PSEUDO-STATIONARY OBLIQUE-SHOCK-WAVE REFLECTIONS
IN LOW GAMMA GASES - ISOBUTANE AND SULPHUR HEXAFLUORIDE

by

J. T. Urbanowicz

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Summary

Nonstationary oblique-shock-wave reflections in sulphur hexafluoride and isobutane were investigated experimentally and numerically. Over 100 experiments were conducted in the UTIAS 10 x 18cm Hypervelocity Shock Tube in order to investigate the RR-MR transition boundary and the features of terminal-double-Mach reflection. Five types of shock-wave reflections, i.e., regular (RR), single-Mach (SMR), complex-Mach (CMR), double-Mach (DMR) and terminal-double-Mach (TDMR) reflections were observed. These were studied with infinite-fringe interferograms and shadowgraphs using a 23-cm dia field of view Mach-Zehnder interferometer.

The analytical transition boundaries were established up to an initial shock-wave Mach number $M_s = 10$. The experimental results agree well with the predicted boundaries for the gases in vibrational equilibrium except for the MR-RR boundary where Mach reflections were observed up to 8 degrees above the detachment criterion line.

Strong dependence of the initial test pressure on the first triple point trajectory angle $\chi$ was noted and investigated analytically. The physical model taking into account the viscous boundary layer is proposed. According to this model there are several RR-MR boundaries corresponding to different initial pressures $p_0$, and the detachment criterion is replaced with the mechanical-equilibrium criterion. A procedure for predicting the MR-RR boundary for various $p_0$ is presented. The existing first and second triple-point trajectory predictions are discussed and compared with the experimental results. The influence of gamma on the oblique-shock-wave reflection process and the viscous boundary-layer effect are discussed.

The instability of incident shock waves and the nonuniformity of the regions behind them were observed in the low gamma gases. An additional study is required for a better understanding of these phenomena.
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NOTATION

\begin{itemize}
  \item \textbf{a} \hspace{1cm} \text{speed of sound in a gas}
  \item \textbf{CMR} \hspace{1cm} \text{complex-Mach reflection}
  \item \textbf{DMR} \hspace{1cm} \text{Double-Mach reflection}
  \item \textbf{f(n)} \hspace{1cm} \text{boundary-layer velocity-distribution function}
  \item \textbf{I} \hspace{1cm} \text{incident shock wave}
  \item \textbf{k} \hspace{1cm} \text{thermal conductivity}
  \item \textbf{kPa} \hspace{1cm} \text{kilopascal}
  \item \textbf{K} \hspace{1cm} \text{kink, degrees Kelvin}
  \item \textbf{l} \hspace{1cm} \text{length}
  \item \textbf{L} \hspace{1cm} \text{horizontal distance from wedge corner}
  \item \textbf{M} \hspace{1cm} \text{Mach stem}
  \item \textbf{M_i} \hspace{1cm} \text{Mach number in region (i)}
  \item \textbf{MR} \hspace{1cm} \text{Mach reflection}
  \item \textbf{p_i} \hspace{1cm} \text{pressure in region (i)}
  \item \textbf{P} \hspace{1cm} \text{reflection point in regular reflection (RR)}
  \item \textbf{Pr} \hspace{1cm} \text{Prandtl number}
  \item \textbf{r(n)} \hspace{1cm} \text{boundary-layer temperature-distribution function for adiabatic wall}
  \item \textbf{R} \hspace{1cm} \text{reflected shock wave}
  \item \textbf{R'} \hspace{1cm} \text{second reflected shock wave in double-Mach reflection}
  \item \textbf{s(n)} \hspace{1cm} \text{boundary-layer temperature-distribution function for nonadiabatic wall}
  \item \textbf{S} \hspace{1cm} \text{slipstream or contact surface}
  \item \textbf{SMR} \hspace{1cm} \text{single-Mach reflection}
  \item \textbf{t} \hspace{1cm} \text{time}
  \item \textbf{T} \hspace{1cm} \text{triple point}
  \item \textbf{T'} \hspace{1cm} \text{second triple point in double-Mach reflection}
\end{itemize}
$T_i$  temperature in region (i)
$u$  flow velocity in the x-direction
$U_i$  flow speed in region (i)
$v$  flow velocity in the y-direction
$\AA$  Angstrom
$\gamma$  specific heat ratio
$\delta$  angle between the incident and reflected shock waves
$\delta_b$  boundary-layer thickness based on 99% freestream velocity
$\varepsilon^*$  boundary-layer displacement thickness
$n$  non-dimensional boundary-layer coordinate
$\theta_{bl}$  boundary-layer flow entry angle
$\theta_i$  flow-deflection angle through the shock wave when entering region (i)
$\theta_m$  maximum possible flow-deflection angle through a shock wave
$\theta_s$  flow-deflection angle through a shock wave which results in sonic flow behind shock
$\theta_v$  characteristic vibrational temperature
$\lambda$  wavelength of light
$\mu$  absolute viscosity
$\nu$  kinematic viscosity
$\rho$  density
$\tau$  shear stress, vibrational relaxation time
$\phi_i$  incidence angle between the flow and shock wave in region (i)
$\chi$  triple-point trajectory angle
$\chi'$  second triple-point trajectory angle
$\psi$  stream function
$\phi',\psi$  reflected-shock-wave angle in DMR and RR
$\Omega$  boundary-layer flow entry angle at nominal distance $x = 1 \text{ mm}$
1 INTRODUCTION

Shock waves appear when the particles in a fluid behind a moving wave flow with a velocity greater than zero. The thickness of the shock front is of the order of a few molecular mean free paths. The occurrence of shock waves is found in processes that generate abrupt compression fronts such as explosions, electrical discharges and supersonic flight. The nearly discontinuous changes in fluid velocity, temperature and pressure across the shock front along with the nonlinearity of the equations of motion create a challenging task for the practical applications of fluid mechanics.

Numerical solutions to the Euler partial differential equations are available [1]. However, solutions to the complex Navier-Stokes equations for the viscous case poses several difficulties in computational analysis. Moreover, the physical processes of such complex phenomena as the influence of the boundary-layer on oblique-shock-wave reflections is up to date not thoroughly known. Therefore, experimental results are necessary both for confirming analytical and numerical data as well as for the advancements in understanding the physics of the phenomena. In the present work a two-dimensional wedge model was mounted on a shock-tube wall to investigate the oblique-shock-wave reflection process. The incident shock wave is generated by breaking a diaphragm. When a planar moving shock wave encounters a sharp wedge the incident shock wave is reflected and the induced nonstationary flow is deflected by the compressive corner.

The study of oblique-shock-wave reflections has been carried out for more than a century by many experimentalists and theoreticians. Although the phenomenon of regular and single-Mach reflections were first observed by the distinguished physicist, Ernst Mach [3], back in 1878, it was not until the early 1940's that von Neumann [4,5] reinitiated the investigation of this problem and explored it systematically.


Finally, Lee and Glass [25] analytically predicted the existence of terminal-double-Mach reflection in low gamma gases and Iikui et al [44] and Hu [45] observed it experimentally. With these five types of reflection being identified, it became necessary to establish their transition criteria and delineate their regions on a shock Mach number-wedge angle ($M_s - \theta_w$) transition map. When a planar shock wave collides with a sharp compressive corner the type of reflection depends on the wedge angle and the incident shock Mach number $M_s$. For an actual case of a viscous flow of an imperfect gas where the excitation of internal degrees of freedom takes place, the initial pressure $p_0$, initial temperature $T_0$ and even the distance of the shock wave from the leading edge are also important (see chapter 2.3).

An analytical transition criterion for the termination of RR in a two dimensional, pseudo-stationary, inviscid gas flow was suggested by
von Neumann [4,5] and is referred to as the "detachment" criterion. Smith [7] found in his experimental results that RR persisted beyond the limit of the detachment criterion and MR did not occur immediately when the theoretical RR limit was exceeded. Bleakney and his students [6,8,9] further studied the disagreement found by Smith, but no fruitful explanation was obtained. The persistence of RR in the MR region was called the "von Neumann paradox".

Henderson and Lozzi [10] investigated the transition problem from RR to MR experimentally, both in a wind tunnel and in a shock tube. They proposed an alternative transition criterion which is referred to as the "mechanical equilibrium" criterion. This criterion was confirmed experimentally in a wind tunnel i.e, the stationary case, but increased the disagreement in the nonstationary case.

Hornung and Kychakoff [12] suggested another criterion called the "sonic" criterion, which in practice yields almost an identical transition boundary prediction as the detachment criterion. Hornung [23] proposed the explanation for the discrepancy in a wind tunnel and shock tube transition boundary data based on the unavailability of a length scale in a nonstationary supersonic flow.

In the present work another explanation based solely on the viscous boundary layer effect is proposed. When RR terminates, four types of Mach reflection can occur in nonstationary flows, and they are SMR, CMR, DMR and TDMR. White [8] investigated the reflection pattern of CMR and DMR in addition to SMR, and suggested a termination criterion for SMR.

Henderson and Lozzi [10] proposed the criteria for termination of SMR and CMR. However, their proposal was neither analytically nor experimentally substantiated. The criteria for transition from SMR to CMR and from CMR to DMR, which were initiated by Law and Glass [15], were established by Ben-Dor [11] and Ben-Dor and Glass [13,14]. An additional criterion between SMR and CMR was later appended by Shirouzu and Glass [16].

Lee and Glass [25] found that the TDMR reflection pattern occurs when \( \chi' = 0 \), for adequately low-gamma gases. Glass [52] mapped the TDMR to DMR transition boundary lines for several values of gamma. This reflection pattern was confirmed by Hu [45] and Ikui et al [44] who referred to it as a pseudo-regular reflection. This type of reflection was investigated experimentally and analytically in the present work. Real-gas effects in shock waves due to the excitation of the internal degrees of freedom (vibration, dissociation, electronic excitation and ionization) have been studied by some researchers. Mach reflections in perfect argon, nitrogen and oxygen, vibrationally excited air and vibrationally excited carbon dioxide were investigated at initial pressures of 12 torr (1.6 kPa) by Gvozdeva et al [21]. They found that while the incident shock velocity increases, the flow deflection angle decreases, and under certain conditions, becomes negative.
Naboko et al [22] investigated the flow parameters behind moving shock waves in relaxing gases. Their results revealed that the vibration of carbon dioxide molecules does not get completely excited for $4.5 < M_S < 11$; nitrogen is not vibrationally excited and oxygen molecules are in equilibrium for $3.7 < M_S < 7.5$.

Hornung et al [23] studied the transition to Mach reflection in steady and pseudo-stationary flow with or without relaxation for dissociating nitrogen and carbon dioxide. Besides real gas effects, they explained that the von Neuman paradox is due to the displacement effect of the boundary layer on the wedge surface in pseudo-stationary flow.

Ben-Dor [11] and Ben-Dor and Glass [13,14] analyzed the domains and boundaries of nonstationary oblique-shock-wave reflections in dissociating nitrogen and ionizing argon under equilibrium conditions. They showed that real-gas effects play an important role in the size of the regions and their boundaries. A wide range of Mach number and wedge angle were covered in their shock-tube experiments. From the comparison between their experiments and calculations in the transition boundary maps they erroneously suggested that nitrogen should be considered as an equilibrium gas and correctly treated argon as a frozen gas in the range of their experiments.

Shirouzu and Glass [16] maintained that, although the perfect carbon dioxide transition boundary map agrees best with the experimental results of Ando [24], the agreement is accidental and the perfect-gas model is fictitious. In their study they showed that CO$_2$ should be treated as an equilibrium gas. Lee and Glass [25] carried out an analytical and numerical study of pseudo-stationary oblique-shock-wave reflection in air and established the transition boundaries up to Mach number 20 for both perfect and imperfect air. No crossing of transition lines were found, and hence they removed the conjecture of possible triple Mach reflection.

Deschambault [26] investigated the transition boundaries in air and in nitrogen. He also studied the pressure histories on and above a specially constructed 40° wedge to obtain more insight into the pressure and density distribution field. In the range of Ben-Dor's and Deschambault's experiments, Lee and Glass [25] and Shirouzu and Glass [16] reported that air and nitrogen are frozen at lower Mach numbers ($M_S < 6$) and tend towards equilibrium at higher Mach numbers. Hu's experiments [45] in SF$_6$ support the equilibrium gas model. Real-gas effects play an important role in the size of the regions as the analytical imperfect boundary lines are different from the perfect-gas boundary lines. They play a deciding role in the case of TDMR which occur only if gamma is sufficiently low.

Therefore, an examination of the TDMR phenomenon requires a polyatomic gas with many internal degrees of freedom that are easily excited. In the present study SF$_6$ with 15 vibrational degrees of freedom was initially chosen as a test gas. SF$_6$ has a specific heat ratio of $\gamma = 1.094$ at room temperature and is non-toxic. To investigate the role of gamma on the RR-MR transition boundary and to improve the stability of the
incident shock waves isobutane was introduced as a test gas. Isobutane, \( \text{CH(CH}_3\text{)}_3 \) has a lower molecular weight - 58.12, higher refractive index \( n = 1.001286 \), 36 vibrational degrees of freedom and significantly lower gamma at higher \( M_s \). However, it is flammable and explosive when mixed with air, but non-toxic.

An examination of the relaxation lengths in \( \text{SF}_6 \) and isobutane led to the assumption that both gases can be treated as in vibrational equilibrium. The appropriateness of this model was confirmed by the agreement of the transition boundaries with the experimental results. The present work is a continuation of a program initiated by Prof. I. I. Glass of studying oblique shock-wave reflections at the University of Toronto, Institute for Aerospace Studies (UTIAS) by Molder [27], Weynants [28], Law and Glass [15], Ben-Dor [11], Ben-Dor and Glass [13,14,29], Ando [24], Ando and Glass [30], Lee and Glass [25], Shirouzu and Glass [16], Hu and Glass [58], Hu and Shirouzu [31], Deschambault [26], Deschambault and Glass [32], Hu [45], Wheeler [43] and Glass [52]. The objectives of the present work were as follows:

(1) To examine the features of TDMR reflection and predict the transition boundary between DMR and TDMR.

(2) To investigate the persistence of MR above the detachment criterion line at higher \( M_s \) as distinct from low \( M_s \) where RR persists below the detachment line, and propose an explanation of this phenomenon.

(3) To study the phenomena that occur in low-gamma gases, such as the instabilities of shock-wave fronts and the nonuniformity of the regions behind them.

To achieve these objectives over 100 experiments were performed in the 10x18 cm UTIAS Hypervelocity Shock Tube in sulfur hexafluoride and isobutane. Experiments were performed at initial pressures ranging from 2.8 to 256 torr at an incident shock wave Mach number of 1.58 < \( M_s < 9.01 \), over a series of sharp steel wedges of \( 10^\circ < \theta_w < 45^\circ \). Laser interferograms and shadowgraphs were used to record the reflection process and were obtained with a 23-cm diameter field-of-view Mach-Zehnder interferometer equipped with a Q-switched ruby laser providing exposure times of 15 ns, fast enough to freeze the fastest shock.

2 DESCRIPTION OF OBLIQUE-SHOCK-WAVE REFLECTION.

2.1 Steady Flow

Oblique shock-wave reflections in a steady, inviscid flow can be simulated in a wind tunnel where the shock wave is reflected off a symmetry plane using two identical wedges (Fig. 1). For regular reflection (Fig. 1a) the incident shock I encountering wedge \( \theta_w \) deflects the flow and a second shock \( R \) is needed to deflect it back by \(-\theta_w \) so the flow is parallel to the surface. This situation can be represented in a \((\theta,p)\)-plane. In Mach
reflection (Fig. 1b) an additional shock, S, is necessary to alter the flow so that it can negotiate the wedge surface by turning subsonically. Regions 3 and 4 are separated by the slipstream V, across which entropy, density, temperature and flow velocity are discontinuous, but the pressure and the flow directions are continuous. In this case no boundary layer influences the reflection process in the vicinity of the confluence point P. However, when only one wedge is mounted in a wind tunnel wall a boundary layer exists. The algebraic Euler equations can be used to obtain the flow properties.

2.2 Nonstationary Flow

Nonstationary flow is usually investigated in a shock tube, which consists of a high pressure driver section separated from a low pressure channel by a diaphragm. A diaphragm pressure ratio greater than unity is required to generate incident shock waves. Experimentalists are limited not only by the maximum allowable pressure in a driver section, but the bursting characteristics of diaphragms, and by the high test section pressure p0, which if too high can cause burning of the test section windows [36,37]. Owing to the foregoing, use is made of lower initial test section pressures, thereby introducing significant viscous boundary-layer effects.

2.3 Pseudo-Stationary Oblique-Shock-Wave Reflection

When a planar incident shock wave encounters a sharp wedge in a shock tube, it moves with a constant velocity along the wedge surface. The entire reflection problem can be considered pseudo-stationary in a frame of reference attached to the confluence point of the shock wave. Hence, instead of three independent variables x, y and t, the phenomenon becomes describable in terms of x/t and y/t and the flow problem is self-similar [19]. The variables x and y can be measured relative to any point moving with constant velocity with respect to the wedge corner. In such a pseudo-stationary self-similar space the "streamlines" in the regions of uniform flow converge into a single "sink" point. Therefore, in the vicinity of the confluence point we can calculate the flow properties using the Euler equations in the inviscid case, or the Navier-Stokes equations for viscous flow. To calculate the flow properties outside this point we need to solve the partial differential equations for the whole flow field.

The concept of pseudo-stationary flow (which has been verified experimentally, see Refs [8,49]) proved to be very useful and accurate for predicting flow properties near a confluence point (point P in Fig. 2a and points T and T' in Fig. 5). However, we must be aware that:

- in a viscous flow the ratio of actual thickness of the boundary layer to a characteristic length such as the Mach stem changes along the wedge so the viscous effects are time-dependent

- if real-gas effects are present the ratio of the relaxation length to a characteristic length may vary with time
the bow shock emanating from the wedge corner influences the flow properties near the confluence point, for example the triple-point trajectory.

2.4 Types of Oblique-Shock-Wave Reflections

The concepts and definitions of regular, single-Mach, complex-Mach and double-Mach reflection are the same as have been used by previous authors. An introduction of a terminal-double-Mach reflection as a separate type of reflection is justified due to its special features being neither the continuation of double-Mach nor regular reflection. The concepts, description and classification criteria of these five types of reflections are stated below.

2.4.1 Regular Reflection (RR).

The regular reflection shock-wave configuration as shown in Fig. 2a denotes the state ahead of and behind the incident shock wave I by (0) and (1), respectively, and the state behind the reflected shock wave R by (2). The frame of reference is attached to the two shock confluence point P which is moving parallel to the surface of the wedge at an angle $\theta_w$, with a constant velocity $U_0 = U_s \csc \phi_0$ or a Mach number $M_0 = M_s \csc \phi_0$ where $\phi_0$ is the incident angle between the flow $U_0$ and the incident shock wave I which is given as

$$\phi_0 = 90^\circ - \theta_w \tag{2.1}$$

When the flow $U_0$ passes through the incident shock wave I, it is deflected towards the wedge surface by an angle $\theta_1$ from its original direction. The flow has a new value $U_1$ and the thermodynamic properties are changed. This supersonic flow $U_1$ is redeflected by the reflected shock wave R through an angle $\theta_2 = \theta_1$, so that the flow $U_2$ is now parallel to the wedge surface, thus satisfying the required boundary condition.

With the frame of reference attached to the reflection point P, the flow configuration becomes stationary and two-shock theory may be used to find the flow properties of the regions around the reflection point P. The method of calculation is discussed in Appendix A.

If the flow $U_2$ behind the reflected shock R is sonic or subsonic, the corner signal can reach the point P and affect the reflected shock R. Consequently, R can be curved near point P (Fig. 2b). Since the velocity with which the point P moves along the wedge surface is very high for a small incident wave angle $\phi_0$, the flow $U_2$ is generally supersonic relative to point P. Therefore a corner signal cannot reach P and there will be a region behind R in which the thermodynamic state of the gas is uniform and there the reflected shock is straight (Fig. 2b). The reflected shock near the wall of the shock-tube can be attached to, or detached from, the wedge corner, depending on the flow Mach number $M_1$ behind the incident shock wave and the wedge angle $\theta_w$ (Fig. 2c).
2.4.2 Mach Reflection (MR)

As the wedge angle $\theta_w$ is decreased, the flow behind the incident shock cannot negotiate the wedge surface through just the reflected shock R. An additional shock is necessary to alter the flow so that it can negotiate the wedge surface again by turning subsonically. This additional shock protrudes forward into state (0) at the lower part of the incident shock wave I and is known as the Mach stem M (Fig. 3).

The resulting irregular reflection configuration is called Mach reflection. It is characterized by a confluence of the three shocks and a fourth discontinuity, the slipstream or contact surface S, across which entropy, density, temperature and flow velocity are discontinuous, but the pressure and the flow direction are continuous.

The confluence of the four discontinuities is called the first triple point $T$. Four types of reflection can occur in nonstationary flows depending on the initial shock Mach number $M_s$ and wedge angle $\theta_w$, namely, single-Mach, complex-Mach, double-Mach and terminal-double-Mach reflections.

One major difference between these Mach reflections and regular reflections is that they all produce a subsonic flow near the wedge surface behind the Mach stem M, while in RR the flow is usually supersonic near the wall behind the reflection point P. As in a regular reflection, the reflected shock R (Fig. 3) can be attached to or detached from the wedge corner.

2.4.3 Single-Mach Reflection (SMR)

When the flow $U_2$ behind the reflected shock R is subsonic relative to the triple point $T$ and the reflected shock R is curved over its entire length up to the triple point $T$, the configuration is a single-Mach reflection (Fig. 3). The frame of reference is attached to the triple-point T and it moves along a straight line at an angle $\chi$ (first triple-point trajectory) with a constant velocity $U_0 = U_s \csc \phi_0$, where $\phi_0$ is the incident angle between the incident shock wave and the oncoming flow, and is given as

$$\phi_0 = 90^\circ - \theta'_w$$  \hspace{1cm} (2.2)

The angle $\theta'_w$ is defined as the angle between the first triple-point trajectory and the surface, and is referred to as the effective wedge angle. They are related by

$$\theta'_w = \theta_w + \chi$$  \hspace{1cm} (2.3)

where the Mach stem M lies in front of the incident shock wave I and is normal to the wedge surface, but not necessarily straight.
The flow in state (0) can reach the region bounded by the shock waves R and M by passing through two shock waves I and R above T, or only one shock wave M below T. From stability consideration, the gas must be compressed to the same pressure, and must move in the same direction. Consequently, this gives rise to a slipstream S dividing the two regions of equal pressure and flow directions. The reflected shock wave R, therefore, needs to redeflect the flow so that \( \theta_3 = \theta_1 - \theta_2 \).

With the frame of reference attached to the triple point T, the flow configuration becomes stationary and three-shock theory may be used to find the flow properties around the triple point T. The method of calculation is discussed in Appendix A.

2.4.4 Complex-Mach Reflection (CMR)

When the flow \( U_2 \) behind the reflected shock R is supersonic relative to triple point T, a segment of the reflected shock R near T becomes straight, and the flow behind it is uniform, whereas the rest, R' has continuous curvature until it finally terminates at the wedge tip on the wall surface. The resulting Mach reflection configuration is referred to as complex-Mach reflection (Fig. 4) which is similar to single-Mach reflection except that instead of a curved reflected shock over its entire length, here the reflected shock wave R develops a reversal of curvature discontinuously. In some cases, a smooth reversal of curvature takes place. The reversal of curvature happens because of the existence of a band of compression waves [10] behind the reflected shock wave and as a result, a kink K is formed in the reflected shock wave. The method of calculations used for SMR applies to CMR.

2.4.5 Double-Mach Reflection (DMR)

When the band of compression waves at the kink of a complex-Mach reflection converges to form a new shock wave M', the kink becomes the second triple point T' and the flow Mach number M_2 behind the reflected shock wave R, in a frame of reference attached to the second triple point T', exceeds unity.

Consequently, two systems of three-shock interactions exist in the flow, hence this configuration is termed double-Mach reflection (Fig. 5). In the second three-shock interaction system, the flow behind the second triple point T', states (4) and (5), can be obtained from state (1) by passing through either the second reflected shock wave R' or through the reflected shock wave R and the second Mach stem M'. A second slipstream S' is formed at the second triple point T', and the second triple point trajectory angle \( \chi' \) is defined as the angle between the wedge surface and the line extending from the wedge corner to the triple point T'. The method of calculations used for SMR applies to the first triple point of DMR. Since the analogy can be made between the first and second triple points, three-shock theory may also be used to find the flow properties around the second triple point T'.
2.4.6 Terminal-Double-Mach-Reflection (TDMR)

At higher wedge angles and higher Mach numbers the flow in region (1) (Figs. 5,6) is pushing the second triple point T' toward the rigid boundary, and the pressure in regions (4) and (5) is pushing T' upstream. When the second triple point approaches the wedge surface the second Mach stem is forced up to join the slipstream above the wedge surface. The second Mach stem is therefore almost parallel to the surface and is very short, because the slipstream lies close to the reflected wave for high incident Mach numbers.

Therefore, region (5) becomes very small, but its existence is necessary to prevent a supersonic flow in region (2) from encountering the wedge surface. This configuration occurs only in low gamma gases and was first observed by Hu [45] and Ikui et al [44]. A simple extension of the calculation method for DMR results in a wedge surface penetration by the second Mach stem, which is impossible. Consequently, calculated values such as angle $\psi'$ disagree with the experimental results. As can be seen from Fig. 6d, if it were not for the triangle BTT', we would have a regular reflection. The flow properties in region (4) correspond to properties in region (2) in RR (Fig. 2a) and $\psi'$ corresponds to $\psi$ in RR. For this reason Ikui et al., named this type of reflection a pseudo-regular reflection - PRR, and concluded that a second Mach stem shrinks to a single T' point. This assumption would bring the supersonic region (2) to the wedge surface which is impossible from a physical point of view. Numerical simulations done by Glaz et al [48,51] have shown that even for a very low gamma, $\gamma = 1.04$, there is always a second Mach stem and a subsonic region (5). Therefore, the name of Terminal-Double-Mach reflection proposed by Prof. I. I. Glass was found to be more accurate, and will be used herein.

2.5 Transition Criteria

With the existence of the five different types of nonstationary oblique-shock-wave reflections, it becomes necessary to determine the transition criteria for one type of reflection to terminate and another type of reflection to occur. Establishment of the transition criteria requires an understanding of the physics of the flow fields, and it is important to the mapping of the transition boundaries.

2.5.1 Transition from RR to MR

There are three most quoted termination criteria for RR in the literature. von Neumann [4] proposed that the transition from RR to MR takes place when the wedge angle $\theta_w$ is lowered to an extent where the flow deflection condition $\theta_1 + \theta_2 = 0$ can no longer be satisfied. This violation occurs when the flow deflection angle $\theta_1$, through the incident shock wave I exceeds the maximum flow deflection angle $\theta_{2m}$, through the reflected shock R. This criterion is referred to as the detachment criterion and the term
detachment stems from steady flows where the oblique shock wave detaches at this angle. The detachment criterion is sometimes called the von Neumann criterion and has the analytical form

\[ \theta_1 + \theta_{2m} = 0 \]  

(2.4)

The criterion can best be illustrated by using the pressure-deflection \((P - \theta)\) shock polars as shown in Fig. 7, where the I and R polars represent the incident and reflected shock waves, respectively. Through RR, the net deflection angle is zero, hence state (2) is at the point where the R polar intersects the \(P/P_0\) axis, say point A on RL (Fig. 7). As the wedge angle \(\theta_w\) decreases, the deflection angle \(\theta_1\) through the incident shock wave I increases, and the R polar moves away from the \(P/P_0\) axis until it is tangent to the \(P/P_0\) axis (point B). By decreasing the wedge angle \(\theta_w\) any further, the R polar no longer intersects the \(P/P_0\) axis and RR is not possible, and MR occurs. Therefore, the detachment criterion corresponds to the reflected-shock polar \(R_d\) where the pressure of state (2) behind the reflected shock wave jumps from point B (RR state) to point C (MR state). Note that there is a discontinuous pressure jump during transition according to the detachment criterion. An alternative criterion was proposed by Henderson and Lozzi [10] based on the assumption that the system should remain in mechanical equilibrium during transition. They argued that a system which develops a pressure discontinuity during transition cannot be in mechanical equilibrium. In order to enable the system to remain in mechanical equilibrium, the transition between RR and MR should occur at point D (Fig. 7) where the R polar intersects the I polar (MR state) at the same point where it meets the \(P/P_0\) axis (RR state). Consequently, the mechanical equilibrium criterion is illustrated by the reflected-shock polar \(R_m\), and analytically expressed as

\[ \theta_1 + \theta_2 = \theta_3 = 0 \]  

(2.5)

where \(\theta_3\) is flow deflection through the Mach stem (Fig. 3). Consider Fig. 7; according to the mechanical equilibrium criterion, MR will take place for all reflected shock polars R above point D, since the transition criterion described by polar \(R_m\) has been exceeded. Yet, according to the detachment criterion, RR will take place for all shock polars R below point B, and the transition criterion described by polar \(R_m\) is not satisfied. Therefore, RR and MR are theoretically possible for all reflected shock polars R lying between polars \(R_d\) and \(R_m\). The dual-solution region in the \((M_w - \theta_w)\)-plane is shown for a perfect gas with specific heat ratio \(\gamma = 1.4\) in Fig. 8, and it can be seen that the area of disagreement between these two criteria is very large. However, experimental results show that the mechanical equilibrium criterion made the agreement between experiment and theory worse for nonstationary oblique-shock-wave reflection. The detachment criterion is the limit for the two-shock theory to have solutions. The mechanical equilibrium criterion is the limit for the three-shock theory to have solutions which correspond to \(\chi = 0\).
Hornung and Kychakoff [12] suggested another criterion in which they argued that in order for a MR to form, a length scale must be available at the reflection point, that is, pressure signals must be communicated to the reflection point. They argued that the transition between RR and MR takes place at the sonic deflection angle $\theta_2$, the deflection angle at which the flow behind the reflected shock R is just sonic relative to the reflection point P.

In the (P-\theta) shock polar diagram of Fig. 7, the transition occurs when the reflected-shock polar intersects the $P/P_0$ axis at the sonic point E. Consequently, this criterion is referred to as the sonic criterion and is expressed by the relations

$$\theta_1 + \theta_2 = 0; \quad M_{2P} = 1$$

where $M_{2P}$ is the flow Mach number in state (2) in a frame of reference fixed to the reflection point P. The sonic criterion is illustrated by the reflected-shock polar $R_s$. The state behind the reflected shock wave jumps from point E (RR state) to point F (MR state) during transition. However, the sonic criterion results in a transition boundary which is too close to the detachment criterion so that the differences are experimentally unresolvable.

2.5.2 Transition from SMR to CMR

In SMR, the flow behind the reflected shock wave R is subsonic in a frame of reference attached to the triple point T, and it can negotiate the wedge surface subsonically. However, when the incident shock-wave Mach number is increased, or in some cases when the wedge angle is raised, the flow Mach number behind the reflected shock wave R is also increased and eventually it exceeds unity. Subsequently, a SMR is no longer sufficient, because there exists again a supersonic flow directed towards the wedge surface. Theoretically, this supersonic flow has to be made either parallel to the wedge surface or subsonic before it reaches the wedge by means of a compression wave or a shock wave, so that it can negotiate the wall subsonically. The flow Mach number $M_2$ behind the reflected shock wave R appears to be reduced by passing through a compression wave and a CMR is formed. As the flow Mach number $M_2$ increases, the compression wave gets stronger and finally this compression wave becomes a shock wave and CMR terminates.

Henderson and Lozzi [10] were the first to suggest that a band of compression waves must exist in CMR and can converge to a shock wave. Unfortunately, they put forth neither analytical nor experimental substantiation for their suggestion. Ben-Dor and Glass [13,14] suggested that the transition between SMR and CMR occurs when the flow behind the reflected shock wave R is sonic with respect to the triple point T. Consequently, the criterion for the termination of SMR and the formation of CMR is
Shirouzu and Glass [16] studied the problem of whether a flow is in equilibrium or frozen from a more fundamental assessment through the appropriate relaxation length as related by the angle $\delta$ (see Fig. 4) between the incident and reflected waves rather than by the $(M_s-\theta_w)$ plots. During the course of their studies, they came up with an additional condition for the existence of CMR. They argued using the results of Law and Glass [15] that the kink $K$ moves with a horizontal flow velocity as for state (1). This means that the flow in state (1) moves parallel to the incident shock wave $I$ and downward with the frame of reference attached to either side of the kink $K$ are the same and the existence of a band of compression waves in CMR implies that:

$$\rho_2 < \rho'_2 \quad \text{and} \quad \phi_{1k} - 90^\circ < \phi_{1k} - 90^\circ \quad (2.7)$$

as shown in Fig. 9b. Of the four possible cases illustrated in Fig. 9c, only configurations A and B satisfy the condition $\beta < \alpha$, and also the angle $\delta$ is larger than $90^\circ$ in both cases. Hence, $\delta > 90^\circ$ is a necessary condition for the existence of a band of compression waves which is required for the formation of CMR. This is only a necessary condition because configuration C also has $\delta > 90^\circ$, although it does not satisfy $\beta < \alpha$. Consequently, the transition from SMR to CMR takes place when both the former criterion Eq. (2.6) and the new necessary condition are satisfied. The new transition criterion from SMR to CMR can be expressed as

$$M_{2T} > 1 \quad \text{and} \quad \delta > 90^\circ \quad (2.8)$$

Shirouzu and Glass [16] showed that the transition boundaries based on this new transition criterion improved the agreement with experiments in Ar, N$_2$, air and CO$_2$, and Glass [52] examining additional data showed that in some cases it is not so helpful.

### 2.5.3 Transition from CMR to DMR

Law and Glass [15] proposed that the onset of the second triple point $T'$ in DMR, resulted from the supersonic flow in state (2) trying to negotiate a compressive wedge when striking the wall. Consequently the flow $U_2$ in a laboratory frame of reference was supersonic with respect to the wedge surface. Gvozdeva et al [34] suggested that DMR is formed as a result of the excitation of the internal degrees of freedom, which leads to the increase of the density ratio across the incident shock front. Hence, the flow Mach number $M_2$ becomes supersonic at lower incident shock wave Mach numbers and smaller wedge angle for real gases than for perfect gases.

According to Semenov et al [35] DMR is formed due to the curling of the primary slipstream $S$ into a vortex. In Mach reflection, the slipstream $S$ can either lie along the wedge surface or curl into a large vortex near the wedge surface and the Mach stem $M$. However, Bazhenova et al [20] indicated that experiments show that DMR can occur when slipstream $S$ does not curl
into the vortex but extends over the wedge surface at large wedge angles. Therefore, the curling of the slipstream S does not establish the onset of DMR as proposed by Semenov et al [35].

Bazhenova et al [20] and Gvozdeva et al [21,40], using piezo-gauges to measure pressure histories on the wedge surface, found that their results did not agree with the formation criterion of DMR suggested by Law and Glass [15]. They considered that the flow $U_2$ behind the reflected shock $R$ becomes supersonic with respect to the triple point $T$ as the necessary but not sufficient condition for the existence of DMR. They argued that when CMR takes place, there is a small secondary pressure rise on the wall, and it is more pronounced for DMR with the second confluence. Ben-Dor and Glass [13,14] suggested that the transition from CMR to DMR occurs when the flow $U_2$ behind the reflected shock $R$ becomes supersonic in a frame of reference attached to the kink $K$ (Fig. 5). Their transition criterion is expressed analytically as

$$M_{2K} = 1$$

Comparisons between the transition boundaries based on this criterion and experiments in nitrogen [13] and air [26] in the $(M_5-\theta_W)$-plane show good agreement except at lower $M_5$ and higher $\theta_W$ near the RR boundary line. In the case of argon [14], the predicted transition line would have to be lowered to take into account the DMR that penetrated into the RR region. However, for CO$_2$ [16,30], the calculated boundary line fails to include many CMR points that lie in the DMR region, and the boundary line must be shifted upward. These discrepancies can be considered as inappropriate for a transition criterion. The criterion $M_{2K} = 1$ is only one of the necessary conditions and the existence of other necessary conditions may improve the agreement.

### 2.5.4 Transition from DMR to TDMR.

When the wedge $\theta_W$ increases, the flow in region (1) becomes stronger and pushes the second triple point toward the wedge (Fig. 6a-d).

At some $\theta_W$ the second Mach stem is forced up and forward to meet the first slipstream above the wedge surface. At this point the reflection process acquires TDMR features. In this reflection pattern the second triple point lies close to the rigid boundary. It is extremely difficult to observe experimentally when the second Mach stem does not behave as predicted by the three shock theory because of its minute size.

To overcome this difficulty a reflection in the present work is classified as a TDMR when the space occupied by region (5) under $T'$ approaches zero. Practically it corresponds to $\chi' < 0.2^\circ$ and can be fairly well estimated analytically by $\chi' = 0$. Therefore, a practical value of $\chi' = 0.2$ was utilized as the transition boundary for a fully developed terminal-double-Mach reflection.
3 INFLUENCE OF VIBRATIONAL EXCITATION

3.1 Vibrational Excitation

If we assume that only translational and rotational degrees of freedom are excited, i.e., a frozen gas case, the specific heat ratio is constant and equal to 4/3 for a polyatomic gas such as SF$_6$ or isobutane. However, this assumption is valid only for low temperatures, and even at room temperature the vibrational degrees of freedom are excited causing a significant decrease of the specific ratio. The Rankine-Hugoniot relation can be applied to a flow across a shock wave only when gamma is constant, that is in a frozen or perfect case. However, if a temperature increase caused by the shock compression changes gamma, there is no closed-form solution as for a frozen or perfect flow. Such cases must be solved numerically using the equations of continuity, momentum, energy and the equation of state. Vibrational excitation, dissociation, and ionization may be present. Dissociation and ionization occur only for very high Mach numbers, well outside the range of experiments in this study.

To determine the effect of vibrational excitation, two factors must be considered; the magnitude of excitation at equilibrium and the rate at which equilibrium is approached. The extent of vibrational excitation depends on the number of vibrational modes which is proportional to the number of atoms in a gas molecule and from the ratio of the characteristic temperature of each mode to the actual temperature. The ratio at which equilibrium is approached is assessed by the relaxation length.

3.2 Role of the relaxation length

Upon passing through a shock wave, the translational and rotational degrees of freedom are fully excited within a few mean free paths, the thickness of the shock. The vibrational degrees of freedom take much longer to reach equilibrium. The size of the relaxation zone is dependent on temperature and pressure. To determine whether an assumption of frozen or equilibrium flow is appropriate, the relaxation length must be compared to a characteristic length of the region being studied behind the shock wave. If the relaxation length is much greater than the characteristic length, then the flow region of interest is in essence frozen. Conversely, if the relaxation length is much smaller than the characteristic length, then an assumption of equilibrium flow is more appropriate. Flows are considered nonequilibrium where the relaxation length is of comparable size to the characteristic length, and must be treated in a more precise way. The relaxation length is a function of temperature and pressure. Vibrational relaxation length behind the shock wave vs incident shock Mach number for various gases are plotted in Fig. B2 (APP. B) for $p_0 = 15$ torr.

It can be seen that gases with more vibrational degrees of freedom have shorter relaxation length and reach the vibrational equilibrium state.
sooner. The relaxation length is directly proportional to the test pressure $P_0$.

### 3.3 Effect on the Boundaries

Both in sulfur hexafluoride and isobutane an examination of the relaxation lengths [2] in the $P_0$ range used in the experiments dictates the use of an equilibrium flow model. Although at room temperature both gases have approximately the same gamma, the influence of vibrational excitation at higher temperatures is more evident in isobutane, which has 14 atoms compared to 7 atoms in an SF$_6$ molecule. In a vibrational equilibrium flow model all transition boundaries on a ($\theta_w$-$M_s$)-plane are significantly lowered compared with a frozen flow model (compare Figs. 10-11 with 35-36).

### 4 INFLUENCE OF VISCOSITY

#### 4.1 Characteristics of Shock-Induced Boundary Layer

When a shock wave passes over a surface, it induces a velocity in the gas behind the shock. A boundary layer grows behind the shock wave, due to the friction between the moving gas and the stationary wedge surface. This boundary layer has a significant effect on the flow at the shock-wave reflection point $P$. In a laboratory-fixed frame of reference, the flow velocity at the wall must be zero. It is convenient though, to attach the reference frame to the point where the shock intersects the surface. In this reference frame, the flow velocity at the wall must be that of the shock wave. It should be noted that in this new shock-fixed reference frame, the boundary-layer displacement thickness, $\delta^*$, has a negative value, or the wall acts as a sink. Instead of impeding the flow, the shear stress at the wall tends to 'sweep' the flow away from the shock wave (see Fig. 12). The boundary-layer profile can be calculated by solving the transformed laminar-boundary-layer equations outlined by Mirels [55].

As a result of the boundary layer a centred expansion is followed by a compression at the foot of the reflected shock $R$ that complicates the wave and flow system [23]. The assumption of laminar flow is appropriate, based on the Reynolds number of the experiments performed [43]. A summary of the equations used and method of solution is contained in Appendix D. Normalized velocity and temperature profiles are shown in Figs. 13a,b for the boundary layer behind a shock wave travelling at $M_s = 2.0$, at $P_0 = 2$ kPa and $T_0 = 300$ K. While the boundary-layer thickness is significantly affected by pressure, and to a lesser extent by temperature (see App. D), the shape of the profiles is scarcely influenced by either. Very rapid changes in velocity and temperature occur close to the wall. Half of the total change in velocity and temperature within the boundary layer occur in the first 12% nearest the wall. It is interesting to note that the temperature and velocity profiles are quite similar, but at the outer edge of the boundary layer, the normalized temperature reaches 0.98 compared to the normalized velocity of 0.99. The most important thing about the equations is the
dependence of the various size parameters on density or initial pressure, and the variation of boundary-layer thickness with distance from the shock wave. The relationships are:

\[ \delta, \delta^* \propto \sqrt{x} \]  

(4.1)

\[ \delta, \delta^* \propto 1/\sqrt{\rho} \text{ or } 1/\sqrt{p} \]  

(4.2)

This implies that if shock-wave reflection is influenced by viscous effects, Mach number \( M_s \) and wedge angle \( \theta_w \) are not sufficient to define the flow; initial pressure or density must also be specified. While viscosity may not have a great influence where the Reynolds number is high (high initial pressures), many shock-tube experiments are done at very low initial pressures where viscous effects are important.

Consider, for example, the results of three first triple point trajectory angles \( \chi \) achieved in the present experiments in SF\(_6\), at \( \theta_w = 37^\circ \) and \( M_s = 6 \pm 0.1 \). At \( p_0 = 11 \text{ torr} \chi = 2.0^\circ \), at \( p_0 = 4 \text{ torr} \chi = 1.2^\circ \) and \( p_0 = 2.78 \text{ torr} \chi = 0 \) and RR occurs. An analytical, inviscid, three-shock theory predicts \( \chi = 2.3^\circ \), this means that if \( \chi \) would decrease monotonically to a zero as \( \theta_w \) increases, which occurs at \( \theta_w = 73^\circ \) for \( M_s = 6 \) according to three-shock theory, a change of 36\(^\circ\) in \( \Delta \theta_w \) corresponds to only a 2.3\(^\circ\) change in \( \Delta \chi \). This illustrates the large effect of \( p_0 \) on the experimentally observed \( \Delta \chi \) (at \( \theta_w = 37^\circ, M_s = 6 \)), for small changes in the initial pressure. Therefore, it is important to consider \( p_0 \) and consequently the boundary-layer effect in predicting \( \chi \) and finally the MR-RR transition.

4.2 Physical Model of the Reflection Process over a Wedge with a Shock-Wave-Induced Boundary Layer.

The complexity of the Navier-Stokes equations together with some uncertainties concerning the boundary-layer effect on the oblique-shock-wave reflection process creates serious problems for a quantitative analysis of the boundary-layer influence. Up to now, most researchers used the inviscid, pseudostationary model of a nonstationary flow over the wedge as presented in Figs. 1-6, neglecting the influence of the boundary layer. We can use the inviscid approximation to a real flow by adding a negative boundary-layer effective thickness profile \( \delta^* \) to the wedge surface (Fig. 14) and using a set of pseudostationary equations (App. A). If the deformed wedge is filled with the boundary layer, a real viscous flow model is obtained.

The distance \( x \) (Fig. 14) at which a tangent line to the boundary-layer profile is drawn is introduced to assess the influence of the boundary-layer effect on a flow in the nearest vicinity of the first triple point T. In this way the set of pseudostationary equations can be used. In this approach the distance \( x \) depends on several factors, such as \( M_s, \theta_w \) and \( \chi \). Therefore, it cannot be explicitly calculated and \( \theta_w \) cannot be predicted analytically.
A solution of the pseudostationary, inviscid set of equations for a flow over a steeper wedge $\theta_w''$ should yield the same results of the first triple-point trajectory angle $\chi$, as observed experimentally in a real viscous flow at $\theta_w$ and the same $M_\infty$. The higher $\theta_w$, the lower $\chi$ is. As $\theta_w''$ is always larger than $\theta_w$, the existence of the boundary layer causes the observed $\chi$ to be smaller than the inviscid prediction for the same $\theta_w$. In other words, the MR-RR transition observed in shock-tube experiments for $\theta_w$ would occur at the higher wedge angle $\theta_w''$ in the inviscid flow. According to the hypothesis presented here the $\theta_w''$ (MR-RR) transition should be predicted by the $\chi = 0$ criterion (mechanical equilibrium criterion), whereas the $\theta_w$ (MR-RR) transition observed in a shock tube would lie close to (for higher gamma gases) or a few degrees above (for a low gamma gases) the detachment criterion line.

The model shown in Fig. 14 serves only as a transposition of a real viscous flow over a real wedge into a relevant inviscid flow over a deformed wedge. The actual physical process taking place may look as in Fig. 12 as proposed by Hornung [23] and used by Shirouzu and Glass [54] and Wheeler [43]. However, Sieller [50] who used a Monte-Carlo method at the molecular level could not observe such a wave system in the boundary layer, nor could it be observed by optical methods.

5 EXPERIMENTAL WORK

5.1 Shock Tube Facility

Experiments were carried out in the UTIAS 10cm x 18cm Hypervelocity Shock Tube (Fig. 15). This facility has the capability of both cold-gas driven and combustion-driven runs. However, combustion driven runs were not attempted owing to the possibility of damaging the test-section windows when operating with large wedge angles. A full description of the facility and its capabilities can be found in Boyer [36] and Ben-Dor and Whitten [38].

By increasing the pressure in the compression chamber on one side of diaphragm until it ruptured, shock waves were sent down the channel. The driver gas was added slowly so that after the diaphragm ruptured, the shock speed would not be influenced during its time of travel by the additional mass flow. Shock speeds were controlled by varying the type of driver gas, the diaphragm thickness and pressure ratio across the diaphragm. The diaphragms consisted of several sheets of mylar polyester stacked together.

The sheets were available in several nominal thicknesses, and by proper combination nearly any desired thickness could be achieved. There is a practical limit to the maximum thickness which can be used since very thick diaphragms do not break properly and their breaking pressure is unpredictable. For this reason, the maximum overall diaphragm thickness used was 1.07 mm.

Initial gas temperature was measured by a mercury-bulb thermometer embedded in the shock tube wall. A minimum of 5 minutes was allowed between
the entry of the test gas and reading the thermometer to allow the temperature to stabilize. The thermometer scale was marked at 0.1°C intervals.

The initial gas pressure was measured by a series of Wallace and Tiernan type FA 160 gauges. The range and maximum error of the gauges are shown below.

<table>
<thead>
<tr>
<th>Range</th>
<th>Absolute Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. 0 - 50 torr (6.67 kPa)</td>
<td>0.2 torr</td>
</tr>
<tr>
<td>2. 0 - 200 torr (26.7 kPa)</td>
<td>1.0 torr</td>
</tr>
<tr>
<td>3. 0 - 400 torr (53.3 kPa)</td>
<td>2.0 torr</td>
</tr>
<tr>
<td>4. 400 - 800 torr (106.7 kPa)</td>
<td>2.0 torr</td>
</tr>
</tbody>
</table>

To guard against possible leakage of air into the shock-tube test section, test-gas pressure was checked at the time of admission, and also 5 minutes after. Leaks which would cause a variation in pressure of greater than 0.05 torr (6.7 Pa) at the time of firing the shock tube were unacceptable and the runs were aborted. This corresponds to a leak rate of approximately 0.01 torr/min. and was found to be extremely rare.

The incident-shock-wave speed was determined by measuring the time interval between a common starting point and several stations distributed down the channel. Time intervals were measured by 5 digital counters using a 1 MHz oscillator as a common time base. Trigger signals from each station were obtained using Atlantic LD-25 blast-wave pressure-transducers. The start signal for all the counters came from station D, and the 5 counters were stopped by signals from stations F, G, H, I and J. A schematic diagram of the set-up is shown in Fig. 15. To allow for deceleration of the shock wave, owing to the sidewall boundary-layer growth, a second-order best-fit curve was applied to the measurements from stations F, G, H and I, and extrapolated to give a velocity at the centre of the test section. The interval at station J was not used because the shock-wave speed changes significantly after it has passed over the wedge.

Shock-wave-reflection patterns were recorded using a 23 cm dia. field-of-view Mach-Zehnder interferometer [39]. The light source for the interferometer was a giant-pulse ruby-laser incorporating a TRG model 104A laser head, a TRG model 2113-1 harmonic generator and TRG Pckels Q-switch.

The flashlamp capacitor bank of the laser and the Q-switch were fired at the appropriate times by delaying the trigger signal from stations F and I respectively. The capacitor bank was fired at 880 ns before the Q-switch in order to get a single giant pulse from the laser. In this mode, the laser delivers 0.8 Joules of light energy in a 15 ns pulse. Interferograms were recorded on Kodak Royal-X Pan film, and developed in a Kodak HC-110 developer. The negatives were developed using twice the recommended time, to increase the effective film speed (nominal speed is 1250 ASA), and contrast. 6943 A light was used to obtain more contrast pictures. The
3971A light would provide twice as many fringes, consequently more accurate density and pressure distribution, but that was not the main objective of the present study.

5.2 Measurement of Data from Interferograms and Shadowgraphs

The one drawback to recording shock-wave reflections photographically is the inevitable problem of interpreting the data. Flaws in the optics and grain of the film limit the resolution of the interferograms, and sometimes can make the determination of the type of reflection difficult and subjective. In the case of transition from RR to DMR, the determination is rather simple. In DMR the two Mach stems are separate and distinct. Transition from RR to SMR or CMR is often difficult to see. The Mach stem in MR may be so small that its size is almost immeasurable, and the fringe distortion caused by the slipstream can be confused with that caused by the boundary layer.

One way of separating the two is to look at the shape of the fringe distortion. A slipstream should slope toward the wedge surface and the distortion from it appears to decrease in size as distance from the Mach stem increases. A boundary-layer-induced distortion, if visible, should increase in size as the distance from the reflection point increases. Measurement of wave angles is also difficult. For optical reasons, the reflected shock wave appears to have a finite, measurable thickness. It also appears to be quite 'fuzzy' near the reflection point. It was found that a precise and accurate measurement of angle could only be made at distances of 1 mm or greater from the reflection point. The angle between the incident and reflected shock waves, was measured by drawing lines tangent to the incident and reflected shock waves at a distance of 1 mm from P, and trigonometrically determining the angle.

As the objective of the present work was to examine the type of reflection, first and second triple points trajectories rather than the density distribution, the shadowgraphs were deemed to provide more useful results.

5.3 Experimental Results for Sulfurhexafluoride

The results of the present experimental work in SF$_6$ are given in Table 1. Hu's results [45] for higher wedge angles (37, 40, 42 and 45 degree wedges) were utilized in further analysis. The thermodynamic properties of SF$_6$ are presented in Appendix B.

5.4 Experimental Results for Isobutane

The experimental results for CH(CH$_3$)$_3$ are presented in Table 2. The thermodynamic properties of isobutane are presented in Appendix C.
6 DISCUSSION OF RESULTS

6.1 Stability of Shock Waves.

Over fifty experiments were performed in SF₆ in order to determine the TDMR region and RR-MR transition boundary in the \( (M_s - \theta_w) \)-plane. A substantial problem was encountered in the evaluation of the experimental results. In order to achieve high flow Mach numbers \( (M_s > 4) \) helium had to be used as a driver gas. In this combination \((\text{He}/\text{SF}_6)\) the incident shock waves are not planar and the regions behind it are nonuniform. It is believed that instability of the shock front (Fig.16) is a consequence of Rayleigh-Taylor instabilities at the contact surfaces caused by the turbulent mixing of the light helium driver and the heavy driven gas. In order to avoid the high discrepancy in the molecular weights between the test gas and the helium, a lighter test gas was introduced. Instead of heavy SF₆ with a molecular weight of 156, a lighter isobutane was used. Isobutane, \((\text{CH}_3)_3\text{CH}\), has a molecular weight of 58.12, with approximately the same gamma at room temperature as SF₆, but with significantly lower gamma at higher temperatures. It also has a high refractive index of 1.001286, which is useful in optical studies.

The disadvantages of using hydrocarbonates as test gases are: a high liquification temperature, 28.12°C for isobutane, flammability and explosion risk when exposed to oxygen. Generally, an introduction of isobutane improved the quality of the interferograms and shadowgraphs. However, in some cases the shock front was unstable and the regions behind the shock front were nonuniform (Fig. 16). Furthermore, both in SF₆ and in isobutane an interesting phenomena of outrunning of region (2) to region (1) was observed at \( M_s > 2.2 \) (Figs. 18, 25 and 28a). In the inviscid flow there should not be any influence of region (2) beyond the reflected shock wave. However, in all pictures taken in SF₆ and isobutane at higher Mach numbers a 'crown' occurs as part of reflected wave. It was observed that at higher \( M_s \), the 'crown' starts closer to the confluence point of the shock waves, and for \( 2 < M_s < 3 \) only the part of the reflected shock wave closest to the wall is influenced by this phenomenon, probably caused by the boundary layer on the walls of the windows. It is worth noting that the whole phenomenon of instability of the incident shock wave is not repeatable, i.e., experiments repeated at the same initial conditions yield different instabilities.

This suggests a strong correlation between the various, unrepeatable ways of diaphragm breaking and the resulting instabilities. On the other hand, in the previous experiments conducted in the UTIAS Hypervelocity Shock Tube, where the same mylar diaphragms were used, this phenomenon was not observed except for Hu's experiments in SF₆ [45]. Therefore, the instabilities are strongly dependent on the differences in molecular weight between the driver and test gas and perhaps even on gamma. The phenomenon of shock-wave instability in low gamma and heavy test gases deserves more careful consideration and future studies.
6.2 **Observed Features of RR, SMR, CMR, DMR and TDMR**

Since it is not possible to analyze and discuss each interferogram and shadowgraph obtained in the present study in detail, only several interesting cases will be presented. As the SF₆ interferograms along with their density distributions are beautifully presented and described by Hu [45], only the isobutane cases are analyzed below. No specific features different from SF₆ were observed, except for a 'toe out' of the Mach stem at higher Mach numbers.

### 6.2.1 Regular Reflection

**Case 1: Exp. 24, Mₛ = 1.58, θ_w = 45°, p₀ = 96 torr, T₀ = 297.3K**

An infinite fringe interferogram of a detached RR in isobutane is shown in Fig. 17. A large number of isopycnics is due to the high molecular weight of the test gas. There exists a small uniform supersonic region immediately behind the reflection point and the reflected shock wave is straight in this region, and immediately the flow becomes subsonic. The reflected wave becomes curved and terminates at the bottom wall of the shock tube as a detached bow shock wave. Its interaction with the wall boundary layer is still weak. If the flow had been supersonic all throughout region (3), the corner signal could not catch up and influence the flow and it would result in an attached reflection, i.e., the reflected wave would terminate at the wedge corner. A relaxation zone approximately equal to the apparent shock wave thickness is visible behind the incident shock wave. A sudden change in the direction of the isopycnics in the vicinity of the wedge surface and at the bottom wall is caused by the cold wall boundary layer.

**Case 2: Exp. 23, Mₛ = 2.2, θ_w = 45°, p₀ = 33 torr, T₀ = 296.9K**

An example of a lambda bow shock-wave configuration in isobutane is shown in Fig. 18, caused by the wall boundary layer. The reflected wave is curved and attached to the wedge corner but another detached wave runs ahead. This configuration happens in a transition case from a detached to an attached reflection. For a still higher Mₛ we would see an attached reflected wave, again with a lambda shock configuration. A weak disturbance ahead of the reflected shock wave is also visible, and is caused by the induced boundary layer on the windows.

### 6.2.2 Single-Mach Reflection

**Case 3: Exp. 14, Mₛ = 1.58, θ_w = 37°, p₀ = 112 torr, T₀ = 298.1K**

An infinite-fringe interferogram of a detached SMR in isobutane is shown in Fig. 19. No uniform region is observed behind the triple point and the reflected shock since the flow is subsonic in region (2). The corner signal can propagate upstream to the triple point and influence the reflected shock wave. As a result, the reflected wave is curved throughout
its entire length from the triple point to the walls of the shock tube, where it terminates as a detached shock. The straight slipstream clearly emanates from the triple point and merges with the boundary layer on the wedge surface. This case is close to CMR - the reflected wave emanating from the triple point is almost straight and the angle between the incident and reflected wave agrees well with analysis.

Case 4: Exp. 20, $M_s = 2.2$, $\theta_w = 10^\circ$, $p_o = 33$ torr, $T_o = 298K$

An example of an infinite-fringe interferogram in isobutane of an attached SMR is presented in Fig. 20. Note the significant increase in the Mach stem length for a less steep wedge angle compared with case 3 (Fig. 19). Another interesting feature is the curling up of the slipstream. It is straight near the triple point but it curls up into a vortex structure near the wedge surface. The pressures across the slipstream are equal, but the flow velocity in region (3) is significantly lower than in region (2). As this difference grows, the point at which the slipstream intersects the wedge surface moves forward toward the Mach stem. It is concave until it reaches the point at the wedge where the tangent which touches the slipstream at the triple point intersects the wedge [44]. For higher $M_s$ we would observe a convex slipstream, and finally a roll up of a slipstream.

6.2.3 Complex-Mach Reflection

Case 5: Exp. 45, $M_s = 1.7$, $\theta_w = 42^\circ$, $p_o = 60$ torr, $T_o = 295.8K$

An interferogram of a detached CMR with a bow shock lambda configuration in isobutane is shown in Fig. 21. A straight portion of the reflected shock wave divides a uniform region from a subsonic region under the curved portion of the reflected wave. The angle between the incident and reflected wave exceeds 90 degrees. However, it is difficult to single out the position where a change of curvature occurs, the straight part of the shock smoothly joins the curved part.

Case 6: Exp. 26, $M_s = 2.25$, $\theta_w = 30^\circ$, $p_o = 66$ torr, $T_o = 296.3K$

In Fig. 22, a transitional case between CMR and SMR is presented. Angle $\delta = 90^\circ$ and the change in curvature lies very close to the triple point. Visible instabilities of the slipstream and flow Mach waves were introduced using a rough wedge. The slipstream is slightly curled up. The bow shock is detached with a weak lambda shock.

Case 7: Exp. 34, $M_s = 1.75$, $\theta_w = 40^\circ$, $p_o = 40$ torr, $T_o = 297.5K$

An excellent interferogram of a CMR showing the incident shock wave just past the upper corner of the wedge is presented in Fig. 23. The kink is quite shallow (above the circular isopycnics) and the curvature of the Mach stem is due to its interaction with the corner-expansion wave. The bow shock shows a weak $\lambda$-interaction.
6.2.4 Double-Mach Reflection

Case 8: Exp. 10, $M_s = 2.2$, $\theta_w = 37^\circ$, $p_0 = 33$ torr, $T_0 = 299.3K$

An infinite-fringe interferogram in isobutane illustrating a DMR is presented in Fig. 24. Visible on the picture are compression waves which converge to a kink as well as a second Mach stem. For a stronger flow, the compression wave turns into a second Mach stem. The kink then becomes a second triple point. The slipstream develops instabilities, forming small ripples before it rolls up at the wedge surface. At the wedge corner an intense lambda-shock interaction is visible.

Case 9: Exp. 5, $M_s = 2.6$, $\theta_w = 37^\circ$, $p_0 = 12$ torr, $T_0 = 296.3K$

An infinite-fringe interferogram of DMR in isobutane is presented in Fig. 25. Here the flow Mach number $M_s$ is greater than for the previous case and a strong second Mach stem emanates from the second triple point, which is driven down compared with the weaker flow in case 8 (Fig. 24). A second Mach stem joins the slipstream above the wedge surface. A weak second slipstream is also visible emanating near the second triple point. The reflected shock wave is attached to the wedge corner with a smaller lambda-shock configuration.

6.2.5 Terminal-Double-Mach Reflection

Case 10: Exp. 31, $M_s = 6.74$, $\theta_w = 30^\circ$, $p_0 = 4.0$ torr, $T_0 = 296.3K$

A shadowgraph of an attached DMR in isobutane is shown on Fig. 26. The second triple point is pressed closer to the wedge surface. The second Mach stem joins the slipstream exactly at the wedge surface. Even for very strong flows the interaction of the second Mach stem and the first slipstream cannot be forced beneath the rigid boundary. Consequently, the second Mach stem is forced out of its usual DMR position. When terminal-double-Mach reflection features begin to appear, the most noticeable feature is a gradual change of the angle $\phi'$ (see Fig. 6a) from a value characteristic for DMR to a $\phi$ corresponding to RR (see Fig. 2a). Unfortunately, it was very difficult to obtain a well-defined wave system using optical methods here and in Ref. 44.

Case 11: Exp. 28, $M_s = 7.75$, $\theta_w = 30^\circ$, $p_0 = 4.4$ torr, $T_0 = 297.3K$

A DMR case in isobutane with partially developed TDMR features is presented in Fig. 27. An additional interesting feature characteristic of high shock Mach number flows in low gamma gases is visible where the first Mach stem is toed out or convex. This confirms the numerical results of Glaz and Lijewski [48] for an inviscid case. This effect is probably caused by the strong second Mach stem pushing region (2) outward.
A shadowgraph of an attached terminal-double-Mach reflection in isobutane is presented in Fig. 28a. It can be seen that the second triple point lies almost directly at the wedge surface. The second Mach stem and the second slipstream are invisible here, and led some experimentalists [44] to the assumption that the second Mach stem does not appear in this type of reflection. Consequently, they further concluded from the fact that \( \phi' \) here corresponds to \( \phi \) for RR that such type of reflection is a bifurcation form of RR and accordingly termed it a pseudo-regular reflection. On the other hand, Lee and Glass [25] predicted that such a configuration of DMR would occur in low gamma gases and consequently proposed the term terminal-double-Mach reflection (TDMR).

During the present work, an analysis was conducted of Fig. 28a and angle \( \omega' \) (see Fig. 6d ) computed with three-shock theory, i.e., the same code used for DMR, which agrees exceptionally well with the experimental results. It confirms the assumption that the first slipstream exists in TDMR and makes the velocity and pressure calculations results trustworthy in regions (1), (2) and (3). For this case, as for all TDMR cases, the flow in region (2) (see Fig. 6d) is clearly supersonic with a flow Mach number exceeding 2. This supersonic flow cannot negotiate the wedge surface without the help of another shock wave. This means that the second Mach stem must exist, although region (2) is so small and the second Mach stem lies almost at the wedge surface so its appearance is extremely difficult to observe (see Figs. 28a-b).

The second Mach stem must join the first slipstream, and that is where computer codes based on a three shock theory start to yield unreasonable results showing that the slipstream joins the second Mach stem below the rigid surface, which is impossible, so that the Mach stem is forced out from the position that would be achieved in DMR. Consequently all numerical results calculated for DMR behind the second triple point are not confirmed experimentally. Accordingly, the most visible feature, angle \( \phi' \), does not agree with the numerical solution for DMR, but agrees very well with the \( \phi \) for the corresponding RR with the same \( M_s \) and \( \theta_w \). This is not surprising, because the influence of the small region (5) (see Fig. 6d), on a large region (4) is extremely small and the situation behind the second triple point in TDMR is almost identical to the RR case behind the reflection point. For example, an experiment conducted in SF\(_6\) at \( M_s = 7.24 \) and \( \theta_w = 40^\circ \) yields \( \phi' = 11^\circ \). A computer code based on three-shock theory, i.e., assumed DMR case yields \( \phi' = 44^\circ \), and for the same initial conditions a code based on two-shock theory, i.e., assumed RR case, yields \( \phi = 12^\circ \). A comparison of experimental \( \phi' \) (TDMR) with \( \phi \) (RR) for \( \theta_w = 40^\circ \) is presented in Fig. 29. Moreover, we can assume that the pressure history for region (4) in TDMR would be the same as for region (2) for RR (see Figs. 2a, 6a), and the highest pressure achieved would be directly behind the second triple point. Therefore, we can accurately predict the pressure distribution and the shock configuration for DMR and TDMR, but there exists quite a large range of \( \theta_w \) and \( M_s \) for which the second Mach stem is still forced out of
the predicted position, but the second triple point lies above the wedge surface. On the \((M_s-\theta_w)\)-plane there is a region of DMR with TDMR features that extends for a few degrees below the DMR-TDMR transition line. We can delineate this region or check if TDMR features are present, using the following procedure illustrated in Fig. 30. We can calculate angle \(\eta\) using the three-shock computer code:

\[
\eta = 180^\circ + \gamma - \beta - \omega - \chi + \chi'
\]  
(6.1)

or

\[
\eta = \gamma + \beta'
\]  
(6.2)

or find it from the experiment \(\eta = \eta_{\text{ex}}\) then compare \(\eta\) with \(\eta_{\text{ex}}\) using the computer code and if \(\eta = \eta_{\text{ex}}\) holds the transition point when the TDMR features begin to appear. If \(\eta\) calculated is greater than \(\eta_{\text{ex}}\), then a clear case of DMR is obtained where the computer code for three-shock theory yields accurate results and if \(\eta\) calculated is smaller than \(\eta_{\text{ex}}\), then DMR with TDMR features exist.

As we can see from Eq. 6.1, we need a very accurate prediction of several angles to draw a \(\eta = \eta_{\text{ex}}\) line on the \((M_s-\theta_w)\)-plane. Unfortunately, the prediction of \(\chi\) and \(\chi'\) is only fair (see the discussion in Chapter 6.3 and 6.5) and for this reason further analysis of the \(\eta = \eta_{\text{ex}}\) case was abandoned in the present work.

6.3 Examination of a First Triple Point Trajectory.

6.3.1 Analytical, Inviscid Prediction

When applying the three-shock theory to MR it is necessary to give a value of \(\chi\) as an initial condition in order that the system of equations be closed. Law and Glass [15] proposed an empirical method for predicting the value of \(\chi\) based on experimental observations that for practical purposes the Mach stem is straight and normal to the wedge surface, and introduced an additional independent geometrical relation

\[
\phi_3 = 90^\circ - \chi
\]

where \(\phi_3\) is the incident flow angle to the Mach stem (see Fig. 4). At \(\theta_w = 0\), \(\chi\) has its maximum value for a fixed \(M_s\). If \(M_s\) is fixed, \(\chi\) is a decreasing function with increasing \(\theta_w\) and approaches zero at the critical wedge angle for the transition from MR to RR. The plot of \(\chi\) as a function of \(M_s\), with the actual wedge angle \(\theta_w\) as a parameter for equilibrium SF\(_6\) is shown in Fig. E3a-d, and in Fig. E4 (Appendix E) for equilibrium isobutane. For a given wedge angle \(\chi\) is a decreasing function with increasing Mach number \(M_s\), except for high \(\theta_w\) at low \(M_s\). The experimental points are also plotted in the figures with the initial \(p_0\) pressure given underneath. A high dependence of \(\chi\) on the initial pressure is clearly visible. Experiments conducted at higher \(p_0\) have higher \(\chi\) than experiments done at lower \(p_0\) and same \(\theta_w\) and \(M_s\). Angles \(\chi\) for the
experiments conducted at higher initial pressures lie closer to the inviscid, three-shock theory for wedge angles exceeding 30 degrees. At $\theta_w = 30^\circ$ experimentally observed $\chi$ agrees quite well with the inviscid, three-shock theory prediction. However, for lower wedge angles such as 20 or 10 degrees the experimental $\chi$ exceed the inviscid, numerical prediction.

Such behaviour at lower wedge angles cannot be explained with the help of three-shock theory. Still, at lower wedge angles, the boundary layer affects the triple point trajectory in the same way as at the higher wedge angles, i.e., the higher $p_0$, the higher is $\chi$ (see Hu's results [45]). The inaccuracy of three-shock theory in the prediction of $\chi$ is caused not only by neglecting the viscous effects. As described in Section 6.2, the Mach stem at the triple point is not perpendicular to the wedge surface because of its curvature. This discrepancy affects the analytical three-shock theory solutions, especially the value of $\chi$.

Besides, the bow shock wave has an effect on the position of the triple point. The reflected shock wave is the result of two processes; one is a reflected shock wave emanating from the triple point and the other is the bow shock wave caused by the flow-deflection process of the flow induced by the incident shock wave over the wedge corner. In the three-shock theory, the interaction of these two processes has not been taken into account [54]. At low Mach numbers or wedge angles, the reflected shock wave from the triple point becomes weaker more rapidly than the bow shock wave. Hence, the position of the reflected shock wave is governed mainly by the bow shock wave. Therefore, a better method for the prediction of $\chi$ in the inviscid flow which takes into account the curvature of the Mach stem and the effects of the bow shock wave is necessary [54].

### 6.3.2 Influence of Viscosity

A strong influence of initial $p_0$ on the actual value of $\chi$ was observed by several researchers [23,54,43] and is confirmed in the present study. The experimental results of $\chi$ are compared with the three-shock theory prediction in Fig. E3a-d for SF$_6$ and in Fig. E4 for CH(CH$_3$)$_3$. It can be seen that for the same $M_s$ and $\theta_w$ a lower $\chi$ is obtained for the lower initial pressure $p_0$. For low enough $p_0$ (Exp 27 in SF$_6$, $M_s = 6.04$, $\theta_w = 37^\circ$, $p_0 = 2.85$ torr) $\chi$ achieves zero value and RR occurs, whereas for same $\theta_w$ and $M_s$, but higher $p_0$, MR takes place. This behaviour is consistent with the assumed character of the boundary-layer effect. The boundary-layer thickness, and presumably its influence, should increase with decreasing pressure according to Eq. D.18. This influence is represented by $\Delta \chi = \chi(\text{inviscid}) - \chi(\text{viscous})$; where $\chi(\text{inviscid})$ is approximated by the inviscid prediction of the triple point trajectory angle $\chi_{an}$ computed according to three-shock theory (see Appendix A) and $\chi(\text{viscous})$ is the experimentally obtained $\chi_{ex}$. In Fig. 31, the relationship between $\Delta \chi$ and $p_0$ at $M_s = 6$ and $\theta_w = 37^\circ$ for SF$_6$ is plotted, together with experimental data for same $\theta_w$ and $M_s = 6 \pm 0.1$. It has the form:

$$\Delta \chi = C_1 + C_2 \ p_0^{-1/2}$$

(6.3)
which could have been anticipated since the boundary layer thickness is affected by the initial pressure in that way (see Appendix D). Theoretically, as the initial pressure gets extremely large, the viscous forces should become negligible and the solution should approach the inviscid case. However, due to the experimental limitations, the results for high $M_S$ could not be obtained at high $p_0$, similarly, for low $M_S$ results at low $p_0$. Moreover, we cannot obtain a universal form of Eq. (6.3) because for transition each $M_S$ and $\theta_w$ the values of $C_1$ and $C_2$ change and the $\chi$ inviscid, predicted by the three-shock theory is not accurate. At lower wedge angles the experimental results exceed the inviscid prediction (for example $\theta_w=20^\circ$). There is also one more obstacle in assessing the $\Delta \chi - p_0$ relationship: the influence of the boundary layer on the triple-point-trajectory angle $\chi$ should be dependent on their proximity to each other. As the Mach stem grows, it gets further from the boundary layer. If the boundary layer is thought of as a sink, its influence on $\chi$ should drop off with distance.

Therefore, as the Mach stem grows, the boundary-layer influence diminishes, so that the growth of the Mach stem, and therefore $\chi$, is inherently non-linear with time. The existence of viscosity eliminates the self-similarity that exists for the inviscid process. Any comparison of the triple-point trajectory angle with inviscid predictions should be done with respect to the initial pressure, especially when the initial pressure and $\chi$ are quite small. Nevertheless, by these obstacles which cause the scatter of data in Fig. 31, it is seen that the trend is quite close to (6.3). Even neglecting $C_1$ the accuracy of Eq. (6.3) is good enough for any qualitative analysis and this simpler form is employed subsequently. Moreover, neglecting $C_1$ guarantees $\Delta \chi = 0$ for $p_0 \rightarrow \infty$ as predicted for the inviscid case.

6.4 Prediction of MR-RR Boundaries for Various $p_0$

During the present investigation of oblique-shock-wave reflection in SF$_6$ and CH(CH$_3$)$_3$ several Mach reflections were observed above the detachment criterion line (Figs. 35-36). Similar results were reported by Ikui et al. [44] and Hu [45]. This fact suggests that the detachment criterion is inappropriate as the MR-RR transition criterion.

This criterion agrees quite well with hundreds of shock-tube experimental results for higher gamma gases (He, Ar, N$_2$, O$_2$, air, CO$_2$) where the detachment criterion line lies considerably higher on a ($M_S$-$\theta_w$) plane than for low gamma-gases. However, in all of these experiments, the analysis of the first triple point trajectory angle $\chi$ shows no jump from the value predicted by the three-shock theory (on the MR side of the transition) to zero (on the RR side) across the transition line. According to the detachment criterion theory this jump would have a finite value of two to three degrees and should be accompanied by a pressure jump behind the shock's confluence point. This behaviour stems from the "von Neumann theory" which allows RR to exist at all deflection angles lower than $\theta_{2m}$ (Fig. 7), cutting off a large part of the MR domain predicted by three-shock
theory [between the detachment criterion and the mechanical equilibrium criterion on the $M_s-\Theta_w$ plane (Fig. 8), or the D-C Mach stems on a shock polar (Fig. 7)].

Such an approach forces the two-shock theory (RR) into the three-shock theory (MR) domain and results in several analytical problems. Consider, for example, the CMR-DMR and SMR-CMR transition lines from Fig. 35a. As one moves up on an $M_s-\Theta_w$ plane along the CMR-DMR line $\chi$ is decreasing and the second triple point (or kink) is approaching the first triple point monotonically. The CMR-SMR transition line intersects the RR-MR transition line at point C. At this point the second triple point converges into the first triple point and becomes a reflection point P. Therefore in the vicinity of point C (Fig. 36a) $M_{2K} = M_{2T} = 1$. In other words the $M_{2T} = 1$ (SMR-CMR) transition line should intersect the RR-MR transition line in the same point as the $M_{2K} = 1$ (CMR-DMR) line does, but according to calculations it joins the detachment criterion line at point B (Fig. 36a). Points C and B are quite apart ($\Delta M_s = 0.3$ and $\Delta \Theta_w = 5^\circ$). Moreover, experimental data [11, 15, 20, 24, 25, 26, 43, 44] supports the assumption that point C and B, i.e., DMR and SMR, are separated in the vicinity of the detachment criterion line by the CMR region. The reason for this inconsistency lies in the fact that the $M_{2T}$ and $M_{2K}$ lines calculated with three-shock theory are suddenly cut off by the two-shock theory detachment criterion line.

The above dilemmas disappear when the mechanical equilibrium criterion is considered as the MR-RR transition line.

The first triple-point trajectory angle $\chi$ is monotonically approaching a zero value, as observed experimentally. The occurrence of Mach reflections above the detachment criterion line is natural, since MR takes place below the mechanical equilibrium line.

The CMR-DMR and SMR-CMR transition lines intersect the RR-MR transition line at one point (Figs. 35-36), where $M_{2T} = M_{2K} = 1$.

The three-shock theory naturally yields RR results for the wedge angles above the $\chi=0$ line, and becomes simply a two-shock theory in this domain. Therefore, three shock theory can be used all over the $(M_s-\Theta_w)$-plane, and it will yield the same results as the two-shock theory above the mechanical equilibrium criterion line and practically the same results for low $M_s$ ($M_s < 1.3$).

Moreover, the MR-RR transition in a nonstationary flow becomes the same as in a steady flow, establishing a universal criterion for both flows.

In a steady-flow case, examined usually in a wind tunnel with two symmetrical wedges the boundary layer does not exist, or its influence is marginal for a one wedge arrangement where $p_0$ is usually high. This is not the case in a shock tube, where the boundary-layer effect forces RR far below the mechanical equilibrium criterion line.
Using the proposed hypothesis we can predict the MR-RR boundaries for various $p_0$.

For a specified value of $M_s$ and $p_0$, a straight line interpolation of $\chi$ to zero (Fig. E1) on a $\chi-\theta_w$ plane yields the MR-RR wedge angle transitional value. However, as the boundary layer effect is related to $\Delta\chi = \chi^{(inviscid)} - \chi^{(viscous)}$, the introduction of $\Delta\chi$ instead of $\chi$ proves to be beneficial in predicting transition points for different initial pressures $p_0$ based on experimental results at one $p_0$.

The boundary-layer effect or lowering the first triple point trajectory angle $= \Delta\chi$ can be fairly well assessed by

$$\Delta\chi = C_l p_0^{1/2} \quad (6.4.1)$$

as discussed in Chapter 6.3.2. We can approximate $\Delta\chi$ as the difference between the analytical, inviscid prediction $\chi_{an}$ and the experimental result $\chi_{ex}$:

$$\Delta\chi = \chi_{an} - \chi_{ex} \quad (6.4.2)$$

For a specified $M_s$ and $p_0$ we can find $\theta_w$ (MR-RR) where the boundary-layer effect $\Delta\chi$ achieves the inviscid value predicted at this wedge angle (Fig. E2). Then $\chi_{an} = \Delta\chi$, which means that $\chi_{ex} = 0$, and we observe RR. This procedure can be repeated for another $M_s$, but generally, due to the shock-tube limitations, we cannot always use the desired $p_0$. In this case, Eq. (6.4.1) was used.

In this way the MR-RR transition points were found for a few values of $M_s$ at $p_0 = 4, 10$ and 100 torr. A best fit curve was interpolated through these points on a $(M_s-\theta_w)$ plane. The transition lines were extended according to the existing trend for higher $M_s > 9$ and intuitively drawn below the mechanical equilibrium line for low $M_s < 1.7$. The results are presented in Figs. 35-36. The MR-RR transition lines for $p_0 = 4, 10, 100$ torr lie below the mechanical equilibrium line, but above the detachment criterion line for higher $M_s$ and slightly below this line for low $M_s$. The initial test pressure $p_0$ is given for RR experimental results. Agreement with the experimental results will be discussed in Chapter 7.

The full lines on these graphs: $\chi = 0$, $M_{2T} = 1$ and $M_{2K} = 1$ were calculated with three-shock theory. A $\chi = 0$ line divides the $(M_s-\theta_w)$ plane into RR and MR (SMR, CMR, DMR and TDMR) domains. The $M_{2T} = 1$ line is a CMR-SMR transition criterion line and the $M_{2K} = 1$ is a CMR-DMR boundary line (Fig. 35a). A $\chi' = 0$ line was added to delineate the TDMR region. This line was calculated according to Eq. (6.5) and quite closely corresponds to the $\eta = \eta_{ex}$ condition (see Chapter 6.2.5), i.e., such DMR configuration that the first slipstream joins the second Mach stem at the wedge surface (Figs. 26, 27). Therefore, above the $\chi' = 0$ line Mach reflections have TDMR features. At this configuration $\chi' = 0.6^\circ$. Unfortunately, we cannot
predict second triple-point trajectory angles \( \chi' \) very accurately (see discussion in Chapter 6.5) and the line \( \chi' = 0 \) in graphs 35-36 does not mean that the second triple point lies on the wedge surface, but serves as an approximation of the \( \eta = \eta_{ex} \) line.

The detachment criterion line was incorporated on graphs 35-36 as a dashed line. This line was calculated with two-shock theory and was generally accepted as the RR-MR transition line in a nonstationary flow. It is worth noting that in a very high \( p_0 \) nonstationary flow (or in inviscid flow) we get the wind-tunnel case (steady flow) from the point of view of the boundary-layer consideration. Consequently, we have one universal criterion for RR termination for both flows.

The RR reflections on Figs. 35-36 that are located below the RR-MR transition line \( \chi = 0 \) for a specific \( p_0 \) (i.e., \( p_0 = 10 \) torr) result from experiments conducted at lower \( p_0 \). Similarly, MR above the transition lines for a specific \( p_0 \) result from experiments conducted at higher \( p_0 \).

In Fig. 35 the initial pressure \( p_0 \) is given for all RR and by tracing all RR below the \( p_0=100 \) torr line, it can be seen that all regular reflections below this line have a lower \( p_0 \) than 100 torr. Similarly, all RR below the \( p_0 = 10 \) torr line have \( p_0 \) below 10 torr, and RR below \( p_0 = 4 \) torr have \( p_0 \) less than 4 torr.

The opposite trend can be observed with experimental data for MR reflections. There are no MR experimental results at \( \theta_w \) above the MR-RR viscous boundaries for 4, 10 or 100 torr for experiments conducted at \( p_0 \) lower than that of the boundary, i.e., MR above the MR-RR boundary for \( p_0=4 \) torr have \( p_0 \) higher than 4 torr, and so on. However, the agreement of the MR results with the predicted boundaries cannot be used to verify the procedure employed (Appendix E) as the boundary lines are forced to fit the MR experimental results.

It can be seen from graphs 35-36 that the influence of initial pressure \( p_0 \) on the MR-RR transition lines is quite high - an increase in \( p_0 \) from 10 to 100 torr can increase \( \theta_w \) at MR-RR boundary for over 10°. It can be also seen directly from the experimental data. Experimental results at \( \theta_w = 40^\circ \) and \( M_S = 4\pm0.3 \) (Fig. 35a) show that experiments conducted at \( p_0=4.0 \) and 4.9 torr are RR, but for \( p_0 = 11.4, 16.8 \) torr - MR.

Another important initial pressure \( p_0 \) for practical implementation of the oblique-shock-wave reflection analysis is the atmospheric pressure, for example, the case of an above-the-ground blast [58]. The extension of the procedure developed in the present work for \( p_0 = 1 \) atm would not yield very reliable results due to a very big difference between the experimental pressures used (4-20 torr) and 1 atm. Nevertheless, the viscous MR-RR boundary for \( p_0 = 1 \) atm on a \( (M_S-\theta_w) \) plane would lie between the line for 100 torr and \( p_0 = \infty \) (inviscid case).
A detailed description of the method of calculation used in constructing Figs. 35-36 is given in Appendix E.

6.5 Examination of a Second-Triple-Point Trajectory

Although the second triple point T' appears in DMR, its trajectory angle $\chi'$ is usually calculated in the same way as the kink position K in CMR. As SMR just terminates, the kink is at the first triple point T and $\chi' = \chi$. As $M_s$ increases, CMR occurs and $\delta$ (Figs. 4-6) is slightly larger than 90°, and $\chi' > \chi$. For a stronger flow $\delta$ is increasing but the distance TT' is increasing as well keeping $\chi'$ greater than $\chi$. At the onset of DMR, the portion of the reflected shock wave between T and T' has to bend further down to ensure a greater density change across the kink.

The pressure in regions (4) and (5) pushes the second triple point T' upstream and $\chi'$ gets closer to $\chi$, and for sufficiently low $\gamma$, becomes smaller than $\chi$ (Fig. 6c). Finally, for certain wedge angles at higher $M_s$ a shock-wave configuration with $\chi' = 0$, TDMR occurs (Fig. 6d).

6.5.1 Comparison with the Analytical Prediction

There are four main analytical methods for predicting the location of a kink on the reflected shock. Gvozdeva [40] suggested that the kink must correspond to the condition that the second Mach stem is a sonic wave. Consequently, this method gives good results for CMR only.

Law's [15] and Ikui's [44] methods differ in the calculation of the distance between T and T' and yield more reliable results for higher $M_s$ (DMR). The fourth, and the most convenient and accurate method for the prediction of $\chi'$, was developed by Ben-Dor [11]. He assumed that the second triple point T' with respect to the first triple point T moves with the same horizontal velocity as the induced flow behind the incident shock wave and derived an empirical relation:

$$\chi' = 90^\circ - \theta_w - \tan^{-1} \frac{1 - \rho_0/\rho_1}{\cot \phi_0 - \rho_0/\rho_1 \cot (\phi_1 - \phi_0)}$$  (6.5)

The three-shock theory can be used to calculate $\rho$, $\phi$ and $\theta$ and compare $\chi'$ (calculated with Eq. 6.5) with $\chi'(ex)$.

During the present investigation of TDMR in SF$_6$ and isobutane, the relation (6.5) was found to be quite accurate, i.e., $\Delta \chi' = \chi'$ (Eq. 6.5), $\chi'(ex)$ is below 0.7°. A $\chi' = 0$ line calculated according to Eq. (6.5) and three-shock theory is presented on the $(M_s-\theta_w)$-plane in Figs. 35-36. Three experiments were conducted at $\theta_w = 30^\circ$ in isobutane (Fig. 36a) to check the accuracy of the $\chi' = 0$ prediction. Based on these experimental results, we can expect the $\chi'$ angle close to 0.6° at the $\chi' = 0$ line for higher $M_s$. Moreover, for these experiments, the second Mach stem joins the first slipstream on,
or very close to, the wedge surface (Figs. 26, 27). Therefore, the \( \chi' = 0 \) line (Eq. 6.5) was used to delineate the TDMR region on Figs. 35-36.

The numerical simulations of Glaz et al. [48,51], who simulated the whole flowfield are very accurate in their first and second triple-point prediction for the inviscid case up to the detachment criterion line. It appears that they forced RR solutions for all cases above this line, and that is where the discrepancies with the experimental results start to appear. They could not find a configuration with the second triple point exactly at the rigid boundary, which agrees with the assumption based on physical analysis, that the second Mach stem and the second slipstream must exist in TDMR.

7 DISCUSSION

The hypothesis that RR terminates at the mechanical equilibrium line for an inviscid flow in a pseudostationary flow is difficult to prove explicitly by an experiment. We can decrease the boundary-layer effect using high initial pressures. For example, an analysis of an air blast, i.e., test pressure \( p_0 = 1 \) atm, should show a high persistence of RR above the detachment criterion line - close to the mechanical equilibrium line. Presently conducted experiments show clearly that an increase in \( p_0 \) lifts the RR-MR transition point on a \((\theta W-M_S)\) plane (Figs. 35-36).

The detachment criterion was considered valid for nonstationary flows for a long time due to good agreement with the shock-tube experimental results. Though hard to believe, this agreement was accidentally caused by the range of initial pressures used in the experiments.

As one can see from Fig. 8 the detachment criterion line lies approximately at \( \theta W = 50^\circ \) for \( \gamma = 1.4 \). The RR-MR boundaries for various \( p_0 \) would probably look qualitatively as in Figs. 35-36. It means that the RR-MR transition line at \( p_0 = 100 \) torr would cross the sulfurhexafluoride (Fig. 35) detachment criterion line at \( M_S = 2.2 \), \( p_0 = 10 \) torr at \( M_S = 3.9 \) and \( p_0 = 4 \) torr at \( M_S = 4.4 \). These initial pressures were used approximately by other researchers at these Mach numbers [45]. Consequently, experimentalists were able to observe RR below the detachment criterion line at lower Mach numbers \( M_S < 2.2 \) ('von Neumann paradox').

In other words, for higher gamma-gases the detachment criterion line coincides quite closely with the RR-MR viscous transition points for the initial pressures used in experiments. This is not the case in low \( \gamma \)-gases where the detachment criterion line is significantly lowered at higher \( M_S \). As one can see from Figs. 35-36 the RR experiments at \( \theta W = 45^\circ \) and \( 1.5 < M_S < 2.5 \) lie below, and at \( M_S > 4.5 \) MR lie above the detachment criterion line.

The accuracy of Figs. 35-36 is limited by the interpolations used in constructing these graphs (see discussion in Appendix E). A higher number of experiments would undoubtedly increase the accuracy. Nevertheless, the
trend is clear - the higher the $p_0$, the closer the RR-MR transition line is to the mechanical equilibrium criterion line. Eventually, for a very high $p_0$, when the boundary layer effect is negligible, i.e., essentially in the inviscid case, we are at the mechanical equilibrium criterion line. This conclusion is a consequence of an experimentally observed fact that MR-RR transition occurs at $\chi = 0$. Consequently, MR-RR transition lines for various $p_0$ (Figs. 35-36) are $\chi = 0$, or mechanical equilibrium, lines for various degrees of viscous effects.

As explained in Appendix E and Chapter 6.4, these lines are constructed by interpolating experimental results of $\chi$ to zero value. Therefore, these lines are forced to fit experimental results of Mach reflections. Consequently, the agreement of MR results with the predicted boundaries can be used only to analyze the accuracy of interpolations used (mainly the accuracy of Eq. 6.4.1)

This is not the case with RR experimental results, which were not used in constructing graphs 35-36. The excellent agreement of RR results with predicted MR-RR transition boundaries proves that the transition occurs at $\chi = 0$.

An analytical, inviscid prediction of $\chi$ is not accurate, so the mechanical equilibrium criterion line can only be approximated by the $\chi = 0$ line calculated with the pseudostationary set of equations (Appendix A). In the steady case the mechanical equilibrium criterion line for the inviscid flow can be found experimentally in the wind-tunnel. According to the hypothesis proposed here, the same criterion governs MR-RR transition for both flows and the $\chi = 0$ line for the inviscid, nonstationary flow can be taken directly from the wind-tunnel experiments.

Hornung and Taylor [23] examined the boundary-layer effect on the MR-RR transition in argon at $M_\infty = 5.5$. They described the influence of $p_0$ on the RR-MR transition and qualitatively explained the RR persistence below the detachment criterion line (von Neumann paradox). However, the inconsistency of their procedure and simplifications used in their calculations make their final conclusion unreliable. Hornung and Taylor interpolated their experimental $\chi$ to zero to determine the MR-RR transition points for several $p_0$ (Fig. 33), i.e., they used the mechanical equilibrium criterion at this stage. Subsequently, they assumed the linear relation:

$$\alpha = \alpha_{\text{inv}} + C_1 \frac{\chi}{\lambda}^{1/2}$$

where $\alpha = 90^\circ - \theta_w$, and found the inviscid RR-MR limit $\alpha = 35^\circ$, as predicted by the detachment criterion (Fig. 34). Consequently, at this limit the Mach stem must behave exactly as at the viscous transition points used in the interpolation, i.e., it must have an infinitely small length on the MR side as stated by the mechanical equilibrium criterion which contradicts their quantitative result. Hornung and Taylor used the same value of velocity in region (3) - $V_3$ (corresponding to $\alpha = 35^\circ$) to calculate the viscous length
scale \( \lambda_v = \mu_3/\rho_3 V_3 \) at the transition points for various \( p_0 \). As the \( \alpha(\text{trans}) \) is different for different \( p_0 \), \( V_3 \) is different as well. Moreover, \( \lambda \) - the smallest resolvable length, which corresponds to distance \( x \) in Fig. 14 - may be different for different values of \( \alpha \) and \( p_0 \). Therefore, the relation \( \alpha_{\text{trans}}(p_0) vs. (\lambda_v/\lambda)^{1/2} \) is not linear and graph 34 cannot yield accurate results. The use of pseudostationary analogy in a nonstationary flow was substantiated both analytically and experimentally. The most noticeable obstacle in applying the steady-flow analysis in a nonstationary case was the accepted existence of different MR-RR transition criteria for steady and nonstationary flows.

The most convincing explanation for this differentiation was proposed by Hornung [23], who claimed that in a nonstationary case, unlike the steady case (Fig. 1b), there exists no subsonic path of information that could justify the growth of the Mach stem along the wedge. That might be true for the RR case, but for the MR case there is a subsonic path (under the first slipstream, the second Mach stem, and the second slipstream) that enables the corner signal to influence the shock wave's confluence point.

The existence of the subsonic path of information in RR could be verified by the experimental analysis of MR-RR and RR-MR transitions on concave and convex wedges. If the information path does not exist in RR, the transition wedge angle would be higher on a concave wedge - MR changes to RR - than on a convex wedge when RR changes to MR. Such experiments were conducted by Ben-Dor et al [17] and Itoh et al [18] to explain the poor agreement of the experimental results of Taub et al [6,9] and Kawamura et al [41] with the detachment criterion. The MR-RR boundary was found to be different than the RR-MR boundary. For example, at \( M_s = 4.0 \), \( \theta_w \) (MR-RR) = 67°, but \( \theta_w \) (RR-MR) = 38°. Itoh et al proposed a method of calculation of MR-RR and RR-MR boundaries based on 'ray shock theory'.

Some authors [17,18] assumed that on a curved wedge a length scale is always available and consequently disregarded the length scale aspect. Moreover, since the experimentally found RR-MR transition was approximately 10° below the detachment criterion line and MR-RR a few degrees above the mechanical equilibrium criterion line they concluded that neither of these criteria governs the RR-MR or MR-RR transition in a truly nonstationary flow. However, the occurrence of RR below the detachment criterion line can be caused by the boundary-layer effect [23,46,43] and the MR existence above the \( \chi = 0 \) line can be caused by inaccurate \( \chi \) prediction by the three-shock theory (see Chapter 6.3). Therefore, Hornung's concept that RR-MR transition occurs at the sonic criterion appears to be more convincing.

8 CONCLUSIONS

An experimental and numerical investigation was conducted on oblique-shock-wave reflections in low gamma gases, sulfur hexafluoride and isobutane. The persistence of Mach reflections above the "detachment criterion line" was confirmed for higher initial shock Mach numbers.
A large influence of boundary-layer effects at low initial pressures in the shock-tube experiments was observed and assessed quantitatively.

Terminal-double-Mach reflection was investigated experimentally and analytically. The following main conclusions were derived from the experimental results and the calculations:

1. The von Neumann detachment criterion is inappropriate and should be replaced with the "mechanical-equilibrium criterion".

The existence of the boundary layer causes a shrinking of the Mach stem in MR and consequently can change the expected pattern of MR to RR. According to the boundary-layer equations the boundary layer thickness is larger for the lower initial pressure. For low-gamma gases the detachment criterion line is significantly lowered compared with higher-gamma gases. Therefore, the Mach stem in MR is not completely swallowed at higher wedge angles above the detachment criterion line. For lower $p_0$, the boundary-layer effect increases and the MR-RR transition line is forced down in the $(M_s-\theta_w)$-plane. For higher $p_0$, the MR-RR boundary moves toward the mechanical-equilibrium criterion line.

Therefore, for the inviscid case, the MR-RR boundary should be the mechanical-equilibrium criterion, the same as in a wind tunnel, i.e., the steady-flow case, thus providing a universal criterion for steady and nonstationary flows.

The viscous boundary-layer effect that causes persistence of RR below the mechanical equilibrium criterion line can be predicted using the procedure described in Chapter 6.4. and Appendix E.

2. Terminal-double-Mach reflection should be introduced as a separate, fifth type of oblique-shock-wave reflection.

As TDMR cannot be considered either as a DMR or as RR it should be treated as a separate type of reflection. TDMR features start to appear when the second Mach stem joins the first slipstream. This can be approximated on a $(M_s-\theta_w)$-plane by the $\chi' = 0$ line, calculated with Eq. (6.5).

A reflection pattern with $\chi' = 0$ cannot occur, based on physical grounds, since a second Mach stem and a second slipstream must exist. The TDMR configuration pattern readily appears only in low gamma gases, but can appear in initially high $\gamma$-gases if real-gas effects reduce $\gamma$ to a low value [25].
REFERENCES


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<td>15.3</td>
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<td>24.5</td>
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</tr>
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<td>24.5</td>
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<tr>
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<tr>
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<td>2.22</td>
<td>33.0</td>
<td>23.2</td>
<td>0.00</td>
<td>RR</td>
</tr>
</tbody>
</table>
Fig. 1a. Regular reflection in steady flow. Physical (left) and $(\theta, p)$-plane (right).

Fig. 1b. Mach reflection in steady flow. Physical (left) and $(\theta, p)$-plane (right).

$W =$ width of wedge surface, $I =$ incident shock wave, $1, 2, 3, 4, 5 =$ physical states, $R =$ expansion wave, $P =$ reflection or confluence point, $S =$ Mach stem, $\theta_w =$ wedge angle, $\phi =$ wave angle, $q_1 =$ initial velocity, $V =$ slipstream, (Ref. 46).
Figure 2a. Schematic diagram of regular reflection (RR)

$\psi$ - reflected-shock-wave angle
Figure 2b. Two possible reflected-shock-wave configurations at point P in RR

(a) straight reflected shock
(b) curved reflected shock
Figure 2c. Possible bow-shock configurations
(a) straight, attached shock
(b) curved, attached shock
(c) curved, detached shock
Figure 3. Schematic diagram of single-Mach reflection (SMR)
I - incident shock wave, R - reflected shock wave,
M - Mach stem, S - slipstream, T - triple point,
$\theta_w$ - wedge angle, $\chi$ - triple-point trajectory angle,
$\phi$ - incidence angle, $\theta$ - deflection angle
Figure 4. Schematic diagram of complex-Mach reflection (CMR)
I - incident shock wave, R - reflected shock wave,
M - Mach stem, S - slipstream, K - kink, T - triple point,
θ_w - wedge angle, X - triple-point trajectory angle,
Figure 5. Schematic diagram of double-Mach reflection (DMR).

\( \theta_w \) - wedge angle, \( I \) - incident shock wave,
\( R, R' \) - first and second reflected shock waves,
\( M, M' \) - first and second Mach stems,
\( T, T' \) - first and second triple points,
\( S, S' \) - first and second slipstreams,
\( \chi, \chi' \) - first and second triple-point trajectory angles
\( \psi' \) - reflected-shock-wave angle
Fig. 6. Schematic diagrams of four patterns of double-Mach reflection.
Fig. 6. (continued) Schematic diagrams of four patterns of double-Mach reflection.
Fig. 7. Shock-polar illustrating three different RR termination criteria. 
I - incident shock wave; 
R₁ - reflected shock wave of RR at A; 
Rₘ, Rₛ, Rₐ - reflected shock wave at RR termination 
( Rₘ - mechanical equilibrium criterion at D, 
Rₛ - sonic criterion at E + F, 
Rₐ - detachment criterion at B + C).
Figure 8. Regular to Mach reflection transition boundary as defined by two theoretical inviscid criteria ($\chi = 1.4$)
flow vector in region I in laboratory frame $U_{1x}$

flow velocity in region I from kink

velocity vector of kink in laboratory frame

Fig 9 (a). Schematic diagram illustrating assumption of position of $K$ or $T'$ and flow direction in region (1) relative to $K$. 

same as $U_{1x}$
Fig. 9(b) Definitions of $\phi_{1k}$, $\phi'_{1k}$, $p_2$ and $p_2'$.

Fig. 9(c) Four possible geometric configurations of reflected shock wave.
Fig. 10 Transition lines for frozen isobutane ($\gamma=4/3$) and experimental results.
Fig. 11 Transition lines for frozen sulfur hexafluoride (γ=4/3) and experimental results (Hu's [45] and present).
Figure 12. Physical model of the RR reflection process at point P in the presence of a laminar boundary layer [23]
Figure 13a. Laminar-boundary-layer temperature profile behind a shock wave travelling at $M_s = 2.0$ into air at $P_0 = 2$ kPa, $T_0 = 300$ K

- based on Mirels [55].
Figure 13b. Laminar-boundary-layer velocity profile behind a shock wave travelling at $M_s = 2.0$ into air at $p_0 = 2$ kPa, $T_0 = 300$ K

- based on Mirels [55].
Fig. 14 Boundary-layer displacement thickness $\delta^*$ in Mach reflection and definition of actual wedge angle $\theta_w$, apparent wedge angle $\theta_w''$ and leading edge distance $x$ of boundary layer which determines the tangent for $\theta_w''$. 
Figure 15: Schematic diagram of test equipment associated with UTIAS 10 x 18 cm Hypervelocity Shock Tube [36-39]
Fig. 14 Boundary-layer displacement thickness $\delta^*$ in Mach reflection and definition of actual wedge angle $\theta_W$, apparent wedge angle $\theta_W''$ and leading edge distance $x$ of boundary layer which determines the tangent for $\theta_W''$. 
Figure 15: Schematic diagram of test equipment associated with UTIAS 10 x 18 cm Hypervelocity Shock Tube [36-39]
Fig. 16 Infinite-fringe interferogram showing instability of incident shock-wave and region behind it in isobutane. $M_s = 4.2$, $\theta_W = 37^\circ$, $p_0 = 11.2$ torr, $T_0 = 24^\circ$C.
Fig. 17 Infinite-fringe interferogram of a detached regular reflection in isobutane. $M_s = 1.58$, $\theta_w = 45^\circ$, $P_0 = 96$ torr, $T_0 = 297.3$ K.
Fig. 18 Infinite-fringe interferogram of a regular reflection with λ-shock-wave configuration in isobutane. $M_s = 2.2$, $\theta_w = 45^\circ$, $p_0 = 33$ torr, $T_0 = 296.9$ K.
Fig. 19 Infinite-fringe interferogram of a detached SMR in isobutane. $M_s = 1.58$, $\theta_w = 37^\circ$, $p_o = 112$ torr, $T_o = 298.1$ K.
Fig. 20 Infinite-fringe interferogram of an attached SMR in isobutane. $M_s = 2.2$, $\theta_w = 10^\circ$, $p_0 = 33$ torr, $T_0 = 298$ K.
Fig. 21 Infinite-fringe interferogram of a detached CMR in isobutane. $M_s = 1.7$, $\theta_w = 42^\circ$, $p_0 = 60$ torr, $T_0 = 296.8$ K.
Fig. 22 Infinite-fringe interferogram of a transitional case between CMR and SMR in isobutane. Rough wedge was used. $M_s = 2.25$, $\theta_w = 30^\circ$, $p_0 = 66$ torr, $T_0 = 296.3$ K.
Fig. 23 Infinite-fringe interferogram of CMR in isobutane past the upper corner of the wedge. $M_s = 1.75$, $\theta_w = 40^\circ$, $p_0 = 40$ torr, $T_0 = 297.5$ K.
Fig. 24 Infinite-fringe interferogram of DMR with visible compression waves and $\lambda$-shock configuration in isobutane. $M_S = 2.2$, $\Theta_W = 37^\circ$, $P_0 = 33$ torr, $T_0 = 299.3$ K.
Fig. 25 Infinite-fringe interferogram of DMR in isobutane. $M_s = 2.6$, $\theta_w = 37^\circ$, $p_0 = 12$ torr, $T_0 = 296.3$ K.
Fig. 26 Shadowgraph of an attached DMR in isobutane; the second Mach stem joins the first slipstream at the wedge surface. $M_s = 6.44$, $\theta_w = 30^\circ$, $p_0 = 4.0$ torr, $T_0 = 296.3$ K.
Fig. 27 Shadowgraph of an attached DMR with convex Mach stem in isobutane. $M_S = 7.75$, $\theta_W = 30^\circ$, $p_0 = 4.4$ torr, $T_0 = 297.3$ K.
Fig. 29a Shadowgraph of an attached TDMR in isobutane. $M_s = 6.77$, $\theta_w = 40^\circ$, $P_0 = 4.0$ torr, $T_0 = 293.3$ K.
Fig. 28b Infinite fringe interferogram of an attached TDMR in isobutane. 
$M_s = 4.84$, $\theta_w = 42^\circ$, $p_o = 15.3$ torr, $T_o = 298$ K.
Fig. 29 Comparison of experimental values of $\psi'$ in TDMR with calculated values of $\psi$ for RR case at $\theta_w=40^\circ$ in sulfur hexafluoride.
Fig. 30 A schematic diagram of DMR configuration where the second Mach stem $M'$ joins the first slipstream $C$ at the wedge surface. Definition of angle $\eta$. 
Fig. 31 Interpolation of $\Delta x$-$p_0$ relationship with $\Delta x=C_1+C_2 p_0^{-1/2}$ and $\Delta x=C_2 p_0^{-1/2}$ curves. Experimental points taken from experiments in SF$_6$ at $\theta_w=37^\circ$, and $M_5=6.0$. 

$\Delta x = 2.4 p_0^{-1/2} = C_2 p_0^{-1/2}$
$M_S, P_0 = \text{const.}$

Fig. 32 Illustration of the procedure used in finding MR-RR transition point $\theta_w(MR-RR)$ at constant $M_S$ and $P_0$. 
Fig. 33. Experimental results of Hornung and Taylor [23] in argon. Straight line interpolation of \( \tan \chi \) to zero value was used to find MR-RR transitions for: ○ \( p_0 = 70 \) torr, □ 25 torr, △ 10 torr, ○ 5 torr.

Fig. 34. Straight line interpolation of experimentally found MR-RR viscous transitions to the inviscid value used by Hornung and Taylor [23]. \( \alpha^* \) is the MR-RR transitional value.
Incident Shock Wave Mach No. $M_s$

Fig. 35 Comparison of predicted domains of various types of reflections and experimental results (initial pressure for RR given in torr) in the $(M_s - \theta_w)$ plane for vibrational equilibrium SF6. Solid lines are for the inviscid case, dashed lines are RR-MR boundaries for $P_0 = 100, 10$ and 4 torr.
Fig. 36 Comparison of predicted domains of various types of reflections and experimental results (initial pressure for RR given in torr) in the $(M_s-\theta_w)$ plane for vibrational equilibrium isobutane. Solid lines are for inviscid case, dashed lines are MR-RR boundaries for $P_0=100$, 10 and 4 torr.
Appendix A

Method of Calculations

Reflections Inside a Shock Tube

Since the planar incident shock wave moves with constant velocity along the wedge surface, the entire reflection phenomenon can be considered as pseudo-stationary in a frame of reference attached to a point which moves with a constant velocity. Therefore, instead of three independent variables \( x, y \) and \( t \), the phenomenon is described in terms of \( x/t \) and \( y/t \) and the flow is self-similar \([19, 49]\). In a pseudo-stationary flow, the shock wave configuration remains self-similar and grows linearly with time from the moment the incident shock wave collides with the wedge corner. The phenomenon is also assumed to be a two-dimensional inviscid flow.

The calculations for obtaining the flow properties of each region in the angular vicinity of a shock confluence point require the formulation of the oblique-shock-wave relations for each shock wave. The reference point is the reflection point \( P \) in \( RR \), and the triple point \( T \) in \( MR \); the region which are divided by shock waves and a slipstream are designated by 0 to 3 as shown in Fig. 3a and 4. The details of the analytical formulations can be found in Law \([59]\). The physical quantities on both sides of each shock wave in the vicinity of the reference point satisfy the following equations of motion:

Conservation of tangential velocity

\[
\rho_i \tan \phi_i = \rho_j \tan (\phi_i - \theta_j) \tag{A.1}
\]

Continuity

\[
\rho_i U_i \sin \phi_i = \rho_j U_j \sin (\phi_i - \theta_j) \tag{A.2}
\]

Normal momentum

\[
P_i + \rho_i U_i^2 \sin^2 \phi_i = P_j + \rho_j U_j^2 \sin^2 (\phi_i - \theta_j) \tag{A.3}
\]

Energy

\[
h_i + \frac{1}{2} U_i^2 \sin^2 \phi_i = h_j + \frac{1}{2} U_j^2 \sin^2 (\phi_i - \theta_j) \tag{A.4}
\]

where \( i \) and \( j \) refer to the states ahead and behind the shock, respectively. The equations of state applied are

\[
\rho = \rho(P, T), \quad h = h(P, T) \tag{A.5}
\]

In \( RR \), use two-shock theory, there are two sets of equations:

For incident shock I: \( i = 0, j = 1 \) \( \tag{A.6} \)

For reflected shock R: \( i = 1, j = 2 \) \( \tag{A.7} \)
These two sets of equations are solved under the boundary condition that the flow deflections across the incident and reflected shocks be equal and opposite with the direction of the flow behind the reflected shock wave being along the wedge surface, i.e.,

\[ \theta_1 = \theta_2 \]  
(A.8)

The calculating procedures for the solution of equation (A.5) through (A.8) are as follows [11]:

1. The initial conditions of the pressure \( P_0 \), the temperature \( T_0 \), the Mach number \( M_s \) and the wedge angle \( \theta_w \) are given.

2. The first set of equations (A.6) is solved for \( P_1, T_1, U_1, \) and \( \theta_1 \). Note that \( U_0 = U_s \sec \theta_w \) and \( \phi_0 = 90^\circ - \theta_w \).

3. Some initial guess for \( \phi_1 \) is made.

4. The second set of equations (A.7) is solved to obtain the values for \( P_2, T_2, U_2 \) and \( \theta_2 \).

5. The calculated results are checked against the boundary condition (A.8).

6. A new value of \( \phi_1 \) is predicted.

7. The procedure is repeated until

\[ |\theta_1 - \theta_2| < 10^{-4} \]

In the case of MR, use three-shock theory, there are three sets of equations for the first triple-point system:

For incident shock I:  
\[ i = 0, \ j = 1 \]  
(A.9)

For reflected shock R:  
\[ i = 1, \ j = 2 \]  
(A.10)

For Mach stem M:  
\[ i = 0, \ j = 3 \] (\( \phi_1 \) is referred to as \( \phi_3 \))  
(A.11)

The above three sets of equations are solved under the boundary conditions that the flow deflection across the incident and reflected shocks be equal to the flow deflection across the Mach stem and the pressure on both sides of the slipstream be identical, i.e.,

\[ \theta_3 = \theta_1 - \theta_2 \]  
(A.12)

\[ P_2 = P_3 \]  
(A.13)

In MR, it is necessary to predict a value of the triple-point trajectory angle \( \chi \) in order that the system of equations be closed. The Mach stem is assumed normal to the wedge surface based on experimental observations, and Law and Glass [15] introduced an additional geometrical relation

A.2
\[ \phi_3 = 90^\circ - \chi \]  

(A.14)

The calculating procedures for the solution of equations (A.5) and (A.9) through (A.14) are as follows [11]:

1. The initial conditions of \( P_0, T_0, M_s \) and \( \theta_w \) are given.
2. Some initial guesses for \( \chi \) and \( \phi_1 \) are made.
3. The first set of equations (A.9) is solved for \( P_1, T_1, U_1 \) and \( \theta_1 \). Note that \( U_0 = U_s \cos \phi_0 \) and \( \phi_0 = 90^\circ - (\theta_w + \chi) \).
4. The second set of equations (A.10) is solved for \( P_2, T_2, U_2 \) and \( \theta_2 \).
5. Using Eq. (A.14), the third set of equations (A.11) is solved for \( P_3, T_3, U_3 \) and \( \theta_3 \).
6. The calculated results are checked against the boundary conditions (A.12) and (A.13).
7. New values of \( \chi \) and \( \phi_1 \) are predicted.
8. The solutions are iterated until

\[ |\theta_3 - \theta_1 + \theta_2| < 10^{-4} \]

and

\[ |(P_3 - P_2)/P_2| < 10^{-4} \]

Details of the analytical formulation of the second triple point \( T' \) is given by Ben-Dor [11]. The flow fields around \( T' \) are shown in Fig. A1 with the prime to denote that the properties are measured with respect to \( T' \). The three set of equations for the system are:

For reflected shock \( R \): \[ i = 1, \ j = 2 \]  

(A.15)

For second reflected shock \( R' \): \[ i = 1, \ j = 4 \] \( \phi_4 \) is referred to as \( \phi_4 \)

(A.16)

For second Mach stem \( M' \): \[ i = 2, \ j = 5 \]

(A.17)

The above three sets of equations are solved under the boundary conditions that the flow deflection across the reflected shock \( R \) and the second Mach stem \( M' \) be equal to the flow deflection across the second reflected shock \( R' \), and the pressure on both sides of the second slipstream \( S' \) be identical, i.e.,

\[ \theta_4 = \theta_2 - \theta_5 \]  

(A.18)

\[ P_4 = P_5 \]  

(A.19)

The calculation involves a transformation of the already calculated values of \( U_1 \) and \( \phi_1 \) from a frame of reference attached to \( T \) to a frame of reference attached to \( T' \). Since thermodynamic properties do not depend on the frame of reference, the prime is omitted from the thermodynamic variables in the equations. The values of \( U_1' \) and \( \phi_1' \) are given as
The calculating procedures for the solution of equations (A.5) and (A.15) through (A.21) are as follows:

1. The initial conditions of \( P_1, T_1, M_1, \phi_0, \phi_1 \) and \( \theta_1 \) are given.
2. Some initial guesses for \( \phi_2 \) and \( \phi_4 \) are made.
3. Using equations (A.20) and (A.21), the first set of equations (A.15) is solved for \( P_2, T_2, U_2 \) and \( \theta_2 \).
4. The second set of equations (A.16) is solved for \( P_4, T_4, U_4 \) and \( \theta_4 \).
5. The third set of equations (A.17) is solved for \( P_5, T_5, U_5 \) and \( \theta_5 \).
6. The calculated values are checked against the boundary conditions (A.18) and (A.19).
7. New values of \( \phi_2 \) and \( \phi_4 \) are predicted.
8. The solutions are iterated until

\[
|\phi_4' - \theta_2' + \theta_5'| < 10^{-4}
\]

and

\[
|P_4 - P_5|/P_5 < 10^{-4}
\]
Fig. A1 The analogy between the two triple-point wave systems in a double-Mach reflection.
APPENDIX B
THERMODYNAMIC PROPERTIES OF SULFUR HEXAFLUORIDE

The polyatomic SF₆ molecule has one sulfur atom surrounded by six fluorine atoms in the form of a regular octahedron with 15 vibrational degrees of freedom. The characteristic modes of vibration of SF₆ with corresponding degeneracy factors and wave numbers are shown in Fig. B1.

The thermodynamic quantities are given by

\[ p = \rho \frac{R}{T/m}, \quad a = \sqrt{\gamma p/\rho}, \quad U = M a \]

In the frozen-gas model, only translational and rotational degrees of freedom are excited to the new equilibrium state and the other degrees are frozen at the initial state, thus

\[ h = \frac{\gamma_0}{(\gamma_0 - 1)} \frac{R}{T/m} \]

where

\[ \gamma_0 = \frac{4}{3} \]

and

\[ \gamma = \gamma_0 \]

In the equilibrium-gas model, all vibrational, translational and rotational modes are excited to the new equilibrium state immediately behind the shock wave. The enthalpy and specific heats ratio are given by

\[ h = \frac{\gamma_0}{(\gamma_0 - 1)} \frac{R}{T/m} + \sum \frac{T_k}{\exp(T_k/T) - 1} \frac{R}{m} \]

\[ \gamma = \frac{\gamma/(\gamma_0 - 1) + V}{1/(\gamma_0 - 1) + V} \]

where

\[ V = \sum \frac{T_k}{\exp(T_k/T) - 1} \]

and

\[ T_k = h\nu_k/K \]

with \( h \) being the Planck constant and \( K \) the Boltzmann constant.

B.1
Fig. B1 Characteristic modes of vibration of sulfur hexafluoride with corresponding degeneracy numbers, wave numbers and characteristic vibrational temperatures.
Fig. B2  Vibrational relaxation length behind shock wave vs $M_s$ at initial conditions of $p_0 = 15$ torr, $T_0 = 300$ K for various gases (after Ref. [45]). The characteristic vibrational temperatures and degeneracy numbers for the different gases are given in the table.
**APPENDIX C**

**PROPERTIES OF ISOBUTANE (2-METHYL PROPANE) - CH(CH₃)₃ [53]**

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<td>Normal fp in air at 101.3 kPa (1 atm), K</td>
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<tr>
<td>Normal bp at 101.3 kPa, K</td>
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<td>Flammability limits at 293.15 K and 101.3 kPa, vol%</td>
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</tr>
<tr>
<td>in air, lower</td>
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</tr>
<tr>
<td>upper</td>
<td></td>
</tr>
<tr>
<td>in oxygen, lower</td>
<td>1.8</td>
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<tr>
<td>upper</td>
<td></td>
</tr>
<tr>
<td>Autoignition temperature at 101.3 kPa, K</td>
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<tr>
<td>in air</td>
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<td>in oxygen</td>
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<tr>
<td>Heat of combustion, net, kJ/mol</td>
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<td>H, kJ/mol</td>
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<td>Density, kg/m</td>
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<td>Vibrational frequencies of isobutane [33, 47]</td>
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<table>
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<td>2950</td>
<td>10</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>36 vibrational modes</strong></td>
</tr>
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</table>

C.1
This appendix outlines the basic technique used in calculating the boundary-layer profile behind a moving shock wave. This method of solution is based on the work by Mirels [55]. Recent more precise methods of solution exist [56,60] but were not used due to their complexity. The reference frame is attached to the shock wave, as shown in Fig. D-1, so that the flow \((U_s - U_1)\) is steady, and the wall moves with a velocity \(U_s\).

The equations which govern the flow in the boundary layer are the same as those for the external flow: continuity, momentum, energy and an equation of state. The boundary-layer equations include terms for viscosity, \(\mu\), and heat conduction, \(k\), and in this solution it is assumed that no pressure gradient exists. For a more general explanation of the equations and their physical meaning, see Schlichting [57].

The boundary layer equations are:

Continuity:
\[
\frac{\partial}{\partial x} (\rho u) + \frac{\partial}{\partial y} (\rho v) = 0 \quad (D.1)
\]

Momentum:
\[
u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \frac{1}{\rho} \frac{\partial}{\partial y} \left( \mu \frac{\partial u}{\partial y} \right) \quad (D.2)
\]

Energy:
\[
\rho c_p \left( u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} \right) = \frac{\partial}{\partial y} \left( k \frac{\partial T}{\partial y} \right) + \mu \left( \frac{\partial u}{\partial y} \right)^2 \quad (D.3)
\]

Equation of State:
\[
p = \rho RT \quad (D.4)
\]

with boundary conditions:
\[
u(x, 0) = u_w \quad (D.5a)
\]
\[
u(x, \infty) = u_e \quad (D.5b)
\]
\[
v(x, 0) = 0 \quad (D.5c)
\]
\[
T(x, 0) = T_w \quad (D.5d)
\]
\[
T(x, \infty) = T_e \quad (D.5e)
\]
Since the flow is continuous, a stream function $\psi$ exists such that

$$\frac{\partial \psi}{\partial y} = \frac{\rho u}{\rho_r} \quad \text{(D.6a)}$$

$$-\frac{\partial \psi}{\partial x} = \frac{\rho v}{\rho_r} \quad \text{(D.6b)}$$

Introduce a similarity parameter:

$$\eta = \sqrt{\frac{u_e}{2xv_r}} \int_0^y \frac{T_r}{T} \, dy \quad \text{(D.7)}$$

and rewrite the stream function:

$$\psi = \sqrt{2u_exv_r} f(\eta) \quad \text{(D.8)}$$

It should be noted that:

$$\frac{u}{u_e} = f' \quad \text{(D.9a)}$$

$$\frac{v}{u_e} = \frac{T}{T_r} \sqrt{\frac{v_r}{2ux_e}} (f + 2xf' \frac{\partial \eta}{\partial x}) \quad \text{(D.9b)}$$

As a simplification, it is assumed that the viscosity and the thermal conductivity are directly proportional to the temperature.

$$\mu = \frac{\mu_r}{T} \quad \text{(D.10a)}$$

$$k = \frac{k_r}{T} \quad \text{(D.10b)}$$
The choice of reference temperature for viscosity and thermal conductivity is somewhat arbitrary at this point. A wise choice of reference temperature may minimize error caused by this simplification [55] and will be discussed later.

When Eqs. (0.8), (0.9) and (0.10) are substituted into the momentum equation, it becomes:

\[ f'' + \sigma f'' = 0 \quad (D.11) \]

with boundary conditions:

\[ f(0) = 0 \quad (D.11a) \]
\[ f'(0) = u_w/u_e \quad (D.11b) \]
\[ f'(\infty) = 1 \quad (D.11c) \]

Substitution of Eqs. (0.8), (0.9) and (D.10) into the energy equation yields:

\[ T'' + \sigma T'' = -\sigma(\lambda-1) M_e^2(f'')^2 \quad (D.12) \]

assuming the Prandtl number \( \sigma \) is constant.

From Eq. (D.12), the temperature distribution can be expressed as:

\[ \frac{T}{T_e} = 1 + \frac{\gamma-1}{2} \left[ \frac{u_w}{u_e} - 1 \right]^2 r(\eta) + \frac{T_w - T_{wi}}{T_e} s(\eta) \quad (D.13) \]

where \( r(\eta) \) satisfies

\[ r'' + \sigma fr' = -\frac{2\sigma}{(u_w/u_e - 1)^2} (f'')^2 \quad (D.14) \]

\[ r'(0) = 0 \quad (D.14a) \]
\[ r(\infty) = 0 \quad (D.14b) \]

\( s(\eta) \) satisfies

\[ s'' + \sigma fs' = 0 \quad (D.15) \]
and
\[ \frac{T_{w,1}}{T_e} = 1 + \frac{\gamma-1}{2} \left[ \frac{u_w}{u_e} - 1 \right]^2 \frac{r(0)}{r(\infty)} \] (D.16)

The temperature distribution is in fact a combination of two solutions: the solution for the case of an insulated wall, plus one for addition of heat transfer through the wall. \( T_{w,1} \) is the temperature which would occur at the wall if it were insulated, and \( r(0) \) is a recovery factor for the recovery temperature \( T_r \) at the wall, which is less than the adiabatic total temperature.

To get \( y \) in terms of \( \eta \), Eqs. (D.7) and (D.13) are combined, giving:
\[
y = \sqrt{2x} \frac{T_e}{T_r} \frac{T_e}{T_r} \left\{ \eta + \frac{\gamma-1}{2} \left[ \frac{u_w}{u_e} - 1 \right]^2 \int_0^\eta r \, d\eta + \frac{T_w - T_{w,1}}{T_e} \int_0^\eta s \, d\eta \right\} \] (D.17)

Another parameter of interest is:
\[
\delta^* = 2x \frac{T_e}{T_r} \sqrt{\frac{\nu}{2xu_e^2}} \left\{ \lim_{\eta \to \infty} (\eta - f) + \frac{\gamma-1}{2} \left[ \frac{u_w}{u_e} - 1 \right]^2 \int_0^\eta r \, d\eta + \frac{T_w - T_{w,1}}{T_e} \int_0^\eta s \, d\eta \right\} \] (D.18)

The distribution of \( \delta^* \) with axial distance is of the form:
\[
\delta^* = C_1 x^{1/2} \] (D.19)

The angle at which flow enters the boundary layer is determined by the rate at which \( \delta^* \) grows. The boundary-layer flow entry angle is defined by:
\[
\tan(\theta_{bl}) = \frac{d}{dx} (\delta^*) = \frac{1}{2} C_1 x^{-1/2} \] (D.20)
Figure D-1: Laminar-boundary-layer velocity distribution in two different reference frames

(a) Laboratory-fixed reference frame
(b) Shock-fixed reference frame
APPENDIX E

PROCEDURE FOR PREDICTING MR-RR TRANSITION LINES FOR VARIOUS INITIAL TEST PRESSURES.

Objective: To find MR-RR transition points for three $p_0$ values ($p_0 = 4, 10, 100$ torr) for a wide range of $M_s$.

A mechanical equilibrium criterion is used to find MR-RR transition points at constant $M_s$ and $p_0$. This method of finding MR-RR boundary is based on the experimentally observed fact that at constant $M_s$ and $p_0$ the first triple-point trajectory angle $\chi$ is a linear function of $\theta_w$ [23] (Fig. 33). Therefore, plotting the experimental results gathered at the same $M_s$ and $p_0$ on a $(\chi-\theta_w)$ plane and interpolating $\chi$ by a straight line to zero yields the MR-RR transition point $\theta_w(\text{MR-RR})$ (Fig. E1).

Another set of experiments is necessary to find MR-RR transition at another $p_0$. In this way, MR-RR transition points at different $p_0$, but for one value of $M_s$ can be found.

Unfortunately, this clearly experimental way of finding $\theta_w(\text{MR-RR})$ requires experimental results at specified initial pressures $p_0 = 4, 10$ and 100 torr for several incident flow Mach numbers $M_s$. Due to the experimental limitations low $p_0$ cannot be used at low $M_s$, similarly, high $p_0$ cannot be used at high $M_s$. Therefore, a way is needed to relate the experimental results achieved at one $p_0$ to predict $\chi_{\text{ex}}$ at another $p_0$, i.e., the method to assess the influence of the boundary layer.

In the present procedure it is done by introducing

$$\Delta \chi = \chi(\text{inviscid}) - \chi(\text{viscous})$$  \hspace{1cm} (E.1)

Then, as shown in Fig. E2, $\Delta \chi$ results are interpolated by a straight line. At the point of intersection with $\chi(\text{inviscid})$, $\chi(\text{viscous}) = \chi(\text{inviscid}) - \Delta \chi = 0$, i.e., the MR-RR transition for viscous flow is obtained. In this approach the relation

$$\Delta \chi = C_1 p_0^{-1/2}$$  \hspace{1cm} (E.2)

can be used to find MR-RR transition (at one $M_s$) for various $p_0$ using one set of experiments. As discussed in Section 6.3.2, this relation is substantiated both analytically and experimentally. $\chi(\text{viscous})$ in Eq. (E.1) can be quite accurately approximated by $\chi_{\text{ex}}$ experimentally observed in a shock-tube. The instabilities (see Section 6.1) and shock-tube window boundary layer limit this accuracy to the practical value of $\Delta \chi_{\text{ex}} = 0.25^\circ$. That is why, for two experiments conducted at the same initial conditions, two different $\chi_{\text{ex}}$ were reported (Tables 1 and 2).
As discussed in Chapter 4, \( \chi \) (inviscid) cannot be observed in a shock-tube (although, according to the present hypothesis it can be taken directly from wind-tunnel experiments in the same test gas) and must be predicted analytically. The best available method was the pseudostationary, three-shock prediction (Appendix A). This method and its accuracy is discussed in Section 6.3.1, and can be assessed as \( \Delta \chi_{an} = 0.6^\circ \).

Therefore, \( \Delta \chi \) used in Figs. E5a-f and Figs. E6a-d, to find MR-RR viscous transition points, were taken from \((\chi - M_s)\)-planes presented in Figs. E3a-d and Fig. E4. In these figures \( \chi \) predicted with three shock theory vs \( M_s \) is plotted along with \( \chi \) experimental values for various wedge angles. \( \Delta \chi \) values for different wedge angles, at one \( M_s \) and \( P_o \), is then located on the \((\chi - \theta_w)\) plane (Figs. E5a-f and Figs. E6a-d). A straight line interpolation yields viscous MR-RR transmission point at constant \( M_s \) and \( P_o \).

It is worthwhile to discuss the effect of this quite low accuracy on predicting the transition wedge angle - \( \theta_w(MR-RR) \). Imagine that \( \chi_{an}(\theta_w) \) is a straight line as in Fig. E2. If \( \chi_{ex} \) is subtracted from the \( \chi_{an} \) line, the \( \Delta \chi \) line is obtained. This line will always cross the \( \chi_{an} \) line at the same \( \theta_w(MR-RR) \) as \( \chi_{ex} \) crosses the \( \chi_{ex} = 0 \) axis in Fig. E1. Neither rotating nor shifting the \( \chi_{an} \) line changes \( \theta_w(MR-RR) \) as presented on Fig. E2. In reality, the \( \chi_{an} \) line is slightly curved (Figs. 32, E5a-f and E6a-d), as is the \( \chi_{ex} \) line, so that only \( \Delta \chi \) is a straight line - this fact does not change the result either. Therefore, the result \( \theta_w(MR-RR) \) is not affected by the accuracy of the \( \chi_{an} \) prediction and depends entirely upon the experimental results - \( \chi_{ex} \) as in Fig. E1. That happens only when \( P_o \) used in the experiment was exactly the same as specified \( P_o \) (4, 10 or 100 torr). For all other experimental data a transformed relation (E.2) has to be used:

\[
\Delta \chi = \left( \frac{P_o}{P_o^*} \right)^{1/2} \Delta \chi \tag{E.2a}
\]

where the underlined quantities are related to \( P_o = 4, 10 \) or 100 torr, and \( \Delta \chi \) and \( P_o \) are experimental data. This relation is non-linear, and \( \Delta \chi \) depends on the accuracy of \( \Delta \chi = \chi_{an} - \chi_{ex} \). As mentioned before, the accuracy of the \( \chi_{an} \) prediction is quite low, so the experimental results at \( P_o \) are related to the closest \( P_o^* \) to ensure the highest accuracy. The mean value of \( \Delta \chi_{an} \) for all experiments taken into account at one wedge angle, one \( M_s \), and one specified \( P_o \), is calculated. Based on this mean value, \( \Delta \chi_{an} \) for the two remaining \( P_o \) are predicted according to the relation (E.2a).

Equation (E.2a) can yield unreasonable results, such as \( \Delta \chi > \chi_{an} \). That is impossible in reality, as \( \chi_{ex} \) cannot be negative, but such results are qualitatively correct and necessary to find the slope of the \( \Delta \chi \) line used to predict \( \theta_w(MR-RR) \). However, if \( \Delta \chi \) is negative, equation (E.2a) cannot be used. In RR cases, \( \Delta \chi = \chi_{an} \) for all specified \( P_o < P_o^* \), consequently, \( \Delta \chi \) ceases to be a function of \( P_o \) and \( \Delta \chi \) for \( P_o > P_o^* \) cannot be predicted. Therefore, RR results cannot be used in this procedure.
Ideally, the experimental data should be gathered at constant $M_s$. In reality, diaphragms do not break at the same driver pressure, so that the experimental data at $M_s = M_s \pm 0.2$ were used. Therefore, $M_s$ was chosen in such a way that at $M_s \pm 0.2$ experimental data existed for at least two wedge angles. Figures 3a-c and 4 were used to find such $M_s$ that sufficient experimental values of $\chi_{ex}$ existed for at least two wedge angles $\theta_w$. The medium value $M_s(\theta_w)$ is calculated for each $\theta_w$, and the mean value for all wedge angles $M_{sm}$ is used to calculate $\chi_{an}$ in graphs E5-E6.

The calculated $\Delta \chi_m$ points generally do not lie in one line (for three or more $\theta_w$ used) and several best fit straight lines can be drawn through them (Figs. E5-E6). An additional condition $\Delta \chi = 0$ at applicable $\theta_w$ is then used. As observed experimentally in the present study (see Section 6.3.1) and by Hu [45], who used the same $\chi_{an}$ prediction method, at $\theta_w = 30^\circ$ the three-shock theory predicts approximately the same $\chi$ values as were observed experimentally. Therefore, $\theta_w(\Delta \chi=0) = 30^\circ$ for a wide range of $M_s$.

The final step involves incorporating the transition points on $M_s-\theta_w$ transition-boundaries plane and plotting a best fit fourth order polynomial curve through them (Figs. 35-36). At high $M_s > 9$ the MR-RR transition lines were extended according to the existing trend, at low $M_s < 1.7$ these lines were intuitively drawn slightly below the mechanical equilibrium line. The accuracy of these plots is checked with the RR results and discussed in Sections 6.4 and 7.

Example

To demonstrate the use of the above described procedure, trace the calculation of $\theta_w$(MR-RR) at $M_{sm} = 9.02$ in SF$_6$.

Three experimental results presented in Table E1 are considered.

<table>
<thead>
<tr>
<th>EXP</th>
<th>$\theta_w$</th>
<th>$M_s$</th>
<th>$P_0$ [torr]</th>
<th>$\chi$</th>
<th>$\Delta \chi$</th>
<th>$\Delta \chi$</th>
<th>$M_s$</th>
<th>$\theta_w$(MR-RR)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>4 torr</td>
<td>10 torr</td>
<td>100 torr</td>
<td></td>
</tr>
<tr>
<td>38</td>
<td>37</td>
<td>9.08</td>
<td>3.8</td>
<td>1.38</td>
<td>.59</td>
<td>.575</td>
<td>.364</td>
<td>.115</td>
</tr>
<tr>
<td>42</td>
<td>40</td>
<td>8.97</td>
<td>3.8</td>
<td>0.9</td>
<td>.5</td>
<td>.488</td>
<td>.308</td>
<td>.098</td>
</tr>
<tr>
<td>49</td>
<td>42</td>
<td>9.0</td>
<td>4.0</td>
<td>0.3</td>
<td>1.2</td>
<td>1.2</td>
<td>.759</td>
<td>.24</td>
</tr>
</tbody>
</table>

(1) Choosing the experimental data.

It can be seen in Figs. E3a-c that the experimental results gathered at three wedge angles = 37°, 40° and 42° are available in the vicinity of $M_s = 9.0$. Experiment number, wedge angle, incident shock Mach number, initial test pressure and experimentally observed $\chi$ are presented in the

E.3
first, second, third, fourth and fifth columns of Table E1, respectively.

(2) Calculating $\Delta \chi(p_0)$.

For each experiment the distance between $X_n$ (plotted as a full line) and $X_e$ (points) is measured from Figs. E3a-c. Results are given in the sixth column of Table E1.

(3) Transforming $\Delta \chi(p_0)$ to $\Delta \chi(p_0)$.

Only one experiment (at $\theta_w = 42^\circ$) was conducted exactly at the specified $p_0 = 4.0$ torr. Therefore, two other $\Delta \chi(p_0 = 3.8$ torr) must be transformed to the $\Delta \chi(p_0 = 4$ torr) according to Eq. (E2a):

$$
\Delta \chi(4 \text{ torr}) = (4/3.8)^{-1/2}\Delta \chi
$$

Results are given in the seventh column of Table E1. Analogically, $\Delta \chi(10$ torr) and $\Delta \chi(100$ torr) are calculated in columns 8 and 9 of Table E1, respectively.

(4) Calculating the mean value of $\Delta \chi(p_0)$ and $\Delta \chi(p_0)$ if more than one experimental result was considered for one wedge angle. As only one experimental result was used for each wedge angle, $\Delta \chi_{\text{m}} = \Delta \chi$ and $M_s = M_s$.

(5) Calculating the final $M_{\text{sm}}$.

$$
M_{\text{sm}} = 1/3[M_s(37^\circ) + M_s(40^\circ) + M_s(42^\circ)] = 9.017
$$

(6) Checking $M_s$ of experiments considered.

Now we can check if $M_s(\theta_w) = M_{\text{sm}} \pm 0.1$ and if $M_s = M_{\text{sm}} \pm 0.2$.

(7) Finding $\theta_w(\text{MR-RR})$ - the MR-RR transition at $M_{\text{sm}}$ and $p_0 = 4$, 10 and 100 torr. The calculated $\Delta \chi(p_0)$ points and the three-shock theory, inviscid prediction $X_n$ line are incorporated in Fig. E5f. The best fit straight $\Delta \chi$ lines (note that $\Delta \chi = 0$ for $\theta_w = 30^\circ$) intersects the $X_n$ line at $\theta_w(\text{MR-RR})$. The results are presented in the last column of Table E1. These results were put on the $\theta_w - M_s$ plane in Figs. 35-36.
Fig. E1 Determining the MR-RR transitional wedge angle at const. \( M_s \) and \( p_0 \). Experimental results of \( \chi \) are interpolated to zero to find the MR-RR transition - \( \theta_w(MR-RR) \).

Fig. E2 Finding the MR-RR transition at constant \( M_s \) and \( p_0 \). Different \( \chi_{an} \) prediction (dashed line) doesn't change \( \theta_w(MR-RR) \).
Fig. E3a Comparison of experimental results of first triple point trajectory angle $\chi$ with analytical, inviscid prediction $\chi_{an}$ for wedge angle $\theta_w=37^0$ in sulfur hexafluoride. Figures near the experimental points denote the initial pressure $p_0$ in torr.
Fig. E3b Comparison of analytical, inviscid prediction of $\chi_{an}$ with experimental results $\chi(p_0)$ for $\theta_w=40^\circ$ in SF6.
Fig.E3c  Comparison of inviscid, analytical prediction of $X_{an}$ with experimental results $X(p_0)$ for $\theta_W=42^\circ$ in SF6.
Fig. E3d Comparison of inviscid, analytical prediction of first triple point trajectory angle $\chi_{an}$ with experimental results $\chi(p_0)$ for $\theta_w=45^0$ in SF$_6$. 

Experimental data for $\phi=45$ degree wedge angle.
Fig. E4 Comparison of analytical, inviscid prediction of first triple point trajectory angle $\chi_{an}$ with experimental results $\chi$ (initial pressure given in torr) for five wedge angles in isobutane.
Fig. E5a  MR-RR transition points for $p_0=100$, 10 and 4 torr, at $M_s=1.73$ in SF$_6$. 
Fig.E5b MR-RR transition points for $p_0=100$, 10 and 4 torr, at $M_s=2.13$ in SF$_6$
Fig.E5c MR-RR transition points for $p_0=100$, 10 and 4 torr, at $M_S=4.17$ in SF$_6$. 
Fig.E5d  MR-RR transition points for $p_0=100$, 10 and 4 torr, at $M_s=6.08$ in SF$_6$. 
Fig. E5e  MR-RR transition points for $p_0=100, 10$ and $4$ torr, at $M_s=8.49$ in SF$_6$. 
Fig. E5f MR-RR transition points for $p_0 = 100$, 10 and 4 torr, at $M_s = 9.02$ in SF$_6$. 

EXPERIMENTAL DATA AT $M_s = 9.02$
INITIAL PRESSURE: □ 100 ○ 10 △ 4 TORR

VIB.EQ.SF6

$\chi_{\alpha n}$
$\Delta X$

ACTUAL WEDGE ANGLE

35.00 40.00 45.00 50.00 55.00

0.00 1.00 2.00 3.00
Fig. E6a  MR-RR transition points for three values of initial pressure $P_0$ at $M_s=1.73$ in isobutane.
Fig. E6b MR-RR transition points for $p_0=100$, 10 and 4 torr at $M_s=4.72$ in isobutane.
VIB.EQ.ISOBUTANE

![Graph showing experimental data for isobutane.](image)

EXPERIMENTAL DATA AT $M_s=5.61$
INITIAL PRESSURE: +100  x 10  x 4 T

Fig. E6c MR-RR transition points for $p_0=100$, 10 and 4 torr at $M_s=5.61$ in isobutane.
Fig. E6d MR-RR transition points for $p_0=100, 10$ and $4$ torr, at $M_s=6.78$ in isobutane.
Nonstationary oblique-shock-wave reflections in sulphur hexafluoride and isobutane were investigated experimentally and numerically. Over 100 experiments were conducted in the UTIAS 10 cm x 10 cm hypervelocity Shock Tube in order to investigate the RR-RR transition boundary and the features of terminal-double-Mach reflection. Five types of shock-wave reflections, i.e., regular (RR), single-Mach (SMR), complex-Mach (CMR), double-Mach (DMR) and terminal-double-Mach (TDMR) reflections were observed. These were studied with infinite-fringe interferograms and shadowgraphs using a 23-cm dia field of view Mach-Zehnder interferometer. The analytical transition boundaries were established up to an initial shock-wave Mach number $M_0 = 10$. The experimental results agree well with the predicted boundaries for the gases in vibrational equilibrium except for the MR-RR boundary where Mach reflections were observed up to 8 degrees above the detachment criterion line. Strong dependence of the initial test pressure on the first triple point trajectory angle was noted and investigated analytically. The physical model taking into account the viscous boundary layer is proposed. According to this model there are several RR-MR boundaries corresponding to different initial pressures $p_0$, and the detachment criterion is replaced with the mechanical-equilibrium criterion. A procedure for predicting the RR-MR boundary for various $p_0$ is presented. The existing first and second triple-point trajectory predictions are discussed and compared with the experimental results. The influence of gamma on the oblique-shock-wave reflection process and the viscous boundary-layer effect are discussed. The instability of incident shock waves and the nonuniformity of the regions behind them were observed in the low gamma gases. An additional study is required for a better understanding of these phenomena.

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Urbanowicz, J. T.

1. Oblique-shock-wave reflections in low-gamma gases 2. A resolution of the von Neumann paradox
3. Interferometric and numerical analyses

UTIAS Technical Note No. 267

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