HEAT TRANSFER IN A LAMINAR BOUNDARY LAYER AT MACH 2.5 FROM A SURFACE HAVING A TEMPERATURE DISTRIBUTION

BY

B. N. PRIDMORE-BROWN

FEBRUARY, 1957

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Especial thanks are due to Mr. B. G. Dawson for constructing the heat transfer model and for his assistance in designing it.

This work was made possible by the financial assistance of the Defence Research Board of Canada.
SUMMARY

Heat transfer measurements were made on the outer surface of a hollow cylinder with a sharp leading edge at a Mach number of 2.5. The surface of the model was divided into half inch segments, the temperature of which could be controlled and measured individually. Heat transfer results for various surface temperature distributions were compared with the theory of Chapman and Rubesin and were found to be up to 100% higher than their paper would predict (Ref. 1). Good agreement was found with the results obtained by Slack on a cooled flat plate (Ref. 3). Boundary layer pitot traverses were made with a rectangular mouthed probe. The increase in boundary layer thickness due to heat transfer from the model to the flow was found to be greater than that predicted by theory.

A temperature recovery factor of .877 was obtained for the first element, in good agreement with other flat plate results.
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NOTATION

<table>
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<tr>
<th>Roman Letters</th>
<th>Description</th>
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<tbody>
<tr>
<td>(a_n)</td>
<td>coefficients in series defining wall temperature</td>
</tr>
<tr>
<td>(C)</td>
<td>constant of proportionality used in the relation (\frac{\mu}{\mu_i} = C \frac{T}{T_i})</td>
</tr>
<tr>
<td>(c_p)</td>
<td>specific heat at constant pressure</td>
</tr>
<tr>
<td>(E)</td>
<td>voltage applied to electrical bridges</td>
</tr>
<tr>
<td>(e_p)</td>
<td>bridge unbalance voltage</td>
</tr>
<tr>
<td>(f(\eta))</td>
<td>Blasius function defined by (f(\eta) = \frac{\psi^<em>}{\sqrt{\chi^</em>}})</td>
</tr>
<tr>
<td>(F)</td>
<td>(F = \frac{e_p}{E})</td>
</tr>
<tr>
<td>(G)</td>
<td>bridge constant (G = \frac{R_4}{R_3}) (see Figs. 6 and 7)</td>
</tr>
<tr>
<td>(h)</td>
<td>local heat transfer coefficient defined as (\frac{q}{T_w - T_e})</td>
</tr>
<tr>
<td>(k)</td>
<td>thermal conductivity</td>
</tr>
<tr>
<td>(K)</td>
<td>thermal coefficient of resistivity for &quot;Nilvar&quot;. (K = \frac{1}{R_r} \cdot \frac{R - R_r}{T - T_r} \left[ ^{\circ}C^{-1} \right])</td>
</tr>
<tr>
<td>(L)</td>
<td>length of surface considered, measured from the leading edge, which is also the normalisation interval for Tchebichef polynomials</td>
</tr>
<tr>
<td>(M)</td>
<td>Mach number</td>
</tr>
<tr>
<td>(Nu)</td>
<td>Nusselt number (\frac{h_x}{k})</td>
</tr>
<tr>
<td>(p)</td>
<td>pressure</td>
</tr>
<tr>
<td>(P_T)</td>
<td>pressure measured at pitot probe</td>
</tr>
<tr>
<td>(q)</td>
<td>rate of heat transfer per unit area</td>
</tr>
<tr>
<td>(Q)</td>
<td>rate of heat transfer</td>
</tr>
<tr>
<td>(r(\eta))</td>
<td>a function of (\eta) defined in Appendix. (r(\dot{\eta}) = 0.845) for (\dot{\eta} = 0.72) and is the temperature recovery factor</td>
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R

resistance

$R_o$

value of $R_1$ for bridge balance (see Figs. 6 and 7)

$R_c$

reference resistance (at 20°C)

Re

Reynolds number \[ \frac{\rho u x}{\mu} \]

t

time in seconds

t(x)

$T_w(x) - T_e$

S

constant in Sutherland's formula

\[ C = \sqrt{\frac{T_w}{T_i}} \left\{ \frac{T_i + S}{T_w + S} \right\} \]

T

temperature

u, v

velocity components in the x and y directions respectively

x

coordinate parallel to model surface

$x_\alpha$

points at which values of t(x) are taken to determine Tchebichef polynomials

y

coordinate perpendicular to model surface

$Y_\eta(\eta)$

functions of $\eta$ and $x$ defined in appendix

$N_\eta(\eta)$

$X(x^*)$

$\gamma$

ratio of specific heats

$\delta$

bridge unbalance \[ \frac{R_1 - R_o}{R_o} \]

dimensionless independent variable occurring in Blasius function $f(\eta)$

$\eta$

coefficient of viscosity

$\mu$

kinematic viscosity

$\nu$

mass density

$\rho$

Prandtl number \[ \frac{c_p \mu}{k} \]

\psi

stream function
Subscripts
0  indicates stagnation conditions
1  free stream conditions
e  equilibrium condition of an insulated surface
w  wall conditions

Superscripts
*  non-dimensional quantity
/  ordinary differentiation
I. INTRODUCTION

The characteristics of a laminar boundary layer at a given point on a surface in supersonic flow are determined both by the character of the flow and by the surface temperature distribution upstream of the point. For this reason the use of a local heat transfer coefficient defined in terms of local temperatures as

\[ h = \frac{q}{T_w - T_e} \]

can lead to apparently anomalous results, particularly in cases where there is a large variation of wall temperature in a streamwise direction.

In 1949 Chapman and Rubesin published a paper (Ref. 1) which considered the effect of an arbitrary surface temperature distribution on heat transfer and on temperature and velocity profiles in a compressible laminar boundary layer. They solved the boundary layer equations with the wall temperature as a boundary condition expressed in the form of power series in \( x \). For practical solutions wall temperature can usually be adequately represented by a polynomial. The Chapman and Rubesin solution for heat transfer coefficient reduces to the conventional one only for constant wall temperature.

There have been a number of experimental determinations of rate of heat transfer in a laminar boundary layer in supersonic flow. Some of these are listed in References 3, 4, 7, 8, 19. The experimental results in all these papers, except those of Reference 3, were correlated with theory which takes no account of the effect of a wall temperature distribution. The results of Reference 3 are for a cooled flat plate with a relatively mild negative temperature gradient. In one case, Reference 4, it is possible that the unexplained high rate of heat transfer near the plate leading edge was partly due to a large positive \( \frac{dT_w(x)}{dx} \). Thus the effects of temperature gradient have not been well explored.

The purpose of the present investigation was to make heat transfer measurements and pressure traverses in a laminar boundary layer on a surface whose temperature could be controlled and to correlate the results using the theory of Reference 1. A hollow cylinder heat transfer model was chosen with a diameter large enough that flat plate boundary layer theory could be compared directly with measurements made on it. A cylindrical model had certain advantages over a flat plate in that corner effects were avoided and heating elements were more easily attached to the surface.

II. EQUIPMENT AND PROCEDURE

1. Wind Tunnel

The measurements were made in the UTIA 16 x 16 supersonic wind tunnel at a Mach number of 2.5. The tunnel is of the blowdown type with an atmospheric inlet and has a maximum running time of about 24
seconds. For a complete description of the installation see Reference 5. Average test conditions were as follows:

\[ p_0 = 1 \text{ atmosphere} \]
\[ T_0 = 300^\circ \text{K} \]
\[ \text{Re}/\text{inch} = 2.5 \times 10^5 \]

2. Model

The model used was a 5\(\frac{1}{2}\) inch outer diameter, 5 inch inner diameter hollow cylinder of laminated phenolic plastic aligned axially with the flow. A sharp steel leading edge was fitted to the front of the model. Aft of the leading edge elements of 1/32" x .002" Nilvar ribbon were wound on to the model in \(\frac{1}{2}\) inch segments, using the thread feed on a lathe at 28 TPI. The ends of each segment were brought through the model wall and secured with brass pegs flush with the outer surface. The pegs protruded on the inside of the model so that lead wires could be soldered to them. The outer surface was coated with temperature resistant silicone varnish. After the varnish had dried it was scraped off the exposed surface of the Nilvar so that spaces between adjacent wires remained filled. See Figure 1.

The model was mounted in the tunnel as shown in Figures 2 and 3. By loosening the clamping screws at the top of the mount the model could be moved in an axial direction to adjust its position with respect to the pitot probe.

3. Heating and Temperature Measuring Equipment

The Nilvar elements served the dual purpose of heating the surface and of providing a means of measuring average wall temperature over a distance of half an inch. Each half-inch heating element, consisting of a number of turns, was connected to a Wheatstone bridge as one arm of the bridge.

The Nilvar wire ribbon used in the heating elements had been calibrated to determine its temperature coefficient of resistivity

\[ K = \frac{1}{R_r} \frac{R - R_r}{T - T_r} \]  \hspace{1cm} (2.1)

In Figure 10, K vs. T is plotted with \(T_r = 20^\circ\text{C}\).

Two types of bridges were constructed; the first (Fig. 6) designed so that the bridge current would be sufficient to heat the surface element; and the second (Fig. 7) designed for a low bridge current and intended for measuring resistance only. Current to the heating bridges could be controlled by changing the amount of resistance in series with them. The resistances used in the bridges were manganin.
wound and of 1% accuracy.

The relay A, Figure 6, was actuated by the main tunnel switch and was used to prevent excessive heating of the model when the tunnel was stopped.

Four heating bridges were made, each with design values of $G = 100, 110, 120, 130$, and design $R_o = 100, 110, 120, 130$ ohms. Design maximum current was 3 amperes. Six unheated bridges were made, each with design values of $G = 1, 1.1, 1.2, 1.3, R_o = 100, 110, 120, 130$ ohms.

Current for the heating bridges was supplied by a General Electric speed variator (part of a Ward-Leonard type motor control system) with a maximum output of 37.5 amperes at 230 volts D.C. Bridge input voltage was measured on a Weston model 341 voltmeter with a full scale accuracy of 0.25%. Input voltage for the unheated bridges was supplied by a 2 volt accumulator and was measured on a Pye potentiometer. Output voltages from all bridges were measured on three Brown potentiometer recorders. $e_p$ for six bridges could be recorded by switching each of the three potentiometers between two bridge outputs with a DPDT relay. The relays were controlled automatically by a flip-flop circuit that switched at intervals of about two seconds.

4. Method for Determining Wall Temperature and Rate of Heat Transfer

The equation governing a Wheatstone bridge in an unbalanced state is

$$F = \frac{\delta}{(G+1) \left[ \frac{G+1}{G} + \delta \right]} \quad (2.2)$$

or

$$\delta = \frac{F(G+1)^2}{G \left[ 1 - F(G+1) \right]} \quad (2.3)$$

All the bridges were calibrated using a precision decade resistance box as $R_1$ and measuring $E$ with the Pye potentiometer. $e_p$ was measured on a Brown potentiometer recorder. It was found that for the correct values of $R_o$ and $G$ (which were generally slightly different from the design values) $F$ was accurately represented by Eq. (2.2) to within $\pm$ 0.4%. Calibrations were checked at various room temperatures and were found to be in good agreement.

A reference resistance value $R_r$ for each element on the model had to be measured. Using Eq. (2.1) the temperature of an element is given by the equation

$$T = -\frac{R_o + R_0 + \delta R_o}{K R_r} + T_r \quad (2.4)$$
Rate of heat transfer was calculated directly from power input to a given element.

\[ Q = \left( \frac{E}{R_1 + R_2} \right)^2 R_1 \]  

5. Boundary Layer Pressure Measuring Equipment

The boundary layer pressure probe was of rectangular form with external tip dimensions .0347" x .0066". A photograph of the probe mouth is shown in Figure 5.

In making a boundary layer traverse the probe datum position was taken as that position in which the tip just touched the lower surface of the model. This position was found optically and checked electrically. The \( y \) position of the probe was taken as the vertical distance from the model surface to the centre of the probe opening, and was measured by a micrometer on the traversing mechanism. The distance of the probe from the leading edge was controlled by moving the model in its mount.

Pressure was measured with a Northam DP-7 transducer with a range of ± 25 mm. Hg. Such a transducer consists of two chambers separated by a thin steel diaphragm. One chamber is set at a reference pressure and the other is connected to the unknown pressure. On either side of the diaphragm is an inductance coil, each of which constitutes an arm of a bridge. With equal pressure on both sides of the diaphragm the bridge output is zeroed. If now a change in the pressure of one of the chambers causes a deflection of the diaphragm the inductance of the coils is changed and the bridge becomes unbalanced. Pressure difference across the diaphragm is proportional to bridge unbalance voltage. Figure 9 shows the pressure measuring system. Valves A and B were actuated when the tunnel started. A time delay was used for the closing of valve B to avoid excessive momentary loadings of the transducer.

6. Method of Finding Recovery Temperature

The maximum running time of the wind tunnel was 24 seconds, and in that time only the first element came close to temperature equilibrium.

Equilibrium temperature on the unheated model was found in the following way: A value of \( T_e \) was guessed and the points represented by \( \ln(T_w - T_e) \) over the last 16 seconds of a run were fitted by a straight line according to the method of least squares. A new value of \( T_e \) was then selected and a new line computed. For each value of \( T_e \) the sum of the squares of the errors was found and compared with that resulting from the previous guess for \( T_e \). The value of \( T_e \) resulting in the least sum of squares of errors was chosen. This iteration procedure obtained in effect, the best match to an exponential cooling law with the initially unknown \( T_e \) as asymptote, thereby determining \( T_e \). These calculations
were done on the Ferranti digital computer FERUT at the University of Toronto Computation Centre.

It was thought that the mass of the leading edge might provide a heat source which could affect the equilibrium temperature of the downstream elements. A run was therefore made with the leading edge pre-cooled to below the theoretical recovery temperature.

7. **Heat Transfer Measurements and Boundary Layer Pressure Traverses**

Heat transfer measurements were made for five different surface temperature distributions as shown in Figures 15 to 19. With each one of these temperature distributions a boundary layer pressure traverse was made. For temperature distributions 1, 2, 3, and 4 the boundary layer traverse was made at 3" from the leading edge and for distribution 5 at 2.3".

8. **Accuracy of Measurements**

The following were the steps necessary to determine the value of $T_w$:

(a) $G$ (bridge constant) obtained from calibration.
(b) $R_o$ (bridge balance point) obtained from calibration
(c) Resistance of element measured at known temperature from which

$$ R_r = \frac{R}{K(T-T_r) + 1} \quad \text{at } 20^\circ\text{C} $$

(d) $E_p$ and $E$ measured during run.
(e) $\delta = \frac{(G+1)\frac{1}{2}F}{G-FG(G+1)} = \frac{R_r - R_o}{R_o}$
(f) $R_r = R_o \delta + R_o$
(g) $T_w = \frac{R_r - R}{KR_r} + 20 \quad [T_w \text{ in } ^\circ\text{C}]$

<table>
<thead>
<tr>
<th>Step</th>
<th>Quantity</th>
<th>Estimated Max. Error</th>
<th>HEATED ELEMENTS</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>$\delta (G,F)$</td>
<td>$\pm 0.4%$</td>
<td>Maximum difference observed between $\delta$ measured &amp; $\delta$ from Eq. (2.2)</td>
</tr>
<tr>
<td>b</td>
<td>$R_o$</td>
<td>$\pm .02$ ohms</td>
<td></td>
</tr>
<tr>
<td>c</td>
<td>$R_r$</td>
<td>$\pm .15$ ohms</td>
<td></td>
</tr>
<tr>
<td>d</td>
<td>$E_p = \frac{E}{E}$</td>
<td>$\pm 2%$</td>
<td>$E_p$ from Brown Pots, $E$ from Weston voltmeter</td>
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UNHEATED ELEMENTS

<table>
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<th>Step</th>
<th>Quantity</th>
<th>Estimated Max. Error</th>
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<tr>
<td>a</td>
<td>$\delta$ (G, F)</td>
<td>$\pm 0.2%$</td>
</tr>
<tr>
<td>b</td>
<td>$R_o$</td>
<td>$\pm 0.01$ ohms</td>
</tr>
<tr>
<td>c</td>
<td>$R_r$</td>
<td>$\pm 0.05$ ohms</td>
</tr>
<tr>
<td>d</td>
<td>$F = \frac{E_p}{E}$</td>
<td>$\pm 0.5%$</td>
</tr>
<tr>
<td>e</td>
<td>$\delta$</td>
<td>$\pm 0.2%$</td>
</tr>
<tr>
<td>f</td>
<td>$R$</td>
<td>$\pm 0.21$ ohms</td>
</tr>
<tr>
<td>g</td>
<td>$T$</td>
<td>$\pm 1.8^\circ C$</td>
</tr>
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The calculated heat leakage to the inner surface of the model is negligible.

Estimated maximum errors for boundary layer pressure measurements were as follows:

Position of probe $\pm 0.003''$

Pressure (near middle of boundary layer) $\pm 1$ mm. Hg.

Pressure (near edges of boundary layer) $\pm 0.2$ mm. Hg.

Calibration of transducer $\pm 0.5$ mm. Hg.

III. RESULTS

1. Extent of Laminar Boundary Layer

Total head traverses through the boundary layer showed that transition on the unheated model began at approximately 6 inches from
the leading edge. Figure 12 shows $\frac{P_T}{P_o}$ vs $\sqrt{\frac{\rho_v}{\rho_o}}$ for traverses at 5, 6 and 6½ inches from the leading edge. At 6 and 6½ inches it can be seen that in the vicinity of the wall $\frac{P_T}{P_o}$ is somewhat higher than would be expected for a laminar boundary layer. Figure 37 is a schlieren photograph of the flow over the unheated model. The boundary layer is clearly becoming turbulent in the vicinity of 6 inches.

Boundary layer total head traverses on the heated model were made at 3 inches from the leading edge for temperature distributions numbers 1, 2, 3 and 4; and at 2.3 inches for temperature distribution 5. It is seen in Figures 32 and 33, which show total head traverses for temperature distributions 3 and 4, that transition has already begun at those stations. The schlieren photographs of the heated wall cases are seen in Figures 38 to 42. The more forward position of transition for temperature distributions 3 and 4 is apparent.

2. Recovery Temperature

Recovery temperatures were obtained only for the first element centred at 9/16 inch from the leading edge. Cooling curves for elements in further aft positions indicated that a greater running time would be required for equilibrium to be approached. Equilibrium temperatures were calculated as described in Section II. 6 and were found to be the same with the leading edge initially at room temperature as when it was pre-cooled to below the theoretical recovery temperature. Cooling curves of $\ln(T_w - T_e)$ are shown in Figure 11. The recovery factor

$$r = \frac{T_e - T_i}{T_o - T_i} = .877$$

which agreed with other flat plate and hollow cylinder results. Slack (Ref. 3) obtained a value of .880. Brinich (Ref. 13) reports a value of about .866 at .2" aft of a .0008" thick leading edge. The leading edge on the present model was of comparable thickness (.0006") and hence the value of $r$ is slightly high in comparison to that of Reference 13. The higher value may be attributable to the probability that the plastic wall of the model could not reach equilibrium in the available running time.

3. Approximation of Wall Temperatures by Polynomials

The five $T_w(x)$ distributions at which measurements were made are shown in Figures 15 to 19. The temperature of an element used for calculation purposes was the average value over the last third of a tunnel run. In Figures 15 to 19 the vertical lines through the points represent the extent of temperature variation of a given element in this time interval.

The theory of Reference 1 requires that the surface temperature be represented in the form of a power series or approximated by a polynomial. In Figures 20 to 24 $t(x)$ vs $x/L$ is shown approximated by Tchebichef polynomials.
If a Tchebichef polynomial is to be fitted to an experimental curve in an interval normalised to \(-1 \leq x \leq 1\), points \( x_\alpha \) at which the polynomial is fitted must be taken at

\[ x_\alpha = \cos \frac{\alpha \pi}{m} \quad (\alpha = 0, 1, 2, \ldots, m) \]

where \( n \) is the degree of the polynomial. So that for a given degree of polynomial the \( x_\alpha \) are predetermined.

In general the \( x_\alpha \) did not coincide with measurement stations, so it was necessary to guess intermediate values of \( t(x) \) from a curve drawn through experimental points.

The rate of heat transfer at a given point predicted by theory is nearly proportional to the value of \( t(x) \) given by the approximating polynomial at that point. Slight changes of \( t(x) \) upstream of the point have relatively little effect on calculated rate of heat transfer. Polynomials which gave the best fit at the stations where \( t(x) \) was measured were therefore chosen, rather than those that gave the best average fit for the curves drawn through the experimental points. These same stations were also used for calculations of theoretical rate of heat transfer.

It was found that polynomials of sixth degree or greater fitted to temperature distributions with large \( \frac{d^2 T(x)}{dx^2} \) tended to have excessively large coefficients of the powers of \( x \) which resulted in loss of significant figures when calculating rates of heat transfer and boundary layer total head profiles. Hence most of the polynomials used are of fifth degree.

From Figures 20 to 24 it is seen that the polynomial approximations for \( t(x) \) were generally good for all stations except the first. The leading edge was always assumed to be at the theoretical recovery temperature.

4. Comparison with Theory

(a) General

Heat transfer results plotted in the form \( \frac{q_x}{k_w \sqrt{Re}} \) vs \( Re \) are presented in Figures 25A to 29A. Two sets of theoretical values are presented for comparison. The higher of the theoretical curves in each case was obtained using the theory of Reference 1 with the wall temperature approximated by a polynomial. The lower theoretical curve was calculated using experimental wall temperatures but neglecting the effect of surface temperature variation in a streamwise direction. In Figures 25B to 29B experimental heat transfer is plotted as \( \frac{q_x}{k_w \sqrt{Re}} \) vs \( Re \) and is compared with theoretical values of \( \frac{Q_x}{k_w \sqrt{Re}} \). This is equivalent to multiplying experimental values of \( \frac{q_x}{k_w \sqrt{Re}} \) by \( \frac{k_w}{k_{w'}} \) which is about .52. There is no theoretical justification for this procedure, but it enables the trends of the two curves to be compared more easily.

Boundary layer pressure traverses are shown in Figures 30 to 34 in the form \( \frac{q}{Re} \) vs \( \frac{1}{x} \sqrt{Re} \). For the traverse taken at 3'' from
the leading edge experimental values obtained on the unheated model are compared with theory in Figure 13, with $\frac{1}{x} \sqrt{R\gamma C}$ of the theoretical curve increased by 11.8% (see Ref. 6). The theoretical values of $\frac{1}{x} \sqrt{R\gamma C}$ on the heated model are likewise increased by 11.8%. The total head traverse at 2.3" on the unheated model is shown in Figure 14 compared with theory with $\frac{1}{x} \sqrt{R\gamma C}$ increased 8.5%. Theoretical $\frac{1}{x} \sqrt{R\gamma C}$ on the heated model at this station is increased by the same amount.

(b) Temperature Distribution No. 1 (Fig. 15) Probe station 3"

L = 1.31"

A single element, no. 1, was heated. Only the temperatures of the leading edge and elements 1 and 2 were considered in making the polynomial approximation for $f(x)$. This interval of normalisation was chosen, as with a larger one it appeared impossible to find a polynomial which did not oscillate excessively. Two polynomials were tried, one of fourth degree and one of sixth. They both predicted about the same rate of heat transfer for element 1, while the sixth degree polynomial gave a higher rate of heat transfer from the fluid to element 2.

Experimental rate of heat transfer at element 1 (Fig. 25A) was about 95% higher than theory. Elements 2 and 3 which were not heated assumed a temperature higher than $T_e$. Theory for constant $T_w$ therefore predicted heat flow from the model to the fluid. The theory of Reference 1 predicts a slight heat flow from the fluid to element 2
\[
\left( \frac{q'x}{k_i T_e \sqrt{C}} = -0.015 \right).
\]
Experimentally the heat flow to elements 2 and 3 was virtually zero.

The boundary layer thickened slightly as a result of heating element 1 as can be seen in Figure 30. It was not possible to calculate a theoretical profile since the probe was outside the interval of normalisation for the Tchebichef polynomial.

(c) Temperature Distribution No. 2 (Fig. 16) Probe station 3"

L = 3"

Elements 2, 3, 4 and 5 were heated, each to a higher temperature than the preceding one. A polynomial of fifth degree gave a good fit to the temperature of all except the first element (Fig. 21).

Experimental rate of heat transfer was about 95% higher than theory (Fig. 26A). Theory gave too high a rate of heat transfer for element 1, but this was evidently due to the high value of $f(x)$ given by the polynomial.

The boundary layer thickened about 25% over the unheated case. The experimental boundary layer thickness was 11% greater than that of the heated theoretical boundary layer (Fig. 31). Theoretical values were adjusted as described in Section III. 4. a.
(d) Temperature Distribution No. 3 (Fig. 17) Probe station 3"
L = 3".
Here an attempt was made to produce a negative \( \frac{dT}{dx} \).
Again a fifth degree polynomial gave a good approximation to the
temperature of all except element 1 (Fig. 22).

Heat transfer results were about 95% higher than theory,
with the discrepancy increasing towards the aft portion of the region measured (Fig. 27A).

The boundary layer thickened by 18% over the unheated case.
The total head profile (Fig. 32) shows that the boundary layer was
becoming transitional at this station.

(e) Temperature Distribution No. 4 (Fig. 18) Probe station 3"
L = 3".
The maximum power that could be used without damaging
the model was applied to elements 2, 3, 4 and 5. A fifth degree
polynomial fitted \( t(x) \) for these elements (Fig. 23).

Heat transfer values were 100% higher than theory with the
discrepancy increasing in the x direction. (Fig. 28A).

The boundary layer pressure profile showed that transition
had begun at 3" (Fig. 33).

(f) Temperature Distribution No. 5 (Fig. 19) Probe station 2.3"
L = 2.3"
Elements 1, 2, 3 and 4 were heated to give a positive \( \frac{dT}{dx} \).
The experimental values of \( t(x) \) were fitted with a sixth degree
polynomial (Fig. 24).

Heat transfer results for elements 1 and 2 were about 75%
higher than theory and for elements 3 and 4 about 80% higher (Fig. 29A).

The boundary layer on the heated model thickened considerably
more than theory predicted as can be seen in Figure 34.

IV. DISCUSSION

The laminar boundary layer in supersonic flow has been
observed to manifest certain differences in behaviour depending on
whether it is generated on a flat plate type model or on a cone or cone-
cylinder. Experimental values for rate of heat transfer measured on
cones and cone-cylinders agree well with theory (see, for example,
Refs. 4, 7, 19; ) while those measured on flat plates are high (Refs.
3 and 4). Further, transition on flat plates and hollow cylinders occurs
earlier. A possible cause of these differences is the geometry of the
forward parts of the two types of model, since a leading edge must
affect the boundary layer flow more than the point of a cone. There is
evidence (e.g. Ref. 13) that a sharp leading edge causes earlier transition than a blunt one, a fact that has been attributed to (a) reduction in boundary layer Reynolds number due to losses through the shock on a blunt leading edge (Refs. 13 and 14); (b) leading edge vibration, a thinner leading edge being more prone to vibrate (Ref. 9).

A reduction in surface Reynolds number would tend to produce a lower rate of heat transfer under the conditions of the present investigation (Ref. 14). Unsteady disturbances resulting from vibration of a thin leading edge might promote turbulent spots which would increase the heat transfer rate. However, if at the probe station the boundary layer were turbulent for a certain percentage of time a distortion of the total heat profile in the direction of a turbulent profile would be expected, as found in Reference 21. In the present results there is some evidence of profile distortion in the vicinity of the wall in Figures 13, 32, 33, but it is not marked enough to conclude that the boundary layer is turbulent for a significant percentage of time.

Reference 3 reports heat transfer measurements made on a flat plate with a negative $\frac{dT(x)}{dx}$ on the forward part followed by a region of fairly constant $T_w$. Heat transfer results from that paper are 100% higher than theory at a station 2" from the leading edge, and at stations farther downstream the discrepancy decreased and disappeared. The discrepancy did not seem to be affected by Reynolds number, but only by the position on the plate at which measurements were taken. The present results agree well with those of Reference 3, at comparable distances from the leading edge, but unfortunately no comparison can be made beyond about 3".

The better downstream agreement with theory in Reference 3 suggests that an effect occurring at the forward part of the model may disappear downstream more rapidly in the stabler laminar boundary layer on a cooled surface. No boundary layer traverses near the forward part of the flat plate are reported in that paper so that no conclusions regarding profile distortion near the leading edge can be reached.

The heat transfer results reported in Reference 4 were measured on a flat copper plate which was heated uniformly before each run. The greater rate of heat transfer near the leading edge probably caused a strong positive $\frac{dT(x)}{dx}$ to form shortly after the run had begun. Results from Reference 4 are compared (Fig. 35) with measurements at elements 1 and 2, and agreement is quite good.

V. CONCLUSIONS

1. On the unheated model the boundary layer pressure profiles exhibit the characteristics of a laminar boundary layer up to six inches from the model leading edge.
2. Schlieren photographs showing transition agree with results from probe traverses.
3. The temperature recovery factor determined at element 1 agrees well with values obtained for flat plates.
4. Transition on the heated model begins at 2½ to 3½ inches from the leading edge depending on the degree of surface heating.
5. Heat transfer results show qualitative agreement with theory of Reference 1, but are up to 100% higher.
6. Heat transfer values agree well with Slack's results obtained on a cooled flat plate in the laminar part of the boundary layer.
7. Total head profiles on the heated model show a greater thickening due to heat addition than predicted by theory.
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APPENDIX

Outline of the Theory of Chapman and Rubesin

In Reference 1 the equations for a compressible laminar boundary layer on a flat plate or axially symmetric surface are solved for the case of an arbitrary analytic surface temperature distribution and zero pressure gradient.

The equations are:

\[
\frac{\partial}{\partial x}(\rho u) + \frac{\partial}{\partial y}(\rho v) = 0 \quad (1)
\]

\[
\rho u \frac{\partial u}{\partial x} + \rho v \frac{\partial u}{\partial y} = \frac{1}{\rho} \frac{\partial p}{\partial y} - \nu \frac{\partial^2 u}{\partial y^2} \quad (2)
\]

\[
\rho u \frac{\partial T}{\partial x} + \rho v \frac{\partial T}{\partial y} = \frac{1}{\rho c_p} \frac{\partial p}{\partial y} + \frac{\partial}{\partial y} \left( \frac{\partial u}{\partial y} \right)^2 \quad (3)
\]

If the fluid properties \( \mu \) and \( k \) are regarded as variables with temperature, as they must be where large differences in temperature occur in the flow, it is necessary to assume some relation between these properties and temperature. A widely used, quite accurate empirical relationship is Sutherland's formula:

\[
\frac{\mu_r}{k_r} = \left( \frac{T}{T_r} \right)^{3/2} \frac{T_r + S}{T_r} \quad (4)
\]

Simpler approximations sometimes used are:

\[
\frac{\mu_r}{k_r} = \frac{T}{T_r}, \quad \frac{\mu_r}{k_r} = \left( \frac{T}{T_r} \right)^{0.76}
\]

In Reference 1 the approximation used is in effect

\[
\frac{\mu}{k_w} = \frac{T}{T_w} \quad (5)
\]

where \( \mu_w \) and \( k_w \) are related to the free stream values by Sutherland's formula. Hence Eq. (5) can be expressed as

\[
\frac{\mu}{\mu_i} = \frac{k}{k_i} = C \frac{T}{T_i}
\]

where

\[
C = \sqrt{\frac{T_w}{T_i} \frac{T_i + S}{T_w + S}}
\]

Thus the viscosity and thermal conductivity are accurately represented at an average wall temperature rather than at free stream conditions.

The solution of Eqs. (1), (2), (3) depends on the introduction of a stream function \( \Psi \) as an independent variable, where \( \Psi \) is the rate of mass flow between a given point and the wall. In terms of \( \Psi \), \( u \) and \( v \) are given by the equations
which satisfy the continuity equation.

Eqs. (2) and (3) in non-dimensional form, after a transformation replacing \( y \) by \( \psi \) as independent variable, become

\[
\begin{align*}
\frac{\partial u^*}{\partial x^*} &= \frac{1}{\partial x}(u^* \frac{\partial u^*}{\partial x})\\
\frac{\partial T^*}{\partial x^*} &= \frac{1}{\partial x}(u^* \frac{\partial T^*}{\partial x}) + \left(1 - 1 \right) M^2 \int \frac{\partial u^*}{\partial x^*}^2
\end{align*}
\]

with the boundary conditions

\[
\begin{align*}
U^* &= 0 \\
T^* &= T_w(x) \\
U^* &= 1 \\
T^* &= \infty
\end{align*}
\]

The momentum equation and the boundary conditions on \( u^* \) are now independent of the energy equation and the boundary conditions on \( T^* \). Density does not appear in the momentum equation so that its solution must be that for incompressible flow, namely the Blasius function

\[
U^* = \frac{1}{2} f'(\eta)
\]

where \( \eta \) is defined by

\[
f(\eta) = \frac{\psi^*}{\sqrt{x^*}}
\]

The energy equation which remains to be solved is transformed to the coordinates \( (x^*, \eta) \) and becomes

\[
\frac{\partial^2 T^*}{\partial \eta^2} + \sigma f' \frac{\partial T^*}{\partial \eta} - 2\sigma f' x^* \frac{\partial T^*}{\partial x^*} = \frac{\partial (\eta - 1) M^2 (f'^2)}{4}
\]

with boundary conditions that can be expressed in the form

\[
\begin{align*}
T^*(x^*, 0) &= T_e^* + t(x^*) \\
T^*(x^*, \infty) &= 1
\end{align*}
\]

A solution of the inhomogeneous equation corresponds to the case of flow over an insulated wall. This solution is

\[
N(\eta) = 1 + \frac{\eta - 1}{2} M^2 r(\eta), \quad \text{and at the wall}
\]

\[
N(0) = 1 + \frac{\eta - 1}{2} M^2 + r(0),
\]

where \( r(0) \) is the recovery factor. In general

\[
r(\eta) = \frac{\sigma}{2} \int_0^\infty \left[ f''(\varphi) \right]^2 \sqrt{\left[ f''(\varphi) \right]^2 - \sigma} d\varphi d\varphi
\]
Solutions to the homogeneous equation can be obtained by separation of variables in the form

\[ T^* = X(x^*) Y(\eta) \]

from which

\[ X_n(x^*) = x_n \eta \]

\[ Y_n'' + \sigma + f Y_n' - 2 \sigma f'(\eta) Y_n = 0 \]

Convenient boundary conditions on the functions \( Y_n(\eta) \) are

\[ Y_n(0) = 1, \quad Y_n(\infty) = 0 \]

The homogeneous equation is linear so that solutions of the form \( \alpha_n x_n \eta Y_n(\eta) \) can be superimposed to give a general solution

\[ \sum \alpha_n x_n \eta Y_n(\eta) \]

which satisfies the boundary conditions

\[ T^*(x^*,0) = \sum \alpha_n x_n \eta \]

\[ T^*(x^*,\infty) = 0 \]

The complete solution of the energy equation is the sum of the particular solution of the inhomogeneous equation and the general solution of the homogeneous equation, and the boundary conditions satisfied are the sum of the boundary conditions imposed on these solutions, namely

\[ T^*_w = T^*(x,0) = T_c^* + \sum \alpha_n x_n \eta \]

\[ T^*(x^*,\infty) = 1 \]

so that the wall temperature is expressed as an infinite power series.

The effect of wall temperature and free-stream Mach number on boundary layer temperature and velocity profiles can be seen in the relation between the physical coordinate \( y \) and the variable \( \eta \):

\[ \frac{y}{2x} \sqrt{\frac{R_e}{C}} = \eta + \frac{\gamma - 1}{2} M^2 \int_0^\eta r(\eta) d\eta + \sum \alpha_n x_n \eta \int_0^\eta Y_n(\eta) d\eta \]

The rate of heat transfer \( q \) is given by

\[ q = -k_w \left( \frac{\partial T}{\partial y} \right)_w \]

and is found to be

\[ q = -k \frac{\gamma}{2x} \sqrt{\frac{R_e}{C}} \sum \alpha_n x_n \eta \int_0^\eta Y_n(\eta) d\eta \]

The conventional definition of heat transfer coefficient

\[ h = \frac{q}{(T_w - T_e)} \]

leads to anomalous infinities in \( h \) when a temperature distribution exists.
since the zeros of the series $\sum a_n x^n Y_n(\xi)$ do not necessarily occur at the same values of $x$ as those of the series $\sum a_n x^n \sim (T_w - T_e)$. It seems therefore more appropriate to present experimental data in terms of the dimensionless quantity $k_i T_e \sqrt{Re_x}$ rather than in terms of the more conventional Nusselt or Stanton numbers, both of which depend on the heat transfer coefficient.

For the case of a wall with constant surface temperature

$$q = - \frac{k_i T_i}{2x} \sqrt{C} \sqrt{Re_x} a_0 Y_0'(0)$$

$$N_u = - \frac{\sqrt{C}}{2} Y_0(0) \sqrt{Re_x}$$

which closely approximates the usual expression

$$N_u = - 0.332 \sigma^{-\frac{1}{3}} \sqrt{Re_x}$$
FIGURE 1
HEAT TRANSFER MODEL

plastic tube
leads to bridges
brass pegs securing Nilvar ribbon
steel leading edge thickness .0006"
Nilvar wound heating elements 1/2" wide
FIGURE 4
PITOT TRAVERSING MECHANISM
FIGURE 6
CIRCUIT FOR HEATING BRIDGES
FIGURE 7

CIRCUIT FOR UNHEATED BRIDGES
PRESSURE MEASURING EQUIPMENT

FIGURE 9

PRESSURE TRANSDUCER TO PITOT

Solenoid Valve N.C.

Solenoid Valve N.O.

REFERENCE PRESSURE

WALLACE AND TIERNAN PRECISION Hg. MANOMETER

VAC. PUMP
FIGURE 10

TEMPERATURE COEFFICIENT OF RESISTIVITY FOR NILVAR

\[
K = \frac{1}{\frac{R - R_r}{T - T_r}}
\]

\(T_r = 20 \, ^\circ\text{C}\)
FIGURE 11
COOLING CURVES FOR ELEMENT 1 UNHEATED
TOTAL HEAD PROFILES ON UNHEATED MODEL

FIGURE 12

DISTANCE FROM LEADING EDGE

5" •

6" ◇

6.5" ○
FIGURE 13

TOTAL HEAD PROFILE AT 3" ON UNHEATED MODEL
FIGURE 14

TOTAL HEAD PROFILE AT 2.3" ON UNHEATED MODEL
FIGURE 15
TEMPERATURE DISTRIBUTION NO. 1
FIGURE 16
TEMPERATURE DISTRIBUTION NO. 2
FIGURE 17

TEMPERATURE DISTRIBUTION NO. 3
FIGURE 18
TEMPERATURE DISTRIBUTION NO. 4
FIGURE 19

TEMPERATURE DISTRIBUTION NO. 5
FIGURE 20

POLYNOMIAL APPROXIMATION FOR $t(x)$, TEMPERATURE DISTRIBUTION NO. 1

EXP. O
$t(x_\alpha)$ △ $n=4$ polynomial
$t(x_\alpha)$ X $n=6$ polynomial
POLYNOMIAL APPROXIMATION FOR $t'(x)$, TEMPERATURE DISTRIBUTION NO. 2
FIGURE 22
POLYNOMIAL APPROXIMATION FOR $t(x)$, TEMPERATURE DISTRIBUTION NO. 3
Figure 23

Polynomial Approximation for $t(x)$, Temperature Distribution No. 4.

$E_X P. \circ$  
$t(x_\alpha) \times \ n=5$ polynomial
FIGURE 24

POLYNOMIAL APPROXIMATION FOR \( \hat{t}(x) \), TEMPERATURE DISTRIBUTION NO. 5

EXP. 
\( t(x) \) 
\( n = 4 \) polynomial 
\( n = 6 \) polynomial

\( t(x_\alpha) \)
FIGURE 25A

HEAT TRANSFER, TEMPERATURE DISTRIBUTION 1
**FIGURE 25B**

HEAT TRANSFER, TEMPERATURE DISTRIBUTION 1
\[ \frac{q_x}{k_1 T_e \sqrt{Re}} \]

**FIGURE 26A**

HEAT TRANSFER, TEMPERATURE DISTRIBUTION 2
FIGURE 26B
HEAT TRANSFER, TEMPERATURE DISTRIBUTION 2
FIGURE 27A

HEAT TRANSFER, TEMPERATURE DISTRIBUTION 3
FIGURE 27B
HEAT TRANSFER, TEMPERATURE DISTRIBUTION 3
FIGURE 28A
HEAT TRANSFER, TEMPERATURE DISTRIBUTION 4
FIGURE 28B

HEAT TRANSFER, TEMPERATURE DISTRIBUTION
FIGURE 29A

HEAT TRANSFER, TEMPERATURE DISTRIBUTION 5
Figure 29B

Heat Transfer, Temperature Distribution 5
FIGURE 31

BOUNDARY LAYER TOTAL HEAD PROFILE, TEMPERATURE DISTRIBUTION 2
BOUNDARY LAYER TOTAL HEAD PROFILE, TEMPERATURE DISTRIBUTION 3
FIGURE 33
BOUNDARY LAYER TOTAL HEAD PROFILE, TEMPERATURE DISTRIBUTION 4
FIGURE 34

BOUNDARY LAYER TOTAL HEAD PROFILE, TEMPERATURE DISTRIBUTION 5
FIGURE 35

NUSELT NUMBER COMPARISON WITH REFERENCE 4
FIGURE 36

SCHLIEREN PHOTOGRAPH OF FLOW OVER UNHEATED MODEL
FIGURE 39
BOUNDARY LAYER, T.D. 3
FIGURE 40
BOUNDARY LAYER, T. D. 4
FIGURE 41

BOUNDARY LAYER, T. D. 5, PROBE AT 2.3"
FIGURE 42

BOUNDARY LAYER, T.D. 5, PROBE AT 6"
Heat transfer measurements were made on the outer surface of a hollow cylinder with a sharp leading edge at a Mach number of 2.5. The surface of the model was divided into half inch segments, the temperature of which could be controlled and measured individually. Heat transfer results for various surface temperature distributions were compared with the theory of Chapman and Rubesin and were found to be up to 100% higher than their paper would predict (Ref. 1). Good agreement was found with the results obtained by Slack on a cooled flat plate (Ref. 3). Boundary layer pitot traverses were made with a rectangular mouthed probe. The increase in boundary layer thickness due to heat transfer from the model to the flow was found to be greater than that predicted by theory.

A temperature recovery factor of 0.877 was obtained for the first element in good agreement with other flat plate results.
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Copies obtainable from: Institute of Aerophysics, University of Toronto, Toronto 5, Ontario