FEASIBILITY STUDY
of a
PROPOSED CONTROL SYSTEM
on a
HYDROFOIL SAILBOAT

by

P. L. Davis

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During the past year I have communicated with several men without whose help and guidance the completion of this report as it now stands would have been impossible. I would therefore like to thank these men and the concerns they represent:

Mr. Christopher Hook (hydrofoil designer)
Mr. Donald J. Nigg (hydrofoil designer)
Mr. Dennis Newman - deHavilland Aircraft of Canada
Mr. A. W. Feprey - Assistant Public Relations Officer United States Navy

I would like to conclude by giving a special note of thanks to the three professors here at the Institute for Aerospace Studies who aided and encouraged me in my work: Dr. Allan Howsmon, Dr. J. B. French and Professor B. Etkin.
DEDICATION

The following report is dedicated to the idea as expressed below by Francis Herreshoff, a noted yacht designer:

"To me the pleasure of sailing is almost in direct proportion to the speed, and wallowing around in some pot-bellied abortion, heeled over and straining under a lapping jib or some other rule cheating windbag, seems quite ridiculous. Some yachtsmen seem to think the sole object of sailing is to beat a brother yachtsman and have adopted certain rules of measurement that insure the wealthy to be winners. They sail around courses perhaps at a rate of five miles an hour and if they have won consider themselves great sailors. But the general public, and particularly the sailor, is getting sick of that game. He sees no sense in trying to force several thousands of dollars worth of lead through the water with several thousand dollars worth of rule cheating sails handled by a big crew. The sailor wants to sail and says to hell with the wealthy, bridge playing sea lawyers who win their races travelling at a rate slower than their ancestors".
SUMMARY

In the Introduction, a number of hydrofoil sailing experiments were enumerated and the basic type of craft in question was defined. The need for a simple and practical control system providing stability and safety was stated.

Special problems of the pitch and roll control of the hydrofoil sailboat were then discussed and the solutions of some other experimenters were shown.

The static equations of equilibrium were written for a hydrofoil sailing craft. The hydrodynamic forces for a particular set and configuration of foils were calculated and at the same time the aerodynamic forces for a particular sail were determined.

A computer program was written to solve the equations of equilibrium under particular sets of conditions. Results were obtained for both the case when no active control system was used and also when the proposed type of control system was used.

It was found that the proposed control system, although it improved on the stability of the uncontrolled case, was still not adequate in providing necessary stability.
NOTATION

\( \mathbf{P}_A \)
aerodynamic sail force with components \([T, H, P]\) in the fixed reference frame

\( T \)
component parallel to both the centre plane and the water surface; thrust (line of action in the positive \( x_F \) direction)

\( H \)
component perpendicular to the centre line and parallel to the water surface; heeling force (line of action in the negative \( y_F \) direction)

\( P \)
component perpendicular to the water surface and perpendicular to the centre line; (line of action in the negative \( z_F \) direction)

\( [\bar{X}, \bar{Y}, \bar{Z}] \)
coordinates of origin of fixed axis system in the body reference frame

\( \mathbf{F}_H \)
hydrodynamic force vector; there are three such vectors one for each of the front foils and one for the rear foil each with components \([D, R, L]\)

\( D \)
component parallel to both the centre plane and the water surface; drag (line of action in the negative \( x_F \) direction)

\( R \)
component perpendicular to the centre line and parallel to the water surface; heel reaction force (line of action in the positive \( y_F \) direction)

\( L \)
component perpendicular to both the water surface and the centre line; left force (line of action in the positive \( z_F \) direction)

\( h \)
height of the mast

\( f_h \)
\( Z \) coordinate of the centre of pressure of the sail (in body axes system)

\( \lambda \)
angle between the boom of the sail and the centre line of the boat (\( x \) axis in body axis system) and hence the angle between the sail and the centre line

\( c \)
length of the boom and also the length of the foot of the sail

\( \epsilon_c \)
distance from the mast to the centre of pressure of the sail

\( r \)
\( Z \) coordinate (in the body axis system) of the centre of pressure of each of the three foil systems

\( b \)
distance between line of action of the lift forces of the two front hydrofoils
\((\psi, \theta, \phi)\) Euler angles for the body axis system

\(\ell\) distance along longitudinal member from stern to mast

\(W\) weight of the overall craft including operators

\(V_b\) boat’s velocity

\(V_A\) apparent wind velocity

\(\psi_A\) angle between the apparent wind velocity vector and the \(x_F\) axis

\(\psi_T\) angle between the true wind velocity vector and the \(x_F\) axis

\(\beta\) angle of slideslip i.e., angle between the boat velocity vector \(V_b\) and the \(x\) axis (body axis)

\(\rho_w\) density of water

\(\rho_A\) density of air

\(\alpha_s\) sail angle of attack

\(\alpha_w, \alpha_L, \alpha_R\) windward, leeward, and rear hydrofoil angles of attack

\(F\) froude number (Defined as \(F = \frac{V}{\sqrt{g c}}\)).

\(V_T\) true wind velocity
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INTRODUCTION

A hydrofoil sailboat basically consists of a "conventional" sailboat of some shape or description to which hydrofoils have been added.

The so-called conventional sailboat may take any one of several different forms. It may be a mono-hull or a multihull (catamaran or trimaran) or it may have no actual hull at all with flotation being provided by suitably positioned buoyancy billets.

The sail plan and rigging can be of any type as long as they are designed with strength in mind since the aerodynamic forces on the sails of such a boat are far higher than those on a conventional boat. For example the heeling force which is the lateral component of the aerodynamic force on the sails is often comparable in magnitude to the total weight of the boat.

In photographs No.1, 2 and 3 one of the first hydrofoil sailboats is shown. Built by the Baker Mfg. Co. for the United States Navy it is a single hulled boat with both main and jib sails. It uses the step-ladder type of hydrofoil configuration where a number of foils are placed on the same strut in venetian-blind fashion. The foils in this boat are on a fairly large dihedral angle.

Photograph No.4 shows the first boat built by a Mr. D. J. Nigg of Kansas. This boat has no actual hull but derives its flotation from the two pontoons running from bow to stern on either side of the boat. For propulsion a single sail is employed. He uses four single "V" shaped foils to operate his boat. Photographs 5 and 6 show the second boat built by Mr. Nigg. He again uses a single sail. This boat is a mono-hull in contrast to his first boat. Again "V" shaped foils have been used but the rear foils have been modified as well by the addition of a third foil section connecting the top of the "V".

From this introduction it may be seen that a number of solutions to the general problem of designing a hydrofoil sailboat have been found each with its own merits; however, the fact that to date no such sailboat has ever been produced for sale to the public should indicate that existing designs fall short of what the public demands in the way of price and performance. If a successful design is one which meets the demands of the public, then no really successful sailing hydrofoil has yet been developed. It is well perhaps to mention here the lament of the editor of the Amateur Yacht Research Society, John Morwood, in his editorial of October 1966: "If just one person can devise a controllable flying hydrofoil boat, I think the A.Y.R.S. (Amateur Yacht Research Society) should seize on it and sell the plans as a basis for a "Development Class" of flying hydrofoils".

Thus it can be seen that the basic problem of the hydrofoil sailing craft is one of control.

The purpose of this report is then to move one step closer to this goal of devising a controllable hydrofoil sailing craft by making a static control analysis of a control system proposed for use in such a craft.

In this report the control parameters which the operators of the craft will be allowed to vary will be the boom angle, the rudder angle, and the x and y coordinates of the centre of gravity. The Z coordinate will be held constant.
Given a particular set of variables, the true wind speed and its direction with respect to the centre line of the boat and the trim angles, $\theta$ and $\phi$, the control parameters are determined for optimum performance still preserving equilibrium and the corresponding boat velocity, and mean water level are calculated.

This procedure is repeated for two cases. In the first, the boat is considered to be "free-flying" (i.e., no active control system is employed). The second analyses the equilibrium when an active control system is used to help provide stability.

I. A DISCUSSION OF THE PROBLEMS RELATED TO CONTROL OF HYDROFOIL CRAFT

Problems of control of hydrofoil craft in a seaway are discussed in great detail in the literature\textsuperscript{5,6}. These reports concern themselves with the problems of longitudinal or pitch control more than with that of lateral or roll control due to the fact that pitch control is usually much harder to achieve.

The reports to date have considered powered hydrofoil craft primarily. Thus they do not consider the effect of a side or heeling force $H$, acting on their craft nor do they make allowance for the fact that the line of action of the aerodynamic or driving forces on a sailboat is not coincident with the centre of gravity as is the case with the thrust line of most hydrofoil power craft.

Because the magnitude of the heeling force $H$ can approach the total weight of the craft including operators, for the sailing hydrofoil the problem of both pitch and roll stability become both equally important as well as equally difficult to achieve.

In a report done by Benson and King\textsuperscript{7} to investigate the dynamic stability characteristics of several hydrofoil configurations, the following conclusions were drawn:

1) Multiplane hydrofoil systems in general offer wider margins of stability than do monoplane systems.

2) Dihedral contributes greatly to the stability of hydrofoil systems principally because a hydrofoil with dihedral will have much less severe discontinuities in lift and drag as it approaches and breaks through the water surface than will a flat hydrofoil.

Henceforth in this report only surface piercing hydrofoils with dihedral will be considered.

(a) Problems of Pitch Control:

The problems relating to pitch control are generally much more severe to a hydrofoil craft than an aircraft. In aircraft the pitching amplitude resulting from a disturbance may cause a change in angle of attack of the wings and tail of a fairly large percent of its equilibrium value. The resulting motion induced in the $Z$ direction\textsuperscript{6} may cause the plane to move up and down through several hundred feet. For an aircraft at its equilibrium altitude this relatively small perturbation in altitude is usually of no consequence. However, for the hydrofoil craft where the same type of perturbations in pitching moment can occur only very small deviations from the equilibrium water level can be
allowed. Similarly if stability is to be maintained, the perturbations to angles of attack must be of relatively small percents.

The phenomena known as "sea-crash" described below occurs to hydrofoil craft that are unstable in pitch. Consider the effect of a nose down pitching moment applied to a hydrofoil craft. In a powered craft this could be due to some perturbation in the throttle setting or in the water surface. In a sailing craft it could be due to perturbations in wind or waves, changes in position of the centre of gravity, etc. As a result of the pitching moment the boat will rotate through some angle about its centre of gravity as shown in Figure 1. If its foils have been set at an angle of attack say $45^\circ$ and the pitching moment causes a change in pitching angle of $-5^\circ$ then the foils will be set at a negative incidence of $-1^\circ$. The resultant hydrodynamic lift forces will all be negative and the boat will "crash" nose first into the water.

A number of ingenious methods have been proposed to prevent "sea crash". On the more sophisticated boats sensitive accelerometers have been combined with automatic control systems to adjust the foil angles of attack to compensate for perturbations in pitching moment. In some boats a "feeler" has been used set in front of the bow foils to sense changes in pitch angle and adjust the angles of attack accordingly. An interesting solution by D. J. Nigg has been to adopt the planform of an iceboat with a single bow foil supporting about 20% of the weight and tandem stern foils supporting the remaining 80%. The bow foil is set very close to its stall angle so that any nose down pitching moment causes only a small percentage change in its angle of attack; however, the stern foils set at a substantially lower angle of attack have a much larger percentage change to their angles of attack. The resultant motion is that the stern sinks lower in the water from the loss in lift due to the angle of attack change. This sinking of the stern increases the pitch angle again towards the equilibrium value.

The solution proposed in this report and explained in detail later is to couple the thrust component of the aerodynamic force on the sails to the hydrodynamic forces so as to preserve a stable equilibrium in the pitching mode.

(b) Problems of Roll Control:

The problems of roll control for the hydrofoil sailboat centre on the magnitude of the rolling moment generated by the heeling force, $H$. As the heeling force can exceed the weight of the craft in magnitude and is displaced by several feet from the centre of gravity a very large rolling moment is usually present and even small perturbations in $H$ cause large absolute changes in the rolling moment. To preserve roll stability relatively large dihedral ($30^\circ$ to $50^\circ$) is used along with a fairly high taper ratio. This allows even small changes in roll angle to cause a fairly large percent increase in wetted area on the leeward side and a proportionate increase in lift force as is shown in Figure 2. Similarly on the windward side the wetted area and hence the lift decreases. The difference between the lift forces on windward and leeward sides of the craft provides a restoring moment to counteract that due to the heeling force, $H$.

The stability to be obtained in this manner is however limited. Once the boat heels to the point where the windward foil is fully out of the water the restoring moment has reached a terminal value and any further increase in heeling moment will cause the boat to capsize. In an attempt to make the restoring moment as large as possible some boats have been designed placing their
foils as far apart as possible. One boat design has gone to the extreme of being wider than it is long which makes the boat relatively unwieldy to try to manoeuvre in close quarters.

The proposed control system couples the heeling force, $H$, to the lift forces on the bow hydrofoils by changing their angle of attacks from their equilibrium values. The leeward hydrofoil has its angle of attack increased while the windward foil has its decreased. This differential in foil angles of attack sets up a restoring moment to counteract the heeling moment imposed on the craft. This is discussed in more detail in the section dealing with the roll control of the boat.

II. STATIC EQUATIONS OF EQUILIBRIUM

From Figures 3 through 7, the general structure of the craft to be analyzed can be seen. The actual values of the various geometric and other parameters to be used later in the program were chosen on a basis of what seemed reasonable for the type of boat considered. They by no means represent an optimized design; on the contrary they have been chosen only to test the effectiveness of the proposed control system.

The equations of equilibrium were formulated for an axis system moving with the vehicle with its origin at the vehicle's centre of gravity and its axes parallel to those of an earth fixed reference frame. The axes of this system are marked with a subscript "F". The vehicle has a body reference frame with origin at the base of the mast and positive $x$, $y$, and $z$ axes coinciding with that portion of the longitudinal member forward of the mast, that portion of the transverse member on the port side of the vehicle and the mast itself respectively. The axes of the body reference frame carry no subscripts.

The moments of the equations of equilibrium were taken about the origin of the reference frame moving with the vehicle with axes parallel to those of an earth fixed frame. The origin of this frame is the centre of gravity of the craft.

The "right hand rule" was used to determine the signs of the various moments.

(a) Non-linearized Force Equations

\[ \sum F_x^F = 0 \]
\[ \sum F_y^F = 0 \]
\[ \sum F_z^F = 0 \]
\[ T - (D_L + D_R + D_W) = 0 \]
\[ -H + R_L + R_W + R_R = 0 \]
\[ L_L + L_W + L_R - P - W = 0 \]
(b) Non-linearized Moment Equations

\[ \sum \mathbf{M}_F = 0 \quad \text{Static Roll Stability} \]

\[ H \cos \phi \{(f_h - \bar{Z}) + (\bar{X} - \text{ec} \cos \lambda)\tan \theta \} \cos \theta \]

\[ + R_L \left\{ (d_L + \frac{b}{2} + \bar{Y}) \tan \phi + \bar{Z} \cos \phi - \bar{X} \tan \theta \right\} \cos \theta \]

\[ + R_w \left\{ (d_w - \frac{b}{2} - \bar{Y}) \tan \phi + \bar{Z} \cos \phi - \bar{X} \tan \theta \right\} \cos \theta \]

\[ + R_r \left\{ (d_r + \bar{Z} - \bar{X} \tan \phi) \cos \phi + (1 - \bar{X}) \tan \theta \right\} \cos \theta \]

\[ - L_L \left( \frac{b}{2} + \bar{Y} - \bar{Z} \tan \phi \right) \cos \phi \]

\[ + L_w \left( \frac{b}{2} - \bar{Y} + \bar{Z} \tan \phi \right) \cos \phi \]

\[ - L_r (\bar{Y} - [d_r + \bar{Z}] \tan \phi) \cos \phi \]

\[ + P (\text{ec} \sin \lambda + \bar{Y}) \cos \phi = 0 \]

\[ \sum \mathbf{M}_F = 0 \quad \text{Static Pitch Stability} \]

\[ T \left\{ f_h - \bar{Z} - (\text{ec} \sin \lambda + \bar{Y}) \tan \phi + (\bar{X} - \text{ec} \cos \lambda) \tan \theta \right\} \cos \theta \cos \phi \]

\[ - L_w \left\{ \bar{X} + d_w \tan \theta + \bar{Z} \tan \phi \right\} \cos \theta \]

\[ + D_w \left\{ d_w + \bar{Z} - \bar{X} \tan \theta - (\frac{b}{2} - \bar{Y}) \tan \phi \right\} \cos \phi \cos \theta \]

\[ + D_L \left\{ d_L + \bar{Z} - \bar{X} \tan \theta + (\frac{b}{2} + \bar{Y}) \tan \phi \right\} \cos \phi \cos \theta \]

\[ + L_r \left\{ 1 - \bar{X} - \bar{Z} \tan \theta \right\} \cos \theta \]

\[ + D_r \left\{ d_r + \bar{Z} + (1 - \bar{X}) \tan \theta + \bar{Y} \tan \phi \right\} \cos \theta \cos \phi \]

\[ + P \left\{ (f_h - \bar{Z}) \tan \theta - (\bar{X} - \text{ec} \cos \lambda) \right\} \cos \theta = 0 \]

\[ \sum \mathbf{M}_F = 0 \quad \text{Static Yaw Stability} \]

\[ - R_w \left\{ \bar{X} + d_w \tan \theta + \bar{Z} \tan \phi \right\} \cos \theta \]

\[ - R_L \left\{ \bar{X} + (d_L + \bar{Z}) \tan \theta \right\} \cos \theta \]

\[ + R_r \left\{ 1 - \bar{X} - \bar{Z} \tan \phi \right\} \cos \theta \]

\[ + D_L \left\{ \frac{b}{2} + \bar{Y} - \bar{Z} \tan \phi \right\} \cos \phi \]
(c) Simplifying Approximations to the Equations of Equilibrium

To simplify the analysis of these equations the small angle approximations for \( \phi \) and \( \theta \) will be used.

\[
\begin{align*}
\sin \phi \cos \phi & \approx \phi \\
\sin \theta & \approx \tan \theta \approx \theta \\
\cos \phi & \approx 1.0 \\
\cos \theta & \approx 1.0
\end{align*}
\]

For the pitch angle \( \theta \) this is a very good approximation since only very small angles of pitch (much less than 10\(^0\)) can be tolerated. The approximation for the roll angle \( \phi \) is not as good because large roll angles occur quite frequently in normal sailing; however, the roll control system employed in this design functions to minimize \( \phi \) and hence the small angle approximation is considered reasonable. These assumptions will be checked against the results to assure their validity.

The three force equations are the same as shown before the linearization:

\[
T - (D_L + D_R + D_w) = 0 \\
-H + R_L + R_R + R_w = 0 \\
L_L + L_R + L_w - P - W = 0
\]

The three moment equations after these approximations become:

\[
\sum M_x = 0
\]

\[
H \left\{ f_h - \bar{Z} \right\} + \left\{ \bar{X} - \text{ec cos} \lambda \right\} \theta
\]

\[
+ R_L \left\{ (d_L + \left( \frac{b}{2} + \bar{Y} \right) \phi + \bar{Z}) - \bar{X} \theta \right\}
\]

\[
+ R_w \left\{ (d_w - \left( \frac{b}{2} - \bar{Y} \right) \phi + \bar{Z}) - \bar{X} \theta \right\}
\]

\[
+ R_R \left\{ d_r + \bar{Z} + \bar{Y} \phi + (1-\bar{X}) \theta \right\}
\]

\[
- L_L \left\{ \frac{b}{2} + \bar{Y} - \bar{Z} \phi \right\}
\]

\[
+ L_w \left\{ \frac{b}{2} - \bar{Y} + \bar{Z} \phi \right\}
\]

\[
- L_R \left\{ \bar{Y} - (d_r + \bar{Z}) \phi \right\}
\]

\[
+ P \left\{ \text{ec sin} \lambda + \bar{Y} \right\} = 0
\]
\[ \sum M_{y_F} = 0 \]
\[ T \left\{ f_h - Z - (ec \sin \lambda + \dot{Y})\phi + (\ddot{X} - ec \cos \lambda) \theta \right\} \]
\[ -L_w \left\{ \ddot{X} + (dw + \ddot{Z})\theta \right\} - L_L \left\{ \ddot{X} + (d_L + \ddot{Z})\theta \right\} \]
\[ +D_w \left\{ dw + \ddot{Z} - \dddot{X} \theta - (\frac{b}{2} - \dddot{Y}) \phi \right\} \]
\[ +D_L \left\{ d_L + \ddot{Z} - \dddot{X} \theta + (\frac{b}{2} + \dddot{Y}) \phi \right\} \]
\[ +L_R \left\{ \ell - \dddot{X} - \dddot{Z} \theta \right\} \]
\[ +D_R \left\{ dR + \ddot{Z} + (\ell - \dddot{X} \theta + \dddot{Y} \phi \right\} \]
\[ + P \left\{ f_h - Z \theta - (\ddot{X} - ec \cos \lambda) \right\} = 0 \]
\[ \sum M_{z_F} = 0 \]
\[ -R_w \left\{ \ddot{X} + (dw + \ddot{Z}) \theta \right\} - R_L \left\{ \ddot{X} + (d_L + \ddot{Z}) \theta \right\} \]
\[ +R_R \left\{ \ell - \dddot{X} - \dddot{Z} \theta \right\} + D_L \left\{ \frac{b}{2} + \dddot{Y} - \dddot{Z} \phi \right\} \]
\[ -D_w \left\{ \frac{b}{2} - \dddot{Y} + \dddot{Z} \phi \right\} + D_R \left\{ \dddot{Y} - (dR + \ddot{Z}) \phi \right\} \]
\[ -T \left\{ \dddot{Y} + ec \sin \lambda \right\} - H \left\{ ec \cos \lambda - \dddot{X} \right\} = 0 \]

(d) **Stiffness**

Stiffness is the term used to assess the tendency of the vehicle once perturbed to return to its original undisturbed equilibrium position.

With positive pitch and roll stiffness any perturbation of the angles \( \theta \) and \( \phi \) must generate moments which will act to reduce the magnitudes of these perturbations to zero. Thus any change in pitch or roll angle causes a restoring moment to reduce these angle changes to zero.

With negative stiffness any change in pitch or roll angle generates moments which act in a direction to further increase the perturbations in the
angles. Thus any equilibrium state with negative stiffness is one where perturbations will result in a motion which diverges from that equilibrium state and so it can be seen how important it is for control purposes to have a positive stiffness.

With reference to Figures 3 and 5 a positive (nose-up) perturbation in pitching angle \( \theta \) can be eliminated if a positive pitching moment (nose down pitching moment) results as a consequence. Thus for the sign convention chosen positive stiffness requires a positive change in pitch angle, \( \Delta \theta \), to introduce a positive change in pitching moment \( \Delta M_{yF} \).

Hence for positive pitch stiffness:

\[
\lim_{\Delta \theta \to 0} \frac{\Delta M_{yF}}{\Delta \theta} = \frac{\partial M_{yF}}{\partial \theta} > 0
\]

Similarly with reference to Figures 4 and 5, a positive perturbation in roll angle \( \phi \) can be eliminated if a negative rolling moment results. Thus for the sign convention chosen positive roll stiffness requires a positive change in roll angle, \( \Delta \phi \), to induce a negative change in rolling moment, \( \Delta M_{xF} \).

Hence for positive roll stiffness:

\[
\lim_{\Delta \phi \to 0} \frac{\Delta M_{xF}}{\Delta \phi} = \frac{\partial M_{xF}}{\partial \phi} < 0
\]

The expressions for pitch and roll stiffness will now be found by taking the first derivative with respect to \( \theta \) and \( \phi \) of the pitching and rolling moments respectively.

(i) Roll stiffness

\[
\frac{\partial M_{xF}}{\partial \phi} = \frac{\partial M}{\partial \phi} \left\{ fh - \tilde{Z} \right\} + \left( \tilde{X} - ec \cos \lambda \right) \theta + R_L \left( \frac{b}{2} + \tilde{Y} \right)
\]

\[
+ \frac{\partial R_L}{\partial \phi} \left\{ \left( d_L \left( \frac{b}{2} + \tilde{Y} \right) \phi + \tilde{Z} \right) - \tilde{X} \theta \right\} - R_w \left( \frac{b}{2} - \tilde{Y} \right)
\]

\[
+ \frac{\partial R_k}{\partial \phi} \left\{ d_w - \left( \frac{b}{2} - \tilde{Y} \right) \phi + \tilde{Z} \right\} + R_R \tilde{Y}
\]

\[
+ \frac{\partial R_R}{\partial \phi} \left\{ dr + \tilde{Z} + \tilde{Y} \phi + \left( \tilde{Z} - \tilde{X} \right) \theta \right\} + R_L \tilde{Z}
\]

\[
- \frac{\partial L_L}{\partial \phi} \left( \frac{b}{2} + \tilde{Y} - \tilde{Z} \phi \right) + \frac{\partial L_w}{\partial \phi} \left( \frac{b}{2} - \tilde{Y} + \tilde{Z} \phi \right)
\]

\[
- \frac{\partial L_R}{\partial \phi} \left\{ \tilde{Y} - (dr + \tilde{Z}) \phi \right\} + L_w \tilde{Z}
\]

\[
- L_R \left( dr + \tilde{Z} \right) + \frac{\partial \rho}{\partial \phi} \left\{ ec \sin \lambda + \tilde{Y} \right\}
\]
In the small angle approximation
\[ \frac{\partial H}{\partial \phi} = 0 \]

For the hydrofoil force derivatives
\[ \frac{\partial F}{\partial \phi} = \frac{\partial F}{\partial z} \frac{\partial z}{\partial \phi} \]
where \( Z \) is the distance from the \( x-y \) plane of the hydrofoil axis system to the water level measured along the \( Z \) hydrofoil axis

hence the following relations hold

(1) windward side
\[ \frac{\partial F_w}{\partial \phi} = \frac{\partial F_w}{\partial z} \left( \frac{b}{2} - \bar{Y} \right) \]
since
\[ \left( \frac{b}{2} - \bar{Y} \right) = Z \]

(2) leeward side
\[ \frac{\partial F_L}{\partial \phi} = \frac{\partial F_L}{\partial z} \left( \frac{b}{2} + \bar{Y} \right) = Z \]
since
\[ \left( \frac{b}{2} + \bar{Y} \right) \phi = Z \]

(3) rear foil system
\[ \frac{\partial F_R}{\partial \phi} = \frac{\partial F_R}{\partial z} (\bar{Y}) \]
since
\[ \bar{Y} \phi = Z \quad \text{here} \]

The expression for \( P \) is
\[ P(\alpha_s \gamma_A) = F_F(\alpha_s \gamma_A) \theta - F_s(\alpha_s, V_A) \phi \]
\[ \frac{\partial P}{\partial \phi} = -F_s(\alpha_s, V_A) \]
\[ = -H(\alpha_s, V_A) \]

With these approximations the equation for roll stiffness becomes
\[ \frac{\partial M_y}{\partial \phi} = R_L \left( \frac{b}{2} + \bar{y} \right) - R_w (b/2 - \bar{y}) + R_R \bar{y} + L_R \, d\phi \]

\[ + \bar{z} (L_L + L_w + L_R) - H(\alpha_s, V_A) \left[ \text{ec } \sin \lambda + \bar{y} \right] \]

\[ + \frac{\partial R_L}{\partial z} \left( \frac{b}{2} + \bar{y} \right) \left\{ (d_L + (b/2 + \bar{y}) \phi + \bar{z}) - \bar{x} \theta \right\} \]

\[ + \frac{\partial R_w}{\partial z} \left( \frac{b}{2} - \bar{y} \right) \left\{ d_w - (b/2 - \bar{y}) \phi + \bar{z} \right\} - \bar{x} \theta \right\} \]

\[ + \frac{\partial R_R}{\partial z} \left( \bar{y} \right) \left\{ \bar{d} + \bar{z} + \bar{y} \phi + (\ell - \bar{x}) \theta \right\} \]

\[ - \frac{\partial L_L}{\partial z} \left( \frac{b}{2} + \bar{y} \right) \left\{ b/2 + \bar{y} - \bar{z} \phi \right\} \]

\[ + \frac{\partial L_w}{\partial z} \left( \frac{b}{2} - \bar{y} \right) \left\{ b/2 - \bar{y} + \bar{z} \phi \right\} \]

\[ + \frac{\partial L_R}{\partial z} \left( \bar{y} \right) \left\{ \bar{y} - (d + \bar{z}) \phi \right\} \]

(ii) Pitch stiffness

\[ \frac{\partial M_y}{\partial \theta} = \frac{\partial T}{\partial \theta} \left\{ f_h - \bar{z} - (\text{ec } \sin \lambda + \bar{y}) \phi + (\bar{x} - \text{ec } \cos \lambda) \theta \right\} \]

\[ + T \left\{ \bar{x} - \text{ec } \cos \lambda \right\} - \frac{\partial L_w}{\partial \theta} \left\{ \bar{x} + (d_w + \bar{z}) \theta \right\} \]

\[ - L_w \left\{ d_w + \bar{z} \right\} - \frac{\partial L_L}{\partial \theta} \left\{ \bar{x} + (d_L + \bar{z}) \theta \right\} \]

\[ - L_L \left\{ d_L + \bar{z} \right\} + \frac{\partial D_w}{\partial \theta} \left\{ d_w + \bar{z} - \bar{x} \theta - (b/2 - \bar{y}) \phi \right\} \]

\[ - D_w \bar{x} + \frac{\partial D_L}{\partial \theta} \left\{ a_L + \bar{z} - \bar{x} \theta + (b/2 + \bar{y}) \phi \right\} - D_L \bar{x} \]

\[ + \frac{\partial L_R}{\partial \theta} \left\{ \ell - \bar{x} - \bar{z} \theta \right\} - L_R \bar{x} + D_R (\ell - \bar{x}) \]

\[ + \frac{\partial D_R}{\partial \theta} \left\{ \bar{d} + \bar{z} + (\ell - \bar{x}) \theta + \bar{y} \phi \right\} + P \left\{ f_h - \bar{z} \right\} \]

\[ + \frac{\partial P}{\partial \theta} \left\{ (f_h - \bar{z}) \theta - (\bar{x} - \text{ec } \cos \lambda) \right\} \]

In the small angle approximation

\[ \partial T / \partial \theta = 0 \]
The expression for $P$ is

$$P(\alpha_s, V_A) = T(\alpha_3, V_A) \theta - H(\alpha_3, V_A) \phi$$

then

$$\frac{\partial P}{\partial \theta} = T(\alpha_s, V_A)$$

For the hydrofoil force derivatives

$$\frac{\partial F}{\partial \alpha} = \frac{dF}{d\alpha}$$

where $\alpha$ is the angle of attack of the force producing foil.

With these assumptions and approximations the equation becomes

$$\frac{\partial M_{yF}}{\partial \theta} = T\left\{ \bar{X} - eccos \bar{\lambda} \right\} - L_w(d_w + \bar{Z}) - L_L(d_L + \bar{Z})$$
$$- L_R \bar{Z} - \bar{X} (D_L + D_w + D_R) + D_R \ell$$
$$+ P(fh - \bar{Z}) - \frac{\partial L_w}{\partial \alpha} \left\{ \bar{X} + (d_w + \bar{Z}) \theta \right\}$$
$$- \frac{\partial L_L}{\partial \alpha} \left\{ \bar{X} + (d_L + \bar{Z}) \theta \right\} + \frac{\partial L_R}{\partial \alpha} \left\{ \ell - \bar{X} - \bar{Z} \theta \right\}$$
$$+ \frac{\partial D_w}{\partial \alpha} \left\{ d_w + \bar{Z} - \bar{X} \theta - (b/2 - \bar{Y}) \phi \right\}$$
$$+ \frac{\partial D_L}{\partial \alpha} \left\{ d_L + \bar{Z} - \bar{X} \theta + (b/2 + \bar{Y}) \phi \right\}$$
$$+ \frac{\partial D_R}{\partial \alpha} \left\{ dr + \bar{Z} + (\ell - \bar{X}) \theta + \bar{Y} \phi \right\}$$
$$+ T(\alpha_s, V_A) \left\{ fh - \bar{Z} \theta - (\bar{X} - eccos \bar{\lambda}) \right\}$$

III. AERODYNAMIC FORCES

(a) Lift and Drag Coefficients for Sails

The aerodynamic forces on the sail $[T, H, P]$ at various relative wind speeds and angles of attack will be formulated using data from two sources:

(a) The Forces on a Yacht's Sail - T. Tanner

(b) The Aerodynamic Characterists of a 2/5th scale Finn Sail and its Efficiency when Sailing to Windward - C. A. Marchaj

Although there is a considerable difference between types of sails tested, the results are consistent within a margin of approximately 15%. The results of Tables II, III, IV and V of source (b) and Table I of source (a) have been assembled into a least squares regression program and polynomial
expressions of the following form have thus been calculated.

\[ C_L(\alpha) = a_0 + a_1 \alpha + a_2 \alpha^2 + a_3 \alpha^3 + \ldots \]
\[ C_D(\alpha) = b_0 + b_1 \alpha + b_2 \alpha^2 + b_3 \alpha^3 + \ldots \]

All data points used have been included in Tables 1 and 2 for the sail lift and drag coefficients respectively. Blank entries in the tables do not represent data points in the calculation.

The order of the polynomial fit was in each case determined by examining the "modified standard error" of estimate which was given by

\[ S_{XY} = \sqrt{\frac{\sum Y^2 - \alpha \sum Y - a_1 \sum XY - \ldots - a_{n-1} \sum x^{n-1} Y}{N - n}} \]

where the regression polynomial equation can be written

\[ Y = a_0 + a_1 X + a_2 X^2 + \ldots + a_{n-1} X^{n-1} \]

and \( N \) = number of data points

\( n-1 \) = order of polynomial

\( S_{xy} \) = modified standard error of estimate of \( y \) on \( x \).

A fifth order polynomial was found to give the best fit in both cases resulting in the following polynomials.

\[ C_L(\alpha) = .0539 + 1.53\alpha + 9.41\alpha^2 - 23.5\alpha^3 + 18.1\alpha^4 - 4.66\alpha^5 \]
\[ C_D(\alpha) = .125 - .804\alpha + 5.33\alpha^2 - 6.83\alpha^3 + 4.06\alpha^4 - .918\alpha^5 \]

Here the equations are non-dimensional. The results giving the coefficients for the other orders of polynomials are included in the appendix containing the computer output, Appendix 3.

(b) Sail Angle of Attack

Because the angle \( \Psi \) between the fixed and body axis systems is arbitrary it will be chosen to be zero. i.e., \( x_F//x \)

The sail angle of attack \( \alpha_s = \Psi_A - \lambda \)

To find an expression for \( \Psi_A \)

\[ V_A = V_B^2 + V_T^2 + 2V_B V_T \cos (\beta + \Psi_T) \]

\[ \frac{V_T}{\sin(\beta + \Psi_A)} = \frac{V_B}{\sin(\Psi_T - \Psi_A)} = \frac{V_A}{\sin(\beta + \Psi_T)} \]
\[
\sin(\beta + \psi_A) = \frac{V_T}{V_A} \sin(\psi_T)
\]

\[
\psi_A = \sin^{-1} \left[ \sin(\beta + \psi_T) \frac{V_T}{V_A} \right] - \beta
\]

\[
\alpha_s = \sin^{-1} \left[ \sin(\beta + \psi_T) \frac{V_T}{V_A} \right] - \beta - \lambda
\]

Thus the sail angle of attack is dependent on the boom angle \( \lambda \), the angle of sideslip \( \beta \), the true wind direction and speed and the apparent wind speed by the above transcendental equation.

**TABLE**

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<th>(b) Table IV</th>
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(c) Sail Forces

Using the approximate expressions for $C_L(\alpha)$ and $C_D(\alpha)$ just calculated, the lift $L$, and drag $D$ on the sail are:

\[
L(\alpha_s, V_A) = C_L(\alpha_s) \frac{1}{2} \rho_A V_A^2 S_A
\]

\[
D(\alpha_s, V_A) = C_D(\alpha_s) \frac{1}{2} \rho_A V_A^2 S_A
\]

where $S_A$ is the sail area

$\rho_A$ is the density
The forward force \( F_F \) in the body axis system is

\[
F_F(\alpha_s, V_A) = L(\alpha_s, V_A)\sin(\Psi_A) - D(\alpha_s, V_A)\cos(\Psi_A)
\]

The side force \( F_S \) in the body axis system is

\[
F_S(\alpha_s, V_A) = L(\alpha_s, V_A)\cos(\Psi_A) + D(\alpha_s, V_A)\sin(\Psi_A)
\]

The aerodynamic force on the sail \( F_A \) has components in the fixed axis system given by:

\[
T(\alpha_s, V_A) = F_F(\alpha_s, V_A)\cos(\theta)
\]

\[
= [L(\alpha_s, V_A)\sin(\Psi_A) - D(\alpha_s, V_A)\cos(\Psi_A)] \cos \theta
\]

\[
H(\alpha_s, V_A) = F_S(\alpha_s, V_A)\cos \phi
\]

\[
= [L(\alpha_s, V_A)\cos(\Psi_A) + D(\alpha_s, V_A)\sin(\Psi_A)] \cos \phi
\]

\[
P(\alpha_s, V_A) = F_F(\alpha_s, V_A)\sin \theta - F_S(\alpha_s, V_A)\sin \phi
\]

\[
= [L(\alpha_s, V_A)\sin(\Psi_A) - D(\alpha_s, V_A)\cos(\Psi_A)] \sin \theta
\]

\[
- [L(\alpha_s, V_A)\cos(\Psi_A) + D(\alpha_s, V_A)\sin(\Psi_A)] \sin \phi
\]

The equations will be linearized making the small angle approximation; hence they become:

\[
T(\alpha_s, V_A) = F_F(\alpha_s, V_A)
\]

\[
= L(\alpha_s, V_A)\sin \Psi_A - D(\alpha_s, V_A)\cos \Psi_A
\]

\[
H(\alpha_s, V_A) = F_S(\alpha_s, V_A)
\]

\[
= L(\alpha_s, V_A)\cos \Psi_A + D(\alpha_s, V_A)\sin \Psi_A
\]

\[
P(\alpha_s, V_A) = F_F(\alpha_s, V_A)\theta - F_S(\alpha_s, V_A)\phi
\]

\[
= [L(\alpha_s, V_A)\sin \Psi_A - D(\alpha_s, V_A)\cos \Psi_A] \theta
\]

\[
- [L(\alpha_s, V_A)\cos \Psi_A + D(\alpha_s, V_A)\sin \Psi_A] \phi
\]

**IV. SOLUTION TO THE EQUATIONS OF EQUILIBRIUM**

As an example to show the effectiveness of the proposed control system, the solutions to the equations of equilibrium will be found for the cases with and without the control system. To evaluate the effect of the control system in each case the static stiffness for pitch and roll will be calculated for a number of different true wind speeds and directions.

The hydrodynamic forces and moments will be derived from the data.
of Appendices 1 and 2 which concern the bow and stern foils respectively. Note that the bow and stern foil characteristics do not represent the optimal in such design. They were chosen only because such foil designs have in the past been used successfully on hydrofoil sailboats and were thought to supply reasonable examples of hydrofoil configurations.

(a) Coefficient Form of Forces and Force Equations

For the aerodynamic forces on the sail, the coefficient representation is:

\[ T(\alpha_s, V_A) = \frac{1}{2} \rho_A V_A^2 S_A C_T = \frac{1}{2} \rho_A V_A^2 S_A \left[ C_D(\alpha_s) - C_D(\alpha_s) \cos \alpha \right] \]

Similarly

\[ H(\alpha_s, V_A) = \frac{1}{2} \rho_A V_A^2 S_A \left[ C_D(\alpha_s) \cos \alpha + C_D(\alpha_s) \sin \alpha \right] \]

\[ P(\alpha_s, V_A) = \frac{1}{2} \rho_A V_A^2 S_A \left[ \left\{ C_D(\alpha_s) \cos \alpha - C_D(\alpha_s) \sin \alpha \right\} \theta \right. \]

The coefficient form of the force equations are written below.

For the hydrodynamic forces on the hydrofoils, the coefficient representation is:

\[ D(\alpha, V_B, Z) = \frac{1}{2} \rho_A V_B^2 A(Z) C_D(\alpha) \]

assuming

\[ C_D = C_D(\alpha) \text{ is approximately valid} \]

\[ L(\alpha, V_B, Z) = \frac{1}{2} \rho_A V_B^2 A_H(\alpha) \]

Assuming linearity

\[ C_\ell(\alpha) = \left( \frac{\partial C_\ell}{\partial \alpha} \right) \alpha \]

Then

\[ L(\alpha, V_B, Z) = \frac{1}{2} \rho_A V_B^2 A_H(\alpha) \left( \frac{\partial C_\ell}{\partial \alpha} \right) \alpha \]

Similarly

\[ R(\alpha, V_B, Z) = \frac{1}{2} \rho_A V_B^2 A_V(\alpha) \left( \frac{\partial C_\ell}{\partial \alpha} \right) \beta \]

The coefficient form of the force equations are written below.
In the above equations

\[
Z_L = Z_m - \frac{b}{2} \phi
\]
\[
Z_w = Z_m + \frac{b}{2} \phi
\]
\[
Z_R = Z_m - \ell \theta
\]

Where \(Z_m\) is the distance of the body axis origin (mast step) above the water surface.

(b) Solution of the Force Equations

The first equation to be solved will be that giving the forces in the \(Z\) direction. From the section of this report dealing with the design of the hydrofoils, the area function for the front foils with respect to \(Z\) is

\[
A(Z) = \frac{Cr(S \sin \Omega - Z)^2}{2S \sin^2 \Omega}
\]

and that for the horizontal front foil area is

\[
A_H(Z) = \frac{Cr(S \sin \Omega - Z)^2 \cos(\Omega - \phi)}{2S \sin^2 \Omega}
\]

The horizontal stern foil area is
\[ A_Z(Z) = Z \left\{ (a-Z \csc \xi) \cos H (a-Z \csc \xi) \right\} \\
+ (b-Z \sin \xi) \csc \gamma H (Z \csc \xi-a) \cos \gamma \]

For a given value of \( V_A, V_B, V, \phi, W \) etc. using Newton's Method, the value of \( Z_m \), the distance from the body axis origin to the water surface to satisfy the force equation can be found.

Writing this equation in the form:

\[ F(Z_m) = \rho \pi V_B^2 \left[ A_\alpha (Z_m) \alpha_L + A_\gamma (Z_m) \alpha_H + A_\phi (Z_m) \alpha_R \right] - (P+W) \]

The solution or zero of this equation \( Z_m = \tilde{Z}_m \) will be found using Newton's iteration technique:

\[ Z_{m_{i+1}} = Z_{m_i} - \frac{F(Z_{m_i})}{F'(Z_{m_i})} \]

where

\[ F'(Z_{m_i}) = \frac{d}{dZ} F(Z) \]

\( Z_{m_{i+1}} \) is the \((i+1)\) approximation to the zero \( Z_m \). The rudder angle, \( \delta_R \), is calculated from the moment equation about the \( Z_F \) axis and is a relatively small angle \( \pm 5^\circ \).

The \( y_F \) force equation is automatically satisfied by the boat's assuming a relatively slight sideslip. Because the angle of sideslip will be relatively small \( \pm 2^\circ \) its effect on augmenting the lift of the foils will be minimal. Therefore the angle of sideslip, \( \delta \), was not calculated nor was any augmentation of lift due to sideslip.

Then the drags associated with each hydrofoil at its particular angle of attack will be calculated and subtracted from the apparent wind velocity. If the residue from this subtraction is positive, a lower boat speed, \( V_B \), will be chosen and the calculations will be repeated until the residue is at a sufficiently small value.

Should the residue be negative the boat speed will be raised until again the residue is approximately zero. Thus the three force equations have now been satisfied.

(c) Solution of the Moment Equations

Writing the linearized moment equations in the form

\[ \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} A \end{bmatrix} \]

where \([A]\) and \([B]\) are matrices
\[ \sum M_{x_F} = 0 \]
\[ \bar{Y} \left[ H_\phi - W + P \right] + \bar{Z} \left[ W_\phi \right] = H[ec \cos \lambda \theta - fh] \]
\[ + b/2 \phi \left[ R_w - R_L \right] - R_L d_w - R_w \left( d_\phi + \ell \theta \right) \]
\[ + b/2 \left[ L_L - L_w \right] - L_r d_\phi - P e \cos \lambda \]

\[ \sum M_{y_F} = 0 \]
\[ \bar{X} \left[ -W - 2P + \theta(2P + \ell \phi) \right] = T \left\{ ec(\phi \sin \lambda + \theta \cos \lambda) - fh \right\} \]
\[ + \theta \left( L_w d_w + L_L d_L \right) - L_R \ell + b/2 \phi \left\{ D_w - D_L \right\} \]
\[ - D_w d_w - D_L d_L - D_R \left( d_\phi + \ell \theta \right) + P (fh \theta + eccos \lambda) \]

\[ \sum M_{z_F} = 0 \]
\[ \bar{Z} \left[ -H \theta + T \phi \right] = \theta \left[ R_w d_w + R_L d_L \right] - R_R \ell \]
\[ + b/2 \left( D_w - d_L \right) + D_R d_\phi \]
\[ + ec \left( T \sin \lambda + H \cos \lambda \right) \]

The moment equations have been written in the form
\[ [A] \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = [B] \]
where \([A]\) and \([B]\) are matrices whose entries are calculated from the various parameters both given and calculated before.

(i) Control of the centre of gravity

In a small boat it is usually necessary for the operators to shift their position to the windward side of the boat so that their weight will cause a moment which will help to balance that of the heeling force. As this is done by the operators so as to better control the boat, this shifting of the centre of gravity may be considered to be a control parameter much as the boom angle, \(\lambda\), and the rudder angle, \(\delta_R\), are control parameters.

Although the \(X\) and \(Y\) coordinates of the centre of gravity are easily varied by shifts in position of the operators, there is usually very little change in the \(Z\) coordinate regardless of the operators' position. For this reason a value for \(Z\), the \(Z\) coordinate of the centre of gravity, will be chosen in line with what is a reasonable estimate for the boat in question. For this value of \(Z\), the corresponding \(X\) and \(Y\) coordinates will be calculated from a reduced form of the moment equations given by:

\[ [A'] \begin{bmatrix} X \\ Y \end{bmatrix} = [B'] \]

where \([A']\) is the \(2 \times 2\) subset of \([A]\) composed of the first and second rows and
the first and second columns and $[B']$ is identical to $[B]$ except that its elements have been augmented by the $Z$ entries of matrix $[A]$. 

With an assumed value for $Z$, the $Z$ moment equation is now used to determine the rudder force required for yaw stability. From the rudder force, the rudder angle, $\delta_R$, is readily calculated.

It should be noted that an approximation is being made here. If the rudder angle, $\delta_R$, is relatively large it will result in a correspondingly large increment to the angle of attack of the rear foil. Thus large rudder angles may induce changes in the rear foil lift resulting in a different equilibrium state being set up. This augmentation in rear foil lift due to rudder angle, $\delta_R$, has been neglected as insignificant and hence to be valid the rudder angle should always be small.

(ii) Weight estimate: permissible range for C.G.

The various dimensions and weights of the component sections of the boat are estimated below. Included in the estimate is the weight of the two operators who are assumed to be capable of moving to the extremities of the boat in order to effect the position of the centre of gravity.

Mast (including sails, rigging and boom) 50 lbs.
Longitudinal Member (including rear foil assembly) 100 lbs.
Length 16 ft.
Centre of gravity 6 ft (from stern)
Transverse Member (including front foils) 100 lbs.
Length 14 ft.
Centre of gravity 7 ft (from either end)
Operators 2 at 175 lbs. 350 lbs.

Using these estimates the permissible C.G. range can be calculated as follows:

![FIGURE 8: Estimated C.G. for various components of boat (plan view)](image_url)
With operator at (0,6)

\[ \bar{X} = \frac{+600}{600} = 1.0 \text{ ft.} \]

\[ \bar{Y} = \frac{350 \times 7}{600} = 4.08 \text{ ft.} \]

With operators at (-10,0)

\[ \bar{X} = \frac{10 \times 350}{600} = 5.83 \text{ ft.} \]

\[ \bar{Y} = 0 \text{ ft.} \]

It is assumed that it is impractical for reasons of controlling the rudder to contemplate having the operators forward of the mast.

The permissible range for the X and Y coordinate of the centre of gravity is from approximately 1 to 6 feet aft of the mast and within 4 feet of the centre line of the boat.

A value for Z of 1.0 ft. above the centre line of the boat was chosen as a reasonable estimate.

Therefore the estimated permissible range for the centre of gravity is

\[ \bar{X}_{\text{max}} = 5.83 \text{ ft.} \]

\[ \bar{X}_{\text{min}} = +1.0 \text{ ft.} \]

\[ \bar{Y}_{\text{max}} = 4.08 \text{ ft.} \]

\[ \bar{Y}_{\text{min}} = -4.08 \text{ ft.} \]

\[ \bar{Z} = 1.0 \text{ ft.} \]

(d) Estimate of Static Stability

To estimate whether the chosen set of parameters \( \theta, \phi, \alpha_r, \alpha_w, \alpha_L \), etc., constitute a stable equilibrium, the values of the rolling and pitching derivatives (\( \frac{\partial M_y}{\partial \phi} \) and \( \frac{\partial M_y}{\partial \theta} \)) were calculated.

Positive pitch stiffness exists if the pitching derivative is positive. Positive roll stiffness exists if the rolling derivative is negative.

V. COMPENSATION BY PROPOSED CONTROL SYSTEM VERSUS FREE RUNNING

The equilibrium force and moment equations of the past pages will be solved in two ways to show the effects of the proposed control system.

(A) "Free Running"

In the first method, the boat is considered to be "free running". That is to say that the static stability of the boat is maintained solely through the movement of the centre of gravity which is accomplished by the operators shifting their weight in response to the various aerodynamic and hydrodynamic loads imposed on the boat.

As has just been shown, the position of the centre of gravity can be shifted only within certain bounds by the operators to maintain the boat's stability. Thus the boat in its "free running" state has corresponding limits placed upon its static stability. Forces can occur of such magnitude that the
simple shift of the operators' weight will not be sufficient to maintain stability
and consequently the boat will pass into a state of unstable equilibrium as shown
by the change in sign of the stability derivatives.

In practice this would result in the following occurrences unless some
non-linearity entered into the system causing a restoring moment.

(a) \( \frac{\partial M_x}{\partial \phi} > 0 \) unstable in roll resulting in capsizing
(b) \( \frac{\partial M_y}{\partial \theta} < 0 \) unstable in pitch resulting in "porpoising" or "sea crash"
(c) \( \frac{\partial M_x}{\partial \phi} > 0 \)
\( \frac{\partial M_y}{\partial \phi} < 0 \)
both capsizing and "porpoising" or "sea crash"

(B) Proposed Control System Compensation: Static Evaluation

The proposed feedback control system couples the hydrodynamic forces
to the aerodynamic force inputs. The hydrodynamic forces are compensated by
amounts proportional to the magnitude of the aerodynamic forces. The method
by which this is accomplished for the two modes of pitching and rolling is as
described below. The diagram giving the frontal and side view of the proposed
craft, Figures 9 and 10 will aid in the explanation.

(i) Pitch compensation

Pitching of the craft can be compensated for by changing the rear foil
angle of attack in a proportionate manner in response to changes in thrust. As
the thrust increases the rear foil angle of attack will decrease and may eventu­
ally go into negative incidence. This change in the rear foil lift force,
\( \Delta L_R \), will cause a change in the moment about the \( y_F \) axis to compensate for
the increased moment caused by the change in thrust, \( \Delta T \).

The method of changing \( \alpha_R \) in response to the thrust will be through
changes in the tension of the backstay which are induced by changes in the
thrust.

As may be see from Figure 11, the windward and leeward shrouds form
a plane perpendicular to the X-Y plane of the boat and parallel to the Z-Y plane.
Hence for no deflection of the mast out of its Z-Y plane, the shrouds can carry
no component of the thrust force. Assuming the mast to be rigidly fixed at its
base, a component of the thrust, \( T \), must be held by the backstay. By fastening
the backstay to the rear foil assembly as shown in Figures 11 and 12 the appro­
priate angle of attack changes can be effected.

As can be seen from Figure 12, the method of reducing \( \alpha_R \) also causes
a reduction in the distance between the lowermost tip of the rear foil and the
longitudinal member. This reduction in length, \( \Delta \ell \), causes an increase in the
pitch angle \( \Delta \theta \) which effectively adds a component to the angles of attack of
all the foils. This increase in the angles of attack \( \Delta \theta \) can be calculated as

\[ \Delta \theta \approx \frac{\Delta \ell}{\ell} \quad \text{if} \quad \Delta \ell \ll \ell \]
where \( l \) is the distance from bow to stern foils

The desired changes in \( \alpha \) are obtained using the tension of the backstay \( S_B \) to compress a spring between the hinged section and the longitudinal member. The compression of the spring which is proportional to the tension in the backstay and ultimately to the thrust on the sails causes the hinged section to swing through an angle, \( \Delta \alpha \), proportional to the thrust, \( T \), the spring constant, \( k \), and the length of the hinged section, \( d \). Since \( k \) and \( d \) are constants the angle of attack change, \( \Delta \alpha \), is directly proportional to the thrust \( T \) which is related to the tension in the backstay \( S_B \) by a constant, \( \kappa \), which is dependent on the geometry of the craft.

\[
S_B = \kappa T
\]

\[
\Delta \alpha(T) = \frac{S_B}{k} \cdot \frac{1}{d} \cos(\xi)
\]

\[
= \left\{ \frac{K \cos(\xi)}{k d} \right\} T
\]

\[
= C_1 T
\]

The increase in pitch angle, \( \Delta \theta \), due to the effective strut length reduction, \( \Delta l \), can be given now as:

\[
\Delta \theta = \frac{\Delta l}{\kappa} = \frac{S_B \cos(\xi)}{k} \frac{1}{l}
\]

\[
= \left\{ \frac{K \cos(\xi)}{k \ell} \right\} T = C_2 T
\]

Thus the form of the rear foil angle of attack, \( \alpha_R(T) \) is

\[
\alpha_R(T) = C_R - C_1 T + C_2 T
\]

\[
= C_R + T(C_2 - C_1)
\]

where \( C_R \) is a preset constant component of the angle of attack

By trial and error the values shown below were determined and used to begin the simulation.

\[
C_R = 5^\circ
\]

\[
C_1 = \frac{1}{200} \text{ ft}/\text{lb.}
\]

\[
C_2 = \frac{1}{50} \text{ ft}/\text{lb.}
\]

As the simulation progressed these values were varied to ascertain their effect on the stiffness in both pitch and roll.

(ii) Heel compensation

Heeling of the craft can be compensated by changing the angles of
attack of the windward and leeward foils in response to the applied heeling force, \( H \). If the windward foil angle of attack \( \alpha_w \) is decreased and the leeward foil angle of attack \( \alpha_L \) increased the resulting decrease in \( L_w \) and increase in \( L_L \) can act to produce a moment to compensate for that due to the heeling force, \( H \).

As the boat itself is geometrically symmetric about the X-Z plane if the angle of roll \( \phi \) is zero then the total lift force of the front foils, \( (L_L + L_L) \) will be constant even after equal angle of attack changes, \( + \Delta \alpha \), to both the leeward and windward foils (i.e., if the leeward and windward hydrofoils are both set to an angle of attack, \( \alpha_L \), when the heeling force is zero; then for any non zero value of \( H \), \( \alpha_L = \alpha_L + C_L H \)

\[
\alpha_L = \alpha_L - C_L H
\]

and the total lift \( (L_L + L_L) \) remains constant for any value of \( H \).

Since the total lift from the bow foils remains constant the heel compensation system will not affect the force equilibrium nor the value of the pitch angle, \( \theta \).

The method by which the heel compensation is effected is similar to that of the pitch compensation. The assumption is again made that the mast is rigidly fixed in the mast step. Because both shrouds are in a plane parallel to that of the heeling force, \( H \), and perpendicular to that of the thrust force, \( T \), they can only carry a component of \( H \), not of \( T \). Thus the tension in each shroud (\( S_L \) the leeward shroud tension and \( S_w \) the windward shroud tension) is independent of the thrust \( T \).

From Figure 9 it can be seen that the shrouds are attached at the upper end to the mast. At the lower end with reference to Figure 13, it can be seen how they are attached to the bow foils:

Both windward and leeward shrouds are in tension when the heeling force is zero (i.e., the rubber material is partially compressed between the upper foil support and the transverse member).

When the heeling force is not zero, the tension in the windward shroud increases in direct proportion to the heeling force by an amount dependent on the geometry of the craft. The rubber material is further compressed allowing the foil to rotate about its hinge thereby decreasing its angle of attack.

On the leeward side the heeling force causes a decrease in the tension in the shroud which in turn relaxes some of the compressive force on the rubber material allowing it to expand. Thus on the leeward side the foil rotates in the opposite direction as compared to the windward side and the angle of attack is increased. In this way the objective of increasing the lift on the leeward side and decreasing that on the windward side may be achieved.

With reference to the pitch control, it was seen that the method of preserving equilibrium was to decrease the rear foil lift with an increase in the thrust. Thus it appears that a net loss in overall lift from the three foils can result from this method of pitch control. This is partially compensated by two effects.

The first of these is that of dihedral. All three foils were designed with dihedral so that the submerged foil area (which is the only foil area
effective in providing lift) is automatically increased with even a small increase in the mean water level. Thus any decrease in lift caused by the pitch control mechanism would be quickly "made up" by the lift from the increased area brought into play with a small rise in the mean water level.

The second effect tending to maintain the total lift constant regardless of the effects of the pitch control is that of the augmentation in pitch angle, $\Delta \theta$, as is illustrated in Figure 12. The increase in pitch angle occurs to all three foils simultaneously and has the same effect as an increase in angle of attack. Because the area of the two bow foils is much greater than that of the stern foil, the increase in pitch angle, $\Delta \theta$, can cause enough of an increase in the lift of the bow foils to compensate for the loss in lift from the rear foil.

Taking these effects into consideration gives a form of the bow foil angles of attack as shown below:

Leeward Foil:

$$\alpha_L(T, H) = C_L + C_2 T + C_3 H$$

Windward Foil:

$$\alpha_W(T, H) = C_W + C_2 T - C_3 H$$

where $C_L$ and $C_W$ are preset constant components of the angles of attack of the leeward and windward foils respectively.

By trial and error values shown below were determined and used in the computer simulation to begin with.

$$C_L = 6^\circ, \quad C_2 = \frac{1}{50^\circ} / \text{lb.}$$

$$C_W = 6^\circ, \quad C_3 = \frac{1}{40^\circ} / \text{lb.}$$

As the simulation continued and the results were analyzed the parameters were varied to ascertain the effects on the stiffness derivatives.

VI. COMPUTER SIMULATION OF PROBLEM

A copy of the "print-out" of program used to analyze the equilibrium is included in Appendix 3.

Basically the program begins by reading in the various parameters of the craft including the boat speed, the true wind speed and the true wind direction. The iteration procedure starts with the true wind speed, 10 m.p.h., and the true wind direction, $\Psi_T$, at 0° (refer to figure 5).

The apparent wind direction, $\Psi_A$, and the apparent wind speed, $V_A$, are then calculated using an assumed value for the boat speed. The thrust on the sails is then calculated using the apparent wind direction, $\Psi_A$, as the angle of attack. The boom angle, $\lambda$, is then increased by an increment and subtracted from $\Psi_T$ to decrease the angle of attack. The value of the thrust is then compared with the last value and if it is greater the procedure is repeated.
until the maximum value of thrust $T_{\text{max}}$ is found for the particular wind and boat conditions. With this maximum value of thrust $T_{\text{max}}$ now found, the corresponding values for the heeling force $H$ and the downhaul force $P$ are calculated. An initial guess is then made for the mean water level.

The angles of attack are then set for each foil. In the "free running" case the three foil angles are preset constants with the leeward foil angle equal to the windward foil angle.

In the case of the proposed control system, the three foil angles are functions of the aerodynamic force components $H$ & $T$.

The values of the lift, drag, and side force etc., are then calculated for each foil for that particular depth of submersion.

Newton's Iteration Method is then used to solve for the mean water level height in the $Z$-force equation. If the mean water level value found is negative, it corresponds to a physically unrealizable situation and the message "No equilibrium possible and $Z$ negative" is printed out. Then the program increases the true wind direction by an increment of $10^\circ$ and the whole procedure is begun again. If after 50 iterations, the mean water level has not converged within the error criterion the message: "50 Iterations and Failed to Converge" is printed out.

The $X$ force equation is then satisfied in the following manner. The drags from the three foils are subtracted from the thrust. If the resulting difference is small enough to be within the error criterion, the boat speed and the error are written according to the format: "Error, Convergent Speed". If the error is too large, a correction is calculated using the magnitude of the error and either added to or subtracted from the boat speed depending upon the sign of the error. If, for example, the error is positive and the thrust is greater than the drags, the boat speed is increased which has the effect of increasing the drags while because the apparent wind automatically shifts forward, the thrust force $T$ is slightly reduced. With this corrected value of boat speed the program returns to the point of calculating the apparent wind and repeats the interim calculations to again satisfy the $Z$-force equation. As the boat speed is iterated in this manner the $X$-force equation is eventually satisfied within the allowable error criterion.

The $Z$ moment equation is then used to solve for the rudder force $R_z$, using the assumed value for $Z$. With the rudder force known, the rudder angle is then calculated.

The entries in matrices $[A']$ and $[B']$ of the moment equations are then calculated. The inverse of matrix $[A']$ is taken using subprogram MLNW and this is multiplied by matrix $[B']$ using subprogram GMFRD. These calculations result in the determination of the $X$ and $Y$ coordinates of the centre of gravity for the equilibrium state.

Then the static pitch and roll stiffness derivatives are calculated.

The true wind direction, $\varpi_T$, is increased by increments of $10^\circ$ up to a limit of $180^\circ$ and on each increment the equilibrium state if any is analyzed as above.
When the true wind direction $\Psi_T$ reaches $180^\circ$ the true wind speed is increased by an increment of 5.0 m.p.h., $\Psi$ is set to zero and the equilibrium calculations are continued at this higher wind speed.

The value of the true wind speed, $V_T$, is thus increased from a low of 10.0 m.p.h. in increments of 5.0 m.p.h. to a maximum of 20.0 m.p.h. In all, the equilibrium states associated with 18 different true wind directions at each of 3 different true wind speeds are analyzed giving the boat speed, the required position of the centre of gravity, the roll and pitch stiffness, the mean water level, the rudder angle and the boom angle for each equilibrium state.

(a) Simulation Results

(i) Permissible zones of sailing

With reference to Figure 14 the zones where foil borne sailing are possible are shown for true wind speeds of 10, 15 and 20 m.p.h.

Although it was possible to carry the simulation further to give zones of sailing for higher values of true wind speed it was felt that at such speeds the assumption upon which the hydrodynamic forces were calculated, that of a flat water surface (without waves) was increasingly invalid even for relatively short stretches of open water. Thus the limiting true wind velocity was chosen at 20 m.p.h.

The direction of the boat in Figure 14 is along the positive axis. The true wind velocity vector makes an angle $\Psi_T$ with the axis as shown.

For a true wind speed of 10.0 m.p.h., the craft can operate on its foils for the range of true wind direction from $\Psi_T = 70^\circ$ to $\Psi_T = 110^\circ$. For a true wind speed of 15.0 m.p.h. this range broadens to $\Psi_T = 70^\circ$ to $\Psi_T = 140^\circ$, a spread of 90°. For an increase in true wind speed to 20.0 m.p.h. the zone for foil borne sailing is only increased by a further 10°.

For all these cases it appears from Figure 14 that the boat's sailing pattern is from a narrow to a broad reach (i.e., the wind is coming from a direction more or less perpendicular to the boat's centre line). This appearance is deceiving since it is the apparent wind direction that determines the name of the course sailed. Because of the boat's velocity, the apparent wind direction is in all cases an angle smaller than that of the true wind direction (i.e., $\Psi_A < \Psi_T$ for $V_B > 0$). Thus the courses sailed more likely vary between a fairly close "tack" at about $\Psi_T = 60^\circ$ to a "reach" at about $\Psi_T = 140^\circ$.

From Figure 14, it can be seen that the increase in true wind speed from 15.0 to 20.0 m.p.h. did not increase $\Psi_T$ from 140° to some higher value. From this one observation it may be inferred that the drop in apparent wind speed at angles of true wind direction exceeding about 140° is so great as to make foil borne sailing impossible. (At angles exceeding 140°, sufficient thrust cannot be derived from the sails to maintain the craft on its foils). Thus a hydrofoil sailboat would never "run" downwind ("run" defined as $\Psi_T \geq 180^\circ$) because it would be impossible for it to stay foil borne. Instead it would "tack" downwind holding $\Psi_T$ to a limit of approximately 140°, maintain itself foil borne, and hence maintain a fairly high rate of speed.
From Figure 14, it is apparent that increases in true wind speed allow the boat to "point" closer to the true wind direction (i.e., the boat can proceed more directly into the true wind). At 20.0 m.p.h. the angle between the boat's velocity and the minimum true wind direction is 40°. This is comparable with the "pointing" ability of conventional sail boats; however, when the true wind speed drops to 15 m.p.h., the boat angle to be made to windward is 50° and when the speed drops to 10.0 m.p.h. the best angle to windward is 70°. The conventional boat maintains a more or less constant ability to go to windward with an angle from 30° to 45° regardless of the true wind speed. Unfortunately the hydrofoil sail boat does not possess this ability. Once the magnitude of the true wind is known only a certain zone is permissible for foil borne sailing and this zone decreases in extent as the true wind speed drops.

Eventually as the wind speed drops the zone of sailing will shrink to a single compass heading along which the boat will just be able to maintain itself foil borne. Any further decrease in wind speed, or any slight deviation in compass heading will cause the boat to "come off its foils" and proceed as a conventional boat.

Implications as to whether the hydrofoil sailing craft is practical in a particular region can be drawn from the above discussion. In the places of the world with average wind velocities during the days of the summer months much lower than 10 m.p.h. the hydrofoil sail boat does not seem to be practical since there would be insufficient winds during much of the time to allow hydrofoil sailing. In Toronto for example approximately 40% to 50% of the daylight hours had winds of 10 m.p.h. or greater in July and August during the two years 1960 and 1961. The judgement as to whether this represents a reasonable enough percentage of the time to render hydrofoil sailing practical here is left to the reader.

(ii) Mean water level variation with boat speed, true wind speed and direction

As can be seen from the entries of Table 3, the mean water level varies directly with the boat velocity. For any particular course sailed the water level increases with increases in the boat velocity. This is as expected since the hydrodynamic forces on the foils are dependent on the square of the boat velocity. Since the hydrodynamic force required in the $Z_F$ direction is approximately constant, being equal to the weight of the boat plus the downward sail force component, $P$, then as the boat speed increases less and less immersed foil area is required. Thus the mean water level increases with speed thereby decreasing the area of the foils immersed.

The true wind direction also has a very significant effect on the boat velocity. The boat first becomes foil borne at small values of $V_T$ and is travelling relatively slowly. For fairly small increases in $V_T$ the boat speed increases dramatically. One should note that while these boat speeds seem to be unreasonably high in relation to the speeds of conventional sail boats, hydrofoil sail boats have already been built which will attain speeds up to 30 knots. Thus while the estimates for drag may be slightly low, the calculated boat speeds are reasonable for the type of boat considered.

The results of Table 3 were calculated for the values of the parameters listed; however, the results are generally consistent with those calculated from corresponding sets of parameters. This particular case was chosen
**TABLE 3: Mean Water Level and Boat Speed vs. True Wind Direction and Velocity**

"Free Flying" Case $\phi = 3^\circ; \theta = 0^\circ$

$\alpha_R = 3^\circ$

$\alpha_w = 6^\circ$

$\alpha_L = 6^\circ$

<table>
<thead>
<tr>
<th>True Wind Direction [°]</th>
<th>True Wind Velocity 10 m.p.h.</th>
<th>True Wind Velocity 15 m.p.h.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Boat Velocity [ft./sec.]</td>
<td>Mean Water Level [ft.]</td>
</tr>
<tr>
<td>0</td>
<td>32.00</td>
<td>2.3816</td>
</tr>
<tr>
<td>10</td>
<td>44.48</td>
<td>2.7501</td>
</tr>
<tr>
<td>20</td>
<td>54.43</td>
<td>2.9533</td>
</tr>
<tr>
<td>30</td>
<td>58.70</td>
<td>3.0847</td>
</tr>
<tr>
<td>40</td>
<td>63.84</td>
<td>3.1096</td>
</tr>
<tr>
<td>50</td>
<td>66.14</td>
<td>3.0938</td>
</tr>
<tr>
<td>60</td>
<td>64.09</td>
<td>3.0921</td>
</tr>
<tr>
<td>70</td>
<td>NSC</td>
<td>58.33</td>
</tr>
<tr>
<td>80</td>
<td>14.68</td>
<td>.8659</td>
</tr>
<tr>
<td>90</td>
<td>14.68</td>
<td>.8646</td>
</tr>
<tr>
<td>100</td>
<td>14.68</td>
<td>.8631</td>
</tr>
<tr>
<td>110</td>
<td>NSC</td>
<td>52.16</td>
</tr>
<tr>
<td>120</td>
<td></td>
<td>2.8876</td>
</tr>
<tr>
<td>130</td>
<td></td>
<td>NSC</td>
</tr>
<tr>
<td>140</td>
<td></td>
<td></td>
</tr>
<tr>
<td>150</td>
<td></td>
<td></td>
</tr>
<tr>
<td>160</td>
<td></td>
<td></td>
</tr>
<tr>
<td>170</td>
<td></td>
<td></td>
</tr>
<tr>
<td>180</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**NSC** - no speed convergence

To show the pertinent trends in the data because in nearly all cases the iterative procedure in the computer program converged (for $\Psi_T = 70^\circ$ and $\Psi_T = 110^\circ$, at $V_T = 10$ m.p.h., and $\Psi_T = 140^\circ$ at $V_T = 15$ m.p.h., there was no speed convergence and hence no values can be given for these entries in the tables).

(iii) Changes in $\Psi_A$, $V_A$, $\lambda$ and $\delta_R$ with $V_T$ and $\Psi_T$

From examining the column of Table 4 containing the rudder angle, $\delta_R$, it can be seen that the values are in all cases small (the maximum being $1.03^\circ$ and the minimum $0.25^\circ$). This verifies the assumption made during the solution of the equations that the rudder angle must be relatively small so that the increment to rear foil lift from the rudder angle can be neglected.
In part (i) it was stated that regardless of the value of the true wind direction most courses sailed would be termed either a "tack" or a "reach". This point is borne out by the results in this table.

**TABLE 4:** Changes in $\Psi_A$, $V_A$, $\phi$, $\delta_R$ with $V_T$ and $\Psi_T$

<table>
<thead>
<tr>
<th>True Wind Direction</th>
<th>True Wind Velocity</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>10 m.p.h.</td>
</tr>
<tr>
<td></td>
<td>15 m.p.h.</td>
</tr>
<tr>
<td>[$^\circ$]</td>
<td>Apparent Wind</td>
</tr>
<tr>
<td></td>
<td>Speed</td>
</tr>
<tr>
<td></td>
<td>[ft./sec.]</td>
</tr>
<tr>
<td></td>
<td>Boom Angle</td>
</tr>
<tr>
<td></td>
<td>Rudder Angle</td>
</tr>
<tr>
<td>0</td>
<td>17.6</td>
</tr>
<tr>
<td>10</td>
<td>16.0</td>
</tr>
<tr>
<td>20</td>
<td>15.2</td>
</tr>
<tr>
<td>30</td>
<td>14.68</td>
</tr>
<tr>
<td>40</td>
<td>14.0</td>
</tr>
<tr>
<td>50</td>
<td>21.0</td>
</tr>
<tr>
<td>60</td>
<td>12.31</td>
</tr>
<tr>
<td>70</td>
<td>47.8</td>
</tr>
<tr>
<td>80</td>
<td>37.9</td>
</tr>
<tr>
<td>90</td>
<td>42.9</td>
</tr>
<tr>
<td>100</td>
<td>47.8</td>
</tr>
<tr>
<td>110</td>
<td>13.64</td>
</tr>
<tr>
<td>120</td>
<td>12.31</td>
</tr>
<tr>
<td>130</td>
<td>19.2</td>
</tr>
<tr>
<td>140</td>
<td>NSC</td>
</tr>
<tr>
<td>150</td>
<td>17.6</td>
</tr>
<tr>
<td>160</td>
<td>NSC</td>
</tr>
<tr>
<td>170</td>
<td>19.2</td>
</tr>
<tr>
<td>180</td>
<td>NSC</td>
</tr>
</tbody>
</table>

In the case of the true wind velocity being 15 m.p.h. as the true wind direction varies from 50° to 130° a total of 80° the apparent wind direction changes only from 17.6° to 19.2° a total of 1.6°. This is due to the very large value of boat speed in relation to true wind speed over this range and any sailor would term this course a "tack", since from his reference frame the wind (relative wind) is from nearly directly ahead of the boat.

It can also be seen how this effects the boom angle. For a boat to "tack", the boom must be hauled in to be more or less parallel to the centre.
line of the boat (termed "running closed-hauled"). Because of this for $V_T = 15$ m.p.h., the boom angle is relatively small.

For $V_T = 10$ m.p.h., the boat is proceeding on a "reach". The angles of the apparent wind direction are in comparison to those of the higher true wind speed relatively large. In turn, the boom angle is also relatively large consistent with normal sailing practice on conventional boats proceeding on a "reach".

(iv) Centre of gravity and stiffness derivatives

Table 5 shows the values of $\bar{x}$ and $\bar{y}$ and the stiffness derivatives $\frac{\partial M}{\partial \phi}$ and $\frac{\partial M}{\partial \theta}$ for various angles and speeds of the true wind considered.

For a true wind of 10 m.p.h., the values of $\bar{x}$ and $\bar{y}$ are well within those considered permissible and natural. They represent positions approximately three feet aft of the mast slightly to leeward.

The values for a true wind speed of 15 m.p.h. are somewhat more extreme. $\bar{x}$ must assume values up to nearly 9 feet aft of the mast while $\bar{y}$ up to 6 feet to windward. These values are outside the range of those considered permissible for the centre of gravity. However with a slight increase in pitching angle $\theta$ and a similar increase in rolling angle $\phi$ it should be possible to reduce these values somewhat.

TABLE 5: Centre of Gravity and Stiffness Derivatives

$\phi = 3^\circ \quad \alpha_R = 3^\circ$

$\theta = 0^\circ \quad \alpha_w = \alpha_L = 6^\circ$ "Free Flying" Case

<table>
<thead>
<tr>
<th>True Wind Direction</th>
<th>True Wind Velocity</th>
<th>$\bar{x}$</th>
<th>$\bar{y}$</th>
<th>$\frac{\partial M}{\partial \phi}$</th>
<th>$\frac{\partial M}{\partial \theta}$</th>
<th>$\frac{\partial M}{\partial \phi}$</th>
<th>$\frac{\partial M}{\partial \theta}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>10 m.p.h.</td>
<td>15 m.p.h.</td>
<td>10</td>
<td>5.62</td>
<td>2.17</td>
<td>28,100</td>
<td>13,300</td>
<td></td>
</tr>
<tr>
<td>20</td>
<td></td>
<td>6.80</td>
<td>3.50</td>
<td>46,300</td>
<td></td>
<td>14,700</td>
<td></td>
</tr>
<tr>
<td>30</td>
<td></td>
<td>7.53</td>
<td>5.17</td>
<td>71,600</td>
<td></td>
<td>12,600</td>
<td></td>
</tr>
<tr>
<td>40</td>
<td></td>
<td>8.78</td>
<td>4.81</td>
<td>77,200</td>
<td></td>
<td>23,500</td>
<td></td>
</tr>
<tr>
<td>50</td>
<td></td>
<td>7.57</td>
<td>4.81</td>
<td>77,200</td>
<td></td>
<td>12,600</td>
<td></td>
</tr>
<tr>
<td>60</td>
<td></td>
<td>8.74</td>
<td>4.81</td>
<td>77,200</td>
<td></td>
<td>23,500</td>
<td></td>
</tr>
<tr>
<td>70</td>
<td></td>
<td>7.84</td>
<td>4.81</td>
<td>77,200</td>
<td></td>
<td>12,600</td>
<td></td>
</tr>
<tr>
<td>80</td>
<td></td>
<td>8.74</td>
<td>4.81</td>
<td>77,200</td>
<td></td>
<td>23,500</td>
<td></td>
</tr>
<tr>
<td>90</td>
<td></td>
<td>7.84</td>
<td>4.81</td>
<td>77,200</td>
<td></td>
<td>12,600</td>
<td></td>
</tr>
<tr>
<td>100</td>
<td></td>
<td>7.84</td>
<td>4.81</td>
<td>77,200</td>
<td></td>
<td>12,600</td>
<td></td>
</tr>
<tr>
<td>110</td>
<td></td>
<td>7.84</td>
<td>4.81</td>
<td>77,200</td>
<td></td>
<td>12,600</td>
<td></td>
</tr>
<tr>
<td>120</td>
<td></td>
<td>7.84</td>
<td>4.81</td>
<td>77,200</td>
<td></td>
<td>12,600</td>
<td></td>
</tr>
<tr>
<td>130</td>
<td></td>
<td>7.84</td>
<td>4.81</td>
<td>77,200</td>
<td></td>
<td>12,600</td>
<td></td>
</tr>
<tr>
<td>140</td>
<td></td>
<td>7.84</td>
<td>4.81</td>
<td>77,200</td>
<td></td>
<td>12,600</td>
<td></td>
</tr>
<tr>
<td>150</td>
<td></td>
<td>7.84</td>
<td>4.81</td>
<td>77,200</td>
<td></td>
<td>12,600</td>
<td></td>
</tr>
<tr>
<td>160</td>
<td></td>
<td>7.84</td>
<td>4.81</td>
<td>77,200</td>
<td></td>
<td>12,600</td>
<td></td>
</tr>
<tr>
<td>170</td>
<td></td>
<td>7.84</td>
<td>4.81</td>
<td>77,200</td>
<td></td>
<td>12,600</td>
<td></td>
</tr>
<tr>
<td>180</td>
<td></td>
<td>7.84</td>
<td>4.81</td>
<td>77,200</td>
<td></td>
<td>12,600</td>
<td></td>
</tr>
</tbody>
</table>

For example at $\psi_T = 50^\circ$, $V_T = 15$ m.p.h. $\theta = 40^\circ$, $\phi = 30^\circ$

$\{\bar{x} = 2.44; \bar{y} = .463\}$ (The full particulars of this case were not given since there was no equilibrium state)
for a true wind magnitude of 10.0 m.p.h. It is assumed that at this much higher value for \( \theta \) and hence a much larger angle of attack the drag of the foils increased to the point where a true wind of 10 m.p.h. was not sufficient to provide the thrust necessary to overcome this drag. At true winds of 15 m.p.h., equilibrium states were present comparable to those calculated when \( \theta = 0^\circ \) except for the centre of gravity position.

For true wind speeds of both 10 and 15 m.p.h., the pitching moment derivative is positive corresponding to a positive stiffness. Therefore it is inferred that the boat as designed is statically stable in pitch and needs no further active control device to either achieve or enlarge this positive stiffness.

The roll stiffness is however negative. The positive value of the derivative indicates that the boat is unstable in roll.

To test the effectiveness of the proposed control system in stabilizing the boat, the following control parameters were used in the simulation:

\[
C_1 = .0008725 \ [lb] \\
C_2 = .000349 \ " \\
C_3 = .000436 \ "
\]

These parameters are one order of magnitude smaller than those first suggested for use with the simulation program and were used to begin the test of the control system.

It was found that the increased drag caused by the alterations in the angles of attack was greater (in all cases of the true wind direction) than the thrust which could be derived from the sails for a true wind speed of 10 m.p.h. Thus for \( V_T = 10 \) m.p.h., there were no foil borne equilibrium states.

Table 6 shows the boat speed, \( \bar{X} \), \( \bar{Y} \), and the two stiffness derivatives for a true wind speed of 15.0 m.p.h.

As can be seen from the table the boat speeds for the controlled cases are somewhat lower than those for the free flying case. This is likely a result of the fact that the foils are now providing a much larger restoring moment to counterbalance the aerodynamic moments of the sail as well as providing the necessary forces to maintain force equilibrium. Thus the drag of the foils will be somewhat larger now in the controlled case although the thrust of the sails has not changed from the "free flying" case. Hence lower values of boat speed occur in the controlled case in comparison the "free flying" case.

In comparing Tables 5 and 6 it is evident that the values for \( \bar{X} \) and \( \bar{Y} \) are smaller for the controlled than for the free flying case. In the controlled case, \( \bar{X} \) and \( \bar{Y} \) now lie within the estimated permissible range for the centre of gravity.

The effect of the control on the pitch stiffness has been to increase this stiffness in all but one case; however as a side effect, the roll derivative has also been increased. Thus with the control system in its present form and the control constants already specified, the boat has an increased pitch stiffness but at the same time a decreased and unstable roll stiffness (for a positive roll stiffness, \( \partial M_x / \partial \phi < 0 \)).
### TABLE 6: Boat Velocity, \( \ddot{x}, \dot{y} \) Stiffness Derivatives vs. True Wind Speed and Direction

<table>
<thead>
<tr>
<th>True Wind Direction ([^\circ])</th>
<th>Boat Velocity ([\text{ft./sec.}])</th>
<th>( \ddot{x} )</th>
<th>( \dot{y} )</th>
<th>( \frac{\partial M_y}{\partial \theta} ) ([\text{ft.-lbs.}])</th>
<th>( \frac{\partial M_x}{\partial \theta} ) ([\text{ft.-lbs.}])</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>14.68</td>
<td>2.16</td>
<td>-1.29</td>
<td>7,540</td>
<td>13,800</td>
</tr>
<tr>
<td>10</td>
<td>41.70</td>
<td>4.57</td>
<td>1.69</td>
<td>60,500</td>
<td>27,000</td>
</tr>
<tr>
<td>20</td>
<td>50.64</td>
<td>5.13</td>
<td>2.47</td>
<td>91,400</td>
<td>27,600</td>
</tr>
<tr>
<td>30</td>
<td>59.17</td>
<td>4.91</td>
<td>4.04</td>
<td>139,000</td>
<td>26,500</td>
</tr>
<tr>
<td>40</td>
<td>62.57</td>
<td>5.07</td>
<td>3.62</td>
<td>127,000</td>
<td>26,400</td>
</tr>
<tr>
<td>50</td>
<td>64.08</td>
<td>5.17</td>
<td>3.36</td>
<td>119,000</td>
<td>26,100</td>
</tr>
<tr>
<td>60</td>
<td>63.18</td>
<td>5.18</td>
<td>2.89</td>
<td>98,600</td>
<td>26,300</td>
</tr>
<tr>
<td>70</td>
<td>55.64</td>
<td>5.98</td>
<td>.80</td>
<td>57,900</td>
<td>26,200</td>
</tr>
<tr>
<td>80</td>
<td>NSC</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>90</td>
<td>NSC</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>100</td>
<td>59.28</td>
<td>5.61</td>
<td>3.46</td>
<td>101,000</td>
<td>25,800</td>
</tr>
<tr>
<td>110</td>
<td>63.08</td>
<td>5.58</td>
<td>2.94</td>
<td>97,600</td>
<td>25,600</td>
</tr>
<tr>
<td>120</td>
<td>56.64</td>
<td>5.96</td>
<td>.82</td>
<td>57,400</td>
<td>25,500</td>
</tr>
<tr>
<td>130</td>
<td>NSC</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>140</td>
<td>NSC</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>150</td>
<td></td>
<td></td>
<td></td>
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<td></td>
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<tr>
<td>160</td>
<td></td>
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<td></td>
<td></td>
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<tr>
<td>170</td>
<td></td>
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<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>180</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The simulation was then re-run changing the control parameters by a factor of 10, so that:

- \( C_1 = .008725 \text{ [lps}^{-1} \text{]} \)
- \( C_2 = .00349 \)
- \( C_3 = .00436 \)

These are the values for the control parameters specified in Section IV. Unfortunately when these values are used, no equilibrium states exist. This is again due to the fact that the drag is increased when the control system is used to the point where the sails in their present configuration cannot provide sufficient thrust. Hence no foil borne equilibrium states exist.
Next it was decided to see the effect of incorporating the heeling moment compensator alone. For this simulation, the constants were as follows:

\[
\begin{align*}
C_1 &= 0 \\
C_2 &= 0 \\
C_3 &= 0.000436 \text{ [lbs}^{-1}\text{]} \\
\end{align*}
\]

The decision to use the heel compensator alone was based on the fact that the boat as designed already had a positive pitch stiffness and thus did not really need the suggested pitch compensator.

The results of the simulation produced the entries of Table 7. Again there were no equilibrium states for a true wind speed of 10.0 m.p.h. The values of \(\bar{X}\) and \(\bar{Y}\) in Table 7 were generally larger than those in Table 6 but were smaller than those in the free flying case shown in Table 5. These values are fairly close to the limit of what was considered permissible for the center of gravity of the craft.

### Table 7: Boat Velocity, X, Y, Stiffness Derivatives vs. True Wind Speed

<table>
<thead>
<tr>
<th>True Wind Direction [°]</th>
<th>True Wind Speed = 15 m.p.h.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(Heel) Controlled Case</td>
</tr>
<tr>
<td></td>
<td>(C_1 = 0.0) (C_2 = 0.0) (C_3 = 0.000436)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>True Wind Direction [°]</th>
<th>Wind Speed and Direction</th>
<th>(C_R = 3°) (C_P = 6°) (C_L = 6°)</th>
<th>(\bar{X}) [ft.]</th>
<th>(\bar{Y}) [ft.]</th>
<th>(\partial M_F^X/\partial \theta) [ft.-lbs.]</th>
<th>(\partial M_F^Y/\partial \theta) [ft.-lbs.]</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td></td>
<td>(\phi = 3°)</td>
<td>5.19</td>
<td>2.80</td>
<td>72,600</td>
<td>21,600</td>
</tr>
<tr>
<td>10</td>
<td></td>
<td>(\theta = 0°)</td>
<td>5.43</td>
<td>4.06</td>
<td>114,000</td>
<td>20,700</td>
</tr>
<tr>
<td>20</td>
<td></td>
<td>(\phi = 0°)</td>
<td>5.64</td>
<td>4.39</td>
<td>134,000</td>
<td>19,100</td>
</tr>
<tr>
<td>30</td>
<td></td>
<td>(\theta = 3°)</td>
<td>5.82</td>
<td>4.34</td>
<td>140,000</td>
<td>17,800</td>
</tr>
<tr>
<td>40</td>
<td></td>
<td>(\phi = 6°)</td>
<td>5.90</td>
<td>4.06</td>
<td>131,000</td>
<td>17,100</td>
</tr>
<tr>
<td>50</td>
<td></td>
<td>(\theta = 6°)</td>
<td>5.81</td>
<td>3.32</td>
<td>97,600</td>
<td>17,900</td>
</tr>
<tr>
<td>60</td>
<td></td>
<td>(\phi = 6°)</td>
<td>5.77</td>
<td>2.59</td>
<td>72,300</td>
<td>19,500</td>
</tr>
<tr>
<td>70</td>
<td></td>
<td>(\theta = 6°)</td>
<td>5.55</td>
<td>0.65</td>
<td>38,900</td>
<td>18,200</td>
</tr>
<tr>
<td>100</td>
<td></td>
<td>(\phi = 3°)</td>
<td>5.90</td>
<td>4.06</td>
<td>131,000</td>
<td>17,100</td>
</tr>
<tr>
<td>110</td>
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<td>(\theta = 6°)</td>
<td>5.81</td>
<td>3.32</td>
<td>97,600</td>
<td>17,900</td>
</tr>
<tr>
<td>120</td>
<td></td>
<td>(\phi = 6°)</td>
<td>5.77</td>
<td>2.59</td>
<td>72,300</td>
<td>19,500</td>
</tr>
<tr>
<td>130</td>
<td></td>
<td>(\theta = 6°)</td>
<td>5.55</td>
<td>0.65</td>
<td>38,900</td>
<td>18,200</td>
</tr>
<tr>
<td>140</td>
<td></td>
<td>(\phi = 3°)</td>
<td>5.90</td>
<td>4.06</td>
<td>131,000</td>
<td>17,100</td>
</tr>
<tr>
<td>150</td>
<td></td>
<td>(\theta = 6°)</td>
<td>5.81</td>
<td>3.32</td>
<td>97,600</td>
<td>17,900</td>
</tr>
<tr>
<td>160</td>
<td></td>
<td>(\phi = 6°)</td>
<td>5.77</td>
<td>2.59</td>
<td>72,300</td>
<td>19,500</td>
</tr>
<tr>
<td>170</td>
<td></td>
<td>(\theta = 6°)</td>
<td>5.55</td>
<td>0.65</td>
<td>38,900</td>
<td>18,200</td>
</tr>
<tr>
<td>180</td>
<td></td>
<td>(\phi = 3°)</td>
<td>5.90</td>
<td>4.06</td>
<td>131,000</td>
<td>17,100</td>
</tr>
<tr>
<td>190</td>
<td></td>
<td>(\theta = 6°)</td>
<td>5.81</td>
<td>3.32</td>
<td>97,600</td>
<td>17,900</td>
</tr>
<tr>
<td>200</td>
<td></td>
<td>(\phi = 6°)</td>
<td>5.77</td>
<td>2.59</td>
<td>72,300</td>
<td>19,500</td>
</tr>
<tr>
<td>210</td>
<td></td>
<td>(\theta = 6°)</td>
<td>5.55</td>
<td>0.65</td>
<td>38,900</td>
<td>18,200</td>
</tr>
<tr>
<td>220</td>
<td></td>
<td>(\phi = 3°)</td>
<td>5.90</td>
<td>4.06</td>
<td>131,000</td>
<td>17,100</td>
</tr>
<tr>
<td>230</td>
<td></td>
<td>(\theta = 6°)</td>
<td>5.81</td>
<td>3.32</td>
<td>97,600</td>
<td>17,900</td>
</tr>
<tr>
<td>240</td>
<td></td>
<td>(\phi = 6°)</td>
<td>5.77</td>
<td>2.59</td>
<td>72,300</td>
<td>19,500</td>
</tr>
<tr>
<td>250</td>
<td></td>
<td>(\theta = 6°)</td>
<td>5.55</td>
<td>0.65</td>
<td>38,900</td>
<td>18,200</td>
</tr>
</tbody>
</table>
The values of the pitch stiffness are marginally higher while those of the roll stiffness are in most cases considerably lower however they are not so low as the values shown in Table 5 (the free flying case).

The values of the boat speed in Table 7 are slightly higher than those of Table 6 showing that higher speeds can be attained when the drag of the pitch compensation mechanism is not present.

The simulation was run a final time using the heeling moment compensator alone and the control constants as follows:

\[
C_1 = 0 \\
C_2 = 0 \\
C_3 = 0.00436 [\text{lbs}^{-1}]
\]

The results of this simulation were that no equilibrium states were found for a true wind speed of 10, 15 or 20 m.p.h. Hence it must be assumed that the drag incorporated in the use of the heel compensation mechanism was large enough to prevent the boat from rising on its foils on any point of sailing. Unfortunately then, it seems that for the particular boat parameters chosen (including sails and foils it is impossible to simultaneously fulfill the conditions of holding the centre of gravity within a permissible range, satisfy the equations of equilibrium and ensure positive pitch and roll stability using the proposed control system.

Since the purpose of this report was to test the effectiveness of the proposed control system in controlling a hydrofoil sailing craft of an arbitrary (but "reasonable") configuration, it must be said that the proposed control system fails. It is still possible that if a hydrofoil craft were designed with optimized components and this control system applied to it, the result could be a reasonably well controlled boat; however for any particular hydrofoil sail boat design, the proposed control system does not seem adequate.

CONCLUSIONS

1. Hydrofoil sail boats offer very much higher speeds than do conventional sail boats (either monohulls or multihulls).

2. Hydrofoil sail boats are limited in use to periods when the wind velocity is approximately 10 m.p.h. or more.

3. Hydrofoil sail boats must sail within permissible angular zones with respect to wind direction if they are to remain foil borne. These zones broaden as the wind speed increases. For the boat considered it becomes increasingly difficult to proceed into the wind as the wind speed drops from 20 m.p.h. At slightly below a wind speed of 10 m.p.h., the permissible zone of sailing contracts to a single compass heading. A further drop in wind speed or a change in direction of this course will cause the boat to come down off its foils and proceed as a conventional boat.

4. The boat considered has in its free flying form a positive pitch stiffness and a negative roll stiffness. The coordinates of the centre of gravity in the free flying case are considered to be somewhat outside the margins of what is considered acceptable.
5. While the proposed control system brings the values of the centre of gravity within those considered permissible, it is unable to provide the boat with a positive roll stiffness. A considerable increase in drag is associated with the operation of the control system.
FOOTNOTES

1. Common Sense of Yacht Design Volume II - L. Francis Herreshoff

2. Shipbuilding and Shipping Record - May 3, 1951. High Speed Sailing - Hugh Barkla


12. Ibid, Pg. 12, The Hook Hydrofin

13. Ibid, Pg. 11, Grunberg.


25. Monthly Meteorological Summary for Toronto Island Airport (published by the Federal Department of Transport)


32. Ibid, Pg. 247


34. Durand - Aerodynamic Theory. Volume II. Pg. 217. Section 2.2 Table 10.

35. Ibid, Pg. 183: Table 3; Section 8

37. Ibid, Pg. 188.

APPENDIX 1: BOW HYDROFOILS

The bow foils will have a triangular planform with the base connected to the transverse member. The foil will be cantilevered downwards and inwards towards the centre line of the boat at an angle of dihedral $\Omega$. Figure 15 shows the general arrangement. The $Z$ coordinate runs from the root chord of the foil perpendicularly downward to the water surface as shown in Figure 15.

The chord function $C(Z)$ is

$$C(Z) = C_r \left[ 1 - \frac{Z}{S \sin \Omega} \right]$$

The aspect ratio function $AR(Z)$ is

$$AR(Z) = \frac{2h^2}{A} = \frac{2(S-Z\csc \Omega)^2}{C(Z) (S-Z\csc \Omega)}$$

$$= \frac{4S}{C_r}$$

The use of a triangular planform here allows a constant aspect ratio at any and all water levels. The factor of 2.0 in the formula for the aspect ratio comes from the assumption that a surface piercing hydrofoil actually has an image reflection above the water surface. Hence the span is effectively doubled to give an aspect ratio comparable to that in aerodynamic usage.

The centre of pressure function $CP(Z)$ is

$$CP(Z) = \frac{(S-Z\csc \Omega)}{3} + Z \csc \Omega$$

$$= \frac{S + 2Z \csc \Omega}{3}$$

$Z$ coordinate

$$CP_Z(Z) = \frac{(S + 2Z \csc \Omega) \sin \Omega}{3}$$

$$= \frac{S \sin \Omega + 2Z}{3}$$

$Y$ coordinate

$$CP_Y(Z) = \frac{Scos\Omega + 2Z \cot \Omega}{3}$$

$X$ coordinate (taken at $1/4$ chord point)

$$CP_X(Z) = \frac{C[CP_Z(Z)]}{4}$$

$$= \frac{C_r}{6} \left[ \frac{S \sin\Omega - Z}{S \sin \Omega} \right]$$

A1
AREA FUNCTION

The effective (submerged) area function \( A(Z) \) is

\[
A(Z) = \frac{C(Z) (S-Z \csc \Omega)}{2} = \frac{Cr(S \sin \Omega - Z)^2}{2 S \sin^2 \omega}
\]

Reynolds Number

It is necessary at this time to have at least an estimate of the Reynolds number of the flow over the bow foils. For the following values of the various parameters \( Re \) will be calculated as \( Re = 3.42 \times 10^6 \).

Cruising Speed:

\[
V = 25 \text{ m.p.h.} = 36.7 \text{ ft./sec.}
\]
\[
\ell = 1.0, \text{ mean chord}
\]
\[
\mu = 0.672 \times 10^{-3} \text{ lbs./ft.sec.}
\]

For take-off speed:

\[
V = 6 \text{ m.p.h.} = 8.8 \text{ ft./sec.} \quad Re = 0.819 \times 10^6
\]

Hydrofoil Sectional Characteristics

The NACA 65-410^27 wing section was chosen for the bow foils. Its characteristics are shown in Figures 16 and 17.

The characteristics agree favourably with reports\textsuperscript{28} on suggested sections for this use.

Over the range -8° to 12° the \( C_l \) vs. \( \alpha \) curve is linear with

\[
\frac{C_l}{C_l \alpha} \approx 2\pi = a_o
\]

Using tables given in Dynamics of Flight\textsuperscript{29} assuming

\[
\beta = 1 - M^2 = 1.0
\]
\[
K = \frac{C_l \alpha}{a_o} = 1.0
\]
\[
AR = A
\]

It is anticipated that the aspect ratio for the foils will be approximately 10, and since \( AR \) does not vary with depth of immersion this \( AR \) will be constant for all water levels.

from the table

\[
\lambda = 0 \quad \text{taper ratio} \quad \lambda = \frac{\text{tip chord}}{\text{root chord}}
\]
\[
\Lambda = 0 \quad \text{angle of sweepback}
\]
\[
C_l \alpha = 0.085/°
\]
\[
= 4.86
\]
Hydrodynamic Force: \( F_H(Z, \alpha, V_B) \)

\[
F_H(Z, \alpha, V_B) = \frac{\rho_w}{2} \frac{V_B^2}{A(Z)} (4.86\alpha)
\]

\[
= 2.43 \frac{V_B^2 \alpha}{S (\sin \omega-Z)^2}
\]

Lift Force: \( L(Z, \alpha, V_B) \)

Side Force: \( R(Z, \alpha, V_B) \)

Drag Force: \( D(Z, \alpha, V_B) \)

With zero angles of pitch and roll, the lift and side forces could be calculated using the angle of dihedral and finding with it the submerged foil areas projected on the horizontal and vertical planes respectively; the roll angle \( \phi \) has the effect of increasing the dihedral angle \( \omega \) and the pitch angle \( \theta \) provides an increment to the angle of attack \( \alpha \). Hence

\[
L(Z, \alpha, V_B) = \frac{2.43V_B^2(\alpha + \theta)C_r(S \sin \omega-Z)^2 \cos(\omega + \phi)}{S \sin^2 \omega}
\]

\[
R(Z, \alpha, V_B) = \frac{2.43V_B^2(\alpha + \theta)C_r(S \sin \omega-Z)^2 \sin(\omega + \phi)}{S \sin^2 \omega}
\]

Drag Force: \( D(Z, \alpha, V_B) \)

The drag force is composed of a number of independent components. The coefficient for each component is evaluated below

\[
C_D = C_{D_o} + C_{D_p} + C_{D_1} + C_{D_w}
\]

(i) \( C_{D_o} \) Profile Drag Coefficient (assuming no dependence on depth of submergence)

\[
C_{D_o} = (C_{D_o})_{\min} + KC_L^2
\]

from the aerodynamic data Abbott and von Doenhoff, pg. 615

\[
(C_{D_o})_{\min} = .004
\]

\[
K = .7 \times 10
\]

\[
C_{D_o} = .004 + .7 \times 10^{-2} C_L^2
\]

But

\[
C_L = 4.86 \alpha
\]

\[
C_{D_o} = .004 + .165 \alpha^2
\]
(ii) \( C_{Dp} \) Parasitic Drag

The boundary layer over the foils is assumed to be completely turbulent. This is a reasonable assumption leading to the use of the following formula for calculation of the skin friction coefficient

\[
C_{Dp} = \frac{0.455}{(\log Re)^{2.58}}
\]

for take-off speed:

\[
C_{Dp} = 0.00463
\]

for cruising speed:

\[
C_{Dp} = 0.00357
\]

(iii) \( C_{Di} \) Induced Drag:

\[
C_{Di} = \frac{C_{L}^2 (1 + \sigma)(1 + \delta)}{\pi AR}
\]

\( \sigma \) The Munk Interference Factor

\( \sigma = 0.29 \)

An interference factor of this magnitude is obtained from classical biplane theory if an image hydrofoil is assumed to exist above the water surface identical to that below the surface. An estimate of the hydrofoil span was taken to be 5.0 ft. and an assumed separation between centres of pressure of the hydrofoil and its image of 4.0 ft. With these approximations the value of \( \sigma \) was determined.

\( \delta \) - the planform correction

This correction factor shows the increase in induced drag due to the use of a planform other than the elliptic planform which gives minimum induced resistance.

Taking a value for \( \lambda' = 1.0 \) the value of \( \delta \) was found to be 0.141. Hence the induced drag becomes;

\[
C_{Di} = \frac{C_{L}^2 (1 + \delta)(1 + \sigma)}{\pi AR}
\]

\[
= \frac{(1.141)(1.29)(4.86 \alpha^2)}{\pi (10)}
\]

\[
= 0.111 \alpha^2
\]

(iv) \( C_{Dw} \) Wave Drag

A two dimensional treatment of wave drag based upon bound vortex circulation is assumed to be adequate for calculating the wave drag coefficient.
The form of the coefficient is:

\[ C_D = 0.5 \frac{Cl^2}{F_c^2} e^{-2/F_h^2} \]

where \( F_c = V_b / \sqrt{gc} \) is the Froude number with respect to foil chord

\( F_h = V_b / \sqrt{gh} \) is the Froude number with respect to foil submergence

For take-off speeds

estimating \( V = 6 \) m.p.h. = 8.8 ft./sec.
\( C = 1.0 \) ft.
\( h = 2.5 \) ft.

\[ C_{D_w} = \frac{0.5(4.86 \alpha)^2}{8.8/\sqrt{32.2} \times 1.0} e^{-2/ \left( \frac{8.8}{\sqrt{32.2} \times 2.5} \right)^2} = 0.611 \alpha^2 \]

For cruising speeds

estimating \( V_B = 25 \) m.p.h. = 36.7 ft./sec.
\( C = 0.5 \) ft.
\( h = 1.5 \) ft.

\[ C_{D_w} = \frac{0.5 (4.86 \alpha)^2}{36.7/\sqrt{32.2} \times 0.5} e^{-2/ \left( \frac{36.7}{\sqrt{32.2} \times 1.5} \right)^2} = 0.132 \alpha^2 \]

Total Drag Coefficient

(i) At take-off speed: \( \alpha + \theta \) being the effective angle of attack

\[ C_D = 0.00863 + 0.887 (\alpha + \theta)^2 \]

(ii) At cruising speed

\[ C_D = 0.00757 + 0.408 (\alpha + \theta)^2 \]

Because \( C_D \) has a significant dependence on speed, values for \( C_D \) will be calculated for each particular boat speed. For this calculation the profile drag coefficient, \( C_{D_p} \), and the induced drag coefficient, \( C_{D_i} \), will be taken to be independent of speed giving:

\[ C_{D_i} + C_{D_p} = 0.004 + 0.276 (\alpha + \theta)^2 \]

The values for wave drag coefficient, \( C_{D_w} \), and parasitic drag, \( C_{D_p} \) will
be calculated for the particular speed in question.

In calculating the parasitic drag, the mean chord (varying with depth of submergence $Z$) will be used as the characteristic length. In calculating the Froude numbers for the wave drag coefficient, the mean depth of submergence $h$ will be taken as the $Z$ coordinate of the centre of pressure and the reference length will again be taken as the mean chord.

**Effect of Water Surface Proximity**

So far in the calculations for the hydrodynamic forces acting, the effect of the proximity of the free water surface has not been taken into account. The hydrofoil tends to lose lift as it approaches the water surface; however, for submersences of greater than one chord length the effect is not particularly significant and may reasonably be ignored in preliminary design calculations.

An estimate of the size of the error made from ignoring the free surface effect can be obtained using the empirical formula:  

$$C_l = C_{l\infty} (1 - 0.422 e^{-1.454 h_m/C})$$

where $C_{l\infty}$ is the value of the lift coefficient at infinite depth (the aerodynamic value)

$hm$ is the mean submergence of the foil

$C$ is the foil chord for a surface piercing foil with dihedral

$$\frac{hm}{C} = \frac{AR}{4} \tan(\Omega) \quad AR = 10$$

$$= 2.5 \quad \Omega = 40^\circ$$

then

$$C_l = C_{l\infty} (1 - 0.422 e^{-1.454 \times 2.5})$$

$$= C_{l\infty} (1 - 0.0111)$$

As can be seen an error of approximately 1% is incurred in the approximation.
APPENDIX 2: STERN HYDROFOIL

The stern foil should carry approximately 30% of the weight when the thrust and heeling force are zero. As well it should be capable of producing a substantial side force, $R_R$, and thus act as the rudder for the boat. The rear lift gradient ($\frac{\partial L_R}{\partial Z}$) should be complementary to those of the front foils so that any changes in pitch angle $\theta$ which occur as the boat rises will not cause the motion of the boat to become unstable.

The chosen configuration is that as shown in the diagram.

To determine the forces produced by the foil system the following assumptions will be made:

(i) The effects of interference between various foil surfaces will be neglected.

(ii) The flow field over the hydrofoils is assumed to be essentially two dimensional.

Assumptions (i) and (ii) allow the forces to be calculated by dealing separately with the projected areas on the horizontal and vertical planes. The angle of attack on the foil area in the horizontal plane is $\alpha$ and for that in the vertical plane is $\delta_R + \beta$ where $\beta$ is the angle of sideslip and $\delta_R$ is the rudder angle defined as the angle between the centre plane of the boat and any chordline of the rear hydrofoil.

The rudder angle $\delta_R$ will be varied such as to eliminate any angle of sideslip. This can be done without any excessive rudder angle by relying on the difference in sideforce produced between the leeward and windward hydrofoils when the boat is slightly heeled to leeward.

Independent Foil Design Parameters

(i) Angles and Blades $\xi, \gamma, a, b$

(ii) Root Chord $C_R$

Note: The rear foil has a taper ratio of 1.0 (i.e. the root and tip chords are equal)

A rear foil axis system $X_Y_Z$ is set up as shown in the diagram. The origin will be in the plane of the 1/4 chordline of the foils comprising the rear configuration at the vortex of the upper triangle.

The lift and side forces thus become
\[ L(\alpha, Z, V_b) = 2 \left\{ (a - Z \csc \xi) \cos \gamma \left\{ a - Z \csc \xi \right\} \right. \\
+ \left. (b - \frac{Z - \sin \xi \csc \gamma}{H(Z \csc \xi - a)} \right\} \cos \gamma \right\} \frac{k}{2} V_b^2 (2\pi) \alpha \]

\[ H(\chi) = 0 \quad \chi \leq 0 \]
\[ = 1 \quad \chi > 0 \]

\[ R(\delta_R + \beta, Z, V_b) = 3\pi C \frac{k}{2} \left( \sin \xi + b \sin \gamma - Z \right) \]

Centre of Pressure

Due to the symmetry of the rear foil configuration in the \( X_R Y_R \) plane, \( Y_R \) coordinate of the C.P. will always be 0.0 (i.e. on the \( X_R \) axis). The \( Z \) coordinate is given by

\[ CP_z(Z) = Z + \frac{(a \sin \xi + b \sin \gamma - Z)}{2} \]

where \( Z \) is the water level.

Effect of Pitch and Roll

The effects on including pitch angle \( \theta \) and roll angle \( \phi \) in the calculation of forces is shown below:

The effect of an increase in pitch angle is to increase the angle of attack of the rear foil. Thus \( \alpha_R \) becomes \( \alpha_R + \theta \).

The effect of an increase in roll angle \( \phi \) is change the lift and side force. They become:

\[ L(\alpha, Z, V_b) = 2C_{\text{sw}} V_b^2 \pi (\alpha_R + \theta) \left\{ (a - Z \csc \xi) \cos \gamma H(a - Z \csc \xi) \right. \\
+ \left. \left( b - \frac{Z - \sin \xi \csc \gamma}{H(Z \csc \xi - a)} \right) \cos \gamma \right\} \frac{k}{2} V_b^2 (2\pi) \alpha \]

\[ R(\alpha, Z, V_b) = 2C_{\text{sw}} V_b^2 \pi (\alpha_R + \theta) \left\{ (a - Z \csc \xi) \cos \gamma H(a - Z \csc \xi) \right. \\
+ \left. \left( b - \frac{Z - \sin \xi \csc \gamma}{H(Z \csc \xi - a)} \right) \cos \gamma \right\} \frac{k}{2} V_b^2 (2\pi) \alpha \]

for small angles \( \phi \) and \( \theta \) to first order.

Drag Force

The drag force coefficient can be calculated here with some modifications in the same way as that found for the bow foils. Given the assumption that two dimensional flow properties exist around this foil there will be no induced drag coefficient; however, the other three forms of drag are present.

The same section is to be used here as was used in the bow foil. Hence an estimate for the drag coefficient is the sum of those given for the bow foil excluding any contribution from induced drag.
\[ C_D = C_D + C_D + C_D \]

For take-off speed

\[ C_{D_{to}} = 0.00863 + 0.766 (\alpha_R + \delta)^2 \]

For cruising speed

\[ C_{D_{cr}} = 0.00757 + 0.297 (\alpha_R + \delta)^2 \]

The area to be used when calculating the drag force is the total "wetted" area.

\[ A_R(Z) = \{(a-Z\csc\xi)H(a-Z\csc\xi) + (a \sin\xi + b \sin\gamma - Z)C + \]
\[ (b - [Z - a \sin\xi] \csc\gamma \left[H(Z\csc\xi - a)]\right) \} \]

Thus the drag force becomes:

\[ D_R(\alpha_R, Z, V_b) = \frac{1}{2} \, \xi \, w \, V_b^2 \, A_R(Z) \, C_D(\alpha) \]
APPENDIX 3: COMPUTER PROGRAMS AND OUTPUT

From Tables 1 and 2 of Section III: AERODYNAMIC FORCES, the program used to calculate the "best fit" polynomial is shown. As well the coefficients for several other orders of polynomials (from 1st to 5th order) are given along with the modified standard error of estimate under the title "SIGMA".

The equilibrium program notation is then given followed by a print-out of the actual program.
EQUILIBRIUM PROGRAM NOTATION

\( l = L \)
\( \alpha = AL \)
\( A(z) = A \) the bow hydrofoil area function
\( \Omega = E \)
\( W = 600.0 \) lbs. = \( W \)
\( V_T = V_T \)
\( \psi_T = SIT \)
\( \beta = B \)
\( V_B = V_B \)
\( \theta = TH \)
\( \phi = FI \)
\( V_A = VA \)
\( \psi_A = SIA \)
\( \lambda = LM \) (real)
\( C_m = CM \) (mean chord)
\( C_r = CR \) (root chord)
\( CP_x = RX \) centre of pressure (X coordinate)
\( CP_y = RY \) centre of pressure (Y coordinate)
\( CP_z = RZ \) centre of pressure (Z coordinate)
\( \rho_w = ROW \) water density
\( \rho_A = ROA \) air density
\( \delta_R = DLR \)
\( b = BB \)
\( \partial R_L / \partial \beta = RLB \)
\( \partial A / \partial z = DERA \) front foil area gradient
\( \partial L_L / \partial z = LLZ \)
\( \partial R_w / \partial \beta = RWB \)
\[ \frac{\partial L_w}{\partial z} = LWZ \]
\[ CP_{x_R} = RRX \]
\[ CP_{y_R} = RRY \]
\[ CP_{z_R} = RRZ \]
\[ CD_{TR} = CDTR \text{ rear foil drag coefficient} \]
\[ \frac{\partial A_R}{\partial z} = ARZ \text{ rear foil area gradient} \]
\[ A_R(z) = AR \text{ rear foil area} \]
\[ \alpha_R = ALR \text{ rear foil } \]
\[ \alpha_w = ALW \text{ windward foil } \text{ angle of attack} \]
\[ \alpha_L = ALL \text{ leeward foil} \]
\[ L_w = LW \text{ windward} \]
\[ L_L = LL \text{ leeward foil lifts} \]
\[ L_R = LR \text{ rear foil} \]
\[ AR_H(z) = ARH \text{ horizontal rear foil "wetted" area} \]
\[ AR_V(z) = ARV \text{ vertical rear foil "wetted" area} \]
\[ \frac{\partial AR_H(z)}{\partial z} = ARHZ \text{ horizontal rear foil "wetted" area gradient} \]
\[ \frac{\partial R_R}{\partial \phi} = RRB \]
\[ RR_{DR} = RRD \]
\[ D_R = DR \]
\[ F' = DERF \]
\[ e = EE \% \text{ distance from leading edge of boom to sail C.P.} \]
\[ c = CB \text{ chord of boom} \]
\[ f = FB \% \text{ distance from base of mast to sail C.P.} \]
\[ h = HH \text{ height of mast} \]
\[ \frac{\partial M}{\partial \phi} = DMDF \]
\[ \frac{\partial m_y}{\partial \theta} = DMDTH \]
\[ dL = RZL \]
\[ dw = RZW \]
\[ dr = RRZ \]
\[ \partial R_L/\partial z = RLZ \]
\[ \partial R_R/\partial z = RRZ \]
\[ \partial R_w/\partial z = RWZ \]
\[ \partial L_w/\partial \alpha = LWAL \]
\[ \partial L_L/\partial \alpha = LLAL \]
\[ \partial L_R/\partial \alpha = LRAL \]
\[ \partial D_w/\partial \alpha = DWAL \]
\[ \partial D_L/\partial \alpha = DLAL \]
\[ \partial D_R/\partial \alpha = DRAL \]
\[ \xi = SI \quad \{ \text{angles of rear foil design} \} \]
\[ \gamma = GA \quad \{ \text{spans of rear foil design} \} \]
\[ a = AR \quad \{ \text{spans of rear foil design} \} \]
\[ b = BR \]
\[ c = C \text{ rear foil chord} \]
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Nose down Foils in negative pitching moment incidence

(a)  (b)

FIGURE 1: SEA CRASH

Windward Leeward Windward Leeward

\[ \Phi = 0 \]
\[ F_W = F_L \]
\[ \Phi \neq 0 \]
\[ F_L > F_W \]

FIGURE 2: STABILIZING EFFECT OF DIHEDRAL IN ROLL
FIG. 3: SIDE VIEW OF HYDROFOIL SAILBOAT SHOWING BODY & FIXED AXIS SYSTEMS

FIG. 4: FRONTAL VIEW OF HYDROFOIL SAILBOAT SHOWING AXIS SYSTEMS
FIG. 5: PLAN VIEW OF HYDROFOIL SAILBOAT SHOWING AXIS SYSTEMS

FIG. 6: SIDE VIEW OF HYDROFOIL SAILBOAT $\theta + \phi = 0$

FIG. 7: FRONTAL VIEW OF HYDROFOIL SAILBOAT $\theta + \phi = 0$
FIG. 9: FRONTAL VIEW SHOWING SHROUDS

FIG. 10: SIDE VIEW SHOWING FORE & BACKSTAYS

FIG. 11: MECHANISM FOR PITCHING MOMENT COMPENSATION
FIG. 12: PITCH COMPENSATION PROVIDING AN INCREASE IN PITCH ANGLE

FIG. 13: MECHANISM FOR ROLLING MOMENT COMPENSATION

FIG. 14: PERMISSIBLE ZONES OF HYDROFOIL SAILING (FOR VARIOUS TRUE WIND SPEEDS & DIRECTION)
FIG. 15: BOW FOIL CONFIGURATION

FIG. 16: NACA 65-410 WING SECTION
FIG. 17: NACA 65-410 WING SECTION

FIG. 18: STERN HYDROFOIL, REAR FOIL CONFIGURATION