A REVIEW OF THE JET FLAP

by

G. K. Korbacher and K. Sridhar

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SUMMARY

A detailed review is presented of the theoretical and experimental advances that have been made in the study of pure jet flaps, jet control and jet-augmented flaps. Great care was taken in defining acting forces and in presenting all equations and graphs in a unified notation. Theories are correlated. Experimental results are quoted, illustrated by relevant graphs and compared with theory. Chapters on jet mixing and on the jet flap's implication on aircraft design are included. Another chapter attempts to assess the jet flap on the basis of its possible merits and de-merits. A list of future research projects is added. More than 150 references have a bearing on the material presented and another 50 in a separate list are related in subject.

It is hoped that this review will convey a well-rounded picture to the "student" of the jet flap and that it brings to attention gaps in knowledge and understanding due to lack of theoretical and experimental work.
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ϕ  Sink Strength
v₀ or u₀  Undisturbed free stream velocity
P₀  Undisturbed free stream static pressure
ρ₀  Undisturbed free stream density
v  Local flow velocity
P  Local flow pressure
ρ  Local flow density
v_J  Jet flow velocity
ρ_J  Jet flow density
Γ, γ  Circulation
Ma  Mach number
T  Static temperature
T_T  Total temperature
C_p  Specific heat at constant pressure
Re  Reynolds number
m  Mass flow (slugs/sec.)

m_J  Jet mass flow (slugs/sec.)
n  Ratio of mass flow rate of free air (n m_J) entrained in jet to original jet mass flow rate (m_J)

v₁  Free stream velocity at the position of mixing

B. GEOMETRIC PROPERTIES

c'  chord of analogous wing inclusive of flap

c  Chord of jet flapped wing = chord of analogous wing from L.E. to the beginning of the flap.
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<td>Xa.c.</td>
<td>Distance in chords of aerodynamic centre from L.E.</td>
</tr>
<tr>
<td>X</td>
<td>Distance in chords from L.E.</td>
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<tr>
<td>X_{L.C.}</td>
<td>Distance in chords of centre of total lift from L.E.</td>
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<tr>
<td>$\alpha$</td>
<td>Angle of incidence $\equiv$ angle between the free stream direction and zero lift line</td>
</tr>
<tr>
<td>$\alpha_g$</td>
<td>Angle of attack $\equiv$ angle between the free stream direction and the chord</td>
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<tr>
<td>$\alpha_{mL}$</td>
<td>Angle of maximum lift</td>
</tr>
<tr>
<td>$\eta$</td>
<td>Angle between chord and zero lift line ($=\alpha_g - \alpha$)</td>
</tr>
<tr>
<td>$\beta$</td>
<td>Angle between mechanical flap chord and wing chord</td>
</tr>
<tr>
<td>$\theta$</td>
<td>Jet deflection angle $\equiv$ angle between zero lift line and the jet direction at T.E.</td>
</tr>
<tr>
<td>$\tau$</td>
<td>Jet angle $\equiv$ angle between the free stream and jet direction at T.E. ($=\theta + \alpha$)</td>
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<td>$\alpha_i$</td>
<td>Induced angle of attack (see Sec. 6.3.2.)</td>
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<td>Downwash angle at infinity downstream</td>
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<tr>
<td>$\tau_\infty$</td>
<td>Jet angle at infinity (three-dimensional wing only)</td>
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<td>Tail setting</td>
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<td>$\delta$</td>
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<td>AR</td>
<td>Wing aspect ratio</td>
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<tr>
<td>$S_W$</td>
<td>Wing area</td>
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<td>$t$</td>
<td>Aerofoil thickness</td>
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<td>$h$</td>
<td>Distance of wing above ground</td>
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b, s  
Wing span

y  
Span coordinate

C. FORCES (see Fig. 9)

TH  
Engine thrust \( = m_J (v_J-v_o) \)

TH_c  
Engine thrust of conventional aircraft

TH_f  
Engine thrust of jet flapped aircraft

T_M  
Balance measured thrust

TH_d  
Force of deflected jet

J  
Jet momentum \( = m_J v_J \) per unit span

D_J  
Jet drag

T_R  
Jet reaction thrust

J_T  
Total jet momentum (three-dimensional)

L_R  
Jet reaction lift

L_T  
Total lift

L_Tc  
Circulation lift \( = L_T - J_T \sin \tau_\infty \)

D_i  
Induced drag as defined by Eq. (6-53)

\( \bar{D}_i \)  
Induced drag \( = D_i - J_T (1-\cos \tau_\infty) \)

D. COEFFICIENTS OF FORCES AND MOMENTS (see Fig. 9)

All forces and moments are subsequently used in their coefficient form, unless otherwise stated. The non-dimensionalizing divisor is \( 1/2 \rho_o v_o^2 S_W \).

\( C_J \) or \( C_\mu \)  
Jet momentum coefficient \( (m_J v_J/\rho_o S_W) \) or \( J_T/\rho_o S_W \)

\( C'_L \)  
Lift of wing without blowing

\( C'_D \)  
Profile drag \( = C'_DF + C'_DFR \) of wing without blowing

\( C'_DF \)  
Form drag of wing without blowing

\( C'_DFR \)  
Skin friction drag of wing without blowing
(xi)

\[ C'D_i \quad \text{Induced drag} \left( = \frac{C'_L^2}{\pi \cdot \text{AR} \cdot e} \right) \text{of wing without blowing} \]

\[ C'D \quad \text{Drag} \left( = C'D\text{DP} + C'D_i \right) \text{of wing without blowing} \]

\[ \Delta C'D_i \quad \text{Change in induced drag due to blowing} \left( = C'D_i - C'D_i' \right) \]

\[ C'D_i \quad \text{Induced drag of jet flapped wing} \]

\[ C'DJ \quad \text{Change in profile drag due to blowing} \left( \Delta C'DP = C'DP - C'DP' \right) \]

\[ C'DP \quad \text{Profile drag of jet flapped wing} \]

\[ C'DJ \quad \text{Jet drag due to mixing} \]

\[ C'DF \quad \text{Form drag of jet flapped wing} \]

\[ C'DFR \quad \text{Skin friction drag of jet flapped wing} \]

\[ C'DT \quad \text{Total drag of jet flapped wing} \]

\[ \Delta C'DT \quad \left( = C'DT - C'D \right) \]

\[ C'L_1; C'L_2; C'L_3 \quad \text{Flat plate loadings (see Sec. 6.6, Eq. 6-67 & 68)} \]

\[ C'LJ \quad \text{Jet induced pressure lift} \]

\[ C'L_P \quad \text{Pressure lift} \left( = C'L + C'LJ \right) \]

\[ C'L_R \quad \text{Jet reaction lift} \left( = C_J \cdot \sin \tau \right) \]

\[ C'L_T \quad \text{Total lift} \left( = C'L_P + C'L_R \right) \]

\[ \Delta C'L_T \quad \left( = C'L_T - C'L \right) \]

\[ C'TP_i \quad \text{Ideal jet induced pressure thrust} \left( = C_J - C'TR \right) \]

\[ C'TR \quad \text{Jet reaction thrust} \left( = C_J \cdot \cos \tau \right) \]

\[ C'TP \quad \text{Jet induced pressure thrust} \left( = C'TP_i - C'DT \right) \]

\[ C'TM \quad \text{Total thrust as obtained from pressure integration} \left( = C'TM + C'DFR \right) \]

\[ C'TM \quad \text{Total (measured) thrust as measured with a balance} \]

\[ \Delta C'TM \quad \text{Total thrust change on balance due to blowing} \left( = C'TM + C'D \right) \]
Pressure thrust as obtained from pressure integration
\( = C_{TP} + C_{DFR} \)

Pressure coefficient

\( = C_J - C_{D_1} \)

Lift efficiency \( = C_{LJ}/C_{LR} \) or lifting effectiveness

Lift gain factor \( = C_{LT}/C_{LR} \)

Thrust recovery \( = C_{TM}/C_J \)

Net thrust recovery \( = \frac{C_{TM} + C'D + \Delta C_{D_1}}{C_J} \)

Thrust gain factor \( = \frac{C_{TM} + C'D}{C_TR} \)

\( \frac{\Delta C_{LT}}{\sqrt{C_J^2 \cdot \sin \theta}} \)

Specific lift increase coefficient

Resultant force on wing due to blowing \( = \Delta C_{LT} \rightarrow C_{TM} \)

Thrust efficiency \( = \frac{C_{TP} + C'D}{C_J (1 - \cos \tau)} \)

Coefficient of the wing polar in the absence of blowing

Mass coefficient \( = \sqrt{C_J \cdot \delta / 2c} \)

Ratio of flap length to aerofoil chord (Sec. 6.4.2)

Total pitching moment coefficient about the A.C., L.E. and mid chord point respectively

E. ABBREVIATIONS USED

A/C Aircraft
A.C. Aerodynamic centre
L.C. Lift centre
L.E. Leading edge
T.E. Trailing edge
AR Aspect ratio
B. L Boundary layer
B. L. C. Boundary layer control
W. T. Wind tunnel
db Decibels
I. INTRODUCTION

The secret behind the versatility of birds in flight is the ability to change their body geometry at will. These changes comprise the position and shape of their wings, span, area, incidence, dihedral, sweepback and even the wing profile to meet any momentary need. Some birds use also their tails in a similarly versatile way and others, in addition, even their feet. All these geometry changes can be made differentially, if required. In other words, the birds flight superiority over the aeroplane is based upon their built-in efficient auto-stabilization system. It enables them to fly with configurations which are inherently unstable, configurations which aircraft designers hardly dared to suggest so far. No doubt no aeroplane could ever be expected to vary its basic geometry even to the extent which the least sophisticated of birds employs.

Variable geometry in the flight of birds or aeroplanes is essential for two reasons:

a) for instantaneous thrust and/or lift adjustments and

b) for stability and control.

In this review, we primarily deal with point a) and case b) is mentioned only as far as stability and control changes are a consequence of a).

An instantaneous thrust and/or lift adjustment—which birds are able to perform—requires the use of a single force, entirely or partially, as either a lift and/or a thrust, whatever the momentary requirement may dictate. Of course this principle has been known for centuries but practical means for its materialization were not found. VTOL and STOL aircraft were the closest answers until, in 1952, H. Constant (Director of N.G. T.E.) suggested the "jet flapped wing" and I.M. Davidson and B.S. Stratford proved its potentialities.

Constant's concept of the jet flapped wing was "a wing with the jet engine exhaust jet issuing from a full span slot along the wing's trailing edge". In this way the high momentum engine exhaust can be used to provide lift as well as thrust. The jet flap potentialities, laid down in the so-called thrust and lift hypotheses, are striking and responsible for intensive jet flap research work, initiated along the following lines:

a) to check the theoretical performance predictions (hypotheses) for the jet flapped wing experimentally.

b) research into supercirculation, the phenomenon underlying the jet flap concept.
c) research into differential jet deflection by means of shrouds (N. G. T. E.) or jet control flaps (O. N. E. R. A.) making use of what is known as the COANDA-effect.

d) research into the benefits of the supercirculation principle if used with the existing hardware (conventional jet engines and aircraft). A vast number of reports resulted from research on all kinds of flaps such as conventional mechanical flaps, slotted flaps, free streamline flaps or either internal or external flow jet augmented flaps. Even possible combinations of the jet flap with the slip stream deflection principles were investigated. In all these cases, wherever the jet exhaust or bleed off air from jet engines is used for jet augmentation, it is done with as little as possible alteration to their conventional way of usage.

e) research into the optimum benefits which the jet flap principle may provide in connection with hardware which is especially designed for its most effective application (integration of the aircraft's propulsive and lifting systems - propulsive wing).

The common entity in all the above fields of research is a jet, the jet of either free stream air (e.g., propeller slip stream), bleed off or secondary air, or of the engine exhaust. Its characteristic parameter is the so-called jet coefficient, being primarily a function of the jet's momentum. Wherever a circular air or exhaust jet or a jet sheet is used to augment lift either direct or indirect (induced forces), the magnitude of the jet coefficient determines whether this lift increase is a consequence of boundary-layer control or of circulation control (hypersustentation). Whereas B. L. C. in its original and concise concept was thought of as a means of raising the lift coefficients of a wing to the theoretically predicted values (natural circulation - KUTTA condition), circulation control (supercirculation, on which the jet flap principle is based) starts where B. L. C. ends and improves the wing's lift performance far above that of the ideal conventional wing.

It is obviously difficult in practice to draw meaningful lines between B. L. C., circulation control and supercirculation. This is the reason for the extended scope of this "jet flap" review. Besides, particularly on this continent, the word jet flap is used in a much wider sense than originally coined by its inventors.

II. THE JET FLAP AND ITS PRINCIPLE

2.1 General Remarks

In Fig. 1 are shown various flap configurations and the names by which they will be referred to in this review. Accordingly, in jet flap configurations, the conventional mechanical flap is replaced
by a jet sheet which, for differential jet deflection control may be combined with a small shroud (N.G.T.E.) or jet control (i.e., deflecting) flap (O.N.E.R.A.). In view of Constant's suggestion to use the exhaust jet of gas turbines and from the evident jet sheet-mechanical flap analogy, the term "JET FLAP" originated.

To give some idea of how, in principle, flaps effect the lift characteristics of wings, Fig. 2 is added. It shows the improvement in lift due to various wing-flap configurations with blowing or suction as compared with the lift curve of a conventional wing.

Shrouds or jet control flaps are used not only for the purpose of jet deflection, but also for lift increase. If the energy available for blowing is small, blowing at the T.E. does not pay dividends unless being used in combination with a mechanical flap and its lift increasing effect. The smaller the available energy for blowing, the larger should be the chord of the flap and as a limit, the conventional B.L.C. by means of blowing tangentially over the flap would result.

2.2 The Jet Flap

Originally, the pure jet flap (Figs. 1E and 3) meant just a thin full span jet sheet which is obtained if air of high momentum or if the entire propulsive jet of a jet engine is ejected from a narrow spanwise slot, located at the T.E. of a wing. The angle $\theta$ under which the jet is issuing with respect to the wing's zero lift line can be any angle, but is unalterable during flight.

An improved version of the fixed angle jet flap is the variable angle jet flap, which employs (see Fig. 1F) a hinged jet shroud or a small flap for the purpose of altering the jet deflection angle $\theta$ during operation either on the ground or in flight. The jet sheet easily follows the shroud or control flap through angle changes of $90^\circ$ and greater by what is commonly known as the "COANDA EFFECT" (Refs. 13, 85, 86 and 10*). As the jet control flap combines the aerodynamic characteristics of the mechanical flap with that of the pure jet flap, their combination can be shown by idealized theory (Ref. 93) to be superior to the pure jet flap in lift augmentation. However, in practice, it is not so for $\text{CJ} > 1$. Still, it is considered so far as the most promising suggestion for the practical application of the jet flap principle to current aircraft design. In addition, it prevents the complete loss of lift control in the case of a failure of the jet flap blowing system.

Figure 4 illustrates a jet control flap and its lift coefficient versus jet coefficient curve. It further shows where B.L. control ends and circulation control begins. The curve A'B represents the optimum lift of the jet flapped wing in an ideal fluid flow and AB is its experimentally obtained counterpart in viscous fluid flow. The difference between both
curves therefore indicates the optimum gain in total lift that can be achieved by B.L. control. This gain in lift is primarily due to energizing the B.L. sufficiently to prevent separation. Curve BC illustrates, how the total lift can be further enhanced by increasing $C_J$ beyond the value at which an attached B.L. is ensured. Now, blowing produces a modification of the circulation around the wing. As this circulation is greater than the optimum "natural" circulation obtained by B.L. control, it is called "supercirculation". In idealized theory, this supercirculation increases ad infinitum with $C_J$. In practice, however, it is limited by flow separation at the L.E. of the wing.

There is something else that is clearly illustrated in Fig. 4. Consider point C, which is at four times the $C_J$-value of point B. The increase in $C_{LT}$, however, is only $0.55 \cdot (\Delta C_{LT})$, the increase in total lift due to B.L. C. This demonstrates that supercirculation lift necessitates significant quantities of energy, which can reasonably be provided only if the propulsive (exhaust) jet is used and integrated with the wing; whence the idea of a propulsive wing, the lift of which - at constant $C_J$ - can be altered instantly to meet any momentary requirements, just by varying the jet deflection angle.

2.3 Its Effect on the Flow Field

The primary effect of the deflected jet sheet on the flow around a jet-flapped aerofoil is shown - oversimplified - in Fig. 5, which is a transcription of an actual streamline photograph (see Ref. 12, Fig. 14C). This flow configuration, characteristic for the jet flap, is fundamentally different from that observed with the classic high lift flap or other conventional high lift devices. Their effectiveness is based on keeping the flow attached to the upper wing surface whereas with the jet flap, the fluid from the main stream flow below the wing plane, approaching the wing from infinity upstream forms a separation bubble by being forced over the upper wing surface, thereby inducing higher velocities and lower pressures on the wing top, and lower velocities with higher pressures on the bottom wing surface. It is the highly asymmetric flow (see Fig. 5) that produces those induced pressure forces on the aerofoil in both the thrust and lift direction which are primarily responsible for the merits - high pressure lift and thrust - of the jet flap. These merits are laid down in the so-called lift and thrust hypothesis which are discussed in Sec. IV.

Figure 6 - also confirmed by Fig. 8 of Ref. 33 and Fig. 3 of Ref. 26* - shows the flow field in the vicinity of the wing's T.E. The main stream flow enters the jet sheet near the T.E. under angles up to and even greater than 90°.
2.4 Its Analogy with the Mechanical Flap

In its effect (see Sec. 2.2) the jet sheet can be compared with either the ZAPP or the FOWLER FLAP depending upon whether the pure or the shrouded jet flap respectively is considered.

Although the curved jet sheet extends downstream to infinity, only its initial almost straight portion was originally (for simplification) considered to affect the flow around the wing (Ref. 11) and to produce the same pressure lift as would some specific straight mechanical flap of finite extent (Fig. 7). Later, however, an attempt was made (Ref. 53) to improve the straight mechanical flap analogy by introducing a curved mechanical flap instead (see Sec. 6.3).

It is this analogy with the straight mechanical flap that, besides providing a basis for an early theoretical treatment of the jet flap problem led also to its descriptive name.

2.5 The Jet Flap Principle

The principle of the jet flap is to create jet-induced pressure lift (supercirculation) by means of a high-momentum jet sheet.

Although the idea of using the entire propulsive jet for the production of a high-momentum jet sheet (i.e., of a high jet coefficient) was new, the method of blowing a portion of the engine working fluid (secondary air) over the top surface of a trailing edge flap was known and already under experimental investigation. But the discovery of supercirculation in principle stems from still earlier tests in connection with B.L.C. However, its potentialities at high $C_f$-values were not realized then. This was due mainly to two reasons:

1) that originally the supercirculation principle was discovered only as a by-product of B.L. control investigations and

2) that the whole idea of blowing secondary air over the top surface of a trailing edge flap was considered uneconomical especially for higher jet coefficients as required for the production of supercirculation.

However, with jet propulsion a reality this side line discovery should have become simultaneously - at least in theory - the hit which it became in 1952, when H. Constant suggested the highly powerful propulsive jet as a natural free energy source for the required high momentum jet sheets. It was this idea which provided the first practical solution to the supercirculation principle and it pointed the way to an airframe-engine integration.
III. HISTORICAL SKETCH OF THE JET FLAP IDEA

As early as 1917, Föttinger (Ref. 1) suggested boundary layer control by blowing a tangential jet sheet over the upper leading edge of a mechanical flap as a means of preventing flow separation. Ten years later Seewald (Ref. 22) and Wieland (Ref. 47) investigated Föttinger's idea. It was, however, not before 1931 that his suggestion, tested by Bamber (Ref. 2) proved its predicted beneficial effect. Many more boundary layer control experiments - in principle based on Föttinger's concept - followed, wherein gradually the momentum of the blown tangential jet sheet was increased.

In 1933, Schubauer (Ref. 3) thought of using a blown flap as a possible means of thrust augmentation. His experiments could have led to the discovery of the induced pressure forces in thrust direction (thrust hypothesis) if he had used blown flaps of higher momentum i.e., larger thrust values. Still, his lately re-evaluated test results may be considered as the first characteristic jet-flap results measured. Some years later (1939) Hagedorn and Ruden (Ref. 4) in the course of boundary layer experiments with blown flaps of higher than conventional strength discovered and correctly analysed the supercirculation principle. Their characteristic jet flap results are shown in Fig. 8.

In 1942, this principle was rediscovered independently by Valenci, Parigi and Borgel (Ref. 5) and more relevant work was published in 1948 by Poisson-Quinton (Ref. 6). Further work which may be quoted here is that of Lyon, Barnes and Adamson in 1941 (Ref. 7), Mettam in 1951 (Ref. 8) and Jousserandot in 1954 (Ref. 9). But none of these researchers realized the potentialities and possible future importance of the supercirculation principle. They more or less referred to it only as a scientific curiosity.

Finally in 1952 an "engine man" H. Constant (Ref. 10), had the crucial idea which turned this scientific curiosity into a vital principle of unforeseen significance. In Ref. 11, Constant's contribution as referred to by Stratford is: "when Constant questioned whether a gain might be achieved by using the ordinary but still highly powerful propulsive jet to induce an external flow that would lift the aircraft. This first suggestion was for boundary layer control by the injection of some of the jet stream flow over a mechanical flap, thus introducing the idea of a two-dimensional propulsive jet being issued from a slot nozzle along the trailing edge of the wing. The prospects looked interesting, but they soon became exciting when the mechanical flap was removed and the whole propulsive jet was brought into play". In Ref. 12, this episode is described by Davidson as: "that (crucial) step was the observation by Constant that, the propulsive jet of a modern aircraft being a very powerful physical entity should be one hundred per cent combined with the wing in flight near the ground".
With to-days knowledge of the jet flap, the jet-flap idea could be best described as: "the complete integration of the propulsive system of an aircraft with its lifting system". If this idea could - in practice - be perfected, man's dream of old "to fly like the birds" may become true after all.

In the U.S.A., the work on the jet flap has reached the point where research aircraft equipped with jet flaps are being considered (Ref. 95). In England and France, actual flight tests of jet flap aircraft have started already or can be expected in the near future.

IV. THE JET FLAP HYPOTHESES

The jet flap hypotheses concern both the total thrust force and the total lift force acting on a jet flapped wing. Their "too good to be true" predictions - now supported in principle also by substantial experimental evidence - were responsible for the keen interest the jet flap was met with all over the world when it was introduced early in 1956 (Ref. 11 and 12).

These hypotheses are based on the following assumptions:

1. the working fluids are ideal fluid i.e.,
   a) there is no mixing between the main stream and the jet stream flow
   b) the profile drag (C_{DP}, see Fig. 9) is zero

2. the jet flapped wing is two dimensional i.e., the induced drag (C_{D1}) is zero.

For the above assumptions it can be shown that the jet sheet issuing from the wing T.E. at any arbitrary jet deflection angle (θ) will be curved by the action of the main stream flow until it becomes finally parallel to it at downstream infinity. Otherwise the entire flow field would eventually deviate from the horizontal and possess an infinite vertical momentum.

4.1 The Thrust Hypothesis

4.1.1 Its Statement

In Ref. 12, Davidson quotes the thrust hypothesis as:

"In an idealized two-dimensional jet flap system the gross thrust (total thrust C_{TM} in our notation, see Fig. 9) is equal to the total jet reaction (jet momentum J or C_{J}) independent of the angle of deflection of the jet".
and Stratford in Ref. 11 states it as:

"Provided that the main stream speed is sufficient to prevent separation near the leading edge, the total forward thrust (total thrust CTM) on the aerofoil will be almost independent of the deflection of the jet".

Independent French work on the jet flap in Ref. 33 refers to the thrust hypothesis as:

"Calculations indicate the existence of a propulsive component (CTM) equal to the impulse of the jet (CTM = CJ), independent of the jet orientation θ...........".

or

"It can be shown (Refs. 12 & 34) that, in the ideal case where there is no parasitic loss of momentum (on the profile or in the diffusion of the jet), the total momentum of the jet may be retained in the form of thrust, whatever its orientation may be".

4.1.2 Arguments for its Justification

With the simplifying assumptions of Sec. IV, semi-theoretical justifications of the thrust hypothesis can quite easily be supplied, provided that the calculation of the resultant force of the flow on the wing does not examine the precise mechanism of force transfer.

In Ref. 11, four arguments are presented of which two will be discussed below but only one, the control volume (momentum box) approach in detail.

Further, Ref. 35 should be consulted in which "it is rigorously proved that in subsonic compressible flow the ideal thrust of the jet (assumed not to mix with the main stream) is independent of the exit angle" (see Sec. 4.1.3).

Finally Ref. 42 has to be mentioned here. If a two-dimensional jet flapped wing is tested in a wind tunnel instead of in free air, the measured thrust has to be corrected for the blockage effect. The appropriate correction equation (Eq. 21 of Ref. 42) shows the independence of the thrust and jet deflection angle.

A. The Control Volume Approach

Imagine a large control volume around the jet flapped aerofoil as shown in Fig. 10A. If this box is large enough it can be assumed that:
1) the jet mass flow can be neglected in comparison with the main stream mass flow

2) the pressure forces acting on the boundaries of the momentum box cancel each other.

Without friction the momentum flux of the jet sheet leaving the control volume is still the same as that at the wing trailing edge. Newton's second law applied now to this momentum box states simply that the aerofoil - the only solid body in it - must experience a thrust force which is equal but opposite in direction to the momentum flux of the jet, \( J \).

B. The Streamline - Solid Boundary Approach

The second argument for the justification of the thrust hypothesis - offered by Stratford - is based on replacing the jet sheet boundaries (a stream surface) by very thin rigid surfaces, which are made integrated parts with the wing (Fig. 10B). Assuming the main stream flow to be an ideal fluid, no external drag will be exerted on this wing configuration. The reaction thrust (see Fig. 9) must therefore be equal to \( J \).

Mathematically, these streamline surfaces could be treated as vortex sheets and, if their local strength be \( \gamma_1 \) and \( \gamma_2 \), the jet main stream equilibrium criterion would be (Ref. 12)

\[
\gamma_1 - \gamma_2 = \frac{J}{\rho_0 \cdot v_0 \cdot R}
\]

where \( R \) is the local mean radius of curvature of the stream surface.

When the solution of this equation is used in place of the Kutta-Joukowski hypothesis, Stratford's thrust hypothesis follows.

In the French work on the jet flap the proof given for the thrust hypothesis is quoted in Ref. 33 as: "The application of the momentum theorem and the calculation - according to Joukowski's law - of the effect that the external flow exercises on the profile contour extended by the jet, which is assumed to form a solid extension of the profile in the region where it presents an appreciable curvature, indicate ..........." that \( C_{TM} = C_J \) also if \( \theta \neq 0 \).

4.1.3 Momentum Theorem Approaches (Refs. 35 and 59)

The control volume approach (Sec. 4.1.2) is based on the assumption that the pressure forces acting on the boundaries of the control volume cancel each other. The following two references prove rigorously that the thrust hypothesis is true for any arbitrary control
surface, for which the pressure forces on its boundaries not necessarily are cancelling one another. But both approaches are again based on the assumption that the jet sheet will be deflected by the main stream flow until it becomes parallel to it at downstream infinity.

Woods (Ref. 35) considers a two-dimensional aerofoil with a jet leaving the trailing edge at some angle. An expression for the (ideal) thrust is derived assuming that there is no flow separation and that the jet is an irrotational stream separated from the main stream by two vortex sheets (see Fig. 11A). The mass flow in the infinite channel (jet) is constant, but the momentum flux is increased within the aerofoil. Across $A_\infty$ $B$ and $A'_\infty$ $B'$ pressure and velocity are continuous while across $CD_\infty$ and $C'D'_\infty$ only pressure is continuous (but not velocity).

We can write for the thrust acting on the two-dimensional aerofoil (Fig. 11).

$$TH = \int_{BFC} P \sin \phi \cdot ds - \int_{B'F'C'} P \sin \phi \cdot ds + \int_{B'E'C'} P \sin \phi \cdot ds - \int_{BEC} P \sin \phi \cdot ds \quad (4-1)$$

where $P$ is the pressure, $s$ is the distance and $\phi$ is the local angle between the aerofoil surface and the free stream direction.

Applying Euler's momentum theorem to the jet channel, we get:

$$\int_{A_\infty BFCD_\infty} P \sin \phi \cdot ds - \int_{A'_\infty B'F'C'D'_\infty} P \sin \phi \cdot ds + \delta' (P_0 + P_0 \cdot v^2) - \delta (P_0 + P_0 \cdot v_J^2) = 0 \quad (4-2)$$

where $\delta'$ and $\delta$ are widths of jet at $A_\infty$ $A'_\infty$ and $D_\infty$ $D'_\infty$ respectively; $v$ and $v_J$ are velocities of main stream and jet respectively.

It can also be shown (Ref. 35) for regions outside the jet that:

$$\int_{A_\infty BECD_\infty} P \sin \phi \cdot ds - \int_{A'_\infty B'E'C'D'_\infty} P \sin \phi \cdot ds = P (\delta - \delta') \quad (4-3)$$

Subtracting Eq. (4-2) from Eq. (4-3) we get

$$TH = \rho_o (\delta \cdot v_J^2 - \delta' \cdot v^2) \quad (4-4)$$

From continuity of mass follows that $\rho_o \cdot \delta v_J = \rho_o \cdot \delta' v$. Using this relationship in Eq. (4-4) and reducing this equation to a non-dimensional form, one obtains

$$C_T = C_J - 2 C_q \quad (4-5)$$
Eqs. (4-4) and (4-5) show that the thrust is independent of the jet deflection angle. In the case (of wind-tunnel tests), where the jet is derived from a source within the aerofoil, \( C_q = 0 \) and therefore

\[
C_T = C_J \tag{4-6}
\]

Next we consider Maskell's and Gates' (Ref. 59) derivation of the expression for thrust who apply the Momentum Theorem to the two-dimensional jet-flapped wing (with a viscous jet). The control surface used is shown in Fig. 11B. The force in the \( x \)-direction, the thrust acting on the wing, is given by:

\[
T_H = - \int_{S_0 + S_J} P \, dy + \int_{S_0 + S_J} \rho \cdot u \cdot (v \, dx - u \, dy) \tag{4-7}
\]

where \( S_J \) is the part of the control surface covering the portion of the jet, \( S_0 \) the part covering the main flow. Define:

\[
P = P_0 + P^1 \\
u = u_0 + u^1 \quad \text{(x-velocity components)} \tag{4-8} \\
v = v^1 \quad \text{(y-velocity component)} \\
Q = \int_{S_0 + S_J} \rho (v \, dx - u \, dy)
\]

Substituting these into Eq. (4-7) we get

\[
T_H = \int_{S_0 + S_J} (P^1 + \rho u_0 u^1) \, dy + Q u_0 + \int_{S_0 + S_J} \rho u^1 (v^1 \, dx - u^1 \, dy) \tag{4-9}
\]

\( Q \) is equal to zero for the case where the jet mass flow is taken from the main flow and equal to \( m_J \) for the case where there is a source within the aerofoil (W.T. tests).

In the limit \( R \to \infty \) (see Fig. 11B) it can be shown by using Bernoulli's equation (Ref. 59) and from the statement that at infinity downstream the jet becomes parallel to the free stream direction that:

\[
T_H = \int_{S_J} \rho \cdot u \cdot (u - u_0) \cdot dy + Q \cdot u_0 \tag{4-10}
\]

As jet mass flow and momentum at infinity downstream can be written as

\[
m_{J_\infty} = \int_{S_J} \rho \cdot u \cdot dy \tag{4-11} \\
J = \int_{S_J} \rho \cdot u^2 \, dy \tag{4-11}
\]

it follows from Eqs. (4-10) and (4-11) that

\[
T_H = J - (m_{J_\infty} - Q) \cdot u_0 \tag{4-11}
\]
which for an inviscid jet, \( \dot{m}_J \infty = m_J \) (no mixing) becomes

\[
\text{TH} = J - (\dot{m}_J - Q)u_o \quad (4-13)
\]

For the case where the jet mass flow originates from a source within the aerofoil \( Q = m_J \)

\[
\text{TH} = J \quad (4-14)
\]

In non-dimensional form Eq. (4-14) becomes

\[
C_T = C_J \quad (4-15)
\]

Both derivations are based on the assumption that the two-dimensional jet sheet is deflected by the ideal main stream flow until it becomes parallel to it at downstream infinity. Generally, two-dimensional conclusions are used to provide a better understanding of what happens with three-dimensional cases. With the theory of jet flapped wings however such an application must be made with caution. This follows if one imagines a three-dimensional jet in a cross flow of ideal fluid. There is no pressure force exerted on the jet to produce a jet deflection unless the cross flow is viscous. In ordinary airfoil theory, the Kutta-Joukowski hypothesis is used to overcome a similar difficulty which arises from the neglect of viscosity. For the jet flapped wing the addition of a "jet turning hypothesis" is suggested by Prof. B. Etkin, UTIA.

Assume the free stream velocity to be \( V_o \) and the induced velocity due to the wing plus jet sheet to be \( \vec{V}_i \). The jet turning hypothesis for either two or three-dimensional jet flaps maintains that "the jet sheet deflects until it becomes parallel to the \( \vec{V}_o + \vec{V}_i \) direction". From this hypothesis follows that in the two-dimensional case \( \vec{V}_i = 0 \) the jet is deflected until it becomes parallel to \( \vec{V}_o \).

4.2 The Lift Hypothesis

Stratford in Ref. 11 states the lift hypothesis as:

"If in a two-dimensional system a jet issues from the trailing edge of an aerofoil, aerodynamic lift will be induced on the aerofoil in addition to the direct vertical component (reaction lift \( C_{LR} \)) in our notation, see Fig. 9C) of the jet thrust (jet momentum \( C_J \)). The usual stall limitations on lift will not apply as there will be a suction pressure peak at the trailing edge of the aerofoil. Further, the centre of the lifting force thus created will be near the 50% chord line and not near the jet".
French researchers (Ref. 33) refer to it as:

"Calculations indicate the existence of a vertical component corresponding to the value of circulation \( \Gamma \) of the general flow on a circle of infinite radius. This vertical component gives a total increase in lift at small jet deflection angles \( \theta \) due to blowing of

\[
\Delta C_{LT} = C'_{LT} = [f(C_J) + C_J] \cdot \theta
\]

Thus it is verified that applying Joukowsky's Law to an imaginary profile - compounded of a wing and a jet sheet which can be visualized as a solid extension of the base profile - is the same as applying it to the profile alone and subsequently adding the vertical component \( C_J \cdot \theta \) of the jet momentum. It is probably possible to extend the linear theory to a larger range of values of \( \theta \) by replacing \( \theta \) by \( \sin \theta \).

Theories, which further strengthen the validity of the lift hypothesis are discussed in Sec. VI.

### 4.3 The "Times Four" Argument

This argument (Ref. 11) is still less scientific than those presented above for the justification of the thrust and lift hypotheses. It is mentioned here only because it provides a rough idea of the lift magnification or gain in lift which could be expected from the jet flap system.

The assumptions in Sec. IV apply again. The force required to turn the jet sheet along CD back into the main flow direction (Fig. 12) must be provided by a pressure difference across the jet sheet, which constitutes a lift force exerted by the main stream flow. This lift is not, however, exerted on the aerofoil. From momentum considerations follows that the lift on CD must be equal in magnitude to the jet reaction lift (see Fig. 9). If the entire flow field is subdivided into regions as shown in Fig. 12, it can be argued that the pressure lift on BC is roughly equal to that on CD. As the centre of lift is at about 50% of the wing chord, it demands that the lift force on BD (which is about twice the jet reaction lift) is equal to that on AB. In other words, the ratio of the total wing lift to the jet reaction lift is of the order of four.

### 4.4 The Jet Flap Hypotheses and Their Practical Significance

One has to realize that the very attractive thrust and lift hypotheses are true only in ideal two-dimensional flow. Further the fact that the total thrust (\( C_{TM} \), see Fig. 9C) is equal to the jet momentum \( C_J \) - independent of \( \theta \) - is due primarily to the high suction peak around the L.E. of the wing (Ref. 12).
In real i.e., viscous fluid flow, however, such high suction peaks cannot exist. The actual flow will separate and the prediction of the thrust hypothesis can no longer be fully realized. In the extreme case of a very thin plane wing at zero angle of attack, the total thrust may not be only smaller than $C_J$ but even as small as $C_{TR} = C_J \cdot \cos \theta$, the horizontal component of the jet reaction force. A well rounded and thickened nose on the other hand may be expected to increase $C_{TM}$. This line of thought led to investigations into the effect of nose flaps (Ref. 31). In general, the discrepancy between the predictions of the jet flap hypotheses and experimental results is attributed to viscosity, in particular to the mixing process between the jet sheet and the main stream flow. The questions to be dealt with in subsequent sections of this review are therefore

1) What are the parameters and to what degree are they responsible for the fact that the predictions of the jet flap hypotheses cannot be fully realized in practice?

2) What theories exist to predict the performance of jet flapped wings?

3) What empirical laws to predict the performance of the actual jet flap can be derived from test results and

4) how do test results compare with the theoretical predictions for the jet flapped wing?

V. DEFINITION OF THE AERODYNAMIC FORCES ACTING ON A JET FLAPPED WING

In this section, the aerodynamic (including propulsive) forces which are known to act on a jet flapped wing are defined. They are subsequently expressed and used in their non-dimensional form only i.e., as coefficients of their components in lift and thrust (drag) direction.

Figure 9A shows a jet flapped wing but without the jet blowing and the conventional lift and drag forces acting on it. Figure 9B demonstrates the forces acting with the jet blowing. These forces may be either direct forces (as e.g. the reaction thrust) or indirect (induced) forces (as e.g. the jet induced pressure lift). Jet induced forces are those which result from a change of the pressure field around the wing due to the effect of the blowing jet. Finally, in Fig. 9C, the most general case of a jet flapped wing at an angle of attack and with a deflected jet sheet is shown.

In all three cases, the lift and thrust (drag) forces of the wing as obtained either by force balance measurements or from pressure distributions for both ideal and viscous flows are given in tables below their respective figures. These tables are added to ease co-ordination of experimental results from various researchers.
5. 1 Jet Momentum and Jet Momentum Coefficient

5.1.1 The Jet Momentum (J)

The jet momentum flux \( J = m \cdot v_J \) is equal to the jet reaction force acting on the wing. It can be determined in three ways:

1) from the measurements of mass flow and jet velocity. As a first approximation, \( v_J \) could be determined from the reservoir pressure and temperature, assuming isentropic expansion. However, as in general the losses (primarily discharge losses) are large with the conventional small size model jet flapped wing, this assumption actually is not correct. On the other hand, the precise measurement of the required pressures and temperatures for the calculation of the true jet velocity is difficult, mainly due to the small scale of the flow passages in model jet flapped wings.

2) from wind tunnel balance measurements. In tests described in Refs. 23, 26 and 32, the jet deflection angle \( \theta \) is first determined by means of wool tufts. Then the model wing is pitched until the jet axis coincides with the balance drag axis. Next, \( T_M \) is measured with the tunnel flow (wind) off. As in this case \( T_M = J - D_J \), \( T_M \) has to be corrected for the jet drag, \( D_J \), to obtain \( J \). The jet drag \( D_J \) is most readily obtained from an integration of the static pressure distribution around the wing in the direction of thrust. Using this \( J \) also for the wind-on tests assumes that \( J \) does not change when the wind tunnel flow is switched on. This is not so, as the back pressure at the slot nozzle exit changes (see Ref. 36).

In Ref. 33 another but similar method was used. The jet flapped wing was mounted at zero angle of incidence on the W. T. balance, the tunnel flow off. Then \( T_M \) was measured as a function of the jet total pressure in the "settling chamber" inside the model wing. As \( T_M \) contains still the jet drag \( D_J \) - which can be obtained as above - \( T_R \) follows in this case from \( T_R = T_M + D_J \). From \( T_R \) and \( L_R \), the jet momentum follows as

\[
J = \sqrt{T_R^2 + L_R^2} \tag{5-1}
\]

The jet deflection angle can then be obtained from \( \tan \theta = L_R / T_R \).

If a convergent nozzle at pressure ratios above the critical (1.89) is used, a post exit expansion takes place which promotes some additional losses (post exit thrust). The measured thrust must therefore differ by a few per cent from thrust values as obtained if isentropic flow expansion is assumed (see also Sec. 3.2.1. of Ref. 33).
3) from a combination of 1) and 2). The theoretical \( (v_J)_{\text{cal.}} \) is calculated using the measured total pressure in the wing settling chamber and the free stream static pressure. This static pressure is in general slightly higher than that at the nozzle exit, which actually should be used. From balance measurements with the wind-off and from mass flow measurements, the relationship (Ref. 39)

\[
k = \frac{J}{m_J} \cdot (v_J)_{\text{cal.}}
\]

(5-2)
can be derived, where \( k \) takes care of the decrease of jet momentum due to all kinds of losses. The jet momentum coefficient and the jet deflection follow then as:

\[
C_\mu = \frac{k \cdot m_J \cdot (v_J)_{\text{cal.}}}{\frac{\rho_o}{2} \cdot v_o^2 \cdot S_w}
\]

and

\[
\Theta = \tan \left( \frac{C_{LR}}{C_{TR}} \right)
\]

(5-3)

where \( k \) was found to be approximately 0.75 in Ref. 39.

This method was applied also in Ref. 39 to truncated wings with T. E. blowing in high speed flows (Ma > 1.0).

5.1.2 The Jet Momentum Coefficient (\( C_J \)) or (\( C_\mu \))

It is defined as

\[
C_J = \frac{J}{\frac{\rho_o}{2} \cdot v_o^2 \cdot c}
\]

for two-dimensional cases

or

\[
C_J = \frac{J_T}{\frac{\rho_o}{2} \cdot v_o^2 \cdot S_w}
\]

(5-4)

for three-dimensional cases

Figure 13 shows that the total lift increase (\( \Delta C_{LT} \)) plotted vs. \( C_\mu \) is almost independent of the slot nozzle width. This indicates that \( C_\mu \) is a good parameter for representing the characteristics of a jet or jet sheet.

5.2 The Aerodynamic Forces in Lift Direction

5.2.1 The Reaction Lift (\( C_{LR} \))

If the jet momentum vector (\( C_J \)) is in the direction of the undisturbed flow (x-direction), its component in lift direction, \( C_{LR} = C_J \cdot \sin \tau \) is zero i.e., the jet cannot "directly" contribute to an increase in lift. If, however, the jet is issuing at an angle \( \tau \) (see Fig. 9C) a component (\( C_J \cdot \sin \tau \)) in lift direction called REACTION LIFT (\( C_{LR} \)) results. It becomes a maximum for \( \tau = 90^\circ \) as it is then equal to the jet momentum (\( C_J \)). This obviously suggests wing lift coefficients which are higher than those obtainable with any other known and practical means.
If it is further realized that these high reaction lifts are gained - in ideal two-dimensional flow at least (see thrust hypothesis) - without a loss in total thrust ($C_J = CTM$), the keen interest with which the jet flap idea was taken up all over the world can be better appreciated.

5.2.2 The Jet-Induced Pressure Lift ($C_{LJ}$)

The jet induced pressure lift ($C_{LJ}$) is the result of the changes in the pressure field around the jet flapped wing due to the effects of the blown jet sheet. These effects are the stronger the larger the jet velocity and the jet deflection. They add to the pressure field which produces the basic lift ($C'L$)

1) a region of reduced static pressure on the upper wing surface, which extends - gradually dying out - from the trailing edge to even as far upstream as the leading edge (Fig. 14)

2) a region of increased static pressure on the lower wing surface in particular close to the trailing edge (Fig. 14).

The resultant of these additional induced pressure forces in the lift direction is called the jet induced pressure lift. It should be realized that it is this force which in B.L.C. raises the lift performance of a wing in viscous flow to its performance in ideal fluid flow. At still higher jet momentum coefficients supercirculation is produced which, if strong enough may even eliminate the usual stall limitation of the wing.

5.2.3 Pressure and Total Lift ($C_{LT}$)

The pressure lift ($C_{LP}$) is simply the sum of all the pressure forces acting on the wing in lift direction i.e., the sum of the jet induced pressure lift and the basic lift ($C_{LP} = C_{LJ} + C'L$).

The total lift ($C_{LT}$) results if the reaction lift ($C_{LR}$) is added to the pressure lift ($C_{LP}$).

5.3 The Aerodynamic Forces in Thrust Direction

5.3.1 The Reaction Thrust ($C_{TR}$)

It is simply the component ($C_J \cos \tau$) of the jet momentum in thrust direction (see Fig. 9C). At a first glance it seems that this force is the only one in thrust direction that matters. But how then could the thrust hypothesis be explained or the fact that even in viscous fluid flow the measured thrust may be greater than $C_{TR}$, at least at jet deflection angles $\tau > 30^\circ$? The force responsible for this phenomenon is called the jet-induced pressure thrust.
5.3.2 The Total (Measured) Thrust \((C_{TM})\)

The total thrust \((C_{TM})\) is the resultant force in thrust (drag) direction as measured by a thrust balance. Or, it can be obtained if the total drag \((C_{DT})\) is subtracted from \(C_J\), the jet momentum coefficient at downstream infinity.

5.3.3 The Jet-Induced Pressure Thrust \((C_{TP})\)

The induced pressure thrust - defined as \(C_{TP} = C_{TM} - C_{TR}\) - primarily originates from the jet-induced changes in the pressure field around the jet flapped wing (see Figs. 15 and 16). It results from the suction forces which are essentially concentrated at the leading and trailing edge of the wing (saddle back pressure distribution). For jet sheets issuing from the T.E. proper, \(C_{TP}\) has been shown to be a function of the L.E. suction peak only (Fig. 17), if ideal fluid flow is assumed. According to the thrust hypothesis (no jet mixing) \(C_{TP}\) should be as large as \(C_{TPi} = C_J - C_{TR}\). In practice, however, high suction peaks at the T.E. - growing faster with \(C_J\) than those at the L.E. - may reduce \(C_{TP}\) to zero or even negative values \((C_{TM} < C_{TR})\).

An idea of the actual magnitude of \(C_{TP}\) may be obtained from results communicated in Ref. 12. For jet deflection angles of \(\theta = 90^\circ\) at \(\alpha_i = 0\), \(C_{TP}\) values of up to 40% of the jet momentum \((C_J)\) are quoted for a two-dimensional wing. Note that in this case, \(C_{TR} = 0\) (see Fig. 9C).

5.3.4 The Pressure Thrust \((C_{TP})\)

The pressure thrust is the force in thrust direction as obtained from an integration of the measured pressure distribution. It is the sum of the jet-induced pressure thrust \((C_{TP})\) and the skin friction drag \((C_{DFR})\) of the jet flapped wing. It can also be expressed as the ideal jet induced pressure thrust (see Sec. 5.3.5) minus the form drag and the induced drag (see Fig. 9C), both of which affect the static pressure distribution.

5.3.5 The Ideal Jet-Induced Pressure Thrust \((C_{TPi})\)

In ideal two-dimensional flow, \(C_{TP}\) becomes equal to \(C_J - C_{TR}\). Since this is the maximum value the jet-induced pressure thrust can ever attain, it is called the ideal jet-induced pressure thrust \((C_{TPi})\). It follows from Fig. 9C, that \(C_{TPi} - C_{DT} = C_{TP} = C_{TM} - C_{TR}\), where \(C_{DT}\) is the total drag of the wing.
5.4 The Aerodynamic Forces in Drag Direction

5.4.1 Drag Forces on Conventional Wing (No Blowing)

There is the basic wing drag ($C'D$) which is the sum of the form drag ($C'DF$), the friction drag ($C'DFR$) and the induced drag ($C'Di$). The sum of $C'DF + C'DFR = C'DP$ is known as the profile drag (see Fig. 9A).

5.4.2 The Profile Drag ($CDP$)

The profile drag of the jet flapped wing $CDP = C'DP + \Delta CDP$, where $\Delta CDP$ is the increase in profile drag due to blowing. On the other hand $CDP$ is the sum of the form drag ($CDF$) and the skin friction drag ($CDFR$).

5.4.3 The Jet Drag ($CDJ$)

Imagine a jet flapped wing operating in fluid almost at rest. The main stream flow entrainment as stimulated by the jet ejector effect locally accelerates the air close to and surrounding the trailing edge, thus producing a suction pressure resulting in a pressure (form) drag (see Fig. 18). The magnitude of this form drag depends upon the rate of entrainment, or more specifically (Ref. 14), on the ratio of the density − velocity products as illustrated in Fig. 19. If e. g., in a cold jet $\rho_J \cdot v_J > \rho_0 \cdot v_0$, a mixing drag would result but when $\rho_J \cdot v_J < \rho_0 \cdot v_0$ (excluding the case of $v_J = 0$), then jet mixing would in effect produce an analogous thrust. This criterion does not apply for a hot jet (see Ref. 36). Besides the changes in form drag the jet drag contains the reduction of thrust (see Ref. 30) between the nozzle exit and downstream infinity due to the finite thickness of the jet and due to mixing. It also contains the skin friction drag due to frictional forces produced in the T. E. vicinity by the jet-induced local acceleration of free stream air as a result of the jet's ejector action. In general the jet drag may be defined as the change in profile drag due to blowing

$$CDJ = \Delta CDP = CDP - C'DP \quad (5-5)$$

This jet drag is difficult to measure directly. However, the following hypothetical experiment serves to illustrate its meaning. Imagine two identical symmetrical wings, one with a conventional circular jet, the other one equipped with a pure jet flap. Set both at zero angle of incidence and zero jet deflection angle. With the wind off, adjust the jet momentum of both, the circular jet and jet sheet, to be equal. With the wind on, measure the thrust produced on both wings. The difference in measured thrust is the jet drag.
5.4.4 The Induced Drag \( (C_{D_i}) \)

The induced drag coefficient \( (C_{D_i}) \) is defined as the drag of a three-dimensional jet flapped wing in ideal flow. It is

\[
C_{D_i} = C_J - C_{TM_i}
\]

(5-6)

where \( C_J \) corresponds to the thrust acting on a two-dimensional jet flapped wing in ideal flow (thrust hypothesis) and where \( C_{TM_i} \) is that of a three-dimensional jet flapped wing in ideal flow. This \( C_{D_i} \) may be calculated as

\[
C_{D_i} = C_{LT}^2 / (\pi AR + 2C_J) \cdot e
\]

(5-7)

which for elliptic loading \( (e=1) \) becomes \( (\text{Ref. 30}) \)

\[
C_{D_i} = C_{LT}^2 / (\pi AR + 2C_J)
\]

(5-8)

5.4.5 The Total Drag \( (C_{DT}) \)

This drag may be obtained experimentally from

\[
C_{DT} = C_J - C_{TM}
\]

(5-9)

and may be calculated from

\[
C_{DT} = C_{DP} + C_{D_i} = C_{DP} + \frac{C_{LT}^2}{(\pi AR + 2C_J) \cdot e}
\]

(5-10)

VI. THEORETICAL TREATMENTS OF THE JET FLAP

6.1 Introductory Remarks

Some of the available theoretical treatments of the jet flapped wing, are briefly described below. The principal lines of attack underlying the theoretical methods are outlined without going into details. Significant results found with each method are summarized, again omitting details. For additional information reference is made to the original papers. The methods considered here are:

1. The Straight Mechanical Flap Analogy Method, see Sec. 6.2 (Ref. 11).

2. The Curved Mechanical Flap Analogy Method, see Sec. 6.3 (Ref. 31).

3. The Thin Aerofoil Theory Method, see Sec. 6.4 (Ref. 16)
4. The Method for the Jet Sheet Issuing From a General Point on the Lower Surface, see Sec. 6.6 (Ref. 100)

5. An Approximate Method for the Calculation of Pressure Distributions, see Sec. 6.7 (Ref. 17)

6. The Electrical Analogy Method, see Sec. 6.8 (Refs. 19 and 33).

7. The Theory of the Finite-Span Blowing Wing, see Sec. 6.9 (Ref. 63).

8. The Source-Type Flow Method, see Sec. 6.10 (Ref. 35)

For all practical purposes, the physical models used by most of the methods are the same. As the names of the methods indicate, they mainly differ in the lines of attack.

6.2 The Straight Mechanical Flap Analogy Method (Ref. 11)

It was mentioned earlier in connection with the origin of the name "jet flap" that the air flow over a jet flapped wing is similar in nature to that of a wing with a mechanical flap and that a jet sheet and a specific mechanical flap of finite size, both causing the same induced lift on the wing, are considered to be analogous.

This method uses the similarity between the mechanical flap and the jet flap to calculate the pressure lift induced by the jet flap from the lift induced by the mechanical flap, which in potential flow can be determined, if the configuration of the mechanical flap is known. The above mentioned condition of analogy is satisfied, if the lifts on the mechanical and the jet flap are equal in magnitude and distribution.

The simplest approximation to the curved jet flap is the straight mechanical flap. The lift magnitude requirement is fulfilled if the lift on the mechanical flap is equal to the lift on the jet sheet produced by the pressure difference across it, or in other words, is equal to the vertical component of the jet reaction force. However, the second condition that the lift distribution along both flaps should be the same cannot be met exactly by this approximation, as it would require the flap shapes to be the same. This could only be achieved by a curved mechanical flap (see Sec. 6.3). The closest to the shape similarity that could be obtained with a straight mechanical flap, is to make the flap angles approximately equal. It is interesting to note that these conditions would also make the total lifts equal.

Stratford applies Glauert's thin aerofoil theory (Ref. 15) to an aerofoil at zero incidence with a mechanical flap and derives the following expressions:

\[(21)\]


\[
[C_L]_{\alpha=0} = 5.0 \cdot \left( \frac{C_T}{C_J} \cdot \frac{1}{\sin \theta} \right)^{1/2} \left( 1 + \frac{\pi}{48} \cdot \frac{C_J}{C_T} \cdot \frac{\sin \theta}{\beta} + \ldots \right)
\]  

(6-1)

\[
\frac{\partial C_L}{\partial \alpha} \bigg|_{\alpha=0} \approx 2 \pi \left\{ 1 + \left( \frac{C_J \cdot \beta}{2 \pi \cdot \sin \theta} \right)^{1/2} \right. \\
+ \frac{\pi}{48} \cdot \frac{C_J}{C_T} \cdot \sin \theta + \left( \frac{2 \pi}{3} \right)^{1/2} \cdot \left( \frac{\sin \theta}{\beta} \right)^{1/2} + \ldots \} \\
\left. \times \frac{1}{\theta \text{ small}} \right.
\]

\[
X_{A.C.} \approx 0.25 \left[ 1 + \left( \frac{C_J \cdot \beta}{2 \pi \cdot \sin \theta} \right)^{1/2} \right]
\]

\[
X_{L.C.} = 0.5 + \frac{\pi}{48} \cdot \frac{C_J}{C_T} \cdot \sin \theta \left\{ 1 - \left( \frac{C_J \cdot \sin \theta}{2 \pi \cdot \beta} \right)^{1/2} - \frac{1}{120} \cdot \frac{C_J}{C_T} \cdot \frac{\sin \theta}{\beta} + \ldots \} \right. \\
\left. \times \times \frac{1}{\theta \text{ small}} \right.
\]

\[
\lambda' = \frac{\pi}{8} \cdot \frac{C_J}{C_T} \cdot \frac{\sin \theta}{\beta} \left( 1 - \frac{\pi}{34} \cdot \frac{C_J}{C_T} \cdot \frac{\sin \theta}{\beta} + \ldots \right)
\]

\[
\text{chord of analogous flap} = \text{chord of aerofoil}
\]

where \( \beta \) is the deflection angle of analogous flap which is approximately equal to \( \theta \). The gain \( G_L \) is defined as:

\[
G_L = \frac{C_{LT}}{C_{LR}} = \frac{C_L}{C_J \cdot \sin \theta}
\]

Fig. 20 shows the variation of \( G_L \) with \( C_J \). Agreement between theory and experimental results can be said to exist in trend only.

### 6.3 The Curved Mechanical Flap Analogy Method (Refs. 31 and 53)

#### 6.3.1 The Two-Dimensional Wing

In order to satisfy the second condition of equivalence, that the pressure distribution along the analogous flap must be the same as that for the jet sheet, a curved mechanical flap is considered. In this method the jet sheet issuing from the trailing edge is replaced by a curved equivalent flap having a shape similar to that of the jet sheet. Hence, the jet sheet shape has to be determined first.
This method considers a two-dimensional airfoil with a jet sheet issuing at an angle $\tau$ and velocity $v_J$ from a slot of width $\delta$ at the trailing edge (Fig. 21). The jet sheet causes a pressure difference between its upper and lower surfaces. The resultant pressure force ($dF_{PR}$) on a small element of length $ds$ is balanced by the centrifugal force ($dF_C$). We can write

\[
dF_{PR} = \Delta P \cdot ds = (K_1 \cdot \sin \tau') \frac{\rho_0 \cdot v_o^2 \cdot ds}{2}
\]

where $(K_1 \cdot \sin \tau')$ is equivalent to the normal force coefficient of a flat plate at an angle of attack of $\tau'$ and $\tau'$ is the local angle between jet sheet and undisturbed flow (see Fig. 21). Note, that Eq. (6-3) is not applicable to a three-dimensional jet sheet. Further,

\[
dF_c = \rho_o \cdot \delta \cdot v_J^2 \cdot d\tau'
\]

By equating the expressions for pressure and centrifugal force and substituting $ds = dx' / \cos \tau'$ (see Fig. 21), we get

\[
dx' = \frac{2 \delta}{K_1} \left(\frac{v_J^2}{v_o^2}\right) \cot \tau' \cdot d\tau'
\]

where $x'$ is positive in downstream direction. From the above equation an expression for the jet shape can be derived as

\[
\tau' = \tau \cdot \exp \left[\frac{K_1 \cdot (c - x)}{c \cdot C_J}\right] \quad (6-6a)
\]

where the factor $K_1$ in the above equation can be assumed equal to unity without significantly affecting its accuracy.

If we rewrite Eq. (6-6a) as

\[
\tau' = \tau \cdot \exp \left[\frac{-K_1 \cdot x'}{c \cdot C_J}\right] \quad (6-6b)
\]

and apply the conventional linear-theory approximations to

\[
\tau' \approx \tan \tau' = \frac{dy'}{dx'}
\]
we get
\[
\frac{dy'}{dx} = \tau \cdot \exp \left[ \frac{-K_1 \cdot x'}{c \cdot C_J} \right]
\] (6-6c)

Integrating Eq. (6-6c) while treating \(\tau\), \(c\) and \(C_J\) as constants, we get
\[
y' = \frac{-c \cdot C_J}{K_1} \left\{ \tau \cdot \exp \left[ \frac{-K_1 \cdot x'}{c \cdot C_J} \right] \right\} + I
\] (6-7)

where \(I\) is the integration constant. The value of \(I\) can be obtained from the condition \(y'(at \ x' = 0) = 0\)

Inserting the value for \(I\), we get the following equation for the jet sheet shape
\[
y' = \tau \cdot \left\{ \frac{c \cdot C_J}{K_1} \left(1 - \exp \left[ \frac{-K_1 \cdot x'}{c \cdot C_J} \right] \right) \right\}
\] (6-7a)

Here the units of \(y'\) are fixed by the units of \(c\), the chord.

Next, the length of the analogous flap has to be known. The length of the curved mechanical flap is fixed by the condition that the lift on it must be equal to the jet reaction lift which is equal to the force produced by the pressure difference across the jet sheet. Knowing the shape and the length of the analogous curved mechanical flap, Glauert's thin aerofoil theory (Ref. 15) can now be applied to the aerofoil-flap combination at zero incidence. The following expressions result:
\[
C_{LT} = \frac{-c'}{c} \cdot 2\tau \int_0^{\gamma_0} \exp \left[ \frac{\cos \gamma - \cos \psi}{2 \cdot c \cdot C_J} \right] (1 + \cos \psi) \, d\psi
\] (6-8)

where \(c'\) (see Fig. 22) is the chord length of wing plus flap and \(\gamma\) and \(\gamma_0\) are defined by the following equations:
\[
\begin{align*}
x &= \frac{c'}{2} \cdot (1 + \cos \psi) \\
c &= \frac{c'}{2} \cdot (1 + \cos \gamma_0)
\end{align*}
\] (6-9)

The pitching moment coefficient about the leading edge of the wing is given by
\[ C_M = -\left( \frac{2}{1+ \cos \gamma_0} \right)^2 \left[ C_{L \nu} \frac{c}{c_f} - \frac{1}{2 \nu_0} \int_{-\gamma_0}^{\gamma_0} \psi(\psi) \cos \psi \sin \psi \, d\psi \right] \]

\[ + C_{J} \cdot \frac{c}{c_f} \sin \tau \left( \frac{1 + \cos \gamma_0}{2} \right) \]  \hspace{1cm} (6-10)

where \( \gamma \) is the local circulation given by Eq. (14) of Ref. 31.

It is interesting to note that appreciable differences exist between the straight and the curved mechanical flap theories in their respective predictions of pitching moment. Results are plotted in Fig. 23, 24, 127 and 129. As intuitively expected and as is shown also in these figures, curved flap analogy gives a better prediction of pitching moment coefficients. But both the curved flap and the straight flap analogy method do not correctly predict the location of the centre of lift and of the aerodynamic centre (Fig. 129). According to the flap analogy theories, location of the A.C. and L.C. depend upon the jet deflection angle, but experiments show (see Fig. 25) that this is not so, at least not in practice (see Sec. 9.2).

### 6.3.2 The Three-Dimensional Wing

As the free vortices of the finite wing induce vertical velocities, the angle of attack of the infinite wing is reduced by the induced angle of attack, \( \alpha_i \), which depends on the strength of the free vortices and their spanwise distribution. Assuming that the free vortices lie in the horizontal wing plane, Prandtl's integral equation for the conventional finite wing can be written as:

\[ \bar{\alpha}(\bar{\gamma}) = \frac{a_\infty \cdot c(\bar{\gamma}) \cdot (\alpha(\gamma) - \alpha_i(\gamma))}{2b} \]  \hspace{1cm} (6-11)

where \( b \) is the wing span, \( a_\infty \) is half the value of the lift curve slope for the two-dimensional conventional wing, \( \bar{\gamma} \) is the non-dimensional span coordinate and \( \bar{\gamma} \) is the non-dimensional circulation. The latter are defined as

\[ \begin{align*}
\bar{\gamma} &= \frac{2 \gamma}{b} \\
\bar{\gamma} &= \frac{T}{b \cdot \nu_0}
\end{align*} \]  \hspace{1cm} (6-12)

Prandtl's equation can be solved, if the geometrical angle of attack along the span and the shape of the wing are known.

The very same procedure is used by Jacobs and Paterson for a jet flapped wing by replacing the jet-induced pressure lift of the two-
dimensional case by a geometrical angle of attack of a conventional wing without jet ($\alpha_g = C_{L_{\alpha}}^{(2)} / (dC'_{L}^{(2)} / d\alpha)$). It can be assumed that the downwash induced at the wing has very little or no effect on the jet reaction lift, but has some effect on the pressure lift and further, that the downwash is only induced by the vortices associated with the pressure lift. Based on these assumptions, results are obtained by Multhopp's method (Ref. 126) which are given in Fig. 26. The main conclusions which can be drawn are that the jet-induced pressure lift decreases with decreasing aspect ratio and that the spanwise-lift distribution for the case of constant $C_J$ and $\tau$ is similar to that of the conventional wing, only the absolute values being different.

6.4 The Thin Aerofoil Theory Method

6.4.1 Application to the Jet Flapped Wing (Refs. 16 and 101)

Spence applies the thin aerofoil theory to the inviscid and incompressible flow past a two-dimensional wing of unit chord at an angle of incidence, $\alpha$, at the trailing edge of which a jet with zero thickness ($\delta \rightarrow 0$) but with finite momentum ($J$) emerges at a small deflection angle $\theta$ (see Fig. 27A). Further, the assumption is made that the jet flow is irrotational and that there is no mixing between the jet and the main stream. Another assumption, that of zero jet thickness eliminates the source effects, which are very small in practical cases, where generally very narrow slots are used.

The jet, along its boundaries, is separated from the main stream by vortex sheets, across which there is equality of static pressures and no flow mixing takes place. Consider an element of the jet shown in Fig. 27B. Defining that,

$$
\begin{align*}
\nu &= 1/2 \cdot (v_1 + v_2) \\
\nu_J &= 1/2 \cdot (v_{J1} + v_{J2})
\end{align*}
$$

(6-13)

it can be easily shown by using Bernoulli's equation and the condition of irrotationality that

$$
\begin{align*}
\nu_{J1} - \nu_{J2} &= \nu_J \cdot \delta / R \\
\nu_1 - \nu_2 &= \rho_J \cdot \nu_J^2 \cdot \delta / \rho_0 \cdot v \cdot R \\
P_1 - P_2 &= - \rho_J \cdot \nu_J^2 \cdot \delta / R
\end{align*}
$$

(6-14) (6-15) (6-16)

The two vortex elements on the jet boundaries can be replaced by an equivalent single vortex located on the centre line of the jet, together with a doublet the strength of which can be derived. The vortex strength is given as
and the doublet strength is

$$\mu = v \cdot \delta \cdot \left(1 - \frac{J \cdot \delta}{4 \cdot \rho_o \cdot v^2 \cdot R^2}\right)$$

For the limiting case of $v_J \to \infty$, assuming the jet momentum to remain finite, it is shown in Ref. 101 that $J = \text{constant}$ and that the mass flow and thickness of the jet approach zero. Therefore, the vortex distribution representing the jet becomes

$$\gamma_J = \frac{v \cdot J}{R \cdot \rho_o \cdot v^2}$$

The doublet distribution vanishes. By using the approximation that $v = v_o$ (small perturbation theory) we can write:

$$\gamma_J = \frac{v_o \cdot C_J}{2 \cdot R} \sim \frac{1}{2} \cdot v_o \cdot C_J \frac{d^2}{dx^2} = \frac{1}{2} \cdot v_o \cdot C_J \cdot \gamma''(x)$$

(Note: Here $C_J = J/\frac{1}{2} \rho_o v_o^2$, because we are considering a two-dimensional wing of unit chord and span).

Similarly, the aerofoil can be replaced by a vortex distribution defined by

$$\gamma_w = v_o \cdot f(x)$$

This representation is illustrated in Fig. 28B.

The downward velocity or downwash $w(x)$, at a point on the axis due to these two vortex distributions (measured positive clockwise) is

$$w(x) = -\frac{v_o}{2 \pi} \int_0^1 \frac{\rho(\xi) d\xi}{\xi - x} + \frac{v_o \cdot C_J}{4 \pi} \int_1^\infty \frac{\gamma''(\xi) d\xi}{\xi - x}$$

where $\xi$ is the running coordinate of integration.
Then the usual boundary conditions are applied. (The vortex distributions along the jet and the aerofoil are so chosen that the flow velocity vector is tangential to either the jet or the aerofoil). From these boundary conditions follows:

\[ w(x) = v_0 \cdot \varepsilon(x) = v_0 \alpha \quad (6-23) \]

which applies to a flat plate or an uncambered aerofoil, see Fig. 28A.

b) for the region defined by \( 1 < x < \infty \)

\[ w(x) = v_0 y'(x) \quad (6-24) \]

Using these equations we can write:

\[
\frac{1}{\pi} \int_0^1 \frac{\xi(x) d\xi}{\xi - x} - \frac{1}{\pi} \int_0^\infty \frac{\xi(x) d\xi}{\xi - x} = \begin{cases} 
-2\alpha & (0 < x < 1) \\
-2y'(x) & (1 < x < \infty)
\end{cases} \quad (6-25)
\]

Note: \( y'(1) = \alpha + \theta = \tau \)

\( y'(\infty) = 0 \)

This is a pair of integro-differential equations, one for the aerofoil region and the other for the jet region. They relate the strength of the vortex sheet of each region to the downwash distribution in that region. By mathematical means the two equations are reduced to the following single integro-differential equation (Refs. 16 and 101), which is valid in the range \( 1 < x < \infty \):

\[
\frac{1}{\pi} \left( \frac{x-1}{x} \right)^{1/2} \int_0^\infty \frac{\eta^2}{\eta^2 - 1} \frac{g(\eta) d\eta}{\eta - x} + \frac{1}{\pi} \left( \frac{x-1}{x} \right)^{1/2} \int_0^1 \frac{\eta^2}{1-\eta} \frac{\varepsilon(\eta) d\eta}{\eta - x} = \lambda g(x) \quad (6-26)
\]

where \( \eta \) is a dummy variable, \( \varepsilon(\eta) \) is the angle of attack of the flat plate and

\[
\begin{align*}
\lambda &= 4 \frac{C_j}{\pi} \\
g(x) &= -\frac{2}{\lambda} \cdot y'(x)
\end{align*} \quad (6-27)
\]
Solving this equation by an approximate Fourier series method the following expressions for total lift coefficient, lift curve slopes, chord wise loading, pitching moment coefficient, centre of lift location etc. are derived.

**Total Lift Coefficient:**

\[ C_{LT} = 4\pi A_0 \theta + 2\pi (1 + 2B_0) \alpha \]  \hspace{1cm} (6-28)

where \( A_0 \) and \( B_0 \) are Fourier coefficients which are functions of \( C_J \) (Fig. 29), tabulated in Ref. 16.

**Lift Curve Slopes:**

The following interpolation values can be used for \( \frac{\partial C_{LT}}{\partial \theta} \) and \( \frac{\partial C_{LT}}{\partial \alpha} \) (see Fig. 30A):

\[ \frac{\partial C_{LT}}{\partial \alpha} = 2\pi + 1.152 C_J^{1/2} + 1.106 C_J + 0.051 C_J^{3/2} \]  \hspace{1cm} (6-29a)

\[ \frac{\partial C_{LT}}{\partial \theta} = 3.54 C_J^{1/2} + 0.325 C_J + 0.156 C_J^{3/2} \]  \hspace{1cm} (6-29b)

These interpolation formulae can be much improved, according to Spence. In a private communication, he quotes the improved versions of the above equations as

\[ \left( \frac{\partial C_{LT}}{\partial \theta} \right)^2 = 4\pi C_J (1 + 0.151 C_J^{1/2} + 0.139 C_J) \]  \hspace{1cm} (6-29b)

\[ \frac{\partial C_{LT}}{\partial \alpha} = 2\pi (1 + 0.151 C_J^{1/2} + 0.219 C_J) \]  \hspace{1cm} (6-29b)

**Chordwise Vorticity Distribution (Loading, Fig. 31B)**

\[ f(x) = \frac{2\theta}{x^{3/2}} \left\{ -\frac{1}{\pi} \log(1-x) + A_0 \left( \frac{2X}{1+X} \right) + \sum_{n=1}^{N-1} A_n X^n \right\} \]

\[ + \frac{2\alpha}{x^{3/2}} \left\{ B_0 \left( \frac{2X}{1+X} \right) + \sum_{n=1}^{N-1} B_n X^n + x (1-x)^{1/2} \right\} \]  \hspace{1cm} (6-30)
where \( A_n \) and \( B_n \) are Fourier coefficients which are functions of \( C_J \); \( N \) is the number of pivotal points and \( X \) is defined as:

\[
X = \frac{1 - (1-x)^{1/2}}{1 + (1-x)^{1/2}} \quad (6-31)
\]

**Pitching Moment Coefficient:**

\[
C_M = \theta (C_J + I_{-1}) + \alpha \frac{n}{2} \sum_{n=0}^{N-1} (\theta A_n + \alpha B_n) \cdot I_n \quad (6-32)
\]

where \( I_{-1} \ldots \ldots I_n \) are integral values, which are given in Ref. 16 and Ref. 101.

**Location of Centre of Lift:**

\[
X_{L.C.} = \frac{C_M}{C_{LT}} \quad (6-33)
\]

For zero incidence, \( X_{L.C.} \) is independent (see Fig. 30B) of \( \theta \). This result is contrary to that predicted by mechanical flap analogy methods (see Sec. 6.3.1).

For a comparison of Spence's theory with experimental evidence, see Chapt. IX.

No doubt, neither Spence's Eqs. 6-28 and 6-33 (nor the corresponding Eqs. 6-84 and 6-85 provided by the electric analog tank method, Sec. 6.8) may be considered as very practical expressions. Spence (Ref. 16) therefore provides approximate expressions for \( C_{LT} \), given by Eq. 6-29. In Ref. 133, the above equations are approximated by analytical expressions for the lift coefficient

\[
C_{LT} = 2 \pi \alpha + (\pi C_J^{1/2} + C_J) \cdot (\theta + \alpha) \quad (6-33a)
\]

and for the location of the centre of the jet induced lift

\[
\overline{X_{L.C.}} = 0.5 - 0.07 C_J^{1/4} \quad (6-33b)
\]

where \( C_{LT} \) is claimed to agree with the experimental evidence of Ref. 136 at least for \( C_J \ll 1 \) and \( \overline{X_{L.C.}} \) for \( C_J > 0.2 \). Note, that the above equations are valid (as, e.g., Eq. 6-28) for small jet deflection angles only. Reference 133 further presents the following equation

\[
C_{LT} = 2 \pi \alpha \cdot \eta + (3.9 C_J^{1/2} + C_J) \sin (\theta + \alpha) \quad (6-33c)
\]

which is valid for all jet deflection angles up to \( 90^0 \). Eq. (6-33c) is
obtained by substituting 3.9 for \( \pi \) in Eq. (6-33a) and introducing a constant \( \eta \), which is to be determined experimentally in a test with no jet blowing.

6.4.2 Application to a Thin Aerofoil with a Jet-Augmented Flap (Refs. 48 and 93)

This is an extension of Spence's jet-flapped aerofoil theory (Sec. 6.4.1) to a thin aerofoil with a jet-augmented flap (sometimes also called a blown flap). The case where a jet is blown tangentially over a mechanical flap from the hinge line may be treated precisely as if the jet emerged tangentially at the trailing edge, provided that there is no flow separation over this flap. The purpose of the jet flow is merely to prevent separation over the flap and the wing pressure distribution is not modified in any other way. (The pressure gradient across the jet is proportional to its curvature, and thus is zero for the part in contact with the flap). Unlike the previous case, \( \varepsilon(x) \) (the aerofoil surface slope) is defined in two regions (see Fig. 32, insert) as:

\[
\varepsilon(x) = \begin{cases} 
\alpha ; & 0 < x < (1 - E) \\
\alpha + \beta ; & (1 - E) < x < 1 
\end{cases}
\]  

(6-34)

where \( E \) is the ratio of flap length to aerofoil chord.

Solving Eq. (6-26) along with Eq. (6-34) by the approximate Fourier series method, the following expressions for the total lift coefficient, chordwise vorticity distribution etc. are obtained.

**Total Lift Coefficient**

\[
C_{LT} = 2\pi B_0 L_0 + 2(\alpha + \sin \alpha + 2\pi D_0) \beta
\]

(6-35)

where

\[
\alpha = 2\sin^{-1}\left(\sqrt{E}\right) = \cos^{-1}(1 - 2E)
\]

(6-36)

\( B_0 \) is the coefficient evaluated as a function of \( C_J \) (see Sec. 6.4.1)(Ref. 16 and 101), and \( D_0 \) is a function of \( C_J \) and \( E \) (Ref. 48).

**Vorticity Distribution (Chordwise Loading)**

\[
\phi(x) = \frac{2.3 X \left(1 - \frac{1}{X}\right)^{\frac{3}{2}}}{\pi} + \frac{2.3 \beta}{\pi} \log \left( \frac{(1 - x)^{\frac{1}{2}}}{x^{\frac{1}{2}}} + \frac{E^{\frac{1}{2}}}{1 - E^{\frac{1}{2}}} \right) + 2\alpha \left(1 - \frac{1}{X}\right)^{\frac{1}{2}}
\]

\[
+ x^{3/2} \left\{ (2\alpha B_0 + 2\beta D_0) \left(\frac{2}{1 - X}\right) + \sum_{n=1}^{N-1} 2(\alpha B_n + \beta D_n) X^n \right\}
\]

(6-37)
Some results are shown in Fig. 32 and 33.

The $C_J$ - value used in this analysis is the effective jet momentum flux at the T.E. of the flap; in practice this will be less than the momentum emerging from the slot at the hinge point, since some momentum is transferred to the boundary layer to prevent it from separating in the adverse pressure gradient on the flap.

From Fig. 32 it can be seen that the effectiveness of the jet flap scheme might be greatly increased by the use of flaps of finite chord. Even at low $C_J$'s, it seems that useful gains can be obtained from the conventional boundary layer control applications. It is interesting to note that a two per cent flap (Fig. 32) is sufficiently different from the pure jet flap to indicate that trailing edge conditions can make a good deal of difference.

For the case without a jet this analysis and its results reduce to that of a wing with a mechanical flap (Glauert's theory, Ref. 15) Spence obtains the results in a more compact form as a result of using the Carlemann inversion formula (Ref. 127) for Cauchy integrals.

NOTE: The following remarks concern a possible relaxation of the two assumptions (Sec. 6.4.1) being made in deriving these results. The first assumption is that of zero jet thickness. For jets of non-zero thickness one would have to add a doublet distribution (Eq. 6-18) along the jet centre-line. The second assumption is that of a "shallow-jet" (replacing the true curvature by $d^2y/dx^2$). This could be improved by using that true curvature instead and allowing for the displacement of the jet below the axis by finding the downwash it induces.

6.4.3 Extension to the Three-Dimensional Wing (Ref. 30)

Maskell and Spence extend the thin-aerofoil method (Sec. 6.4.1) to the three-dimensional wing using the known idea that the downwash field associated with a finite wing is produced by elementary horseshoe vortices lying in the plane $z = 0$.

It may be shown that in the case of a three-dimensional wing the pressure difference across the jet (analogous to Eq. 6-16) is given by

$$
\Delta P = \left( \frac{v_I}{R_1} + \frac{v_J}{R_2} \right) \rho J \cdot \delta
$$

(6-38)

where $R_1$ and $R_2$ are radii of principal curvature of the jet sheet and $v_{1J}$ and $v_{2J}$, the components of velocity along the lines of curvature. Assume that the flow within the jet takes place in planes parallel to the undisturbed stream with no transverse transport of momentum. This condition may be expected to be closely true over most of the jet sheet.
near the wing and can be made exact at the trailing edge by design. This condition also implies that principal sections of maximum curvature would be sections by the planes $y = \text{const}$. This will not be true near the edges of the jet and in the regions of the sheet far downstream. This condition simplifies Eq. (6-38). Set $R_1 = R_2$, $v_{1J} = v_{xJ}$, and $v_{2J} = 0$. With this simplification we reduce Eq. (6-38) to the form of Eq. (6-16)

$$\Delta P = \frac{1}{R_x} \cdot \frac{v_{xJ}^2}{J(y)} = \frac{1}{R_x} \cdot J(y) \quad (6-39)$$

where $J(y)$ is the momentum flux per unit span.

Next, suppose the downwash field to be that produced by a distribution of elementary horseshoe vortices lying in the plane $z = 0$. The circulation around each element \( \gamma'(x, y) dy \) is related to $\Delta P$ at its origin $(x, y)$ by

\[
\Delta P = \rho_0 v_0 \gamma'(x, y) \quad (6-40)
\]

From Eqs. (6-39) and (6-40) it can be shown that for the region behind the wing

$$\rho_0 v_0 \gamma'(x, y) = -J(y) \frac{d}{dx} \left\{ \frac{w(x, y)}{v_0} \right\} \quad (6-41)$$

An induced downwash $w_i(x, y)$ in plane $z = 0$ may be defined as

$$w_i(x, y) = w(x, y) + \frac{1}{2\pi} \int_{-\infty}^{\infty} \frac{\gamma'(\xi, y)}{\xi - x} d\xi \quad (6-42)$$

where $\xi$ is the dummy variable in the $x$ direction, $w(x, y)$ is the total downwash in the plane $z = 0$ and the integral term represents the downwash corresponding to a two-dimensional distribution $\gamma'(\xi)$ of spanwise vorticity equal to the local section distribution $\gamma'(x, y)$.

By integrating over the horseshoe vortices one gets (see Ref. 148)

$$w(x, y) = \frac{1}{4\pi} \frac{\partial}{\partial \gamma} \int_{-b_x}^{b_x} \int_{\gamma - b_y}^{\gamma + b_y} \left\{ 1 - \frac{r^2}{(x - \xi)^2 + (y - \eta)^2} \right\} d\xi d\eta \quad (6-43)$$

where $\xi, \eta$ are dummy variables along $x$ and $y$ and

$$r^2 = (x - \xi)^2 + (y - \eta)^2 \quad (6-44)$$
From Eqs. (6-43) and (6-41) we can write
\[
\begin{align*}
  w_i(x,y) &= \frac{1}{4\pi} \frac{\partial}{\partial y} \int_{-b_x^2}^{b_x^2} \left[ \frac{\Gamma(\eta)}{\eta - y} \right] d\eta - \frac{1}{4\pi} \frac{\partial}{\partial y} \int_{-b_x^2}^{b_x^2} \left[ (r + |\eta - y|) \chi(\xi,\eta) \right] d\sigma d\eta \tag{6-45}
\end{align*}
\]

where
\[
\Gamma(\eta) = \int_{\xi}^{\infty} y(\xi,\eta) \cdot d\xi
\]

In principle, \( y \) can be determined from these equations. But the last term in Eq. (6-45) cannot be calculated except when \( x \) tends to \( \infty \). We get
\[
\begin{align*}
  w_i(x,\infty, y) &= \frac{1}{2\pi} \frac{\partial}{\partial y} \int_{-b_x^2}^{b_x^2} \left[ \frac{\Gamma(\eta)}{\eta - y} \right] d\eta \tag{6-46}
\end{align*}
\]

The induced downwash \( w_i(x,y) \) in the plane \( z = 0 \) cannot be calculated at finite distances \( x \) from the wing. Maskell and Spence use an approximation for replacing \( w_i(x,y) \) by an interpolation formula having the correct values \( v_0 \alpha_i \) at the wing and \( v_0' \alpha_i'(\infty) \) or \( v_0' \alpha_i(h) \) far downstream. Interpolation is so chosen as to make use of the known two-dimensional solution to find the loading in the streamwise direction of a three-dimensional wing. The approximation requires that the error in \( w_i \) is a small fraction of \( w \), which implies a large AR and that its minimum permissible value increases with jet strength.

Assume for simplicity that the loading and downwash distribution depend only on \( x/c \) (\( x \) measured from the leading edge and \( c \) the local chord) which requires both \( c(y) \) and \( J(y) \) to be elliptic and both \( \alpha \) and \( \tau \) to be constant over the span. Based on this, Maskell and Spence derive the following expression for the total lift coefficient, \( C_{LT}^{(3)} \), of a three-dimensional wing (Ref. 30)
\[
\begin{align*}
  \frac{C_{LT}^{(3)}}{C_{LT}} &= B + \frac{2\sigma B^2}{AR + (2/\pi) \cdot C_J} \tag{6-47}
\end{align*}
\]

where
\[
B = \frac{AR + (2/\pi) \cdot C_J}{AR + (2/\pi)(\partial C_{LT} / \partial \alpha) - 2} \tag{6-48}
\]
\[
\sigma = \frac{(1 - \lambda'')(C_J / \pi AR)}{\lambda'' - (1 - \lambda'')(C_J / \pi AR)} \tag{6-49}
\]
where
\[ \lambda'' = \frac{\tau_\infty}{\theta + \alpha} = \frac{2C_{LT}/(\theta + \alpha)}{\pi AR + 2(\theta C_{LT}/\partial \alpha)} - 2\pi (1 + \sigma) \] (6-50)

In the limit \( C_J = 0, \sigma = 0, B = AR/(AR + 2) \) we get the familiar relation
\[ \frac{C_{LT}^{(3)}}{C_{LT}} = \frac{C_L^{(3)}}{C_L' \frac{AR}{AR + 2}} \] (6-51)

Results calculated with Eq. (6-47) are plotted and compared with experimental data in Fig. 34.

6.5 The Induced Drag

6.5.1 A Lift-Induced Drag Relationship (Ref. 30)

Maskell and Spence also derive an exact overall relation between the total lift and the induced drag by a method which does not require \( w_i \) to be known. This method is described below.

For simplicity, consider the special case in which \( w_i = (v_0, \tau_\infty) \) is constant and \( \Gamma(y) \) is distributed elliptically over the span. From Eq. (6-46) we get the following expression for \( \Gamma(y) \).
\[ \Gamma(y) = v_0 \cdot \tau_\infty \cdot b \left\{ 1 - \left( \frac{y}{b/2} \right)^2 \right\}^{1/2} \] (6-52)

The thrust force per unit span is equal to \( J \) for a two-dimensional wing in ideal flow. But in the three-dimensional case, the thrust is less than the total jet momentum flux (\( J_T \)) because the jet does not ultimately become parallel to the undisturbed stream. Maskel and Spence define the difference between total jet momentum flux and the thrust in ideal flow as the induced drag.

\[ D_i = J_T - T_{Mi} \] (6-53)

Both the total lift and the thrust force, acting on a three-dimensional jet flapped wing can be split into contributions of the jet and contributions of the main stream. The jet contribution results from the horizontal and vertical components of \( J_T \) far behind the wing. The main stream contribution is obtained by considering the flow in a Treftz plane normal to the undisturbed main stream and far behind the wing (Refs. 30 and 2*). Expressions for \( L_T \) and \( T_{Mi} \) are
\[ L_T = \tau_\infty J_T + \frac{1}{4} \rho_0 v_0 \pi b^2 \omega_i \infty \] (6-54)
\[ T_{M_i} = (1 - \frac{1}{2} \tau_\infty^2) \frac{1}{T} - \frac{1}{8} \rho_0 \pi b^2 w_\infty^2 \]  
(6-55)

From Eq. (6-55) and (6-53) we obtain \( C_{D_i} \) by substituting for \( w_\infty \) and dividing by \( \frac{1}{2} \rho_0 v_0 S_W \)

\[ C_{D_i} = \frac{1}{4} \tau_\infty^2 (\pi AR + 2 C_J) \]  
(6-56)

Similarly from Eq. (6-54) one gets

\[ C_{LT} = \frac{1}{2} \tau_\infty (\pi AR + 2 C_J) \]  
(6-57)

From Eqs. (6-56) and (6-57)

\[ C_{Di} = \frac{C_{LT}^2}{\pi AR \left[ 1 + \frac{2 C_J}{\pi AR} \right]} = \frac{C_{LT}^2}{\pi AR \cdot \Omega} = \frac{C_{LT}^2}{\pi AR E} \]  
(6-58)

When \( C_J = 0 \) we get the well known formula

\[ C_{Di} = \frac{C_L^2}{\pi AR} \]  
(6-59)

For the case of non-elliptic loading, Eq. (6-58) becomes

\[ C_{Di} = \frac{2}{\pi AR + 2 C_J} C_{LT} \]  
(6-58a)

where \( e \) is the aerodynamic efficiency of a jet flapped wing.

The expression for \( C_{Di} \), Eq. (6-58), has also been derived independently by Berndt (Ref. 157) who further shows that the minimum induced drag for given values of \( C_{LT} \) and \( AR \), occurs when the spanwise load distribution is elliptic. This shows that in this respect, there is no difference between a conventional wing and a jet flapped wing.

6.5.2 Explanation of an Apparent Discrepancy

The main purpose of this section is to explain an apparent discrepancy which arises when the change in effective aspect ratio due to blowing, as predicted by the theoretically derived expression for the induced drag coefficient (Maskell and Spence, Ref. 30 or Sec. 6.5.1, Eq. 6-58) is compared with what one intuitively expects from the general flow picture. If the jet sheet is considered merely as a chordwise
extension of the wing (mechanical flap analogy), the general flow picture suggests that the effective aspect ratio of a wing without blowing is reduced by blowing. Maskell's and Spence's relation for the induced drag coefficient (see Eq. (6-58)), however, indicates that the effect of jet blowing is to increase the effective aspect ratio and experimental results verify their prediction. It will be shown that this contradiction evidently results from an improper choice of lift for this comparison. (We are indebted to Prof. B. Etkin for suggesting this idea).

In Eq. 6-58, the induced drag of a jet flapped wing is related to the total lift which is the sum of the circulation lift \( L_C \) and the impulse lift \( J_T \sin \tau_\infty \). Imagine now that the jet sheet of this wing is replaced by vortex sheets or by a chordwise extension (analogous mechanical flap) of the wing. It is known that such a wing produces circulation lift only. Therefore, in order to make a proper comparison an expression for \( C_{D_i} \) in terms of \( C_{LC} \) (instead of \( C_{LT} \)) has to be derived. From Eq. (6-54) follows

\[
L_C = L_T - J_T \sin \tau_\infty = \frac{1}{4} \pi b \rho_0 v_\infty^2 \tau_\infty
\]

which, in non-dimensional form is

\[
C_{LC} = \frac{1}{2} \pi \frac{AR}{\tau_\infty}
\]

Rewriting Eq. (6-56) as

\[
C_{Di} = \frac{1}{4} \pi AR \tau_\infty^2 \left( 1 + \frac{2C_J}{\pi AR} \right)
\]

and combining it with Eq. (6-61), we get

\[
C_{Di} = \frac{C_{LC}^2}{\pi AR \left[ \frac{1}{1 + 2C_J/\pi AR} \right]} = \frac{C_{LC}^2}{\pi AR \cdot \frac{1}{\Omega}} = \frac{1}{\pi AR_E}
\]

This expression shows that the effective aspect ratio \( AR_E \) decreases with increasing \( C_J \) and that no discrepancy results for the \( AR_E \) change as predicted by either Maskell and Spence or the mechanical flap analogy method provided that the circulation lift \( C_{LC} \) is used for this comparison.

The interpretation of Eqs. (6-58) and (6-63), can be summarized as follows. Let \( C'_L \) and \( C'_{Di} \) denote the lift and the induced drag coefficient of a conventional wing. If we compare the jet flapped wing with a conventional wing under the condition \( C'_L = C_{LC} \) we get \( C'_{Di} < C_{Di} \), i.e., the effective aspect ratio of the jet flapped wing is smaller than that of the conventional wing. However, a comparison under the condition \( C'_L = C_{LT} \) furnishes \( C'_{Di} > C_{Di} \), which means that the effective aspect ratio of
the jet flapped wing is greater than that of the conventional wing. This improved performance is due to the additional impulse lift term in $C_{LT}$. The experimental results - based on $C_{LT}$ - verify the increase in effective aspect ratio, since the test results only reflect the total lift effect.

The induced drag can be obtained from either Eq. (6-58) or Eq. (6-63). However, Eq. (6-58) is the more practical one to use as $C_{LT}$ is easier to obtain, both theoretically or experimentally. The circulation lift follows from $C_{LC} = C_{LT}/\Omega$ if $C_{LT}$ is known.

An interesting expression for $C_{Di}$ is obtained from Eqs. (6-58) and (6-63)

$$C_{Di} = \frac{C_{LT}.C_{LC}}{\pi AR}$$

(6-63a)

6.6 The Method for the Jet Sheet Issuing From a General Point on the Lower Wing Surface (Ref. 100)

Consider the two-dimensional, steady motion of an ideal incompressible fluid past a thin symmetrical aerofoil at zero incidence. When a jet issues from a general point on its lower surface at an angle $\theta$ (see Fig. 35). The motion in both the mainstream and the jet is taken to be irrotational. The aerofoil and the jet are replaced by a plane and by curvilinear vortex sheets respectively. The strength of these curvilinear sheets is proportional to their curvature (see Sec. 6.4.1).

Let $\gamma$ and $\gamma_j$ represent the strength of the vortex sheets which replace the aerofoil and the jet respectively. One of the boundary conditions of the flow in the mainstream is that the velocity normal to the aerofoil surface is zero. This condition can for $(0 \leq x \leq c)$ be written as:

$$\frac{1}{2\pi} \int_0^c \frac{\gamma(l) \, dl}{x - l} + \frac{1}{2\pi} \int_{BC} \frac{\gamma_j(x - x') \, ds}{(x-x')^2 + \gamma'^2} = 0$$

(6-64)

where $l$ is the distance of a general point on the chord from the leading edge and $x'$, $y'$ are the coordinates of a general point on BC corresponding to an arc of length $s$ measured from B. The second term in this equation may be interpreted as the downwash on the aerofoil due to the jet and will be denoted by $-(\sqrt[2]{\pi}) \cdot w_x$.

The inversion of the integral equation (Eq. 6-64) is given by the following equation (see Ref. 100):

$$\gamma(\lambda) = \frac{1}{\pi \lambda^{1/2} (c-\lambda)^{1/2}} \left\{ \pi \int_0^c \gamma(l) \, dl - \int_0^c \frac{w(x) \lambda^{1/2} (c-x)^{1/2}}{\lambda - x} \, dx \right\}$$

(6-65)
The constant $\int_0^C (l) dl$ is determined from the condition that B (see Fig. 35) is a stagnation point of the mainstream. (Since $B_1$ and $B_2$ - referring to Fig. 35 - are stagnation points of the mainstream, the pressure across the jet must be equal to the stagnation pressure of the mainstream). The condition that the velocity normal to AD is zero has been satisfied above (see Eq. 6-64). Hence it is sufficient to write down the condition that the x-component of the velocity is zero at point B, the x-coordinate of which is $x_B$. This condition can be stated as

$$-v_0 + \frac{f(x_B)}{2} + \frac{1}{2\pi} P \int_{BC} \frac{f(x_B') ds}{(x_B - x')^2 + y'^2} = 0$$  \hspace{1cm} (6-66)

where $P$ denotes the Cauchy principal value of the integral and the second term represents the velocity contribution of the aerofoil vortex sheet. Thus a value for $f(x_B)$ may be obtained which, if substituted in Eq. 6-65 together with $x = x_B$ provides a value for $\int_0^C (l) dl$, if $w(x)$ is known. However, to determine $w(x)$, the equation of the line BC (shape of the jet) is needed. It can be obtained by means of iteration... The first step requires the assumption of an approximate jet shape (for details see Ref. 100).

If $\int_0^C (l) dl$ is known, the total lift follows as:

$$L_T = \rho_0 v_0 \left\{ \int_0^C (l) dl + \int_{BC} f_J \cdot ds \right\}$$ \hspace{1cm} (6-67)

By substituting

$$f_J = \frac{1}{2} v_0 C_J \cdot \frac{C}{R}$$ \hspace{1cm} (6-68)

and

$$ds = R \cdot d\tau'$$

the total vorticity on the jet boundaries may be written as:

$$\int_{BC} f_J ds = \frac{1}{2} C_J \cdot \tau \cdot v_0 \cdot C$$ \hspace{1cm} (6-69)

Similarly, expressions for $C_M$ and $C_P$ can be derived (see Ref. 100).

It is of interest to note that the significant difference between the models used in this method and in the thin aerofoil theory method (Refs. 16 and 101) is the location of the jet exit. It changes the boundary condition required for the determination of the constant in the solution of the integral equation for the vorticity on the aerofoil. However, it can be shown that the solution obtained for the jet issuing from the trailing edge (as in Refs. 16 and 101) is the limiting case of the solution as presented in this section.
6.7  An Approximate Method for the Calculation of Pressure Distributions
(Ref. 17)

6.7.1 The Infinite-Aspect-Ratio Jet-Flapped Wing

Kuchemann considers a two-dimensional aerofoil in inviscid, incompressible flow with the jet emerging from the lower surface somewhere near the T.E. Assuming that $C_{LP}$ and $C_J$ are known he derives a simple method for the determination of the chordwise pressure distribution over a jet flapped aerofoil. Spence's equation for $C_{LT}$ (Eq. 6-29a) is used in the derivation of the following equation:

$$\frac{C_{LP}}{\theta} = 3.54 C_J^{1/2} - 0.675 C_J + 0.156 C_J^{3/2} \quad (6-70)$$

This method is very suitable for design purposes and represents a useful supplement to the more complicated method of Spence. Further, Kuchemann's method can be used for cambered aerofoils of non-zero thickness, for wings of finite aspect ratio and for particular cases of swept wings. This method brings out the fact that a saddleback-chordwise loading is typical for jet flapped aerofoils. It is this saddle-back loading which is the basic reason for most of the aerodynamic advantages of the jet flap (see Fig. 36A).

Based on the idea of the saddleback-chordwise loading, Kuchemann derives two methods. The first and simpler of the two accounts only for the total lift, but not for the jet induced (external) pressure thrust. This is because the method assumes a symmetrical chordwise loading (fore and aft of the mid chord point). This symmetrical chordwise loading consists of two flat plate loadings $\ell_1(x)$ and $\ell_2(x)$ one producing a uniform upwash and the other a uniform downwash (Fig. 36A).

$$\ell(x) = \ell_1(x) + \ell_2(x) \quad ; \quad \ell_2(x) = \ell_1(1-x) = \frac{2}{\pi} C_{L2} \left( \frac{x}{1-x} \right)^{1/2}$$

$$C_{LP} = C_{L1} + C_{L2} = \theta \cdot (3.54 C_J^{1/2} - 0.675 C_J + 0.156 C_J^{3/2}) \quad (6-71)$$

A more refined solution, without sacrificing the simplicity of the above, is presented by Kuchemann in his second method. In it, the chordwise loading is made asymmetric fore and aft of the mid chord point, thereby satisfying the jet induced pressure thrust condition as well. Simplicity is retained by the fact that the flat-plate loadings $\ell_1(x)$ and $\ell_2(x)$ are left the same and a third flat plate loading $\ell_3(x)$ is added (Fig. 36B). The third flat-plate loading produces a uniform downwash which exactly compensates for the upwash induced on the aerofoil surface by the jet. This upwash is assumed to be constant along the chord of the jet.

The following relations (Ref. 17) can be written down:

$$C_{L1} = C_{L2} \quad ; \quad C_{LP} = C_{L1} + C_{L2} + C_{L3}$$

$$C_{L1} = C_{L2} = \frac{1}{2} \left( C_{LP} - C_{L3} \right) \quad (6-72)$$
and further:

\[
\ell (x) = \ell_1(x) + \ell_2(x) + \ell_3(x)
\]

\[
\ell_1(x) = \frac{2}{\pi} C_{L1} \left( \frac{1-x}{x} \right)^{1/2}
\]

\[
\ell_2(x) = \frac{2}{\pi} C_{L2} \left( \frac{x}{1-x} \right)^{1/2}
\]

\[
\ell_3(x) = \frac{2}{\pi} C_{L3} \left( \frac{1-x}{x} \right)^{1/2}
\]

(6-73)

The expression for \( C_{L3} \) is derived from the external pressure thrust condition for the ideal case where

\[
C_{TP} = C_{TP'} = C_J (1 - \cos \theta)
\]

(6-74)

By equating the algebraic sum of the suction forces at the trailing and leading edge - which are now no longer equal - the following equation for \( C_{L3} \) follows:

\[
C_{L3} = 2 \pi \frac{C_J}{C_{LP}} (1 - \cos \theta)
\]

(6-75)

with \( C_{L3} \) known, \( \ell(x) \) for a two-dimensional wing can be calculated.

6.7.2 The Finite Aspect Ratio Jet Flapped Wing

The above derived expression, \( \ell(x) \), for chordwise loading of a two-dimensional wing in inviscid, incompressible flow is now extended to allow for finite aspect ratio effects (see Fig. 37). The altered expression for \( \ell(x) \) is derived in Ref. 17 as:

\[
\ell(x) = \frac{\sin \pi n}{2 \pi n} \left\{ \left( \frac{1-x}{x} \right)^n + \left( \frac{1}{1-x} \right)^n \right\} C_{LP}^{(3)}
\]

(6-76)

where \( C_{LP}^{(3)} \) is the \( C_{LP} \) of a wing of finite aspect ratio and \( n \) is the index parameter which depends on the aspect ratio. The value of \( n \) lies between 1/2 and 1 (\( n = 1/2 \) for \( AR = \infty \); \( n = 1 \) for \( AR = 0 \)).

A comparison of Kuchemann's theories for the two-dimensional and three-dimensional jet flapped wing indicates that its basic characteristics are not affected by the aspect ratio. Only the numerical values of the various parameters are of course different. For instance, the chordwise loading still remains saddleback in principle, its L.E. suction peak-largest at \( \theta = 90^\circ \) - gradually decreasing with decreasing \( \theta \). It is interesting to note that for the special case of a wing with an elliptic spanwise loading and a 90-degree jet, \( n = 1/2 \). Another noteworthy point, mentioned by Kuchemann is (see Fig. 38A) that the jet flap principle is
better suited for a wing of moderate aspect ratio than for a low aspect ratio wing.

6.7.3 Aerofoil Thickness and Camber Effects

Thick and cambered aerofoils are widely used (e.g., the camber for delaying L.E. separation). The infinite suction peaks, which are present on their flat plates are reduced by thickness. In this section Kuchemann’s theory is modified to allow for non-zero thickness and camber of jet flapped wings. Making use of the method described by Weber (Ref. 20) for the correction of non-zero thickness, and of Brebner’s family of camber lines (Ref. 21) to correct for non-zero camber, the following expression for the pressure distribution over a thick cambered aerofoil can be derived:

\[
C_p = 1 - \frac{1}{1 + (S(1)(x))^2} \left\{ \cos \alpha' \left(1 + \frac{V_{xJ}}{V_0} \right) \pm \frac{\gamma(x)}{2 V_0} \right\}
\]

\[
+ \left[ \left( \frac{C_{L1} + C_{L3} \sin \alpha'}{2 \pi} \right) \left( \frac{1 - x}{x} \right)^{\frac{1}{2}} + \frac{C_{L2}}{2 \pi} \left( \frac{x}{1 - x} \right)^{\frac{1}{2}} \right] \left(1 + S(3)(x) \right) \right\}^2
\]  

(6-77)

where \(S(1)(x), S(2)(x)\) and \(S(3)(x)\) are functions of shape of the aerofoil. They can be determined with the help of Ref. 20. The value for \(\gamma(x)\) can be taken from Brebner’s family of camber lines (Ref. 21). The angle of incidence \(\alpha'\) of the aerofoil in two-dimensional flow may be interpreted as the effective incidence of a section of an unswept wing of finite span, provided the aspect ratio is not too small. The following equation furnishes \(\alpha'\).

\[
\alpha' = \frac{1}{2 \pi} (\alpha - \alpha_i) \left( \frac{\partial C_{LT}}{\partial \alpha} \right)
\]  

(6-78)

where \(\frac{\partial C_{LT}}{\partial \alpha}\) is the sectional lift curve slope (to be determined for thin aerofoils from Spence’s theory - Ref. 16 and Ref. 101); and \(\alpha_i\) is the induced angle of attack, determined by the relation:

\[
\alpha_i = C_{LT} / (XAR + 2C_J)
\]  

(6-79)

where \(C_{LT}\) is the lift coefficient of the finite aspect ratio wing.

Equation (6-78) can also be used for an unswept wing of moderate aspect ratio when \(C(3)\)\(_P\), \(C_J\) and \(\tau\) are known. \(C_{L1}, C_{L2}\) and \(C_{L3}\) are to be determined from equations (6-68) and (6-71) and \(C_{LP}\) follows from:
where
\[ \alpha_e = \alpha - \alpha_i \]  

and \( [C_{LP}]_{\alpha_e=0} \) is to be determined from Spence's theory, Eq. (6-71).

Further, \( \frac{v_{xJ}}{v_0} \), the velocity increment due to the jet (mainly mixing, see Chapt. V, p. 100 of Ref. 129) is given by:

\[ \frac{v_{xJ}}{v_0} = 0.005 \left( \left( \frac{C_T}{2 \alpha_e c} \right)^{\frac{1}{2}} - 1 \right) \]  

This method as demonstrated in Fig. 38B, can be applied to aerofoils of reasonable shape. A noteworthy point is that the approximate Eq. (6-77) for the non-zero thickness wing becomes exact (Eq. 38 of Ref. 17) for aerofoils of elliptic section shape.

### 6.7.4 Sweep Effects

Finally, sweep effects will be briefly discussed. In general, swept wings have a non-uniform loading along the span (the lift curve slope is higher and the chordwise loading is peakier near the tips). This non-uniformity, due to essentially three-dimensional flow phenomena, is the cause of many of the undesirable features of sweptback wings - premature flow separation, higher drag and unfavourable pitching moment characteristics. In combination with the jet flap, jet angle and momentum can be varied to remedy some of these unfavourable properties (e.g., stronger blowing at higher angle in the centre region than near the tips could be used for making the loading more uniform).

For a swept wing with a jet flap, a loading given by \( \lambda(x) = \lambda_1(x) + \lambda_2(x) + \lambda_3(x) \) would be needed where \( \lambda_3(x) \) is to compensate for the upwash induced by the jet. A calculation of \( \lambda_3(x) \) has not been carried out yet.

### 6.8 The Rheoelectric Analogy Method (Refs. 19 and 33)

This method is based on a model of the jet flapped wing which is practically identical with that of Spence. Further, as in the methods previously discussed, a linear approximation is used. By mathematical means and physical reasoning the problem is reduced to one of finding a harmonic function for the perturbation velocity potential (\( \phi(x, y) \)), set up by the presence of the aerofoil and the jet sheet.
For the determination of the velocity potential \( \phi(x, y) \), the perturbation velocity potential has to be known. It can be measured as an electric potential \( \psi(x, y) \) in a plane electrolytic tank of large size, being set up such as to represent a half plane. This analog representation includes the factors of shape, the full range of angles of attack and also the boundary conditions (Ref. 19), see Fig. 39.

The measurement of the electric potential at the point corresponding to the trailing edge, \( \psi_{T.E.} \), furnishes the circulation about the aerofoil which in turn gives the lift increase due to the jet sheet (Fig. 40A). This circulation about the aerofoil \( \Gamma_c \) is equal to \( 2 \cdot \phi_{T.E.} \), where \( \phi_{T.E.} \) is obtained from the measurement of \( \psi_{T.E.} \). So we have

\[
C_{LP} = 4 \frac{\phi_{T.E.}}{v_0 c} \tag{6-83}
\]

\[
C_{LT} = C_{LP} + C_J \sin \theta \approx C_{LP} + C_J \theta \tag{6-84}
\]

The determination of the distribution of the electric potential along the chord gives the pressure distribution on differentiation. Then the centre of pressure can be determined, after taking the jet reaction into account (Fig. 40B) from:

\[
x_{L.C.} = \frac{C_J + \phi(C_J) \cdot x_1}{\phi(C_J) + C_J} \tag{6-85}
\]

where

\[
x_1 = 1 - \frac{\int_0^1 \phi \, dx}{\phi_{T.E.}} \tag{6-86}
\]

Measurement of the electric potential variation over the jet portion proves the shape of the jet. A noteworthy advantage of this method is that a thin aerofoil of any shape (arbitrary lifting surface) and problems like ground interference can be investigated without any difficulty. But this method does not permit an easy determination of the streamlines. A slightly modified tank, however, eliminates this difficulty.

6.9 Theory of the Finite-Span Blowing Wing (Ref. 63)

In this section we shall consider Helmbold's theory of a finite-span blowing wing. In the original paper (Ref. 63) he also briefly discusses the two-dimensional case which is omitted here. The three-dimensional model considered below is an aerodynamically untwisted wing of elliptical plan form with \( C_J \) and \( \tau \) assumed to be constant in the spanwise direction. The lift distribution along the span is then elliptical. The jet sheet will move downstream in the flow like a solid body. Its cross section will remain elliptical \((\delta/c = \text{const})\), if no cross sectional deformations occur between infinity downstream and the T.E. of the wing. Unlike the two-dimensional case, the jet sheet is not completely turned to a direction parallel to that of the undisturbed flow upstream of the wing.
This results in a loss of horizontal jet momentum flux \((m_J v_J - m_J v_J \cos \tau_{\infty})\) at infinity downstream. The lift can be shown, applying the momentum theorem, to be

\[
L_T = m_J v_J \sin \tau_{\infty} + m v_o \sin \tau_{\infty}
\]

(6-87)

where \(m\) is the virtual mass flux of the trailing vortex system (the cylindrical air stream with span as its diameter). The assumption made in deriving the above equation is that the static pressure at downstream infinity is everywhere equal to \(P_o\) and that the magnitudes of the velocity of the main stream is \(v_o\). But its direction is different from that of the undisturbed main stream (Fig. 41). Expressions for \(m\) and \(m_J\) are given assuming \(\rho_J = \rho_o\) as:

\[
\begin{align*}
\rho_o \frac{\pi}{4} b^2 v_o \\
\rho_o \frac{\pi}{4} b \delta_o v_J
\end{align*}
\]

(6-88)

where \(\delta_o\) is the thickness of the jet sheet at midspan. From these equations we can derive

\[
\sin \tau_{\infty} = \frac{L_T}{m v_o + m_J v_J}
\]

\[
= \frac{L_T}{b} \frac{1}{\rho_o \left(\frac{\pi}{4}\right) \left(\delta_o v_J^2 + b v_o^2\right)}
\]

(6-89)

It is interesting to note for the limiting cases that:

1) for two-dimensional wings, \(\tau_{\infty}\) is zero

2) for the static case (i.e., \(v_o = 0\), \(\tau_{\infty} = \tau\)

It should be noted also that the first term on the right hand side of Eq. 6-87 is an impulse force term and that the second term associated with the main flow may be called a circulation lift, \(L_C\). This circulation lift causes an induced drag, \(D_i\) (see Fig. 41). In the usual way or from Fig. 41 follows:

\[
\begin{align*}
\bar{\alpha}_i &= \frac{\tau_{\infty}}{L_C} \\
F_C &= \bar{\alpha}_i + D_i
\end{align*}
\]

(6-90)

Where \(\bar{\alpha}_i\) is the induced angle of attack which in this three-dimensional case is the inclination to the vertical of the resultant pressure force \(F_C\) acting on the wing-jet combination.
Breaking down $F_C$ into its physical elements, using relevant information of the two-dimensional case and making the following assumptions, Helmbold derives an equation for $\sin \bar{\alpha}_i$. One of these assumptions is that that part of the turning of the main stream flow which causes the jet-induced lift, is practically completed close enough to the wing where its inclination is still not yet appreciably different from $\bar{\alpha}_i$. Another is that the subsequent turning of the main stream flow from $\bar{\alpha}_i$ to $T_{\infty}$ occurs at such far distances from the T.E. that its effect on the wing can be neglected. The equation for the determination of $\sin \bar{\alpha}_i$ is:

$$2 \pi (\sin \alpha \cdot \cos \bar{\alpha}_i - \sin \bar{\alpha}_i \cdot \cos \alpha) - C_E \sin \bar{\alpha}_i + (C_E + f(C_E))(\sin \tau \cdot \cos \bar{\alpha}_i - \cos \tau \cdot \sin \bar{\alpha}_i)$$

$$- f(C_E) \left\{ \frac{\sin^{n+1} \bar{\alpha}_i}{\sin^n (\tau \sigma \bar{\alpha})} \right\} = \pi AR \cdot \sin \bar{\alpha}_i \quad (6-91)$$

where $n \geq 2$ and $C_E$, the excess jet momentum coefficient is

$$C_E = 2 \left( \frac{v_J^2 - v_o^2}{v_o^2} \right) \frac{\delta}{c}$$

In the limiting case where $\tau_{\infty}$ tends to $\tau$, $C_E$ tends to infinity and an approximate expression for $f(C_E)$ is given by $\sqrt{C_E}$. 

Equation (6-91) can be solved graphically. The following approximate equation sufficient for moderate values of $\sin \bar{\alpha}_i$, say, smaller than 0.4, is obtained by omitting all terms of higher than the second degree in $\sin \bar{\alpha}_i$

$$M \left(1 - \sin^2 \bar{\alpha}_i\right) = N \cdot \sin \bar{\alpha}_i \quad (6-92)$$

where

$$M = 2 \pi \sin \alpha + (C_E + f(C_E)) \cdot \sin \tau$$

$$N = 2 \pi \cos \alpha + \pi AR + C_E (1 + \cos \tau) + f(C_E) \cdot \cos \tau$$

The solution of Eq. (6-92) is:

$$\sin \bar{\alpha}_i = \frac{M}{(M^2 + N^2)^{1/2}} \quad (6-93)$$

From the above equations we can derive the following expressions.
The force in the direction of thrust acting on a finite jet flapped wing in an ideal flow is given by:

\[
[T_M] = m_J (v_J \cdot \cos \tau_\infty - v_0) - D_i
\]

which in coefficient form is

\[
[C_{TM}] = C_J (1 - 2 \sin^2 \alpha_i) - 2 C_q - C_{Di}
\]

where \( C_q = \frac{v_J \delta}{v_o c} \equiv \text{mass coefficient.} \)

It should be realized that the results derived above for the three-dimensional jet flapped wing are approximate (first order) only because of simplifying assumptions such as constant static pressure at infinity downstream, undeformed jet sheet and the account for the overall effect of loss of curvature due to the finite aspect ratio. It should also be noted that the induced drag, as defined by Helmbold, does not include the loss of horizontal momentum flux, \( C_J(1 - \cos \tau_\infty) \), at infinity downstream. This definition is different from that used by Maskell and Spence (see Sec. 6.5). Helmbold gets the term \( 2 C_q \) in Eq. (6-97) because he considers the case where the jet mass flow is taken from the main stream.

Next we proceed to derive the relationship between the induced drag \( (D_i) \) and circulation lift \( (L_C) \) as defined by Helmbold for a three-dimensional jet flapped wing. (This derivation is not given in Ref. 63). His definitions of \( L_C \) and \( D_i \) are:

\[
L_C = L_T - J_T \cdot \tau_\infty
\]
\[
D_i = D_i - 1/2 J_T \cdot \tau_\infty^2
\]

From Eqs. (6-54) and (6-98) one obtains

\[
L_C = \frac{1}{4} \pi \rho \mathbf{v}_0 \cdot \mathbf{b} \cdot v_0 \cdot \tau_\infty
\]

and from Eqs. (6-56) and (6-99)

\[
D_i = \frac{1}{8} \pi \rho \mathbf{v}_0 \cdot \mathbf{b} \cdot v_0 \cdot \tau_\infty
\]

In non-dimensional form
\[ C_{LC} = \frac{1}{2} \pi m \alpha \tau_\infty \]  \hfill (6-102)

\[ \bar{C}_{D_i} = \frac{1}{4} \pi m \alpha \tau_\infty \]  \hfill (6-103)

Therefore

\[ \bar{C}_{D_i} = \frac{C_{LC}^2}{\pi mAR} \]  \hfill (6-104)

Hembold's \( \bar{C}_{D_i} \) will be compared with Maskell and Spence's \( C_{D_i} \) in Sec. 6.11.2.

6.10 Source-Type Flow Method (Ref. 35)

Considering the two-dimensional, irrotational, steady motion of an infinite ideal incompressible fluid past a thin symmetrical aerofoil with a jet issuing from the trailing edge, Woods derives an expression for the lift by extending the analysis for a source-type flow to that of a jet-type flow (see Fig. 42).

Before describing the method, the definitions of the terms, source and jet-type flow have to be given. The source-type flow can be defined as a flow in which a source located on or within an aerofoil produces fluid at the same total head as the main stream flow (Fig. 42A). Stream lines \( \text{D}H_\infty \) and \( \text{D}'H'_\infty \) separate the source fluid from the main stream, across which pressure and velocity are continuous. In the case of jet-type flow, the source produces fluid at a different total head and across \( \text{F}H_\infty \) and \( \text{C}H'_\infty \), only pressure is continuous. Since the velocity is not continuous, the lines \( \text{F}H_\infty \) and \( \text{C}H'_\infty \) represent vortex sheets (see Fig. 42B).

Let us consider the source-type flow first. Three stagnation points \( \text{(B, D & D')} \) are to be fixed in order to define an unique flow. Assuming D to coincide with F - Joukowski's trailing edge condition - and using the values of the circulation and the source strength, the location of the stagnation points B and D' follows. But for a given value of circulation, there is only one source strength that makes the duct exit, C, a stagnation point. Woods derives the following expression for the total lift and pitching moment coefficients (about the mid-chord point) in the case of a source-type flow. These expressions are approximate and valid only for small values of \( C_q \) and \( \alpha \). This limitation in magnitude permits linear superposition of their effects.

\[ C_{LT} = 2 \pi \alpha + 4 \theta \left( \frac{2 C_q}{\pi} \right)^{1/2} \]  \hfill (6-105)

\[ [C_{M}]_x = 0.5 = \left( \frac{\pi}{2} \right) \cdot \alpha \]  \hfill (6-106)
If one assumes that a jet of a given momentum and deflection angle has essentially the same effect on the main stream as a source-type flow of the same momentum and deflection angle, the above results can be extended to the jet-type flow. In other words, it is suggested that the lift and moment coefficients for the jet-type flow tend continuously to the source type flow values given by Eqs. (6-105) and (6-106), as the total head of the jet flow approaches that of the main stream. The solution for the jet-type flow can be deduced from the above equations by substituting $C_J$ for $2C_q$ and by adding a further term which originates from the vorticity distribution induced on the profile by the vortex sheets bounding the jet. Considering the source-type flow as a special case of jet-type flow, the expressions for $C_{LT}$ and $C_M$ in this special case are

$$C_{LT} = 2\pi \alpha + 4 \theta \left(\frac{C_J}{\pi}\right)^{\frac{1}{2}}$$  \hspace{1cm} (6-107)

$$\left[C_M\right]_{x=0.5} = \frac{\pi}{2} \cdot \alpha$$  \hspace{1cm} (6-108)

Next we have to find the additional term due to the vortex distribution in the jet. Taking this term into consideration and assuming $C_J$ to be constant, we can derive the following equation

$$C_{LT} = 2\pi \alpha + 4 \theta \left(\frac{C_J}{\pi}\right)^{\frac{1}{2}} + \frac{1}{2}(C_J - 2C_q)$$ \hspace{1cm} (6-109)

the function, $(f)$, has been determined by Woods (see Ref. 35) for the case of small $(C_J - 2C_q)$-values on the assumption that the jet shape remains unchanged during the increase of the total head of the jet. We finally get:

$$C_{LT} = 2\pi \alpha + 4 \theta \left(\frac{C_J}{\pi}\right)^{\frac{1}{2}} \left\{ 1 + 0.76 \left(1 - \frac{2C_q}{C_J}\right) \right\}$$ \hspace{1cm} (6-110)

$$\left[C_M\right]_{x=0.5} = \frac{\pi \alpha}{2} + \frac{0.76 \theta}{(\pi)^{\frac{1}{2}}} \left(1 - \frac{2C_q}{C_J}\right)$$ \hspace{1cm} (6-111)

$$\left[\frac{x_{L,C}}{C}\right]_{x=0} = 0.5 - 0.19 \left(1 - \frac{2C_q}{C_J}\right) \frac{1 - 2C_q}{1 + 0.76 (1 - 2C_q/C_J)}$$ \hspace{1cm} (6-112)

Equations (6-111) and (6-112) are derived in a similar way as Eq. (6-110).
6.11 Comparative Remarks

6.11.1 Comparison of Theories

In this section an attempt is made to present a comparative study of the various methods discussed. The authors are of the opinion that of all the methods discussed, Spence's thin-aerofoil theory method (Sec. 6.4) is the most gratifying as far as numerical analysis is concerned. His method has the very useful feature of following the widely used and accepted methods of thin-aerofoil theory. In a somewhat similar way, this applies also to flap-analogy theories. But preference is given to Spence's method (Sec. 6.4.1) as his theory predicts the location of the L.C. and A.C. in better agreement with experimental results than flap-analogy methods (see Sec. 6.3.1).

If one is interested only in the pressure distribution round a jet-flapped wing, Kuchemann's approximate method is considered most useful (see Sec. 6.7). It should, however, be pointed out, that the first calculation of the pressure distribution around a thick and cambered aerofoil section with a jet flap would be fairly lengthy. Subsequent calculations for the same aerofoil section under different settings would be less laborious.

In a report published later than Spence's paper (Ref. 16), Woods (Ref. 92) shows that his source-type flow method leads to the same integro-differential equation and results as those derived by Spence. We believe, however, that the thin-aerofoil theory method is preferable to the source-type flow method because the former is more practical for numerical solutions and is based on more familiar ground. Another reason is that Woods does not give any theoretical reasoning for the validity of replacing \( 2C_q \) by \( C_J \) (see Sec. 6.10). Woods himself says that Spence has carried the numerical work of solving the integral equation and deriving values for the lift and moment to a stage which renders his work (showing the equivalence of the integro-differential equations (Ref. 92)) obsolete.

6.11.2 Comparison of Induced Drag Expressions

This comparison deals with treatments of the induced drag and downwash by Helmbold (Ref. 63, see Sec. 6.9), by Maskell and Spence (Ref. 30, see Sec. 6.5) and by Jacobs and Paterson (Ref. 31, see Sec. 6.3.2). Helmbold relates the induced drag to the circulation lift; Maskell and Spence relate it to the total lift, Jacobs and Paterson relate it to the pressure lift. First let us compare Refs. 30 and 63.
Maskell and Spence define the induced drag \( D_i \) as the difference between the total jet momentum flux \( J_T \) and the measured thrust in ideal flow. Helmbold defines the induced drag \( D_i \) as the difference between the actual horizontal jet momentum flux at infinity and the measured thrust in ideal flow. These two induced drag definitions:

\[
D_i = J_T - T_{Mi} \tag{6-113}
\]

\[
\bar{D}_i = J_T \cos \tau_\infty - T_{Mi} \tag{6-114}
\]

are related as:

\[
D_i = \bar{D}_i + J_T (1 - \cos \tau_\infty) \tag{6-115}
\]

Since \( \tau_\infty = 2 \bar{\alpha}_i \)

\[D_i \text{ becomes} \]

\[D_i = \bar{D}_i + J_T \sin 2 \bar{\alpha}_i \tag{6-116}
\]

Writing Eq. (6-117) in non-dimensional form, one gets

\[C_{D_i} = \frac{\bar{C}_{D_i}}{\beta} + 2 C_J \sin 2 \bar{\alpha}_i \tag{6-118}
\]

We know from Eq. (6-95), that \( \bar{C}_{D_i} = \pi \beta \sin 2 \bar{\alpha}_i \)

Therefore,

\[C_{D_i} = (\pi \beta + 2 C_J ) \sin 2 \bar{\alpha}_i \tag{6-119}
\]

Helmbold's expression for \( C_{LT} \) is given by (see Eq. (6-94))

\[C_{LT} = (\pi \beta + 2 C_J ) \sin \bar{\alpha}_i \cos \bar{\alpha}_i \tag{6-94}
\]

Therefore,

\[C_{D_i} = C_{LT} / (\pi \beta + 2 C_J ) \cos 2 \bar{\alpha}_i \tag{6-120}
\]

When \( \bar{\alpha}_i \) is small, Eq. (6-120) reduces to that derived by Maskell and Spence (Eq. 6-58). From Eqs. 6-58, 6-63 and 6-104 follows

\[C_{D_i} / \bar{C}_{D_i} = C_{LT}/C_{LC} = \Omega \tag{6-121}
\]
Next, let us consider the method used by Jacobs and Paterson (Ref. 31), who correlate the induced drag to the pressure lift. Their reasoning is based on the assumption that the vortices originating from the jet sheet have less effect on the downwash than those from the wing. This is attributed to their finite distance below the wing. They also postulate that the downwash at the wing has very little or no effect on the jet reaction force. Another reason they give for using $C_{LP}$ instead of $C_{LT}$ is the increase in effective aspect ratio due to the outward movement of the tip vortices (Ref. 31) caused by the jet.

It will be shown that Jacob's and Paterson's $C_{D_i} = C_{LP} / \pi AR$ follows from Eq. (6-58). First assume that $\pi AR \gg 2 C_J$. Then Eq. (6-58) becomes

$$C_{D_i} \propto C_{LT}^2 / \pi AR$$

(6-122)

Next, assume that $C_J$ and $\theta$ are so small that $C_J \cdot \sin \theta$ (in $C_{LT} = C_{LP} + C_J \cdot \sin \theta$) can be neglected. Then $C_{LT} \propto C_{LP}$ and

$$C_{D_i} \propto C_{LP}^2 / \pi AR$$

(6-123)

The following expressions for the induced drag have been also suggested (from Ref. 28):

$$C_{D_i} = \frac{C_{LT} \cdot C_{LP}}{\pi AR \cdot e}$$

(6-124)

where $e$ accounts for non-elliptic spanwise loading and also for the effect of blowing. The other expression is (from Ref. 71):

$$C_{D_i}^* = C_{TP_i} - C_{D_i} - \Delta C_{DP} = \frac{C_{LJ}^2}{\pi AR \cdot e^*}$$

(6-125)

This $C_{D_i}^*$ has no longer any resemblance with the induced drag of Maskell and Spence. The factor $e^*$ has to account for non-elliptic loading, for the ideal jet induced pressure thrust ($C_{TP_i}$) and for the variation in profile drag ($\Delta C_{DP}$) with lift.

These expressions are not as easy to use as Eq. (6-58). We are of the opinion that the expression for $C_{D_i}$ derived by Maskell and Spence is the most useful and practical one.
We proceed now to examine the limitations of the assumption used in Refs. 33 and 138 (see also Ref. 142). The assumption is:

\[ \frac{dC_{DT}}{dC_{LT}^2} = \frac{dC_{D}'}{dC_{LT}^2} \]  

(6-126)

Since \( C_{DT} \) can be written as

\[ C_{DT} = \left[ C_{DT} \right]_{C_{LT}=0} + \frac{C_{LT}^2}{(x_{AR} + 2C_J) \cdot \bar{e}} \] 

(6-127)

then

\[ \frac{dC_{DT}}{dC_{LT}^2} = \frac{1}{x_{AR}(1 + \frac{2C_T}{x_{AR}}) \cdot \bar{e}} + \frac{dC_{D}'}{dC_{LT}^2} = \frac{1}{x_{AR} \cdot \bar{e}} \] 

(6-128)

But if \( x_{AR} \gg 2C_J \) we get

\[ \frac{dC_{DT}}{dC_{LT}^2} \approx 1/(x_{AR} \cdot \bar{e}) = \frac{dC_{D}'}{dC_{LT}^2} \] 

(6-129)

From this equation we see that \( \frac{dC_{DT}}{dC_{LT}^2} \) can be considered equal to \( \frac{dC_{D}'}{dC_{LT}^2} \) only for quasi two-dimensional wings or for three-dimensional tests at very small \( C_J \)-values.

Summarizing, we have presented two theoretically derived expressions for the induced drag coefficient, \( CD_i \) and \( \bar{CD}_i \). They are (see Eq. (6-115)) related as

\[ CD_i = \bar{CD}_i + C_J \cdot (1 - \cos \tau_\infty) \] 

(6-130)

It is known that the induced drag of a conventional wing originates from the inclination to the vertical of the pressure force on the wing. In the case of a jet flapped wing, the induced drag results from two sources. The first one (analogous to the conventional wing) is due to the inclination of the resultant pressure force on the wing - jet sheet combination \( (\bar{CD}_i = CLC. \sin \alpha_i) \) which may be called "vortex drag". The second one \( (C_J(1 - \cos \tau_\infty)) \) originates from the loss in thrust due to a non-zero downwash angle at infinity \( (\tau_\infty) \) and could be called "induced loss of thrust". We see, that Helmbold's \( CD_i \) only accounts for the pressure force contribution to the induced drag. Comparing \( CD_i \) and \( \bar{CD}_i \) as to their usefulness in practice, it can be said that \( CD_i \) is (due to \( C_{LT} \)) easier to determine, incorporates all finite aspect ratio effects, is best suited for the evaluation of balance measurements and the interpretation of the total drag. In the light of the aforesaid, we suggest that the induced drag of a three-
We have received Ref. 167, a lifting line theory for jet flapped wings.

VII. THEORETICAL ANALYSIS OF INTERFERENCE EFFECTS AND LEADING EDGE MODIFICATIONS (Refs. 31, 54, 55, 124 and 139)

7.1 Introduction

A brief account of theoretical treatment of the ground effect by Paterson (Refs. 31 and 55) and Hugget (Refs. 124 and 139) will be presented first. Next, work on fuselage interference effects and on effects of leading edge modifications by Jacobs and Paterson (Refs. 31 and 54) will follow. However, as mentioned already in Sec. 6.8 these problems can be tackled also with the help of the rheoelectric analogy method without any difficulty (Refs. 19 and 33). A study of the ground interference is important because the jet flap's high lift characteristics can best be used for take-off and landing, which are close-to-the-ground operations.

7.2 Paterson's Method for Ground Effect (Refs. 31 and 55)

For a jet-flapped aerofoil the influence of the proximity of the ground is somewhat different and more serious than that for a conventional wing mainly due to higher lift coefficients and the modification of the jet sheet. Further, the changes in velocity at the wing change the jet coefficient which in turn alters the lift and pitching moment coefficient. For the analysis of this complex problem the method of images is used to determine the influence of the ground (under the condition of zero normal velocity at the ground). The forces on the tail need only be considered for the determination of the ground effect on the pitching moment.

7.2.1 Two-Dimensional Case

Let the wing be at a distance from the ground such that the total circulation may be considered to be concentrated at just one point. The change in velocity at the real wing due to the image is then given by:

\[ \Delta v = v_1 - v_0 = - \frac{\Gamma}{4 \pi h} \]  

(7-1)
where $\Gamma$ is the constant circulation. The new total lift, $L_{T1}$, is:

$$L_{T1} = \rho_0 v_1 \frac{\Gamma}{2} = C_{LT1} \rho_0 v_0 \frac{c}{2}$$

(7-2)

and the total lift coefficient referred to $v_0$ is:

$$C_{LT1} = C_{LT0} \left(1 + \frac{\Delta v}{v_0}\right) = C_{LT0} \left(1 - \frac{\Gamma}{4\pi h}\right)$$

(7-3)

where $C_{LT0}$ is defined as equal to $\rho_0 v_0 \frac{\Gamma}{2} \frac{1}{2} c$.

The new jet coefficient is then

$$C_{J1} = C_{J0} / \left(1 + \frac{\Delta v}{v_0}\right)^2$$

(7-4)

For this value of $C_{J1}$ we obtain a new lift coefficient $C'_{LT1}$ which, when referred to the velocity $v_0$ gives another new value for the lift coefficient, $C_{LT2}$, given by

$$C_{LT2} = C'_{LT1} \left(\frac{C_{LT1}}{C_{LT0}}\right)^2$$

(7-5)

By iteration the real lift in the presence of the ground can be calculated. Figure 43 compares calculated and experimental results. Agreement is only fair in the region of interest, i.e., for $h/c \leq 1$. A similar analysis for a model where the total circulation is not concentrated at just one point would give the pitching moment coefficient in the presence of the ground.

7.2.2 Three-Dimensional Case

The main effect of dimensionality is a reduction of the induced vertical velocity at the real wing due to the trailing vortices of the image vortex system. Wieselsberger (Ref. 64) obtained the following expressions for the resulting change in angle of attack and drag coefficient.

$$\Delta \alpha_i = -C_{LP} (\sigma - (57.3)) / \pi AR$$

(7-6)

$$\Delta C_D = C_{LP}^2 \sigma / \pi AR$$

(7-7)

where the subscript $G$ denotes the effect of the ground and $\sigma$ is Prandtl's interference coefficient and is given by the following approximation.
These changes are equivalent to those produced by a change in aspect ratio and the "EFFECTIVE" aspect ratio, $(AR)_{EFF}$, of the wing near the ground can be obtained from

$$(AR)_{EFF} = AR/(1-\sigma)$$

Finally, the change in lift coefficient can be calculated from

$$\left(\Delta C_{LT}\right)_G = \left(\Delta \alpha\right) \cdot \frac{dC_{LT}}{d\alpha}$$

if the lift curve slope is known.

7.3 Huggetts Theory for Ground Interference (Refs. 124 and 139)

Huggetts method introduces the idea of blockage (defined below) of the main stream flow by the jet and by the ground. This condition of blockage imposes an upper limit on the pressure lift.

Consider a jet issuing from a slot AB (see Fig. 44). CAD and EBF are the mixing regions. If PP' is a streamline, then AQ may be drawn such that

$$(m)_{across AP} = (m)_{across QP'}$$

and therefore

$$(m)_{across AQ} = 0$$

"Blockage" is defined as the condition under which Q becomes a point on YY'. It should be noted that the net mass flow across a line drawn from any point on XA to any point on YY' will be zero, when the condition of blockage occurs. Using this idea and the principle of the image vortex, a maximum pressure lift coefficient can be determined.

Now consider an aerofoil represented by a uniform distribution of vorticity along the chord (neglecting the effect of jet vorticity) in two-dimensional flow and at a distance $h$ from the ground. Let $\Gamma_c$ be the density of vortex distribution per unit length of chord. The presence of the ground is taken into account by the image vortex distribution (see Fig. 45A).

The net downstream velocity at point P is given by:
By considering the mass flow between the aerofoil and the ground and equating it to zero, we get

$$\Gamma = \frac{2\pi h}{v_o \cdot \frac{4h}{c} \cdot \tan^{-1} \frac{c}{4h} + \frac{1}{2} \cdot \ln \left[ \left( \frac{4h}{c} \right)^2 + 1 \right]}$$  (7-13)

Further we know that the pressure lift

$$L_P/\text{unit span} = C_{LP} \cdot \frac{\rho}{2} \cdot v_o \cdot c = \frac{\rho}{\rho_o} \Gamma \cdot v_{\text{mid chord}}$$  (7-14)

From Eqs. (7-12) and (7-14) one gets

$$C_{LP} = \frac{\frac{\Gamma}{\rho}}{2} \left( 1 - \frac{1}{\frac{\rho}{\rho_o} \cdot \frac{\Gamma}{v_o} \cdot \frac{\tan^{-1} \frac{c}{4h}}{4h} } \right) \cdot \frac{\Gamma}{v_o}$$  (7-15)

Using the $\Gamma$-value given by Eq. (7-13), $C_{LP_{\text{max}}}$ can be determined. In Fig. 134 comparison is made between the experimental and theoretical results. The agreement is good for $C_J$-values less than one.

An improved theory was presented in a recent paper (Ref. 149) by Hugget in which he accounts for the non-uniformity of the vortex distribution along the wing and the jet. In this method the jet is replaced by a vortex distribution (along the mean line) which, as we know, depends on the shape. The presence of the ground is taken into account by the image vortex distribution. The model used is shown in Fig. 45B. The unknown vortex distributions along the aerofoil and the jet, and the jet shape are determined by the following three conditions:

1) Blockage between the aerofoil and the ground.

2) Blockage between the jet and the ground.

3) Equilibrium across a thin jet.

A numerical solution of the equations describing the above three conditions is given in Ref. 149. In Fig. 135 four calculated points are shown and compared with curves obtained from experiments and the simple theory (Ref. 139).
In Refs. 145, 146, 147 and 150, additional work on the ground effect on the jet flap is presented.

7.4 Fuselage Interference (Ref. 31)

In this section some interference effects between fuselage and jet-flapped wing are briefly described. Consider a high-wing configuration at an angle of yaw (Fig. 46A). The side component \( v_{oy} \) causes a flow around the fuselage, thus inducing additional angles of attack which are asymmetrical along the wing span. Hence a rolling moment is produced. Assuming that the flow around the fuselage in the wing root region is two-dimensional, the additional angles of attack can be obtained by conformal transformation, if the case is limited to slender fuselages.

Figure 46B shows the variation of the induced rolling moment coefficient with the vertical location (mid to high) of the wing. This moment is independent of \( \alpha \), but varies with fuselage section, ratio of span to diameter of fuselage and wing planform. For a low-wing configuration the influence of the fuselage is equal and opposite to that of the high-wing and for the mid-wing no sidewash is induced.

The induced lift distribution may also influence the directional stability due to the fact that asymmetrical lift distributions induce sidewash which affects the efficiency of the vertical tail (Ref. 130). All these effects though not peculiar to the jet flap are enhanced in the case of jet-flapped wings due to their inherently higher lift curve slope as compared with the conventional wing (see Fig. 46B).

7.5 Leading-Edge Modifications (Refs. 31 and 54)

Jet-flap test results show (see Sec. 8.5.3) a downstream movement of the centre of pressure with increasing \( C_J \) and the tendency of leading edge separation. This leading edge separation will produce a loss in thrust. In order to counteract these short comings, leading-edge modifications were suggested.

Calculations done by Jacobs and Paterson to determine the effects of leading-edge modifications are based on Glauert's linearized theory which allows superposition of effects. In Ref. 31, an aerofoil with a droop nose and a straight trailing edge flap is considered. Using a method very similar to the one described in Sec. 6.3, the following expressions are derived for the lift coefficient and the additional pitching moment coefficient about the leading edge for the leading edge flap (Ref. 31):
where (see Fig. 22B)

\[ C_{LT} = \frac{4}{1 + \cos \gamma_0} \left\{ -\frac{\tau_1}{(1 + \cos \gamma_0)} \left( \pi - \gamma_1 - \sin \gamma_1 \right) + \frac{\tau}{\gamma_0 + \sin \gamma_0} \right\} \] (7-16)

\[ \left[ \text{Additional } C_M \right]_{L.E.} = \frac{2 \tau_1}{(1 + \cos \gamma_0)^2} \left\{ \pi - \gamma_1 - \sin \gamma_1 (\cos \gamma_1 + 2) \right\} \] (7-17)

where \( \tau_1 \equiv \text{angle of L.E. flap} \)

\( \gamma_1 \) and \( \gamma_0 \) are angle coordinates of leading and trailing edge flap hinge lines defined as follows

\[ x_1 = \frac{c'}{2} \left( 1 + \cos \gamma_1 \right) \]

and

\[ x_0 = \frac{c'}{2} \left( 1 + \cos \gamma_0 \right) \]

where \( c' \) is the chord of the wing including both, the leading and the trailing edge flap.

Exact calculations, using curved trailing edge flaps show that the droop nose procures a decrease in lift, an increase in pitching moment and a rearward shift in the centre of lift position. To compensate for these defects, an extensible type leading edge flap, which increases the chord and the area of the aerofoil, must be applied. Such a leading edge flap or droop nose would have the advantage of producing a forwardly inclined lift force. Its component in flight direction adds to the jet thrust component (see Fig. 47B). In addition, especially at \( C_J \)-values greater than unity, it causes the L.C. to move forward by an appreciable amount even at \( \tau_1 = 90^\circ \) (see Fig. 47A).

VIII. EXPERIMENTAL JET FLAP RESULTS

In this chapter, experimental information about the aerodynamic behaviour and characteristics of jet flapped wings is presented and analyzed.
8.1 The Separation Bubble Phenomenon

8.1.1 Its Formation

The formation of a closed full chord separation bubble on the upper wing surface - as shown schematically in Fig. 5, a transcription of an actual streamline photograph - is a phenomenon characteristic of the jet flap only. This bubble - the result of the tremendous upstream effect of the jet mixing and entrainment region (see Sec. 2.3) - is due to strong suction pressures produced by the ejector effect of the high momentum jet sheet on the main stream flow surrounding the wing.

In Ref. 28, the bubble phenomenon was investigated on a three dimensional wing at CJ-values up to 2.1. At zero incidence, no flow separation on the upper wing surface due to the inherent adverse pressure gradient at the L.E. could be detected. As \( \alpha \) was increased, a small bubble of laminar B.L. separation appeared at the inboard wing sections first and close to the L.E. with subsequent reattachment as a turbulent B.L. Without blowing (\( C_J = 0 \)), the separation bubble expanded only slightly chordwise until \( \alpha > 10^5 \) was reached. With the jet blowing the bubble behaviour was similar, but the angle of incidence at which the bubble began to expand chordwise decreased somewhat as \( C_J \) was increased.

In Fig. 48 - another transcription of actual streamline photographs - the changes in separation bubble size and shape with incidence are schematically illustrated. It shows also in what way the separation bubble is linked with stall.

8.1.2 The Separation Bubble and Stall

If the bubble burst open into a wake (Fig. 48) at the wing T.E., stall occurs which may be initiated by an increase either in \( \alpha \) or in \( \theta \). As \( \alpha \) is raised, the jet B.L.C. effect may reach the point where it is no longer strong enough to keep the bubble bottled up at the T.E. If, however, the jet coefficient \( C_J \) is increased and simultaneously \( \alpha \) is decreased, bursting of the separation bubble can be prevented (see Fig. 49). A more comprehensive discussion of the stall is given in Sec. 8.2.6.

8.2 The Lift Forces

8.2.1 Their Relative Magnitude

If a jet flapped wing is mounted on a lift-drag balance at zero incidence, the basic lift \( (C_{L'}) \) will be zero whether or not the jet is blowing (see Fig. 9A and B). In this case, the total lift \( (C_{LT}) \) is the sum of the jet induced pressure lift \( (C_{LJ}) \) and the jet reaction lift \( (C_{LR}) \).
In Fig. 50, typical lift coefficients for a symmetrical jet flapped wing at zero incidence are plotted versus the jet coefficient, for the jet sheet issuing at a jet deflection angle $\theta = 58.1^\circ$. It is obvious that it is not - as might be expected - the reaction lift which primarily contributes to the amazingly high total lift coefficients. It is the jet induced pressure lift which is by far the greater, and in itself is even larger than lift coefficients which can be generated by any other presently known and practical means.

From the above it follows that - at least in theory - the reaction lift alone is decidedly too small to make a pure jet lift system (deflected circular jet) a workable and economical suggestion (see Fig. 50). This assertion can still better be illustrated with the so-called lift gain factor ($G_L$), defined as $C_{LT}/C_{LR} = C_{LT}/C_J \sin \theta$. If plotted versus $C_J$ (see Fig. 51), it shows that one pound of reaction lift can be amplified by a factor of three (for higher $C_J$-values) or up to factors as high as 25 at small $C_J$-values. These amplifications can be obtained only if a wing and its inherent pressure lift ($C_{LP} = C_{LJ} + C'L$) are made to participate in the total lift generating system or if in other words, a jet flapped wing is used.

8.2.2 The Total Lift Coefficient

As mentioned previously in Sec. 2.2, there is - theoretically at least - no limitation for the rise of $C_{LT}$ (i.e., supercirculation) with increasing $C_J$. In practice, however, $C_{LT}$ is limited by flow separation at the L.E.

Total lift coefficients of the order of 12 were found in early jet flap experiments for $C_J = 4$ (see Fig. 52B). In later investigations, $C_{LT}$-values above 70 were recorded (Ref. 71) for pure jet flapped wings at $C_J$-values as high as 56.75 (Fig. 53).

8.2.3 Lift at Incidence

The change in lift coefficient of a two-dimensional jet flapped wing as a function of incidence and jet coefficient for a given jet deflection angle are shown in Fig. 52. This figure conveys two facts of interest:

a) that the slope of the lift curve increases slowly with $C_J$ and

b) that the stalling incidence decreases as $C_J$ increases.

Further, three-dimensional test results are reported in Refs. 71 and 28, where $C_{LT}$ is plotted as a function of $\alpha$ for various $C_J$-values and jet deflection angles. In Figs. 54 and 55, pure jet flap results are shown for two different aspect ratios and different ranges of $C_J$. Like the two-dimensional jet flapped wing, the lift curve slope increases slowly with $C_J$. The stalling incidence, however, seems to increase with $C_J$. 
Similar tests were also conducted with internal flow jet augmented flaps (Ref. 71), which gave about the same lift and drag coefficients as the pure jet flap.

8.2.4 Lift Curve Slope

If the test results from Ref. 23, presented in Figs. 52A and B are evaluated for the total lift curve slope \( \frac{dC_{LT}}{d\alpha} \) and plotted vs \( C_J \) for various \( \theta \)-values, Fig. 56 is obtained for the two-dimensional jet flapped wing. The \( \frac{dC_{LT}}{d\alpha} \) was found to vary roughly as \( \sqrt{C_J} \). (The tests of Ref. 23 were conducted with transition wires, the removal of which had only little effect on \( \frac{dC_{LT}}{d\alpha} \), except for \( C_J = 0 \) (no blowing)).

Ref. 28 quotes the following relation

\[
\frac{dC_{LT}}{d\alpha} = \frac{dC_{LR}}{d\alpha} + \frac{dC_{LP}}{d\alpha}
\]

\[
= 0.0175 \cdot C_J \cdot \cos (\theta + \alpha) + \frac{dC_{LP}}{d\alpha} \quad (8-1)
\]

where the angles are measured in degrees.

In Fig. 57, \( C_{LT} \) vs \( \alpha \) for \( AR \approx 20 \) is shown. No wall correction (Ref. 33) - due to a closed test section - was applied to \( \alpha \). It would have had the effect of increasing the effective incidence angle and thereby decreasing the lift curve slope. For a constant jet coefficient \( C_J = 0.5 \), Fig. 57 shows (see also Fig. 56) that \( \frac{dC_{LT}}{d\alpha} \) is practically independent of the jet deflection angle \( \theta \). This fact is further emphasized in Fig. 58 which shows this independence to apply also for jet flapped wings with symmetrical blowing and \( C_J \)-values up to 1.5. (Fig. 11 of Ref. 33 proves this to be true for the pure jet flapped wing). Tests conducted with a wing-body combination (\( \theta = 90^0; AR = 3; C_J < 0.2 \)) also indicate an increase of lift curve slope with increasing \( C_J \) (see Fig. 12A and B of Ref. 33).

8.2.5 The Stalling Envelopes

Figures 52 and 57 demonstrate for a two-dimensional wing that the stalling incidence decreases if either the jet coefficient or the jet deflection angle is increased. This experimental evidence is confirmed also by French test results (Ref. 33).

If transition wires are used on a three-dimensional jet flapped wing (\( AR = 6.8 \)), hardly any decrease in stalling incidence can be detected (see Fig. 5 of Ref. 28) at \( C_J < 1 \). At \( C_J \)-values greater than unity, even an increase seems to be indicated. Evidence obtained with a three-dimensional wing-body model aeroplane (\( AR = 3, \theta = 90^0, C_J < 0.2 \), very thin straight wings) confirms (see Fig. 12A of Ref. 33) the results of Ref. 28 for \( C_J \leq 1 \). Note that longitudinal trimming of the model complete with tail does not reduce the stalling incidence.
Tests undertaken with a jet control flap and blowing conducted over its upper and lower surface (see Fig. 59) show definitely again a decrease in stalling incidence with increasing jet deflection angle at constant \( C_J \) and \( AR \approx 20 \).

If e.g., in Figs. 52A and B, the stalling incidences are connected (broken lines), a stalling envelope results. Fig. 49 shows stalling envelopes for three jet deflection angles.

8.2.6 Lift and the Stall Behaviour

As we know, the lift coefficient of a conventional aeroplane of fixed geometry can be altered by one parameter only, the angle of incidence (or attack). The \( C_{LT} \) of a jet flapped wing, however, can be changed substantially, at constant incidence, by changing either the jet deflection angle \( \theta \) or the jet coefficient \( C_J \). Stability considerations of the jet flapped wing have to deal therefore with three derivatives, the conventional \( \partial C_{LT}/\partial \alpha \) (see Sec. 8.2.4), the \( \partial C_{LT}/\partial \theta \) and the \( \partial C_{LT}/\partial C_J \).

A. The Stability Derivative (\( \partial C_{LT}/\partial \theta \))

This derivative is quite critical. Figure 49 indicates that, especially under high lift conditions, a jet flapped wing can easily be stalled if the jet deflection angle \( \theta \) is increased indiscriminatively. Stall definitely occurs if the operational point A (Fig. 49), after \( \theta \) being increased at constant \( \alpha \) and \( C_J \) happens to fall on the stalling envelope for this particular jet deflection angle. An increase of \( \theta \) e.g. during take-off as a means of enhancing the lift is therefore a dangerous control manipulation, which possibly could lead to a catastrophic stall. A reduction of \( \theta \) during take-off, however, if no longer the maximum lift is required, is feasible and quite safe.

B. The No-Stall Aircraft

Another significant fact can be deduced from Fig. 49. The stalling envelopes obviously asymptote a specific angle of incidence, below which the jet flapped wing seems to be unstallable. This feature suggests the concept of a no-stall aircraft with (at least in theory) unlimited lift coefficients being produced by a combination of large jet deflection angles and high jet coefficients.

C. The Stability Derivative (\( \partial C_{LT}/\partial C_J \))

This derivative is less critical due to the fact (see Fig. 49) that at high \( C_{LT} \)-values it is not possible to stall a jet flapped wing merely by increasing \( C_J \). This statement may be supplemented as follows:
a) the danger of an incidence stall at any \( C_{LT} \) is diminished if \( C_J \) is increased (see line a - a in Fig. 52A)

b) compare two jet flapped wings at the same zero-incidence lift, but their jet deflection angles being different. The wing with the larger \( \theta \) will require the smaller \( C_J \)-value and hence be more prone to incidence stall (compare line b - b in Fig. 52B with line a - a in Fig. 52A).

8.2.7 Lift Gain Factor and Lift Efficiency

These factors establish the jet flap's superiority in creating lift over pure jet lift systems (flying bedstead, Ryan vertijet and other direct jet lift aircraft). They indicate by how much one pound of direct jet lift (jet reaction lift \( C_{LR} \)) can be amplified if a jet flapped wing is used.

A. The Lift Gain Factor (\( G_L \))

Davidson, in Ref. 12 uses the factor \( G_L = C_{LT}/C_{LR} \), which Dimmock (Refs. 24 and 32) introduced earlier as "magnification factor". In Fig. 51, \( G_L \) is plotted vs. \( C_J \) for two-dimensional jet flapped wings and various jet deflection angles. It seems that \( G_L \) is independent of \( \theta \). French research work (Ref. 33) verifies this statement (see Fig. 60). Figure 51 further shows that at \( C_J \)-values of about 5, \( G_L \) is of the order of three which asymptotically approaches the value of unity for \( C_J \) tending to infinity (\( v_o = \) zero, VTOL). At practical \( C_J \)-values, the lift gain factor is about 4 (times four argument, Sec. 4.3) and increases to values of the order of 25 at small \( C_J \)-values (\( C_J < 0.1 \), which is still larger than the \( C_J \)-range in general encountered in cruising flight).

In case of the three-dimensional jet flapped wing, however, Fig. 61 indicates that \( G_L \) is dependent of \( \theta \). If the jet deflection angle increases, the lift gain factor increases too, at least for \( \theta \) up to 90°. Figure 61 shows also, that in VTOL application of the jet flap system (\( C_J \rightarrow \infty \)), no gain in lift can be expected and only the jet reaction lift can be obtained. In this case, the jet flap provides no advantage over conventional (circular) jet deflection systems (e.g., flying bedstead, Ryan vertijet) unless perhaps, the deflection of the jet sheet can be achieved more efficiently by means of the COANDA - effect.

In Ref. 33, a "SPECIFIC LIFT INCREASE" is defined as \( \Delta C_{LT}/(C_J^{1/2} \cdot \sin \theta) \). It is shown that - as a first approximation - it is proportional to the lift curve slope \( dC_{LT}/d\alpha \).
B. The Lift Efficiency ($E_L$)

The lift efficiency $E_L = \frac{C_{LJ}}{C_{LR}}$ is related to the lift gain factor simply by $E_L = G_L - 1 - \frac{C_L'}{C_{LR}}$. For a symmetrical jet flapped wing at $\alpha_0 = 0$ ($\alpha_0 = \infty$), $C_L'$ is zero and $E_L$ reduces to $E_L = G_L - 1$. In this case the results - except for the constant of unity - and the curves are identical.

In Fig. 60, $E_L$ for the two-dimensional jet-flapped wing appears to be independent of $\theta$. Figure 62, however, presenting $E_L$ for a three-dimensional jet flapped wing with tip tanks indicates that $E_L$ decreases with increasing jet deflection angle. This finding contradicts the general conclusion that can be drawn from test results of Ref. 71 as shown in Fig. 61. If the comparison of Figs. 61 and 62 is limited to $C_J < 0.2$, the evidence of both figures seems to be in agreement. As the $C_J$-range in question is of practical importance, the curves in Fig. 61 for $\theta < 86^\circ$ should be extended to cover the range of $C_J < 0.15$.

Based on the evidence of Fig. 62, Ref. 33 concludes that "too large a deflection of the jet is therefore harmful, and as in the case of the two-dimensional flow it is not profitable to go beyond a deflection of order of 60$^\circ$: beyond these values the lift ceases to increase proportionately to sin $\theta$ and the loss of momentum in the propulsive component becomes marked". That beyond $\theta = 60^\circ$, the lift - at least for $C_J > 0.5$ - no longer increases proportional to sin $\theta$ is demonstrated in Fig. 63A. Further, if quasi two-dimensional test results (of Ref. 33, Fig. 8A) are plotted as $C_{LT} \cdot C_{TM}$ vs $\theta$, an optimum value for $\theta$ - at which $C_{LT} \cdot C_{TM}$ is a maximum - can be shown to occur at about 60$^\circ$ (see Fig. 64). This maximum value increases with $C_J$.

In Fig. 65, the variation of $E_L$ with aspect ratio is presented. The lift efficiency drops appreciably with decreasing aspect ratio, especially at $C_J < 0.2$.

C. Evidence for the Lift Hypothesis

It is the jet-induced pressure lift that is responsible for the high total lift coefficients of jet flapped wings. Since both the lift gain factor and the lift efficiency are functions of $C_{LJ}$, evidence for the lift hypothesis is provided if $G_L > 1$ or $E_L > 0$.

8.2.8 The Pressure Lift ($C_{LP}$)

The variation of $C_{LP}$ with $C_J$ for a two-dimensional jet flapped wing is shown in Fig. 50 and for a three-dimensional wing in Fig. 66.
Tests conducted on a three-dimensional jet flap model wing (Ref. 28) at $\alpha = 0$ indicate that the spanwise load distribution curves of the pressure lift differ little from those calculated with the lifting line theory (see Fig. 67). Furthermore, they have the characteristic shape of $C_{LP}$-curves as obtained if the wing is set at an angle of incidence, but without jet blowing.

8.3 The Thrust and Drag Forces

8.3.1 Their Relative Magnitude

Dimmock, in Ref. 23, compares the thrust as measured by a balance ($C_{TM}$) with that calculated from pressure distributions ($C_{TM}$) for various jet deflection angles as obtained with a two-dimensional jet flapped wing. Figure 68 shows this comparison.

Take for example the $C_{TM}/C_J$ and $\bar{C}_{TM}/C_J (=C_{TM}/C_J + C_{DFR}/C_J)$ curves for $\theta = 31.4^\circ$. According to the thrust hypothesis $C_{TM}/C_J$ and $\bar{C}_{TM}/C_J$ should be equal to unity. In reality, they are smaller than unity due to form drag and skin friction drag ($C_{D_j} = 0$ for the two-dimensional wing). The downward displacement of the $C_{TM}$-curve measured from unity is the consequence of the form drag and the distance between the $C_{TM}$ and the $C_{TM}$ curve is due to the skin friction drag. It is obvious that the form drag is by far the larger especially at higher jet deflection angles and after L.E. separation has taken place. Note also how strongly the skin friction drag is reduced when L.E. flow separation occurs. The jet drag ($C_{D_J}$)-due to mixing between the jet and the free stream flow - is contained in the profile drag of the jet flapped wing.

8.3.2 The Jet Induced Pressure Thrust ($C_{TP}$)

The total (measured) thrust as obtained from balance measurements on a two-dimensional jet flapped wing at zero incidence is

$$\frac{C_{TM}}{C_J} = \frac{C_{TP} + C_J \cdot \cos \theta - C_{DT}}{C_J} = \frac{C_{TP} + \cos \theta}{C_J} = 1 - \left( \frac{C_{DF} + C_{DFR}}{C_J} \right) (8-2a)$$

and that as obtained from pressure distributions is

$$\frac{\bar{C}_{TM}}{C_J} = \frac{\bar{C}_{TP} + C_{DFR} + \cos \theta}{C_J} = \frac{\bar{C}_{TP} + \cos \theta}{C_J} = 1 - \frac{C_{DF}}{C_J} \quad (8-2b)$$
In Fig. 68, the existence of the jet induced pressure thrust $C_{TP}$ for a two-dimensional jet flapped wing is only clearly established for $\theta = 58.1^\circ$ and $\theta = 90^\circ$. At $\theta = 31.4^\circ$, $C_{TP} \propto 0$ indicating that $C_{DT} = C_{DP} = C_{DF} + C_{DFR} \approx C_{TP} = C_{J} - C_{TR}$, i.e., that $C_{DT}$ nulls the ideal induced pressure thrust. For $\theta = 0^\circ$, $C_{TP} = C_{TP} = 0$ and for $\theta = 90^\circ$, the jet induced pressure thrust $C_{TP}$ becomes equal to $C_{TM}$ and $C_{TP}$ equal to $C_{TM}$.

Further evidence for $C_{TP}$ is given in Ref. 24. Figure 69 for the case of $\theta = 90^\circ$, indicates $C_{TP}$ to be up to 37% of $C_{J}$ for a two-dimensional wing at $\alpha = 0$. Theoretically (thrust hypothesis), $C_{TP}$ should be equal to $C_{TP} = C_{J}$. The loss of 63% of $C_{J}$ is believed (Ref. 24) to be due to the high mixing losses in the region of about one inch length adjacent downstream of the slot nozzle, in which the main stream flow enters perpendicularly into the jet sheet (see Fig. 5).

For a three-dimensional jet flapped wing ($AR = 3$) at various incidences $\alpha$ and at a constant jet deflection angle $\theta = 90^\circ$, the total measured thrust, $C_{TM}$, (by balance) was always negative i.e., it is a force in drag direction as shown in Fig. 12B of Ref. 33. At $\alpha = 0$ and $C_{\mu} = 0$, $C_{LT}$ is zero and therefore $C_{Di}$ is zero too. The $C_{TM}$ measured in this case is then equal to the profile drag $C_{DP}$. As $\alpha$ is increased the jet angle $\tau = \alpha + \theta$ may become greater than $90^\circ$ and $C_{TM}$ becomes more negative due to two additional forces to the profile drag, the induced drag $C_{Di}$ and the jet reaction thrust $C_{TR}$. If $C_{\mu}$ is raised at $\theta = 90^\circ$ and $\alpha = 0^\circ$, the jet reaction lift $C_{LR}$ has to be equal to $C_{\mu}$ and the total lift e.g., for $C_{\mu} = 0.025$ follows from Fig. 58A to be 0.25. A lift efficiency $E_{L} = (\Delta C_{LT} - C_{LR})/C_{LR} \approx 9$ would result at such a small jet coefficient.

The pressure thrust ($C_{TP}$) as obtained from an integration of the pressure distribution is found in Ref. 28 at zero incidence and $C_{J} < 2$ to satisfy the following relationships:

**AR = 2.75**

$$- (C_{TP})_{2.75} = 0.013 + 0.14 \cdot C_{LP} \quad \text{for } 0 < C_{J} < 0.5$$

$$- (C_{TP})_{2.75} = 0.013 + 0.16 \cdot C_{LP}^2 \quad \text{for } 0.5 < C_{J} < 2$$  \hspace{1cm} (8-3a)

**AR = 6.8**

$$- (C_{TP})_{6.8} = 0.015 + 0.068 \cdot C_{LP} \quad \text{for } 0 < C_{J} < 2$$

$$- (C_{TP})_{6.8} = 0.015 + 0.068 \cdot C_{LP}^2 \quad \text{for } 0 < C_{J} < 2$$  \hspace{1cm} (8-3b)

In the above equations, the negative sign accounts for the pressure thrust being actually a negative thrust or a drag force.
The total drag $C_{DT} = C_{DP} + C_{Di}$. Maskell and Spence (Ref. 30) have calculated $C_{DP}$, using Eq. (6-58) and the test results of Refs. 28 and 32. They found that $C_{DP}$ is almost independent of aspect ratio as shown in Fig. 10 of Ref. 30.

A. The Thrust Recovery

In Ref. 33, a factor is used which could be called a net thrust recovery factor. It is defined as

$$R_N = \frac{C_{TM} + C'_{Di} + \Delta C_{Di}}{C_{\mu}} = \frac{C_{TP} + C_{TR} + C_{Di} + C'DP}{C_{\mu}}$$

and plotted vs $C_{\mu}$ in Figs. 70A and B for both a three-dimensional and a quasi two-dimensional jet flapped wing respectively. The induced drag $C_{Di}$ can be calculated (see Sec. 6.5). Both, $\Delta C_{Di}$ and $C'D$ are added in order to present the true jet-induced pressure thrust $(C_{TP} + C_{Di} + C'DP) = C_{TP} - \Delta C_{DP}$ unobscured by the drag due to lift $(C_{Di})$, which at high lift coefficients and small aspect ratios becomes the predominant contribution to the total drag.

Another more practical thrust recovery factor is defined in Ref. 32 as:

$$R = \frac{C_{TM}}{C_{J}} = \frac{C_{TP} + C_{TR}}{C_{J}}$$

and two and three-dimensional test results are shown in Figs. 68 and 71. Note that $R$ increases with increasing $C_{J}$ in the three-dimensional case. Note further, that $R$ tends to zero for $\theta \geq 60^\circ$ (Fig. 71).

Both factors, however, do not properly reflect the influence of the jet deflection angle on the magnitude of the jet induced pressure thrust $(C_{TP})$.

B. The Thrust Efficiency ($E_T$)

It is believed that this quantity less intricately illustrates, how the thrust superiority of jet flap systems over direct or pure jet lift systems is based on the jet induced pressure thrust $C_{TP}$ and its variation with the jet deflection angle $\theta$. The thrust efficiency is defined (see Fig. 72) as:
For the evaluation of already available test results (see Fig. 73), the following definition is more suitable

\[
E_T = \frac{C_{TP} + C_D'}{C_{TP_i}} = \frac{C_{TM} - \cos \tau + C_D'}{C_J (1 - \cos \tau)}
\]  

(8-5a)

It should be realized, however, that omitting \( C'D \) means to ignore that before blowing starts, a negative thrust \( (C'D) \) is acting on the wing. Blowing changes this negative thrust to a positive force which the balance measures as \( C_{TM} \). The actual thrust force due to blowing is therefore \( C_{TM} + C'D \) (see Fig. 72).

C. Force Gain Factor \( (G_R) \)

In order to assess the gain produced by the chances of the lift \( \Delta C_{LT} \) and thrust \( \Delta C_{TM} \) forces due to blowing in the case of a jet flapped wing, we suggest a force gain factor defined as

\[
G_R = \frac{C_R}{C_J} = \frac{(\Delta C_{LT}^2 + \Delta C_{TM}^2)^{1/2}}{C_J}
\]  

(8-6)

In this expression, \( G_R \) can be interpreted as the ratio of the resultants of the changes in the lift and thrust forces due to blowing with the main stream flow on and off.

D. Evidence for the Thrust Hypothesis

If for a jet flapped wing under idealized conditions (Sec. 4.1) \( C_{TM} \) is equal to \( C_J \), independent of the jet deflection angle \( \theta \) (thrust hypothesis), the jet induced pressure thrust \( C_{TP} \) must be equal to \( C_{TP_i} = C_J (1 - \cos \theta) \). For the case of \( \theta = 90^0 \), \( C_{TP} \) would be equal to \( C_{TP_i} = C_J \).

For an actual jet flapped wing, however, \( C_{TP} \) is smaller than \( C_{TP_i} \). But as long as \( C_{TP} \) exists as a positive (thrust) force it is an unmistakable proof for the thrust hypothesis. Since the thrust efficiency \( E_T \) is based on \( C_{TP} \), \( E_T \) may provide evidence in support of the thrust hypothesis. However, from test results presented in Fig. 73 it follows that in practice only the two-dimensional jet flapped aerofoils can produce a positive jet induced pressure thrust.
Another way to verify the thrust hypothesis experimentally is to measure $C_{TM}$ and to calculate from test results $C_{DT}$. Both should add up to $C_J$ (see Ref. 142).

### 8.3.3 The Induced Drag

The induced drag of the three-dimensional jet flapped wing is a very important and critical quantity primarily due to its strong response to high lift coefficients. Besides, there is not even full agreement on its definition for a discussion and comparison of theoretically derived $C_{D_1}$-values, see Sec. 6.11.2. In this section, some sideline comments on the induced drag and on what lift (the total lift, the pressure lift or their product) should be used for its calculation are added.

In Ref. 28, $C_{TP} = C_{TP_1} - C_{DF} - C_{D_1}$ is defined as

$$C_{TP} = \text{const.} + \frac{C_{LT} \cdot C_{LP}}{\pi \cdot AR \cdot e'}$$

and values for the factor $e'$ are plotted against $C_J$ (see Fig. 74) for a jet flapped wing at $\alpha = 0$ and for two aspect ratios (AR = 2.75 and 6.8). An increasing $e'$ is in effect the same as increasing the aspect ratio and $e'$ may therefore be considered to be a function of $C_J$, $\theta$, wing planform and aspect ratio. It should be realized, however, that $C_{LT} \cdot C_{LP} / \pi \cdot AR \cdot e'$ in Eq. (8-7) cannot be the induced drag as e.g., defined by Eqs. (6-124) or (6-58) and that $e' \neq e$. This point is not clearly brought out in Ref. 28.

In Ref. 71, another induced drag definition is introduced (see also Sec. 6.11.2, Eq. (6-125) as

$$C_{D_1}^* = \frac{C_{LJ}^2}{\pi \cdot AR \cdot e^*}$$

A curve for $e^*$ vs. $C_J$ is shown in Fig. 18 of Ref. 71, but $e^*$ is too obscure to promote any better insight into the induced drag, theoretically or experimentally.

In Ref. 33, the induced drag equation is presented as:

$$\Delta C_{D_1} = C_{D_1} - C_{D_1}' = k_i (C_{LP} - C_{L'})^2 = k_i C_{LJ}^2$$

for a jet flapped wing of constant aspect ratio. It is further suggested that the induced drag arising either from the effects of angle of incidence without blowing, or from blowing at zero incidence are similar. The limitations of this hypothesis are stated in Sec. 6.11.2 (see Eqs. (6-123) and (6-129)).
8.3.4 The Jet Drag \((C_{DJ})\)

Stratford (Ref. 14) derived theoretically that the real criterion for the jet drag \((C_{DJ})\) is the ratio of the density-velocity products (Fig. 19). If a cold air jet is used so that \(\rho_J = \rho_O\), the magnitude of the jet drag \((C_{DJ})\) becomes a function of the velocity ratio of the two fluids only. That this is so is shown in Fig. 75, where a large leading edge spoiler on a two-dimensional wing is used to keep the transition point fixed. From Fig. 9B follows, that \(C_{DT} = C_{DJ} + C_{DP'} = C_J - C_{TM}\) and that the slope \(dC_{DT}/dC_J = dC_{DJ}/dC_J\) of all these curves is about the same. Their average slope \(dC_{DJ}/dC_J = 0.060\) indicates that the jet drag (sink effect) - a combined friction and suction drag caused by a low pressure region in the close vicinity upstream of the trailing edge - amounts to about 6% of \(C_J\).

In Fig. 19, the sink effect \((\rho_J \cdot v_J > \rho_O \cdot v_O)\) is shown by the contraction of the main stream which is substantially higher than what could be justified by potential flow theory. In Fig. 76, the source effect \((\rho_J \cdot v_J < \rho_O \cdot v_O)\) is specifically demonstrated. These results were obtained with a hydrogen jet sheet, the velocity of which was progressively raised. Figure 76 also indicates that there is definitely a region where the jet drag \((C_{DJ})\) is negative (a thrust) and peaked at about \(\rho_J \cdot v_J/ \rho_O \cdot v_O = 0.5\).

Note further that the jet drag increases rapidly with increasing jet deflection angle.

The jet drag is easily defined but is difficult to obtain as a single quantity from actual tests. If pressure distribution measurements are employed, \(C_{DJ}\) cannot be obtained directly, but it could be obtained from force balance measurements as follows. Imagine for instance, a symmetrical jet flapped wing at \(\alpha = 0\) and \(\theta = 0\), three tests would have to be conducted:

a) a W.T. run with the jet flapped wing, but without blowing to obtain \(C'D = C'DP\)

b) the same run but with blowing, measuring the total thrust \(C_{TM} = C_J - C_{DT} = C_J - C_{DP}\) (as \(C_{D1} = 0\), because \(\alpha = 0\) and \(\theta = 0\)) and

c) the same run, but the W.T. flow at rest, measuring the jet momentum \((J)\). Assume now that there is no change in thrust of a jet (the reservoir pressure of which is constant) whether it exhausts into a moving fluid or into one at rest. With this assumption as used by Dimmock (Ref. 24, the jet drag would result as
\[ C_{DJ} = C_J - C'D' - C_{TM} \]  
\[ = C_J - C'DP - CJ + CDP = \Delta CDP \]  
which confirms the original jet drag definition as introduced in Ref. 24 and used e.g., in Ref. 12. It is maintained now, however, that the thrust and therefore \( C_J \) for a jet flap test cannot be obtained from a static test, i.e., a test with the main stream flow at rest - without applying certain corrections. It is shown in Ref. 36 that the thrust (at constant reservoir pressure) may increase with increasing main stream velocity. Taking this change in \( C_J \) not properly into account for the determination of the jet drag, \( C_{DJ} \) would be falsely considered to have decreased by that amount by which the thrust increased above its static value.

For a symmetrical wing with \( \alpha = 0 \) and \( \theta = 0 \), \( C_{DP} \) is simply equal to \( C_{DT} = C_J - C_{TM} \) (see Fig. 9B). If \( C_{DT} \) is plotted vs. \( C_J \), Fig. 75 is obtained which indicates that for \( C_J = 0 \), \( C_{DT} = CDP = C'DP \) and that with increasing \( C_J \), \( C_{DP} \) increases linearly as:

\[ C_{DP} = C'DP + a \cdot C_J \]  
(8-11)

Neither an artificial increase (L. E. spoiler) nor decrease (plasticine L. E. fairing) of \( C'DP \) affected the rate of increase of \( C_{DP} \) with \( C_J \) in any significant way.

As from definition \( C_{DJ} = \Delta CDP = CDP - C'DP \), Eq. (8-11) suggests for a jet flapped wing of above specification that

\[ C_{DJ} = a \cdot C_J \]  
(8-12)

which is plotted in Fig. 77. In the range of \( C_J \) up to about 0.1, the jet drag is best approximated by 0.06 \( C_J \), but beyond jumps to the reduced rate of increase of \( C_{DJ} = 0.017 \cdot C_J \). If, however, a full span L. E. spoiler is added to enhance \( C'D' \), W. T. tests at 50 fps (Re = 2.13 x 10^5) proved that the \( C_{DJ} = 0.06 \cdot C_J \) law holds up to \( C_J \)-values of 0.25, being reduced however to \( C_{DJ} = 0.0104 \cdot C_J \) at larger \( C_J \)-values. As these tests were performed with a cold air jet, the jet drag criterion \( \rho_J \cdot v_J / \rho_o \cdot v_o \) was reduced to merely the velocity ratio.

If Stratford's jet drag criterion is used for the prediction of \( C_{DJ} \) it should be realized that he ignored temperature effects in his analysis. Payne (Ref. 36) shows that Stratford's conclusion (see Sec. 10.2.5) that THF = THC when \( \rho_J \cdot v_J = \rho_o \cdot v_o \) is correct only for the cold air jet and that the theoretically possible (THF - THC)/THC > 1 (if \( \rho_J \cdot v_J < \rho_o \cdot v_o \)) is appreciably higher (see Fig. 78) for the hot jet than
what is predicted by Stratford's cold jet analysis. Payne further concludes
that when his equation indicates a jet thrust \( \text{TH}_F - \text{TH}_C / \text{TH}_C > 1 \), this
thrust can not be realized in practice if the secondary air has not a suit­
able surface on which to generate this thrust increment (see Fig. 79).

Finally, since the jet drag must cause a jet momentum loss, the
question of how this loss affects the total lift is of interest. No experi­
mental evidence can be called upon to answer this question directly. It is
suspected that the jet drag losses are caused primarily in the highly curved
region of the jet sheet adjacent to the jet nozzle exit.

8.4 Pressure Distributions

8.4.1 Saddle-Back Distribution

Pressure distributions of jet flapped wing configurations are
apt to help in understanding the following characteristic jet flap phenomena:

a) the nose suction peak, which - via the induced thrust - is
considered to be responsible for the validity of the thrust
hypothesis.

b) the T.E. suction peak, increasing in strength with increasing
\( C_J \) and shifting the separation point on the upper flap surface
rearward.

c) the saddle-back pressure distribution (see Fig. 80) which is
responsible for a rearward shift of the centre of the pressure
lift from its location on the wing without blowing. Besides,
such a pressure distribution produces a more uniform free
stream velocity distribution by reducing the velocity peaks
at the L.E. and T.E. to about half the value which would be
obtained at the L.E. of a conventional wing at incidence, at
the same total lift coefficient.

8.4.2 Suction Pressure Peaks

Some representative chordwise pressure distributions for
two spanwise locations of a wing \((\theta = 31.3^\circ, \ AR = 2.75)\) at zero incidence,
equipped with transition wires are shown in Fig. 81. It indicates lower
peak suction pressures at both the L.E. and T.E. for the section close to
the wing tip. All suction pressure peaks increase of course in magnitude
with \( C_J \). In Fig. 82 the variation of the peak suction pressure near the
L.E. and T.E. with incidence is shown for two \( C_J \)-values. As the inci­
dence increases at constant \( C_J \) (see Figs. 10A, B, and C of Ref. 28) the
L.E. suction peaks of the inboard section grow faster than those of the
outboard section, suggesting flow separation to start inboard first.
Whereas the T.E. suction peak of the outboard section grows with \( \alpha \),
the inboard peak varies little at smaller $\alpha$ and decreases at high incidences. Furthermore, as $C_J$ increases at constant $\alpha$, the T.E. suction pressure peaks grow more rapidly than those near the L.E., partly because the L.E. suction peaks (two-dimensional upwash) are reduced by three-dimensional downwash effects.

The above observations on the three-dimensional wing ($AR = 2.75$) can be compared with those obtained after its conversion into a "quasi two-dimensional" one ($AR = 6.8$). In general (Ref. 28), the trend of these peak suction curves is the same as that of the inboard section curves of the three-dimensional model as demonstrated in Fig. 83. The Maximum values for the suction peak ($-C_{p_{\text{max}}} \approx 4.5$) of the $AR = 6.8$ wing was found to occur at smaller $\alpha$ -values than that for the wing of $AR = 2.75$. The L.E. peak suction for the quasi two-dimensional model in general exceeds that measured on the three-dimensional model.

In Ref. 33, the pressure distribution is given for a NACA 0018 truncated wing section with jet control flap, with and without blowing, as shown in Fig. 84 for $\alpha = 0$ and $\theta = 67^\circ$.

Finally, in Ref. 94, pressure distributions including the T.E. flap are presented. Fig. 85 shows such a case for $\theta = 45^\circ$ and at $\alpha = 0$.

8.5 Pitching Moment and Shift of the Lift Centre and Aerodynamic Centre

8.5.1 The Total Pitching Moment

The total pitching moment can be split up in several ways to bring out those forces which contribute to it. In Ref. 32, $C_M$ is subdivided into three parts:

1) the first part is due to the integrated pressure forces acting on the aerofoil in the direction of lift, the mid chord point being the moment centre.

2) the second part comes from the integrated pressure forces in thrust direction with the moment centre being at the mid chord point again. This contribution is in general neglected (Ref. 32) as the resulting error is only 1.5% of the sum of the other two moments at $C_J = 0.15$, and 0.23% at $C_J = 1.5$. Expressed in rearward shift of the centre of the lift, this error corresponds to only 0.04% of the chord.

3) the third moment contribution is due to the jet reaction force.

In Ref. 71, the total pitching moment about the A.C. (at 27.6% c) is split up into three contributions again. The principal part (see Fig. 86) is identical with that described under 3) above. The second
one is due to the integrated pressure forces as induced by the blowing jet and the third part is the moment acting on the wing if there is no jet blowing.

In Fig. 87 the pitching moment $C_M$ plotted vs. $C_{LT}$, was obtained from tests with transition wires. The pivot point was located at $a = c/2$. This figure also shows that the mean slope $dC_M/dC_L$ of the curves for constant, but low $C_J$-values is about 0.25, indicating that the aerodynamic centre is close to the quarter-chord position. As $C_J$ increases, $dC_M/dC_L$ decreases, i.e., the aerodynamic centre moves slowly aft. This is due to the growing jet induced pressure lift which forces the total lift centre to move aft of its initially about 50% chord position when $C_J$ is increased at constant $\alpha$ or when $\alpha$ is decreased at constant $C_J$ (Fig. 88). In Ref. 33, this increase in nose down pitching moment with increasing $C_J$ is quoted as being proportional to the lift, i.e., proportional to $\sqrt{C_J}$ for the two-dimensional wing. For an AR = 3 wing (Fig. 89A), the same tendency can be observed. Besides, the curves straighten with increasing $C_J$. Figure 89B demonstrates that the tail setting angle $\beta$ required for trim is merely displaced towards more negative settings to compensate for the large nose down $C_M$ with increasing $C_J$.

For comparison, actual flight tests of a jet engine powered METEOR aircraft, equipped with a fixed angle (60°) jet deflector may be mentioned here (Ref. 89). They showed that the stability and control of the METEOR were adequate for test flying at speeds down to 70% of the normal (power-off) stalling speed. But a new technique of using the throttle had to be mastered - since this control affected both the longitudinal and the normal forces on the aircraft at the same time - especially when a reduction of engine power was required.

8.5.2 Shift of Lift Centre and Aerodynamic Centre

A. Shift at Zero Incidence

In Fig. 25 the shift of the aerodynamic centre (A.C.) past the quarter chord point, as observed on a two-dimensional jet flapped wing at zero incidence is shown to be as high at 15% of the chord for high lift coefficients. A similar shift, but a little smaller in magnitude (about 13% of the chord) was observed for the centre of lift (L.C.) past the 50% chordal station.

In Ref. 28, a shift of the A.C. of about 6% of the chord was found for a "quasi two-dimensional" wing (AR = 6.8) as $C_J$ was raised from 0.2 to unity. In another series of tests with a wing which was tested either as a two or a three-dimensional wing, an L.C. shift of 24% and 31% respectively was observed past the mid chord position at $C_J$-values of about 2.1. The effect of aspect ratio on the A.C. or L.C. shift is discussed in detail in Sec. 8.7.
An A.C. shift of 6% is also quoted in Ref. 33 if $C_J$ is raised from zero to unity. However, the aspect ratio was as high as 20 for this case.

It could be concluded from Fig. 25 that the shift of both the A.C. and L.C. is almost independent of the jet deflection angle, if $\alpha = 0$; and further, that the pitching behaviour of the jet flapped wing is better illustrated by the moment and force centre shifts than by the conventional $C_M$ vs $C_{LT}$ presentation of ordinary aerofoils, where both centres remain fixed.

Note that the A.C. and L.C. shifts are almost independent of the jet deflection angle only if plotted against $C_J$ but not, if plotted against $C_{LT}$. The independence of the L.C. shift of $\theta$ is confirmed also by Ref. 33, as shown e.g., in Fig. 58.

### B. Shift at Incidence

The L.C. variation as obtained from test results at various angles of incidence is shown in Fig. 90. It indicates that even a forward shift is possible at small $C_J$-values and positive incidences. At negative incidences, however, high $C_J$-values show the least backward L.C. shift. Note that for $C_J < 0.2$, the L.C. is moved to a position even downstream of the wing trailing edge. Plottings of $(\theta/d)/\alpha$ are available for the $\theta = 31.4$, 58.1 and 90$^\circ$ jet flapped wings (two-dimensional) in Refs. 24 and 32.

### C. Shift Due to Ground Interference Effects

In Ref. 32, the effect of ground interference on the L.C. shift is investigated on a two-dimensional wing at zero incidence and at $\theta = 58.1^\circ$. Figure 91 shows this effect to be appreciable for ground distances smaller than one wing chord (6") but especially for ground clearances below one half of the chord. Note the change of inflection of all curves at $h/c \approx 0.375$.

### 8.6 Thickness, Nose Radius and Camber Effect

In Ref. 112, the effect of thickness, camber and nose radius on the performance of jet flapped aerofoils is investigated.

First, pressure distributions for six aerofoils were calculated (Kuchemann's method, Ref. 17) and the efficacy of thickness, camber and nose radius in preventing flow separation near the nose was studied qualitatively. The conclusions arrived at - useful for the selection of suitable aerofoils for use with jet flaps - are that:
a) thicker aerofoils have not only a higher normal force coefficient, but attain it with less risk of flow separation, because of the more favourable pressure gradient near the L.E.

b) an increase in camber results in an increased lift and a slightly more favourable pressure gradient aft of the L.E. suction peak.

c) a larger nose radius produces a pressure gradient near the nose which is more favourable to attached flow.

d) for a given $C_{LT}$, lower jet deflection angles ($\theta$) and consequently higher $C_J$-values should result in more desirable L.E. pressure distributions.

It should be realized, that conclusions b) and c) cannot by themselves make a thin aerofoil practical for high lift coefficients. They should be regarded only as a means of improving the performance of very thick (20%) aerofoils.

8.7 The Jet Flap and Aspect Ratio

There is no doubt that the jet flap is able to produce large lift coefficients also on low aspect ratio wings. In this case, however, the jet flap is no exception to the rule that large lift forces obtained from small span wings are paid for by large induced drag coefficients.

In assessing the potentialities of the jet flap in its application to finite aspect ratio wings, Maskell's and Spence's work (Ref. 30), providing a basis for induced drag estimates, has to be emphasized. It is claimed in Ref. 29 that such estimates are in substantial agreement with the limited experimental results.

Nicholson, in Ref. 29, touches on another interesting point. In order to get high lift without excessive drag, a large quantity of air has to be operated on. A jet sheet issuing sideways from the tips of a very low aspect ratio wing might increase the effective aspect ratio appreciably and consequently allow a higher lift coefficient to be used without increasing the induced drag. This concept has been tested with a conventional wing (Ref. 103). But a similar test with a jet flapped wing has not been done so far.

In analysing the aspect ratio effects, two questions have to be considered here

a) in what way does aspect ratio physically affect the jet sheet shape and

b) what is the effect of AR on the aerodynamic characteristics of jet flapped wings.
8.7.1 The Physical Effects

It seems that basically aspect ratio effects originate in the combined wing and jet sheet vortex fields, but what actually happens here is not yet analyzed satisfactorily.

Davidson (Ref. 12) assumes that the conventional wing tip vortices may partly, if not completely, be entrained by the jet sheet. He supposes further that "in practice of course, the shed vorticity (of tip vortex) will go more or less directly into the jet, so that it will never really appear as a tip vortex.

Davidson concludes that at higher $C_J$, this entrainment effect (see Fig. 92) - as it would reduce the effective strength of the tip vortices - should lead to a reduction in magnitude of the usual negative induced incidence and hence of the downwash. Such a reduction in downwash would also suggest a reduction in induced drag as compared with that of a conventional wing under the same conditions of loading (same $C_{LT}$). Theory (Ref. 30) also predicts a reduction in $C_{Di}$.

Experimental evidence of Ref. 73, however, does not seem to confirm all of Davidson's above assumptions. Figures 93 and 94 suggest that the wing tip vorticity is increased in strength by the presence of the jet sheet, i.e., that the jet sheet is "feeding" energy into the wing tip vorticity. This is what plausibly follows if the line vortex fields of both the wing and the jet sheet are considered. The same horseshoe vortex system which develops into the wing tip vortices behind a conventional wing exists on the jet sheet as well and adds its vortex filaments to those already gathered in the wing tip vortices, as long as the jet sheet has to be considered as a discontinuity surface, i.e., a surface across which pressures and velocities are different. More work, both theoretical and experimental, should be done to help to a better understanding of what happens behind the jet flapped wing, which could also add to a better interpretation of the induced drag.

8.7.2 Aerodynamic Effects

It seems reasonable to assume that the components ($C_{TR}$ and $C_{LR}$) of the jet reaction force are unaffected by dimensionality.

As, however, $C_{LR}$ is affecting the main stream flow as a consequence of the pressure difference across the jet sheet, it thereby contributes, Davidson argues, to the induced incidence (upwash at L.E.). If this were not so, the conventional formula for the correction of pressure lift for aspect ratio should apply for the jet flapped wing also, which it does not.

In Ref. 28, results for one and the same wing section tested as both a three-dimensional (AR $\approx 2.75$) or a "quasi two-dimensional" (AR $\approx 6.8$) wing indicate that for $C_J$-values up to about 4, the curves of
total lift, pitching moment and thrust are generally similar in character. If, for completeness, the experimental results of Ref. 24, obtained with a truly two-dimensional wing of the same section as that tested in Ref. 28 are added and compared with empirical or theoretical predictions, the conclusions, discussed in Sec. 9.6 can be drawn.

In Ref. 72, the effect of AR and end plates was investigated on the aerodynamic characteristics of an unswept, untapered jet flapped wing \( \theta = 85^\circ; C_j \leq 17.5 \). The results indicate that various end plates increased the effective aspect ratio (e.g., \( AR = 2.8 \) being increased to 3 or 4) depending on the particular end plate configuration used.

8.7.3 The Lift Efficiency \((E_L)\)

Figure 65 shows the dependence of \( E_L \) on the aspect ratio. The jet-induced pressure lift obviously is higher for a two-dimensional than for a three-dimensional jet flapped wing. This evidence is further supported by test results of Ref. 72. In Fig. 95, it is shown that \( C_{L_J} \) has a maximum when plotted vs. \( C_\mu \). If these \( (C_{L_J})_{\text{max}} \) values are plotted vs. AR, Fig. 96 is obtained which shows a rapid growth of \( (C_{L_J})_{\text{max}} \) with AR.

8.8 Reynolds Number Effects

Reynolds number effects are quite appreciable. It is therefore strongly recommended to conduct model experiments at full scale Reynolds numbers wherever possible (also see Ref. 151).

In Ref. 24, results are recorded which show the effect of tripping the boundary layer, creating artificially the state of boundary layer which would exist on a full-scale jet-flapped wing in flight. In Fig. 21 of Ref. 24 the effect of the character of the boundary layer on the total lift coefficient \( (C_{L_T}) \) is shown and its importance is even more emphasized in Fig. 97, which demonstrates its influence on the lift gain factor. Dimmock (Ref. 24) advocates that the increase in \( C_{L_T} \) is due probably to the benefits bestowed by a laminar boundary layer over the main part of the aerofoil surface, being turned turbulent at the trip wires. Then, the flow may not at all separate and if it does, separation would occur very close to the T.E.

Wall interference, in particular jet - wall interference - if \( T \) and \( C_L \) are larger - are the limiting factors to the size of a jet flapped model wing to be tested in a closed wind tunnel. The test Reynolds number is then primarily governed by the test section speed except in variable density tunnels. As the test section speed cannot be increased arbitrarily without running into Mach number effects, it seems that model tests in general have to be conducted at lower than full scale Reynolds numbers.
Ref. 26 states that the determination of what constitutes a desirable value of Re for model testing is not yet possible. But some guidance at least can be given: in model tests where low momentum air is ejected over the top surface of mechanical flaps, a test Reynolds number of \(4 \times 10^6\) can be expected in most cases to give reasonably close approximation to results obtained at the full scale \(Re = 20 \times 10^6\), unless there is appreciable flow separation at the nose due to high \(\alpha\) values (sectional \(C_{LT} > 4\), say). In jet-flap tests, however, where \(C_L\)-values of 10 with slow flight speeds will be encountered, the test Reynolds number (Ref. 26) will probably be less than \(1 \times 10^6\) as compared with a full scale Reynolds number of perhaps \(5 \times 10^6\). This Re disparity is considered to be critical. The model wing may show a substantial laminar leading edge separation which might not at all exist with full scale wings or at least to a lesser degree. No doubt that e.g., differences in chordwise pressure distribution, affecting perhaps the centre of lift position, the form drag etc. would require much care in the interpretation of such results.

In Fig. 68 (Ref. 23), the effect of Re on the balance measured thrust is shown at \(Re = 2.12 \times 10^5\) and at \(Re = 4.25 \times 10^5\). The measured thrust is higher for the higher Reynolds number, at least for the \(\theta = 0\) wing.

Tests in Ref. 33 indicated that a Reynolds number change between \(1.25 \times 10^6\) and \(6.6 \times 10^6\) has very little influence on the lift for both, upper surface or upper and lower surface blowing, as shown in Fig. 98.

8.9 Mach Number Effects

8.9.1 The Jet Flapped Wing

At O.N.E. R.A. (Ref. 33) a straight wing half model (AR = 3.4) was tested up to Mach numbers of 0.91, whereby locally on the t = 10% thick NACA 64A 010 aerofoil, shock waves were observed at wind tunnel Mach numbers of about 0.75. The test Reynolds number was \(1.5 \times 10^6\). The results obtained at very small jet coefficients (\(C_J = 0.023\), due to high W.T. speeds) are shown in Fig. 99. The lift increases with Mach number for all three jet deflection angles investigated (\(\theta = -2^0, +29^0\) and \(76^0\)) up to a Mach number of approximately 0.85. The drag begins to rise already at about \(Ma = 0.82\) for \(\theta = -2^0\) and at \(Ma = 0.75\) for the \(\theta = 76^0\) case. The theoretical lift curves were obtained by the Prandtl-Glauert theory for a straight wing of AR = 3.4.

The results indicate that compressibility effects cannot be eliminated by the jet flap, but the lifting effectiveness can be increased even in the presence of local shock waves. On the other hand, as blowing creates increased circulation due to a more even velocity distribution along the chord, a higher critical Mach number for a given lift and Mach number may be obtained.
Later tests at O.N.E.R.A. (Ref. 39) confirm the above observations on the NACA 64 A 010 wing section with a pure jet flap (Fig. 100B) and compare them with those obtained on a wing with internal jet augmented flaps using the same $C_J \approx 0.02$ and $\theta$-values in both cases. Figure 100A shows that the lift increment in the latter case decreases progressively due to separation of the flow behind shock waves on the upper flap surface, which appear at $Ma > 0.6$. At $Ma = 0.9$, blowing has practically lost its effectiveness. These tests also proved (Ref. 39) the practical impossibility of controlling this separation as long as the shock waves appear ahead of the nozzle slot, as they do in this case of the internal-flow jet-augmented flap.

Another report is dealing with Mach number effects of various jet flap configurations in combination with unswept rectangular wings in Ref. 87. The Mach numbers investigated covered the range from 0.40 to 1.10, the momentum coefficient range was from 0 to 0.30. It was observed that the lift efficiency $E_L$ for model 3 and 4 (see Figs. 3 and 4 of Ref. 87) increased with Mach number, reaching a maximum value at high subsonic speeds and then decreasing through the transonic speed range. Further, that a more rearward chordwise location of the blowing holes produced a considerably increased jet induced lift (see Fig. 101).

Model configuration 1 loses its improved characteristics due to blowing at $Ma > 0.6$, and model 7 showed a drag reduction about equivalent to the jet momentum coefficient throughout the $Ma$-range investigated. Models 5 and 6, however, showed smaller drag reductions all through the transonic $Ma$-range (see Fig. 102).

8.9.2 The Truncated Aerofoil

Further investigations in Ref. 39 deal with aerofoils of rectangular planform, $AR = 3$ and sharp leading edges. They are either truncated (85% of c) for T.E. blowing or provided with a jet control flap (11.5% of c) for symmetrical blowing. Figure 6A of Ref. 39 confirms Busemann's theory (Ref. 40) which, when compared with the full length chord wing - predicts a decrease of the drag for the truncated aerofoil (no blowing) especially in the transonic speed range and also, but less pronounced at supersonic flow velocities. At subsonic speeds, however, this drag is up to many times greater than that of the full length chord (lenticular) aerofoil. More details about e.g. lift curve slope, shift of aerodynamic centre and drag change as compared with theory are given in Ref. 39 for the truncated aerofoil without and with a T.E. blade for the prevention of the Karman vortex street.

Tests on truncated aerofoils were extended in Ref. 39 up to Mach numbers of 2 and the results are shown in Figs. 103, 104 and 105. The beneficial effect of the small jet control flap (with or without blowing) is reduced rapidly as soon as shock waves occur on the upper wing sur-
face at $\text{Ma} \approx 0.8$ (critical Mach number). In Fig. 103, the lift increase due to blowing is shown to be still appreciable even in the transonic speed range as compared with the case of no blowing or blowing at the T. E. The drop in lift at $\text{Ma} \approx 0.8$ is of course due to shock waves on the aerofoil's upper surface. At supersonic flow speeds, the lift reduces to just its reaction force component $C_{LR}$, since the induced pressure lift approaches zero because the jet can no longer produce any upstream effects.

Figure 104 at $\text{Ma} = 0.65$ and $\text{Ma} = 2.04$ illustrates the changes in lift as a function of the deflection angle of the jet control flap. In the supersonic case, the jet control flap effect is not very significant as the deflection angles are too small to get the short flap out of the dead water zone behind the truncation. Even blowing achieves only a negligible lift increase. In the case of zero deflection of the jet control flap, the truncated aerofoil with a blown blade is compared with the symmetrically blown jet control flap. Their drag curves are almost identical as shown in Fig. 105A and B. In Fig. 105B, the change in total drag $\Delta C_{DT}$ due to blowing is shown as the difference between the curves for $C_J = 0$ and $C_J = 0.002$ for Mach numbers up to 2.0.

8.9.3 Summary of Results

In summarizing Mach number effects on pure jet flapped and truncated wings, it can be said:

1) local shock waves on the upper wing surface at higher than critical Mach numbers reduce the lifting effectiveness of the jet flapped wing less than that of a wing with an internal flow jet augmented flap (Fig. 100A and B).

2) the critical $\text{Ma}$ - depending primarily on the aerofoil section shape - is not appreciably increased by blowing in general. In case of T. E. blowing, due to improving the velocity distribution over the upper wing surface, a slightly raised critical $\text{Ma}$ was observed (Fig. 100B).

3) the strength of local shock waves can be reduced and the critical $\text{Ma}$ increased - as on conventional aerofoils - by avoiding any large changes in surface curvature (see Fig. 100A and B).

4) the truncated aerofoil with a blown blade ($\theta = 0$) shows a substantial reduction of the minimum drag in the subsonic speed range by eliminating the periodic shedding of T. E. vortices. In the supersonic speed range, a blown blade of low intensity produces still a drag reduction (Fig. 105).
5) the truncated aerofoil with a blown spoiler blade (θ = 90) in subsonic flow induces an important increase in circulation, which decreases gradually to the solid spoiler effect as Ma approaches unity.

8.10 Some Special Jet Flap Applications

8.10.1 The Jet Flap in a Centrifugal Field

Investigations of this kind are relevant for e.g. jet flapped helicopter, turbine and compressor blades.

In Ref. 45, results are reported on the influence of a centrifugal field of up to 15 g on the total lift of a jet flap model wing. The change in lift characteristics was measured and compared with the lift of the same model wing, when tested in a wind tunnel.

It was found that at constant CJ-values greater than 1.5 the lift coefficients decreased with increasing spanwise acceleration, but that this rate of decrease in lift \( \frac{\partial C_{LT}}{\partial g} \) drops almost to zero at g-values of about 15 (see Fig. 106). Since in such full scale applications of the jet flap, centrifugal fields of up to 500g (Ref. 45) may be encountered, the decrease in jet flap lift to be expected will be at least as large as that obtained in model tests.

8.10.2 Jet Flapped Compressor Blades

In Refs. 153 and 154, jet flapped axial flow compressor blades are tested in a two-dimensional cascade. In comparison with the performance of conventional compressor blades, jet flapped compressor blades were found to provide larger stall and flow turning angles and higher pressure rises. In addition to these improvements, a more uniform velocity profile and an increase in axial flow velocity in the initial stages helps to prevent rotating stall in subsequent stages.

8.11 The Delta Wing-Jet Flap Performance

In Ref. 33, a jet flapped delta wing of 60° sweep at the L.E., 6% symmetrical airfoil section, AR = 1.42, \( \theta = 70^\circ \), Re = 1.4 x 10^6 at a Mach number, Ma = 0.3 and a maximum Mach number of 0.93 was tested. Due to the small thickness of the model wing, CJ at high free stream speeds was very small, its maximum value being only about 0.1.

The aerodynamic characteristics of the jet flapped delta wing does not differ in principle from that of the straight wing. The lift change at \( \alpha = 0 \) is shown in Fig. 107 and was found to follow the relationship.
8.12 Ground Interference Effects

During take-off and landing (high $C_{LT}$, large $\theta$) a change in path and shape of the jet sheet of a jet flapped wing takes place, if the distance of the wing from the ground becomes smaller than about one wing chord. The loss as caused e.g. in lift is physically explained by Davidson (Ref. 12) as being due to the jet entrainment which draws an appreciable ventilating flow through the clearance space.

Dimmock’s experimental results, shown in Fig. 108 indicate the magnitude of the loss in lift due to the ground interference effect. At ground clearance below 0.5$c$, Dimmock suspects the appreciable drop in lift to be in error. It was found that the supposedly two-dimensional model was actually of low aspect ratio due to gaps between the wing tips and the wind tunnel walls. Nevertheless, low ground clearances have to be avoided if the jet flap system is required to be fully proficient at take-off and landing.

An extensive experimental investigation into the ground effect on lift, thrust and pitching moment of a shrouded jet flap is presented in Ref. 73 from $h/c$ values of 0.5 up to ground distances, where the loss in lift (see Fig. 109) becomes zero. The following conclusions can be drawn from the evidence presented, when compared with "no ground" results.

1) Lift: an increasing loss in lift with decreasing ground distance or increasing jet deflection angle or increasing jet coefficient.

2) Thrust: a gain in thrust with decreasing ground distance (see Figs. 5 and 7 of Ref. 73) or decreasing jet deflection angle ($\theta = 85^\circ$ and $55^\circ$). For increasing $C_J$ values, either a gain or a loss occurs depending upon the values of $\theta$ and $h/c$.

3) Pitching Moment: a reduction in nose down (negative) pitching moment (about 1/4 chord point) with decreasing ground distance or decreasing $C_J$ or decreasing $\theta$. 

\[ \Delta C_{LT} = 1.46 \sqrt{C_J} \text{ (NO TIP TANK)} \] 
\[ \Delta C_{LT} = 1.64 \sqrt{C_J} \text{ (WITH TIP TANK)} \]
Attention may also be drawn to the high downwash observed in the region where tail surfaces on conventional aircraft are located. (For test results on downwash angles of a pure jet flapped wing as a function of $h/c$ and $C_J$, consult also Figs. 7 and 8 of Ref. 111). Further, an increase in strength of the wing tip vortices with increasing jet coefficients and large (20°) angles of upflow were found in the tail plane behind the wing tips (Ref. 73).

In Ref. 31, additional evidence to that of Ref. 73 is provided. In Fig. 110A, the decrease in ground influence on $C_{LT}$ is shown for decreasing jet deflection angles and in Fig. 110B, that the $C_{LT}$-variation with $\theta$, e.g., for $C_J = 4.0$ indicates an optimum $\theta$, at which $C_{LT}$ is a maximum for any chosen value of $h/c$. This optimum jet deflection angle, however, also varies with $C_J$. Finally, in Fig. 111, the change in the lift position is given in the presence of the ground. All these effects, although severe, are not considered prohibitive (Ref. 31) provided the ground clearance is not too small and the jet deflection angles used do not exceed their optimum values.

Further tests of the effect of the ground on $C_{LT}$ are reported in Ref. 110. They indicate that for any given ground distance there is a max. $C_{LT}$ which may be obtained no matter how much $C_J$ is increased. Fig. 112 shows this clearly and how the pressure lift increases up to a maximum, then decreases and may become even negative (see $h/c = 0.25$ at $C_J > 2.7$). Figure 112 indicates also that the curves for each ground distance $h/c$ branch off from a common line, after which the rate of increase of $C_{LT}$ with $C_J$ falls off rapidly. It is shown in Ref. 110 that the point of branching off is closely related to the instant when a streamline of the jet sheet first hits the ground, i.e., has a stagnation point on the ground.

Also in Ref. 110, the ground effect on the pressure distribution of the aerofoil and along the ground is investigated. Fig. 113 shows that the pressure lift losses occur due to a suction peak on the lower wing surface near the T. E., resulting also in a reduction of the inherent strong nose down pitching moment of jet flapped wings. The pressure distribution measured on the ground indicates that the stagnation point of the stagnation streamline moves upstream with increasing $C_J$, even beyond the point where the undeflected jet sheet would have hit the ground.

For an indication of how the ground will affect the performance of an aircraft at low speed near the ground, Fig. 114 should be consulted. It shows that the ground effect becomes the more significant the higher the thrust/weight ratio is chosen.

It may be also of interest to compare the ground interference effects of the jet flapped wing with those induced by a circular jet, emerging centrally from the wing, of a light-weight lifting engine. Figure 13 of Ref. 29 shows that with the wing too close to the ground, the whole direct jet lift may be lost. But still worse in its consequences may be potential pitching moment instabilities. Better schemes should be those based on annular jet sheets of either circular or rectangular shape. However, a circular jet (conventional exhaust jet) was deflected 60° downwards on an aircraft, which was flown down the length of a runway with its wheels just clear of the ground. No ill-effects due to the ground were detected (Meteor, Ref. 89, Discussion: A. R. Howell).
In Ref. 111, the ground effect on the pitching moment, is investigated. An appreciable reduction of the large nose-down pitching moment-inherent with jet flapped wings - was observed when the wing approached the ground. The phenomenon, shown in Fig. 115, results from a significant reduction in pressure on the wing under-side close to the T.E., which is the greater, the larger \( C_J \). Figure 115 also shows that for each \( C_J \) there is a critical \( h/c \) value above which \( C_M \) hardly changes, but below which the nose-down pitching moment falls off rapidly. If, however, the jet deflection angle is changed from \( \theta = 58.10^\circ \) to \( 31.40^\circ \), the change in \( C_M \) is no longer so large nor so abrupt. The evidence of Ref. 111 suggests that for \( h/c \leq 1 \) and \( C_J \leq 2.0 \), there will be no stability problem due to ground effect.

In view of the significant changes in \( C_M \), it is surprising that the position of the total lift centre varies only little (Ref. 111) with \( h/c \) at a given \( C_J \), as shown in Fig. 116. In Fig. 46 of Ref. 32, these results are extended to ground distances down to \( h/c = 0.175 \), indicating stronger L.C. shifts particularly at \( C_J \)-values above 2 (see also Fig. 111).

In Ref. 110 and 139, the effect of aspect ratio on the ground interference effects are discussed qualitatively. It is argued that compared with the two-dimensional wing, the three-dimensional wing may alter the ground effect due to the spanwise velocity component of the main stream which is a characteristic of finite aspect ratio wings. It is believed that this velocity component

a) facilitates the jet sheet penetration into the main stream flow and

b) provokes a spanwise upstream and downstream spreading of the jet sheet, when it hits the ground.

Both effects (Ref. 110) seem to oppose each other and it is concluded that at least for high aspect ratios, three-dimensional effects are small. This conclusion has, however, to be proved or disproved by experimental evidence.

8.13 Jet Sheet Features

8.13.1 Jet Sheet Shape

In Ref. 24, Dimmock reports on measurements of the jet path and jet velocity profiles taken at various stations downstream of the T.E. of a two-dimensional wing. He also presents the penetration of the jet stream into the main stream as a function of \( C_J \), the distance from the slot nozzle being the parameter. His final graph as shown in his Fig. 10, illustrates the jet path as measured for the \( \theta = 31.4^\circ \) jet at varying \( C_J \)-values.
In Ref. 28 the same wing but now three-dimensional (AR = 2.75) is used. The jet path and shape was found as shown in Fig. 117. Note the rapid increase in wake width - 1 inch at 1/4 chord (=2 inch) downstream of T. E. - and the corresponding turn in jet direction of about 15° from the initial θ = 31.4°. It was further found that at CJ = 0.5, the jet sheet shape varies spanwise only beyond the 80% span point, a result which in principle is confirmed also in Ref. 73. If the jet shape as measured with the same but "quasi two-dimensional" wing (AR = 6.8) is added in Fig. 117 for comparison, one sees that for the aspect ratios considered, there is hardly any detectable effect of dimensionality on the jet shape, but it changes considerably with CJ.

In Ref. 31, an equation is derived for the calculation of the centre line of a two-dimensional jet stream. This equation is integrated in Sec. 6.3.1. The centre line of the jet stream, as calculated with this equation for the conditions of Fig. 117, is added for comparison. It seems that the aspect ratio effect on the jet shape is appreciable. More work has to be done on this point.

Another jet shape equation, as derived in Ref. 33, can be solved either by a conformal transformation (mapping the jet slot into a circle) or by the method of electrical analogy (see Sec. 6.8 for further details). It is claimed in Ref. 33, that there is good agreement with experimental results.

If results of Ref. 73 are replotted, Fig. 118 is obtained which shows the jet centre line (i.e., the locus of maximum total head) as a function of CJ for a AR = 8.3 jet flapped wing of θ = 55°. Ref. 73 also supports the findings of Ref. 28 as to the variation in jet sheet shape in spanwise direction and indicates that even for CJ-values up to 6.46 (θ = 55°; α = 0), an appreciable variation takes place only beyond the 75% span point. At θ = 85°, however, the jet sheet shapes at 25% and 50% are no longer similar, especially at larger distances (3 chords) downstream of the T. E.

For calculated jet shapes (two-dimensional) in the presence of a ground, see Figs. 4 to 7 of Refs. 149.

8.13.2 The Jet Sheet Induction Effect

In Fig. 6, - also confirmed by Fig. 2F of Ref. 33 and Figs. 10, 11 and 12 of Ref. 110, it is shown how the main stream flow enters the jet sheet in the vicinity of the wing T. E. at angles up to and even greater than 90° to the jet sheet. A more detailed experimental study of this induction effect is given in Ref. 99. Investigations of the entrainment process with flap blowing (see Refs. 97 and 98) have revealed similar "OVER-TURNING" or sink effects as those observed by Davidson (Ref. 12) and Stratford (Refs. 14 and 46) with the jet flap. Reference 99
concludes that the entrainment does not arise from tangential inter-mixing at the jet-air stream interface, but involves primarily the following two processes:

a) a jet deflection due to the COANDA effect

b) the production of large negative pressures along the convex flap upper surface, which 'attract' the low velocity main stream flow inwards towards the surface. This causes the main stream fluid to flow almost normal into the high velocity jet in the mixing region close to the jet slot.

Note that his concept of non-parallel mixing is contrary to the simplifying assumption of parallel mixing used in the analysis of jet flap mixing processes (see Sec. X).

8.14 The Jet Flap and Aerodynamic Noise

It should be remembered that theoretically, the sound intensity of a circular jet issuing into air at rest is proportional to the 8th power of its velocity, the square of its diameter, and its density, etc. In practice, however, when a jet exhausts into an airstream of velocity \( v_o \), it is the shear velocity \( (v_J - v_o) \) to which the sound intensity is proportional, but to a somewhat reduced power (about 6th close to the jet). It follows that the jet noise problem and that of structural loads induced by jet noise is primarily one of ground running and take-off \( (v_J - v_o \approx v_J) \) and not so much of cruising flight, except if the jet is over- or under expanded (over choked).

The jet flapped wing promises to reduce the jet noise appreciably (Ref. 113). This is illustrated in Fig. 119, where the jet sheet (two-dimensional jet) is compared with a circular jet of equal mass flow and jet exit velocity. The frequency spectrum for the jet sheet is shifted to higher frequencies (Ref. 12*) which in the atmosphere - as known (see Ref. 13*) - attenuate faster with distance than the lower frequencies. Possible noise reductions up to 30 db are indicated in Fig. 119 in all but the highest frequencies.

Another application of the jet flap as a jet engine noise suppressor is described in Ref. 27*, indicating again very desirable noise reduction characteristics. It reports on a far field noise comparison of a circular jet (see also Ref. 41*) with various aspect ratio slot nozzle configurations of equal area. The results obtained with cold air jets at pressure ratios just below choking can be summarized as follows:

a) **Nozzles Without Flaps**

with increasing aspect ratio nozzle slots (thinner jet sheets),
the radiation patterns become radially unsymmetrical and the peak of the noise spectrum is shifted to higher frequencies.

b) Nozzles With Flaps

1) the addition of a flap (below exit) to a slot nozzle may change both its radiation pattern and frequency spectrum.

2) turning of the jet sheet causes the noise pattern to be turned an equal amount. The below-exit flap thereby tends to shield off noise radiation directly downward, but jet deflection plates (above exit flaps) increase noise radiation particularly in downward direction.

3) the jet flap used as a noise shield may have some potential even as a jet noise suppressor.

IX. COMPARISON: THEORY AND EXPERIMENT

This chapter is intended to show the practical usefulness of available jet flap theories (see Sec. VI) in predicting jet flap performance. Therefore, wherever possible, experimental evidence is compared with theoretically calculated results. In case that empirical laws are known, they are included in this comparison.

9.1 The Lift Forces

9.1.1 The Total Lift Coefficient

A. Two-Dimensional Wing

Stratford derived the following equations (see Ref. 11, Eqs. 6 and 10).

\[ C_J = \frac{2 \gamma^2}{\pi} \frac{\sqrt{3}}{\sin \theta} \cdot \frac{2}{1 + \cos \gamma} \]  \hspace{1cm} (9-1)

and

\[ C_{L,T} = \frac{C_{L,T}}{C_J \cdot \sin \theta} = \frac{\pi}{\gamma} \cdot \left( 1 + \frac{\sin \gamma}{\gamma} \right) \]  \hspace{1cm} (9-2)

where \( \gamma \) is the flap size parameter. Dimmock (Ref. 23) shows that these equations fit his experimental results quite well (see Fig. 120).
Based on Eqs. 7a and 11 of Ref. 11, an expression for the total lift coefficient of a wing at an angle of incidence can be derived (see also Eq. 8 of Ref. 24) as

\[
C_{LT} = C_{LT, \alpha=0} + \alpha \cdot 2\pi \left[ 1 + \frac{k}{\sqrt{2}\pi} \cdot C_T^{1/2} + \frac{\pi}{24} \cdot C_T + \frac{1}{24\pi k} \left( \frac{\pi C_T}{2} \right)^{3/2} + \ldots \right] \tag{9-3}
\]

where \(k = (\beta / \sin \theta)^{1/2}\). In Fig. 121, test results of \(C_{LT}\) vs \(\alpha\) are shown and compared with values calculated with the above equation.

In Fig. 29, the total lift coefficient variation at zero incidence as calculated by Spence's theory (Ref. 16) - discussed in Sec. 6.4 - is compared with Dimmock's experimental results. A similar comparison but extended to various incidences is shown in Fig. 122. The straight lines account for a wing thickness correction.

In Ref. 33, another relationship for \(\Delta C_{LT}\) is derived (with the aid of the electrical analog tank) as

\[
\Delta C_{LT} = \left[ f(C_{\mu}) + C_{\mu} \right] \cdot \sin \theta \tag{9-4}
\]

where \(f(C_{\mu}) = C_{\mu} \cdot E_{L}\). Values calculated with this equation are compared with experimental results, as obtained with an AR \(\alpha = 20\) jet flapped wing, in Fig. 63A. Agreement can be considered as good at least for the range \(\theta \leq 65^\circ\) and \(C_{\mu} \leq 1.0\). The above equation can also be written as

\[
\frac{\Delta C_{LT}}{\sin \theta} = f(C_{\mu}) + C_{\mu} = \frac{C_{LT}}{\sin \theta} + C_{\mu} \tag{9-5}
\]

which is used to calculate the theoretical curves in Fig. 63B. It is indicated that the experimental \(\Delta C_{LT}\)-curve can be best approximated by the empirical relationship

\[
\Delta C_{LT} = 3.9 \cdot \sqrt{C_{\mu}} \cdot \sin \theta \tag{9-6}
\]

for the range considered.

Finally, Dimmock's results (Refs. 24 and 32) are compared in Ref. 31 with total lift coefficients predicted by the straight and curved mechanical flap analogy method (Refs. 11 and 31) as discussed in Secs. 6.2 and 6.3. Both theories give satisfactory results. Agreement, however, is slightly better for the curved flap analogy method (see Fig. 23).

For the effect of the ground on lift, see Sec. 9.5.1.
B. Three-Dimensional Wing

In Ref. 28, the total lift coefficients for a three-dimensional wing as shown in Fig. 123 are given for \( \theta = 31.3^\circ \) and \( \alpha = 0^\circ \) and compared with the empirical relationship

\[
C_{LT} = 1.4 \cdot \sqrt{C_J} = 2.7 \cdot \sqrt{C_J} \cdot \sin \theta
\]  
(9-7)

Agreement is good for \( C_J \leq 1 \) but is no longer satisfactory at \( \sqrt{C_J} > 1.2 \). Note, that transition wires at 0.2·c behind the L.E. hardly affect the \( C_{LT} \)-values at least up to \( C_J = 1.0 \).

Further tests (Ref. 33) on a complete model aircraft were conducted at \( AR = 3 \), a W.T. speed of 75 ft./sec. and a Reynolds number of \( 8.5 \times 10^5 \). The wing profile was symmetrical and of 4.25% thickness, the jet deflection angles used were \( \theta = 29^\circ \) and \( 90^\circ \). Jet coefficients were rather small, the maximum value being \( C_J \approx 0.2 \) only. The horizontal tail placed below the wing plane and tip tanks could be added or removed. If the total lift coefficients as measured are expressed by empirical relationships the following comparison can be made:

- two-dimensional wing (\( C_J \leq 1 \))

\[
\Delta C_{LT} = 3.9 \cdot \sqrt{C_J} \sin \theta
\]  
(9-6)

- three-dimensional model (\( C_J \leq 0.2; \ \theta = 90^\circ \)) straight wing

  a) without tail and tip tanks

\[
\Delta C_{LT} = 1.6 \cdot \sqrt{C_J}
\]  
(9-8)

  b) with tip tanks (equivalent to increasing AR)

\[
\Delta C_{LT} = 1.9 \cdot \sqrt{C_J}
\]  
(9-9)

- three-dimensional model (\( C_J \leq 0.1; \ \theta = 90^\circ \)) delta wing

  a) without tip tanks

\[
C_{LT} = 1.46 \cdot \sqrt{C_J}
\]  
(9-10)

  b) with tip tanks

\[
C_{LT} = 1.64 \cdot \sqrt{C_J}
\]  
(9-11)

Summarizing the above evidence it can be said that for two and three-dimensional jet flapped wings the total lift can be expressed in the general form.
for $C_J$-values less than unity. The parameter $X$ depends upon aspect ratio and $\theta$.

In Fig. 34 total lift coefficients for two and three-dimensional jet flapped wings as calculated with Maskell and Spence's theory (Ref. 30) are compared with experimental results of Refs. 24 and 28. Agreement is very good. The same is true for Fig. 124, in which the total lift variation with $C_J$ and $\alpha$, as obtained from Eq. (6-47) (Ref. 30) is compared with measured values (Ref. 28).

9.1.2 Lift Curve Slope

In Refs. 24 and 32, an equation for the total lift coefficient is given as

$$C_{LT} = C_{LT\alpha=0} + \alpha \left[ \frac{\partial C_{LT}}{\partial \alpha} \right]_{\alpha=0}$$

(9-13)

where \[ \left[ \frac{\partial C_{LT}}{\partial \alpha} \right]_{\alpha=0}, \] derived in Ref. 11 is

\[
\frac{\partial C_{LT}}{\partial \alpha} \bigg|_{\alpha=0} = \frac{2\pi}{1 + \frac{k}{2\pi} C_J^{1/2}} + \frac{\pi}{24} \cdot \frac{C_J}{k^2} + \frac{1}{24\pi k} \cdot \left( \frac{\pi C_J}{2} \right)^{3/2}
\]

(9-14)

The broken lines in Fig. 121 are calculated with these relationships and compared with experimental results. Agreement between experimental and theoretical curves at a first glance seems to be reasonably close. Actually this does not mean much unless at least the stalling angle is specified as well.

It seems only natural to take the lift curve slope at $\alpha=0$. But if this point $\alpha=0$ is close to or even greater than the stalling incidence (see Fig. 121), the slope at $\alpha=0$ becomes meaningless. This point is illustrated in Fig. 125, where the upper range limits of $dC_L/d\alpha$ correspond to the tangents at the experimental points of the $C_{LT}$ vs. $\alpha$ curves just before L.E. separation occurs and the lower range limits to the tangents to $C_{LT}$ at $\alpha=0$. It seems, that tangents drawn to $C_{LT}$ before stall occurs, provide lift curve slopes which agree reasonably well with the theoretically predicted ones as shown in Fig. 56.

The lift curve slope may also be predicted by Spence's thin aerofoil theory method. After applying a wing thickness correction to it, agreement with Dimmock's measured results can be considered as fair (see Fig. 126).
Finally, the lift curve slope calculated with the curved flap theory of Ref. 31 seems to be (see Fig. 127) a function of $\theta$ whereas tests of Ref. 23 indicate (Fig. 56) that $dC_{LT}/d\alpha$ is practically independent of $\theta$ and almost in agreement with the theoretical curve obtained with the straight mechanical flap analogy method (see Eq. (9-14) and Ref. 11).

### 9.1.3 The Lift Efficiency and Gain Factor

The variation of $E_L$ with $C_\mu$ as obtained from experiments and as predicted by theory is shown in Fig. 60. Agreement is quite remarkable. It should be noted also that $E_L$ seems to be almost independent of the jet deflection angle. As

$$E_L = \frac{C_{LJ}}{C_\mu \sin \theta} = \frac{f(C_\mu)}{C_\mu} \tag{9-15}$$

and

$$\Delta C_{LT} = \left[ f(C_\mu) + C_\mu \right] \cdot \sin \theta \tag{9-4}$$

it follows, assuming independence, that

$$\frac{C_{LJ}}{\sin \theta} = \frac{\Delta C_{LT}}{\sin \theta} \cdot C_\mu = C_\mu \cdot E_L = f(C_\mu) \tag{9-16}$$

This curve is added on Fig. 63B, where also $C_{LJ}/\sin \theta$ as obtained from the empirical relationship

$$\frac{C_{LJ}}{\sin \theta} = 3 \cdot q \cdot \sqrt{C_\mu} - C_\mu \tag{9-17}$$

is shown for comparison.

In Fig. 20, the lift gain factor $G_L$ as calculated by Stratford's straight flap theory (Ref. 11) is compared with early jet flap experimental results. Agreement can be said to exist only in trend. A similarly poor agreement is obtained if curved flap analogy predictions are compared.

### 9.2 Shift of Lift Centre and Aerodynamic Centre

A comparison of the shift of the A.C. and L.C. as obtained (Ref. 23) experimentally with a two-dimensional wing and calculated is shown in Fig. 128. The L.C. shift was found from the equation
which was derived in Ref. 11. Agreement between experiment and theory is poor.

The A.C. shift was calculated from the equation (see Ref. 11)

\[ a = \frac{k}{4} \frac{C_{J}^{1/2}}{\sqrt{2\pi}} \]  

(9-19)

Again, agreement between the experimental and calculated shift is poor.

Another comparison of experimental results with theory is shown in Ref. 33 for the L.C. shift. Agreement is very good as indicated in Fig. 40. The theoretical values were obtained from

\[ x_{L.C.} = \frac{C_{\mu} + f(C_{\mu}) \cdot x_{1}}{f(C_{\mu}) + C_{\mu}} \]

(9-20)

where \( f(C_{\mu}) = C_{\mu} \cdot E_{L} \) (see Sec. 9.1.3) and \( x_{1} \) is defined by Eq. (6-86).

Finally in Ref. 31, the experimentally obtained L.C. and A.C. shifts (Dimmock) are compared with that predicted by the curved flap analogy method. Agreement is very poor as shown in Fig. 129.

Reference 133 investigates the location of the centre of the jet induced pressure lift, \( C_{LJ} \), (as represented by the second term of Eq. 6-33b) and concludes - after a comparison of the calculated location with that from British tests - that the experimental L.C. of the jet induced pressure lift does not vary much. It seems to remain stationary at about the 45% chord position, at least as long as \( C_{J} < 1 \) (see Fig. 6 of Ref. 133).

9.3 The Pitching Moment Coefficient

If the pitching moment coefficient as calculated by both the straight and the curved flap theory (see Secs. 6.2 and 6.3) is compared with experimentally obtained \( C_{M} \)-values (Refs. 24 and 32), the curved flap theory provides the better agreement as illustrated in Fig. 24.
9.4 The Pressure Distribution

In Fig. 130, the pressure distribution for a 12.5% thick ellipse as calculated with Spence's thin aerofoil theory (Ref. 16) is compared with the measured one. The local differences are believed to be due to chordwise velocity components of the main stream flow, being induced by the jet sheet.

Pressure distributions calculated with Eq. 38 of Kuchemann's report (Ref. 17) are compared with experimental results obtained with a two-dimensional wing of elliptic section, for which that equation is an exact solution. The result is shown in Fig. 131. Note, that the effect of wing thickness is quite appreciable as shown in Fig. 131C.

French results obtained with an AR \( \propto 20 \) NACA 0018 wing, being shortened to 90\% of the basic chord, the jet issuing at \( \theta = 58\text{o} \), are compared with the pressure distribution calculated by Kuchemann's method. In general, agreement is good and Kuchemann's prediction of a saddle-back pressure distribution (see Fig. 8 of Ref. 17) is again well confirmed.

In Fig. 38A, the theoretical pressure distribution (Kuchemann) is compared with measured ones (from Ref. 28) on a wing of finite aspect ratio (AR = 2.75). Agreement is also fairly good in this case.

For the effect of the ground on the pressure distribution, see Sec. 9.6.2.

9.5 Total Drag of Two-Dimensional Wings

In Ref. 142, it is shown that two-dimensional test results follow the empirical relationship (see Fig. 132) given by:

\[
C_{DT} = C'_{DP} + a(\theta) C_{\mu}
\]  \hspace{1cm} (9-21)

A similar relationship was found by Dimmock (see Fig. 32 of Ref. 24) to apply for the profile drag, \( C_{DP} \). From all the available (quasi) two-dimensional test results plotted in Fig. 133, the value for \( a(\theta) \) can be obtained approximately as

\[
a(\theta) = 0.63 \cdot \sin^2 \theta
\]  \hspace{1cm} (9-22)

Therefore,

\[
C_{DT} = C'_{DP} + C_{\mu} \cdot 0.63 \cdot \sin^2 \theta
\]  \hspace{1cm} (9-23)

No doubt this is a very straight forward and useful expression for the total drag of a (quasi) two-dimensional jet flapped wing.
9.6 Ground Interference Effects

9.6.1 On Lift

Equations to predict the ground effect on the lift and pitching moment for the two-dimensional as well as the three-dimensional jet flapped wing are derived in Ref. 31 and discussed Sec. VII. Figure 43 presents a comparison of the ground effect on the lift as measured and calculated. Agreement is poor where it matters, i.e., for ground clearances smaller than one wing chord and for $C_J$-values smaller than 2.5. This is due to simplifying assumptions made in the theoretical treatment (see Sec. VII.).

A much better agreement between the theoretically calculated and measured maximum pressure lift is furnished by the theory of Ref. 139 as demonstrated in Fig. 134. In Ref. 149, Huggett presents a more refined - but less practical - theory which, as shown in Fig. 135 provides still better agreement with experimental results especially, it seems, at $C_J < 1.0$.

9.6.2 On Pressure Distribution

In Ref. 149, a theory is presented which allows the prediction of the pressure distribution around two-dimensional jet flapped wings in the proximity of a ground. In Fig. 136, calculated and experimental results for a ground distance of $h/c = 0.5$ and high $C_J$-values are compared. Note, that the $C_J$-values for this comparison are not the same. Agreement is quite good.

9.7 Aspect Ratio Effects

9.7.1 Total Lift Coefficient

For $C_J < 1$, $C_{LT}$ at $AR = 2.75$ and 6.8 was found in Ref. 28 to be about 60% and 70% respectively of the $C_{LT}$-values obtained with the two-dimensional version of the same wing. The appropriate conventional aspect ratio correction factors (from $[1 + a_o / \pi AR \cdot e]^{-1}$ assuming elliptical loading ($e = 1$) and the section lift curve slope $a_o = 2x$ (flat plate value)) are 58% and 77% respectively.

The aspect ratio effects on total lift coefficients of jet flapped wings as measured (Refs. 24 and 28) and calculated (Ref. 30) are shown and compared in Fig. 34.

9.7.2 Lift Centre Shift

The shift in centre of lift at $C_J = 2.08$ was found in Ref. 28 to amount to 24% and 31% of the chord past the mid chord point for the
"quasi two-dimensional" and three-dimensional wing respectively, both at zero angle of attack. The two aspect ratio curves added on to Fig. 128 give an idea of the aspect ratio effect on the L.C. shift.

9.7.3 Concluding Remarks

As the geometric aspect ratio has no meaning in the case of quasi two-dimensional tests and for wing configurations with e.g., wing tip tanks, the lift curve slope $dC_L/d\alpha$ - being directly related to the effective AR - may prove a better choice for correlating AR - effects on the performance of the jet flapped wing.

Assuming that the fluid flap behaves like a mechanical flap - the effectiveness $(dC_L/d\beta)$ of which is proportional to $dC_L/d\alpha$ of the wing to which it is attached - the lift increase for a given $\theta$ and $C_J$ should then be proportional to $dC_L/d\alpha$ of the wing without the fluid flap (no blowing). Further, the presence of a fuselage can be accounted for by tabulated calculations for the mechanical flap as given in Refs. 37 and 38. The lift will be decreased (Ref. 33) in the ratio of

\[
\frac{[\Delta C_{LT}]_{W+F}}{[\Delta C_{LT}]_W} = k < 1
\]  

(9-24)

In Fig. 137, all results of Ref. 33 for various AR are plotted as $\Delta C_{LT}/k \cdot \sin \theta$ vs $(dC_L/d\alpha)_{c_u = 0}$ for two values of $C_J$ (0.02 and 0.1) and $\alpha = 0$. Note that in spite of very different test conditions ($4 < t < 18\%$; $0.3 \times 10^6 < Re < 2.4 \times 10^6$; etc.) all points lie on practically straight lines. Since these results were obtained at low jet coefficients, the linearity of these curves at higher $C_J$-values should not be taken for granted without proof. If, however, linearity were an established fact, the two-dimensional results could be used for the preliminary design of jet flapped aeroplanes (Ref. 33), for which the $(dC_L/d\alpha)_{c_u = 0}$ and $k$ are known. As the experimental jet flap curve satisfies the equation (see Fig. 63)

\[
\Delta C_{LT} = 3.9 \cdot \sqrt{C_{\mu}} \cdot \sin \theta
\]  

(9-25)

and $(dC_L/d\alpha)_{c_u = 0} = 0.092$ (see Fig. 9 of Ref. 33), an approximate formula to be used for a preliminary design could be

\[
\Delta C_{LT} = 42.5 \cdot \sqrt{C_{\mu}} \cdot \sin \theta \cdot (dC_L/d\alpha)_{c_u = 0}
\]  

(9-26)
X. JET MIXING (REFS. 11, 46, 58, 14 and 36)

10.1 Introductory Remarks

10.1.1 The Thrust Discrepancy Between Theory and Experiment

While experimental results do agree fairly well with the predictions of the lift hypothesis, they do not agree too well with the predictions of the thrust hypothesis. Experimentally, the measured thrust \( (C_{TM}) \) is found to be less than the full jet reaction force \( (C_J) \), although for large jet deflection angles it generally exceeds the jet reaction thrust \( (C_{TR}) \), if \( v_o > 0 \). The differences can partly be attributed to the rapid entrainment of the main stream air into the jet (mixing between the jet and the main stream).

We know that the thrust hypothesis is stated for a two-dimensional jet-flapped wing in an ideal flow. In reality, wings are finite and the flow is viscous. This accounts for the following additional forces which act on a real jet flapped wing:

1) induced drag ("Potential" Drag of Finite Wings)

2) skin friction drag

3) form drag including the jet-mixing effect.

If these forces are properly taken into account in a theoretical approach, better agreement between the theory and experiments should result. As a first step in this direction, the forces and effects of jet mixing on the jet flapped wing have to be investigated.

10.1.2 Jet Mixing and its Effect on the Jet-Flapped Wing

A circular jet or a jet sheet, mixes with the surrounding air due to viscous effects. The consequences, however, are different on account of the location of the mixing region. With a circular jet, the dimensions of the jet are such that most of its mixing occurs so far downstream of the wing as not to affect its pressure distribution. Hence, the thrust acting on the aircraft is equal to the thrust of the engine. In the jet flap case, the jet is spread out all along the trailing edge of the wing and its thickness is greatly reduced. As a result the mixing occurs within a much shorter distance from the wing. It therefore can affect the wing's pressure distribution and hence its pressure thrust. This fact will be better appreciated when it is remembered that, at large deflection angles, the jet-flap system primarily relies for its thrust on the wing's pressure forces \( (C_{TR} \rightarrow 0) \).
How can this jet mixing effect be isolated? Consider two model aircraft which are exactly similar except for the fact that one has the conventional circular jet and the other one is jet-flapped. Wings and jets are parallel to the undisturbed main stream far ahead ($\alpha = 0$ and $\theta = 0$). With both jets having the same flow properties (mass flow, total pressure and temperature), the respective thrust on the two models is measured in a wind tunnel. Let $TH_f$ be the thrust acting on the jet-flapped aircraft and $TH_C$ the thrust on the conventional aircraft. The difference $|TH_f - TH_C|$ is due to the jet mixing effect on the jet flapped wing.

This result can be explained as follows: in ideal flow one expects no loss in thrust on either model, because there is neither profile nor induced drag (no lift) acting. In real flow the induced drag would still be zero (no lift). But there would be a loss in thrust due to skin friction and from drag. If one assumes that the skin friction drag of both model aircraft is the same, one has to conclude that any difference in thrust is due to a change in form drag, which has to originate from the changes in the pressure field of the wing due to mixing. But there are only negligible mixing effects in the case of the conventional jet aircraft.

The assumption that there is no difference between the skin friction drag of the two similar aircraft needs checking. The authors believe that in case of jets of high momentum, this assumption may no longer be correct. This belief is based on the fact that the sink effect of the jet alters the flow speed over the jet flapped wing especially close to the wing's T.E., thereby altering the skin friction drag.

The above assumption, however, does not hold in the case of blowing over a shroud or flap. This is because of the skin friction drag produced by the high velocity jet sheet flowing over the flap surface. This drag is larger than that which would be produced by the low velocity main stream flow.

Next, two theoretical studies of jet mixing in connection with jet flaps will be considered.

10.2 Stratford's Analysis (Ref. 14)

Stratford used the following three methods for his theoretical study of the jet flap mixing process:

1) The Energy Method

2) The Momentum Method

3) The Sink Effect Method
10.2.1 The Energy Method (Ref. 46)

Imagine the engine power to be expended in two ways, one for producing the thrust and the other for compensating the dissipation due to jet mixing (neglecting kinetic heating and the wake of the wing). Or in other words:

Useful power expended on the aircraft

\[ \text{Useful power} = \text{Engine power} - \text{power dissipated in jet mixing} \] (10-1)

Since the

Useful power = thrust \times \text{aircraft speed} \hspace{1cm} (10-2)

it follows that

\[ \text{TH} = \frac{\text{Engine Power} - \text{Power Dissipated in Mixing}}{v_o} \] (10-3)

This equation applies to both the conventional and the jet flap system. If the conventional aircraft is taken as a standard and compared with the jet flapped aircraft of equal engine power, one gets the change of thrust (\( \Delta \text{TH} \)) as

\[ \Delta \text{TH} = -\left( \frac{\text{change of power dissipated in jet mixing}}{v_o} \right) \] (10-4)

or

\[ \frac{\text{TH}_f - \text{TH}_C}{\text{TH}_C} = -\left( \frac{\text{change of power dissipated in jet mixing}}{\text{TH}_C \cdot v_o} \right) \] (10-5)

Now the expression for the change of power dissipated in jet mixing has to be found.

In the conventional system almost all the mixing occurs in a region where the static pressure is equal to the undisturbed pressure \( P_0 \). In the jet flap system, however, the initial stages of mixing occur where the pressure, \( P_1 \), is different from \( P_0 \). To facilitate theoretical study the following model is utilized. Assume the jet to mix with \( n \) times its own mass flow in a region of pressure \( P_1 \), and the remainder of the mixing to occur at pressure \( P_0 \). This change in static pressure causes a change in power dissipation. It is further assumed that \( (P_1 - P_0) \) is sufficiently small for the use of differentials \( \delta P \) and \( \delta v \) and that the velocities of the jet and main stream are locally parallel in the mixing region.
It is shown in Ref. 46 that
\[
\text{dissipated power} = \frac{1}{2} m_J v_d^2 \quad \text{(for } n \to \infty) \tag{10-6}
\]
where \( v_d \) is the velocity difference between jet and main stream and \( m_J \) is the mass flow of the jet. Then,

the change in dissipated power \( = m_J v_d \delta v_d \) \tag{10-7}

and

\[
m_J v_d \sim m_J (v_J - v_o) = TH_C \tag{10-8}
\]

Therefore from Eq. (10-5)
\[
\frac{TH_f - TH_C}{TH_C} = - \frac{\delta v_d}{v_o} \tag{10-9}
\]

Generalising the above result for finite values of \( n \), one gets:
\[
\frac{TH_f - TH_C}{TH_C} = - \frac{\delta v_d}{v_o \left( 1 + \frac{v_J}{n \cdot v_o} \right)} \tag{10-10}
\]

One can show using Bernoulli's equation in differential form \( (d \rho = - \rho \cdot v \cdot d v) \) that
\[
\delta v_d = (v_1 - v_o) \left( \frac{\rho_o \cdot v_o}{\rho_J \cdot v_J} - 1 \right) \tag{10-11}
\]

therefore
\[
\frac{TH_f - TH_C}{TH_C} = \frac{(v_1/v_o - 1)(1 - \rho_o/v_o)}{(1 + v_J/n \cdot v_o)} \equiv \text{primary change in thrust} \tag{10-12}
\]

10.2.2 The Momentum Method (Refs. 14 and 58)

The physical model considered in this method is the same as the one used in the energy method. Further, the assumptions that the flow is incompressible and that the jet has a uniform square velocity profile before and after mixing, are made.

This method is based on the theorem that the thrust acting on the aerofoil \( (TH = m_J(v_J - v_o)) \) is equal to the change of momentum of the flow between upstream and downstream infinity. It follows that
\[ \text{TH}_C = m_J \cdot v_d, o, n=0 \]  

(10-13)

where \( m_J \) is the jet mass flow before mixing. Suffixes \((n)\) & \((n = 0)\) indicate whether or not mixing has occurred and suffix \((o)\) designates that the static pressure there is equal to \( P_o \). Further,

\[ \text{TH}_f = (n + 1) \cdot m_J \cdot v_d, o, n \]  

(10-14)

Therefore

\[ \frac{\text{TH}_f - \text{TH}_C}{\text{TH}_C} = \frac{(n + 1) \cdot v_d, o, n}{v_d, o, n = 0} - 1 \]  

(10-15)

The jet velocity before mixing \( v_{J, 1, n = 0} \) at pressure \( P_1 \) is given by Bernoulli's equation in terms of the jet velocity before mixing at pressure \( P_0 \) as:

\[ v_{J, 1, n = 0} = v_{J, o, n = 0}^2 + \frac{P_o}{\rho_J} \cdot (v_{1}^2 - v_o^2) \]  

(10-16)

and the jet velocity after mixing is given by:

\[ (n + 1) \cdot m_J \cdot v_{J, 1, n = 0} = m_J \cdot v_{J, 1, n = 0} + n \cdot m_J \cdot v_1 \]  

(10-17)

But

\[ v_{J, 1, n = 0} = v_1 + v_d, 1, n = 0 \]  

(10-18)

and

\[ v_d, 1, n = \frac{v_d, 1, n = 0}{(n + 1)} \]  

(10-19)

The density of the jet after mixing can be obtained from:

\[ \frac{\rho_o - \rho_J, n}{\rho_J, n} = \frac{1}{n + 1} \cdot \frac{\rho_o}{\rho_J, n} \]  

(10-20)

(Note: It is shown in Ref. 36, that Eq. (10-20) applies only for a cold jet, see subsequent Sec. 10.3).

The velocity of the jet at infinity is given by Bernoulli's equation as:

\[ v_{J, o, n}^2 = v_{J, 1, n = 0}^2 - \frac{P_o}{\rho_J, n} \cdot (v_{1}^2 - v_o^2) \]  

(10-21)

From Eqs. (10-15, 19, 20 and 21), Stratford derives two approximate equations for the primary change in thrust. The first order approximate equation is:
The second order approximate equation is:

\[
\frac{TH_F - TH_C}{TH_C} = \frac{\frac{V_i}{V_o} - 1}{1 + \frac{V_J}{n\cdot V_o}} \cdot \left(1 - \frac{\rho_o \cdot V_o}{\rho_J \cdot V_J}\right)
\]  

Equation (10-22) is the same as that derived previously by the energy method (Eq. (10-12)).

10.2.3 The Sink Effect Method (Refs. 14 and 58)

Mixing of a jet with its surrounding air, which otherwise would be either stationary or moving parallel to the jet stream is due to entrainment of this air which in effect is similar to that of a line of aerodynamic sinks. The resulting suction at the boundaries of the jet is therefore known as sink effect of the jet. Induced suction extends upstream past the T.E. even as far as the L.E. However, the suction thrust near the L.E. is so much smaller than the T.E. suction drag that the resultant suction force causes an increase in the drag of the wing.

An analysis of the sink effect of the jet becomes somewhat more complex when the surrounding air is moving. Stream tube areas would have to be considered as the mixing of the jet with the main stream air causes a contraction of the external flow (Fig. 138). A proper distribution of aerodynamic sinks along the jet axis could be used to simulate this contraction. New boundary conditions would have to be established for the external flow which alone can transfer the mixing effects to the aerofoil. The sink distribution is considered to be equivalent in effect to the jet mixing if the contraction caused by the sinks is the same as that caused by mixing.

Consider the model of the previous case. The contraction resulting when a jet of mass flow \(m_J\) mixes with \(n\) times its own mass flow can be shown to be

\[
-\Delta W = \frac{m_J}{\rho_i \cdot V_i} \left[1 - \frac{\rho_i \cdot V_i}{\rho_J \cdot V_J, n=0} \cdot \frac{V_J, n=0}{V_i} - 1\right] \left[1 + \frac{V_J, n=0}{n \cdot V_i}\right]
\]  

and the equivalent sink strength follows from

\[
q = v_1 (-\Delta W)
\]
The force on a body in a body-sink system is \( \rho q (v_1 - v_o) \). (The sink is positioned where the velocity, but for the sink, would have been \( v_1 \)). Based on the above relationships, Stratford derived:

\[
\frac{TH_p - TH_c}{TH_c} = \frac{(\frac{v_1}{v_0} - 1)(1 - \frac{\rho_i v_1}{\rho_j v_J, n=0})(\frac{v_J, n=0}{v_1} - 1)}{(1 + \frac{v_J, n=0}{n \cdot v_i})(\frac{v_J, n=0}{v_0} - 1)} \tag{10-26}
\]

This expression agrees basically with Eqs. (10-12) or (10-23) except for the second order difference resulting from the inclusion of the values corresponding to pressure \( P_1 \) instead of those corresponding to \( P_0 \). Another second order correction, taking into account the sink effect in the region where the mixed fluids undergo the change in pressure from \( P_1 \) to \( P_0 \) could be applied, but would render results less suitable for practical use.

10.2.4 Estimation of Mass Flow Ratio (Ref. 46)

First consider the jet mixing with air at rest. Due to the entrainment, the external air must have a velocity, \( v_{ent} \), called entrainment velocity - which is perpendicular to the jet boundary. From semi-empirical theory, one gets:

\[
v_{ent} = \frac{v_J}{25} \quad ( \text{for } \rho_j = \rho_0 \text{ and } v_i = 0 ) \tag{10-27}
\]

Integrating \( v_{ent} \) along both sides of the jet, a value for \( n \) can be obtained. If the air is not at rest (\( v_1 \neq 0 \)):

\[
v_{ent} = \frac{v_J - v_1}{25} \quad ( \text{for } \rho_j = \rho_0 ) \tag{10-28}
\]

No standard theory is available accounting for the effects of density. Stratford suggests the following generalized equation based on experiments conducted with hydrogen and cold air:

\[
v_{ent} = (\frac{\rho_j}{\rho_0})^{\frac{1}{2}} \cdot \frac{v_J - v_1}{25} \tag{10-29}
\]

It should be noted that this entrainment velocity has been neglected (parallel flow assumption) in the three methods discussed above. Also a correction to the value of \( n \), obtained as indicated above, would have to be applied before this \( n \) is used in Eqs. (10-12, 22 and 23). The reason for this correction is the assumption of rectangular velocity profiles made in the derivation of these equations. A factor of the order of 0.7 has been suggested.
10.2.5 Discussion of Stratford's Results

The expression for \( \frac{\text{TH}_f - \text{TH}_c}{\text{TH}_c} \) gives the "primary change" in thrust. It is termed "primary" because of the assumption of parallel flow in the mixing region. In actual cases, however, the jet and mainstream velocities are not parallel since the main-stream has an additional velocity component normal to the jet boundary. Losses due to this component are termed secondary losses.

The thrust expressions (Eqs. 10-2 or 10-22) indicate that when the term \( \left( 1 - \frac{\rho_o v_o}{\rho_j v_j} \right) \) is positive, a thrust gain may be achieved, if the mixing is allowed to take place at a pressure lower than that of the free stream (i.e., at \( P_1 < P_o \) and \( v_1 > v_o \)). But drag would result if the mixing occurs at a pressure higher than that of the free stream. One way to allow mixing to take place at a pressure lower than \( P_o \) is e.g., to blow the jet over a mechanical flap. Stratford concludes that in theory such a controlled mixing will produce only a limited thrust gain as compared with a pure jet flapped wing, since the required rapid rate of mixing with its inherent secondary losses and the skin friction between the jet and the flap would counteract much of the otherwise possible thrust gain. In practice, however, the thrust in both cases will be less than that of a conventional aircraft. Stratford further expresses the opinion that it would be difficult to surpass at large deflection angles the prediction of the thrust hypothesis which effectively implies that \( \text{TH}_f \) is always equal to \( \text{TH}_c \).

The expression (Eq. 10-12) for the primary change in thrust also shows that the density of the jet has an important influence on the thrust. If the density-velocity products of the jet and of the undisturbed mainstream are equal, there should be no thrust or drag increment in whatever regions mixing is assumed to occur, neglecting skin friction drag and assuming a cold jet (see Sec. 10.3). Generally in cruising flight such a condition is closely satisfied (\( \rho_j v_j \approx \rho_o v_o \)). Hence, the jet flapped aircraft in cruising flight can be expected to experience about the same thrust as a conventional aircraft (for cold propulsive jets).

It can also be seen from the equation for \( \frac{\text{TH}_f - \text{TH}_c}{\text{TH}_c} \) that for large values of \( v_j/n v_o \), the primary change in thrust is proportional to the amount of mixing that takes place at pressure \( P_1 \) or more specifically to the ratio \( v_j/n v_o \).

In this analysis the secondary losses due to the entrainment angle and entrainment velocity are not considered. They are proportional to the velocity difference \( (v_j - v_o) \) between the jet and mainstream. Therefore any device which reduces \( (v_j - v_o) \) will reduce the secondary losses. Such a device is e.g., a by-pass engine which would allow lower jet speeds for a given thrust. Moreover, because of a reduced mixing rate, the use of by-pass engines should reduce the primary losses also.
10.2.6 Boundary Layer Mixing with the Jet (Ref. 46)

In the boundary layer over the wing surface, velocities are small compared with the main stream velocity. Hence the entrainment velocity which is proportional to the difference in velocities between the jet and the external air will be increased where the boundary layer mixes with the jet. Another undesired consequence is the increased rate of mixing. Also the entrainment angle (for a given mixing rate) of the boundary layer will be greater than that of the main stream due to the lower velocities in the boundary layer. This will increase secondary losses due to the large entrainment angles.

The logical conclusion would be that, if mixing is allowed to take place at pressures lower than that of the free stream, the primary and the secondary losses would be reduced because suction should increase the boundary layer velocities. Such suction pressure could be achieved by means of a shrouded jet flap. The total drag should then be less than that of a conventional aircraft, if the increase in skin friction drag due to blowing over the flap surface is ignored. This effect can be considered as a reduction in or even a removal of the form drag. In the pure jet flap case, however, the form drag will be greater than that of the shrouded jet flap since some of the mixing would take place at a higher pressure on the bottom surface of the jet sheet which otherwise would be partly sheltered by the shroud.

10.3 Payne's Analysis (Ref. 36)

10.3.1 Introduction

This analysis is intended to be an extension of Stratford's work (Ref. 46) and his equation (Eq. (10-20)) for the jet density after mixing. Payne introduced a factor $K$ which is primarily a function of jet temperature. His extended momentum analysis based on $K$ as a variable shows that the condition $\rho_j v_j = \rho_o v_o$ does not necessarily mean zero jet drag. Payne claims also that the most important factor which determines the direction of the suction force due to jet mixing is the surface slope near the jet exit. Zero slope is equivalent to zero force. Payne arrives at conclusions, based on his results, which are mentioned later.

Finally, Payne implies that the jet drag cannot be deduced from static (wind-off) jet momentum measurements at different reservoir pressures and total thrust force measurements with the wind-on (see Sec. 8.3.4).

10.3.2 The Jet Density After Mixing (Ref. 36)

Consider a model of jet mixing as shown in Fig. (139A). The pressure $P_1$ in the mixing region is assumed (as in Ref. 46) to be
constant. Considering the flow at the jet exit and where the jet is fully mixed, we have the following relationships,

From conservation of mass:

\[
\left[ \rho \cdot A \cdot v \right]_{J, l, n=0} + \left[ \rho A \cdot v \right]_l = \left[ \rho \cdot A \cdot v \right]_{J, l, n}
\] (10-30)

and the momentum equation \((P_1 = \text{const.})\):

\[
\left[ \rho A \cdot v^2 \right]_{J, l, n=0} + \left[ \rho A \cdot v^2 \right]_l = \left[ \rho A \cdot v^2 \right]_{J, l, n}
\] (10-31)

and the energy equation:

\[
\left[ \rho A \cdot v \cdot T \cdot c_p \right]_{J, l, n=0} + \left[ \rho A \cdot v \cdot T \cdot c_p \right]_l = \left[ \rho A \cdot v \cdot T \cdot c_p \right]_{J, l, n}
\] (10-32)

and finally the equation of state \((\text{const. pressure})\):

\[
\left[ \rho \cdot T \right]_{J, l, n=0} = \left[ \rho \cdot T \right]_l = \left[ \rho \cdot T \right]_{J, l, n}
\] (10-33)

From these four equations, four unknowns could be determined; but only the equations for velocity and density are stated here.

\[
v_{J, l, n} = \frac{v_{J, l, n=0} + n \cdot v_l}{(1 + n)}
\] (10-34)

Equation (10-34) is the same as Stratford's Eq. (10-17) (see Sec. 10.2.2) and \(v_{J, l, n}\) is the velocity of the jet at pressure \(P_1\) after mixing with \(n\)-times as much air at constant pressure \(P_1\)

\[
\frac{\rho_{J, l, n}}{(1 + n)} = \left\{ \left( \frac{T}{T_l} \right)_{J, l, n} \frac{(c_p \cdot T)}{\rho \cdot T} \right\}_{J, l, n=0} \left( \frac{1}{C_{p_{J, l, n}}} \right)
\]

\[+ \left( \frac{n}{\rho_l} \right) \left( \frac{T_l}{T} \right) \left( \frac{c_{p_1}}{C_{p_{J, l, n}}} \right) \right\}^{-1}
\] (10-35)

as compared with

\[
\frac{\rho_{J, l, n}}{(1 + n)} = \left[ \left( \rho_{J, l, n=0} \right)^{-1} + n / \rho_1 \right]^{-1}
\] (10-36)

which is a modified form of Stratford's Eq. (10-20). Stratford's derivation implies incompressible flow and constant specific heat. In Fig. (139B),
\( \rho_{J,1,n} \) is plotted against the local main stream velocity, using both Stratford's theory and Payne's analysis for a typical jet flap application. It is seen that Payne's density after mixing is considerably greater than Stratford's values. In order to retain the basic form of Stratford's expression, which is to be used in the momentum analysis of the next section, a correction factor \( K \) is introduced as

\[
\rho_{J,1,n} = K \cdot \rho_{J,1,n=0} \cdot (1+n) \left[ 1 + \left( n \cdot \rho_{J,1,n=0} / \rho \right) \right]^{-1} \tag{10-37}
\]

This correction factor \( K \) chiefly depends upon the temperature of the jet (see Fig. 140).

10.3.3 The Momentum Method (Ref. 36)

From Stratford's Eqs. (10-13, 14 and 15) Payne after applying Bernoulli's incompressible flow equation derives the following expression for \( (TH_f - TH_C) / TH_C \) (i.e., for \( \rho_0 = \rho_1 = \rho ; \rho_{J,1,n} = \rho_{J,0,n} \) and \( \rho_{J,n} = \rho_{J,1,n=0} = \rho_{J,0,n} = 0 \)).

\[
\frac{TH_f - TH_C}{TH_C} = \frac{1+n}{v_J - v_0} \left[ \frac{(p_0 + n)(v_o^2 - v_i^2)}{K(1+n)} \right. \\
+ \left. \left\{ \left( \frac{p_0 + n}{v_J + v_i} \right)^{1/2} + n \cdot v_i \right\}^{1/2} \right] - 1 
\tag{10-38}
\]

The form of this equation is different from Stratford's approximate expressions (Eqs. (10-22) and (10-23)). Let us consider now a few special cases.

A) \( v_0 = 0 \); Eq. (10-38) reduces to

\[
\frac{TH_f - TH_C}{TH_C} = \left[ -\left( 1+n \right) \left( \frac{\rho_{J} + n}{K \cdot (v_J/v_i)^2} \right) + \left( \frac{v_i^2}{v_J^2} \right)^{1/2} + n \cdot v_i \right]^{1/2} \tag{10-39}
\]

which for \( \rho_0 = \rho_J \) and \( K = 1.0 \) becomes

\[
\frac{TH_f - TH_C}{TH_C} = \left[ -\left( 1+n \right) \frac{v_i^2}{v_J^2} + \left( \frac{v_i^2}{v_J^2} + 1 \right)^{1/2} + n \cdot v_i \right]^{1/2} \tag{10-40}
\]
Stratford's equations (10-22) and (10-23) may be simplified for \( v_0 = 0 \) to:

\[
\frac{TH_f - TH_C}{TH_C} = n \cdot \frac{v_1}{\sqrt{v_J}}
\]

(10-41)

and

\[
\frac{TH_f - TH_C}{TH_C} = \frac{n \cdot v_1}{v_J} \left\{ \frac{1}{2} = \frac{1}{a} n \cdot (\frac{v_1}{v_J}) \right\}
\]

(10-42)

These equations are compared with each other in Fig. 141. Their divergence results from the fact that Stratford's solutions are series type solutions. A negative value for \((TH_f - TH_C)/TH_C\) is meaningless.

For \( n > 0 \) and \( v_1 > 0 \) \((P_1 < P_o)\), there will always be an increase in the momentum of the jet (see Eq. 10-40). Payne is of the opinion that such a potential thrust increase is not realized in practice because the low pressure region has no surface on which to generate this thrust increment and that a thrust gain without altering \( P_1 \) could only be obtained by means of a suitably shaped shroud, which provides a mechanism for transmitting such a momentum gain. It seems to us that Payne's reasoning contradicts the momentum theorem on which his derivation is based (see Eqs. (10-13) and (10-14)). According to this theorem, the momentum change between upstream and downstream infinity determines the force on the body in the momentum box, irrespective of the body shape. In Fig. 140, the variation of \((TH_f - TH_C)/TH_C\) with the density ratio \( P_o/P_J \) is shown.

B) \( v_0 > 0 \).

The following limiting cases will be considered:

For \( v_1 = v_0; (TH_f - TH_C)/TH_C = 0 \)

and for \( n = 0; (TH_f - TH_C)/TH_C = 0 \)

If \( \rho_o v_0 = \rho_J v_J \) and substituting \( v_J = \rho_o v_0 / \rho_J \) in Eq. (10-38), one gets

\[
\frac{TH_f - TH_C}{TH_C} = \frac{1 + n}{\rho_o \rho_J - 1} \left\{ \frac{\rho_J \rho_o + n}{K(1 + n)} \cdot \frac{1 - \frac{v_1}{v_o}}{v_0} \cdot \left\{ \left[ \frac{(\rho_J \rho_o) (\frac{v_1}{v_o} - 1) + \frac{\rho_J \rho_o}{\rho_J} \cdot \frac{1}{v_o} \right]^{1/2} + n \cdot \frac{v_1}{v_o} \right\} \right\}^{1/2} - 1 \right\} - 1
\]

(10-44)

This equation (see Fig. 78) obviously disproves Stratford's conclusion that the jet drag is necessarily zero \((TH_f = TH_C)\), when \( \rho_J v_J = \rho_o v_o \). It demonstrates that Stratford's approximate equations agree well only with Payne's exact solution for \( K = 1 \) (i.e., for the cold jet). As the jet temperature rises, \( K \) becomes greater than unity (see Fig. 140) and agreement between Stratford's and Payne's results deteriorates except for large values of \( v_J/v_o \) (see Figs. 8 and 9 of Ref. 36). In Fig. 140, the effect of the density ratio \( \rho_o / \rho_J \) is illustrated.
Payne also suggests that unless a thrust augmentation is obtained in the static case \((v_o = 0)\), there will be no thrust augmentation at any forward (flight) speed even if a kind of shroud is employed.

10.3.4 Sink Distribution Method (Ref. 36)

The source and sink effect of the jet was discussed previously in Sec. 10.2.3. Payne's sink distribution method, which is different from the one described in Sec. 10.2.3 is dealt with next.

Consider a sink of strength \(q\) in a main stream velocity \(v_o\). The resultant velocity at any point \(x, y\) is (see Fig. 142):

\[
V_R = \left[ \left( v_o + \frac{q}{2\pi x} \right)^2 + \frac{q\cdot y}{2\pi x^2} \right]^{1/2}
\]  

(10-45)

Assuming incompressible flow, we get, using Bernoulli's Equation:

\[
\Delta P_1 = P_o - P_1 = \frac{1}{2} \cdot \rho \left( V_R^2 - V_o^2 \right)
\]  

(10-46)

It can be shown that a streamline is defined by

\[
\gamma = \frac{t - 2v_o}{v_o + \frac{q}{2\pi x}}
\]  

(10-47)

The elemental drag force due to the sink is then

\[
d(\Delta D) = 2 \cdot dy \cdot \Delta P_1
\]  

(10-48)

Next from Eqs. (10-46, 47 and 48), Payne derives the following expression for the jet drag

\[
C_{DJ} = -\left( \frac{\gamma}{t} \right) \left[ \frac{b_1^2}{X} \left( X + b_1 \right) \right]^{X_1}
\]  

(10-49)

where \(b_1 = \frac{q}{2\pi v_o \cdot t}\) and \(X = \frac{x}{t}\).

Note: In deriving the above expressions for \(C_{DJ}\), Payne neglects the vertical velocity component which is small except near the trailing edge.

The parameter \(b_1\) can be related to the mixing ratio as follows:

\[
b_1 = \frac{q}{2\pi v_o \cdot t} = \frac{n \cdot m}{2\pi \rho_o \cdot v_o \cdot t} = \left( \frac{n}{2\pi} \right) (\frac{\delta}{t}) (\frac{\rho_f \cdot v_f}{\rho_o \cdot v_o})
\]  

(10-50)

In Fig. 143A, \(C_{DJ}\) minus the integration constant is plotted versus \(X\) indicating that the jet drag occurs principally over a
small region close to the trailing edge, and that no effect can be felt near the "leading edge". Moreover, the addition of the neglected vertical velocity component would make it even more localized (see Fig. 143B) and Payne concludes that the slope of the trailing edge is the dominating factor in jet drag (see Fig. 79).

10.3.5 Effect of Jet Slot Near the Leading Edge (Ref. 36)

Payne concluded (see Sec. 10.3.4) that the important factor in the jet drag is the surface slope near the trailing edge (see Fig. 79). This hypothesis is expected to hold good also when the jet emerges elsewhere on the aerofoil. To obtain a negative jet drag, i.e., a thrust augmentation, the section slope of a wing should be negative upstream of the location of the jet slot. Such a negative slope exists on conventional wing sections forward of its maximum thickness position. A thrust increase, however, is not realized in practice, since this simplified model neglects the increased skin friction of the wing immersed in a higher speed jet stream. The theoretical thrust gain is more than compensated for by the increased skin friction. Payne derives the following expression for this drag increase.

\[ \Delta C_{D,F} = 2 \xi C_f \left[ \frac{C_f}{\frac{1}{2}(\frac{\partial}{\epsilon})} - 1 \right] \]  

(10-51)

where \( \xi \) = Ratio of wing area covered by the jet sheet to total wing area.

and \( C_f \) = Coefficient of skin friction.

It is interesting to note that e.g., with the jet emerging at the midchord point, the skin friction jet drag could be fifteen times as great as the static pressure jet drag.

10.3.6 Payne's Conclusions

a) Stratford's equation for jet density has to be corrected for jet temperature.

b) The condition \( \rho_J V_J = \rho_0 V_0 \) does not imply the condition of zero jet drag.

c) A thrust gain could only be obtained if a mechanism for transmitting the gain in momentum exists. If no thrust augmentation occurs in the static case, thrust augmentation cannot occur at any forward flight speed.

d) The slope of the trailing edge is the dominating factor in jet drag.
e) In practice it is not possible to realize the theoretical thrust gain because of a simultaneous increase in skin friction when the jet slot is located upstream of the maximum thickness point.

f) One implication of this work is that a jet flap shroud (suggested by Davidson and Stratford) would actually be less efficient and give higher jet drag than any reasonable alternative.

A second implication is that the jet drag cannot be deduced from static jet momentum \( (C_J) \) measurements at different reservoir pressures and from "wind-on" measurements of the total thrust force acting on the aerofoil. Due to increasing depression at the T.E. with higher wind tunnel speeds, the jet momentum may increase for a given and constant reservoir pressure, leading to an apparent reduction in jet drag, if the static thrust force measurement is subtracted from the thrust force measured with the tunnel wind on.

XI. JET AUGMENTED FLAP ARRANGEMENTS

11. 1 The Jet Control Flap

11. 1. 1 Its Purpose

The unquestioned adoption of the pure jet flap principle for today's aircraft and jet power units is met with reservation (Ref. 33) due mainly to technical reasons, for example:

1) the jet deflection angle must be controllable during flight of the aeroplane.

2) a failure in jet blowing means a complete loss of the control of the lift of the aeroplane.

If, however, a mechanical flap is used in combination with jet blowing the following advantages can be obtained:

1) the flap deflection produces the required changes in operational jet deflection angle (COANDA EFFECT).

2) the deflected mechanical flap contributes additional lift.

3) B.L. control exerted on the flap by means of the blowing jet prevents flow separation over its upper surface, keeping the flap effectiveness up to values as high as theoretically possible, even at very large flap deflection angles.
4) in case of failure of the jet flap blowing system (jet engines), the mechanical flap upholds an effectiveness sufficient to control the aeroplane.

The B.L. control (prevention of flow separation) on such mechanical flaps can be obtained in two ways:

1) by tangential blowing, which ensures a proper flow over the flap at any desired deflection (COANDA effect)

2) by suction (Fig. 144) which can be combined with T.E blowing, using ejectors (see Fig. 17 of Ref. 33) in connection with either conventional or truncated aerofoils.

In Ref. 33, test results are presented for some of the above mentioned mechanical flap - jet flap combination schemes. It should be realized, however, that some of these schemes have to be considered as intermediate rather than ultimate solutions to the adoption of the jet flap principle, if compressor bleed air is suggested instead of the high momentum propulsive jet itself, the total energy of which is directly available.

11.1.2 The Superiority of the Jet Flap - Mechanical Flap Combination at Low CJ-values

In Fig. 144, the lift vs. $C_{j}$ for such a blowing flap with and without suction is shown and compared with a pure jet flapped wing of the same jet deflection angle $\tau = 76^\circ$. This graph obviously indicates a liftwise superiority of the jet flap - mechanical flap combination over the pure jet flap at low jet coefficients and zero angle of incidence. It is interesting to note that e.g., for $C_{\mu} = 0.1$, the value of $C_{L} \approx 3.3$ corresponds approximately to the theoretical lift of the 30% mechanical flap as calculated by Glauert's linearized theory.

This apparent superiority is slightly misleading. First compare the pure jet flap with the "without suction" curve. At take-off $C_{\mu}$ -values ($C_{\mu} > 1.0$), where any high lift principle is most effectively utilized, the pure jet flap becomes superior. In the "with suction" case, the shown advantage of the jet flap-mechanical flap combination may be appreciably reduced in practice, if the required equipment for producing this suction (weight, bulk, power etc.) is taken into account in evaluating overall performance.

11.1.3 Ejector Blowing and Suction

In Fig. 17 of Ref. 33, a technically possible way of combining suction and blowing by means of ejector pumps, distributed span-wise along the axis of rotation of the flap is shown for both a conventional
and a truncated aerofoil. On the latter, suction eliminates its inherent high drag by preventing the cyclic separation of the flow (vortex shedding) from the trailing edge of the truncated wing.

11.1.4 The O. N. E. R. A. Jet-Control Flap with Surface Blowing

A. Symmetrical Blowing

Jet-control flaps of various sizes used by O. N. E. R. A. (Ref. 33) and the total lift coefficients experimentally obtained are shown and compared with the pure jet flap theory in Fig. 145. As indicated, blowing is provided over the upper and the lower flap surface.

In spite of a somewhat smaller lift increase than that obtained with upper surface blowing only (see Fig. 98), symmetrical blowing is of practical interest in so far as it can supply positive as well as negative lift changes more effectively. This is obviously an advantage if such a flap is used as a tail surface elevator or as a gust alleviating device in cruising flight.

From Fig. 98, the following empirical equation for the calculation of the total lift coefficient

\[
\Delta C_{LT} = 5.4 \sqrt{C_J} \cdot \sin \theta
\]  

(11.1)

was derived. If \( C_{LT} \) is calculated by Glauert's flap theory for various flap sizes and compared with corresponding experimental results plotted vs the jet deflection angle \( \theta \), Fig. 145 results. It should be noted, however, that in Glauert's method, the aerofoil is replaced by a thin flat plate, while the actual experiments were done with up to 22% thick wings. Nevertheless, agreement is satisfactory.

Attention has also to be drawn to the pressure distribution curves for such a wing as shown in Fig. 84. Note that at higher \( C_{\mu} \)-values the centre of pressure seems to move forward, thereby reducing the high nose down pitching moments inherent with the jet flapped wings (see Fig. 81 for comparison). With increasing jet deflection angles \( \theta \), however, the longitudinal static stability curves (\( C_{LT} \) vs \( C_M \)) are shifted in the direction of nose down pitching moments.

B. Upper Flap Surface Blowing

If blowing is restricted to the upper flap surface only (note the characteristic inflexion of the flap, which makes possible jet deflection angles of at least \( \pm 60^\circ \)) a further lift increase can be verified as shown in Fig. 98. For instance, at \( C_J = 4 \), an increase in \( C_{LT} \) from 11.1 to 15.1 is indicated. About one quarter, however, of this lift increase is due to the larger jet deflection angle (\( \theta = 78^\circ \) instead of 67°),
which increases $C_{LR} = C_J \cdot \sin \theta$. Ref. 33 quotes the empirical relationship

$$
\Delta C_{LT} = 6.5 \sqrt{C_J} \cdot \sin \theta \quad (11-2)
$$

for the total lift coefficient. It is derived from the above figure and can be compared with a similarly derived equation

$$
\Delta C_{LT} = 3.9 \sqrt{C_J} \cdot \sin \theta \quad (11-3)
$$

for the pure jet flap which furnishes much smaller $C_{LT}$-values.

C. Blowing at High Flow Speeds

In Ref. 39, the above described investigations are extended to higher flow speeds ($0.65 \leqslant Ma \leqslant 2$). Symmetrical blowing from nozzle slots of 0.3% chord on a 70% chord truncated wing of AR = 3 is investigated.

D. Supercirculation by Blowing and Sucking

If supercirculation is to be obtained with, e.g., a blown flap, a high momentum, i.e., high energy jet sheet is required. If the exhaust jet of a jet propulsion unit can be used, this high energy is available at least in theory at no extra cost.

Reference 33 suggests another method by which to establish supercirculation, a combination of suction on the upper with blowing from the lower wing surface near the T.E. as shown in Fig. 146A. It is expected that the amount of energy required to produce equivalent suction on the upper surface by solely blowing from the lower surface slot would be essentially larger.

The theory presented in Ref. 33 is claimed to be confirmed by experiments. Accordingly, the effects of suction and blowing add linearly (see Fig. 146) and the increase in lift in both cases is proportional to $C_q$ (since $C_{\mu} = \frac{\Delta C_{LT}}{\Delta \theta}$ for a given jet-slot-width to chord ratio $(\frac{\Delta}{C})$, $\Delta C_{LT} \propto \sqrt{C_{\mu}} \propto C_q$).

11.1.5 Size of Jet Control Flaps

Mechanical flaps have chord lengths up to and greater than 30% of the wing chord to which they are attached. By contrast, typical jet shroud sizes are mentioned in Ref. 12 to be of the order of 3 to 10%, depending on many diverse factors.

In Ref. 33, the jet control flap used was 12.5% of the wing chord and systematic tests with flaps of 17.7% and 22.2% produced no further appreciable lift increase for a given jet deflection angle and
given \( C_J \) of unity. This is shown in Figs. 145 and 147. Note that theory predicts, as expected, a higher lift increase for the larger jet control flap, whereas all the test points fall practically on the theoretical curve for the 12.5% flap. This seems to suggest that there is no point in increasing the jet control flap chord beyond 12.5% except if \( C_J \) is zero (case of no blowing). But even then (see Fig. 147) the lift increases only slightly with larger flap chords.

Davidson (Ref. 12) mentions that the effects of the shroud and jet will clearly be not additive since the lift of a small flap (shroud) varies as the square root of its chordal extent. A similar investigation of French researchers (Ref. 33) using the electrolytic tank method came up with the same observation.

In Ref. 39, a flap chord of 11.75% of the wing chord was used, in Ref. 138 it was only 7% of the basic chord (O.N.E.R.A. type flap).

11.2 The Truncated Aerofoil

11.2.1 General Remarks

As described below, extensive work was done at O.N.E.R.A. (Ref. 39) on truncated aerofoils.

To start with, truncated aerofoils were compared theoretically (Busemann's Theory) and experimentally with the full length chord (lenticular) aerofoil. Then T.E. blowing was investigated in subsonic, transonic and supersonic (Ma \( \leq 2.5 \)) flows. Further work compared the \( \theta = 90^\circ \) deflected jet sheet with a solid spoiler. Finally, more experimental evidence was added to the results of Ref. 33 by testing a truncated aerofoil with a jet control flap. Only the last three projects are briefly discussed below.

11.2.2 The Truncated Aerofoil with T.E. Blowing at High Speeds

The purpose of these tests was two-fold: the first objective was to determine whether the jet sheet can be used to eliminate T.E. vortex shedding as does, e.g., a solid thin blade. The following experimental results were obtained and are reported in Ref. 39.

In subsonic flow and at very low jet coefficients (\( C_J < 0.002 \)), the jet prevents the formation and periodic shedding of T.E. vortices. In supersonic flow, it reduces the expansion around the T.E. and hence reduces the base drag as predicted in Ref. 41 and shown in Fig. 148B (see also Figs. 10A and B of Ref. 39). If the gain in base drag reduction due to blowing (\( \Delta C_{D_s} \)) is plotted against Ma, Fig. 148A
is obtained. In the subsonic flow range the base drag reduction is spectacular, but less in the transonic and supersonic range. The maximum base drag occurs at $Ma = 1$.

The second objective of these tests was the investigation of the effect of the jet sheet on the transonic and supersonic flow around the aerofoil in the proximity of the T.E.

The experimental results of Ref. 39 cover two cases: the blowing jet being deflected at $\theta = 30^\circ$ and $\theta = 90^\circ$. At $\theta = 30^\circ$ and $Ma \leq 0.65$, the lift increases approximately with $\sqrt{C_J}$ (Fig. 149) as in the case of the jet flap or jet control flap. At higher Mach numbers, $\Delta \alpha_{CLT}$ becomes approximately proportional to $C_J$. As soon as shock waves occur at the T.E. (at $Ma > 0.85$), the $\Delta \alpha_{CLT} = C_{LT} - C'L$ decreases abruptly indicating that the jet no longer produces what could be called supercirculation. But in spite of the loss in $E_L$ (lifting effectiveness) with Mach numbers approaching unity, there is still a lift gain available, as the pressure induced lift is still of the order of the direct jet reaction lift. A vectorial presentation of $C_R = \Delta \alpha_{CLT} \rightarrow (C_{TM} + C'D)$ proves that up to $Ma = 1$, $C_R$ is always greater than $C_J$ but that both magnitude and direction $(\tan^{-1} \Delta \alpha_{CLT} / \Delta C_{TM})$ of $C_R$ decrease considerably with $Ma$, changing from subsonic to transonic (see Fig. 13 of Ref. 39).

At $\theta = 90^\circ$, the change in $\Delta \alpha_{CLT}$ with $Ma$ is shown and compared with the case for $\theta = 30^\circ$ in Fig. 14 of Ref. 39. The sharp decrease in supercirculation occurs at a $Ma$ as high as 0.85 and the asymptotic value which $E_L$ approaches at $Ma > 1.6$ is still worth noting. The superiority of the jet at $\theta = 90^\circ$ over that at $\theta = 30^\circ$ is due to its induced action on the flow upstream of the jet nozzle. In Fig. 150 the aerodynamic characteristics of the truncated aerofoil with $\theta = 90^\circ$ as a function of $C_J$ are compared with those for $\theta = 0^\circ$ and $\theta = 30^\circ$ at $\alpha = 0^\circ$ and $Ma = 2.04$. This figure indicates a lift curve slope ($d\alpha_{LT}/dC_J$) for $\theta = 90^\circ$ which is quite favourable at low $C_J$-values ($C_J \leq 0.02$) and remains always greater than that for $C_{LR} = C_J \sin \theta$. At $\theta = 30^\circ$, the lift contribution of the blowing jet has been reduced to merely the jet reaction lift $C_{LR}$: The variation in thrust ($C_{TM} + C'D$) was found to be proportional to the jet reaction thrust $C_{TR}$ and the nose down pitching moments indicate a lift increment in the proximity of the wing's T.E.

11.2.3 The $90^\circ$ Deflected Jet and the Solid Spoiler

The configuration used in these tests and the wing's aerodynamic characteristics vs Mach number are shown in Fig. 151 for $\alpha = 0^\circ$ and $C_J = 0.02$.

Whereas at subsonic and transonic speeds the lift is about the same for both configurations, it remains almost constant at supersonic speeds for the blown spoiler but increases for the solid spoiler.
blade. Practically the same applies for the changes in pitching moment with \( \text{Ma} \), except that the pitching moment decreases for the solid spoiler.

The minimum drag is much smaller for the blown spoiler but both curves are similar in trend. The decrease of \( \Delta C_D T \) at about \( \text{Ma} = 1 \) is due to the decrease of the induced drag, a consequence of the drop in the lift curve at transonic speeds.

It should be noted that theory and measurements in the supersonic speed range are in good agreement. Further, that the \( \theta = 90^\circ \) wing tip blowing appears to be a promising way for lateral control of high speed aircraft or missiles.

### 11.3 Related High Lift Devices

#### 11.3.1 The Free Streamline Flap

The free streamline flap (Refs. 81 and 82) has to be mentioned here as a device for improving the low speed characteristics of thin wings for high speed (e.g., transport) aircraft. Two possible configurations are shown in Fig. 1, C and D. Further versions, combining the jet flap principle with the free streamline flap - whereby the forward facing flap may be equipped if necessary with L.E. blowing for a more effective B.L. control - can easily be contemplated.

The two-dimensional model free streamline flap - as tested in Ref. 81 - showed remarkable performance characteristics, which are worth further investigation. Test results indicate very similar features as those observed with the pure jet flap or with jet control flaps. At small values of \( \alpha \) and at \( C_J > 0.15 \), the form drag was found to become positive (a thrust) as shown in Fig. 152. The total drag to lift ratio vs \( \alpha \) at \( \alpha < 20^\circ \) and \( C_J > 0.20 \) is smaller than 0.1 and the undesired but inherent high nose down pitching moments of the pure jet flap are less pronounced.

It may be of further interest to know that over the experimentally available Reynolds number range practically no scale effect was detected for this novel wing configuration and that B.L. control blowing (see Fig. 1D) at an angle \( \xi = 90^\circ \) produced higher lift values than blowing at \( \xi = 135^\circ \). This is due to reattachment of the flow to the L.E. of the forward facing flap at smaller \( C_J \)-values. Finally in Fig. 153, the \( C_{LT} \) vs \( C_J \) curve of the free streamline flap is compared with jet control flap and pure jet flap results. It is obvious that this comparison is not fair and - as the authors express it - "would be less favourable if it were made for equal values of incidence. This was not done as suitable results for the jet flap and the jet control flap at large incidences were not available".
In summarizing, the attractive features of the two-dimensional free streamline flap it should be realized that it represents an effective scheme for reattaching the flow to the upper surface of a sharp nosed (high speed) wing. Since the flap leads to a large increase in the chordwise loading primarily between the L. E. of the wing and L. E. of the flap, it should make it possible to design wings so that the centre of pressure is near the quarter chord point. This would result in smaller nose down pitching moments.

It would be of interest to see what effect dimensionality might have on the very attractive features of the two-dimensional free-streamline flap.

11.3.2 Exhaust Jet Deflection and Lift Augmentation

There is a vast variety of lift augmentation devices which, by deflecting all or part of the (in general) circular exhaust jet provide a reaction force component in lift direction. This component is

\[ C_{LR} = C_J \sin \tau \]  

(11-4)

for a jet being deflected through an angle \( \tau \). Whereas these lift augmentation devices can easily be applied to the conventional gas turbine exhaust jet, they do not provide either a noteworthy jet-induced thrust or lift and neither the thrust nor the lift hypotheses of the jet flapped wing apply.

A theoretical investigation of the relative merits of fixed and variable jet-deflection angles and separate lift and thrust engines (as referred to by A. R. Howell in Ref. 89) indicates the variable deflection angle system to allow the biggest reduction in approach speed and the separate lift and thrust engines the least reduction for a given available thrust/weight ratio.

A. Jet Deflection Devices

In so far as jet deflection devices are - aside from directional control - used for lift augmentation, they should at least be mentioned here.

In Fig. 154, a number of tested (Ref. 83) jet deflection devices are listed in order of their deflecting efficiency, being defined as the loss in axial thrust encountered in obtaining a given deflected force. Further, the range of possible deflection is expressed as the maximum ratio of the deflected force (\( TH_d \)) to the undeflected jet thrust (\( TH \)). This range determines the possible application - whether for lift augmentation, aircraft control or trim - of these jet deflection devices.

It is seen that the swivelled nozzle provides lift augmentation
most efficiently, followed by the swivelled tail pipe which, however, has the advantage of being able to convert the full thrust force into lift.

Reference 89 reports about flight tests with jet deflection of a converted METEOR aircraft. The deflector of the fixed type (deflection angle of 60°) was developed by N. G. T. E. and is shown in Fig. 1 and 2 of Ref. 89.

B. The Annular Nozzle

Whereas a circular jet impinging perpendicularly on the ground suffers a loss in thrust at close distances to the ground (Ref. 84) an annular jet provides a thrust (lift) augmentation as shown in Fig. 155. Unfortunately this increase in thrust (lift) is limited to ground distances smaller than the base diameter of the annular jet.

C. The Coanda Nozzle

In Refs. 83, 85 and 86, Coanda nozzles using single or multiple flat plate and curved plate deflection surfaces are reported on. A single flat deflection plate is shown in Fig. 156 together with the lift-to-thrust ratios vs total plate deflection angle for 1, 2 and 3. flat plate surfaces. The sine curve represents again the optimum values for the lift-to-thrust ratios. As expected, the curved plate (infinite number of flat plate surfaces) comes closest to the optimal sine curve (at 90° deflection, (CLR is between 81 to 88% of CJ). The thrust component C_TM is essentially zero (no jet induced pressure thrust C_TP).

11.3.3 External-Flow Jet-Augmented Slotted Flaps

The external-flow jet-augmented slotted flaps (see Fig. 1 D. b )) should also be mentioned here, as the results of investigations" are considered generally applicable for any type of jet flap system" (Ref. 76) as well.

Such flap arrangements have been found to be fairly easily applied to model configurations having pod-mounted engines. Wind tunnel investigations on flap-extended models (see also Refs. 74, 75, 69, 128, 141 and 144) covered a range of lift coefficients from 0.6 to about 12.5 and a range of angles of attack from -12° to about +13°. A downwardly directed nose jet had to be used to supplement the trimming power of the horizontal tail for tests at very high C_L-values (Ref. 76).

11.3.4 Blowing over Flaps From Nacelles Mounted Above the Wing

This subject is investigated in Ref. 43, using either circular or fish-tailed nacelles, blowing over the upper surface of flaps behind a delta wing at AR = 3 at zero angle of attack and jet coefficients in the range of 0<C_J<3.
The results obtained show that the $C_{LT}$ values are larger than the jet reaction lift $C_{LR}$ alone, which proves the existence of jet-induced lift. This induced lift was found to increase when a) the gaps between the wing and single or double slotted flaps were sealed and b) the circular jet was converted into a two-dimensional (fish-tail nacelle) jet.

For additional information, see also Ref. 141.

XII. THE JET FLAP AND ITS IMPLICATION ON AIRCRAFT DESIGN

12.1 Basic Considerations

From the definition of $C_{LT}$ and $C_J$ it follows that for a jet-flap aeroplane in steady level flight:

$$\frac{C_J}{C_{LT}} = \frac{TH}{L} = \frac{J}{W} \quad (12-1)$$

In Fig. 157, experimental curves (Ref. 33) of $C_{LT}$ and $V_0$ vs $C_J$ for various $\beta$ are superimposed to the straight lines at $J/W = \text{const}$. In Fig. 158, the general relation of functions $C_J$, $C_{LT}$, $J/W$ and $W/S_W$ are shown and how to work with such a graph. A change in lift can be performed either:

a) by changing $\theta$ ($\beta$), keeping $J$ constant

or

b) by changing $J$, but keeping $\theta$ constant

Whereas at small $V_0$-values, $C_J$ is large, $C_J$ becomes very small at flight speeds close to sonic (e.g., for $J/W = 0.5$, $C_J = 5$ at $V_0 = 66 \text{ fps}$, but reduces to $C_J = 0.02$ at $V_0 = 990 \text{ fps}$.)

12.2 Alternatives in the Generation of Thrust

As $TH = m(v_J - v_0)$, a certain jet reaction force can be obtained either by a high mass flow and low $(v_J - v_0)$ or vice versa.

12.2.1 Large Mass Flow

It can be bled off the compressor (cold air) or the total exhaust flow of the jet engines can be used (hot air).

Obtaining large cold mass flows at small pressures from the compressor is attractive but in practice would require large ducting inside the wing to keep flow losses small enough. Large hot mass flows on the other hand present difficult problems of installation and heat control and is thought to be feasible only where many small jet engines -
partially buried in the wing - spread their exhausts out into the T.E. slot. The by-pass jet engine with its rather low exhaust temperatures is a promising solution, especially for subsonic aeroplanes. On the other hand, supersonic aeroplanes would require very large jet velocities which can be achieved only with high gas temperatures.

12.2.2 Large Jet Velocities

Small mass flows at high pressures could for example be bled off from the later stages of the compressor. This method is not very economical if compared with the free energy available from the hot exhaust jets, but it facilitates a practical solution for its spanwise distribution due to smaller ducting cross sections required for flows at high pressure (density). If however the required exhaust velocity is still too small, Ref. 33 suggests - as an intermediate solution - to increase the jet velocity by adding heat to the air just upstream of the T.E. slot. In this case, only the T.E. part of the wing would have to be heat resistant.

12.3 High Speed Control

High speed control based on the jet flap principle has proved promising from both tunnel tests and some flight testing. Wind tunnel investigations with a blown spoiler blade (Fig. 159) - known to be effective over a large subsonic speed range - proved to be adequately effective at supersonic speeds (Refs. 27, 39 and 66). Some flight testing on rolling moment control was conducted at ONERA (Ref. 50). The inboard mechanical flaps at zero deflection angle were used to eject about 1% bleed air of the turbojet engines from their trailing edges.

The jet control flap with symmetrical blowing (ONERA) may be mentioned here for use instead of elevator flaps. Its aerodynamic characteristics up to $Ma = 2.5$ is covered in Ref. 39.

Another way of lateral control by means of wing tip jets - jet sheets ejected from the wing tip edges which also increase the effective wing aspect ratio - is recommended in Ref. 39 as a practical solution for high speed aircraft and missiles.

12.4 Integration of the Propulsive and Lifting Systems in Future Aircraft

12.4.1 The Idea

Küchemann and Maskell (Ref. 67) mention, amongst others, as one obvious advantage of the jet flap system "that it can lead to an aircraft where the means of producing lift and propulsion are integrated".
In the ideal case of such an integration, part or all of the propulsive force should be convertible either automatically or at the pilot's command into additional lift for take-off or landing of such an aircraft. In cruising flight a small portion of the propulsive power could then be used for drag reduction by means of B.L. control, resulting in an improved lift/drag ratio.

A proposal of how a conventional by-pass turbojet with reheat can be used for such a lift-thrust integrated system is discussed in Ref. 30* and shown in Fig. 160. If 58% of the total air is used (see Fig. 160) as secondary(by-pass) air and heated to 1900°C, it will provide a thrust force \( TH_2 \) equal to the max. possible forward thrust (100%) as obtained for the case of no by-pass air, while the remaining main flow air furnishes still 16% of the total forward thrust \( TH_T \). After 20 seconds, the aircraft will have reached a resultant speed of about 150 ft/sec. By progressive valve regulation, the pilot may now reduce the by-pass air to the lifting jets in favour of increased wing blowing but ultimately reducing until the total air mass flow is exhausted through the turbojet afterburner nozzles only.

12.4.2 The Problems Involved

At both very low and very high flight speeds, conventional controls leave much to be desired. It seems feasible, however, to use jet reaction forces instead of conventional aerodynamic forces for the control of aircraft. In short, the propulsive force would have to provide thrust, augment lift, supply adequate control forces and reduce the aerodynamic drag of the aircraft or increase it by thrust reversal (braking), whatever the momentary requirement may be.

Reference 27 discusses where we stand to-day with respect to the above described hypothetical aircraft of the future in the light of late developments and tested practical solutions to various possible systems suggested. For cruising flight, flow laminarization - capable of reducing, e.g., a 230,000 pound aircraft to 65,000 pounds, keeping payload and range constant - and the means to achieve it are considered. For low speed flight, mechanical and blown flaps, slats etc. as means of lift augmentation and boundary layer control are discussed. The use of propulsive power for the control of an aeroplane is dealt with under two aspects:

a) using propulsive power to increase the effectiveness of conventional controls (ailerons, elevators or rudders)

b) replacing these controls and their induced forces by direct propulsive forces, especially desirable for high speed flight (where exceptionally large loads - requiring boost systems - act on conventional controls) and at very low speeds (when the conventional aerodynamic control forces are not large enough).
In Fig. 161, the favourable effect of blowing over a conventional control on the yawing moment is shown. Since the lift can also be altered by trailing edge suction on either the T.E. of the wing or of the flap, a combination of both systems (Ref. 27) would supply the flap for low speed flight and the use of suction alone for high speed flight control. Another method for a lift decrease or increase (force control) is to blow an air jet directed upward or downward respectively from the wing surface.

Based on the jet flap idea, jet energy has been suggested as the ideal power source for an adequate aerodynamic control system - without mechanical control surfaces - for the hypothetical aircraft of the future.

In Ref. 29, the problem of "engine-airframe integration" is discussed in a very broad way. The reasons which make this integration a necessity in future aircraft can be divided into two groups: increasing flight speeds and engine lift. It is a fact that:

a) in high speed aircraft (supersonic), interference of engine inflow and jet with the airframe through shock and pressure waves becomes so serious that positioning of intake and exhaust nozzle have to be incorporated in the aerodynamic design of the future high speed aircraft.

b) the size of intakes and exhaust nozzles at flight Mach numbers above about 2.5 becomes significant with respect to e.g., the lifting surfaces of the aircraft itself, resulting in an appreciable part of the aerodynamic forces being provided by the fairing of the jet engine. Thus, the engine has to be incorporated in the total lift and drag balance of the aircraft and will probably become also an inherent member of its structure.

Further, the propulsive system may be called upon to help, especially at low flight speeds in carrying the total weight of the aircraft.

c) either by using additional light-weight lifting engines

d) or by deflecting the conventional circular jet

e) or by means of a deflected jet sheet (jet flap) to obtain very high $C_L$ values.

In case e), where the engine thrust is not only overcoming the aerodynamic drag but provides additional lift and controls the flow over the lifting surfaces, the need for airframe-engine integration becomes inevitable.
In Ref. 29, a conventional jet aircraft with jet deflection or added lifting engines is compared with the jet flapped aeroplane, the comparison being based on theoretical work (Ref. 30) for the estimation of the induced drag. It is shown in Fig. 162 that the conventional aircraft with jet deflection (which in principle does not require the airframe-engine integration) is almost identical with the jet-flap aeroplane as far as the thrust required - at least at low flight speeds - is concerned. One would expect a lower thrust requirement for the jet flap wing due to the additional jet induced lift. However, it seems that a possible gain in thrust is almost compensated for by an increased induced drag resulting from the high lift values of jet flapped wings.

The conclusion drawn in Ref. 29 is that, as the jet flap cannot evade the large induced drags \( C_{D_i} \approx C_{LT} \frac{2}{\pi AR} \) resulting especially from high lift coefficients at low aspect ratios (high speed flight) the jet flap's most promising field of application is for high aspect ratios, i.e., for low speed flight or else wherever very high lift coefficients can be justified.

At low aspect ratios (Ref. 29) or for very low landing speeds, the direct jet lift from a conventional (circular) but deflected jet requires little or no extra power compared with the jet flap and therefore the former may often be preferred due to its lesser complexity.

12.5 The Jet Flap and the "Lifting Engine"

In Ref. 29, Nicholson suggests the use of light-weight lifting jet engines or downward deflected jets to provide direct lift for take-off and landing in addition to conventional engines which are to supply the propulsive force (thrust). These lifting engines would have to be mounted at or close to the aircraft's centre of gravity or in such a way as not to produce additional moments around it.

The argument which led to the lifting engine idea is affiliated with the induced drag. The jet flap produces high lift forces, (lift hypothesis, Sec. 4.2), primarily due to the pressure lift, \( C_{LP} \), (see Fig. 50). Since the induced drag varies as \( C_{LT}^2 \) (see Sec. 6.5) and approximately as \( 1/AR \), the jet flap, particularly if used on high speed small span wings, will experience large induced drags which in turn reduce the thrust available for propulsion. Direct additional lift forces, however, such as are provided by fast jets of small mass flow from light-weight lifting jet engines are believed hardly to affect the pressure lift. These comments in favour of lifting engines equally apply to conventional jet engines with deflected jets which in Ref. 29 are claimed to contribute a small but significant advantage in high speed flight.
If, however, in order to produce direct lift more efficiently (lift gain factor) slower jet speeds and large mass flows are used, serious interference with the air flow over the wing is to be expected, which results in desired (induced lift and thrust) and undesired forces (induced drag).

Nicholson concludes, that in cruising flight of low aspect ratio aircraft, the deflected jet should have the edge over the jet flap economy-wise, (see Ref. 29, Fig. 10) and that the jet flap no longer shows to advantage, as high lift coefficients are of little value and the induced drag becomes a problem. At low speed flight (take-off and landing), the direct jet lift requires little, if any, extra power compared with the jet flap and may be preferred by conservative aircraft designers. With the moderately high aspect ratio aircraft at speeds below 60 mph, the "jet flap can be a very potent and valuable high lift device".

XIII. THE JET FLAP MODEL WING

13.1 The Jet Flap Model Wing and Its Wind Tunnel Testing

Problems which arise in tunnel tests on such models are discussed in detail in Ref. 26. They concern the choice of the model scale in relation to the test section size, the conventional tunnel wall interference and its correction (see also Ref. 166), the test Reynolds number, model accuracy, necessary number of pressure taps for pressure distribution measurements, interference of tunnel floor with the jet sheet (ground effect) etc.

In the following sections some of the above mentioned points are discussed in the light of Ref. 26 and other references.

13.2 Accuracy of Model

An overall tolerance of manufacture is very hard to specify for such a model since there are regions as, e.g., around the L.E. which under higher angles of attack are very critical to imperfections, chord and spanwise. Reference 26 recommends to keep tolerances near the nose to within 0.002 of an inch on a 30 inch chord aerofoil.

High accuracy in manufacture is also required for the slot width. Reference 26 states that wider tolerances can be allowed for in three-dimensional models. However, for two-dimensional work, the required higher accuracy may be difficult to verify on smaller models. For a 36 inch chord model, the jet slot width might be only of the order of 0.015 inch and a 0.001 inch spanwise variation in width would cause a 7% variation in the jet coefficient. Only if such a 0.001 inch variation is random, it could probably be tolerated.
13.3 Jet-Slot-Width to Wing Chord Ratio

In Refs. 28 and 65, the model (c = 8 in., b = 12 in.) was equipped with a slot width \( \delta = 0.025 \) in., i.e., a \( \delta/c = 0.003 \). The spanwise width variation was found to be less than 0.0025 in.

In Ref. 33, tests showed that the effective jet deflection angle \( \theta \) differs from its geometric angle the more, the larger the slot width, because of less effective guidance of the jet at the inherent small scales of jet flap model wings. Slot widths were \( \delta/c = 0.0068 \) for \( \theta = 78^\circ \) and \( 0.0189 \) at \( \theta = 63^\circ \).

In Ref. 39, truncated aerofoils with slots of \( \delta/c = 0.0065 \) were used at \( \theta = 0^\circ, 30^\circ \) and \( 90^\circ \) in combination with T.E. blowing. In combination with symmetrical blowing jet control flaps, a slot width of 0.003 \( c \) was employed.

The latest tests published (Ref. 138) used a \( \delta/c = 0.0047 \) ratio.

13.4 Pressure Plotting Requirements

In early tests at N. G. T. E. and N. P. L., only 26 static pressure tappings were used per wing section for pressure distribution investigations. If, however, forces and wing moments on a wing section as e.g., lift, thrust (drag) or pitching moment have to be obtained from wing surface pressure integrations also, the number and positioning of the pressure holes becomes critical. Reference 26 suggests to calculate the expected pressure distribution and to choose the stations for the required pressure taps accordingly. Forty to fifty stations round the aerofoil with closer spacing near the leading edge and at the upper surface of the trailing edge is considered a minimum. Reference 26 further recommends to place tubes spanwise at each station in order to be able to change the spanwise position for pressure traversing in case that e.g., due to surface faults, readings at a selected station are not satisfactory.

In Ref. 94, the static pressure tappings are extended to include also the whole flap.

13.5 So-Called Two-Dimensional Jet-Flapped Wing Models

It should be realized that operational jet flap models are never truly two-dimensional if they are provided for balance force measurements. In most practical cases they are either quasi-two-dimensional or worse. In the quasi-two-dimensional case, the effective aspect ratio is of such an order (15 to 20) that downwash effects (induced drag) exist, but are still small enough to be neglected. This may be the case when the model spans the entire tunnel test section width. If, however, the height/width ratio of the test section is relatively small as, e.g., in conventional wind tunnels or if the model strength or the air supply available limits the span, separate end-plates have to be used.
These end-plates can be either plates which bridge the tunnel completely from roof to floor (model horizontal), dividing the test section into 3 separate main stream regimes or small end-plates, which actually at best produce quasi two-dimensional conditions (see Sec. 8.7). Whereas in the first case the effective aspect ratio may be considered as large (order of 15 to 20), that is not so in the second case, because downwash effects may reduce the lift curve slope appreciably below its two-dimensional value and provide an additional induced drag to the pressure drag which no longer can be neglected. The effective AR can be found from the lift curve slope of the wing measured without the jet blowing or by using, e.g., Mangler's end plate theory (Ref. 11*).

In Ref. 26, the advantages and disadvantages of half and complete models are compared on the bases of:

a) forces and moments which can be measured

b) mounting and air-feed problems

c) tunnel speed calibration and boundary layer control on tunnel walls.

Further, in Appendix IV of this reference, values for end-plate sizes relative to the wing chord are suggested as:

1.5 c upstream of L.E.
4 c downstream of T.E.
1.5 c above chordline
3 c below chordline

13.6 Clearance Effects (Model-Wall)

There are two possible sources of flow disturbance, wall boundary layer separation and flow through the gap from the high pressure below to the low pressure region above the wing.

The wall boundary layer separation, enhanced by very large adverse pressure gradients on the upper wing surface at high lift coefficients spreads spanwise, resulting in a non-uniform distribution of lift, loss in lift, high profile and induced drag. Even pressure traverses at or near mid-span can be affected by these separations. This trouble can be substantially cured by boundary layer control - suction or blowing - at the walls themselves (Ref. 26). Fortunately, these effects are less pronounced in jet flap experiments due to the re-energizing of the wall boundary layer by the jet flow.
Gap leakage produces similar effects (Ref. 26) at high-lift tests, as described above. There is no need for an air gap in pressure plotting work nor will there be detrimental effects if small end plates are fitted integral parts with the model wing. In all other cases, leakage has to be controlled by various kinds of seals as, e.g., inflatable rubber seals, soft felt, etc.

13.7 Subsonic or Supersonic Jet Coefficients

The wide range of $C_J$-values required in jet flap test work can be obtained by variation of the tunnel speed, the nozzle width or by variation of the jet flow pressure ratio.

A decrease in wind tunnel speed - in order to increase $C_J$-unfortunately decreases the Reynolds number, which in most cases (see Sec. 8.8) complicates the comparison and interpretation of the results. A variation of the slot nozzle width - a desirable feature as a mass flow control - however complicates the model design.

The last alternative, the change of the jet speed by means of varying its pressure ratio is very inviting, especially for slender wing models with little room to spare for ducts which are big enough to allow for higher mass flows. In this case spanwise duct losses can be kept small. At pressure ratios above 1.89 (critical) the jet will be supersonic, (i.e., overchoked) but there is no conclusive evidence (according to Ref. 26) to show that aerodynamic effects on a jet flapped wing, produced by either a subsonic or a supersonic jet differ as long as their jet coefficients are the same. One advantage of the choked or overchoked jet should be noted: its mass flow is independent of the inherent small fluctuations in the external static pressure field along the wing span.

13.8 Moisture in Jet Flow

The moisture content of the jet supply air has to be kept small enough to avoid condensation. This can be achieved at lower pressure ratios by preheating the air. At higher pressure ratios, however, the air has to be dried or the air may be cooled after leaving the compressor and after draining off the condensed water, be re-heated again electrically. It was found that condensation alters the effective $C_J$-value. It further may falsify static pressure readings by blocking pressure holes etc.

13.9 Use of Transition Wires

Trip wires close to the aerofoil T.E. on the upper and/or lower wing surface can be used if the test Reynolds numbers are so small as to produce laminar boundary layer separation near the T.E. at low $C_J$-values. The pressure recovery at the T.E. obtained after fitting of trip wires is illustrated for $C_J$-values up to 0.07 in Figs. 25 and 26 of Ref. 32.
In Refs. 24 and 32, test results using transition wires of 0.034 inch diameter, fitted at a station 82% of the chord (c = 8 in.) from the L.E. on both the upper and lower surface are reported and compared with tests at the same Reynolds number (Re $= 4.25 \times 10^5$) but without trip wires. The arguments how these results may be interpreted are given in Ref. 24 and a summary on the main points of agreement or divergence is provided in Ref. 32.

XIV. ASSESSMENT OF THE JET FLAP

The jet flap, essentially a means of producing increased lift at constant wing incidence by provoking an extremely asymmetric flow around the wing without recourse to mechanical devices such as large T.E. flaps has obvious inherent advantages. Its shortcomings, however, are strongly relative and should be assessed not on the basis of conventional aircraft design practice and todays state of art.

It is contemplated to critically assess the jet flap in a subsequent and separate paper. This section will therefore deal only with statements and conclusions which follow directly from the material presented in the previous chapters.

14.1 Possible Benefits of the Jet Flap

The benefits which can be gained by the adaptation of the jet flap principle to future aircraft design - advantages which can only be attained fully by designing these aeroplanes unprejudiced by conventional design methods - are discussed below. Imagine an aircraft of given weight. The jet flap - due to higher lift coefficients - must lead to a reduction of one or more of the following:

1) wing area
2) take-off run \{ shorter runways and flight decks
3) landing run
4) engine size

and an increase in the

5) stalling angle (unstallable aircraft).

Other merits of jet flap application to conventional or future aircraft would be:

6) lower noise levels (aerodynamic noise)
7) gust alleviation (the jet flap provides a fast acting lift control in combination with automatic gust sensing devices)

8) use of jet stabilization instead of stabilization by the conventional tail

9) increase of the effective aspect ratio by means of sideways ejection of jet sheets from the wing tips.

10) use of wing tip blowing for lateral control especially of high speed aircraft (Ma > 1.5)

11) the chordwise saddleback loading which allows

   a) a more powerful and efficient augmentation in lift than that possible with an increase in angle of incidence.

   b) the use of larger values for wing thickness and camber close to the L.E. than those which are conventional for ordinary aerofoils.

   c) higher L.E. suction peaks without stall as long as the separation bubble can be kept bottled-up.

   d) shift of the lift coefficient spectrum from $C_{LT} \leq 2.0$ (conventional A/C) to $C_{LT} \geq 3.5$ for the jet flapped aircraft.

12) flight at smaller angles of incidence and therefore reduced drag, as high lift can be gained by methods other than increasing $\alpha$.

13) the effectiveness of the jet flap phenomena up to and far into the critical (transonic) Mach number range.

14) a large drag reduction due to fullspan jet flaps on supersonic aircraft especially if used in connection with aerofoils having their maximum thickness at the T.E. As a result,

   a) the supersonic lift/drag ratio will be increased due to a reduction up to 75% in wave drag.

   b) the maximum design gross weight of such an aeroplane can be increased due to the higher take-off and landing lift coefficients possible.
If the jet flap is shrouded, additional advantages can be listed such as:

15) simplicity of jet deflection control under all operating conditions.

16) better take-off and landing characteristics than that of the pure jet flap.

17) reduced form drag as compared with the pure jet flap (see 20) below) and possibly even with the conventional low speed wing (smaller angles of incidence, see also 12).

18) a reduction of the induced drag if an engine fails at take-off or landing (due to the decrease in lift) and the action of the jet shroud as a conventional flap for producing lift and control.

19) an increased lift efficiency in cruising flight with the jet issuing at zero jet deflection angle.

20) the prevention of jet mixing in the high pressure region (at lower jet surface close to T.E.) and its promotion on the upper wing surface in the low pressure region.

14.2 Disadvantages of the Jet Flap

These disadvantages are:

1) that blowing at the T.E. produces an enormous (negative) nose down pitching moment, which has to be trimmed. With conventional aircraft configurations (tail to the rear of wing), trimming means a loss in tail efficiency. Ref. 33 suggests the use of the CANARD configuration and to produce the required tail lift augmentation by blowing from the tail T.E. also. Another remedy to the inherent large pitching moments (in the case where only part of the jet engine exhaust is used for the jet flap) could be the use of conventional, but detached jet engines which are pivoted around the centre of gravity of the whole aircraft. A small angle tilt could compensate the jet flap pitching moment.
In Ref. 133, the location of the jet flap on swept back wings with part span jet flaps is recommended to be as far inboard as possible. In this way, the moment arm would be kept smallest. Exceptionally large lift coefficients at take-off or landing with no increase in horizontal tail surface area could be realized if the L.C. of the jet induced lift could be moved as far forward as the 25% mean aerodynamic chord point. Finally, Davidson's suggestion (Ref. 12) has to be mentioned here. He recommends for future designs to place the centre of gravity at approximately the mid chord point instead of at the quarter chord. Such aeroplanes would always fly with the centre of pressure very close to their centre of gravity.

2) that, as it appears at present, there is only a very small range of angle of attack associated with high lift (see Fig. 49)

3) the shrouded jet flap adds surface friction and heat transfer between the jet and the flap.

4) high induced drag especially with high speed (low aspect ratio) A/C configurations.

5) that, as with all high lift devices, the jet-flap's proper use and the benefits obtained from it are limited to primarily the take-off and landing operation only. This means that optimum use of the jet flap is curtailed to about 5 - 10% of the operational time of such an aeroplane. But it should be kept in mind that not all of the jet flap benefits are lost in cruising flight.

6) that structural problems (engines in wing, fuel tank location, hot ducts, etc.) and heat transfer problems add to the complexity of the wing design.

7) that from the viewpoint of exhaust jet energy distribution to the T.E. slot and for safety reasons a new type of multiunit (small gas turbine) jet power source has to be favoured instead of conventional large gas turbines.

8) that for the same jet exit area, lower nozzle discharge coefficients are to be expected for the slot nozzle than for a circular one.
14.3 Application of the Jet Flap

The most promising application of the jet flap as suggested by researchers in this field is:

1) for passenger planes (Ref. 68) with high subsonic speeds. Here, also the use of turboprops with a jet flap is a possibility, the exhaust efflux being used to deflect the propeller slipstream so that the whole becomes the jet flap. On supersonic airplanes, the extreme thinness of the wings for \( Ma \sim 2 \) would preclude the use of buried engines in all but delta wings.

2) for STOL aircraft, i.e., aircraft with circulation control (Ref. 33). Here its field of usefulness is so vast and its possibilities for the future so great that it merits more profound study.

3) for propeller planes (Ref. 12). Here the jet flap appears even more promising than its application to pure jet aircraft. This is due to the fact that many turboprops at take-off give about twice their cruising power. With a variable jet slot, most of this extra jet thrust could be used to increase \( C_J \), thereby providing extra lift and propulsive force at take-off.

4) for cutting take-off speeds by half and the length of required runways to a quarter for all conventional types of jet aircraft. It was found (Ref. 160) that the \( C_J \)-values for all types of jet aircraft in operation, at take-off are very close to 0.5. To produce the take-off lift at half the speed, \( C_J \) would be 4 x 0.5 = 2. Lift gain figures for jet flapped wings (see, e.g., Fig. 51) indicate that at \( C_J = 2.0 \), the conventional lift is still augmented by a factor of 4 due to the jet induced pressure lift. It seems, however, that a take-off speed reduction to one half is about the limit, which the jet flap is able to provide efficiently.

Other fields, to which the jet flap idea was applied and may turn out to be useful are:

5) helicopter blades (Refs. 113 and 25*)

6) axial flow compressor blades (Refs. 152, 153 and 154)
Another possible and herewith suggested use for the jet flap is for
7) turbine blades. Here it may be used
   a) to enhance turning of the hot working fluid and
   b) to cool the turbine blades.

XV. FUTURE RESEARCH PROJECTS

In this section, possible future research projects are listed as they were encountered either as specified by other researchers or as lack of present day knowledge in understanding certain phenomena.

1) An increase in lift without excessively increasing drag can also be achieved fundamentally by operating on a large quantity of air by, e.g., issuing jet sheets sideways from the wing tips. The resulting rise in effective aspect ratio would allow to use higher $C_{LT}$-values without increasing the induced drag. This concept (Ref. 29) has to be assessed.

2) Additional experimental work is suggested to bring out the limitations in the ranges of application of the different expressions proposed for the induced drag (see Sec. 6.11.2).

3) The aerodynamic efficiency factor, $e$, for a three-dimensional jet flapped wing is to be determined experimentally for various wing planforms with non-elliptic load distribution.

4) The drag hypothesis (Ref. 142) has to be further investigated experimentally in order to establish its range of applicability.

5) An experimental study of the effect of sweep on three-dimensional jet flapped wings is suggested.

6) Theoretical (see Sec. 7.3) and experimental work (see Sec. 8.12) of the ground effect on two-dimensional jet flapped wings should be extended to the three-dimensional case.

7) The effect of the proximity of the ground on the induced drag of a three-dimensional jet flapped wing needs investigation, both theoretical and experimental.
8) Because of the two opposing effects (see a) and b) at the end of Sec. 8.12 and Ref. 110) caused by the spanwise velocity component of the main stream on high aspect ratio wings, three-dimensional effects are small. This qualitative conclusion has, however, to be proved or disproved by experimental evidence.

9) The practical $C_J$-range of present day aircraft is $C_J \leq 1$ where $C_J$-values of the order of 0.5 apply for take-off and $C_J$-values smaller than 0.025 for cruising flight. Figure 61 lacks information in the $C_J < 0.1$ range for the jet deflection angles smaller than 86°.

10) More work, both theoretical and experimental should be done to help to a better understanding of what happens behind a three-dimensional jet flapped wing which could also help to a better interpretation of the induced drag.

11) Further experimental evidence has to be provided for the effect of aspect ratio on jet shape.

12) For a given value of $C_J$, the jet can either be subsonic or supersonic. Conclusive experimental evidence is needed to show that aerodynamic effects on a jet flapped wing produced by either jet are the same.

13) In theory (Refs. 14 and 36) it is assumed that the friction drag for the jet flapped wing without or with the jet blowing does change only insignificantly. Experiments should be conducted to prove or disprove this assumption.

14) In mixing theory (Ref. 46) the amount of air entrained into the jet (mass flow ratio, $n$, see Chap. X) is an important factor. For a quantitative evaluation of the jet drag, empirical values of $n$ must be known. To obtain values of $n$, experiments would have to be conducted (with slot nozzles of various width to height ratios) to determine the amount of free air entrained between the jet exit and the plane where the jet static pressure just becomes the undisturbed free stream pressure.
15) Stratford's and Payne's conclusions (Sec. 10.2.5 and 10.3.6 respectively) need experimental verification. More specifically, the changes in jet drag due to changes:

1) in the ratio of the density velocity product \( \rho_o \cdot v_o / \rho_J \cdot v_J \) of the jet and main stream

2) in the static pressure in the mixing region

3) in the surface slope of the wing near the jet exit

4) in the jet momentum caused by the change in the exit static pressure due to mainstream flow

have to be investigated experimentally.

16) Since the theories so far available for the jet drag consider only the case of zero jet deflection, they should be extended to non-zero deflection angles.

17) The expectation that the jet drag increases rapidly with increasing jet deflection angle has to be checked experimentally.

18) Experimental evidence is required to establish, in what way the jet momentum loss caused by the jet drag affects the total lift of a jet flapped wing.

19) Information is required about the change, if any, of the form drag with AR.

The use of jet flaps for fast acting lift control as a means of gust alleviation needs study of problems in unsteady dynamics of jet sheets, such as blowing over an oscillating jet shroud or control flap. This study would provide information on

20) The effect of magnitude and frequency of oscillation on

a) the jet sheet shape

b) the jet deflection due to the Coanda principle

c) the lift, drag and pitching moment of the jet flapped wing.
21) The effect of a sudden change in jet deflection angles (analogous to the sudden change in angles of attack of a conventional wing - Wagner effect).
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<thead>
<tr>
<th></th>
<th>Authors</th>
<th>Title</th>
<th>Source Details</th>
</tr>
</thead>
</table>
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   Tolhurst, W. H.
   Maki, R. L.

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A. CONVENTIONAL MECHANICAL FLAPS

PLAIN-FIXED

PLAIN-VARIABLE

SLIDING FLAP (Ref. 107)

ZAPP

SPLIT FLAP (Ref. 90)

COMBINED SLIDING AND FOWLER FLAP (Ref. 109)

FIG. 1. DESIGNATION OF FLAPS
B. SLOTTED FLAPS

SINGLE SLOTTED FLAP

SINGLE SLOTTED FOWLER FLAP

SINGLE SLOTTED FLAP WITH L.E. FLAP
(REF. 44, 106, 115)

DOUBLE SLOTTED FOWLER FLAP
(REF. 108)

C. FREE STREAM-LINE FLAPS
(REFS. 81 and 82)

FIG.1 (con'd) DESIGNATION OF FLAPS
b) EXTERNAL FLOW JET-AUGMENTED FLAPS

SLOTTED FLAP (REF. 75, 76, 74)

DOUBLE SLOTTED FLAP (Ref. 69, 128)

UPPER SURFACE NACELLE & FLAPS
(REF. 141)

FIG. 1 (con'd) DESIGNATION OF FLAPS
D. JET AUGMENTED FLAPS
(Conventional Flaps, Greater than 20% of Wing Chord)

a) INTERNAL FLOW JET-AUGMENTED FLAPS

BLOWING FLAP (FIXED)
(O.N.E.R.A., Ref. 33)

Suction

INTERNAL FLOW JET-AUGMENTED FLAPS
(Ref. 71, 79, 131, 37*)

FREE STREAMLINE FLAP (Refs. 81 and 82)

AREA - SUCTION FLAP (Ref. 106)

FIG. 1 (con'd) DESIGNATION OF FLAPS
E. THE PURE JET FLAP (BLOWN FLAP)
   (N.G.T.E., REF. 12)

F. THE SHROUDED JET FLAP
   (Flap, Less Than 10% of Wing Chord)
   (N.G.T.E., REF. 12)

FIG. 1 (cont'd) DESIGNATION OF FLAPS
G. JET CONTROL FLAPS
(Flaps of about 10-20% of Wing Chord)

SYMETRICAL BLOWING
(O. N. E. R. A., Ref. 33)

SYMETRICAL SUCTION
(O. N. E. R. A., Ref. 33)

UPPER SURFACE BLOWING
(O. N. E. R. A., Ref. 33)

UPPER SURFACE SUCTION
(O. N. E. R. A., Ref. 33)

H. TRUNCATED AEROFOILS

SOLID BLADE

BLOWN BLADE (REF. 39)

SOLID SPOILER BLADE

BLOWN SPOILER BLADE (REF. 39)

FIG. 1 (con'd) DESIGNATION OF FLAPS
FIG. 2 EFFECT OF FLAPS AND SUCTION AND BLOWING ON THE WING LIFT (REF. 34*)

FIG. 3 A JET FLAPPED AIRCRAFT (REF. 12)
FIG. 4 LIFT AS A FUNCTION OF $C_{\mu}$ (CIRCULATION CONTROL)  
(REF. 33) (SUBSCRIPT $r$ DENOTES REATTACHMENT OF B. L.)
FIG. 5  THE SEPARATION BUBBLE, A CHARACTERISTIC OF THE JET FLAP (REF. 12)
FIG. 6  THE FLOW ENTRAINMENT NEAR THE T.E.
FIG. 7 GEOMETRY FOR THE JET FLAP-SOLID FLAP ANALOGY (REF. 11)

\[ m = \frac{1 - \cos \psi}{1 + \cos \psi} \cdot \frac{c_A}{c} = \frac{2}{1 + \cos \psi} \]

FIG. 8 THE FIRST CHARACTERISTIC JET FLAP RESULTS (HAGEDORN & RUDEN) (REF. 12)
<table>
<thead>
<tr>
<th>Viscous Flow</th>
<th>3-Dim. Lift Drag</th>
<th>[ C'<em>L = C'<em>D + C'</em>{DF} + C'</em>{DFR} ]</th>
<th>[ C'<em>L = C'<em>D + C'</em>{DF} + C'</em>{DFR} ]</th>
</tr>
</thead>
<tbody>
<tr>
<td>2-Dim. Lift Drag</td>
<td>[ C'<em>{DP} ] as [ C'</em>{D1} = 0 ]</td>
<td>[ C'<em>L ] [ (C'<em>D - C'</em>{DFR}) = C'<em>DF + C'</em>{D1} ] as [ C'</em>{DFR} ] is not included</td>
<td>[ C'<em>L ] [ (C'<em>D - C'</em>{DFR}) = C'<em>DF ] as [ C'</em>{DFR} ] is not included and [ C'</em>{D1} = 0 ]</td>
</tr>
<tr>
<td>Ideal (Inviscid) Flow</td>
<td>3-Dim. Lift Drag</td>
<td>[ C'_L ] [ C'<em>D ] as [ C'</em>{DFR} ] and [ C'_DF = 0 ]</td>
<td>[ C'<em>L ] [ C'</em>{D1} ] as [ C'_{DFR} ] and [ C'_DF = 0 ]</td>
</tr>
<tr>
<td>2-Dim. Lift Drag</td>
<td>No Forces in x-Direction as [ C'<em>{D1} ] [ C'</em>{DFR} ] and [ C'_DF = 0 ]</td>
<td>No Forces in x-Direction as [ C'_{DFR} ] [ C'<em>DF ] and [ C'</em>{D1} = 0 ]</td>
<td></td>
</tr>
</tbody>
</table>

FIG. 9a THE FORCES ACTING ON A CONVENTIONAL SYMMETRICAL AEROFOIL
### The Forces Acting on a Symmetrical Jet Flapped Wing at Zero Incidence

#### Table of Forces

<table>
<thead>
<tr>
<th>Viscous Flow</th>
<th>3-Dim Lift Thrust</th>
<th>Forces as Measured with Balance</th>
<th>Forces as Obtained from Pressure Integration</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lift</td>
<td>$C_L = 0$</td>
<td>$C_L = 0$</td>
<td>$C_L = 0$</td>
</tr>
<tr>
<td>Thrust</td>
<td>$C_{TM} = 0$</td>
<td>$C_{TM} = C_J - C_{DT} = C_J - (C'D + \Delta C_{DP})$</td>
<td>$C_{TM} = C_J - C_{DT} = C_J - (C'D + \Delta C_{DP})$</td>
</tr>
<tr>
<td></td>
<td>$C_{D_i} = 0$</td>
<td>$C_{D_i} = 0$</td>
<td>$C_{D_i} = 0$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Ideal (Inviscid) Flow</th>
<th>2-Dim Lift Thrust</th>
<th>Forces as Measured with Balance</th>
<th>Forces as Obtained from Pressure Integration</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lift</td>
<td>$C_L = 0$</td>
<td>$C_L = 0$</td>
<td>$C_L = 0$</td>
</tr>
<tr>
<td>Thrust</td>
<td>$C_{TM} = 0$</td>
<td>$C_{TM} = C_J - C_{DT} = C_J - (C'D + \Delta C_{DP})$</td>
<td>$C_{TM} = C_J - C_{DT} = C_J - (C'D + \Delta C_{DP})$</td>
</tr>
<tr>
<td></td>
<td>$C_{D_i} = 0$</td>
<td>$C_{D_i} = 0$</td>
<td>$C_{D_i} = 0$</td>
</tr>
</tbody>
</table>

| 3-Dim Lift Thrust     | $C_L = 0$         | $C_L = 0$                        | $C_L = 0$                                   |
|                       | $C_{TM} = C_J$    | $C_{TM} = C_J$                    |                                              |
|                       | as $C_{DT} = 0$   |                                 |                                              |

| 2-Dim Lift Thrust     | $C_L = 0$         | $C_L = 0$                        | $C_L = 0$                                   |
|                       | $C_{TM} = C_J$    | $C_{TM} = C_J$                    |                                              |
|                       | as $C_{DT} = 0$   |                                 |                                              |

**FIG. 9b**

---

*The image contains a diagram of the forces acting on a symmetrical jet flapped wing at zero incidence, illustrating the forces measured with a balance and those obtained from pressure integration.*
\[ C_{DP} = C'_{DP} + \Delta C_{DP} \]
\[ C_{DT} = C_{DP} + C_{D1} \]
\[ C_{D1} = \frac{(C_{LT})^2}{AR + 2C_{J}} \]
\[ C_{TP1} = C_{J}(1-\cos \tau) \]

<table>
<thead>
<tr>
<th></th>
<th>3-Dim Lift Thrust</th>
<th>2-Dim Lift Thrust</th>
<th>3-Dim Lift Thrust</th>
<th>2-Dim Lift Thrust</th>
</tr>
</thead>
<tbody>
<tr>
<td>Viscous Flow</td>
<td>C_{LT} = C_{LR} + C_{LJ} + C'_{L}</td>
<td>C_{LT} = C_{LR} + C_{LJ} + C'_{L}</td>
<td>C_{LT} = C_{LR} + C_{LJ} + C'_{L}</td>
<td>C_{LT} = C_{LR} + C_{LJ} + C'_{L}</td>
</tr>
<tr>
<td></td>
<td>C_{TM} = C_{J} - C_{DT} = C_{TR} + C_{TP}</td>
<td>C_{TM} = C_{J} - C_{DP} = C_{TR} + C_{TP}</td>
<td>C_{TM} = C_{J} - C_{DP} = C_{TR} + C_{TP}</td>
<td>C_{TM} = C_{J} - C_{DP} = C_{TR} + C_{TP}</td>
</tr>
<tr>
<td></td>
<td>C_{LP} = C_{LJ} + C'_{L}</td>
<td>(C_{TP} + C_{DFR}) = C_{TP1} - (C_{DF} + C_{D1})</td>
<td>CLR and C_{TR} do not contribute</td>
<td>CLR and C_{TR} do not contribute</td>
</tr>
<tr>
<td></td>
<td>C_{LR} = C_{LJ} + C'_{L}</td>
<td>CLR and C_{TR} do not contribute</td>
<td>CLR and C_{TR} do not contribute</td>
<td>CLR and C_{TR} do not contribute</td>
</tr>
<tr>
<td></td>
<td>CLR and C_{TR} do not contribute</td>
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<td></td>
<td>CLR and C_{TR} do not contribute</td>
<td>CLR and C_{TR} do not contribute</td>
<td>CLR and C_{TR} do not contribute</td>
<td>CLR and C_{TR} do not contribute</td>
</tr>
</tbody>
</table>

**FIG. 9c**  THE FORCES ACTING ON A SYMMETRICAL JET FLAPPED WING AT INCIDENCE (SEE ALSO FIG. 9d)
THE 3-DIM. JET FLAPPED WING

BALANCE: MEASURED FORCES

FORCES FROM PRESSURE DISTRIBUTION

\[ C_{TP} + C_{DFR} = \overline{C_{TP}} \]

\[ C_{TM} + C_{DFR} = \overline{C_{TM}} \]

FIG. 9d  THE FORCES IN THRUST (DRAG) DIRECTION ACTING ON A THREE-DIMENSIONAL JET FLAPPED WING.
FIG. 10

THE MOMENTUM BOX (a) AND SOLID BOUNDARY (b) APPROACH TO PROVE THE THRUST HYPOTHESIS (REF. 11)

The flow pattern.

Flow in a channel.

FIG. 11a

THE FLOW PATTERN (REF. 35)
FIG. 12

FIG. 11b  THE FLOW PATTERN (REF. 59)

FIG. 12  SUB-REGIONS FOR THE "TIMES FOUR" LIFT ARGUMENT (REF. 11)
FIG. 13  THE TOTAL LIFT, ESSENTIALLY INDEPENDENT OF THE JET SLOT WIDTH
($A_R \approx 20; \alpha = 0; \text{REF. 33}$)
(1) Space to be occupied
(2) Hence fluid drawn down
(3) Therefore exhausting to "low" pressure
(4) High and low pressure regions established
(5) Fluid diverted upwards while still upstream

FIG. 14 THE MECHANISM BEHIND THE INDUCED PRESSURE LIFT (REF. 11)

JET COEFFICIENT = 0.38
JET ANGLE = 90°
REACTION THRUST = 0
MEASURED THRUST = 0.37 J

FIG. 15 THE PRESSURE INDUCED THRUST (REF. 12)
FIG. 16 THE STREAMLINES AROUND A JET FLAPPED WING AT HIGH LIFT AND JET COEFFICIENTS (REF. 12)
FIG. 17 
THE VARIATION OF THE L.E. AND T.E. SUCTION PEAKS 
AT LOW JET COEFFICIENTS
FIG. 18

PHYSICAL INTERPRETATION OF THE JET DRAG ACTING ON A WING AT REST. (REF. 12)

FIG. 19

STRATFORD'S JET DRAG CRITERION (FOR COLD JETS, (REF. 12))
FIG. 20  COMPARISON OF THE STRAIGHT FLAP ANALOGY METHOD WITH EXPERIMENTAL RESULTS
(2 - dim.; $\alpha = 0; \Theta = 48.5^\circ$; Ref. 11)
FIG. 21 MODEL FOR CURVED FLAP ANALOGY METHOD (REF. 31)
FIG. 22  DEFINITION OF ANGLE COORDINATES (REF. 31)
FIG. 23  TOTAL LIFT COEFFICIENT COMPARISON WITH FLAP ANALOGY PREDICTIONS
(2-dim.; $\alpha = 0^\circ$; Ref. 31)
FIG. 24  PITCHING MOMENT MEASURED AND CALCULATED (2-dim.; Θ = 31.4°; Ref. 31)

FIG. 25  SHIFT OF AERODYNAMIC AND LIFT CENTRE AS A FUNCTION OF THE JET COEFFICIENT AND JET DEFLECTION ANGLE (2-dim; α = 0; Ref. 12)
FIG. 26a  EFFECT OF ASPECT RATIO (3-dim; $\alpha = 0^\circ$; Ref. 31)
FIG. 26b  VARIATION OF $CL_T/CD_i$ WITH ASPECT RATIO AND JET COEFFICIENT (RECTANGULAR WING, TAPER RATIO TR = 1; $\Theta = 57.3^\circ$; Ref. 31)

FIG. 27 a) MODEL USED IN SPENCE'S THEORY (REF. 101)

b) FLOW IN POLAR ELEMENT OF JET (REF. 101)
(a) Forces acting on aerobill and jet.
(b) Distributions of vorticity and downwash on x-axis.

FIG. 28 REPRESENTATION OF THIN SHALLOW JET
FIG. 29 COMPARISON OF EXPERIMENTALLY (REF. 24) AND THEORETICALLY (REF. 16) OBTAINED TOTAL LIFT COEFFICIENTS.
(2-dim.; $\alpha = 0^\circ$; $\Theta = 31.4^\circ$ & $58.1^\circ$; Ref. 16)
FIG. 30  COMPARISON OF EXPERIMENTAL RESULTS WITH SPENCE'S THEORY
(2-dim.; \( \alpha = 0 \); \( \Theta = 31.4^\circ \) and 58.1\(^{\circ}\); Ref. 101)

Experimental points: \( \bullet, \Theta = 31.4^\circ; \Delta, \Theta = 58.1^\circ \).
FIG. 31  JET SHAPES AND VORTICITY DISTRIBUTIONS
(2-dim.; $\alpha = 0$; $\tau = 30^\circ$; Ref. 101)
FIG. 32 $\frac{\partial C_{LT}}{\partial \beta}$ FOR THIN WINGS WITH BLOWN FLAPS
(2-dim. ideal flow; $\alpha = 0$; Ref. 48)
Blowing over a 30 per cent flap. Vorticity distribution.

Blowing over a 30 per cent flap. Non-dimensional jet shape.

FIG. 33  THEORETICALLY OBTAINED RESULTS FOR A THIN AEROFOIL WITH A JET AUGMENTED FLAP (see Sec. 6.4.2) (2-dim.; $\alpha = 0^\circ$; $E = 30\%$; Ref. 93)
FIG. 34  VARIATION OF $C_L T^{(3)}$ WITH $C_J^{1/2}$

( $\infty = 0$; $\Theta = 31.4^\circ$; Refs. 30, 32 & 28)

$\bigcirc$ - Ref. 24;  $\bigtriangleup$, $\nabla$ - Ref. 28; — Theory, Ref. 30

ASS. : 1) $B_1$ AND $B_2$ = STAGNATION PTS.
2) STATIC PRESS. AT $B =$ TOTAL PRESS. IN MAIN STREAM.

FIG. 35  AEROFOIL WITH JET SHEET ISSUING FROM A GENERAL POINT ON LOWER WING SURFACE
(2-dim.; $\alpha = 0^\circ$; Ref. 100)
FIG. 36  THE TWO FLAT PLATE SYMMETRICAL LOADING (a)
AND THE THREE FLAT PLATE ASSYMMETRICAL LOADING (b)
(2-dim.; $\alpha = 0^\circ$; Ref. 17)
FIG. 37  DISTRIBUTION OF BOUND VORTICITY VECTOR AND OF DOWNWASH FOR WING OF FINITE SPAN
Fig. a) Pressure distribution over a section of a three-dimensional wing from tests by Williams and Alexander

Fig. b) Shapes and minimum pressure coefficients for some cambered aerofoils with constant-load camber-lines, \( t/c = 0.125 \).
FIG. 39  ANALOG REPRESENTATION OF THE JET FLAPPED WING

FIG. 40  COMPARISON OF JET FLAP RESULTS OBTAINED BY THE "RHEOELECTRIC-ANALOGY" METHOD AND FROM W. T. TESTS
(2-dim.; α = 0°; Θ = 33°; Ref. 33)
FIG. 41 HELBOLD'S INDUCED DRAG ($D_{\text{IND}}$) DEFINITION
(3-dim.; Ref. 63)
FIG. 42  FLOW TYPE MODELS USED IN ANALYSIS OF REF. 35

Both, velocity and pressure are continuous

a) SOURCE - TYPE FLOW

Only pressure is continuous

b) JET - TYPE FLOW
FIG. 43 GROUND EFFECT ON LIFT AS CALCULATED (REF. 31) AND MEASURED (REF. 32) (2-dim.; \( \Theta = 57.3^\circ \); Ref. 31)

FIG. 44 JET AND MAIN STREAM NEAR THE GROUND (REF. 139)
FIG. 45

a) THEORETICAL MODEL OF REF. 139

b) THEORETICAL MODEL OF REF. 149
FIG. 46

THE EFFECT OF FUSELAGE INTERFERENCE ON ROLLING MOMENT OF A JET FLAPPED HIGH-WING CONFIGURATION AT AN ANGLE OF YAW.

(AR = 5; b/D = 5; Θ = 45°; Ref. 31)
FIG. 47  
EFFECT OF LEADING-EDGE MODIFICATION ON LIFT CENTRE SHIFT AND ON THRUST.  
(2-dim.; droop nose; Ref. 31)
FIG. 48  
SEPARATION BUBBLE BEHAVIOUR WITH INCREASING ANGLE OF ATTACK (REF. 12)

FIG. 49  
STALLING ENVELOPES FOR THREE JET DEFLECTION ANGLES (θ)  
(2-dim.; C_J ≤ 4; Ref. 12)
FIG. 50

**LIFT VARIATION WITH JET COEFFICIENT**

(2-dim.; $\alpha = 0^\circ$; $\Theta = 58.1^\circ$; Ref. 12)

$L = C_L + C_J \sin \Theta$

**TOTAL**

**REACTION**

**PRESSURE**

**JET DEFLECTION = 58.1°**
FIG. 51 LIFT GAIN FACTOR FOR A PURE JET FLAP
(2-dim.; $\alpha = 0^\circ$; Refs. 24 and 32)
FIG. 52  TOTAL LIFT COEFFICIENTS AS A FUNCTION OF INCIDENCE AND JET DEFLECTION ANGLE (2-dim.; $\Theta = 31.4^\circ$ and $58.1^\circ$; Ref. 12)
FIG. 53  VARIATION OF TOTAL LIFT COEFFICIENT WITH JET COEFFICIENT FOR A JET FLAPPED WING AND VARIOUS JET DEFLECTION ANGLES
(AR = 8.4; \( \alpha = 0^\circ \); Ref. 71)
FIG. 54  
THE TOTAL LIFT AS A FUNCTION OF INCIDENCE AND JET COEFFICIENT OF A PURE JET FLAP  
($AR = 8.4; \Theta = 57^0; \text{Ref. 71}$)
FIG. 55 VARIATION OF TOTAL LIFT WITH INCIDENCE AT CONSTANT JET COEFFICIENTS
(AR = 2.75; Θ = 31.3°; Ref. 28)
FIG. 56 LIFT CURVE SLOPE AS A FUNCTION OF JET COEFFICIENT AND JET DEFLECTION ANGLE.
(2-dim.; \( \alpha = 0 \); Ref. 23)

FIG. 57 TOTAL LIFT COEFFICIENT AS A FUNCTION OF INCIDENCE AND JET DEFLECTION ANGLE.
\( AR \approx 20 \); Ref. 33)
FIG. 58  VARIATION OF THE CENTRE OF LIFT LOCATION AND THE LIFT CURVE SLOPE WITH $C_\mu$
(AR $\approx$ 20, SYMMETRICAL BLOWING, Ref. 33)
FIG. 59  TOTAL LIFT COEFFICIENT AS A FUNCTION OF INCIDENCE AND JET DEFLECTION (AR ≈ 20, JET CONTROL FLAP WITH UPPER AND LOWER SURFACE BLOWING, REF. 33)
COMPARISON OF THE EXPERIMENTAL AND THEORETICALLY PREDICTED LIFT EFFICIENCY

$E_L = \frac{\Delta C_{Lr} - C_L \cdot \sin \theta}{C_L \cdot \sin \theta}$

$\alpha \leq 0^\circ$

- $\circ \theta = 0^\circ$
- $\triangle \alpha = 33^\circ$
- $\diamond \alpha = 55^\circ$
- $\square \alpha = 63^\circ$

$\alpha = 0^\circ$

$\theta = 0^\circ$

$\alpha = 33^\circ$

$\alpha = 55^\circ$

$\alpha = 63^\circ$

FIG. 60

( $\alpha = 0$; $C_J \leq 1.25$; $AR = 20$; Ref. 33)
FIG. 61  LIFT GAIN FACTOR FOR VARIOUS JET DEFLECTION ANGLES
(AR = 8.4;  \( \alpha = 0^\circ \); Ref. 71)
FIG. 62  LIFT EFFICIENCY (LIFTING EFFECTIVENESS) AT ZERO INCIDENCE
(AoA = 0°; AR = 3; C_{\mu} \leq 0.2; Ref. 33)
FIG. 63  LIFT VARIATION AS A FUNCTION OF THE JET MOMENTUM COEFFICIENT AND THE JET DEFLECTION ANGLE
(AR≈20; α = 0°; Θ ≤ 90°; Cμ ≈ 1; Ref. 33)
FIG. 64
THE OPTIMUM JET DEFLECTION ANGLE
(AR = 20; $\alpha = 0; C_{\mu} \ll 1$)
FIG. 65 VARIATION OF THE LIFT EFFICIENCY WITH ASPECT RATIO AND JET COEFFICIENT
( $\alpha = 0^\circ$; $\theta = 90^\circ$; Ref. 18)
FIG. 66  VARIATION OF TOTAL AND PRESSURE LIFT WITH $C_J$
($AR = 8.4; \alpha = 0; \Theta = 85^\circ$; Ref. 72)

\[ C_{LT} = C_L' + C_{LJ} \frac{\Theta}{85} + C_\mu \sin (\Theta + \alpha) \]
FIG. 67 SPANWISE DISTRIBUTION OF JET PRESSURE LIFT
( $\alpha = 0; \ AR \approx 3.0; \ \theta = 31.3^\circ; \ Ref. \ 28$)
PRESSURE DISTRIBUTION INDICATES THE ONSET OF LEADING EDGE SEPARATION IN THIS RANGE OF C_j.

FIG. 68 COMPARISON OF THE THRUST FROM BALANCE AND PRESSURE DISTRIBUTION MEASUREMENTS
(2 dim.; \( \alpha = 0^\circ; \Theta \leq 90^\circ; \) Ref. 23)
FIG. 69  VARIATION OF THE JET INDUCED PRESSURE THRUST WITH JET COEFFICIENT
(2-dim; $\alpha = 0^\circ$; $\Theta = 90^\circ$; $C_J < 0.6$; Ref. 24)
FIG. 70  THE NET THRUST RECOVERY AS A FUNCTION OF $C_\mu$

($AR = 3$ & $\simeq 20; \alpha = 0^\circ$, Ref. 33)
FIG. 71 THE THRUST RECOVERY FOR TWO- AND THREE-DIMENSIONAL PURE JET FLAPPED AEROFOILS.
FIG. 72

THRUST AND DRAG FORCES ACTING ON A TWO-DIMENSIONAL JET FLAPPED WING
The thrust efficiency for various $C_T$-ranges and two- and three-dimensional pure jet flapped aerofoils.
VARIATION OF JET INDUCED PRESSURE THRUST
(FROM PRESSURE INTEGRATION) WITH $C_{LT}$, $C_{LP}$
AND ASPECT RATIO
($AR = 2.75 & 6.8; \alpha = 0^\circ; \Theta = 31.3^\circ; \text{Ref. 28}$)
FIG. 75  
THE JET DRAG (SINK EFFECT) AS A FUNCTION OF THE JET COEFFICIENT AND L.E. CONFIGURATION (2-dim.; \( \alpha = 0^\circ \); \( \Theta = 0^\circ \); Ref. 23)

FIG. 76  
THE JET DRAG VARIATION AS A FUNCTION OF THE VELOCITY-DENSITY RATIO. (HYDROGEN JET TEST RESULTS, REF. 12)
SYMBOLS AS IN FIG. 75
ADDITION OF TEST X FOR MODEL
AS AT + BUT WITH A CHORDAL REYNOLDS
NUMBER OF 2.12 x 10^5

FIG. 77 VARIATION OF THE JET DRAG WITH JET COEFFICIENT
(2-dim.; \( \alpha = 0^\circ \); \( \Theta = 0^\circ \); Ref. 24)
FIG. 78 VARIATION OF THE THRUST AUGMENTATION PARAMETER WITH THE DENSITY RATIO (Ref. 36)

(a) Aerofoil section giving high jet drag
(b) Aerofoil section giving negligible jet drag
(c) Aerofoil section giving negative jet drag

FIG. 79 THE JET DRAG AND ITS DEPENDENCE UPON THE T.E. SHAPE (Ref. 36)
FIG. 80  TYPICAL SADDLE BACK PRESSURE DISTRIBUTION AS OBTAINED FROM ACTUAL TESTS OF REF. 24 (2-dim; $\alpha = 0^\circ$; $\Theta = 31.4^\circ$; Ref. 17)
FIG. 81 PRESSURE DISTRIBUTION FOR 2 CHORDWISE SECTIONS

(\(AR = 2.75; \ \Theta = 31.3^\circ; \text{Ref. 28}\))
FIG. 82 PEAK SUCTION PRESSURE VARIATION AT L.E. AND T.E. FOR 2 CHORDWISE SECTIONS (AR = 2.75; $\theta = 31.3^\circ$; Ref. 28)

$C_f = 0.42$

$C_f = 0.91$

\[ \frac{C_f}{(U_0 = 100 \text{ ft/sec})} \]

\[ \frac{C_f}{(U_0 = 50 \text{ ft/sec})} \]
FIG. 83  PEAK SUCTION PRESSURE COMPARISON OF THREE AND TWO DIMENSIONAL WING 
($\theta = 31.3^\circ$; Ref. 28)
FIG. 84  PRESSURE DISTRIBUTION OF A TRUNCATED WING AND JET CONTROL FLAP WITH SYMMETRICAL BLOWING (AR \approx 20; \alpha = 0; \Theta = 67^\circ; C_{p} \leq 4.8; \text{Ref. 33})
FIG. 85  THE MODEL WING SECTION AND THE VARIATION OF THE PRESSURE DISTRIBUTION FOR VARIOUS JET COEFFICIENTS (2 dim.; $\alpha = 0$; $\Theta = 45^\circ$; Ref. 94)
FIG. 86 THE THREE-PART CONTRIBUTION TO THE TOTAL PITCHING MOMENT

(AR = 8.4; \(\alpha = 0\); \(\Theta = 86^\circ\); \(C_M \leq 7\); Ref. 71)
FIG. 87  VARIATION OF TOTAL PITCHING MOMENT WITH TOTAL LIFT
(AR = 2.75 & 6.8; Θ = 30°; Ref. 28)
**FIG. 88** NOSE DOWN PITCHING MOMENT VARIATION WITH JET COEFFICIENT AND INCIDENCE ANGLE (2-dim.; $\Theta = 31.4^\circ$; Ref. 12)

**FIG. 89** LIFT vs. PITCHING MOMENT COEFFICIENT VARIATION WITH INCIDENCE ANGLE AND JET COEFFICIENT (AR = 3; $\Theta = 90^\circ$; $t/c = 4.25\%$; Ref. 33)
FIG. 90 CENTRE OF LIFT VARIATION WITH INCIDENCE
(2-dim; \( \Theta = 58.1^\circ \), \( C_J \approx 4 \); Ref. 32)
FIG. 91 GROUND INTERFERENCE EFFECT ON THE CENTRE OF LIFT POSITION
(2-diml; Θ = 58.1°; α = 0; Ref. 32)
A GUESS AT HOW THE WING TIP VORTEX COMBINES WITH THE JET SHEET SINK EFFECT. (REF. 12)
PHOTOGRAPHS OF TUFT GRID MOUNTED 2.5 CHORDS
BEHIND TRAILING EDGE OF 7.25-INCH-CHORD WING
WITH JET-AUGMENTED FLAP DEFLECTED 55°.
$\alpha = 0^\circ$; MAIN STREAM DYNAMIC HEAD = 2 lb/sq. ft., Ref. 73)
FIG. 94  PHOTOGRAPHS OF TUFT GRID MOUNTED 2.5 CHORDS BEHIND TRAILING EDGE OF 7.25 INCH-CHORD WING WITH JET AUGMENTED FLAP DEFLECTED 55°  
(C_{L_e} = 3.28; MAIN STREAM DYNAMIC HEAD = 2 lbs/sq. ft.; RÉF. 73)
FIG. 95 THE MAXIMUM JET INDUCED PRESSURE LIFT
(AR = 2.8; \( \alpha = 0 \); \( \Theta = 85^\circ \); Ref. 72)
FIG. 96  
VARIATION OF THE MAXIMUM JET INDUCED PRESSURE LIFT WITH ASPECT RATIO
($\alpha = 0^\circ; \theta = 85^\circ$; Ref. 72)
FIG. 97  EFFECT OF TRANSITION WIRES ON THE LIFT GAIN FACTOR
(2-dim.;  $\alpha = 0^\circ$; $G = 30^\circ$; Ref. 24)
FIG. 98  THE O. N. E. R. A. JET CONTROL FLAP WITH
UPPER SURFACE AND SYMMETRICAL BLOWING
(AR \( \approx \) 20; \( \alpha = 0^0; \Theta = 67^0; \) Ref. 33)
FIG. 99
TOTAL LIFT AND MEASURED THRUST VARIATION
WITH FLOW MACH NUMBER
($AR = 3.4; \alpha = 3^\circ; \text{Ref. 33}$)
FIG. 100  COMPARISON OF THE TOTAL LIFT OBTAINED BY JET AUGMENTED FLAPPED (a), AND JET FLAPPED (b), WING AT LOW CJ-VALUES (SEE ALSO FIG. 15 OF REF. 33) (AR = 3.4; Θ ≈ 30°; CJ ≈ 0.02; Ref. 39)
FIG. 101  VARIATION OF JET INDUCED LIFT WITH MACH NUMBER
(AR = SEE ABOVE, $\alpha = 0^\circ$; $\theta = 90^\circ$; $C_{\mu} < 0.06$; REF. 87)
FIG. 102 EFFECT OF MACH NUMBER ON THE CHANGE IN DRAG COEFFICIENT DUE TO BLOWING
(AR = SEE ABOVE; $\alpha = 0^\circ$; $\Theta = 0^\circ$; $C_{\mu} < 0.3$; REF. 87)
FIG. 103 TOTAL LIFT VARIATION OF TRUNCATED WING (AND JET FLAPPED WING FOR COMPARISON) (AR = 3.4; $\alpha = 0^\circ$, $\Theta = 30^\circ$, $\beta = 30^\circ$; Ref. 39)
FIG. 104  TOTAL LIFT VARIATION AS A FUNCTION OF THE JET CONTROL FLAP ANGLE $\beta$ AT ONE SUBSONIC AND ONE SUPersonic MACH NUMBER

$(AR = 3.4; \alpha = 0^\circ; \rho = 0.02; \text{Ref. 39})$
FIG. 105  COMPARISON OF THE TRUNCATED AND JET FLAPPED AEROFOIL
(AR = 3; \( \zeta = 0^\circ \); \( \Theta = 0^\circ \); Ref. 39)
FIG. 106  THE EFFECT OF VARIOUS STRENGTH CENTRIFUGAL FIELDS ON THE LIFT GAIN FACTOR ($G_L$)
($AR = \infty$; $\alpha = 0; \Theta = 30^\circ$; Ref. 45)
FIG. 107 LIFT VARIATION OF A DELTA-WING WITH AND WITHOUT A TIP TANK
(AR = 1.42; $\alpha = 0^\circ$; $\Theta = 70^\circ$; Ref. 33)
FIG. 108  LOSS IN TOTAL LIFT DUE TO GROUND INTERFERENCE EFFECTS
(2-dim.; $\varphi = 0$; Ref. 23)
FIG. 109 LOSS IN TOTAL LIFT DUE TO GROUND INTERFERENCE EFFECTS
(AR = 8.3; $\alpha = 0^\circ$; Ref. 73)
FIG. 110  THE GROUND EFFECT ON LIFT AS A FUNCTION OF THE JET DEFLECTION ANGLE, JET COEFFICIENT AND GROUND DISTANCE.
(2-dim.; DIMMOCK'S RESULTS (REFS. 24 &32); REF. 31)
FIG. 11

THE LIFT CENTRE SHIFT DUE TO THE GROUND EFFECT
(2-dim.; DIMMOCK'S RESULTS (REFS. 24 & 32); REF. 31)
FIG. 112
THE MAXIMUM OBTAINABLE TOTAL LIFT FOR A GIVEN GROUND DISTANCE.
(2-dim.; φ = 0°; Θ = 58.1°; REF. 110)
FIG. 113  THE CHANGES IN PRESSURE DISTRIBUTION DUE TO THE GROUND EFFECT
(2-dim.; \( \alpha = 0^\circ \); \( \Theta = 58.1^\circ \); REF. 110)
FIG. 114

THE LOW SPEED PERFORMANCE OF A JET FLAPPED AIRCRAFT IN CLOSE PROXIMITY TO THE GROUND.

(2-dim.; $\alpha = 0^\circ$; $\Theta = 58.1^\circ$; REF. 110)
THE GROUND INTERFERENCE EFFECT ON THE PITCHING MOMENT.
(2-dim.; $\alpha = 0^\circ$; $\Theta = 58.1^\circ$; REF. 111)
FIG. 116  THE TOTAL LIFT CENTRE SHIFT DUE TO GROUND INTERFERENCE EFFECTS.
(2-dim.; $\alpha = 0^\circ$, $\Theta = 58.1^\circ$; REF. 111)
THE EFFECT OF THE JET COEFFICIENT AND ASPECT RATIO ON THE JET SHEET SHAPE.

FIG. 117

(A.R = 2.75 and 6.8; C_J = 0.5 and 0.18; C_l = 0; \( V_o \) = 100 fps;
C = 8 in.; b = 12 in. Ref. 28)
FIG. 118

THE JET CENTRE LINE SHAPE AS A FUNCTION OF $C_J$ (JET CENTRE LINE $\equiv$ LOCUS OF MAX. TOTAL HEAD PRESSURE) AT 50% SEMI-SPAN STATIONS

($AR = 8.3; \alpha = 0^\circ; \Theta = 55^\circ$)
FIG. 119  REDUCTION IN AERODYNAMIC NOISE TO BE EXPECTED WITH THE JET FLAPPED WING AS A FUNCTION OF FREQUENCY (REF. 18)

FIG. 120  THE TOTAL LIFT AT ZERO INCIDENCE (2-dim.; \( \alpha = 0 \); \( C_J \leq 2.0 \); REF. 23)
FIG. 121  TOTAL LIFT VARIATION WITH INCIDENCE
(2-dim.; \( \Theta = 58.1 \); Ref. 32)
EXPERIMENTAL RESULTS.

- $C_J = 0.2$
- $C_J = 0.5$
- $C_J = 1$
- $C_J = 2$
- $C_J = 3$
- $C_J = 4$

Straight lines are 1/8 above values given by Eq. 108 (see Ref. 16)

**FIG. 122 COMPARISON OF EXPERIMENTALLY (REF. 24) AND THEORETICALLY (REF. 16, Eq. 108) OBTAINED LIFT COEFFICIENTS**

(2-dim.; $\Theta = 31.4^\circ$; Ref. 16)
FIG. 123  VARIATION OF TOTAL LIFT WITH $\sqrt{C_J}$
(AR = 2.75; $\alpha = 0^\circ; \Theta = 31.3^\circ$, Ref. 28)
VARIATION OF $C_{LT}$ WITH $C_J$ AND $\alpha$
(AR = 6.8, $\Theta = 31.4$, REFS. 28 AND 30)
FIG. 125 VARIATION OF LIFT CURVE SLOPE WITH $C_J$
(2-dim.; $\theta = 58.1^\circ$; Ref. 32)

- Tangent to $C_L$ vs $\alpha$ curves
- AT $\alpha = 0^\circ$
- Tangent at the exp' point before L.E. separation occurs.

Extreme range of experimental interpretation thus.

Theoretical curve putting $\alpha = 1.0$.
FIG. 126    LIFT CURVE SLOPE COMPARISON (REF. 16)
FIG. 127  VARIATION OF THE LIFT CURVE SLOPE AS CALCULATED BY THE CURVED FLAP ANALOGY. (2-dim.; $\alpha = 0$; Ref. 31)

$\frac{dC_l}{d\alpha}$

$\Theta = 0$

$\Theta = 31.4^\circ$

$\Theta = 45.4^\circ$

$\Theta = 57.3^\circ$

$\Theta = 68.8^\circ$

$\Theta = 90^\circ$

30° MODEL  90° MODEL  60° MODEL

THEORETICAL CURVES FROM REF. PUTTING

QUARTER-CHORD POINT

MID-CHORD POINT

AR = $\infty$  6.8  3.75

FIG. 128  SHIFT OF THE AERODYNAMIC AND LIFT CENTRE (2-dim.; $\alpha = 0$; Refs. 23 and 28)
FIG. 129
LIFT CENTRE AND AERODYNAMIC CENTRE SHIFT
AS PREDICTED BY CURVED FLAP ANALOGY
(THE EXPERIMENTAL CURVES WERE OBTAINED FROM
FIGS. 2 AND 3 OF REF. 23; 2-dim.; \( \alpha = 0 \); REF. 31)
Fig. 130 Comparison between theoretical and experimental pressure distribution
(2-dim.; \( \alpha = 0^\circ; \Theta = 30^\circ; \) Ref. 16)
FIG. 131 COMPARISON OF THEORETICALLY (REF. 17) AND EXPERIMENTALLY (REF. 24) OBTAINED PRESSURE DISTRIBUTIONS (2-dim.; $\alpha = 0^\circ$; $\Theta = 31.4^\circ$, REF. 17)
FIG. 132  THE TOTAL DRAG vs. JET COEFFICIENT
(2-dim.; $\alpha = 0$, REFS. 24, 32 and 142)
Fig. 133

The function $a(\theta)$

(2-dim.; $\alpha = 0$, Refs. 24, 32, 33, 138, 142)
FIG. 134  
CALCULATED AND EXPERIMENTAL MAXIMUM PRESSURE LIFT COEFFICIENTS FOR A JET FLAPPED WING NEAR THE GROUND,  
(2-dim.; \( \alpha = 0 \); REF. 139)
FIG. 135  COMPARISON OF THE THEORETICAL AND EXPERIMENTAL VALUES OF THE MAXIMUM PRESSURE LIFT COEFFICIENT NEAR THE GROUND
(2-dim.; $\alpha = 0$; REF. 149)
FIG. 136 COMPARISON OF CALCULATED AND EXPERIMENTAL PRESSURE DISTRIBUTIONS AROUND JET FLAPPED WINGS CLOSE TO THE GROUND (2-dim.; $\alpha = 0$; REF. 149)
FIG. 137  THE TOTAL LIFT CHANGE DUE TO ASPECT RATIO VARIATION
( $\alpha = 0; \ 30^\circ < \theta < 63^\circ; \ k_{\alpha} \leq 0.10; \text{REF 33}$)
THE MIXING ALONG XY CAUSES THE SAME
CONTRACTION OF THE EXTERNAL FLOW AS
WOULD A SINK OF STRENGTH $v_1'(-\delta \omega)$

FIG. 138  CONTRACTION OF THE MAIN STREAM
FLOW CAUSED BY THE JET (REF. 14)
FIG. 139 PAYNE'S MODEL (a) AND HIS JET DENSITY COMPARISON AFTER MIXING. (REF. 36)
FIG. 140  VARIATION OF THE THRUST AUGMENTATION PARAMETER WITH $\rho_0 / \rho_T$ FOR $n = 3$ (REF. 36)
VARIATION OF THE THRUST AUGMENTATION PARAMETER WITH \( \frac{v_1}{v_j} \) AND \( n \) FOR THE MAIN FLOW AT REST (REF. 36)
FIG. 142  FLOW OVER AN AEROFOIL WITH A SINGLE SINK AT THE T. E. (REF. 36)

FIG. 143  THE JET DRAG COEFFICIENT MINUS THE INTEGRATION CONSTANT PLOTTED AGAINST X (REF. 36)
FIG. 144 COMPARISON OF $C_{LT}$ FOR A BLOWING FLAP WITH T. E. BLOWING WITH THAT FOR THE PURE JET FLAP AT LOW JET MOMENTUM COEFFICIENTS
(2-dim.; $\theta \approx 76^\circ$; $\alpha = 0^\circ$; REF. 33)
FIG. 145  EFFECT OF JET CONTROL FLAP SIZE ON THE TOTAL LIFT
(AR = 20; $\alpha = 0^\circ; C_{L_{\mu}} = 1$; REF. 33)
FIG. 146  THE TOTAL LIFT COEFFICIENT AS A FUNCTION OF THE JET AND THE SINK VOLUME COEFFICIENT
(2-dim.; $\chi = 0$; $\Theta = 58^\circ$; REF. 33)
FIG. 147  EFFECT OF JET CONTROL FLAP SIZE ON THE TOTAL LIFT
(AR ≈ 20; ζ = 0°; θ = 67°; REF. 33)
FIG. 148  THE BASE DRAG AND ITS REDUCTION (GAIN) DUE TO BLOWING AT A TRUNCATED AEROFOIL AT SUBSONIC AND SUPERSONIC SPEEDS.
(AR = 3; $\alpha = 0^\circ$; $\Theta = 0^\circ$; REF. 39)
FIG. 149  THE CHANGE IN LIFT WITH MACH NUMBER OF A TRUNCATED JET FLAPPED WING
(A R = 3; \( \infty = 0^\circ \); \( \Theta = 30^\circ \); REF. 39)
FIG. 150  THE LIFT, THRUST AND PITCHING MOMENT CHANGES OF A TRUNCATED JET FLAPPED WING WITH JET DEFLECTION ANGLE
(AR = 3; \( \alpha_i = 0^\circ \); \( Ma = 2.04 \); REF. 39)
FIG. 151 AERODYNAMIC CHARACTERISTICS OF A TRUNCATED AEROFOIL WITH A SOLID OR BLOWN SPOILER BLADE.

(AR = 3; $\alpha = 0^\circ$; $0.6 \leq Ma \leq 2.6$; REF. 39)
FIG. 152  THE DRAG VARIATION OF A "FREE-STREAM LINE FLAP" AS A FUNCTION OF THE JET COEFFICIENT.
(2-dim.; $\Theta = 130^\circ$ and $\tau = 30^\circ$ (SEE NEXT FIG.); REF. 81)
FIG. 153 THE "FREE STREAMLINE FLAP" LIFT COEFFICIENT AS COMPARED WITH A JET CONTROL FLAP AND THE PURE JET FLAP.

(NOTE: COMPARISON UNFAIR DUE TO DIFFERENT $\alpha$)
DEFLECTION DEVICES IN ORDER OF DECREASING EFFICIENCY

(LEAST AXIAL THRUST LOSS FOR GIVEN DELECTED FORCE)

<table>
<thead>
<tr>
<th>CONFIGURATION</th>
<th>$\left( \frac{TH_a}{TH} \right)_{max}$</th>
<th>POSSIBLE APPLICATION</th>
</tr>
</thead>
<tbody>
<tr>
<td>Swiveled Nozzle</td>
<td>0.35</td>
<td>Control or Lift Augmentation</td>
</tr>
<tr>
<td>Movable Plug</td>
<td>0.10</td>
<td>Control or Trim</td>
</tr>
<tr>
<td>Swiveled Tailpipe</td>
<td>-1.00</td>
<td>Control or Lift</td>
</tr>
<tr>
<td>External Flap</td>
<td>0.40</td>
<td>Control or Lift Augmentation</td>
</tr>
<tr>
<td>Swiveled Shroud (Hi Dia. Ratio)</td>
<td>&gt;0.30</td>
<td>Control or Lift Augmentation</td>
</tr>
<tr>
<td>Internal Flap (Low Angle)</td>
<td>0.20</td>
<td>Control or Trim</td>
</tr>
<tr>
<td>Swiveled Shroud (Low Dia. Ratio)</td>
<td>0.35</td>
<td>Control or Lift Augmentation</td>
</tr>
<tr>
<td>Cylindrical Thrust Reverser</td>
<td>0.20</td>
<td>Control or Trim</td>
</tr>
<tr>
<td>Internal Flap (Hi Angle)</td>
<td>0.17</td>
<td>Control</td>
</tr>
<tr>
<td>Swiveled Primary, Fixed Shroud</td>
<td>0.16</td>
<td>Control</td>
</tr>
<tr>
<td>90° Side Bleed Nozzle</td>
<td>-1.00</td>
<td>Control</td>
</tr>
</tbody>
</table>

FIG. 154  DEFLECTION DEVICES IN ORDER OF DECREASING EFFICIENCIES. (REF. 83)
a) ANNULAR NOZZLE CONFIGURATION FOR JET SUPPORTED AIRCRAFT (REF. 83)

b) THRUST AUGMENTATION WITH ANNULAR NOZZLE (PRESSURE RATIO = 2.1; REF. 83)
FIG. 156  a) COANDA NOZZLE USING SINGLE FLAT PLATE FOR JET DEFLECTION (REF. 83)

b) LIFT-TO-THRUST RATIOS OBTAINED WITH 2- AND 3- FLAT PLATE SURFACES
(PRESSURE RATIO = 2.1; REF 83)
FIG. 157 EXPERIMENTAL JET CONTROL FLAP RESULTS

\[ \beta = 60^\circ = \text{Const and } C_{w} / C_{LT} = 0.5 = \text{Const.} \]

SUPERIMPOSED TO A FIELD OF JET FLAP CHARACTERISTICS.

(2-dim.; \( \alpha = 0; W / \dot{S}_w = 51 \text{ lb/ft}^2; \text{REF 33} \))
FIG. 158  GENERAL RELATIONS OF THE FUNCTIONS
\( C_\mu \), \( C_{LT} \), \( \left( \frac{g}{W} \right) \), AND \( \left( \frac{W}{\Sigma W} \right) \)
FIG. 159  LIFT CONTROL BY A BLOWN SPOILER BLADE AT HIGH SUBSONIC MACH NUMBERS (REF. 18)
FIG. 160  A LIFT-THRUST INTEGRATED SYSTEM, BASED ON CONVENTIONAL BY-PASS TURBO-JET. (REF. 30*)
FIG. 161 LATERAL CONTROL BY MEANS OF DIFFERENTIAL BLOWING OVER CLASSICAL AILERONS. (REF. 27)
FIG. 162  THE THRUST REQUIREMENTS OF THE JET FLAP AIRCRAFT AS COMPARED WITH ONE, USING CONVENTIONAL BUT DEFLECTED JETS (REF. 29)
A detailed review is presented of the theoretical and experimental advances that have been made in the study of pure jet flaps, jet control, and jet-augmented flaps. Great care was taken in defining acting forces and in presenting all equations and graphs in a unified notation. Theories are correlated. Experimental results are quoted, illustrated by relevant graphs and compared with theory. Chapters on jet mixing and on the jet flap's implication on aircraft design are included. Another chapter attempts to assess the jet flap on the basis of its possible merits and de-merits. A list of future research projects is added. More than 150 references have a bearing on the material presented and another 50 in a separate list are related in subject.
A detailed review is presented of the theoretical and experimental advances that have been made in the study of pure jet flaps, jet control and jet-augmented flaps. Great care was taken in defining acting forces and in presenting all equations and graphs in a unified notation. Theories are correlated. Experimental results are quoted, illustrated by relevant graphs and compared with theory. Chapters on jet mixing and on the jet flap's implication on aircraft design are included. Another chapter attempts to assess the jet flap on the basis of its possible merits and de-merits. A list of future research projects is added. More than 150 references have a bearing on the material presented and another 50 in a separate list are related in subject.