PRANDTL-MEYER FLOW OF DISSOCIATED AND IONIZED GASES

by

I. I. Glass and H. Kawada

JUNE, 1962
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SUMMARY

Prandtl-Meyer flows in oxygen, nitrogen, air and argon at high-temperatures have been computed for a number of cases that may be suitable for experimental investigation in the UTIA 4 in. x 7 in. hypervelocity shock tube. For the diatomic gases, the two extremes of complete chemical equilibrium and frozen vibration and dissociation have been treated. For oxygen, the case of vibrational equilibrium for partially frozen flows has also been computed. For the monatomic gas, argon, the frozen ionization and the chemical equilibrium cases have been calculated. Estimates of the expected fringe shifts in an interferometric investigation of the foregoing flows and of the expected chemical recombination lengths are included in appendices. It should be possible to obtain reasonable values of the recombination rate constants for dissociated and ionized gases from an interferometric study of Prandtl-Meyer flows. However, owing to the conflicting requirements between an adequate fringe shift and a reasonable relaxation length the interferometric measurements might prove to be quite difficult. An experimental verification is therefore desirable.
# TABLE OF CONTENTS

<table>
<thead>
<tr>
<th>NOTATION</th>
<th>ii</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. INTRODUCTION</td>
<td>1</td>
</tr>
<tr>
<td>2. THEORETICAL CONSIDERATIONS</td>
<td>2</td>
</tr>
<tr>
<td>2.1 Solution for Equilibrium Flows</td>
<td>3</td>
</tr>
<tr>
<td>2.2 Solutions for Frozen Flows</td>
<td>6</td>
</tr>
<tr>
<td>2.3 Solutions for Partially Frozen Flows</td>
<td>9</td>
</tr>
<tr>
<td>3. COMPUTED THERMODYNAMIC AND DYNAMIC FLOW QUANTITIES</td>
<td>11</td>
</tr>
<tr>
<td>3.1 Dissociated Gases: Oxygen, Nitrogen, and Air</td>
<td>11</td>
</tr>
<tr>
<td>3.2 Ionized Gases: Argon</td>
<td>13</td>
</tr>
<tr>
<td>4. DISCUSSION OF RESULTS</td>
<td>14</td>
</tr>
<tr>
<td>5. CONCLUSIONS</td>
<td>17</td>
</tr>
<tr>
<td>REFERENCES</td>
<td>19</td>
</tr>
</tbody>
</table>

**TABLES**

**FIGURES**

**APPENDIX A:** Sample Calculations for Equilibrium and Frozen Flows

**APPENDIX B:** Computed Fringe Shifts Through Shock Waves and Expansion Waves for Interferometric Flow Studies of Dissociated and Ionized Gases.

**APPENDIX C:** Characteristic Chemical and Relaxation Times and Lengths for Recombination of Dissociated Oxygen and Ionized Argon.
(ii)

**NOTATION**

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C_p$</td>
<td>specific heat at constant pressure (ft. lb/slug °R, cal/gm °K)</td>
</tr>
<tr>
<td>$C_v$</td>
<td>specific heat at constant volume</td>
</tr>
<tr>
<td>$k_R$</td>
<td>chemical recombination rate (mole$^{-2}$cm$^6$ sec$^{-1}$)</td>
</tr>
<tr>
<td>$k_D$</td>
<td>chemical dissociation rate (mole$^{-1}$cm$^3$ sec$^{-1}$)</td>
</tr>
<tr>
<td>$K$</td>
<td>Gladstone-Dale constant</td>
</tr>
<tr>
<td>$K_c$</td>
<td>equilibrium constant based on concentration ($K_c = k_D / k_R$)</td>
</tr>
<tr>
<td>$L$</td>
<td>optical length in test section</td>
</tr>
<tr>
<td>$M$</td>
<td>Mach number; molecular weight</td>
</tr>
<tr>
<td>$M_f$</td>
<td>$= U/a_f$, based on frozen sound speed</td>
</tr>
<tr>
<td>$M_e$</td>
<td>$= U/a_e$, based on equilibrium sound speed</td>
</tr>
<tr>
<td>$M_r$</td>
<td>$= U/a_T$, based on vibrational equilibration</td>
</tr>
<tr>
<td>$N_A$</td>
<td>Avogadro number</td>
</tr>
<tr>
<td>$R$</td>
<td>gas constant per unit mass (ft. lb/slug °R, cal/gm °K)</td>
</tr>
<tr>
<td>$R_o$</td>
<td>gas constant per mole (ft. lb/slug mole °R, cal/gm mole °K)</td>
</tr>
<tr>
<td>$S$</td>
<td>specific entropy (ft. lb/slug °R, cal/gm °K)</td>
</tr>
<tr>
<td>$T$</td>
<td>temperature (°R, °K)</td>
</tr>
<tr>
<td>$W$</td>
<td>shock wave speed (ft/sec, m/sec)</td>
</tr>
<tr>
<td>$a$</td>
<td>speed of sound (ft/sec, m/sec)</td>
</tr>
<tr>
<td>$c$</td>
<td>speed of light (mi/sec, km/sec)</td>
</tr>
<tr>
<td>$c_o$</td>
<td>speed of light in vacuum (mi/sec; km/sec)</td>
</tr>
<tr>
<td>$e$</td>
<td>specific internal energy (ft. lb/slug, cal/gm)</td>
</tr>
<tr>
<td>$h$</td>
<td>specific enthalpy (ft. lb/slug, cal/gm)</td>
</tr>
<tr>
<td>$l$</td>
<td>relaxation length (ft, cm)</td>
</tr>
<tr>
<td>$l_R$</td>
<td>characteristic recombination length (ft, cm)</td>
</tr>
</tbody>
</table>
(iii)

\[ m \] \quad \text{mass fraction}

\[ n \] \quad \text{refractivity}

\[ p \] \quad \text{pressure (lb/ft}^2, \text{ dynes/cm}^2)\]

\[ r \] \quad \text{radial distance (ft, cm)}

\[ r_c \] \quad \text{chemical radial length at head of expansion wave (ft, cm)}

\[ s \] \quad \text{fringe shift}

\[ t \] \quad \text{time (sec)}

\[ U \] \quad \text{magnitude of flow velocity vector (ft/sec, m/sec)}

\[ v \] \quad \text{velocity component (ft/sec, m/sec)}

\[ x \] \quad \text{degree of ionization}

\[ z \] \quad \text{compressibility factor}

\[ \alpha \] \quad \text{degree of dissociation}

\[ \beta \] \quad \text{vibrational excitation parameter, defined in Eq. (39)}

\[ \gamma \] \quad \text{isentropic index of perfect gas or specific heat ratio}

\[ \Gamma \] \quad \text{isentropic index for equilibrium flow of an imperfect gas}

\[ \Gamma_f \] \quad \text{isentropic index for frozen flow}

\[ \Gamma_{fp} \] \quad \text{isentropic index for partially frozen flow}

\[ \theta \] \quad \text{angular position of a characteristic line (degrees, radians)}

\[ \Phi \] \quad \text{angle between tail of expansion wave and deflected wall}

\[ \lambda \] \quad \text{wavelength of light (ft, angstrom)}

\[ \mu \] \quad \text{moles/mole of original gas as defined in Eq. (20); Mach angle} = \sin^{-1} \frac{1}{M}

\[ \nu \] \quad \text{flow deflection angle; vibrational frequency}

\[ \overline{\nu} \] \quad \text{vibrational frequency in wave numbers}
\( \rho \) density (slugs/ft\(^3\), gm/cm\(^3\))

\( \tau \) relaxation time (sec)

\( \tau_r \) characteristic chemical time for recombination (sec)

\( \omega_p \) plasma frequency (sec\(^{-1}\))

Subscripts

\( o \) standard state

\( 1 \) state (1) in front of shock wave

\( 2 \) state (2) behind the shock wave

\( 3 \) state (3) behind the expansion wave

\( s \) shock wave

\( r \) radial

\( \theta \) circumferential

\( e \) equilibrium

\( f \) frozen

\( \bar{f} \) partially frozen

\( v \) vibrational

\( d \) dissociational

\( i \) ionizational

\( a \) active

Important Constants (Ref. 1)

\( \theta_d = 59,000^0\text{K} \): characteristic dissociation temperature of oxygen

\( \theta_d = 113,000^0\text{K} \): characteristic dissociation temperature of nitrogen

\( \theta_i = 183,000^0\text{K} \): characteristic ionization temperature of argon
Molecular Weights

\[ M_{O_2} = 32.000 \]
\[ M_{N_2} = 28.016 \]
\[ M_{Air} = 28.967 \]
\[ M_{NO} = 30.008 \]
\[ M_A = 39.944 \]

\[ R_o = 8.314 \times 10^7 \text{erg/mole degK} \] \hspace{1cm} \text{gas constant per mole}
\[ R_o = 1.98717 \text{cal/mole degK} \]
\[ N_A = 6.023 \times 10^{23} \text{/mole} \] \hspace{1cm} \text{Avogadro number}

Conversion Factor

1 cal/gm = 4.185 x 10\(^7\) cm\(^2\)/sec\(^2\)
1. INTRODUCTION

With the development of hypervelocity flight a number of new aerodynamic problems have arisen. One that is of considerable importance deals with the question whether or not a gas flow after passing through a shock wave or an expansion wave is in a state of chemical equilibrium, non-equilibrium, or if the active degrees of freedom only are in equilibrium and the inert degrees are frozen at their upstream values. Each type of flow will give rise to different aerodynamic pressure distributions and heat transfer rates on wing or body surfaces or the thrust that may be obtained from a rocket nozzle. The actual flow that will exist will depend on the collision processes or the dissociation and recombination rate constants of the various gas reactions that are taking place in the flow.

Many analyses involving flows through shock waves and expansion waves have been done during the past decade (see for example the numerous authors listed in Refs. 1, 2, and 3). A well-defined steady flow that lends itself to a theoretical treatment is the one through a centred expansion wave or the so called Prandtl-Meyer flow. Such flows have been considered by Heims (Ref. 4), Hayes and Probstein (Ref. 4), Cleaver (Ref. 5), Appleton (Ref. 6) and Napolitano (Ref. 7). In the present report an attempt is made to simplify and extend the solutions. Only equilibrium and frozen flow cases are treated. It is hoped that the nonequilibrium flows will be the subject of a separate report.

In the case of dissociated oxygen, 15 combinations of initial channel conditions and shock Mach number are considered. In this instance, partially frozen flow, where the degree of dissociation is frozen at its initial value but molecular vibration is in equilibrium with the active degrees, has also been calculated. Quite a number of cases were considered for centred expansion waves in oxygen since it is a very suitable gas for experimental studies. Its rate constants are reasonably well established and it is a gas that can be readily dissociated. It is planned to investigate such flows in some detail in the UTIA 4 in. x 7 in. hypervelocity shock tube by using a Mach-Zehnder interferometer, piezo-pressure gauges, heat transfer gauges, and line-reversal temperature measurements.

For comparison with the oxygen cases, calculations also were performed for dissociated nitrogen and air for one set of initial conditions of $M_s = 14$ and $p_1 = 10$ mm Hg.

In view of current interest in magnetogasdynamics and plasma physics, 9 combinations of shock Mach number and initial channel conditions were considered for the case of ionized argon.

The variation with expansion angle of temperature, pressure, density, flow velocity, Mach number and characteristic slopes are plotted and also listed in tables. In all cases considered, the results show that the temperature is the most sensitive physical quantity to be measured in order to
determine the state of the gas. The other flow quantities in order of sensitivity are Mach number, pressure, and density, respectively.

A characteristic "chemical time" for recombination can be defined from the molar concentration, the degree of dissociation and the recombination rate coefficient (Ref. 5). The "flow time" of a particle along a streamline through a centred rarefaction wave increases with increasing radial distance from the corner. Consequently, the ratio of the chemical time to the flow time decreases with radial distance as the flow passes from a frozen state (large ratio) to an equilibrium state (small ratio). The Prandtl-Meyer type of flow is therefore very suitable for the study of the three types of flow noted previously by appropriately adjusting the initial conditions, for example, pressure and shock Mach number, which have a strong influence on whether or not a flow is in a frozen or equilibrium state.

It may be possible to determine the recombination rate coefficient from an interferometric study of the expansion wave head itself or from the distribution of physical quantities in the entire flow field (Refs. 5 and 6). In addition, there are some interesting side problems such as the determination of the index of refraction when dealing with an ionized gas (Refs. 8 and 9). An interferometric study of the foregoing flows will undoubtedly contribute to a better understanding of the aerophysical properties of dissociated or ionized gas flows at high temperature. Such a study will be the subject of a report by A.G. Boyer using the 4 in. x 7 in. UTIA Hypersonic Shock Tube.

2. THEORETICAL CONSIDERATIONS

A Prandtl-Meyer type of expansion is illustrated in Fig. 1, as it might appear in an actual shock-tube experiment. Neglecting any diffraction or reflection patterns, for simplicity, it is seen that the shock wave as it diffracts around the corner at O, induces a quasi-steady centred expansion. If state (1) and the shock Mach number $M_s$ before the corner $O$ are given, then state (2) is completely determined assuming equilibrium flow (Ref. 1). The problem is to find the flow within the expansion wave and in state (3). If it is assumed that the degree of dissociation $\alpha_2$ adjusts immediately to its appropriate local value $\alpha$ on each characteristic line within the expansion wave until it has the value $\alpha_3$ appropriate to state (3), then the flow is in equilibrium. If $\alpha_2$ and the vibrational energy remain fixed throughout the expansion process as well as in state (3), then the flow is frozen. If $\alpha_2$ alone remains a constant but the vibrational mode is in equilibrium with the translational and rotational modes then the flow is defined as partially frozen. In the case of an ionized flow, only the conditions of frozen and equilibrium flows without electronic excitations are considered. In a nonequilibrium flow, none of these limiting flows exist. Consequently the entire flow field within the expansion wave as well as in state (3), is nonuniform and quite complex.
It is worth noting that only in nonequilibrium flow is there a production of entropy along a streamline. The entropy results from the heat addition on release of the dissociation or ionization energy during atom-atom or electron-ion recombination. For the case of equilibrium flow, where the free-enthalpy (Gibbs free-energy) is a constant, and for the case of frozen flow, where the degree of dissociation or ionization is a constant, both flows remain isentropic or irrotational (Ref. 7).

2.1 Solution for Equilibrium Flows

The Prandtl-Meyer type of expansion naturally lends itself to the use of polar coordinates. For a two-dimensional, steady, irrotational flow the continuity, momentum, and energy equations can be written as (Ref. 10):

\[
\frac{V_r}{\rho} \frac{\partial \rho}{\partial r} + \frac{V_\theta}{\rho} \frac{\partial \rho}{\partial \theta} + \frac{\partial V_r}{\partial r} + \frac{\partial V_\theta}{\partial \theta} + \frac{V_r}{r} = 0 \tag{1}
\]

\[
V_r \frac{\partial V_\theta}{\partial r} + V_\theta \frac{\partial V_r}{\partial \theta} - \frac{V_r^2}{r} + \frac{1}{\rho} \frac{\partial p}{\partial \theta} = 0 \tag{2}
\]

\[
V_r \frac{\partial h}{\partial r} + V_\theta \frac{\partial h}{\partial \theta} - \frac{V_r V_\theta}{\rho} + \frac{1}{\rho} \frac{\partial p}{\partial \theta} = 0 \tag{3}
\]

Since in an isentropic planar flow, all the physical quantities are constant along a characteristic line the terms involving a rate of change with \( r \) are zero. Consequently, all flow quantities depend only on the angle \( \theta \) of the characteristic lines. This simplification reduces the above set of nonlinear partial differential equations to ordinary differential equations as follows:

\[
\frac{V_\theta}{\rho} \frac{d\rho}{d\theta} + \frac{dV_\theta}{d\theta} + V_r = 0 \tag{5}
\]

\[
\frac{dV_r}{d\theta} - V_\theta = 0 \tag{6}
\]

\[
V_\theta \frac{dV_\theta}{d\theta} + V_\theta V_r + \frac{1}{\rho} \frac{dp}{d\theta} = 0 \tag{7}
\]
Combining Eqs. (5), (6) and (7), and writing \( dp = a_e^2 d\rho \) for an isentropic flow

\[
\left( \frac{dV_\theta}{d\theta} + V_r \right) \left( V_\theta - \frac{a_e^2}{V_\theta} \right) = 0
\]

(9)

From Eq. (9) either

\[
\frac{dV_\theta}{d\theta} + V_r = 0 \quad \text{or} \quad V_\theta = a_e
\]

(10)

The former result is inadmissible, since from Eq. (7) the pressure would remain a constant. From Eqs. (6), (7), (8) and (10),

\[
V_\theta \frac{dV_\theta}{d\theta} + V_r \frac{dV_r}{d\theta} + \frac{dh}{d\theta} = 0
\]

or

\[
\frac{1}{2} (V_r^2 + a_e^2) + h = h_0, \quad \text{a const.}
\]

(11)

where,

\[
h_0 = \frac{1}{2} U_2^2 + h_2
\]

Equation (11) is the usual relation for constant enthalpy flow and could have been stated at once instead of using the differential form, Eq. (8).

From Eq. (11),

\[
V_r = \left( 2(h_0 - h) - a_e^2 \right)^{\frac{1}{2}}
\]

(12)

and from Eq. (6),

\[
d\theta = \frac{dV_r}{a_e(h)}
\]

(13)

It is seen from Eqs. (12) and (13) that the variation of the characteristic slope \( \theta \) is a function only of the thermodynamic quantities \( a_e \) and \( h \), and these in turn could be expressed in terms of other state variables if required. However, for convenience one chooses the values of the enthalpy \( h \) along an isentrope from a Mollier chart or tables and all the other state properties are determined simultaneously at that point. Therefore, Eq. (13) can be integrated to give the variation of the characteristic slope with the flow properties as

\[
\theta = \int_{h_2}^{h} \frac{dV_r(h)}{a_e(h)} + \theta_2
\]

(14)
The integration is best performed graphically, unless analytic functions for the thermodynamic quantities are available to fit sections of the required range.

Since
\[ U^2 = V^2_r + V^2_\theta = V^2_r (h) + a^2_e (h) = U^2 (h) \]

therefore,
\[ U(h) = \left( V^2_r (h) + a^2_e (h) \right)^{1/2} \] \hspace{1cm} (15)

and
\[ M_e (h) = \frac{U(h)}{a_e (h)} \] \hspace{1cm} (16)

From Fig. 1, the flow deflection angle (\( \nu \)) can be determined from geometric considerations as shown or,
\[ \nu (h) = \theta (h) - \cos^{-1} \left( \frac{1}{M_e (h)} \right) \] \hspace{1cm} (17)

Therefore, all the flow parameters are known as functions of the Mach line angle (\( \theta \)) or in terms of the deflection angle (\( \nu \)). It should be noted that if the flow is expanded from \( M = 1 \) to \( M = M_2 \), then a specific starting angle (\( \nu_2 \)) is required to achieve this flow acceleration. Consequently, the total deflection angle (\( \nu_T \)), which is necessary to increase the Mach number from \( M = 1 \) to \( M = M_3 \), is given by the sum \( \nu_T = \nu_2 + \nu_3 \). The maximum expansion angle is obtained by expanding the flow from \( M_2 = 1 \) to \( M_3 \rightarrow \infty \).

In this case, from Eqs. (14) and (17),
\[ \nu_{\text{max}} = \int_{h_{M=1}}^{h_{M=\infty}} \frac{dV_r (h)}{a_e (h)} - \frac{\pi}{2} \] \hspace{1cm} (18)

For a perfect gas this expression reduces to
\[ \nu_{\text{max}} = -\frac{2A}{2 \sqrt{\gamma R}} \int_{T=T_2}^{T=0} \frac{dT}{\sqrt{BT + A T^2}} - \frac{\pi}{2} \]

where \( A = -\gamma R \left( \frac{\gamma + 1}{\gamma - 1} \right), \) \( B = U^2_2 + 2C_p T_2, \) where \( U^2_2 = \sqrt{\gamma RT_2} \)

The integral is of standard form (see for example Eq. 29, Sec. 2.2) and for these limits has the value \( \frac{\pi}{2} \sqrt{(\gamma + 1)/(\gamma - 1)} \)

or
\[ \nu_{\text{max}} = \frac{\pi}{2} \left( \sqrt{\frac{\gamma + 1}{\gamma - 1}} - 1 \right) \] \hspace{1cm} (18a)

Equation (18a) is the usual result for a perfect gas and for a diatomic gas \( \nu_{\text{max}} = 130.5 \) degrees (Ref. 11).
2.2 Solutions for Frozen Flows

It is assumed that in a frozen flow the inert degrees of freedom such as molecular vibration and the degree of dissociation retain their upstream values in a Prandtl-Meyer expansion of a dissociated gas. Similarly in a monatomic gas, the translational degrees are in equilibrium and it is assumed that electron-ion recombination does not occur.

Under the preceding assumptions, the thermodynamics of the expansion flow is very much simplified and it is possible to obtain an exact solution to the ordinary differential equation, Eq. (14), that relates the slope of a characteristic line to the thermodynamic quantities. For a frozen flow the internal energy per unit mass of the gas can be considered as the sum of two parts, the frozen and active energies, or,

\[ e = e_a + e_f \]  

(19)

where, \( e_f \) is now taken as a constant. The active internal energy is made up of translational and rotational degrees each contributing \( \frac{1}{2} \) \( R \) \( T \) per mole. Consequently, all of the contributions can be written as

\[ e_a = \frac{R_o T}{2} \sum_j \frac{m_j D_j}{M_j} = \frac{RT}{2} \sum_j \mu_j : D_j \]  

(20)

The summation is performed over all existing species and the value of \( D \) is 3 for an atom or electron and 5 for a diatomic molecule. The sum \( \sum_j \mu_j D_j \) is a constant for a frozen flow and may be obtained from thermodynamic tables such as those given in Ref. 11 for air, for example.

Similarly, the enthalpy per unit mass can be expressed as

\[ h = e + \frac{p}{\rho} = e_a + e_f + ZRT \]

or

\[ h = e_f + \frac{RT}{2} (2Z + \sum_j \mu_j D_j) \]  

(21)

The sound speed for a frozen flow is given by

\[ a_f = \left( \frac{\partial p}{\partial \rho} \right)_{S,D_j} = \sqrt{\frac{\gamma \rho}{\rho}} = \sqrt{\gamma RT} \]  

(22)

The isentropic index \( \gamma \), can be found in the following manner (Ref. 12). Since the flow is frozen \( Z \) is constant and the gas behaves like a perfect gas. From Eqs. (20) and (21) therefore,
\[ C_v = \frac{d_e}{dT} = \frac{R}{2} \sum_j \mu_j D_j \]  
\[ C_p = \frac{dh}{dT} = \frac{R}{2} \left( 2Z + \sum_j \mu_j D_j \right) \]  
\[ \gamma = \frac{C_p}{C_v} = 1 + \frac{2Z}{\sum_j \mu_j D_j} \]  
\[ = 1 + \frac{2Z}{\sum_j \mu_j D_j} = 1 + \frac{ZR}{C_v} \]

It is worth noting that for a pure diatomic gas the summation \( \sum_j \mu_j D_j \) is given by the following:

\[ C_p = \frac{dh}{dT} = \frac{R}{2} \left( 2Z + \sum_j \mu_j D_j \right) \]

\[ \gamma = \frac{C_p}{C_v} = 1 + \frac{2Z}{\sum_j \mu_j D_j} \]

Similarly for a monatomic gas consisting of atoms, ions, and electrons,

\[ m_1 = (1 - \alpha) \quad m_2 = \alpha(1 - \xi) \quad m_3 = \xi \]

\[ D_1 = D_2 = D_3 = 3 \quad M_1 = M_0 \quad M_2 = M_0 (1 - \xi) \quad M_3 = \xi M_0 \]

\[ \mu_1 = (1 - \alpha) \quad \mu_2 = x \quad \mu_3 = x \]

\[ \sum_j \mu_j D_j = 3 \times (1 - x + x + x) = 3(1 + x) \]

As expected, the value of \( \gamma \) does not change from the monatomic value of \( \gamma \) for a perfect gas.

The well-known isentropic relations also apply, for a frozen flow, that is,
\[ \frac{P}{\rho_{r}} = \left( \frac{\rho}{\rho_{r}} \right)^{n} \]
\[ \frac{\rho}{\rho_{r}} = \left( \frac{T}{T_{r}} \right)^{\frac{1}{n-1}} \]
\[ \frac{P}{P_{r}} = \left( \frac{T}{T_{r}} \right)^{\frac{n}{n-1}} \]

Referring to Eq. (12), and substituting for \( h_{o}, h \) and \( a_{f}^{2} \),

\[ V_{r} = \left( U_{2}^{2} + (T_{2} - T) \left( R \sum_{j} \mu_{j} D_{j} + 2 ZR \right) - \Gamma ZRT \right)^{\frac{1}{2}} \]

or

\[ V_{r} = \left( B + A T \right)^{\frac{1}{2}} \]

where

\[ B = U_{2}^{2} + T_{2} \left( R \sum_{j} \mu_{j} D_{j} + 2 ZR \right) \]
\[ A = - \left( R \sum_{j} \mu_{j} D_{j} + 2 ZR + \Gamma ZR \right) \]

From Eq. (14),

\[ \frac{\text{d}V_{r}}{a} = \frac{A}{2 \left( \Gamma ZR \right)^{\frac{1}{2}} \left( B + A T \right)^{\frac{1}{2}}} \]

or

\[ \theta - \theta_{2} = \frac{A}{2 \left( \Gamma ZR \right)^{\frac{1}{2}}} \int_{T_{2}}^{T} \frac{\text{d}T}{\left( B T + A T^{2} \right)^{\frac{1}{2}}} \]

This is a standard integral of the form,

\[ \theta - \theta_{2} = \frac{A}{2 \left( \Gamma ZR \right)^{\frac{1}{2}}} \left[ \frac{1}{(-A)^{\frac{1}{2}}} \left\{ \sin^{-1} \left( \frac{\Gamma+1}{1+\frac{\Gamma-1}{2} M_{2f}^{2}} \right) - \sin^{-1} \left( \frac{\Gamma+1}{1+\frac{\Gamma-1}{2} M_{2f}^{2}} \right) \right\} \right] \]

where \( A \) is negative as noted in Eq. (28). Substituting the correct sign and the values of \( \Gamma \) and \( M_{2} \), then

\[ \theta - \theta_{2} = \frac{1}{2} \left( \frac{\Gamma+1}{\Gamma-1} \right)^{\frac{1}{2}} \left[ \sin^{-1} \left( \frac{\Gamma+1}{1+\frac{\Gamma-1}{2} M_{2f}^{2}} \right) - \sin^{-1} \left( \frac{\Gamma+1}{1+\frac{\Gamma-1}{2} M_{2f}^{2}} \right) \right] \]

or

\[ \frac{\Gamma+1}{1+\frac{\Gamma-1}{2} M_{2f}^{2}} \cdot \frac{T}{T_{2}} - 1 \]
It is of interest to note that when $M_2 = 1$, and $T = 0$, one obtains the expression for the maximum inclination of the characteristic line where the gas escape speed is attained, that is

$$\theta_{\text{max}} = \frac{\pi}{2} \left( \frac{n+1}{n-1} \right)^{\frac{1}{2}}$$

Equation (31) is the usual result for a perfect gas with a constant specific heat ratio or isentropic index ($\Gamma$). The flow Mach number is given by,

$$M_f = \frac{U}{a_f} = \left( \frac{V_r^2 + a_f^2}{a_f^2} \right)^{\frac{1}{2}}$$

Substituting Eq. (27) in the above equation yields,

$$M_f = \left\{ \left( \frac{M_r^2}{2} + \frac{2}{\Gamma - 1} \right) \frac{T_2}{T} - \frac{2}{\Gamma - 1} \right\}^{\frac{1}{2}}$$

As for the equilibrium case, from Fig. 1, the expansion angle,

$$\psi = \theta - \cos^{-1} \frac{1}{M_f}$$

Consequently, the entire flow can be determined analytically once the initial state (2) is completely defined.

2.3 Solutions for Partially Frozen Flows

It was noted in the preceding section that a partially frozen flow can be defined for diatomic or polyatomic molecules for which it is assumed that the vibrational excitation is in equilibrium with the translational and rotational modes. However, the degree of dissociation is considered as frozen. This is a reasonable assumption since for oxygen for example the vibrational relaxation time is short compared to that of dissociation (Ref. 2). Consequently, a partially frozen flow in such a case may under certain conditions be more realistic than the assumption of a frozen flow.

The internal energy now consists of a vibrational portion in equilibrium with the active degrees and a frozen dissociational contribution expressed as,

$$e = e_a + e_f$$
where
\[ e_a = \frac{RT}{2} \sum_j \mu_j D_j + RT \sum_i g_i \frac{Z}{e^Z - 1} \]

\[ Z = 1.438 \quad \frac{\nu}{T} = \theta_v/T, \text{ a temperature ratio (Ref. 1)} \]
\[ \nu = \text{vibrational frequency in wave numbers} \]
\[ \theta_v = \text{characteristic vibrational temperature (1.438 \( \nu \))} \]
\[ g_i = \text{degeneracy factors (number of repeated frequencies)} \]

The second term is the vibrational contribution to the internal energy. The specific heat at constant volume is given by
\[ C_{v_a} = \frac{R}{2} \sum_j \mu_j D_j + R \sum_i g_i e^Z \left( \frac{Z}{e^Z - 1} \right)^2 \]  
(37)

For a pure diatomic gas (with a single vibrational mode) Eq. (37) reduces to,
\[ C_{v_a} = \frac{R}{2} (5 + \alpha) + R e^Z \left( \frac{Z}{e^Z - 1} \right)^2 \]  
(38)

In the case of oxygen, the value of \( \nu = 1570 \text{ waves/cm, and } \theta_v = 1.438 \times 1570 = 2260^0\text{K} \). For example, when \( T = 2200^0\text{K} \) the value of the second term in Eq. (38) is already 0.9R and the maximum possible as \( T \to \infty \) is a value of R. Therefore, for higher temperatures (2000 - 4000^0\text{K}) occurring in the present calculations the value of the second term may be considered as a constant during the expansion process. Consequently, a new isentropic index \( \bar{n} \) may be defined for a partially frozen pure diatomic gas, like oxygen, as,
\[ \bar{n} = \frac{C_{pa}}{C_{v_a}} = \frac{7 + 3 \alpha + 2 \beta (1 - \alpha)}{5 + \alpha + 2 \beta (1 - \alpha)} \]  
(39)

where,
\[ \beta = e^Z \left( \frac{Z}{e^Z - 1} \right)^2 \]

The value of \( \bar{n} \) is therefore a constant depending on the upstream conditions before the Prandtl-Meyer expansion. All the previous analytical results for a frozen flow apply except that the new value of \( \bar{n} \) must be used where applicable. Computations have been performed for oxygen in the partially frozen condition and are discussed in Section 3.
3. COMPUTED THERMODYNAMIC AND DYNAMIC FLOW QUANTITIES

The present section contains a summary of the various calculations for dissociated and ionized gas flows in an equilibrium, a frozen, or a partially frozen state. The computations are based on the analysis presented in the previous section.

3.1 Dissociated Gases: Oxygen, Nitrogen, and Air

Although calculations were done for the Prandtl-Meyer flow in all of the three diatomic gases noted above, the most extensive are those for oxygen. The choice was dictated by the facts that oxygen is a pure gas that is not too difficult to dissociate in a hypervelocity shock tube and its dissociation rate constant has been studied quite extensively both theoretically and experimentally (Refs. 13, 14, 15). Consequently, it should be possible now to provide a good estimate of the recombination rate constant, which might be obtained experimentally from a study of Prandtl-Meyer flows in oxygen. The use of a pure diatomic gas also simplifies the problem of interferometric measurements (see Appendix B) and eases the difficulty of a theoretical computation of a nonequilibrium expansion flow.

Table 1 gives a summary of the various initial conditions used in calculating the Prandtl-Meyer flows for oxygen, nitrogen and air. The shock Mach numbers and initial pressures fall within the capabilities of the UTIA 4 in. x 7 in. hypervelocity shock tube. Conditions outside of this range are impractical for the following reasons: the combustion chamber pressures \( p_4 \) would exceed the maximum operating combustion pressure of 10,000 psi; the channel pressure \( p_1 \) would be too low to give a significant fringe shift for interferometric observations; the channel pressure would be too high to give a significant amount of dissociation \( \chi_2 \). As can be seen, there are fifteen cases treated for oxygen, one case for nitrogen, and one case for air. The latter case \((M_S = 14, \; p_1 = 10 \text{ mm Hg})\) is common to all three gases in order to provide a basis for subsequent comparison.

When the initial state \((1)\) in the channel is specified and the shock Mach number \((M_S)\) is given, then all of the flow quantities in state \((2)\) behind a plane shock wave can be determined from tables, Mollier diagrams, and other pertinent graphs (Refs. 16, 17, 18, 19, 20 and 21). It is assumed that state \((2)\) is in thermodynamic and chemical equilibrium. Consequently, the initial state \((2)\) in which the Prandtl-Meyer expansion is generated is completely determined.

Table 2 lists the values of the thermodynamic and dynamic properties of state \((2)\) for oxygen, nitrogen and air. The following quantities are included for a given set of initial conditions, pressure \(p_2\), temperature \(T_2\), enthalpy \(h_2\), entropy \(S_2\), particle velocity \(U_2\), density \(\rho_2\), degree of dissociation \(\chi_2\); the equilibrium Mach number \((M_{2e} = U_2/a_{2e}\)), the frozen Mach number \((M_{2f} = U_2/a_{2f}\)), the partially frozen Mach number \((M_{2p} = U_2/a_{2p}\)).
where it is assumed that the vibrational energy is in equilibrium and the vibrational specific heat has a constant value $\beta_R$; the angular position of a characteristic line for equilibrium, frozen, and partially frozen flow ($\theta_{2e}$, $\theta_{2f}$, $\theta_{2p}$), the corresponding isentropic indices ($s_2$, $\gamma_2$, $\Gamma_2$); and the vibrational specific heat parameter $\beta$.

The details of a sample calculation are given in Appendix A, for oxygen, $M_s = 14$, $p_1 = 10$ mm Hg and $T_1 = 273^0K$. All the results are tabulated in Table 3 and shown graphically in Figs. 2 to 29, inclusive. Not all of the quantities shown in Table 3 have been plotted. However, the data is tabulated so that additional curves may be drawn if required. All of the results have been rounded to three figures. A greater apparent accuracy was not justified since the basic data was read from graphs or interpolated from tables.

It will be noted that all quantities are plotted against the expansion or flow deflection angle ($\phi$). Consequently, the graphs show the values of a given flow quantity in the uniform region behind the tail of the expansion wave as a function of the wall deflection.

The results for oxygen appear in Figs. 2 to 23, inclusive. The temperatures are plotted for the entire 15 cases (Figs. 2 to 7; 13 to 20; 23) found in Table 3, as a ready reference, since this quantity is the most sensitive indicator of whether or not the flow behind the Prandtl-Meyer fan is partially frozen, frozen or in equilibrium. The degree of dissociation ($\alpha$) for the equilibrium temperature curve is also shown at a number of intervals of the expansion angle ($\phi$). It should be noted that the equilibrium flow quantities do not extend beyond $\phi \sim 60-70^0$, owing to the lack of readily available thermodynamic data.

For one case only, $M_s = 14$, $p_1 = 10$ mm Hg and $T_1 = 273^0K$, all of the flow quantities of interest are shown in Figs. 7 to 12. The case of $M_s = 20$, $p_1 = 1$ mm Hg and $T_1 = 273^0K$ is of special interest since the degree of dissociation ($\alpha$) has its highest value, and the curves of pressure and density have been added for convenience (Figs. 20 to 22).

In the case of nitrogen, only one set of initial conditions were considered at present for the purpose of comparison with the identical conditions in oxygen ($M_s = 14$, $p_1 = 10$ mm Hg, $T_1 = 273^0K$). The results are shown in Figs. 24 to 26. It should be noted that the density spacing of the graphs in Ref. 17, which were used in the present calculations, were coarser, leading to less reliable results. In this particular case, only four points were available along the equilibrium isentrope.

A similar calculation was done for air (with the exception that $T_1 = 300^0K$) using some of the basic thermodynamic data given in Refs. 1, 19 and 20. The Mollier charts (Ref. 20) proved to be very useful for equilibrium flow calculations and provided more accurate data than for oxygen or nitrogen. The isentropic index ($s_2$) was obtained graphically from Ref. 21; while $M$ was calculated from the data given in Ref. 19. The results are shown on Figs. 27 to 29.
3.2 Ionized Gases: Argon

The solution of a Prandtl-Meyer flow in an ionized monatomic gas is very similar to a pure diatomic gas. However, in this case the only contributions to the internal energy come from the translational modes (including those resulting from ionization), electronic excitation, and the added ionization energy (Ref. 1). The energy of electronic excitation was neglected in the present analysis. Consequently, the partition functions in the Saha equation can be considered as constants. The present results therefore will be inaccurate for degrees of ionization greater than approximately twenty-five percent (x > 0.25). An inspection of Table 3, for the case of argon shows that x < 0.32 for all cases concerned. Consequently, the error should not be very significant.

Argon was chosen as a convenient gas for the study of the Prandtl-Meyer flow of an ionized gas since it is quite economical, readily available commercially, and has a relatively low ionization potential (15.75 electron volts). Some of the other monatomic gases might be more suitable for an interferometric study owing to their high index of refraction (neon or xenon, for example), but they do not meet the foregoing requirements. Argon also has a molecular weight comparable to that of oxygen. Consequently, it is possible to produce approximately the same shock Mach numbers for the same diaphragm pressure ratio in the shock tube. As a result, nine combinations were chosen for analysis on a basis similar to that for oxygen (see sub-Sec. 3.1). The initial conditions are shown in Table 1.

The physical quantities in state (2), behind the normal shock wave, were obtained from the tables and Mollier diagram of Refs. 22 and 23. The Mollier diagram in this case is not as detailed as that for air (Ref. 20), and the results are therefore not as accurate. The state properties are listed in Table 2.

The equilibrium sound speed was calculated using the relation (Ref. 1),

\[
a^2_e = \gamma_e \frac{p}{\rho} = \gamma_e (1 + x) \frac{R_0 T}{m}
\]

where,

\[
\gamma_e = \frac{C_p}{C_v} = \frac{2}{(1 + x) (2 - x)}
\]

\[
C_v = \frac{3}{2} \frac{R_0}{m} (1 + x) + \frac{R_0 x}{m} \frac{(1 - x)}{(2 - x)} \left( \frac{3 + \theta_i}{2} \right)^2
\]

\[
C_p = \frac{5}{2} \frac{R_0}{m} (1 + x) - \frac{R_0 x}{2m} \frac{(1 - x^2)}{(2 - x)} \left( \frac{5 + \theta_i}{2} \right)^2
\]

\[
\theta_i = \frac{1}{k} = 183,000^\circ K
\]
\( E_1 = 15.75 \, \text{eV} \) (ionization energy), \( k = 86.16 \times 10^{-6} \, \text{eV/}^0\text{K} \) (Boltzmann constant)

The previous remark regarding constant partition functions or negligible electronic excitation should be noted.

The frozen flow calculations follow the methods outlined in Sec. 2. Since the electronic excitation is neglected, a partially frozen flow state is not possible in this case.

The quantities in state (3) for the cases of equilibrium and frozen flows are listed in Table 3. Only the temperatures have been plotted as a function of the expansion angle (\( \psi \)) in all cases (Figs. 30 to 34; 40 to 42; 45). For one representative case \( M_s = 20, \, p_1 = 1 \, \text{mm Hg}, \, T_1 = 300^0\text{K} \), where the highest degree of ionization occurs, the curves of pressure and density versus expansion angle have been added as a reference.

4. DISCUSSION OF RESULTS

An examination of all the temperature plots for the diatomic gases (Figs. 2 to 7; 13 to 20; 23, 24, 27) shows that they all follow similar trends. That is, for a given flow deflection angle the equilibrium curve has the highest temperature owing to the heat addition resulting from recombination, and the frozen flow curve has the lowest temperature as a result of the energy bound up in the dissociated molecules. In the case of oxygen, the partially frozen curve lies increasingly closer to the frozen flow curve as the degree of dissociation becomes increasingly greater. Physically, the above result shows that when the degree of dissociation is high the vibrational contribution to the internal energy is not very significant. However, it is seen that very large temperature differences exist between the equilibrium and frozen flow states. Therefore, a direct measurement of temperature would give a very positive indication of the state of the gas.

The other thermodynamic and dynamic flow quantities for the case of \( M_s = 14, \, p_1 = 10 \, \text{mm Hg} \) and \( T_1 = 273^0\text{K} \) are shown in Figs. 8 to 12. It can be seen that all the curves of pressure and density lie close together for the three different states that might be encountered in an actual flow. Consequently, a measurement of pressure or density will not be nearly as sensitive as a measurement of temperature to indicate the state of the gas. It is also worth noting that the curves cross over at different values of \( \psi \) for pressure and density. This result can also be shown analytically (Ref. 4). It is seen that the pressure curves and density curves possess their own optimum \( \psi \), where the difference between the frozen and equilibrium flow values are a maximum. For example, in this particular case the maximum pressure difference occurs at \( \psi \sim 30^0 \) (Fig. 8), and the maximum density difference at \( \psi \sim 10^0 \) (Fig. 9). Similar remarks apply to Figs. 21 and 22.
Since the speed of sound is highly temperature dependent, it can be expected that it will show a variation similar to that of temperature as seen in Fig. 10. The equilibrium and frozen values of the particle velocity (\( U = \)) differ by small amounts at the smaller expansion angles (\( \gamma \sim 30^\circ \), Fig. 11). At greater angles the energy of recombination also increases the kinetic energy so that the equilibrium particle velocity, becomes increasingly greater than that for the frozen and partially frozen flows. The Mach number (Fig. 12), which involves both \( U \) and \( T \), therefore becomes an extremely sensitive indicator of the state of the gas for expansion angles \( \gamma > 30^\circ \). It is seen that for this particular case a cross-over occurs at \( \gamma \sim 15^\circ \), and at that angle this parameter has nearly the same value for all three states. At high temperatures and sizeable degrees of dissociation the pressure and density curves (Figs. 21 and 22) show that at low expansion angles it will be very difficult to distinguish one type of state from another. On the other hand, it does not help to go to large flow deflection angles (of 30°, say), where the differences in these flow quantities do become significant, because the tail of the wave lies close to the deflected wall and consequently the region of uniform flow (in the frozen or equilibrium state) is very much reduced. This is a significant disadvantage in interferometric studies. For example, the angle \( \Phi \) between the tail of the expansion wave and the wall is given by \( \Phi = \left( \frac{\pi}{2} - \theta \right) + \gamma \). When \( M = 1 \), \( \gamma = 0 \), \( \left( \frac{\pi}{2} - \theta \right) = \mu \) or \( \Phi = \mu \), a maximum; when \( M \rightarrow \infty \) (say for a frozen flow)

\[
\Phi = \frac{\pi}{2} - \frac{\pi}{2} \left( \frac{\pi + 1}{\pi - 1} \right)^{\frac{1}{2}} + \frac{\pi}{2} \left( \frac{\pi + 1}{\pi - 1} \right)^{\frac{1}{2}} - \frac{\pi}{2} = 0
\]

(see Eqs. 31 and 34), a minimum.

The pressure and density curves for nitrogen and air \((M = 14, p_1 = 10 \text{ mm Hg})\) show somewhat similar trends to that of oxygen (Figs. 25, 26, 28 and 29). It is worth noting that for air (Fig. 29) the density curves show that an interferometric study would be rather difficult owing to the small density changes between the frozen and equilibrium states. The values that were computed by Heims (Ref. 4), for one set of initial conditions in air, predicted similar trends.

The previous discussion of the diatomic gases applies with some slight modification to the monatomic gas argon. An examination of Figs. 30 to 45, reveals similar trends. Again, large temperature differences are noted. The change of curvature of the equilibrium lines for temperature corresponds to practically complete recombination of ions and electrons.

The pressure curves (Figs. 35 and 43) do not exhibit a cross-over point, while those of density (Figs. 36 and 44) do. The sound speed (Fig. 37), particle velocity (Fig. 38) and Mach number after the expansion wave (Fig. 39), show trends quite similar to the diatomic gases. For \( \gamma > 20^\circ \), the Mach number becomes a sensitive quantity in indicating the state of the gas. The technique of artificially generating Mach waves behind the expansion wave by using small roughness elements would in this case be a very useful device for detecting the state of the gas (Ref. 1).
Figures 46 and 47 summarize the temperature variations behind a Prandtl-Meyer expansion wave generated by a 15° wall deflection with different initial shock Mach numbers and channel pressures in oxygen and argon. All other quantities can be plotted in a similar manner from the tabulated results, if required. The large difference in temperature, for a given set of initial conditions, between frozen and equilibrium flows is well illustrated.

The various appendices at the end of the text should be of some interest to those who wish to become familiar with the details of the various calculations. Appendix A describes the method used to derive the data for Figs. 2 to 45.

Appendix B deals with the important practical problem of expected fringe shifts when considering an interferometric study of dissociated and ionized expansion flows. Figure B. 5 illustrates the fringe shifts across the shock wave for the diatomic gases considered previously. The frozen values for oxygen increase slightly with shock Mach number as expected. However, this increase for the equilibrium cases is quite marked as the density ratio rises for the strong shock waves. The difference in density between the two cases can result in fringe shifts two to three times greater for the equilibrium flows.

The fringe shifts for a given shock Mach number $M_s$, across expansion waves are not widely different for the frozen flows (dashed line) and equilibrium flows. This reflects the small change in density noted previously and emphasizes again the difficulty in using an interferometer as a major diagnostic instrument for determining the states of the gas in dissociated flows.

When it comes to ionized gas flows, the interferometer becomes a very useful instrument. This stems from the fact that in an ionized gas two indices of refraction exist, the phase index (monochromatic light) and the group index (white light). The striking difference in the size and sign of the monochromatic fringe shift in argon at the higher initial pressures and shock Mach numbers is well illustrated in Fig. B. 7. The negative fringe shifts result from the contributions to the phase index of refraction by the electron gas. Figure B. 8 shows the same trend through 15° expansion waves in argon. This property of the negative monochromatic fringe shift in an ionized gas should prove to be very useful in an interferometric study of frozen, nonequilibrium and equilibrium flows.

Similar results are obtained for white light interferograms. It is seen from Fig. B. 7a that large positive shifts result this time from the contributions of the electron gas. For example, at $M_s = 16$ and $p_1 = 10$ mm Hg., the equilibrium monochromatic fringe shift across the shock wave is a negative shift of approximately (-4) or 4 fringes below the original position in front of the shock wave. Whereas, the white light fringe shift is positive or about 12 fringes above the pre-shock condition. Similarly, the equilibrium monochromatic fringe shift across a 15 degree expansion behind the shock gives a negative
shift of about 3 fringes, whereas the white light shift has a positive value of 7 fringes. Similar results hold for frozen flows. Consequently, both types of interferograms should be used in the study of high-temperature gas flows. Further details can be found in Appendix B.

Relatively little experimental information is available to-day regarding flows of dissociated and ionized gases. A considerable effort could also be profitably expended in understanding some of the interesting interferometric problems before tackling the more complex flow problems at high temperatures. A discussion of possible experimental procedures is also given at the conclusion of Appendix B, which may be helpful in obtaining and interpreting experimental data of this nature.

In Appendix C, the chemistry of the flow is considered in some detail. The rate equations for dissociated oxygen and ionized argon are developed and are later applied to the determination of characteristic recombination lengths along the expansion wave head and along the wall behind the expansion wave that may lend themselves to interferometric or other measurements. It should then be possible to find values of the recombination rate constants from these characteristic lengths. Accurate measurements of these rate constants would constitute a significant contribution to the chemistry of high-temperature gas flows. However, it is shown in Appendix B, that it will be rather difficult to determine the rate constants from interferometric measurements owing to the small changes in fringe shift, which would have to be determined with a high degree of precision in order to get meaningful results. This situation arises mainly from the conflicting requirements of high densities to attain large fringe shifts and low densities in order to obtain reasonable relaxation lengths in nonequilibrium flows. Three optimum experiments are considered for this purpose for dissociated oxygen and three cases for ionized argon for the UTIA 4 in. x 7 in. hypervelocity shock tube. Complete details are given at the end of Appendix B.

The characteristic equations for nonequilibrium flows are also developed as a possible extension of the present analysis to such complex flows as well as to illustrate the concept and significance of the frozen sound speed in the propagation of characteristic lines. The results are applied to dissociating and ionizing gas flows which have been simplified through certain assumptions in order to clearly illustrate some of the physical properties of such flows.

5. CONCLUSIONS

Prandtl-Meyer flows in dissociated oxygen, nitrogen and air as well as in ionized argon have been computed for a considerable number of cases that may be suitable for experimental investigation in the UTIA 4 in. x 7 in. hypervelocity shock tube. The results show that the temperature is the most sensitive physical quantity to be measured in order to determine the state of the gas. The other physical quantities in order of sensitivity are Mach
number, pressure and density. It is very likely that heat transfer rates will also be very useful but this has not been investigated in the present report. Owing to the large differences in flow temperature between frozen and equilibrium flows, the heat transfer gauge may be very suitable for finding relaxation distances on the wall behind the expansion fan.

Special attention has been paid to possible interferometric studies of such dissociated and ionized gas flows. From the analysis it becomes evident that not only is the interferometric problem of considerable interest in itself especially in ionized gases, but that it may be possible to determine from interferometric experiments an entire portion of the density flow field, as well as the more important measurements of the recombination rate constants in dissociated and ionized gases. However, owing to the conflicting requirements between an adequate fringe shift and a reasonable relaxation length the interferometric measurements might prove to be quite difficult. An experimental verification is therefore desirable.

Details of the various calculations are given in separate appendices, tables and figures. Additional discussions and conclusions may also be found in the various appendices regarding the computed results and possible experimental procedures.

The experimental investigations of such dissociated and ionized flows will be the subject of separate UTIA reports.
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TABLE 1
INITIAL CONDITIONS FOR PRANDTL-MEYER FLOWS OF DISSOCIATED AND IONIZED GASES

1. **Oxygen:**

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<th>$p_1$ (mm Hg)</th>
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<th>30</th>
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<td>$M_s$</td>
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($T_1 = 273^\circ K$)

2. **Nitrogen:**

$M_s = 14, \ p_1 = 10 \text{ mm Hg}$

($T_1 = 273^\circ K$)

3. **Air:**

$M_s = 14, \ p_1 = 10 \text{ mm Hg}$

($T_1 = 300^\circ K$)

4. **Argon:**

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3. \( \frac{P_1}{\rho_0}, \rho_0 = \text{NTP density} \)
4. \( p_2 \) atm
5. \( h_2 \) cal/gm
6. \( T_2 \) deg. K
7. \( \frac{P_2}{\rho_0}, \rho_0 = \text{NTP density} \)
8. \( u_2 \)

Row

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**Column**
- **9** $S_2$ cal/gm deg. K
- **10** $U_2$ m/sec.
- **11** $M_{2e}$
- **12** $\theta_{2e}$ deg.
- **13** $M_{2r}$
- **14** $\theta_{2r}$ deg.
- **15** $M_{2r}$ not calculated for air or $N_2$, and not applicable to argon.
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Column

17 $r$

18 (see 15 and 16)

19 $\beta_2$

20 $C_p/2/R$

21 $C_v/2/R$
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**Column**
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2. Temperature, \( T^{0K} \)
3. Temperature ratio, \( T/T_2 \)
4. Pressure ratio, \( p/p_2 \)
5. Density ratio, \( \rho/\rho_2 \)
6. Dissociation or ionization degree, \( \alpha \) or \( \chi \)
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O$_2$, $M_S = 12$, $p_1 = 10$ mm Hg

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O$_2$, $M_S = 14$, $p_1 = 1$ mm Hg

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| .947  | 1.110 | 2.69  | 81.7   |
| .915  | 1.165 | 2.94  | 91.2   |
| .880  | 1.220 | 3.19  | 100.0  |
| .848  | 1.270 | 3.44  | 108.5  |
| .811  | 1.327 | 3.77  | 118.7  |
### TABLE 3 (continued)

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(Fig. 41)

| A, $M_S = 20$, $p_1 = 1$ mm Hg | 0.0 | 13280 | 1.000 | 1.000 | 1.000 | .310 |
|   | 2.7 | 13130 | .988 | .836 | .857 | .300 |
|   | 8.0 | 12750 | .960 | .631 | .676 | .280 |
|   | 12.2 | 12380 | .932 | .447 | .500 | .260 |
|   | 17.5 | 12000 | .903 | .302 | .354 | .240 |
|   | 21.5 | 11700 | .880 | .219 | .268 | .220 |
|   | 26.7 | 11330 | .853 | .145 | .186 | .200 |
|   | 31.2 | 11000 | .828 | .110 | .144 | .180 |
|   | 37.0 | 10520 | .792 | .068 | .097 | .160 |
|   | 41.5 | 10230 | .770 | .045 | .067 | .140 |
|   | 46.5 | 9990 | .752 | .029 | .046 | .120 |

(Figs. 42 to 44)

| A, $M_S = 20$, $p_1 = 3$ mm Hg | 0.0 | 14140 | 1.000 | 1.000 | 1.000 | .296 |
|   | 4.5 | 13840 | .978 | .785 | .815 | .280 |
|   | 9.7 | 13390 | .946 | .557 | .608 | .260 |
|   | 14.7 | 12980 | .917 | .376 | .429 | .240 |
|   | 19.0 | 12630 | .893 | .279 | .332 | .220 |
|   | 24.2 | 12200 | .863 | .197 | .248 | .200 |
|   | 28.5 | 11880 | .840 | .137 | .179 | .180 |
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|   | 38.7 | 11020 | .780 | .059 | .087 | .140 |

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TABLE 3

FROZEN FLOW PARAMETERS IN A PRANDTL-MEYER EXPANSION

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O₂, $M_s = 14$, $p_1 = 3$ mm Hg

(Fig. 5)

(Fig. 6)
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(O2, $M_S = 14$, $p_1 = 10 \text{ mm Hg}$)

(Original figures 7 to 12)

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| 8.0| 3320   | .800   | .512   | .640   |
| 12.2| 2905  | .700   | .343   | .490   |
| 16.7| 2490  | .600   | .216   | .360   |
| 21.2| 2075  | .500   | .125   | .250   |
| 27.0| 1660  | .400   | .064   | .160   |
| 32.7| 1245  | .300   | .027   | .090   |
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(O2, $M_S = 14$, $p_1 = 30 \text{ mm Hg}$)

(Fig. 13)
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O₂, \( M_s = 14 \), \( p_1 = 10 \) mm Hg

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**Table 3** (continued)

O$_2$, $M_s = 16$, $p_1 = 3$ mm Hg

(Fig. 14)

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O$_2$, $M_s = 16$, $p_1 = 10$ mm Hg

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O₂, Mₛ = 16, p₁ = 3 mm Hg

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**O₂, Ms = 18, p₁ = 1 mm Hg**

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**O₂, Mₛ = 18, p₁ = 3 mm Hg**

(Fig. 18)

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**O₂, Mₛ = 18, p₁ = 10 mm Hg**

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| .707 | 1.092 | 4.59 | 95.2 |
| .632 | 1.109 | 5.21 | 101.2|
| .548 | 1.126 | 6.11 | 107.5|
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| .894                      | 1.062| 3.05 | 79.7 |
| .837                      | 1.092| 3.35 | 86.2 |
| .775                      | 1.121| 3.71 | 93.2 |
| .707                      | 1.150| 4.17 | 100.5|
| .632                      | 1.178| 4.77 | 108.0|
| .548                      | 1.205| 5.64 | 116.5|
| .447                      | 1.231| 7.05 | 126.5|
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(Figs. 42 to 44)

| A, $M_s = 20$, $p_1 = 1$ mm Hg |

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**TABLE 3**

PARTIALLY FROZEN FLOW PARAMETERS IN A PRANDTL-MEYER EXPANSION

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**Column**

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8 Flow speed ratio, \( U/U_2 \)
9 Flow Mach number, \( M \)
10 Angular position, \( \theta \) degrees
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O$_2$, $M_S = 14$, $p_1 = 1$ mm Hg

(Fig. 5)

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O$_2$, $M_S = 14$, $p_1 = 3$ mm Hg

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O₂, Ms = 14, P₁ = 3 mm Hg
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(Figs. 7 to 12)
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O₂, Mₛ = 16, P₁ = 3 mm Hg

(Fig. 14)

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O₂, Mₛ = 16, P₁ = 10 mm Hg

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|    | 0.894 | 1.046 | 3.52  | 81.7   |
|    | 0.837 | 1.069 | 3.85  | 87.5   |
|    | 0.775 | 1.091 | 4.24  | 93.2   |
|    | 0.707 | 1.112 | 4.74  | 100.0  |
|    | 0.632 | 1.133 | 5.40  | 107.0  |
|    | 0.548 | 1.154 | 6.56  | 114.2  |
|    | 0.447 | 1.174 | 7.91  | 123.0  |
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O₂, \( M_\Sigma = 16 \), \( p_1 = 30 \) mm Hg

(Fig. 16)

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O₂, \( M_\Sigma = 18 \), \( p_1 = 1 \) mm Hg

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O₂, Mₛ = 16, p₁ = 30 mm Hg

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O₂, Mₛ = 18, p₁ = 3 mm Hg

(Fig. 18)

O₂, Mₛ = 18, p₁ = 10 mm Hg

(Fig. 19)
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$O_2$, $M_S = 18$, $p_1 = 3$ mm Hg

$O_2$, $M_S = 18$, $p_1 = 10$ mm Hg
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| 8.5 | 3365 | .700  | .399  | .569  |
| 12.2| 2885 | .600  | .268  | .446  |
| 16.0| 2405 | .500  | .167  | .334  |
| 20.0| 1925 | .400  | .094  | .235  |
| 24.7| 1445 | .300  | .045  | .149  |
| 30.0| 960  | .200  | .016  | .079  |
| 36.7| 480  | .100  | .003  | .027  |
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(Figs. 20 to 22)

(Fig. 23)
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O₂, Mₛ = 20, p₁ = 1 mm Hg

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O₂, Mₛ = 20, p₁ = 3 mm Hg
FIG. 1 PRANDTL-MEYER FLOW IN A SHOCK TUBE.
FIGURE 2

VARIATION OF FLOW TEMPERATURE (T) VS EXPANSION ANGLE (\(\gamma\)) IN A PRANDTL-MEYER FLOW FOR A FIXED CHANNEL PRESSURE \(p_1\) AND SHOCK MACH NO. \(M_s\).

OXYGEN, \(M_s = 10, p_1 = 10\) mm Hg.
FIGURE 3

VARIATION OF FLOW TEMPERATURE ($T$) VS EXPANSION ANGLE ($\gamma$) IN A PRANDTL-MEYER FLOW FOR A FIXED CHANNEL PRESSURE ($p_1$) AND SHOCK MACH NO. ($M_s$), OXYGEN, $M_s = 12$, $p_1 = 10$ mm Hg.
FIGURE 4

VARIATION OF FLOW TEMPERATURE ($T$) VS EXPANSION ANGLE ($\psi$) IN A PRANDTL-MEYER FLOW FOR A FIXED CHANNEL PRESSURE ($p_1$) AND SHOCK MACH NO. ($M_s$), OXYGEN, $M_s = 12$, $p_1 = 30$ mm Hg
FIGURE 5

VARIATION OF FLOW TEMPERATURE (T) VS EXPANSION ANGLE (\(\psi\)) IN A PRANDTL-MEYER FLOW FOR A FIXED CHANNEL PRESSURE (p₁) AND SHOCK MACH NO. (Mₛ), OXYGEN, Mₛ = 14, p₁ = 1 mm Hg.
VARIATION OF FLOW TEMPERATURE ($T$) VS EXPANSION ANGLE ($\psi$) IN A PRANDTL-MEYER FLOW FOR A FIXED CHANNEL PRESSURE ($p_1$) AND SHOCK MACH NO. ($M_s$), OXYGEN, $M_s = 14$, $p_1 = 3$ mm Hg.
FIGURE 7

VARIATION OF FLOW TEMPERATURE (T) VS EXPANSION ANGLE (\( \alpha \)) IN A PRANDTL-MEYER FLOW FOR A FIXED CHANNEL PRESSURE (p₁) AND SHOCK MACH NO. (Mₛ), OXYGEN, Mₛ = 14, p₁ = 10 mm Hg.
FIGURE 8

VARIATION OF FLOW PRESSURE ($p/p_2$) VS EXPANSION ANGLE ($\gamma$) IN A PRANDTL-MEYER FLOW FOR A FIXED CHANNEL PRESSURE ($p_1$) AND SHOCK MACH NO. ($M_s$), OXYGEN, $M_s = 14$, $p_1 = 10$ mm Hg.
FIGURE 9

VARIATION OF FLOW DENSITY ($\rho / \rho_2$) VS EXPANSION ANGLE ($\psi$) IN A PRANDTL-MEYER FLOW FOR A FIXED CHANNEL PRESSURE ($p_1$) AND SHOCK MACH NO. ($M_s$), OXYGEN, $M_s = 14$, $p_1 = 10$ mm Hg.
FIGURE 10

VARIATION OF FLOW SOUND SPEED \( (a/a_2) \) VS EXPANSION ANGLE \( (\gamma) \) IN A PRANDTL-MEYER FLOW FOR A FIXED CHANNEL PRESSURE \( (p_1) \) AND SHOCK MACH NO. \( (M_s) \), OXYGEN, \( M_s = 14 \), \( p_1 = 10 \) mm Hg.
FIGURE 11

VARIATION OF FLOW PARTICLE VELOCITY ($U/U_2$) VS EXPANSION ANGLE ($\psi$) IN A PRANDTL-MEYER FLOW FOR A FIXED CHANNEL PRESSURE ($p_1$) AND SHOCK MACH NO. $M_s$, OXYGEN, $M_s = 14$, $p_1 = 10$ mm Hg.
VARIATION OF FLOW MACH NO. (M) VS EXPANSION ANGLE (\( \psi \)) IN A PRANDTL-MEYER FLOW FOR A FIXED CHANNEL PRESSURE (p_1) AND SHOCK MACH NO. (M_s) OXYGEN, M_s = 14, p_1 = 10 mm Hg.
FIGURE 13

VARIATION OF FLOW TEMPERATURE ($T$) VS EXPANSION ANGLE ($\psi$) IN A PRANDTL-MEYER FLOW FOR A FIXED CHANNEL PRESSURE ($p_1$) AND SHOCK MACH NO. ($M_s$), OXYGEN, $M_s = 14$, $p_1 = 30$ mm Hg.
FIGURE 14

VARIATION OF FLOW TEMPERATURE (T) VS EXPANSION ANGLE ($\psi$) IN A PRANDTL-MEYER FLOW FOR A FIXED CHANNEL PRESSURE ($p_1$) AND SHOCK MACH NO. ($M_s$). OXYGEN, $M_s = 16$, $p_1 = 3$ mm Hg.
FIGURE 15

VARIATION OF FLOW TEMPERATURE (T) VS EXPANSION ANGLE (v) IN A PRANDTL-MEYER FLOW FOR A FIXED CHANNEL PRESSURE (p1) AND SHOCK MACH NO. (Ms), OXYGEN, Ms = 16, p1 = 10 mm Hg.
FIGURE 16

VARIATION OF FLOW TEMPERATURE (T) VS EXPANSION ANGLE (ψ) IN A PRANDTL-MEYER FLOW FOR A FIXED CHANNEL PRESSURE (p₁) AND SHOCK MACH NO. (Mₛ).

OXYGEN, Mₛ = 16, P₁ = 30 mm Hg.
FIGURE 17

VARIATION OF FLOW TEMPERATURE (T) VS EXPANSION ANGLE (\( \psi \)) IN A PRANDTL-MEYER FLOW FOR A FIXED CHANNEL PRESSURE (p₁) AND SHOCK MACH NO. (Mₛ), OXYGEN, Mₛ = 18, p₁ = 1 mm Hg.
FIGURE 18
VARIATION OF FLOW TEMPERATURE (T) VS EXPANSION ANGLE (\( \psi \)) IN A PRANDTL-MEYER FLOW FOR A FIXED CHANNEL PRESSURE (\( p_1 \)) AND SHOCK MACH NO. (\( M_s \)), OXYGEN, \( M_s = 18, \ p_1 = 3 \) mm Hg.
FIGURE 19

VARIATION OF FLOW TEMPERATURE (T) VS EXPANSION ANGLE (Ψ) IN A PRANDTL-MEYER FLOW FOR A FIXED CHANNEL PRESSURE (p₁) AND SHOCK MACH NO. (Mₛ), OXYGEN, Mₛ = 18, p₁ = 10 mm Hg.
VARIATION OF FLOW TEMPERATURE ($T$) VS EXPANSION ANGLE ($\psi$) IN A PRANDTL-MEYER FLOW FOR A FIXED CHANNEL PRESSURE ($p_1$) AND SHOCK MACH NO. ($M_s$), OXYGEN, $M_s = 20$, $p_1 = 1$ mm Hg.
VARIATION OF FLOW PRESSURE ($p/p_2$) VS EXPANSION ANGLE ($\psi$) IN A PRANDTL-MEYER FLOW FOR A FIXED CHANNEL PRESSURE ($p_1$) AND SHOCK MACH NO. ($M_s$) OXYGEN, $M_s = 20$, $p_1 = 1$ mm Hg.
FIGURE 22

VARIATION OF FLOW DENSITY ($\rho_1 / \rho_2$) VS EXPANSION ANGLE ($\psi$) IN A PRANDTL-MEYER FLOW FOR A FIXED CHANNEL PRESSURE ($p_1$) AND SHOCK MACH NO. ($M_s$). OXYGEN, $M_s = 20$, $p_1 = 1$ mm Hg.
FIGURE 23

VARIATION OF FLOW TEMPERATURE (T) VS EXPANSION ANGLE (\( \gamma \)) IN A PRANDTL-MEYER FLOW FOR A FIXED
CHANNEL PRESSURE (\( p_1 \)) AND SHOCK MACH NO. (\( M_s \)),
OXYGEN, \( M_s = 20, \ p_1 = 3 \) mm Hg.
VARIATION OF FLOW TEMPERATURE (T) VS EXPANSION ANGLE (\( \psi \)) IN A PRANDTL-MEYER FLOW FOR A FIXED CHANNEL PRESSURE (\( p_1 \)) AND SHOCK MACH NO. (\( M_s \)), NITROGEN, \( M_s = 14 \), \( p_1 = 10 \) mm Hg.
FIGURE 25

VARIATION OF FLOW PRESSURE ($p_1/p_2$) VS EXPANSION ANGLE ($\gamma$) IN A PRANDTL-MEYER FLOW FOR A FIXED CHANNEL PRESSURE ($P_1$) AND SHOCK MACH NO. ($M_s$) NITROGEN $M_s = 14$, $P_1 = 10$ mm Hg.
FIGURE 26

VARIATION OF FLOW DENSITY \( \left( \frac{\rho}{\rho_2} \right) \) VS EXPANSION ANGLE \( (\psi) \) IN A PRANDTL-MEYER FLOW FOR A FIXED CHANNEL PRESSURE \( (p_1) \) AND SHOCK MACH NO. \( (M_s) \) NITROGEN \( M_s = 14 \), \( p_1 = 10 \) mm Hg.

\[ \text{Density ratio} \left( \frac{\rho}{\rho_2} \right) \text{ vs expansion angle} (\psi) \]
FIGURE 27

VARIATION OF FLOW TEMPERATURE (T) VS EXPANSION ANGLE (\( \psi \)) IN A PRANDTL-MEYER FLOW FOR A FIXED CHANNEL PRESSURE (p_1) AND SHOCK MACH NO. (M_s)

AIR, M_s = 14, p_1 = 10 mm Hg.
FIGURE 28

VARIATION OF FLOW PRESSURE \( (p/p_2) \) VS EXPANSION ANGLE (\( \psi \)) IN A PRANDTL-MEYER FLOW FOR A FIXED CHANNEL PRESSURE \( (p_1) \) AND SHOCK MACH NO. \( (M_s) \)

AIR, \( M_s = 14 \), \( p_1 = 10 \text{ mm Hg} \).
FIGURE 29

VARIATION OF FLOW DENSITY ($\rho_1/\rho_2$) VS EXPANSION ANGLE ($\nu$) IN A PRANDTL-MEYER FLOW FOR A FIXED CHANNEL PRESSURE ($p_1$) AND SHOCK MACH NO. ($M_s$) AIR, $M_s = 14$, $p_1 = 10$ mm Hg.
FIGURE 30

VARIATION OF FLOW TEMPERATURE ($T/T_2$) VS EXPANSION ANGLE ($\psi$) IN A PRANDTL-MEYER FLOW FOR A FIXED CHANNEL PRESSURE ($p_1$) AND SHOCK MACH NO. ($M_s$) ARGON, $M_s = 14$, $p_1 = 1$ mm Hg.
FIGURE 31

VARIATION OF FLOW TEMPERATURE ($T/T_2$) VS EXPANSION ANGLE ($\psi$) IN A PRANDTL-MEYER FLOW FOR A FIXED CHANNEL PRESSURE ($p_1$) AND SHOCK MACH NO. ($M_s$) ARGON, $M_s = 14$, $p_1 = 3$ mm Hg.
VARIATION OF FLOW TEMPERATURE ($T/T_2$) VS EXPANSION ANGLE ($\psi$) IN A PRANDTL-MEYER FLOW FOR A FIXED CHANNEL PRESSURE ($p_1$) AND SHOCK MACH NO. ($M_s$) ARGON, $M_s = 14$, $p_1 = 10$ mm Hg.
FIGURE 33

VARIATION OF FLOW TEMPERATURE ($T/T_2$) VS EXPANSION ANGLE ($\nu$) IN A PRANDTL-MEYER FLOW FOR A FIXED CHANNEL PRESSURE ($P_1$) AND SHOCK MACH NO. ($M_s$) ARGON, $M_s = 14$, $P_1 = 30$ mm Hg.
FIGURE 34

VARIATION OF FLOW TEMPERATURE (T/T_2) VS EXPANSION ANGLE (ψ) IN A PRANDTL-MEYER FLOW FOR A FIXED CHANNEL PRESSURE (p_1) AND SHOCK MACH NO. (M_s).

ARGON, M_s = 10, p_1 = 3 mm Hg.
VARIATION OF FLOW PRESSURE ($\frac{p}{p_2}$) VS EXPANSION ANGLE ($\psi$) IN A PRANDTL-MEYER FLOW FOR A FIXED CHANNEL PRESSURE ($p_1$) AND SHOCK MACH NO. ($M_s$) ARGON, $M_s = 16$, $p_1 = 3$ mm Hg.
FIGURE 36

VARIATION OF FLOW DENSITY ($\rho_1/\rho_2$) VS EXPANSION ANGLE ($\gamma$) IN A PRANDTL-MEYER FLOW FOR A FIXED CHANNEL PRESSURE ($p_1$) AND SHOCK MACH NO. ($M_s$). ARGON, $M_s = 16$, $p_1 = 3$ mm Hg.
FIGURE 37

VARIATION OF FLOW SOUND SPEED $a/a_2$ VS EXPANSION ANGLE $(\psi)$ IN A PRANDTL-MEYER FLOW FOR A FIXED CHANNEL PRESSURE $(p_1)$ AND SHOCK MACH NO. $(M_s)$ ARGON, $M_s = 16$, $p_1 = 3$ mm Hg.
VARIATION OF FLOW PARTICLE VELOCITY ($u/u_2$) VS EXPANSION ANGLE ($\gamma$) IN A PRANDTL-MEYER FLOW FOR A FIXED CHANNEL PRESSURE ($p_1$) AND SHOCK MACH NO. ($M_s$) ARGON, $M_s = 16$, $p_1 = 3$ mm Hg.
FIGURE 39

VARIATION OF FLOW MACH NO. (M) VS EXPANSION ANGLE (\(\theta\)) IN A PRANDTL-MEYER FLOW FOR A FIXED CHANNEL PRESSURE \(p_1\) AND SHOCK MACH NO. \(M_s\) ARGON, \(M_s = 16\), \(p_1 = 3\) mm Hg.
FIGURE 40
VARIATION OF FLOW TEMPERATURE ($T/T_2$) VS EXPANSION ANGLE ($\psi$) IN A PRANDTL-MEYER FLOW FOR A FIXED CHANNEL PRESSURE ($p_1$) AND SHOCK MACH NO. ($M_s$) ARGON, $M_s = 16$, $p_1 = 10$ mm Hg.
FIGURE 41

VARIATION OF FLOW TEMPERATURE \( \left( \frac{T}{T_2} \right) \) VS EXPANSION ANGLE \( (\psi) \) IN A PRANDTL-MEYER FLOW FOR A FIXED CHANNEL PRESSURE \( (p_1) \) AND SHOCK MACH NO. \( (M_s) \) ARGON, \( M_s = 18, p_1 = 3 \) mm Hg.
FIGURE 42

VARIATION OF FLOW TEMPERATURE ($T/T_2$) VS EXPANSION ANGLE ($\psi$) IN A PRANDTL-MEYER FLOW FOR A FIXED CHANNEL PRESSURE ($p_1$) AND SHOCK MACH NO. ($M_s$) ARGON, $M_s = 20$, $p_1 = 1$ mm Hg.
FIGURE 43

VARIATION OF FLOW PRESSURE ($p/p_2$) VS EXPANSION ANGLE ($\psi$) IN A PRANDTL-MEYER FLOW FOR A FIXED CHANNEL PRESSURE ($p_1$) AND SHOCK MACH NO. ($M_S$) ARGON, $M_S = 20$, $p_1 = 1$ mm Hg.
VARIATION OF FLOW PRESSURE ($p_1 / p_2$) VS EXPANSION ANGLE ($\psi$) IN A PRANDTL-MEYER FLOW FOR A FIXED CHANNEL PRESSURE ($p_1$) AND SHOCK MACH NO. ($M_s$). ARGON, $M_s = 20$, $p_1 = 1$ mm Hg.
FIGURE 45

VARIATION OF FLOW TEMPERATURE ($T/T_2$) VS EXPANSION ANGLE ($\psi$) IN A PRANDTL-MEYER FLOW FOR A FIXED CHANNEL PRESSURE ($p_1$) AND SHOCK MACH NO. ($M_s$) ARGON, $M_s = 20$, $p_1 = 3$ mm Hg.
FIGURE 46

VARIATION OF FLOW TEMPERATURE (T) BEHIND A PRANDTL-MEYER EXPANSION WAVE VS CHANNEL PRESSURE ($p_1$) FOR A FIXED EXPANSION ANGLE $\psi = 15^\circ$, FOR VARYING SHOCK MACH NO. ($M_s$), FOR OXYGEN ($T_1 = 273^\circ K$).
FIG. 46  (CONCLUDED)
FIGURE 47

VARIATION OF FLOW TEMPERATURE (T) BEHIND A PRANDTL-MEYER EXPANSION WAVE VS CHANNEL PRESSURE (p₁) FOR A FIXED EXPANSION ANGLE $\beta = 15^\circ$, FOR VARYING SHOCK MACH NO. ($M_s$), FOR ARGON (T₁ = 300°C).
APPENDIX A

Sample Calculations for Equilibrium and Frozen Flows

In order to illustrate how the calculations were done for the diatomic and monotomic gases, a specific example will be considered as follows:

Gas: oxygen
Shock Mach number: $M_S = 14$
Channel Pressure: $p_1 = 10$ mm Hg
Channel temperature: $T_1 = 273^\circ K$

From above values obtain,

Density: $\rho_1 = 0.01316$ NTP density
Speed of sound: $a_1 = 3.142 \times 10^4$ cm/sec.

Enthalpy: $h_1 = 59.25$ cal/gm

From Ref. 18, Fig. 5 find,

$$\frac{P_2}{P_1} = 252 \quad \text{and} \quad p_2 = 2520 \text{ mm Hg} = 3.315 \text{ atm} \quad (A.1)$$

and from Fig. 2

$$\alpha_2 = 0.324 \quad (A.2)$$

The above result is obtained by interpolation using a logarithmic plot of $\alpha$ vs $p_2$ for the range $1 \text{ atm} \leq p_2 \leq 10 \text{ atm}$.

In a similar way find using Figs. 9 and 11, respectively,

$$T_2 = 3905^\circ K \quad (A.3)$$

and

$$\frac{\rho_2}{\rho_1} = 13.2 \quad \text{or} \quad \rho_2 = 0.1735 \text{ NTP} \quad (A.4)$$

From Ref. 1,

$$h_2/h_1 = 1 + \frac{\gamma_1 - 1}{2} \left[ 1 - \left( \frac{\rho_1}{\rho_2} \right)^2 \right] = 39.95$$

or

$$h_2 = 2369 \text{ cal/gm} = 139.8 \text{ RT}_1 \quad (A.5)$$

This gives a very good value for $h_2$ since the effect of $(\rho_1/\rho_2)^2$ is very small.

Using the foregoing values of $h_2$ and $p_2$, other quantities can be obtained from the tables in Ref. 16 as follows:
\[ \begin{align*}
\mathcal{A}_2 &= 0.338 \quad (A.2) \\
T_2 &= 3940^\circ K \quad (A.3) \\
\mathcal{P}_2 &= 0.1715 \text{ NTP} \quad (A.4)
\end{align*} \]

They are in good agreement with the graphical values A2 to A4. The discrepancies are about 1% for \( T_2 \) and \( \mathcal{P}_2 \), and about 4% for \( \mathcal{A}_2 \) and results from the difficulty in reading the graphical values to the same accuracy.

The foregoing values have been found as follows:

To find \( T_2 \) and \( \mathcal{P}_2 \):

(a) Plot \( h \) versus \( T \) for four values of \( \log \mathcal{P} / \mathcal{P}_0 = -0.4, -0.6, -0.8 \) and \(-1.0\) in the neighbourhood of \( \mathcal{P}_2 \) \( (\mathcal{P}_2 / \mathcal{P}_0 = -0.942) \) obtained above: Fig. A.1. (use Table XIII, Ref. 16).

(b) Plot \( p \) versus \( T \) for the same four values of \( \mathcal{P} / \mathcal{P}_0 \): Fig. A.2 (use Table XIV).

(c) Read \( T \) at \( h = h_2 \) from Fig. A.1 and plot \( T \) versus \( \mathcal{P} \): Fig. A.3. Read \( T \) at \( p = p_2 \) from Fig. A.2 and plot \( T \) versus \( \mathcal{P} \) on the same graph. The intersection of the two curves gives the required values of \( T_2 \) and \( \mathcal{P}_2 \).

The degree of dissociation \( \mathcal{A}_2 \) is also obtained graphically,

(a) Plot \( \mathcal{A} \) versus \( T \) for four values of \( \mathcal{P} \) in the neighbourhood of \( \mathcal{P}_2 \): Fig. A.4. (use Table XIII, wherein \( z = 1 + \mathcal{A} \)).

(b) Read \( \mathcal{A} \) at \( T = T_2 \) from Fig. A.4 and plot \( \mathcal{A} \) versus \( \mathcal{P} \): Fig. A.5. Read \( \mathcal{A} \) at \( \mathcal{P} = \mathcal{P}_2 \), which gives \( \mathcal{A}_2 \).
The entropy \( S_2 \) is found in a similar manner, (see Figs. A.6, and 7).

\[
S_2 = 2.507 \text{ cal/gm ^0K} = 40.35 \text{ R} \tag{A.6}
\]

The values of \( \alpha_2, T_2 \), and \( \rho_2 \) found from the thermodynamic data of Ref. 16, (A.2)', (A.3)', and (A.4)' will be used throughout.

The flow velocity in state (2) is (Ref. 1),

\[
U_2 = M_s (1 - \frac{\rho_1}{\rho_2}) a_1 \\
U_2 = 4.08 \times 10^5 \text{ cm/sec} \tag{A.7}
\]

The initial state (2) of the Prandtl-Meyer flow has now been completely determined. The procedure for calculating the equilibrium flow pertinent to these initial conditions is as follows:

The equilibrium sound speed is given by,

\[
a_2^e = 1.257 \times 10^5 \text{ cm/sec} \tag{A.8}
\]
and is obtained from Table XV, Ref. 16 (See Figs. A.8 and 9).

Therefore the equilibrium flow Mach number has the value,

$$M_{2e} = \frac{U_2}{a_{2e}} = 3.24$$  \hspace{1cm} (A.9)

The angular position of the first Mach line (measured clockwise from the vertical line) is

$$\theta_{2e} = \cos^{-1}\left(\frac{1}{M_{2e}}\right) = 72.0 \text{ deg.}$$  \hspace{1cm} (A.10)

As the flow starts at $P_2 = 0.1715 \text{ NTP}$ or at $\log \left( \frac{P_2}{P_0} \right) = -0.742$, where $P_0$ is the standard density in Ref. 16, equal to 0.957 NTP, there are five more points available from the tables down along the isentrope, i.e., the points corresponding to $\log \left( \frac{P}{P_0} \right) = -0.8, -1.0, -1.5, -2.0$ and $-3.0$, respectively. All the thermodynamic quantities needed in the calculation can be obtained by linear interpolation from the tables, along the isentrope $S = S_2 = 2.507 \text{ cal/gm}^0\text{K}$ all the way. The results are given in Table A1.

In order to find $(\theta_2 - \theta_{2e})$, $1/a_2e$ is plotted against $v_T$ (see Eq. 14, Sec. 2.1) (Fig. A.10) and a graphical integration is performed. The velocity, flow Mach number and deflection angle are calculated using the equations in Sec. 2.1, to obtain the thermodynamic and dynamic quantities at six points along the isentrope. (See Tables III, 1 to 26 for all other tabulated results).
Note: To obtain the foregoing tabulated values apply the following relations:

\[
1 \text{ cal/gm} = 4.185 \times 10^7 \text{ erg/gm} = 4.185 \times 10^7 \text{ cm}^2/\text{sec}^2,
\]

\[a_0 = 3.237 \times 10^4 \text{ cm/sec}; \text{ standard speed in Ref. 16},\]

\[h_2 = 9.92 \times 10^{10} \text{ cm}^2/\text{sec}^2,\]

\[U_2 = 16.61 \times 10^{10} \text{ cm}^2/\text{sec}^2,\]

\[h_0 = 18.23 \times 10^{10} \text{ cm}^2/\text{sec}^2,\]

and

\[
v_r = \sqrt{2(18.23 - h/10^{10}) - \left(\frac{a_e^2}{10^5}\right) \times 10^5}
\]

(A.11)

where

\[h \text{ is in cm}^2/\text{sec}^2\]

and

\[a_e \text{ is in cm/sec}\]

For the frozen and partially frozen flows, the relations for \(\Gamma\), \(M_{2f}\), \(\bar{M}_{2f}\) and \(M_{2f}\) developed in Sections 2.2 and 2.3 are used as follows:

\[
\Gamma = \frac{7 + 3 \alpha^2}{5 + \alpha^2} = 1.501,
\]

\[
a_{2f} = \sqrt{\Gamma R (1 + \alpha^2) T_2} = 1.437 \times 10^5 \text{ cm/sec},
\]
\[
M_{2f} = \frac{u_2}{a_{2f}} = 2.84, \quad \theta_{2f} = 69.4 \text{ deg.}
\]

\[
\Gamma_\infty = \frac{7 + 3 \chi_2 + 2 (1 - \chi_2) \beta_2}{5 + \chi_2 + 2 (1 - \chi_2) \beta_2} = 1.404
\]

\[
a_{2f} = \sqrt{\frac{\Gamma_\infty R (1 + \chi_2) T_2}{1 - 0.829}} = 1.390 \times 10^5 \text{ cm/sec},
\]

\[
M_{2f} = \frac{u_2}{a_{2f}} = 2.94, \quad \text{and} \quad \theta_{2f} = 70.1 \text{ deg.}
\]

where \( \chi_2 = 0.338 \) and \( \beta_2 = 0.973 \).

Therefore, in Eq. (30),

\[
\frac{1}{2} x \sqrt{\frac{\Gamma + 1}{\Gamma - 1}} = 1.117 \quad \frac{\Gamma + 1}{1 + \frac{\Gamma - 1}{2} M_{2f}^2} = 0.829
\]

and so we have, for the frozen flow,

\[
(\theta - 69^\circ.4) \text{ deg} = 1.117 \left[ \sin^{-1} (1 - 0.829) \frac{T}{T_2} - \sin^{-1} (1-0.829) \right]
\]

\[
(\theta - 69^\circ.4) \text{ deg} = 1.117 \left[ \sin^{-1} (1 - 0.829) \frac{T}{T_2} - 9.8 \text{ deg} \right] \quad (A.12)
\]

Also, in Eq. (33),

\[
M_{2f}^2 + \frac{2}{\Gamma - 1} = 12.06 \quad \text{and} \quad \frac{2}{\Gamma - 1} = 3.99.
\]

Therefore,

\[
M_f = \sqrt{12.06 \frac{T_2}{T} - 3.99}.
\]

Similarly, for the partially frozen flow

\[
\frac{1}{2} x \sqrt{\frac{\Gamma + 1}{\Gamma - 1}} = 1.220 \quad \frac{\Gamma + 1}{1 + \frac{\Gamma - 1}{2} M_{2f}^2} = 0.868
\]

\[
M_{2f}^2 + \frac{2}{\Gamma - 1} = 13.59 \quad \text{and} \quad \frac{2}{\Gamma - 1} = 4.95
\]
Therefore,

\[ (\theta - 70.1) \text{ deg.} = 1.220 \left\{ \sin^{-1} \left(1 - 0.868 \frac{T}{T_2} \right) - \sin^{-1} (1-0.868) \right\} \]

\[ = 1.220 \left\{ \sin^{-1} \left(1 - 0.868 \frac{T}{T_2} \right) - 7.5 \text{ deg} \right\} \quad (A.13) \]

and

\[ M_f = \sqrt{13.59 \frac{T_2}{T} - 4.95} \]

Note:

The integral

\[ I = \int_{t}^{T_2} \frac{dT}{T \sqrt{BT + AT^2}} \quad (A < 0, \, B > 0) \]

can alternatively evaluated as follows:

By means of the transformation:

\[ t = \sqrt{T}, \quad \text{dt} = \frac{1}{2\sqrt{T}} \text{d}T \]

we get

\[ I = \left[ \frac{1}{\sqrt{-A}} \left( \frac{1}{\sqrt{-B}} - \frac{1}{\sqrt{A}} \right) - \frac{1}{\sqrt{-A}} \right]_{t}^{T_2} \frac{dT}{\sqrt{B} - A^2} \]

\[ = \frac{2}{\sqrt{-A}} \left( \sin^{-1} \sqrt{\frac{A}{B}} t \right)_{t}^{T_2} - \frac{2}{\sqrt{-A}} \left( \sin^{-1} \sqrt{\frac{A}{B}} T \right)_{T}^{T_2} \]

\[ = \frac{2}{\sqrt{-A}} \left( \sin^{-1} \sqrt{\frac{A}{B}} T_2 - \sin^{-1} \sqrt{\frac{A}{B}} T \right) \]
Or, making use of the relation:

\[
\sin^{-1} \sqrt{a} = \frac{\pi}{2} - \sin^{-1} \sqrt{1 - a}
\]

\[
I = \frac{2}{\sqrt{-A}} \left[ \sin^{-1} \sqrt{1 + \frac{A}{B} T} - \sin^{-1} \sqrt{1 + \frac{A}{B} T_2} \right]
\]

Accordingly, Eq. (3) can alternatively written

\[
\theta - \theta_2 = \sqrt{\frac{\Gamma + 1}{\Gamma - 1}} \left[ \sin^{-1} \sqrt{1 - \frac{\Gamma + 1}{2 + (\Gamma - 1) M_2^2} \frac{T}{T_2}} - \sin^{-1} \sqrt{1 - \frac{\Gamma + 1}{2 + (\Gamma - 1) M_2^2} \frac{T}{T_2}} \right]
\]

Thus an alternative form of Eq. (A.12) is

\[
(\theta - 69.4) \text{ deg} = 2.234 \left( \sin^{-1} \sqrt{1 - 0.414} \frac{T}{T_2} - \sin^{-1} \sqrt{1 - 0.414} \right)
\]

In the same way, for the partially frozen flow, we have

\[
(\theta - 70.1) \text{ deg} = 2.440 \left( \sin^{-1} \sqrt{1 - 0.434} \frac{T}{T_2} - \sin^{-1} \sqrt{1 - 0.434} \right)
\]
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TABLE A1
PARAMETERS REQUIRED FOR EVALUATING \( \Delta \theta \)
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Source: Eq. (12), Eq. (14), Table 14, Ref. 16
FIG. A.1  \((1/a)\) vs. \(V_r\) FOR EQUILIBRIUM PRANDTL-MEYER EXPANSION; OXYGEN, \(M_s = 14, p_1 = 10\) mmHg.
APPENDIX B

Computed Fringe Shifts Through Shock Waves and Expansion Waves for Interferometric Studies of Dissociated and Ionized Gas Flows

In this section the use of interferometry for the study of Prandtl-Meyer flows in dissociated and ionized gases will be discussed. The computed fringe shifts through shock and expansion waves for cases considered will be presented.

B1. Monochromatic Interferograms

The light beam from a monochromatic source is represented by its amplitude:

$$I = A \cos 2\pi (\nu t - \kappa Z)$$  \hspace{1cm} (B1)

where, \(t\) is the time, \(Z\) the distance along the path of propagation, \(\nu\) the frequency (cycles/sec) and \(\kappa\) the wave number (cycles/cm). The reciprocal of the wave number,

$$\kappa = \frac{1}{\lambda}$$  \hspace{1cm} (B2)

is the wavelength (cm) of the light beam. The frequency of the light beam remains unchanged as it travels through any medium, viz.

$$\nu = \nu_o$$  \hspace{1cm} (B3)

(the suffix \(o\) will be used throughout this section to denote quantities in vacuum). The wave number or the wavelength of the light beam depends on the medium through which it is travelling, viz. for a pure gas,

$$\kappa = \kappa (\nu, T, \rho), \quad \lambda = \lambda (\nu, T, \rho)$$  \hspace{1cm} (B4)

The 'phase velocity' of the light beam, i.e. the speed of propagation of individual intensity crests, is given by,

$$c = \nu / \kappa = \nu \lambda$$  \hspace{1cm} (B5)

The ratio of the phase velocity of the light beam of a given frequency in vacuum to the same quantity in a material medium:

$$c_o / c = n = \lambda_o / \lambda$$  \hspace{1cm} (B6)

is called the 'phase index of refraction' or simply 'refractivity' of the medium and it depends on the thermodynamic state of the gas (see subsections B.3 and B.4).
A conventional interferometric system is sketched below.

**SKETCH 1**

Mach-Zehnder Interferometer

In the following, quantities without a suffix are those in the test section, and those with suffixes 'a' and 'c' denote conditions in the atmosphere and compensating chamber, respectively.

The collimated beam is split into two at the splitter plate $S_1$, one passes through the compensating chamber (A) (p--A--s--r) and the other through the test section (B) (p--q--B--r). The beams are recombined at the splitter plate $S_2$ giving a certain resultant brightness at a point on the screen. In the case of adjustment for the so-called 'infinite fringe method' all mirrors and splitters are made parallel, and consequently the path difference between the two rays (in cycles) under this condition is given by

$$\frac{L_1}{\lambda} - \frac{L_c}{\lambda_c}$$

The resultant brightness then is,

$$I = A \cos \frac{2\pi L_1}{\lambda_o} \left( \frac{1}{\lambda} - \frac{1}{\lambda_c} \right)$$

or, in terms of the phase indices,

$$I = A \cos 2\pi \frac{L}{\lambda_o} (n - n_c)$$
Since $\lambda_0$ is fixed by the light source and $n_c$ is a known constant, it is seen from Eq. (B9) that the phase index $n$ of a point $(x, y)$ in the test section determines the brightness of the corresponding point on the screen. When $\frac{L}{\lambda_0}(n - n_c)$ is an integer, the brightness is a maximum. Curves with integer values of the said quantity appear on the screen as distinct bright lines (fringes). Generally, the flow field is mapped with bright and dark fringes. If a uniform region (wherein $n = \text{const.}$) exists, it will show up as a uniformly illuminated region free of fringes. The difference in $n$ between adjacent fringes $\Delta n$ is given by,

$$\frac{L}{\lambda_0}(n + \Delta n - n_c) = \frac{L}{\lambda_0}(n - n_c) + 1$$

or

$$\Delta n = \frac{\lambda_0}{L}$$

Consequently, the distribution of $n$ over the entire flow field can be determined if the value of $n$ is known on one of the fringes, as shown in the sketch below.

![Sketch of Fringe Formation](image-url)
A modification of the previous method is the 'parallel fringe method'. In this case the mirror $M_2$ is rotated about the $x$-axis through a small angle $\varepsilon$ (Sketch 1).

Let,

$$l = pq + qr, \quad l_c = ps + sr,$$

then,

$$l = l_c + 2 \varepsilon y$$

and when $y = 0$ (optical axis), $l = l_c$.

The resultant intensity becomes

$$I = A \cos 2\pi \left( \frac{L}{\lambda} - \frac{L}{\lambda_c} + \frac{1}{\lambda_a} - \frac{l_c}{\lambda_a} \right)$$

Which gives

$$I = A \cos 2\pi \left( \frac{L}{\lambda_o} (n - n_c + 2 n_a \varepsilon \frac{y}{L}) \right)$$

When $n$ is constant, $I$ is a periodic function of $y$, and there now exists a row of parallel fringes aligned to $x$-axis at a constant interval given by,

$$d = \frac{\lambda_o}{2 n_a \varepsilon}$$

This results from the following relation which express the fact that for a bright fringe the quantity in brackets in Eq. (B12) must differ by unity,

$$\frac{L}{\lambda_o} (n - n_c + 2 n_a \varepsilon \frac{y + d}{L}) = \frac{L}{\lambda_o} (n - n_c + 2 n_a \varepsilon \frac{y}{L}) + 1$$

Generally, the flow field will be mapped with non-parallel fringes (Sketch 3).

![SKETCH 3](Parallel Fringe Flow)
Suppose that points \( P(X_p, y_p) \) and \( Q(x_Q, y_Q) \) are on the same fringe, then

\[
\frac{L}{\lambda_0} (n_p - n_c + 2 n_a \varepsilon \frac{y_p}{L}) = \frac{L}{\lambda_0} (n_Q - n_c + 2 n_a \varepsilon \frac{y_Q}{L}) \quad (B14a)
\]

or

\[
\frac{L}{\lambda_0} (n_Q - n_P) = \frac{2 n_a \varepsilon}{L} \frac{y_p}{L} = \frac{1}{d} (y_p - y_Q) \quad (B14b)
\]

The above equation relates the \( y \)-deflection, i.e., the fringe shift in "fringe numbers" \((y_p - y_Q)/d\), to the refractivity difference between \( P \) and \( Q \), \((n_Q - n_P)\). Since \( \lambda_0 \) and \( L \) are fixed, one can determine the distribution of \( n \) over the entire unknown flow field in state (3) provided \( n \) is known in a given state (2) (since \( d \) is a constant).

Suppose that the uniform state in the test section is identical to that of the compensating chamber, viz. \( n = n_c \). The intensity distribution in this case ('no-flow interferogram') is, from Eq. (B12),

\[
I = A \cos \left( 2\pi \frac{L}{\lambda_0} \cdot 2 n_a \varepsilon \frac{y}{L} \right) \quad (B15)
\]

which again gives a horizontal row of fringes. It is seen that the subtraction of the right-hand side of Eq. (B15) from that of Eq. (B12) gives the right-hand side of Eq. (B9), which represents the brightness of the infinite-fringe interferogram. Thus one can obtain the constant refractivity lines by the superposition of parallel-fringe interferograms with and without flow. This technique is used extensively in interferometric flow studies.

### B.2 White-Light Interferograms

When the light is not monochromatic but consists of an infinite number of infinitesimal components of different frequencies, distributed over the frequency range \( \nu_1 \leq \nu \leq \nu_2 \), it is called a 'white light'. Then the expression for the amplitude, Eq. (B1), is replaced by

\[
I = \int_{\nu_1}^{\nu_2} a(\nu) \cos 2\pi (\nu t - \nu Z) d\nu \quad (B16)
\]

where \( a(\nu) \) is the amplitude distribution function.

Consider a rectangular distribution over the frequency range \( \nu_1 \) to \( \nu_2 \) given by \( a(\nu) = A/(\nu_2 - \nu_1) \). Then the intensity is given by

\[
I = \frac{A}{\nu_2 - \nu_1} \int_{\nu_1}^{\nu_2} \cos 2\pi (\nu t - \nu Z) d\nu \quad (B17)
\]
Denote the mean frequency by

\[ \bar{\nu} = \frac{\nu_1 + \nu_2}{2} \]  

(B18)

and the bandwidth by

\[ 2\Delta\nu = \nu_2 - \nu_1 \]  

(B19)

Equation (B17) can be rewritten as

\[ I = \frac{A}{2\Delta\nu} \int_{\nu-\Delta\nu}^{\nu+\Delta\nu} \cos 2\pi (\nu t - k Z) \, d\nu \]

\[ I = \frac{A}{2\Delta\nu} \int_{0}^{\Delta\nu} \cos 2\pi (\nu t - k Z) \, d\nu \]

\[ I = \frac{A}{2\Delta\nu} \int_{0}^{\Delta\nu} \cos 2\pi \left[ (\nu + \nu')t - (k + k')Z \right] \, d\nu' \]

\[ + \frac{A}{2\Delta\nu} \int_{0}^{\Delta\nu} \cos 2\pi \left[ (\nu - \nu')t - (k - k')Z \right] \, d\nu' \]

\[ I = A \cos 2\pi (\bar{\nu} t - k Z) \cdot F(\Delta\nu, t - \Delta k, Z) \]  

(B20)

where

\[ F(\Delta\nu, t - \Delta k, Z) = \frac{\sin 2\pi(\Delta\nu t - \Delta k Z)}{2\pi(\Delta\nu t - \Delta k Z)} \]  

(B21)

Equation (B20) represents a sinusoidal oscillation at the mean frequency \( \bar{\nu} \) just as in Eq. (B1) for the monochromatic light. However, in this case the oscillation is modulated by the function \( F(t, Z) \), such that it decays inversly with \( Z \) or \( t \) (see Sketch 4).

SKETCH 4
Decay of White Light Fringes
The faster oscillation in Eq. (B20) at frequency $\nu$, propagates at the speed,

$$ c = \frac{\nu}{\kappa} = \frac{\nu}{\kappa} $$  \hspace{1cm} (B22)

i.e. at the same 'phase velocity' as for the monochromatic light of frequency $\nu$ (Eq. (B5)). Let us assume that the bandwidth is small so that $\Delta \kappa / \Delta \nu$ may be replaced by $\partial \kappa / \partial \nu$. Then the slower oscillation in Eq. (B20), at frequency $\Delta \nu$, has the wavelength $\lambda_g = 1 / \Delta \kappa$ and travels at the speed

$$ c_g = (\partial \kappa / \partial \nu)^{-1} = \left( \frac{\partial (\nu / c)}{\partial \nu} \right)^{-1} = c \left( 1 - \frac{\nu}{c} \frac{\partial c}{\partial \nu} \right)^{-1} $$  \hspace{1cm} (B23)

The velocity $c_g$ defined by Eq. (B23) is called the 'group velocity' of light in the medium in question at the frequency $\nu$.

The 'group index of refraction' is defined as

$$ n_g = \frac{c_0}{c_g} \hspace{1cm} (B24) $$

In vacuum, however, the phase velocity of light is constant regardless of frequency and consequently $c_{go} = c_0$. Thus we may write

$$ n_g = \frac{c_0}{c} \hspace{1cm} (B25) $$

When the phase index $n$ is given as a function of the wavelength in vacuum, $n = n (\lambda, \nu)$, $n_g$ can be related to it, from Eqs. (B6), (B23) and (B25), as

$$ n_g = n - \lambda_0 \frac{\partial n}{\partial \lambda_0} \hspace{1cm} (B26) $$

Consider the parallel-fringe interferogram described in the previous subsection with a "white light" source instead of a monochromatic source. Faster oscillations (with wavelength $\lambda$) from $p-\rightarrow A-\rightarrow S-\rightarrow r$ and from $p-\rightarrow q-\rightarrow B-\rightarrow r$ (Sketch 1) interfere and as before in a uniform region, a row of horizontal fringes with constant intervals $d = \lambda / 2 n_g \epsilon$ will appear. Slower oscillations (with wavelength $\lambda_g$) from the two paths interfere with each other in the same way and reinforcement will occur at intervals of $d_g = \lambda_g / 2 n_g \epsilon$, which is much larger than $d$. Thus the periodic change of brightness (with intervals $d$) in the $y$-direction is modulated by a slower (oscillatory but of decreasing amplitude) change in $y$-direction (see Sketch 4). However, because of the decreasing peak heights of the slower oscillation Eq. (B21) the reinforcement between them is distinct only at one position (at the central fringe, $y = y_0$, say) and only a few distinct fringes with decreasing brightness (at intervals $d$) will show up throughout the entire field. Individual fringes are shifted in the same way as for the monochromatic interferogram Eq. (B12). The central fringe group is also shifted according to Eq. (B14b), whereby $n_p$, $n_Q$, $n_a$ and $n_c$ are to be replaced by the corresponding group indices $n_{gP}$, $n_{gQ}$, $n_{ga}$ and $n_{gc}$. Thus the $y$-shift of the location of the central fringe group is given by
Since \( n_{\text{ga}} = n_a \), in an ionized atmosphere, Eq. (B27) becomes,

\[
\frac{y_p - y_Q}{2n_{\text{ga}}} = \frac{L}{\xi} (n_{gQ} - n_{gp})
\]

which is of the same form as Eq. (B14), and relates the shift of the central fringe group to the difference in group indices between P and Q.

**SKETCH 5**

Formation of White Light Fringes

The appearance of the central fringes can be explained in a more intuitive way as follows (see Sketch 6). Regard the white light as a superposition of several monochromatic sources with wavelengths ranging from \( \lambda_1 \) to \( \lambda_2 \). Consider the uniform state (1), each monochromatic source will generate on the screen a row of parallel fringes at intervals \( d = \frac{\lambda}{2n_a} \xi \) (Eq. B13). Suppose at some vertical position the fringes generated by all monochromatic components coincide on a line (central fringe (a)), giving a very sharp bright fringe. Since each monochromatic component has a different spacing, the horizontal fringes constructed as superpositions will be less sharp and less bright as we move away from the central fringe. The \( N \)-th fringe will have a width: \( N(\frac{\lambda_2 - \lambda_1}{2n_a}) \).
Now assume that the state of the flow field changes from (1) to another uniform state (2) through a transition region. The fringe shift of each monochromatic component is governed by Eq. (B14), and the fringe pattern in (2) is again given as a superposition of shifted fringes of monochromatic components. Assume that the refractivity change $\Delta n$ is such that all individual fringes are shifted downward, and that $\Delta n$ is an increasing function of the wavelength $\lambda$ (which is the case for the transition through a shock wave in an ionized gas). Then a component with longer wavelength is shifted more downward than a component with shorter wavelength. Consequently, the zero fringe in (1) widens as it is shifted through the transition region. Also a fringe below the central fringe in (1), which has a finite width, widens further because its lower boundary (longest wavelength component) is shifted more downward than its upper boundary (shortest wavelength component), whereas one above the central fringe narrows because its upper boundary is shifted more downward than its lower boundary. The resulting pattern shown in Sketch 6 indicates that the net result is an upward shift in the central fringe even though individual fringes are shifted downward.

**Phase shifts:**

$$\frac{L}{2n_0 \varepsilon} \Delta n(\lambda_1) < \frac{L}{2n_0 \varepsilon} \Delta n(\lambda_2)$$

**Spacings:**

$$\frac{\lambda_1}{2n_0 \varepsilon} < \frac{\lambda_2}{2n_0 \varepsilon}$$

**SKETCH 6**

Shift of White Light Fringes
B 3. Dissociated Gases

The phase index of a nonionized gas mixture can be expressed as,

\[ n - 1 = \sum_j K_j \rho_j \]  

(B29)

where, \( K_j \) = Gladstone-Dale constant (specific refractivity) of the jth species (cm\(^3\)/gm, ft\(^3\)/lb.)

\( \rho_j \) = partial density (gm/cm\(^3\), lb/ft\(^3\)) of the jth species.

In the case of a single dissociating diatomic gas Eq. (B3) can be written as

\[ n - 1 = \rho \left[ K_{\text{molecule}} (1 - \alpha) + K_{\text{atom}} \alpha \right] \]  

(B30)

For air at temperatures \( T \sim 6000^\circ K \) and \( \rho \sim 10^{-1} \) NTP (of interest in the present computations), the expression for the phase refractive index is given by (disregarding minor species),

\[ n - 1 = (K \cdot \rho)_{O_2} + (K \cdot \rho)_{N_2} + (K \cdot \rho)_{O} + (K \cdot \rho)_{N} + (K \cdot \rho)_{NO} \]  

(B31)

The empirical values of \( n \) at 5180 Å accepted at present are (expressed in ft\(^3\)/lb).

\[ K_{O_2} = 3.050 \times 10^{-3}, \quad K_{N_2} = 3.832 \times 10^{-3} \text{ (Ref. 24)} \]

\[ K_{NO} = 3.575 \times 10^{-3}, \quad K_A = 2.550 \times 10^{-3} \text{ (Ref. 24)} \]

\[ K_O = 2.8 \times 10^{-3} (+12\%), \quad K_N = 5.0 \times 10^{-3} (+6\%) \text{ (Ref. 8)} \]

The values of \( K \) depend on the wavelength of the source and at high temperature may also vary with \( T \). This temperature variation is now known theoretically or experimentally. In the present calculations \( K \) was assumed to be a constant. The values of \( K \) from Ref. 8 were obtained in a shock tube and those from Ref. 24 at room temperature. The values of \( K_{O_2} \) and \( K_{O} \) did not differ significantly (Ref. 8). Consequently, the fringe shift for dissociated oxygen depends only on density to a first approximation.

The group index of a nonionized gas mixture can be obtained from Eq. (B26) as

\[ n_g = n - \lambda_0 \frac{\partial n}{\partial \lambda_0} \]

or

\[ n_g = n - \lambda_0 \sum_j \frac{\partial K_j}{\partial \lambda_0} \rho_j \]  

(B32)
The difference between the phase and group indices is

\[ n - n_g = n_0 \sum_j \frac{\partial K_j}{\partial \lambda_0} \rho_j \]  

For \( \text{O}_2 \) at wavelengths 4000 Å to 6000 Å, from the table below

\[ \frac{\partial K}{\partial \lambda_0} < \frac{0.10 \times 10^{-3}}{2000 \times 10^{-8}} = \frac{10^{-4}}{2 \times 10^{-5}} = 5 \text{ ft}^3/\text{lb cm}. \]

Therefore

\[ n_0 \frac{\partial K}{\partial \lambda_0} < 5000 \times 10^{-8} \times 5 = 2.5 \times 10^{-4} \text{ ft}^3/\text{lb}. \]

This is one order of magnitude smaller than the value of \( K \) in the wavelength range in question. Similar results apply to other molecular species. It is expected that the same is true of atomic species (Ref. 8) although this has not been verified yet.

**TABLE**

Values of the Gladstone-Dale Constants for Molecular Oxygen in the Range 4000 Å to 6000 Å

<table>
<thead>
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<th>( \lambda_0 ) (Å)</th>
<th>( K_{\text{O}_2} ) (ft(^3)/lb)</th>
</tr>
</thead>
<tbody>
<tr>
<td>4000</td>
<td>3.11 \times 10^{-3}</td>
</tr>
<tr>
<td>5000</td>
<td>3.06</td>
</tr>
<tr>
<td>6000</td>
<td>3.03</td>
</tr>
</tbody>
</table>

The above argument shows \( n - n_g \ll n - 1 \) for nonionized gases.

Thus the group index \( n_g \) is assumed to be equal to the phase index \( n \) to a first approximation. Consequently, in the white light interferogram the central fringe group is shifted by the same amount as the individual fringes, and through the transition from one uniform region to another the same few fringes maintain their brightness throughout as distinct from an ionized gas.
B 4: Ionized Gases

To compute the fringe shift in an ionized monatomic gas (argon) the contributions to the phase index of refraction by the atom, ion, and electron must be determined as follows (Ref. 8),

\[ n - 1 = \rho \left[ K_{atom} (1 - x) + K_{ion} x \right] - \frac{\omega_p^2}{2\omega^2} \]  \hspace{1cm} (B34)

where, \( \omega_p \) = plasma frequency (Ref. 25)

or \( \omega_p = (\frac{4\pi N_e e^2}{m_e})^{\frac{1}{2}} \)

\( \omega \) = angular frequency (\( \omega = 2\pi \nu = 2\pi \frac{c_0}{\lambda_0} \))

\( N_e \) = electron density (electrons/cm\(^3\))

\( m_e \) = electron mass (9.107 \times 10^{-28} \text{ gm})

\( e \) = charge of proton (4.803 \times 10^{-10} \text{ esu})

\( c_0 \) = light speed in vacuum (2.9979 \times 10^{10} \text{ cm/sec})

\( \lambda_0 \) = wavelength of light in vacuum (cm or \( \text{Å} \))

Substituting the numerical values in \( \omega_p \) and \( \omega_p^2/2\omega^2 \) yields,

\[ \omega_p = 5.64 \times 10^4 \sqrt{N_e} \]  \hspace{1cm} (B35)

\[ \frac{\omega_p^2}{2\omega^2} = 4.46 \times 10^{-14} \lambda_0^2 N_e \]  \hspace{1cm} (B36)

The Gladstone-Dale constants for an argon plasma are given in Ref. 8. For the atom (\( K_A \)),

\[ K_A = 2.550 \times 10^{-3} \text{ ft}^3/\text{lb} \text{ or } 1.59 \times 10^{-1} \text{ cm}^3/\text{gm} \]

For the ion (\( K_{A^+} \)),

\[ K_{A^+} = 0.67 \times K_A \text{ for } \lambda_0 = 5180 \text{ Å} \]

The electron number density is the same as the ion number density and may be taken as the product of the degree of ionization and gas density divided by the mass of the ion (assuming the contribution of the electrons to be negligible).
where, $m_{A^+} = m_A = 9.107 \times 10^{-28} \times 1836 \times \frac{39.94}{1.008} = 6.66 \times 10^{-23} \text{ gm}$

A summation of the three terms given in Eq. (B6) using the values indicated above yields, for $\lambda_o = 5180 \text{ Å}$

$$n - 1 = (2.84 - 33.1 x) \times 10^{-4} \frac{\rho}{\rho_o}$$

where

$$\rho_o = 1.78 \times 10^{-3} \text{ gm/cm}^3 \text{ (NTP density)}$$

It can be seen from Eq. (B10) that the value of $(n - 1)$ will be negative for all values of $33.1 x \geq 2.84$. When that occurs the index is less than one and the fringe shift for monochromatic light behind the shock wave will be negative (downward) despite the fact that the density increases across the wave. The above result is due to some influence of the ions but arises mainly from the electrons in the plasma (Eq. B34).

From Eq. (B34)

$$\lambda_o \frac{\partial n}{\partial \lambda_o} = \lambda_o \frac{\partial}{\partial \lambda_o} \left(-\frac{\mu_p^2/2}{\omega^2}\right) = -\lambda_o \frac{\partial}{\partial \lambda_o} \left(4.46 \times 10^{-14} \lambda_o^2 \text{ Ne}\right)$$

$$= -2 \times 4.46 \times 10^{-14} \lambda_o^2 \text{ Ne}$$

Therefore

$$n_g - 1 = \int \left( K_{\text{atom}} (1-x) + K_{\text{ion}} x \right) + 4.46 \times 10^{-14} \lambda_o^2 \text{ Ne}$$

The contributions from $K$ in the right-hand side of Eq. (B34) to

$$\lambda_o \frac{\partial n}{\partial \lambda_o}$$

is small, and has been neglected.

A substitution of the numerical constants yield, for $\lambda_o = 5180 \text{ Å}$,

$$n_g - 1 = (2.84 + 31.3 x) \times 10^{-4} \frac{\rho}{\rho_o}$$

where $\rho_o = 1.78 \times 10^{-3} \text{ gm/cm}^3 \text{ (NTP density)}$. 

$$N_e = \frac{\rho x}{m_{A^+}}$$

(B13)

(B37)
It is to be noted that the value of \((n_g - 1)\) is always positive and that
the central fringe group is always shifted in the normal sense, \textit{i.e.}, in the same
sense as for nonionized gases.

**B 5: Computed Results**

The calculations are summarized in Tables B1 to B3, inclusive. Table B1 contains the quantities in the uniform flow region behind the expansion wave, state (3), for a wall deflection angle of 15 degrees for of the 26 cases that have been calculated. The 15 degree corner flow will be investigated in detail and was chosen as a practical compromise for the UTIA 4 in x 7 in. hypervelocity shock tube. Physical quantities in oxygen are given for the three types of possible expansions, equilibrium flow (e) frozen flow (f) and a flow with vibration in equilibrium with the active degrees (\(f^*\)). For nitrogen and air, only the states (e) and (f) were computed; similarly for argon.

The dimensionless 'fringe shift' \(S_{ij}\) in number of fringes from region (i) to region (j) is defined by,

\[
S_{ij} = \frac{(y_i - y_j)}{d}
\]

where, \(y_i\) and \(y_j\) are \(y\)-coordinate of a fringe line in the parallel-fringe type of interferogram in regions (i) and (j), respectively, and \(d\) is the spacing of the parallel fringes in a known uniform region. From Eq. (B14) the fringe shift is given by,

\[
S_{ij} = \frac{L}{\lambda_o} (n_j - n_i)
\]

Table B2 lists the computed fringe shifts for oxygen, nitrogen and air, across the incident shock wave and those following the expansion corresponding to a 15 degree wall deflection. It will be noted that the fringe shifts for states (e), (f), and (f\(^*\)) are very similar since there is little difference in the final densities of these three cases. The fringe shifts across shock waves are shown on Fig. B5. As expected, the largest fringe shifts occur for an initial high pressure and Mach number at equilibrium. Figure B6 summarizes the results for expansion waves and as noted above there is only a small difference between the equilibrium and frozen flow cases.

Table B3 gives the corresponding results for ionized argon. Columns 15 to 18 give fringe shifts for monochromatic interferograms \((\lambda_o = 5180 \text{ A})\). The initial jump across a shock front gives an upward fringe shift \(S_{12}\) owing to the increase in the index of refraction. As ionization occurs, this jump can be nullified by the negative contributions to the refractive index and can fall below the initial value in front of the shock as noted previously (Fig. B3).

The results are summarized in Fig. B7. It is seen that because of the high initial Mach number the initial density ratio jump across the shock wave has a value \((\rho_2/\rho_1)_{f} = 4\) for the entire range \(14 \leq M_s \leq 20\). Consequently,
only one curve results, which displays the variation of fringe shift with initial density. However, for the equilibrium cases, Mach number plays a very important role. For a Mach number of approximately $M_s = 14$, the net fringe shift is practically zero for the given pressure range. As the Mach number increases and the number density of electrons rises with increasing initial pressures, large negative (downward) fringe shifts occur (see for example, the curve marked $M_s = 20$), because the index of refraction falls below unity.

Figure B8 summarizes the results for the Prandtl-Meyer expansion of ionized argon over a 15 degree wall deflection. It is seen that at high shock Mach number ($M_s^*$) and at the higher initial density (or pressure $p_1 \sim 5$ mm Hg) large numbers of electrons per unit volume are available in the uniform state (2) behind the shock wave. Consequently, the phase refractive index is less than unity (Eq. B38). However, during the expansion the density and consequently the electron number density falls, so that the index of refraction becomes less negative behind the expansion wave (3) than in front of it and consequently the net fringe shift is negative. This is opposite to what usually happens in a nonionizing gas. If vertical fringes were used to investigate the flow, then they would shift to the right in a unionized gas; but to the left (negatively) in an ionized gas (Figs. B2 and B4). In the case of an equilibrium flow the degree of ionization is also dropping as the density is decreasing, resulting in a refractive index in state (3) that is even less negative than in the corresponding frozen flow. Consequently, the net fringe shift is even more negative for the equilibrium flow than the frozen flow. This conclusion is well illustrated in Fig. B8. It appears that for an ionized-gas flow the fringe shift would be an excellent indicator of the state of the gas even though the frozen and equilibrium densities do not differ greatly.

The above argument applies to all cases treated except for two at the lowest Mach number and at higher pressures, viz. $M_s = 14$, $p_1 = 10$ mm Hg and 30 mm Hg. In these cases, the term $(n - 1)$ is positive before expansion due to the low degree of ionization $33.1 < 2.84$. See Eq. (B38). The frozen flow yields a positive shift (to the right for vertical fringes) because $(n - 1)$ is rendered less positive by the density decrease, giving a net positive fringe shift. For the equilibrium flow one obtains a negative shift (to the left) because the falling ionization degree makes $(n - 1)$ more positive behind the expansion wave than in front.

When a white-light source is used instead, interferograms as shown in Fig. B3a (for a shock wave) and Fig. B4a (for an expansion wave) will be obtained. It is assumed that the light intensity has a narrow symmetrical distribution on the wavelength scale (around the mean wavelength $\lambda_o$). In the present calculations $\lambda_o = 5180$ Å. As demonstrated in Subsection B3, the zero fringe is shifted by a distance different from that by which individual monochromatic fringe lines are shifted. In contrast to the fringe pattern in a dissociated (nonionized) gas, the brightness varies along an individual fringe through the transition region (see Figs. B3b, B4b). Define in a similar way,
where, \( y_i \) and \( y_j \) are the \( y \) coordinates of the zero fringes in the uniform regions (i) and (j), respectively, and \( d \) the fringe spacing in a uniform region for a monochromatic light with \( \lambda_0 = \overline{\lambda}_0 \), as given by Eq. B13. (The subscript \( w \) refers to a white light interferogram).

From Eq. B28,

\[
S_{w, ij} = \frac{L}{\overline{\lambda}_0} (n_g, j - n_g, i)
\]  

(B45)

The zero fringe shift at the shock front (frozen compression) is the same as the monochromatic fringe shift, since both (1) and (2f) are non-ionized states. The zero fringe shifts between (1) and (2) are listed in Column 19, Table B3, and are also plotted in Fig. B7a. An increase in \( n_g \) is brought about by the density rise from (1) to (2f), and since the additional density rise and ionization in (2) both act to increase \( n_g \) (Eq. B41) the zero fringe is further shifted upward behind the shock front. Due to the strong electron contribution greater fringe shifts from (1) to (2) result in argon than in oxygen despite the smaller density rise (Fig. B7a). This is especially conspicuous at higher shock Mach numbers.

The central fringe shifts across 15 degree expansion waves are listed in Columns 20 (equilibrium flow) and 21 (frozen flow), and are also plotted in Fig. B8a. The density decrease acts to decrease \( n_g \) and makes the central fringe shift positive (in the normal sense). In the equilibrium case recombination (i.e., the reduction of electron density) brings about an additional decrease in \( n_g \), i.e., a further positive shift. Since the density behind the expansion wave is very nearly the same for two cases for a deflection angle \( \psi = 15 \) degrees, the fringe shift is consistently larger for equilibrium flow than for frozen flow. Note that through transitions (1) to (2) and (2) to (3e) or (3f) the location of the zero fringe is shifted (except for a few cases) in opposite direction to individual fringes (see Figs. B7 and B7a; B8 and B8a).

B 6: Experimental Procedure

Combinations of shock Mach number \( (M_s) \) and channel pressure are required that would be suitable for the interferometric study of nonequilibrium, equilibrium and frozen flows. One important requirement is that in order to obtain sufficient measuring accuracy the expected fringe shift across the expansion wave should be at least two. To observe a nonequilibrium flow and to be able to estimate the recombination rate constants, combinations of \( (M_s, P_1) \) have to be chosen such that the relaxation lengths \( (r_c \text{ and } l_R) \) are suitable to the size of the viewing area. For the present working section, having an elliptical viewing area with a 9-inch major diameter and a 7-inch minor diameter, the relaxation lengths should lie between 1 cm and 10 cm. In the present study a 15-degree deflection angle is used throughout and \( l_R \) as well as \( S_{23} \) is estimated for this value only. A corner deflection of 15°
was dictated by practical consideration such as sufficient thickness for mounting transducers but could not be too thick so as to reduce the flow region behind the expansion wave. In addition, the 15-degree deflection is not a good choice from the point of view of measuring the Mach number of the frozen or equilibrium flow behind the expansion wave from Mach waves, since for this angle the two Mach lines are nearly coincident (see Figs. 12, 39). For this purpose a deflection of about 40-degrees would have been very useful.

Figure B9 shows the \((M_s, p_1)\)-plane for oxygen divided into non-equilibrium, equilibrium and frozen-flow regions. The curves \(C_1, C_2, C_3\) and \(C_4\) represent combinations of \((M_s, p_1)\) that yield \(1_R = 1 \text{ cm}\), \(1_R = 10 \text{ cm}\), \(r_c = 1 \text{ cm}\) and \(r_c = 10 \text{ cm}\), respectively. The nonequilibrium regions defined in terms of \(1_R\) (between \(C_1\) and \(C_2\)) and in terms of \(r_c\) (between \(C_3\) and \(C_4\)) do not coincide. Within these regions the relaxation process along the wall and along the wave head should be observed. Above \(C_1\) the whole flow field will be virtually in equilibrium, whereas below \(C_4\) it will be virtually frozen at the initial dissociation degree. Curves giving fringe shifts \(S_{23e} = 1, 2,\) and \(5\) are also plotted in Fig. B9. It is seen that to satisfy this condition it is necessary to work at higher channel pressures (5 to 30 mm Hg). The following combinations have been chosen as optimum for the three types of flows in oxygen:

(I) \(M_s = 14, p_1 = 30 \text{ mm Hg} : \) Equilibrium flow

(II) \(M_s = 10, p_1 = 10 \text{ mm Hg} : \) Frozen flow

(III) \(M_s = 12, p_1 = 20 \text{ mm Hg} : \) Nonequilibrium flow

The same argument applies for argon. Figure B10 shows the \((M_s, p_1)\)-plane for argon containing similar curves \(C_1, C_2, C_3\) and \(C_4\). When these curves are compared with the corresponding curves for oxygen it is seen that they have been shifted to lower pressures. The reason being that \(k_R\) is about 3 to 4 orders greater for argon than oxygen and therefore all the border curves \(C_1\) to \(C_4\) are shifted to lower pressures. However, this causes a decrease in the fringe shift which is very disadvantageous in the study of such flows in argon. Curves showing fringe shifts \(S_{23e} = 1\) and \(2\) are also plotted. Reasonable fringe shifts can be obtained for equilibrium flow but the channel pressure that can be used with a 10,000 psi driver to obtain high \(M_s\) is limited for example, to 10 mm Hg at \(M_s = 16\), and even here the shift is small. The results are much worse for frozen and nonequilibrium flows. The following combinations have been chosen as optimum for the three types of flow in argon:

(IV) \(M_s = 16, p_1 = 10 \text{ mm Hg} : \) Equilibrium flow

(V) \(M_s = 14, p_1 = 1 \text{ mm Hg} : \) Frozen Flow

(VI) \(M_s = 17, p_1 = 2.5 \text{ mm Hg} : \) Nonequilibrium flow

Flow quantities of interest as well as the expected fringe shifts for cases (I) to (VI) are listed in Table B4.
In order to verify the calculations of the quantities in state \(\beta\), an interferogram should be taken of the transition across the incident shock wave for the six combinations of \(M_S\) and \(p_1\). Since the density changes abruptly across the shock front, white light fringes are most useful to measure such changes. The expected fringe shifts across the shock wave for combinations of \(M_S\) and \(p_1\) chosen above are listed in Table B4. (See Figs. B1, B3a and B3b for the notation.) These values are based on the theoretical values of \(\mathcal{F}_2\) and \(\alpha_2\) or \(x_2\) from the shock relations and on the available values of the Gladstone-Dale constants \((K_{O_2}, K_O, K_A\) and \(K_{A^+}\), which are assumed to be temperature-independent) and, for argon the theoretical electron-contribution term (with the theoretical value of the electron density \(N_e\) from the shock relations) is included.

In view of the fact that the values of the Gladstone-Dale constants at high temperatures are not too well-established (see Subsection B3), the following procedure would be useful to check the value of the constants.

For oxygen, choose the combination (I) \((M_S = 14, p_1 = 30 \text{ mm Hg})\) and measure the fringe shift \(S_{12}\), which can be expressed as (Eqs. B30 and B43),

\[
S_{12} = \frac{L}{\lambda} \left[ \mathcal{F}_2 \left( K_{O_2} (1 - \alpha_2) + K_O \alpha_2 \right) - (n_0 - 1) \right] \quad \text{(B46)}
\]

If one assumes that the shock relations are correct, then

\[
\mathcal{F}_2 = 0.428 \text{ NTP} = 0.0381 \text{ lb/ft}^3, \quad \alpha_2 = 0.321
\]

Since \(L/\lambda = 1.963 \times 10^5\) and \(n_0 - 1 = 1.075 \times 10^{-5}\) (for \(p_1 = 30 \text{ mm Hg}\)) are known constants, one can substitute the measured value of \(S_{12}\) on the left-hand side and obtain an equation for the unknowns \(K_{O_2}\) and \(K_O\),

\[
a_1 K_{O_2} + b_1 K_O = c_1
\]

where,

\[
a_1 = 0.0259 \text{ lb/ft}^3, \quad b_1 = 0.0122 \text{ lb/ft}^3, \quad c_1 = 0.510 \left( S_{12} + 2.11 \right) \times 10^{-5}
\]

Choose another combination of \(M'\) and \(p'_1\) which gives the same temperature \(T_2\) but different density \(\mathcal{F}'_2 \neq \mathcal{F}_2\) and dissociation degree \(\alpha'_2 \neq \alpha_2\). For example, choose the combination (I'): \(M' = 16, p'_1 = 6.7 \text{ mm Hg}\), which yields \(T_2 = 4150 \text{OK}, \mathcal{F}'_2 = 0.129 \text{ NTP} = 0.0115 \text{ lb/ft}^3, \alpha'_2 = 0.507\), and measure the fringe shift \(S'_{12}\). Then one obtains another linear equation for the unknowns \(K_{O_2}\) and \(K_O\), according to Eq. (B46): \((n'_0 - 1 = 0.240 \times 10^{-5}\) for \(p'_1 = 6.7 \text{ mm Hg}\)).

\[
a_2 K_{O_2} + b_2 K_O = c_2
\]

where,

\[
a_2 = 0.00567 \text{ lb/ft}^3, \quad b_2 = 0.00583 \text{ lb/ft}^3, \quad c_2 = 0.510 \left( S'_{12} + 0.47 \right) \times 10^{-5}
\]
By solving Eq. B47, B48, \( K_0 \) and \( K_0^2 \) at temperature \( T_2 = 4150^\circ K \) may be obtained. In this manner it would be possible to determine if there is any variation of the Gladstone-Dale constants with temperature.

The same procedure can be used for argon to find the Gladstone-Dale constants. With a white light source one can measure two fringe shifts \( S_{12} \) and \( S_{w, 12} \) which are expressed as (Eqs. B40, B45),

\[
S_{12} = \frac{L}{\lambda} \left[ \rho_2 \left( K_A (1 - x_2) + K_A + x_2 - c x_2 \right) - (n_1 - 1) \right] \tag{B49}
\]

\[
S_{w, 12} = \frac{L}{\lambda} \left[ \rho_2 \left( K_A (1 - x_2) + K_A + x_2 + c x_2 \right) - (n_1 - 1) \right] \tag{B50}
\]

where \( c x = 4.46 \times 10^{-14} \lambda^2 x_2/\rho_m \) \( \lambda \) \( \rho_m \) \( \text{ft}^3/\text{lb} \) is the electron contribution term. If the two equations are added,

\[
\frac{1}{2} (S_{12} + S_{w, 12}) = \frac{L}{\lambda} \left[ \rho_2 \left( K_A (1 - x_2) + K_A + x_2 \right) - (n_1 - 1) \right] \tag{B51}
\]

Choose the combination (IV) \( (M_s = 16, P_1 = 10 \text{ mm Hg}) \) and measure both fringe shifts. Then, assuming that the shock relations are correct, \( \rho_2 = 0.0868 \text{ NTP} = 0.00964 \text{ (lb/ft}^3 \text{)}, x_2 = 0.140 (n_1 - 1 = 0.340 \times 10^{-5} \text{ for } P_1 = 10 \text{ mm Hg}) \). Consequently,

\[
a_1 K_A + b_1 K_A^+ = c_1 \tag{B52}
\]

where,

\[
a_1 = 0.00829 \text{ lb/ft}^3, \quad b_1 = 0.00135 \text{ lb/ft}^3, \quad c_1 = 0.510 \times 10^{-5} \quad \left( \frac{S_{12} + S_{w, 12}}{2} + 0.668 \right)
\]

Choose another combination that yields the same temperature \( T_2 = 13380^\circ \text{K} \). For example, take the combination (IV'): \( M_s^\prime = 15, P_1^\prime = 21 \text{ mm Hg} \). Then \( \rho_2^\prime = 0.172 \text{ NTP} = 0.0191 \text{ (lb/ft}^3 \text{)}, x_2^\prime = 0.1045, \quad (n_1^\prime - 1 = 0.714 \times 10^{-5} \text{ for } P_1^\prime = 21 \text{ mm Hg}) \).

\[
a_2 K_A + b_2 K_A^+ = c_2 \tag{B53}
\]

where,

\[
a_2 = 0.01712 \text{ lb/ft}^3, \quad b_2 = 0.00197 \text{ lb/ft}^3, \quad c_2 = 0.510 \times 10^{-5} \quad \left( \frac{S_{12}^\prime + S_{w, 12}^\prime}{2} + 1.401 \right)
\]

From Eqs. (B52) and (B53), the unknowns \( K_A \) and \( K_A^+ \) can be solved.
Fringe-Shift Measurement Across and Behind Expansion Waves

For the cases of equilibrium and frozen flows (I, II, IV and V), the flow quantities within the expansion wave will depend only on $\theta$, the angular position of any Mach line and the flow field behind the wave will be uniform ($3e$, $3f$ or $3f'$).

For oxygen (I and II) monochromatic and white light fringe shifts should be essentially the same, $S(\theta) = S_w(\theta)$. Due to the very small difference between $K_0^2$ and $K_0$ the fringe shift $S(\theta)$ will be essentially proportional to the density change (Eq. B30), since the terms in $\alpha$ cancel. Thus the curve $\rho$ versus $\theta$ can be obtained directly from an interferogram. The calculated density variation through the expansion wave for an equilibrium or frozen (or partially frozen) flow (Table 3) can then be checked.

For argon (IV and V) two different fringe shifts through the expansion wave, $S(\theta)$ (negative) and $S_w(\theta)$ (positive) will be obtained (Fig. B4b). When the flow is frozen (V), $S$ as well as $S_w$ will be proportional to the density change (Eqs. B38 and 41). When the flow is in equilibrium (IV), both the ionization degree ($\alpha$) and density ($\rho$) will be changing with $\theta$. They can be determined simultaneously from the experimentally measured $S$ and $S_w$ and applying Eqs. B38 and 41. The calculated density and ionization degree for frozen and equilibrium flows (Table 3) can be verified experimentally.

For nonequilibrium flows (III and VI) the flow quantities will also depend on the radial distance ($r$). Therefore, the region ($3$) behind the expansion wave will not be uniform and a transition will occur from ($3f'$) or ($3f$) to ($3e$). Consequently, one should use monochromatic fringes that cover the whole field of view to study such flows.

The following steps should be taken to determine the relaxation lengths from an interferogram. Measure the variation of fringe shift $S$ at the frozen wave head (from the angular position $\theta_{2f}$ or $\theta_{2e}$ to $(\theta_{2f} + \Delta \theta)$ or $(\theta_{2e} + \Delta \theta)$ with the radial distance $r$. $\Delta \theta$ should be smaller than $(\theta_{2e} - \theta_{2f})$. From Table B4 it is seen that for oxygen this is about $2^\circ$ and for argon it is about $5^\circ$. $\Delta S$ between these two Mach lines $(\Delta \theta)$ will start from a certain finite value $(\Delta S)_0$ pertinent to the frozen flow and decrease to zero as $r$ increases. $\Delta S$ thus measured will be proportional to $(\partial \rho/\partial \theta)$. If this quantity is plotted against $r$, a curve similar to the one shown below is obtained. Following the discussion in Subsection C6 the characteristic radial distance $r_c$ and the recombination rate constant $k_R (T_2)$ can be calculated from Eqs. C89 (to obtain $r_c$) and C9 (to obtain $k_R$) for oxygen. Similarly use Eqs. C96 and C19 for argon.
Measure the variation of the fringe shift $S(s)$ along the wall behind the expansion wave (with respect to the state (2)) with the streamline distance $s$. $S(s)$ will start from a value pertinent to the frozen flow ($S_{23f}$) and tend to another value corresponding to the equilibrium flow ($S_{23e}$) far enough downstream. A plot of $S$ vs $s$ (see Sketch 8) will exhibit an exponential transition similar to that in $\chi$ or $x$ (Fig. C5), as predicted by the linear theory (Eqs. C63 and C64). Then, following the discussion in Subsection C5, the characteristic relaxation length ($l_R$) or time ($\tau_c$) and then the recombination rate constant $k_R$ ($T_{3f}$) can be deduced (Eqs. C60a and C9 for oxygen and C60b and C19 for argon).
(iii) Discussion of Results Shown in Table B4

Case I:

Large fringe shifts $S_{12f}$ and $S_{12}$ (columns 9 and 10) are available across the incident shock wave. This case may be most suitable for the determination of $K_0$ and $K_1$ as noted previously [see (i)]. Except in the small neighbourhood of the corner the flow is in equilibrium (Col. 4 and 5). The head and tail of the expansion wave are inclined at $\theta_{2e} = 71.3^0$ and $\theta_{3e} = 89.0^0$, respectively. The total fringe shift, $S_{23e} = 13.6$. However, the corresponding values of $\theta_2$, $\theta_3$ and $S_{23}$ for frozen or partially frozen flows are very close to those predicted above. (Col. 6 to 8, 21 to 23, 24 to 26). Consequently, if an interferogram shows no variation in fringe shift along the wave head or along the wall it would indicate that the flow is in equilibrium. However proof that this was the case must come from pressure measurements along the wall or temperature measurements in (3e) (see Col. 15 to 17 and 12 to 14).

Case II:

The fringe shifts at the shock wave are reasonably large (Col. 9 and 10). The flow may be regarded as entirely frozen with respect to recombination (Col. 4 and 5). At the head of the wave observe $\theta_{2f} = 68.2^0$; at the corner behind the wave observe $\theta_{3f}$ quickly becoming $\theta_{3f}$ (Col. 21 to 23). The fringe shift across the wave $S_{23f} = 3.9$ (Col. 26). The foregoing remarks regarding equilibrium flows apply here as well.
Case I:

The fringe shifts at the shock wave are again quite large (Col. 9 and 10). In the expansion wave the transition from (partially) frozen flow to equilibrium flow will be observed over the lengths of 2 cm (at the frozen wave head) and 9 cm (along the wall) (Col. 4 and 5). Draw $\Delta \theta = 1.5$ degrees, since in this case (Col. 6 to 8) $\theta_{2e} - \theta_{2f} = 1.8$ degrees. As $\rho$ vs $\theta$ is nearly linear, one can estimate $(\Delta S)$ as $S_{23f} \times \Delta \theta (\theta_{3f} - \theta_{2f}) = 8.5 \times 1.5/20 \approx 0.6$ fringe. Great care will have to be taken to detect this small quantity at the wave head near the corner and its disappearance about 2 cm away along the wave head. In this case we cannot employ $I_R$ to find $k_R$ since $S_{23f} \approx S_{23e}$ (Col. 24 to 26).

Case IV:

By using a white light source the fringe pattern across the shock wave as shown in Fig. B3b can be observed. Fringe shifts are reasonably large (Col. 9 to 11). The expansion is completely in equilibrium (Col. 4 and 5) and $\theta_{2e} = 64.2$ degrees, $\theta_{3e} = 83.7$ degrees may be observed. The central fringe is shifted in a direction opposite to the individual fringes (Col. 24 and 27) and the fringe shifts are reasonably large. If the expected phase and white light fringe shifts are obtained this would give an indication that equilibrium flow exists. However, as noted previously the significant changes in pressure and temperature are needed to establish this fact (Col. 24 to 24, 15 and 16, 12 and 13).

Case V:

This is the only case that will give frozen flow in the $(M_s, p_1)$-range considered. The fringe shifts through the shock wave as well as through the expansion wave are too small for significant measurements. However, the wave-head slopes $\theta_{2e}$ and $\theta_{2f}$ differ by 6 degrees and may be a good indication as to whether the flow is frozen or in equilibrium.

Case VI:

The fringe shifts at the shock wave (Col. 9 to 11) in this case are small but detectable. The nonequilibrium flow region is limited to a small neighbourhood at the corner (Col. 4, 5) and therefore will make experimental observations very difficult.

The wave head the transition from frozen to equilibrium flow will be observed over the length of 0.5 cm. $\Delta \theta$ can be taken, say, as 5 degrees, since $\theta_{2e} - \theta_{2f} \approx 6$ degrees (Col. 6, 7) $(\Delta S)$ is estimated as

$$S_{23f} \times \frac{\Delta \theta}{\theta_{3f} - \theta_{2f}} \approx 0.9 \times \frac{5}{24} \approx 0.2$$ fringe,

a value which is rather small to detect.
Along the wall, behind the expansion wave, the transition takes place over the length of 1.2 cm. The phase shifts \((S_{23e}, S_{23f})\) are rather small and will make a measurement of \(l_R\) difficult. The pressure differences are also rather small and a temperature measurement would be the best method for determining the state of the gas.

From the above results it can be concluded that it would be preferable to investigate oxygen first in view of the larger pressures and fringe shifts. The results for equilibrium and frozen flows would substantiate the calculations and the values of the Gladstone-Dale constants at high temperature for the oxygen molecule and atom.

It is possible that reasonable results for the recombination rate coefficient will be obtained, but it will be a difficult measurement in view of the accuracy that might be available. It presupposes a no-flow set of fringes over the field that are uniform and straight to high precision (1/10 fringe at least).

In argon these measurements will be much more difficult owing to the reduced fringe shift at the head of the wave and along the wall. However, the unique roles of group and phase indexes in an ionized flow makes it desirable to perform these experiments in order to verify the calculations despite the experimental difficulty that may be encountered.

(iv) Summary Discussion

The calculation of relaxation lengths, is based on the semi-empirical data of the recombination-rate constants described in Subsection C4. As noted previously, there is some uncertainty in the absolute values of the recombination-rate constants as well as in their temperature dependence (especially for argon for which little experimental work has been done). It is seen from the formulations in Appendix C that both \(l_R\) and \(r_c\) are proportional to \(\tau_c\) which in turn is inversely proportional to the recombination rate constant \(k_R\). Thus, if \(k_R\) had been overestimated (or underestimated), say, by a factor of three, \(l_R\) and \(r_c\) would then be underestimated (or over-estimated) by the same factor. It is possible (especially for argon) that one might in such a case have an entirely different result than what has been predicted.

For the calculation of \(l_R\), the assumption of small perturbation from equilibrium as well as of small changes of flow quantities during relaxation was used. This will probably not lead to any significant error for the 15-degree deflection. If this is not fulfilled then the relaxation lengths will be larger. To calculate the variation of flow quantities along the wall, especially for larger deflection angles, nonlinear equations would have to be formulated and solved.
For the calculation of \( r_c \), the assumptions of an "ideal dissociating gas" for oxygen and of an "ideal ionizing gas" for argon were used. The former assumption could be eliminated, but the resulting expressions would be very complicated, and, in view of the expected small error introduced by this assumption, the exact numerical calculation was not attempted. The assumption for argon is less certain for degrees of ionization \( x > 0.25 \) but is regarded as acceptable since the maximum degree of ionization \( x \approx 0.31 \).

The range of the \((M_s, p_1)\)-combinations (Table 1) was decided from a plot of diaphragm pressure ratio versus shock Mach number \((M_s vs p_{41})\) in a combustion driven shock tube (Ref. 1). In the present shock tube \( p_4 \) (chamber pressure) should not exceed 10,000 p.s.i. Therefore, for a given channel pressure \( p_1 \), there is a maximum permissible Mach number corresponding to \( p_{41} = 10,000 \) p.s.i. /\( p_1 \) mm Hg. However, in view of the presence of the attenuation due to viscous effects, it is desirable to work at relatively low \( M_s \) and high \( p_1 \) if possible. This fact was considered in determining cases (I) to (IV). The viscous effects also bring about a nonuniformity in the flow in State (2). This is especially undesirable for the study of non-equilibrium expansion, since it would make the interpretation of relaxation lengths more difficult. Careful measurements should be done of the attenuation and nonuniformity in the shock tube before the study of Prandtl-Meyer flow is made.

The actual experiments in the UTIA 4 in. x 7 in. hypervelocity shock tube will be performed on two types of models, that is, on a deflection of the wall of the shock tube, and on a double wedge airfoil model in the free stream. In the first case, the viscous effects of the nonstationary boundary layer behind the shock wave will tend to modify the sharp corner and will affect the location of the focal point of the centred expansion wave and its flow gradients. Consequently, the relaxation lengths along the head of the wave \((r_c)\) and along the wall \((l_c)\) may also be changed. In the second case a similar but smaller change will occur since the quasi-steady boundary layer thickness will be thinner. However, in this case, the airfoil will suffer from some shock wave diffraction effects before the flow becomes uniform. Therefore, the decision as to which model is most suitable for this experiment will only be reached during the course of the actual investigation.
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7. α₃f ≡ ξ₃f
8. not computed or not applicable to an ionized gas
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**TABLE B 2**

**FRINGE SHIFTS ACROSS SHOCK AND EXPANSION WAVES IN A DISSOCIATED GAS**

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**Column**

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3. $p_1$ mm Hg
4. $f_2f/\rho_0$, $\rho_0 = 1$ NTP density
5. $\rho_3e/\rho_0$
6. $f_3f/\rho_0$
7. $(n_1 - 1) \times 10^5$
8. $(n_2 - 1) \times 10^5$
9. $(n_2f - 1) \times 10^5$
10. $(n_3e - 1) \times 10^5$
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**Column**

11 \((n_{3f} - 1) \times 10^5\)

12 \((n_{g, 2} - 1) \times 10^5\)

13 \((n_{g, 3e} - 1) \times 10^5\)

14 \((n_{g, 3f} - 1) \times 10^5\)

15 \(S_{12}\)

16 \(S_{12f}\)

17 \(S_{23e}\)

18 \(S_{23f}\)

19 \(S_{w, 12}\)

20 \(S_{w, 23e}\)

21 \(S_{w, 23f}\)
### TABLE B4
Flow Quantities for Optimum Interferometric Experiments for Prandtl-Meyer Flows Over a 15-degree Corner in Oxygen and Argon

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FIG. B 1: SHOCK WAVE IN A DISSOCIATED GAS

FIG. B 3a: SHOCK WAVE IN AN IONIZED GAS (MONOCHROMATIC FRINGES)

FIG. B 4a: EXPANSION WAVE IN AN IONIZED GAS (MONOCHROMATIC FRINGES)

FIG. B 3b: SHOCK WAVE IN AN IONIZED GAS (WHITE LIGHT FRINGES)

FIG. B 4b: EXPANSION WAVE IN AN IONIZED GAS (WHITE LIGHT FRINGES)
FIG. B. 5
FRINGE SHIFTS THROUGH SHOCK WAVES IN DIATOMIC GASES FOR EQUILIBRIUM AND FROZEN COMPRESSIONS.
FIG. B. 6
FRINGE SHIFTS THROUGH 15 DEG.
EXPANSION WAVES IN DIATOMIC GASES FOR EQUILIBRIUM AND FROZEN FLOWS (— —)
FIG. B. 7
FRINGE SHIFTS THROUGH SHOCK WAVES IN ARGON FOR EQUILIBRIUM AND FROZEN COMPRESSIONS
MONOCHROMATIC INTERFEROGRAM
FIG. B 7a: FRINGE SHIFTS THROUGH SHOCK WAVES IN ARGON FOR EQUILIBRIUM AND FROZEN COMPRESSIONS (WHITE LIGHT INTERFEROGRAM)
FIG. B. 8
FRINGE SHIFTS THROUGH 15 DEG.
EXPANSION WAVES IN ARGON FOR
EQUILIBRIUM AND
FROZEN FLOWS
MONOCHROMATIC
INTERFEROGRAM
FIG. B 8a: FRINGE SHIFTS THROUGH 15 DEGREE EXPANSION WAVES IN ARGON FOR EQUILIBRIUM AND FROZEN FLOWS (WHITE LIGHT INTERFEROGRAM)
FIG. B 9: OPTIMUM CONDITIONS FOR INTERFEROMETRIC STUDIES OF FROZEN, NONEQUILIBRIUM AND EQUILIBRIUM PRANDTL-MEYER FLOWS OVER A 15-DEGREE CORNER IN OXYGEN IN THE UTIA 4 IN. X 7 IN. HYPERVELOCITY SHOCK TUBE.
FIG. B 10: OPTIMUM CONDITIONS FOR INTERFEROMETRIC STUDIES OF FROZEN, NONEQUILIBRIUM AND EQUILIBRIUM PRANDTL-MEYER FLOWS OVER A 15-DEGREE CORNER IN ARGON IN THE UTIA 4 IN. x 7 IN. HYPERVELOCITY SHOCK TUBE
Two lengths which characterize the linear dimensions of the nonequilibrium flow region in a Prandtl-Meyer flow will be calculated. To do so, it will be necessary to derive the rate equation and the energy equation for steady, plane, nonequilibrium flow of a dissociating pure diatomic gas and an ionizing pure monatomic gas. The (overall) continuity equation and the two momentum equations remain unchanged. The characteristics of the above system of equations will then be found and they will be used for the determination of a radial length at the head of the expansion wave that may be useful for the study of nonequilibrium flows.

**C.1: Rate Equations**

a) Pure Diatomic Gas

\[
\left( \mathcal{P} \lambda U + \frac{\partial}{\partial s} (\mathcal{P} \lambda U) \right) \, ds\left( dn + \frac{\partial n}{\partial n} \, dn \, ds \right)
\]

**FIG. C1**

FLOW THROUGH A STREAMTUBE SEGMENT

Consider a small streamtube segment fixed in the plane of flow (Fig. C1).

Dissociation and recombination are assumed to be described by the process,
where, \( X \) is the third body of the reaction.

The mole concentration of \( O_2 \) is given, in terms of macroscopic quantities, as

\[
\left[ O_2 \right] = \frac{\rho (1 - \alpha)}{2m} \text{ moles/cc} \quad (1)
\]

where, \( m \) is the atomic weight (grams/mole) and \( 2m \) is the molecular weight (grams/mole) of the species of gas considered. Since one mole of a gas contains \( N_A \) particles (\( N_A \) denotes the Avogadro number), the number density of \( O_2 \) is expressed as

\[
n_{O_2} = N_A \left[ O_2 \right] = \frac{N_A \rho (1 - \alpha)}{2m} \text{ molecules/cc} \quad (1)'
\]

Similarly, for \( O \),

\[
\left[ O \right] = \frac{\rho \alpha}{m} \text{ moles/cc} \quad (2)
\]

\[
n_O = N_A \left[ O \right] = \frac{N_A \rho \alpha}{m} \text{ atoms/cc} \quad (2)'
\]

Also, as \( X \) can be either \( O_2 \) or \( O \),

\[
n_X = n_{O_2} + n_O = \frac{N_A \rho (1 + \alpha)}{2m} \text{ particles/cc} \quad (3)
\]

\[
\left[ X \right] = \left[ O_2 \right] + \left[ O \right] = \frac{\rho (1 + \alpha)}{2m} \text{ moles/cc} \quad (3)'
\]

The number of forward reactions in unit time (or their frequency) is proportional to the number of collisions between an oxygen molecule \( O_2 \) and a third body \( X \), and

\[
\frac{\partial [O]}{\partial t} \bigg|_x \sim n_{O_2} n_X \quad (4)
\]

or

\[
\frac{\partial [O]}{\partial t} = 2k_D \left[ O_2 \right] \left[ X \right] \quad (4)'
\]
The dissociation rate constant \( k_D \) (mole\(^{-1}\)cc sec\(^{-1}\)), which was defined as the proportionality factor in Eq. (4) above, is a function of molecular size and the field of force and depends only on temperature \( T \). The factor two appears since in the dissociation of a molecule two atoms are formed.

The number of backward reactions in unit time (or their frequency) is likewise proportional to the number of simultaneous encounters of two atoms of oxygen (O) and a third body (X), and

\[
- \frac{\partial [O]}{\partial t} \propto n_O \cdot n_X
\]

or

\[
- \frac{\partial [O]}{\partial t} = 2 k_R \left[ O \right]^2 \left[ X \right]
\]

The factor two appears since two oxygen atoms disappear to form an oxygen molecule. Summing up the two contributions yields the rate equation,

\[
\frac{\partial [O]}{\partial t} = 2 k_D \left[ O_2 \right] \left[ X \right] - 2 k_R \left[ O \right]^2 \left[ X \right]
\]

The recombination rate constant \( k_R \) (mole\(^{-2}\)cc\(^2\)sec\(^{-1}\)) is defined in Eq. (5) and it again is a function of temperature \( T \).

Substituting relations (1), (2) and (3) in Eq. (6), yields,

\[
\frac{\partial (\rho \alpha)}{\partial t} = \frac{k_R}{m^2} \int_1^{\frac{3}{m}} \left( 1 + \alpha \right) \left( \frac{m}{2 \rho} K_c (1 - \alpha) - \alpha^2 \right) \text{gm cc}^{-1} \text{sec}^{-1}
\]

where \( K_c^* = k_D / k_R \) is the equilibrium constant (based on concentration) and is a function of \( T \) only. (The factor \( m \) was cancelled on either side of the equation since it is a constant).

The right-hand side of Eq. (7), when multiplied by \( ds \cdot dn \cdot 1 \), gives the rate of increase of the mass of dissociated O in the streamtube segment as a result of the chemical reaction (in gm/sec). For the flow to be steady this must be equal to the net outflow rate across the ends of the streamtube segment of dissociated O. Referring to Fig. 1 the following equation is obtained,

\[
K_c(T) = 2 \frac{J_D}{m} e^{-\theta_D/T} \text{ (mole cc}^{-1}) \text{ (Ref. 5)}
\]

where, \( J_D \) and \( \theta_D \) are characteristic dissociation density and temperature, respectively.

* NOTE: \( K_c(T) \) for oxygen and nitrogen are given in tabular form in Refs. 16 and 17, respectively. For an ideal dissociating gas it is given simply as

\[
K_c(T) = 2 \frac{J_D}{m} e^{-\theta_D/T} \text{ (mole cc}^{-1}) \text{ (Ref. 5)}
\]
Dividing ρ gives,
\[
\left[ \left( \rho U \right)_s + (\rho U) Y_n \right] \frac{\alpha}{\rho} + U \alpha_s = \frac{k R \rho^3 (1 + \alpha)}{m^2} \left[ \frac{m}{2 \rho} K_c (1 - \alpha) - \alpha^2 \right]
\]

The terms in the square brackets on the left-hand side vanish by virtue of the overall continuity equation, resulting in
\[
U \alpha_s = \frac{k R \rho^2 (1 + \alpha)}{m^2} \left( \frac{m}{2 \rho} K_c (1 - \alpha) - \alpha^2 \right)
\]

The quantity \( T_c \) (sec) is known as the 'characteristic chemical time'.

b) Pure Monatomic Gas

It is assumed that ionization and recombination (de-ionization) can be described by the single process,

\[
A + e^- \rightarrow A^+ + e^- + e^-
\]

where, the contribution of A or A\(^+\) as the third body is neglected as ineffective. It's also assumed that atoms and ions have a common temperature, but the electron temperature \( T_e \) is generally different, that is,

\[ T_A = T_{A^+} = T, \quad T_e \neq T \]

The mole concentrations of \( A, A^+, \) and \( e^- \) are obtained from

\[
\begin{align*}
[A] &= \frac{\rho (1 - x)}{m} \quad \text{moles/cc} \\
[A^+] &= \frac{\rho x}{m} \quad \text{moles/cc}
\end{align*}
\]

It is also assumed that the atom-ion-electron mixture (plasma) is electrically neutral and that the ion and the neutral atom have the same mass (electron mass is negligible)

The number densities of these species are given by

\[
n_A = N_A \frac{N_A \rho (1 - x)}{m} \quad \text{atoms/cc}
\]
respectively.

The number of forward reactions per unit time in a fixed unit volume is proportional to the collision frequency between A and e\(^{-}\), i.e., to the product \(n_A n_e\),

\[
\frac{\partial \left[ e^{-} \right]}{\partial t} \propto n_A n_e
\]

and

\[
\frac{\partial \left[ e^{-} \right]}{\partial t} = k_I \left[ A \right] \left[ e^{-} \right] (15)
\]

Here the ionization rate constant \(k_I\) (mole\(^{-1}\)sec\(^{-1}\)) has been defined in an analogous manner to \(k_D\), and the factor of unity occurs since only one electron is formed in the reaction. Under ordinary conditions the mobility of electrons is much greater than that of atoms because of large difference in their masses, and the collision mechanism is almost exclusively governed by the motion of electrons. Thus \(k_I\) is supposed to be a function of \(T_e\) only to a good approximation.

Similarly, for the backward reaction,

\[
- \frac{\partial \left[ e^{-} \right]}{\partial t} = k_R \left[ e^{-} \right]^3 (16)
\]

The recombination rate constant \(k_R\) (mole\(^{-2}\)sec\(^{-1}\)), like \(k_I\) depends only on \(T_e\). The factor of unity occurs since only one electron disappears on recombination.

Combining both reactions gives the rate equation,

\[
\frac{\partial \left[ e^{-} \right]}{\partial t} = k_I \left[ A \right] \left[ e^{-} \right] - k_R \left[ e^{-} \right]^3 (17)
\]

Therefore the rate equation for a pure monatomic gas is given by

\[
(\rho x U) + (\rho x U)_{ii} = \frac{k_R \rho^2 x}{m^2} \left[ \frac{m}{\rho} K_c (1 - x) - x^2 \right] (18)
\]

where, \(K_c = k_I/k_R\) is the equilibrium constant (mole/cc)
or, by virtue of the overall continuity equation

\[
U \rho = \frac{k_{\text{R}} \rho^2}{m^2} \left( \frac{m}{\rho} K_{\text{C}} (1 - x) - x^2 \right)
\]

\[
U x = \frac{1}{T_{\text{c}}} \left( \frac{m}{\rho} K_{\text{C}} (1 - x) - x^2 \right) = W
\]

(19)

In this case as well \( T_{\text{c}} \) is the 'characteristic chemical time'.

C.2: Energy Equation

a. Pure Diatomic Gas

The equation of conservation of energy in steady, plane flow can be expressed as,

\[
U \left( \frac{e_S}{\rho} + p \left( \frac{1}{\rho} \right)_S \right) = 0
\]

(20)

The internal energy \( e \) of a pure diatomic gas is given by,

\[
e = e_{\text{O}_2} + e_{\text{O}} + e_{\text{dissociation}}
\]

(21)

or

\[
e = \frac{1}{\rho} \left( n_{\text{O}_2} kT \left( \frac{5}{2} + f_1(Z) \right) + n_{\text{O}} kT \cdot \frac{3}{2} + \frac{n_{\text{O}}}{2} k \theta_D \right)
\]

(22)

where, \( f_1(Z) = Z/(e^Z - 1) \) \( (Z = \theta_V / T, \theta_V = \text{vibrational temperature}) \) is a parameter giving the contribution from the vibrational mode of molecules, \( \theta_D \) is the characteristic temperature for dissociation, and the number densities of \( \text{O}_2 \) and \( \text{O} \), \( n_{\text{O}_2} \) and \( n_{\text{O}} \), can be replaced, by \( \rho (1 - \alpha) \) and \( \rho \alpha \frac{m_{\text{O}_2}}{2} \) respectively, and \( m_{\text{O}_2} \) is the mass of an oxygen molecule.

The Boltzmann constant \( k \) is related to the gas constant \( (R) \) of \( \text{O}_2 \) by,

\[
k = m_{\text{O}_2} R
\]

Consequently, the internal energy can be written in terms of the macroscopic quantities as,

\[
e = (1 - \alpha) RT \left( \frac{5}{2} + f_1(Z) \right) + \alpha RT \frac{6}{2} + \alpha R \theta_D
\]

or

\[
e = RT \left( \frac{5}{2} + f_1(Z) + \frac{1}{2} - f_1(Z) \right) \alpha + R \theta_D \alpha
\]

(23)
The first and third terms of Eq. (20) are consequently reduced to
\[
U e_s = \left[ \frac{5}{2} + \frac{\alpha}{2} + (1 - \alpha) f_2(Z) \right] R U T_s
+ \left[ \frac{\theta_D}{T} + \frac{1}{2} - f_1(Z) \right] R T U \alpha_s
\]  
(24)

Here
\[
f_2(Z) = f_1(Z) + Z f_1'(Z) = \frac{Z^2 e^Z}{(e^Z - 1)^2}
\]

(It is worth noting that \(f_2(Z) = \) vibrational specific heat divided by \(R\)).

It is convenient to choose \(\alpha\), \(p\) and \(T\) as the basic (independent) thermodynamic flow variables and to write the energy equation in terms of their derivatives. To do this, use is made of the equation of state,
\[
\frac{1}{p} = \frac{R (1 + \alpha)}{\rho}
\]
and the remaining terms of Eq. (20) can be written as,
\[
U p \left( \frac{1}{p} \right)_s = U \left( - \frac{1}{\rho} p_s + R T \alpha_s + R (1 + \alpha) T_s \right)
\]  
(25)

The final form of the energy equation is,
\[
A T_s - \frac{1}{\rho} p_s + B \alpha_s = 0
\]  
(26)

where
\[
A = \left[ \frac{7}{2} + \frac{3}{2} + (1 - \alpha) f_2(Z) \right] R
\]  
(27)
\[
B = \left[ \frac{\theta_D}{T} + \frac{3}{2} - f_1(Z) \right] R T
\]  
(28)

Note that for an ideal dissociating gas, an assumption is made that the vibrational energy is equal to \(\frac{1}{2}RT\), therefore,
\[
f_1(Z) = \frac{1}{2}, \quad f_2(Z) = \frac{1}{2},
\]
\[
A = (4 + \alpha) R
\]
\[
B = \left( \frac{\theta_D}{T} + 1 \right) R T
\]
b) Pure Monatomic Gas

The internal energy $e$ of a pure monatomic gas can be expressed as,

$$e = e_A + e_{A^+} + e_e + e_{\text{ionization}}$$

(29)

where, $\theta_I$ is a characteristic temperature for ionization. The energy arising from electronic states has been neglected. This assumption is reasonable for $x < .25$ (Ref. 1)

If the following substitutions are made,

$$n_A = \frac{\rho (1 - x)}{m_A}, \quad n_{A^+} = n_e = \frac{\rho x}{m_A}, \quad k = m_A R$$

where, $m_A$ is the mass of the argon atom, then the expression of $e$ in terms of macroscopic quantities becomes,

$$e = (1 - x) \frac{RT}{2} + x RT \frac{3}{2} + x \frac{RT_e}{2} + x R \frac{\theta_I}{T_e}$$

(30)

Near equilibrium $T_e \rightarrow T$ and the relation for $e$ is simplified.

However, in general $T = T(T_e, x)$ which is derived from the consideration of the energy balance for electrons (see note at the end of the section), and it is convenient to choose $x$, $p$ and $T_e$ as basic thermodynamic variables and write the energy equation in terms of their derivatives as follows.

Let,

$$\frac{\partial T}{\partial T_e} = F_1 (T_e, x), \quad \frac{\partial T}{\partial x} = F_2 (T_e, x)$$

(31)

The equation of state,

$$\frac{1}{p} = \frac{R(1 + x) T}{p}$$

(32)

is used for the reduction as above for a diatomic gas. The final equation is

$$A T_{es} - \frac{1}{p} p_s + B x_s = 0$$

(33)
where
\[ A = \left\{ \left( \frac{5}{2} + x \right) F_1(T_e', x) + \frac{3}{2} x \right\} R \]
\[ B = \left\{ \left( \frac{5}{2} + x \right) F_2(T_e', x) + \frac{3}{2} T_e \right\} R + T(T_e, x) + \theta_1 \]

For equilibrium or near equilibrium flows,

\[ T \approx T_e' \]
\[ F_1(T_e', x) = 1, \]
\[ F_2(T_e, x) = 0. \]

and
\[ A = \frac{5}{2} (1 + x) R, \]
\[ B = \left( \frac{\theta_1}{T} + \frac{5}{2} \right) RT. \]

C.3: Equations in Characteristic Form

The set of equations for steady plane flow of a pure diatomic gas can now be written as,

Rate equation: \[ U \alpha_s = W \quad \text{(Eqs. 9, 19)} \] (I)

Continuity equation: \[ \int U U_s + U \int \rho \alpha_s + U \int \rho p p_s + U \int \rho T T_s \]
\[ + \int U \nu_n = 0 \quad \text{(Eq. 8, } \alpha = \text{ const)} \] (II)

s-momentum equation: \[ U U_s + p_s = 0 \] (III)

n-momentum equation: \[ U^2 \nu_s + n_n = 0 \] (IV)

Energy equation: \[ A U T_s - \frac{1}{\rho} U p_s + B U \alpha_s = 0 \] (Eqs. 26, 33) (V)

The dependent variables are \( U, \nu, \alpha, p, \) and \( T, \) and all the coefficients \( W, A \) and \( B \) (Eqs. 9, 27 and 28) appearing in the equations can be expressed as functions of these five variables.

The corresponding system for a pure monatomic gas is formally the same except that \( \alpha \) and \( T \) are to be replaced by \( x \) and \( T_e \),
respectively. The coefficients in this case are given by Eqs. (19), (34), (35) and the equation of state.

To find the characteristics of the system of equations (I) to (V), the following general procedure for a higher-order system (Refs. 26, 27) can be used. Introduce the characteristic parameter $\beta$. Then

$$U_s s_\beta + U_n n_\beta = U_\beta$$  \hspace{1cm} (VI)

$$v_s s_\beta + v_n n_\beta = v_\beta$$  \hspace{1cm} (VII)

$$\alpha_s s_\beta + \alpha_n n_\beta = \alpha_\beta$$  \hspace{1cm} (VIII)

$$p_s s_\beta + p_n n_\beta = p_\beta$$  \hspace{1cm} (IX)

$$T_s s_\beta + T_n n_\beta = T_\beta$$  \hspace{1cm} (X)

Write Eqs. (I) to (X) inclusive in matrix form:

$$D \cdot v = a$$  \hspace{1cm} (36)

$$D = \begin{bmatrix}
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
2 & f & 0 & 0 & \rho U & U_f & 0 & 0 & 0 & 0 \\
3 & f U & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
4 & 0 & 0 & \rho U^2 & 0 & 0 & 0 & 0 & 0 & 1 \\
5 & 0 & 0 & 0 & 0 & BU & 0 & 0 & 0 & 0 \\
6 & s_\beta & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
7 & s_\beta & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
8 & 0 & 0 & 0 & 0 & s_\beta & 0 & 0 & 0 & 0 \\
9 & 0 & 0 & 0 & 0 & 0 & s_\beta & 0 & 0 & 0 \\
10 & 0 & 0 & 0 & 0 & 0 & 0 & s_\beta & 0 & 0 \\
\end{bmatrix}$$
Then the determinant equation,

\[ |D| = 0 \quad (37) \]

is the singularity condition that yields (generally) five characteristic directions in the \((n, s)\)-plane,

\[ \beta_1, \beta_2, \beta_3, \beta_4, \beta_5 \]

For this system only three unique characteristic directions are found to exist, since three of them are identical, that is

\[ |D| = s^5 \rho^3 \rho^2 A U^4 \left[ \left( \frac{\rho_p + \rho_T}{\rho_A} \right) U^2 - 1 \right] \phi^2 - 1 = 0, \]

where \( \phi = \frac{n \beta}{s \beta} \quad (37) \)

\[ s: \frac{\delta n}{\delta s} = 0 \quad \text{(triple root)} \quad (38) \]

\[ \xi: \frac{\delta n}{\delta s} = \frac{1}{\sqrt{\left( \frac{\rho_p + \rho_T}{\rho_A} \right) U^2 - 1}} \quad (39) \]

\[ \eta: \frac{\delta n}{\delta s} = -\frac{1}{\sqrt{\left( \frac{\rho_p + \rho_T}{\rho_A} \right) U^2 - 1}} \quad (40) \]
The first characteristic is identified with the streamline. Thus s denotes the distance measured along the streamline, where \( n = \text{const} \) and Eq. (38) is automatically satisfied.

The angles \((\mu, \nu)\) that the second and third characteristics make with the streamline are found to be (Eqs. 39 and 40),

\[
\tan \mu = \pm \frac{1}{\sqrt{(\rho_p + \rho_T) U^2 - 1}}
\]

(41)

\[
\tan \nu = \frac{1}{\sqrt{(\rho_p + \rho_T) U^2 - 1}}
\]

(42)

By analogy with non-reacting flows these should be the two Mach directions, and the right-hand sides of (41) and (42) should be equal to

\[
\tan \mu = \frac{1}{\sqrt{M^2 - 1}} \quad \text{where, } M = \frac{U}{a}
\]

Or, conversely, the speed of sound (speed of propagation of a disturbance) a in nonequilibrium flow can now be defined from a gasdynamical viewpoint as,

\[
a = \frac{1}{\sqrt{ \rho_p + \rho_T } a}
\]

(43)

Some special cases where a reduces to known form will be given subsequently.

Identify \( \xi \) and \( \eta \) with the distances measured along the respective characteristic directions (Mach lines, see sketch). Thus

along \( \xi \) : \( \frac{d \xi}{ds} = \frac{ds^2}{ds^2 + dn^2} \)

along \( \eta \) : \( \frac{d \eta}{ds} = \frac{ds^2}{ds^2 + dn^2} \)

and the derivatives along three characteristic directions can be written as
An inspection of Eqs. (I), (III) and (V) shows that they are already in characteristic form, i.e., differentiation occurs only with respect to the s-characteristic

\[ U \frac{\delta \alpha}{\delta s} = W \]  
\[ U \frac{\delta U}{\delta s} + \frac{\delta p}{\delta s} = 0 \]  
\[ AU \frac{\delta T}{\delta s} - \frac{U}{F} \frac{\delta p}{\delta s} + BU \frac{\delta \alpha}{\delta s} = 0 \]  

This leaves Eqs. (II) and (IV), of which the first is reduced, by virtue of (I), (III) and (V), to a convenient form containing the space derivatives \( p_s \) and \( \nu_n \) only

\[
- \frac{1}{U} p_s + W \rho_\alpha + U \rho_p p_s + \frac{T U}{A} p_s = 0
\]

or
\[
\rho U^2 \nu_n + \left( \rho_p + \frac{\rho T_\rho}{A \rho} \right) U^2 - 1 \right) p_s = \left( \frac{B \rho T_\rho}{A} - \rho_\alpha \right) W U
\]

Linear combinations of (II)' and (IV) yield

\[
\frac{\delta \nu}{\delta \xi} + C \frac{\delta \rho}{\delta \xi} = V \]  
\[
\frac{\delta \nu}{\delta \eta} - C \frac{\delta \rho}{\delta \eta} = -V \]  

where,

\[
C(\alpha, p, T, U) = \sqrt{\frac{M^2 - 1}{\rho U^2}}
\]

\[
V(\alpha, p, T, U) = \frac{1}{\rho U M} \left( \frac{B \rho T_\rho}{A \rho} - \rho_\alpha \right) W
\]
Equations (CI) to (CV) now constitute the complete set of characteristic equations corresponding to the original system.

It is worth noting that from the equation of state,

\[ \rho = \frac{p}{R (1 + \alpha) T} \]

or

\[ \frac{d\rho}{\rho} = \frac{dp}{p} - \frac{d\alpha}{1 + \alpha} - \frac{dT}{T} \]

giving,

\[ \rho_p = \frac{\rho}{p} = \frac{1}{R(1 + \alpha) T} \]

\[ \rho_\alpha = - \frac{\rho}{1 + \alpha} = - \frac{p}{R (1 + \alpha)^2 T} \]

\[ \rho_T = - \frac{\rho}{T} = - \frac{p}{R (1 + \alpha) T^2} \]

Therefore,

\[ a = \sqrt{\sqrt[4]{\frac{AR (1 + \alpha)}{A - (1 + \alpha) R}}} T \quad (43a) \]

As noted above, for an ideal dissociating gas \( A = (4 + \alpha) R \)

and

\[ a = \sqrt{\frac{4 + \alpha}{3} (1 + \alpha)} \frac{RT}{(4 + \alpha) R} \quad (43b) \]

It is worth noting that for a gas with frozen composition \( a = \sqrt[3]{\frac{R (1 + \alpha) T}{(1 + \alpha) R}} \), and \( \Gamma = C_p/C_v = (4 + \alpha)/3 \), that is, the sound speed to be used in nonequilibrium flow is the frozen sound speed.

For an ionized gas \( A = \frac{5}{2} (1 + x) R \)

\[ a = \sqrt[5]{\frac{5}{3} (1 + x) RT} \quad (43c) \]

that is, the frozen sound speed for a monatomic ionizing gas.

The factor in \( V \) (Eq. 45) can be evaluated as

\[ \left( \frac{B \rho_T}{A} - \rho_\alpha \right) = -\rho \left( \frac{B}{AT} - \frac{1}{1 + \alpha} \right) \]
and so

\[ V = - \frac{1}{\text{UM}} \left( -\frac{B}{AT} - \frac{1}{1 + \alpha} \right) W \]  

(45a)

For an ideal dissociating gas \( (A = (4 + \alpha) R) \), \( B = \left( \frac{\theta_D}{T} + 1 \right) RT \)

\[ V = - \frac{1}{\text{UM}} \left( \frac{\theta_D}{T} + \frac{1}{4 + \alpha} - \frac{1}{1 + \alpha} \right) W \]

(45b)

whereas for an ionizing gas \( (A = \frac{5}{2} (1 + x) R) \), \( B = \left( \frac{\theta_I}{T} + \frac{5}{2} \right) \frac{RT^2}{T} \)

\[ V = - \frac{1}{\text{UM}} \left( \frac{\theta_I}{5} \frac{T + 5}{1 + x} - \frac{1}{1 + x} \right) W \]

(45c)

C. 4: Recombination Rate Constant Data

The calculation of characteristic lengths of recombination (next two subsections), and more generally of nonequilibrium flow field, requires knowledge of the numerical value of the recombination rate coefficient \( (k_R) \) of the gas in question.

Theoretical prediction of this rate constant must stand upon the microscopic picture of the collision mechanism, where the potential field and size of particles have to be taken into account, and usually a number of assumptions have to be introduced to allow a mathematical treatment. Thus any theoretical prediction may be regarded as reliable only after good agreement with experimental results is seen.

Experimentally, we can deduce the dissociation and ionization rate coefficients \( (k_D, k_I) \) at high temperatures from the measurement of physical quantities through a shock front. Since we have well-established data of equilibrium constants \( K_c \), \( k_R \) can be determined from \( k_D \) or \( k_I \). On the other hand \( k_R \) at room temperature can be obtained directly by flash photolysis which is a technique very extensively used for variety of gases. But since we are interested in \( k_R \) at elevated temperatures, they merely serve as a check.
For oxygen, Heyman (Ref. 13) recently performed a simplified calculation and came up with a formula: \( k_R(T) \) which checks with experimental data with reasonable agreement. It is a statistical treatment based on the assumptions:

1. The potential is of Lennard-Jones type (using the value of \( \sigma_{12} \), zero energy distance, estimated by Lees).

2. Recombination occurs when the total collision energy is equal to \( E^* \) or lower (\( E^* \) is not a sensitital parameter).

3. There is no ionization. (Valid at \( T < 10,000 \text{°K} \)).

He considered \( \text{O} \) as well as \( \text{O}_2 \) as the third body. Thus formulation was made for two rate constants \( k_{R1} \) and \( k_{R2} \). However the numerical estimate was made only with \( \text{O}_2 \) as the third body, and the presented result is to be regarded as correct at comparatively low dissociation degrees \( \alpha \) (say less than 0.5).

We will adopt his formula for \( k_R \) for oxygen, which is written:

\[
k_R = 9.58 \frac{T}{3500} \times 10^{-33} \text{ molecule}^{-2} \text{ cc}^{2} \text{ sec}^{-1}
\]

(Eq. 55, Ref. 13) (46a)

\[
k_R = 4.14 \left( \frac{T}{3500} \right)^{\frac{1}{6}} \left[ 1 + 0.11 \left( \frac{T}{3500} \right)^{\frac{1}{5}} + 0.62 T \right] \times 10^{14} \text{ mole}^{-2} \text{ cc}^{2} \text{ sec}^{-1}
\]

This is plotted in Fig. C.2 and Fig. C.3 (detail), where the experimental values of Matthews (Ref. 15a) and Byron (Ref. 14) and also the inverse square law curve proposed in Ref. 6 are included for comparison, (see also experimental work of Wilson, Ref. 31)

\[
k_R = 8.4 \times 10^{14} \left( \frac{T}{35000} \right)^{-2} \text{ mole}^{-2} \text{ cc}^{2} \text{ sec}^{-1}
\]

(Petschek and Byron (Ref. 28), in connection with their study of strong shock waves in argon, derived a rate equation for ionization in the shock front (second stage) where the ionization degree has reached a certain value through atom-atom or atom-impurity collisions (first stage) and the following reaction is predominant,

\[
A^- + e^- \rightarrow A^+ + e^- + e^-
\]

The major assumptions are,
(C17)

(1) Coulomb potential field between charged particles.

(2) Use of simple empirical relation for the inelastic cross-section ($\sigma_i'$), (Eq. 12, Ref. 28).

(3) Ionization probability identified with probability of excitation to the first electronic level. (It is assumed that an atom excited to that level will eventually be ionized).

(4) Negligible recombination.

(5) Ionization degree small. Their expression for the production term in the rate equation (see Eq. 19 above),

$$\frac{Dx}{Dt} = 4.4 \times 10^{-6} N_e \left( \frac{2(kT_e)^3}{\pi m_e} \right)^{\frac{1}{2}} \left( \frac{T_{exc}}{T_e} + 2 \right) \exp \left( -\frac{T_{exc}}{T_e} \right)$$

(47a)

(All quantities are in c.g.s. units; see Ref. 28, p. 278).

This equation was in reasonable agreement with their experimental results. However, because of the complex structure of shock waves in argon an accurate determination of $k_I$ or $k_R$ from shock-front measurement was made considerably more difficult than for oxygen.

Bray and Wilson (Refs. 29, 30) applied the above result to the calculation of nonequilibrium expansion flow in nozzles. The assumption (5) can be eliminated by simply multiplying Eq. (47a) by the factor $(1 - x)$. The recombination coefficient $k_R$ is found from the relation $K_c = k_I/k_R$, where $K_c$ is expressed in terms of $x$, by using the Saha equation for equilibrium ionization degree (Ref. 1) and the existing production term (Eq. 47a).

From Ref. 29, Eqs. 23 and 24,

$$\frac{Dx}{Dt} = r_I - r_R$$

(47b)

where

$$r_I = 3.35 \times 10^{16} \left( \frac{\rho}{\rho_I} \right) x (1 - x) \left( \frac{T_e}{\theta_I} \right)^{\frac{3}{2}} \left( \frac{T_{exc}}{T_e} + 2 \right) \exp \left( -\frac{T_{exc}}{T_e} \right)$$

(48)

$$r_R = 3.35 \times 10^{16} \left( \frac{\rho}{\rho_I} \right)^2 x 3 \left( \frac{T_e}{T} \right)^{\frac{3}{2}} \left( \frac{T_{exc}}{T_e} + 2 \right) \left( \frac{T}{T_e} - \frac{T_{exc}}{T_e} \right)$$

(49)

where from Saha's equation, the relation $K_e = \frac{1}{m} \left( \frac{T}{\theta_I} \right)^{\frac{3}{2}} \exp \left( -\frac{\theta_I}{T} \right)$ has been used.
In order to perform the numerical calculation in the same way as for oxygen it is convenient to have the expression: $k_R(T_e)$. Since the right-hand side of (Eq. 49) is equal to (see also Eq. 19),

$$k_R \rho^2 x^3 \frac{m}{m^2}$$

we can write down

$$k_R = \frac{m^2}{\rho^2 x^3} r_R$$

or

$$k_R = 3.35 \times 10^{16} \times \frac{m^2}{\rho^2 T_e^2} F(T, T_e)$$

or

$$k_R = 2.33 \times 10^{15} F(T, T_e) \text{ mole}^{-2} \text{cc}^2 \text{sec}^{-1}$$

where

$$F(T, T_e) = \left( \frac{T_e}{T} \right)^{3/2} \left( \frac{T_{\text{exc}}}{T_e} + 2 \right) \exp \left( \frac{\theta_1 - T_{\text{exc}}}{T} \right)$$

The relation between $T$, $T_e$ and $x$ was derived from the consideration of electron-energy balance for inelastic and elastic collisions with atoms (Ref. 28). This relation is plotted in detail by Bray and Wilson (Ref. 40). However, in the present calculations it is assumed that $T = T_e$. This is correct at the frozen wave head (see the calculation of $r_c$, in subsection C.6) where the gas is in equilibrium, but is approximately correct only for small deviation from equilibrium on the wall behind the expansion fan (see the calculation of $r_c$ in subsection C.5).

The dependence of $k_R$ on the temperature $T_e$ (Eq. 52) is plotted in Fig. C.5. The factor

$$F(T) = \left( \frac{T_{\text{exc}}}{T} + 2 \right) \exp \left( \frac{\theta_1 - T_{\text{exc}}}{T} \right)$$

is also plotted against $T$ in Fig. C.4, for convenience.
It is expected that at the corner O of the Prandtl-Meyer flow (Fig. C5) the expansion takes place in a frozen manner and that close behind O, on the inclined wall, we will have a state (3f) with the dissociation or ionization degree 'frozen' at the initial value \( \alpha_{3f} = \alpha_2 \) or \( x_{3f} = x_2 \). Flow quantities of interest of state (3f) for the deflection angle of 15 degrees are listed in Table B.1 for a range of upstream conditions.

Along the wall the gas will relax as it undergoes molecular, atomic or electron collisions until sufficiently downstream it attains equilibrium. The exact nature of this relaxation process can only be obtained by solving the system of nonequilibrium flow equations over the entire flow field. However, an estimate of the relaxation distance involved in this process can be made in the following way.

The equations which give the rate of change of the degree of dissociation and ionization along a streamline are (Eqs. 9 and 19 above)

\[
\frac{U}{\tau_c} \frac{\partial \alpha}{\partial s} = \frac{1}{\tau_c} \left[ \frac{m}{2 \rho} K_c(T) (1 - \alpha) - \alpha^2 \right] 
\]  

\[
\frac{U}{\tau_c} \frac{\partial x}{\partial s} = \frac{1}{\tau_c} \left[ \frac{m}{\rho} K_c(T_e) (1 - x) - x^2 \right] 
\]

where, the characteristic chemical time \( \tau_c \) (sec) was defined as

\[
\tau_c = \frac{m^2}{k_R(T) \rho^2 (1 + \alpha)} 
\]

Since in equilibrium state the term in the square brackets on the right-hand side of Eqs. (54) and (55) must vanish, therefore,

\[
K_c(T) = \frac{2 \rho}{m} \frac{\alpha_e^2}{1 - \alpha_e} 
\]

and

\[
K_c(T_e) = \frac{\rho}{m} \frac{x_e^2}{1 - x_e} 
\]

where, \( \alpha_e \) (T, \( \rho \)) and \( x_e \) (T_e, \( \rho \)) are the equilibrium dissociation degree and ionization degree corresponding to a given temperature and density.
Therefore,

\[ U \frac{\partial \alpha}{\partial s} = \frac{1}{\tau_c} \left( \frac{\alpha_e^2}{1 - \alpha_e} (1 - \alpha) - \alpha^2 \right) \]  

\[ U \frac{\partial x}{\partial s} = \frac{1}{\tau_c} \left( \frac{x_e^2}{1 - x_e} (1 - x) - x^2 \right) \]  

Now assume that the deviation from equilibrium in (3f) is small.

Then

\[ \alpha - \alpha_e \ll \alpha \sim \varepsilon \]
\[ x - x_e \ll x \sim \varepsilon \]

where, \( \varepsilon \) is a small quantity. Substituting in Eq. 57 and neglecting second order terms in \( \varepsilon \), the following are obtained

\[ U \frac{\partial (\alpha - \alpha_e)}{\partial s} = -\frac{1}{\tau_R} (\alpha - \alpha_e) \]  

\[ U \frac{\partial (x - x_e)}{\partial s} = \frac{1}{\tau_R} (x - x_e) \]  

where new time scales for the diatomic and monatomic cases

\[ \tau_R = \frac{1 - \alpha}{\alpha(2 - \alpha)} \tau_c \]  

\[ = \frac{1 - x}{x(2 - x)} \tau_c \]  

have been introduced. These will be called 'characteristic relaxation times'. Alternatively, we can write

\[ \frac{\partial (\alpha - \alpha_e)}{\partial s} = \frac{-1}{1_R} (\alpha - \alpha_e) \]  

\[ \frac{\partial (x - x_e)}{\partial s} = \frac{-1}{1_R} (x - x_e) \]  

where \( 1_R = U \tau_R \)

and \( U \) is the frozen particle velocity behind the expansion wave.
The solution of Eqs. 61 and 62 are given by,

\[(\alpha - \alpha_e) = (\alpha - \alpha_e) \cdot e^{-\frac{t}{\tau_R}} \]  

(63)

\[(\alpha - \alpha_e) = (\alpha - \alpha_e) \cdot e^{-\frac{s}{l_R}} \]

(64)

and

\[(x - x_e) = (x - x_e) \cdot e^{-\frac{t}{\tau_R}} \]

\[(x - x_e) = (x - x_e) \cdot e^{-\frac{s}{l_R}} \]

Therefore for small changes from the initial state the dissociation or ionization degree as well as all other flow quantities approach their equilibrium values exponentially with the time constant \(\tau_R\) (see Fig. C5). Alternatively at the distance \(l_R\) from O all the physical quantities will have attained \((1 - 1/e)\) of their difference between their respective frozen and equilibrium values as shown on Fig. C5.

Numerical values of \(\tau_R\) and \(l_R\) for oxygen and argon for various initial conditions are listed in Tables C1 and C2, respectively.
Another characteristic length that can be defined in connection with the transition from frozen to equilibrium flow was suggested by Appleton (Ref. 6).

Fig. C6

Wave-Head Properties of a Prandtl-Meyer Flow in a Relaxing Gas

Along the wall streamline (a) in Fig. C6 the corner expansion starts at the first frozen Mach line which is inclined at an angle,

\[ \theta_{2f} = \cos^{-1} \left( \frac{1}{M_{2f}} \right) \]

On this line the angular derivative of density, \( (\partial \rho / \partial \theta) \), jumps from zero to a finite value \( (\partial \rho / \partial \theta)_{2f} \). The derivatives of all other flow quantities except \( \rho \) or \( x \) (which remains frozen) also jump discontinuously to finite values.

Along a streamline far off from the wall where the flow is in equilibrium, the changes in all quantities at (c) start at the head of the wave.
(H_e) which lies beyond the frozen wave head (H_f), as shown in Fig. C6. The equilibrium wave head is inclined at the angle $\theta_2^{e} = \cos^{-1} \left(1/M_2^{e}\right)$, and since $M_2^{e} > M_2^{f}$, $\theta_2^{e} > \theta_2^{f}$. Therefore at $\theta = \theta_2^{f}$, at point (b), the derivative \( \frac{\partial f}{\partial \theta} \) and derivatives of all other quantities are zero, since the flow undergoes no change until it reaches point (c) or H_e.

Between (a) and (b) \( \frac{\partial f}{\partial \theta} \) will decrease with the radial distance \( r \) from O.

Making use of the characteristic equations derived in subsection C. 3, it is possible to examine the behaviour of the thermodynamic variables at the frozen wave head. Referring to Fig. C7, take a point P on the frozen wave head, which is a straight line passing through O and inclined at an angle $\theta_2^{f}$ with respect to the vertical axis OY. Thus, if $\overline{OP} = r$, the Cartesian coordinates of P (x, y) are $(r \sin \theta_2^{f}, r \cos \theta_2^{f})$. Note that the frozen wave head (\( \xi_1 \)) is a characteristic curve on the \( \xi \)-family. Draw another characteristic curve of the \( \xi \)-family (\( \xi_2 \)) adjacent to the wave head. Draw the streamline (s_1) passing through P (a horizontal line since \( \nu = 0 \) at P), and let it meet \( \xi_2 \) at R. If $\overline{PR} = \delta s$, the Cartesian coordinates of R are $(r \sin \theta_2^{f} + \delta s, r \cos \theta_2^{f})$. Take another point on the wave head (Q) such that the $\eta$-characteristic passes through R and makes an angle \( (\pi - \theta_2^{f}) \) with OY. It is seen that the Cartesian coordinates of Q are $(r + \delta r \sin \theta_2^{f}, (r + \delta r) \cos \theta_2^{f})$, where $\delta r = \overline{PQ} = \delta s/2 \sin \theta_2^{f}$. Draw the streamline S_2 (horizontal) from Q to the point T on \( \xi_2 \). The Cartesian coordinates of T are found to be $((r + \delta r) \sin \theta_2^{f} + \delta s', (r + \delta r) \cos \theta_2^{f})$, where $\theta_2^{f} + \delta \theta$ is the angle RT makes with OY at R. Finally, draw the line (\( \eta_2 \)) parallel to QR.

Equation (CIV) is applied along RT to give

\[
\left(\nu_T - \nu_R\right) + C_R (p_T - p_R) = V_R \tag{65}
\]

where,

\[
\delta \xi = \overline{RT} = \frac{\delta s}{2 \sin \theta_2^{f}} \left(1 + \tan \theta_2^{f}\right) \tag{66}
\]

Apply (CV) along QR and TW,

\[
(\nu_R - \nu_Q) - C_Q (p_R - p_Q) = V_Q \tag{67}
\]

or

\[
\nu_R - C_2 (p_R - p_2) = 0 \tag{68}
\]

or

\[
\nu_T - C_2 (p_T - p_2) = 0 \tag{68}
\]
It should be noted that \( \nu_Q = \nu_W = \nu_2 = 0; \) \( p_Q = p_W = p_2; \) \( C_p = C_Q = C_W = C_2, \) \( V_p = V_Q = V_W = 0, \) since the flow is in equilibrium (\( W \) in Eq. 9 is zero).

Equations (65), (67) and (68) yield \[
\frac{2C_2}{V_p} = \nu \xi \]

(69)

Since \( V_p = 0, \) \( V_R \) can be obtained from Eq. 45 as follows,

\[
V_R - V_F = \nu \alpha \delta \alpha + \nu \delta p + V_T \delta T + V_U \delta U
\]

(70)

Apply (C1) along streamline PR; (P is in uniform state 2)

\[
\nu \delta \alpha = U(\alpha_R - \alpha_2) = W \delta s = 0
\]

(1 for equilibrium flow)

Also apply (CII) and (CIII) along PR;

\[
\int_2 U_2 (U_R - U_2) + (p_R - p_2) = 0
\]

or

\[
\delta U = (U_R - U_2) = \frac{1}{\int_2 U_2} (p_R - p_2)
\]

(72)

or

\[
\delta T = (T_R - T_2) = \frac{1}{A_2 \int_2} (p_R - p_2)
\]

(73)

These relations are to be substituted in Eq. 70.

Now let us denote the pressure gradient in a horizontal direction evaluated at the wave head by \( p'(r). \) This will be a decreasing function of the distance (r) from O.

Therefore along \( S_1, \)

\[
p_R - p_2 = p'(r) \delta s
\]

(74)

and along \( S_2, \)

\[
p_T - p_2 = p'(r + dr)
\]

(75)
Note that, \( \delta \theta = \theta_p \delta p + \theta_T \delta T + \theta_U \delta U + \theta_\alpha \delta \alpha \)

or \( \delta \theta = (\theta_p \frac{\theta_T}{\rho_2 A_2} - \frac{\theta_U}{\rho_2 U_2} + C_2 \theta_\nu) \delta S \)

and

\[ V_R = (V_p + \frac{V_T}{\rho_2 A_2} - \frac{V_U}{\rho_2 U_2}) \delta S \]

Write,

\[ p'(r + dr) = p'(r) + \frac{dp'}{dr} dr \]

Substitute in Eq. (69) for \( (p_R - p_T) \) from Eqs. (74) and (75) and for \( V_R \) and \( \delta \theta \). The values noted above, to obtain a differential equation for \( p'(r) \) of the form,

\[ \frac{dp'}{dr} = A_1 p' + A_2 p'^2 \]

where

\[ A_1 = \frac{\rho_2 U_2^2}{2\sqrt{M_2^2 - 1}} \left[ V_p + \frac{V_T}{\rho_2 A_2} - \frac{V_U}{\rho_2 U_2} \right] > 0 \]

\[ A_2 = -M \left( \theta_p + \frac{\theta_T}{\rho_2 A_2} - \frac{\theta_U}{\rho_2 U_2} + C_2 \theta_\nu \right) \]

The solution of Eq. 77 is,

\[ |p'| = \frac{A_1/A_2}{e^{A_1 r} - 1} \]

This gives the change of horizontal pressure gradient at the frozen wave head with the radial distance. It has a value \( p' = -\infty \) at \( r = 0 \) (at \( 0 \) where the change from \( p_2 \) to \( p_3 \) occurs in a vanishing distance) and decreases to 0 asymptotically

\[ r \rightarrow \infty \]

It is more convenient to rewrite Eq. (80) in terms of the angular pressure gradient,

\[ p_\theta = \frac{\partial p}{\partial \theta} \bigg|_{r = \text{const.}} \]
at the frozen wave head. To do this refer to Fig. C7, and obtain,

\[ p_\theta = \frac{r}{\cos \theta} \frac{p'}{2f} \]  

(81)

Substituting Eq. (80), obtain,

\[ |p_\theta| = \frac{1}{A_2 \cos \theta} \frac{A_1 r}{e^{A_1 r} - 1} = \frac{(p_\theta)^0}{e^{A_1 r} - 1} \]  

(82)

On the wave head, when \( r = 0 \), \( p_\theta \) assumes a finite limiting value, \( (p_\theta)^0 \), and decreases with the radial distance \( r \) such that \( r \to \infty, \ p_\theta \to 0 \), asymptotically.

Let,

\[ F(A_1 r) = \frac{A_1 r}{e^{A_1 r} - 1} \]

Then,

\[ F(0) = 1, \ F(0.1) = .952, \ F(1) = .583, \]

and \( F(10) = .0022 \) (see sketch)

Define,

\[ r_c = 1/A_1 \quad \text{(cm)} \]  

(83)

as the length along the wave head over which the attenuation of the pressure gradient due to recombination is occurring. This will be called the characteristic radial length for chemical recombination.

Since from Eq. (73) the temperature derivative is proportional to \( p_\theta \) and the density derivative also is a linear combination of the two by virtue of the equation of state, \( (\xi = \text{const}) \), their decay along the wave head follows the same law.

In the following the expression for \( r_c \) will be found for an ideal dissociating gas and an ionizing gas. All the quantities are evaluated at the wave head and the suffix (2) will be omitted throughout for simplicity.
i) Ideal Dissociating Gas

From subsection C.3,

\[ V = \frac{1}{M} \left( \frac{B}{AT} - \frac{1}{1+\alpha} \right) \frac{1}{l_c} \left[ \frac{m (1+\alpha) RT}{2p} K_c (1 - \alpha) - \alpha^2 \right] \]  \hspace{1cm} (45a)

where, the substitutions

\[ \frac{1}{\mathcal{P}} = \frac{(1 + \alpha) RT}{p}, \quad K_c = 2 \frac{D}{m} e^{-\frac{\theta_D}{T}} \quad l_c = U \tau_c \]

can be made. For the calculation of \( r_c \)

\[ V_p = \left( \frac{\partial V}{\partial p} \right)_{\alpha, T, U}, \quad V_T = \left( \frac{\partial V}{\partial T} \right)_{\alpha, p, U}, \quad \text{and} \]

\[ V_U = \left( \frac{\partial V}{\partial U} \right)_{\alpha, T, p} \]

are now required. The differentiations are simplified by the substitution arising from the fact that the square bracket in Eq. (45a) is zero at the wave head because state (2) is an equilibrium state. Therefore,

\[ V_p = \frac{1}{M} \left( \frac{B}{AT} - \frac{1}{1+\alpha} \right) \frac{1}{l_c} \left[ \frac{m (1+\alpha) RT}{2p} K_c (1 - \alpha) - \alpha^2 \right] \]  \hspace{1cm} (84)

\[ V_T = \frac{1}{M} \left( \frac{B}{AT} - \frac{1}{1+\alpha} \right) \frac{1}{l_c} \left[ \frac{m (1+\alpha) RT}{2p} K_c (1 - \alpha) - \alpha^2 \right] \]  \hspace{1cm} (85)

Note, \[ \frac{d}{dT} \left( TK_c \right) = TK_c \left( \frac{1}{T} + \frac{\theta_D}{T^2} \right) = K_c \left( 1 + \frac{\theta_D}{T} \right) \]

since \[ \ln \left( TK_c \right) = \ln T - \frac{\theta_D}{T} + \text{const.} \]

\[ V_U = 0 \]  \hspace{1cm} (86)
Therefore

\[
V_p + \frac{V_T}{\rho_A} - \frac{V_U}{\rho_U} = \frac{1}{M} \left( \frac{B}{AT} - \frac{1}{1 + \alpha} \right) \frac{\alpha^2}{l_c} \tag{87}
\]

and

\[
x \left( \frac{\theta_D/T + 1}{\rho(4 + \alpha) \text{RT}} - \frac{1}{\rho(1 + \alpha) \text{RT}} \right) = \frac{1}{\rho M \text{RT}} \frac{\alpha^2}{l_c} \left( \frac{\theta_D/T (1 + \alpha) - 3}{(4 + \alpha)(1 + \alpha)} \right) \tag{88}
\]

and, finally,

\[
r_c = \frac{1}{l_c} \frac{M^2 \frac{4 + \alpha}{3} \text{RT}}{2 \sqrt{M^2 - 1}} \left( \frac{V_p + \frac{V_T}{\rho_A} - \frac{V_U}{\rho_U}}{\rho} \right) \tag{89}
\]

It can be verified that this expression coincides with the one given by Appleton (Ref. 6).

Equation (89) derived above for an 'ideal dissociating gas' was used here for the calculation of \( r_c \) in view of the complexity of the more general formula. It is to be noted that \( r_c / l_c \) is of the order unity and appears like a reasonable result considering that the two quantities stem from the same process.

ii) Ionizing Gas

If the assumption is made that \( T = T_e \), then from subsection C3,

\[
V = \frac{1}{M} \left( \frac{B}{AT} - \frac{1}{1 + x} \right) \frac{1}{l_c} \left( \frac{m(1 + x) \text{RT}}{p} K_c (1 - x) - x^2 \right) \tag{90}
\]

Since the square bracket on the right-hand side of Eq. (90) is zero at the wave head, therefore,
\[
V_p = \frac{1}{M} \left( \frac{B}{AT} - \frac{1}{1 + x} \right) \frac{1}{l_c} \frac{x^2}{p} 
\]

\[
V_T = + \frac{1}{M} \left( \frac{B}{AT} - \frac{1}{1 + x} \right) \frac{1}{l_c} \frac{1}{T} \frac{d}{dT} \frac{(TK_c)}{TK_c} x^2 
\]

\[
= + \frac{1}{M} \left( \frac{B}{AT} - \frac{1}{1 + x} \right) \frac{1}{l_c} \frac{1}{T} \left( \frac{\theta_I}{T} + \frac{5}{2} \right) x^2 
\]

\[
V_U = 0 
\]

\[
and \quad V_p + V_T = \mathcal{A}A = \frac{1}{M} \frac{\theta_I}{T} \frac{x^2}{l_c} \left( \frac{\theta_I}{T} \frac{5}{2} (1 + x) \right)^2 
\]

\[
= \frac{1}{\rho MRT} \frac{x^2}{l_c} \left( \frac{\theta_I}{T} \frac{5}{2} (1 + x) \right)^2 
\]

\[
A_1 = \frac{M^2}{2} \frac{5}{3} (1 + x) RT \frac{x^2}{l_c} = \frac{4}{25} \frac{(\theta_I/T)^2}{(1 + x)^2} \frac{1}{\rho MRT} 
\]

\[
A_1 = \frac{2}{15} \frac{1}{\sqrt{1 - \frac{1}{M^2}}} \frac{x^2}{1 + x} \left( \frac{\theta_I}{T} \right)^2 \frac{1}{l_c} 
\]

Therefore, 

\[
r_c = \frac{15}{l_c} \frac{\sqrt{1 - \frac{1}{M^2}}}{2x^2} \left( \frac{T}{\theta_I} \right)^2 
\]

Again \( r_c / l_c \) is of the order unity and the previous remarks apply here as well.
C.7: Results

The characteristic relaxation length along the wall behind the expansion fan ($l_{R}$), as formulated in subsection C.5, was calculated for 15 combinations of shock Mach number and channel pressure for oxygen and 9 combinations for argon at a fixed deflection angle $\psi = 15$ degrees. The results are listed in Tables C1 and 2 together with the values of the characteristic chemical recombination length ($l_{c3}$). All the data are plotted in Figs. C9 & 11.

The characteristic radial length along the frozen wave head of the expansion fan ($r_{c}$), as formulated in subsection C6, was calculated for the same combinations of shock Mach number and channel pressure as for $l_{R}$ for both oxygen and argon. The results are listed in Tables C2 and 3 together with the values of the chemical length ($l_{c2}$), and are plotted in Figs. 10 & 12.

As expected, the characteristic lengths decrease with increasing channel pressure (roughly inversely proportional) and also decreased with increasing shock Mach number. Both parameters are indicative of increasing collision frequency or approach to equilibrium.

It is seen that, for oxygen, $l_{R}$ has a reasonable (<10 cm) experimental observation length only at higher channel pressures (>10 mm Hg). Also $r_{c}$ can be several times shorter than $l_{R}$, and has a reasonable dimension at channel pressures $p_{1}>1$ mm Hg.

For argon, on the other hand, $l_{R}$ and $r_{c}$ are of the same order of magnitude and are 1 to 1.5 orders of magnitude shorter than $l_{R}$ for oxygen at same shock Mach number and channel pressure. Thus both characteristic lengths are within laboratory dimension only at lower channel pressure ($\sim 1$ mm Hg).

These general tendencies can be explained as follows. The recombination rate constant ($k_{R}$) for oxygen as given by Heyman's formula (see Fig. C5) varies only slightly over the whole temperature range (regions (2) and (3), 2000 0 K to 4000 0 K). Since the chemical length ($l_{c}$) is proportional to $\rho^{-2}$ and $f_{3} \sim 0.5 f_{2}$ for the deflection angle $\psi = 15$ degrees, we have $l_{c3} \sim 4 l_{c2}$. This accounts for the magnitudes of $l_{R}$ and $r_{c}$ for oxygen.

The recombination rate constant ($k_{R}$) for argon as given by Petschek-Bray-Wilson's formula (see Fig. C5) is highly temperature-dependent. $k_{R3}$ behind the frozen expansion fan at the deflection angle $\psi = 15$ degrees ($T = 7000$ to 8000 0 K) is about 1 order of magnitude greater than $k_{R2}$ in front of it ($T = 10000$ to 14000 0 K). By the mutual canelling of this factor and the factor $\rho^{2}$, $l_{c2}$ and $l_{c3}$ are brought to the same order of magnitude.

It is worth comparing the orders of magnitudes of $l_{c2}$ for oxygen and the same quantity for argon at same shock Mach number and channel pressure. $k_{R}$ for argon is 3 orders of magnitude greater than that for oxygen; or $(1/k_{R})_{\text{argon}}$
However, (see Eqs. 9 and 19)

\[ (m^2)_{\text{oxygen}} \sim (m^2)_{\text{argon}} \]

and

\[ (\rho^2)_{\text{oxygen}} \sim 4 (\rho^2)_{\text{argon}}. \]

Besides, the factor \( x \) occurs in \( l_c \) for argon in place of \( (1 + \alpha) \) in \( l_c \) for oxygen, and

\[ (1 + \alpha)_{\text{oxygen}} \sim 15 (x)_{\text{argon}}. \]

Thus, roughly, the chemical length \( (l_c^2) \) for argon is only 1 order of magnitude shorter than that for oxygen in spite of vast differences in \( k_R \). This accounts for the relative orders of magnitudes of characteristic lengths for two gases.

Finally, it will be appropriate to quote some results from Appleton's numerical solution of the whole nonequilibrium flow field (Ref. 6). His initial state (2) is given by the conditions:

\[ p_2 = 0.21 \text{ atm,} \quad \rho_2 = 1.05 \times 10^{-2} \text{ NTP density} \]

\[ T_2 = 3540^\circ \text{K,} \quad \alpha_2 = 0.524, \quad U_2 = 2.9 \times 10^5 \text{ cm/sec,} \quad M_2 = 2.0. \]

Thermodynamically, this corresponds to the state generated by a combination of shock Mach number \( M_s = 15 \) and channel pressure \( p_1 = 0.5 \) mm Hg. But the flow velocity and flow Mach number are lower than the values for the shock tube flow.

He made the calculations for the wall deflection angles 10 degrees, 20 degrees and 30 degrees. Variation of dissociation degree \( \alpha \) along the wall was obtained (Fig. 9, Ref. 6), which exhibits an exponential approach to equilibrium. Appleton estimated the wall distance,

\[ x_w = (\alpha - \alpha_e) / (d \alpha / ds)_{\text{tip}} \]

which corresponds to our \( l_R \). Appleton's \( x_w \) for three values of \( \gamma \) are entered (rectangles) in Fig. C9 for comparison. Note that there the flow velocity is lower than the shock tube flow on which our calculation is based and that Matthew's value of \( k_R \) (assumed constant) is adopted.

Variation of density, pressure, etc. with angular position \( \theta \) at different radial distances are also shown (Figs. 1 to 6, Ref. 6). The value of \( r_c \) corresponding to this particular initial state can be estimated from there, and is plotted in Fig. 10 for comparison. The results of Ref. 6 appear to be in reasonable good agreement with the present calculations.
### TABLE C1

Characteristic Times and Lengths for Chemical Recombination

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Column:

1. Shock Mach number, $M_s$
2. Channel pressure, $p_1$ mm Hg
3. Recombination rate constant, $k_{R3}$ mole$^{-2}$ cm$^2$ sec$^{-1}$ x 10$^{-15}$
4. Characteristic chemical time, $\tau_{c3}$ microsec
5. Characteristic relaxation time, $\tau_{R3}$ microsec
6. Flow speed behind expansion wave, $U_{3f}$ cm/microsec
7. Characteristic chemical length, $l_{c3}$ cm
8. Characteristic relaxation length, $l_{R3}$ cm
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Column:

1. Shock Mach Number, $M_s$
2. Channel pressure, $p_1$ mm Hg
3. Recombination rate constant, $k_{R3}$ mole$^{-2}$ cc$^2$ sec$^{-1}$ x 10$^{-19}$
4. Characteristic chemical time, $\tau_{c3}$ microsec
5. Characteristic relaxation time, $\tau_{R3}$ microsec
6. Flow speed behind expansion wave, $U_{3f}$ cm/microsec
7. Characteristic chemical length, $l_{c3}$ cm
8. Characteristic relaxation length, $l_{R3}$ cm
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<td>3) Recombination rate constant, $k_{R2}$ mole$^{-2}$cc$^2$sec$^{-1}$x$10^{-15}$</td>
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<td>6) Characteristic radial length, $r_c$ cm</td>
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<td>7) Characteristic Radial Length</td>
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### TABLE C3

**Characteristic Radial Length**

Oxygen, $\psi = 15$ degrees

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<th>$M_s$</th>
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Column:

1. Shock Mach number, $M_s$
2. Channel pressure, $p_1$ mm Hg
3. Recombination rate constant $k_{R2}$ mole$^{-2}$cc$^2$sec$^{-1}$
4. Characteristic chemical time, $\tau_{c2}$ microsec
5. Characteristic chemical length, $l_{c2}$ cm
6. Characteristic radial length, $r_c$ cm
FIG. C 2: VARIATION OF THE RECOMBINATION RATE COEFFICIENT $k_R$ WITH TEMPERATURE ($T$) FOR OXYGEN AND ($T_e$) FOR ARGON

See FIG. C 3 for the detail of this portion.

1: Matthews
2: Appleton
3: Heyman
4: Byron
5: Reeves et al.
6: Wilson
A1: Bray (Petschek)

(3f) Temperature in state (3), frozen flow
(2) Temperature in state (2)

Temperature Range ($O_2$)  

Temperature Range ($A$)  

$10^{12}$  $10^{13}$  $10^{14}$  $10^{15}$  $10^{16}$  $10^{17}$  $10^{18}$  $10^{19}$  $10^{20}$

$0$  $2000$  $4000$  $6000$  $8000$  $10000$  $12000$  $14000$

$T$ or $T_e$ °K

$14000$  $2000$  $4000$  $6000$  $8000$  $10000$  $12000$  $14000$
FIG. C 3: ENLARGED VIEW OF $k_R$ VS. $T$ FOR OXYGEN

1: Matthews
2: Appleton
3: Heyman
4: Byron
5: Wilson
\[ F(T) = \left( \frac{T_{\text{exc}}}{T} + 2 \right) \exp \left( \frac{\theta_I - T_{\text{exc}}}{T} \right) \]

FIG. C4: VARIATION OF THE FUNCTION \( F(T) \)
FIG. C 5: DEFINITION OF THE CHARACTERISTIC RELAXATION TIME ($\tau_R$) OR LENGTH ($l_R$)
\[ P( r \sin \theta_{2f}, r \cos \theta_{2f} ) \]
\[ R( r \sin \theta_{2f} + \delta s, r \cos \theta_{2f} ) \]
\[ Q( (r + dr) \sin \theta_{2f}, (r + dr) \cos \theta_{2f} ) \]
\[ T( (r + dr) \sin \theta_{2f} + \delta s', (r + dr) \cos \theta_{2f} ) \]

\[ \delta r = \frac{\delta s}{2 \sin \theta_{2f}} = \delta \eta \]

\[ \delta s' = \delta s \left( 1 + \tan \theta_{2f} \frac{\delta \theta}{\sin 2\theta_{2f}} \right) \]

**FIG. C 7:** DEFINITIONS OF VARIOUS QUANTITIES AT THE WAVE HEAD OF A PRANDTL-MEYER FLOW IN A RELAXING GAS
FIG. C 8: VARIATION OF THE RATIOS OF RELAXATION TIMES AND LENGTHS WITH THE DEGREE OF DISSOCIATION IN OXYGEN AND THE DEGREE OF IONIZATION IN ARGON
FIG. C 9: CHARACTERISTIC RELAXATION LENGTH FOR CHEMICAL RECOMBINATION ($l_R$) VS. CHANNEL PRESSURE ($p_1$) FOR VARYING SHOCK MACH NO. ($M_s$) IN OXYGEN, $T_1 = 273^\circ K$, $\gamma = 15^\circ$
FIG. C 10: CHARACTERISTIC RADIAL LENGTH FOR CHEMICAL RECOMBINATION ($r_c$) VS. CHANNEL PRESSURE ($p_1$) FOR VARYING MACH NO. ($M_s$) FOR OXYGEN, $T = 273^\circ$K
FIG. C 11: CHARACTERISTIC RELAXATION LENGTH FOR CHEMICAL RECOMBINATION ($l_R$) VS. CHANNEL PRESSURE ($p_1$) FOR VARYING SHOCK MACH NO. ($M_s$) IN ARGON, $T_1 = 300^\circ$K, $\theta = 15^\circ$. 
FIG. C 12: CHARACTERISTIC RADIAL LENGTH FOR CHEMICAL RECOMBINATION ($r_c$) VS. CHANNEL PRESSURE ($P_1$) FOR VARYING SHOCK MACH NO. ($M_s$) IN ARGON, $T_1 = 300^\circ K$. 