von KARMAN INSTITUTE

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INCIPIENT SEPARATION OF A COMPRESSIBLE
TURBULENT BOUNDARY LAYER

Cyriel APPELS

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RHODE SAINT GENESE BELGIUM
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SUMMARY

Results are presented of an investigation of turbulent boundary layer incipient separation in a compression corner. The experiments were carried out on the walls of two VKI blowdown facilities at Mach numbers of 3.5 and 5.4 and at unit Reynolds numbers between $10^7$ and $10^8$/m. Profile measurements showed that when undisturbed, the turbulent boundary layer was fully developed in all cases examined.

Small separated flow regions were detected at relatively low wedge angles, using liquid line and schlieren methods. The effect on the separated flow behaviour of Mach and Reynolds number, state of the turbulent boundary layer and the influence of the incipient separation detection method was investigated and the results were compared with previous experimental data.

An approximate method was developed for predicting incipient separation, using as a basis the observed existence of very small separation bubbles.
NOTATION

$C_f$ skin friction coefficient

$C_P$ pressure coefficient

$k$ von Karman constant

$L_s$ distance between the separation point and the hinge line

$M$ Mach number

$n$ power-law velocity profile exponent

$P$ pressure

$R$ radius of curvature of axi-symmetric models

$Re(\cdot)$ Reynolds number based on quantity ( )

$Re_\infty$ unit Reynolds number

$T$ temperature

$T_r$ recovery temperature

$U$ velocity component parallel to surface

$U_f$ friction velocity $\frac{\tau}{\sqrt{\rho}}$

$y$ distance normal to the surface

$\alpha$ compression corner angle

$\beta$ $\frac{\sqrt{M^2 - 1}}{M}$

$\delta$ boundary layer thickness

$\nu$ kinematic viscosity

$\rho$ mass density

$\tau$ local shear stress
Subscripts

0  free stream conditions

i  conditions at the edge of the boundary layer

i  incipient separation conditions

L  conditions at the edge of the sublayer

t  total conditions

w  conditions at the wall
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1. INTRODUCTION

The separation of turbulent boundary layers caused by adverse pressure gradients has been studied in detail during the last 15 years. Since the onset of separation changes the flow field entirely, one of the most important parameters of interest is incipient separation.

The first extensive investigation in this field was that of Kuehn (Ref.1, 1959) who used the appearance of a kink in the wall pressure distribution to detect incipient separation. Since then, several detection methods have been developed and the amount of experimental data has strongly increased; however, with it, the disagreement between results has become larger.

The following influences on the angle for incipient separation are studied in the work presented here: the effect of Mach and Reynolds number, the state of the turbulent boundary layer (flat plate or nozzle wall boundary layer, fully developed or near-transitional boundary layer) and the method used to detect incipient separation. In this study the effect of the ratio of wall-to-recovery temperature is neglected, because previous investigations indicated that it is small and most of the experimental data considered are at approximately adiabatic conditions.

In his early study, Kuehn already indicated that when incipient separation conditions were reached, according to his detection method, some evidence for the existence of small separation bubbles was found. He mentioned that these small separated regions were "primarily of academic interest", since they did not perceptibly affect the wall pressure distribution. Although this seems to be correct for the thin boundary layers prevalent in ground facilities, it is possible that on full scale models (e.g. re-entry vehicles) with very thick boundary layers these relatively small separated regions can become so
large, that their effects can no longer be ignored. Furthermore, the understanding of the existence of these regions is of importance for any theoretical approach to the problem.

The aim of the present investigation was to explain the reasons for the lack of agreement between several experimental investigations on incipient separation so as to develop a better understanding of the phenomenon. Special care is taken in showing the existence of very small separation bubbles using oil flow and schlieren methods.
2. EXPERIMENTAL PROCEDURE

2.1 Wind tunnels and test conditions

All the tests were carried out in two VKI blowdown facilities, both having two-dimensional contoured nozzles.

The S4 supersonic tunnel with a Mach number of 3.5 has an 80 mm x 100 mm test section. The stagnation temperature is normally 6°C and the stagnation pressure can be varied between 2 and 17 kg/cm² giving unit Reynolds numbers from $10^7$ to $10^8$/m. The thickness of the boundary layer on the nozzle wall at the exit is between 6 and 7 mm resulting in values for $Re_6$ of $9.10^4$ to $7.10^5$.

The H1 pebble bed heated hypersonic 140 mm x 145 mm tunnel (Ref. 2) with a Mach number of 5.4 can be operated at stagnation pressures varying from 13 to 33 kg/cm². All the tests reported herein were carried out at a temperature of approximately 200°C giving a range of unit Reynolds numbers from $10^7$ to $3.10^7$/m. The boundary layer thickness at the wall was somewhat less than 20 mm and hence $Re_6$ could be varied between $2.10^5$ and $6.10^5$.

A list of the free stream conditions used in this study is given in Table 1.

2.2 Model configurations

For these tests, wedges of different angles ($\alpha$) were mounted at the same position on the tunnel wall; i.e., at the nozzle exit in the Mach 5.4 tunnel, and downstream of the nozzle exit in the Mach 3.5 tunnel. The test set-up for both tunnels is shown in Fig. 1. The leading edge thickness of the...
wedge was kept small enough to avoid the existence of a step at the hinge line which could cause small separation bubbles.

To avoid an interaction between the flow over the wedge and the boundary layers on the tunnel side walls, the span of the wedge was smaller than the width of the test section. The ratio of the span of the wedge to the undisturbed boundary layer thickness was 14 for the supersonic tests. However, this ratio was only 6.5 for the hypersonic tests so that it was impossible in this case to study large separated regions which could be classed as two-dimensional because of three-dimensional effects due to outflow.

2.3 Experimental techniques

The angle of incipient separation was found by extrapolating the length ($L_s$) between the separation line and the hinge line for different wedge angles, to zero. The separation length was obtained from photographs of flow visualisation using an oil flow method and a Toepler-schlieren method.

2.3.1 Oil_flow_visualisation_method

A liquid, consisting of oil, dye and oleic acid, was smeared on the wall or the wedge before the test. During the test, pictures of the resulting oil pattern were recorded. For large separated regions the oil film is expected to have a negligible interference on the separation behaviour and therefore oil was put on the hinge line and upstream of the separated region. The oil pattern before and during a test is shown in Fig.2. It is possible to see the complete separation line coincident with the accumulation of oil.

For small separated regions, however, one has to be very careful not to disturb the separation bubble by the introduction of the oil itself. Therefore only a very small
quantity of oil was put ahead of the expected position of the separated region but not on the hinge line itself, so that during the test individual streamwise streaks of oil were formed, which stopped (at different times and spanwise positions) when encountering a region of reversed flow. Fig.3 helps to illustrate that with these precautions the interaction between the separation bubble and the oil flow will be negligible. The separation line was then visualised by only a few discrete points in any one test, however, exactly the same position was detected in repeat tests at different spanwise locations.

2.3.2 Schlieren method

In both tunnels a single pass schlieren system with a horizontal knife edge was used. Again the length between the separation point and the hinge line was measured and not the length between separation and re-attachment shocks as measured by other investigators for reasons explained in Section 4.1.2. The position of the separation point was determined by extrapolating the separation shock to the wall. The error on this extrapolation is relatively small as the shock is formed within the boundary layer and hence very close to the wall (see Fig.4). However, this method could not be used at very low Reynolds numbers, because the density was too small to give a clear shock pattern in the boundary layer.
3. UNDISTURBED BOUNDARY LAYERS

3.1 Instrumentation

Pitot pressure traverses were made through the undisturbed tunnel wall boundary layer at the position of the wedge for all relevant test conditions in both tunnels. The wall static and pitot pressures were measured using very sensitive Ultradine and Validyne transducers* respectively, and the pitot pressure profile was directly recorded on an X-Y plotter through the use of a linear transducer.

Different pitot probes were used in the two tunnels (Fig. 5): a circular probe with an external diameter of 0.75 mm for the M = 5.4 tunnel and a flat probe with a height of 0.6 mm for M = 3.5 tunnel. Both probes were calibrated against large circular probes in the free stream.

3.2 Measurement results

Figs 6 and 7 show for both tunnels the Mach number profiles at two typical Reynolds numbers. The velocity profiles were obtained using the quadratic law

\[
\left( \frac{U}{v_e} \right)^2 = \frac{T_t - T_w}{T_{te} - T_w}
\]

which is often used in connection with tunnel wall boundary layers as an alternative to the linear Crocco law. The results from these calculations are also given in Figs 6 and 7, illus-

* Ultradine, type 41S4-10-1D
Validyne, model DP 15 TL
trating that the power law exponent $n$ in the relation

$$\frac{U}{U_e} = \left( \frac{y}{\delta} \right)^{1/n}$$

increases with increasing Reynolds number.

A Van Driest transformation (Ref.3) was applied to these compressible profiles in order to compare them with generalised incompressible profiles. This particular method was chosen since, according to Ref.4 it gives the best correlation for the law-of-the-wall region. As the skin friction coefficient, required for these transformations, could not be measured, it was calculated using the Spalding and Chi (Ref.5) and the Van Driest (Ref.6) methods. The transformed profiles were compared in Fig.8 with Coles law-of-the-wall (Ref.7) and with the following logarithmic law:

$$\frac{U}{U_\tau} = \frac{1}{k} \ln \frac{yU_\tau}{v} + C$$

The deviation from these two laws in the outer part of the boundary layer shows that in all the profiles a strong developed wake component exists, which is in agreement with other tunnel wall data. The low values of $U/U_\tau$ close to the wall in the Mach 3.5 profiles are due to probe-wall interference; this effect is of no importance in the $M = 5.4$ profiles because in that case the probe size is relatively much smaller, as is shown in Figs 6 and 7.
4. RESULTS AND DISCUSSION

4.1 Separation length

The main objective of this study was to visualise very small separation bubbles in order to reduce the error involved in extrapolating the separation length to L_s = 0 and hence locating the angle of incipient separation.

4.1.1 Oil flow method

Examples of the experimental results obtained in the present study are now presented. Fig. 9 shows the flow pattern for different wedge angles at a particular Mach and Reynolds number condition; while Fig. 10 shows the flow pattern for a given wedge angle and Mach number, but different Reynolds numbers.

The separated lengths for all wedge angles and flow conditions tested are plotted in Figs 11 and 12. For M = 3.5 the circles in Fig. 11 indicate the length measured using the oil flow technique, as a function of the wedge angle. This figure shows that for all the different conditions, there is a difference in growth, for small or for large separated regions and small separation bubbles exist at relatively low wedge angles.

Fig. 12 gives the results of tests at Mach 5.4. However, in this case the separation length is not represented by a discrete point because the quality of the oil pattern was not as good as at M = 3.5, therefore, an error bar is given to indicate the separation length. As in this case a wide range of wedge angles was tested, one can see very clearly the difference in growth of the separation length for small and large separation regions. Again, for low deflection angles, very small separation bubbles exist (as small as 1/10 of the boundary layer thickness). In order to prove that the stagnation of the oil close to the hinge line for small wedge angles is due to a reversal in skin friction (and hence to a separation line), and not due to insuf-
ficient skin friction, several tests were carried out in which oil was put on the wedge close to the hinge line. The result, of which a typical photograph is shown in Fig.13, is that the oil flows forward and stagnates at exactly the same position as found if the oil is placed upstream of the separated region. The effect of wedge leading edge thickness on the oil stagnation was proved to be negligible since no stagnation at all was detected in tests on compression corners at low deflection angles in which attached flow was expected.

4.1.2 Schlieren method

Figs 14 and 15 show schlieren pictures of separated regions for different wedge angle, at a constant Reynolds number and Mach number. The separation length obtained by extrapolating the separation shock to the wall is presented in Figs 11 and 12. As an uncertainty is involved in this extrapolation, the error bar indicated in these Figures is somewhat larger than for the oil flow results. The schlieren pictures of small separated regions clearly show that the usual criterion of defining the separation length as the distance between the separation and re-attachment shocks is incorrect for small separation bubbles because in this case no re-attachment shock exists. It is therefore necessary to use the distance from the separation line to the hinge line for determining the incipient separation angle.

4.2 Incipient separation

The angle of incipient separation was found by extrapolating the separation length, as a function of $\alpha$ to zero. However, because of the difference in growth of $L_s$, a different curve of extrapolation was used for the small and large separated regions, as shown in Figs 11 and 12. For the same Mach number all these curves have roughly the same shape (Figs 16 and 17). The angles of incipient separation are indicated on these figures as well.
In this section a comparison is made between the results of the present study and most of the experimental data obtained by other investigators, which are summarised in Table 2. In Fig. 18, which shows the incipient separation angle for two-dimensional configurations, the lack of agreement between the different results, for both the magnitude of incipient separation angle and the trend with Reynolds number change, is striking. Fig. 19 shows the same kind of disagreement for axi-symmetric separated regions (for comparison purposes a few two-dimensional results are added as well). These disagreements will now be discussed in detail.

4.2.1. Disagreement in magnitude of incipient separation angle

In general it is believed that the lack of agreement is mainly due to differences in boundary layer properties (e.g. boundary layers on nozzle walls, boundary layers on flat plates, boundary layers at different stages of development etc.) and to the use of different methods of detecting incipient separation. The importance of each of these factors is still under discussion. Concerning the detection method Fig. 18 shows that some investigators (Refs 8 and 9) found extremely low angles, using liquid line or similar techniques (most commonly used is the oil flow method). As these latter results were considered by some workers to be doubtful (e.g. Refs 9 and 12), one of the purposes of this present study was to test the reliability of this method.

The main arguments against the use of liquid line methods are: the unsteady character of the separated region, the interaction between the liquid and the separation bubble due to their comparable thicknesses and finally the "buoyancy" effect (Ref. 10). As illustrated in Fig. 20 which shows a high speed schlieren picture (5000 frames per second) of a small separated region, the effect of the first factor is negligible. During the test, the separation length remains constant and the shape of the separation shock hardly changes. The two latter
factors can only be of importance in small separated regions and as indicated in Section 2.3.1 in these experiments, special care was taken to avoid any kind of interaction. Further the measurements were made before accumulation of the oil, so that the effect of the strong adverse pressure gradient on the oil (i.e. buoyancy) was negligible. In that case the driving force of the oil is the wall shear stress and not drag forces or pressure gradients. A few tests with much longer running times and strong accumulation of oil were carried out and they showed that after a while, at spanwise positions where the oil accumulation was greatest, entrainment of the oil into the separated region occurred and the separation bubble was filled up with oil (Fig. 21). This shows the existence of a close contact between the separation region and the oil accumulation line and so, even in this case, the accumulation is the result of a reversed flow and not of the balance between wind and buoyancy forces.

Another proof of the reliability of this oil flow method, lies in the comparison with other methods. Spaid and Frishett (Ref. 8) found good agreement between their oil flow data, the results of a powder deposition technique and the extrapolation of the shock to the wall. In the axi-symmetric case Rose et al (Ref. 11) used an alcohol injection technique and found values of \( \alpha_i \) which are comparable with the two-dimensional liquid line data of Spaid and Frishett (Ref. 8) and the present study. Finally, for the present study, Figs 11 and 12 show the very good agreement between oil flow data and results obtained by extrapolating the shock to the wall.

The large differences between most of the methods of detecting incipient separation can now be explained in terms of the existence of these very small separation bubbles, which in the beginning grow very slowly when increasing the deflection angle. The accuracy of each method depends on the smallest
One of the most commonly employed methods has been the detection of the kink in the pressure distribution (Kuehn Ref. 1) and in the original report, Kuehn indicated that possibly some of his so-called attached flows could have had very small separation regions. More recently, several investigators (Refs 8, 11 and 12) showed that indeed the separated region has to reach a certain minimum size before an inflection in the pressure distribution occurs, even when very dense pressure instrumentation is used. (It is also obvious that the spacing of the pressure taps changes the accuracy of detecting this inflection point).

Because these small separated regions hardly affect the pressure distribution, the method of Roshko and Thomke (Ref. 13), in which at a certain position close to the hinge line an inflection in the curve $P$ versus $\alpha$ is observed, cannot be accurate either.

In other methods, Refs 11 and 13, probes or dams are introduced into the flow (surface pitots, orifice dams, etc.). The size of these objects relative to the size of the very small separation regions is not negligible, so that here this methods are likely to be inaccurate as well.

For hypersonic flows, the appearance of a pressure overshoot at re-attachment is used to detect incipient separation (Refs 14 and 15) but as this method is based on the existence of a double shock compression again it cannot be expected to detect small separation bubbles.

The extrapolation of the length of relatively large separated regions, measured on schlieren pictures, to $L_s = 0$ (Refs 14 and 16) can result in large errors in $\alpha_i$. This is illustrated in Figs 11 and 12, where the extent of separation size of separated region that can be detected.
present at the incipient separation angles obtained using such an extrapolation, is shown actually to be between 0.15 and 0.40 times the boundary layer thickness.

Finally, there is the more positive method of measuring the reversal in skin friction, using floating element balances (Ref. 12), which is theoretically the ideal approach to the problem. However, the development of these rather complicated measuring devices is in some cases still in an early stage, and apart from the fact that these gauges are often rather large compared to small separated regions, the reliability of these balances when used in complex flow-fields like separation and re-attachment regions, is still in question.

As a conclusion, one can say that the incipient separation angles indicated in Fig. 18 in most cases, do not represent incipient separation conditions, but the conditions at which a certain extent of separation occurs. This "certain extent" is a function of the detection method used. This explains why agreement between the data can be found when the same or similar methods are used. As an example, the results obtained by detecting a kink in the pressure distribution are in rather good agreement except for some disagreements caused probably by a lack of densely spaced instrumentation.

4.2.2 Disagreement in the effect of Reynolds number

Figs 18 and 19 show that the angle of incipient separation changes with Reynolds number. No obvious trend is apparent. Until recently, experiments indicated that \( \alpha_1 \) decreased with increasing Reynolds number except at very high Reynolds numbers where Rhosko and Thomke found an opposite trend. Recent experiments (Refs 8 and 9) showed that for similar free stream conditions completely different trends are found. These differences can again be explained using the following arguments: different stages in development of the turbulent boundary layers.
and the way of detecting incipient separation.

Spaid and Frishett proved in their experiments (Ref.8, see Fig.18) that the latter factor can indeed influence the effect of Reynolds number on \( \alpha_i \). This is probably due to the fact that these values of \( \alpha_i \) represent a certain degree of onset of separation and if the growth of the separated region is a function of Reynolds number, the methods which detect small regions will suggest different Reynolds number trends than methods which detect only large regions.

This argument of course, does not explain the different trends observed when using the same technique and therefore it is believed that the most important factor is the stage of development of the turbulent boundary layer. Elfstrom (Ref.14) indicated that the reversal in the \( \alpha_i \) trend with \( Re_\delta \) closely follows the development of the wake component in the velocity profile and hence the power law exponent \( n \) in the relation \( U/U_e = (y/\delta)^1/n \). A stronger developed wake (higher \( n \), fuller velocity profile) results in a more energetic boundary layer which is more resistant to separation. Studies in turbulent boundary layers on flat plates (Ref.17) proved that close to transition \( n \) decreases with increasing Reynolds number (so-called overshoot in \( n \)) and at high Reynolds number, when the boundary layer is fully developed, \( n \) increases with \( Re \). Up to now no experiments on turbulent separation (with the probable exception of one very recent study by Holden, Ref.18) have been carried out in this high Reynolds number region. Ref.17 also shows that this overshoot in \( n \) does not exist in boundary layers on nozzle walls, because of the favourable pressure gradients, and therefore, in this case \( n \) (and hence \( \alpha_i \)) increases even at low Reynolds numbers. This trend was also shown in the undisturbed boundary layer profiles in section 3.2.
4.2.3 Correlation method

Several of the correlation methods for incipient separation conditions are given in Ref. 15 and some of these (e.g. that of Roshko and Thomke, Ref. 13, shown in Fig. 22) gave rather good agreement for most of the experimental results. However, this figure also shows that the very low incipient separation conditions detected in the present study, cannot be correlated in the same way. In order to account for these very small separated regions, new methods have to be developed. As these small separation bubbles are probably a result of the flow reversal occurring initially only in the laminar sublayer, parameters based on this sublayer should be considered.

In order to prove that this is the correct approach to the problem, a very simple method for predicting these low values of $\alpha_1$ was developed. The laminar sublayer was considered to have the separation characteristics of a laminar boundary layer, assuming that the conditions at the edge of the sublayer are the free stream conditions for the laminar boundary layer; hence, the influence of the outer part of the boundary layer is neglected. This assumption can be made as a first approximation since as the wedge angle is increased from a low angle, the separation will be restricted to a very small region (at the hinge line) deep in the turbulent boundary layer. A prediction method for laminar separated flows was then used to calculate incipient separation in that boundary layer.

The edge of the laminar sublayer was defined as being the position at which, in the transformed velocity profile:

$$\frac{yu_1}{v} = 15 \quad \text{(or: } \frac{U}{u_1} = 11.5).$$
The appropriate skin friction coefficient to be applied to this method was calculated from the actual skin friction coefficient assuming constant static pressure through the laminar sublayer, hence:

\[
\frac{C_f^0}{C_f} = \frac{M_L^2}{M_0^2}
\]

For predicting laminar separation the correlation of Hakkinen et al. (Ref. 19) was used:

\[
\frac{P_s - P_\infty}{P_\infty} = 1.0 \sqrt{\frac{M_\infty^4 \cdot C_f}{\beta_\infty}}
\]

where the index \( \infty \) denotes free stream conditions which are, for this application, the conditions (L) at the edge of the laminar sublayer.

The method was applied to the undisturbed boundary layer profile data of the present study and of the investigation of Spaid and Frishett. The latter was the only work found that contained enough information to apply the method. The results are compared with the experimental data in Fig. 18 and the agreement is shown to be highly dependent on the Reynolds number. This is connected with the thickness of the sublayer \( \delta_L \) relative to the overall boundary layer thickness \( \delta \). As indicated in Ref. 20, the sublayer is much thinner at higher Reynolds numbers and a factor of 10 difference was found for \( \delta_L / \delta \) between low Reynolds number experiments of Spaid and Frishett (Ref. 8) and the high Reynolds number data of the present study. So, for comparison with the experimental results of the liquid line technique, this very crude prediction method, in which the interaction between the laminar sublayer and the rest of the turbulent
boundary layer is neglected, can only be used with relative confidence at relatively low Reynolds numbers, when the sublayer forms a fairly large portion of the boundary layer.

It is also encouraging to note that in the present study the Mach number trend of $\alpha_i$ is the same for the experiments and the prediction method, however, this trend is so small that one can only conclude that the Mach number dependence of $\alpha_i$, when taking into account the very small separated regions, is almost negligible.
CONCLUSIONS

The present study shows that, with care, the liquid line method is a reliable technique for detecting incipient separation.

The two visualisation methods (oil flow and schlieren) gave very similar values of the length of turbulent separated regions for each condition examined.

For all test conditions, the results indicate that very small separated regions exist at low wedge deflection angles and that when the wedge angle is increased, a difference in growth exists between these small separated regions and larger ones.

The disagreement between the present results and those obtained using other detection techniques (e.g. a kink in the pressure distribution) arises from the fact that each method detects a different degree of onset of separation.

The general change in angle of incipient separation with Reynolds number follows closely the trend of the development of the wake component in the undisturbed boundary layer (as suggested by Elfstrom).

The results of an approximate correlation method, consisting of a laminar incipient separation criterion applied to the laminar sublayer in the turbulent boundary layer, are in good agreement with liquid line data, especially at relatively low Reynolds number.

Both the experiments and the correlation method seem to indicate, when taking into account $\alpha_i$ as determined in the case of small separated regions that the Mach number dependence of the incipient separation angle is very weak. This would mean that the strong trends found using other detection methods,
appear to represent the Mach number dependence of the growth of the small separated regions.
REFERENCES


### TABLE 1: SUMMARY OF TEST CONDITIONS

<table>
<thead>
<tr>
<th>$P_t$ (kg)</th>
<th>$T_t$ (°K)</th>
<th>$M$</th>
<th>$T_w/T_R$</th>
<th>$Re_\infty$ /m</th>
<th>$\delta$ (mm)</th>
<th>$Cf_0$</th>
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<tbody>
<tr>
<td>2</td>
<td>278</td>
<td>3.46</td>
<td>1.0</td>
<td>$1.28 \times 10^7$</td>
<td>7.3</td>
<td>1.64 $10^{-3}$</td>
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<td>TEST CONFIGURATION</td>
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<td>1. Appels</td>
<td>- Disappearance of the overshoot in the wall pressure distribution</td>
<td>Flat plate - wedge 6.5 mm</td>
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<td>2. Batham</td>
<td>- Distance between separation and re-attachment shocks measured on schlieren pictures</td>
<td>Flat plate - wedge 4.8 - 6.5 mm</td>
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<td>Ref.21</td>
<td>- Appearance of a kink in the wall pressure distribution</td>
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<td>3. Coleman</td>
<td>- Distance between separation and re-attachment shocks measured on schlieren pictures. Linear extrapolation of ( L_p ) to zero</td>
<td>Flat plate - wedge 7.6 - 8.2 mm</td>
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<td>4. Drougge</td>
<td>- Appearance of a kink in the wall pressure distribution</td>
<td>Flat plate - wedge at low Reynolds numbers</td>
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<td>Ref.22</td>
<td>- Change in angle of shock-wave, due to the formation of a double shock system</td>
<td>Tunnel wall - wedge at high Reynolds numbers</td>
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<td>5. Elfstrom</td>
<td>- Disappearance of the overshoot in the wall pressure distribution</td>
<td>Flat plate - wedge 8.0 mm</td>
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<td>- Change in shape of pressure distribution at a certain position on the wedge</td>
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| 6. Holden Ref. 18 | - Separation length measured on schlieren pictures  
- Detection of flow reversal using skin friction balances | Flat plate - wedge |
| 7. Kessler et al. Ref. 23 | No information available |   |
| 8. Kuehn Ref. 1 | - Appearance of a kink in the wall pressure distribution | Flat plate - wedge  
Use of boundary layer trip  
δ = 2 - 4 mm |
| 9. Law Ref. 9 | - Appearance of a kink in the wall pressure distribution  
- Change in shape of the pressure distribution at a certain position in front of the wedge  
- First appearance of the deflection of the boundary layer measured on schlieren pictures  
- Oil flow method | Flat plate - wedge  
Leading edge of flat plate far upstream of nozzle exit  
δ = 4 mm |
| 10. Rhosko & Thomke Ref. 13 | - Change in shape of the pressure distribution at a certain position on the wedge  
- Detection of separation and re-attachment using orifice dams | Tunnel wall - wedge  
δ = 75 - 140 mm |
| 11. Spaid & Frishett Ref. 8 | - Oil flow method  
- Powder deposition technique  
- Extrapolation on schlieren pictures of the separation shock, to the wall | Tunnel wall - wedge  
δ = 8 mm |
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<th>Change in shape of the pressure distribution at a certain position in front of the wedge</th>
<th>Appearance of a kink in the wall pressure distribution</th>
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<td>- Appearance of a kink in the wall pressure distribution</td>
<td>Flat plate - wedge with and without roughness trips δ = 4 - 8 mm</td>
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<td>- Oil flow method</td>
<td>Tunnel wall - wedge δ = 6 &amp; 20 mm</td>
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<td>- Extrapolation, on schlieren pictures, of the separation shock, to the wall</td>
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<td>14. Coleman</td>
<td>Similar techniques as Flfstrom's and Coleman's two-dimensional tests, see No 3 and 5.</td>
<td>Hollow cylinder-flare δ = 4 mm R/δ = 8</td>
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<td>Ref.25</td>
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<td>15. Kuehn</td>
<td>- Appearance of a kink in the wall pressure distribution</td>
<td>Cone-cylinder-flare δ = 4.5 mm R/δ = 3.65</td>
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<td>Ref.26</td>
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<td>16. Rose et al.</td>
<td>- Injection of oil in the separated region (liquid line method)</td>
<td>Axi-symmetric tunnel wall</td>
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<tr>
<td>Ref.11</td>
<td>- Detection of separation and re-attachment using orifice dams</td>
<td>Shock induced separation δ = 5 mm R/δ = 5</td>
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<td>- Appearance of a kink in the wall pressure distribution</td>
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<td>- Appearance of a kink in the surface pitot pressure distribution</td>
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Fig. 1 a: COMPRESSION CORNER ON THE WALL OF THE $M = 5.4$ TUNNEL

Fig. 1 b: COMPRESSION CORNER ON THE WALL OF THE $M = 3.5$ TUNNEL
Fig. 2 OIL FLOW PATTERN BEFORE AND DURING THE TEST, FOR LARGE SEPARATED REGIONS
Fig. 3: OIL FLOW PATTERN DURING THE TEST, FOR SMALL SEPARATED REGIONS

Fig. 4: SCHLIEREN PICTURE OF A SEPARATED FLOW
Fig. 5 a: PITOT PROBE USED IN THE $M=5.4$ TUNNEL

Fig. 5 b: PITOT PROBE USED IN THE $M=3.5$ TUNNEL
Fig. 6 MACH NUMBER AND VELOCITY PROFILES FOR THE TURBULENT BOUNDARY LAYER ON THE M=5.4 TUNNEL WALL
Fig. 7 MACH NUMBER AND VELOCITY PROFILES FOR THE TURBULENT BOUNDARY LAYER ON THE M = 3.5 TUNNEL WALL
Fig. 8 COMPRESSIBILITY TRANSFORMATION OF VAN DRIEST APPLIED TO BOUNDARY LAYER VELOCITY PROFILES.
Fig. 9: OIL FLOW PATTERN OF A SEPARATED REGION FOR DIFFERENT COMPRESSION CORNER ANGLES

\[ \alpha = 21^\circ \quad 25^\circ \]

\[ 23^\circ \quad 30^\circ \]
Fig. 10: OIL FLOW PATTERN OF A SEPARATED REGION FOR DIFFERENT REYNOLDS NUMBERS

\[ \text{Re}_6 = \begin{array}{c|c|c} 0.93 & 3.23 & 10^5 \\ \hline 2.07 & 3.23 & 10^5 \\ \hline 4.37 & 4.37 & 10^5 \end{array} \]
FIG. 11: SEPARATION LENGTH AS FUNCTION OF THE WEDGE ANGLE, \( M = 3.5 \)
\[ \text{Re}_\delta = 6.43 \times 10^5 \]
FIG. 11: CONTINUED
$Re \delta = 5.31 \times 10^5$
\[ L_5 \text{ (mm)} \]

\[ \alpha \text{ (DEG)} \]

**FIG. 11 : CONTINUED**

\[ Re_5 = 4.37 \times 10^5 \]
FIG. 11 : CONTINUED

\[ \text{Re} \delta = 3.23 \times 10^5 \]
FIG. 11: CONTINUED
Re_6 = 2.07 \times 10^5
FIG. 11: CONCLUDED

\[ \text{Re} \delta = 0.935 \times 10^5 \]
FIG. 12: SEPARATION LENGTH AS FUNCTION OF THE WEDGE ANGLE, \( M = 5.4 \)
\[ \text{Re}_\delta = 6.67 \times 10^5 \]
FIG. 12 : CONTINUED

$$Re_\delta = 3.96 \times 10^5$$
FIG. 12: CONCLUDED

Re_δ = 2.88 \times 10^5
Fig. 13: OIL FLOW PATTERN OF A VERY SMALL SEPARATED REGION
Fig. 14: SCHLIEREN PICTURES OF A SEPARATED REGION FOR DIFFERENT WEDGE ANGLES. $M = 3.5$
Fig. 15: SCHLIEREN PICTURES OF A SEPARATED REGION FOR DIFFERENT WEDGE ANGLES. \( M = 5.4 \)
FIG. 16: DETECTION OF INCIPIENT SEPARATION BY EXTRAPOLATING THE SEPARATION LENGTH

$M = 3.5$
FIG. 17: DETECTION OF INCIPIENT SEPARATION BY EXTRAPOLATING THE SEPARATION LENGTH

$M = 5.4$
Fig. 18 INCIPIENT SEPARATION ANGLES FOR TWO-DIMENSIONAL CONFIGURATIONS
Fig. 19 INCIPIENT SEPARATION ANGLES FOR AXI-SYMMETRIC CONFIGURATIONS
Fig. 20: HIGH SPEED SCHLIEREN PICTURE OF A SMALL SEPARATED REGION
Fig. 21: OIL FLOW PATTERN SHOWING THE ACCUMULATION OF OIL AFTER LONG RUNNING TIMES
Fig. 22 CORRELATION METHOD FOR THE PRESSURE RISE TO INCIPIENT SEPARATION.

\[
\frac{P_{\text{inc}}}{P_0} - 1 = 62.5 \cdot M_0^3 \cdot C_{f_0}
\]

Roshko & Thomke (1966)