FIRST-ORDER SUPERDIRECTIONAL ACOUSTIC ZOOMING IN THE PRESENCE OF DIRECTIONAL INTERFERENCES

R.M.M. Derkx

Philips Research, Eindhoven
High Tech Campus 36, 5656 AE, Eindhoven
phone: +31 402746138, email: Rene.Derkx@philips.com

ABSTRACT

Acoustic zooming aims at increasing and decreasing the perceived distance of the sound-image by varying a zoom-parameter. Previously proposed superdirective acoustic zooming techniques focus on controlling the directivity-factor of the constructed beampattern by varying this zoom-parameter. As a result, these zooming techniques are only consistent in the case of (spherically) isotropic interferences. In practical situations however, often directional interferences (mainly coming from a single direction) are present. To have a consistent behaviour of the acoustic zooming, we will propose a new zooming technique that is based on a novel first-order beampattern construction. The beampattern is constructed in such a way that for every angle, the response is monotonically increasing/decreasing in a consistent way with the zooming-parameter.

Index Terms — microphones, acoustic zooming

1. INTRODUCTION

Given that we capture a desired sound-source coming from a predefined angle, we can define acoustic zooming as increasing and decreasing interfering sounds coming from other directions. Zooming-in means that we decrease interferences, while zooming-out\(^1\), means that we increase the interferences. In [1] the interferences are assumed to be reverberation (i.e. diffuse noise), while in this paper also directional interferences (not necessarily related to the desired sound-source) are considered.

Throughout this paper, we will focus on first-order superdirective microphones techniques [2], where we use two omnidirectional microphones in end-fire with a spacing of \(d\) meters, where \(d \ll \lambda\), with \(\lambda\) the wavelength of interest. We note however, that the techniques described in this paper are not limited to first-order superdirectivity (although it is known that a second- and higher-order superdirectivity is difficult to realize in practice when using omnidirectional microphones). Also it is noted that the acoustic zooming technique presented in this paper can be easily extended for a steerable superdirective microphone as described in [3].

In this paper, we will first describe a straightforward/standard method for acoustic zooming using superdirective microphone techniques. Next, we will describe our newly proposed acoustic zooming method, and we explain the advantages over the standard acoustic zooming method. Also, we will relate our zooming method to zooming methods that are known from the literature.

\(^1\)In this paper, we use two omnidirectional microphones and define the maximum zoom-out as being the omnidirectional response.

2. STANDARD ACOUSTIC ZOOMING

For the construction of a first-order superdirective microphone response by means of a microphone array of two omni-directional microphones, we use the scheme as shown in Fig. 1.

![Fig. 1. Standard acoustic zooming using two microphones.](image)

When we take the middle of two microphones as a reference, we can write the two omnidirectional microphone responses with index \(i = 1, 2\) as:

\[
E_{m} = e^{-j(-1)^{i} \frac{\pi}{2} \frac{\omega}{\omega_{c}}} \cos \phi \sin \theta, \tag{1}
\]

with \(\phi\) being the azimuthal angle and \(\theta\) the elevation angle, \(\omega = 2\pi f\) with \(f\) the frequency, \(d\) the distance between the two microphones and \(c = 340 \text{ m/s}\).

The monopole response can be easily constructed by means of summation and scaling with a factor \(\frac{1}{2}\):

\[
E_{m} = \frac{1}{2} [E_{x1} + E_{x2}]. \tag{2}
\]

When assuming that \(\frac{\omega d}{2\pi} \ll 1\), we obtain \(E_{m} \approx 1\) having an omni-directional response that is not dependent on the azimuthal angle \(\phi\) and elevation angle \(\theta\).

The dipole response is constructed by means of subtraction and integration with a factor \(\frac{1}{d\omega_{c}}\) \([3]\):

\[
E_{d} = \frac{1}{d\omega_{c}} [E_{x1} - E_{x2}]. \tag{3}
\]

Again assuming that \(\frac{\omega d}{2\pi} \ll 1\), we obtain \(E_{d} \approx \cos \phi \sin \theta\) having a dipole response, i.e. a figure of eight pattern in the azimuthal plane.

By combination of the monopole and dipole response by means of a weighting of \(\alpha\) and \(1 - \alpha\) respectively, we can construct any first-order superdirective response, given by:

\[
E_{\theta}(\theta, \phi) = \alpha E_{m} + (1 - \alpha) E_{d} \approx \alpha + (1 - \alpha) \cos \phi \sin \theta. \tag{4}
\]
It is noted that for frequencies where \( \frac{\pi}{AB} \ll 1 \), we obtain superdirectional beampatterns that have a shape that is independent of the frequency.

Using the scheme as shown in Fig. 1 for acoustic zooming, only allows us to vary the parameter \( \alpha \) to modify the directivity-index. This is known from e.g. [2]. A zoom-example using this scheme is shown in Fig. 2, where we zoom-in from monopole (\( \alpha = 1 \)) to hyper-cardioid response (\( \alpha = \frac{1}{3} \)).

![Fig. 2. Magnitude-response \( |E_y(\pi/2, \phi)| \) for the standard acoustic zooming by variation of \( \alpha \).](image)

For analyzing this zoom example, we will look at the directivity-factor \( Q \) given by [2] [4]:

\[
Q = \frac{4\pi}{\int_{\phi=0}^{\pi} \int_{\theta=0}^{\pi} E_y^2(\theta, \phi) \sin \theta d\theta d\phi}.
\]

(5)

If we combine Eq. (4) with Eq. (5) and perform the integration, we see that the directivity-factor \( Q \) is expressed as:

\[
Q = \frac{3}{1 - 2\alpha + 4\alpha^2}.
\]

(6)

In Fig. 3, it can be seen that the directivity-index \( 10 \log_{10}[Q] \) decreases monotonically for increasing values of \( \alpha \), where \( \alpha > 1/4 \) (hyper-cardioid).

In Section 1, we defined the zooming as a monotonic function of the rejection of interferences from angles other than 0 degrees. In this respect, it is not sufficient to look at the directivity-factor \( Q \). Hence, we will also look at the rejection of directional interferences, e.g. at the rejection of an interference coming from 180 degrees. To analyze the back-rejection, we first compute the back-response, given by:

\[
|E_y(\pi/2, \pi)|^2 = (2\alpha - 1)^2.
\]

(7)

The back-rejection (being the inverse of the back-response) is shown in Fig. 3. It can be seen that for monotonically increasing values of \( \alpha \) in the range \( 1/4 < \alpha < 1/2 \), the back-rejection is increasing, while for monotonically increasing values of \( \alpha \) in the range \( 1/2 < \alpha < 1 \) the back-rejection is decreasing. This is a little bit awkward, as for zooming-out, we want a continuous monotonically decreasing back-rejection.

In [1] an acoustic zooming method is proposed that has the above mentioned disadvantages for the first-order directivity patterns, where a back-to-back cardioid configuration is used. For the second-order directivity patterns however, the above mentioned disadvantages are overcome due to a different way of beampattern construction where also a frequency dependent equalizer function is applied. Although this way of beampattern construction is comparable with our newly proposed method discussed in this paper, the authors in [1] do not give a solution for the first-order directivity pattern.

**3. NEW PROPOSED ACOUSTIC ZOOMING**

To enable an acoustic zoom that decreases and increases monotonically for zooming-in and zooming-out, we will introduce a variable offset in the first-order response.

In [3] it was found that the depth of the null that is generated by the scheme for the construction of a superdirective response is influenced by sensitivity-differences on the two microphones. This principle will be used as basis for our new proposed acoustic zooming algorithm.

Furthermore, it was found in [3] that for a certain (frequency-independent) sensitivity-difference between the microphones, the null depth is frequency-dependent (smaller null depth for lower frequencies). To compensate for this frequency-dependency, we apply a differentiator function \( \frac{\pi}{2} j\omega \) on each of the microphones, apply a symmetric level-difference of a factor \( \pi/2 \) to these differentiated microphone signals and add the result to the original microphone signals (see Fig. 4). This results in two modified (but still omnidirectional) microphone signals, indicated by \( \tilde{x}_1 \) and \( \tilde{x}_2 \).

![Fig. 4. New acoustic zooming scheme with two microphones.](image)

As the differentiator function introduces a 90 degrees phase-shift, we apply the same phase-shift on the original microphone signals, before the addition takes place. This 90 degrees phase-shift can be constructed via a Hilbert transformer \( j \). In the digital domain, the ideal differentiator impulse-response \( h_D[n] \) is given by:

\[
h_D[n] = \begin{cases} 
0 & \text{for: } n \text{ odd} \\
\frac{2}{\pi n} & \text{for: } n \text{ even} 
\end{cases}
\]

(8)

where \( F_s \) is the sampling-rate of the digital signal and \( n \in \mathbb{Z} \) is the sample-index. The impulse-response \( h_H[n] \) of the ideal Hilbert transform in the digital domain is given by:

\[
h_H[n] = \begin{cases} 
0 & \text{for: } n = 0 \\
\frac{-1}{\pi} & \text{for: } n \text{ odd} \\
\frac{1}{\pi} & \text{for: } n \text{ even} 
\end{cases}
\]

(9)
Similarly as in the technique shown in the introduction, the resulting monopole and the dipole responses can be combined via a weighting-factor of \( \alpha \) and \( 1 - \alpha \). Instead of controlling the parameter \( \alpha \), the acoustic-zooming takes place by modifying the factor \( \epsilon \) and \( S \) (how to choose \( \epsilon \) and \( S \) is explained in Section 4).

To analyze the new proposed acoustic zooming method as shown in Fig. 4, we will compute the modified microphone signals that include the frequency-dependent level-differences by using the differentiator function \( 2c \frac{d}{d\phi} \):

\[
E_{\delta 1} = je^{i \frac{\omega d}{2c} \cos \phi \sin \theta} + \epsilon \frac{d}{2c} j e^{i \frac{\omega d}{2c} \cos \phi \sin \theta} \sin \theta, \tag{10}
\]

\[
E_{\delta 2} = je^{-i \frac{\omega d}{2c} \cos \phi \sin \theta} - \epsilon \frac{d}{2c} j e^{-i \frac{\omega d}{2c} \cos \phi \sin \theta}. \tag{11}
\]

From these two signals, we compute the difference to construct the dipole response:

\[
E_{d} = E_{\delta 2} - E_{\delta 1} = j \left( e^{i \frac{\omega d}{2c} \cos \phi \sin \theta} - e^{-i \frac{\omega d}{2c} \cos \phi \sin \theta} \right) + \frac{d}{c} \epsilon \frac{d}{2c} j e^{i \frac{\omega d}{2c} \cos \phi \sin \theta} \overset{\text{Term w.r.t. \( \phi \)}}{=} -2 \sin \left( \frac{\alpha d}{2c} \cos \phi \sin \theta \right) + \frac{d}{c} \epsilon \frac{d}{2c} j e^{i \frac{\omega d}{2c} \cos \phi \sin \theta}. \tag{12}
\]

After applying the scaling and the integrator function\(^2\), we get the dipole response given by:

\[
E_{d} = -2S \frac{c}{d} \frac{1}{j \omega} \left( \frac{\alpha d}{2c} \cos \phi \sin \theta \right) + \epsilon S \left( \frac{\alpha d}{2c} \cos \phi \sin \theta \right). \tag{13}
\]

When assuming that \( \frac{\omega d}{2c} \ll 1 \), we obtain:

\[
E_{d} \approx S \left( \frac{1}{j \omega} \right) \left( \frac{\alpha d}{2c} \cos \phi \sin \theta + \epsilon \right). \tag{14}
\]

For the monopole response we have due to the Hilbert transform:

\[
E_{m} \approx j. \tag{15}
\]

The combined superdirectional response is now given by:

\[
E_{\tilde{d}}(\theta, \phi) = \alpha E_{m} + (1 - \alpha) E_{d} \approx j \left[ \alpha + (1 - \alpha) S \cos \phi \sin \theta \right] + (1 - \alpha) \epsilon S. \tag{16}
\]

From Eq. (16), it can be seen that the offset \((1 - \alpha) \epsilon S\) is real-valued while the monopole and the dipole response are imaginary-valued\(^3\). As a result, the term \((1 - \alpha) \epsilon S\) will only result in an offset in the square magnitude-response of the superdirectional beampattern given by:

\[
\left| E_{\tilde{d}}(\theta, \phi) \right|^2 \approx \left[ \alpha + (1 - \alpha) S \cos \phi \sin \theta \right]^2 + (1 - \alpha)^2 \epsilon^2 S^2. \tag{17}
\]

The next question is how to choose \( \epsilon \) and \( S \) to obtain a certain zoom-effect and to normalize the front response to unity. Clearly, this will depend on \( \alpha \). This will be discussed in the next section.

\(^2\)In the digital-domain, such an integrator can be easily implemented as an IIR filter \([5]\).

\(^3\)Knowing that the only requirement is that the offset is 90 degrees out-of-phase w.r.t. the first-order response, it is clear that several alternative schemes can be proposed for the acoustic zooming.

4. Analysis of the New Proposed Acoustic Zooming

We compute \( \epsilon \) and \( S \) in such a way that the minimum square magnitude-response (for any angle) yields \( \psi^2 \).

We start with the square magnitude-response of Eq. (17). To find the minima we first differentiate this function w.r.t. the variable \( \phi \):

\[
\frac{\partial |E_{\tilde{d}}(\theta, \phi)|^2}{\partial \phi} = -2 \left[ \alpha + (1 - \alpha) S \cos \phi \sin \theta \right] (1 - \alpha) S \sin \phi \sin \theta. \tag{18}
\]

The solutions where \( \frac{\partial |E_{\tilde{d}}(\theta, \phi)|^2}{\partial \phi} = 0 \) yield the maximum at \( \phi = 0 \) and the minimum at:

\[
\phi_{min} = \arccos \left[ \frac{\alpha}{S \sin \theta (1 - \alpha)} \right]. \tag{19}
\]

Filling in \( \phi = \phi_{min} \) in Eq. (17) yields:

\[
\left| E_{\tilde{d}}(\theta, \phi_{min}) \right|^2 = (1 - \alpha)^2 \epsilon^2 S^2 \triangleq \psi^2. \tag{20}
\]

Whereas Eq. (20) defines the minimum response of the beam-shape at the null-angle \( \phi_{min} \) as function of the variables \( \epsilon \) and \( S \), we also need to normalize the response to unity for \( \phi = 0 \) and \( \theta = \pi/2 \) (where a maximum is obtained). This yields:

\[
\left| E_{\tilde{d}}(\pi/2, 0) \right|^2 \approx \left[ \alpha + (1 - \alpha) S \right]^2 + (1 - \alpha)^2 \epsilon^2 S^2 = 1. \tag{21}
\]

By using the equation of Eq. (20) that defines the minimum response and Eq. (21) that defines the maximum response, we can compute the variables \( \epsilon \) and \( S \) as:

\[
S = \sqrt{1 - \psi^2 - \alpha}, \tag{22}
\]

\[
\epsilon = \frac{\psi}{\sqrt{1 - \psi^2 - \alpha}}. \tag{23}
\]

In order to guarantee that the dipole response does not switch in polarity, we need to ensure that the scaling value \( S \) stays positive. Using Eq. (22), we can therefore constrain the value of \( \psi \):

\[
\psi < \sqrt{1 - \alpha^2}. \tag{24}
\]

To normalize the zooming-parameter such that it is independent of the variable \( \alpha \), we introduce the normalized zooming parameter \( \psi \) defined as:

\[
\tilde{\psi} = \frac{\psi}{\sqrt{1 - \alpha^2}}. \tag{25}
\]

Similar to the results of the standard zooming method by means of variation of the the parameter \( \alpha \), shown in Fig. 2, we show the results of the new proposed acoustic zooming by variation of the parameter \( \tilde{\psi} \) in Fig. 5. To select the hyper-cardioid (having the optimal directivity-factor \( Q \)), we use \( \alpha = 1/4 \). We zoom-in from monopole (\( \tilde{\psi} = 1 \)) to hyper-cardioid response (\( \tilde{\psi} = 0 \)).

From Fig. 5, it can be clearly seen that the null-depth of the beampattern at angle \( \phi_{min} = \arccos [-1/3] \) is controlled by the zoom-parameter \( \tilde{\psi} \).

Just as for the standard zooming method discussed in Section 2, we compute the directivity-factor \( Q \) as function of \( \epsilon \) and \( S \):

\[
\tilde{Q} = \frac{16}{1 + 3S^2 + 9S^2 \epsilon^2}. \tag{26}
\]
Section 2, the rejection of sounds coming from the back at the back-response, given as:

\[ E_{\phi}(\pi/2, \psi) \]

By variation of \( \alpha \) from Eqs. (22)-(23) and using Eq. (24) gives:

\[ \tilde{Q} = \frac{3}{4 \alpha^2 - 2 \psi \alpha^2 + 2 \psi - 1 - 2 \alpha \sqrt{1 - \psi^2 + \psi^2 \alpha^2}} \quad (27) \]

Similar to the results of the standard zooming method in Fig. 3, we plot the directivity-index in Fig. 6 for the new proposed acoustic zooming method, where the directivity-index \( 10 \log_{10} [\tilde{Q}] \) also decreases monotonically for increasing values of \( \psi \).

Also shown in Fig. 6 is the back-rejection that is computed from the back-response, given as:

\[ |E_{\phi}(\pi/2, \psi)|^2 = 4 \alpha^2 + 1 - 4 \alpha \sqrt{1 - \psi^2 + \psi^2 \alpha^2} \quad (28) \]

In contrast to the results of the zooming method discussed in Section 2, the rejection of sounds coming from the back at \( \phi = \pi \) and \( \theta = \pi/2 \) show a monotonically decreasing behaviour for increasing values of \( \psi \).

Fig. 6. Directivity-index \( 10 \log_{10} [\tilde{Q}] \) and back-rejection \(-10 \log_{10} [|E_{\phi}(\pi/2, \pi)|^2] \) as function of zoom-parameter \( \psi \) for the new proposed acoustic zooming method.

To show the behaviour of the new proposed acoustic zooming method for other types of beam-shapes, we also show the situation for \( \alpha = 0 \), for different values of \( \psi \) in Fig. 7.

Fig. 7. Magnitude-response \( |E_{\phi}(\pi/2, \psi)| \) for the new acoustic zooming with \( \alpha = 0 \) by variation of \( \psi \).

Again, it can be seen that the parameter \( \psi \) defines the depth of the null at angle \( \phi_{\text{min}} = \pi/2 \) in the dipole beampattern.

5. Measurements in the Anechoic Room

Two microphones (\( d = 0.02 \text{ m} \)) are placed in an anechoic room with a far-field sound-source at different azimuthal angles \( \phi \). We validate the new zoom-technique using an FIR filter for the differentiator and Hilbert transform based on Eq. (8) and Eq. (9) and an IIR filter based on the Trapezoidal-rule [5] for the integrator. We simulate the case for \( \alpha = 1/4 \) with different zoom-parameters \( \psi \).

Fig. 8. Magnitude-responses for the new zoom technique: from inner to outer curve, the parameter \( \psi \) ranges from 0 to 1 with steps of 0.25.

In Fig. 8, the result is shown for 630 and 1250 Hz, which are well below the spatial aliasing frequency. As can be seen, the results coincide with the theoretic results shown in Fig. 5.

6. Conclusions

We presented a new method for acoustic zooming where an offset is added to the ideal first-order superdirectional response. This offset is related to the desired zoom-factor. As the offset is 90 degrees out-of-phase with respect to the ideal response, we obtain a response that increases/decreases monotonically as function of the zoom-factor. In this way, the zooming is consistent not only for diffuse noise, but also for directional interferences.

7. References