FILM COOLING WITH EJECTION FROM
A ROW OF INCLINED CIRCULAR HOLES
AN EXPERIMENTAL STUDY FOR THE
APPLICATION TO GAS TURBINE BLADES

Christian LIESS

MARCH 1973
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1 This note contains the main parts of a doctoral thesis presented at the Université Libre de Bruxelles, 1972.
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SUMMARY

Film cooling with ejection of the coolant from a row of holes is the only practical way to apply this cooling method to gas turbine blades. A literature survey shows that information on this type of cooling is very limited. Measurements of the adiabatic wall effectiveness and the heat transfer coefficient on a flat plate downstream of a row of ejection holes are described. The effect of the principal flow parameters is studied for ejection of hot air into a cold air main flow. Laterally averaged values are measured for an ejection angle of 35° and a spacing-diameter ratio of 3. Ejection at a stagnation point is investigated also.

The main test conditions, such as main flow Mach numbers, pressure gradients and boundary layer properties are determined according to experiments on a cascade of typical turbine blade profiles. Application of the results to film cooled gas turbine blades is, therefore, possible.

It is found that the main flow Mach number and Reynolds number in the range covered by the tests have no measurable effect on the adiabatic wall effectiveness and the heat transfer coefficient. The importance of the mass velocity ratio on the film cooling parameters is confirmed. The effect of main flow pressure gradient and boundary layer thickness is found to be of the same order as the effect of the mass velocity ratio. A strong favorable pressure gradient or a thick boundary layer can reduce the effectiveness by a factor of two.
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<tr>
<td>$A$</td>
<td>area $m^2$</td>
</tr>
<tr>
<td>$c_w$</td>
<td>specific heat of copper elements $J/kg , ^oK$</td>
</tr>
<tr>
<td>$d$</td>
<td>diameter of ejection holes $m$</td>
</tr>
<tr>
<td>$D$</td>
<td>$= 2R_N$ leading edge diameter of model $m$</td>
</tr>
<tr>
<td>$D_C$</td>
<td>total cross-sectional area of ejection holes $m^2$</td>
</tr>
<tr>
<td>$G^*$</td>
<td>$= \frac{\rho_{SC} V_c}{\rho_{SG} V_G}$ mass velocity ratio</td>
</tr>
<tr>
<td>$h$</td>
<td>local heat transfer coefficient $J/m^2 , sec , ^oK$</td>
</tr>
<tr>
<td>$\bar{h}$</td>
<td>laterally averaged heat transfer coefficient $J/m^2 , sec , ^oK$</td>
</tr>
<tr>
<td>$H_{12}$</td>
<td>$= \delta/\delta$ shape parameter of boundary layer velocity profile</td>
</tr>
<tr>
<td>$k$</td>
<td>thermal conductivity $J/m , sec , ^oK$</td>
</tr>
<tr>
<td>$K$</td>
<td>constant</td>
</tr>
<tr>
<td>$k_{T*}$</td>
<td>thermal conductivity evaluated at $T^*$ $J/m , sec , ^oK$</td>
</tr>
<tr>
<td>$l$</td>
<td>curvilinear coordinate starting at the stagnation point $m$</td>
</tr>
<tr>
<td>$m$</td>
<td>mass flow $kg/sec$</td>
</tr>
<tr>
<td>$M$</td>
<td>Mach number</td>
</tr>
<tr>
<td>$Nu_D$</td>
<td>$= hD/k_1$ Nusselt number based on leading edge diameter</td>
</tr>
<tr>
<td>$\bar{Nu}_x$</td>
<td>$= \bar{h}x/k_{T*}$ laterally averaged Nusselt number</td>
</tr>
<tr>
<td>$Nu^*$</td>
<td>ratio of Nusselt number with ejection to Nusselt number without ejection</td>
</tr>
<tr>
<td>$p$</td>
<td>pressure $N/m^2$</td>
</tr>
<tr>
<td>$q$</td>
<td>heat flux per unit surface $J/sec , m^2$</td>
</tr>
<tr>
<td>$\dot{q}$</td>
<td>heat flux $J/sec$</td>
</tr>
<tr>
<td>$r$</td>
<td>recovery factor</td>
</tr>
<tr>
<td>$Re_{C}$</td>
<td>$= \frac{V_{C} \rho_{SC} s'/\mu_{SC}}{}$ Reynolds number of coolant in slot</td>
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<tr>
<td>$Re_D$</td>
<td>$= \frac{V_{1} \rho_{S1} D/\mu_{S1}}{}$ Reynolds number based on inlet conditions and leading edge diameter</td>
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<tr>
<td>$R_N$</td>
<td>leading edge radius of blade or turbulent model nose $m$</td>
</tr>
<tr>
<td>Symbol</td>
<td>Description</td>
</tr>
<tr>
<td>--------</td>
<td>-------------</td>
</tr>
<tr>
<td>( s )</td>
<td>spacing of ejection holes ( \text{m} )</td>
</tr>
<tr>
<td>( s' )</td>
<td>slot height for slot ejection ( \text{m} )</td>
</tr>
<tr>
<td>( t )</td>
<td>time sec</td>
</tr>
<tr>
<td>( T )</td>
<td>temperature ( \text{oK} )</td>
</tr>
<tr>
<td>( T_{\text{aw}} )</td>
<td>adiabatic wall temperature ( \text{oK} )</td>
</tr>
<tr>
<td>( T_{\text{rG}} )</td>
<td>recovery temperature of main flow ( \text{oK} )</td>
</tr>
<tr>
<td>( T_{w} )</td>
<td>wall temperature ( \text{oK} )</td>
</tr>
<tr>
<td>( T^* )</td>
<td>reference temperature (see section IV.7) ( \text{oK} )</td>
</tr>
<tr>
<td>( V )</td>
<td>velocity ( \text{m/sec} )</td>
</tr>
<tr>
<td>( x )</td>
<td>distance downstream of ejection (Fig. 2) ( \text{m} )</td>
</tr>
<tr>
<td>( y )</td>
<td>distance normal to cooled surface (Fig. 2) ( \text{m} )</td>
</tr>
<tr>
<td>( y )</td>
<td>distance defined in section II.4 ( \text{m} )</td>
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<tr>
<td>( z )</td>
<td>lateral distance from hole center line (Fig. 2) ( \text{m} )</td>
</tr>
<tr>
<td>( z_{1/2} )</td>
<td>distance defined in section II.4 ( \text{m} )</td>
</tr>
<tr>
<td>( \beta )</td>
<td>ejection angle (Fig. 2) ( \text{o} )</td>
</tr>
<tr>
<td>( \gamma )</td>
<td>ratio of specific heats</td>
</tr>
<tr>
<td>( \delta_{0.995} )</td>
<td>boundary layer thickness where ( V/V_G = 0.995 \text{ m} )</td>
</tr>
<tr>
<td>( \delta_{w} )</td>
<td>thickness of copper elements ( \text{m} )</td>
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\[
\delta_1 = \left\{ \begin{array}{ll}
\delta_{0.995} \\
(1 - \frac{\rho_S V}{\rho_{SG} V_G})
\end{array} \right. \text{dy boundary layer displacement thickness \( \text{m} \)}
\]

\[
\delta_2 = \left\{ \begin{array}{ll}
\delta_{0.995} \\
\frac{\rho_S V}{\rho_{SG} V_G} (1 - \frac{V}{V_G})
\end{array} \right. \text{dy momentum loss thickness of boundary layer \( \text{m} \)}
\]

<table>
<thead>
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<th>Symbol</th>
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<tr>
<td>( \varepsilon )</td>
<td>turbulent diffusivity ( \text{m}^2/\text{sec} )</td>
</tr>
<tr>
<td>( \eta )</td>
<td>local film cooling effectiveness defined in section II.2</td>
</tr>
<tr>
<td>( \eta_r )</td>
<td>local film cooling effectiveness for high main flow velocities (section II.2)</td>
</tr>
</tbody>
</table>
\( \bar{\eta}_r \) laterally averaged value of \( \eta_r \)

\( \theta = T_S - T_{SG} \) temperature difference used in section II.4 °K

\( \theta_2 = T_{SC} - T_{SG} \) temperature difference used in section II.4 °K

\( \mu \) dynamic viscosity kg/m sec

\( \xi \) correlation parameter in section II.3

\( \rho \) density kg/m\(^3\)

\( \rho_w \) density of copper elements kg/m\(^3\)

**SUBSCRIPTS**

C coolant, ejected flow

G main flow

S static conditions

o total, stagnation conditions

l inlet
I. INTRODUCTION

It is well known that the overall efficiency of gas turbine engines can be improved considerably by an increase of the maximum temperature of the thermodynamic cycle, i.e., the turbine inlet temperature. However, the augmentation of the turbine inlet temperature is limited by the properties of the blade materials. The allowable temperature limits for 1000 hours operation of present day blade materials are 1400°C for inlet guide-vanes and 1300°C for rotor blades (Brunetaud, 1971). The maximum turbine inlet temperature is thus of the order of 1400°C for uncooled blades. Operation at higher temperatures is possible if the blades are cooled down to the allowable temperatures. The efficiency of the blade cooling system is then very important because it determines the allowable increase of the turbine inlet temperature and, thereby, the overall efficiency of the whole engine.

A review of the large number of cooling systems proposed and realized in production engines (see Fig. 1) indicates that the systems employing a liquid coolant are very efficient, but mechanically very complicated. Practically all cooled production engines use air as coolant. The cooling air is taken at the outlet of the compressor and led directly to the cooled blades. The temperature difference of about 700°C between cooling air and hot flow makes it possible to keep the blades at temperatures which are below the above mentioned limits.

In internal air cooling systems, such as convection cooling, the air is passed through more or less complicated passages within the blades. The heat transport from the blade to the coolant occurs by convection. The temperature reduction is, therefore, limited. In external air cooling systems, such as film and transpiration cooling, the cooling air is ejected from the blade through discrete holes or porous blade walls, respectively. The ejected air forms a thin layer along the blade
surface and reduces the heat transfer from the hot gas to the blade considerably.

It is obvious, that the transpiration cooling system is the most efficient air cooling system, because the coolant is ejected continuously through a porous blade wall. But its realisation is mechanically very complex and costly. Film cooling is less efficient, but mechanically simple, poses less severe manufacturing problems and is, therefore, comparable to transpiration cooling, if the overall costs of the engine are taken into account.

The high efficiency of film cooling - compared to convection cooling - was found already very early in the history of blade cooling (Küppers, 1944; Destival, 1949; Arne et al, 1951). The first investigations used mostly continuous slots for the ejection of the coolant, because ejection through slots is more efficient than ejection through holes, due to the higher mixing between hot gas and coolant in the latter case. However, it was found quite early that the high thermal stresses encountered on turbine blades do not allow the use of long slots. This is why the modern industrial film cooled turbine blades are provided with rows of holes or rows of very small slots.

At present there exists only limited information on this type of film cooling, due to the complex three-dimensional mixing of cool jets and hot gas and the large number of parameters involved. The present study is intended to furnish some information on the effect of the main flow parameters on the adiabatic wall effectiveness and the heat transfer coefficient downstream of a row of inclined ejection holes.
II. REVIEW OF PREVIOUS FILM COOLING RESEARCH

II.1 General considerations

The geometry and the flow field of the two main types of film cooling are sketched in Fig. 2. The ejection through slots can be done parallel or inclined to the cooled surface. The surface is covered by a continuous film of coolant. Mixing of coolant and hot gas occurs only at the outer edge of the film and is therefore not very intensive, especially if the velocities of gas and coolant are approximately the same. This type of film cooling is called also two-dimensional film cooling.

The flow pattern of film cooling with ejection through holes is more complicated. The coolant leaves the holes always at an angle to the cooled surface. Mixing between hot gas and coolant occurs over the whole surface of the jets and is, therefore, very intensive. This mixing leads to a large spreading of the jets and to the formation of a closed layer of coolant. The interaction of main flow and jets causes the formation of vortices, as sketched in Fig. 2, which are preserved within the closed layer of coolant over relatively long distances downstream. The presence of these vortices is indicated by flow visualisation studies described later and by the measurement of the velocity profiles downstream of the ejection (Liess et al., 1971). The intensive mixing of hot gas and coolant, increased additionally by the presence of the streamwise vortices, is the reason for the lower effectiveness of hole film cooling, if compared to slot film cooling.

Most experimental investigations use "film heating" instead of "film cooling", because of the experimental complexity involved with high temperature gas flows. The flow can be considered as a constant property flow if the temperature differences are small. The dimensionless temperature distribution in the boundary layer depends then only on the temperature of the main flow and the
ejected flow, but not on the direction of the heat flux (Goldstein, 1971). This has been verified by experiments (Metzger et al., 1968; Papell et al., 1959), where identical results were obtained from film cooling and film heating tests, even for large temperature differences between main flow and coolant. The ejection of the coolant causes apparently such a strong distortion of the main flow boundary layer that the stabilizing effect of wall cooling on the boundary layer becomes negligible. The present investigation was done also with film heating, i.e. a hot film was ejected into a cold main flow. The following equations are arranged accordingly. The term "film cooling" will be used in the following for both, film heating and film cooling, because the target of most investigations is the cooling of hot surfaces.

II.2 Basic relations

The heat transferred from a wall to a constant property flow can be expressed by (Goldstein, 1971)

\[ \dot{q} = h A (T_w - T_{aw}) \]

where \( \dot{q} \) is the heat per unit time, \( h \) the heat transfer coefficient, \( A \) the surface area, \( T_w \) the wall temperature and \( T_{aw} \) the adiabatic wall temperature of a thermally insulated surface. This equation can be applied also to a film cooled wall, provided that the heat transfer coefficient \( h \) and the adiabatic wall temperature are known for film cooling conditions.

The adiabatic wall temperature in the presence of film ejection depends on the gas temperature and the coolant temperature. This dependence can be eliminated by the use of the so-called film cooling effectiveness:
\[ \eta = \frac{T_w - T_{SG}}{T_{SC} - T_{SG}} \]

where \( T_{SG} \) and \( T_{SC} \) are the static temperatures of the gas and the coolant, respectively. This equation is used for low speed flows. For high speed flows it is more practical to use

\[ \eta_r = \frac{T_w - T_{rG}}{T_{oC} - T_{rG}} \]

where \( T_{rG} \) is the recovery temperature of the main stream and \( T_{oC} \) the stagnation temperature of the coolant.

The two parameters \( h \) and \( \eta \), or \( \eta_r \), have to be determined by analysis or experiment.

An important parameter in film cooling is the mass velocity ratio

\[ G^* = \frac{\rho_{SC} V_C}{\rho_{SG} V_G} \]

where \( \rho_{SC} V_C \) represents the density and velocity of the coolant at the slot or hole exit, respectively, \( \rho_{SG} V_G \) represents the density and velocity of the main flow upstream of the ejection, respectively. It will be shown later that the film cooling effectiveness \( \eta_r \) depends strongly on the mass velocity ratio \( G^* \).

II.3 Two-dimensional film cooling

Film cooling with ejection from a parallel or inclined slot has found considerable attention in the past. A survey of the main investigations is presented by Goldstein (1971). Most of the
theoretical and experimental work is concerned with the determination of the film cooling effectiveness \( n \), because it has been found quite early that the heat transfer coefficient is influenced only very moderately by the presence of film cooling.

**Heat transfer coefficient** \( h \)

Experiments of Hartnett et al. (1961) and Seban (1960) (1) showed that for distances \( \frac{x}{s'} > 50 \) and for ejection rates \( G^* < 1.0 \) the heat transfer coefficient with film cooling is equal to the heat transfer coefficient without film cooling. The symbols \( x \) and \( s' \) are defined in Fig. 2. These results were obtained for tangential ejection of the coolant. The experimental results of Metzger et al. (1968) for angled slots show the same tendency for \( G^* < 1.0 \). This means that the heat transfer coefficient is determined mainly by the mainstream boundary layer and not influenced by the ejection provided that the ejected mass flow is relatively small. The theoretical investigations of Leont'ev (1966) confirm this conclusion.

At high ejection rates \( (G^* > 1.0) \) and also close to the ejection slot the heat transfer coefficient can be determined from relations for the ejected flow alone, if the ejection is parallel to the surface (Seban 1960 (1), Seban et al., 1961). In this case, the wall jet is predominant and determines the heat transfer coefficient.

High ejection mass flow from angled slots seems to increase the heat transfer coefficient considerably, probably due to increased mixing of the flows (Metzger et al., 1971).

**Film cooling effectiveness** \( n \) or \( \eta \)

The theoretical investigations of the film cooling effectiveness use mainly two types of models, the heat sink model and the boundary layer model. The heat sink model assumes a line heat sink at ejection location. The temperature field in downstream direction produced by this heat sink is then calculated. The
ejected mass flow is neglected. The boundary layer model balances the ejected mass and the mass entrained from the mainflow. Both models lead to equations of the following type for the film cooling effectiveness at low speeds:

\[ \eta = \frac{K_1}{K_2 + K_3 \xi^{0.8}} \]

with

\[ \xi = \frac{x}{s'} \frac{1}{G^*} \left( \frac{\mu_{SC}}{\mu_{SG}} \right) \left( \frac{\text{Re}_C}{\text{Re}} \right)^{-0.25} \]

where \( K_1, K_2, K_3 \) are constants, \( \mu_{SC} \) and \( \mu_{SG} \) the viscosity of the coolant and the mainflow, respectively, and \( \text{Re}_C \) is the Reynolds number of the coolant based on the slot height. The agreement of this type of formula with experimental results is not always very good.

A combination of the two models is described by Eckert (1971) where the assumptions necessary for the boundary layer model are formulated in such a way that the values of the heat sink analysis are obtained for vanishing ejection. The effect of angled ejection is included also. The final relation is in excellent agreement with experimental results.

The experimental investigations showed also that the film cooling effectiveness depends mainly on the distance from slot exit \( \frac{x}{s'} \) and the mass velocity ratio \( G^* \) (Fig.3). The effectiveness decreases with \( \frac{x}{s'} \) continuously. It increases with \( G^* \) as long as the ejection is parallel to the surface, but there is a maximum for \( G^* = 1.0 \) if the ejection slot is inclined to the surface (Wieghardt, 1946). This is apparently due to a lifting of the jet from the surface at higher values of \( G^* \) and a corresponding increased mixing with the main stream.
Some experiments covering main stream Mach numbers from 0.1 to 0.7 showed that there is no influence of Mach number on the film cooling effectiveness in this range (Papell et al., 1959; Goldstein et al., 1965). Fig. 3 shows the results of one investigation.

The effect of variable main stream velocity on the film cooling effectiveness has been investigated for tangential ejection (Carlson et al., 1968; Escudier et al., 1968; Pai et al., 1970; Seban et al., 1962). The effect of pressure gradient is in general rather small and depends on the position of the gradient relative to the slot. If the acceleration of the main flow occurs close to the slot exit, i.e., in the mixing region of jet and main stream, then the effectiveness is decreased by increasing favorable gradients. Favorable gradients further downstream, as well as adverse gradients have only very small influence on $n$. The heat transfer coefficient is generally slightly reduced by favorable pressure gradients.

If the ejection is done from slots which are inclined to the direction of the main flow, the film cooling effectiveness is reduced progressively with increasing ejection angle (Metzger et al., 1971; Papell et al., 1959).

The thickness of the main stream boundary layer influences the effectiveness only very slightly (Burns et al., 1969; Kacker et al., 1967; Mabuchi, 1965; Seban, 1960 (2)). The effectiveness decreases with increasing boundary layer thickness. This is explained by the higher deflection of the jet caused by a thin boundary layer. But a tenfold increase of the boundary layer thickness resulted only in 5% reduction of the effectiveness (Seban, 1960 (2)).

This short review of slot film cooling research shows that the effect of the main flow parameters on the heat transfer is approximately known.
II.4 Three-dimensional film cooling

Film cooling with ejection from holes has found much less attention than two-dimensional film cooling, due to the complicated flow pattern described above. The presence of the streamwise vortices causes a variation of the adiabatic wall effectiveness and the heat transfer coefficient in downstream direction $x$ and in lateral direction $z$ (Fig. 2).

**Heat transfer coefficient $h$**

The only measurements of the heat transfer coefficient downstream of a row of ejection holes are reported by Metzger et al. (1971). An average value of the heat transfer coefficient in lateral and downstream direction over various downstream distances was measured with film cooling and compared to the corresponding value without ejection. The results show a similar trend as the corresponding results for slot ejection, i.e., an increase of the heat transfer coefficient close to the ejection, which is reduced with increasing distance from the hole exit. After a certain distance a constant value of $h$ is reached which is up to 10 percent higher than the "uncooled" value.

**Film cooling effectiveness $n$ or $n_r$**

An analytical approach to the single hole problem has been reported (Ramsey et al., 1970). The method is based on the solution for a point heat source moving in a semi-infinite medium. The final relation for the temperature field downstream of the ejection hole is given as

$$\frac{\theta(x,y,z)}{\theta_2} = \frac{G^* V_G d}{8\varepsilon(x/d)} e^{-0.693 \left[ \left( \frac{y}{y_{1/2}} \right)^2 + \left( \frac{z}{z_{1/2}} \right)^2 \right]}$$

$$\theta = T_S - T_{SG}; \quad \theta_2 = T_{SC} - T_{SG}$$
where \( T_s \) is the local static temperature, \( V_G \) the main flow velocity, \( \varepsilon \) the turbulent diffusivity and \( x, y, z \) the coordinates in downstream, vertical and lateral direction, respectively (Fig. 2). \( y_{1/2} \) and \( z_{1/2} \) are defined by

\[
\frac{\theta(x, y_{1/2}, 0)}{\theta(x, 0, 0)} = \frac{1}{2}; \quad \frac{\theta(x, 0, z_{1/2})}{\theta(x, 0, 0)} = \frac{1}{2}
\]

The thermal diffusivity has to be determined from experiments and is shown to be a function of the mass velocity ratio \( G^* \) and the downstream distance \( x \). Introduction of the corresponding diffusivity gives very good agreement with experiments. This calculation method is claimed to be applicable also to rows of holes by superposition of the effect of a number of single ejection holes.

The disadvantages of this method are the necessarily experimental determination of the turbulent diffusivity \( \varepsilon \) and the limited application range. Since it neglects the effect of mass addition to the main flow boundary layer, it is applicable only to small ejection ratios. A further limit is given by the ejection rate at which the jets start to lift off the wall, i.e., at \( G^* = 0.5 \).

Local measurements of the film cooling effectiveness downstream of a row of ejection holes showed a variation of the effectiveness in lateral direction increasing with the spacing to diameter ratio \( \frac{s}{d} \) (Goldstein et al, 1969; Liess et al, 1971; Eckert, 1971).

The mass velocity ratio \( G^* \) has an important influence on the effectiveness, because all the tests showed that a maximum of \( \eta \) exists at \( G^* = 0.5 \) for all downstream distances. Apparently, the jets start to lift off from the surface at this value allowing the main flow to slip in between jets and surface. Laterally
averaged values of Metzger et al (1969) and Jones et al (1971) show the same tendency close to the ejection. Further downstream the effectiveness increases continuously with increasing mass velocity ratio.

The effect of upstream boundary layer thickness was studied by Goldstein et al (1969) for single hole ejection and found to be quite remarkable, especially at a blowing rate of 0.5. Doubling the displacement thickness of the main flow boundary layer caused a decrease of the film cooling effectiveness by 50% at $G^* = 0.5$. This effect was attributed to the higher bending of the jet for the thinner boundary layer.

The effects of the main stream Mach number, main stream boundary layer, and velocity gradient in the main stream on the heat transfer coefficient and the adiabatic wall effectiveness downstream of a row of ejection holes are still unknown and are, therefore, investigated in the present study.

Film cooling with ejection at a stagnation point has also not yet been investigated. Some results will be presented in the following.
III. EXPERIMENTAL FLOW CONDITIONS

The present investigation was intended to provide information on film cooling which can be used to the design of film cooled gas turbine blades. This means that the non-dimensional similarity parameters, such as Reynolds number, Mach number, Nusselt number, of the tests should be as close as possible to the corresponding values in a real turbine.

The flow conditions on gas turbine blades were determined experimentally by testing a cascade of a typical advanced gas turbine profile in a high speed wind tunnel at V.K.I. The Reynolds number and Mach number at the inlet of the cascade corresponded to the working conditions in an actual turbine. These tests provided information on the range of local Mach numbers on the blade, the velocity gradients, the type of the boundary layer, and the range of local boundary layer displacement thickness.

The film cooling tests were done with one fixed ejection geometry. The ejection angle and the spacing diameter ratio correspond to mean values realized on existing film cooled blades.

The final test conditions, listed below, represent a compromise between the similarity requirements and the possibilities of the experimental apparatus. The only parameter, which could not be reproduced accurately in the film cooling tests is the local Reynolds number based on the local boundary layer displacement thickness. The test values differ by about 50 percent from the actual turbine values. However, it will be shown later, that within the investigated range of flow conditions, the Reynolds number has no measurable effect on the heat transfer parameters.

The actual test conditions are the following:

- 12 -
Varied parameters:

- Main flow Mach number
- Main flow pressure gradient
- Main flow boundary layer upstream of ejection
- Ratio of main flow boundary layer displacement thickness to ejection hole diameter
- Mass velocity ratio
- Ejection pressure ratio for stagnation point ejection

Constant parameters:

- Ejection angle (flat plate)
- Ejection angle (stagnation point)
- Ejection hole spacing
- Ejection stagnation temperature
- Main flow stagnation temperature
- Inlet flow Mach number for stagnation point ejection

The ratio of main flow boundary layer displacement thickness to ejection hole diameter $\delta_1/d$ was varied by the effect of the Mach number and the type of the boundary layer on the boundary layer thickness and by varying the ejection hole diameter from 1 mm to 4 mm. The length of the ejection holes corresponded approximately to the hydrodynamic starting length for turbulent pipe flow.

The adiabatic wall temperatures and the heat transfer coefficient were measured over a maximum downstream distance of $x/d = 80$. This covers the interesting range of downstream distances on gas turbine blades.
IV. EXPERIMENTAL APPARATUS AND PROCEDURE

IV.1 General test set up

A sketch of the complete test set-up is shown in Fig. 4. The tunnel is a blowdown facility with a maximum running time of fifty minutes. The air is taken from high pressure reservoirs of 60 m$^3$ which are continuously filled with dry air up to a maximum pressure of 40 atmospheres. The temperature of the air in the reservoirs is kept constant by 5 tons of sheet metal stored inside the reservoir.

The main flow air enters the settling chambers through a backwards bent perforated pipe to suppress any inlet swirl. It passes through a honeycomb screen and wooden contours into a rectangular channel. The side walls of the channel are formed by perspex plates, the upper and lower wall are formed by straight wooden blocks. The flow channel has a width of 50 mm and a height of 100 mm. The model is situated in the test section at the end of the straight flow channel.

The secondary air is taken from the high pressure supply system of the Institute which provides air at a pressure of $\frac{1}{4}$ atmospheres and at about room temperature. It passes first through a regulation valve and a flow meter. The air is then heated up in a heater by electrical resistances, passes through a steel pipe and flexible rubber hoses into the perspex feed tubes of the ejection element. The heater and the subsequent steel tube were well insulated with glass wool. This, together with the low thermal conductivity of the rubber hoses and the perspex feed tubes, minimized the heat losses to the surroundings.

IV.2 Test section and model

The test section is shown in detail in Fig. 5. The model has a symmetrical form and is placed in the centre of the straight flow channel. This is the only possible arrangement for the study of film cooling with ejection at the stagnation point.
In addition, it allows the control of the boundary layer upstream of the ejection and reduces possible heat losses to the surroundings to a minimum.

The thickness of the model was chosen as 20 mm. This is a compromise between the manufacturing requirements and the aerodynamic requirements. The quite complicated ejection elements are easier to manufacture if the model is thick and the elements large. On the other hand, the main flow area has to be large enough so that the main flow properties, such as Mach number and Reynolds number, are not sensibly altered by the ejection of the secondary flow. The highest value of the ratio of ejection mass flow to main mass flow was about 0.075 for the maximum ejection area and the maximum ejection mass flow. This ratio was much smaller for all the other test conditions. During the tests practically no effect of the ejection on the main-stream Mach number was observed.

The wooden blocks forming the upper and lower wall of the flow channel (see Fig. 5) could be replaced by contoured blocks for the study of the effect of pressure gradients. Two favorable gradients were chosen as corresponding to the gradients measured on the cascade blades. The wooden contours were designed and fabricated to produce an approximately linear variation of static pressure just downstream of the ejection holes.

The leading edge of the model was designed in such a way that either a laminar or a turbulent boundary layer could be produced upstream of the ejection holes by replacement of the nose part.

The nose for the turbulent boundary layer had simply a circular leading edge. The local velocity on a cylinder in a cross flow increases from zero at the stagnation point to a value which is about twice the free flow velocity and decreases then again. The maximum velocity occurs at the point where the cylinder surface is parallel to the free flow direction (Schlichting
et al., 1967). The same happens on a plate with a cylindrical leading edge. In that case there appears in the region where the flat plate starts a strong deceleration from the maximum velocity to the flat plate velocity. The laminar boundary layer which has developed from the stagnation point along the cylindrical surface cannot support the sudden pressure rise and separates locally. This small separated region causes transition of the laminar boundary layer.

To obtain a laminar boundary layer at the ejection point, it is necessary to have a steady increase of the velocity from the stagnation point to the ejection location. A series of symmetrical airfoils with a laminar boundary layer over almost the whole profile length was developed by NACA (Abbott, et al., 1959). The front part of such a profile was taken as nose contour for the model. Since it was important to avoid any deceleration of the flow along the surface, the velocity distribution was checked for both noses by Martensen's method for the calculation of incompressible potential flow (Van den Braembussche, 1970). A corresponding method for compressible flow was not available at the time when the model was designed. The laminar form was changed until a steady velocity increase from zero to the flat plate value was obtained. The final nose profile corresponds to the front part of the NACA 16-015 basic thickness form and is shown in Fig. 5a together with the turbulent nose form. A comparison of the velocity distributions calculated with Martensen's method and measured are shown in Fig. 6 for both model noses. The maximum velocity on the cylindrical nose is 2.1 times the inlet velocity. The curvilinear coordinate $\ell$ is made non-dimensional by the nose radius of the turbulent nose corresponding to one half of the model thickness.

The ejection elements are indicated also in Fig. 5 for the flat plate ejection and for the nose ejection tests. The heated secondary air enters in both cases the elements from both sides by feed tubes passing through the large hole of the ejection element.
In the flat plate element it passes then through a small settling chamber into the ejection holes. The stagnation point elements were too small for a settling chamber. The air passes here directly from the main feed hole into the ejection holes.

All ejection elements, as well as all other model parts are manufactured from perspex. This material is chosen because of its low conductivity and relatively high strength. The model noses and ejection elements are interchangeable.

IV.3 Heat transfer test plate

The test plate for the heat transfer measurements is sketched in Fig. 5. The test plate is made of perspex with a thermal conductivity of \( k_{\text{perspex}} = 0.174 \, (\text{J/sec m} \cdot \text{°K}) \) which is only 0.05% of the thermal conductivity of copper. 53 copper strips with a cross section of 1 x 2 mm are inserted in grooves directed perpendicular to the flow direction. The length of the copper elements is 36 mm. The elements are separated by 1 mm of perspex left between the grooves. Additional grooves of 1 x 1 mm cross section are fabricated below the copper elements. They provide space for the thermocouples fixed to the bottom side of the elements and serve also as insulation. The thermocouples are made of copper-constantan wires of 0.2 mm thickness and soldered to the copper elements at half span. They are led along the elements in order to avoid conduction errors due to the presence of the thermocouples. The copper elements are fixed with glue in the grooves flush to the plate surface. After mounting of the elements the plate was polished until a completely smooth surface was obtained. The thickness of the copper elements which is important for the calculation of the heat transfer coefficient, was measured before mounting. The distance from the bottom side of the plate to the outer surface of each element was determined before and after polishing of the test surface, so that the final thickness of the mounted elements could be determined with
an accuracy of 0.01 mm corresponding to 1 percent of the element thickness.

The test plate can slide laterally in slots in the perspex side walls of the test section in such a way that the copper elements can be placed easily inside or outside of the test section.

The test section with the instrumented heat transfer plate is shown in Fig. 7. During the tests at constant main stream velocity the static pressure along the test plate was slightly below the atmospheric pressure. The tolerances of the slots in the side walls allowing the shifting of the model were kept as small as possible in order to reduce leakage of ambient air into the test section. Leakage could be prevented completely for the test with constant main flow velocity by covering one slot with a closed box. This slot was on the non-instrumented side of the test plate. The other slot could not be closed in the same way because of the presence of the thermocouple wires. Leakage on the upper side of the test plate was prevented here by a metallic end plate fixed to the test plate (see Fig. 7). This metallic end plate was covered with a thin layer of plasticine at the inner side so that the slot was efficiently closed when the model was injected.

For the tests with variable main stream velocity and nose ejection the closed box on the non-instrumented side of the plate was not mounted because the static pressures in the low velocity regions were much higher than the atmospheric pressure and would have caused increased leakage from the high pressure region through the box into the low pressure region. However, the interesting parts on the test plate were for these investigations the regions with the high static pressure in the main flow, where leakage occurred only from the test section to the atmosphere so that the thermal conditions in the test section were not affected.
With this test plate it was possible to measure local values of temperatures in the downstream direction $x$ and average values in the lateral direction $z$. The neglect of the lateral variations was necessary because only a limited number of measuring values could be recorded at the same time. The downstream variation of adiabatic temperatures and heat transfer coefficients was considered to be more important for engineering purposes than the variation in lateral direction, where identical conditions are repeated periodically at very close distances.

**IV.4 Instrumentation**

The local main flow velocities on the models are obtained by the measurement of the static pressure distribution on the model. For these measurements the model shown in Fig. 5 is replaced by a solid version of identical shape instrumented with static pressure tappings.

The side walls of the tunnel are instrumented also with two rows of static pressure tappings for the control of the uniformity of the inlet flow and the measurement of the main flow conditions during the film cooling tests. One vertical row of static pressure tappings spaced at 10 mm is placed close to the settling chamber (see Fig. 4) covering the height of the flow channel. The second line is placed horizontally over the whole length of the side plates at 10 mm above the upper model surface and spaced at 20 mm. These static pressure tappings are shown also in Fig. 7.

A combined total temperature-total pressure probe is placed close to the settling chamber somewhat below the centre of the channel at the same position as the vertical static pressure line. The main flow conditions at inlet are determined with this probe and the corresponding static pressure tappings. All static pressure tappings as well as the total inlet pressure are read on a mercury multi-manometer. The inlet stagnation temperature is recorded on a recording galvanometer described
below. The cold junction of the thermocouple of the stagnation temperature probe is kept in an ice bath.

The velocity profiles of the boundary layer on the models were measured at the ejection location and 130 mm further downstream. This was done in order to be sure about the actual boundary layer displacement thickness and the nature of the boundary layer at ejection. The probe is shown in Fig. 8. It was introduced laterally into the test section through a slot in the side wall and could be moved in the vertical direction by means of a carriage. The vertical movement could be controlled with a precision of 0.01 mm. Since the thickness of the boundary layers on the relatively short models was small, it was necessary to reduce the size of the probe end as much as possible. A stainless-steel tube of 0.6 mm outer diameter and a wall thickness of 0.15 mm was flattened at the end to the dimensions shown in Fig. 8. The probe end was controlled and measured under a microscope. The centre of the probe opening was only 0.16 mm from the model wall if the probe touched the model.

The secondary mass flow was regulated by a valve upstream of the calibrated flow meters. The total temperature of the cold ejection flow in the flow meter was measured by the same type of probe as the stagnation temperature of the main flow, i.e. a shielded thermocouple.

The secondary flow was heated in the heater to a temperature between 50° C and 100° C, depending on the ejected mass flow. The electric power supplied to the heater could be varied by a variable resistance. A by-pass valve allowed a continuous operation of the heater without the danger of pre-heating model and test section by continuous ejection of hot secondary flow.
The stagnation conditions of the hot ejection flow were measured upstream of the perspex feed tubes in small perspex blocks providing the connection between rubber hoses and feed tubes (see Fig. 7). The stagnation pressure was measured by a pitot tube placed in a small settling chamber in one connecting block upstream of the feed tube. The stagnation temperature was measured by a shielded thermocouple placed at the corresponding location in the other connecting block. This separation of the measuring stations was necessary in order to avoid different flow rates in both feed tubes caused by blockage of the flow in one feed branch due to the presence of the probes. Equality of the flow in both branches was assured by geometrically identical realization of the feed branches. The stagnation pressure of the ejection was read on the mercury multi-manometer, as well as the static pressure in the flow meters.

The thermocouples used for the measurement of the stagnation temperature of the main flow, the cold secondary flow and the hot secondary flow were all copper-constantan thermocouples. The cold junctions were kept isolated from one another in a well stirred ice bath. The temperature of the ice bath was controlled by means of a precision mercury thermometer with the accuracy of 0.1° C.

The copper-constantan thermocouples fixed to the bottom side of the copper elements on the test plate were led out through small grooves in the plate. These grooves were filled with cement after mounting of the copper elements. The "cold" junctions of the thermocouples were kept in a well stirred bath at room temperature insulated from one another. The copper lines of the hot junctions and of the cold junctions were connected to a switch box. The recorders were connected to the switch box by two multi-channel plugs. The switch box allowed the rapid connection of different sets of thermocouples to the recorders. This was necessary because the number of recorders available at the Institute was limited, so that each
recorder had to be used for two thermocouples. Each test was therefore performed twice for the two different sets of thermocouples.

Two types of recorders were used for the measurements of the adiabatic wall temperature and the heat transfer coefficient. A series of 11 graphispot recorders was used as galvanometers, the voltage output of the thermocouples being measured directly without any intermediate resistance. The graphispos were used only for the measurement of the copper element temperatures, due to their high sensibility.

In addition, a multichannel CEC galvanometric recorder was used for the measurement of the stagnation temperature of main flow and heated secondary flow and for additional measures of plate temperatures. The recorder contained eleven galvanometric elements of two different sensibilities. Five elements of the highest sensibility were used for the measurement of the copper element temperatures, two less sensible galvanometric elements were used for the determination of the stagnation temperatures of main flow and heated secondary flow.

Schlieren pictures of the flow with ejection of hot air were obtained with a conventional schlieren system allowing continuous observation of the flow and spark-light photographs on fast developing polaroid.

Oil flow visualization of the flow close to the surface was done for a better understanding of the actual flow behaviour downstream of the ejection by applying a mixture of medium grade oil and titanium dioxide to the surface following ejection.
IV.5 Test procedure

As mentioned before, each test was done twice in order to obtain data from two sets of copper elements on the plate. The impossibility to obtain identical values of all test conditions for two separate runs is taken into account by proper non-dimensionalization of the final data.

For the measurements of the adiabatic wall temperatures the model was placed with the copper elements in the test section. The main flow was started at the required Mach number and the secondary flow was regulated to the required mass flow rate. The temperatures of the copper elements were recorded when thermal equilibrium was reached. The stagnation temperatures of the main flow and the secondary flow were recorded also.

For the measurements of the heat transfer coefficients the plate was placed with the copper elements outside of the test section until it reached a uniform temperature corresponding to the ambient temperature of about 18°C. The main flow and the secondary flow were then started at the required values and the copper elements were injected laterally into the test section. The transient variation of the element temperatures was recorded, as well as the stagnation temperatures of main flow and secondary flow. The temperature gradient of the copper elements was used then for the evaluation of the heat transfer coefficient (see section IV.7).

IV.6 Additional verifications

The accuracy of the technique for the measurement of the heat transfer coefficient was determined by measuring the heat transfer coefficient at the stagnation point of a cylindrical leading edge and by comparing the results with existing analytical solutions.
The leading edge of a model with a cylindrical nose was instrumented with a copper element and two thermocouples in the same way as the test plate. Instrumentation and test procedure were the same as for the main test series.

The analytical solutions for incompressible air flow provide the following relations for the stagnation point Nusselt number:

\[ \text{Nu}_D = 0.994 \text{ Re}_D^{0.5} \]  
\[ \text{Nu}_D = 0.945 \text{ Re}_D^{0.5} \]

(Van Driest, 1956)  
(Frössling, 1958)

with \( \text{Nu}_D = \frac{h_D}{k}, \text{Re}_D = \frac{V \rho s D}{\mu s l}, \ D = 2R_N \)

The test results can be summarized as

\[ \text{Nu}_D = (0.78 - 0.85)\text{Re}_D^{0.5} \]

This means that the absolute value of the heat transfer coefficient measured by the present technique is 15 to 20 percent lower than the real value, due to heat conduction from the perspex plate to the copper elements. This systematic error disappears, however, if the results are presented in the non-dimensional form described below.

The losses in the ejection elements were determined in a separate test series without main flow. The cooling of the ejection elements due to the presence of the main flow was taken into account separately. The losses of stagnation pressure and temperature of the secondary flow occur between the measuring station and the exit of ejection holes and are due to friction, disturbance of the flow by changes in flow area and direction, and heat transfer from the feed tubes to the ambient air. The stagnation pressure and temperature at the exit of the ejection holes were measured with a small total temperature-total pressure
probe. Average values of pressure and temperature were obtained from the measured pressure and temperature profiles and related to the conditions at the measuring stations. This allows the use of the actual conditions at the outlet of the ejection holes for the correlation of the final data, so that the loss characteristics of the different ejection geometries does not affect the final results.

The periodicity of the ejection flow was checked at the same time as the total pressure and temperature losses in the ejection elements. The periodicity was found to be quite acceptable in the central part of the row of ejection holes, where the copper element thermocouples were located. The maximum differences over the whole row of holes were found to be 0.7% of the corresponding total pressure or temperature. In general they were much smaller.

The assumption of uniform temperature in the copper elements was checked by injecting the model for one set of test conditions once in the normal position and once to a position which differed from the first one by one half of the hole spacing. The hole diameter of 4 mm with a spacing of 12 mm was chosen for this check in order to have the most severe conditions. The temperature gradients obtained for the two positions agreed within the accuracy of the measurements.

IV.7 Data reduction

The reduction of the data was performed with the IBM 1130 computer of the von Karman Institute. The principle of the data reduction will be outlined in the following.

The main flow conditions at inlet are determined from the measured stagnation pressure and temperature and the static inlet pressure. The same is done for the main flow conditions upstream of ejection. The ejected mass flow is obtained
from the measured flow meter data and the calibration curves of the flow meters. The mass velocity ratio can be obtained from the main flow conditions and the ejected mass flow together with the ejection cross-sectional area $F_C$

$$G^* = \frac{m_C}{F_C} \frac{1}{\rho_S G}$$

The stagnation conditions of the ejected flow at hole outlet are obtained from the measurements by means of the loss curves for the individual ejection elements. The static conditions and the velocity of the ejection flow at the outlet of the ejection holes are obtained from these stagnation conditions and the mass flow rate.

The local adiabatic wall effectiveness averaged in lateral direction is calculated as

$$\bar{\eta_r} = \frac{T_{aw} - T_{rG}}{T_{aw} - T_{SG}}$$

The adiabatic wall temperature is obtained from the tests by means of the calibration curve of the individual copper element. The local recovery temperature of the main flow is calculated from the measured velocity distribution on the model by

$$T_{rG} = T_{SG} \left(1 + r \frac{Y-1}{2} M^2_G\right)$$

The local heat transfer coefficient averaged in lateral direction is calculated from the transient temperature variation and the properties of the copper elements as

$$\bar{h} = \frac{\rho_C c_w}{T_w - T_{aw}} \frac{dT_w}{dt}$$
The density and specific heat of the copper elements were obtained from the manufacturer as

\[ \rho_w = 8930 \text{ kg/m}^3 \quad \text{and} \quad c_w = 385.2 \frac{J}{\text{kg grd}} \]

The thickness of the copper elements \( \delta_w \) on the plate was measured as described in section IV.3. The thickness of the elements on the rounded elongation of the plate for the nose ejection tests (see Fig. 5b) was measured after completion of the test series and subsequent demounting of the elements.

The temperature gradient \( \frac{dT}{dt} \) and the wall temperature at ejection \( T_w \) were obtained from the measurements and the calibration curves of the individual copper elements. The local adiabatic wall temperature \( T_{aw} \) in the presence of film cooling was calculated from the adiabatic wall effectiveness \( \eta_r \) at the same mass velocity ratio and local properties of the main flow by

\[ T_{aw} = T_{rg} + \eta_r (T_{OG} - T_{rg}) \]

The heat transfer coefficient can be expressed non-dimensionally as Nusselt number:

\[ \bar{Nu}_x = \frac{h_x}{k_T*} \]

where the thermal conductivity \( k_{T*} \) is evaluated at the reference temperature \( T^* \) defined by

\[ T^* = 0.5 (T_w + T_{SG}) + 0.22 (T_{aw} - T_{SG}) \]

This expression is proposed by Hartnett et al. (1961) for the application to film cooling.
The influence of systematic errors can be reduced by use of the ratio of Nusselt number with ejection to the Nusselt number without ejection:

\[ \frac{(\text{Nu}_x)_{\text{ejection}}}{(\text{Nu}_x)_{\text{no ejection}}} \]

A representation of the results in this form allows the direct evaluation of the Nusselt number and hence the heat transfer coefficient on a film cooled turbine blade if the Nusselt number without ejection is known.

The Nusselt number without ejection was obtained from measurements of the heat transfer coefficient without secondary flow ejection. The ejection element was replaced for these tests by a solid element providing a closed smooth surface upstream of the test plate.

IV.8 Accuracy of the measurements

The maximum possible errors of the final parameters was estimated from the maximum errors of the individual measured values, such as temperatures, pressures, mass flows and temperature gradients.

The maximum error in the measurement of temperatures is 0.5°C, the maximum error of the pressure measurements is 3 mm Hg. The mass flow can be determined with a precision of ±3 percent, as well as the temperature gradients. With these individual errors it is possible to determine the maximum errors of the final parameters.
The maximum relative error of the mass velocity ratio

\[ \left( \frac{\Delta G^*}{G^*} \right)_{\text{max}} = 0.041 \]

The maximum relative error of the adiabatic wall effectiveness

\[ \left( \frac{\Delta \bar{n}_r}{\bar{n}_r} \right)_{\text{max}} = 0.16 \]

Additional errors occurred due to conduction in the test plate caused by the different temperatures on both sides of the test plate and by the temperature differences between the copper elements. This error was estimated for some extreme cases by establishing a heat balance for the elements and by solution for the real adiabatic wall temperature. The resulting maximum possible error due to heat conduction was found to be of the order of 1 percent in \( \bar{n}_r \). These losses are much less for most of the test conditions and copper elements.

The maximum relative error in heat transfer coefficient is

\[ \left( \frac{\Delta h}{h} \right)_{\text{max}} = 0.19 \quad \text{for ejection} \]
\[ \left( \frac{\Delta h}{h} \right)_{\text{max}} = 0.11 \quad \text{without ejection} \]

The error is larger in the case of ejection due to the use of \( \bar{n}_r \) for the determination of \( T_{aw} \) and the resulting introduction of the \( \bar{n}_r \)-error.

Additional errors were introduced by conduction effects in the perspex material surrounding the copper elements. This error was evaluated experimentally and found to be of the order
of 20% (see section IV.6). However, it does not appear in the final results, which are presented in non-dimensional form. The final maximum error for the ratio of the Nusselt number with ejection to the Nusselt number without ejection is given as

$$({\Delta N_u}^*)_{\text{max}} = 0.30$$

As mentioned before, the final errors of $\bar{\eta}_r$ and $N_u^*$ represent the maximum errors, i.e. the cases where all individual measuring errors influence the final result in the same way. A mean value of the final error can be estimated from the scatter in the final results. Each curve of $\bar{\eta}_r$ or $N_u^*$ in function of $x/d$ was obtained in two runs. In the first test the values of the copper elements number 1, 3, 5, 7, ... were recorded, in the second test the values of the numbers 2, 4, 6, 8, ... The smoothness of the $\bar{\eta}_r$ curves and the fact that the $N_u^*$ curves represent the data points within ± 5% indicate that the actual errors were much smaller than the estimated ones. The mean errors are estimated as

$$({\Delta \bar{\eta}_r})_{\text{mean}} = 0.02 \quad \text{and} \quad ({\Delta N_u}^*)_{\text{mean}} = 0.15$$
V. EXPERIMENTAL RESULTS WITHOUT EJECTION

V.1 Velocity distributions on models

The velocity of the main flow along the model was measured by two different sets of static pressure tappings: the tappings on the model and the tappings on the side wall. The results of the two measurements are in good agreement.

Constant main flow velocity

The detailed velocity distributions measured on the laminar and turbulent nose are already shown in Fig. 6 together with the predictions for incompressible flow. The velocity distributions obtained for the lowest Mach number are in good agreement with the prediction. The differences at higher Mach numbers can be attributed to the combined effect of compressibility and boundary layer development, because the boundary layer on model and channel walls was not taken into account for the prediction.

The complete velocity distribution on the model is shown in Fig. 9 for the laminar nose. The values measured by the side wall pressure tappings are indicated also for comparison. The scatter in the data for the lowest Mach number is due to the precision in reading the values on the mercury manometer. The readings of the values in the constant velocity region differ by about 1 mm Hg.

The curves show that a maximum in the velocity existed on the model for the Mach numbers \( M_0 = 0.6 \) and \( 0.9 \) at \( \frac{L}{R_N} = 6.0 \). The slight deceleration of the flow downstream of the maxima could cause the beginning of boundary layer transition. The state of the boundary layer at the point of ejection was therefore checked by measurements of the boundary layer profiles and by
visualization of the transition point.

The velocity distribution on the model with the turbulent nose is shown in Fig. 10 for the three test Mach numbers. The scatter in the data points at $M_a = 0.3$ is again due to reading uncertainties. The values obtained from the wall tappings are indicated again and are shown to be in agreement with the values measured on the model.

Variable main flow velocity

As mentioned above, two sets of wooden contours were manufactured providing favorable pressure gradients of 75 mm Hg/cm and 150 mm Hg/cm (see Fig. 5a). The velocity distributions on the model with the turbulent nose in the presence of these pressure gradients are shown in Fig. 11. The values measured by the wall static tappings are indicated also. The agreement between the values measured on the wall and on the model is not as good as for the constant velocity configuration. This is due to the vertical distance between model and wall tappings and the non-uniform vertical velocity distribution in the accelerated flow.

The main flow velocity at ejection or just upstream of it is about $M_a = 0.3$. The flow is then accelerated to $M_a = 0.9$ and remains close to this value along the rest of the model.

Model for ejection at stagnation point

The shape of the model used for the ejection of the coolant from the stagnation point corresponded to the turbulent model nose with the cylindrical leading edge. This geometry was necessary in order to house the ejection system. The object of the investigation was the effect of the stagnation point ejection in the region close to the leading edge and also further downstream. The presence of the separated region on the cylindrical
model nose was undesirable because it would influence the behaviour of the flow along the flat plate. Since the separation was caused by the deceleration following the velocity overshoot the only way to suppress this separation was to impose a strong counteracting acceleration in the region where the deceleration was present. The velocity distribution shown in Fig. 12 was obtained after numerous attempts. The applied pressure gradient contour is depicted in Fig. 5b. The values measured by the wall static tappings are indicated also. The discrepancy between the results of the two measuring stations is again due to the difference in position and the nonuniformity of the flow field. The inlet velocity of the flow is $M_1 = 0.22$. The maximum velocity is $M_G = 0.92$. The curve demonstrates that the flow is accelerated continuously from the leading edge to the flat plate velocity. Additional checks with oil flow visualization confirmed the conclusion that the separation region was effectively suppressed.

V.2 Boundary layers on models

Calculations from measured velocity distributions

The development of the boundary layer along the models was calculated from the measured velocity distributions shown in Figs. 9 and 10. The calculation was performed with the method of Walz (1969) for compressible flow.

Fig. 13 shows a comparison of the calculated boundary layer displacement thickness on models and blade. The displacement thickness and the curvilinear coordinate are made non-dimensional by the leading edge radius of the turbulent model nose or of the blade leading edge. This dimension represents the thickness of the model and of the blade downstream of the leading edge and is the only comparable geometrical dimension. Fig. 13a shows that the boundary layer displacement thickness on the models is essentially of the same order as on the blade,
i.e. \(0.01 < \delta_1/R_N < 0.04\) for the turbulent boundary layer and \(0.005 < \delta_1/R_N < 0.025\) for the laminar boundary layer.

The tendency of the turbulent boundary layer curves is very different, due to the different velocity gradients on blade and models. The strong acceleration on the blade pressure side causes a decreasing boundary layer displacement thickness. Similar flow conditions were produced by the pressure gradient contours mentioned above, as one can see from Fig. 13b, where the velocity distribution on the model in the presence of the pressure gradients is compared to the velocity distribution on the blade. The figure shows also that the development of the turbulent boundary layer displacement thickness is very similar.

Measurement of velocity profiles

Fig. 14 shows two representative sets of boundary layer velocity profiles obtained at ejection and downstream. The distance from the model surface is non-dimensionalized by the boundary layer thickness defined by the height where the local velocity reaches 99.5 percent of the free stream velocity. The local velocity is expressed as a fraction of the free stream velocity. The profiles measured at ejection have the typical shape of laminar and turbulent boundary layer profiles. The profiles obtained at different main flow Mach numbers are very similar. The displacement thickness of the laminar profile is only half the value of the turbulent profile. The laminar and turbulent nature of the boundary layers is confirmed by the values of the shape parameters \(H_{12}\) (1.73 for the turbulent nose and 2.38 for the laminar nose at ejection) which are within the range of turbulent and laminar \(H_{12}\)-values.

The velocity profiles measured 130 mm downstream of the ejection are identical and apparently both turbulent. This means that transition of the laminar boundary layer has occurred between the two measuring stations. This result was already indicated by the slight deceleration found in the velocity
distributions on the model with the laminar nose.

A comparison of the measured and predicted displacement thickness of the model boundary layers at ejection (Fig. 13a) shows that the measured values are 10 to 40% higher than the predicted values of both types of boundary layers. This phenomenon can be explained for the turbulent boundary layer by an increase of the real boundary layer thickness due to the small separation region close to the leading edge which is not taken into account by the boundary layer calculation. The values measured further downstream agree within 10 percent with the prediction.

The transitional nature of the laminar boundary layer can be the explanation for the thicker boundary layer in this case. The values measured further downstream on the model with the laminar nose agree within 15 percent with the calculations if the instability point is taken as transition point.

Visualization of transition

An attempt to determine the transition points on the models was done by the application of a visualization technique. A sublimation technique was applied which makes use of the difference of the heat transfer and the mass transfer process in laminar and turbulent boundary layers.

The transition on the turbulent nose was very clearly indicated at $\frac{L}{R_N} = 1.7$ which corresponds to the location of the strong deceleration of the local boundary layer on the model (see Fig. 6b). The transition of the boundary layer on the laminar model was not so clearly determined, which may be due to an extended transition range. However, the tests indicated the beginning of transition at $\frac{L}{R_N} = 6.0\rightarrow 7.0$. 

V.3 Adiabatic wall temperatures

The adiabatic wall temperatures on the test plate were measured without ejection in order to check the type of the boundary layer on the plate. The ejection element was replaced for these tests by a solid element without ejection holes but with the same outer shape as the ejection elements. The measured temperatures obtained with the turbulent nose differed in general by 1° K from the recovery temperature calculated with the local velocity of the main flow.

The wall temperature obtained with the laminar nose was compared also with the theoretical adiabatic wall temperature using the laminar recovery factor of $r = 0.85$ or the turbulent recovery factor $r = 0.88$. The differences were up to 2.7° K for the laminar recovery factor and only 1.2° K for the turbulent recovery factor. This confirms the conclusion from the boundary layer investigations described above that the transition of the boundary layer starts approximately at the ejection location and that the boundary layer on the test plate is turbulent also for the laminar nose.

The differences between the measured and calculated adiabatic wall temperatures are partly due to the limited running time of the wind tunnel. It would have required a testing time of more than one hour to obtain absolute thermal equilibrium of the tunnel and the model while the size of the high pressure reservoirs allowed only a maximum running time of about 50 minutes.

This limitation did not exist for the measurements of the adiabatic wall temperatures with ejection of hot air. The difference between the adiabatic wall temperature with ejection and the model temperature before the test was much smaller than without ejection. The necessary time for the achievement of equilibrium conditions was therefore shorter and within the
available range of testing time. The accuracy of the adiabatic wall temperature measurements with ejection is, therefore, higher than without ejection.

V.4 Heat transfer coefficients

The distribution of the heat transfer coefficient along the test plate was measured without ejection for both model noses. The results are corrected for the conduction effects and compared in Fig. 15 with the predictions for laminar boundary layers by Walz, 1969. The laminar nose test results are an order of magnitude higher than the predictions for laminar boundary layers and agree reasonably well with the turbulent nose results.

The indications on the state of the boundary layer can now be summarized as follows: The boundary layer on the model with the turbulent nose is clearly turbulent. The boundary layer downstream of the laminar nose is laminar up to the ejection location or shortly upstream of it. Transition occurs in the region of the ejection and the boundary layer is then fully turbulent along the test plate. This means, that ejection into a fully laminar boundary layer could not be realized, as it was intended.
VI. ADIABATIC WALL EFFECTIVENESS

VI.1 General discussion of results

The basic test results concerning the adiabatic wall effectiveness downstream of a row of 2 mm diameter ejection holes are presented in Figs. 16, 17, 18 for the three Mach numbers $M_g = 0.31, 0.61, 0.89$ and constant main flow velocity.

The relative position of the curves for the different ejection mass flows or mass velocity ratios, indicates the importance of this parameter on the adiabatic wall effectiveness. At low ejection rates the effectiveness decreases continuously with increasing downstream distance $X_d$ from the ejection. This trend is reversed at high ejection rates, where the effectiveness increases with the downstream distance $X_d$. This can be seen particularly well on Fig. 16 where the test results obtained at the lowest Mach number are shown. The mass velocity ratio was varied for this test condition from $G^* = 0.15$ to $G^* = 2.0$. The effectiveness increases continuously over the whole test surface for $G^* = 2.0$.

An explanation for this reversal of the trends of the adiabatic wall effectiveness due to the mass flow rate can be found by consideration of the behaviour of the ejected flow. At low mass velocity ratios the ejected flow has a low velocity at hole exit and the velocity component normal to the main flow or normal to the test surface is small. The jets are, therefore, deflected by the main flow close to the surface, providing a high adiabatic wall effectiveness close to the hole exit. The subsequent mixing of the ejected flow with the main flow causes the continuous decrease in adiabatic wall effectiveness.

At high ejection rates the ejected flow leaves the holes with a high velocity and, therefore, with a large velocity component in direction normal to the surface. The jets
penetrate into the main flow and allow the cold main flow to enter the dead water region downstream of the ejection holes. This causes the low effectiveness close to the ejection. The jets are then deflected by the main stream to the direction of the main flow. The progressive spreading of the jets due to the mixing with the main flow results in an increase of the adiabatic wall effectiveness with increasing downstream distance.

The region close to the ejection is cooled most efficiently at low mass velocity ratios $G^*$ while the cooling of the regions far downstream increases with increasing mass velocity ratio. This result differs from the results obtained with ejection from one single hole, where the effectiveness decreases over the whole downstream region if the mass velocity ratio exceeds a value of $G^* = 0.5$. In the case of ejection from a row of holes the optimum mass velocity ratio is not the same for the whole downstream region, but depends on the downstream distance $\frac{X}{d}$. This will be illustrated further in the next section.

A comparison of the effectiveness distribution obtained with the laminar nose and the turbulent nose shows that the effectiveness values obtained with the laminar nose are in general higher than those obtained with the turbulent nose. This phenomenon will be discussed in section VI.4.

The adiabatic wall effectiveness is shown in Figs. 19, 20, 21 for a constant Mach number of $M_C = 0.61$ and for different ratios of boundary layer displacement thickness in the main flow to ejection hole diameter. This ratio was varied by using ejection elements with different ejection hole diameters. The maximum in the effectiveness curves for higher ejection mass flow in Figs. 20 and 21 may be an indication for the penetration of the main flow into the dead water region downstream of the jets. But the fact that only the values measured by the first copper strip are lower than expected indicates the possibility of a measuring error at this point.
A general comparison of the curves in Figs. 19 to 21 shows an increase of the effectiveness with increasing ejection hole diameter or decreasing ratio of boundary layer thickness to hole diameter $\frac{d_1}{d}$. This phenomenon will be discussed in detail in section VI.4.

The adiabatic wall effectiveness tends towards zero if the adiabatic wall temperature approaches the recovery temperature of the main flow. This will be approximately the case in the region where main flow and ejected flow are completely mixed, i.e., far downstream of the distances which are of interest for the design of film cooled gas turbine blades, $0 < \frac{x}{d} < 80$, and one can see from Figs. 16 to 21 that the region of complete mixing is far beyond this range.

VI.2 Effect of mass velocity ratio on effectiveness

The test results shown in Figs. 16 to 21 indicate already the importance of the mass velocity ratio $G^*$ for the adiabatic wall effectiveness. The effectiveness is plotted in Fig. 22 in function of the mass velocity ratio in order to demonstrate this effect. The curves in Fig. 22 are for one main flow Mach number and for different downstream stations $\frac{x}{d}$.

The curves for the different downstream stations differ not only in level, but also in shape. The curves corresponding to small distances $\frac{x}{d}$ show a pronounced maximum at $G^* = 0.4$ to 0.5. With increasing downstream distance $\frac{x}{d}$ the curves become flatter and the maximum is shifted to higher values of $G^*$.

The optimum effectiveness at $G^* = 0.4$ to 0.5 close to the ejection is in agreement with the measurements of
Goldstein et al., 1968, where investigations of the adiabatic wall effectiveness downstream of a single ejection hole are described. This agreement means that the jets behave close to the ejection in a similar way as single jets in a deflecting stream. Further downstream the jets start to interact with one another forming a closed film along the surface. This is indicated by the shifting of the optimum effectiveness towards \( G^* = 1.0 \), the position of the optimum effectiveness for inclined two-dimensional slot ejection (see Fig. 3). At larger distances \( \frac{x}{d} \) the jets behave apparently like a closed two-dimensional jet, with respect to the average adiabatic wall effectiveness. The absolute level of the effectiveness for hole ejection is, however, considerably lower than for slot ejection, due to the intensive mixing of main flow and secondary flow in the first case.

The fact that the effectiveness curves in Fig. 22 become flatter with increasing downstream distance is due to the mixing of ejection flow and main flow. This was discussed already in the previous section. The maximum of the effectiveness disappears completely for downstream distances \( \frac{x}{d} > 60 \). The flow behaviour corresponds then to the ejection from a parallel slot, where the effectiveness increases steadily with increasing mass velocity ratio.

The presence of the maximum in the effectiveness curves can be explained again by the behaviour of the ejected flow. The behaviour can be observed directly on schlieren pictures obtained at different mass velocity ratios. Fig. 23 shows photographs obtained with the schlieren system of the tunnel for ejection through 2 mm diameter holes and a main flow Mach number of \( M_g = 0.6 \). The jets are still completely attached at the mass velocity ratio \( G^* = 0.5 \), they are slightly detached at \( G^* = 1.0 \) and completely detached at \( G^* = 1.5 \).
The complex mixing process of the jets with the main flow and the interaction of the jets with one another lead to a dependence of the adiabatic wall effectiveness on the mass velocity ratio which is much more complicated than in the case of the two-dimensional ejection, where the effectiveness is a direct function of the mass velocity ratio (see section II.3).

VI.3 Effect of main flow Mach number and Reynolds number

The variation of the main flow Mach number causes, in general, a variation of the main flow Reynolds number and of the boundary layer thickness. The measurements of the boundary layer at ejection location without ejection showed that the displacement thickness of the boundary layer produced by the laminar leading edge profile was approximately independent of the main flow Mach number or Reynolds number. An increase in main flow Reynolds number leads in general to a decrease of the boundary layer displacement thickness. But at the same time, transition starts earlier resulting in a thicker boundary layer at a given station. The combination of these two effects caused probably the constant boundary layer displacement thickness at ejection.

This fact makes it possible to separate the effects of main flow Reynolds number and Mach number from the effect of boundary layer displacement thickness. A comparison of the effectiveness distribution downstream of the ejection at 3 Mach numbers is shown if Fig. 24 for constant ratios of boundary layer displacement thickness to ejection hole diameter. One can see that there is practically no influence of main flow Reynolds number or Mach number, or that the effects of these two parameters cancel each other. The maximum variations from a mean line through the data are \pm 1 percent in \eta_z which is within the accuracy of the measurements.
The fact that there is no influence of main flow Mach number on the effectiveness agrees with the results of Papell et al., 1959, for two-dimensional slot ejection shown in Fig. 3.

It is apparently not the main flow Reynolds number alone which influences the adiabatic wall effectiveness, but the ratio of the Reynolds numbers of main flow and ejected flow. This ratio is represented in our case approximately by the mass velocity ratio which has a very important influence on the effectiveness.

The independence of the adiabatic wall effectiveness from the main flow Mach number and Reynolds number makes it easier to evaluate the effect of the boundary layer thickness on the effectiveness because test data obtained at different Mach numbers can be compared with one another.

VI.4 Effect of main flow boundary layer

The boundary layer thickness at ejection, non-dimensionalized by the ejection hole diameter, is one of the major parameters governing the adiabatic wall effectiveness. This was already observed from the Figs. 16 to 21, where a general increase in efficiency was observed for a decreasing ratio of boundary layer displacement thickness to ejection hole diameter.

A more detailed demonstration of this influence is shown in Fig. 25, where the adiabatic wall effectiveness is plotted in function of the ratio $\frac{\delta_l}{d}$ for an approximately constant mass velocity ratio $G^*$. The data cover the whole range of Mach numbers and ejection hole diameters for both model noses. There is no difference between the test results obtained with the transitional boundary layer with the laminar velocity profile and the turbulent boundary layer. This shows that the displacement thickness of the boundary layer is more important than the actual velocity profile.
Fig. 25 shows that the influence of the boundary layer thickness is small for small values of $\frac{\delta_1}{d}$. But for values above $\frac{\delta_1}{d} = 0.2$ the effectiveness is considerably reduced with increasing $\frac{\delta_1}{d}$. At a boundary layer thickness ratio of $\frac{\delta_1}{d} = 0.6$ the effectiveness is less than half of its value at $\frac{\delta_1}{d} = 0.1$.

This result is in qualitative agreement with the measurements of Goldstein et al., 1969, concerning the influence of the upstream boundary layer thickness on the effectiveness downstream of one single ejection hole. In these investigations the center line effectiveness decreased to half of its initial value for a doubled boundary layer displacement thickness.

This phenomenon can be explained by the higher deflection of the jets by the main flow in the case of a thin boundary layer. The velocity of the main flow at a given height above the surface is higher for a thin boundary layer than for a thick boundary layer, provided that this station is within the range of the boundary layer thickness. This higher velocity causes a higher deflection of the jets and the corresponding reduction of the jet penetration into the main flow results in a higher adiabatic wall effectiveness.

In a thick boundary layer the jets can penetrate further into the main flow before they are deflected parallel to the main flow. This causes a higher mixing of jets and main flow and leads to a decreased effectiveness.

This effect becomes less important for very thin boundary layers. This is why the effectiveness shown in Fig. 25 stays constant below a value of $\frac{\delta_1}{d} = 0.2$.

These results indicate the importance of the exact knowledge of the boundary layer thickness distribution along the
turbine blade to be cooled by film cooling. Neglecting the
effect of the boundary layer thickness can cause an overesti­
mation of the film cooling effectiveness by a factor of two
or three.

VI.5 Effect of variable main flow velocity

The test results obtained with the contoured channel
blocks are shown in Fig. 26. The velocity distributions produced
by the pressure gradient contours are shown in Fig. 11. The
contours are shown also in their position relative to the test
plate in Fig. 26.

The tests were done with the 2 mm diameter ejection
holes and the turbulent nose. The curves shown in Fig. 13b
indicate that the turbulent displacement thickness at ejection
decreases with increasing pressure gradient. The displacement
thickness during the pressure gradient tests was smaller than
during the constant velocity tests. However, as all values of
δ /d are below 0.17, this difference does not affect the adiabatic
wall effectiveness.

The resulting distributions of adiabatic wall effecti­
veness show a general decrease in effectiveness due to the pressure
gradients. In the constant velocity region the curves have the same
shape as the constant velocity curves. In the accelerated region the
curves are lower than corresponding constant velocity curves. The
small "bump" in the curves marks the border between accelerated and
constant velocity flow.

A comparison of the pressure gradient results with cor­
responding results for constant main flow velocity is shown in Fig.
27b for one value of the mass velocity ratio G* = 1.0. For these
conditions, the general shape of the effectiveness curves is only
slightly altered by the presence of the pressure gradients.
The curve corresponding to the gradient of \( \frac{dp}{dx} = 75 \text{ mm Hg/cm} \) is
almost identical with the constant velocity curve, while the curve corresponding to the high pressure gradient is about 3 percent lower.

The picture changes, however, for other values of $G^*$, as one can see by plotting the effectiveness for given downstream stations $\bar{x}$ in function of the mass velocity ratio $G^*$ (Fig. 27a). The figure shows that the mass velocity ratio $G^* = 1.0$ is the only value where the effectiveness is about the same for $\frac{dp}{dx} = 0$ and $\frac{dp}{dx} = 75$ mm Hg/cm. For the other values of $G^*$ the effectiveness is quite different. In particular the maxima of the pressure gradient curves are all at $G^* = 1.0$, even for locations close to the ejection.

A similar effect of pressure gradients on the adiabatic wall effectiveness downstream of two-dimensional slot ejection was already mentioned in chapter II.3. It was found that the effectiveness decreases with increasing favorable pressure gradient, if the gradient is applied close to the ejection. This is the case in the present investigation and happens also on gas turbine blades if the cooling air is ejected into an accelerated main flow.

This reduction in effectiveness due to the application of favorable pressure gradients is caused probably by an increased mixing of main flow and ejected flow. This is indicated also by the schlieren photographs shown in Fig. 28. The behaviour of the jets is shown for constant main flow velocity and the two pressure gradients. The arrows on the pictures indicate the exit of the ejection holes.

The pictures show that the jets are pressed down by the pressure gradients. This "flattening" of the jets together with the related increased mixing leads to the similarity of the jet behaviour with that of a two-dimensional jet.
The "flattening" of the jets by the pressure gradient producing contours is a result of the related general contraction of the main flow area. It occurs at a certain distance downstream of the hole exit and affects the effectiveness, therefore, in a different way than a thin boundary layer. In the latter case the jets are deflected strongly immediately at hole exit and the decreased penetration results in a higher effectiveness. In the case of pressure gradients the contraction of the flow area leads to a more intensive mixing of main flow and ejected flow and the resulting decrease in effectiveness.

VI.6 Stagnation point ejection

For the tests with ejection at the stagnation point only a small number of the highly sensible graphis spot recorders was available. They were connected to the copper elements close to the ejection. For the rest of the test plate the CEC-recorders with galvanometric elements of a much lower sensibility had to be used. The accuracy of the measurements was therefore not very good. Leakage of the main flow close to the stagnation point through the slot for the test plate caused probably cooling of the feed tubes and corresponding uncertainties about the accurate value of the ejection temperature. But since there is no other information available on film cooling with ejection at a stagnation point, the results of two test series are presented in order to give some qualitative indications on the problem.

Fig. 29 shows the adiabatic wall effectiveness with nose ejection for two ratios of hole diameter to nose radius. The parameter representing the ejected mass flow is in this case the ratio of ejection to main flow stagnation pressure.

The curves indicate a strong influence of the ratio of hole diameter to nose radius. The effectiveness obtained with the 3 mm diameter holes is about twice as high as the correspond-
ing values for the 2 mm diametre holes. The effect of the ejection pressure ratio is not very clear. However, there seems to be a slight decrease in effectiveness for increasing pressure ratio. This can be attributed to a higher penetration of the ejected flow into the main flow, causing a higher mixing of both flows.

The general shape of the curves is similar to the curves obtained with flat plate ejection. This indicates that the mixing of main flow and secondary flow occurs in a similar manner.

VI.7 Comparison of results with published data

There are only two publications up to now containing information on the adiabatic wall effectiveness averaged in spanwise direction, but not in the downstream direction.

Goldstein et al. (1969) measured the local adiabatic wall effectiveness downstream of a row of ejection holes with the same hole geometry as used in the present investigation, that is an ejection angle of 35° and a hole spacing of three diameters. The tests were done at low speeds with an ejection temperature of about 50° C above the main flow temperature. Lateral averaged values of the local measurements were obtained by integration of the measured effectiveness over one spacing at different downstream stations.

A comparison of the results is presented in Fig. 30a for three mass velocity ratios and for a similar ratio of displacement thickness to ejection hole diameter. The curves representing the lowest blowing rate of $G^* = 0.34$ and $G^* = 0.5$ are in excellent agreement except close to the ejection. The curves for $G^* = 1.5$ are similar in shape but different in level while the curves for $G^* = 2$ are again in reasonable agreement. The differences are probably due to the different way
in obtaining the laterally averaged values. The values of Goldstein represent simply integrated values of the spanwise adiabatic wall temperature distribution. The averaging done by the copper elements in the present study is influenced also by the distribution of the heat transfer coefficient in the lateral direction since one can write for the heat entering the element:

$$\frac{1}{s} \int_{-s/2}^{s/2} q \, dz = \frac{1}{s} \int_{-s/2}^{s/2} h(T_w - T_{aw}) \, dz = 0$$

for adiabatic conditions. The constant wall temperature is then obtained from this equation as

$$T_w = \frac{\frac{1}{s} \int_{-s/2}^{s/2} (h \, T_{aw}) \, dz}{\frac{1}{s} \int_{-s/2}^{s/2} h \, dz}$$

This equation reduces to the relation employed by Goldstein et al., 1969):

$$T_w = \frac{1}{s} \int_{-s+2}^{s+2} T_{aw} \, dz$$

for a constant heat transfer coefficient $h$.

However the heat transfer coefficient is not constant in lateral direction in the case of film cooling by means of a row of holes. The mean values of the temperatures are therefore necessarily different.
At low blowing rates or far from the ejection the heat transfer coefficient is close to the value without ejection. This is probably the reason for the good agreement between the curves in Fig. 30a at $G^* = 0.34$ and 0.5 and far downstream. The general shape and the trends of the curves are very similar.

The measurements reported by Jones et al., (1971) were obtained in a manner similar to the present investigation. Laterally averaged values were measured by thin film heat transfer gauges at several stations downstream of the ejection. The tests were done in a shock tunnel with an ejection geometry differing somewhat from the present investigations. The ejection angle was 30° and the spacing to diameter ratio was $\frac{s}{d} = 2.5$. The tests were done at a main flow stagnation temperature of 500° K and a coolant temperature of 300° K. The main flow Mach number was $M_G = 0.40$.

The comparison of the results is shown in Fig. 30b for three downstream stations $\frac{X}{d}$ over a range of mass velocity ratios up to $G^* = 2.0$. The general shape and the order of magnitude of the results is again the same for both investigations. The differences are most probably due to the different ejection geometries used for both tests. The smaller ejection angle and the closer spacing of the ejection holes lead to the higher values of the effectiveness measured by the authors.
VII. HEAT TRANSFER COEFFICIENT

The laterally averaged heat transfer coefficient downstream of a row of ejection holes was measured at the same flow conditions and for the same test configuration as the adiabatic wall effectiveness described in the preceding chapter. The heat transfer coefficient was determined from the transient variation of the copper element temperatures immediately after injection of the model into the test section.

VII.1 General discussion of results

Basic test results of the reduced Nusselt number $N^*$ are shown in Fig. 31. The data points are omitted in the figures in order to avoid confusion. The lines in the figures represent the individual data points to within ± 5%. The effects of the principal flow parameters on the heat transfer coefficient described in the following are partly of the order of the measuring error. However, the data show distinct tendencies so that conclusions on the effect of the different parameters can be drawn.

Direct comparison with other investigations is not possible because the heat transfer coefficient was investigated up to now only for two-dimensional slot ejection (Goldstein, 1971) or averaged in lateral and downstream direction for ejection from a row of holes (Metzger et al., 1969). Comparison with these results shows qualitative agreement, especially in the region close to the ejection (see section II.4).

The curves show a general increase of the heat transfer coefficient due to the ejection. The heat transfer coefficient stays approximately constant along the test surface, except close to the ejection, where it is up to 60 percent higher than without ejection. This increase in heat transfer coefficient close to the ejection was observed also in the case of two-dimensional
slot ejection (Goldstein, 1971). But in that case the heat transfer coefficient with film cooling reached the value of the coefficient without cooling after a rather short downstream distance. Fig. 31 shows that this is not the case for the ejection from a row of holes. The disturbance of the flow field by the presence of the downstream vortices results in an increased heat transfer coefficient. This increase is low at low blowing rates and increases with increasing mass velocity ratio $G^*$. The result agrees with the measurements of the effect of streamwise vortices on the local heat transfer rate in lateral direction in supersonic flow (Gautier, 1972). These tests showed a periodic variation of the heat transfer rate in lateral direction $z$ around a mean value. This mean value corresponded approximately to the heat transfer rate without vortices for small disturbances of the flow. At high disturbance and corresponding stronger vortices the mean value of the heat transfer rate was found to be considerable increased.

A similar flow pattern is present downstream of a row of ejection holes, where strong streamwise vortices are created by the interaction of the main flow with the ejected jets, as it is shown by the surface visualization in Fig. 32.

VII.2 Effect of mass velocity ratio

The effect of the mass velocity ratio on the reduced heat transfer coefficient is indicated already in Fig. 31. A cross plot of the curves for the hole diameters $d = 2$ mm and $4$ mm is presented in Fig. 33. The reduced Nusselt number $\frac{\text{Nu}}{\text{Nu}^*}$ is plotted for two downstream stations $\frac{x}{d}$ in function of the mass velocity ratio $G^*$. The reduced Nusselt number increases with increasing mass velocity ratio at low values of the mass velocity ratio. For values $G^* > 0.4$ there is practically no influence of $G^*$ on the reduced Nusselt number except very close to the ejection. This indicates that the increase in heat transfer
The coefficient due to the ejection is constant in the range of the optimum adiabatic wall effectiveness, i.e. for $0.5 < C^* < 1.0$.

As mentioned above, the increase in heat transfer coefficient caused by an increasing mass velocity ratio is due to the higher disturbance of the main flow by the ejected jets and the resulting stronger streamwise vortices.

VII.3 **Effect of main flow properties**

The Mach number and Reynolds number of the main flow have no measurable influence on the reduced Nusselt number. The results for different Mach numbers agree within the accuracy of the measurements, as in the case of the adiabatic wall effectiveness.

The boundary layer thickness has practically no influence on the heat transfer coefficient, as one can see by comparison of the results presented in Fig. 31. An increased reduced Nusselt number was found only for the smallest ratio of boundary layer thickness to hole diameter, i.e., $\delta_1/d = 0.045$. The increase is certainly due to the higher distortion of the flow by the ejected jets and the related strong streamwise vortices.

VII.4 **Effect of variable main flow velocity**

The results for variable main flow velocity are shown in Fig. 34 together with the corresponding curves for constant main flow velocity. The contours of the wooden pressure gradient blocks are indicated also.

The presence of the pressure gradient causes a reduction of the heat transfer coefficient compared to the constant velocity case. This reduction is, however, only present in the region of the accelerated flow. The heat transfer coefficient
has a maximum at the point where the constant velocity regime is reached and decreases then again to the constant velocity value.

The effect of the mass velocity ratio $G^*$ on the reduced Nusselt number is suppressed in the region of accelerated main flow as one can see from the curves on Fig. 34. The effect seems to be reversed in the constant velocity region downstream of the strongest gradient. However, the difference are so small that finite conclusions cannot be drawn.

From Fig. 34 and the visualization shown in Fig. 32, one can explain the effect of the main flow acceleration in the following way: The development of the streamwise vortices is suppressed in the accelerated region and the heat transfer coefficient is reduced compared to the constant velocity case. As soon as the pressure gradient is released the vortices develop suddenly and cause an increased heat transfer coefficient which reduces then slowly to the constant velocity value.

Similar reduction in heat transfer coefficient due to accelerated flow was observed by Carlson et al. (1968) for two-dimensional slot ejection.

VII.5 Ejection at the stagnation point

The results for the stagnation point ejection are shown in Fig. 35. The main parameters are the ejection hole diameter to nose radius ratio $\frac{d}{R_n}$ and the stagnation pressure ratio $P_{ac}^{\frac{\alpha}{\gamma}}$. For the small hole diameter and low ejection pressure ratios $P_{0G}^{\frac{\alpha}{\gamma}}$ the influence of the ejection is restricted to the region close to the ejection. At the larger ejection hole diameter and for high ejection pressure ratios, the influence is similar to the flat plate ejection, i.e., extending far downstream. The explanation for this phenomenon is again a higher disturbance of the main flow and the resulting streamwise vortices at high ejection pressure ratios and large hole diameters.
VIII. CONCLUSION

The influence of the principal flow parameters on the laterally averaged adiabatic wall temperature and heat transfer coefficient downstream of a row of inclined film cooling holes is investigated experimentally under conditions which allow the application of the results to the design of film cooled turbine blades.

The so far unknown effects of main flow Mach number, Reynolds number, pressure gradient and boundary layer thickness on the adiabatic wall effectiveness and the heat transfer coefficient downstream of a row of inclined ejection holes are investigated. Additional information on the detailed influence of the mass velocity ratio is provided, as well as first results on film cooling with ejection at a stagnation point. All tests are done for one ejection angle and for a constant value of the spacing-diameter ratio of the ejection holes.

It is found that the Mach number and the Reynolds number have no measurable effect on the film cooling parameters. It is therefore possible to apply the results of low speed tests to turbine blades with local Mach numbers up to $M_a = 0.9$.

The test results confirm the important influence of the mass velocity ratio $G^*$ on the adiabatic wall effectiveness. The effectiveness has a pronounced maximum at $G^* \approx 0.5$ for regions close to the ejection. This maximum becomes flatter and is shifted towards $G^* \approx 1.0$ for increasing downstream distance $\frac{x}{d}$. The maximum disappears completely for distances $\frac{x}{d} > 60$ in the considered range of mass velocity ratios. The heat transfer coefficient increases considerably with increasing $G^*$ close to the ejection and stays approximately constant over the whole downstream distance investigated. Close to the ejection the heat transfer coefficient can be up to 60% higher than without ejection, further downstream the maximum increase is about 25%. 
The ratio of main flow boundary layer displacement thickness to ejection hole diameter is found to have practically no influence on the heat transfer coefficient, but a high influence on the adiabatic wall effectiveness. The effectiveness decreases considerably for \( \frac{\delta_1}{d} > 0.2 \). Doubling the displacement thickness can cause a decrease of the effectiveness from \( \bar{n}_r = 0.22 \) to \( \bar{n}_r = 0.14 \) close to the ejection and from \( \bar{n}_r = 0.10 \) to \( \bar{n}_r = 0.05 \) further downstream.

The presence of a favorable pressure gradient in the main flow can cause also a considerable reduction of the adiabatic wall effectiveness, particularly at low mass velocity ratios. The effectiveness optimum is found for all downstream distances at \( G^* = 1.0 \). At a mass velocity ratio \( G^* = 0.5 \) the effectiveness can be decreased close to the ejection from \( \bar{n}_r = 0.22 \) to \( \bar{n}_r = 0.09 \) and further downstream from \( \bar{n}_r = 0.11 \) to \( \bar{n}_r = 0.08 \) if the main flow is strongly accelerated. The heat transfer coefficient is somewhat decreased in the presence of a main flow acceleration, but increases again slightly when constant velocity regions are reached.

It is found further that the results of ejection at the stagnation point are similar to those obtained with flat plate ejection, but at a lower level. Increasing the ratio of ejection to main flow stagnation pressure leads to an increase in heat transfer coefficient and a slight decrease in effectiveness.
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forced convection  natural convection  spray cooling  rim cooling

LIQUID COOLING SYSTEMS

- simple hollow blade
- blade with passages
- passages with inserts
- internally finned blade
- convection cooling

AIR COOLING SYSTEMS

FIG. 1 SURVEY OF BLADE COOLING METHODS
FIG. 2 FILM COOLING GEOMETRIES
tangential ejection at $G^* = 1.0$ (Papell et al., 1959) for three mainflow Mach numbers

Inclined slot (Wieghardt, 1946, taken from Eckert, 1971)

FIG. 3 FILM COOLING EFFECTIVENESS WITH SLOT EJECTION
FIG. 4 SKETCH OF WIND TUNNEL
a. Flat plate ejection

FIG. 5 TEST SECTION
predicted for design nose form
predicted for real nose form
measured

$M_G$
- 0.28
- 0.62
- 0.90

FIG. 6 COMPARISON OF CALCULATED AND MEASURED VELOCITY DISTRIBUTIONS
predicted for design
nose form

predicted for real
nose form

measured

\[ \frac{V}{V_1} \]

\[ M_g \]

- 0.30
- 0.62
- 0.90

b. turbulent nose

FIG. 6 (continued)
FIG. 7 PHOTOGRAPH OF TEST SECTION
FIG. 8 BOUNDARY LAYER PROBE

All measures in mm
FIG. 9  VELOCITY DISTRIBUTIONS ON MODEL WITH LAMINAR NOSE
FIG. 10 VELOCITY DISTRIBUTIONS ON MODEL WITH TURBULENT NOSE

Side wall Model $M_g$

- $0.29$
- $0.62$
- $0.90$

$\frac{V}{V_1}$

$0.8$ $1.0$ $1.2$ $1.4$ $1.6$ $1.8$ $2.0$ $2.2$
FIG. 11 VELOCITY DISTRIBUTIONS ON MODEL WITH PRESSURE GRADIENTS
(TURBULENT NOSE)

Side wall Model Gradient

- 75 mmHg/cm

- 150 mmHg/cm

ejection

\( \frac{V}{V_1} \)
FIG. 12 VELOCITY DISTRIBUTION ON MODEL (CONDITIONS FOR STAGNATION POINT EJECTION)
a. Constant main flow velocity on model

FIG. 13 BOUNDARY LAYER DISPLACEMENT THICKNESS ON MODELS AND BLADES - COMPRESSIBLE CALCULATION FROM MEASURED VELOCITY DISTRIBUTIONS
Turbulent boundary layer

Displacement thickness distribution

Blade pressure side

Ejection on model

Model

1/R_N

Velocity distributions

Blade suction side

Model

Blade pressure side

b. Variable main flow velocity on model

FIG. 13 (cont'd)
FIG. 14 BOUNDARY LAYER VELOCITY PROFILES AT EJECTION AND 130 MM DOWNSTREAM
FIG. 15 HEAT TRANSFER COEFFICIENT WITHOUT EJECTION
FIG. 16 ADIABATIC WALL EFFECTIVENESS FOR FLAT PLATE EJECTION

\[ d = 2 \text{ mm}; M_G = 0.31, \frac{dp}{dx} = 0 \]
FIG. 17 ADIABATIC WALL EFFECTIVENESS FOR FLAT PLATE EJECTION

$d = 2 \text{ mm}; M_G = 0.61, \frac{dp}{dx} = 0$
FIG. 18 ADIABATIC WALL EFFECTIVENESS FOR FLAT PLATE EJECTION
\[ d = 2 \text{ mm}; \quad M_G = 0.89, \quad \frac{dp}{dx} = 0 \]
FIG. 19 ADIABATIC WALL EFFECTIVENESS FOR FLAT PLATE EJECTION
d = 1 mm; \( M_G = 0.62 \), \( \frac{dp}{dx} = 0 \)
Laminar nose

\[ M_G = 0.61 \]
\[ d = 3 \text{mm} \]
\[ \delta/d = 0.060 \]

Turbulent nose

\[ M_G = 0.62 \]
\[ d = 3 \text{mm} \]
\[ \delta/d = 0.115 \]

FIG. 20 ADIABATIC WALL EFFECTIVENESS FOR FLAT PLATE INJECTION
\[ d = 3 \text{mm}; M_G = 0.62, \frac{dp}{dx} = 0 \]
**Fig. 21: Adiabatic Wall Effectiveness for Flat Plate Injection**

Laminar nose

- $M_g = 0.63$
- $d = 4\text{mm}$
- $\delta/d = 0.045$
- $G^*$ values: 0.12, 0.32, 0.56, 1.01

Turbulent nose

- $M_g = 0.62$
- $d = 4\text{mm}$
- $\delta/d = 0.087$
- $G^*$ values: 0.14, 0.33, 0.57, 1.01

For $d = 4\text{mm}$, $M_g = 0.62$, $\frac{dP}{dx} = 0$
FIG. 22 ADIABATIC WALL EFFECTIVENESS VERSUS MASS VELOCITY RATIO, $M_g = 0.31$
FIG. 23 - EFFECT OF MASS VELOCITY RATIO ON BEHAVIOUR OF JETS
FIG. 24 EFFECT OF MAIN FLOW MACH NUMBER ON ADIABATIC WALL EFFECTIVENESS
FIG. 25 EFFECT OF MAIN FLOW BOUNDARY LAYER DISPLACEMENT THICKNESS AT EJECTION ON ADIABATIC WALL EFFECTIVENESS
FIG. 26 ADIABATIC WALL EFFECTIVENESS IN THE PRESENCE OF PRESSURE GRADIENTS IN THE MAIN FLOW
FIG. 27 EFFECT OF MAIN FLOW PRESSURE GRADIENTS ON ADIABATIC WALL EFFECTIVENESS
FIG. 28 - EFFECT OF PRESSURE GRADIENTS ON BEHAVIOUR OF JETS  $d = 2\, \text{mm}$
FIG. 29 ADIABATIC WALL EFFECTIVENESS FOR STAGNATION POINT EJECTION, $M_1 = 0.22$
FIG. 30 COMPARISON OF TEST RESULTS WITH PUBLISHED INVESTIGATIONS

\[ \eta^* \]

\begin{tabular}{ccc}
\hline
& \( G^a \) & \( M_G \) & \( \delta / \Delta \) \\
\hline
& 0.34 & & \\
\hline
& 1.43 & 0.31 & 0.171 \\
\hline
& 1.98 & & \\
\hline
& 0.5 & & \\
\hline
& 1.5 & 0.09 & 0.124 \\
\hline
& 2.0 & & \\
\hline
\end{tabular}

\text{Liess}

\text{Goldstein}
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<th>$x/d$</th>
<th>$T_{0G}$</th>
<th>$T_{0C}$</th>
<th>$M_g$</th>
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<th>$\beta$</th>
<th>Author</th>
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**FIG. 30 (continued)**
FIG. 31 REDUCED NUSSELT NUMBER FOR FLAT PLATE EJECTION

\[ M_G = 0.61, \frac{dp}{dx} = 0 \]
Laminar nose

\( \bar{M}_G = 0.61 \)
\( d = 3\text{mm} \)
\( \delta / d = 0.060 \)

\( \bar{M}_G = 0.62 \)
\( d = 4\text{mm} \)
\( \delta / d = 0.045 \)

FIG. 31 (cont'd)
FIG. 32 VISUALIZATION OF STREAMWISE VORTICES, $d = 2\ mm$
FIG. 33 EFFECT OF MASS VELOCITY RATIO ON REDUCED NUSSELT NUMBER
FIG. 34 EFFECT OF MAIN FLOW PRESSURE GRADIENT ON REDUCED NUSSELT NUMBER (d = 2 mm)
**FIG. 35** REDUCED NUSSELT NUMBER FOR STAGNATION POINT EJECTION, $M_1 = 0.22$