Influence of Repetitive Light Stimulation on Alpha Dynamics

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Abstract: A relative high power in the alpha band (8 – 13Hz) at a particular EEG position is believed to indicate an active inhibitory state of the corresponding cortical site. According to this hypothesis an active inhibitory state corresponds to a lower engagement of the cortical site and is accompanied by a higher engagement of other sites (possibly neighboring ones). In this project the influence of a repetitive visual stimulation on the alpha band was investigated. A special attention was paid to the difference between the individual alpha frequency and alpha power before and after the stimulation. Data was analyzed by using the matching pursuit time–frequency distribution that provides better resolution in time and frequency than STFT or Wigner–Ville transforms.
Conclusions:

- The alpha peak frequency changes when a subject is presented with a repetitive visual stimulation
- Alpha peak frequency decreases during the repetitive light stimulation in the 5–45Hz frequency band
- Alpha peak frequency increases following repetitive light stimulation in the range 0.1 to 0.5Hz and 5 to 45Hz
- No significant correlations between alpha power and SSVEP power were found
- Slight correlations between stimulation signal at infra–frequencies and alpha peak frequency were found

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Section 1

Introduction

1 Introduction

1.1 Goals and motivation.

The purpose of this project was to determine the influence of repetitive visual stimulation on alpha band. More precisely, a correlation between flicker frequency and alpha power and between flicker frequency and alpha frequency was investigated.

There have been many experiments regarding alpha changes caused by different tasks, e.g. Osaka (1984). Because alpha frequency is correlated with cognitive performance (Klimesch et al., 1993; Anokhin & Vogel, 1996; Osaka, 1984; Pfurtscheller, 1992), it is important to know whether it is possible to influence the alpha waves with external stimuli, so that, in the future, cognitive performance itself could be manipulated. Klimesch et al. (2003) showed that it is possible to enhance task performance and task–related alpha desynchronization using repetitive transcranial magnetic stimulation at specified frequency. This project concentrates on repetitive visual stimulus.

1.2 Summary of main conclusions

The results presented in this report show that it is possible to modulate alpha frequency by repetitive light stimulation. The largest change is observed in infra–frequencies (0.1Hz–0.5Hz). Alpha peak frequency decreases during the stimulation and increases after.

1.3 Organization

This report consists of 7 chapters. The second part contains basic introduction to brain physiology and EEG measurements. The third part presents basic facts about alpha rhythm. The fourth chapter deals with some methods of time–frequency analysis: short time Fourier transform, Wigner-Ville distribution and matching pursuit decomposition. Next section provides some information about SSVEP: physiology and different representation with time–frequency (TF) methods. Results of the experiment are outlined in chapter 6. The discussion of results is presented in chapter 7. Some definitions regarding signal and statistical analysis are described in appendices.
Section 2

Brain physiology and EEG

1 Physiology of EEG

This section introduces electroencephalography (EEG) basics. It starts with a short description of the cerebral cortex. Basis of neural bioelectricity are presented in order to explain origins of EEG signals.

1.1 Cerebral cortex

The cerebral cortex is a symmetrical and bilateral structure – a furrowed gray matter covering the cerebral hemispheres [Kandel et al., 2000]. It is divided into four distinct lobes: frontal, parietal, occipital, and temporal (see fig. 2.1). It is on the cortex where occur brain operations responsible for a cognitive abilities: frontal lobe is concerned with controlling movement and planning future actions; parietal lobe is related to somatic sensation, creating a body image, relating this body image with extrapersonal space, integration of sight with movement and under-
standing abstract and geometrical expressions; occipital lobe is responsible for vision: analysis of color, movement, shape and depth and visual associations; temporal lobe is concerned with hearing, smell and speech.

Gray matter consists of cortical columns – interconnected sets of neurons (fig. 2.5 A). Each column receives information from different parts of the nervous system. Signals are then processed, a unique response is returned and forwarded to further brain paths. Columns are organized into cell layers lying parallel to surface of hemispheres (vide fig. 2.5 B). The most typical form of cortex contains six layers numbered from outer surface (pia matter) of cortex to white matter (Kandel et al., 2000) – a set of myelinated axons (cf. section 1.2) connecting gray matter with other structures of nervous system. Because of layer structure, one cortical column can receive information from several parts of the brain. Each input is processed in different way and outputs, arising from different population of neurons, are directed to different regions of nervous system.

1.2 Neurons

![Diagram of a neuron](image)

Figure 2.2: A typical neuron. Figure taken from Wikipedia

Nerve cells – neurons – are basic signaling units of a nervous system. A typical neuron consist of four parts: dendrites, cell body (soma), the axon and synapses (vide fig. 2.2). The cell body is the metabolic center of the cell life. It contains genes carrying nucleus and other organelles necessary to sustain the cell. Short, tree–like dendrites receive information from other cells. The axon transfers processed information to another neurons. In the brain, the information is carried within and between neurons mainly by electrical signals. These electrical signals are possible due to temporary changes in ionic current flow into and out the cell. Myelin sheath, made by Schwann cells, acts like an isolator causing signal to travel...
along the axon from one Ranvier node to another (where signal is regenerated), speeding up the transfer process. The ionic current is passed by to another neurons by synapses.

### 1.3 EEG recordings

As the ionic current travels along the neuron, it generates an electric potential around the neuron. A cell can be treated than as an electric dipole. In a cortical column axons of neurons lie parallel to each other and perpendicular to cortex surface. (see fig. 2.5). When a signal reaches a cortical layer, all neurons in it simultaneously act like electric dipoles, creating an electric potential in surrounding space – a sum of potentials from each neuron. This electric field can be recorded using electrodes. As seen in figure 2.3 the recorded potential depends on the electrode position with respect to the dipole layer orientation. Electroencephalography (EEG) uses sensors placed on the scalp. The signal recorded by a particular electrode is usually referred to as a channel (see fig. 2.4 right). In this project a set of 32 sensors was used. An electrode placement scheme and naming of electrodes is presented on fig. 2.4 left.

![Figure 2.3: Recorded with electrodes (here denoted as P₁ and P₂) potential, depends on a position from dipole layer.](image)

![Figure 2.4: Left: electrode placement. Right: during EEG recording each electrode records separate signal.](image)
Figure 2.5: (A) Neurons of cerebral cortex are organized into columns. Blue fibers on the picture represent inhibitory neurons – neurons that decrease cellular current flow – controlling other neurons in cortex (in red) and preventing from epilepsy. From http://www.coloradocoll.edu/IDProg/Neuroscience/Cellular1-21.html

(B) The appearance of layers depends on what is used to stain it. Golgi stain reveals neuronal cell bodies and dendritic trees, the Nissl method reveals cell bodies and proximal dendrites and Weigert stain reveals axon distribution. The layers are: L1 – molecular layer, L2 – external granular cell layer, L3 – external pyramidal cell layer, L4 – internal granule cell layer, L5 – internal pyramidal cell layer, L6 – multiform layer.
Section 3

Alpha rhythm.

1 Alpha rhythm and dynamics.

1.1 Individual alpha frequency

In an awake state, the dominant frequency in human scalp EEG is alpha (Klimesch, 1999a). It is manifested by a peak in signal spectrum in between 7Hz and 14Hz (see fig. 3.1 (C)). This peak, also called individual alpha frequency (IAF), varies to a large extent as a function of age, neurological diseases, memory performance brain volume and task demands (Klimesch, 1999a,b). Figure 3.1 (A) and (B) illustrates this aspect.

Alpha frequency is positively correlated with the speed of information processing and memory performance (Klimesch, 1999a,b; Klimesch et al., 1993) and intelligence (Anokhin & Vogel, 1996). In general, alpha frequency can be viewed as an indicator of cognitive performance.

The detection of alpha peak frequency can be implemented in many ways:

Fourier transform

The easiest way is to calculate the spectrum using Fourier transform. As the signal is digitalized, the alpha peak frequency will be the frequency bin that corresponds to largest Fourier coefficient.

Autoregressive models

For acquiring better resolution, autoregressive models are used. Each point of autoregressive time series \( x_i \) can be explained by weighted previous points \( \{x_{i-1}, x_{i-2}, \ldots, x_{i-p}\} \) with some weights \( \{\phi_{i-1}, \ldots, \phi_{i-p}\} \), and a random component \( \epsilon_i \) with zero mean and \( \sigma^2 \) variance, i.e.:

\[
x_{i+1} = \sum_{n=0}^{p-1} \phi_{i-n} x_{i-n} + \epsilon_{i+1}
\]

(3.1)

The number \( p \) is the order of the model.

For a given time series (e.g. the EEG data), coefficients of the model can be estimated by, for example, Yule–Walker method. Having this coefficients, the signal spectrum estimation is produced by a formula:

\[
p(f) = \frac{\sigma^2}{|1 - \sum_{n=0}^{p-1} \phi_n e^{-2\pi ifn}|^2}
\]

(3.2)
Figure 3.1: Interpersonal differences in individual alpha frequency (IAF) depend on many factors. (A) IAF dependence on age. Frequency first increases with age, than decreases. The decrease might be due to neurological diseases and lack of “mental training”. From Klimesch (1999a). (B) IAF dependence on memory performance and neurological diseases. $M^+$ denotes good- and $M^-$ bad memory performance. From Klimesch (1999a). (C) A typical power spectrum. With respect to IAF two alpha bands can be defined: a low alpha band (from $IAF - 4Hz$ to $IAF$) and a high alpha band (from $IAF$ to $IAF + 2Hz$)
It is a continuous function that strongly depends on the model order (see fig. 3.2). Determining the alpha peak frequency is only matter of finding the maximum of $p(f)$ function in desired boundaries.

**The gravity frequency**

Both methods presented above do not consider the shape of the spectrum. One solution is to define a peak alpha frequency as a gravity frequency $\hat{f}$, i.e.

$$\hat{f} = \frac{\int_{-\infty}^{\infty} f p(f) df}{\int_{-\infty}^{\infty} p(f) df} \quad (3.3)$$

where function $p(f)$ is a spectrum acquired with Fourier transform or AR model.

![Figure 3.2: Detecting alpha peak frequency using Fourier transform (FT) and Yule–Walker power estimation. Changing model order causes peak to shift.](image)

**1.2 Individual alpha peak shift**

Studies have shown that individual alpha frequency is not a static value and can shift. Osaka (1984) showed that this shift is positively correlated with task difficulty. She investigated difference between alpha frequency in resting state and during mental activities that included simple and complex arithmetic addition tasks, visual imaginary task and mental rotation task. Not only did she find that alpha peak is significantly higher when tasks are performed, but also that parallel processing (like imaginary rotations and complex additions) increase IAF more than simple
Osaka et al. (1999) used magnetoencephalogram (MEG) to detect changes in alpha frequency during memory tasks. They reported that the peak alpha frequency, averaged over 74 MEG sensors, increased when subjects were involved in a listening span test. They also showed that the shift was dominant in the left hemisphere, in fronto–temporal regions.

It is also possible to induce better task performance using individual alpha frequency as stimulation marker. Transcranial magnetic stimulation (TMS) is often used to interfere with cortical functions (Hallet, 2000). One of the parameters that are important when using TMS for altering brain functions is a frequency of delivering single magnetic pulses (repetitive transcranial magnetic stimulation – rTMS). Klimesch et al. (2003) showed that rTMS at frequency IAF+1Hz can enhance task performance (in this: case imaginary rotation).

![Time frequency map](image)

Figure 3.3: A time frequency map obtained with matching pursuit (vide section 4). During the stimulus a significant decrease in signal power in both high and low alpha band (with IAF = 10Hz) can be seen.

### 1.3 Alpha bands

Beside alpha frequency, there is a need of investigating the dynamics of neighboring to IAF regions in spectrum, i.e. there is a need of defining alpha bands. Because of various inter–individual differences, alpha bands have to be adjusted to personal physiology. Klimesch (1999a) suggested using IAF as a starting point for defining bandwidth. Frequencies above alpha frequency are called high alpha, below – low alpha. The dynamics of signal power in each of these bandwidths is correlated with different process like attention, memory performance and information processing (Klimesch, 1999a).

The signal power in a specified band as a result of performed task can either increase or decrease. The former is also referred to as event–related synchronization (ERS), the latter – as
event–related desynchronization (ERD).

It is well known fact that alpha power decreases when a cognitive task is performed (cf. fig. 3.3). A simple method of quantifying ERD/ERS was presented in Clochon et al. (1996): alpha envelopes, obtained by Hilbert transformation of bandpass filtered epochs (vide appendix A), are averaged over trials. A reference period is chosen and its power calculated. Alpha power change in an averaged epochs $s(t)$ can be defined as:

$$ERD/ERS_i = \frac{s_i - R}{R} \times 100\%$$

where $R$ is power of reference period and $s_i$ is power in $i^{th}$ point of the analyzed signal. The function $ERD/ERS$ takes positive values when alpha power is greater than reference; if the power is lesser, then the function takes negative values.

### 1.4 Alpha bands – correlations

Klimesch (1999a) presented comprehensive review of correlations between power in different alpha bands and behavioral factors like attention. Below is a short summary of his conclusions:

**Desynchronization in lower alpha reflects attention and expectancy**

To determine the dependence of attention and expectancy to lower alpha, a simple experiment was conducted. In each trail a warning signal preceded the appearance of a visual stimulus. Two types of stimuli were presented – targets and non–targets. Subject was asked to count only targets. The main assumption was that the warning signal increased alertness. As the presentation of targets and non–targets was not random, after some time subject was able to anticipate the appearance of a specific stimuli.

The results demonstrate that after the warning signal there is a significant desynchronization only in lower the alpha band reflecting the alertness. This effect is greater only when after the signal target is presented – this reflects expectancy.

**Desynchronization in higher alpha reflects semantic processing**

Another experiment was conducted to test the hypothesis that the desynchronization in higher alpha reflects semantic memory performance. Subjects had to judge, whether sequentially presented pairs feature–concept (“claws—eagle”, “wings—banana”, “seeds—cucumber”) are semantically congruent. Results indicated that alpha power in higher band decreases significantly, but only when a semantic information processing takes place, i.e. after the concept word had been said.

**Alpha and sleep**

A good cognitive performance is correlated with alpha frequency and power. This suggests that during hypnagogic state (a transition period from walking to sleeping) alpha power decreases. This is the case, as it is shown for example in Klimesch (1999a) or Cajochen et al. (1996). The decrease is especially visible in lower alpha bands. Higher alpha, on the other hand, first decreases than increases. This is due to the fact that upper alpha reflects frequency range activity where sleep spindles occur.

What is interesting, during a state of sustained wakefulness and sleep deprivation lower alpha
is increased and reflects growth of sleepiness. This might be correlated with difficulties with sustaining state of alert wakefulness.
Section 4

Spectral methods

Time-frequency decomposition is a basic tool for the analysis of a signal. Methods like short time Fourier transform, Wigner transform are bounded by low resolution. Matching pursuit algorithm gives higher resolution. It was first proposed in Mallat & Zhang (1993) and then improved in Durka et al. (2001).

1 Short time Fourier transform

1.1 STFT: definition

The Fourier transform (FT) $F_s(f)$ of a signal $s(t)$, is defined as:

$$F_s(f) = \int_{-\infty}^{\infty} s(t) e^{-2i\pi ft} dt$$ (4.1)

$$s(t) = \int_{-\infty}^{\infty} F_s(f) e^{2i\pi ft} df$$ (4.2)

It can be viewed as a signal decomposition on infinite waves (sinusoids). Because of this time delocalization, Fourier transform is not well adapted for an analysis of non–stationary signals (e.g. EEG signals).

An intuitive and simple solution is to use a short time Fourier transform. The basic idea is to window processed signal around a particular time and then calculate its FT. As the signal is suppressed outside the window, the result is a local spectrum. This procedure is repeated for each time $t$.

In other words, short time Fourier transform (STFT) $F_s(t, f; h)$, of a signal $s(t)$ and using a window $h(t)$, is defined as:

$$F_s(t, f; h) = \int_{-\infty}^{\infty} s(u)\bar{h}(u-t)e^{-2i\pi fu} du$$ (4.3)

Using equations 4.3 and 4.2 signal can be rewritten as:

$$s(t) = \frac{1}{E_h} \int_{-\infty}^{\infty} F_s(t, f; h)h(t-u)e^{2i\pi ft} dt df$$ (4.4)

where $E_h = \int_{-\infty}^{\infty} |h(t)|^2 dt$ is an energy of a window. Relation 4.3 suggest, that signal can be seen as a weighted sum of special waveforms – atoms $h_{t,f}(u) = h(t-u)e^{2i\pi ft}$ (see also section...
The set of atoms $h_{t,u}$ is obtained from original window $h(u)$ by translation in time and frequency (modulation).

### 1.2 Heisenberg principle

The resolution of a STFT is limited by an uncertainty rule, or Heisenberg’s principle. Explicitly it states that:

$$\sigma_t \sigma_f \geq \frac{1}{2}$$  \hspace{1cm} (4.6)

where

$$\sigma_t^2 = \frac{1}{|s(t)|^2} \int_{-\infty}^{\infty} (t-u)|s(t)|^2 \, dt$$  \hspace{1cm} (4.7)

$$u = \frac{1}{|s(t)|^2} \int_{-\infty}^{\infty} t|s(t)|^2 \, dt$$  \hspace{1cm} (4.8)

$$\sigma_f^2 = \frac{1}{|s(t)|^2} \int_{-\infty}^{\infty} (f-\xi)|F_s(f)|^2 \, dt$$  \hspace{1cm} (4.9)

$$\xi = \frac{1}{|s(t)|^2} \int_{-\infty}^{\infty} f|F_s(f)|^2 \, dt$$  \hspace{1cm} (4.10)

The more concentrated the $s(t)$ is, the more spread out $F_s(f)$ is. Because of this, the resolution of STFT is bounded by window length (see fig. 4.1 and 4.2).

### 2 Wigner-Ville transform

Although a window length in STFT can be set to acquire good time–frequency representation, it is usually very hard to do this for unknown signals. A possible solution to this problem can be a Wigner-Ville transform (WVT). It can be derived from a Wiener–Khintchine theorem that states that a square modulus of a Fourier transform $F_s$ is equal to a Fourier transform of the corresponding autocorrelation function, i.e:

$$|F_s(f)|^2 = \int_{-\infty}^{\infty} e^{-2\pi i ft} \left( \int_{-\infty}^{\infty} s(t) \overline{s(t+\tau)} \, d\tau \right) \, dt$$  \hspace{1cm} (4.12)

Removing time integer from the right side of an equation 4.12 and centering around 0 produces a function that is explicitly dependent on time and frequency – a Wigner-Ville transform:

$$W_s(t,f) = \int_{-\infty}^{\infty} s \left( t + \frac{\tau}{2} \right) \overline{s \left( t - \frac{\tau}{2} \right)} e^{-2\pi i f \tau} \, d\tau$$  \hspace{1cm} (4.13)

The Wigner–Ville distribution conserves the time and frequency support, i.e.

$$s(t) = 0, \quad |t| > T \Rightarrow W_s(t,f) = 0, \quad |t| > T$$

$$F_s(f) = 0, \quad |f| > B \Rightarrow W_s(t,f) = 0, \quad |f| > B$$

As it is a quadric distribution, Wigner–Ville transform is negatively affected by cross–terms. For example, a Wigner-Ville transformation of a sum of two signals $l = x + y$ is:
Figure 4.1: A STFT map of a signal consisting of a 32Hz sinusoid with gauss envelope. Signal was sampled with 128Hz rate. The window used in STFT had 257 points length. A good localization in frequency causes poor time localization. cf. fig. 4.2
Figure 4.2: A STFT map of a signal consisting of a 32Hz sinusoid with gaussian envelope. Signal was sampled with 128Hz rate. The window used in STFT had 11 points length. A good localization in time results in poor frequency localization. cf. fig. 4.1
\[ W_t(t, f) = W_x(t, f) + W_y(t, f) + 2\Re(W[x,y](t,f)) \] (4.14)

where

\[ W[x,y](t,f) = \int_{-\infty}^{\infty} x(t + \frac{\tau}{2}) \bar{y}(t - \frac{\tau}{2}) e^{-2i\pi ft} d\tau \]

There is a third component corresponding to an interference between two basic parts of the signal. This can be generalized to \( N \) components. Figure 4.3 shows crossterms on a simple signal.

Figure 4.3: A signal consists of two parts: a sum of sinuses (10Hz and 43Hz) and a sinus (30Hz). Beside main components, Wigner-Ville transform presents additional atoms - crossterms.

### 2.1 Pseudo-Wigner–Ville transform and smoothened pseudo-Wigner–Ville transform

Calculation of the Wigner–Ville distribution requires a knowlege of a value \( q_s = s(t + \frac{\tau}{2}) \bar{s}(t - \frac{\tau}{2}) \) from \(-\infty\) to \(\infty\). As this practically is not possible, a windowed version is used:

\[ PW_s(t,f) = \int_{-\infty}^{\infty} h(\tau) s(t + \frac{\tau}{2}) \bar{s}(t - \frac{\tau}{2}) e^{-2\pi if\tau} d\tau \] (4.15)

where \( h(\tau) \) is a window similar to STFT. Equation 4.15 presents a new distribution called pseudo Wigner–Ville distribution or PWVD. Due to a convolution theorem, the definition can be rewrit-
ten as:

\[ PW_s(t, f) = F_{h,q_s}(f)(t) = F_h(f) \ast F_{q_s}(f)(t) = F_h(f) \ast W_s(t, f) \]  

(4.16)

This is an equivalent of a frequency smoothening. Because of their oscillatory nature, some crossterms will be attenuated due to this operation. Windowing, however, causes the distribution to lose some of its properties, frequency support, for example. Thus the frequency resolution is worse than in WV distribution (see fig. 4.4).

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The PWVD is controlled only by a time dependent window. Providing an additional degree of freedom, separating smoothing for time domain and frequency domain, it is possible to control the crossterms in both planes. This produces a new distribution:

\[ SPW_s(t, f) = \int_{-\infty}^{\infty} h(\tau) \int_{-\infty}^{\infty} g(\xi - t) s(\xi + \frac{\tau}{2}) \bar{s}(\xi - \frac{\tau}{2}) d\xi e^{-2\pi i f\tau} d\tau \]  

(4.17)

called smoothened pseudo Wigner–Ville distribution or SPWVD. As seen on figure 4.5, most crossterms are not visible, but the time and frequency resolution is poorer than in WV. This enforces a compromise between the level of interference and time–frequency resolution (Auger et al., 1996; Durka, 2004). Mallat & Zhang (1993) showed that when an iterative algorithm is applied to the signal, interference in Wigner–Ville transform can be removed without any effect on resolution. This procedure, called matching pursuit (MP) is described in next section.

Figure 4.4: A signal consists of two parts: a sum of sinuses (10Hz and 43Hz) and a sinus (30Hz). Smoothening window reduces some crossterms (cf. fig. 4.3), but at the same time frequency resolution gets poorer.

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\[ SPW_s(t, f) = \int_{-\infty}^{\infty} h(\tau) \int_{-\infty}^{\infty} g(\xi - t) s(\xi + \frac{\tau}{2}) \bar{s}(\xi - \frac{\tau}{2}) d\xi e^{-2\pi i f\tau} d\tau \]  

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called smoothened pseudo Wigner–Ville distribution or SPWVD. As seen on figure 4.5, most crossterms are not visible, but the time and frequency resolution is poorer than in WV. This enforces a compromise between the level of interference and time–frequency resolution (Auger et al., 1996; Durka, 2004). Mallat & Zhang (1993) showed that when an iterative algorithm is applied to the signal, interference in Wigner–Ville transform can be removed without any effect on resolution. This procedure, called matching pursuit (MP) is described in next section.
3 Matching Pursuit: a brief description.

Matching pursuit is an algorithm that approximates signal $s(t)$ with a set of waveforms taken from a redundant dictionary. Finding an optimal approximation by minimizing an error $\epsilon$ with $M$ functions $g_{\gamma_i}$, i.e.

$$\epsilon = \|f(t) - \sum_{i=1}^{M} w_i g_{\gamma_i}\|$$  \hspace{1cm} (4.18)

is a NP-problem. This can be overcome by applying an iterative solution. In each iteration a function from a dictionary $\Delta = g_{\gamma_1}, g_{\gamma_2}, \ldots, g_{\gamma_n}$ (a set of functions that signal will be decomposed to, the same way as an expressions can be explained by words from a vocabulary) is chosen so that it satisfies the equation:

$$g_{\gamma_i} = \arg \max_{g_{\gamma_i} \in \Delta} |\langle R^i s, g_{\gamma_i} \rangle|$$  \hspace{1cm} (4.19)

where $R^i s$ is a residual left after subtracting the results of previous iterations, i.e.

$$R^0 s = s$$  \hspace{1cm} (4.20)

$$R^i s = \langle R^i s, g_{\gamma_i} \rangle g_{\gamma_i} + R^{i+1} s$$  \hspace{1cm} (4.21)

The dictionary $\Delta$ is constructed from real Gabor functions:

$$g_{\gamma}(t) = K(\gamma)e^{-\frac{(t-u)^2}{\sigma^2}} \cos(2\pi f(t-u) + \phi)$$  \hspace{1cm} (4.22)

The argument $\gamma = \{u, f, \sigma, \phi\}$ denotes the set of parameters of $g$ function: $u$ is the translation in time, $f$ is sinus frequency, $\sigma$ is the spread in time, $\phi$ is the phase. Parameter $K(\gamma)$ is used to normalize functions.

If the dictionary $\Delta$ is complete, it is proven (Mallat & Zhang, 1993) that:

$$s(t) = \sum_{i=1}^{\infty} \langle R^i s, g_{\gamma_i} \rangle$$  \hspace{1cm} (4.23)

$$\|s(t)\|^2 = \sum_{i=1}^{\infty} |\langle R^i s, g_{\gamma_i} \rangle|^2$$  \hspace{1cm} (4.24)

In practice finite dictionaries are used. A signal $s(t)$ is then approximated by the decomposition:

$$s(t) \approx \sum_{i=1}^{M} \langle R^i s, g_{\gamma_i} \rangle g_{\gamma_i}$$  \hspace{1cm} (4.25)

3.1 Stochastic dictionaries

Mallat & Zhang (1993) proposed fixed sub–sampling parameter space for obtaining dictionaries, but this leads to a statistical bias. A stochastic solution was presented in Durka et al. (2001). To define a dictionary $\Delta$, a space of parameters $\{t, f, s\}$ is divided into cubes of size $\Delta s$, $\Delta t$ and $\Delta f$, where the resolution $(\Delta s, \Delta t, \Delta f)$ is set before the procedure. From each of these cubes one atom is chosen by drawing a parameters set $(t, s, f)$ from a flat distribution within the given range. First iteration reduces the dictionary $\Delta$ to a subset $\Delta_0$ constructed by choosing an arbitrary percentage of atoms having largest correlations with the original signal. An atom $g_{\gamma_i}$ chosen in each iteration from dictionary $\Delta_0$ or $\Delta$, is optimized by a search in a dictionary constructed of neighboring parameters of chosen $\gamma_i$ set.
3.2 Atoms

Atoms can be seen as basic bricks building signal. They are functions from one class, but differ from each other by a set of parameters (like frequency, shift, scale, etc.). Different decomposition techniques present different definitions of atoms. For example, as equation 4.4 states, short time Fourier transform creates atoms

$$h_{t, f}(u) = h(t - u)e^{2\pi ft}.$$ 

Matching pursuit decomposes signal on Gabor functions that are taken from the dictionary. Each iteration then produces one atom of the signal.

3.3 Energy distribution

A matching pursuit decomposition gives a simple method of clearing the Wigner–Ville distribution from crossterms.

From atomic decomposition, eq. 4.23 of \( s(t) \) it can be derived that:

$$W_s(t, f) = \sum_{i=1}^{\infty} |\langle R^i s, g_{\gamma i} \rangle|^2 W_{g_{\gamma i}}(t, f) +$$

$$+ \sum_{i=1}^{\infty} \sum_{j=0,j\neq i}^{\infty} \langle R^i s, g_{\gamma i} \rangle \langle R^j s, g_{\gamma j} \rangle W[g_{\gamma i}, g_{\gamma j}](t, f)$$  \hspace{1cm} (4.26)

The double sum corresponds to WVD cross-terms. Removing them is simply a matter of defining a distribution:

$$E_s(t, f) = \sum_{i=1}^{\infty} |\langle R^i s, g_{\gamma i} \rangle|^2 W_{g_{\gamma i}}(t, f)$$  \hspace{1cm} (4.27)

As the WVD satisfies

$$\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} W_g(t, f) dt df = \| g \| = 1$$  \hspace{1cm} (4.28)

and eq. 4.24 implies conservation of energy:

$$\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} E_s(t, f) dt df = \| s \|$$  \hspace{1cm} (4.29)

\( E_s(t, f) \) can be interpreted as an energy density of \( s \) in the time-frequency plane. Figure 4.6 illustrates that cross-terms disappear when a matching pursuit algorithm is applied.
Figure 4.5: A signal consists of two parts: a sum of sinuses (10Hz and 43Hz) and a sinus (30Hz). Smoothening window reduces most crossterms (cf. fig. 4.3 and 4.4), but also frequency and time resolution gets poorer.
Figure 4.6: A signal consists of two parts: a sum of sinuses (10Hz and 43Hz) and a sinus (30Hz). No crossterms are visible.
Section 5

Brain activity during repetitive visual stimulation and its presentation

1 Visual evoked potential and Steady–state visual evoked potential

A visual evoked potential (VEP) is a brain response to a single and short-lasting light stimulus (a flash). It is presented in the EEG as a sudden change in signal amplitude, time-locked with a beginning of the flash. Steady state visual evoke potential (SSVEP), on the other hand, is a brain response to a repetitive light stimulus. It is presented in an EEG signal as a oscillation with a frequency corresponding to the on of the stimulus. It is observed within flicker frequency from 2Hz up to 70Hz (Pastor et al., 2003).

Although the power of SSVEP can change drastically between subjects (even to a point when no SSVEP can be recorded from some people), some consistencies were discovered. Research has shown, that highest response is acquired with stimulus frequency in between 10Hz and 15Hz (Pastor et al., 2003; Silberstein et al., 2001) (see figure 5.1).

Skosnik et al. (2006) and Krishnan et al. (2005) presented evidence of gender difference in the SSVEP. Females tend to have larger response than males. Silberstein et al. (2001) and Perlstein et al. (2003) have shown that SSVEP power, recorded over prefrontal cortex, can increase with increasing memory task performance or memory task difficulty. In both experiments the stimulus frequency was within the alpha range. SSVEP power is also positively correlated with the attention paid to the stimulus (Regan, 1977; Müller et al., 1998; Morgan et al., 1996).

The response on repetitive light stimulation reflects also the physiological state of the brain.
Krishnan et al. (2005) have shown, for example, that the power of SSVEP is decreased in patients with schizophrenia when compared to healthy subjects. Skosnik et al. (2006) and Thompson et al. (2000) investigated the influence of some stimulants on SSVEP. First showed that cannabis usage can influence the power of SSVEP, most notably a positive correlation between the age of first cannabis use and SSVEP power (i.e. subjects that consumed cannabis in earlier age showed lower power values) has been presented. Second investigated the influence of nicotine on the response on flickering light, showing an increase of power in central and right parietal regions with extent to frontal and right temporal regions.

2 Presentation of SSVEP

The easiest way to analyze the SSVEP response is to filter EEG signal with a notch filter. This, however, disregards harmonics (responses that frequency is a multiplication of original SSVEP) or subharmonics (frequencies that are below the original frequency in a ratio $1/x$). One possible solution is to apply a comb filter. In this project, however, time–frequency maps (as described in section 4) were used enabling a better overview of a whole signal.

Below is presented a SSVEP response at 19Hz, averaged over 30 trials, acquired on three ways: with short time Fourier transform (cf. section 1), Wigner-Ville distribution (see section 2) and matching pursuit decomposition (see section 3).

STFT

Figures 5.2 and 5.3 present short time Fourier transform maps of brain response to a flickering light at frequency 19Hz. Different window lengths were used in both cases.

As expected, different window lengths produce different maps. Short, one second window (fig. 5.2), ensures good time resolution: the beginning and the end of SSVEP response is clearly present. On the other hand, the frequency of SSVEP is not very clear – the response visible on the map spans from 17Hz to 20Hz.

A good frequency resolution is produced by a long window (see fig. 5.3). This, on the other hand, causes not very accurate time positioning of SSVEP.

Wigner–Ville transform

Figures 5.4 and 5.5 show averaged, over 30 maps, Wigner–Ville time–frequency distributions of a brain response to flickering light a stimulation (at 19Hz).

The Wigner–Ville distribution presents multiple cross–terms – artifacts of the method, present on the map, but not in the signal (see fig. 5.4). The SSVEP, however, is strongly visible, with good frequency and time distribution. One of the possible methods of clearing the map of interferences is using a smoothening windows as described in section 2.1. Figure 5.5 presents PWV distribution; some of the interference is not visible, but the frequency resolution is poorer.

Matching Pursuit

A map acquired by a matching pursuit algorithm gives a good time and frequency resolution (see fig. 5.6). There is no need of applying any windows. Although the shape of SSVEP component, being Gaussian, is not realistic, the MP gives a good time positioning of the beginning of the response.
Figure 5.2: Averaged 30 maps acquired by short time Fourier transform. White vertical lines denote the beginning and the end of stimulation. Window length was one second. Oz electrode was used. See also fig. 5.3.

Figure 5.3: Averaged 30 maps acquired by short time Fourier transform. White vertical lines denote the beginning and the end of stimulation. Window length was 4.5 seconds. Oz electrode was used. See also fig. 5.2.
Figure 5.4: Averaged 30 maps acquired by Wigner–Ville transform. White vertical lines denote the beginning and the end of stimulation. Clearly visible are crossterms (present on the map but not in the signal) e.g. between SSVEP and its first harmonic and between ERPs. See also fig. 5.5. Oz electrode was used.

Figure 5.5: Averaged 30 maps acquired by pseudo Wigner–Ville transform. White vertical lines denote the beginning and the end of stimulation. Vertical crossterms are not present on the map, but when compared to fig. 5.4, the frequency resolution is worse. Oz electrode was used.
Figure 5.6: Averaged 30 maps acquired by MP algorithm. White vertical lines denote the beginning and the end of stimulation. Map provides good resolution in both time and frequency. Cf. figures 5.2, 5.3, 5.4 and 5.5. Oz electrode was used.
Section 6

Experimental validation and discussion

1 Measurement protocol

The main purpose of this project was to record EEG response under repetitive light stimulation. The frequency of stimulus was in two ranges: infra–frequencies, i.e. frequencies below 1Hz and frequencies that can produce SSVEP, i.e. between 5Hz and 70Hz. Recording an alpha response is not an easy task. As Klimesch (1999b) suggested, expectancy of an incoming stimulus influences subject’s alpha power. In order to acquire recordings without that artifact, time between stimuli was extended to 20 seconds for SSVEP frequencies and to 60 second for infra–frequencies.

1.1 Recording protocol

The subject was seated in front of a source of a flickering light. Data was collected by 32 electrodes placed on subject’s scalp (for electrode placement see fig. 2.4 left). The experiment took place in a dimmed room; the time of the experiment was the same for every subject to reduce influence of the fatigue on EEG.

The subjects were four males. Recordings consisted of two parts: in first the stimulation light was flickering with infra–frequencies, i.e. 0.1Hz, 0.2Hz, 0.3Hz, 0.4Hz and 0.5Hz. Second part consisted of 22 SSVEP frequencies: 5Hz, 7Hz, 9Hz, 10Hz, 11Hz, 12Hz, 13Hz, 15Hz, 19Hz, 20Hz, 24Hz, 25Hz, 27Hz, 29Hz, 30Hz, 32Hz, 34Hz, 35Hz, 37Hz, 39Hz, 40Hz, 45Hz. Two subjects, S004 and S005, participated in both parts. Subject S006 participated in SSVEP part only and subject S003 – in infra–frequencies part.

Each recording consisted of 30 (in SSVEP part) or 10 (in infra– part) repetitions of flickering light. The flicker lasted for 4 seconds (SSVEP) or 60 seconds (infra–). To reduce the influence of expectation, the time interval between stimulation was extended to between 15 and 20 second (SSVEP) or 30 and 35 seconds (infra–) and each stimulation was preceded by a warning sound signal. The basic scheme is presented on figure 6.1.

1.2 Equipment

Figure 6.2 presents the basic scheme of equipment placement. It consisted of two parts. In first part was equipment responsible for producing the light stimulus: controlling computer, function generator, amplifier and light panel. Electrodes, amplifier, light detector and second computer were responsible for recording.

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Figure 6.1: The basic scheme of experiment protocol. For infra-frequencies each stimulation lasted 60s and distance between stimulations was 25-30 seconds.

Figure 6.2: Basic scheme of equipment placement.
Control computer and function generator.

To produce a sinusoidal flicker an Agilent 33220A function generator, controlled by a computer, was used. The voltage \( U(t) \), produced by function generator, was correlated with diodes luminescence by a nonlinear function:

\[
U(t) = 0.69752 \left( 1 + e^{4.0524 \times 10^{-7} \phi(t)^{1.80537}} \right) + 1 - e^{-0.12098 \phi(t)^{0.45011}}
\]  

(6.1)

where \( \phi(t) \) is desired luminescence function. In this project luminescence was a sinusoidal function with values ranged between 0 and 400 cd/m², i.e.:

\[
\phi_f(t) = 200 \left( 1 + \sin (2\pi ft) \right)
\]

(6.2)

where \( f \) denotes stimulation frequency.

For each frequency \( f \) computer produced a voltage function defined in equation (6.1). This function was send to function generator by an USB cable. Enhanced by the amplifier voltage was sent to the diodes’ panel that produced flickering function \( \phi_f(t) \).

Recording equipment.

BioSemi (www.biosemi.com) recording equipment was used. 32 electrodes (fig. 6.3a) were placed on subject’s scalp using a cap (fig. 6.3b) for proper placement. A conductive gel was placed between skin and sensors, to reduce impedance and improve signal quality. Electrodes were connected to analog–digital converter and amplifier. An external light detector was also connected here, so that the exact time position of stimulus would be known.

Digitalized with sampling frequency 2048Hz signal was driven to computer via a fiber optic cable and a USB converter, where recordings were saved to files.

2 Measurements and results

2.1 Determining individual alpha frequency (IAF)

The conditions under which recordings for determining IAF are conducted can be unclear. For example, Klimesch et al. (2003) suggests performing a separate recording, proceeding the experiment, during which subject were to relax and keep their eyes closed for about 3 minutes.

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Figure 6.4: A single epoch decomposed with MP algorithm. Each epoch consisted of 14 seconds: 5 seconds before the begin of the stimulus, 4 seconds of stimulation and 5 seconds after the end of stimulus. Atoms from first group were chosen to calculate IAF. Atoms from second group were chosen to calculate alpha frequency shift.

This experiment concentrates on an occipital area of the scalp. Closing eyes causes synchronization of alpha as no processes occur in visual cortex. At the same time, peak alpha frequency shifts to lower frequencies compared to the level obtained with eyes opened (cf. Osaka (1984)). This can lead to errors when analyzing shift obtained by repetitive light stimulation.

Here, IAF was obtained not with separate recording before proper experiment (a priori), but as a statistic value acquired from all recordings (a posteriori). More precisely, every recording from Oz (reference Cz) electrode was divided into epochs, each containing a stimulus, 5 seconds before the stimulus and 5 seconds after the stimulus – 14 seconds in all (see fig. 6.4). Epochs were then decomposed with matching pursuit algorithm.

From each trial atoms, that position in time was between first and third second, frequency position was between 7 and 15 Hz and lasted for more than 0.1 seconds, were chosen. For each epoch mean atoms’ frequency, weighted with atoms’ amplitude, was calculated (gravity frequency). For each stimulation frequency a mean gravity center $\tilde{f}$, with variance $\sigma^2$, was calculated.

An individual alpha frequency was calculated as a weighted with variances mean of alpha frequencies obtained for each stimulus frequency:

$$IAF_{subject} = \frac{\sum_{j=1}^{N} \frac{\tilde{f}_j}{\sigma_j^2}}{\sum_{j=1}^{N} 1/\sigma_j^2}$$ (6.3)
where $s$ means a single subject, and $N$ is the number of stimulation frequencies. The error estimator is bigger of the two values:

$$s_{int} = \sqrt{\frac{1}{\sum_{j=1}^{N} \frac{1}{\sigma^2_j}}}$$  \hspace{1cm} (6.4)$$

$$s_{ext} = \sqrt{\frac{s_{int}^2}{N-1} \sum_{j=1}^{N} \frac{(\hat{f}_j - IAF)^2}{\sigma^2_j}}$$  \hspace{1cm} (6.5)$$

Calculated IAFs with errors are presented in table 6.1.

<table>
<thead>
<tr>
<th>Subject</th>
<th>S004</th>
<th>S005</th>
<th>S006</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frequency</td>
<td>11.02</td>
<td>10.42</td>
<td>10.21</td>
</tr>
<tr>
<td>Error</td>
<td>0.08</td>
<td>0.05</td>
<td>0.09</td>
</tr>
</tbody>
</table>

Table 6.1: Calculated individual alpha frequencies for all subjects.

### 2.2 Ratio of alpha power before and after the stimulation

To determine the influence of light stimulation signals from electrode Oz (reference Cz) were filtered in two bands – lower alpha (between $IAF - 4Hz$ and $IAF$) and higher alpha (between $IAF$ and $IAF + 2Hz$). Envelopes were then obtained as explained in the appendix A. From each trial $i$ maximal alpha power from between seven and two seconds before the stimulation $B_i^\alpha$ and from between 0.1 and five seconds after the stimulation $A_i^\alpha$ were chosen. Ratios $A_i^\alpha / B_i^\alpha$ were calculated, and for each stimulation frequency Wilcoxon test was performed comparing acquired ratios with unity.

Tables 6.2, 6.3 and 6.4 present the median of ratios for each stimulation frequency with standard deviation. $p$-values are the result of Wilcoxon test. Figures 6.5a, 6.5b and 6.5c show the ratios with standard deviation. Red color denotes that data is statistically significant at family-wise significance level 0.01 – with Bonferroni corrections single result significance level is $\alpha = \frac{0.01}{22} = 4 \cdot 10^{-4}$ (cf. 4).

There is only one statistically significant result in lower alpha. It shows alpha power decline after stimulus. This might suggest a rise in subjects attention after the end of the stimulus (see section 1.4). Statistically significant increase in higher alpha band suggests more relaxed state of the subject. It is important to notice, that high alpha power influences alpha peak (see section 2.3).

### 2.3 Alpha peak shift

To determine alpha peak shift, epochs were decomposed with matching pursuit method, as described in section 2.1. The gravity frequency for each trial $f_i$ was calculated from atoms which position in time was between 9 and 14 seconds (cf. second group on fig. 6.4). For each stimulus frequency a ratio $\hat{f} / IAF$ of mean gravity frequency and individual alpha frequency was calculated with error estimate:

$$s = \frac{\hat{f}}{IAF} \sqrt{\left(\frac{\sigma f}{\hat{f}}\right)^2 + \left(\frac{\sigma IAF}{IAF}\right)^2}$$  \hspace{1cm} (6.6)$$
<table>
<thead>
<tr>
<th>Higher alpha</th>
<th>Lower alpha</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frequency(Hz)</td>
<td>Ratio (STD)</td>
</tr>
<tr>
<td>5</td>
<td>1.2732 (0.61154)</td>
</tr>
<tr>
<td>7</td>
<td>1.0839 (0.55113)</td>
</tr>
<tr>
<td>9</td>
<td>0.9376 (0.51784)</td>
</tr>
<tr>
<td><strong>10</strong></td>
<td><strong>1.2870 (0.64970)</strong></td>
</tr>
<tr>
<td>11</td>
<td>1.2331 (0.58293)</td>
</tr>
<tr>
<td>12</td>
<td>1.0771 (0.56114)</td>
</tr>
<tr>
<td>13</td>
<td>1.2775 (0.84799)</td>
</tr>
<tr>
<td><strong>15</strong></td>
<td><strong>1.5739 (0.80795)</strong></td>
</tr>
<tr>
<td><strong>19</strong></td>
<td><strong>1.2596 (0.87535)</strong></td>
</tr>
<tr>
<td>20</td>
<td>1.2521 (0.76015)</td>
</tr>
<tr>
<td>24</td>
<td>1.3434 (0.67919)</td>
</tr>
<tr>
<td>25</td>
<td>1.0504 (0.64294)</td>
</tr>
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<td><strong>27</strong></td>
<td><strong>1.5176 (0.69427)</strong></td>
</tr>
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<td><strong>29</strong></td>
<td><strong>1.3959 (0.56037)</strong></td>
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<tr>
<td>30</td>
<td>1.1147 (0.34150)</td>
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<td><strong>32</strong></td>
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<td><strong>34</strong></td>
<td><strong>1.2939 (0.76733)</strong></td>
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<td>35</td>
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<td>1.0091 (0.59127)</td>
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<tr>
<td>39</td>
<td>1.1577 (0.53613)</td>
</tr>
<tr>
<td><strong>40</strong></td>
<td><strong>1.1699 (0.64040)</strong></td>
</tr>
<tr>
<td>45</td>
<td>1.2115 (0.65005)</td>
</tr>
</tbody>
</table>

Table 6.2: Ratios of maximal alpha power before to after the stimulation. Data for subject S004. Statistically significant data in bold.

where $\sigma_f$ and $\sigma_{IAF}$ are standard deviation of mean gravity frequency and error estimate of individual alpha frequency (cf. table 6.1).

Table 6.5 presents calculated ratios with errors, and p-values of Wilcoxon test comparing ratios for each trial to unity. Figure 6.6 presents data for three subjects. Red color denotes statistically significant ratios at family–wise 0.01 level (Bonferroni corrected single case level is $4.54 \cdot 10^{-4}$).

For subject S004 some of statistically significant shifts are correlated with statistically significant maximal alpha power shifts (cf. fig. 6.5a). The large increase of alpha power in higher band should cause a shift in alpha peak, as the alpha peak is calculated as the gravity center of frequencies (i.e. mean frequency weighted with power). What is interesting is that this regularity appear in only few cases. In fact, Kendall coefficient shows no statistically significant correlation between alpha power shift and peak alpha frequency shift (at 0.01 significance level). That might suggest that mechanisms responsible for these two processes are different. Further, alpha peak shift will only be considered significant if corresponding alpha power shift is not. Table 6.6 frequencies that produced significant shift in peak alpha frequency.

Most of the statistically significant data shows increase in alpha peak frequency. Only one measurement presents decrease, but it is probably because of the increase in lower alpha power (cf. fig 6.5b). This behavior is consistent with experiment results presented by Osaka (1984) and Osaka et al. (1999): the shift is caused by increased difficulty (appearance of the stimulus) of
<table>
<thead>
<tr>
<th>Frequency (Hz)</th>
<th>Ratio (STD)</th>
<th>p-value</th>
<th>Frequency (Hz)</th>
<th>Ratio (STD)</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>1.0200 (0.8624)</td>
<td>0.1543</td>
<td>5</td>
<td>0.7137 (0.2884)</td>
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</tr>
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<td>45</td>
<td>0.9660 (0.2862)</td>
<td>0.1642</td>
</tr>
</tbody>
</table>

Table 6.3: Ratios of maximal alpha power before and after the stimulation. Data for subject S005. Statistically significant data in bold.

### 2.4 Time dependent alpha peak shift.

To calculate how gravity frequency changes in time, averaged over trials time–frequency maps, obtained by matching pursuit algorithm, were analyzed. Each map, \( m(t, f) \), contained an epoch as described in section 2.1. For every point in time \( T \), a gravity frequency was calculated as a center of gravity of a function \( m_T(f) \)

\[
m_T(f) = \frac{\int_{7Hz}^{15Hz} m(t, f) \, dt}{\int_{7Hz}^{15Hz} m(t, f) \, df}
\]

A mean gravity frequency \( \hat{f}_i(t) \) was calculated for each stimulation frequency \( i \) that showed statistically significant rise in alpha peak frequency (see table 6.6). To compare the results with IAF a similar method to ERD/ERS was used. Obtained mean was transformed according to equation:

\[
\tilde{f}_i(t) = \left( \frac{\hat{f}_i}{IAF} - 1 \right) \cdot 100\%
\]
### Table 6.4: Ratios of maximal alpha power before and after the stimulation. Data for subject S006. Statistically significant data in bold.

<table>
<thead>
<tr>
<th>Frequency (Hz)</th>
<th>Higher alpha</th>
<th>Lower alpha</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Ratio (STD)</td>
<td>p-value</td>
</tr>
<tr>
<td>5</td>
<td>0.9680 (0.55684)</td>
<td>0.2781</td>
</tr>
<tr>
<td>7</td>
<td>0.9838 (0.53857)</td>
<td>0.5000</td>
</tr>
<tr>
<td>9</td>
<td>0.8494 (0.58276)</td>
<td>0.3652</td>
</tr>
<tr>
<td>10</td>
<td>1.1518 (0.65471)</td>
<td>0.0131</td>
</tr>
<tr>
<td>11</td>
<td>0.9684 (0.34743)</td>
<td>0.1592</td>
</tr>
<tr>
<td>12</td>
<td>0.9456 (0.32469)</td>
<td>0.1799</td>
</tr>
<tr>
<td>13</td>
<td>0.9184 (0.31370)</td>
<td>0.1746</td>
</tr>
<tr>
<td>15</td>
<td>1.3714 (0.60991)</td>
<td>0.0007</td>
</tr>
<tr>
<td>19</td>
<td>1.1197 (0.96334)</td>
<td>0.0595</td>
</tr>
<tr>
<td>20</td>
<td>1.0018 (0.60524)</td>
<td>0.1400</td>
</tr>
<tr>
<td>24</td>
<td>1.1657 (0.56073)</td>
<td>0.0057</td>
</tr>
<tr>
<td>25</td>
<td>1.0683 (0.47137)</td>
<td>0.1799</td>
</tr>
<tr>
<td>27</td>
<td>1.0572 (0.55634)</td>
<td>0.1400</td>
</tr>
<tr>
<td>29</td>
<td>1.0153 (0.81247)</td>
<td>0.2989</td>
</tr>
<tr>
<td>30</td>
<td>1.0122 (0.85478)</td>
<td>0.1400</td>
</tr>
<tr>
<td>32</td>
<td>0.8925 (0.43232)</td>
<td>0.4596</td>
</tr>
<tr>
<td>34</td>
<td>0.8620 (0.55563)</td>
<td>0.4197</td>
</tr>
<tr>
<td>35</td>
<td>0.9917 (0.84554)</td>
<td>0.3805</td>
</tr>
<tr>
<td>37</td>
<td>1.0644 (0.48801)</td>
<td>0.0621</td>
</tr>
<tr>
<td>39</td>
<td>1.0642 (0.85658)</td>
<td>0.2323</td>
</tr>
<tr>
<td>40</td>
<td>1.0788 (1.30401)</td>
<td>0.1355</td>
</tr>
<tr>
<td>45</td>
<td>0.9991 (1.03216)</td>
<td>0.0701</td>
</tr>
</tbody>
</table>

where $IAF$ is the individual alpha frequency of corresponding subject. The resulting function $\tilde{f}_i(t)$ has basically similar shape to $\hat{f}_i(t)$, but is rescaled so that it has positive values when is greater than IAF and negative values when it is smaller than IAF. Figures [6.7][6.8][6.9] present $\tilde{f}_i(t)$, for signals after the stimulus, i.e. after 9th second; red dots denote points that are statistically significant according to Wilcoxon test that compared data to 0 (at 0.05 family–wise level; single trial level was adjusted by FDR algorithm).

As the result show, in most cases the maximal increase of alpha peak is within 2 seconds after the end of the stimulus. This might be the time required for the brain (and the subject) to adjust itself to the situation when there is no light. As the experiment was taken in a dimmed room, the transition is rapid – when the light goes off the surrounding of the subject gets more visible and brain starts to process them.

The increase of the alpha peak can also be caused by afterimages. This is an optical phenomenon that refers to an image appearing in one’s vision after the presentation of an original image ceased. In this case, a bright light might cause to produce a “glow” after the stimulus has ended. As this effect lasts for a few seconds, it is possible that it is responsible for attenuating the alpha peak right after the end of the flickering.
Figure 6.5: Mean ratios of maximal alpha power, in a period of 5 seconds, after the stimuli to maximal alpha power, in the period of 5 seconds, before the stimuli with standard deviations. Signal filtered in two bands: higher alpha (from $IAF$ to $IAF + 2Hz$) and lower alpha (between $IAF - 4Hz$ and $IAF$). Red color denotes statistical significance on family-wise $\alpha_F$ level.

2.5 Alpha and SSVEP

Correlation between alpha and SSVEP power

The correlation between SSVEP power and alpha power was tested. All the stimulation frequencies were above 15Hz, so that SSVEP, or its harmonics, was not included in alpha power calculations.

For each trial, three groups of atoms (from MP decomposition), with time positions during the stimulations, were chosen. First group consisted of atoms whose frequency corresponded to that of the stimulation with 0.5Hz error. Second and third group consisted of atoms whose frequency corresponded to higher and lower alpha band (see fig. 6.10). For each trial amplitudes of atoms were summed, producing an estimate of power. For each stimulation frequency of each subject, Kendall correlation tests (see appendix 5.1) of SSVEP power against lower alpha power and SSVEP power against higher alpha power, were performed.
S004 $\alpha_F = 4.54 \cdot 10^{-4}$

Ratios

S005 $\alpha_F = 4.54 \cdot 10^{-3}$

Ratios

S006 $\alpha_F = 4.54 \cdot 10^{-4}$

Ratios

Flicker frequency (Hz)

Figure 6.6: Ratios of mean alpha frequency for each stimulus frequency and individual alpha frequency. Red color denotes statistical significance with $\alpha_F$ level.
Table 6.5: Ratios of mean alpha frequencies and IAFs. Statistically significant data in bold.

<table>
<thead>
<tr>
<th>Frequency</th>
<th>S004</th>
<th></th>
<th>S005</th>
<th></th>
<th>S006</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Ratio (STD)</td>
<td>p-value</td>
<td>Ratio (STD)</td>
<td>p-value</td>
<td>Ratio (STD)</td>
<td>p-value</td>
</tr>
<tr>
<td>5</td>
<td>1.0367 (0.02662)</td>
<td>0.0000</td>
<td>1.0011 (0.02009)</td>
<td>0.3500</td>
<td>1.0155 (0.04718)</td>
<td>0.0366</td>
</tr>
<tr>
<td>7</td>
<td>1.0175 (0.02607)</td>
<td>0.0008</td>
<td>1.0044 (0.01835)</td>
<td>0.0821</td>
<td>1.0399 (0.04178)</td>
<td>0.0000</td>
</tr>
<tr>
<td>9</td>
<td>1.0169 (0.02812)</td>
<td>0.0004</td>
<td>0.9950 (0.01869)</td>
<td>0.0759</td>
<td>1.0207 (0.03185)</td>
<td>0.0001</td>
</tr>
<tr>
<td>10</td>
<td>1.0279 (0.02391)</td>
<td>0.0000</td>
<td>1.0054 (0.01913)</td>
<td>0.0853</td>
<td>1.0227 (0.03508)</td>
<td>0.0010</td>
</tr>
<tr>
<td>11</td>
<td>1.0212 (0.03529)</td>
<td>0.0004</td>
<td>1.0068 (0.01378)</td>
<td>0.0882</td>
<td>1.0248 (0.04012)</td>
<td>0.0006</td>
</tr>
<tr>
<td>12</td>
<td>1.0229 (0.01888)</td>
<td>0.0000</td>
<td>0.9986 (0.01585)</td>
<td>0.2919</td>
<td>1.0176 (0.03693)</td>
<td>0.0261</td>
</tr>
<tr>
<td>13</td>
<td>1.0208 (0.02740)</td>
<td>0.0002</td>
<td>1.0081 (0.01688)</td>
<td>0.0073</td>
<td>1.0055 (0.03148)</td>
<td>0.2022</td>
</tr>
<tr>
<td>14</td>
<td>1.0258 (0.02892)</td>
<td>0.0000</td>
<td><strong>1.0180 (0.01716)</strong></td>
<td><strong>0.0000</strong></td>
<td>1.0229 (0.03684)</td>
<td>0.0009</td>
</tr>
<tr>
<td>15</td>
<td>1.0143 (0.02872)</td>
<td>0.0050</td>
<td>0.9971 (0.01925)</td>
<td>0.2646</td>
<td>0.9866 (0.04060)</td>
<td>0.0236</td>
</tr>
<tr>
<td>16</td>
<td>1.0154 (0.02798)</td>
<td>0.0027</td>
<td>1.0095 (0.01820)</td>
<td>0.0038</td>
<td><strong>1.0300 (0.04258)</strong></td>
<td><strong>0.0004</strong></td>
</tr>
<tr>
<td>17</td>
<td>1.0254 (0.02658)</td>
<td>0.0000</td>
<td><strong>1.0119 (0.01659)</strong></td>
<td><strong>0.0001</strong></td>
<td>0.9943 (0.03538)</td>
<td>0.1543</td>
</tr>
<tr>
<td>18</td>
<td>1.0274 (0.01848)</td>
<td>0.0000</td>
<td>0.9982 (0.01628)</td>
<td>0.4677</td>
<td><strong>1.0354 (0.04544)</strong></td>
<td><strong>0.0001</strong></td>
</tr>
<tr>
<td>19</td>
<td>1.0192 (0.03308)</td>
<td>0.0014</td>
<td>1.0081 (0.01714)</td>
<td>0.0082</td>
<td>0.9956 (0.03655)</td>
<td>0.2646</td>
</tr>
<tr>
<td>20</td>
<td>1.0174 (0.03255)</td>
<td>0.0022</td>
<td><strong>1.0188 (0.02031)</strong></td>
<td><strong>0.0000</strong></td>
<td>1.0216 (0.03021)</td>
<td>0.0002</td>
</tr>
<tr>
<td>21</td>
<td>1.0396 (0.02367)</td>
<td>0.0000</td>
<td><strong>1.0219 (0.01373)</strong></td>
<td><strong>0.0000</strong></td>
<td>1.0157 (0.03233)</td>
<td>0.0117</td>
</tr>
<tr>
<td>22</td>
<td>1.0178 (0.02487)</td>
<td>0.0003</td>
<td>0.9925 (0.01674)</td>
<td>0.0249</td>
<td>1.0186 (0.04499)</td>
<td>0.0147</td>
</tr>
<tr>
<td>23</td>
<td>1.0407 (0.02555)</td>
<td>0.0000</td>
<td><strong>0.9870 (0.01567)</strong></td>
<td><strong>0.0000</strong></td>
<td>1.0097 (0.02912)</td>
<td>0.0225</td>
</tr>
<tr>
<td>24</td>
<td>1.0286 (0.02273)</td>
<td>0.0000</td>
<td><strong>1.0158 (0.01628)</strong></td>
<td><strong>0.0000</strong></td>
<td>1.0003 (0.04047)</td>
<td>0.4919</td>
</tr>
<tr>
<td>25</td>
<td>1.0161 (0.02964)</td>
<td>0.0012</td>
<td>1.0005 (0.02278)</td>
<td>0.4356</td>
<td>0.9987 (0.03318)</td>
<td>0.3576</td>
</tr>
<tr>
<td>26</td>
<td>1.0194 (0.02383)</td>
<td>0.0000</td>
<td>1.0082 (0.01917)</td>
<td>0.0041</td>
<td>1.0180 (0.03986)</td>
<td>0.0104</td>
</tr>
<tr>
<td>27</td>
<td>1.0218 (0.02951)</td>
<td>0.0001</td>
<td>1.0116 (0.02025)</td>
<td>0.0012</td>
<td>1.0176 (0.03171)</td>
<td>0.0007</td>
</tr>
<tr>
<td>28</td>
<td>1.0146 (0.02777)</td>
<td>0.0057</td>
<td>1.0041 (0.01425)</td>
<td>0.1065</td>
<td>1.0159 (0.03586)</td>
<td>0.0068</td>
</tr>
</tbody>
</table>

As seen in table [C], no significant results (at Bonferroni corrected $0.01$ level, i.e. $p \leq 7.14 \cdot 10^{-4}$) were found.

**Correlation between alpha peak frequency and SSVEP power.**

The correlation between alpha peak frequency during the stimulation and SSVEP power was tested.

For each trial SSVEP atoms were chosen (see above) and atoms in both alpha bands. An alpha peak frequency was calculated as a mean frequency of atoms from alpha bands weighted with their amplitudes. SSVEP power was calculated as a sum of amplitudes of corresponding atoms. Table [A] presents correlations for each stimulation frequency (above alpha band). With family-wise significance level at 0.01 (i.e. with single significance corrected to $7.14 \cdot 10^{-4}$) no statistically significant results were found for subject S004.

For subjects S005 and S006 there are statistically significant results at family-wise level 0.01 ($7.14 \cdot 10^{-4}$ with Bonferroni corrections). What is interesting is that all of the significant results are negative. It means that when SSVEP power increases, alpha peak frequency decreases and vice versa. According to Osaka (1984), decrease of alpha peak frequency is observed with the decrease of task difficulty. The results presented above suggest that for the visual cortex SSVEP is not a complex process; in fact, the larger the brain response is, the less complex it seems.
Table 6.6: Frequencies producing statistically significant alpha peak shift when alpha power shift is considered. Actual shift with error in parentheses.

<table>
<thead>
<tr>
<th>S004 Frequency (Hz)</th>
<th>Ratio (STD)</th>
<th>S005 Frequency (Hz)</th>
<th>Ratio (STD)</th>
<th>S006 Frequency (Hz)</th>
<th>Ratio (STD)</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>1.037 (0.027)</td>
<td>15</td>
<td>1.018 (0.017)</td>
<td>7</td>
<td>1.040 (0.048)</td>
</tr>
<tr>
<td>9</td>
<td>1.017 (0.028)</td>
<td>24</td>
<td>1.012 (0.017)</td>
<td>9</td>
<td>1.021 (0.032)</td>
</tr>
<tr>
<td>11</td>
<td>1.021 (0.035)</td>
<td>29</td>
<td>1.019 (0.020)</td>
<td>20</td>
<td>1.030 (0.043)</td>
</tr>
<tr>
<td>12</td>
<td>1.023 (0.019)</td>
<td>30</td>
<td>1.022 (0.014)</td>
<td>25</td>
<td>1.035 (0.045)</td>
</tr>
<tr>
<td>13</td>
<td>1.021 (0.027)</td>
<td>35</td>
<td>1.016 (0.016)</td>
<td>29</td>
<td>1.022 (0.030)</td>
</tr>
<tr>
<td>25</td>
<td>1.027 (0.018)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>30</td>
<td>1.040 (0.024)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>35</td>
<td>1.029 (0.023)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>39</td>
<td>1.019 (0.024)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Alpha peak frequency during the stimulation**

To test how alpha peak shifts during the stimulation, for each epoch a gravity frequency from stimulation period was calculated using methods presented in previous sections. Next, ratios of gravity frequencies and IAF were calculated for each subject. For each stimulation frequency, Wilcoxon test was used to test whether resulting values are different from unity. Figure 6.11 shows ratios for three subjects and table 6.9 shows values. Red color denotes statistically significant values (0.01 family–wise level with Bonferroni corrections). Most of significant results show decrease in alpha peak frequency. This suggests that SSVEP process is less complicated for the subject’s visual cortex than observing dimmed room in relaxed state. During the stimulation the light is so bright that nothing else can be seen. During relaxed state between flickering, subject can recognize his surroundings. As above results reflect the behavior of primary visual cortex, that is responsible for basic pattern and movement recognition, it is possible that this decrease is not caused by SSVEP per se, but by a lack of visual objects to process during the stimulation.

**Time dependent alpha frequency during the stimulation**

The time course of alpha peak frequency during the stimulus was tested. The SSVEP profile was acquired by first filtering each signal with peak filter at stimulation frequency, and then taking the envelope of the result. Then the median of signal parts corresponding to the stimulus was calculated. The alpha peak frequency was calculated from averaged time–frequency map as described in [24], but without transforming the mean from $\hat{f}$ to $\tilde{f}$. Figures 6.12, 6.13 and 6.14 present calculated courses.

In all cases there is a clearly visible drop of alpha frequency at the beginning of each stimulation. This is due to evoked potential caused by appearing light. It is also visible at the end of the stimulus, but with much smaller amplitude.

The Kendall correlation coefficient of SSVEP power time course and alpha peak frequency time course (filtered with running average filter of 0.42s.) was calculated. To remove the evoked potential artifact, correlation was made for time between 0.5 second after beginning of the stimulus and to the end of the stimulus. The results are presented in 6.10 table.

Most of results of subjects S005 and S006 show that correlation between SSVEP power and alpha peak frequency is negative along the time. Again it suggests that SSVEP is not a complex...
### Table 6.7: Kendall correlation coefficients between SSVEP power and alpha power in two bands for stimulation frequencies \( f \) above 15Hz. P-values in parentheses. With family-wise alpha of 0.01 and Bonferroni corrections (14 tests), single alpha level equals \( \alpha = 7.14 \cdot 10^{-4} \). Statistically significant data in bold.

<table>
<thead>
<tr>
<th>( f (\text{Hz}) )</th>
<th>S004 High alpha</th>
<th>S004 Low alpha</th>
<th>S005 High alpha</th>
<th>S005 Low alpha</th>
<th>S006 High alpha</th>
<th>S006 Low alpha</th>
</tr>
</thead>
<tbody>
<tr>
<td>19</td>
<td>0.1724 (0.1884)</td>
<td>0.0851 (0.5239)</td>
<td>-0.0023 (1.0000)</td>
<td>-0.1126 (0.3950)</td>
<td>0.2092 (0.1088)</td>
<td>0.0897 (0.5011)</td>
</tr>
<tr>
<td>20</td>
<td>0.1770 (0.1766)</td>
<td>0.0575 (0.6712)</td>
<td>0.0897 (0.5011)</td>
<td>0.1172 (0.3755)</td>
<td>-0.0713 (0.5955)</td>
<td>0.3057 (0.0176)</td>
</tr>
<tr>
<td>24</td>
<td>0.0253 (0.8596)</td>
<td>0.0759 (0.5712)</td>
<td>0.0345 (0.8045)</td>
<td>0.0437 (0.7502)</td>
<td>-0.0115 (0.9436)</td>
<td>0.2414 (0.0631)</td>
</tr>
<tr>
<td>25</td>
<td>0.1770 (0.1766)</td>
<td>0.1494 (0.2560)</td>
<td>-0.2414 (0.0631)</td>
<td>0.0345 (0.8045)</td>
<td>0.2920 (0.0236)</td>
<td>0.1954 (0.1348)</td>
</tr>
<tr>
<td>27</td>
<td>-0.1770 (0.1766)</td>
<td>-0.0529 (0.6972)</td>
<td>-0.1862 (0.1547)</td>
<td>0.1908 (0.1445)</td>
<td>-0.0023 (1.0000)</td>
<td>0.0621 (0.6455)</td>
</tr>
<tr>
<td>29</td>
<td>-0.0575 (0.6712)</td>
<td>0.0943 (0.4788)</td>
<td>-0.0483 (0.7235)</td>
<td>-0.1034 (0.4358)</td>
<td>0.0621 (0.6455)</td>
<td>0.0299 (0.8320)</td>
</tr>
<tr>
<td>30</td>
<td>-0.3396 (0.0105)</td>
<td>0.0613 (0.6545)</td>
<td>0.0851 (0.5239)</td>
<td>0.1310 (0.3207)</td>
<td>0.1586 (0.2272)</td>
<td>0.0759 (0.5712)</td>
</tr>
<tr>
<td>32</td>
<td>0.0023 (1.0000)</td>
<td>0.1770 (0.1766)</td>
<td>-0.1724 (0.1884)</td>
<td>-0.0299 (0.8320)</td>
<td>0.2828 (0.0286)</td>
<td>0.0667 (0.6203)</td>
</tr>
<tr>
<td>34</td>
<td>0.1402 (0.2871)</td>
<td>0.1540 (0.2413)</td>
<td>0.0805 (0.5473)</td>
<td>0.1310 (0.3207)</td>
<td>-0.0483 (0.7235)</td>
<td>0.1080 (0.4151)</td>
</tr>
<tr>
<td>35</td>
<td>-0.1264 (0.3384)</td>
<td>0.0069 (0.9718)</td>
<td>0.1172 (0.3755)</td>
<td>0.0805 (0.5473)</td>
<td>0.2598 (0.0450)</td>
<td>0.0483 (0.7235)</td>
</tr>
<tr>
<td>37</td>
<td>0.0805 (0.5473)</td>
<td>-0.0897 (0.5011)</td>
<td>0.2598 (0.0450)</td>
<td>-0.2046 (0.1170)</td>
<td>-0.0391 (0.7772)</td>
<td>0.0529 (0.6972)</td>
</tr>
<tr>
<td>39</td>
<td>0.1080 (0.4151)</td>
<td>0.2322 (0.0742)</td>
<td>0.0621 (0.6455)</td>
<td>0.2230 (0.0868)</td>
<td>0.0437 (0.7502)</td>
<td>0.1356 (0.3036)</td>
</tr>
<tr>
<td>40</td>
<td>0.0049 (0.9852)</td>
<td>0.3251 (0.0131)</td>
<td>-0.1816 (0.1654)</td>
<td>0.0391 (0.7772)</td>
<td>0.3747 (0.0033)</td>
<td>-0.1218 (0.3567)</td>
</tr>
<tr>
<td>45</td>
<td>0.1084 (0.4233)</td>
<td>0.2857 (0.0301)</td>
<td>-0.0069 (0.9718)</td>
<td>0.2184 (0.0937)</td>
<td>0.1770 (0.1766)</td>
<td>0.2736 (0.0344)</td>
</tr>
</tbody>
</table>

process; what is more, it seems that in most of the cases, SSVEP favors the decrease of alpha peak. On the other hand some of statistically significant data from subject S006 and most of the data from subject S004 show positive correlation between SSVEP power and alpha peak frequency course during the stimulus. As seen on figure 6.12, some of this results can be explained by a poor SSVEP response in subjects brain (see frequencies 34Hz, 35Hz, 37Hz). In those cases visual cortex is excited by a light, but no apparent response is acquired. With no SSVEP, alpha peak increases, as the presence of the stimulus increases the difficulty of task (i.e. processing visual data). The positive correlation of remaining cases suggests that with some special frequencies it is possible to enhance the alpha peak frequency by a flickering light.
Table 6.8: Correlation between SSVEP power and alpha peak frequency. p–values in parentheses. Statistically significant data in bold.

<table>
<thead>
<tr>
<th>Frequency (Hz)</th>
<th>S004</th>
<th>S005</th>
<th>S006</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Ratio (STD)</td>
<td>p–value</td>
<td>Ratio (STD)</td>
</tr>
<tr>
<td>19</td>
<td>-0.1494 (0.2560)</td>
<td>0.0001</td>
<td>0.2192 (0.0136)</td>
</tr>
<tr>
<td>20</td>
<td>-0.1218 (0.3567)</td>
<td>0.0000</td>
<td>-0.0859 (0.3355)</td>
</tr>
<tr>
<td>24</td>
<td>0.0851 (0.5239)</td>
<td>0.0000</td>
<td>-0.1130 (0.2044)</td>
</tr>
<tr>
<td>25</td>
<td>-0.0621 (0.6455)</td>
<td>0.0000</td>
<td>-0.3763 (0.0000)</td>
</tr>
<tr>
<td>27</td>
<td>-0.1126 (0.3950)</td>
<td>0.0000</td>
<td>-0.3051 (0.0006)</td>
</tr>
<tr>
<td>29</td>
<td>-0.0437 (0.7502)</td>
<td>0.0000</td>
<td>-0.1537 (0.0839)</td>
</tr>
<tr>
<td>30</td>
<td>0.0613 (0.6545)</td>
<td>0.0000</td>
<td>-0.2825 (0.0016)</td>
</tr>
<tr>
<td>32</td>
<td>0.0299 (0.8320)</td>
<td>0.0000</td>
<td>-0.3695 (0.0000)</td>
</tr>
<tr>
<td>34</td>
<td>0.2092 (0.1088)</td>
<td>0.0000</td>
<td>-0.2113 (0.0174)</td>
</tr>
<tr>
<td>35</td>
<td>-0.0989 (0.4570)</td>
<td>0.0000</td>
<td>-0.2881 (0.0012)</td>
</tr>
<tr>
<td>37</td>
<td>-0.2230 (0.0868)</td>
<td>0.0000</td>
<td>-0.5062 (0.0000)</td>
</tr>
<tr>
<td>39</td>
<td>-0.2874 (0.0260)</td>
<td>0.0000</td>
<td>-0.3582 (0.0001)</td>
</tr>
<tr>
<td>40</td>
<td>-0.0640 (0.6420)</td>
<td>0.0000</td>
<td>-0.1315 (0.1430)</td>
</tr>
<tr>
<td>45</td>
<td>0.2808 (0.0332)</td>
<td>0.0000</td>
<td>0.0427 (0.6378)</td>
</tr>
</tbody>
</table>

Table 6.9: The ratios of alpha peak frequency during the stimulation and IAF with p–values acquired by Wilcoxon test for each stimulation frequency $f$. Standard deviation in parentheses. Statistically significant data in bold.

<table>
<thead>
<tr>
<th>$f$(Hz)</th>
<th>S004 (STD)</th>
<th>p–value</th>
<th>S005 (STD)</th>
<th>p–value</th>
<th>S006 (STD)</th>
<th>p–value</th>
</tr>
</thead>
<tbody>
<tr>
<td>19</td>
<td>0.9652 (0.0448)</td>
<td>0.0001</td>
<td>0.9512 (0.0352)</td>
<td>0.0000</td>
<td>1.0018 (0.0360)</td>
<td>0.3805</td>
</tr>
<tr>
<td>20</td>
<td>0.9505 (0.0469)</td>
<td>0.0000</td>
<td>0.9999 (0.0325)</td>
<td>0.4118</td>
<td>1.0155 (0.0466)</td>
<td>0.0547</td>
</tr>
<tr>
<td>24</td>
<td>0.9614 (0.0354)</td>
<td>0.0000</td>
<td>0.9643 (0.0372)</td>
<td>0.0000</td>
<td>0.9850 (0.0456)</td>
<td>0.0402</td>
</tr>
<tr>
<td>25</td>
<td>0.9798 (0.0331)</td>
<td>0.0011</td>
<td>0.9558 (0.0229)</td>
<td>0.0000</td>
<td>1.0341 (0.0403)</td>
<td>0.0001</td>
</tr>
<tr>
<td>27</td>
<td>0.9771 (0.0277)</td>
<td>0.0001</td>
<td>0.9962 (0.0331)</td>
<td>0.0001</td>
<td>0.9932 (0.0451)</td>
<td>0.1799</td>
</tr>
<tr>
<td>29</td>
<td>0.9622 (0.0376)</td>
<td>0.0000</td>
<td>0.9705 (0.0284)</td>
<td>0.0000</td>
<td>1.0106 (0.0395)</td>
<td>0.0790</td>
</tr>
<tr>
<td>30</td>
<td>1.0007 (0.0336)</td>
<td>0.4596</td>
<td>0.9653 (0.0360)</td>
<td>0.0000</td>
<td>1.0149 (0.0426)</td>
<td>0.0420</td>
</tr>
<tr>
<td>32</td>
<td>0.9701 (0.0360)</td>
<td>0.0000</td>
<td>0.9463 (0.0319)</td>
<td>0.0000</td>
<td>1.0065 (0.0393)</td>
<td>0.2919</td>
</tr>
<tr>
<td>34</td>
<td>0.9867 (0.0396)</td>
<td>0.0303</td>
<td>0.9386 (0.0387)</td>
<td>0.0000</td>
<td>1.0013 (0.0327)</td>
<td>0.4356</td>
</tr>
<tr>
<td>35</td>
<td>0.9841 (0.0311)</td>
<td>0.0060</td>
<td>0.9757 (0.0429)</td>
<td>0.0019</td>
<td>1.0193 (0.0337)</td>
<td>0.0050</td>
</tr>
<tr>
<td>37</td>
<td>0.9966 (0.0278)</td>
<td>0.3204</td>
<td>0.9461 (0.0402)</td>
<td>0.0000</td>
<td>1.0073 (0.0402)</td>
<td>0.1965</td>
</tr>
<tr>
<td>39</td>
<td>0.9758 (0.0345)</td>
<td>0.0008</td>
<td>0.9614 (0.0311)</td>
<td>0.0000</td>
<td>1.0087 (0.0387)</td>
<td>0.1447</td>
</tr>
<tr>
<td>40</td>
<td>0.9995 (0.0346)</td>
<td>0.4758</td>
<td>0.9781 (0.0482)</td>
<td>0.0041</td>
<td>0.9930 (0.0370)</td>
<td>0.1854</td>
</tr>
<tr>
<td>45</td>
<td>0.9928 (0.0446)</td>
<td>0.1355</td>
<td>0.9490 (0.0270)</td>
<td>0.0000</td>
<td>1.0190 (0.0407)</td>
<td>0.0164</td>
</tr>
</tbody>
</table>
Figure 6.7: The behavior of gravity frequency for 5 seconds after the stimulus in correspondence to IAF. Data for statistically significant stimulation frequencies of subject S004. Red color denotes parts with statistically significant change of peak when compared to IAF with family-wise alpha at 0.05. A single case significance is set by FDR algorithm (see appendix B).
Figure 6.8: The behavior of gravity frequency for 5 seconds after the stimulus in correspondence to IAF. Data for statistically significant stimulation frequencies of subject S005. Red color denotes parts with statistically significant change of peak when compared to IAF with family-wise alpha at 0.05. A single case significance is set by FDR algorithm (see appendix [4]).
Figure 6.9: The behavior of gravity frequency for 5 seconds after the stimulus in correspondence to IAF. Data for statistically significant stimulation frequencies of subject S006. Red color denotes parts with statistically significant change of peak when compared to IAF with family-wise alpha at 0.05. A single case significance is set by FDR algorithm (see appendix [4]).
Figure 6.10: An epoch decomposed with MP algorithm. For determining correlations between SSVEP power and alpha power three groups of atoms were chosen: first group consists of SSVEP atoms, second and third of higher and lower alpha atoms.

<table>
<thead>
<tr>
<th>Frequency (Hz)</th>
<th>S004</th>
<th>S005</th>
<th>S006</th>
</tr>
</thead>
<tbody>
<tr>
<td>19</td>
<td>-0.4472 ( 0.0000)</td>
<td>0.1913 ( 0.0000)</td>
<td>-0.4406 ( 0.0000)</td>
</tr>
<tr>
<td>20</td>
<td>0.7036 ( 0.0000)</td>
<td>-0.0908 ( 0.0000)</td>
<td>-0.0068 ( 0.7602)</td>
</tr>
<tr>
<td>24</td>
<td>0.2996 ( 0.0000)</td>
<td>-0.2119 ( 0.0000)</td>
<td>-0.2469 ( 0.0000)</td>
</tr>
<tr>
<td>25</td>
<td>0.2194 ( 0.0000)</td>
<td>-0.0675 ( 0.0025)</td>
<td>-0.0303 ( 0.1743)</td>
</tr>
<tr>
<td>27</td>
<td>0.2778 ( 0.0000)</td>
<td>-0.6216 ( 0.0000)</td>
<td>-0.2096 ( 0.0000)</td>
</tr>
<tr>
<td>29</td>
<td>0.2961 ( 0.0000)</td>
<td>-0.5995 ( 0.0000)</td>
<td>0.1702 ( 0.0000)</td>
</tr>
<tr>
<td>30</td>
<td>0.4311 ( 0.0000)</td>
<td>-0.3587 ( 0.0000)</td>
<td>0.1876 ( 0.0000)</td>
</tr>
<tr>
<td>32</td>
<td>0.1158 ( 0.0000)</td>
<td>-0.5968 ( 0.0000)</td>
<td>0.2531 ( 0.0000)</td>
</tr>
<tr>
<td>34</td>
<td>0.2156 ( 0.0000)</td>
<td>-0.6934 ( 0.0000)</td>
<td>-0.3865 ( 0.0000)</td>
</tr>
<tr>
<td>35</td>
<td>0.3279 ( 0.0000)</td>
<td>-0.1085 ( 0.0000)</td>
<td>-0.2805 ( 0.0000)</td>
</tr>
<tr>
<td>37</td>
<td>0.3684 ( 0.0000)</td>
<td>0.1225 ( 0.0000)</td>
<td>-0.1438 ( 0.0000)</td>
</tr>
<tr>
<td>39</td>
<td>-0.1924 ( 0.0000)</td>
<td>-0.3735 ( 0.0000)</td>
<td>0.5627 ( 0.0000)</td>
</tr>
<tr>
<td>40</td>
<td>0.0118 ( 0.5968)</td>
<td>-0.4406 ( 0.0000)</td>
<td>-0.4151 ( 0.0000)</td>
</tr>
<tr>
<td>45</td>
<td>-0.1578 ( 0.0000)</td>
<td>-0.1092 ( 0.0000)</td>
<td>-0.3172 ( 0.0000)</td>
</tr>
</tbody>
</table>

Table 6.10: Kendall correlation between SSVEP power time course and alpha peak frequency time course. p–values in parentheses. Statistically significant data in bold.
Figure 6.11: Ratios of alpha peak frequency calculated during stimulation and individual alpha frequency. Data for subjects S004, S005, and S006. Red color denotes values that are statistically significant at 0.01 family-wise level (7.14 · 10^{-4} with Bonferroni corrections).
Figure 6.12: The time course of alpha peak frequency (blue) and the running mean of alpha peak frequency (of 0.42s) (red). Vertical lines denote the beginning and end of the stimulus. Green line denotes the shape of SSVEP power course acquired by peak filter (in normalized units). Data for subject S004.
Figure 6.13: The time course of alpha peak frequency (blue) and the running mean of alpha peak frequency (of 0.42s) (red). Vertical lines denote the beginning and end of the stimulus. Green line denotes the shape of SSVEP power course acquired by peak filter (in normalized units). Data for subject S005.
Figure 6.14: The time course of alpha peak frequency (blue) and the running mean of alpha peak frequency (of 0.42s) (red). Vertical lines denote the beginning and end of the stimulus. Green line denotes the shape of SSVEP power course acquired by peak filter (in normalized units). Data for subject S006.
3 Infra frequencies

3.1 Individual alpha frequency determination

Individual alpha frequency for each subject was determined \textit{a posteriori}, in a similar way as described in section 2.1. The signal for every stimulation frequency, for Oz electrode (reference Cz), was divided into 10 epochs, each containing 13 seconds before the beginning of the stimulation, 15 seconds after the end of the stimulation and 60 seconds of the stimulation itself. Epochs were then decomposed with MP algorithm (see section 3). Atoms, that frequency position was between 7 and 15Hz and time position was between 3 and 6 seconds before the stimulation, were chosen for the IAF calculation. As previously, for each epoch a gravity frequency was calculated; for every stimulation frequency a mean gravity center \( \hat{f} \), with variance \( \sigma^2 \), was calculated. The IAF was estimated as a mean of each \( \hat{f} \) weighted by the variance.

Calculated values and estimates of errors are presented in table 6.11 Comparing these values with those from table 6.1 shows a good coherence between two sets of data. Larger error estimates are due to smaller number of trials and stimulation frequencies.

3.2 Ratio of alpha power before and after the stimulation

Signals acquired from electrode Oz (reference Cz) were filtered in two bands: lower alpha (between \( IAF - 4Hz \) and \( IAF \)) and higher alpha (between \( IAF \) and \( IAF + 2Hz \)). Again, envelopes were calculated and for each trial \( i \) a maximal alpha power from time interval in between 3 and 8, \( B_i \), seconds before stimulation and 0.1 and 5 seconds after stimulation, \( A_i \) was found. Then, for each frequency, a Wilcoxon test was performed comparing ratios \( B_i/A_i \) with unity.

Tables 6.12, 6.13, 6.14 and figure 6.15, 6.15b, 6.15c present the each frequency median of ratios for three subjects. Red color denotes data that is statistically significant at family–wise level 0.01 (Bonferroni corrected 0.002 level).

<table>
<thead>
<tr>
<th>Subject</th>
<th>S003</th>
<th>S004</th>
<th>S005</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frequency</td>
<td>10.54</td>
<td>11.09</td>
<td>10.43</td>
</tr>
<tr>
<td>Error</td>
<td>0.34</td>
<td>0.32</td>
<td>0.19</td>
</tr>
</tbody>
</table>

Table 6.11: Calculated IAFs for infra–frequency stimulations

<table>
<thead>
<tr>
<th>Frequency(Hz)</th>
<th>Higher alpha</th>
<th>Lower alpha</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Ratio (std)</td>
<td>p–value</td>
</tr>
<tr>
<td>0.1</td>
<td>1.4824 (1.2180)</td>
<td>0.0020</td>
</tr>
<tr>
<td>0.2</td>
<td>1.6282 (1.3785)</td>
<td>0.0078</td>
</tr>
<tr>
<td>0.3</td>
<td>1.0485 (1.6989)</td>
<td>0.1113</td>
</tr>
<tr>
<td>0.4</td>
<td>0.8128 (0.9102)</td>
<td>0.2461</td>
</tr>
<tr>
<td>0.5</td>
<td>0.9470 (0.5860)</td>
<td>0.3340</td>
</tr>
</tbody>
</table>

Table 6.12: Ratios of maximal alpha power before to after the stimulation. Data for subject S003. Statistically significant data in bold

The significant change in alpha power is present in both bands. That suggests an increase in the power of the whole alpha band after the stimulation.
Figure 6.15: Median ratios of maximal alpha power, in a period of 5 seconds, after the stimuli to maximal alpha power, in the period of 5 seconds, before the warning signal with standard deviations. Signal filtered in two bands: higher alpha (from $IAF$ to $IAF + 2Hz$) and lower alpha (between $IAF - 4Hz$ and $IAF$). Red color denotes statistical significance on family–wise $\alpha_F = 0.01$ level.

3.3 Alpha peak shift

From each epoch, decomposed with MP algorithm, atoms that position in time was in between 5 seconds after the end of the stimulus and frequency position was in between 7 and 15Hz, lasting for more than 0.1 second, were chosen. The gravity frequency was calculated and averaged over trials. A ratio of acquired mean frequency and IAF was calculated for each stimulation frequency. Wilcoxon test was performed, comparing ratios to unity. The results are shown in tables 6.15, 6.16 and 6.17 and on figures 6.16, 6.16b and 6.16c. The family–wise significance level was 0.01 (Bonferroni corrected 0.002 level).

The transition between stimulation and no–light period is more gentle than in the SSVEP part, though, due to the fact that used frequencies are much smaller and instead of rapid end there is a slow fade. Because of that no after images are present and the adjustment to dimmed surroundings is quicker. However the shift in alpha frequency, considered to be correlated to
signal processing in cerebral cortex (see section 1.1), is still present. This might suggest, that beside of adjusting optical sensors to new environment, there are residual processes connected to stimulation itself that rise alpha frequency.

### 3.4 Time dependent alpha shift after the stimulation

The procedure was described in section 2.4. Investigated was time period in between 10 second after the end of the stimulus. Figures 6.17, 6.18 and 6.19 show the time course of alpha frequency change.

Although most of the results are not statistically significant, the maximal change occurs during first three seconds after the end of the stimulus.

### 3.5 Alpha peak frequency shift during the stimulation

From each epoch atoms that time position was between 13 and 73 seconds and frequency position was between 7 and 15Hz, and lasted for more than 0.1 second, were chosen. A gravity frequency was calculated and averaged over trials. The ratio of the mean gravity frequency and IAF was calculated; Wilcoxon test was performed to comparing the ration to unity. Tables 6.18, 6.19 and 6.20 and figures 6.20, 6.20b and 6.20c show the results.

As expected from experiments performed by other researchers (Osaka, 1984; Osaka et al., 1999), every significant result show that alpha frequency increases.

### 3.6 Correlation between the gravity alpha frequency and stimulation signal

The mean alpha gravity frequency time course was acquired as described above. A Kendall correlation coefficient was calculated between the alpha peak frequency and the stimulation signal.
Figure 6.16: Ratios of mean gravity frequency 5 seconds after the end of the stimulus and IAF. Red color denotes statistical significance at 0.01 family-wise level (0.002 with bonferroni corrections).

signal in time. Table 6.21 presents results with corresponding p-values. With the family-wise error set to 0.01, after Bonferroni corrections single case level was 0.002, there are significant results, each denoting positive correlation. This means that, for very low frequencies, alpha frequency can react on each light modulation. This can be used to determine the reaction time of the cortex.
Figure 6.17: Time course of alpha frequency shift for subject S003. Red color denotes statistical significance at 0.01 family-wise level (with FDR correction).

Figure 6.18: Time course of alpha frequency shift for subject S004. Red color denotes statistical significance at 0.01 family-wise level (with FDR correction).
### Table 6.15: Alpha gravity frequency ratio. Subject 003. Statistically significant data in bold.

<table>
<thead>
<tr>
<th>Frequency</th>
<th>Ratio (STD)</th>
<th>p–value</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1</td>
<td>0.9892 (0.0475)</td>
<td>0.1494</td>
</tr>
<tr>
<td>0.2</td>
<td>1.0251 (0.0611)</td>
<td>0.0557</td>
</tr>
<tr>
<td>0.3</td>
<td>1.0251 (0.0557)</td>
<td>0.0186</td>
</tr>
<tr>
<td>0.4</td>
<td>1.0626 (0.0832)</td>
<td>0.0059</td>
</tr>
<tr>
<td>0.5</td>
<td>1.0181 (0.0607)</td>
<td>0.1113</td>
</tr>
</tbody>
</table>

### Table 6.16: Alpha gravity frequency ratio. Subject 004. Statistically significant data in bold.

<table>
<thead>
<tr>
<th>Frequency</th>
<th>Ratio (STD)</th>
<th>p–value</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1</td>
<td>1.0343 (0.0591)</td>
<td>0.0186</td>
</tr>
<tr>
<td>0.2</td>
<td>1.0371 (0.0569)</td>
<td>0.0234</td>
</tr>
<tr>
<td>0.3</td>
<td><strong>1.0488 (0.0394)</strong></td>
<td><strong>0.0010</strong></td>
</tr>
<tr>
<td>0.4</td>
<td>1.0193 (0.0696)</td>
<td>0.0674</td>
</tr>
<tr>
<td>0.5</td>
<td>1.0359 (0.0504)</td>
<td>0.0107</td>
</tr>
</tbody>
</table>

### Table 6.17: Alpha gravity frequency ratio. Subject 005. Statistically significant data in bold.

<table>
<thead>
<tr>
<th>Frequency</th>
<th>Ratio (STD)</th>
<th>p–value</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1</td>
<td><strong>1.0532 (0.0568)</strong></td>
<td><strong>0.0010</strong></td>
</tr>
<tr>
<td>0.2</td>
<td><strong>1.0312 (0.0251)</strong></td>
<td><strong>0.0010</strong></td>
</tr>
<tr>
<td>0.3</td>
<td><strong>1.0349 (0.0341)</strong></td>
<td><strong>0.0020</strong></td>
</tr>
<tr>
<td>0.4</td>
<td><strong>1.0291 (0.0312)</strong></td>
<td><strong>0.0010</strong></td>
</tr>
<tr>
<td>0.5</td>
<td>1.0256 (0.0311)</td>
<td>0.0059</td>
</tr>
</tbody>
</table>

Figure 6.19: Time course of alpha frequency shift for subject S005. Red color denotes statistical significance at 0.01 family-wise level (with FDR correction).
Figure 6.20: Ratios of mean gravity frequency during the stimulation and IAF. Red color denotes statistical significance at 0.01 family-wise level (0.002 with Bonferroni corrections).

Table 6.18: Alpha gravity frequency ratio – during the stimulation. Subject 003. Statistically significant data in bold.
<table>
<thead>
<tr>
<th>Frequency</th>
<th>Ratio (STD)</th>
<th>p–value</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1</td>
<td><strong>1.0127 (0.0310)</strong></td>
<td><strong>0.0020</strong></td>
</tr>
<tr>
<td>0.2</td>
<td>1.0092 (0.0315)</td>
<td>0.0107</td>
</tr>
<tr>
<td>0.3</td>
<td>1.0044 (0.0327)</td>
<td>0.1113</td>
</tr>
<tr>
<td>0.4</td>
<td><strong>1.0205 (0.0334)</strong></td>
<td><strong>0.0020</strong></td>
</tr>
<tr>
<td>0.5</td>
<td>1.0049 (0.0320)</td>
<td>0.0674</td>
</tr>
</tbody>
</table>

Table 6.19: Alpha gravity frequency ratio – during the stimulation. Subject 004. Statistically significant data in bold.

<table>
<thead>
<tr>
<th>Frequency</th>
<th>Ratio (STD)</th>
<th>p–value</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1</td>
<td>0.9975 (0.0237)</td>
<td>0.2461</td>
</tr>
<tr>
<td>0.2</td>
<td><strong>0.9765 (0.0236)</strong></td>
<td><strong>0.0010</strong></td>
</tr>
<tr>
<td>0.3</td>
<td><strong>0.9733 (0.0225)</strong></td>
<td><strong>0.0020</strong></td>
</tr>
<tr>
<td>0.4</td>
<td>0.9865 (0.0257)</td>
<td>0.0234</td>
</tr>
<tr>
<td>0.5</td>
<td><strong>0.9792 (0.0209)</strong></td>
<td><strong>0.0010</strong></td>
</tr>
</tbody>
</table>

Table 6.20: Alpha gravity frequency ratio – during the stimulation. Subject 005. Statistically significant data in bold.

<table>
<thead>
<tr>
<th>Frequency</th>
<th>0 S003</th>
<th>S004</th>
<th>S005</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Kendall coefficient</td>
<td>p–value</td>
<td>Kendall coefficient</td>
</tr>
<tr>
<td>0.1</td>
<td>-0.0132</td>
<td>0.6055</td>
<td><strong>0.0820</strong></td>
</tr>
<tr>
<td>0.2</td>
<td>-0.0164</td>
<td>0.5208</td>
<td><strong>0.1304</strong></td>
</tr>
<tr>
<td>0.3</td>
<td>0.0459</td>
<td>0.0712</td>
<td>0.0558</td>
</tr>
<tr>
<td>0.4</td>
<td>0.0104</td>
<td>0.6839</td>
<td>-0.0107</td>
</tr>
<tr>
<td>0.5</td>
<td>0.0484</td>
<td>0.0573</td>
<td>-0.0116</td>
</tr>
</tbody>
</table>

Table 6.21: Kendall correlation coefficients between alpha gravity frequency during the stimulation and stimulation signal. Statistically significant data in bold.
Section 7

Discussion

1 Comparing results

Presented results differ from subject to subject and from one stimulation frequency to another. As this is common to the EEG recordings and analysis, it makes drawing conclusions a difficult task. Gathered data, however, show some results are common to all subjects.

It is possible to manipulate alpha peak frequency via repetitive light stimulation. The highest increase, with respect to IAF, is observed with infra–frequencies after the stimulation. The best responses acquired with SSVEP frequencies were in between 20 and 35Hz (also after the end of the stimulation).

Alpha peak frequency changes during the stimulation and few seconds after, when compared to IAF. Many researches (e.g. Osaka [1984, Osaka et al. 1999]) showed that the change is positively correlated to the difficulty of task; gathered results show, however, that during the SSVEP alpha frequency tends to decrease. Calculated correlations between the alpha frequency and the power of the SSVEP or SSVEP envelope amplitude present a negative dependence. This suggests that, in fact, the SSVEP is less complex for the visual cortex, than sitting in relaxed state in a dimmed room. The results acquired by repetitive light stimulation at infra–frequencies (where there is no SSVEP response) are not that unambiguous, although ratios are lower than one acquired after the end of the stimulus. Alpha peak frequency measured in that conditions increases for subject S004 but decreases for subject S005; there are also some significant positive correlations between stimulation signal and alpha peak frequency for subject participating in both infra– and SSVEP frequency sessions.

The alpha behavior is different after the stimulation. Results show an increase of the peak frequency in both infra– and SSVEP frequency range when compared to IAF. The maximal change is in first three seconds after the end of the stimulus. Various effects might cause this phenomenon. At SSVEP frequencies the transition between light and no–light was quite drastic: at the end of the stimulus an ERP is visible in time–frequency; also after when the stimulus ended sometimes afterimages appeared. Although during the infra– sessions those effects were not present, the adjustment of the brain from light to dimmed conditions could have increased the alpha peak frequency. In both cases the magnitude of shift is similar (mean shift for subject S004 is 1.0351 ± 0.0105 for infra–frequencies and 1.0231 ± 0.0078 for SSVEP frequencies; for subject S005: 1.0348 ± 0.003 and 1.0056 ± 0.0083).
The matching pursuit algorithm proved to be very useful in EEG analysis. Due to its good time–frequency resolution it was possible to calculate the time course of alpha peak frequency shift during and after the stimulus without the necessity of choosing windows. Using calculated atoms from decomposition it was possible to estimate the peak frequency exactly within specified bands. In Fourier transform, for example, there is always a possibility of influences from outside of desired frequencies.

2 Further work

There are still many unclear aspects of alpha dynamics during repetitive light stimulation. The reasons of the increase of alpha peak frequency is not certain. The question remains whether the this effect was caused by the experiment conditions, i.e the transition between the bright and dark surroundings, or was it caused by the stimulation itself. There is a need of determining the influence of ERP and afterimages on alpha peak shift.

The experiment conditions might have also caused the decrease of alpha peak shift during the stimulation.

The correlation between stimulation signal at infra–frequencies and the time course of alpha peak frequency might provide some information about the reaction time of the brain. The significant correlation shows that the modulation of the stimulus is slow enough to be perceived as a set of single (or almost single) flashes. That way the stimulation frequency would reflect the reaction speed.

Finally, the connection between decrease of alpha peak frequency during the stimulation and relaxation should be investigated following the observation made by Klimesch (1999a), Cajochen et al. (1996) regarding the decrease of alpha power during the hypnagagic state.
References


ANOKHIN, ANDREY, & VOGEL, FRIEDRICH. 1996. EEG alpha rhythm frequency and intelligence in normal adults. Intelligence, 23, 1–14.


Appendix A

Analytical signal and an amplitude envelope

A known property of a real signal \( s(t) \) is that its Fourier transform \( F_s(\omega) \) is symmetrical i.e.

\[
F_s(\omega) = F_s(-\omega)
\]  
(A.1)

This means that negative components of a signal \( s(t) \) spectrum are, in fact, superfluous. A corresponding function \( z_s(t) \) that satisfies:

\[
F_z(\omega) = F_s(\omega) + \text{sign}(\omega) F_s(\omega)
\]  
(A.2)

has the same power as \( s(t) \) but is complex. \( z_s \) is called an analytic representation of a real signal \( s(t) \).

An explicit form of \( z_s \) can be acquired by an inverse Fourier transform of equation (A.2):

\[
(z_s(t)) = F^{-1}(F_s(\omega) + \text{sign}(\omega) F_s(\omega))
\]  
(A.3)

\[
(z_s(t)) = F^{-1}(F_s(\omega)) + F^{-1}(\text{sign}(\omega) F_s(\omega))
\]  
(A.4)

\[
(z_s(t)) = s(t) + F^{-1}(\text{sign}(\omega) F_s(\omega))
\]  
(A.5)

where \( F^{-1} \) denotes inversed Fourier transform.

The convolution theorem states that:

\[
f(t) = (g \ast h)(t) \iff F_f(\omega) = F_g(\omega) F_h(\omega)
\]  
(A.6)

That leads to:

\[
F^{-1}(\text{sign}(\omega) F_s(\omega)) = F^{-1}(\text{sign}(\omega)) \ast F^{-1}(F_s(\omega))
\]  
(A.7)

\[
F^{-1}(\text{sign}(\omega)) \ast F^{-1}(F_s(\omega)) = F^{-1}(\text{sign}(\omega)) \ast s(t)
\]  
(A.8)

\[
F^{-1}(\text{sign}(\omega)) \ast s(t) = \frac{i}{\pi t} \ast s(t)
\]  
(A.9)

\( z_s \) is then equal:

\[
z_s(t) = s(t) + \frac{1}{\pi t} \ast s(t)
\]  
(A.10)
The imaginary part of equation (A.10) is called Hilbert transform. Explicitly, Hilbert transform is defined as:

\[ H_s(t) = \frac{1}{\pi} P \int_{-\infty}^{\infty} \frac{s(\tau)}{t-\tau} d\tau \]  

(A.11)

where \( P \) denotes Cauchy principal number. In other words, Hilbert transform means that phase of positive part of signal spectrum is shifted by \(-\frac{\pi}{2}\), and negative part - by \(\frac{\pi}{2}\).

Any filtered signal, e.g. EEG filtered in alpha bands, can be written as (Clochon et al., 1996):

\[ EEG(t) = A(t) \cos(\phi(t)) \]  

(A.12)

where \( A(t) \) is a signal envelope.

With an analytic representation of a real signal it is easy to obtain an envelope:

\[ z_s(t) = s(t) + iH_s(t) = A(t)e^{i\phi(t)} \]  

(A.13)

\[ A(t) = |z_s(t)| = \sqrt{s^2(t) + H_s^2(t)} \]  

(A.14)
Appendix B

Statistical testing

In order to verify significance of acquired results, i.e. whether observed effect is larger than it can be expected on the basis of a chance alone, a statistical test is required. The classical, or conventional, tests like a Student’s t-test or ANOVA, might provide a simple answer, but they deal with multiple comparison problem and are bounded by assumptions of data distribution [Burgess & Gruzelier, 1999; Maris & Oostenveld, 2007].

A simple, distribution-free permutation test, based on Karniski et al. (1994), is described below.

1 A permutation test

To determine whether the statistical difference between two conditions, under which the signal was recorded, e.g. a part of a signal before and after the stimulus, before and during the stimulus etc., a null hypothesis is presented:

\( H_0 \) : in a single case, there is no difference between the conditions

A case can be defined differently for each test: it can be a flicker frequency, a subject or a single trial. The null hypothesis leads to \( 2^N \) permutations of the basic data set (where \( N \) denotes number of cases). Indeed, if there is no difference between conditions, replacing, in each case, the value recorded under the first condition, with the one recorded under the second condition should produce a similar data set; \( N \) cases with two possible conditions result in \( 2^N \) permutations.

For each permutation a test statistic is calculated. It can be a two-sample t-value computed between two vectors one consisting of values from first condition across all cases and second consisting of values from second condition across all cases. The two-sample t-value is defined as:

\[
t = \frac{\bar{X}_1 - \bar{X}_2}{S_{X_1X_2} \sqrt{\frac{2}{n}}}
\]

(B.1)

where \( \bar{X}_i \) is mean of \( i^{th} \) vector, \( n \) is the length of vector and \( S_{X_1X_2} \) is a grand standard deviation:

\[
S_{X_1X_2} = \sqrt{\frac{S_{X_1}^2 + S_{X_2}^2}{2}}
\]

(B.2)

where \( S_X \) is a standard deviation of \( i^{th} \) vector.

A simpler test statistic can be a squared difference between means or medians of two vectors.
A p value of the data (significance level) is the number of test statistics greater or equal to the test statistic acquired for not permutated, basic data. It is an exact probability of obtaining such a result if the null hypothesis is true. As seen above, this procedure depends on the number of cases. For example, for three cases significance level can not be smaller than 0.125. The greater the number of cases is, the more accurate the test is.

2 Wilcoxon signed rank test

While permutation test gives an exact probability of the null hypothesis being true, it might cause computational problems. With $N = 30$ cases there is $2^{30} \approx 10^9$ permutations to test. One solution to this problem is using Monte Carlo methods as described in [Ernst 2004]. It is possible to approximate the exact probability $p$ with a probability value $\hat{p}$ acquired with a smaller number of permutations (e.g. $10^6$), randomly chosen from all possibilities. The more permutations is chosen, the closer $\hat{p}$ is to $p$.

A simpler approach, where choosing an appropriate number of permutations is not required, is using Wilcoxon signed-rank test ([Wilcoxon 1945]). This test computes the probability of null hypothesis that states:

$$H_0 : \text{difference between two data sets is symmetrically distributed around 0.}$$

As it was mentioned above recorded data can be divided into two cases - data sets. Each data set consists of $N$ cases. To calculate the value of the test statistic, first a vector of length $N$ is defined as a difference between two conditions. Then, vector is sorted according to the absolute value of differences. Each sorted value is substituted with a rank beginning with 1 for first, nonzero value, and ending at $N$ for the last value. Equal values are substituted by a mean of sum of corresponding ranks.

Two test statistics can be defined. The first is the sum of ranks corresponding to positive differences and the second - the sum of ranks corresponding to negative differences. The final test statistic is the smaller of the two.

For example if columns are conditions $C_1$ and $C_2$ and rows are cases $S_1...S_N$, a simple table can be constructed. Table B.1 an exemplary set of 10 pairs of numbers. In each case first number in pair is a random number drawn from $[20, 50]$ and second - a random number drawn from $[40, 70]$.

The distribution for comparing obtained test statistic can be calculated with exact methods or, for $N > 25$, approximated by a normal distribution.

For $N$ cases, the exact calculations simply determine how many ways there are to acquire each one of $\frac{N(N+1)}{2}$ statistics using different possible ranks. A simple algorithm that produces the distribution is presented in appendix 3.

If the null hypothesis is true each of the statistics is equally probable to appear. For the exemplary set, there are 5 possibilities of producing a statistic equal or more extreme than $S = 3$, and $2^{10}$ outcomes (each of 10 ranks can be added to $S_-$ or $S_+$). The p-value is then $p = \frac{5}{1024} = 0.0049$.

3 Ranks distribution.

As discussed in section 2 obtaining a significance level needs a ranks distribution.

For $N$ cases there are $\frac{N(N+1)}{2} + 1$ statistics possible. Most of these statistics can be obtained on several ways, for example for $N = 4$ statistic 3 just from a single rank 3 or from two ranks 3=2+1. Table B.2 shows on how many ways can statistics be obtained when $N = 6$.
Table B.1: Rank-sum table for exemplary data sets. In \( C_1 \) column are numbers randomly drawn from [20 50] and in \( C_2 \) are numbers randomly drawn from [40 70].

A distribution for \( N \) cases can be derived from a distribution for \( N - 1 \) cases: if \( V_{N-1} \) is distribution vector for \( N - 1 \) cases, than \( V_N \) is equal:

\[
V_N = \begin{pmatrix}
0 \\
\vdots \\
0 \\
V_{N-1}
\end{pmatrix}^T + \begin{pmatrix}
V_{N-1} \\
0 \\
\vdots \\
0
\end{pmatrix}^T
\]  

(B.3)

This is due to the fact that increasing cases number from \( N - 1 \) to \( N \) means that each of previous ranks can be increased by \( N \) (producing distribution vector \((0 \ldots 0 V_{N-1}^T)\)) or left without change (producing distribution vector \((V_{N-1}^T 0 \ldots 0)\)).

Figure [B.1] presents distributions for \( N = 5, 10, 15, 20, 25, 100 \).

<table>
<thead>
<tr>
<th>Statistic</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of occurrences</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>2</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>4</td>
<td>5</td>
<td>5</td>
<td>5</td>
</tr>
</tbody>
</table>

Table B.2: Statistics distribution for \( N = 7 \)

4 Multiple hypothesis testing

Setting significance level at \( \alpha = 0.01 \) means that one out of 100 test is expected to be significant just by a chance alone. Having multiple tests of a data of unknown dependency, for example EEG recordings of a single subject, increases the probability of observing significant results to \( 1 - 0.99^N \), where \( N \) is the number of tests. Indeed, if alpha level is set to \( \alpha = 0.01 \), then a probability of observing a non-significant result is 0.99; with \( N \) tests probability decreases to \( 0.99^N \), thus the probability of observing a significant result is \( 1 - 0.99^N \). With 24 tests there is a 21.43% chance of declaring one or more result to be significant under the null hypothesis.
Figure B.1: Wilcoxon statistics distribution for different number of cases.
In general, a probability $\alpha_F$ of rejecting at least one result under the whole family $\{H_1, \ldots, H_N\}$ of hypotheses is equal to:

$$\alpha_F = 1 - (1 - \alpha)^N$$ (B.4)

$\alpha$ is a probability of rejecting a result in a single test (i.e. a significance level). It’s the same for all tests. $\alpha_F$ is also called family–wise or experiment–wise alpha.

A simple transformation of eq. B.4 produces a Šidák equation Abdi (2007a):

$$\alpha = 1 - (1 - \alpha_F)^\frac{1}{N}$$ (B.5)

It means that to apply a $\alpha_F$ level family–wise, $\alpha$ level must be changed for each test.

A simple approximation of eq. B.5 can be obtained by applying only linear term of Taylor expansion:

$$\alpha \approx \frac{\alpha_F}{N}$$ (B.6)

Equation B.6 is called a Bonferroni correction Abdi (2007a). It means that to reach an $\alpha_F$ level, each test significance level must be decreased $N$ times.

Bonferroni correction is a very conservative approach. For example, to reach an $\alpha_F$ level of 0.01 for 24 tests, a single test significance must be decreased to 0.00042. A more liberal approach is assessing a false discovery rate (FDR) as described in Benjamini & Yekutieli (2001).

The false discovery rate is a ratio of the number of falsely rejected null hypothesis to the number of all hypothesis. Controlling this quotient at a level $q = 0.01$ means that not more than 1% of results declared as significant is declared falsely.

A simple procedure controlling FDR was presented in Benjamini & Yekutieli (2001):

If $\{p_1, \ldots, p_N\}$ are p-levels of $N$ tests sorted ascending order, and $\{H_1, \ldots, H_N\}$ are corresponding hypothesis, than rejecting all hypothesis with index higher than

$$k = \max \left\{ i : p_i \leq \frac{i}{N \sum_{j=1}^{N} j^{-1} q} \right\}$$

controls FDR at $q$ level.

5 Correlation Coefficient

Having two sets of data it is sometimes necessary to test their behavior in correspondence to each other. The covariation provides information on similarities between two functions $x(t)$ and $y(t)$. It is defined as

$$R_{xy} = \int_{-\infty}^{\infty} (x(t) - \bar{x}) (y(t) - \bar{y}) dt$$ (B.7)

where $\bar{x}$ and $\bar{y}$ are means of corresponding functions. A strong positive covariation means that with an increase of function $x$ is mostly accompanied by increase of $y$. On the other hand, strong negative covariation means that an increase of $x$ is accompanied by a decrease of $y$ (Durka, 2004). Zero correlation means that there are no similarities between functions. Dividing the covariation by variation provides normalization – normalized covariation is called correlation:

$$\text{Cor}r_{xy} = \frac{R_{xy}}{\sigma_x^2 \sigma_y^2}$$ (B.8)

where $\sigma_x^2$ and $\sigma_y^2$ are variations of signals $x$ and $y$. 
5.1 Kendall rank correlation coefficient

A different way of measuring the correlation (the similarities between two sets) is Kendall correlation coefficient. For each ordered set of magnitude \( N \), there are \( \frac{1}{2}N(N-1) \) possible ordered pairs (i.e. pairs that first element is bigger than second). The symmetric distance between two ordered sets is a number of ordered pairs that are only in one set. The Kendall correlation coefficient, called Kendall’s \( \tau \), is a normalized symmetric difference. It takes values between -1 and +1 with -1 corresponding to the largest possible distance (obtained when one order is the exact reverse of the other order) and +1 corresponding to the smallest possible distance (equal to 0, obtained when both orders are identical) (Abdi, 2007b). The maximal number of pairs that can be different in two sets with \( \frac{1}{2}N(N-1) \) elements is \( N(N-1) \). This leads to explicit equation for Kendall’s \( \tau \):\[ \tau = \frac{\frac{1}{2}N(N-1) - \Delta(O_1, O_2)}{\frac{1}{2}N(N-1)} \] (B.9)

where \( \Delta(O_1, O_2) \) is the symmetric distance between sets \( O_1 \) and \( O_2 \).

5.2 Statistical testing of Kendall’s \( \tau \)

There is a need of testing whether values provided by Kendall’s method are statistically significant. The exact solution would be a calculation of \( \tau \) coefficient for every possible permutation of one rank set. With \( N \) elements there are \( N! \) permutations. The significance of the \( \tau \) that corresponds to the not permuted set of ranks is the number of coefficients larger than this \( \tau \) divided by the number of all coefficients. This procedure can cause computational problems for large \( N \). Kendall’s \( \tau \) distribution, however, converges towards normal distribution with zero mean and variance equal to:

\[ \sigma^2 = \frac{2(2N + 5)}{9N(N-1)} \] (B.10)

For \( N \) larger than 10, transforming \( \tau \) into \( Z = \frac{\tau}{\sigma_{\tau}} \) produces a value that is normally distributed with zero mean and unity variance (Abdi, 2007b).