AN EXPERIMENTAL AND THEORETICAL INVESTIGATION
OF PROPELLERS OPERATING IN TURBULENCE

by

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SUMMARY

The effect of atmospheric turbulence on the forces and moments on a propeller has been studied both experimentally and theoretically. The experimental study consists primarily of measuring the normal force and pitching moment responses of a model propeller operating in a turbulent field generated in a wind tunnel. No publications could be found of any prior measurements of this kind. The experimental measurements are obtained over the full frequency range of interest in flight dynamic applications. The theoretical part of the present work includes the development of a new, simple method of calculating the response of a propeller to turbulence. This method, referred to as the derivative method, yields expressions for the power spectra of the propeller forces in terms of the turbulence spectra and the aerodynamic derivatives. Based on a comparison of the experimental and theoretical results, the following conclusions are made: (i) The derivative method may be expected to yield accurate results for the propeller response, provided the longitudinal scale of the turbulent field is larger than ten times the propeller diameter; (ii) for propellers operating in a small scale turbulence, a conservative estimate of the response may be obtained by multiplying the results of the derivative method by a factor of 2.
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**NOTATION**

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<tr>
<th>Symbol</th>
<th>Definition</th>
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<td>A</td>
<td>Real part of the frequency response Matrix G; a constant</td>
</tr>
<tr>
<td>a</td>
<td>Speed of sound</td>
</tr>
<tr>
<td>B</td>
<td>Imaginary part of the frequency response Matrix G</td>
</tr>
<tr>
<td>c</td>
<td>Non-dimensional force or moment coefficient; amplitude response matrix</td>
</tr>
<tr>
<td>$C_{xy}$</td>
<td>Co-spectrum of random variables x and y</td>
</tr>
<tr>
<td>D</td>
<td>Diameter of the propeller</td>
</tr>
<tr>
<td>d</td>
<td>Axial separation of the two unbalance masses used to generate sinusoidal pitching moments (Sec. 5.3.2)</td>
</tr>
<tr>
<td>$E_1, E_2$</td>
<td>The front and rear LVDT outputs</td>
</tr>
<tr>
<td>F</td>
<td>Any force or moment on the propeller</td>
</tr>
<tr>
<td>Fn</td>
<td>Function</td>
</tr>
<tr>
<td>f</td>
<td>Frequency; natural frequency of the balance</td>
</tr>
<tr>
<td>G</td>
<td>Frequency response matrix of the balance; dynamic matrix of the balance</td>
</tr>
<tr>
<td>H</td>
<td>Inverse of the frequency response matrix. $H = G^{-1}$</td>
</tr>
<tr>
<td>I</td>
<td>Moment of inertia of the propeller about its own axis</td>
</tr>
<tr>
<td>J</td>
<td>Advance ratio; instantaneous advance ratio</td>
</tr>
<tr>
<td>$\bar{J}$</td>
<td>Mean advance ratio</td>
</tr>
<tr>
<td>$J_1$</td>
<td>Component of $\bar{J}$ along the x-axis</td>
</tr>
<tr>
<td>$J_3$</td>
<td>Component of $\bar{J}$ along the z-axis</td>
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* Some symbols used in this report are not defined here. They are defined locally in the text.*
\[ J_A \]
Instantaneous 'advance ratio' along the x-axis

\[ J_L \]
Instantaneous 'advance ratio' in the propeller plane

\[ j_1, j_2, j_3 \]
Fluctuating 'advance ratios' in the x, y and z directions

\[ K \]
Stiffness

\[ K_t \]
Torsional stiffness

\[ L \]
Turbulence scale (denotes the longitudinal scale unless otherwise mentioned)

\[ L, M, N \]
Moments on the propeller in the x, y and z directions respectively (i.e., rolling moment, pitching moment and yawing moment)

\[ M \]
Any moment on the propeller; Mach number; mass matrix of the balance

\[ m \]
Mass of unbalance used in dynamic calibration

\[ m_i \]
Masses in the lumped parameter model of the balance \((i = 1 \text{ to } 6)\)

\[ n \]
Propeller speed \((\text{rps})\)

\[ Q \]
Torque \(= -L\)

\[ Q_{xy} \]
Quad-spectrum of random variables x and y

\[ R \]
Reynolds number; propeller tip radius

\[ R_{xx} \]
Auto-correlation of random variable x

\[ R_{xy} \]
Cross-correlation of random variables x and y

\[ r \]
Radial location of the mass of unbalance in dynamic calibration

\[ S \]
Non-dimensional frequency \((S = \frac{fD}{U})\); amplitude of the shaft frequency component of \(V_1\) or \(V_2\)

\[ S_0 \]
Amplitude of the shaft frequency component of \(V_1\) or \(V_2\) at \(J = 0\)

\[ S_F \]
Vector representing the sinusoidal signal obtained from the front speed transducer output

\[ S_R \]
Vector representing the sinusoidal signal obtained from the rear speed transducer output

\[ T \]
Time

\[ t \]
Time, thickness of cantilever spring
\( \bar{U} \)  
Mean speed of the propeller; mean speed of the air in the wind tunnel

\( u, v, w \)  
Random turbulent velocities in the propeller body axis

\( u', v', w' \)  
Turbulent velocities in the wind axes

\( u^*, v^*, w^* \)  
Non-dimensional turbulent velocities

\( \left( u^* = \frac{u'}{U}, v^* = \frac{v'}{U} \text{ and } w^* = \frac{w'}{U} \right) \)

\( V_1, V_2 \)  
Voltages related to \( E_1 \) and \( E_2 \) (Eq. 5-2). The DC values of these signals give the steady force and moment

\( V_{1F}, V_{2F} \)

\( V_{1Z}, V_{2Z} \)

\( V_{1M}, V_{2M} \)

\( x, y, z \)  
Propeller body axes

\( x, y, x_1, x_2, x_3 \)  
Random variables

\( x', y', z' \)  
Wind axes; coordinates of the turbulence measurement station in the wind axes

\( \alpha \)  
Angle of attack; instantaneous angle of attack

\( \bar{\alpha} \)  
Mean angle of attack

\( \alpha, \alpha'_1, \alpha'_2, \alpha'_3, \alpha' \)  
Phase differences defined in Fig. 5.4

\( \beta \)  
Phase angle defined in Fig. 5.4

\( \delta_{ij} \)  
Influence coefficient (defined in Sec. 3.5)

\( \lambda \)  
Eigenvalues

\( \theta \)  
Phase response; phase response matrix

\( \mu \)  
Mean value of a random variable; viscosity

\( \sigma \)  
Standard deviation

\( \tau \)  
Time delay
One-sided power spectrum of random variable $x$

One-sided cross-spectrum of random variables $x$ and $y$

Density of air

**Superscripts**

* Complex conjugate

- Inverse

- Mean value

T Transpose

**Subscripts**

ij Refers to the element of a matrix in the $i^{th}$ row and the $j^{th}$ column

J Partial differentiation w.r.t. $J$

$\ell$ Rolling moment

m Pitching moment

n Yawing moment

q Torque

X Thrust

Y Side force

Z Normal force

$\alpha$ Partial differentiation w.r.t. $\alpha$
I. INTRODUCTION

Problems of flight in turbulence have been under intensive investigation over the past two decades. The complexity of atmospheric turbulence, the tremendous variety of aircraft designs and missions, and the numerous ways in which aerodynamic subsystems are employed on the aircraft, all add to the difficulty of the problem. Methods of estimating the response of aerodynamic systems to turbulence are needed to optimize the aircraft design, i.e., to improve its riding and handling qualities and safety. However, so numerous are the assumptions that have to be made in such methods that there is a distinct need for experimental verification of theories. By the same token, so complex and time consuming are dynamic measurements that one can hardly expect to make them in a routine manner. This combination of difficulties in the theoretical and experimental approaches to the problem forces the researcher to seek a compromise solution, viz., (i) to perform a carefully planned experiment in a known turbulent field and (ii) to develop a theory that fits the experimental findings. The subject matter of this report is an experimental and theoretical investigation of the response to turbulence of a propeller, which is one of the most common and the most complex aerodynamic subsystems on low speed aircraft and hovercraft.

Propellers have several distinct advantages over other propulsive systems for use on V/STOL and STOL aircraft and most hovercraft designs to date employ propellers for propulsion. On all such vehicles the propeller has a decided influence on the stability and control characteristics, on the response to atmospheric turbulence, and hence on the handling qualities and safety. V/STOL aircraft often have difficult flight requirements such as operation in congested airspaces, flight close to buildings, hovering, landing on small airstrips etc. These requirements have to be met while operating under the severe-atmospheric conditions that exist at low altitudes, viz., intense small scale turbulence, large velocity gradients, turbulence in the wakes of buildings, trees and other aircraft etc. There is obviously a great need for a detailed study of propellers operating in turbulence. Such a study may be expected to lead to V/STOL designs with better handling qualities.

In recognition of the need for such a study, the subsonic aerodynamics laboratory of UTIAS, under the supervision of Professor B. Etkin, embarked on a research program comprising extensive theoretical and experimental investigations. Two doctoral research programs were undertaken, one primarily theoretical and the second primarily experimental. The theoretical investigation was carried out by J. B. Barlow and the results are published in UTIAS Report No. 155 (Ref. 1). Results of the experimental investigation as well as some additional theoretical work are given here.

The experiments described in this report were carried out on a model propeller operating in a wind tunnel. Turbulence was generated in the UTIAS low speed wind tunnel by means of a wooden grid. The propeller model was installed in the turbulent flow behind the grid and operated at various angles of attack and advance ratios. Spectra of the random in-plane forces and moments on the propeller were measured. The measured values are then compared with the predictions of the derivative method developed by the author. Based on this comparison, the range of validity of the derivative method is examined. Finally, several recommendations are made for future work on propellers.
II. A REVIEW OF RELATED WORK

The problem of calculating the response of an aircraft to turbulence is, in general, very complex. It encompasses a wide variety of subjects such as turbulence, meteorology, system theory, random process theory and aero-dynamics. It is considered appropriate here to offer some comments on the role of these topics in flight dynamics work, with special reference to the present investigation. Previous work in the above fields is discussed briefly from the flight dynamics point of view and some useful references are given.

2.1 Random Processes

Turbulence is a random phenomenon i.e., velocities and pressures in a turbulent field are random functions of time and spatial coordinates. As a result of this, the responses (motion, forces etc.) of an aerodynamic system operating in turbulence are random processes too. One must therefore resort to the principles of random process theory in dealing with turbulence and its effects. Random processes can be described only in a statistical sense. The statistical quantities that are normally employed in flight dynamics work are: means, variances, auto and cross-correlations, power and cross spectra and probability distributions. A single time history representing a random phenomenon is called a sample function (or a sample record when observed over a finite time interval). The collection of all possible sample functions which the random phenomenon might have produced is called a random process. The collection of sample functions is also called the ensemble. The statistical quantities of a random process are defined for the complete ensemble.

Random processes may be classified as stationary or nonstationary processes. For the purpose of this report, a stationary process may be defined as one for which the mean and autocorrelation are independent of time. (Strictly speaking this ensures only what is known as weak stationarity). All other processes are nonstationary. Stationary random processes may be further classified as ergodic or nonergodic processes. If the time averaged statistical properties of a single sample function are equal to the corresponding ensemble averaged properties of the complete random process, then the process is termed ergodic. The assumption of ergodicity greatly simplifies measurement of random processes. References 2, 4, 52 and 56 give a detailed treatment of random process theory and measurement.

The following relations are useful in the measurement and analysis of ergodic random processes. They apply to random processes \{x(t)\} and \{y(t)\}.

\[
\mu_x = \lim_{T \to \infty} \frac{1}{T} \int_0^T x(t) \, dt \quad (2-1)
\]

\[
\sigma_x^2 = \lim_{T \to \infty} \frac{1}{T} \int_0^T [x(t) - \mu(x)]^2 \, dt \quad (2-2)
\]

\[
R_{xx}(\tau) = \lim_{T \to \infty} \frac{1}{T} \int_0^T x(t) x(t+\tau) \, dt \quad (2-3)
\]
\[
R_{xy}(\tau) = \lim_{T \to \infty} \frac{1}{T} \int_{0}^{T} x(t) y(t+\tau) \, dt \quad (2.14)
\]

\[
R_{xy}(-\tau) = R_{yx}(\tau) \quad (2.5)
\]

\[
\Phi_{xx}(f) = 4 \int_{0}^{\infty} R_{xx}(\tau) \cos 2\pi f \tau \, d\tau \quad (2.6)
\]

\[
C_{xy}(f) = 2 \int_{0}^{\infty} [R_{xy}(\tau) + R_{yx}(\tau)] \cos 2\pi f \tau \, d\tau \quad (2.7)
\]

\[
Q_{xy}(f) = 2 \int_{0}^{\infty} [R_{xy}(\tau) - R_{yx}(\tau)] \sin 2\pi f \tau \, d\tau \quad (2.8)
\]

\[
\Phi_{xy}(f) = C_{xy}(f) - i Q_{xy}(f) \quad (2.9)
\]

\[
C_{yx}(f) = C_{xy}(f) \quad (2.10)
\]

\[
Q_{yx}(f) = -Q_{xy}(f) \quad (2.11)
\]

\[
\Phi_{yx}(f) = \Phi_{xy}(f) \quad (2.12)
\]

If

\[
y(t) = A x(t) \quad (2.13)
\]

where \(A\) is a constant, it can be shown from Eq. (2-3) and (2-6) that

\[
R_{yy}(\tau) = A^2 R_{xx}(\tau) \quad (2.14)
\]

and

\[
\Phi_{yy}(f) = A^2 \Phi_{yy}(f)
\]

Also, if

\[
y(t) = x_1(t) + x_2(t) + x_3(t) \quad (2.15)
\]

where \(x_i(t), i = 1, 2, 3\) are independent random variables,

\[
R_{yy}(\tau) = R_{x_1x_1}(\tau) + R_{x_2x_2}(\tau) + R_{x_3x_3}(\tau) \quad (2.16a)
\]

and

\[
\Phi_{yy}(f)_{1} = \Phi_{x_1x_1}(f) + \Phi_{x_2x_2}(f) + \Phi_{x_3x_3}(f) \quad (2.16b)
\]
If $x_1$, $x_2$, and $x_3$ in Eq. (2-15) are not independent,

$$R_{yy}(\tau) = R_{x_1x_1}(\tau) + R_{x_2x_2}(\tau) + R_{x_3x_3}(\tau)$$

$$+ R_{x_1x_2}(\tau) + R_{x_2x_1}(\tau)$$

$$+ R_{x_2x_3}(\tau) + R_{x_3x_2}(\tau)$$

$$+ R_{x_3x_1}(\tau) + R_{x_1x_3}(\tau)$$

(2-17)

and using Eq. (2-17) and (2-12), it can be shown that

$$\Phi_{yy}(\tau) = \Phi_{x_1x_1}(\tau) + \Phi_{x_2x_2}(\tau) + \Phi_{x_3x_3}(\tau)$$

$$+ 2 \Phi_{x_1x_2}(\tau) + 2 \Phi_{x_2x_3}(\tau)$$

$$+ 2 \Phi_{x_3x_1}(\tau)$$

(2-18)

2.2 Atmospheric Turbulence

As mentioned earlier, velocities and pressures in a turbulent field are random functions of time and space. In the case of atmospheric turbulence, several assumptions are usually made as to the nature of turbulence in order to simplify analysis. Among the assumptions commonly made are homogeneity and Taylor's hypothesis. The latter states that the velocity patterns in the atmosphere are time invariant in a reference frame moving with the mean velocity of the atmosphere. This means that the variations of turbulent velocities at an aircraft can be deduced from the spatial variations in the moving frame. Turbulence is specified by means of correlations, spectra and probability distributions of velocities. A detailed description of turbulence and its nature is given by Batchelor (Ref. 10) and Hinze (Ref. 9). The subject of atmospheric turbulence is treated in detail by Lumley and Panofsky (Ref. 40), while a brief account of the subject, from the flight dynamicist's point of view, is given by Etkin (Ref. 3). A review of experimental data on turbulence at high altitudes (approximately 5000 to 40,000 ft) is given by Houbolt et al (Ref. 32). The discussion in the following paragraphs is limited to low altitude turbulence, which is of prime interest for V/STOL flight.

Flight dynamicists are interested in the nature of turbulence in the earth's boundary layer for several reasons. The boundary layer usually extends up to an altitude of about 2000 feet. Critical operations such as landing and take-off, which are known to give rise to most aircraft accidents, take place in the boundary layer. Also V/STOL aircraft spend a large part of their flight times in the boundary layer and often have difficult assignments to perform. Hence it is essential for the flight dynamicist to understand the characteristics of the boundary layer and study its effect on aircraft.

The earth's boundary layer is very difficult to characterize. Its nature depends on a number of factors the most important of which are wind
atmospheric stability, altitude and surface roughness. An enormous number of measurements has been carried out in the boundary layer in recent years in an attempt to understand its nature. The data presented by Gunter et al (Ref. 55) are among the most comprehensive of such measurements. Teunissen (Ref. 42) recently reviewed the published data on the characteristics of earth's boundary layer. He suggests a simple mathematical model for the boundary layer, which gives the mean velocity profiles, Reynolds's stresses, power and cross spectra and integral scales.

Atmospheric turbulence is frequently assumed to be a Gaussian random process. This assumption simplifies the aircraft response problem considerably and enables the flight dynamicist to obtain all the necessary information by means of power spectral techniques. However, measurements show that low altitude turbulence is not Gaussian. Jones (Ref. 43) discusses the importance of considering both the non-Gaussianess and intermittency found in the measurements at R.A.E. These features were shown to have important effects on the handling and ride qualities of an airplane. In order to take these into account in a simple manner, Jones suggests the combined use of a power spectral technique and a new discrete gust technique.

2.3 System Theory

Systems approach is becoming increasingly popular in flight dynamics work. This approach is used both in the experimental and theoretical parts of the present investigation. System theory presents a conceptually simple approach to complex problems such as aircraft response to turbulence. Etkin (Ref. 3) illustrates the application of system theory to various problems of dynamics of atmospheric flight.

Depending on the nature of the equations describing their behaviour, systems may be classified as linear or non-linear systems. Linear systems may further be classified as constant parameter or variable parameter systems. Aerodynamic systems on an aircraft are often treated as linear systems. The assumption of linearity is valid for most aerodynamic systems in most regimes of normal flight. This assumption permits the use of powerful and relatively simple transfer function methods of analysis. It can be shown that the response of a linear system is Gaussian if the input is. In such a case power spectral techniques yield all the information needed about the response. It can also be shown that the response of a constant parameter system is stationary if the input is stationary. The response of a variable parameter system, such as a propeller, is non-stationary even when the input is stationary. Barlow (Ref. 1) gives a method of analyzing variable parameter systems with random inputs.

2.4 Flight in Turbulence

Estimation of the force and motion response of aircraft to atmospheric turbulence is a first step in analyzing the effects of turbulence on the pilot, passenger, structure, cargo and the mission. Such an estimation is often done by treating the aircraft (and its sub-systems) as a linear aerodynamic system operating in a homogeneous, frozen turbulent field. Isotropy of turbulence is also frequently assumed. In spite of all these simplifying assumptions, which limit the validity of results, the problem is still a very difficult one. Flight dynamicists often make two more assumptions - point approximation and quasi-steady aerodynamics, to simplify the problem still further. It is not hard to see that none of the above assumptions is, strictly speaking, valid. The question
then is, under what conditions is each of these assumptions useful? Alternately, given a particular problem, say a tilt-wing aircraft operating at an altitude of 1000 feet, what set of assumptions would give simple and yet meaningful results? Also, for a given problem, how accurate is a solution obtained with a given set of assumptions? The answers to such questions can be obtained only by extensive measurements in the atmosphere, and exhaustive theoretical and experimental investigations of the behaviour of aerodynamic systems. It is hoped that the results of the present report will help answer some of these questions in relation to flight of propellers in turbulence.

A large body of information exists in the literature on problems of flight in turbulence and related areas. Some papers are of fundamental or historical importance and some others indicate the present state of the art. A brief review is given below.

Sears (Ref. 62) presented a solution to the problem of a thin airfoil moving through a one-dimensional downwash field. Taylor (Ref. 63) and von Karman (Ref. 64) introduced statistical concepts to the theory of turbulence. Leipmann (Refs. 25 and 26) pioneered the use of statistical techniques for buffeting and gust response problems. Diederich (Ref. 30) developed a method that includes the effect of spanwise variations of turbulence on an aircraft. Ribner (Ref. 27) presented a unified spectral approach to the problem of flight in turbulence. In his method, turbulence is considered as a superposition of sinusoidal shearing waves of all orientations and wavelengths. Etkin (Refs. 28 and 29) presented a power series approach, which yields aircraft response to turbulence in terms of a set of aerodynamic derivatives like those used in stability and flutter analyses. Filotas (Refs. 35, 36, 37, 38) developed closed form expressions for aerodynamic transfer functions for infinite and finite wings and calculated their responses to turbulence. Barlow (Ref. 1) developed a method of calculating the response of a propeller to turbulence. (This is discussed in more detail in section 2.6). References 31, 32, 33, 34, 39, 43 and 48 are among the other noteworthy contributions to the subject of flight in turbulence. VTOL and V/STOL flight in turbulence is discussed in Refs. 44, 45, and 47. Neuman and Foster (Ref. 46) investigated the performance of a landing system in turbulence. References 49 and 51 investigated the effect of turbulence on flying qualities.

2.5 Propeller Aerodynamics

Most work on propellers has been concentrated on the prediction of thrust and torque on a propeller in axial flight. Description of momentum, blade element and vortex theories of propellers and improvements to vortex theory by Goldstein and Theodorsen are given in Refs. 11, 12, 13, 14 and 15.

A method of predicting in-plane forces on a propeller in yaw is given by Ribner (Refs. 16, 17). The method is limited to small angles of yaw. De Young (Ref. 21) gives empirical relations for propeller characteristics at high angles of attack. These relations are based on Ribner's theory and experimental data. Barlow (Ref. 1) modified Ribner's method and obtained good agreement with experimental data at low advance ratios, where Ribner's method seems to fail. Barlow's method, like Ribner's, is restricted to low angles of yaw. Lack of enough knowledge about airfoils executing large pitching oscillations (well into the non-linear range) is one of the stumbling blocks in the development of a theory of propellers at large angles of attack. It must, however, be noted that the validity of the blade element approach itself remains
to be investigated for propellers at large angles of attack.

A large body of experimental data exists for the thrust and torque of a propeller in axial flight. Charts of thrust and torque coefficients are given, among others, in Refs. 11,12 and 13. References 18,19,20 and this report present experimental measurements of forces and moments on a propeller at angle of attack.

2.6 Propeller in Turbulent Flow

Only recently has this problem received the attention of flight dynamists. Barlow's work (Ref. 1) seems to be the only previous investigation that dealt with this problem directly. References 24,53 and 54 describe researches in some related areas.

Barlow (Ref. 1) presented a blade element approach to the problem of a propeller operating in a turbulent flow. In his method, each blade is considered as a quasistatic lifting line. The aerodynamics of the blade element is assumed to be linear. Unsteady flow effects are estimated by applying a Sears function factor (Ref. 62) based on the propeller rotational speed. No phase correction is made. Three-dimensional effects are considered negligible. Turbulent velocity variations over the propeller disk are fully taken into account. The method requires a knowledge of the three-dimensional power spectrum of the turbulence as an input. It is noted that the basic aerodynamic problem is that of calculating the impulse response matrix of a blade element at an arbitrary radius and azimuth. The elements of the matrix are the contributions of the blade element forces to the propeller in-plane forces, due to impulses in u, v and w at the blade element. The propeller transfer function is derived from the impulse response matrix by using systems theory relations and carrying out certain integrations. It is stated that the dependence of the blade element response on the azimuth position makes the propeller a system with periodic parameters. This implies that the propeller response is non-stationary even when it is moving uniformly in an isotropic, frozen turbulent field. The response is Gaussian, however, if the input is. Expressions for the measurable time-averaged normal force spectra are presented. Expressions for the special case of point approximation (i.e., no variation of turbulent velocities over the propeller disk) are derived from the general relations. Finally, plots of normal force spectra are presented for specific one, two and three bladed propellers. These spectra were obtained by using the von Karman turbulence spectrum and the point approximation.

III. DESIGN OF THE EXPERIMENT AND APPARATUS

3.1 Objective of the Experiment

The experiments were conducted on a model propeller operating in a turbulent field generated in the UTIAS low speed wind tunnel. The aim of the present experimental study was to obtain data that would (i) permit the testing of existing theories, (ii) provide a basis for comparison of such theories and (iii) assist in understanding the aerodynamics of propellers operating in conditions that are not at present amenable to theoretical analysis. It was hoped that the experimental results would provide some physical insights that would lead to better and possibly simpler theories in future. They might also enable the formulation of simple empirical methods of propeller response calculation.
3.2 Dimensional Analysis

Dimensional analysis forms an important part of the design of any experiment. The variables to be considered in the present experiment are:

\[ \Phi_{FF}, f, \rho, \bar{U}, n, D, \bar{a}, \mu, L, u_{\text{rms}} \text{ and } a. \]

Dimensional analysis gives,

\[ \frac{f \Phi_{FF}(f)}{(\rho n D)^2} = F_n (J, \bar{a}, R, M, \frac{L}{D}, \frac{u_{\text{rms}}}{\bar{U}}, S) \quad (3-1) \]

If an experiment is to be designed to simulate the operation of a full scale propeller in atmospheric turbulence, the following conditions must be met: 1) The model and the prototype must be geometrically similar, 2) The turbulence generated in the wind tunnel must be kinematically similar to the atmospheric turbulence, 3) All the non-dimensional quantities of Eq. (3-1) must be identical for the model and the prototype.

3.3 Simulation of Non-Dimensional Parameters

If the objective of the experiment were to simulate the case of a specific propeller operating in a specific turbulence field, it would have been desirable, if not necessary, to simulate all the non-dimensional quantities of Eq. (3-1) in the wind tunnel. However, this is not the case in the present investigation. It is shown below that the difficult requirement of simulating all the dimensionless parameters in the wind tunnel need not be met in order to achieve the objectives stated in section 3.1.

(i) Advance Ratio and Angle of Attack

A propeller on a propeller V/STOL aircraft operates over a large range of advance ratios (J) and angles of attack (\(\alpha\)). These parameters have a direct and strong influence on the aerodynamics of the propeller. Hence it is considered desirable to conduct the experiment over a range of values of J and \(\alpha\) of realistic magnitudes. Experimental data is especially needed at large \(\alpha\) where some of the assumptions used in propeller theories are invalid. In the present investigation J is varied from 0 to 0.8 and \(\alpha\) is varied from 0° to 60°.

(ii) Reynolds Number

Simulation of full scale Reynolds number is not possible in most wind tunnel experiments. However, in the present case this is not considered to be a serious drawback. The basic effect of Reynolds number on a propeller is its influence on the aerodynamics of individual blade elements. Such an effect, which is not expected to be very large, can be conveniently included in the theoretical evaluation of propeller responses. Thus an experiment conducted at any convenient Reynolds number meets the objectives of section 3.1 reasonably well. The derivative method described in chapter 8 uses the measured average forces as input to the theory and hence takes the Reynolds number fully into account. The validity of such a method may be checked by means of
experiments conducted at any convenient Reynolds number. Therefore the lack of simulation of full scale Reynolds number is not considered to be very important in the present experiment. The tests were carried out at Reynolds numbers of the order of \(0.7 \times 10^5\).

(iii) Tip Mach Number

Tip Mach numbers used in the test varied from 0.13 to 0.19. These are considerably smaller than those for full scale propellers which operate a tip Mach numbers of the order of 0.5 to 0.9. Simulation of a high Mach number requires a very high rotational speed and was not possible in this experiment due to practical difficulties. However, by the same reasoning as applied to the Reynolds number, it can be shown that the objectives of the experiment can be met by means of tests conducted at any convenient tip Mach number.

(iv) Scale-Diameter Ratio

The ratio of turbulence scale to propeller (L/D) may be expected to have a strong influence on the propeller response. The following points must be noted in determining the necessity of simulating L/D in the wind tunnel:

(a) In the atmosphere, the scale of turbulence (longitudinal or lateral) is a strong function of several variables (see Refs. 42 and 55) and varies widely. A complete simulation of the propeller operation in atmosphere would involve simulation of several L/D ratios. It does not seem necessary to perform such a simulation to meet the objectives of section 3.1. It may be noted here that simulation of large L/D ratios in the wind tunnel would be very difficult, if not impossible. On the other hand, theory shows conclusively that this case is least in need of experiment.

(b) The theoretical approach to the problem is greatly simplified if L/D is very large. (This point is discussed in detail later in this report). From the aerodynamics point of view, L/D of the order of unity (which is representative of flight close to ground) is the most interesting and most difficult case. A very general theory is needed to calculate the response of a propeller to small scale turbulence of this scale and such a theory would contain large scale turbulence as a special case.

(c) L/D of the order unity represents unsteady, non-uniform flow over the propeller disk. The general aerodynamic transfer function obtained for such a case is needed for the analysis of flight at low altitude and flight in the wakes of other aircraft, buildings and trees.

In view of the above discussion it is felt that the small scale turbulence (L/D of order 1) obtainable in the wind tunnel is not only satisfactory but is in fact necessary to check the validity of general transfer functions such as the one proposed by Barlow (Ref. 1). In the present experiment, longitudinal scale of turbulence was about 5" and the propeller diameter was 8". 
(v) Turbulence Intensity

Turbulence intensity in the atmosphere, like the scale, is a function of several variables and varies considerably. In the earth's boundary layer it ranges between a few percent to as high as 20%. In the present experiment u-intensity was about 13% and the v- and w-intensities were about 10%. It was not considered necessary to conduct the experiment with several different turbulence intensities.

(vi) Non-Dimensional Frequency

The non-dimensional frequency* S is an important dimensionless parameter for the present experiment. It is intended to make the response measurements over a large enough frequency range to cover the flight dynamic and, if possible, aeroelastic applications of the results. A full-scale frequency range of 0-5 Hz. covers most flight dynamic applications. The fundamental bending frequency of a propeller blade is also likely to be in this range. For a 20' diameter propeller travelling at 200 ft/sec, this represents a non-dimensional frequency range of 0 - 0.5. This range must be covered by the experimental measurements. For the model propeller, 8" in diameter in the present experiment, operating in a wind of about 40 fps (corresponding to a fairly large advance ratio of 0.8), this amounts to a frequency range of approximately 0 -30 Hz. Experimental results are obtained over a frequency range of 0 - 55 Hz.

3.4 Measurement Requirements

Based on the discussion in Sec. 3.3, it is found that the following measurements are needed in order to meet the objective stated in sec. 3.1.

(i) Steady forces and moments on the propeller operating in a uniform flow. These measurements are to be made at several angles of attack and advance ratios.

(ii) Spectra of in-plane forces and moments on the propeller operating in a turbulent flow. These again are to be made at several angles of attack and advance ratios.

(iii) Measurement of the turbulence field in which tests (ii) were carried out. This involves measurement of power and cross-spectra of turbulent velocities at several points in the field.

Measurements (i) above are needed for three reasons: (a) to enable comparison of the aerodynamic behaviour of the model propeller with that of typical aircraft propellers. It is possible that such a comparison may reveal, among other things, Reynolds number effects, if any. (b) The steady force measurements at angle of attack are by themselves of value, since such data is relatively scarce. (c) These measurements are needed as input to

* The non-dimensional frequency is given by $S = fD/U$ and is denoted by $S$ because of the close analogy with Strouhal number.
certain theoretical methods of prediction of propeller response to turbulence. Measurements (ii) above i.e., response of the propeller forces to turbulence, form the main experimental results of the present investigation. To the author's knowledge, no such measurements have been made previously. The need for the measurement of the turbulence field is rather obvious - both the input and the response are needed to analyze the behaviour of a system. Also, a knowledge of the turbulence field is needed in order that the experimental results be compared with theory.

3.5 Experimental Apparatus

The experimental apparatus (Fig. 3.1) consists of a 3 bladed nylon propeller, 8" in diameter, driven by a DC electric motor through a flexible wire coupling. A two-component force balance and two speed transducers were installed to enable the measurement of the forces and moments on the propeller and its rotational speed. The two-component balance was used to measure the average as well as the random in-plane forces and moments on the propeller. The rear speed transducer was used to measure the propeller speed while both the speed transducers were used in the measurement of the average torque and sinusoidal components in the propeller responses. (The latter measurements were carried out by means of special techniques described in Chapter 7). The balance and the motor are enclosed in an axisymmetric cowling. A picture of the apparatus is given in Fig. 3.2. A close-up view of the balance and the front speed transducer is given in Fig. 3.3.

The selection of the propeller, motor and the transducers as well as the design of the balance and the coupling involve careful consideration of several criteria, some of which are conflicting. It is not intended here to go into the details of the design procedure, which involved several compromises, much iteration and some optimization. A description of the individual components of the apparatus is given below along with a mention of the most important design criteria and the major development difficulties. The balance design is described in a little more detail.

(i) Propeller

Propeller selection was based on the following criteria:
(a) Magnitude of the aerodynamic forces needed for accurate measurement
(b) Interference from wind tunnel walls and cowling
(c) L/D ratio
(d) effect of propeller size on the balance frequency response and
(e) cost.

Based on these considerations it was decided to use a 3 bladed model aircraft propeller, 8" in diameter. The blade angle at 3/4 R is about 16°. The geometry of the propeller was measured by means of a photographic technique described in Appendix A. Figures A-2 and A-3 describe the geometry of the propeller.

(ii) Motor

The motor selection was based on the requirements of (a) horsepower (b) desired rotational speed (c) small size and (d) adjustable constant speed
operation. These considerations led to the selection of an Elinco Type 680 separately excited DC motor. Specifications of the motor are: H.P.: 1/25, rated speed: 6000 rpm. (Speed is continuously variable from 0 to 9000 rpm). size: 3-3/8" diameter x 4-1/2" long. The motor could be operated at 100% overload continuously, when properly cooled. The speed of the motor changes only by about 2% when the load is varied from zero to full load.

(iii) Flexible Coupling

A flexible coupling is needed between the propeller and motor shafts for the following reasons: (a) in order that the sensitivity of the two-component balance (described later in this section) be high, it is necessary that the coupling should react a negligible part of the propeller forces. This means that the stiffness of the coupling should be much smaller than that of either balance spring. (b) Small transverse and angular misalignments of the propeller and motor shafts are unavoidable. Due to these misalignments, the rear end of the propeller shaft is subjected by the coupling to periodic forces which give rise to spurious balance outputs. A coupling with a small stiffness is needed to minimize such 'noise'. The development of a suitable coupling turned out to be one of the major design and development problems. Several different flexible couplings available in the market were tried and found to be unsatisfactory. Finally, the following deceptively simple solution was adopted. It consists simply of using a 4" long, 0.05" diameter music wire as a flexible coupling. The bending stiffness of the coupling, as a cantilever, is 0.2 lbs/in. The torsional stiffness of the coupling is 0.318 oz.in/deg.

(iv) Two-Component Balance

The design of the two-component force balance required a careful consideration of several factors. Among these are linearity, sensitivity, resolution, range, insensitivity to temperature changes, frequency response, separation of individual force components, ability to measure all four in-plane force components, size, etc. Figure 3.4 shows the configuration of the balance. The propeller shaft passes through two ball bearings supported on springs. The springs are designed to be soft in one direction and very stiff in the two perpendicular directions. In Fig. 3.4 the springs deflect in the vertical direction only and are thus sensitive to the normal force and pitching moment on the propeller only. The deflections of the springs give a measure of their reactions from which the normal force and pitching moment may be estimated. In the present experiment, LVDT's (Linear Variable Differential Transformers) were used as displacements transducers. LVDT's were chosen because they satisfied all the balance requirements and the necessary electronic equipment was already available in the laboratory.

The balance is a complex dynamic system and must be designed to have a high enough fundamental natural frequency to cover the frequency range of interest. (cf. Sec. 3.3 under 'non-dimensional frequency'). A mathematical model of the system is shown in Fig. 3.5. In this model, the propeller is represented by a mass $m_1$ and moment of inertia $I$, the shaft and bearings are represented by concentrated masses. The stiffness of the shaft is taken into account and the springs are assumed to be massless. An influence coefficient method (described in Ref. 60) is well suited to determine the natural frequencies of the balance. The influence coefficient $\delta_{ij}$ is defined as the static deflection at station $i$ due to a unit force applied at station $j$. If $[\delta_{ij}]$ is the matrix
of influence coefficients and M is the mass matrix, the dynamic matrix G of the system is given by

$$ G = \begin{bmatrix} b_{ij} \end{bmatrix} M $$

(3-2)

It can be shown (Ref. 60) that the natural frequencies of the system are given by the relation

$$ f_i = \frac{1}{2\pi} \sqrt{\frac{1}{\lambda_i}} \quad i = 1, 2, \ldots, n $$

(3-3)

where $\lambda_i$'s are the eigenvalues of G and $n$ is the order of the system.

If the gyroscopic effect on the propeller is ignored*, the mass matrix is given by

$$ M = \begin{bmatrix}
    m_1 & 0 & 0 & 0 & 0 & 0 \\
    0 & m_2 & 0 & 0 & 0 & 0 \\
    0 & 0 & m_3 & 0 & 0 & 0 \\
    0 & 0 & 0 & m_4 & 0 & 0 \\
    0 & 0 & 0 & 0 & m_5 & 0 \\
    0 & 0 & 0 & 0 & 0 & m_6
\end{bmatrix} $$

(3-4)

A computer program was developed to compute the dynamic matrix and its largest eigenvalue. Using Eq. (3.3), the fundamental natural frequency was calculated. The computations were carried out with several combinations of shaft and bearing dimensions, lengths $a$, $b$ and $c$ (Fig. 3.5) and spring stiffnesses as inputs. The effect of these parameters on the fundamental natural frequency was noted.

This computer study gave some useful guidelines for the design of the balance. For example, the study brought to light the necessity of meeting two conflicting requirements, in order to optimize the natural frequency. The shaft had to have a fairly large outside diameter to provide enough stiffness, while the bearing size had to be so small that it would not fit on the shaft. This led to the shaft design shown in Fig. 3.6. Such a design enabled the use of a fairly stiff shaft in conjunction with small bearings, which also helped reduce the size of the balance. This, of course, resulted in additional complexities in fabrication and assembly. In the assembly shown in Fig. 3.6, parts 3 and 6 were press fitted into parts 1 and 2 respectively. The bearing 4 was push fitted on part 3 and then the latter was threaded into part 6 and cemented in place with epoxy cement.

The spring, LVDT and cowling assembly is shown in Fig. 3.7. (The cantilever spring design shown in the figure was adopted after some initial unsuccessful trials with a different design. The latter design consisted of a leaf spring with a bearing housing in the middle and both ends bolted to the cowling. The

* It was realized, after the apparatus was built, that this effect should have been included in the design calculations. There would be no point in repeating the calculations, however, since experimentally determined frequency response (Chapter 5) is used for data reduction purposes, and hence the results of this report are unaffected by the omission.
stiffness of such a spring was found to be quite sensitive to bolt tensions and temperature fluctuations and hence the design was abandoned). As shown in the figure, the bearing housing forms an integral part of the spring. The core rod of the LVDT is threaded into the free end of the cantilever spring. The double leaf feature of the spring insures that an application of load at its free end would not result in an angular deflection of the core rod. This insures that the core rod stays clear of the transformer walls during operation. Leaf thicknesses and stiffnesses of the two balance springs are: Front spring \( t = 0.03'' \), \( K = 440 \text{ lbs./in.} \); Rear spring \( t = 0.0246'' \), \( K = 230 \text{ lbs./in.} \).

The fundamental natural frequency of the balance was calculated to be 91 cps. After the apparatus was built, the natural frequency was measured and found to be about 75 cps. (The discrepancy between the two is attributed partially to a few minor design changes made during the fabrication and partially to the use of inadequate mathematical model especially the omission of the propeller moment of inertia. Unfortunately, a long series of diagnostic tests had to be carried out on the balance, during which the balance had to be disassembled completely once and partially several times. The unavoidable reduction in stiffnesses of the joints during these operations reduced the fundamental natural frequency to 60 c/s.

The balance and the motor are enclosed in an axisymmetric cowling, as shown in Fig. 3.1. The cowling is made in three parts fastened together. The middle part, which encloses the balance, consists of two halves bolted together. Such a design enables easy access to the LVDT's and springs for alignment and adjustment. The central part of the cowling, together with the balance, may be rotated through 90° and secured in the new position, when necessary. This feature enables the measurement of all four in-plane force components using the two component balance.

Static and dynamic calibrations of the balance are described in Chapter 5.

(v) Speed Transducers

Two speed transducers were installed - one on the propeller shaft, between the two springs, and the other on the motor shaft behind the motor (Fig. 3.1). Both the transducers are of the photoelectric type, each with a light chopper rotating between a photodiode and a light bulb. The front transducer was designed to give three pulses per shaft revolution while the rear one gives one pulse per revolution. The rear transducer is used to measure the propeller speed. The use of the two speed transducers in measuring the average torque and the sinusoidal components in the propeller responses is described in Chapter 7. A view of the front speed transducer is shown in Fig. 3.3.

IV. INSTRUMENTATION AND MEASUREMENT TECHNIQUES

4.1 Wind Tunnel

An aerodynamic outline of the wind tunnel used for the present experiments is shown in Fig. 4.1. The test section has an octagonal cross-section and is 48" x 32" x 4 ft. long. When the tunnel is empty, test section speeds of up to 200 fps. are obtainable. The wind speed may be varied continuously in the lower half of the speed range and in eleven discrete steps in the upper half. The dynamic pressure in the test section is measured by means of a Betz manometer.
connected to static pressure taps at the entrance to the test section and in the stagnation chamber. The wind speed in the test section is determined from the measured dynamic pressure.

4.2 LVDT System

The LVDT (Linear Variable Differential Transformer) is an electro-mechanical transducer, whose output varies linearly (within limits) with the displacement of the core. In the present experiment, Schaevitz 010 XS-AC LVDT's were used. They are 3/8" diameter by 5/8" long in size. The core is 0.1" diameter by 0.25" long and is mounted on a core rod with 1-72 threads. The linear range of the LVDT's is 0.01".

A custom built Schaevitz CAM - 929 LVDT System was used to provide excitation to the primary windings and demodulate the differential output of the secondary windings. Among the special features of the CAM - 929 System are high sensitivity, good stability, low noise, flat frequency response (to within 1/2%) to 200 Hz., and practically zero sensitivity to temperature changes in the range of interest for the present experiment. The system also includes zero and gain adjustments. More details on the CAM - 929 system are given by Nettleton (Ref.58), who investigated the system characteristics in detail. All experiments were done with a gain setting of 7.5 which corresponds to a sensitivity of about 5 volts/mil.

4.3 Analog Computer

An EAI 221 R analog computer was used for on-line processing of transducer signals. The 221 R is a 100 volt system equipped with a digital voltmeter with a 10 mv resolution. Only amplifiers and potentiometers were used in the present experiment. Both these components were found to have excellent frequency response (with the 3 db points in the KHz. range) far surpassing the requirements of the present experiment. Typically, the output noise of a single amplifier with grounded input is less than 1 mv. rms. Since most of this noise is at frequencies well above 100 Hz., the components of the computer may be considered to be noise free for the purposes of this experiment.

4.4 Variable Filters

Multimetrics AF - 400 and Krohn-Hite variable active filters were used extensively in the present experiment to solve several signal analysis problems. Both of these are two-channel units and can be operated in a low pass, high pass, band pass or band reject mode. The cut-off frequencies on the Multimetrics unit are digitally adjustable from 0.01 Hz to 99.9 KHz, while those on the Krohn-Hite are continuously variable from 20 Hz to 2 MHz. Both these units have fourth order Butterworth filters. The attenuation slope of each filter is 24 db/octave.

In the present experiment, the filters were used for the following purposes: (i) To measure the DC component of a random signal (low pass mode with a low cut-off frequency). (ii) To remove the DC part of a signal (high pass operation with a low cut-off frequency). (iii) Qualitative spectral analysis of a random signal (band pass operation). (iv) Improving the accuracy of spectral measurements in a desired bandwidth by filtering out the undesirable parts of the spectrum (low pass, high pass or band pass operation).
4.5 Correlation and Spectral Analysis System

Auto and cross-correlation of random force and turbulence signals were obtained by means of a Princeton Applied Research Model 100 Signal Correlator. For any two inputs, this instrument gives one hundred equally spaced points on the correlation curve over a time-delay range set by the user. A wide choice of time-delay ranges, between 0.1 msec. and 10 sec., is available. The correlation values are stored in analog fashion in capacitive memories. Several different output devices may be used for readout purposes. In the present experiment, an oscilloscope, DVM and a card punch were used for readout.

Power and cross-spectra of random signals were obtained using the spectral analysis system shown in Fig. B-1. The correlator output is connected to a CIMRON model 6840 Data Logging System, which acts as an A/D converter and drives an IBM 526 Summary Punch to provide punched cards of correlations. The punched card output is then Fourier transformed on an IBM 1130 to obtain spectra. A brief description of the spectral analysis technique is given in Appendix B. More details are given by Surry (Ref. 8) and Teunissen (Ref. 61).

4.6 Tape Recorder

All the analog tape recording was performed using an Ampex SP 300 four track AM/FM tape recorder. Only the FM mode was used in the present experiment, and all recording and playback was done at a tape speed of 7-1/2 ips. At this speed, the tape recorder has a useable frequency range of 0 to 1250 Hz. Each channel has a level setting, which enables continuous adjustment of its gain. In this experiment, the gains of all channels were set to unity at 5 Hz before recording. Amplitude response measurement showed that the gain at all frequencies in the range of interest, i.e., below 100 Hz, lies within 2% of the gain at 5 Hz. A simple frequency response correction was applied to the results, reducing this error to less than 1/2%. Phase response of the tape recorder is important only in cross-correlation measurement between two data channels. In such a measurement, it is the difference in the phase shifts of the two channels that affects the results, and this was found to be negligibly small.

4.7 Hot Wire Anemometry

All turbulent velocity measurements in this experiment were made with two channels of DISA type 55 D01 constant temperature hot wire anemometers and type 55D10 linearizers. Type 55A30 miniature cross-wire probe was used for measuring both the longitudinal and lateral components. Temperature compensation was provided with the help of a compensating probe mounted on the wind tunnel wall. The characteristics of the hot wire anemometer system were thoroughly investigated by Teunissen (Ref. 61) who also described the method of calibration and use.

V. CALIBRATIONS

Static and dynamic calibration of the two component balance as well as static calibration of the torque balance are described in this chapter. The static calibrations are relatively straightforward and are described briefly. The dynamic calibration is much more complex and is described in greater detail.
5.1 Static Calibration of the Two Component Balance

A description of the two component force balance is given in section 3.5. The purpose of static calibration is to obtain equations relating the steady force and moment on the propeller with the DC values of the LVDT outputs $E_1$ and $E_2$. The calibration was done by applying known values of normal force and pitching moment in the propeller plane. The results are equally valid for measuring the side force and yawing moment. Normal force is applied to the system by suspending the desired weight from the propeller shaft in the propeller plane. (The nose cone and the propeller are removed). Pitching moment is applied by means of a special arrangement shown in Fig. 5.2. The results of the normal force and pitching moment calibrations were plotted to yield $E_1$ vs. $Z$, $E_2$ vs. $Z$, $E_1$ vs. $M$ and $E_2$ vs. $M$. All these plots (not shown) were found to be straight lines passing through the origin, thus proving the linearity of the system. The slopes $\frac{\partial E_1}{\partial Z}$, $\frac{\partial E_2}{\partial Z}$, $\frac{\partial E_1}{\partial M}$ and $\frac{\partial E_2}{\partial M}$ are obtained from the plots. It can be easily shown that

\[
\begin{bmatrix}
Z \\
M
\end{bmatrix}
= \begin{bmatrix}
\frac{\partial E_1}{\partial Z} & \frac{\partial E_1}{\partial M} \\
\frac{\partial E_2}{\partial Z} & \frac{\partial E_2}{\partial M}
\end{bmatrix}^{-1}
\begin{bmatrix}
E_1 \\
E_2
\end{bmatrix}
\]  

(5-1)

The matrix inversion yielded the following calibration equations

\[
Z = V_1 = 1.436 \left( E_1 + 0.494 E_2 \right)
\]

\[
M = V_2 = 4.18 \left( 0.91 E_1 + E_2 \right)
\]  

(5-2)

Plots of $Z$ vs. $V_1$ and $M$ vs. $V_2$ are shown in Fig. 5.1. An experimental check revealed that $\frac{\partial V_1}{\partial M} = \frac{\partial V_2}{\partial Z} = 0$, as required.

5.2 Torque Calibration

Torque calibration consists of determining the torsional stiffness of the flexible wire coupling (Fig. 3.1). This was done with the wire coupling installed in the apparatus. With the propeller shaft held fixed, weights were added to a torque arm attached to the rear end of the motor shaft. This applies a known amount of torque to the wire coupling. The angular deflection of the coupling is equal to the rotation of the arm, which was measured. Torque versus angular deflection of the coupling is plotted in Fig. 5.1. The slope of the plot gives the torsional stiffness of the coupling which was found to be 0.318 oz.-in/degree.

5.3 Dynamic Calibration of the Two Component Balance

In order that an accurate measurement of the dynamic forces on the propeller be made, it is necessary to determine the frequency response of the balance and apply the necessary corrections to the measured spectra. This is
particularly necessary because the fundamental natural frequency of the balance is only 60 Hz and hence the results could be in considerable error above 20 Hz, if no frequency response corrections are made. This section describes the technique of dynamic calibration and the method of applying frequency response corrections.

5.3.1 Input-Output Relations for the Balance

In order to decide what dynamic calibrations were needed, the following analysis of the balance system was performed. The balance may be considered as a two input, two output linear system - the two inputs being the force and the moment on the propeller while the two outputs are the voltages \( V_1 \) and \( V_2 \) (Fig. 5.3). The problem is to determine the power and cross-spectra of the force and the moment, knowing the power and cross-spectra of the signals \( V_1 \) and \( V_2 \).

Using the general relations given by Etkin (Ref. 3, Eq. 3.4, 48) it can be shown that

\[
\begin{bmatrix}
\Phi_{V_1 V_1} & \Phi_{V_1 V_2} \\
\Phi_{V_2 V_1} & \Phi_{V_2 V_2}
\end{bmatrix}
= G^* G^T \begin{bmatrix}
\Phi_{FF} & \Phi_{FM} \\
\Phi_{MF} & \Phi_{MM}
\end{bmatrix}
\]  

(5-4)

where the frequency response matrix \( G \) is given by,

\[
G(i\omega) = \begin{bmatrix}
G_{V_1 F}(i\omega) & G_{V_1 M}(i\omega) \\
G_{V_2 F}(i\omega) & G_{V_2 M}(i\omega)
\end{bmatrix}
\]  

(5-5)

(\( G_{yx}(i\omega) \) denotes the frequency response of output y to an input x. \( G^* \) in Eq. (5-4) denotes the complex conjugate of \( G \) and \( G^T \) denotes the transpose of \( G \)).

For simplicity, denote the spectral matrix on the left hand side of Eq. (5-4) by \( \Phi_V \) and the one on the right hand side by \( \Phi_F \), i.e.,

\[
\Phi_V = \begin{bmatrix}
\Phi_{V_1 V_1} & \Phi_{V_1 V_2} \\
\Phi_{V_2 V_1} & \Phi_{V_2 V_2}
\end{bmatrix}
\]  

(5-6)

and

\[
\Phi_F = \begin{bmatrix}
\Phi_{FF} & \Phi_{FM} \\
\Phi_{MF} & \Phi_{MM}
\end{bmatrix}
\]  

(5-7)

Then Eq. (5-4) may be rewritten as

\[
\Phi_F = [G^*]^{-1} \Phi_V [G^T]^{-1}
\]  

(5-8)

where the superscript "-1" denotes inverse matrix.

Noting that

\[
[G^*]^{-1} = [G^{-1}]^*
\]  

(5-9)
and
\[
[G^T]^{-1} = [G^{-1}]^T
\]
and letting
\[
H = G^{-1}
\]
Eq. (5-8) may be re-written as,
\[
\Phi_F = H^* \Phi_V H^T
\]
(5-11)

Equation (5-10) and (5-11) show how the force spectral matrix \( \Phi_F \) may be obtained from the voltage spectral matrix \( \Phi_V \) and the frequency response matrix \( G \). Thus it is seen that the objective of the dynamic calibration should be to obtain the frequency response matrix \( G \), given by Eq. (5-5). It must be noted that each element of the \( G \) matrix is a complex function of frequency. In terms of the real and imaginary parts, it may be written as,
\[
G = A + iB
\]
(5-12)

If \( C \) is the matrix of amplitude responses and \( \Theta \) the matrix of phase responses it can be shown that
\[
A_{ij} = C_{ij} \cos \theta_{ij} \quad i = 1, 2
\]
and
\[
B_{ij} = C_{ij} \sin \theta_{ij} \quad j = 1, 2
\]
(5-13)

In the dynamic calibrations described below, the elements of the matrices \( C \) and \( \Theta \) are measured, from which \( G \) may be evaluated. Knowing \( G \), and using Eq. (5-10) and (5-11), \( \Phi_F \) may be evaluated. Measurement of \( C \) and \( \Theta \) involves the determination of the amplitude as well as the phase response of the following: (a) \( V_1 \) due to \( F \) (b) \( V_1 \) due to \( M \) (c) \( V_2 \) due to \( F \) (d) \( V_2 \) due to \( M \).

It may be noted that both \( V_1 \) and \( V_2 \) are linear combinations of the LVDT outputs \( E_1 \) and \( E_2 \). The static calibration (Sec. 5.1) ensures that, at zero frequency, \( V_1 \) directly gives the force \( F \) in ounces and \( V_2 \) gives the moment \( M \) in oz.-in. That is,
\[
C_{V_1 F} (0) = 1
\]
\[
C_{V_1 M} (0) = 0
\]
\[
C_{V_2 F} (0) = 0
\]
\[
C_{V_2 M} (0) = -1
\]
(5-14)

and
\[
\theta_{V_1 F} (0) = 0
\]
\[
\theta_{V_1 M} (0) = 0
\]
\[
\theta_{V_2 F} (0) = 0
\]
\[
\theta_{V_2 M} (0) = 0
\]
(5-15)
5.3.2 **Amplitude Response**

(i) **Force Input**

In order to measure the amplitude responses to force input, it is necessary to apply a sinusoidally varying normal force in the propeller plane and then measure the $V_1$ and $V_2$ responses. This must be done over a range of frequencies. Such a forcing function was obtained by simply attaching a mass of unbalance to the propeller hub and rotating the propeller at several speeds. The amplitude of the force input may be calculated knowing the mass of unbalance $m$, radial location of the unbalance $r$, and propeller rotational speed. The corresponding amplitudes of $V_1$ and $V_2$ may then be measured and the amplitude responses calculated over a range of frequencies.

One measurement problem must be overcome, however, to obtain results with the desired accuracy. Even without the additional unbalance, sinusoidal forces do exist on the balance. These are due to (a) residual unbalance of the propeller and (b) forces exerted on the rear end of the propeller shaft by the coupling, due to misalignments. These forces give rise to sinusoidal $V_1$ and $V_2$ signals. Also, when the propeller rotates, certain natural modes of the balance are excited and this gives rise to some additional sinusoids. Necessary corrections must be made to remove or reduce the effect of these "noise" signals. This makes the frequency response measurement quite complex.

Briefly, the method adopted was as follows:

(a) A circuit is patched up on the EAI 221 R analog computer to obtain the signal $V_1$ from the LVDT outputs $E_1$ and $E_2$ (Eq. 5-3).

(b) With no addition of mass unbalance to the propeller, the motor is turned on and its speed is adjusted to the desired speed $n$.

(c) The signal $V_1$ will contain sinusoids at shaft frequency as well as at the natural frequencies of the system. The latter frequencies were observed to be generally high ($\sim 200$ Hz). The $V_1$ signal is passed through a band pass filter (with cut-off frequencies $0.75n$ and $1.5n$) to remove DC offsets and high frequency noise. In practice, this filtering was not found adequate. Therefore, the output of the band pass filter is passed through a pair of low pass filters connected in series. The result of this filtering is an almost pure sinusoidal signal at shaft frequency. It must be noted, however, that the filters introduce attenuation and phase shift, which must be corrected for.

(d) The rms value of the filtered $V_1$ signal (referred to below as $V_{1F}$) is measured. The rms value of the shaft frequency component of $V_1$ is obtained by applying the filter gain correction.

(e) Let the outputs of the front and the rear speed transducers be denoted by vectors $S_F$ and $S_R$. These along with the vectors $V_1$ and $V_{1F}$ are shown in Fig. 5.4. Using the method described in Appendix C, the phase difference

$\theta_{V_1M}(o)$ and $\theta_{V_2F}(o)$ are undefined.
The phase difference $\alpha_1$, between $V_1$ and $S_F$ (Fig. 5.4) is measured. The phase difference $\alpha_2$ between $S_R$ and $S_F$ at speed $n$ is also measured. Knowing these and the total phase response of the filters $\alpha_3$, the phase difference $\alpha$ between $V_1$ and $S_F$ is obtained. Now the vector $V_1$ is known both in magnitude and direction with respect to the reference vector $S_F$.

(f) Now a known mass $(m)$ of unbalance is attached to the propeller hub, at a known radius $(r)$. (The phase response technique used in the present experiment requires that the mass be attached at the same azimuth position for all rotational speeds). The propeller is turned at speed $n$. Now the rotating mass of unbalance causes a sinusoidal normal force of amplitude given by,

$$Z_{rms} = \sqrt{2} \frac{n^2 m \pi}{\sqrt{2}}$$

(5-16)

(g) Let $V_1'$ be the rms value of the shaft frequency component of the $V_1$ signal, and $\alpha_1'$ its phase angle relative to $S_F$ (Fig. 5.4). These are measured using the same method as was used for measuring $V_1$ and $\alpha$.

(h) The effect of the rotating mass of unbalance alone is given by the vector difference $V_1 = V_1' - V_1$. Fig. 5.4 shows a graphical method of obtaining $V_1$. Let the phase lead of $V_1$ with respect to the reference vector $S_F$ be denoted by $\beta$.

(i) The amplitude response of $V_1$ to force input is given by

$$C_{V_1F}(n) = \frac{V_1}{Z_{rms}} = \frac{\sqrt{2} V_1}{4\pi^2 mn^2}$$

(5-17)

Measurement of the amplitude response of $V_1$ to force input is done in an exactly identical manner. In the above experiment, the largest possible unbalance mass is used at each speed to obtain the best accuracy. This is limited by the LVDT linear range and the noise level. Masses of up to 11 gms (consisting of carefully shaped modelling clay) were used at low speeds (~ 5 rps). At high speeds (50-70 rps.), small pieces of self adhesive tapes weighing 0.1 to 0.2 gm. were adequate. These masses were attached on the propeller hub at a radius of about 5/8".

(ii) Moment Input

A sinusoidal pitching moment is applied to the propeller shaft in the following manner. The nose cone of the propeller was removed and a U-shaped thin aluminum strip was bolted to the propeller hub (Fig. 5.5). Care is taken to see that the two arms of the U-frame are equidistant from the propeller axis. Two identical masses, each of mass $m$, were attached to the opposite arms of the U-frame at equal radii $(r)$ but with an axial separation $(d)$. When the propeller
rotates at a speed \( n \), each mass is subjected to a centrifugal force \( 4\pi^2 mr^2 n^2 \). The two forces acting a distance \( d \) from each other give rise to a sinusoidal pitching moment. The rms value of the pitching moment is given by

\[
M_{\text{rms}} = \frac{4\pi^2 mr^2 n^2}{\sqrt{2}}
\]

(5-18)

The measurement of responses of \( V_1 \) and \( V_2 \) to moment input is carried out in a manner quite similar to the force input case. Let \( V_{1M} \) and \( V_{2M} \) be the responses. Then,

\[
C_{V_{1M}}(n) = \frac{V_{1M}}{M_{\text{rms}}} \quad (5-19)a.
\]

and

\[
C_{V_{2M}}(n) = \frac{V_{2M}}{M_{\text{rms}}} \quad (5-19)b.
\]

5.3.3 Phase Response

The method of obtaining phase response to force input is described below. The moment input case may be treated in a similar manner. Figure 5.4 shows the vector \( V_{1Z} \) and its phase relation to the reference vector \( S_F \). \( V_{1Z} \) is the sinusoidal voltage \( V_1 \) caused by a sinusoidal force \( Z \). The phase response of \( V_1 \) is given by the angle between the \( Z \) and \( V_1 \) vectors. However, the direction of the \( Z \) vector is not known and must be determined. The following is a description of the method of finding the phase response.

(i) The angle \( \beta \) (Fig. 5.4) is measured at various propeller speeds making sure that the mass of unbalance is always applied at the same azimuth position. This ensures that the vector always has the same (though unknown) phase relationship with the reference vector \( S_F \).

(ii) Assume that the phase response remains constant over the frequency range 0 - 5 Hz. (This is a very reasonable assumption since the balance is very lightly damped and has a fundamental natural frequency of 60 Hz). The static calibration ensures that the phase response at zero frequency is zero. Therefore, under the above assumption, the \( Z \) vector coincides with the \( V_{1Z} \) vector for a speed of 5 rps. (Fig. 5.4). When the speed is changed, the orientation of \( Z \) remains unchanged because of (i), but \( V_{1Z} \) rotates through an angle given by the phase response. Thus it may be seen that the phase response (phase lag) at a frequency \( n \) is given by

\[
\theta_{V_{1Z}}(n) = \beta(5) - \beta(n) \quad (5-20)
\]

The phase response of \( V_2 \) is obtained in a similar fashion. The phase responses of \( V_1 \) and \( V_2 \) to moment input were obtained using \( \beta(20) \) as the reference \( \beta \). This was done because 20 Hz was the lowest frequency for which responses to
moment input were measured. An observation of the results shows that the phase responses are nearly constant at frequencies below 40 Hz and hence no significant errors are likely to result.

5.4 Results

The amplitude and phase responses to force input were measured over a frequency range of 5 to 70 Hz. Two replicates of these results were obtained. Three replicates of responses to moments input were obtained over a frequency range of 20 to 70 Hz. The vector calculations indicated in Fig. 5.4 were done on a digital computer. The static calibration (Sec. 5.1) gives the responses at zero frequency. All these results are shown in Fig. 5.6 to 5.9. As may be seen from these plots, the repeatability of measurements is fairly good in spite of the complexity of the method. As may be expected the off-diagonal terms in the amplitude response matrix, i.e., \( V_1 \) due to moment and \( V_2 \) due to force, are small at low frequencies. This results in large inaccuracies in the corresponding phase response measurements. (The reason for this may be seen easily by referring to Fig. 5.4. Imagine a situation in which \( V_1 \) and \( V_1' \) are nearly identical so that \( V_1 \) is small. Then, small errors in measurement of \( V_1 \) and \( V_1' \) lead to large errors in \( \beta \) and hence in the phase response. Also, small errors in \( \alpha \) and \( \alpha' \) can conceivably reverse the \( V_1 \) vector and cause a 180° error). A certain amount of judgement was used in drawing the off-diagonal phase response curves at low frequencies. It must be noted, however, that, at low frequencies, the off-diagonal elements of the frequency response matrix have a negligible influence on the final results due to their small amplitudes.

The complex frequency response matrix \( G \) is obtained from the amplitude and phase responses by the method described in Sec. 5.3.1. It is inverted to obtain the matrix \( H \) required for the spectral calculation.

VI. GENERATION AND MEASUREMENT OF TURBULENCE IN THE WIND TUNNEL

6.1 Generation of Turbulence

Turbulence was generated in the wind tunnel by placing a wooden bi-planar grid in the first diffuser. The grid has a mesh size of 1/4" x 1/4" and is made of bars 1-7/8" x 3/4" in cross-section. The geometry of the grid is shown in Fig. 6.1. The locations of the grid and the two test stations are shown in Fig. 6.1. Experiments in turbulent flow were conducted at test station 2 behind the grid. Figure 6.3 shows the apparatus mounted behind the grid. Surry (Ref. 8) discussed the principles of design and development of a turbulence generating grid, with special emphasis on certain aspects that affect its use in the UTIAS Low Speed Wind Tunnel. The particular grid used in the present experiment was designed by Nettleton (Ref. 58). The grid generated turbulence has a longitudinal scale of ~ 5" and a lateral scale of ~ 2-1/2", at the propeller station. The u-intensity is about 13% while the v- and w-intensities are about 10%.

6.2 Measurement of Turbulence

The following measurements were made at Test Station 2, with the
propeller model removed. (i) Mean velocities, rms values, power and cross spectra of turbulence velocities at the locations of the centre of the propeller corresponding to various angles of attack. (ii) Horizontal and vertical surveys of mean velocity profiles and turbulence intensities. All these measurements were made at two test section speeds corresponding to advance ratios of 0.8 and 0.04. These measurements were found to be adequate for most purposes in the present project. When necessary, reference is made to the detailed measurements obtained by Nettleton (Ref. 58).

A brief description of the instrumentation used for turbulence measurements is given in Sec. 4.7. A DISA type 55A38 miniature cross wire probe was used for measuring the mean velocity as well as the three turbulent velocity components. Figure 6.4 shows the mounting arrangement of the probe. A schematic of the data handling system is shown in Fig. 6.5. The anemometer outputs are linearized by means of DISA linearizers to yield outputs proportional to the turbulence velocities. The linearizer outputs are then processed on the PACE 221 R Analog Computer to yield the mean velocity \( \bar{U} \) and turbulent velocities \( u' \) and \( v' \) (or \( w' \)) directly in fps. The mean velocity is measured by means of a digital voltmeter. The rms values of turbulence velocities are obtained by using Bruel and Kjaer model 2417 random noise volt meters. The turbulence velocity signals were recorded on an Ampex SP-300 Tape Recorder (Sec. 4.6) for later processing to obtain spectra. The spectral analysis system is described in Appendix B. The outputs of the tape recorder were first passed through a pair of low pass filters, with properly chosen cut-off frequencies, in order to practically eliminate the aliasing problem (Appendix B). The filter outputs were then processed on the Correlation and Spectral Analysis System to yield the required correlations and spectra.

To co-ordinate system used in turbulence measurement is shown in Fig. 6.2. The probe mounting arrangement is shown in Fig. 6.4. The power spectra of \( u^* \), \( v^* \) and \( w^* \) at the locations of the propeller centre corresponding to angles of attack of 0°, 30° and 60° are shown in Fig's. 6.6 to 6.8. Each of the spectral plots shown are obtained from one test run only. The correlator noise introduces some little 'bumps' in the spectra which may be removed by taking several samples and averaging. Due to lack of time this was not done. The bumps were removed simply by drawing smooth curves through the spectral points. Only the smoothed versions of the spectra are shown in Fig. 6.6 to 6.8. The effects of correlator noise on spectral measurement is discussed in Chapter 7 in greater detail. The variations of the mean velocity and turbulence intensities with \( y' \) are shown in Fig's. 6.6 to 6.8. The variations of the mean velocity and turbulence intensities with \( z' \) are shown in Fig's. 6.10 and 6.11.

VII. FORCE AND MOMENT MEASUREMENTS

The following force measurements were made: (i) steady forces* \( (Y, Z, M, N \) and \( Q) \) on the propeller operating in a uniform flow. (ii) Power spectra of the random forces on the propeller \( (F_{zz} \) and \( F_{mm} \) ) operating in a turbulent flow. (iii) Sinusoidal components of the normal force and pitching moment at

* The words 'steady forces' and 'steady moments' are used in this and the following chapters to denote the dc components of the force and moment responses of the propeller, which contain sinusoidal and/or broad band components as well.
the shaft frequency. The steady force measurements were made with the model installed at Test Station 1 while the random and sinusoidal forces were measured with the model installed at Test Station 2 (Fig. 4.1). The measurements were obtained at several advance ratios and angles of attack. Steady forces on a propeller at angle of attack have been measured by several investigators before (Ref's. 18, 19 and 20). The need for the present steady force measurements is discussed in Sec. 3.4. The random force measurements reported here are believed to be the first such measurements. Both the steady and random in-plane forces were measured by means of the two-component balance described in Sec. 3.5. The torque was measured by means of a simple technique that utilizes the flexible coupling and the two speed transducers.

7.1 Steady In-Plane Force Measurement

Steady force measurements were carried out at Test Station 1. The location of the test station as well as the cross section of the wind tunnel at that location are shown in Fig. 4.1. The 9:1 contraction between the settling chamber and the test station ensures an uniform flow of very low turbulence at the test station. Figure 3.2 shows the mounting arrangement of the apparatus. The vertical strut on which the apparatus is mounted passes through a clearance hole in the test section floor and is supported on a turntable. The turntable may be rotated in a horizontal plane through an angle of 60°. This provides a convenient means of yawing the propeller. Static calibration of the two component balance is described in Sec. 5.1. The calibration equations (Eq. 5.2) give the relationship between the LVDT outputs and an in-plane force and moment. The data handling system for the steady force measurement is shown in Fig. 7.1. The LVDT outputs E1 and E2 are processed on the PACE analog computer (using the relations given in Eq. (5.2)) to obtain the signals V1 and V2. When the propeller is in operation the signals V1 and V2 were found to contain AC as well as DC parts, the former consisting primarily of sinusoids at shaft frequency and the natural frequencies of the system. In the present measurement, the AC part is of no interest and hence is filtered out by means of a pair of low pass filters set at about 1 Hz. The outputs of the filters give the steady values of V1 and V2 which are measured by digital voltmeters. V1 gives the in-plane force in ounces while V2 gives the in-plane moment in oz.-in.

The following procedure was used for measuring the steady forces and moments. First the balance was set up for the measurement of normal force and pitching moment. This is done by rotating the central part of the cowling together with the balance (Fig. 3.1) to bring the cantilever springs into a horizontal position. The balance cowling was then secured to the motor cowling with bolts. By comparing the present orientation of the balance with that during the static calibration, proper signs for the calibration equations were established. An appropriate circuit was patched on the analog computer (Fig. 7.1). With the propeller set at the required angle of yaw, the following measurements were made.

1) With the wind off, the propeller was turned at a low speed (≈ 3 rps) and the DC values of V1 and V2 were measured. These give the zero readings V01 and V02. (The zero readings must be obtained while the propeller is rotating for the following reason. When the propeller is stationary the outputs V1 and V2 depend on the azimuthal position of the propeller. This is due to the force exerted by the coupling on the propeller shaft. The force is caused by the small but unavoidable misalignment between the coupling and the
two shafts it joins. The direction of the force depends on the azimuth angle of the propeller shaft. When the propeller rotates at 3 rps., the coupling misalignment gives rise to a sinusoid at 3 Hz, which is filtered out to obtain the true zero reading).

2) Now the propeller was turned at full speed (100 rps). The tunnel was turned on and the wind speed adjusted to obtain a value corresponding to a pre-determined advance ratio. \( V_1 \) and \( V_2 \) were measured. These measurements were made for advance ratios of 0.4, 0.6, 0.7 and 0.8.

3) The normal force on the propeller is given by \( Z = V_1 - V_1^0 \) and the pitching moment is given by \( M = V_2 - V_2^0 \).

The above measurements were made at several angles of yaw over a range of \(-60^\circ\) to \(+60^\circ\), at \(10^\circ\) intervals. Four replicates of all the above measurements were obtained. As expected, the forces at negative angles of attack were found to be equal in magnitude (within the scatter of the results) and opposite in sign to those at positive angles of attack. Therefore, the measurements were treated as eight replicates of data for angles of yaw ranging from 0 to \(60^\circ\). The means and standard deviations of the eight replicates were calculated.

Next the balance was rotated through \(90^\circ\) and secured in the new position. It was now set up for measuring the side force and yawing moment. All the previous steps were repeated to obtain eight replicates of side force and yawing moment measurements over an angle of attack range of \(0^\circ\) to \(60^\circ\) and an advance ratio range of 0.4 to 0.8.

The eight replicates of the force and moment data were averaged and nondimensionalized using the following equations:

\[
C_F = \frac{F}{\rho n^2 D^4} \\
C_M = \frac{M}{\rho n^2 D^5}
\]

where \( F \) stands for a force and \( M \) stands for a moment. The sign conventions for the forces on a propeller in yaw and a propeller in pitch are shown in Fig. 7.2. Using a simple coordinate transformation, the forces on a propeller in pitch may be deduced from those on a propeller in yaw. Such a conversion of the experimental data was made and the results are shown in Fig. 7.3 to 7.6. The variation of the force and moment coefficients with \( \alpha \) is shown in Figs. 7.3 and 7.4 while the variation with \( \beta \) is shown in Figs. 7.5 and 7.6. The reason for presenting the steady force data for a propeller in pitch rather than a propeller in yaw is to enable easy correlation with the spectral data, which were obtained for a propeller in pitch. Typical standard deviation bars are also shown on the force and moment curves.

7.2 Discussion of Steady Force Measurements

From the steady force measurements shown in Fig. 7.3 to 7.6 the
following conclusions may be made.

1) For a given advance ratio, the normal force, pitching moment, and yawing moment increase monotonically with \( \alpha \) in the range \( \alpha = 0^\circ \) to \( 60^\circ \). The side force shows an increase up to an angle of about \( 50^\circ \) and then starts decreasing.

2) At a given \( \alpha \), all the forces and moments show an increase with an increase in advance ratio.

3) The yawing moment is the larger of the two moments and is nearly twice the size of the pitching moment. The normal force is about five times as big as the side force. At \( \alpha = 60^\circ \) and \( J = 0.8 \), the yawing moment and normal force coefficients are approximately equal to 0.02 and 0.03 respectively, compared to an estimated value of static thrust coefficient equal to 0.2, i.e., they are respectively 10% and 15% of the thrust.

4) Even though the variations of the forces with \( \alpha \) are non-linear over the full range of \( \alpha \), variations over ranges as large as \( 20^\circ \) can be considered linear in most cases. This fact is made use of in the derivative method of propeller response calculation described in Chapter 8.

5) Figures 7.5 and 7.6 show that the variation of in-plane forces with \( J \) is nearly linear over the advance ratio range 0.4 to 0.8.

The trends of variation of in-plane forces with \( \alpha \) and \( J \) as well as the relative magnitude of the force components agree well with the data published by Wickens (Ref. 18) and Yaggy and Rogallo (Ref. 19). The derivatives \( \partial C_x/\partial \alpha \) and \( \partial C_y/\partial \alpha \) were determined by means of graphical differentiation and compared with those predicted by Ribner's method (Ref. 17) for zero angle of attack. The results are shown in Table 7.1, and the agreement is seen to vary from fair to good. Ribner's method predicts zero values for the derivatives \( C_y \) and \( C_m \), but the present experimental data as well as those given in Ref. 18 \( \aleph \) and 19 show otherwise. This discrepancy is believed to be due to an inaccurate treatment in Ribner's method of the unsteady blade element aerodynamics (involving a neglect of phase lags) of a propeller at angle of attack.

**TABLE 7.1**

**COMPARISON OF MEASURED AERODYNAMIC DERIVATIVES WITH THOSE PREDICTED BY RIBNER'S THEORY**

<table>
<thead>
<tr>
<th>( C_{za} ) at ( \alpha = 0 )</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>ADVANCE RATIO</strong></td>
<td>0.4</td>
</tr>
<tr>
<td><strong>EXPERIMENTAL</strong></td>
<td>0.0204</td>
</tr>
<tr>
<td><strong>THEORETICAL (Ref. 17)</strong></td>
<td>0.0164</td>
</tr>
<tr>
<td><strong>PERCENTAGE DIFFERENCE</strong></td>
<td>19.6%</td>
</tr>
</tbody>
</table>

27
The aerodynamic derivatives provide important data for the stability analysis of a propeller V/STOL aircraft, especially during the transition phase. Also, the derivative method of calculating the propeller response to turbulence (Chapter 8) is based on the availability of such data. Table 7.2 gives the measured aerodynamic derivatives at several angles of attack and advance ratios.

**TABLE 7.2**

**AERODYNAMIC DERIVATIVES**

(i) Derivatives with Respect to \( \alpha \)

<table>
<thead>
<tr>
<th>( J )</th>
<th>( \alpha )</th>
<th>( C_{y \alpha} )</th>
<th>( C_{z \alpha} )</th>
<th>( C_{m \alpha} )</th>
<th>( C_{n \alpha} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.4</td>
<td>0°</td>
<td>0.0032</td>
<td>-0.0204</td>
<td>0.0061</td>
<td>-0.0112</td>
</tr>
<tr>
<td>0.4</td>
<td>30°</td>
<td>0.0010</td>
<td>-0.0155</td>
<td>0.0080</td>
<td>-0.0104</td>
</tr>
<tr>
<td>0.4</td>
<td>60°</td>
<td>-0.0049</td>
<td>-0.0077</td>
<td>0.0099</td>
<td>-0.0090</td>
</tr>
<tr>
<td>0.8</td>
<td>0°</td>
<td>0.0122</td>
<td>-0.0442</td>
<td>0.0079</td>
<td>-0.0150</td>
</tr>
<tr>
<td>0.8</td>
<td>30°</td>
<td>0.0072</td>
<td>-0.0354</td>
<td>0.0090</td>
<td>-0.0174</td>
</tr>
<tr>
<td>0.8</td>
<td>60°</td>
<td>-0.0065</td>
<td>-0.0203</td>
<td>0.0194</td>
<td>-0.0214</td>
</tr>
</tbody>
</table>

(ii) Derivatives with Respect to \( J \)

<table>
<thead>
<tr>
<th>( J )</th>
<th>( \alpha )</th>
<th>( C_{y J} )</th>
<th>( C_{z J} )</th>
<th>( C_{m J} )</th>
<th>( C_{n J} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.4</td>
<td>30°</td>
<td>0.010</td>
<td>-0.028</td>
<td>0.003</td>
<td>-0.007</td>
</tr>
<tr>
<td>0.4</td>
<td>60°</td>
<td>0.008</td>
<td>-0.043</td>
<td>0.008</td>
<td>-0.021</td>
</tr>
<tr>
<td>0.8</td>
<td>30°</td>
<td>0.009</td>
<td>-0.030</td>
<td>0.002</td>
<td>-0.006</td>
</tr>
<tr>
<td>0.8</td>
<td>60°</td>
<td>0.016</td>
<td>-0.058</td>
<td>0.005</td>
<td>-0.021</td>
</tr>
</tbody>
</table>

7.3 Torque Measurement

Depending upon the operating condition of the propeller, the flexible wire coupling (Sec. 3.5) joining the propeller and motor shafts twists by as
much as 30°. The coupling is made of a steel wire 4" long and makes a good linear torsional spring. It is therefore possible to determine the propeller torque by measuring the torsional deflection of the coupling. The torque calibration consists simply of determining the torsional stiffness of the wire coupling and was described in Sec. 5.2. Details of torque measurement technique are given below.

The two speed transducers (Sec. 3.5) provide a convenient means of measuring the torsional deflection of the coupling and hence the propeller torque. The front speed transducer gives three pulses per shaft revolution, while the rear one gives one pulse per revolution. Figure 7.7 shows an oscilloscope picture of typical speed transducer outputs. The three pulses obtained from the front speed transducer during every revolution have noticeable differences in shape. This makes it possible to measure the phase difference between the two pulse trains even though they have different frequencies. The phase difference θ is measured accurately (to within ±2%) by expanding the time scale of the signals on the oscilloscope.

The following procedure was used to measure torque. First, with the wind tunnel off, the propeller is set at the desired angle of yaw and turned at a low speed (~3 rps). The phase difference between the speed transducer signals θ₀ is measured. θ₀ corresponds to a nearly zero torque on the propeller. Now the propeller speed is increased to the desired speed n. The tunnel is turned on and the wind speed is adjusted to give the required advance ratio. The aerodynamic torque on the propeller causes the wire coupling to twist, thereby changing the phase difference θ. Let θ be the phase difference in the new condition. The twist of the coupling Δθ is given by

\[ Δθ = θ - θ₀ \]  \hspace{1cm} (7-2)

If \( K_t \) is the torsional stiffness of the wire coupling, the torque on the propeller is given by

\[ Q = -L = K_t Δθ \]  \hspace{1cm} (7-3)

The torque coefficient is given by

\[ C_q = \frac{Q}{ρn^2D^5} \]  \hspace{1cm} (7-4)

The propeller torque was measured for angles of yaw ranging from 0° to 60° and advance ratios of 0 to 0.8. Eight replicates of the data were obtained and the means and standard deviations calculated. The mean data are shown in Fig. 7.8. (The figure gives the data for a propeller in pitch. It can easily be shown that the torque on a propeller in pitch is equal to that on a propeller in yaw for the same angle of attack). A typical scatter bar is also shown in Fig. 7.8.

7.4 Measurement of Propeller Response to Turbulence

This section describes the method used for measuring the random forces and moments on the model propeller operating in a turbulent flow. As mentioned in Chapter 6, turbulence was generated in the wind tunnel by means of a grid. The apparatus was installed at Test Station 2, 11 feet behind the grid. Figure 4.1 shows the locations of the grid and the test station.
Turbulence at the test station is fairly homogeneous and intense. The results of turbulence measurements at Test Station 2 are given in Chapter 6.

Figure 6.3 shows the apparatus mounted at Test Station 2 behind the grid. The apparatus is supported at the centre of the tunnel by means of two hollow steel pipes bolted to the motor cowling on either side. The pipes have an O.D. of 1-1/4" and an I.D. of 3/4". They pass through clearance holes in the tunnel walls and are secured to steel support blocks at the outer ends (Fig. 7.9). The support blocks are bolted to 1/4" thick steel plates which in turn are secured to a heavy steel structure. Also shown in Fig. 7.9 is an arrangement used to measure the angle of attack of the propeller and to change it to the desired value.

To measure the propeller response, the grid is installed in place and the propeller is set at the desired angle of attack. A schematic of the data recording phase of the data handling system is shown in Fig. 7.10. The necessary electrical connections are made and the instruments are adjusted for operation in the desired modes. The $V_1$ and $V_2$ signals are passed through two high pass filters with their cut-offs set at 1 Hz, in order to remove the DC components. The latter are of no interest in the present experiment and their removal enables spectral measurements with better accuracy. The $V_1$ and $V_2$ signals and the two speed transducer signals were recorded on the Ampex tape recorder (Sec. 4.6) which was operated in the FM mode at a tape speed of 7-1/2 i.p.s. The speed of the propeller was monitored by connecting the rear speed transducer signal to a frequency counter. The propeller was run at a speed of 70 rps. The wind tunnel was turned on and the wind speed adjusted to obtain the desired advance ratio. The signals were recorded on a tape for a period of 3 minutes. Such recording was done at three angles of pitch ($0^\circ$, $30^\circ$ and $60^\circ$) and two advance ratios ($0.4$ and $0.8$).

The next phase of data handling (Fig. 7.11) is the determination of the power and cross-spectra of $V_1$ and $V_2$ signals. The spectral analysis technique described in Appendix B is used for this purpose. Auto and cross-correlations were obtained first, and then Fourier transformed to obtain the spectra. Figure 7.12 shows typical $V_1$ and $V_2$ signals along with the speed transducer signals. It is clear from this figure that much of the energy in the $V_1$ and $V_2$ signals is at shaft frequency (70 Hz) and higher frequencies. These high frequency components come from mechanical sources such as unbalance, resonance etc. and are of no interest in the present experiment. In order to obtain an accurate measurement of the aerodynamic forces in the frequency range of interest, this high frequency 'noise' must be filtered out. This was done with the help of Multimetrics and Krohn-Hite filters (Sec. 4.4) operated in the low pass mode with their cut-offs set at 50 Hz. For cross-correlation measurements, the filters were carefully phase matched. The filtering also made it possible to reduce and practically eliminate the aliasing problem (Appendix B). The filtered $V_1$ and $V_2$ signals are amplified and shown in Fig. 7.13 on a contracted time scale, along with the rear speed transducer signal. It is clear from the figure that the signals now contain most of their energy in the desired frequency range. The auto and cross-correlations of the filtered signals are obtained by means of a PAR correlator (Sec. 4.5). Two minutes of recorded data is used to obtain each correlation. All correlations are obtained with the time delay range ($\tau_{\text{max}}$) set to 0.2 sec. This corresponds to a Nyquist frequency (Appendix B) of 250 Hz. Fourier transformation of the
correlations yields $\Phi_{V_1 V_1}$, $\Phi_{V_2 V_2}$, $C_{V_1 V_2}$, and $Q_{V_1 V_2}$.

The third and final phase of data handling (Fig. 7.14) consists of calculating the spectra of forces and moments on the propeller from those of $V_1$ and $V_2$. The relationship between the two sets of spectra is given by Eq. 5.11. The necessary calculations are carried out on an IBM 1130 Computer to obtain the power spectra of the normal force and pitching moment. Frequency response corrections are now applied for the tape recorder and the filters. The corrected spectra are then non-dimensionalized using the following equations.

$$\Phi_{C_F C_F} = \frac{\Phi_{FF}}{(\rho n^2 D^4)^2}$$

$$\Phi_{C_m C_m} = \frac{\Phi_{MM}}{(\rho n^2 D^5)^2} \quad (7-5)$$

Using the procedure described above, the spectra of normal force and pitching moment were obtained at angles of attack of 0°, 30° and 60° and advance ratios of 0.4 and 0.8. Four replicates of data were obtained corresponding to each operating condition. Typical variability of the spectral data is illustrated by Fig. 7.15, where four replicates of $\Phi_{V_2 V_2}$ and their average are plotted. These results correspond to a specific operating condition of the propeller but the variability is typical of all spectral data. To determine the effect of correlator noise on the variability, a single two minute long taped signal was processed eight times on the correlation and spectral analysis system. The eight replicates of $\Phi_{V_2 V_2}$ so obtained are shown in Fig. 7.16. A comparison of Figs. 7.15 and 7.16 shows that much of the variability in the spectral data is due to correlator noise. Therefore better spectral data may be obtained by either (i) increasing the correlator time constant and thus reducing the noise (Ref. 7) or (ii) repeated processing of each data record and averaging. Neither of these two methods is adopted in the present experiment. Since four separate data records are available corresponding to each operating condition of the propeller, it is considered quite adequate to process each record once and take the average of the four spectra so obtained. Such a method reduces the effect not only of the correlator noise but also of other random experimental errors. All the force and moment spectra presented in this report, unless otherwise mentioned, are averages of four replicates.

Figures 7.17 to 7.24 show the power spectra of normal force and pitching moment coefficients. (Also shown in these figures are theoretical spectra which were obtained using a method described in Chapter 8). The power spectra are given for a frequency range of 5 to 55 Hz. From these plots the following conclusions may be drawn.

1) As may be expected the power spectra of normal force and pitching moment roughly follow the same trend as the turbulence spectra (Fig. 6.6 to 6.8).
2) The angle of pitch appears to have a relatively small effect on the power spectra.

3) The advance ratio has a decided influence on the power spectra of the normal force and pitching moment. The spectra are larger at the larger advance ratio. (In the wind tunnel, the rms values of turbulence velocities are roughly proportional to the wind speed so that the non-dimensional intensities based on the flow velocity are independent of the advance ratio. For a full scale propeller operating in atmospheric turbulence, however, the rms values of turbulent velocities are independent of the propeller forward speed. In other words, the non-dimensional turbulence intensities based on the propeller forward speed decrease with increasing advance ratio. Therefore, the above conclusion based on the wind tunnel measurements is not applicable to the flight situation. If

\[ \sigma = \left( \frac{u^*}{\text{rms}} + \frac{v^*}{\text{rms}} + \frac{w^*}{\text{rms}} \right) / 3 \]

it is reasonable to expect \( \phi \frac{C_Z C_Z}{C_m C_m} \) and \( \phi \frac{C_Z C_Z}{J^2} \) to vary linearly with \( \sigma^2 \) for a given \( J \). Using this assumption, it can be shown that the variations of \( \phi \frac{C_Z C_Z}{C_m C_m} \) and \( \phi \frac{C_Z C_Z}{J^2} \) with \( J \) in the flight situation are identical to those of \( \phi \frac{C_Z C_Z}{J^2} \) and \( \phi \frac{C_m C_m}{J^2} \) with \( J \) in the wind tunnel situation. Even though these latter variations are not plotted, it can be seen from Figs. 7.21 to 7.24 that the advance ratio has a relatively small effect on

\[ \frac{\phi \frac{C_Z C_Z}{C_m C_m}}{J^2} \text{ and } \frac{\phi \frac{C_m C_m}{J^2}}{J^2} \]

Since the spectra of turbulence at the propeller station vary somewhat with the operating condition (\( J \) and \( \alpha \)) of the propeller (Chapter 6), the effects of \( J \) and \( \alpha \) on the propeller response cannot be assessed well by observing the experimental results only. The experimental results are analyzed in more detail in Chapter 9, where they are compared with theoretical results.

7.5 Measurement of Shaft Frequency Components of the Force and Moment Responses

The propeller being a rotating system, one would expect to find sinusoidal components in the propeller response, possibly at the shaft frequency and/or multiples of that frequency. For a three bladed propeller operating in a small scale turbulence, Barlow's theory (Ref. 1) predicts spikes in the propeller response at blade frequency and multiplies of that frequency. It would therefore be desirable to measure these components and observe their variations with \( J \) and \( \alpha \). Since the fundamental natural frequency of the balance is much smaller than the blade frequency, no meaningful measurements of these components were possible. However the shaft frequency components of \( V_1 \) and \( V_2 \) responses were measured using the cross-correlation technique described in Appendix C. The cross-correlation technique yields the amplitude \( S \) of the sinusoidal component as well as its phase lag \( \theta \) relative to the rear speed transducer signal. Since \( S \) and \( \theta \) contain contributions from mechanical sources such as unbalance, it is their variation with \( J \) and \( \alpha \) rather than their absolute values that can be
expected to throw some light on the propeller aerodynamics. The contribution from mechanical sources to $S$ and $\theta$ may be expected to be independent of $J$ and $\alpha$ while that from aerodynamic sources, if any, may be expected to vary with these parameters. In order to study these variations, $S/S_0$ and $(\theta-\theta_0)$ were plotted against $J$ for $\alpha = 0^\circ$, $30^\circ$ and $60^\circ$ (Fig. 7.25 and 7.26). $S_0$ and $\theta_0$ stand for the values at $J = 0$.

Based on a limited number of measurements (not presented here), it is believed that the experimental variability in $S/S_0$ could be as much as $\pm 25\%$ while that in $(\theta-\theta_0)$ could be $\pm 20^\circ$. Such large errors are inevitable since the shaft frequency (70 rps) is quite close to the fundamental natural frequency of the balance (60 Hz). Therefore the variations in $S/S_0$ and $(\theta-\theta_0)$ shown in Figs. 7.25 and 7.26 could very well be due to experimental errors rather than the propeller response to turbulence. Therefore it is concluded that the shaft frequency components of the normal force and pitching moment, if present, are too small to be detected by the present measurements.

**VIII. THEORY OF RESPONSE OF A PROPELLER TO TURBULENCE**

A new method is developed for calculating the response of a propeller to turbulence. This uses the measured aerodynamic derivatives of the propeller and will be referred to as the derivative method. The method and the assumptions underlying it are discussed in Sections 8.1 and 8.2 respectively. Methods of relaxing the assumptions are suggested in Sec. 8.3. The merits and demerits of the derivative method are compared with those of Barlow's method in Sec. 8.4. The use of the derivative method is illustrated in Sec. 8.5, where it is used to calculate the response of the model propeller to turbulence generated in the wind tunnel.

**8.1 Derivative Method**

This method is based on the following assumptions

1) **Point Approximation:** The propeller diameter is assumed to be much smaller than the smallest turbulence wavelength of interest. This condition is met if $L >> D$.

2) **Quasi-Steady Aerodynamics:** The propeller aerodynamics is assumed to be quasi-steady, i.e., the forces and moments on the propeller at any given instant are dependent only on the turbulence velocities at that instant.

3) **Linearity:** The propeller forces are assumed to vary linearly with $J$ and $\alpha$.

4) **Isotropy:** The turbulence field is assumed to be isotropic.

The last two assumptions are not basic to the method and can be relaxed at the expense of additional computational time.

The co-ordinate system used is shown in Fig. D-1. $x'$, $y'$, $z'$ are the wind axes and $x$, $y$, $z$ are the body axes of the propeller. Turbulence measurements (Chapter 6) were carried out in the wind axes system. For the present purpose these measurements must be converted into the body axes system. The following equations give the relationships between the velocities in the two axes systems.
\begin{align}
    u &= u' \cos \alpha - w' \sin \alpha \\
    v &= v' \\
    w &= w' \cos \alpha + u' \sin \alpha
\end{align} \quad (8-1)

Under the assumption of isotropy \( u' \), \( v' \) and \( w' \) are statistically independent. It can be then shown, using Eq. 2.14 and 2.16 that

\begin{align}
    \Phi_{uu} &= \Phi_{u'u'} \cos^2 \alpha + \Phi_{w'w'} \sin^2 \alpha \\
    \Phi_{vv} &= \Phi_{v'v'} \\
    \Phi_{ww} &= \Phi_{w'w'} \cos^2 \alpha
\end{align} \quad (8-2)

All velocities are non-dimensionalized by dividing them by \( nD \) and are referred to as 'advance ratios'. The following equations give the definitions of the advance ratios:

\begin{align}
    J &= \frac{\bar{u}}{nD} \\
    J_1 &= J \cos \alpha \\
    J_3 &= J \sin \alpha \\
    \bar{J}_1 &= \frac{u}{nD} \\
    \bar{J}_2 &= \frac{v}{nD} \\
    \bar{J}_3 &= \frac{w}{nD} \\
    J_A &= J_1 + J_1 \\
    J_L &= \sqrt{\bar{J}_2^2 + (J_3 + J_3)^2} \\
    J &= \sqrt{J_A^2 + J_L^2}
\end{align} \quad (8-3)

Under the point approximation, the turbulent velocities \( u, v \) and \( w \) are uniform over the propeller disk at any instant. The quasi-steady assumption enables us to treat the situation at each instant separately without regard to the past history. Using these two assumptions, one can calculate the instantaneous forces on a propeller in turbulent flow by treating the instantaneous situation as one of a steady translation of the propeller in atmosphere at rest. The effective speed and direction of such a motion depend on the mean advance ratio and angle of attack and the instantaneous values of \( u, v \) and \( w \). This fact is illustrated in Fig. 8.1, where a simpler problem, viz. a propeller moving through a sinusoidal \( w \)-gust is shown.
This figure shows how the motion through the w-gust causes periodic variations in the effective $J$ and $\alpha$. When all the turbulence velocities are present simultaneously, the situation is of course more complex.

Now consider a propeller moving through an isotropic turbulence field with a mean advance ratio of $J$ and mean angle of attack $\alpha$. For this problem, the linearity assumption ensures that $\partial F/\partial J$ and $\partial F/\partial \alpha$ are constant. Therefore $\partial F/\partial j_1$, $\partial F/\partial j_2$ and $\partial F/\partial j_3$, which are functions of $\partial F/\partial J$ and $\partial F/\partial \alpha$ (Appendix D), may also be treated as constants. Then the assumptions of point approximation and quasi-steady aerodynamics lead to the following expression for the instantaneous value of any propeller force $F$.

$$F = \bar{F} + \left[ \frac{\partial F}{\partial j_1} \right]_{j_1} j_1 + \left[ \frac{\partial F}{\partial j_2} \right]_{j_1, \alpha} j_2 + \left[ \frac{\partial F}{\partial j_3} \right]_{j_1, \alpha} j_3 \quad (8-4)$$

where $\bar{F}$ is the mean propeller force and the force derivatives are those corresponding to the mean operating condition of the propeller. Since the turbulence is isotropic, $j_1$, $j_2$ and $j_3$ are statistically independent random variables. Ignoring the mean force $\bar{F}$ and using Eq. 2.16, it can be shown that the power spectral density of the random force $F$ is given by

$$\Phi_{FF}(f) = \left( \frac{\partial F}{\partial j_1} \right)^2 \Phi_{j_1 j_1}(f) + \left( \frac{\partial F}{\partial j_2} \right)^2 \Phi_{j_2 j_2}(f) + \left( \frac{\partial F}{\partial j_3} \right)^2 \Phi_{j_3 j_3}(f) \quad (8-5)$$

or in non-dimensional form

$$\Phi_{\eta F, \eta F}(f) = \left( \frac{\partial F}{\partial j_1} \right)^2 \Phi_{J_1 J_1}(f) + \left( \frac{\partial F}{\partial j_2} \right)^2 \Phi_{J_2 J_2}(f) + \left( \frac{\partial F}{\partial j_3} \right)^2 \Phi_{J_3 J_3}(f) \quad (8-6)$$

where

$$\Phi_{J_1 J_1} = \frac{\Phi_{uu}}{(nD)^2} J_1^2 \left\{ \Phi_{u^*u^*}\cos^2 \alpha + \Phi_{w^*w^*}\sin^2 \alpha \right\}$$

$$\Phi_{J_2 J_2} = \frac{\Phi_{vv}}{(nD)^2} J_2^2 \Phi_{v^*v^*}$$

$$\Phi_{J_3 J_3} = \frac{\Phi_{ww}}{(nD)^2} J_3^2 \left\{ \Phi_{w^*w^*}\cos^2 \alpha + \Phi_{u^*u^*}\sin^2 \alpha \right\} \quad (8-7)$$
Equation 8.6 relates the power spectrum of any propeller coefficient with the derivatives \( \partial C_f / \partial \eta \) and the power spectra of turbulence velocities. The derivatives \( \partial C_f / \partial \eta \) and \( \partial C_f / \partial \alpha \) for the propeller used in the present experiment are tabulated in Table 7.2. The derivatives \( \partial C_f / \partial \eta_1 \), \( \partial C_f / \partial \eta_2 \) and \( \partial C_f / \partial \eta_3 \) may be obtained from these using the relations given in Appendix D.

8.2 Discussion of the Assumptions

The implications of the assumptions made in the previous section and their validity are discussed below.

(i) Point Approximation: This assumes that the turbulence wavelengths are so large compared to the propeller diameter that the propeller may be considered as a point and the velocity variations over the disk may be ignored. This simplifies the problem greatly. The entire propeller may now be considered as a single aerodynamic system with three inputs \( u, v, w \). There is no need to consider the non-linear, unsteady aerodynamics of the individual blade elements. By enabling the treatment of the entire propeller as a single system, the point approximation allows one to use the measured steady force characteristics of the propeller. This avoids several errors and approximations inherent to blade element approaches to the problem. It must be noted however that all the above advantages have been obtained at a certain cost. For the point approximation to be valid, it is necessary that the turbulence scale \( L \) be much larger than the propeller diameter \( D \). In the atmosphere, at low altitudes, the longitudinal scale of turbulence is of the same order as the altitude (Ref. 42). Therefore for full scale propellers \( D \approx 20' \) the point approximation may be expected to be valid at altitudes above 1000 ft. At lower altitudes, the validity of the approximation becomes progressively more uncertain. The validity at small \( L/D \) can be established only by comparing the theoretical predictions with experimental data. Such a comparison is given in Chapter 9. The experimental results (Sec. 7.4) were obtained in a turbulence field whose longitudinal scale is smaller than the propeller diameter \( L/D \approx 0.6 \). Therefore the comparison of Chapter 9 is a very severe test. Even so, the agreement between the theoretical and experimental spectra is fairly good in most cases indicating that the point approximation is not as restrictive as it might seem to be.

(ii) Quasi-steady Aerodynamics: Quasi-steadiness is assumed for the entire propeller - not for the individual blade elements. When a propeller is at an angle of attack, the effective angles of attack of its blade elements are subjected to periodic variations at the rotational frequency. Quasi-steady assumption applied to the blade element aerodynamics may be expected to cause considerable error unless proper corrections are applied. In the present method the need for such an assumption is eliminated by using the experimentally measured average forces in non-turbulent flow, in which the unsteadiness at rotational frequency is automatically included.

The validity of the assumption of quasi-steady propeller aerodynamics may be analyzed as follows. This analysis is based on the assumptions of point approximation and linearity. For simplicity consider first a propeller moving through a one-dimensional \( g \)-gust (Fig. 8.1). As mentioned earlier, the gust causes sinusoidal changes in the effective values of \( \eta \) and \( \alpha \). This in turn causes sinusoidal changes in the propeller forces. It can easily be seen that the frequency of such changes is given by

\[
f = \frac{\bar{U}}{\lambda}
\]  

(8-5)
where $\lambda$ is the wavelength of the gust. At small $U$ and/or large $\lambda$, the frequency of variations of $J$ and $\alpha$ are small and conditions close to those of steady flow exist. In such a case, quasi-steady aerodynamics may be expected to be a valid assumption. At large values of $f$, both the amplitude and phase of the calculated forces will be in error. When the propeller is operating in a three dimensional turbulent field, the situation is much more complex. However, since the longitudinal scale $L$ represents a characteristic wavelength of the turbulent field, one would expect the quasi-steady assumption to be valid if

$$f = \frac{U}{L} \approx 0 \quad (8-9)$$

or, in terms of the non-dimensional parameter $S$, if

$$S = \frac{fD}{U} = \frac{D}{L} \ll 1 \quad (8-10)$$

i.e., if the propeller diameter is much smaller than the turbulence scale. Thus, it may be seen that the quasi-steady assumption is valid in situations where the point approximation is valid. At small $L/D$, the point approximation is not valid and the turbulent velocities are not uniform over the propeller disk. Therefore the propeller cannot be treated as a single aerodynamic system and one must consider the blade element aerodynamics in detail. In such a situation, it is not possible to talk about the validity of quasi-steady propeller aerodynamics from a theoretical standpoint. However it is still possible to check its validity by comparing the theoretical results with the experimental results. This is done in Chapter 9. The results of the comparisons seem to suggest that the assumption is valid throughout the frequency range considered, viz., 0 to 55 Hz. This corresponds to a non-dimensional frequency range of 0 to 1.8 at $J = 0.4$ and 0 to 0.9 at $J = 0.8$.

(iii) Linearity: Like the previous assumption, the linearity assumption pertains to the aerodynamics of the propeller as a whole - not to that of the individual blade elements. By using the measured average forces as an input to the calculations, non-linear effects such as those due to blade stalling are fully taken into account. Also, the steady force characteristics of the propeller presented in Figs. 7.3 to 7.6 indicate that the forces vary linearly with $\alpha$ and $J$ over a fairly large range of these parameters. For intensities up to about 15%, the changes in $\alpha$ and $J$ caused by the turbulence velocities fall within the linear range. Thus the nonlinearities in the propeller aerodynamics may be taken into account adequately by using the measured aerodynamic derivatives evaluated at the mean $\alpha$ and $J$ and assuming 'local linearity'. It must be noted, however, that the assumption of linearity is not essential to the method. It is possible to take the nonlinearities fully into account by means of digital simulation. However this refinement seems to be unwarranted.

(iv) Isotropy: The turbulence field is assumed to be isotropic in order to simplify the calculations. If the turbulence is non-isotropic, several terms will have to be added to Eq. 8.6 giving the force power spectra. The turbulence generated in the wind tunnel is not quite isotropic but the errors introduced by the assumption were estimated to be quite small. The assumption can be easily relaxed provided the cross-spectra of the turbulence velocities are known. A method of estimating the response of a propeller to non-isotropic turbulence is given in Sec. 8.3.3.
8.3 Relaxing the Assumptions

Methods of relaxing the assumptions of point approximation, linearity and isotropy are discussed below. Not enough is known about the unsteady aerodynamics of the propeller to enable relaxation of the assumption of quasi-steady aerodynamics. However, if a blade element approach is used to the problem, as was done by Barlow (Ref. 1) suitable corrections may be applied to account for the unsteady aerodynamics of the blade elements.

8.3.1 Point Approximation

The point approximation of Sec. 8.1 is based on the condition that the turbulence scale is much larger than the propeller diameter. This restriction may be removed partially by using either Barlow's (Ref. 1) or Etkin's method (Refs. 28 and 29). Some problems are encountered, however, in using these methods. For example, Barlow's method requires the use of certain additional assumptions which are of limited validity while Etkin's approach is based on the availability of certain experimental data. This section is included mainly to discuss the difficulties encountered in relaxing the point approximation.

(i) Barlow's Method: A brief summary of Barlow's method is given in Sec. 2.6. Full details of the method may be found in Ref. 1. The basic problem in Barlow's approach is to determine the aerodynamic transfer functions of the propeller. Such transfer functions yield the forces and moments on the propeller due to arbitrarily inclined sinusoidal shear waves. Figure 8.2 shows the effect of the wavelength of the shear wave on the velocity variations over the propeller disk. As shown in the figure, large wavelengths give rise to practically uniform velocities over the propeller disk, while small wavelengths give rise to sinusoidal spatial variations. Barlow uses a blade element approach to the problem. The blade element aerodynamics is assumed to be linear, two-dimensional and quasi-steady. Using these assumptions, the impulse response matrix of the blade element is derived. Using this as the basic aerodynamic input and following a difficult and lengthy mathematical procedure, Barlow derives the aerodynamic transfer functions of the propeller. (The lifting line model of the propeller blades used by Barlow limits the validity of the transfer functions to gusts whose wavelengths are much larger than the blade chord). Expressions for the response of the propeller to turbulence are then derived using the transfer functions and carrying out several integrations. The resulting expressions are several orders of magnitude more complex than those given in Sec. 8.1 and numerical calculation of the response would require extensive computational effort. It appears that even a partial relaxation of the point approximation can be done only at a heavy cost in terms of computational effort. Further, there seems to be no guarantee that the results so obtained will be more accurate than those of the derivative method. This is so because the derivative method does not suffer from several of the errors and approximations caused by the blade element approach used in Barlow's method. A detailed comparison of the two methods is given in Sec. 8.4.

(ii) Etkin's Method: Application of Etkin's power series approach (Refs. 28 and 29) to the propeller problem yields a method that retains the advantages of the derivative method and improves its range of validity. However, Etkin's method requires the knowledge of several aerodynamic derivatives of the propeller. Many of these are not available at present and are difficult to obtain. In this respect, discussion of the method is of somewhat academic importance. Nevertheless, a brief discussion of the method is given below with the hope that future research in this area may make this method useable.
A detailed description of Etkin's power series method is given in Refs. 28 and 29. In this method, turbulent velocity variations over the aero-
dynamic system (propeller in the present case) are taken into account in an
approximate way. It is shown that the response of any aerodynamic system to
turbulence may be expressed as an infinite series whose terms consist of the
effects of the spatial derivatives of the turbulent velocities at a point on the
system. A truncated series that consists of only the Oth through nth order terms
is referred to as the nth power series method. A special case is the Oth order
power series method which can easily be seen to be the same as the derivative
method. In this method, only the velocities at the centre of the propeller are
considered. In the first order power series method, in addition to the velocities
themselves, their first order spatial gradients i.e., ∂u/∂x, ∂u/∂y, ∂u/∂z, ∂v/∂x etc. are also considered. Retaining the assumptions of quasi-steady aero-
dynamics, linearity and isotropy and using the first order power series method,
the following expression may be derived for the propeller response.

\[
F_{CF} = \left( \frac{\partial C}{\partial u} \right)^2 \Phi_{uu} + \left( \frac{\partial C}{\partial v} \right)^2 \Phi_{vv} + \left( \frac{\partial C}{\partial w} \right)^2 \Phi_{ww}
\]

\[
+ \left( \frac{\partial C}{\partial u} \right)^2 \Phi_{uy} u_y + \left( \frac{\partial C}{\partial u} \right)^2 \Phi_{uz} u_z
\]

\[
+ \left( \frac{\partial C}{\partial v} \right)^2 \Phi_{vy} v_y + \left( \frac{\partial C}{\partial v} \right)^2 \Phi_{vz} v_z
\]

\[
+ \left( \frac{\partial C}{\partial w} \right)^2 \Phi_{wy} w_y + \left( \frac{\partial C}{\partial w} \right)^2 \Phi_{wz} w_z
\]

(8-11)

If the propeller is considered as a planar body, the gradients of the
velocities with respect to x have no effect on the propeller forces. Therefore
terms containing such gradients are not included in Eq. 8.11. The power spectra
of the velocity gradients - \( \Phi_{uy}, \Phi_{uz} \) etc. - may be obtained if the three-
dimensional power spectrum of the turbulent field is known (see Refs. 28 and 29).
The derivatives of propeller forces with respect to \( u_y \) and \( u_z \) may be obtained by
testing the propeller in a linear shear flow that can be generated in a wind
tunnel using a shear grid. Generating steady gradients \( v_y, v_z, w_y, w_z \) in a
wind tunnel is difficult if not impossible. A theoretical estimation of such
derivatives would require the use of several assumptions of questionable validity.

8.3.2 Linearity: The assumption of linearity of propeller aerodynamics may be
relaxed by resorting to digital simulation. It is possible to generate on a
computer three time series to represent the random velocities u, v and w. These
can be arranged to have predetermined auto and cross-correlations and power and
cross-spectra. At any time t, the instantaneous \( J \) and \( \alpha \) may be calculated using
the values of u, v, and w and the mean operating conditions of the propeller
(\( \bar{J} \) and \( \bar{\alpha} \)). Information on the nonlinear steady force characteristics of the
propeller (Figs. 7.3 to 7.6) may be fed to the computer through a set of regression
equations. Now all the information necessary to calculate the instantaneous
forces on the propeller is known. Such calculations may be done at each sampling
point to yield a time for each propeller force. The correlations and spectra of the random forces may then be calculated. The author feels that the inaccuracies introduced by the linearity assumption are not large enough to warrant the complexities introduced by digital simulation.

8.3.3 Isotropy:

In the derivative method described in Sec. 8.1, the turbulent field is assumed to be isotropic in order to simplify the calculations. This simplification results from the fact that the cross-spectra of the turbulence velocities at any point in the field are zero. When the field is non-isotropic (but homogeneous) the propeller response equation will contain not only the terms given in Eq. 8.6 but also some additional terms. It can be shown that the power spectrum of any propeller force is given by

\[
\Phi_{CFF}(f) = \left( \frac{\partial C_F}{\partial j_1} \right)^2 \Phi_{j_1j_1} + \left( \frac{\partial C_F}{\partial j_2} \right)^2 \Phi_{j_2j_2} + \left( \frac{\partial C_F}{\partial j_3} \right)^2 \Phi_{j_3j_3} + 2 \left( \frac{\partial C_F}{\partial j_1} \right) \left( \frac{\partial C_F}{\partial j_2} \right) \Phi_{j_1j_2} + 2 \left( \frac{\partial C_F}{\partial j_2} \right) \left( \frac{\partial C_F}{\partial j_3} \right) \Phi_{j_2j_3} + 2 \left( \frac{\partial C_F}{\partial j_3} \right) \left( \frac{\partial C_F}{\partial j_1} \right) \Phi_{j_3j_1}
\]  

(8.12)

8.4 Comparison of the Derivative Method with Barlow's Method

The merits and demerits of the derivative method (Sec. 8.1) and Barlow's general method (Ref. 1) are discussed below. Barlow's point approximation method suffers from the disadvantages of both these methods and hence is not considered in this discussion. The discussion below is based on the assumptions made in the methods and not on comparisons with experimental results. A comparison of the results of the derivative method with experimental results is given in Chapter 9. It would be desirable to compare the results obtained by Barlow's method also with the experimental results. However Barlow's expressions for propeller response are so complex and require so much computational effort that such a comparison was not possible within the scope of this investigation. (It may be noted here that Barlow himself did not make any calculations using his general method and presented only his point approximation results).

1) The derivative method is based on the assumption that the turbulence scale is much larger than the propeller diameter. Its validity is doubtful when the propeller is operating in small scale turbulence and must be established either by comparison with a better theory or experiment before use. Barlow's method is based on the assumption that the scale of turbulence is considerably larger than the propeller blade chord. (This assumption is implicit in the mathematical model of the propeller which treats the blades as lifting lines). Barlow's assumption
may therefore be expected to have a somewhat higher range of validity.

2) In the derivative method, the propeller forces are obtained from the measured steady force characteristics of the propeller. This results in a better accuracy than that obtained by any theoretical means. Also, the calculation procedure is greatly simplified. However, the method depends on the availability of experimental steady force data on the specific propeller under consideration. While such data are not always available, the author feels that reasonably accurate estimates of the forces may be obtained from the published data (Refs. 18, 19, 20 and the present report) on several propellers of different geometries. In Barlow's method, the propeller forces are calculated using a blade element approach. This requires a considerable amount of computation. Also, because of the nature of the assumptions made in this method, its validity is restricted to propellers operating at small angles of attack, and small thrust coefficients. Further the method predicts zero side force and pitching moment on a propeller in pitch while the experimental data (Refs. 18, 19 and the present report) show otherwise.

3) In the derivative method, quasi-steadiness is assumed for the propeller aerodynamics. This could result in an inaccurate estimation of the effects of unsteadiness associated with turbulence. However, the basic unsteadiness introduced by the angle of attack is fully taken into account by using the experimental steady force data. In Barlow's method, the blade element aerodynamics is assumed to be quasi-steady. This is much more restrictive and results in a greater error. While Barlow does apply an approximate correction to account for the unsteadiness, it seems to be inadequate, for his method fails to predict the side force and pitching moment on a propeller in pitch (see item 2 above). This is believed to be due to the fact that no phase corrections are made in his method.

4) In the derivative method, the propeller aerodynamics is assumed to be linear. In specific terms the forces and moments on the propeller are assumed to vary linearly with $J$ and $\alpha$, within the range of variations of these parameters caused by turbulence. As mentioned in Sec. 8.2, this assumption may be expected to be valid for turbulence intensities up to about 15%. It must be noted that no restriction is placed on the blade element aerodynamics. Non-linear effects such as blade stall are fully taken into account. Barlow's method assumes linear aerodynamics of the blade element and is therefore limited to small angles of attack (of the propeller) as well as operation at small thrust coefficients.

5) The derivative method is a relatively simple method and yields simple expressions for the propeller response. In comparison, Barlow's method is much more complex and is difficult to use.

6) Barlow's method predicts spikes at multiples of blade frequency in the propeller response. The point approximation method does not take the periodicity of the propeller characteristics into account and is not capable of predicting the existence of any such spikes.

In view of the points discussed in items 1 to 5 above, the following conclusions can be made:

(i) For $L/D \gg 1$, the derivative method may be expected to yield better results than Barlow's method, in spite of (and probably because of) the former's simplicity.
(ii) The derivative method is the only valid theoretical method at present for propellers operating at high thrust coefficients and large angles of attack.

(iii) For $L/D \approx 1$, $\alpha = 0$ and $C_x \approx 0$, it is hard to say which method is superior. Ignoring the turbulent velocity variations over the propeller disk makes the derivative method inaccurate while incorrect aerodynamic modelling of the propeller introduces errors in Barlow's method. Only a comparison of the two methods with experimental results will determine which method is more valid.

8.5 Results and Implications for STOL Aircraft

In an attempt to evaluate the validity of the derivative method, $\Phi_{C_x}$ and $\Phi_{C_m}$ were calculated for the model propeller operating in the wind tunnel turbulence. The calculations were done for the three angles of attack ($\alpha = 0^\circ$, $30^\circ$, and $60^\circ$) and the two advance ratios ($J = 0.4$ and $0.8$) for which the spectra were measured experimentally (Chapter 7). The measured turbulence spectra (Chapter 6) and the measured aerodynamic derivatives (Table 7.2) were used as input data for the calculations. First the aerodynamic derivatives with respect to $J_1$, $J_2$, and $J_3$ were calculated using the data given in Table 7.2 and the relations given in Appendix D. Then the power spectra of the normal force and the pitching moment were calculated using Eq. (8-6) for a frequency range of 5 - 55 Hz. All these calculations were carried out on an IBM 1130 computer. The results of these calculations are shown in Figs. 7.17 to 7.24. Figures 7.17 to 7.20 show the variations of the spectra with $\alpha$ while Figs. 7.21 to 7.24 show their variations with $J$. The changes in the response spectra with $\alpha$ and $J$ are due partly to the changes in the turbulence spectra and partly to those in the aerodynamic derivatives. A detailed comparison of the theoretical and experimental results is given in Chapter 9.

The following analysis was carried out to determine the order of magnitude of the contribution of the propellers alone to the response to turbulence of a typical STOL configuration (Ref. 3, Secs. 9.5 and 9.8). The aircraft is of a twin propeller deflected slipstream type. The following are some of the main characteristics of the aircraft:

- Weight: 40,000 lb.
- Wing Area: 1,000 sq.ft.
- Mean aerodynamic chord: 12.4 ft.
- Aspect ratio: 6.5

The aerodynamic derivatives of the aircraft are given in Ref. 3. The propellers are assumed to have a diameter of 20 ft and a rotational speed of 720 rpm. Their derivatives are assumed to be identical to those of the present investigation. The contributions of the propellers alone to the derivatives $C'_x$ and $C'_m$ of the aircraft were calculated. (Note: $C'_n = N/qSb$ and $C'_m = M/qSc$ where $q$ is the dynamic pressure, $S$ the wing area, $c$ the mean aerodynamic chord and $b$ the wing span). The contribution of each propeller comes partly from the yawing (or pitching) moment and partly from the side (or normal) force. The calculations were done for a forward speed of 86 fps, which was the lowest speed for which the aerodynamic data on the aircraft were available. Table 8.1 shows the results of the calculations. It may be seen that the propellers make substantial contributions to the derivatives. Their contributions increase with decreasing speed.
and therefore they have a greater effect on the stability as well as the response to turbulence at lower speeds.

**TABLE 8.1**

**CONTRIBUTION OF PROPELLERS TO THE AERODYNAMIC DERIVATIVES OF STOL CONFIGURATION**

Aircraft forward speed = 86 fps.

<table>
<thead>
<tr>
<th>Aircraft</th>
<th>Propellers</th>
<th>% contribution of propellers</th>
<th>Aircraft</th>
<th>Propellers</th>
<th>% contribution of propellers</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.185</td>
<td>-0.047</td>
<td>25%</td>
<td>0.57</td>
<td>0.3</td>
<td>53%</td>
</tr>
</tbody>
</table>

To evaluate the influence of propellers on the response to turbulence of an aircraft, it must be noted that, for a propeller, $C_{n_{B}}$ and $C_{m_{B}}$ are quite large (in fact they are nearly twice as large as $C_{n_{B}}$ and $C_{m_{B}}$, respectively) and must be taken into account in the response calculations. If the turbulent field is assumed to be isotropic, it may be shown, using the derivative method of Sec. 8.1, that, the rms yawing moment response of the propeller (at $\alpha = 0$) is proportional to $P_{n}$, where

$$P_{n} = \sqrt{(C'_{n_{B}})^2 + (C'_{m_{B}})^2}$$ (8-13)

Also, the rms yawing moment response of the aircraft is roughly proportional to $(C'_{n_{B}})$. Therefore the quantity $P_{n}/(C'_{n_{B}})$ gives a rough indication of the contribution of propellers to the yawing moment response of an aircraft. Similarly, if

$$P_{m} = \sqrt{(C'_{m_{B}})^2 + (C'_{m_{B}})^2}$$ (8-14)

the ratio $P_{m}/(C'_{m_{B}})$ gives a rough indication of the propeller contribution to the pitching moment response. For the STOL configuration described above, the contribution of the propellers to the yawing and pitching moment responses were calculated to be 27% and 55% respectively. At lower forward speeds, these contributions can be substantially higher.
A detailed comparison of the experimental measurements (Sec. 7.4) and the theoretical calculations (Sec. 8.5) of the propeller response is given in this chapter. Before any conclusions are drawn from such a comparison, one must keep in mind the limitations of the derivative method discussed in Sec. 8.2. The most important of these is that it is based on the assumption that $L/D \gg 1$. For the wind tunnel experiments described in Sec. 7.4, $L/D \approx 0.6$. Clearly the derivative method is not applicable for the experimental situation. Even so, a comparison of the theory and the experiment may be expected to throw some light on the validity of the derivative method. Since the experimental situation represents the worst case for a full scale propeller, which usually operates at a relatively large $L/D$, such a comparison may be expected to indicate the largest possible error caused by an application of the derivative method to the flight situation. A conservative estimate so obtained of the inaccuracies of the derivative method at small $L/D$ is very useful in view of the fact that relaxation of the point approximation would unduly complicate the estimation of the propeller response (Sec. 8.3.1). (It may be noted here that the derivative method gives complete agreement with experiment at $L/D = \infty$. A turbulent field with an infinitely large scale is nothing but a uniform non-turbulent flow field. Since the derivatives obtained in such a flow field are used as input to the derivative method, it follows that the method is exact for $L/D = \infty$).

Two kinds of comparisons of the theoretical and experimental results are given here. Firstly, a direct quantitative comparison of the results is given in Figs. 9.1 to 9.4. Figures 7.17 to 7.24 present the data in a different manner to enable a study of the trends of variation of the theoretical and experimental spectra with the parameters $\alpha$ and $\beta$. The following conclusions are made from a study of these figures.

1. In view of the invalidity of the derivative method to the experimental situation, the agreement between the theory and the experiment must be termed good. Barring the two worst cases - pitching moment spectra at $J = 0.8$, $\alpha = 0^\circ$ and $30^\circ$ - typical disagreement between the theoretical and experimental spectra is about 50%. This amounts to an error of less than 25% in the rms values. This is a relatively small error for a problem as complex as the response of a propeller to turbulence treated by a method as simple as the derivative method.

2. Ignoring the 'bumps' in the experimental spectra, the trends of variations of the theoretical and experimental spectra with frequency are surprisingly similar. Though the information available is insufficient, this may be interpreted to mean that the quasi-steady assumption made in Sec. 8.1 is valid through the frequency range considered ($S = 0$ to 1.8). It must be noted that the point approximation and the quasi-steady assumption of Sec. 8.1 are inter-related and the latter cannot be made in situations where the former is invalid (such as in the present experimental case). Therefore the conclusion about the validity of the quasi-steady assumption needs to be verified by further investigation.

3. The disagreement between the theoretical and experimental spectra has several features worthy of note. (i) The theory over-estimates the spectra in some cases and under-estimates them in some others. (ii) The agreement is, in general, better for $J = 0.4$ than for $J = 0.8$. (iii) At $J = 0.8$, the agreement is good for $\alpha = 60^\circ$ and poor for the other angles of attack. These three
observations are true for both the normal force spectra and the pitching moment spectra. The author believes that most of the disagreement is caused by the invalid point approximation. Since there is no simple way of relaxing the point approximation (Sec. 8.3.1), it is not possible at present to estimate the nature of errors introduced by it. However, it is quite conceivable that the errors are dependent on \( \alpha \) and \( J \), as the observations seem to indicate.

4. A study of Figs. 7.17 to 7.24 leads to the following conclusions about the trends of variation of the spectra with \( \alpha \) and \( J \). (i) The angle of attack does not seem to have much effect on experimental spectra while it has a decided influence on the theoretical spectra. (note from Figs. 6.6 to 6.8 that the turbulence spectra at the propeller centre vary somewhat with \( \alpha \). Therefore Figs. 7.17 to 7.24 indicate the combined effect of \( \alpha \) and the turbulence input). (ii) The advance ratio has a significant effect on both the theoretical and experimental responses. (Much of this is caused by the dependence of the rms values of turbulence velocities on \( J \)). The experimental spectra are more sensitive to \( J \) than the theoretical ones.

Based on the above observations and a careful analysis of the assumptions made in the derivative method, the following general conclusions are made: (i) The derivative method may be expected to yield accurate results for the propeller response (at all values of \( \alpha \) and \( J \)) in situations where \( L/D > 10 \). For a propeller operating in the atmosphere, this condition is usually satisfied at altitudes above 10D. (ii) At smaller values of \( L/D \), a reasonably conservative estimate of the force spectra may be obtained by multiplying the results of the derivative method by a factor of 2. Only in two out of the ten cases presented in Figs. 9.1 to 9.4 would such a correction prove insufficient.

X. CONCLUSIONS AND RECOMMENDATIONS

The following are the main conclusions of the present investigation:

1. The use of propellers on V/STOL aircraft and hovercraft operating in the severe small scale turbulence at low altitudes necessitates a detailed study of the behaviour of propellers in such an environment.

2. A combined theoretical and experimental approach offers the best solution to the problem of a propeller operating in turbulence. Any theoretical approach to the problem must necessarily be based on several assumptions of questionable validity and therefore requires experimental verification. Experiments are difficult, time consuming and are of limited validity. They must therefore be used only to verify theories or to gain an understanding of the physics of the problem.

3. Noting this limitation on the role of experimental methods, it is shown that a meaningful experiment can be designed without having to meet the difficult requirement of simulating all the non-dimensional parameters in the wind tunnel.

4. The following conclusions are made from the measurements of the steady forces on a model propeller in pitch.

(a) A propeller in pitch develops a normal force, side force, pitching moment and a yawing moment. These are functions of both the advance ratio and angle of attack. At angles of attack below 60°, it is observed that the normal
force is approximately five times as large as the side force and that the yawing moment is twice as large as the pitching moment.

(b) The variations of the in-plane forces and moments with $\alpha$ and $J$ are smooth and can be considered linear over limited ranges of these parameters.

(c) The measured aerodynamic derivatives $\frac{\partial C_z}{\partial \alpha}$ and $\frac{\partial C_n}{\partial \alpha}$ at $\alpha = 0$ agree fairly well with those predicted by Ribner's theory (Ref. 17).

(d) The trends of variation of the forces with $\alpha$ and $J$ as well as their relative magnitudes agree with those of the experimental data published by Wickens (Ref. 18) and Yaggy and Rogallo (Ref. 19). These data were obtained on propellers which were much larger than the one used in the present experiment.

5. The derivative method described in this report offers a simple means of estimating the response of a propeller to turbulence. It is restricted to propellers operating in large scale turbulence ($L/D > 10$) but is applicable to all advance ratios and angles of attack. In spite of its relative simplicity, it is believed to be superior in many respects to the method proposed by Barlow (Ref. 1).

6. The spectra of the normal force and pitching moment were measured on a model propeller operating in small scale turbulence ($L/D \approx 0.6$). These were then compared with the spectra estimated by the derivative method. The comparison shows that:

(a) In spite of the severity of the test, the errors in the estimated spectra are not excessively large. The error varies with the operating condition of the propeller but is less than 100% for most cases. This amounts to an error of less than 50% in the rms values. A typical error is about 25% in the rms values.

(b) The trends of variation of the measured and estimated spectra with frequency are remarkably similar. Pending further study, this may be tentatively interpreted as showing that the propeller aerodynamics can be considered to be quasi-steady in the non-dimensional frequency range of 0 to 1.8.

7. The derivative method may be expected to yield accurate results for $L/D > 10$. At smaller values of $L/D$, a conservative estimate of the response spectra may be obtained by multiplying the results of the derivative method by 2.

Based on the experience gained during the present work, the author wishes to make a few recommendations for future work on propellers. It is hoped that such work would produce results that would help optimize the design of propellers for V/STOL aircraft and hovercraft.

1. It is suggested that the aerodynamics of a propeller at a large angle of attack be studied in detail. Such a study must include an investigation of the forces on the individual blade elements, as well as the effect of aerodynamic devices such as vanes, flaps, etc.

2. The effect of the propeller geometry on its response to turbulence needs to be investigated. This can be done by using the derivative method described in this report and the steady force data available in the literature (Refs. 18, 19, 20 and the present report). Using the results of the investigation
suggested under item 1 above, the effect of aerodynamic devices on the propeller response may be investigated. The response of propellers of advanced design such as variable camber and cyclic pitch propellers also needs to be investigated.

3. It is recommended that the results of the present investigation be applied to study the influence of propellers on the response to turbulence of a typical STOL aircraft such as the De Havilland DHC-7 or the Canadair CL-84.

4. The pusher type propellers employed on most of the present day hovercraft operate in a turbulent wake. The flow field and its influence on the forces on the propeller need to be investigated.

5. The effects of wing and fuselage on the response of a propeller to turbulence need to be studied.

6. The derivative method may be used to study the effect of turbulence on ducted fans and helicopter rotors.

7. It is suggested that the experiments described in this report be repeated in a turbulent field with a larger scale. A turbulence scale four to five times the present value of 5" is easily obtainable in a large wind tunnel such as the NAE 30 ft. wind tunnel. The data obtained from tests conducted in such a wind tunnel on a smaller propeller may be used to check the validity of the derivative method.
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APPENDIX A: MEASUREMENT OF PROPELLER GEOMETRY

The propeller used in the experiments (henceforth referred to as the test propeller) is a Tornado model aircraft propeller. It is a three bladed nylon propeller with a diameter of 8" and a pitch of 6". Measurement of the propeller geometry forms an essential part of the experiment. A destructive method was chosen for this purpose for reasons of simplicity, expediency and accuracy. The method consists of the following steps: (i) The blades of the propeller are cut at a pre-determined radius. (ii) It is then mounted on a special rig shown in Fig. A-1. The arrangement shown provides a reference surface, with a graph sheet pasted on it, in the plane of the blade section. A picture of the blade section and the graph sheet is taken. All the necessary measurements may now be obtained from the picture. The chord and thickness of the blade section can be measured directly from an enlargement of the picture and properly scaled down. The blade angle is given by the angle between the chord and any of the vertical lines on the graph sheet, which were arranged to be perpendicular to the propeller axis. The steps (i) and (ii) are repeated for all the radii at which measurements are required.

The measurements described above were made on three propellers which are from the same batch of production as the test propeller. A statistical analysis of the data so obtained showed that the peak to peak variations of the measurements (both the blade to blade and propeller to propeller variations) are small. Typically the peak to peak variations are as follows: (a) chord: 2% (b) thickness: 2% (c) blade angle: 3.5%. Much of this variation is expected to be due to measurement errors so that the three propellers as well as the three blades on each of the propellers may be expected to be nearly identical in geometry. The average of the geometries of the three propellers may therefore be expected to be a good approximation to that of the test propeller. Figure A-2 shows a typical blade section while Fig. A-3 shows the variations of the chord, blade angle and thickness with radius.
APPENDIX B: SPECTRAL ANALYSIS

A block diagram of the correlation and spectral analysis system used in this experiment is shown in Fig. B-1. Power and cross-spectra of random signals are obtained by Fourier transforming auto and cross-correlations, which were obtained by using a Princeton Applied Research (PAR) correlator. The system shown in Fig. B-1 was first developed and used at UTIAS by D. Surry who described it in detail in Ref. 8. The reader is also referred to Teunissen (Ref. 61) for some additional comments on the frequency range of validity of spectral estimates. A brief description of some essential features of the spectral measurement method is given below.

The PAR correlator computes auto or cross-correlations of the input signal \( s \) at 100 values of \( \tau = (k + 1/2) \Delta \tau \); \( k = 0,1,2, \ldots 99 \). The maximum time delay \( \tau_m = 100 \Delta \tau \) can be selected over a wide range of values depending on the frequency range and resolution of interest. Using the information given in Ref. 7, and analysis procedures described by Blackman and Tukey (Ref. 4), Surry (Ref. 8) derived the following equations for estimating power and cross-spectra from the correlations.

Raw spectral estimates are given by

\[
\hat{\phi}_{ij}(k \Delta f) = 4 \Delta \tau \sum_{n=0}^{99} R_{ijn} \cos \left( \frac{\pi k}{100} (n + 1/2) \right)
\]

\[
\hat{c}_{ij}(k \Delta f) = 2 \Delta \tau \sum_{n=0}^{99} \left[ R_{ijn} + R_{j in} \right] \cos \left( \frac{\pi k}{100} (n + 1/2) \right)
\]

\[
\hat{q}_{ij}(k \Delta f) = 2 \Delta \tau \sum_{n=0}^{99} \left[ R_{ijn} - R_{j in} \right] \sin \left( \frac{\pi k}{100} (n + 1/2) \right)
\]

where the subscripts \( i \) and \( j \) refer to the two inputs to the correlator and \( R_{ijn} \) is the correlation at time delay \( \tau = (n + 1/2) \Delta \tau \).

Two corrections are applied to the raw estimates of Eq. (B-1) to obtain the final estimates. The first of these is necessitated by the fact that only a finite length of correlation curve is available for Fourier transformation. This results in raw estimates that are equal to the true spectra convolved with the so-called zeroth spectral window (Ref. 4). The raw estimates can be significantly improved by an operation known as 'Hanning'. The second correction is required due to the fact that the nth point on the correlation curve is actually the average correlation over the interval \( n \Delta \tau \) to \( (n + 1) \Delta \tau \). Surry (Ref. 8) has shown that the corrected spectral estimates may be obtained by means of the following equations.

\[
\hat{\phi}_{ij}(k \Delta f) = 4 \Delta \tau \sum_{n=0}^{99} R_{ijn} \cos \left( \frac{\pi k}{100} (n + 1/2) \right)
\]

\[
\hat{c}_{ij}(k \Delta f) = 2 \Delta \tau \sum_{n=0}^{99} \left[ R_{ijn} + R_{j in} \right] \cos \left( \frac{\pi k}{100} (n + 1/2) \right)
\]

\[
\hat{q}_{ij}(k \Delta f) = 2 \Delta \tau \sum_{n=0}^{99} \left[ R_{ijn} - R_{j in} \right] \sin \left( \frac{\pi k}{100} (n + 1/2) \right)
\]

\[
\hat{\phi}_{ij}(k \Delta f) = 4 \Delta \tau \sum_{n=0}^{99} R_{ijn} \cos \left( \frac{\pi k}{100} (n + 1/2) \right)
\]

\[
\hat{c}_{ij}(k \Delta f) = 2 \Delta \tau \sum_{n=0}^{99} \left[ R_{ijn} + R_{j in} \right] \cos \left( \frac{\pi k}{100} (n + 1/2) \right)
\]

\[
\hat{q}_{ij}(k \Delta f) = 2 \Delta \tau \sum_{n=0}^{99} \left[ R_{ijn} - R_{j in} \right] \sin \left( \frac{\pi k}{100} (n + 1/2) \right)
\]
The spectral estimates of Eq. (B-2) are still subject to error due to what is known as 'aliasing'. This problem arises due to the fact that only a finite number of points on the correlation curve are measured. If $f_o$ is the Nyquist frequency (given by $f_o = 1/2\Delta f$), this results in spectral estimates given by

$$\Phi_{ii}(f') = \Phi_{ii}(f') + \sum_{n=1}^{\infty} \Phi_{ii}(2nf_o + f')$$

(B-3)

where $\Phi$ is the estimated spectrum, $\Phi$ is the true spectrum and $f' < f_o$. From Eq. (B-3), it is obvious that aliasing (represented by the summation on the R.H.S.) can be reduced by making sure that the input signal (s) have negligible energy above the Nyquist frequency. This can be achieved by applying a low-pass filter(s) to the input signal(s) before feeding them to the correlator, and later correcting for the filter response. All the spectral measurements in this experiment were carried out in this fashion, so that aliasing effects may be neglected. (Signals to be cross-correlated were passed through a pair of phase-matched filters before feeding them to the correlator. This avoids the necessity of applying phase corrections to the cross-spectra).

Out of the one hundred spectral estimates obtained at frequencies $k\Delta f$, $k = 0$ to 99, some are not acceptable data. Teunissen (Ref. 61) has shown that only the points corresponding to $k = 2$ to 60 are valid and accurate estimates. All the spectral plots shown in this report belong to this range of frequencies.

It can be shown (Ref. 4) that, for stationary data, the statistical variability of the spectral estimates obtained by the above method is given by

$$\epsilon = \frac{\sigma}{\mu}(\Phi) = \left(\frac{3}{4} \frac{T_m}{T'}\right)^{1/2}$$

(B-4)

where $T'$ is the effective length of the record used in obtaining the correlations. Almost all the spectra in this report were measured with $T'_m = 0.2$ sec and $T' = 40$ sec with the result that $\epsilon = 6.1\%$. In order to decrease $\epsilon$, most spectral measurements were repeated four times and the averages calculated. This effectively increases $T'$ by a factor of four and hence $\epsilon$ is reduced to 3.05\%.
A cross-correlation technique was devised for measuring accurately the amplitudes and phases of sinusoidal force components at shaft and blade frequencies. A special technique was needed for this purpose due to the fact that the force signals contained wide band and other sinusoidal components as well.

The technique is based on the following statements, which can be proved by using random process theory. (i) The cross-correlation of a sinusoid with a random signal is zero, for all $\tau$. (ii) The cross-correlation of two sinusoids of unequal frequencies is zero, for all $\tau$. (iii) If two sinusoids of frequency $f$ are given by

$$x(t) = X \sin 2\pi ft$$

and

$$y(t) = Y \sin 2\pi f(t-t')$$

Their cross-correlation is given by

$$R_{xy}(\tau) = \frac{XY}{2} \cos 2\pi f(t-t')$$

$$= X_{\text{rms}} Y_{\text{rms}} \cos 2\pi f(t-t')$$

This last point is illustrated in Fig. C-1.

As mentioned in Section 3.5, two photo-electric speed transducers were installed in the apparatus. One of them is on the propeller shaft and gives a pulse train at blade frequency while the other is on the rear part of the motor shaft and gives a pulse train at shaft frequency. Sinusoids at blade and shaft frequencies may be obtained by filtering the pulse trains with appropriate band-pass filters. Cross-correlations of such sinusoids with a force (or moment) signal yield the necessary information to calculate the amplitude and phase (relative to the speed transducer sinusoid) of blade and shaft frequency components of the force.

For example, if $X$ is the amplitude of the blade frequency sinusoid and $C$ the amplitude of its cross-correlation with the force signal, then the amplitude of the blade frequency component in the force signal is given by

$$F = \frac{2C}{X}$$

Also if the first peak in the cross-correlation occurs at a time delay $\tau = \tau_p$, then the phase lag ($\phi$) of the blade frequency component of force (relative to the blade frequency sinusoid) is given by

$$\phi = \frac{\tau}{2\pi f_b}$$

where $f_b$ = blade frequency in Hz.
APPENDIX D: DERIVATION OF EXPRESSIONS FOR PROPELLER FORCE DERIVATIVES

The 'derivative method' (described in Chapter 8) of calculating the response of a propeller to turbulence requires the knowledge of propeller force and moment derivatives with respect to the turbulence velocity components. Equations relating these derivatives with the experimentally determined derivatives are derived below. For convenience, the notation used in this appendix is also given below.

**NOTATION**

- D: Propeller diameter (ft)
- \( C_X, C_Y, C_Z \): Force coefficients \( C_F = \frac{F}{\rho n^2 D^4} \)
- \( C_L, C_m, C_n \): Moment coefficients \( C_F = \frac{F}{\rho n^2 D^5} \)
- \( F \): Any in-plane force or moment
- \( F_1 \): Instantaneous in-plane force or moment in the \( J_L \) direction
- \( F_2 \): Instantaneous in-plane force or moment perpendicular to the \( J_L \) direction (see Fig. D-1)
- \( J \): Instantaneous total 'advance ratio' = \( \sqrt{J_L^2 + J_A^2} \)
- \( J_A \): Instantaneous 'advance ratio' along the propeller axis = \( J_1 + J_2 \)
- \( J_L \): Instantaneous 'advance ratio' in the propeller plane
  \[ = \sqrt{J_2^2 + (J_3 + J_3)^2} \]
- \( J = \text{Mean 'advance ratio'} = \frac{J_1^2 + J_3^2}{nD} = \frac{\bar{U}}{nD} \)
- \( J_1 \): Component of mean 'advance ratio' along x-axis = \( \bar{J} \cos \alpha \)
- \( J_3 \): Component of mean 'advance ratio' along z-axis = \( \bar{J} \sin \alpha \)
- \( J_1 \): Fluctuating 'advance ratio' along x-axis = \( u/nD \)
- \( J_2 \): Fluctuating 'advance ratio' along y-axis = \( v/nD \)
- \( J_3 \): Fluctuating 'advance ratio' along z-axis = \( w/nD \)
- \( L, M, N \): Moments on the propeller in the x, y and z directions
- \( n \): Propeller speed (rps)
- \( \bar{U} \): Mean flow velocity in the x' direction (fps)
- \( u, v, w \): Turbulence velocities in the propeller body axes
- \( u', v', w' \): Turbulence velocities in the wind axes
Propeller body axes

Wind axes

Forces on the propeller in the x, y and z directions

Instantaneous angle of attack \( \alpha = \tan^{-1} \left( \frac{J_L}{J_A} \right) \)

Mean angle of attack \( \bar{\alpha} = \tan^{-1} \left( \frac{J_3}{J_1} \right) \)

Angle between z axis and \( J_L \) (see Fig. D-1)

Density of air

Figure D-1 shows the co-ordinate axes, 'advance ratios', in-plane forces and moments for a propeller in pitch.

The following relations are known from definitions:

\[
J = \sqrt{(J_1 + j_1)^2 + (J_2 + j_2)^2 + (J_3 + j_3)^2}
\]

\[
\alpha = \tan^{-1} \left\{ \frac{\sqrt{(J_3 + j_3)^2 + (J_2 + j_2)^2}}{(J_1 + j_1)} \right\}
\]  

\[
= \tan^{-1} \left( \frac{J_L}{J_A} \right)
\]

It is also known that the forces (or moments) \( F_1 \) and \( F_2 \) are functions only of \( J \) and \( \alpha \). (Quasi-steady aerodynamics is assumed in this section; i.e., forces and moments on the propeller at any instant are dependent only on the instantaneous aerodynamic conditions).

\[
F_1 = F_1 (J, \alpha)
\]

\[
F_2 = F_2 (J, \alpha)
\]

Normal Force Derivatives with Respect to \( j_1 \):

\[
\frac{\partial F_1}{\partial j_1} = \frac{\partial F_1}{\partial J} \cdot \frac{\partial J}{\partial j_1} + \frac{\partial F_1}{\partial \alpha} \cdot \frac{\partial \alpha}{\partial j_1}
\]

\[
= \frac{\partial F_1}{\partial J} \cdot \frac{1}{2J} \cdot 2(J + j_1) + \frac{\partial F_1}{\partial \alpha} \cdot \frac{1}{1+(J_L/J_A)^2} \cdot \left( -\frac{J_L}{(J_1 + j_1)^2} \right)
\]

\[
= \frac{\partial F_1}{\partial J} \cdot \frac{J_1 + j_1}{J} - \frac{\partial F_1}{\partial \alpha} \cdot \frac{J_L}{J_A^2 + J_L^2}
\]
Similarly
\[
\frac{\partial F_2}{\partial j_1} = \frac{\partial F_2}{\partial J} \cdot \frac{J_1 + j_1}{J} - \frac{\partial F_2}{\partial \alpha} \cdot \frac{J_L}{J_A^2 + J_L^2} \quad (D-6)
\]

From Fig. D-1, it can be seen that
\[
Z = F_2 \sin \beta - F_1 \cos \beta \quad (D-7)
\]

\[
\therefore \frac{\partial Z}{\partial j_1} = \frac{\partial F_2}{\partial j_1} \sin \beta + F_2 \cos \beta \frac{\partial \beta}{\partial j_1} - \frac{\partial F_1}{\partial j_1} \cos \beta + F_1 \sin \beta \frac{\partial \beta}{\partial j_1} \quad (D-8)
\]

Also
\[
\beta = \tan^{-1}\left\{ \frac{j_2}{j_3 + j_3} \right\} \quad (D-9)
\]

The 'derivative method' of Chapter 8 assumes that the propeller forces vary linearly with \(j_1, j_2\) and \(j_3\). This implies constancy of force derivatives over the range of turbulence velocities. In what follows, force derivatives are evaluated for the reference condition (i.e., \(j_1 = j_2 = j_3 = 0\)).

In the reference condition,
\[
\beta = 0, \quad J_A = J_1 = J \cos \alpha, \quad J_L = J_3 = J \sin \alpha, \quad J = J
\]

\[
F_1 = -Z \text{ and } F_2 = -Y
\]

From Eq. (D-9)
\[
\frac{\partial \beta}{\partial j_1} = 0
\]

\[
\therefore \text{ From Eq. (D-5) and (D-8)}
\]
\[
\frac{\partial Z}{\partial j_1} = - \frac{\partial F_1}{\partial j_1}
\]
\[
= - \frac{\partial F_1}{\partial J} \cos \alpha + \frac{\partial F_1}{\partial \alpha} \cdot \frac{\sin \alpha}{J}
\]
\[
= \frac{\partial Z}{\partial J} \cdot \cos \alpha - \frac{\partial Z}{\partial \alpha} \cdot \frac{\sin \alpha}{J} \quad (D-11)
\]

**Normal Force Derivative with Respect to \(j_2\):**
Using Eq. (D-7),

\[ \frac{\partial Z}{\partial j_2} = \frac{\partial F_2}{\partial j_2} \sin \beta + F_2 \cos \beta \frac{\partial \theta}{\partial j_2} - \frac{\partial F_1}{\partial j_2} \cos \beta + F_1 \sin \beta \frac{\partial \theta}{\partial j_2} \]  

(D-14)

From Eq. (D-9),

\[ \frac{\partial \psi}{\partial j_2} = \frac{1}{j_2^2} \cdot \frac{1}{(j_3 + j_3)^2} = \frac{1}{j_3^2} \]  

(D-15)

In the reference condition Eq. (D-10) holds. Also from Eq. (D-15),

\[ \frac{\partial \psi}{\partial j_2} = \frac{1}{j_3} \]

Substitution of these relations in Eq. (D-14) yields

\[ \frac{\partial Z}{\partial j_2} = F_2 \frac{\partial \psi}{\partial j_2} - \frac{\partial F_1}{\partial j_2} \]

\[ = \frac{F_2}{j_3} \]

(D-16)a

\[ = - \frac{Y}{\bar{J} \sin \bar{\theta}} \]

(D-16)b
Normal Force Derivative with Respect to $j_3$:

By proceeding in a similar manner, it can be shown that

$$\frac{\partial Z}{\partial j_3} = \frac{\partial Z}{\partial \bar{\alpha}} \sin \bar{\alpha} + \frac{\partial Z}{\partial \bar{\alpha}} \frac{\cos \bar{\alpha}}{J} \quad (D-17)$$

**Special Case: $\bar{\alpha} = 0$**

Expressions for $\partial Z/\partial j$, and $\partial Z/\partial j_3$ for this special case are obtained by simply substituting $\bar{\alpha} = 0$ in Eq. (D-11) and Eq. (D-17). Evaluation of $\partial Z/\partial j_2$, however, presents a difficulty. This arises because, for $\bar{\alpha} = 0$, both the numerator and the denominator in Eq. (D-16)b are zero. This means that $\partial Z/\partial j_2$ will have to be evaluated as a limit. This is done below:

$$\frac{\partial Z}{\partial j_2} = - \frac{Y}{J \sin \bar{\alpha}}$$

$$= - \frac{Y}{J \bar{\alpha}} \cdot \frac{\bar{\alpha}}{\sin \bar{\alpha}}$$

$$\therefore \lim_{\bar{\alpha} \to 0} \frac{\partial Z}{\partial j_2} = - \frac{1}{J} \left[ \frac{\partial Y}{\partial \bar{\alpha}} \right]_{\bar{\alpha} = 0} \quad (D-18)$$

The normal force derivatives are reduced to the coefficient form and summarized below.

**Summary of Expressions for Normal Force Derivatives**

(i) **General Case**:

$$\frac{\partial C_z}{\partial j_1} = \frac{\partial C_z}{\partial J} \cos \bar{\alpha} - \frac{\partial C_z}{\partial \bar{\alpha}} \frac{\sin \bar{\alpha}}{J}$$

$$\frac{\partial C_z}{\partial j_2} = - \frac{C_y}{J \sin \bar{\alpha}}$$

$$\frac{\partial C_z}{\partial j_3} = \frac{\partial C_z}{\partial J} \sin \bar{\alpha} + \frac{\partial C_z}{\partial \bar{\alpha}} \cdot \frac{\cos \bar{\alpha}}{J}$$

(D-19)
(ii) $\ddot{\theta} = 0$:

\[
\frac{\partial c_z}{\partial j_1} = 0
\]

\[
\frac{\partial c_z}{\partial j_2} = -\frac{1}{j} \frac{\partial c_y}{\partial \alpha}
\]

\[
\frac{\partial c_z}{\partial j_3} = \frac{1}{j} \frac{\partial c_z}{\partial \alpha}
\]  

Side Force Derivatives

From Fig. D-1,

\[
Y = -(F_2 \cos \beta + F_1 \sin \beta)
\]

Using the above relation, expressions for side force derivatives can be derived in a manner quite similar to that of normal force derivatives. The following is a summary of the formulae.

Summary of Expressions for Side Force Derivatives

(i) General Case:

\[
\frac{\partial c_y}{\partial j_1} = \frac{\partial c_y}{\partial j} \cos \alpha - \frac{\partial c_y}{\partial \alpha} \frac{\sin \alpha}{j}
\]

\[
\frac{\partial c_y}{\partial j_2} = \frac{c_z}{j \sin \alpha}
\]  

\[
\frac{\partial c_y}{\partial j_3} = \frac{\partial c_y}{\partial j} \sin \alpha + \frac{\partial c_y}{\partial \alpha} \frac{\cos \alpha}{j}
\]

(ii) $\ddot{\alpha} = 0$:

\[
\frac{\partial c_y}{\partial j_1} = 0
\]

\[
\frac{\partial c_y}{\partial j_2} = \frac{1}{j} \frac{\partial c_z}{\partial \alpha}
\]

\[
\frac{\partial c_y}{\partial j_3} = \frac{1}{j} \frac{\partial c_y}{\partial \alpha}
\]  

D6
Pitching and Yawing Moment Derivatives:

From Fig. D-1, it may be seen that the pitching moment vector coincides with the side force vector and yawing moment vector coincides with the normal force vector. Hence the expressions for M and N derivatives are similar to those for Y and Z respectively. A summary of the formulae is given below.

Pitching Moment Derivatives

(i) General Case:

\[
\frac{\partial C_M}{\partial J_1} = \frac{\partial C_M}{\partial \alpha} \cos \alpha - \frac{\partial C_M}{\partial \alpha} \cdot \frac{\sin \alpha}{J}
\]

\[
\frac{\partial C_M}{\partial J_2} = \frac{C_N}{J_3}
\]

\[
\frac{\partial C_M}{\partial J_3} = \frac{\partial C_M}{\partial \alpha} \sin \alpha + \frac{\partial C_M}{\partial \alpha} \cdot \frac{\cos \alpha}{J}
\]

(ii) \( \alpha = 0 \):

\[
\frac{\partial C_M}{\partial J_1} = 0
\]

\[
\frac{\partial C_M}{\partial J_2} = \frac{1}{J} \frac{\partial C_N}{\partial \alpha}
\]

\[
\frac{\partial C_M}{\partial J_3} = \frac{1}{J} \frac{\partial C_M}{\partial \alpha}
\]

Yawing Moment Derivatives

(i) General Case:

\[
\frac{\partial C_N}{\partial J_1} = \frac{\partial C_N}{\partial \alpha} \cos \alpha - \frac{\partial C_N}{\partial \alpha} \cdot \frac{\sin \alpha}{J}
\]

\[
\frac{\partial C_N}{\partial J_2} = -\frac{C_M}{J \sin \alpha}
\]

\[
\frac{\partial C_N}{\partial J_3} = \frac{\partial C_N}{\partial \alpha} \sin \alpha + \frac{\partial C_N}{\partial \alpha} \cdot \frac{\cos \alpha}{J}
\]
(ii) $\bar{a} = 0$:

\[
\frac{\partial c_N}{\partial j_1} = 0
\]

\[
\frac{\partial c_N}{\partial j_2} = -\frac{1}{j} \frac{\partial c_M}{\partial \alpha} \tag{D-27}
\]

\[
\frac{\partial c_N}{\partial j_3} = \frac{1}{j} \frac{\partial c_N}{\partial \alpha}
\]
FIG. 3.1 EXPERIMENTAL APPARATUS
FIG. 3.2 A VIEW OF THE APPARATUS MOUNTED AT TEST STATION 1

FIG. 3.3 A CLOSEUP VIEW OF THE BALANCE AND FRONT SPEED TRANSDUCER
FIG. 3.4 SCHEMATIC REPRESENTATION OF THE TWO COMPONENT FORCE BALANCE
FIG. 3.5 MATHEMATICAL MODEL OF THE TWO COMPONENT BALANCE
FIG. 3.7 SPRING, LVDT AND COWLING ASSEMBLY

Spring Thickness
Front Spring: $t=0.030''$
Rear Spring: $t=0.024''$
FIG. 4.1 AN AERODYNAMIC OUTLINE OF THE WIND TUNNEL SHOWING THE GRID AND TEST STATION LOCATIONS
FIG. 5.1 STATIC CALIBRATIONS

(a) Normal Force Calibration

(b) Pitching Moment Calibration

(c) Torque Calibration

TORSIONAL STIFFNESS $= 0.318 \text{ Oz.in/deg}$
FIG. 5.2 A SCHEMATIC OF THE ARRANGEMENT USED FOR PITCHING MOMENT CALIBRATION
FIG. 5.3 REPRESENTATION OF THE FORCE BALANCE AS A TWO-INPUT, TWO-OUTPUT SYSTEM

FIG. 5.4 VECTOR DIAGRAM ILLUSTRATING THE METHOD OF OBTAINING $V_{L2}$
FIG. 5.5 ARRANGEMENT USED FOR GENERATING SINUSOIDAL MOMENTS
FIG. 5.6 FREQUENCY RESPONSE OF BALANCE - $V_1$ DUE TO FORCE

(a) Amplitude Response

(b) Phase Response
FIG. 5.7  FREQUENCY RESPONSE OF BALANCE - $V_2$ DUE TO FORCE
FIG. 5.8 FREQUENCY RESPONSE OF BALANCE - $v_1$ DUE TO MOMENT
FIG. 5.9  FREQUENCY RESPONSE OF BALANCE - $V_2$ DUE TO MOMENT
FIG. 6.1 DETAILS OF TURBULENCE GENERATING GRID

FIG. 6.2 CO-ORDINATE SYSTEM FOR TURBULENCE MEASUREMENTS
FIG. 6.3 A VIEW OF THE APPARATUS MOUNTED AT TEST STATION 2 BEHIND THE GRID

FIG. 6.4 MOUNTING ARRANGEMENT OF THE HOT-WIRE PROBE
FIG. 6.5 DATA HANDLING SYSTEM FOR TURBULENCE MEASUREMENT
FIG. 6.6 POWER SPECTRA OF $u^*$ AT TEST STATION 2

Measurement Locations:
1. $y' = 0''$, $z' = 0''$ (corresponds to $\alpha = 0^\circ$)
2. $y' = 0''$, $z' = 4.8''$ (corresponds to $\alpha = 30^\circ$)
3. $y' = 0''$, $z' = 8.0''$ (corresponds to $\alpha = 60^\circ$)
(a) $\bar{U} = 41$ Fps., Velocity corresponds to $J = 0.8$

(b) $\bar{U} = 20.5$ Fps., Velocity corresponds to $J = 0.4$

FIG. 6.7 POWER SPECTRA OF $v^*$ AT TEST STATION 2

Measurement Locations:

1. $y' = 0''$, $z' = 0''$ (corresponds to $\alpha = 0^\circ$)
2. $y' = 0''$, $z' = 4.6''$ (corresponds to $\alpha = 30^\circ$)
3. $y' = 0''$, $z' = 8.0''$ (corresponds to $\alpha = 60^\circ$)
FIG. 6.8 POWER SPECTRA OF $w^*$ AT TEST STATION 2

Measurement Locations:
1. $y' = 0''$, $z' = 0''$ (corresponds to $\alpha = 0^\circ$)
2. $y' = 0''$, $z' = 4.6''$ (corresponds to $\alpha = 30^\circ$)
3. $y' = 0''$, $z' = 8.0''$ (corresponds to $\alpha = 60^\circ$)
FIG. 6.9  SPANWISE VARIATION OF MEAN VELOCITY AND TURBULENCE INTENSITY

FIG. 6.10  VARIATION OF MEAN VELOCITY WITH $z'$
\textbf{FIG. 6.11} VARIATION OF TURBULENCE INTENSITY WITH $z'$

- $V_t = 35.9$ (\(J \approx 0.4\))
- $V_t = 70.4$ (\(J \approx 0.8\))

Turbulence Intensity

\begin{align*}
& \frac{w'_{\text{rms}}}{\overline{U}} \\
& \frac{u'_{\text{rms}}}{\overline{U}} \\
& \frac{v'_{\text{rms}}}{\overline{U}}
\end{align*}
FIG. 7.1 A SCHEMATIC OF THE DATA HANDLING SYSTEM FOR STEADY FORCE MEASUREMENT
(a) Propeller in Yaw

(b) Propeller in Pitch

FIG. 7.2 SIGN CONVENTION
FIG. 7.3 VARIATION OF NORMAL AND SIDE FORCE COEFFICIENTS WITH $\alpha$
FIG. 7.4 VARIATION OF PITCHING AND YAWING MOMENT COEFFICIENTS WITH $\alpha$
FIG. 7.5 VARIATION OF SIDE AND NORMAL FORCE COEFFICIENTS WITH $J$

FIG. 7.6 VARIATION OF PITCHING AND YAWING MOMENT COEFFICIENTS WITH $J$
FIG. 7.7  TYPICAL SPEED TRANSDUCER SIGNALS

Top:  Output of the front speed transducer
Bottom:  Output of the rear speed transducer
Typical Scatter Bar

(± σ)

FIG. 7.8 VARIATION OF TORQUE COEFFICIENT WITH α
FIG. 7.9 A VIEW OF THE END SUPPORT AND THE ARRANGEMENT USED FOR ADJUSTING THE ANGLE OF ATTACK
FIG. 7.10 DATA HANDLING SYSTEM FOR PROPELLER RESPONSE MEASUREMENT
PHASE I DATA RECORDING
FIG. 7.11 DATA HANDLING SYSTEM FOR PROPELLER RESPONSE MEASUREMENT

PHASE II DATA REDUCTION
FIG. 7.12 TYPICAL $V_1$, $V_2$ AND SPEED TRANSDUCER SIGNALS

Propeller in Turbulent Flow.
$n = 70 \text{ rps.}, \ \alpha = 30, \ J = 0.8$

Signals from top to bottom:
(i) $V_1$
(ii) $V_2$
(iii) Output of the rear speed transducer
(iv) Output of the front speed transducer

FIG. 7.13 SIGNALS OF FIG. 7.12 AFTER FILTERING AND AMPLIFICATION

Signals from top to bottom:
(i) Filtered $V_1$
(ii) Filtered $V_2$
(iii) Output of the rear speed transducer
CORRECTIONS FOR BALANCE FREQUENCY RESPONSE
Eq. 5.10 & 5.11

CORRECTIONS FOR FILTER AND TAPE RECORDER FREQUENCY RESPONSES AND NON-DIMENSIONALIZATION

CORRECTED SPECTRA OF FORCE AND MOMENT COEFFICIENTS

$\Phi_{V_1V_1}, \Phi_{V_2V_2}$

$C_{V_1V_2}, Q_{V_1V_2}$

BALANCE FREQUENCY RESPONSE MATRIX $G(t)$

FIG. 7.14 DATA HANDLING SYSTEM FOR PROPELLER RESPONSE MEASUREMENT

PHASE III CORRECTIONS FOR FREQUENCY RESPONSES AND NON-DIMENSIONALIZATION
FIG. 7.15 TYPICAL VARIABILITY OF SPECTRAL DATA
FIG. 7.16 EFFECT OF CORRELATOR NOISE ON THE VARIABILITY OF SPECTRAL DATA
FIG. 7.17 VARIATION OF NORMAL FORCE POWER SPECTRA WITH $\alpha$

$J = 0.4$
FIG. 7.18 VARIATION OF NORMAL FORCE POWER SPECTRA WITH $\alpha$

$J = 0.8$
FIG. 7.19 VARIATION OF PITCHING MOMENT POWER SPECTRA WITH $\alpha$

$J = 0.4$
FIG. 7.20 VARIATION OF PITCHING MOMENT POWER SPECTRA WITH $\alpha$

$J = 0.8$
FIG. 7.21 VARIATION OF NORMAL FORCE POWER SPECTRUM WITH $J$

$\phi_{CZCZ}$

Experimental

$J = 0.8$

$J = 0.4$

Theoretical

Frequency (Hz)

$\alpha = 30$
FIG. 7.22 VARIATION OF NORMAL FORCE POWER SPECTRA WITH J
\[ \Phi_{cz_cz} = \begin{cases} \text{Experimental} & \text{for } J = 0.8 \\ \text{Theoretical} & \text{for } J = 0.4 \end{cases} \]
FIG. 7.23 VARIATION OF PITCHING MOMENT POWER SPECTRA WITH $J$

$\alpha = 30$
FIG. 7.24 VARIATION OF PITCHING MOMENT SPECTRA WITH $J$

$\Phi C_m C_m$

$10^{-6}$ $10^{-7}$ $10^{-8}$ $10^{-9}$

$J = 0.8$

$J = 0.4$

Experimental

Theoretical

Frequency (Hz)

$\alpha = 60$
FIG. 7.25 VARIATION OF THE SHAFT FREQUENCY COMPONENT OF $V_1$ WITH $J$
FIG. 7.26 VARIATION OF THE SHAFT FREQUENCY COMPONENT OF \( V_2 \) WITH \( J \)
(a) Propeller advancing through a one-dimensional sinusoidal $w$-gust

\[ J = \frac{W}{nD} \]

\[ \alpha = \tan^{-1}\left(\frac{W}{U}\right) \]

(b) Figure showing the instantaneous situation

**FIG. 8.1** ILLUSTRATION OF THE EFFECT OF A SINUSOIDAL GUST ON THE EFFECTIVE $J$ AND $\alpha$ OF A PROPELLER
FIG. 8.2 ILLUSTRATION OF THE EFFECT OF $\lambda/D$ ON THE VELOCITY VARIATION OVER THE PROPELLER DISK
FIG. 9.1 COMPARISON OF EXPERIMENTAL AND THEORETICAL NORMAL FORCE SPECTRA $J = 0.8$
FIG. 9.2 COMPARISON OF THEORETICAL AND EXPERIMENTAL NORMAL FORCE SPECTRA \( J = 0.4 \)
FIG. 9.3 COMPARISON OF EXPERIMENTAL AND THEORETICAL PITCHING MOMENT SPECTRA  \( j = 0.8 \)
FIG. 9.4 COMPARISON OF EXPERIMENTAL AND THEORETICAL PITCHING MOMENT SPECTRA  \( J = 0.4 \)
FIG. A-1  ARRANGEMENT USED FOR PHOTOGRAPHING BLADE SECTIONS

FIG. A-2  TYPICAL SHAPE OF A BLADE SECTION (r = 3"")
FIG. A-3 PROPELLER GEOMETRY

[Graph showing propeller geometry with axes labeled: Radius (in.), Blade Angle (Degrees), c/D, t/c, c/D, and c/D.]

- Blade Angle (Degrees) plotted against Radius (in.)
- Lines and markers indicating different parameters such as c/D, t/c, and others.

The graph illustrates the relationship between the blade angle and the radius, highlighting the geometric characteristics of the propeller.

FIG. A-3 PROPELLER GEOMETRY
FIG. B-1 CORRELATION AND SPECTRAL ANALYSIS SYSTEM
FIG. C-1 CROSS-CORRELATION OF TWO SINUSOIDS OF IDENTICAL FREQUENCY
FIG. D-1 FIGURE SHOWING CO-ORDINATE AXES, 'ADVANCE RATIOS' ETC., FOR A PROPELLER IN PITCH
The effect of atmospheric turbulence on the forces and moments on a propeller has been studied both experimentally and theoretically. The experimental study consists primarily of measuring the normal force and pitching moment responses of a model propeller operating in a turbulent field generated in a wind tunnel. No publications could be found of any prior measurements of this kind. The experimental measurements are obtained over the full frequency range of interest in flight dynamic applications. The theoretical part of the present work includes the development of a new, simple method of calculating the response of a propeller to turbulence. This method, referred to as the derivative method, yields expressions for the power spectra of the propeller forces in terms of the turbulence spectra and the aerodynamic derivatives. Based on a comparison of the experimental and theoretical results, the following conclusions are made: (1) The derivative method may be expected to yield accurate results for the propeller response, provided the longitudinal scale of the turbulent field is larger than ten times the propeller diameter, (11) For propellers operating in a small scale turbulence, a conservative estimate of the response may be obtained by multiplying the results of the derivative method by a factor of 2.

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