THE DESIGNING OF DYNAMIC PRESSURE STAGES FOR HIGH-PRESSURE/HIGH-VACUUM SYSTEMS

by

B. W. Schumacher

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THE DESIGNING OF DYNAMIC PRESSURE STAGES

I. PURPOSE AND PROBLEMS

It is well known that beams of charged particles like electrons or ions can only be formed, accelerated, or analysed in a sufficiently evacuated space. The same is true for soft X-rays and partly for ultra-violet and infrared light. The beams may persist, however, in a gaseous space for some time or over a certain range if they are fired from the vacuum into the gas filled space. This may be desirable for many reasons which shall not concern us here.

The transfer from the vacuum to the high pressure gas was long ago accomplished by two different methods:

(a) the use of a thin foil as a "window" which can be penetrated by beam particles of sufficient energy but which is impervious to the gas,

(b) the use of small open holes in the walls of a series of continuously evacuated chambers.

The use of a foil window was first suggested by Heinrich Hertz after he had observed that cathode-rays will penetrate thin metal foils placed in a vacuum tube. P. Lenard (1894) realized the idea. The foil window has three limitations: (i) The energy of the particles has to be sufficiently high so they may penetrate the foil, (ii) The particles are scattered and slowed down in the foil, (iii) The current density in the beam must be low enough not to melt the foil. The great advantage of foils is that, by using a supporting mesh underneath the foil, one can build windows of very large area. This was of special importance in the earlier days of the art when focussing systems for the beams were not available.

The use of open holes and intermediate pumping chambers was first practised by W. E. Pauli (1920). He stressed already the beauty of this system: "The possible exit of all the rays produced in the tube" with an "absorption in the transition range of only 1/7000 compared with a foil window". The vacuum pumps of his time, however, were less beautiful, although he called them "excellent". Therefore he had to use pinholes of about 0.01 mm diameter in platinum foils 0.05 mm thick when using one intermediate pumping chamber and a discharge type cathode-ray tube. He suggested the use of two intermediate chambers to permit larger holes but did not attempt any calculation of the gas flow and the pressures in the system. Due to the lack of focussing devices, the beams thus obtained were weaker than those from tubes with large foil windows. The method remained in the trial and error stage, if tried at all.
A system which is described in U.S. Patent No. 2,640,948 of September 1950 shows just the most unfavourable arrangement of the structural elements one can obtain if the design is not based on calculations. Adjacent to the atmosphere, a tube, several times longer than wide, is used, and no consideration is given to the shape of the gas flow. In the system shown, the gas will likely be channeled through the various auxiliary vacuum chambers. Performance data are not mentioned. It is not impossible that the system will work - with vacuum pumps of sufficiently vast size.

Since 1948, the writer has used and further developed the system of open-holes and intermediate chambers or "dynamic pressure stages", as we called it (for reasons which will be seen later).

Before going into theoretical discussions of the design, we want to show in Fig. 1 a dimensional drawing of one of the systems built by the author. It consists of two stages (two nozzles, two chambers with reduced pressure, the second one of which is the high vacuum space for an electron gun). Note the intermediate chamber and its pump connection. Note the close spacing of the two nozzles resulting in a short transition range from high vacuum to high pressure, and resulting in a large angular aperture of the free passage.

Since, for us, the use of those devices was more important than the thorough development, the latter is still incomplete. However, the design principles have been established clearly, as we believe. More than ten systems have been built to satisfaction. A publication of the design procedure was made (B. Schumacher, 1953)*. But it is not readily available and not too clearly written. Some additional experience has been gained in the meantime. To our knowledge, no other discussion has appeared as to the transition from "friction controlled" flow to "enthalpy controlled" flow, which is so very important in the design calculation of dynamic pressure stages. We think therefore, a new complete presentation of the subject is justified.

We are obviously entering the field of vacuum systems engineering. From the discussion of the very special subject of dynamic pressure stages, we will also get some insight into other problems of this field which, to some extent, is still an art rather than a science. Wherever we deal with flow, rather than with a stationary vacuum system, we have dynamic pressure stages of a more or less complex nature. A numerical or graphical calculation can, then, be greatly simplified by our assumption $p_2 \ll p_1$ for which the diagrams are given. This nice and widely applicable simplification has only partly been discussed in textbooks.

*) Optik 10, 116, 1953
Because this report is intended to be self-sufficient, and since the books on vacuum technique are not written for easy reference, we will start out discussing some of the relations which govern the flow of gases. We will follow the familiar line using the concepts of free molecule-, laminar-, and turbulent flow. As some of the later remarks will show, this traditional approach may not be the best one because we are always confronted with the limits of the validity of those specialized concepts, but nothing better can be offered. One could imagine starting the whole discussion on the gas dynamic approach, as partly used in chapter II-c-iii. While this is nice for smoothly shaped ducts, without friction, and with perfect isentropic conditions, as soon as it comes to practical design calculations, we are not much further ahead but forced to go back to only experimentally determinable critical Reynolds' numbers, coefficients of friction, etc. Another limit of this approach is encountered when we move towards free molecular flow conditions. Rarefied-gas dynamics is still in the development stage.

For our practical purposes, the calculations presented here are quite good enough and can readily replace trial and error methods. The scholarly readers will find many of the discussions superfluous, some always do - but we were thinking of the student reader as well and wanted to offer him something usable and some basis for an understanding of the matter, not only a compilation of formulae and literature references.

II. FLOW OF GAS THROUGH AN ORIFICE OR A TUBE

(a) General Remarks

If two spaces are separated by a wall but interconnected through an orifice or tube in that wall, and if the gas pressure is different, gas will flow from the high pressure side to the low pressure side. This is a basic "primitive" experience. That under certain conditions, flow will also occur at equal pressures but different temperatures, or a pressure difference will not cause any flow, is not as "primitive", but just as basic as the first case*.

*) It is, for instance, an often neglected source of error in ionization gauge measurements. Flow due to differences in temperature changes the gas density in the hot gauge bulb. A calibration in terms of pressure can only be valid for a specified gauge temperature. And this has nothing to do with outgassing, gettering, etc.
For the rate and shape of the flow, we have in no case any ready judgements or "feeling".

In former times, the similar flow of water was used to explain the flow of electric current under an applied potential difference or voltage (pressure) through a resistor (pipe). To-day, the laws for the flow of electricity are generally better known than the laws which control the flow of matter through ducts, partly because in the latter case the simple facts of the first order approximation (Ohm's Law) were hidden by a greater number of complicated "correct" formulae and corrections which are of interest in the kinetic theory of gases, but not of too much importance in the calculations for a practical piece of equipment.

The flow of gas through any narrow passage can be described by the equation:

\[ Q = F \times (p_1 - p_2) \]

where \( Q \) is the mass passing the narrow per unit time, \( F \) is a factor called conductance, \( p_1 \) and \( p_2 \) are the pressures in front of and behind the narrow respectively. This equation is exact and correct by definition. (This is the descriptive side of mathematical formulae). In practice, the problem is to get accurate values for \( F \).

We measure the flow \( Q \) in:

- Number of molecules/second,
- Grams/second,
- Pressure x Volume units/second
  at any specified temperature
- Volume units/second
  at any specified temperature and pressure

It is advisable to use different names if the flow is measured in different units because the temperature dependence may become the reverse. There are no accepted standard terms. To keep our own usage clear, we will use the following terms:

\[ Q^* \text{ in } \{ \text{ molecules/sec, grams/sec, standard-cm}^3/\text{sec } \} = \text{mass-flow} \]

\[ Q \text{ in } \{ \text{Torr x litre/sec, mm Hg x litre/sec, atmospheric cm}^3/\text{sec} \} = \text{throughput} \]

Standard - cm\(^3\) means the specified temperature and pressure are \(0^\circ\text{C}\) and 760 Torr.
We find the following conversion factors:

\[3.240 \times 10^{19} \text{ molecules/sec}\]
\[= 5.40 \times 10^{-5} \times M \text{ gram/sec}\]
\[= 1 \text{ Torr x litre/sec}\]
\[= 1.316 \text{ atm cm}^3/\text{sec}\]

(for 25°C) (at any temp.)

\((M = \text{molecular weight}).\)

We measure the pressure \(p\) in:

- Atmospheres; mm Hg or Torr; \(\mu\) Hg; lbs./sq. in;
- kg/cm²; etc.

We will use mainly the unit 1 Torr = 1 mm Hg (abbreviation for Torricelli, the inventor of the mercury barometer), in accordance with the suggestion of the Standards Committee of CVT*.

Once the pressure and flow units are chosen, we get the

Conductance: \(F\) in litre/sec

or:

\(F^*\) in g/Torr, sec

with the following conversion factor:

\[F^* = 5.40 \times 10^{-5} \times M \times (298/T) \times F\]
\[\text{g/Torr, sec}\]
\[(T = \text{abs. temp.}.)\]

The reciprocal value of the conductance is called:

Resistance \(R = 1/F\) in sec/litre

(\(R\) sec to pass 1 Torr x litre of gas of 1 Torr through the resistance).

The simplest case of flow conductance we can find is:

\(F = \text{const.}\)

Then (1) is equivalent to Ohm's Law for the electric current. But just as Ohm's Law is the lucky exception of more complicated cases where we have voltage dependent resistors, temperature or load dependent resistance, etc., the conductance for gas flow may depend on the pressure, the temperature, etc.

*) CVT = Committee on Vacuum Techniques; see "1954 Vacuum Symposium Transactions", page 137.
Yet, most unfortunate is the fact that the conductance \( F \) is usually not in any simple way related to the geometry of the duct. It is, therefore, by far, the best to work with a graphical representation of equation (1)*. At least three different relations between \( F \) and the geometry exist. They are represented later by the three auxiliary diagrams of Fig. 24.

In the next paragraph, we shall list the formulae for and values of the conductance \( F \) for a number of orifices and tubes under the conditions which are of interest in connection with dynamic pressure stages.

(b) Free Molecular Flow

This is the flow we find at very low pressures. The term means that collisions between the gas molecules are negligible compared with collisions of the gas molecules with the walls of the duct.

The degree of approximation to this condition can be expressed by the Knudsen number

\[
Kn = \frac{\lambda}{d}
\]

(\( \lambda \) = mean free path of the gas molecules, \( d \) = diameter of the duct)**

For air at 20°C, we find \( \lambda = 5 \times 10^{-3}/p \) cm (\( p \) in Torr), hence (with \( d \) in cm):

\[
Kn = \frac{5 \times 10^{-3}}{d \times p}
\]

Since \( \lambda \) is approximately \( \propto T \), it follows that \( Kn \propto T \). The higher the Knudsen-Number, the better the conditions for free molecular flow are fulfilled and the conductance is independent of pressure (no matter which order of magnitude the numerical value of the conductance is). In other words, we find:

\[
\text{for } Kn > 1, \quad mF = \text{const.}
\]

The index \( m \) on \( F \) shall make it unmistakably clear that it is only valid in the molecular flow region.

For an orifice of area \( A \) cm\(^2\) in a thin wall, the kinetic gas theory gives us an exact value for the conductance. It is:

\[
\]
\[ m_{F_0} = \frac{1}{4} Av = 3.638 \text{ A } \sqrt{T/M} \text{ litre/sec} \]  

\[(7)\]

\[ v = 14551 \sqrt{T/M} \text{ cm/sec, mean molecular speed,} \]

\[ T \text{ in } ^{\circ}\text{K, } M = \text{molecular weight}\]

Numerical values are listed in Table 1. Knudsen called this flow through an orifice "molecular effusion".

We find for the throughput in the case of \( P_2 \ll P_1 \)

\[ m_{Q_0} = 3.638 \text{ A } \sqrt{T/M} P_1 \text{ Torr litre/sec} \]  

\[(8)\]

\( P_1 \text{ in Torr, A in cm}^2\)

The conductance of a tube of length \( \lambda \) with cross section \( A \) can be found by multiplying \( m_{F_0} \) by an empirical factor \( k \), known as Clausing's factor. Hence, conductance of a tube in the free molecular flow region:

\[ m_F = k \times m_{F_0} \]  

\[(9)\]

Values of \( k \) as a function of \( \lambda/d \) are listed in Table 2. For large values of \( \lambda/d \), i.e. capillaries, the conductance is proportional to \( 1/\lambda \), and it is still proportional to the area of cross section of the tube. For other kinds of flow, we find other dependences on the geometry.

The conductance and, hence, for a constant pressure difference, the throughput (flow in pv-units) increases with increasing temperature; it is proportional to \( \sqrt{T} \); with (8) we get:

\[ m_Q = k m_{F_0} (p_1-p_2) = k \times 3.638 A \sqrt{T/M} (p_1-p_2) \text{ Torr litre/sec.} \]  

\[(10)\]

However, if we measure those pv-units at the higher temperature, as we must, and since we have a density \( \rho \propto M/T \), the mass-flow decreases with increasing temperature. We find:

\[ m_Q = k \times 5.833 \times 10^{-2} A \sqrt{M/T} (p_1-p_2) \text{ g/sec} \]  

\[(11)\]

whether there is a dependence of the Clausing factor \( k \) on \( T \) has to our knowledge not been discussed in the literature. A slight difference of the Clausing factor \( k \) for different gases is to be expected as well. We will ignore it and use in the following just one value of \( k \) for all gases.

For completeness, we should mention that several theoretically derived formulae for molecular flow through tubes of various length exist, yet apart from the case of very long tubes, they represent rather experimental mathematics than theory.
(c) **Viscous Flow**

Due to collisions between the molecules, the conductance becomes dependent upon the pressure. Three different cases are to be considered.

(i) **For slow flow speeds and long tubes**, a so-called laminar flow develops, which shows a parabolic velocity profile across the tube and parallel lines of flow. It requires a tube with \( l/a \) of at least 40 to develop its profile*.

(ii) **For higher flow speeds**, the laminar profile is no longer stable and turbulence begins. The explanation of this phenomenon is well known. The transition occurs when the transfer of momentum perpendicular to the direction of the flow (due to thermal movement) is no longer sufficiently large compared with the momentum of a fluid element in the direction of the flow. "Neighbouring" flow lines lose "contact" and, hence, lose any correlation of the movement. The critical dimensions and speeds are given by the Reynolds' number.

(iii) **For an aperture in a thin wall, or even short tubes**, we can no longer use the concepts of the two cases mentioned before. We have what we may call viscous effusion flow of the gas through the hole. This case is simpler than the two before and the mass flow is easily calculated from thermodynamic relations. We will see it is "enthalpy controlled" rather than "friction controlled". It is of prime importance for our purposes.

We shall list here the relations which exist, but without deriving them.

(i) **Laminar Flow:**

For long tubes we have one of the few exact solutions of the Navier-Stokes equations. It is the following expression, well known as Hagen-Poiseuille's Law for compressible media of viscosity \( \eta \):

\[
\frac{Q}{l} = \frac{\pi}{8\eta} \frac{a^4}{l} \frac{p_1 + p_2}{2} (p_1 - p_2)
\]

We can deduce

\[
\frac{F}{l} = \frac{\pi}{8\eta} \frac{a^4}{l} \frac{p_1 + p_2}{2}
\]

We see that here, in the laminar flow range, the conductance \( I_F \) is pressure dependent. For \( p_2 \ll p_1 \), it becomes proportional to \( p_1 \), the "driving" pressure. It follows that for \( p_2 \ll p_1 \), the flow becomes proportional to the square of the "driving" pressure \( p_1 \); we get:

\[
I_Q = \frac{\pi}{16\eta} \frac{a^4}{l} p_1^2 \quad I_F = \frac{\pi}{16\eta} \frac{a^4}{l} p_1
\]

(14)

For air of 25°C:

\[
I_Q = 1.42 \times 10^3 \frac{a^4}{l} p_1 = 89 \times d^3 d/l \ p_1^2 \quad \text{Torr x litre/sec} \quad (14a)
\]

\[
I_F = 1.42 \times 10^3 \frac{a^4}{l} p_1 = 89 \times d^3 d/l \ p_1 \quad \text{litre/sec}
\]

(a, d, l in cm, \( p_1 \) in Torr) \( (14b) \)

Note also that the velocity \( u = \frac{I_Q}{\pi a^2 \rho} \), considering \( \rho = \rho_0 x p/p_0 \), becomes proportional the pressure.

For the transition region toward molecular flow, correction formulae for "slip" were developed. We will not use them, but assume that both types of flow exist together and can be added* which also means the two conductances can be added like parallel conductances.

At one point there is even a slight "hindrance" of the two flows: for at \( d \approx 0.6 \lambda \), the total conductance if \( F = 0.95 I_F \), which is less than for pure free molecular flow, but much higher than the formula for viscous flow would suggest. Knudsen found this first experimentally (see e.g. ***).

Numerical values for the factor \( \pi/16\eta \) are listed in Table 1. As for the temperature dependence of the viscosity \( \eta \), we find it increases with temperature, hence, \( I_F \) decreases when the temperature increases. We find (among other formulae)**

\[
\eta_T = \alpha T^x
\]

(15)

(numerical values for \( \alpha \) and \( x \) in Table 1; we find roughly \( x \approx \frac{1}{2} \)).

---

* Dushman, Loc. cit. p. 112
** " " " p. 3437
Again, if we convert the flow figures from pv-units to mass-flow, the temperature effect is even more pronounced than in the case of free molecular flow (see (11)). We get with $\rho \propto 1/T$ and $\gamma \propto T$ as an approximation:

$$Q \propto T^{-3/2}$$  \hspace{1cm} (16)

(ii) Turbulent Flow:

In tubes, a transition from laminar to turbulent flow occurs, as is well known, if the Reynolds' number

$$Re = \frac{\rho ud}{\eta}$$  \hspace{1cm} (17)

($\eta =$ viscosity, $\rho =$ density, $d =$ diameter of tube, $u =$ flow speed) exceeds a critical value

$$Re_c \approx 2000$$  \hspace{1cm} (17')

This critical value is found to vary with such factors as roughness of the wall of the tube, inflow geometry, etc. The Reynolds' number is only then a valid criterion of dynamical similarity if $(\omega \eta)_{10} = 0$ i.e. if this term vanishes in the Navier-Stokes equations*. Hence, the dependence of the critical value $Re_c$ can be used as a guide in our design calculations.

Related to (17') is a critical flow speed $u_c = Re_c (\gamma / a \rho)$ or, more interest in our case, a critical mass-flow

Since

$$Q = \frac{\pi}{4} d^2 \rho u$$  \hspace{1cm} (18)

we get

$$Q_c = Re_c \gamma \frac{\pi}{4} d$$  \hspace{1cm} (19)

For Air

$$Q_c = 0.292 d \text{ g/sec}$$

(25°C)

$$= 187 d \text{ Torr x litre/sec (d in cm)}$$  \hspace{1cm} (20)

For a given viscosity $\eta$, the critical mass flow $Q_c$ is a function of the tube diameter $d$ only. The values of $Q_c$ for various tube diameters $d$ are marked in the graphs, Figures 5, 6, and 24.

Assuming a turbulent flow pattern is encountered, then we want to know the conductance \( F \) for the turbulent range, especially its dependence upon the driving pressure. For narrow tubes where the effects of the boundary layers become especially important, no references were found in the literature. We are not certain whether the following formulae, valid for larger tubes, can be used in our case. Numerical values should be treated as approximate only, as well as the critical values discussed above.

The following experimental formula for the flow resistance in tubes is valid \( \phi \) for compressible media and turbulent flow, if average values for \( \rho \) and \( u \) are used:

\[
\frac{p_1 - p_2}{l} = \lambda \frac{I}{d} \frac{1}{\rho} \bar{u}^2
\]

(21)

Here, \( \lambda \) is the so-called hydraulic friction coefficient, \( d \) the tube diameter. At \( Re = 2000 \), one finds \( \lambda = 0.05 \) for smooth tubes. Formula (21) states that the force (pressure differential) is proportional to the square of the velocity, or that the velocity goes with the square root of \( \Delta p \).

Experimentally, \( (p_1 - p_2) \alpha u \) was found. (24)

For a discussion of this, see e.g. Sommerfeld, loc. cit. Note that for laminar flow, we had \( (p_1 - p_2) \alpha u \).

The relation between \( \lambda \) and \( S \) indicates that \( \lambda \) will increase proportionally to \( T \). This is of importance for our purpose.

Since the pressure builds up with the square of the speed, the turbulent flow is reduced compared with the laminar flow. In using

\[
Q = \pi a^2 \rho u \quad \text{and} \quad \rho = \rho_0 \frac{P_1}{P_0} \quad \rho_0 / \rho = RT/M \quad \text{and assumed } P_2 \ll P_1,
\]

1 Schlichting, loc. cit.
2 W. Frössel, Forschung Bd. 7 (1936) S. 75
we find from (21):

\[ t_Q = \frac{\pi d^2}{4} \sqrt{\frac{d}{\lambda \Delta}} \sqrt{\frac{RT}{M}} p_1 \]  

(25)

The proper choice of the average values of \( \rho \) and \( u \) is not quite clear; in (25) we used \( \bar{\rho} = \frac{1}{2} \rho_0 p_1/p_2 \), and \( u = Q / \pi a^2 \bar{\rho} \). (\( \bar{\rho} = \frac{1}{2} \rho_1 \))

We may define a conductance \( t_F \) for the turbulent region, (and \( p_2 \ll p_1 \));

\[ t_F = \frac{\pi d^2}{4} \sqrt{\frac{d}{\lambda \Delta}} \sqrt{\frac{RT}{M}} \]  

(26)

For air \( t_F = 1.03 \times 10^2 \, d^2 \sqrt{\frac{d}{\lambda \Delta}} \) litre/sec (d in cm) (27)

This is independent of the pressure; a most important fact. Considering (24) \( t_Q \) may increase slightly faster than determined by (25).

Assuming \( \lambda \) is proportional to the temperature, as discussed above, then \( t_Q \) would be independent of temperature. This would be of importance in case high energy beams heat the gas (flowing through the orifice) to high temperatures. The temperature dependence of \( Re_c, Q_c \) would have to be considered as well. We will not do this here, since we will seldom encounter turbulent, friction controlled flow.

(iii) Viscous Effusion or Enthalpy Controlled Flow;

Let us once more assume two spaces with pressure \( p_1 \) and \( p_2 \) were separated by a wall with an aperture of area \( A \), the wall thickness being negligible compared with the diameter of the aperture, and \( p_2 \ll p_1 \).

The flow of gas through the aperture is limited by the fact that the kinetic energy of any mass element of the gas at any one point in the flow cannot become higher than the difference in enthalpy between that point and the stagnating gas. This applies to any type of flow through tubes as well.

The theory has been worked out long before vacuum systems were being engineered, for instance for turbine nozzles.

We have the following relations*:

* see: any text book on Gas Dynamics
\[ \frac{\dot{E}}{\dot{Q}} = f \times \rho \times u \]  
\[ \text{g/sec, mass flow} \]
\[ \text{E}_{\text{kinetic}} / \rho = \frac{1}{2} u^2 \]  
\[ \text{erg/g, kinetic energy} \]
\[ I = U + \frac{p}{\rho} = J (c_p T + i_0) \]  
\[ \text{erg/g, enthalpy} \]
\[ \rho v^k = \text{const or } T^k / p^{k-1} = \text{const} \]  
\[ \text{Adiabatic expansion**} \]

Solving these equations for \( u \), leads us to the well-known formula:
\[ u^2 = \frac{2 k}{k-1} \frac{p_i}{\rho_i} \left[ 1 - \left( \frac{p}{p_i} \right)^{k-1} \right] = \dot{u}_{\text{max}}^2 \left[ 1 - \left( \frac{p}{p_i} \right)^{k-1} \right] \]
\[ \dot{u}_{\text{max}}^2 = \frac{2 k}{k-1} \frac{p_i}{\rho_i} = \frac{2 k}{k-1} \frac{R T_i}{M} \]

\[ f = \text{cross section of the flow; for } p = p_1, f = A; \]
\[ J = 4.187 \times 10^7 \text{ erg/cal; } c_v, c_p = \text{spec. heat at constant volume or pressure, } k = \frac{c_p}{c_v}; \]
the index \( e \) on \( \dot{E}/\dot{Q} \) shall denote that the formula holds only for effusion flow.

The maximum flow speed \( u_{\text{max}} \) depends only on \( T_1 \) and is attained by an expansion into vacuum \( p = p_2 = 0 \).

We shall list a few more relations for easier reference:

The velocity of sound is given by:
\[ C = \sqrt{\frac{k p}{\rho}} = \sqrt{\frac{k-1}{2} \left( \dot{u}_{\text{max}}^2 - u^2 \right)} = \sqrt{\frac{k R T_i}{M}} \]

going to \( C = 0 \) for \( p = 0 \).

The flow speed and the velocity of sound become equal, i.e.
\[ u^* = c^* = \frac{\sqrt{2 k / k+1}}{p_i / \rho_i} = \frac{\sqrt{k-1}}{k+1} \dot{u}_{\text{max}} = \frac{2 k}{k+1} \frac{R T_i}{M} \]

if the pressures, the density, etc. reach the so-called critical values \( p^* / \rho^* \), etc. as listed in Table 6. For \( p / p_1 > p^* / p_1 \), the flow is sub-sonic; for \( p / p_1 < p^* / p_1 \) the flow is super-sonic. Note that \( p^* / p_1 \) lies, in every case, much higher than \( p_2 / p_1 \) as encountered in the dynamic pressure stages.

Since the velocity of sound is also the speed for the propagation of any signal in the gas, a change of the flow downstream of \( p^* \) will have no effects further up. We can understand, therefore, that the mass flow reaches its maximum at \( p_2 = p^* \) and does not increase if \( p_2 \) is reduced further.

** For non-adiabatic expansion, \( k \) has to be replaced by the proper polytropic coefficient \( k > m > 1 \).
Another consequence thereof is that the mass-flow is completely independent of \( p_2 \). In this range, the total flow is not any more the sum of a forward and a backward flow as Knudsen treated it in the free molecular flow range.

Another quite important practical consequence is the exact validity of the equation:

\[
Q = eF \times P_1
\]

the value of \( p_2 \) does really not matter as long as \( p_2 < p^* \).

We find the mass-flow:

\[
q_{\text{max}} = \frac{Q_{\text{max}}}{f^*} = \rho^* u^* = \left( \frac{2}{k+1} \right)^{\frac{k}{k-1}} \left( \frac{M}{R T_1} \right)^{\frac{1}{2}} p_1
\]

(28)

or with \( k = 1.40 \), \( M = 28.98 \) (air), \( T_1 = 298^0 \text{K} \)

\[
q_{\text{max}} = 2.38 \times 10^{-5} \text{ p}_1 \quad \text{g/sec cm}^2; \quad (\text{p}_1 \text{ in dyn/cm}^2)
\]

\[
q_{\text{max}} = 3.165 \times 10^{-2} \text{ p}_1 \quad \text{g/sec cm}^2; \quad (\text{p}_1 \text{ in Torr})
\]

\[
q_{\text{max}} = 20.5 \text{ p}_1 \quad \text{Torr 1/sec}; \quad ** (\text{p}_1 \text{ in Torr})
\]

To get the total flow \( Q_{\text{max}} \), we must know the cross-section \( f^* \) of the flow at the point of the critical values \( p^* \), \( \rho^* \), etc. For the flow through an orifice in an infinitely thin wall, we can expect \( f^* = A \), since friction effects on the edge of the orifice should be infinitely small.

For cylindrical tubes, we have an experimental investigation by Frossel*. He investigated the profile of the flow, as well as the reduction of \( Q_{\text{max}} \) due to friction on the wall of the tube. He used tubes of 10, 20, 25, and 30 mm diameter. We assume here that his results may also be used for tubes with diameters of the order of 0.1 to 1 mm; this may not be completely correct.

Figure 2 shows the profile of the flow measured at the end of a circular cylindrical tube by means of a Pitot-probe. For \( \lambda/d > 6 \), it is nearly rectangular (\( f^* = A \)) and it approaches a final shape for \( \lambda/d \approx 36 \).

Figure 3 represents Frossel's measurements of the reduction \( \nu = \frac{Q_{\text{max}}}{Q_{\text{max}}} \) of the mass-flow per unit area as a function of \( \lambda/d \) and \( p_2/p^* \). We deal always with the case \( p_2 << p^* \), or \( p_2/p_1 << p^*/p_1 \). The effect of the tube length on \( p^* \) can be seen in the graph. Figure 4 shows (for the case \( p_2 < p^* \)) the value of \( \nu (\lambda/d) \) which is, now, only a function of \( \lambda/d \). This is a universal curve and was used to prepare the auxiliary diagram for \( eQ = eQ (\lambda/d, d) \) in Fig. 24. Note that \( eQ \) is very little reduced if \( \lambda/d \) is increased. We assume here that \( \nu \) is to a first approximation independent of \( T \) and \( M \).

* W. Frossel, loc. cit.

** Torr 1/sec = Torr litre/sec. Hereafter, this unit is used in either form.
Hence, we get for the total flow under effusion conditions the following formula:

\[ eQ = 20.5 \times \sqrt{\frac{\lambda}{d}} \times Ap_1 \quad \text{Torr litre/sec} \]  

or

\[ eF = 20.5 \times \sqrt{\frac{\lambda}{d}} A \quad \text{litre/sec} \]

(A in cm\(^2\), \(p_1\) in Torr, air 20°C, \(\sqrt{\frac{\lambda}{d}}\) from Fig. 4 or 24)

As to the temperature dependence of the mass-flow, we see from (28) that it is \(\propto \sqrt{T}\) just as in the case of the free molecular effusion.

Frossel's coefficient \(\gamma = \frac{q}{\sqrt{\gamma}}\) takes care of the transition from effusion - which is completely enthalpy controlled - to turbulent flow in long tubes which is, in addition, controlled by friction. In our later diagrams (Figs. 5, 6, 23, 24, 25), we have indicated the critical mass-flow \(Q_C\) where we have to expect turbulence. As we can see, the laminar flow changes into effusion flow long before the turbulence criterion is reached. We do not know whether we were justified in discriminating between a laminar type and a turbulent type of effusion flow.

III. THE COMPLETE CONDUCTANCE CHARACTERISTIC OF AN ORIFICE OR TUBE

Comparison with Measurements

For the design of dynamic pressure stages, we have to know the conductance of the tubes or holes over the full range of pressures encountered. A graphical presentation in the form of a \(pQF\)-diagram is most suitable. There are a number of ways in which we may arrange the graphic net representing the quantities \(p\), \(Q\), and \(F\). The use of logarithmic scales is indicated. In a former publication*, we have plotted \(Q(d, \lambda)\) vs. \(p\). The lines for \(F = \text{const}\) are then under 45° to the abscissa. It is more economical as far as space is concerned to plot \(F(d, \lambda)\) vs. \(p\), the \(Q\)-lines being under 45°. This is done in Fig. 5 and 6** for two nozzles with \(d = 0.3\) mm, \(\lambda = 2\) mm and \(d = 0.35\) mm, \(\lambda = 50\) mm, respectively.

We start by drawing \(mF, \lambda F, tF, eF\) as derived in the previous chapter; the pertinent formulae are listed once more in the following table 3. To save numerical calculations, the formulae can also be presented in a diagram, preferably with the same scale for \(F\) as used in our \(pQF\)-diagram. This is done in Fig. 24** where three auxiliary diagrams are included showing \(F\) as a function of \(\lambda/d\) with \(d\) as parameter. We simply

* B. Schumacher, loc. cit.
** For Figs. 5, 6, and 24, look at the rear.
have to read off the three F values for a given tube and transfer them to the proper pressure range in the main diagram. \( \mathcal{L} F \) will intersect \( eF \) and \( mF \).

To get the proper F-line for the transition regions, where one kind of flow changes into another, we have to use some approximations. At C (Figs. 5 and 6) where \( \mathcal{L} F \) and \( mF \)-line meet, we have to add both laminar and molecular flow component. This, in other words, means multiplying \( F \) by 2 and we obtain \( C' \). Then we join \( \mathcal{L} F \) and \( mF \) by a smooth curve through \( C' \). The points \( C' \) are marked on all the subsequent diagrams. The error we make may be different for different \( \mathcal{L}/d \) ratios, but we are working with a practical approximation.

Following the \( \mathcal{L} F \) line to higher pressures, we come to the intersection with \( eF \), and henceforth one flow is determined by \( eF \). We do not know how much the transition from \( \mathcal{L} F \) to \( eF \) should be "rounded".

The turbulent conductance \( tF \) for the short tubes we are interested in will be higher than \( eF \). In so far, it has not meaning for us. One would expect the \( \mathcal{L} F \), \( tF \) and \( Qc \) line to intersect in one point. We see that they don't, which shows the uncertainties in either \( ReC \) or in Eq. (21). From that point on where \( Qc \) and \( eF \) intersect, we would expect the effusion flow to become turbulent. We do not know whether this is the case. For the two nozzles of Fig. 5 and 6, we measured the flow rate as a function of \( P1 \). The values obtained are listed in Table 4 and shown in the figures as well*. The measurements were made as shown in Fig. 7. Time and available means did not permit us to make the measurements very accurately and to extend them into the molecular flow region. However, we could establish the transition from \( \mathcal{L} F \) to \( eF \). Previously, nobody has drawn attention to the importance of that transition.

In Fig. 5, we see that for a short tube the Hagen-Poiseuille part is nearly missing. The experimental points are lower than the theoretical curves, most likely due to the errors in the measurements of the diameter and length of the tube. In Fig. 6, we see the Hagen-Poiseuille part with \( F \) \( P1 \) well developed and a sharp transition to the other pressure dependence long before turbulence, i.e., \( Qc \) is reached. The differences between experimental and theoretical values are again most likely due to inaccuracy of the figures for the tube dimensions. We see in both graphs that in the turbulent or the effusion range, the conductance is not exactly constant but increases slightly with the pressure. This would be in agreement with Formula (24).

* The same values were presented as Fig. 2 in the earlier publication, B. Schumacher, 1953, loc. cit.
A few more words are necessary concerning the dependence of the flow $Q$ on the geometry of the tubes. We want, in our case, to reduce the flow by all means, but we want, in addition, a short transition range from high vacuum to high pressure.

We see from Fig. 4 and Formula (26) that the flow rate $\dot{Q}$ or $Q$ is only very slightly dependent upon the length $\ell$ of the duct (as long as $\ell < 10 \text{ cm}$). Hence, for a dynamic pressure stage with $p_1 \approx 1$ atmosphere, it is the best to use a very thin diaphragm wall with an orifice of the desired size. The flow cannot be reduced appreciably by attaching a capillary. The latter would only increase the length of the transition range, and would also reduce the angular aperture.

For a tube in the laminar flow range, we may cut $\dot{Q}$ in half by doubling the length $\ell$. However, this reduces the angular aperture considerably. If possible, we should - in the laminar range - rather reduce the diameter $d$, as is well known.

In the range of free molecular flow, we find for long tubes

$$m_F = k \times m_F_0 \propto \frac{d^3}{\ell}$$

Since we are in the low pressure range, we may make $\ell$ very long without adding much more to the weight per unit area of the transition range. This may be of importance if we have to use large values of $d$ to get a sufficiently large angular aperture, for instance, with a 3-stage system.

Because of the different dependence of $F$ on $d/\ell$, the $F(p)$ lines in Figs. 23 and 24, cross over one another.

IV. THE SHAPE OF THE EMERGING GAS JETS

So far, we have discussed the amount of gas streaming through an orifice into a space with low gas pressure which is in some way maintained by continuous evacuation, i.e. continuous pumping. We can readily calculate the pressure obtainable with a certain pump, as we will see in the following chapter. Before doing that, we have to see whether any such calculation makes sense, in case we have two or more orifices closely spaced in series to reduce the pressure in several stages. We have to know, in other words, the distribution of the gas flowing into the evacuated space, and especially the dynamic pressure produced in front of a wall with a second orifice which may be placed in that gas flow.

When we built our first systems, the existing relations were not investigated in detail, only up to the point of the necessary working
knowledge. We will report here what we found. Recently, Schrüfer pub-
lished some experimental investigations into this subject*. Unfortunately,
the most important case of high values of $p_1$ and small diameters $d$ is not
covered. A thorough study of gas flow patterns in the molecular flow range
has been published recently by Dayton **.

Whatever we do, the gas emerging through a small orifice
into an evacuated space forms a jet. By our new methods of electron
shadowgraphs*** and afterglow excitation ****, we made the flow patterns
visible. Figures 8a and b are two examples.

For us, the flow from a short cylindrical tube, as in Fig. 8b,
is the most important case. For a certain range $p_2 < p_1$, which was not
determined in detail, a "loop" $H$ (in Fig. 8b) is always formed, at the right
of which loop, we see a characteristic standing shock front vertical to the
direction of flow. At the shell $S$, the gas has apparently supersonic speed and
at a certain distance $L$, a standing shock front is found, followed downstream
by an apparently subsonic region with little or no structural pattern. For
cylindrical nozzles and high pressure ratios, we did not see the multitude of
loops that appeared in the pictures of Schrüfer. (Note, however, the structure
in the case of Fig. 8a where the nozzle was conical and where up to 20 "loops"
and "knots" were seen downstream). This region maintains its coherence
and is visible over a very long distance (compared with its diameter).

The coherence of the gas jet is once more demonstrated by
Fig. 9. A special technique was used to show the mixing region between
the gas of the jet and the quiescent gas. The jet is argon; metastable argon
atoms are produced by means of an electron beam fired through the gas jet
perpendicular to the direction of flow. The metastables are carried with the
flow. The afterglow due to the metastables in pure argon is weak. Here,
however, the quiescent gas is CO and in an A-CO mixture the "afterglow"
is strong, since CO is excited by energy transfer from the metastable A
atoms. Therefore, we see in Fig. 9 the mixing region - that is the boundary
of the A jet - as a bright band on both sides of the jet. The coherence is
evident.

The position $L$ of the shock front can be shifted when the jet
is directed against a wall.

In the series of pressure stages which we have built, we used
as the second nozzle a cone with the orifice in the apex. We found that the

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* E. Schrüfer, Z. angew. Phys. 9, (1957), 88-95
** B. B. Dayton, Transactions, 1956 National Symposium on Vacuum
Technology: Pergamon Press
1527
A. E. Grün, Z. Naturforschung, 9a (1954), 1017
*** B. Schumacher: Annalen der Physik 6, 13 (1953), 404
spacing $S_p$ should be such as to bring the orifice into the "loop" of the gas jet from the first nozzle, as shown schematically in Fig. 10.

From the dimensions given in Fig. 8, we see that this will result in a very close spacing of the two nozzles. We know further from the shadowgraphs that the gas density $\rho_v$ in the "loop" is not appreciably higher than the average density outside the jet. Hence, the pressure in front of the second nozzle placed at $H$ is, in fact, very nearly equal to the pressure $p_2$ obtained by the pump of the first stage.

We see further from the dimensions given in Fig. 8 that the spacing $S_p$ is not very critical. However, it is definitely wrong to increase $S_p$ over $L$. The pressure $p_3$ as a function of $S_p$ follows qualitatively the pattern shown in Fig. 11. The increase of pressure for larger $S_p$ is not very high but in the order of 2 to 5.

We could not make quantitative measurements for lack of time and means.

The pattern of Fig. 11 reveals itself also if $L$ (in Fig. 8) is reduced by reducing $P_1$. We get the paradoxical effect that $p_3$ increases if $P_1$ is reduced*. Figure 12** gives the explanation. The pressure $p_2 = 3.2$ Torr is kept constant. The energy of the electrons producing the shadowgraphs is kept constant as well. $P_1$ is varied from 600 Torr (at the left) to 200 Torr (at the right). $L$ decreases from about 9 mm to 2 mm. Accurate values for the density cannot be obtained from the picture. The mass-flow is reduced with $P_1$.

For comparison, we show in Fig. 13 the decrease in $L$ with increase in $P_2$ (throttling of the pump), $P_1$ is kept constant. The mass-flow should not be changing with $P_2$, since $P_2 < P_{***}$.

In Fig. 14***, we can see that similar flow patterns with "loops" are produced by a slit aperture 0.1 mm wide, 10 mm long.

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* B. Schumacher and A. E. Grün, unpublished. In spite of the fact that the system of Fig. 1 worked against $P_1 > 1$ atm., we got (for certain diameters of the nozzle 1) a discharge in space II if we reduced $P_1$, e.g. to take spectra.

** B. Schumacher, Annalen der Physik 13, (1953), 404, Abb. 12

*** B. Schumacher, loc. cit. Fig. 11
As far as an explanation of the observed flow pattern goes, the experts kept telling us they are well understood. It may be noted, however, that only very recently, separation effects of the components of a gas mixture were found in a jet similar to the one in Fig. 8. They were used for the separation of isotopes.

The existence of a shock front in a flow as shown in Figs. 8 and 9 would give us a possibility for a partial pressure recovery by using a diffuser and, hence, pumping off the gas at a pressure \( p_2' > p_2 \). We would have to use a proper duct surrounding the conical second nozzle. No experiments have been made to this point. For large diameters of the nozzles, say 6 mm, it may become very important.

We have shown the "loops" in the flow as density patterns, Schüfer measured Pitot-pressure. (As normal, he had no flow through the Pitot-nozzle). Obviously, both findings are not immediately applicable to dynamic pressure stages where flow through the Pitot-nozzle occurs. We assume, however, the differences are small and insignificant within the accuracy of our design calculation. A detailed study may be worthwhile, studying also the temperature distribution in the gas jets.

Let us now review the recent results of Schrüfer*. With the system shown in Fig. 15, he measured the Pitot-pressure in the air jet coming from the nozzle d. Figure 16 gives some typical results. We see the Pitot pressure in the "loop" (at about \( S_p = 7 \text{ mm} \)) is close to the pressure \( p_2 (1.2 \ldots 1.5 x p_2) \) achieved by the pump, but it rises for larger values of \( S_p \) to about 2.5 x the minimum value. Schrüfer does not give the \( l/d \) of his nozzles, and the Pitot-probe is not a cone as our nozzle in Fig. 1 or 10, hence, we cannot readily apply Fig. 16 to our design. The pressure ratio \( 720/23 \) is not as high as the one at which we have operated. For completeness, we reproduce in Figs. 17 to 21 all the measurements published by Schrüfer. The necessary explanations are given in the captions. From Figs. 17, 20, and 21, Schrüfer shows that the "gas dynamic flow" which exhibits "waves" changes to the molecular flow without "waves" at a Knudsen number of \( \overline{x}/d = 0.01 \) (\( \overline{x} \) being calculated for \( p_1 \), the high pressure side). We show, for reference, the Knudsen numbers 0.01 / 0.1 / 1 / 10 / 100 on all the F-curves for the various nozzles. Schrüfer has further shown that the wavelength for the "loops" and "knots" (e.g. Fig. 18) is in agreement with the calculations based on reflected rarefaction waves from the edge of the nozzle.

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* loc. cit.
For the free molecular flow *, entrance and exit patterns show a pronounced "beaming" effect for increasing ratios \( l/d \) of the tube. We do not know whether it has been discussed how we may calculate the Pitot -pressure or the flow through a "probing nozzle" as a function of these patterns, of the spacing \( S_p \), and of the average pressure, in order to arrive at curves as shown in Fig. 19. Such curves would be of value in our design calculations.

V. THE CHARACTERISTICS OF VACUUM PUMPS

In order to calculate the pressures in our system of holes and chambers, we have to know the suction speed \( S \) of the pump as a function of pressure, the pump characteristic. If it is dependent upon a fore-pressure, we have to know this dependence in detail. Normally, all this information is available from the manufacturer's catalogues. Here we will only discuss a few important points.

The pressure, or rather vacuum, \( p_p \) that can be obtained at the suction side of a given pump with speed \( S(p) \) litre/sec in the presence of a gas flow \( Q \) is determined by the equation

\[
p_p = \frac{Q}{S(p_p)}
\]

We can plot \( S(p) \) in our \( pQF \)-diagram like a (negative) conductance, as is often done. (The electrical analogy would be the transconductance of a radio tube). Fig. 23 is an example. Let us consider a typical rotary pump with characteristic \( S^R \) and a typical mercury diffusion pump \( S_E \). Measured values \( S'X \) for the rotary pump plus a certain vacuum line with valves are also shown. \( S_M \) is another commercial mercury diffusion pump, E.S. + S5 denotes the system built by us. We may use this diagram as follows:

Assume the throughput of gas is given to be \( Q_{11} = 20 \) Torr x litre/sec. (It may, e.g., be determined by an orifice with 0.3 mm diameter, 0.3 mm long, as used in the system of Fig. 1, having a pressure of 2 atmospheres at the high pressure side**). The pump combination E.S. + S5 will achieve a vacuum of \( p_{11} = 2.0 \) Torr in the presence of this flow \( Q_{11} \). The pressure at the forepump S5 will be \( p_{11}^* = 20 \) Torr. (This would be the upper limits of the working conditions of the system Fig. 1).

Similarly, with a constant throughput of \( Q_{12} = 5.5 \times 10^{-3} \) Torr x litre/sec (nozzle F 0.4 x 4; \( p_{12} = 2 \) Torr) a pump with the characteristic \( E \) will give a vacuum of \( 3.8 \times 10^{-4} \) Torr. If \( S'X \) represents the

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* Dayton, loc. cit.
** The F-line for this nozzle is shown for reference.
the forepump, the forepressure will be \( p_{12}^* = 0.45 \) Torr. \( \Delta p_E \) is the pressure reduction maintained by the pump \( S_E \).

As is well known, \( p_{12} \) must be lower than a certain value \( p_{\text{max}} \), the highest fore-pressure for the pump \( E \). Usually a forepressure of 10 to 20 Torr can be tolerated for a mercury diffusion pump. In case we would use an oil diffusion pump, we would have to get a larger forepump to reduce \( p_{12}^* \) to about 0.2 Torr even if \( S_E \) oil would be the same.

The low fore-pressure required for oil-diffusion pumps and their failure in the range > 0.1 Torr was one major difficulty encountered in our first design of a 2-stage system (for large angular aperture). We built, therefore, our own mercury jet pump with 32 jets and 6 KW power input. Its measured characteristic is the one shown in Fig. 23, denoted E.S. + S5. It is nothing spectacular, but it was sufficient for the first stage of the system shown in Fig. 1, whereas \( E \) represented the pump for stage 2. XME3 (in Fig. 23) is a mercury booster pump developed recently by Edwards. The speed is not very high, but a fore-pressure of 100 Torr can be tolerated.

To-day, there are no more difficulties because of the fore-pressure limit, since the Roots-type pumps became available which are ideally suited for our purpose. They cover the old gap between rotary oil pumps and oil diffusion pumps. They can be cascaded to get even a high vacuum of 10^-4 Torr or better. For this purpose, however, the diffusion pumps are much cheaper and smaller. As is evident from Fig. 23, the pumps required for a system like the one in Fig. 1 are not of the large type, but of a convenient "laboratory size". In Chapter VI, we will discuss what we may achieve with the biggest pumps that are available.

With reference to Fig. 23, we want to draw attention to an important point of the characteristics of diffusion pumps. Towards higher pressures, the speed \( S \) drops and may follow a line of constant throughput. If we operate close to this part of the \( S \) curve, and if we have a continuous flow of gas (rather than a vessel to be evacuated), then the system is likely to be unstable. Assuming the throughput is \( Q_{13} = 3 \) Torr litre/sec which gives, with the curve \( M \), a pressure \( p_{13} = 0.125 \) Torr, then we are close to this constant throughput part of the characteristic \( S_M \). If \( Q \) is increased only slightly, for an instant, we miss the \( S_M \) line completely; the result is the same as if there were no pump at all and the system runs full of gas until the forepump pressure is reached. In contrast, the system E.S. + S5 is safe and stable with a throughput of \( Q_{13} \).

We may analyse a system starting at the high pressure side, or we may start at the high vacuum side and ask for the maximum pressure that can be tolerated at the high pressure side. Note: if a system works
against a pressure of 1 atmosphere it can, in principle, be used for any higher pressure as well, since the gas flow from the stages with higher pressures can be ejected directly into the atmosphere.

In choosing the pumps for a set of dynamic pressure stages, another aspect becomes very important as soon as gases, other than air, are to be pumped, especially He and H₂. Mechanical pumps have a constant suction speed \( S \) (in litre/sec) independent of the gas. This is simply because the displacement by the piston is always the same. Diffusion pumps, however, change their speed with any change in the diffusion constant of the gas. The latter is, in the free molecular flow range, proportional to \( \frac{1}{\sqrt{M}} \) as seen from equation (7). On the other hand, the influx of gas through the nozzles changes with a change in gas composition. It is also roughly proportional to \( \frac{1}{\sqrt{M}} \). Note, however, the exception for He in the viscous flow range (\( \pi/16\eta \) in Table 1). It follows that we are generally better off with diffusion pumps than with mechanical pumps, since we get for any increased influx of gas an increase in pumping speed as well.

VI. THE DESIGN DIAGRAM FOR DYNAMIC PRESSURE STAGES

Examples of Various Systems

To obtain the design data for a given system of dynamic pressure stages, we take the \( \rho QF \)-diagram, Fig. 24, with the speed or \( S \) curves for the pumps which we want to use. They can be taken from Fig. 23 or other sources. We draw the conductance curves for the nozzles we intend to use. The \( mF, L_F, \) and \( eF \) values for any \( d \) and \( L/d \) ratio is given in the auxiliary diagrams in Fig. 24, and we proceed as discussed in Chapter III to get the complete \( F \) curve. With reference to Fig. 24, we shall give a few examples of systems which are in use or which are of interest for certain experiments.

If so-called booster pumps or jet-pumps are used, we have to know the shape of the speed curve \( S \) for the particular gas, especially the pressure where the \( S \) curve has its peak (the characteristic feature of the jet-pump as compared to the diffusion pump). The selection of pumps shown in Fig. 23 and 24 was made to show one or two for every decade of the \( S \) scale. Other types from other manufacturers can be found and used as well. For all the numerical data mentioned in the following examples, the corresponding lines are shown in Fig. 24, but without indexing the \( p'\)'s and \( Q'\)'s. The designations of the characteristic curves of the pumps are listed in Table 5.

Example 1:

The two-stage system of Fig. 1 with nozzles \( D_1 = 0.3 \times 0.3 \) mm, \( D_2 = 0.4 \times 0.4 \) mm, and pumps E. S. + S5, E has been discussed in Chapter V, page 21, with the reference to the diagram Fig. 23. The
upper pressure limit at the high pressure side is approximately 2 atmospheres, in agreement with experimental results.

The system was used extensively with high current beams. After some time, the nozzles were burnt out, reducing the high pressure limit. Under those conditions, it was observed several times that the pressure could be raised to, say, 2 atmospheres as long as an electron beam (0.5 mA, 60 KV, or 30 Watt) was passing through the nozzles, but the limit was, say, 1.2 atmospheres if the beam was off. Our explanation is that the heating of the gas by the beam reduced the gas flow. Quantitative measurements could not be made; the heating effect in the gas was readily observed by optical Schlieren-photographs.

Example 2:

In Fig. 25, we show a three-stage system, designed to work with very small pumps. It was attached to a 1 MeV accelerator to get low energy protons into the air.

At the high vacuum side, we required a pressure \( \leq 2 \times 10^{-5} \) Torr, to be maintained by an oil diffusion pump 035. Using a nozzle 0.2 x 2 mm, we get a tolerable pressure \( \leq 1.3 \) Torr. (With a nozzle 0.2 x 0.2, this pressure would be considerably lower, namely 0.35 Torr.) If a pressure of 1.3 Torr is to be maintained by a pump with characteristic S'XI, e.g. a pump S2 or S5 over long pipes, and if we use another nozzle 0.2 x 2 mm, we arrive at a tolerable pressure \( \leq 70 \) Torr. With a shorter nozzle, 0.2 x 0.2 mm, the difference in pressure would be slight: we would find \( \leq 55 \) Torr instead of 70 Torr. Since we are now in the viscous flow range, these values mean Pitot-pressure (see Chapter IV).

For the final stage, we use once more a nozzle 0.2 x 2 mm or 0.2 x 0.2 mm. The difference in the conductance F is negligible. We use once more a pump S'XI. Let us assume it must provide a pressure of half the Pitot-pressure found above, i.e. 35 Torr and 24 Torr. Then the maximum throughput is found to be 36 Torr x litre/sec or 28 Torr x litre/sec, and the corresponding high pressure must be \( \leq 6 \times 10^3 \) Torr (\( \approx 7.8 \) atm) or \( \leq 4.5 \times 10^3 \) Torr (\( \approx 5.9 \) atm) respectively.

When we connected this system to the 1 MeV accelerator, we found the "leakage" through the dynamic pressure stages to be much smaller than the one through real leaks.

Example 3:

In Fig. 26 we show the system used by A. E. Grün*. It has the particular feature that the electron beam does not touch the walls of the nozzles of the pressure stages which is accomplished by the electron-optical

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* A. E. Grün Z. Naturforschung, 12a (1956) 89
system shown. The data given are as follows:

Nozzles: \[ D_1 = 0.6, \quad D_2 = 0.8, \quad D_3 = 1.0 \text{ mm diameter}. \]

Pumps: \[ p_1 : S \, 150^*, \quad p_2 : R \, 150 + S \, 10, \quad p_3 : D \, 250. \]

Let us assume the length of the nozzles to be 0.6 / 0.8 / 4 mm respectively, since they are not mentioned in the publication; the conductances of the pipes to the pumps are also not mentioned. With a pressure \( P_0 = 700 \text{ Torr} \) we find \( p_1 = 0.8 \text{ Torr} \) from our diagram. The measured value was 0.7 Torr. The difference may be due to a slightly smaller diameter of the nozzle or due to the pump running slightly faster than the design speed.

Assuming a Pitot-pressure of \( p_1 = 2 \text{ Torr} \), we find with \( D_2 \) and \( R \, 150, p_2 = 5 \times 10^{-3} \text{ Torr} \). The measured value was \( 5 \times 10^{-2} \text{ Torr} \). The difference is most likely due to a reduction in pumping speed in the connecting pipes. We see from the diagram that a pipe of, e.g. \( 25.6 \times 1060 \) (1" x 40") would account for this pressure drop.

With \( p_2 = 5 \times 10^{-2} \text{ Torr} \) and \( D_3 \), \( p_3 \), we find \( p_3 = 5 \times 10^{-6} \text{ Torr} \). The measured value was \( 2 \times 10^{-5} \text{ Torr} \). Again the reduction in pumping speed by baffles, etc., was not taken into account. From the diagram, we can estimate that a connection of \( 51.2 \times 200 \text{ mm} \) or \( 102.4 \text{ mm} \) may be the cause of this pressure difference. The line for \( p_3 = 2 \times 10^{-5} \text{ Torr} \) and the line for \( Q = 1.2 \times 10^{-3} \text{ Torr} \times 1/\text{sec} - \text{nozzle } 1.0 \times 4; p_2 = 5 \times 10^{-2} \text{ Torr} - \text{intersect at } S = 60 \text{ litre/sec} \). From the \( mF \)-diagram, we see that a tube \( 51.2 \times (3.9 \times 51.2) \) would just show \( S \) or \( F = 60 \text{ l/sec} \).

Example 4:

The system of Fig. 27 is used with the electron gun of the UTIA wind tunnel. It is unusual because of the long drift tube.

Assuming a vacuum of \( 2 \times 10^{-4} \text{ Torr} \) is required, with the pump 203, baffled, a throughput of \( Q = 7.7 = 10^{-3} \text{ Torr} \times 1/\text{sec} \) can be tolerated. Due to this flow, we get a pressure drop along the drift tube for which we may get an approximate figure from the intersection of the \( Q \)-line with the \( F \)-line for a tube 16 x 320. We find \( 5.5 \times 10^{-2} \text{ Torr} \). Assuming the pressure in the wind tunnel is not higher than that value, we should not need a further flow reduction by another nozzle.

With a nozzle of \( 0.8 \times 3 \text{ mm} \) and the above \( Q \), we find a maximum allowed pressure of \( 0.8 \text{ Torr} \). Behind the nozzle, the pressure will likely drop to \( 5.5 \times 10^{-2} \text{ Torr} \) found for the end of the drift tube. However, due to the formation of gas jets, the pressure distribution along the drift tube may be fairly complicated.

* Very close to E 135; we use E 135 line.
We have not yet made any detailed measurements on this system.

The forepump (plus its connections) must produce a vacuum of 0.2 Torr in the presence of the flow $Q = 7.7 \times 10^{-3}$ Torr x 1/sec. In other words, its speed must be 0.04 litre/sec at 0.2 Torr.

Assuming we make a connection from the oil diffusion pump (fore-pressure side) back to the wind tunnel, then this connection must carry $7.7 \times 10^{-3}$ Torr 1/sec at a pressure difference of 0.2 Torr - 0.055 Torr = 0.145 Torr. We may use this as the "$p_1$" value in our diagram ($p_2 \ll p_1$ is the condition for accurate results; for the case $p_2 \approx p_1$ see Dushman, loc. cit.). We find the conductance required for this connection is $F = 5.5 \times 10^{-2}$ l/sec. The fore-vacuum connection of the 203 pump has a diameter of ½". Using a pipe with 12.8 mm diameter, we find, from the auxiliary diagram of Fig. 24, that the maximum length it may have is 400 x its diameter, or 510 cm.

We can still operate the electron gun at a pressure of $6 \times 10^{-4}$ Torr. We find for the flow $Q = 2.4 \times 10^{-4}$ Torr x 1/sec. With the tube 16 x 320, the maximum pressure would be 0.13 Torr; with the nozzle 0.8 x 8, we find 1.7 Torr. Assuming the fore-pressure may be as high as 0.4 Torr (0.5 Torr is the limit given by the manufacturer), then the pumping system would have to have a speed of $6.5 \times 10^{-2}$ 1/sec. (More than was required before). The smallest size fore-pump would do; but a rubber tube ¼" x 2 ft. as a connection will not be good enough, as is evident from the diagram. We want to show with these figures how important it is to include the fore-vacuum side of the system in the calculations.

Example 5:

A system of dynamic pressure stages shall work with any kind of gas to about the same high pressure limit.

To achieve that, it is indicated to use even for the first stage a jet or a diffusion pump. Let us investigate what could be attained using the mercury booster pump XME3, and the mercury diffusion pump 6M3.

The XME3 will carry safely a throughput of $Q = 20$ Torr 1/sec of air at an inlet pressure of 1 Torr. With a nozzle 0.4 x 0.4 mm, a pressure of 850 Torr can be tolerated on the high pressure side. A suitable forepump would be the type K20 which gives with $Q = 20$ Torr 1/sec a fore-pressure of 4.5 Torr (air). In case $H_2$ is being pumped with $Q_{H_2} \approx 3.8 Q_{air}$ (the maximum increase to be taken into account) the fore-pressure may rise to approximately 20 Torr which is still safe.
Allowing for a Pitot-pressure of 4 Torr in the second stage, and using a nozzle 0.8 x 0.8 mm (for high angular aperture), the pump 6M3 will give a vacuum of 6.5 x 10^{-4} Torr. If baffles have to be used, reducing the pumping speed, another stage may be added using another pump 6M3.

Example 6:

Let us look for the system with the largest possible aperture using the biggest pumps available. It shall operate against a pressure of 2000 Torr (2.6 atmospheres) of H$_2$.

The largest throughput is obtained by the pump-set RG 25030 + RG 6020 + W 1300. These are displacement pumps having the same throughput for all gases. (The pumps 30" Jet-Vac or 20" C-R have higher speeds, but we have not found throughput data for higher pressures). With a flow of Q = 30 000 Torr x 1/sec, the pumps will achieve a pressure of 5 Torr. For air, a nozzle of 10 x 10 mm could be used. For hydrogen, the conductance has to be 3.8 x lower, hence we must use a smaller nozzle of 5.3 x 5.3 mm.

Note: So far the flow pattern for the intake side of the nozzles was not considered; at rates of the order of 10$^4$ Torr x 1/sec, it may well be of importance and may reduce the flow.

If we assume for the next stage a Pitot-pressure of 15 Torr and use the same set of pumps once more, a pressure of 5 x 10^{-2} Torr is obtained. For H$_2$, two pumps type 1863 may be sufficient; for air, six of this type were required. We may also use the pump type 20"-C-R (compare Fig. 23).

The final stage is easily designed; another nozzle 10 x 10 mm, a pressure in front of it of 5 x 10^{-2} Torr, and a pump 1603 will give a final pressure of 6 x 10^{-5} Torr.

Note: This calculation is rough. We wanted to show that "holes" of 10 mm diameter in a high vacuum system are not impossible. The set of pumps gives no problems. The spacing of the nozzles in such an arrangement is more difficult, since we need an unrestricted gas flow in the low pressure ranges. "Intake reduction" may be to our advantage. We may also make use of a partial pressure recovery in the fast flow. A system with 4 mm holes would be fully sufficient for the high temperature experiments which we have mentioned elsewhere.*

Example 7:

Let us discuss the system shown in Fig. 28. It is used in connection with an X-ray spectrometer. Along the axis $X_1$, an electron beam is fired through the nozzles $f$, $d$, and $a$ onto the target $O$. X-rays from $O$ enter through the nozzles $b$, $c$, and $g$ into the spectrometer space $R_3$. The spectrometer, as well as the electron gun $R_E$, have each an independent oil diffusion pump 403. (Suction speed with baffle and connections approximately equal to 203 unbaffled). The pressures are $p_{3R}$ and $p_{3E}$ respectively.

Obviously, we have a 3-stage system with the nozzles $a-b$ and $e-d$ in parallel. $Z_0$ is a flat chamber with a pressure $p_0$ sufficiently above atmospheric pressure to prevent target substance from $O$ being sucked in through the nozzles. The chamber $Z_1$ is evacuated to a pressure $p_1$ by a pump $R_1$. The chamber $Z_2$ is connected to a pump $R_{150}$ through a fairly narrow passage of approximately $25 \times 50$ mm which restricts the speed of the pump to one-half.

The diameters of the nozzles are determined by requirements not to be discussed here. They are as follows:

$$
a = 0.2 \times 0.2 \quad / \quad b = 0.4 \times 0.2 \quad / \quad d = 0.2 \times 1.2
\quad e = 0.1 \times 1 \quad / \quad f = 0.6 \times 12 \quad / \quad g = 0.8 \times 4
$$

We want to know the pressure distribution in case of air and hydrogen, assuming a pressure $p_0 = 1000$ Torr in each case. From Fig. 24, we find the following values:

Nozzle $a$: $Q = 5.8$ Torr x $1/\text{sec}$ (air)
" b: $Q = 24$ " " " "
Total $Q \approx 30$ " " " "

For $H_2$, the throughput is $3.8 \times$ higher, which gives

$Q = 114$ Torr x $1/\text{sec}$ (H$_2$)

We find: $p_1 = 1.2$ " (air)
$\quad = 5$ " (H$_2$)

Assuming the Pitot-pressure in front of the nozzles $d$ and $e$ is 2 Torr for air and 10 Torr for $H_2$, then we find the following values for throughput and pressure in $Z_2$:

Nozzle $d$: $Q = 1.7 \times 10^{-3}$ Torr x $1/\text{sec}$ (air)
" e: $Q = 2.3 \times 10^{-4}$ " " "
Total $Q \approx 2 \times 10^{-3}$ " " "
or for $H_2$: $Q \approx 7.6 \times 10^{-3}$ " " " (H$_2$)

In $Z_2$, the pressure will be:

$P_2 = 3.3 \times 10^{-4}$ Torr for air
or

\[ p_2 \approx 8 \times 10^{-4} \text{ Torr for } H_2 \]

This pressure is already so low that the vacuum in the next chambers (electron gun and spectrometer) is rather determined by the end-vacuum of the oil diffusion pumps than by the flow through the nozzles \( f \) and \( g \).

**Example 8:**

For some years there has been a commercial model of an electron gun available that works with dynamic pressure stages. It is the "Electron Generator E 200" by W. C. Heraeus GmbH., Hanau, Germany. It shows a three-stage system and uses Roots-pumps exclusively, which were not yet in use, when we built our first system. The Heraeus generator has free apertures of 0.5 mm diameter and can work against a pressure of 10 atmospheres. Beam power of 200 KeV and 50 mA (or 4 KW) is available. For further details, see the Heraeus catalogue.

**VII. THE MECHANICAL CONSTRUCTION OF DYNAMIC PRESSURE STAGES**

In the design of an apparatus with dynamic pressure stages, we should obviously follow the good practices established in vacuum engineering. Apart from that, only a few points need to be mentioned.

The material used for the nozzles should be preferably copper. With powerful beams striking the walls of the nozzles occasionally, there is always the danger the nozzles will melt. We experience that with beams of 50 KV and 0.5 mA (25 watt), well focused, and hitting a steel nozzle. The better heat conduction of copper reduces this danger. It is advisable but not necessary to gold-plate the copper to avoid oxidation. As can be seen from Fig. 1 and 25, the nozzle proper forms just an insert, and is exchangeable.

If more than two nozzles are to be aligned, it is advantageous if one of them is adjustable in two directions perpendicular to the axis.

In systems with the axis vertical, there is the danger of the nozzles becoming clogged by dust or foreign matter like a broken end piece of the tungsten cathode. To reduce this danger, dust traps as shown in Fig. 29 are useful.

The gaskets which should give the tight seal between the nozzle insert and the body of the apparatus need not be pressed very firmly since the pressure differences are usually low (1:100 or so). "O"-rings slipped over the outside of the joint, as shown in Fig. 29, are often sufficient. An alternative are ground joints held by a ring nut as shown in Fig. 1.
Adjustments for the beam forming system are most important since the axis of the system is determined already by the pressure stages. Arrangements for systematic adjustments of a beam to a given axis are described in the literature and should be used.
## Often Used Numerical Values

<table>
<thead>
<tr>
<th>Gas</th>
<th>H₂</th>
<th>He</th>
<th>Air</th>
<th>A</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>Molecular weight M</td>
<td>2.016</td>
<td>4.003</td>
<td>28.98</td>
<td>39.94</td>
<td></td>
</tr>
<tr>
<td>Viscosity γ (25°C)</td>
<td>0.89x10⁻⁴</td>
<td>2.0x10⁻⁴</td>
<td>1.854x10⁻⁴</td>
<td>2.2x10⁻⁴</td>
<td>dyn cm⁻² sec⁻¹ = g cm⁻¹ sec⁻¹ = poise</td>
</tr>
<tr>
<td>Coefficients of equation 15</td>
<td>1.860x10⁻⁶</td>
<td>4.894x10⁻⁶</td>
<td>N₂/O₂ (3.3x10⁻⁶)</td>
<td>2.782x10⁻⁶</td>
<td></td>
</tr>
<tr>
<td>Factor π/16γ (25°C)</td>
<td>2.21x10³</td>
<td>0.982x10³</td>
<td>1.06x10³</td>
<td>0.89x10³</td>
<td>dyn⁻¹ cm⁻² sec⁻¹</td>
</tr>
<tr>
<td>Specific conductance mF_o/A of orifice of area A in a thin wall; kn &gt; 1 (25°C)</td>
<td>44.24</td>
<td>31.39</td>
<td>11.67</td>
<td>9.94</td>
<td>litre sec⁻¹ cm⁻²</td>
</tr>
<tr>
<td>Specific conductance mF_o/σ² of round orifice with radius a in a thin wall</td>
<td>139.0</td>
<td>98.58</td>
<td>36.66</td>
<td>31.23</td>
<td>litre sec⁻¹ (cm²)</td>
</tr>
<tr>
<td>( \sqrt{RT/M} ) (for T = 298 K)</td>
<td>11.12x10⁴</td>
<td>7.86x10⁴</td>
<td>2.93x10⁴</td>
<td>2.49x10⁴</td>
<td>cm/sec</td>
</tr>
</tbody>
</table>

**TABLE 1**
Clausing's Factor $k$ for Various Tubes*

**Circular Tube of Diameter $d$**

<table>
<thead>
<tr>
<th>$l/d$</th>
<th>$k$</th>
<th>$l/d$</th>
<th>$k$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1</td>
<td>5.0</td>
<td>0.1973</td>
</tr>
<tr>
<td>0.25</td>
<td>0.8013</td>
<td>10</td>
<td>0.1135</td>
</tr>
<tr>
<td>0.5</td>
<td>0.6720</td>
<td>20</td>
<td>0.0613</td>
</tr>
<tr>
<td>1.0</td>
<td>0.5136</td>
<td>30</td>
<td>0.0420</td>
</tr>
<tr>
<td>2.0</td>
<td>0.3589</td>
<td>40</td>
<td>0.0319</td>
</tr>
<tr>
<td>4.0</td>
<td>0.2316</td>
<td>50</td>
<td>0.0258</td>
</tr>
</tbody>
</table>

$k \to \frac{8 \delta}{3l}$

**Rectangular Tube of Cross Section $a \times b$; $a \neq b$, $a l$**

<table>
<thead>
<tr>
<th>$l/b$</th>
<th>$k$</th>
<th>$l/b$</th>
<th>$k$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1</td>
<td>2</td>
<td>0.5417</td>
</tr>
<tr>
<td>0.2</td>
<td>0.9096</td>
<td>4</td>
<td>0.3999</td>
</tr>
<tr>
<td>0.4</td>
<td>0.8362</td>
<td>10</td>
<td>0.2457</td>
</tr>
<tr>
<td>0.8</td>
<td>0.7266</td>
<td>$\infty$</td>
<td>b/l $\ln (l/b)$</td>
</tr>
<tr>
<td>1</td>
<td>0.6848</td>
<td>$\infty$</td>
<td>b/l $\ln (l/b)$</td>
</tr>
</tbody>
</table>

* Dushman, loc. cit.
## TABLE 3

Summary of the Formula for the Calculation of Conductances

<table>
<thead>
<tr>
<th>Orifice</th>
<th>General Formula</th>
<th>Air (25°C)</th>
<th>H₂(25°C)</th>
<th>He</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tube, k₁ &gt; 1</td>
<td>[ m_F = 3.638 \times 10^{-2} \sqrt{T/M} \times \lambda ]</td>
<td>= 11.67 A</td>
<td>= 44.24 A</td>
<td>= 31.39 A</td>
<td>litre/sec</td>
</tr>
<tr>
<td>Tube, k₁ &lt; 1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Tube, Laminar</td>
<td>[ Q_c = \frac{\pi}{16} \gamma \frac{d}{\lambda} \rho ]</td>
<td>= 89 ( d^3 \frac{d}{\lambda} ) ( p_1 )</td>
<td>185 ( d^3 \frac{d}{\lambda} ) ( p_1 )</td>
<td>82 ( d^3 \frac{d}{\lambda} ) ( p_1 )</td>
<td>litre/sec</td>
</tr>
<tr>
<td>Tube, Turbulent</td>
<td>[ t_F = \frac{\pi}{4} \frac{d^2}{\lambda} \frac{\sqrt{T}}{\sqrt{\gamma M}} ]</td>
<td>= 103 ( d^2 \sqrt{\frac{d}{\lambda}} )</td>
<td></td>
<td></td>
<td>litre/sec</td>
</tr>
<tr>
<td>Effusion Flow</td>
<td>[ e_F = \frac{2\sqrt{\frac{2k}{k+1}} - \frac{2k}{k+1}}{\frac{m}{R T_i}} ]</td>
<td>= 20.5 A</td>
<td>= 77.5 A</td>
<td>= 56.5 A</td>
<td>litre/sec</td>
</tr>
<tr>
<td>Orifice Tube</td>
<td>[ e_F = \nu (\lambda/d) e_{F_0} ]</td>
<td>= 20.5A ( \nu (d/\lambda) )</td>
<td>77.5A ( \nu )</td>
<td>56.5A ( \nu )</td>
<td>litre/sec</td>
</tr>
<tr>
<td>F in litre/sec ( \rightarrow ) Q = F ( p_1 ) ( p_1 ) in Torr litre/sec</td>
<td>A = cross sectional area in ( \text{cm}^2 )</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>F* in gram/Torr, sec ( \rightarrow ) Q = F ( p_1 ) ( p_1 ) in g/sec</td>
<td>d = diameter of tube in cm</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>F* = 5.40 \times 10^{-5} \times M \times (298/T) \times F</td>
<td>( \lambda ) = length of tube in cm</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \nu ) = viscosity in poise (g cm(^{-1}) sec(^{-1}))</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Experimental Values</td>
<td>Theoretical Values</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>---------------------</td>
<td>--------------------</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>P₁ = Torr</strong></td>
<td><strong>Q = Torr x 1/sec</strong></td>
<td><strong>Nozzle:</strong></td>
<td><strong>0.3 mm diameter</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td><strong>2 mm long</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>2.9 x 10⁻²</td>
<td>m_{F₀} = 8.25 x 10⁻³ litre/sec</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>2.45 x 10⁻²</td>
<td>l/d = 2/0.3 = 6.66</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>22</td>
<td>9.3 x 10⁻²</td>
<td>k = 0.16</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>29</td>
<td>1.65 x 10⁻¹</td>
<td>m_{F} = 1.32 x 10⁻³ litre/sec</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>60</td>
<td>0.37</td>
<td>l_{F} = 3.59 x 10⁻⁴ litre/sec</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>105</td>
<td>0.73</td>
<td>t_{F} = 3.6 x 10⁻² litre/sec</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>200</td>
<td>1.55</td>
<td>Q_{C} = 5.54 Torr litre/sec</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>200</td>
<td>1.70</td>
<td>e_{F} = 1.41 x 10⁻² litre/sec</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>400</td>
<td>3.60</td>
<td>ν(l/d) = 0.93</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>750</td>
<td>7.2</td>
<td>e_{F} = 1.31 x 10⁻² litre/sec</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>770</td>
<td>6.8</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1000</td>
<td>9.0</td>
<td></td>
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<tr>
<td>1500</td>
<td>14.5</td>
<td></td>
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<td></td>
</tr>
<tr>
<td>2000</td>
<td>19.5</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>30</td>
<td>2.5 x 10⁻²</td>
<td>m_{F₀} = 9.98 x 10⁻³ litre/sec</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>50</td>
<td>7.2 x 10⁻²</td>
<td>l/d = 143</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>100</td>
<td>0.3</td>
<td>k = 0.00933</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>200</td>
<td>1.20</td>
<td>m_{F} = 9.3 x 10⁻⁵ litre/sec</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>400</td>
<td>3.2</td>
<td>l_{F} = 2.10 x 10⁻⁵ litre/sec Torr</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>750</td>
<td>6.4</td>
<td>t_{F} = 1.00 x 10⁻² litre/sec</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>850</td>
<td>7.0</td>
<td>Q_{C} = 6.4 Torr litre/sec</td>
<td></td>
<td></td>
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<td>1000</td>
<td>9.0</td>
<td>e_{F₀} = 1.71 x 10⁻² litre/sec</td>
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<tr>
<td>1500</td>
<td>15.0</td>
<td>ν(l/d) = 0.36</td>
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<td>2000</td>
<td>20.0</td>
<td>e_{F} = 6.1 x 10⁻³ litre/sec</td>
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### TABLE 5

References to Fig. 23 and 24

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<th>Designation</th>
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<td>D 36 000</td>
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<td>D 250</td>
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<td>Oil diffusion</td>
<td>Edwards</td>
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<td>&quot;</td>
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<td>18 B 3</td>
<td>Oil, booster</td>
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<td>&quot;</td>
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<td>18B 3, 6X</td>
<td>Oil, booster</td>
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<td>(6 in parallel)</td>
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<td>Hg diffusion</td>
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<td>XME3</td>
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<td>Article by Maslach</td>
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<td>Natl. Sym. on Vac.</td>
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<td>used for U of C. Wind</td>
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<td>1.67</td>
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<td>$T^*/T_1 = \frac{2}{k+1}$</td>
<td>0.750</td>
<td>0.834</td>
<td>0.870</td>
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<td>$c^*/c_1 = \left(\frac{2}{k+1}\right)^{1/2}$</td>
<td>0.866</td>
<td>0.913</td>
<td>0.933</td>
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<td>$\rho^*/\rho_i = \left(\frac{2}{k+1}\right)^{1/(k-1)}$</td>
<td>0.760</td>
<td>0.644</td>
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<td>$\rho^*/\rho_i = \left(\frac{2}{k+1}\right)^{k/(k-1)}$</td>
<td>0.488</td>
<td>0.528</td>
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<td>$u^*/u_{\text{MAX}} = \left(\frac{k-1}{k+1}\right)^{1/2}$</td>
<td>0.434</td>
<td>0.373</td>
<td>0.337</td>
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Fig. 1

Electron Gun with dynamic pressure stages for the exit of the beam into the atmosphere (Two-stage system).
Fig. 2

(After Frössel)
Profile of the Flow in the end cross-section of the tube.

\[ w = \text{velocity at the distance } y \text{ from the wall of the tube}. \]

\[ W = \text{max. velocity} \]

---

Fig. 3

(After Frössel) Mass-flow per unit area and related pressure drop \( p_x \) along the axis \( x \) of the tube of diameter \( d \). (The \( x \)-values appear as parameters along the cut \( AA \)). For \( x = l \) (length of the tube) we get \( p_x = p_2 \).

---

Fig. 4

(After Frössel)
\[ \nu = \nu(1/d) \text{ for} \]
\[ p_x = p_1 = p_2 \]
\[ p_2 \ll p^* \]
Conductances of Nozzles, Experimental and Theoretical (Air 25°C)

WORKING SIZE GRAPHS CAN BE OBTAINED AT COST FROM THE ONTARIO RESEARCH FOUNDATION, 43 QUEEN'S PARK, TORONTO 5, ONTARIO, CANADA
FIG. 7
Fig. 8a

Afterglow picture of the gas jet emerging from a conical nozzle into an evacuated space.
Gas A + 8% N₂; \( p_1 = 600 \text{ Torr} \); \( p_2 = 0.5 \text{ Torr} \); 
\( d_0 = 0.35 \text{ mm} \); \( d_1 = 1.3 \text{ mm} \); \( l = 6 \text{ mm} \).

Fig. 8b

As above but cylindrical nozzle.
\( d = 0.6 \text{ mm} \); \( l = 2.7 \text{ mm} \).

Both pictures from A. E. Grön, loc. cit.
Fig. 9

Argon-jet excited by an electron beam and passing through a CO-Atmosphere; conical nozzle; $p_1 = 600$ Torr; $p_2 = 5$ Torr. The mixing region of the gases is visible by a bright luminescence. (Taken from A. E. Grün, loc. cit.)

Fig. 10

Fig. 11
**Fig. 12***

Variation of the shape of the jet of air from a cylindrical nozzle 1.0 x 10 mm, showing \( L = L(p_1) \). \( p_2 = 3.2 \text{ Torr} \) = const.; \( p_1 = 600/500/500/300/200 \text{ Torr} \) (from a to e). Electron shadowgraph.

---

**Fig. 13**

Electron Shadowgraph showing the shape of the flow (air) from a cylindrical orifice 0.3 x 0.5 mm, showing \( L = L(p_2) \).

\( p_1 = 740 \text{ Torr} \) = const; \( p_2 = 2.0(a)/2.3(b)/2.5(c) \text{ Torr} \). Energy of the electrons 3.7/6.0/8.6 kev.

---

**Fig. 14***

Electron Shadowgraph of the flow-pattern from a slit 0.1 x 10 mm.

\( p_1 = 740 \text{ Torr}; p_2 = 5 \text{ Torr} \).  
a: view at the edge; b: view at the long side of the slit.  
(Electrons 17.4 kev)

---

** B. Schumacher, unpublished.
System of Schröfer to measure Pitot-pressure $p$
$P$ adjustable Pitot-probe
d nozzle

Diameter of nozzle: 1.5 mm
$p_1 = 720$ Torr; $p_2 = 23$ Torr
Diameter of Pitot-probe:
$\Delta = 1.35/\div = 0.6/0 = 0.25$
$\Delta = 0.14/\div = 0.08$ mm
Influence of probe diameter becomes negligible at larger distances.

Diameter of nozzle: 2 mm.
$p_1 = 720$ Torr, $p_2 = 188$ Torr
Note: Flow is of Hagen-Poiseuille type.
Pressure distribution perpendicular to the flow; much lower pressure than in Fig. 18
Diameter of nozzle: 2 mm
\(p_1 = 0.5 \) Torr, \(p_2 = 0.02 \) Torr
No loops are formed.
Note: Flow is of the free molecular type.

Pressure distribution along the axis at low pressures
Diameter of nozzle: 2 mm
\(p_1/p_2 = \) const = 25
"Waves" disappear for \(p_1 \approx 2 \) Torr
Note: The type of flow is changing.

As Fig. 20, but \(p_1 = \) const = 0.4 Torr
Note: Type of flow is free molecular.
The fact that for zero distance \(p\) is less than \(p_1\) is explained by a pressure drop in the nozzle (length not given).

Pressure distribution along the axis if the diameter of the nozzle is varied.
\(p_1 = \) const = 10 Torr
\(p_2 = \) const = 0.1 Torr
Note the "paradoxon", with a distance of 9 mm a system with a nozzle of 2 mm diameter may work, whereas a system with a nozzle of 1 mm may fail. This shows that the term "differential pumping" does not cover systems like that.
 Characteristics of Vacuum Pumps (Air, 25°C)  

WORKING SIZE GRAPHS CAN BE OBTAINED AT COST FROM THE ONTARIO RESEARCH FOUNDATION, 43 QUEEN'S PARK, TORONTO 5, ONTARIO, CANADA
Design Diagram for Dynamic Pressure Stages (Air, 25°C)

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5, ONTARIO, CANADA
Fig. 25
3-stage system of dynamic pressure stages for a 1 Mev accelerator

- Adjustments
- High-vacuum chamber
- Water cooled aperture

---

(B. Schumacher, unpublished)

Fig. 26
Grün's system of dynamic pressure stages

- L: magnetic lens

The parts A, D_1, D_2 are electrically insulated.

(A. E. Grün, loc. cit.)
WIND TUNNEL

DRIFT TUBE
16 x 320 mm

PUMP 203

NOZZLE
0.8 x 8 mm

FIG. 27
Fig. 29

- Particle Beam
- Sharp Edge
- Annular Space Forming Dust Trap
- Gasket
- Exchangeable Nozzle