AERODYNAMIC FORCES ON AN AIRSHIP HULL
IN ATMOSPHERIC TURBULENCE

by

Mario J. B. Lagrange

April, 1984
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ABSTRACT

The aerodynamic forces acting on an airship placed in a turbulence flow field inside the UTIAS boundary layer wind tunnel were measured. This provided a data base upon which theoretical simulation of the airship's response to turbulence could be evaluated. The slender body theory, as used in this report, has failed to predict the behaviour of a bare airship hull in turbulence. An extended three-dimensional slender body theory is also included. Although slightly better predictions were obtained, the extended theory is still far from providing accurate simulation. In general the slender body theory is an extremely conservative means of evaluating modern airship behaviour in atmospheric turbulence. The method of simulation should be reevaluated.

The addition of fins on the hull significantly changed the shape of the response curves. This variation is attributed to the wave number dependency of the fins and hull force vector which creates a variable phase angle between the two.

The enormous wake of the hull was measured and was found to be invariant with respect to Reynolds number for the limited range of velocity used in this experiment. The non-linearity of the aerodynamic forces was found to be negligible over the range of angles of attack used in this experiment.
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<td>balance dynamic transfer function</td>
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<tr>
<td>C</td>
<td>transducer force-voltage constant</td>
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<tr>
<td>$C_L$</td>
<td>lift coefficient</td>
</tr>
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<td>$C_N$</td>
<td>normal force coefficient</td>
</tr>
<tr>
<td>$C_M$</td>
<td>pitching moment coefficient</td>
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<tr>
<td>$C_{L\alpha}$</td>
<td>lift curve slope</td>
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<td>D</td>
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<tr>
<td>$E_F$</td>
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<td>$E_R$</td>
<td>rear transducer voltage reading</td>
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<tr>
<td>F</td>
<td>force (general)</td>
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<tr>
<td>$f$</td>
<td>frequency (Hz)</td>
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<tr>
<td>G</td>
<td>&quot;transfer function&quot; of the airship</td>
</tr>
<tr>
<td>H</td>
<td>tunnel height</td>
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<tr>
<td>h</td>
<td>neutral point</td>
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<tr>
<td>J</td>
<td>hull's inertia about the Y axis</td>
</tr>
<tr>
<td>k</td>
<td>arbitrary constant</td>
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<tr>
<td>$K_F$</td>
<td>front spring constant</td>
</tr>
<tr>
<td>$K_R$</td>
<td>rear spring constant</td>
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<tr>
<td>L</td>
<td>longitudinal integral scale</td>
</tr>
<tr>
<td>$\ell$</td>
<td>hull length</td>
</tr>
<tr>
<td>$\ell_F$</td>
<td>distance nose-fins' 1/4 chord</td>
</tr>
<tr>
<td>M</td>
<td>pitching moment about buoyancy center</td>
</tr>
<tr>
<td>m</td>
<td>mass of the model</td>
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</table>
$M$ momentum

$N$ total normal force

$N_H$ hull's normal force contribution

$N_F$ fins' normal force contribution

$q$ dynamic pressure

$S$ surface area

$T, T'$ integrals in slender body theory

$U$ free stream velocity along X axis

$U_c$ hot wire calibration horizontal velocity

$V$ free stream velocity (in appendix A only)

$V$ hull's volume

$U, V, W$ velocity components of the turbulence

$W_c$ hot wire calibration vertical velocity

$X, Y, Z$ frame of reference components (X along hull symmetry line)

$Y_f$ front transducer displacement

$Y_r$ rear transducer displacement

$\alpha$ angle of attack

$\alpha_g$ non-dimensional turbulence

$\lambda$ turbulence's wave length

$\Omega$ non-dimensional frequency (wave number)

$\hat{\Omega}$ non-dimensional wave number

$\Omega_P$ wave number of the peak of the response PSD

$\Omega_H$ higher value of the wave number bandwidth

$\Omega_L$ lower value of the wave number bandwidth

$\phi$ spectral density

$\hat{\phi}$ one-sided spectral density

$\phi$ phase angle between hull and fins forces
\[ \rho \] density of the air
\[ \sigma \] intensity of the turbulence
\[ \hat{\sigma} \] intensity of the truncated turbulence PSD
\[ \theta \] angle of the flow in hot wire calibration

Acronyms

BC buoyancy center
CSD cross spectral density
FFT fast fourier transform
fps feet per second
GAA,GBB one sided power spectrum of channel A & B
GAB one sided cross spectrum of channel A & B
HLA heavy lift airship
Im imaginary part of a complex number or function
LTA lighter than air
LVDT linear variable differential transformer
PSD power spectrum density
Re real part of a complex number or function
RMS root mean square
RPM revolutions per minute
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CHAPTER 1
INTRODUCTION

In the past few years interest in airship transportation has increased substantially, leading to a revival in the design and construction of modern airships. Contrary to their predecessors, most new designs are based on non-rigid structures which are not so slender as their predecessors. One viable application is in the lifting and transportation of large payloads, up to 100 tons. These airship designs may use hulls in conjunction with helicopters or similar power plants and are referred to as Heavy Lift Airships (HLA). The size of these mastodons as well as their relatively slow forward velocities make them an easy target for mother nature. The loading and unloading task of the HLA requires a high degree of precision and controllability. This becomes particularly critical when the HLA is used as a free floating crane in construction applications (as for example the CN tower in Toronto). Atmospheric turbulence is a major obstacle for airships. The response to turbulence is critical in high precision manoeuvres and should be studied before risking catastrophes.

Analytical and computer simulation are now used to estimate the behavior of airships in atmospheric turbulence. It was felt here at UTIAS that experimental testing was needed to backup these analytical techniques. This experiment was designed to create a benchmark upon which the theoretical results could be assessed. At the same time, the
The experiment would determine the level of the forces and moments acting on an airship hull operating in turbulence.

The experiment consisted of the measurement of normal forces and pitching moments acting on an airship hull model placed in a subsonic wind tunnel in which a turbulent flow field was created. The 30 inches airship model (76 cm), based on the Goodyear's HLA shape was principally tested as a bare hull. The addition of four relatively large fins permitted us to observe their influence on the airship's response to turbulence. A wake measurement was performed to demonstrate the flow attachment on our bare hull model. The experimental results are compared with simple computer estimation using the Munk's slender body theory. An extension of this theory was developed by Dr B. Etkin and is included in this report. The experimental non-dimensional wave number range was also compared to a typical atmospheric turbulence wave number bandwidth for a full scale hull. Finally, proposals are made for a more complete theory that might give better estimations of the airship's response to turbulence.
2.1 TURBULENCE.

Atmospheric turbulence is one of the obstacles that man must face when he uses air as a medium of transportation. The theory dealing with this phenomenon has been well developed by Etkin in references 1 and 2. The velocity vector of atmospheric turbulence is normally a random function of time and space. However, in our experiment we will only consider the vertical component of the gust which will be referred to as \( W_g \).

The assumption of stationary process combined with the homogeneity of the turbulence permits the experimentalist to replace ensemble averages by time averages. This property is known as ergodicity. It can also be assumed that the turbulence is isotropic. Therefore its intensity will be equal to the mean-square velocity of any of the three velocity components of the turbulence. For a one-dimensional spectrum function the mean-square value of any velocity component, hence its intensity, is defined as:

\[
\sigma_g^2 = \langle W_g^2 \rangle = \int_{-\infty}^{\infty} \omega \Phi_{W_g W_g}(\omega) \ d \omega
\]  

(2.1-1)

For the purpose of comparison of turbulence intensity we will define a truncated intensity that will simply be the same integral but...
over a defined bandwidth as shown in equation 2.1-2. The limits of the integral represent the smallest and the largest frequencies of the experimental bandwidth, and a one-side spectrum is used, so that $\hat{\phi} = 2 \phi$.

$$\hat{\phi}^2 = \int_{\Omega_L}^{\Omega_H} \hat{\phi}_W \hat{W} (\Omega) \, d\Omega$$  \hspace{1cm} (2.1-2)

The most frequently used one-dimensional spectrum model that simulates the vertical component of the atmospheric turbulence was developed by von Karman; its mathematical representation can be found in appendix B. For large wave numbers this spectrum function predicts that $\phi_3(\Omega) \sim \Omega^{-5/3}$ as $\Omega \to \infty$. This will be verified in chapter 4 of this report.

The integral scale, which is a measurement of the "size" of the turbulence, is directly related to the peak of the one-dimensional spectrum by:

$$\Omega_{pL} = 1.33$$  \hspace{1cm} (2.1-3)

The scale value must therefore be chosen with precaution, since it defines the value of the the dominant frequency of the turbulence.

2.2 SLENDER BODY THEORY.

The slender body theory was conceived by Max Munk in 1924 for application to rigid airships (ref 8). Although new non-rigid airship
designs tend to be less slender than the early designs, the theory is still extensively used in turbulence response simulations.

This theory is based on the change of momentum of the air resulting from the change of cross-sectional area as the slender body moves across a perpendicular imaginary fixed plane in the surrounding fluid. In this plane the Wg component of the velocity is assumed to be constant to infinity. Because the airship is a closed body, the theory gives a zero $C_{N\infty}$ in steady flow. To obtain a non-zero value, the body is first truncated, usually at the fin-hull intersection (reference 3) then the slender body theory is applied. In such a case, the $C_{N\infty}$ value depends only on the base area where the truncation has been performed. Where the bare hull is tested, the location of the truncation, for simulation comparison, will be at $0.77 \ell$; the imaginary fin-hull intersection. Simulation of the normal force and pitching moment created by atmospheric turbulence is derived in appendix C. The application of this theory to the bare airship hull model used in this experiment is shown in figure 1. The curves show the effect of truncation and the importance of choosing its appropriate location.

The assumption of constant Wg in the fixed plane is questionable and therefore an extension of the slender body theory was developed by Dr. B. Etkin, as included in appendix A of this report.
3.1 WIND TUNNEL AND TURBULENCE.

Details concerning the UTIAS boundary layer wind tunnel are found in reference 4. The flow in its 1.12 m high by 1.68 m wide testing section is driven by two power plants. An axial fan connected to a 45 kW motor are located in the return section of the tunnel. This motor can be set at eleven fixed RPM values, providing flow velocity up to 30 m/sec in the test section. Also, a 56 kW blower supplies 96 jets located at the exit of the contraction cone. These jets are grouped in sets of three and controlled by 32 independent valves. Because the blower is operated at constant RPM, the flow velocity of the jets are controlled by the servo-valves. The blower provides enough pressure rise to reach a jet nozzle velocity of 80 m/sec, which corresponds to a test section mean speed of 17 m/sec. The axial fan, used by itself, creates a fairly smooth non-turbulent flow. Combined with the jets, the fan can create different velocity profiles.

One of the most traditional ways of creating turbulence in a wind tunnel is using flow through a grid. Using this method, Liu (reference 5) created isotropic turbulent flow field having high intensity as well as large integral scale size. This was possible using a coarse grid consisting of four barriers close to the tunnel walls in conjunction with proper jet settings. The grid is located at 1 tunnel height (1.12 m) downstream from the jet's exhaust plane. The best
uniform test section was found to lie between 7 and 8 tunnel heights downstream (figure 2).

3.2 HOT WIRE ANEMOMETRY.

In order to verify the spectrum density of the turbulent flow field, a measurement of the vertical velocity component was performed. A DISA probe type 55 P61 was used. This 45 degree X-wire probe was properly oriented in the X-Z plane so that the difference between the measured signals would give the Z-axis velocity component \( W_z \). To provide against temperature effect, each wire of the cross probe was paired with the compensator of a DISA 55 P81 temperature compensated probe. The theory for hot wire anemometry is well discussed in reference 6.

Each channel consisted of one DISA 55D01 anemometer unit, including the bridge connected to a DISA 55D10 linearizer (figure 3). These provided measurement capabilities for flows from 1 to 90 m/sec with a frequency response up to 100 kHz. The output of each channel was fed to a simple electronic summing circuit of which the outputs were connected to a spectrum analyzer (section 3.5).

3.3 HULL SHAPE AND FINs.

The shape of the hull was based on modern airships. The hull has a fineness ratio of 4.0 \((\ell/D)\) and maximum thickness location at 0.4\(\ell\) from the nose. The envelope was built using Goodyear's Heavy Lift Airship (HLA) shape with some modifications. Some analysis indicated
that larger models were preferable to our experiment in terms of sensitivity to turbulence per weight unit of the envelope. The factor that put restrictions on the model size was the clearance between the envelope and the horizontal walls of the tunnel. The final length was chosen to be 30 inches (76 cm) and hence 7.5 inches (19 cm) for maximum diameter. Appendix B details the hull parameters. These dimensions gave a minimum tunnel-model clearance of twice the maximum diameter up to a 20 degree angle of attack with tunnel blockage of less than 2.5%. A picture of the bare hull showing well its shape is included as figure 4a.

The fins were designed with a NACA 0012 cross-section and taper ratio of 0.6. Each fin had a surface area of about 10 square inches (65 cm²). They were intentionally designed larger than that of the fins of an HLA vehicle for experimental purposes. The projection plan along the X-axis of the hull shows a half inch extension of the fins above the maximum diameter (outside the hull's wake). For the sake of practicality the fins were increased from 3 in number, as in an HLA, to 4. Figure 4b shows the size of the fins compared to the hull.

3.4 FORCE AND MOMENT BALANCE DESIGN,

The design of a two degree of freedom force balance is a complex procedure involving a number of factors and parameters. The aerodynamic forces created by the turbulence on the model were calculated to be relatively small. Sensitivity was therefore a major factor in the choice and design of the spring-transducer components of
the balance. On the other hand, the frequency bandwidth of interest forced us to keep resonance frequencies at high values. To reach both objectives it was decided to minimize the weight of the envelope without changing its dimensions. Styrofoam SM, used in building insulation, was found to be the most suitable material, having low density and fairly high stiffness. The hull was cut from a single block of styrofoam using a nickel-chromium hot wire then hollowed out leaving an envelope of about one inch thick. This reduced considerably the weight of the model and permitted us to design a force balance that would fit inside the envelope thereby lessening outside disturbances affecting the flow field. To gain access to the balance components, once assembled the hull was cut into three sections (see figure B-1). Two pairs of aluminium bulkheads were used to insure proper force transmission from nose and tail to the center body, the bulkheads of which were secured to the moving parts of the springs (figure 5). This system including the bulkheads, the foam, and the glue weighed a little less than 1.0 lb (454 g.).

The aforementioned frequency bandwidth was a critical factor. To begin, it was important that we include the peak of the turbulence spectrum which occurs at \( \omega_p L = 1.33 \) (reference 1, chap 13). For an integral scale assumed to be around 1/2 ft (0.152 m) (the actual value was found to be 1.10 ft (0.335 m)). With a speed of 80 fps (24.4 m/sec), the frequency of the peak reached 34 Hz. Of particular interest was the turbulence wave length being of the order of the hull length (\( \lambda/L = 1 \)). Again, for the same above speed, this happened at 32 Hz. We therefore decided that the minimum acceptable resonance
Two spring balance systems were designed and built for this experiment. The original system was based on two proving rings mounted at the front and rear of the center body. The rings were carefully machined from mild steel to avoid hysteresis and to provide good linearity. The moving parts of the springs were attached to the top of the center body bulkheads while the other ends were solidly fixed to a heavy steel base placed inside the hull. This base was grounded to the floor of the lab by two heavy struts of which the vertical position could be changed allowing for variation in the angle of attack of the model. The drag and side forces were countered by four internal steel wires running from the bulkhead horizontally to the base so that negligible vertical restriction would be imposed. During calibration, rolling moment was found to substantially influence the readings. The introduction of eight external steel wires running from the bulkheads to the side walls of the tunnel provided against rolling motion without restricting the vertical displacements. Tunnel vibrations, however, were transmitted to the hull via the bracing wires, hence to the transducers creating unwanted peaks on the spectral densities. Despite this problem a few runs were made using the proving ring balance system. These early measurements showed a higher resolution of the transducer than expected. In order not to overload the signal, only 25 % amplification was used. It was then decided to design a new set of springs compatible with the other components of the first balance system.
Because of the experience gained during the first design we opted for a parallel beam type of spring to replace the proving ring. This allowed displacement in the vertical plane with extremely high resistance to any force in the horizontal plane. In conjunction with a redesigned bulkhead attachment the new parallel beam countered the rolling motion without the need for external or internal wires. Due to the high resolution of the transducers, the Z-axis spring stiffness was increased to 700 lbs/in (125 Kg/cm) per spring, therefore raising the lowest natural frequency of the system from 52 Hz to about 110 Hz. The second design of the force balance system, using parallel beams, is shown in figures 5 & 6 and will henceforth be referred to as the balance.

3.5 LVDT SYSTEM.

The high resolution transducers mentioned earlier are known as Linear Variable Differential Transformers (LVDT). In our experiment we used the Schaevitz LVDT model 010 MHR (Miniature High Resolution). They are 3/8 inch (9.53 mm) in diameter by 1/2 inch (12.7 mm) in length and the core is approximately 0.1 inch (2.54 mm) by 0.23 inch (5.84 mm). This electromechanical transducer has a linear range of +0.010 inch (0.254 mm). The coil or main body of the transducer was mounted on a non-magnetic frame fixed to the base inside the airship hull. The core was attached to the moving end of the spring at a precise known location. The magnetic core being aligned at the center inside the coil permitted a frictionless movement between the two devices. The core displacement was transformed into an output signal.
by taking the difference of the signals from the two secondary
windings. The choice of this type of transducer was based on numerous
excellent performances in the history of UTIAS.

An available custom built Schaevitz CAM-929 system was used to
provide excitation to the primary windings and to demodulate the
differential output of the secondary windings. Some of the features of
this system are high sensitivity, good stability, low noise, flat
frequency response of less than 1/2 % to 200 Hz, and practically zero
sensitivity to temperature change in our range of interest. The system
also included gain and zero adjustments. Two Sorenson QRB 15-2 were
used as a ±14 volt DC power supply for the CAM-929 system.

3.6 BALANCE SYSTEM DYNAMICS.

Because of the stiff nature of the springs, the balance's
resonance peaks are located at frequencies outside our range of
interest (around 110 Hz). However, resonance effects can be noticed at
frequencies considerably smaller than that of the natural frequencies.
The equations of motion of the balance system were derived using a two
degree of freedom mass spring model (appendix E). The model does not
account for damping since the only damping present in our experiment
was aerodynamic, found to be quite negligible. The curves of figure
7 represent the dynamic transfer function of the balance system. One
should notice the close location of the two resonance frequencies
creating a single wider peak.
In order to obtain the power and cross spectral densities of the output signal of both the LVDT system and the hot wire anemometry the SD 375 Dynamic Analyzer II was used. This instrument, from Spectral Dynamics Division of Scientific-Atlanta, is a microprocessor-based Fast Fourier Transform (FFT) analyzer and signal processor that analyzes frequencies up to 100 kHz with 400 lines per channel resolution. The frequency range for our experiment was set at 400 Hz, therefore providing a 1 Hz resolution on the machine. Each run produced an ensemble average of 500 transforms calculated at a rate of 2.56 per second. Each transform utilized 400 samples of the random time function. A series of tests and double-checks were performed so that the behavior of the machine was familiar to the operator. The Dynamic Analyzer II was used only in the "power" mode. The PSD and CSD of both front and rear LVDTs for a given run were obtained from a unique ensemble average by changing the menu from GAA & GBB to GAB (Re & Im). It was found more practical to measure the CSD in real and imaginary parts than to use amplitude and phase angle. The results were transmitted to an X-Y plotter model HP 7046B. Numerical data in the low frequencies were hand recorded by the operator directly from the CRT display of the Analyzer for later use (section 3.9). Figure 8 shows a schematic of the LVDT data acquisition setup.
3.8 EXPERIMENTAL TUNNEL SETUP.

The best location for uniform turbulence as well as high intensity and integral scale was found to be at 7 H downstream from the jet exit plane (reference 5). With the model centered at that location the test section was then considered to lie between 6.6 H and 7.4 H. The model was placed in the middle of the cross-section plane of the test section for subsequent angles of attack. To begin a X-wire probe was installed six inches in front of the nose to check for turbulence change due to the presence of the model in the test section. The probe was removed and a series of runs were made at various speeds and angles of attack. The angle was set by lowering and raising the front and rear struts joining the base to the ground. The speed was changed using a different setting of the axial fan controller.

The same procedure was used for both the bare airship hull and the model with fins. The only difference being that fewer runs were performed on the latter.

Before attaching fins to the airship hull, a U-wire probe was installed behind the model. This hot wire probe, mounted on a traversing rig, was moved vertically as well as horizontally in a Y-Z plane at various distances behind the end of the tail section. By taking measurements at different positions, it was possible to see the shape of the wake behind the airship without fins.
3.9 DATA PROCESSING.

UTIAS computer facilities are based on a Perkin Elmer 3250 System. This 2 mega-byte system was a recent acquisition of the Institute. Since the experiment involved a relatively small number of data points it was then more efficient to use manual interfacing between the dynamic analyzer and the UTIAS main computer system. As stated before, the numerical values of the spectral densities were recorded by the operator for 70 different frequencies from 1 Hz to 100 Hz. The front transducer and the rear transducer PSD values, along with the real and imaginary parts of their CSD, were then put into a data file. Individual data files were created for each run. The first line of each file included the run identification number, the mean wind velocity, and the angle of attack of the airship.

The mean normal force and pitching moment for each run were measured by passing the output signal of the LVDT system through a low pass active filter with a 0.5 Hz cut off frequency. The modified signal was then recorded on an analog plotter with a total sweep of 100 second full scale. It was then possible to draw a mean line through the trace and take its value to be the average output voltage for each specific run.

In order to verify the accuracy of the computer calculations of the normal force and pitching moment PSD, an active summing circuit was built. This circuit was placed between the LVDT system and the dynamic analyzer. Its function was to electronically add and subtract the
output signal so that the FFT of the normal force and pitching moment could be processed by the analyzer (figure 8, dotted line). The results are compared in section 5.3.

3.10 Gust Response of Model.

The conventional way of expressing the response of a system to a certain disturbance is via the transfer function \( G \). Our model consisted of a system with one input, the vertical component of the turbulence velocity \( W_g \), and two outputs known as normal force \( N \) and pitching moment \( M \).

\[
\begin{array}{c}
W_g \rightarrow G \rightarrow N \rightarrow M \\
\text{Airship}
\end{array}
\]

The values of the force and moment are not directly read from the experiment but are obtained from another transfer function, the inputs of which are the voltages of the front and rear transducers \((E_f \text{ and } E_r)\). This dynamic transfer function \( B \) accounted for the dynamic resonance of the two degrees of freedom balance system.

\[
\begin{array}{c}
N \rightarrow B \rightarrow E_f \rightarrow E_r \\
\text{Balance}
\end{array}
\]

The redesigned balance was sufficiently stiff that it was unnecessary to allow for its dynamic response (see section 4.3). Consequently the data analysis was much simplified. The static equations for the normal force and pitching moment can then be used in
the response to turbulence calculations. The constants relating the force to the voltage are defined for each spring component, $C_f$ for the front and $C_r$ for the rear. The absolute distance between the front transducer and the center of buoyancy is defined by "$a$" while "$b$" is the distance from the rear transducer.

$$N = C_f E_f + C_r E_r$$  \hspace{1cm} (3.10,2a) \hspace{1cm}

$$M = a C_f E_f - b C_r E_r$$  \hspace{1cm} (3.10,2b) \hspace{1cm}

Since we are dealing with the spectral densities of the LVDT voltages, it can then be shown that the normal force and pitching moment PSD are functions of the front and rear voltage PSDs as well as their CSD. Hence the need to measure the front-rear voltages CSD. Using equation 3.4,48 of reference 1, we can derive the following equations.

$$\phi_{NN} = C_f^2 \phi_{E_f E_f} + 2 C_r C_f \phi_{E_f E_r} + C_r^2 \phi_{E_r E_r}$$  \hspace{1cm} (3.10,3a) \hspace{1cm}

$$\phi_{MM} = a^2 C_f^2 \phi_{E_f E_f} - 2 a b C_r C_f \phi_{E_f E_r} + b^2 C_r^2 \phi_{E_r E_r}$$  \hspace{1cm} (3.10,3b) \hspace{1cm}

The response of an airship in turbulence is a function of three velocity components of gust $U_g$, $V_g$ and $W_g$. Our experiment is dealing only with the normal force and pitching moment induced by the turbulence flow field. Although some of the measured gust responses may be associated to $U_g$ and $V_g$, the main gust velocity component
responsible for the airship behavior in this experiment is $W_g$. At this point we are assuming that only the latter is creating the normal force and pitching moment acting on the airship. This permitted us to define the transfer function $G$ as a $2 \times 1$ matrix of the following form.

$$G = [C_{NW_g}, C_{MW_g}]^T \quad (3.10,4)$$

The spectral densities of $E_f$ and $E_r$ were measured using the dynamic analyzer. The PSD of the gust velocity $W_g$ was also measured in the same manner, making the transfer function $G$ the only unknown variable. Its two components were calculated by a computer program that used as inputs the PSD of $E_f$, $E_r$, $W_g$ and the CSD of $E_f$ & $E_r$. The values of $B$, the dynamic transfer function, were inserted in the program and the output was given in the form of two non-dimensionalized components of the transfer function $G$.

The non-dimensionalization of these variables is a simple yet tricky process. It is general practice to use the hull volume to characterize the size of the body. Instead of surface area, the volume $V^{2/3}$ was therefore used in the non-dimensionalization process. The normal force and pitching moment coefficients are then defined as.

$$C_N = \frac{N}{\frac{1}{2} \rho U^2 V^{2/3}} \quad (3.10,5a)$$

$$C_M = \frac{M}{\frac{1}{2} \rho U^2 V} \quad (3.10,5b)$$
The non-dimensional spectrum is defined by.

\[ \phi_{CN_{N}}(f) = \frac{\phi_{NN}(f)}{\frac{1}{4} \rho U^4 V^{4/3}} \]  \hspace{1cm} (3.10,6a)

\[ \phi_{CM_{M}}(f) = \frac{\phi_{MM}(f)}{\frac{1}{4} \rho U^2 V^2} \]  \hspace{1cm} (3.10,6b)

In order to non-dimensionalize the turbulence vertical component \( W_g \) we simply divided by the mean forward velocity \( U \). The PSD of the \( W_g \) becomes.

\[ \phi_{g g} = \frac{\phi_{WgWg}}{U^2} \]  \hspace{1cm} (3.10,7)

The calculations requiring a bit more caution were the non-dimensionalization of the frequency or the abscissas of the spectral densities. Conformal transformation states that the area under the spectral density curves must remain constant. In order to change the frequency (Hz) into omega (1/ft) one must divide by \( U/2\pi \). Hence, the ordinate must be multiplied by the same parameter.

\[ \phi_{g g} (\Omega) = \frac{\phi_{WgWg}(f)}{2\pi U} \]  \hspace{1cm} (3.10,9)
\[ \Phi_{CNN}(\Omega) = \frac{\phi_{NN}(f)}{\frac{\pi}{2} \rho^2 U^2 V^{4/3}} \]  
(3.10,10a)

\[ \Phi_{CMC}(\Omega) = \frac{\phi_{MM}(f)}{\frac{\pi}{2} \rho^2 U^2 V^2} \]  
(3.10,10b)

Note that the dimensions of the spectrum are in length units since the abscissas are in length. An additional step can be taken to provide for non-dimensional units by dividing the ordinate and multiplying the abscissa by the airship model length. This step must be taken if one wishes to apply the results to a full scale airship or a different length model (see figure 29).
4.1 HOT WIRE ANEMOMETER CALIBRATION.

The calibration of the hot wire probe was performed using a custom built jet-calibration rig shown in figure 9. The jet velocity at the nozzle was deduced from the dynamic pressure measured with an AVAG Original Betz Manometer. The flow was regulated by an electrically operated valve. The rig was equipped with a manually adjusted probe mounting that permitted different probe angles relative to the jet.

The X-wire probe calibration was done at two different velocities corresponding to the experimental velocities. The probe angle was changed from -20 to +20 degree using 5 degree increments. The probe was aligned in such a way that the signal difference of the two channels would give the vertical velocity component voltage $E_w$ relative to the probe axis velocity voltage $E_u$. The corresponding velocities $U_c$ and $W_c$ were derived from the dynamic pressure readings and angle settings.

\[
U_c = \left( \frac{2q}{\rho} \right)^{1/2} \cos(\theta) \tag{4.1-1}
\]

\[
W_c = \left( \frac{2q}{\rho} \right)^{1/2} \sin(\theta) \tag{4.1-2}
\]

During calibration the hot wire system was set as for experimental measurements. This included the temperature compensator, anemometer
unit, linearizer and summing circuit. The calibration curves for the U and W velocity components are shown in figure 10. Slopes show no dependence on the jet exhaust velocity of the calibration rig.

4.2 BALANCE STATIC CALIBRATION.

The first step was to measure the stiffness of the springs and compare them with the design values. This was done using a 0.0005 inch (0.013 mm) resolution dial gauge aligned with the core position on the moving part of the parallel beams. The displacement readings for known applied weights yielded spring constants of 715 lbs/in (128 Kg/cm) for the front and 695 lbs/in (124 Kg/cm) for the rear spring. The design was calculated to produce 700 lbs/in (125 Kg/cm).

Once the LVDT transducers were put in place and carefully aligned to give zero core-coil friction, the CAM 929 gains were adjusted to measure the same voltage for identical loads on each of the two springs. Transducer voltages were measured for different loads. Static calibration curves for both the front and rear spring-transducer systems are shown in figure 11. Note the excellent linearity and identical response of the transducers.

Resolution tests were performed by adding or subtracting small weights to different known loads. We found the system to have a very high resolution, limited by the voltmeter accuracy as well as the system environment (i.e. laboratory's floor vibrations, air movement in around the hull, etc.)
The system's response to drag and side forces as well as yawing and rolling moments were found to be practically nil, therefore not considered in the data processing.

Before each experimental run, the CAM 929 zero potentiometers were reset to give zero readings on the front and rear voltmeters when the air in the test section was still. This procedure was necessary after a new angle of attack was set, since the static load distribution over the springs was altered. Very little voltage drift occurred between two consecutive runs using the same angle setting.

4.3 BALANCE DYNAMIC RESPONSE CHECK.

The natural frequencies of the balance system, being outside the bandwidth of interest, enabled us to bypass the dynamic calibration. However, a few tests were made by simply exciting the model through an elastic collision with a small mass (about 50 grams). Spectral analysis showed, as expected, high resonance peaks at the natural frequencies of the system which were both around 110 Hz. An unexpected peak appeared near 90 Hz which afterward was associated to the strut-bracing structure of the balance system. It was decided at this point to consider data valid up to 60 Hz only, since time did not permit further modification of the system.
CHAPTER 5
EXPERIMENTAL RESULTS

5.1 TURBULENCE INPUT.

The turbulence PSDs are shown in figure 12. These were measured with the hot wire X-probe and processed by the dynamic analyzer. Comparison of the lower and middle graphs permits us to conclude that there are pratically no effects on the turbulence flow field due to the presence of the model in the tunnel test section. The uppermost graph is the result of a cubic spline fit on the PSD of the turbulence with the airship present in the test section. This smoothing was processed using the UTIAS computer facilities and the result was stored in a data file that was used as turbulence input whenever transfer functions were to be plotted. The turbulence intensities were calculated by numerical integration of the spectrum curves. Because the spectra do not range from 0 to infinity, the intensities shown on the graphs of figure 12 and 13 are only for the truncated part of the PSDs and will be refered to as \( \hat{\sigma} \). A comparison of the intensities shows very little loss due to the presence of the model in the tunnel. From these non-dimentionalized turbulence PSDs we note that the high velocity curve has a greater intensity. When the turbulence field was created by Liu (ref 5) only the high speed setting of the axial fan was used, the turbulence intensity was therefore maximized. By changing the axial fan RPM without resetting the jets' valves, we drift from the optimal conditions resulting in a slightly lower non-dimentionalized intensity.
The turbulence integral scale is determined using two lines tangent to the low frequency and high frequency asymptotes. The intersection defines a value of that is then converted to integral scale using \( L \approx \text{peak} = 1.33 \) (ref 1, chap 13). Both speeds give a common value of \( L = 1.1 \) ft.

The shape of the turbulence PSD is compared to the von Karman one-dimensional PSD in figure 13. The theoretical spectrum uses the integral scale previously defined along with a true intensity value, adjusted so that the surfaces under both the theoretical and experimental truncated PSD are the same for a given test section velocity. The high frequency asymptote is in agreement with the Kolmogorov law which states that \( \phi \sim \Omega^{-5/3} \) as \( \Omega \to \infty \). On the other hand, the long wave length components of the turbulence were being dampened by the restriction of the tunnel boundaries. This resulted in a steeper low frequency asymptote which created a more pronounced peak of the experimental tunnel turbulence PSDs. Over the range corresponding to the turbulent fluctuations, the aerodynamic forces can be reasonably assumed to be linear. Therefore the difference between the von Karman turbulence model and the measured spectrum did not affect the experimental result which is the ratio of output / input, the input being the tunnel turbulence.

5.2 AVERAGE NORMAL FORCE & PITCHING MOMENT.

The traces of the voltages from the front and rear force transducers passed through a low pass filter enabled us to measure the
mean values of normal force and pitching moment. Figures 14 and 15 show these mean force and moment coefficients values plotted for different angles of attack. The slopes of the lines give the $\alpha$ derivatives $C_{N\alpha}$ and $C_{M\alpha}$. The slope indicated in the left corner of each graph is for the zero $\alpha$ region. The difference in the slopes for different speeds indicates a tunnel Reynolds number effect, especially in the normal force plot. The slope of the curves tend to increase with the absolute value of the angle of attack showing some non-linearity, especially in the $C_N$ plot. The RMS value of the turbulence input discussed in the previous section can be associated to an angle of attack range of about 6 degrees. The amount of non-linearity over that $\alpha$ range is practically negligible. The whole method of response to turbulence depends crucially on the linearity of the aerodynamic forces which is reasonably confirmed by this experiment. The non-zero value of the normal force at zero angle of attack can be attributed to asymmetry of the airship envelope or to non-horizontal flow in the tunnel test section or both. The presence of struts is another possible explanation of this offset. However, previous experiments in the UTIAS boundary layer wind tunnel have shown similar results for axisymmetrical bodies.

The values of $C_{N\alpha}$ and $C_{M\alpha}$ can be roughly compared with the results of a round, streamlined body with a tapered afterbody in laminar flow (Hoerner, ref 7). For a fineness ratio of 4.0 the values from the Hoerner are $C_{L\alpha}=0.0021$ deg and $C_{M\alpha}=0.0022$ deg. For small angles the difference between normal force and lift is minimal and can be neglected. For a comparison of these with the experimental values
of $C_{N_a}$ and $C_{M_a}$, adjustments were made for the differences in the non-dimensionalization process (Hoerner used $d_1$ and $d_2^2$ instead of $V^{2/3}$ and $V$). In this experiment we were interested in the moment about the center of buoyancy which was located at 0.45$t$ from the nose. The necessary adjustments and conversions were made and the resulting values are shown in table 1a. Considering the difference in shape of the two models, especially in the afterbodies, the agreement of the compared figures seems quite acceptable. Using the experimental values of $C_{N_a}$ and $C_{M_a}$ the neutral point location was easily determined for each of the two speeds, also shown in table 1a. The negative sign indicates that the location of the neutral point is in front of the nose. The value of -0.3$t$ given by Hoerner is for blunt-base parabolic-arc bodies. Also by inspection of Hoerner's results it can be deduced that turbulence tends to reduce the static margin which, in our case, would bring the reference neutral point closer to the nose.

The same parameters are also given for the hull with fins and can be found in table 1b.

5.3 NORMAL FORCE & PITCHING MOMENT SPECTRA.

The PSD of the normal force and pitching moment coefficients are shown in figure 16 using two different calculation procedures. The turbulence PSD with model in the test section is also shown. The first set of curves, referred to as "digitally computed", is calculated by the UTIAS computer using the LVDT output voltages spectral densities along with equations 3.10,3a and 3.10,3b. The second set consists of a reproduction by the same computer of the PSDs measured and plotted...
using an electronic summing circuit that adds and subtracts the LVDT output voltages. This last procedure is called "analog summation". The former was used throughout the experiment, the correlation of the two sets of curves proves the validity of our method. We are assured the results can be reproduced due to the fact both sets of curves were derived from two different runs with the same setting in a two week interval.

The sharp peaks at the extreme right of the plots are caused by mechanical resonance of the balance system. The different wave number locations of these peaks indicates their independence of the turbulence input and therefore can be omitted in the gust response of the airship (see section 5.4). The same applies to the small sharp peaks in the middle of the C_M PSD curves, attributed to some tunnel system resonance.

The shape of the C_N PSD shows a nice rounded peak close to \( \Omega = 1 \). This peak, well defined at both speeds, is related to the turbulence response of the airship hull. A less pronounced equivalent type of peak appears in the C_M PSD but at a lower wave number. Again we note the Reynolds number effect, more apparent in the force and moment PSD than in the turbulence PSD (upper graph).

5.4 DATA TRUNCATION AND SMOOTHING.

The resonance of the strut bracing system limited the validity of the experimental data up to 60 Hz. The first alteration made to the
data was simply a truncation of all data points above 60 Hz. Afterwards all plots showed results up to that cut-off frequency or its equivalent wave number depending on the mean velocity of the run. This truncation left us with 50 data points of different frequencies ranging from 1 Hz to 60 Hz inclusive.

The smoothing process used in the next section involved two additional steps in addition to truncation. The first eliminated the mechanical resonance, which can be traced to the basic LVDT output spectral densities. Since these sharp resonance peaks are independent of the behavior of the hull in turbulence, it is totally acceptable to round off these peaks in the calculation of the airship response to turbulence. This task was performed by simply replacing the data points belonging to such a peak by interpolation points creating a smooth line at the base of the peak. Figure 17 is an example of the rounding off and the truncation processes used in this experiment, the dashed lines representing the new corrected curves. The final step in the smoothing process was based on the response to turbulence calculated using the roundoff and truncated data as input. These rough and generally noisy curves were then smoothed by passing a cubic spline fit through the turbulence response curves. The smoothing factor used by the spline fit was adjusted so as not to destroy the basic shape of the curves (figure 21 to 25). This process was used whenever a smooth curve was needed.
Because the frequency bandwith of our experiment did not include the mechanical resonance of the balance system, we can obtain a close estimate of the turbulence response from the ratio \( \frac{C_N C_N}{C_{\alpha g} C_{\alpha g}} \) and \( \frac{C_N C_N}{C_{\alpha g} C_{\alpha g}} \), as done in figures 18 and 19. To be more exact, one can include the dynamic transfer function of the balance system in the calculation of the turbulence response. The results of this last procedure are shown in figure 20. When compared to the previous results from simple ratios, hardly any distinction can be made. This is due to the cut off frequency of the experimental data being well below any natural frequency of the balance system. The mechanical resonance frequencies of the tunnel can still be noticed on these plots, since only the frequency truncation had been applied thus far. These peaks appear only on the pitching moment curves, meaning that only one mode was being excited.

Using the smoothing process we eliminated these mechanical resonance peaks as well as the noise, the result being the smoothed curves shown in figures 21 to 25. Comparison with the uncorrected response curves shows the effectiveness of the smoothing process in keeping the general shape and magnitude of the turbulence response curves. The effect due to tunnel Reynolds number is very apparent, especially for the normal force response curves. The same effect was present on the mean C_N and C_M plots of figures 14 and 15. This Reynolds number effect seems not only to increase the magnitude of the response, but to move the peak of the normal force response to a
slightly higher wave number. For the bare hull case, a 30% increase in Reynolds number created a 5% increase in the wave number location of the peak as well as a 7% rise in its magnitude. The peak location for the high velocity curve occurs at $\Omega = 1.2$. This corresponds to a turbulence wave length of almost exactly twice the length of the hull. One should also note that there is no peak on the pitching moment response and therefore the maximum happens at a zero $\Omega$ value.

Theory predicts the zero frequency values of the response curves should match the values of $C_{N\alpha}$ and $C_{M\alpha}$ given in table 5.1. The difference between these values and the interpolation from the lower $\Omega$ limit of our graph is no more than 30%. Considering that this left hand side boundary has not yet reached the zero wave number some disagreement between these figures is to be expected.

The non-linearity experienced in the calculation of the mean values of $C_N$ and $C_M$ are reflected in the response curves shown for different angles of attack in figure 22. The increasing lower limit response values can be matched with the increasing slope of $C_{N\alpha}$ and $C_{M\alpha}$ of figure 14. The magnitude of the peak is not affected by a change in angle of attack, but its location moves to smaller wave number as alpha gets larger. The effect on the moment response appears to be a smoothing of the curve as well as an increased lower wave number response, where the maximum moment response to turbulence is experienced. We can also observe that the non-linearity disappears as the wave number increases. This is indicated by the joining of all curves at the right hand side of the graph.
The addition of fins changed considerably the airship's response to the Wg component of the turbulence. The peak that was present in the normal force response of the bare airship hull had disappeared and it was now the moment response curve that showed a peak. One can note from figure 25 an increase in \( C_{Na} \) due to the fins. The \( C_{Ma} \) is still positive but much lower than the value without fins. These changes are due to the presence of added vertical forces located at the aerodynamic center of the fins. These forces are basically oriented in the same direction as the local gust velocity \( Wg \), thus counteracting the opposite force created by the reduction in hull cross-sectional area of the afterbody. This increases the total normal force acting on the airship as well as counteracting the high contribution of the nose force to the pitching moment about the center of buoyancy. The crossing pattern of the responses of the bare hull and of the hull with fins (figure 25) is mostly due to the wave number's dependence of the phase angle between the hull and fin components of the force or moment. Section 6.3 as well as appendix D will cover that matter more deeply.

5.6 AIRSHIP HULL'S WAKE.

Measurement of the wake behind the model was performed along both the Y and Z axis. Figure 26 illustrates the wake measured by moving the probe horizontally. The sketch at the top of the figure shows the probe location on the axis of symmetry of the airship. As expected the wake dies out quickly as the location of the probe moves downstream. This is mostly caused by the high intensity turbulence in which the airship was located. The same measurements were performed moving the
probe vertically. Figure 27 shows similar results in the upper half of the graphs, however the wake from the lower part of the hull was affected by the presence of the struts supporting the model. The non-dimensionalized change of velocity of the wake is shown in figure 28. No change in the wake's profile can be detected over the small variation of Reynolds number involved in this experiment (about 30%). Considering the boundary of the wake to be defined as a 5% drop from the free stream velocity, the result indicates a wake of the same magnitude as the maximum diameter of the hull. This is not an unrealistic case for a full scale airship in atmospheric turbulence.
CHAPTER 6
THEORETICAL COMPARISON & DISCUSSION

6.1 SLENDER BODY ESTIMATION.

As mentioned in chapter 2, slender body theory is used extensively to predict airship response to atmospheric turbulence. Figure 1 shows three estimations from this theory, the first one is using the entire airship body and the others, truncated bodies to provide for a non-zero value of $C_{Na}$. The recommended location of the truncation was at 0.77% from the nose, this would normally be the forwardmost fin-body intersection point. Note that the location of that truncation defines a base area that is very critical in the estimation of $C_{Na}$ and $C_{Ma}$. One can also observe from the plots that truncation reduces the amplitude of the estimated peak, as well as moving its location to a higher wave number.

When compared with the experimental curve (figure 29), the estimation for the normal force response appears to agree reasonably well with the first few data points. After $\Omega = 0.5$ the theory overestimates the response to vertical gust. The estimated peak is, above all, much too high in magnitude and its predicted location is twice the wave number value obtained for the experimental results. At large values of $\Omega$, the mean value of the theoretical estimate tends to converge towards the experimental response curve. The only common factor about the pitching moment estimations and the experimental results seems to be the similarity in the shape of the curves. Over
the frequency range covered by our experiment, the estimations tend to be fairly constant in their double magnitude prediction of the experimental pitching moment response.

To conclude we do not know whether the difference between the simulation and the experimental results is because of the presence of the hull's boundary layer, or inadequacy of potential flow theory, or both.

6.2 DRAWBACKS OF THE SLENDER BODY THEORY.

Munk's slender body theory was first developed for axisymmetric bodies of fineness ratios of 6 and above. Modern non-rigid airships are more likely to have fineness ratios of the order of 4 or even less. Hence, the slenderness assumption of new generations of hulls is put into question. None the less, most if not all computer simulations of the turbulence response of modern airships uses this slender body theory.

The theory also assumes that the Wg component of the turbulence is constant to infinity in the Y-Z plane containing the hull cross-section normal to the mean velocity vector (see appendix C). This is possibly unnecessary if one uses an inclined wave of shearing motion as done by Etkin in appendix A. Taken into consideration is the fact that Wg varies not only along the X axis but also along the Y and Z axis. Using the extended theory of appendix A, we found a slight improvement over the previous estimations, but the predictions were still quite
different from the experimental results.

The boundary layer present in the real case is not included in our slender boundary theory estimation. This could account for some of the differences between the predictions and the experimental results.

Finally, the most significant problem with this theory is probably the weight carried by the truncation location when calculating the airship response using slender body theory. Even this truncation does not explain the shift in the peak location between experimental and theoretical results. The only rule of thumb that seems to exist is to truncate the body where the fins and hull intersect. This is quite arbitrary for such a dominant parameter.

6.3 FINS EFFECT ON RESPONSE.

The addition of fins changed drastically the shape of the turbulence response curve. Because of the rear location of the fins there is a lag in their gust response. The fins' reaction to the vertical turbulence component differ from the hull's reaction not only in magnitude but also by a phase angle $\phi$. This phase shift is a function of $\Omega$ as developed in appendix D. When plotted as a function of $\Omega$, as in figure 30, the result shows a same type of pattern as in figure 25 which is the experimental response of the airship with and without fins. Depending on the value of $\Omega$ the fins increase or decrease the total value of the force and moment. For some specific values of $\Omega$ the fins do not influence the magnitude of the response.
Since this theoretical development involves the slender body theory, the same drawbacks mentioned previously are still present.

6.4 FULL SCALE TURBULENCE REPRESENTATION.

The airship hull used in this experiment represents a 1:120 scale model of a 300 ft airship based on the Goodyear HLA shape (appendix B). The Reynolds number experience in the high speed run is 120 times smaller than that of the full scale model flying at 30 knots in atmospheric turbulence. From reference 1 figure 13.6 we see an example of the vertical component of atmospheric turbulence using the one-dimensional von Karman model. When we multiply the wave number by the length of the body one can make direct comparison as to where the airship response stands with respect to a typical atmospheric turbulence PSD. The normal force and pitching moment along with the vertical component PSD of the atmospheric turbulence are shown in figure 31. The uppermost graph illustrates two turbulence spectrum representing a typical high altitude atmospheric turbulence of \( L/\ell = 6.7 \) along with an earth boundary layer turbulence of \( L/\ell = 1.7 \). This permits us to observe that for this particular model length and velocity, the peak of the normal force response is at a wave number for which the high altitude atmospheric turbulence input is already at a low level. However the earth boundary layer can still have large enough turbulence magnitude at the peak response location to create rather noticeable forces and moments on the hull.
CHAPTER 7

CONCLUSION & RECOMMENDATIONS

This report provides a data base with which theoretically-based simulations can be compared. It was not our objective to develop a new theory, nor to improve the existing theories that simulate the turbulence response of airships. However, the slender body theory was extended to eliminate the assumption of constant Wg in the cross-sectional plane. Simulations using the extended theory, still showed discrepancies with the experimental results although they were closer than first estimates. The slender body theory is an ultraconservative tool when used to predict the response to turbulence of airships of \( \ell/d = 4 \). Because of their relative location in the bandwidth of atmospheric turbulence, the effects of the hull's response peaks on the airship survivability are probably not too critical. However due to the implications of turbulence in airship transportation, especially for HLA operating at low altitude, the accuracy with which their responses are simulated should be reevaluated. There is an obvious need for a more accurate theory to simulate turbulence behavior of conventional low-finess-ratio hulls. The presence of the fins induces major changes in the behavior of airships in turbulence. Considerable analysis should be dedicated to the size of the fins in the design process. When doing this analysis the designer should account for the magnitude of the wake created by the airship.
An exact potential flow analysis is possible using modern panel methods for a body alone or body-fin combinations. Such analysis carried out for inclined waves of shearing motion would yield to the correct potential-flow results, thus superceding the slender body theory. However, one should not expect the result to agree well with experiment because of the large boundary layer effect over the afterbody. In order to get accurate results, one would expect to have to include the boundary layer in the analysis. As a continuation of this experiment, an attempt should be made to suck in the flow on the aft part of the hull, reducing the width of the wake. This would permit association of the results with higher Reynolds numbers; closer to those of full scale airships. At the same time, an increase in the tunnel turbulence scale by using a larger wind tunnel would make the experiment more representative of natural turbulence. Using different type and size of fin, one can "flatten" the response curve. This, however, may well result in large structural stresses and complications for non-rigid airships. The best way of obtaining data from which parameters can be derived to evaluate the theory would be via full scale flight testing, would it not?
REFERENCES


### Table 1A - Parameter Comparison (Bare Hull)

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### Table 1B - Parameter for Hull with Fins

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APPENDIX A

THREE DIMENSIONAL THEORY

(unpublished theory by Dr. B. Etkin)

In this development we extend Munk's slender-body theory (Ref. 8) to the case of an axisymmetric slender body flying through three dimensional isotropic turbulence. We consider the axis of symmetry of the body to be the x axis of a body-fixed frame, and the body to be travelling in the x direction with steady speed \( V \) (Fig. A-1). The turbulence is assumed to be frozen - i.e. unchanging in a space-fixed frame of reference.

The analysis is formulated along the lines of Appendix B of Ref. 2. The ideas presented in that report are a prerequisite to this analysis. We begin by considering a single spectral component of the turbulence, described by

\[
\frac{d\mathbf{g}}{d\mathbf{g}} = \begin{bmatrix}
\frac{du_g}{d\mathbf{g}} \\
\frac{dv_g}{d\mathbf{g}} \\
\frac{dw_g}{d\mathbf{g}}
\end{bmatrix} = e^{-i\mathbf{\Omega} \cdot \mathbf{r}}
\begin{bmatrix}
dU \\
dV \\
dW
\end{bmatrix}
\]  

(A-1)

Here \( \mathbf{\Omega} \) is the wave number vector, \( \mathbf{r} \) is the position vector in space-fixed coordinates, and \( [dU \ dV \ dW] \) is the amplitude of the velocity in the wave. The fluid velocity and wave number vectors are perpendicular - i.e., the fluid motion is a shear wave. (the term "wave" is used even though there is no propagation involved, because of the sinusoidal structure of the velocity field).
Figure A-1 illustrates the situation, with choices of geometry such that

\[ r = iVt + r' \]  \hspace{1cm} (A-2)

We now consider the plane containing \( \Omega \) and \( ox \), as in Fig. A-2, and define the reference frame \( F' \) by the following sequence of rotations; begining with \( F \):

1. A rotation \( \phi \) about \( ox \), yielding \( oxy'z' \).
2. A rotation \( \theta \) about \( oy' \), yielding \( ox'y'z' \).

Figure A-2
It follows that vectors are transformed between $F$ and $F'$ according to the laws

$$v' = L v, \quad v = L^T v' \quad (A-3)$$

where

$$L = L_2 L_1$$

$$L_1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos \phi & \sin \phi \\ 0 & -\sin \phi & \cos \phi \end{bmatrix} \quad (A-4)$$

$$L_2 = \begin{bmatrix} \cos \theta & 0 & -\sin \theta \\ 0 & 1 & 0 \\ \sin \theta & 0 & \cos \theta \end{bmatrix}$$

To arrive at aerodynamic force, we consider the flow in an infinite slice of the inclined wave, as shown in Fig. A-2. This slice is fixed in space, and in view of its orientation, and the assumption of frozen turbulence, the velocity in it remote from the body is $\mathbf{u}$ constant. The essential idea of slender body theory is that the flow in the slice is treated as 2 dimensional, past a solid cylinder whose cross-section is the intersection of the slice with the body, Fig. A-3.

Figure A-3

$$\frac{d}{|\cos \theta|} \quad dw' = \frac{1}{g} \quad dv' = \frac{1}{g} \quad (A-5)$$
We assume the body is sufficiently slender that the section may be regarded as an ellipse, with principal axes as shown. The value \( d(x) \) is the diameter of the body at \( Q \). The velocity at every point of \( S \) remote from the body is equal that at the point \( Q \) when the body is absent, i.e.,

\[
\begin{bmatrix}
\frac{dU}{dV} \\
\frac{dV}{dW} \\
\frac{dW}{dV}
\end{bmatrix}
= \begin{bmatrix}
0 \\
\frac{dU}{dV} \\
\frac{dV}{dW}
\end{bmatrix}
\begin{bmatrix}
i\Omega_1(Vt+x) \\
e \\
i\Omega_1(vT+x)
\end{bmatrix}
\]  
(A-6)

The components of \( dg \) in the plane \( S \) are obtained from

\[
\begin{bmatrix}
\frac{dU}{dV} \\
\frac{dV}{dW} \\
\frac{dW}{dV}
\end{bmatrix}
= \begin{bmatrix}
\frac{dU}{dV} \\
\frac{dV}{dW} \\
\frac{dW}{dV}
\end{bmatrix}
\begin{bmatrix}
i\Omega_1(vT+x) \\
e \\
i\Omega_1(vT+x)
\end{bmatrix}
\]  
(A-7)

Associated with the velocities \( dV' \) and \( dW' \) are virtual masses \( dm_y' \) and \( dm_z' \) and corresponding fluid momenta given by (Ref. 11)

\[
\begin{align*}
\frac{dM_y'}{dV'} &= \rho dV'(\pi d^2/4\cos^2\theta)dx' = \rho dV'A(x)dx/|\cos\theta| \\
\frac{dM_z'}{dW'} &= \rho dW'(\pi d^2/4)dx' = \rho dW'A(x)|\cos\theta|dx
\end{align*}
\]  
(A-8)

where

\[
A(x) = \frac{\pi}{4} d^2(x) \quad \text{and} \quad dx' = dx|\cos\theta|
\]

These momenta can change as the body penetrates the slice by virtue of changing value of \( A(x) \):

\[
\frac{dA(x)}{dt} = -\frac{dA}{dx} V = -A'(x)V
\]  
(A-9)
Thus the body experiences reactive forces

\[
dF' = \rho V A'(x) \cdot \frac{dv'}{g} \cdot dx \cdot \cos \theta
\]

\[
dF' = \rho V A'(x) \cdot |\cos \theta| \cdot dw' \cdot dx
\]

By symmetry these act through the point Q.

To get the total force and moment on the body, (A-10) are first transformed into frame F and then integrated. In frame F we have

\[
\begin{bmatrix}
\frac{dF_y}{y} \\
\frac{dF_z}{z}
\end{bmatrix} = \frac{L}{T} \begin{bmatrix}
\frac{dF'_y}{y} \\
\frac{dF'_z}{z}
\end{bmatrix}
\]

(A-11)

and over the whole body

\[
F_z = \int_{x=-L}^{0} \frac{dF_z}{dx} \cdot dx
\]

(A-12)

In view of the isotropic nature of the turbulence, and the symmetry of the body, the mean-square values of \(F\) and \(F'\) must be equal, so we need to calculate only one of them.

When (A-7) is substituted into (A-10), (A-10) into (A-11), and (A-11) into (A-12) the result is

\[
F_z = \rho V e^{i\Omega t} \int_{-\ell}^{0} \left[ G_{zu} \cdot dU + G_{zv} \cdot dV + G_{zw} \cdot dW \right] \int_{-\ell}^{0} \frac{dA}{dx} \cdot e^{i\Omega x} \cdot dx
\]

(A-13)
Where

\[ G_{zu} = \sin \theta \sin \phi |\cos \theta| \]
\[ G_{zv} = \sin \phi \cos \theta \left( 1 - \cos^4 \theta \right) \]
\[ \frac{1}{|\cos \theta|} \left( \sin^2 \phi + \cos^4 \theta \right) \]

We denote the integral in \((A-13)\) by

\[ T(\Omega_1) = \int_{-\ell}^{\ell} \frac{dA}{dx} e^{i\Omega_1 x} \] \hspace{1cm} \((A-15)\)

and denote

\[ X_{zu}(\Omega) = \rho V T(\Omega_1) G_{zu}(\Omega), \text{ etc.} \]

so that

\[ F_z = \left[ X_{zu} dU + X_{zv} dV + X_{zw} dW \right] e^{i\Omega_1 V t} \] \hspace{1cm} \((A-16)\)

Equation \((A-16)\) is then in the same form as the equation following B.11 of Ref. 2.

The pitching moment about the nose is given by a similar formula, i.e.,

\[ M_y = \left[ X_{mu} dU + X_{mv} dV + X_{mw} dW \right] e^{i\Omega_1 V t} \]

where

\[ X_{mu}(\Omega) = \rho V T'(\Omega_1) G_{zu}(\Omega), \quad T'(\Omega_1) = \int_{-\ell}^{\ell} \frac{dA}{dx} e^{i\Omega_1 x} \] \hspace{1cm} \((A-17)\)
In order that the $G_{zu}$, etc., be expressed in terms of $\Omega$, we want the trigonometric functions in (A-14) to be expressed in terms of $\Omega_i$. The appropriate relations are derived from the pair of equations

$$[0 0]_T = L[\Omega_1 \Omega_2 \Omega_3]_T$$

and its transpose. From them we get

$$\sin \varphi = \Omega_2/(\Omega_2^2 + \Omega_3^2)^{1/2}$$

$$\cos \varphi = -\Omega_3/(\Omega_2^2 + \Omega_3^2)^{1/2}$$

(A-18)

$$\sin \theta = (\Omega_2 \sin \varphi - \Omega_3 \cos \varphi)/\Omega$$

$$\cos \theta = \Omega_1/\Omega$$

The foregoing relations permit the calculation of the three-dimensional "transfer functions" - the $X$'s - of (A-16) and (A-17), as functions of $\Omega$. The theory of Ref. 2, Appendix B then yields the 3-D spectral density of $F_z$ as

$$\Theta_{zz}(\Omega) = |X_{zu}|^2 \Theta_{uu} + |X_{zv}|^2 \Theta_{vv} + |X_{zw}|^2 \Theta_{ww}$$

$$+ 2\text{Re} \left[ X_{zu} X_{zv}^* \Theta_{uv} + X_{zv} X_{zw}^* \Theta_{vw} + X_{zw} X_{zu}^* \Theta_{wu} \right]$$

(A-19)

The $\Theta_{ij}$ in (A-19) are the 3-d spectrum functions of the turbulence, given by (2.17) of Ref. 2:

$$\Theta_{ij}(\Omega) = \frac{E(\Omega)}{4\pi^4} (\Omega^2 \delta_{ij} - \Omega_i \Omega_j)$$

(A-20)
Where $E(\Omega)$ is the energy density of the turbulence, given for the von Karman spectrum by

$$
E(\Omega) = \frac{55}{9\pi} \sigma_L^2 \frac{(aL\Omega)^4}{[1 + (aL\Omega)]^{17/6}} \quad (A-21)
$$

The one-dimensional spectrum is obtained from $\Theta$ by integration

$$
\phi_{zz}(\Omega_1) = \iint_{-\infty}^{\infty} \Theta_{zz}(\Omega_1, \Omega_2, \Omega_3) d\Omega_2 d\Omega_3 \quad (A-22)
$$

It is that last quantity that can be compared with the experiment reported herein.

Limitations of the Theory

The theory is approximate in two main respects. Firstly it is a potential-flow theory, and hence does not take account of boundary layer effects. This is known to be a major source of error at low wave number, since in the limit $\Omega_1 \to \infty$ it predicts zero lift on a closed body, whereas experiments yield finite lift.

Second the assumption that the sections of the body by the plane $S$ are ellipses is violated whenever the plane approaches the nose or the tail of the body, and this is the case for large angles $\theta$. A refinement of the theory could readily take this into account. However, the derivative $dA(x)/dt$ is then large, impairing the validity
of the 2-D cross-flow assumption. It seems reasonable, therefore, to
limit the integration in (A-22) to a finite range of \(\Omega_2\) and \(\Omega_3\) so that
\(|\theta|\) is always less than some fixed value. Reference to (A-18) shows
that if \(|\Omega_2|, |\Omega_3| < c|\Omega_1|\) then

\[ |\theta| \leq \tan^{-1}\sqrt{2c} \]

For \(\theta_{\text{max}} = 60^\circ, c = 1.22\), and this seems to be a reasonable value to
use. The theory then does not account for the effects of all wave
numbers outside the indicated domain. These are the spectral
components that may expected to contribute little to the lift and
moment on the body.

A further point to consider in comparing the above theory with
experiment is the spectrum of the incident turbulence. The energy
function \(E(\Omega)\) should of course represent the actual condition of the
experiment. Although the von Karman spectrum (A-21) is known to be
good for many realizations of isotropic turbulence, the spectrum
observed in the present wind tunnel experiment departs significantly
from the von Karman at low wave number. A corresponding disagreement
between the predicted and measured spectra of lift and moment should be
expected.
Most of the system specifications were taken from the design drawings from which the balance was built. However, the spring constants were directly measured from the machined parts during calibration procedures (Chap 4.2). The weight and inertia, along with the location of buoyancy center and center of gravity, were computed using a 60 point approximation of the mass distribution along the axis of symmetry (X-axis). Figure B-1 is a sketch of the locations of the hull's principal parameters.

FIGURE B-1 MODEL BALANCE DIMENSIONS
Table B-1 gives the major specifications of the airship balance system as well as some hull characteristics.

**TABLE B-1 AIRSHIP MODEL PARAMETERS**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>MODEL LENGTH:</td>
<td>30.0 in (76 cm)</td>
</tr>
<tr>
<td>MAXIMUM DIAMETER</td>
<td>7.5 in (19 cm)</td>
</tr>
<tr>
<td>MAXIMUM DIAMETER LOCATION:</td>
<td>12.0 in (30 cm)</td>
</tr>
<tr>
<td>FINENESS RATIO (l/d):</td>
<td>4.0</td>
</tr>
<tr>
<td>FRONT TRANSDUCER LOCATION:</td>
<td>6.25 in (16 cm)</td>
</tr>
<tr>
<td>REAR TRANSDUCER LOCATION:</td>
<td>21.75 in (55 cm)</td>
</tr>
<tr>
<td>FRONT BULKHEAD LOCATION:</td>
<td>6.00 in (15 cm)</td>
</tr>
<tr>
<td>REAR BULKHEAD LOCATION:</td>
<td>22.00 in (56 cm)</td>
</tr>
<tr>
<td>TOTAL DYNAMIC WEIGHT:</td>
<td>1.00 lbs (454 g)</td>
</tr>
<tr>
<td>INERTIA (Y axis):</td>
<td>55.00 lbs in (0.634 Kg m)</td>
</tr>
<tr>
<td>CENTER OF GRAVITY LOCATION:</td>
<td>13.77 in (35 cm)</td>
</tr>
<tr>
<td>BUOYANCY CENTER LOCATION:</td>
<td>13.56 in (34 cm)</td>
</tr>
<tr>
<td>ENVELOPE VOLUME:</td>
<td>856. in (0.014 m)</td>
</tr>
<tr>
<td>ENVELOPE SURFACE:</td>
<td>579. in (0.374 m)</td>
</tr>
<tr>
<td>FRONT SPRING CONSTANT:</td>
<td>715. lbs/in (1250 N/cm)</td>
</tr>
<tr>
<td>REAR SPRING CONSTANT:</td>
<td>695. lbs/in (1215 N/cm)</td>
</tr>
<tr>
<td>STYROFOAM SM DENSITY:</td>
<td>0.0011 lbs/in (0.030 g/cm)</td>
</tr>
</tbody>
</table>
Figure B-2 gives the shape of the airship envelope, as well as the interior clearance, in the form of radius in function of the position along the X-axis (X=0 at the nose). The scale on the X-axis was reduced for convenience and accuracy. The shaded area represents the styrofoam wall of the envelope as cut using hot nickel-chromium wire.
APPENDIX C
MUNK'S SLENDER BODY THEORY

The lift and moment on an airship hull flying through turbulence is generally approximated using Munk's slender body theory. The theory is based on the changes of momentum in a fixed plane perpendicular to the axis of symmetry of a moving body.

In that Z-Y plane containing the slice A(x) we assume constant turbulence velocity $W_g$ to infinity in the Z direction. No Y component of the velocity is considered.

The turbulence is assumed to be frozen and is defined by:

\[ dW_g = W_g e^{i\Omega_1(Ut+x)} \]  

(C-1)
Associated with dWg is a vertical momentum perturbation (caused by the body) of amount

\[ dM = \rho A(x) dWg \, dx \]  
\text{(C-2)}

Therefore the rate of change of momentum of the surrounding air is:

\[ d\dot{M}_z = dWg \frac{dA(x)}{dt} \, dx \]  
\text{(C-3)}

If one replaces \( \frac{dA}{dt} = \frac{dA}{dx} \frac{dx}{dt} = U \frac{dA}{dx} \), the reaction from the body to this change of momentum is then:

\[ dF_z = \rho UdWg \frac{dA(x)}{dx} \, dx \]  
\text{(C-4)}

And the force over the whole body of length \( l \) is:

\[ F_z = \rho U \int_{-l}^{0} dWg \frac{dA(x)}{dx} \, dx \]  
\text{(C-5)}

Replacing \( dWg \) by equation C-1 gives:

\[ F_z = \rho U W_0 e^{i\Omega l U t} \int_{-l}^{0} \frac{dA(x)}{dx} e^{i\Omega x} \, dx \]  
\text{(C-6)}
Similarly the pitching moment about the nose of the airship is given by:

\[ M_z = \rho U W g_0 e^{-\frac{i\Omega_1 U t}{\ell}} \int_{-\ell}^{0} \frac{xdA(x)}{dx} e^{i\Omega_1 x} dx \]  

(C-7)

If we define the expression for the integral as follows,

\[ T(\Omega_1) = \int_{-\ell}^{0} \frac{dA(x)}{dx} e^{i\Omega_1 x} dx \]  

(C-8a)

\[ T'(\Omega_1) = \int_{-\ell}^{0} x dA(x) e^{i\Omega_1 x} dx \]  

(C-8b)

then we can derive the expression for the lift and moment one-dimensional power spectral density (Appendix B, ref 2).

\[ \phi_{LL}(\Omega_1) = \rho^2 U^2 \phi_{WgWg}(\Omega_1) |T(\Omega_1)|^2 \]  

(C-9a)

\[ \phi_{MM}(\Omega_1) = \rho^2 U^2 \phi_{WgWg}(\Omega_1) |T'(\Omega_1)|^2 \]  

(C-9b)

\[ \phi_{WgWg} \text{ is generally approximated by the one-dimensional Von Karman spectrum.} \]

\[ \phi_{33}(\Omega_1) = \frac{\frac{2}{2\pi} \frac{1 + \frac{8}{3}(aL\Omega_1)^2}{[1 + (aL\Omega_1)^2]^{11/6}}}{a = 1.339} \]  

(C-10)
Directly from C-9 the transfer function for lift and moment can be obtained; \( T \) and \( T' \) being complex values.

\[
G_{NWg} = \rho U |T(\Omega_1)| \quad (C-11a)
\]

\[
G_{MWg} = \rho U |T'(\Omega_1)| \quad (C-11b)
\]

Let's now change the transfer function into their respective non-dimensional forms. By definition we get:

\[
G_{NWg} = \frac{N}{Wg} = \frac{C_N \frac{1}{2} \rho U^2 S}{\frac{\alpha U}{g}} \quad (C-12)
\]

Hence the non-dimensionalized normal force transfer function:

\[
G_{CN} = \rho U \frac{1}{\frac{1}{2} \rho U^2 S} |T(\Omega_1)| \quad (C-13)
\]

Which can be simplified to:

\[
G_{CN} = \frac{2}{S} |T(\Omega_1)| \quad (C-14a)
\]

Similarly the moment coefficient transfer function becomes:

\[
G_{CM} = \frac{2}{V} |T(\Omega_1)| \quad (C-14b)
\]
This appendix will derive basic equations that will predict qualitatively the changes on the airship's response due to the addition of the fins on the hull. This development will be using the simple slender body theory of appendix C in conjunction with a simplified lifting theory.

Let us assume that the force acting on the fins can be grouped in one unique vector \( N \) located at the 1/4 chord of the root of the fin. At the same time, for zero \( \alpha \) this force \( N_F \) is a function of the vertical gust component \( W_{g_F} \).

\[
N_F = k W_{g_F} \quad \text{(D-1)}
\]

\[
W_{g_F} = W_{g_o} e^{i\Omega (Ut - \ell_F)} \quad \text{(D-2)}
\]

Hence:

\[
N_F = kW_{g_o} e^{i\Omega \ell_F} e^{i\omega t} \quad \text{(D-3)}
\]
Which will be defined as:

\[ N_F = B(\Omega) e^{i\omega t} \]  

Similarly, from appendix C, the force acting on the airship hull can be expressed as:

\[ N_H = A(\Omega) e^{i\omega t} \]

The total normal force on the airship is then a simple vector summation of the two forces \( N_F \) and \( N_H \) as shown here.

\[ \vec{N} = \vec{N}_H + \vec{N}_F = [\vec{A}(\Omega) + \vec{B}(\Omega)] e^{i\omega t} \]

Given that:

\[ \vec{A}(\Omega) = |A| e^{i\phi_A} \]  

\[ \vec{B}(\Omega) = |B| e^{i\phi_B} \]
The magnitude of the force vector \( N \) is then:

\[
|N|^2 = |A|^2 + |B|^2 + 2|A||B|\cos(\phi_A - \phi_B) \quad (D-9)
\]

The phase angle of each force components is a function of the wave number and can be expressed by the following:

\[
\phi_B = -\Omega \ell_F \quad (D-10)
\]

\[
\phi_A = \phi[T(\Omega_1)] \quad (D-11)
\]

Where \( T(\Omega_1) \) is defined in equation C-8a of the previous appendix.

Since the magnitude of \( N \) depends on the phase angle between \( N_H \) and \( N_F \) which are functions of \( \Omega \), change of the turbulence wave number will generate variations in the amplitude of the turbulence response. This can be noticed in figure 25.
It is also possible to predict the values of $\Omega_s$ for which the presence of the fins will not affect either the normal force or the pitching moment response. Equating equation D-9 to the modulus square of the hull's force component results in the following relation.

$$\phi_A - \phi_B = \cos^{-1} \left( \frac{-|B|}{2|A|} \right)$$  \hspace{1cm} (D-13)

A similar development can be applied to the pitching moment response. The application of this qualitative estimation for both the normal force and pitching moment can be found in figure 30 plotted as phase angle versus wave number.
Let us define a dynamic transfer function $B$ that represents the two degree of freedom balance system. The system's inputs are the normal force and pitching moment created by the reaction of the hull to the gust. The outputs are the front and rear transducer voltages.

$$N(t) \quad M(t) \quad B \quad E_F(t) \quad E_R(t)$$

Let $X$ be the input force vector and $Y$ be the transducer voltage readings. The $B$ matrix is a 2x2.

$$X = [N \ M]^T \quad (E-1)$$

$$Y = [E_F \ E_R]^T \quad (E-2)$$

$$Y = B \ X \quad (E-3)$$

If, on the other hand, we are given the values of $E_F$ and $Er$ as in our case, then the vector $X$ can be found using the inverse of the matrix $B$.

$$X = B^{-1}Y \quad (E-4)$$
Since we are dealing mainly with the spectral components of the signals and forces, from reference 1 we can derive the equivalent expression for the spectral densities.

\[ \phi_x = B^*^{-1} \phi B^{-1} \]  

(E-5)

where:

\[ \phi_x = \begin{bmatrix} \phi_{NN} & \phi_{NM} \\ \phi_{MN} & \phi_{MM} \end{bmatrix} \]

\[ \phi_y = \begin{bmatrix} \phi_{E_fE_f} & \phi_{E_fE_r} \\ \phi_{E_fE_r^*} & \phi_{E_rE_r} \end{bmatrix} \]

In order to derive the components of \( B^{-1} \) one must consider the following mass-spring system with no damping. The only damping present in our experiment was aerodynamic, which was found to be negligible.

![Free body diagram](image)

The free body diagram:
the general equilibrium equations are:

\[ F_i + F_f + F_r = N \quad (E-6) \]
\[ M_i + aF_f - bF_r = M \quad (E-7) \]

the particular equations of inertia:

\[ F_i = \frac{m}{g_c} \ddot{Y} \quad (E-8) \]
\[ M_i = \frac{J}{g_c} \ddot{\theta} \quad (E-9) \]

The center of gravity displacement and rotation can be expressed as a function of front and rear transducers displacements.

\[ \ddot{Y} = \frac{a\ddot{Y}_f + b\ddot{Y}_r}{a + b} \quad (E-10) \]
\[ \ddot{\theta} = \frac{\ddot{Y}_f - \ddot{Y}_r}{a + b} \quad (E-11) \]

The particular equations for the spring force are given below, with \( K_f \) and \( K_r \) being front and rear spring constants.

\[ F_f = K_f Y_f \quad (E-12) \]
\[ F_r = K_r Y_r \quad (E-13) \]
Introducing E-10 and E-11 into E-8 and E-9 then inserting these into the equations E-6 and E-7 along with E-12 and E-13 leads to the following equations of motion.

\[
\frac{m}{g_c} \frac{a}{a+b} \ddot{Y}_f + \frac{m}{g_c} \frac{b}{a+b} \ddot{Y}_r + K_f Y_f + K_r Y_r = N \quad (E-14)
\]

\[
\frac{J}{g_c} \frac{1}{a+b} \ddot{Y}_f - \frac{J}{g_c} \frac{1}{a+b} \ddot{Y}_r + aK_f Y_f - bK_r Y_r = M \quad (E-15)
\]

These equations are a function of the front and rear displacements $Y_f$ and $Y_r$. Since we are dealing with the LVDT voltages as inputs, we must express the $Y$ values as the voltage variable $E$. The displacement of the transducer can be expressed by the ratio of the force-voltage constant, $C$, over the spring constant, $K_f$ and $K_r$.

\[
Y_f = \frac{C}{K_f} E_f \quad (E-16)
\]

\[
Y_r = \frac{C}{K_r} E_r \quad (E-17)
\]
Introducing the above expressions into equations (E-14) and (E-15) as well as changing from time to Laplace domain leads to equation (E-18). The B matrix can be obtained by inspection from this equation.

\[
\begin{bmatrix}
N \\
M
\end{bmatrix} = \begin{bmatrix}
\frac{m}{g_c} & \frac{b}{a+b} \frac{C}{K_f} & s^2 + C \\
\frac{J}{g_c} & \frac{1}{a+b} \frac{C}{K_f} & s^2 + aC
\end{bmatrix} \begin{bmatrix}
\frac{m}{g_c} & \frac{a}{a+b} \frac{C}{K_r} & s^2 + C \\
\frac{-J}{g_c} & \frac{1}{a+b} \frac{C}{K_r} & s^2 + bC
\end{bmatrix} \begin{bmatrix}
E_f \\
E_r
\end{bmatrix}
\]

(E-18)

For convenience let us define the matrix \(H\) as being a 2*2 inverse of the B matrix.

\[
H = B^{-1} = \begin{bmatrix}
h_{ff} & h_{fr} \\
h_{fr}^* & h_{rr}
\end{bmatrix}
\]

(E-19)

Similarly to equation E-5 we have:

\[
\phi_x = H^* \phi H^T
\]

(E-20)

Since the components of \(H\) are all real values (no damping) then \(H = H^H\) and equation E-20 can be written as:

\[
\begin{bmatrix}
\phi_{NN} & \phi_{NM} \\
\phi_{NM}^* & \phi_{MM}
\end{bmatrix} = \begin{bmatrix}
h_{ff} & h_{fr} \\
h_{fr}^* & h_{rr}
\end{bmatrix} \begin{bmatrix}
\phi_{EfE_f} & \phi_{EfE_r} \\
\phi_{ErE_f} & \phi_{ErE_r}
\end{bmatrix} \begin{bmatrix}
h_{ff} & h_{fr} \\
h_{fr}^* & h_{rr}
\end{bmatrix}
\]

(E-21)
Using the following conditions,

\[
\begin{align*}
\text{Im}(\phi_{EfEf}) &= \text{Im}(\phi_{ErEr}) = 0 \\
\text{Re}(\phi_{EfEr}) &= \text{Re}(\phi^*_{EfEr}) \\
\text{Im}(\phi_{EfEr}) &= -\text{Im}(\phi^*_{EfEr})
\end{align*}
\]  

(E-22)

we can express each component of the output matrix

\[
\begin{align*}
\phi_{NN} &= h^2_{ff} \phi_{EfEf} + 2h_{ff} h_{fr} \text{Re}(\phi_{EfEr}) + h^2_{fr} \phi_{ErEr} \\
\phi_{NM} &= h^2_{fr} \phi_{EfEf} + 2h_{fr} h_{rr} \text{Re}(\phi_{EfEr}) + h^2_{rr} \phi_{ErEr} \\
\text{Re}(\phi_{NM}) &= h_{ff} h_{rf} \phi_{EfEf} + (h_{fr} h_{rf} + h_{ff} h_{rr}) \text{Re}(\phi_{EfEr}) + h_{fr} h_{rr} \phi_{ErEr} \\
\text{Im}(\phi_{NM}) &= (h_{ff} h_{rr} - h_{fr} h_{rf}) \text{Im}(\phi_{EfEr})
\end{align*}
\]

(E-23a) (E-23b) (E-23c) (E-23d)

where the values of \( H \) are taken from E-18.

\[
\begin{align*}
h_{ff} &= \frac{m}{g_c} \frac{b}{a+b} \frac{C}{K_f} S^2 + C \\
h_{fr} &= \frac{m}{g_c} \frac{a}{a+b} \frac{C}{K_r} S^2 + C \\
h_{rf} &= \frac{J}{g_c} \frac{1}{a+b} \frac{C}{K_f} S^2 + aC \\
h_{rr} &= \frac{J}{g_c} \frac{1}{a+b} \frac{C}{K_r} S^2 + bC
\end{align*}
\]

(E-24a) (E-24b) (E-24c) (E-24d)
The dynamic response transfer functions are derived from equation E-18 using Cramer's rule. These four dynamic responses are plotted in figure 7 for the parameters of appendix B.

\[
\begin{align*}
\frac{E_f}{N} &= \frac{h_{rr}}{\Delta} \\
\frac{E_r}{N} &= \frac{h_{fr}}{\Delta} \\
\frac{E_r}{M} &= \frac{h_{rf}}{\Delta} \\
\frac{E_r}{M} &= \frac{h_{ff}}{\Delta}
\end{align*}
\]

(E-25)

Where \( \Delta = h_{ff}h_{rr} - h_{fr}h_{rf} \).

By equating the determinant to zero one can easily compute the natural frequencies of the 2 degree of freedom balance system. Using appendix B values we get the following natural frequencies.

\[f_{n1} = 110 \text{ Hz}, \quad f_{n2} = 115 \text{ Hz}\]  

(E-26)
Fig. 3 Hot Wire Anemometer Set-up
FIGURE 4a  BARE HULL.

FIGURE 4b  HULL WITH FINS.
FIGURE 5  FORCE AND MOMENT BALANCE SYSTEM.
FIGURE 6  AIRSHIP BALANCE COMPONENTS.
DYNAMIC RESPONSE OF AIRSHIP BALANCE

FIGURE 7
Outputs on X-Y Plotter and Manual Interfacing with P.E. 3200 System

**Fig. 8 Schematic of LVDT Data Acquisition System**
FIGURE 9  HOT WIRE CALIBRATION APPARATUS.
CALIBRATION OF HOT WIRE X-PROBE

○ JET SPEED = 64.7 FPS
× JET SPEED = 83.5 FPS
\( \alpha = -20^\circ, -15^\circ, -10^\circ, \ldots +20^\circ \)

FORWARD VELOCITY COMPONENT (FPS)

VERTICAL VELOCITY COMPONENT (FPS)

FIGURE 10
CALIBRATION OF LVDT TRANSUCERS

RESULTS: FRONT CHANNEL = 0.1510 LBS/VOLT
REAR CHANNEL = 0.1510 LBS/VOLT

FIGURE 11
INPUT TURBULENCE POWER SPECTRA

- TUNNEL REYNOLDS NUMBER = 1051000
- TUNNEL REYNOLDS NUMBER = 1344000

Smooth $\Phi_\infty \propto \Omega$

- $\Phi_\infty \propto \Omega$ Model IN
  - $\Phi_\infty \propto \Omega$ Model IN
  - $\Phi_\infty \propto \Omega$ Model OUT

$\Phi_\infty \propto \Omega$

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FIGURE 13
MEAN $C_n$ & $C_m$ vs ANGLE OF ATTACK

BARE HULL - NO FINS

- TUNNEL REYNOLDS NUMBER = $1.04 \times 10^6$
- TUNNEL REYNOLDS NUMBER = $1.34 \times 10^6$

PITCHING MOMENT COEF.
ABOUT BUOYANCY CENTER (0, 0.45, 0)

$C_n \alpha_2 = 0.792 \text{ RAD}^{-1}$
$C_m \alpha_2 = 0.827 \text{ RAD}^{-1}$

NORMAL FORCE COEF.

$C_n \alpha_2 = 0.320 \text{ RAD}^{-1}$
$C_m \alpha_2 = 0.994 \text{ RAD}^{-1}$

FIGURE 14
MEAN $C_n$ & $C_m$ vs ANGLE OF ATTACK

HULL WITH FINS (NACA 0012)

- TUNNEL REYNOLDS NUMBER = $1.06 \times 10^6$
- TUNNEL REYNOLDS NUMBER = $1.35 \times 10^6$

$C_n \alpha = 0.274 \text{ RAD}^{-1}$
$C_m \alpha = 0.309 \text{ RAD}^{-1}$

$C_n \alpha = 0.782 \text{ RAD}^{-1}$
$C_m \alpha = 0.868 \text{ RAD}^{-1}$

FIGURE 15
MEASURED SPECTRA OF AIRSHIP BALANCE

RUN 2  BARE HULL  $\phi = 0.0$ DEG  $U = 65.6$ FPS  7/05/83

![Graphs of measured spectra with various scales and frequency ranges.](image)

FIGURE 17
UNCORRECTED OUTPUT-INPUT RATIOS

RUNS 144 15  BARE HULL - NO FINS  26/04/83

- Re = 1.02 x 10^6  CC = 12. deg
- Re = 1.34 x 10^6  CC = 12. deg

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FIGURE 18
UNCORRECTED OUTPUT-INPUT RATIOS

RUNS 22 & 23  HULL WITH FINS (NACA 0012)  3/08/83

- Re = 1.37 x 10^6  CC = 0. DEG
- Re = 1.37 x 10^6  CC = 8. DEG

FIGURE 19
UNCORRECTED SYSTEM GUST RESPONSE

RUNS 2 & 3  BARE HULL - INCLUDING DYNAMIC T. F.  26/04/83

- Re = 1.04 x 10^6  θθ = 0. DEG
- Re = 1.34 x 10^6  θθ = 0. DEG

FIGURE 20
SMOOTHED SYSTEM GUST RESPONSE

RUNS 2 & 3  BARE HULL - CONSTANT CC  26/04/83

- Re = 1.04 x 10^6  CC = 0.0 DEG
- Re = 1.34 x 10^6  CC = 0.0 DEG

Figure 21
SMOOTHED SYSTEM GUST RESPONSE

RUNS 11, 13 & 15  BARE HULL - CONSTANT SPEED  26/04/83

- $Re = 1.33 \times 10^6$  $\alpha = 4.0$  DEG
- $Re = 1.34 \times 10^6$  $\alpha = 8.0$  DEG
- $Re = 1.34 \times 10^6$  $\alpha = 12.0$  DEG

FIGURE 22
SMOOTHED SYSTEM GUST RESPONSE

RUNS 21 & 22  HULL WITH FINS - CONSTANT \( \alpha \)  3/08/83

- \( Re = 1.04 \times 10^6 \)  \( \alpha = 0 \) deg
- \( Re = 1.37 \times 10^6 \)  \( \alpha = 0 \) deg

\[ \frac{C_m}{C_n} \]

\[ \Omega \text{ (ft}^{-1}) \]

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FIGURE 23
SMOOTHED SYSTEM GUST RESPONSE

RUNS 22 & 23  HULL WITH FINS - CONSTANT SPEED  3/08/83

- Re = 1.37 x 10^6  \alpha = 0.  deg
- Re = 1.37 x 10^6  \alpha = 8.  deg

The graph shows the response of the system under different conditions.

FIGURE 24
SYSTEM GUST RESPONSE COMPARISON

RUNS 3 & 22 BARE HULL & HULL WITH FINS

- $Re=1.34 \times 10^6$ $\phi_{\alpha} = 0.\ deg$ BARE HULL
- $Re=1.37 \times 10^6$ $\phi_{\alpha} = 0.\ deg$ HULL WITH FINS (NACA 0012)

Figure 25

Mario Lagrange 30 June 1983
VERT. WAKE BEHIND AIRSHIP HULL

![Diagram of airship hull wake with graph showing probe position vs. velocity for low and high Reynolds numbers.]

LOW REYNOLDS NUMBER

HIGH REYNOLDS NUMBER

VELOCITY U (FPS)

FIGURE 27
FIGURE 28
THEORETICAL PHASE ANGLE vs $\Omega$

PHASE ANGLE BETWEEN THE HULL AND THE FINS CONTRIBUTIONS TO THE TOTAL FORCE AND MOMENT

\[ N_H \quad N \quad N_F \]

\[ \phi_{N_H - N_F} \]

\[ \phi_{N_H - N_F} \]

\[ \phi_{N_H - N_F} \]

\[ \Omega \text{ (FT}^{-1}) \]

FIGURE 29
FULL SCALE TURBULENCE COMPARISON

RUN NO 3  MODEL SCALE = 1:120  26/04/83

\[ \omega \text{ Re} = 1.34 \times 10^6 \quad \theta = 0 \quad \text{deg} \quad \text{BARE HULL} \]

- HIGH ALTITUDE V.K. TURB. \( L/\lambda = 6.7 \)
- LOW ALTITUDE V.K. TURB. \( L/\lambda = 1.7 \)

\[ \frac{2\Omega C_{D,0}^2}{\sigma C_0} \]

\[ \frac{C_{Ch} C_{Nh}}{C_{Ch} C_{Nh}}^{1/2} \]

\[ \frac{C_{Ch} C_{Nh}}{C_{Ch} C_{Nh}}^{1/2} \]

\[ \Omega \]

\[ 10^3 \quad 10^2 \quad 10^1 \quad 1 \quad 10^1 \quad 10^4 \]

FIGURE 31
The aerodynamic forces acting on an airship placed in a turbulence flow field inside the UTIAS boundary-layer wind tunnel were measured. This provided a data base upon which theoretical simulation of the airship's response to turbulence could be evaluated. The slender body theory, as used in this report, has failed to predict the behavior of a bare airship hull in turbulence. An extended three-dimensional slender body theory is also included. Although slightly better predictions were obtained, the extended theory is still far from providing accurate simulation. In general the slender body theory is an extremely conservative means of evaluating modern airship behavior in atmospheric turbulence. The method of simulation should be re-evaluated.

The addition of fins on the hull significantly changed the shape of the response curves. This variation is attributed to the wave number dependency of the fins and hull force vector which creates a variable phase angle between the two.

The enormous wake of the hull was measured and was found to be invariant with respect to Reynolds number for the limited range of velocity used in this experiment. The nonlinearity of the aerodynamic forces was found to be negligible over the range of angles of attack used in this experiment.