^2 - (kR)^2}{2 \cos^2 \left[ (\theta - \theta_0)/2 \right]} \left[ \cos(\theta - \theta_0) - \cos(\theta + \theta_0) \right] \right\}^{1/2}
\]

If the Fresnel number \( N \),

\[
N = \frac{2}{\lambda} (R_1 - R) = \frac{1}{\pi} (kR_1 - kR)
\]

(3)

is introduced there are only four independent variables, \( N, kR, \theta, \theta_0 \).

It is known from Eqs. (1) and (2) that the coordinates of source and observer relative to barrier, \( r, r, z-z_0, \theta \) and \( \theta_0 \) define the above four parameters. Note that some of them appear in \( N, kR \) in groups, namely they are combined into two groups \( (r + r_0)^2 + (z - z_0)^2 \) and \( rr_0 \). It means that the attenuation depends on these groups, except for \( \theta \) and \( \theta_0 \), not on these three coordinates individually. Therefore for different values of
$r, r_0$ and $z - z_0$ with the value of two groups $(r + r_0)^2 + (z - z_0)^2$ and $r r_0$ and hence the value of $kR$ and $N$ remaining constant, a given degree of attenuation can be achieved. A sample is illustrated in Fig. 2, where the plane $\theta_0$ is unfolded to the plane $\theta$. The nondimensional shortest distance therefore is a straight line in this unfolded plane.

Before analysing the effect of the parameters on the attenuation it should be noted that the value of parameters $N$, $kR$, $\theta - \theta_0$, and $\theta_0$ cannot be taken arbitrarily for there are restrictions in geometry. For example, for given $kR$, $\theta$, $\theta_0$ there is a maximum value of $N$. It can be shown that $N_{\text{max}}$ occurs at $z-z_0 = 0$ and $r/r_0 = 1$. Hence, from Eq. (3), we have

$$N_{\text{max}} = \frac{kR}{\pi} \left( \sqrt{\frac{2}{1 - \cos(\theta - \theta_0)}} - 1 \right)$$  \hspace{1cm} (4)

Similarly, for given $N$, $\theta$ and $\theta_0$ there is a minimum value of $(kR)_{\text{min}}$

$$(kR)_{\text{min}} = \frac{\pi N}{\sqrt{\frac{2}{1 - \cos(\theta - \theta_0)}} - 1}$$ \hspace{1cm} (5)

and

$$|\theta - \theta_0|_{\text{min}} = \cos^{-1} \left[ 1 - \frac{2}{\left( \frac{\pi N}{kR} + 1 \right)^2} \right]$$ \hspace{1cm} (6)

for given $N$, $kR$, $\theta_0$. These limitations are indicated in some of the later figures.

For the purpose of showing a brief view of the effect of four parameters on sound attenuation some numerical results of the rigorous solution are plotted in Figs. 3 to 5. It is clear that the secondary parameters $kR$, $\theta_0$, $\theta - \theta_0$ have a noticeable effect on attenuation besides the strong parameter $N$, the Fresnel number. The attenuation predicated according to Maekawa's curve [2] is also plotted in Fig. 3. It shows that there is the possibility of producing noticeable error if the effect of secondary parameters is ignored.
After looking at a large quantity of numerical results of the rigorous solution we found that the total attenuation $A$ can be broken down into three parts $A_1$, $A_2$ and $A_3$ as shown in Fig. 6:

$$A = A_1 + A_2 + A_3 \quad (7)$$

$A_1$ is the attenuation corresponding to $kR \to \infty$ and $\theta_0 = 0$ (or $\theta = 360^\circ$). The source or observer is located on the barrier. From Fig. 4 it can be seen that for $kR$ greater than 1000 the attenuation remains almost constant with increasing $kR$, and $A_1$ (according to definition now $A = A_1$) is only a function of the Fresnel number $N$. Actually Maekawa's curve is just 3 dB above the value of $A_1$. Later it will be seen that 3 dB is the average value of the angular correction $A_3$.

$A_2$ is the correction for finite $kR$, i.e. the difference of attenuation between the finite and infinite $kR$ for given Fresnel number $N$ and $\theta_0 = 0$ (Fig. 4). It was found that $A_2$ depends on $R_1/R = (1 + (\pi N/kR))$, i.e. on both $kR$ and $N$. Because for the same path difference $R_1 - R$, namely the same Fresnel number $N$, the shorter the nondimensional direct distance $kR$ the larger the increase of spherical divergence of the diffracted ray relative to the direct ray. $A_2$, therefore, increases with decreasing $kR$ and increasing $N$ and it can be very large when both source and observer are near the barrier. The reverse holds. If $kR$, the nondimensional direct distance between source and observer, is very large (say $kR > 100$ for $N < 5$), it has no significant effect on the sound attenuation (refer to Fig. 4).

$A_3$ is the angular correction to the attenuation at $\theta_0 = 0^\circ$ (or $\theta = 360^\circ$), that is, either source or observer is on the barrier. Here, because of strong reflection due to the barrier, the diffracted field will have the highest sound level (lowest sound attenuation) (Fig. 6), angular correction $A_3 = 0$; and when both source and observer are not on the barrier the correction
will be positive and reaches its maximum (6 dB) at \( \theta - \theta_o = 180^\circ \) (at the interface between the shadow and bright zone). Figure 7 shows typical features of the angular correction \( A_3 \). It was found that the angular correction \( A_3 \) can be treated as independent of \( kR \) with high accuracy; moreover, it almost remains invariable for \( N \geq 1 \). For that case \( A_3 \) is only a function of angles \( \theta_o \) and \( \theta \), while it also depends on Fresnel number \( N \) for \( N < 1 \) (Fig. 7).

3. APPROXIMATE SOLUTION

After dividing the total sound attenuation \( A \) into three parts, \( A_1 \), \( A_2 \), and \( A_3 \), and giving them physical interpretation, we try to find the approximate expression for every term. As mentioned in Section 1 we found that Keller, by means of his geometrical theory of diffraction, has presented an approximate form of the rigorous solution valid for \( N \geq 1 \). Kurze and Anderson have rewritten this formula by introducing the path-length difference \( \delta = R_1 - R \) [Ref. 3, Eq. (14)]. Here, by using our notation (Fig. 1), we rewrite Kurze's form and then rearrange it for further developing and physical interpretation.

\[
A = \left\{ \begin{align*}
20 \log \frac{1}{2\pi (\delta \lambda)^{\frac{3}{2}}} - 20 \log \frac{R}{R_1} & - 20 \log \frac{\frac{1}{2}}{(1 + \frac{R}{R_1})^{\frac{1}{2}}} - 20 \log \frac{1}{12} \left[ 1 + \frac{\cos \left( \frac{\theta - \theta_o}{2} \right)}{\cos \left( \frac{\theta + \theta_o}{2} \right)} \right] \right\} \text{(dB)} \\
10 \log 2\pi N + 20 \log \frac{R_1}{R} \left[ 1 + \frac{R}{R_1} \right]^{-3} - 20 \log \left[ 1 + \frac{\cos \left( \frac{\theta - \theta_o}{2} \right)}{\cos \left( \frac{\theta + \theta_o}{2} \right)} \right] + 3 \right\} \text{(dB)} \\
10 \log N + 10 + 20 \log \frac{R_1}{R} \left[ 1 + \frac{R}{R_1} \right]^{-3} - 20 \log \left[ 1 + \frac{\cos \left( \frac{\theta - \theta_o}{2} \right)}{\cos \left( \frac{\theta + \theta_o}{2} \right)} \right] + 6 \right\} \text{(dB)}
\]

Here \( \frac{R_1}{R} \) can be deduced from parameters \( N \) and \( kR \):

\[
\frac{R_1}{R} = 1 + \frac{\pi N}{kR}
\]
With Fresnel number \( N \) given, \( kR \) approaching infinity and \( \theta_0 = 0 \), the attenuation is equal to \( A_1 \) (from the definition in Section 2) and we have

\[
20 \log \frac{R_1}{R} \sqrt{1 + \frac{R}{R_1}} - 3 = 20 \log \sqrt{2} - 3 = 0
\]

and

\[
-20 \log \left[ 1 + \frac{\cos \left( \frac{\theta - \theta_0}{2} \right)}{\cos \left( \frac{\theta + \theta_0}{2} \right)} \right] + 6 = -20 \log 2 + 6 = 0
\]

Therefore, from Eq. (8) we have

\[
A = A_1 = 10 \log N + 10 \quad \text{(dB)}
\]  \hspace{1cm} (9)

and it is very clear that from Eq. (8)

\[
A_2 = 20 \log \frac{R_1}{R} \sqrt{1 + \frac{R}{R_1}} - 3 \quad \text{(dB)}
\]  \hspace{1cm} (10)

\[
A_3 = 6 - 20 \log \left[ 1 + \frac{\cos \left( \frac{\theta - \theta_0}{2} \right)}{\cos \left( \frac{\theta + \theta_0}{2} \right)} \right] \quad \text{(dB)}
\]  \hspace{1cm} (11)

According to the physical meaning of the attenuation components, \( A_1, A_2 \) and \( A_3 \), we obtained the numerical results of the rigorous solution and compared them with Eqs. (9), (10), (11) correspondingly.

The numerical results of \( A_1 \) are plotted in a semi-logarithmic plane (Fig. 8) which shows that for \( N \geq 1 \) it is nearly a straight line to which formula (9) is a very close approximation with an error less than 0.1 dB for \( N \geq 1.5 \). But for \( N < 1 \), the formula (9) will be invalid, for rigorous solution is no longer a straight line, and an empirical formula is suggested:

\[
A_1 = 10.38 + 9 \log N + 2.23(\log N)^2 \quad \text{(dB)}
\]  \hspace{1cm} (12)
for \( N < 1.5 \), which is in excellent agreement with the exact solution down to \( N = 0.01 \) (Fig. 8).

Formula (10) also gives a very close approximate solution \( A_2 \) to the numerical value of the rigorous one with error less than 0.1 dB for most solutions. The iso-error curve for \( A_2 \) is shown in Fig. 9, from which it is seen that for very small \( kR \) with Fresnel number \( N \) near 0.05 ~ 0.5, the error will be somewhat larger. But we can safely say that when \( kR > 5 \) the error will be less than 0.2 dB for any value of \( N \). The region where the error is larger than 0.5 dB has no practical importance.

Formula (1) which gives the angular correction \( A_3 \) demonstrates that \( A_3 \) is only a function of \( \theta_o \) and \( \theta \), independent of the other two parameters, \( N \) and \( kR \). It is true for \( N > 1 \) and is a very close approximate solution for this case. Actually the value of \( A_3 \) calculated from formula (11) is almost exactly the same as the numerical results of the rigorous solution for \( N = 2 \). However, it can cause definite error for \( N < 1 \) since in this case \( A_3 \) is no longer independent of \( N \) and decreases with decreasing \( N \) (Fig. 7). Therefore formula (11) should be modified by a correction factor \( f \):

\[
f = 0.98 + [0.27 \log N - 0.44(\log N)^2] \cdot \frac{\cos\left(\frac{\theta - \theta_o}{2}\right)}{\cos\left(\frac{\theta + \theta_o}{2}\right)} \quad (13)
\]

Hence

\[
A_3 = \begin{cases} 
6 - 20 \log \left[ 1 + \frac{\cos\left(\frac{\theta - \theta_o}{2}\right)}{\cos\left(\frac{\theta + \theta_o}{2}\right)} \right] \times f \quad (\text{dB}) 
\end{cases} \quad (14)
\]

for \( N \leq 1 \).

It is very clear that the modified formula (14) still follows the reciprocal theorem, that is the sound level (or attenuation) at the observation
point is still the same by exchanging the source and observer position with each other. Comparing with the exact solution, the error is less than $0.1 \sim 0.2$ dB for $N$ down to 0.05. In practice, very small $N$ is always indicative of a small diffraction angle, i.e. $\theta - \theta_0$ is not far from $180^\circ$. Therefore in this case formula (14) still gives a good approximation for $N$ down to 0.01.

The total attenuation $A$ obtained from the above formulas has been compared with both the numerical results of the rigorous solution and experiment measurements which were conducted by using a pure tone point source in the anechoic room of the University of Toronto, Institute for Aerospace Studies [7]. Part of the comparison is presented in Fig. 10, which shows excellent agreement between the approximate and exact solution down to very low $N$ and quite good agreement between the theoretical and experimental results.

4. APPLICATION TO PROBLEMS INVOLVING THE INTERFERENCE OF SOUND WAVES

Many sound shielding problems involve the interference of sound waves. In these cases the phase of the diffracted wave must be taken into account. Keller has derived an approximate solution based on his geometrical theory of diffraction [5]:

$$\psi = \frac{1}{R} \cdot 10^{-\frac{A}{20}} \exp \left[ i \left( k R_1 + \frac{\pi}{4} \right) \right]$$

(15)

Here $A$ is the sound attenuation by the barrier which is given by Eq. (8). But as mentioned above Eq. (8) is valid only for $N \geq 1$, and for $N < 1$ the modified Eqs. (12), (14) should be used instead.

In this paper the approximate solution of Eq. (15) was applied to solve two problems of practical interest and compared with experimental results.
Isei, Embleton and Piercy have used different theories, one of them is Keller's, to calculate the diffraction due to a point source behind a barrier on a ground of finite impedance and made comparisons with measurements [8]. The particular configuration of source, receiver and barrier which they used, had a Fresnel number \( N \) ranging from less than 1 all the way down to 0.006 for the lowest frequency 100 Hz. Surely, in this case, good agreement between Keller's theory and measurement results cannot be expected. In this paper, according to the value of Fresnel number \( N \), Keller's formula (8) and our empirical formulas (11), (13) were used to calculate the diffracted field of the particular configuration as shown in Fig. 11, which is taken from Ref. 8.

With indices as defined in Fig. 12, application of the principle of superposition permits the total sound field \( \phi \), at the receiver, to be written as

\[
\phi = \phi_{SR} + \phi_{TR} + \phi_{SX} + \phi_{TX}
\]  

First, the diffracted field \( \psi \) based on the equivalent geometry, as shown in the second column of Fig. 12, was calculated by Eq. (15). Then the corrections for pressure reflection coefficient \( P \) and the ground wave \( F(w) \) were made. Both \( F(w) \) and \( P \) (\( P_S \) and \( P_R \) represent the coefficients at the source or receiver side) depend on the real geometry, as shown in the first column of Fig. 12, and the impedance of ground which can be presented as a function of sound frequency \( f \) and specific resistance \( \sigma \) [9].

The attenuation spectrum obtained here was compared with Isei's measured curve [8], which shows that there is good agreement between theory and experiment (Fig. 11). It was indicated in Ref. 8 that the random fluctuation about a mean position, particularly in high frequency, is due to atmospheric turbulence, for the results were measured outdoors.
(b) Point source shielding by a finite planar barrier

Consider a finite planar barrier which has a finite length in the x-direction and infinite length in the y-direction (Fig. 13). A point source is located above the barrier. There are two principal diffraction paths in this configuration, via the right edge and left edge, which interfere with each other. Since the ratio of the width to wavelength $C/\lambda$ is large enough (3.6 for $f = 4000$ Hz and 5.4 for $f = 6000$ Hz) to ignore the effect of double diffraction, so the diffracted field $\varphi$ at the receiver can be obtained by simply adding the diffracted field $\varphi_{l}$ via the left edge and $\varphi_{r}$ via the right edge (Fig. 14).

From Eq. (15) we have

$$\varphi_{l} = 10^{-20} \frac{A_{l}}{20} e^{ikR} e^{i\pi(N_{l} + \frac{1}{2})}$$

$$\varphi_{r} = 10^{-20} \frac{A_{r}}{20} e^{ikR} e^{i\pi(N_{r} + \frac{1}{2})}$$

and finally the total attenuation $A_{t}$ at the receiver by the finite planar barrier is

$$A_{t} = -20 \log \left| \frac{\varphi_{l} + \varphi_{r}}{\varphi_{o}} \right|$$

$$= -20 \log \left| 10^{-20} \frac{A_{l}}{20} e^{i\pi N_{l}} + 10^{-20} \frac{A_{r}}{20} e^{i\pi N_{r}} \right|$$

Here $A_{l}$ and $A_{r}$ are the attenuation by the equivalent semi-infinite barriers.

Two theoretical approaches are used to predict the sound attenuation. Both of them ignore the double diffraction; but approach 1 is based on numerical results of the rigorous solution introduced in Section 2 and approach 2 on the approximate solution presented in this paper. The agreement between these two approaches is very good (Fig. 13). The comparisons between theory and measured results are also made and are
presented in Fig. 13. They show quite good agreement. The measurement results and results of the theoretical method (1) are from Ref. 7.

5. CONCLUSION

Carlaw's rigorous solution for the diffracted sound field behind a barrier shows that sound reduction depends on four parameters: Fresnel number $N$, nondimensional distance $kR$, incident angle $\theta_0$ and angle $\theta$ where the receiver is located. The secondary parameters, $kR$, $\theta_0$, $\theta$, have a noticeable effect on sound reduction even though they are dominated by the parameter $N$. This is brought to light by the numerical results of the rigorous solution.

After looking at the numerical solution closely, we divided the total attenuation $A$ into three components, $A_1$, $A_2$ and $A_3$, then gave them physical interpretation and explicit analytic expressions. $A_1$ is the sound attenuation for the case in which the source (or receiver) are located on the barrier and the direct distance from the source to the receiver is infinite. It is due to the difference of the shortest distance $R_1$ across the edge of the barrier and the direct distance $R$ from the source to the receiver and is usually the major part of the total attenuation.

$A_2$ and $A_3$ are correction terms for finite nondimensional direct distance $kR$ and angle $\theta_0$ and $\theta$. $A_2$ is evaluated by Eq. (10), which shows that for a given Fresnel number $N$, $A_2$ decreases with increasing $kR$, and it can be ignored for relatively large $kR$, say $kR > 100$ (for $N < 5$). $A_3$ equals 0 dB if the source or receiver is located on the barrier and increases to its maximum when the receiver is at the boundary of the shadow region.

Based on these equations the total sound attenuation is obtained which agrees within $0.3 \sim 0.4$ dB with rigorous solution down to $N = 0101$. Besides the accuracy, this approximate method is very convenient for application
and expresses the effect of four parameters on attenuation, explicitly.

REFERENCES

Fig. 1 Geometry used to describe diffraction of sound waves from a point source $P_o$ by a half plane.

Fig. 2 Combination of source-receiver position for identical attenuation provided by semi-infinite barrier (i.e. in each case all independent parameters are kept constant. $N = 1.0$, $kR = 30$, $\theta_o = 30^\circ$, $\theta - \theta_o = 270^\circ$).
Fig. 3  Plots of attenuation (calculated by rigorous solution) vs

diffraction angle $(\theta - \theta_o)$ for different values of Fresnel

number $N$ and source angle $\theta_o$ ($\forall \theta_o = 15^\circ$, $\blacktriangle \theta_o = 30^\circ$, $\circ \theta_o = 60^\circ$,

$\triangledown \theta_o = 90^\circ$, $\blacktriangledown \theta_o = 0^\circ$), $\cdots$ attenuation from Maekawa's

curve [2]).
Fig. 4 Plots of attenuation (calculated by rigorous solution) vs $kR$ as function of Fresnel number $N$. --- the limitation of geometry.

Fig. 5 Plots of attenuation (calculated by rigorous solution) vs Fresnel number $N$ for different values of $kR$. 
Fig. 6 A typical result of sound attenuation which shows that the total attenuation can be broken into three parts, $A_1$, $A_2$, and $A_3$.

Fig. 7 Plots of angular correction $A_3$ for sound attenuation vs diffraction angle for different values of $N$. 
**Fig. 8** The comparison of attenuation component $A_1$ between numerical results of rigorous solution and approximation from Eq. (9) for $N \geq 1.5$ and Eq. (12) for $N < 1.5$.

**Fig. 9** The error of approximate formula (10) for attenuation correction $A_2$ compared with rigorous solution.
Fig. 10 Comparison of sound attenuation by a half-plane between the rigorous approximate solution and experimental results.

Fig. 11 Comparison between theoretical and measured results for the configuration as at the top of this figure (the measured results are after Ref. 8).
**Fig. 12** Ray path diagrams and equations used for calculating the sound level of diffracted field. Column 1 shows the actual ray path. Column 2 shows the equivalent ray path used for calculation of the diffracted field $\psi$. The equations are as follows:

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<th>ACTUAL RAY PATH</th>
<th>EQUIVALENT RAY PATH</th>
<th>CORRECTION</th>
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<tr>
<td>$S \rightarrow R$</td>
<td>$S \rightarrow R$</td>
<td>$\phi_{SR} = \psi_{SR}$</td>
</tr>
<tr>
<td>$S \rightarrow \text{image source} \rightarrow R$</td>
<td>$S \rightarrow \text{image source} \rightarrow R$</td>
<td>$\phi_{TR} = \psi_{TR} \left[ P_{T} + (1-P_{T})F(W_A) \right]$</td>
</tr>
<tr>
<td>$S \rightarrow \text{image receiver} \rightarrow X$</td>
<td>$S \rightarrow \text{image receiver} \rightarrow X$</td>
<td>$\phi_{SX} = \psi_{SX} \left[ P_{R} + (1-P_{R})F(W_D) \right]$</td>
</tr>
<tr>
<td>$S \rightarrow \text{image receiver} \rightarrow X$</td>
<td>$S \rightarrow \text{image receiver} \rightarrow X$</td>
<td>$\phi_{TX} = \psi_{TX} \left[ P_{T} + (1-P_{T})F(W_A) \right]$ $\left[ P_{R} + (1-P_{R})F(W_D) \right]$</td>
</tr>
</tbody>
</table>
Fig. 13 Comparison of experimental and theoretical results for finite rectangular shield.

Fig. 14 The equivalent configuration used for calculation of diffracted field around two edges (valid for ratio of width to wavelength much larger than 1).
The approximate diffraction formalism of Keller and others is re-examined and cast into three explicit terms, each of which is given a physical interpretation. The new approximate forms are valid for Fresnel numbers as small as 0.05. In this regime the often neglected secondary parameters make significant contributions. The scheme is applied to several test cases and performs very well.

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