A comparison of FBP and BPF reconstruction methods for circular X-ray tomography with off-center detector

Dirk Schäfer, Michael Grass
Philips Technologie GmbH Forschungslaboratorien,
Röntgenstrasse 24–26, D–22335 Hamburg, Germany.

Peter van de Haar
Philips Healthcare, Interventional X-ray Innovation,
Veenpluis 6, 5684 PC Best, The Netherlands.
Abstract

**Purpose:** Circular scanning with an off-center planar detector is an acquisition scheme that allows to save detector area while keeping a large field of view (FOV). Several filtered back-projection (FBP) algorithms have been proposed earlier. The purpose of this work is to present two newly developed back-projection filtration (BPF) variants and evaluate the image quality of these methods compared to the existing state-of-the-art FBP methods.

**Methods:** The first new BPF algorithm applies redundancy weighting of overlapping opposite projections before differentiation in a single projection. The second one uses the Katsevich-type differentiation involving two neighboring projections followed by redundancy weighting and back-projection. An averaging scheme is presented to mitigate streak artifacts inherent to circular BPF algorithms along the Hilbert filter lines in the off-center trans-axial slices of the reconstructions. The image quality is assessed visually on reconstructed slices of simulated and clinical data. Quantitative evaluation studies are performed with the Forbild head phantom by calculating root mean squared deviations to the voxelized phantom for different detector overlap settings and by investigating the noise resolution trade-off with a wire phantom in the full detector and off-center scenario.

**Results:** The noise-resolution behaviour of all off-center reconstruction methods corresponds to their full detector performance with the best resolution for the FDK based methods with the given imaging geometry. With respect to RMSD and visual inspection the proposed BPF with Katsevich-type differentiation outperforms all other methods for the smallest chosen detector overlap of about 15 mm. The best FBP method is the algorithm that is also based on the Katsevich-type differentiation and subsequent redundancy weighting. For wider overlap of about 40-50 mm these two algorithms produce similar results outperforming the other three methods. The clinical case with a detector overlap of about 17 mm confirms these results.

**Conclusion:** The BPF-type reconstructions with Katsevich differentiation are widely independent of the size of the detector overlap and give the best results with respect to RMSD and visual inspection for minimal detector overlap. The increased homogeneity will improve correct assessment of lesions in the entire field of view.

Keywords: computed tomography, large FOV, finite Hilbert filtering
I. INTRODUCTION

Circular X-ray tomography with off-center detector means that the detector is positioned asymmetrically in fan direction with respect to the central ray passing through the iso-center. The asymmetric positioning leads to truncated projections in fan direction and causes artifacts when using filtered back-projection (FBP) reconstruction algorithms. These artifacts can be reduced by projection extension with data measured at the opposite side of the circular scan. On a full 360° circular scan, the overlapping detector part measures redundant data in the central axial plane on opposite source positions. The concept of redundancy is extended as an approximation to higher cone-angles. Rays from opposite direction with same cone angle will be called complementary rays in the following.

Cho et al. adapted the well known FDK algorithm for off-center cone beam acquisitions and proposed to weight the complementary data in the overlap region before filtering similar to the Parker weighting for circular short scans. This method is called pre-weighting FDK in the following. The pre-weighting FDK method was reinvented by Wang and later generalized to helical scanning. Yu et al. applied the pre-weighting FBP concept to circular acquisitions with rebinning to parallel beam geometry and improved numerical properties. The methods described in this paper are purely focused on image reconstruction from off-center projections in native divergent geometry and methods based on rebinning to parallel beam geometry are not included.

Cho et al. also proposed a method for divergent geometry to approximate the truncated projection data from the views measured at the opposite sides of the circle before the filtering step in the FDK algorithm. The ramp filtering is applied to a much smoother profile around the projected rotational axis and reconstructions improve at higher cone angles. The weighting of complementary rays is applied after the filtering and only data corresponding to physical detector pixels are back-projected. Hence, this method is called post-weighting FDK in the following and proved to mitigate cone artifacts especially for small detector overlap. However, the weighting of complementary data in the FDK algorithm after the filtering is incorrect from a theoretical point of view and already Cho et al. reported slight shading artifacts, though they used almost symmetric objects. Zamyatin et al. adapted the concept of approximating truncated data by complementary rays for helical CT with off-center detector.

A more recently developed FBP algorithm has also been applied to the off-center detector scenario. Clackdoyle et al. investigated the fan-beam reconstruction algorithm proposed by Noo et al. in case of truncated data for region-of-interest reconstructions. This algorithm
has been generalized by Yu and Wang\textsuperscript{11} to the full detector cone-beam case and is the special
case of a Katsevich-type reconstruction\textsuperscript{12} for circular acquisitions. One important feature of
this FBP algorithm in divergent geometry is that complementary data is weighted after the
two-step filtering of differentiation and Hilbert filtering. The combination of the approxima-
tion of truncated data by complementary rays\textsuperscript{7} with a Katsevich-type FBP algorithm that
handles complementary data after filtering in a correct manner has been proposed for the
off-center problem simultaneously by Kunze and Dennerlein\textsuperscript{13} and Schäfer and Grass\textsuperscript{14}. The
redundancy problem is correctly handled with this algorithm but still approximated data is
filtered with a long range filter in the projection domain.

Back-projection filtration methods apply only a short range differentiation filter in the pro-
jection domain and perform the long range Hilbert filtering in the image domain. Leng
et al.\textsuperscript{15} applied a back-projection filtration (BPF) algorithm to the fan-beam case of an
asymmetric half-size detector and Li et al.\textsuperscript{16} presented a cone-beam variant of this concept.
In this paper, we present two new variants of BPF-type algorithms applied to off-center de-
tector acquisitions. The first one is based on the circular BPF algorithm with differentiation
in a single projection and redundancy weighting for short scan acquisitions by Yu et al.\textsuperscript{17}.
Here, it is modified for the off-center scenario and combined with the method of inverse
Hilbert filtering presented by You and Zeng\textsuperscript{18}. The second one uses the Katsevich-type dif-
f erentiation to calculate the differentiated back-projection (DBP)\textsuperscript{16,17} and combines it with
the inverse Hilbert filtering of You and Zeng\textsuperscript{18}. Additionally, we present a simple but effec-
tive method to mitigate the streak artifacts along the Hilbert filter lines in the reconstructed
volume inherent to circular BPF algorithms by averaging several reconstructions along dif-
ferent filter line directions. Reconstruction algorithms are presented, and the resulting image
quality is compared by means of reconstructions of simulated and clinical projection data.

II. RECONSTRUCTION METHODS

A schematic view of the acquisition geometry is shown in Fig. 1. The planar detector and
the X-ray source are rotated around the $y$-axis. The distance between source and detector is
given by $D$. The distance from the source to the rotation axis is denoted $R$, and $I$ represents
the iso-center of the imaging system. The circular orbit is parameterized by the path length
$\lambda \in \Lambda = [0, 2\pi R)$. The projected iso-center on the detector is located at $D(\lambda)$ and defines
the origin of the detector system. The normalized vector $\hat{d}(\lambda)$ points from $D(\lambda)$ to the
iso-center. The detector $v$-axis is parallel to the rotational axis. Accordingly, the $u$-axis is
parallel to the trajectory tangent vector with $u_{\text{min}} \leq u \leq u_{\text{max}}$. The cone beam projection
data is denoted by $X(u, v, \lambda)$:

$$X(u, v, \lambda) = \int_0^\infty f(S(\lambda) + l\hat{e}(u, v, \lambda)) dl,$$

(1)

where $\hat{e}(u, v, \lambda)$ is the unit vector from the source position $S(\lambda)$ to the detector element $E(u, v, \lambda)$. The corresponding length is denoted by $SE$. The flat detector is positioned in off-center geometry. The overlap region $O(\lambda) = \{(u, v) \in \mathbb{R}^2 | u_{-o} \leq u \leq u_{+o}, v_{min} \leq v \leq v_{max}\}$ is defined as the symmetric region around $D(\lambda)$ with measured projection values $X(u, v, \lambda)$. The width of the overlap region is $\Delta u = u_{+o} - u_{-o}$.

Following the idea first introduced by Cho et al., a redundancy weight $w(u)$ is introduced according to:

$$w(u) = \begin{cases} 0, & u_{min} \leq u < u_{-o} \\ \sin^2\left(\frac{\pi}{2}\frac{u - u_{-o}}{\Delta u}\right), & u_{-o} \leq u \leq u_{+o} \\ 1, & u_{+o} < u \leq u_{max}. \end{cases}$$

(2)

Five different reconstruction methods are used in this article and presented in the following.

A. Pre-weighting FDK: $F_{FBP1}$

Cho et al. have used a simple pre-weighting of the projection data with a redundancy weight similar to Eq. 2, followed by standard FDK reconstruction. The weighting function mitigates artifacts that are caused by ramp filtering across the truncated projection data. These pre-weighted projections are reconstructed according to the modified FDK formula given in Eq. 3:

$$F_{FBP1}(x) = \frac{D}{|x - S| \cdot \hat{d}|^2} \int_{-\infty}^\infty w(u') \frac{D}{SE(x)} X(u', v, \lambda) h_R(u - u') du'd\lambda$$

(3)

with: $h_R(\rho) = \int_{-\infty}^\infty |P| e^{j2\pi P} dP$.

The pre-weighting FDK is exact in the axial mid-plane, where filtering and redundancy are correctly handled. At higher cone-angles no really redundant data exist due to the divergent geometry. Combined with the approximative FDK reconstruction, this leads to significant artifacts especially for asymmetric objects.
B. Post-weighting FDK: $f^{FBP_2}$

Redundancy weighting after filtering combined with the FDK algorithm leads to a non-exact reconstruction algorithm in the axial mid-plane but mitigates the artifacts at higher cone angles compared to the pre-weighting FDK. First, the truncated projections from off-center geometry are re-binned to a complete projection data set using complementary rays. The fan angle $\alpha$ of a specific ray $u$ is given by $\alpha(u) = \arctan(u/D)$ and the source angle by $\beta = \lambda/R$. Rewriting the projection data with these coordinates gives $\tilde{X}(\alpha, v, \beta) = \mathcal{X}(\arctan(u/D), v, \lambda/R)$. The projections are extended with the complementary rays in the region $u_{\text{min}} \leq u < u_{\text{o}}$ with $u_{\text{min}} = -u_{\text{max}}$:

$$\tilde{X}_1(\alpha, v, \beta) = \tilde{X}(-\alpha, v, \beta + \pi \pm 2\alpha)$$

for $\alpha(u_{\text{min}}) \leq \alpha \leq \alpha(u_{\text{o}})$,

where the sign depends on the rotation direction. To guarantee a smooth transition of the extended data and the originally measured data, a faded additive offset correction is applied:

$$X_2(u, v, \lambda) = \begin{cases} 
  X_1(u, v, \lambda), & u_{\text{min}} \leq u < (u_{\text{o}} - \Delta) \\
  X_1(u, v, \lambda) + \delta \cos \left( \frac{u_{\text{o}} - u}{\Delta} \right), & (u_{\text{o}} - \Delta) \leq u \leq u_{\text{o}} \\
  X(u, v, \lambda), & u_{\text{o}} < u \leq u_{\text{max}},
\end{cases}$$

where $\delta = X(u_{\text{o}}, v, \lambda) - X_1(u_{\text{o}}, v, \lambda)$ defines the offset and the fading region is chosen as $\Delta = u_{\text{o}} - u_{\text{o}}$. The post-weighting FDK with extended projections as presented in Eq. 6 has been proposed by Cho et al.$^{1,7}$.

$$f^{FBP_2}(x) = \int \frac{D}{\Lambda} \frac{w(u)}{\|x - S\|} \int_{-\infty}^{\infty} \frac{D}{SE(x)} \mathcal{X}_2(u', v, \lambda) h_R(u - u') \, du' \, d\lambda. \quad (6)$$

C. Katsevich-type FBP: $f^{FBP_3}$

The problem of incorrect handling of redundancy with the post-weighting FDK method can be removed by using an algorithm that correctly applies redundancy weights after the filtering step$^{14}$:

$$f^{FBP_3}(x) = \frac{1}{2\pi} \int \frac{w(u)}{R - \|x - I\|} \int_{-\infty}^{\infty} \frac{D}{SE(x)} \mathcal{X}_2^{KD}(u', v, \lambda) h_H(u - u') \, du' \, (1/R) \, d\lambda, \quad (7)$$
with \( h_H(\rho) = -\int_{-\infty}^{\infty} i \, \text{sgn}(P) e^{i 2\pi \rho P} dP \).

\[
\mathcal{X}^{KP}_2(u, v, \lambda) = \left( \frac{\partial \mathcal{X}_2}{\partial \lambda} + \frac{\partial \mathcal{X}_2}{\partial u} \frac{\partial u}{\partial \lambda} + \frac{\partial \mathcal{X}_2}{\partial v} \frac{\partial v}{\partial \lambda} \right)
\]

is the Katsevich-type derivative along the source trajectory with fixed ray direction\(^{12}\). This derivative is computed using the blended chain rule for arbitrary detector orientations derived by Noo et al.\(^{19}\).

### D. BPF with differentiation in a single projection

A BPF algorithm with differentiation in a single projection for circular acquisitions has been proposed by Yu et. al\(^{17}\). Let \( \mathcal{L}(t, s, \hat{m}) \) be the line in direction of \( \hat{m} \) through \( s \). Then,

\[
x(t, s, \hat{m}) = s + t\hat{m}
\]

is a parametrization of the points on this line, with \( t \in (-\infty, \infty) \). For those lines \( \mathcal{L} \) intersecting the object support \( \Omega \), there is a finite interval \([t_{\text{min}}, t_{\text{max}}]\) corresponding to this intersection.

The differentiated back-projection (DBP) onto parallel lines \( \mathcal{L}(t, s, \hat{m}) \) is expressed as:

\[
l^{SD}(x(t, s, \hat{m}), \lambda_1, \lambda_2) = \int_{\lambda_1}^{\lambda_2} \frac{D^2}{|R - (x - I) \cdot d|^2} \cdot w_{\text{sig}}(\hat{m}, u, \lambda) \cdot \left( \frac{1}{R} \right) d\lambda
\]

where in the case of a full circular scan with offset detector \( \lambda_1 = 0 \) and \( \lambda_2 = 2\pi \), so the second term in eq. 9 cancels out. The redundancy weight \( \overline{w}(\lambda, u) = w(u) \) is given by the off-center weighting function from Eq. 2. The source positions \( \lambda(\hat{m}_1), \lambda(\hat{m}_2) \) correspond to the intersections of the line \( \mathcal{L}(t, s, \hat{m}) \) (projected in the central axial plane) and the source trajectory. The weight \( w_{\text{sig}} \) accounts for the change of sign when the back-projected ray
crosses the direction $\hat{m}$:

$$w_{\text{sig}}(\hat{m}, u, \lambda) = \begin{cases} +1, & \text{if } 0 \leq \cos(\hat{m} \cdot \hat{S}(\lambda)) \\
- \tan(u/D(\lambda)) < \pi, & \text{else.} \end{cases} \quad (10)$$

Finally, the object function $f^{SD}$ is obtained from the DBP $b^{SD}$ by finite inverse Hilbert transformation $\hat{H}^{18}$:

$$f^{SD}(x(t, s, \hat{m})) = \frac{1}{2\pi} \hat{H}[b^{SD}] \quad (11)$$

E. BPF with Katsevich-style differentiation

The DBP of the Katsevich-type differentiated projections $\mathcal{X}^{KD}$ onto parallel lines $\mathcal{L}(t, s, \hat{m})$ is expressed as$^{16,17}$:

$$b^{KD}(x(t, s, \hat{m}), \lambda_1, \lambda_2) = \int_{\lambda_1}^{\lambda_2} \frac{w(u) \cdot w_{\text{sig}}(\hat{m}, u, \lambda)}{|x - S(\lambda)|} \mathcal{X}^{KD}(u, v, \lambda) \frac{1}{R} d\lambda, \quad (12)$$

where $\mathcal{X}^{KD}$ is the Katsevich-type differentiation as in Eq. 7 but applied only to the measured detector pixels. Likewise, the object function $f^{KD}$ is obtained from the DBP $b^{KD}$ by finite inverse Hilbert transformation:

$$f^{KD}(x(t, s, \hat{m})) = \frac{1}{2\pi} \hat{H}[b^{KD}] \quad (13)$$

F. Finite inverse Hilbert transform of DBP

The DBP for a non-truncated full scan or minimum data schemes is equivalent to the Hilbert transform $\mathcal{H}_{\hat{m}}[f(x)]$ of the object function along a set of lines $\mathcal{L}(t, s, \hat{m})^{17,20}$. Short scan or off-center detector redundancies can also be incorporated in the BPF scheme$^{10,17,21}$.

$$\mathcal{H}[f](x(t, s, \hat{m})) = \int_{-\infty}^{\infty} \frac{1}{\pi(t - t')} f(x(t, s, \hat{m})) dt'$$

$$= \frac{1}{2\pi^2} b(x(t, s, \hat{m}), \lambda_1, \lambda_2). \quad (14)$$

The DBP $b$ (which can be either $b^{KD}$ or $b^{SD}$) has to be known along the line $\mathcal{L}(t, s, \hat{m})$ within an interval $[t_{L2}, t_{U2}]$, that is slightly larger than the support $\Omega$ of $f$ on this line, namely $[t_{\min}, t_{\max}]$, such that the following condition holds$^{18}$:

$$t_{L2} < t_{L1} < t_{\min} < t_{\max} < t_{U1} < t_{U2}. \quad (15)$$
In practice, it is sufficient to choose a spacing of two grid points from $t_{L2}$ to the intermediate point $t_{L1}$ and another two grid points to $t_{\text{min}}$ and respectively between $t_{U1}$, $t_{U2}$ and $t_{\text{max}}$. Then the object $f$ can be recovered by computing the finite inverse Hilbert Transform $\mathcal{H}^{18}$:

$$f(x(t, s, \hat{m})) = \frac{1}{2\pi} \mathcal{H}[b] = \frac{1}{2\pi^2 [k(t, t_{L2}, t_{U2}) - k(t, t_{L1}, t_{U1})]} \times \int_{t_{L2}}^{t_{U2}} \frac{dt'}{t' - t} \left[ k(t', t_{L2}, t_{U2}) - k(t', t_{L1}, t_{U1}) \right] \times b(x(t', s, \hat{m}), \lambda_1, \lambda_2)$$

with

$$k(t, t_{L}, t_{U}) = \begin{cases} \sqrt{(t - t_{L})(t_{U} - t)}, & \text{if } t_{L} < t < t_{U} \\ 0, & \text{else.} \end{cases}$$

The reconstruction is denoted $f^{SD}$ and $f^{KD}$ when using $b^{SD}$ and $b^{KD}$ for the DBP. Typically, the resulting reconstruction suffers from streak artifacts along the filter line direction in regions of sharp intensity transitions. Therefore, it is beneficial to reconstruct the same object $N$-times along different filter directions $\hat{m}_i$ and average the results to suppress the streak artifacts:

$$f_N(x) = \frac{1}{N} \sum_{i=1}^{N} f_i(x(t, s, \hat{m}_i)).$$

The object function is resampled at the final grid positions using tri-linear interpolation.

### III. RESULTS

Simulated projection data of the Forbild head phantom\textsuperscript{22} positioned offcenter ($x = 40$ mm) have been generated using 600 source positions equally sampled on 360 degree. The detector is symmetric with respect to the projected rotation axis and has a width of 756.6 mm and a height of 397.3 mm with $975 \times 512$ pixels. The distance from the source to the rotation axis is $R = 881.8$ mm and to the detector $D = 1325.1$ mm. The influence of the number of reconstructions with filter lines along different directions is shown for a full detector acquisition in Fig. 2. In the axial slice with an offset in y-direction of 26.5 mm strong streak artifacts in the horizontal filter direction can be observed for $f_1^{SD}$, i.e. when using only one reconstruction with a single filter line. In the central sagittal slice the streaks originating from the ear and the nose are visible. All these artifacts are significantly reduced by averaging reconstructions over 9 different filter line directions with a constant angular separation of 20 degree each.
A. Simulation with minimal detector overlap $u_{-\sigma} = -15.5 \text{ mm}$

The off-center detector acquisitions are generated by setting all projection data with $u < u_{-\sigma} = -15.5 \text{ mm}$ to zero. Reconstructions of the phantom using the three different FBP algorithms and the two BPF algorithms are shown in Figures 3,4 for the central axial and coronal slices. The central sagittal slices can be seen in the first column of Figure 5. The BPF reconstructions are averaged over 9 reconstructions with a constant angular separation of 20 degree for the filter line directions. In general, the BPF methods show less aliasing due to the interpolation in the averaging step of the reconstructions. The central axial slice is almost identical for all methods except for $f^{FBP2}$ using the incorrect combination of ramp filtering and redundancy weighting after filtering in divergent geometry leading to low frequent shading artifacts. The alternating artifacts for the FBP methods reported in Ref.14 for different objects can be retrieved in the central coronal and especially in the central sagittal slice (Fig. 5 left column). The best FBP method clearly is $f^{FBP3}$ with reduced artifacts at higher cone angles and without shading artifacts as observed for $f^{FBP2}$ (see Figs. 3-5).

Interestingly, the BPF reconstructions $f^{SD9}$ using the off-center detector redundancy weighting before the differentiation in a single projection similar to FBP 1 show the same type of artifacts at higher cone angles as $f^{FBP1}$ (see Fig. 5, row 1 and 4). This artifact is not present in the central sagittal slice of the full detector reconstruction $f^{SD9}$ (see Fig. 2). The best image quality is obtained using the BPF algorithm with Katsevich-type differentiation and off-center detector redundancy weighting after the differentiation, i.e. for $f^{KD9}$. No alternating artifacts are observed, neither in the central coronal nor in the sagittal plane. Only a slightly increased low intensity drop in the coronal plane at highest cone angles close to the calotte is visible compared to the FBP methods.

B. Simulations with varying detector overlap

Different detector overlaps are generated from the simulated projections of the shifted $(x = 40 \text{ mm})$ Forbild head phantom by setting $u_{+\sigma}$ to 15.5, 31.0, 46.6, 52.1, 77.6 and 93.1 mm, which corresponds to 20, 40, 60, 80, 100 and 120 pixels. All other parameters are identical to those used before. The central sagittal slices of the corresponding reconstructions for all five methods are shown in Figure 5. The central axial and coronal slices are omitted for the sake of brevity and artifacts are most prominent in this (sagittal) orientation as can be seen in the previous section (IIIA). The most important result is that the image quality
for BPF algorithm with Katsevich differentiation (BPF-KD) is almost independent of the overlap size. Only slight variations in the shape of the low intensity drop artifact at high cone angles are visible. The FBP with Katsevich differentiation (FBP 3) shows increasing image quality with increasing detector overlap until it reaches a constant quality at 52 mm overlap that is even superior to the BPF-KD method. The very strong artifacts of the algorithms with redundancy pre-weighting (FBP 1, BPF-SD) show improving image quality with increasing detector overlap, however even for 93 mm the quality level of BPF-KD or FBP 3 is not reached. The low frequent shading artifact of the redundancy post-weighting method FBP 2 is smoothed out with increasing overlap, but still significant with 93 mm overlap.

Line plots crossing the head phantom at relatively high cone angle are shown in Fig. 7 for two different detector overlaps. The line position is indicated in Fig. 6 by the red line. The strong asymmetric artifact for the pre-weighting methods (FBP 1, BPF-SD) extends to ±200 HU on this line for the minimal detector overlap of 15.5 mm and decreases for wider overlap as already discussed for the slice images in Fig. 5. The slightly stronger low intensity drop at high cone angles for BPF-KD (and BPF-SD) can also be recognized in this line plots as lower values on the bone voxels for both detector overlaps.

The line plot in Fig. 8 shows a cross section through the soft tissue contrast region at moderate cone angle as indicated by the green line in Fig. 6. The algorithms with differentiation in a single projection (FBP-1, FBP 2, BPF-SD) show in the line plots significant deviations as well, whereas the algorithms with Katsevich-type differentiation are very close to the the reference FDK reconstruction from full detector projections.

C. Evaluation of RMSD

The root-mean-squared-deviation (RMSD) of the algorithms under investigation is computed for a quantitative assessment of image quality. The RMSD is shown in Fig. 9 for all voxels in the FOV of the central sagittal slice. The RMSD for the full detector FDK reference reconstruction is shown as a line for comparison. The BPF-KD method shows the lowest RMSD of all methods for small detector overlaps and this value is quite constant also for wider overlaps. The other methods show decreasing RMSDs for wider overlaps, and the RMSD of the FBP methods falls below the RMSD of the BPF-KD algorithm, which is due to the stronger variations on the bone voxels especially at higher cone angles as can be seen in the line plots of Fig. 7.

To assess the spatial distribution of the RMSD in more detail, the RMSD has been computed
in the central sagittal slice for the object’s soft tissue voxels only (see Fig. 10). A soft tissue voxel mask is identified by those voxel in the object which are below 700 HU and above −1000 HU and then eroded by two voxels. The RMSD of the soft tissue voxels is significantly lower compared to the full FOV, and the BPF-KD algorithm has again the lowest RMSD for minimal detector overlap. This value stays again constant and below all other methods except FBP 3. The lower RMSD at higher detector overlaps for FBP 3 compared to BPF-KD is obviously due to the smaller low intensity drop at high cone angles for FBP 3 (see Fig. 5). The lower RMSD at higher detector overlaps for FBP 3 compared to the reference FDK reconstruction from full detector projections is probably due to the smoothing effect of the additional interpolation step involved in the Katsevich-type differentiation in FBP 3 compared to the differentiation in a single projection in FDK.

D. Noise versus spatial resolution

As further quantitative image metrics the spatial resolution and noise is determined in a simulation study using projections with Poisson noise. Silver wires ($\mu = 2.874 / \text{mm}$) with a radius of 0.2 mm and a length of 50 cm are placed parallel to the rotational axis in a water cylinder ($\mu = 0.0183 / \text{mm}$) of 30 cm diameter. The 17 wires are positioned at different radial offsets, eight wires at $r = 5$ cm, eight at $r = 10$ cm and one almost central at $r = 0.23$ cm. The projections are simulated using the same geometry as described in the beginning of Sec. III with an oversampling using 3x3 sub-pixels. Poisson noise is added to the projection data assuming a tube current of 100 mA and an acquisition period of 1 ms corresponding to an average flux of $6.74 \times 10^4$ photons per detector element. The projection data is smoothed with Gaussian filter kernels of different sizes ($\sigma = 0.1, 0.3, 0.5, 0.7, 1.0, 1.3, 1.6, 2.0, 2.5, 3.0$ pixels), and reconstructions are performed from the full detector and with a detector overlap of $u_{+o} = 15.5$ mm for each resulting projection data set. A transaxial slice through the reconstructed phantom from the full detector simulation with the Katsevich-type FBP is shown in Fig. 11 ($\sigma = 0.7$ pixels). The FDK and related methods use the Ram-Lak filter kernel. The differentiation in BPF-SD is calculated by a multiplication in Fourier space and the Katsevich-type differentiation along the source trajectory is realized by finite differences. Reconstructions with an isotropic voxel size of 0.25 mm are evaluated in the central axial plane ($y = 0 \text{ cm}$) and in an offcenter plane at ($y = 10 \text{ cm}$) to determine the modulation transfer function (MTF) and the noise level in the background. The MTF is calculated for every wire by projecting the measured point spread function (PSF) in one dimension (Radon transform), averaging the Radon transformed PSFs from several directions, computing the
discrete Fourier transform, and dividing by the frequency content of the wire. The MTF’s for all wires with the same radial offset are averaged and the 50% MTF value is chosen to represent the spatial resolution. The corresponding standard deviation of the 50% MTF values is calculated to represent their variation. The noise in the water background is measured as the standard deviation from the mean water HU value in a region of interest (ROI) of 100x100 pixels for every wire, while ignoring the central peak region of 40x40 pixels. This noise values are averaged for all eight wires of the same radial offset and their corresponding standard deviation is calculated. The noise resolution trade-off curves for the full detector acquisition and with a detector overlap of $u_o = 15.5$ mm are shown in Fig. 12 and Fig. 13, respectively. The error bars indicate a region of three standard deviations.

The best spatial resolution in the full detector study for the central axial plane and 5cm radial offset (Fig. 12a) is achieved for the FDK algorithm with 6.5 cycles/cm. The BPF-SD algorithm performs slightly worse due to the averaging of nine reconstructed volumes. The algorithms involving differentiation along the trajectory using two projections (Katsevich FBP and BPF-KD) show significantly reduced resolution caused by the much coarser sampling of source positions ($\approx 9$ mm) compared to the detector pixel sampling ($\approx 0.8$ mm). However, all methods show a similar noise resolution trade-off, i.e. the noise ($n$) increases with a similar power $a$ with increasing resolution ($r$): $n = cr^a$. The power $a$ is calculated from least square fitting of the corresponding linear regression problem: $\ln n = a \ln r + \ln c$.

Li et al. derive for high resolution kernels a lower bound for the power $a$ with $a \leq 2$ and report experimentally measured values on wires and beads of $2.05 \leq a \leq 3.3$. The lines connecting the data points in Figs. 12, 13 are the best fits in the above described sense. For the radial offset of 5 cm in the central axial plane, this power is with $a = 2.6 - 2.7$ well in the expected range.

For a radial offset of 10 cm the FDK resolution slightly increases to 6.6 cycles/cm due to the finer sampling at higher fan angles. However, the algorithms based on the Katsevich-type differentiation suffer from a significant resolution decrease due to the ”faster motion through the sinogram” for objects with higher radial offset. The resulting asymmetry in the PSF and decrease of resolution is also visible in the close up in Fig. 11 and was already reported by Dennerlein et. al. The results for the plane at $y = 10$ cm are almost identical to those in the central axial plane for a radial offset of 5 cm (see Fig. 12c) and also for a radial offset of 10 cm, which are omitted for the sake of brevity.

The results for the off-center detector simulation study in Fig. 13 correspond nicely to the full detector results. The two FDK based methods (FBP 1, FBP 2) are plotted directly onto each other. Therefore, the black line is missing and the data points are additionally marked.
by black circles. Also the other methods show the same behaviour for both radial and axial offsets, only with increased noise level, because the available projection data is reduced to almost the half.

In summary, the off-center detector acquisition does not change the noise-resolution behaviour of all presented methods.

E. Clinical case

Clinical projection data has been acquired using 720 source positions equally sampled on 360 degree. The detector is positioned asymmetrically with respect to the projected rotation axis and has a width of 397.3 mm and a height of 298.0 mm resampled to 512 × 396 pixels. The distance from the source to the rotation axis is $R = 880$ mm and to the detector $D = 1325$ mm. The short side of the detector determining the maximal overlap is located at $u_{-o} = -17.8$ mm. Reconstructions of a neck and shoulder case with all five methods of the central axial and sagittal slices are shown in Fig. 14 and the central coronal slices are shown in Fig. 15. No significant difference can be perceived in the central axial and sagittal slices, but the central coronal slice shows the same type of artifacts observed in the simulation study. The redundancy pre-weighting methods (FBP 1, BPF-SD) show a giant asymmetric artifact towards the lungs. The post-weighting FDK (FBP 2) and the Katsevich-type FBP 3 provide a mitigation of this artifact but homogeneous reconstruction is only achieved with the BPF-KD algorithm.

IV. SUMMARY

Two cone-beam BPF algorithms for off-center detector acquisitions have been presented and compared to three existing FBP algorithms. Those algorithms that apply the off-center detector redundancy weights after the filtering show less shading artifacts. This holds for FBP and BPF algorithms. The FBP algorithms using the redundancy weights after the filtering, however, need approximated (re-binned) projection data from the opposite side of the circular trajectory for the filtering step. This drawback is overcome using BPF methods, which apply only the short range differentiation on the projection and the long range Hilbert filter on the DBP volume, and hence do not use any re-binned or approximated projection data. Surprisingly, the BPF framework in the off-center detector setting is also sensitive to the order of differentiation and redundancy weighting. The off-center detector acquisition does not change the noise-resolution behaviour of all presented methods with
the best resolution for the FDK based methods with the given imaging geometry. The best
image quality for small detector overlaps in terms of RMSD and visual inspection is obtained
using the BPF algorithm with Katsevich-style differentiation (BPF-KD) for simulated and
clinical data. In the Forbild head simulation study, the Katsevich-style FBP (FBP3) gives
similar results to BPF-KD for a detector overlap of about 47 mm and outperforms all other
methods for even larger overlaps.

Averaging of several reconstructions using different filter lines mitigates the streak artifact
problem inherent to the BPF methods.

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22 http://www.imp.uni-erlangen.de/forbild/deutsch/results/head/head.html


Figures
FIG. 1: Geometry for off-center circular X-ray tomography.
FIG. 2: Reconstructions from full detector projections of shifted (x=40 mm) Forbild head phantom with 1,3 and 9 reconstructions along different filter line directions, Level/Window=0/100 HU,
FIG. 3: Central axial (y) slice: Reconstructions from off-center detector projections with $u_o = -15.5$ mm of shifted (x=40 mm) Forbild head phantom, Level/Window=0/100 HU.
FIG. 4: Central coronal (z) slice: Reconstructions from off-center detector projections with $u_o = -15.5$ mm of shifted (x=40 mm) Forbild head phantom, Level/Window=0/100 HU.
FIG. 5: Central sagittal (x) slice: Reconstructions of shifted (x=40 mm) Forbild head phantom with different detector overlaps, Level/Window=0/100 HU.
FIG. 6: Central sagittal slice of voxelized phantom. The red and the green line indicate the positions of the line plots shown in Fig. 7 and Fig. 8, respectively.
FIG. 7: Line plots through reconstructions with different detector overlaps of 15.5 mm (top) and 77.6 mm (bottom). The line position is indicated by the red line in Fig. 6.
FIG. 8: Line plots through soft tissue region of the phantom for detector overlap of 15.5 mm. The line position is indicated by the green line in Fig. 6.
FIG. 9: RMSD for all voxels inside FOV in the central sagittal slice.
FIG. 10: RMSD for soft-tissue voxels inside FOV in the central sagittal slice.
FIG. 11: Central transaxial slice of the spatial resolution phantom reconstructed using the Katsevich-type FBP from noisy full detector projections smoothed with a Gaussian kernel of $\sigma = 0.7\text{ pixel}$, Level/Window $= 1500/4000\text{ HU}$. 


FIG. 12: Noise resolution trade-off curves for reconstruction algorithms from simulated full detector acquisition.
FIG. 13: Noise resolution trade-off curves for reconstruction algorithms from simulated acquisition with detector overlap of $u+o = 15.5$ mm.
FIG. 14: Clinical data acquired with $u_\text{off} = -17.8$ mm overlap of the off-center detector, Level/Window = -50/600 HU.
FIG. 15: Clinical data acquired with $u_{-o} = -17.8$ mm overlap of the off-center detector, central coronal slice, Level/Window = -100/400 HU.