AUTOMATIC LANDING THROUGH THE TURBULENT PLANETARY BOUNDARY LAYER

by

Shangxiang Zhu

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Abstract

In this report a statistical approach to automatic landing in turbulent planetary boundary layer is presented which is valuable for use in aircraft design and the analysis of terminal operation safety, also useful in design of autoland systems.

The linearized equations of motion for an aircraft following a glide slope and an exponential curved path in the presence of wind shear and turbulence are developed. The effects of gust gradients on the motion were considered.

The wind is modeled as a multi-dimensional random process, characterized by the mean wind shear and turbulence. In the planetary boundary layer the mean wind is adequately described by a power law and by Weibull wind speed distribution. The turbulence is assumed locally isotropic. The modified von Kármán model adequately represents correlations along an approach-flare trajectory. The forces and moments are considered to depend linearly on uniform gust and gust gradient components which are obtained by Etkin's four-point method which utilizes the air velocity at 4 points on the aircraft. As a test base, an autopilot of a jet transport for the approach and flare was designed. The "Random Choice Direct Search" technique was employed to find a set of optimal feedback gains for the auto-flare control system.

A method to calculate the covariance matrix, as a function of time, of a linear system perturbed by the atmospheric turbulence, which is described as a Gauss-Markov process, is presented.

Having found the perturbations of the state variables due to mean wind shear and turbulence, a hard landing probability analysis was carried out.

The primary purpose of the proposed method is to establish a structure containing the system elements, disturbances and the certification limits in
an analytical framework. With it, the relative effects of changes in the various system elements, mean wind and turbulence parameters and operational limitations on precision of control and available margins of safety can be estimated.

A numerical example calculation for a large jet transport with autopilot was conducted to demonstrate the proposed analysis method.

In the case of a piloted aircraft with the automatic system model replaced by a human pilot model, this analysis approach is still valid.
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<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A$</td>
<td>aerodynamic forces</td>
</tr>
<tr>
<td>$A_1, A_2$</td>
<td>system matrices</td>
</tr>
<tr>
<td>$B$</td>
<td>control matrix</td>
</tr>
<tr>
<td>$C$</td>
<td>coefficient of aerodynamics, or correlation function</td>
</tr>
<tr>
<td>${c, k, F_0}$</td>
<td>Weibull parameter set</td>
</tr>
<tr>
<td>$D$</td>
<td>weighting matrix for state vector</td>
</tr>
<tr>
<td>$D_1, D_2$</td>
<td>wind influence coefficient matrix</td>
</tr>
<tr>
<td>$d$</td>
<td>aircraft m.c. position perpendicular to glide slope, positive above</td>
</tr>
<tr>
<td>$e(t)$</td>
<td>error index</td>
</tr>
<tr>
<td>$F$</td>
<td>external force applied to aircraft</td>
</tr>
<tr>
<td>$F_0$</td>
<td>probability of observing zero wind speed</td>
</tr>
<tr>
<td>$F(w)$</td>
<td>cumulative distribution function of mean wind</td>
</tr>
<tr>
<td>$f(\xi)$</td>
<td>longitudinal correlation function in von Kármán model</td>
</tr>
<tr>
<td>$G(\omega)$</td>
<td>system transfer function</td>
</tr>
<tr>
<td>$q$</td>
<td>gravity vector or aerodynamic forces, moments due to ground effect</td>
</tr>
<tr>
<td>$g(\xi)$</td>
<td>traverse correlation function</td>
</tr>
<tr>
<td>$H$</td>
<td>theoretical C.G. height profile in flare</td>
</tr>
<tr>
<td>$\dot{H}$</td>
<td>theoretical vertical speed profile in flare, $\dot{H} = dH / dt$</td>
</tr>
<tr>
<td>$h$</td>
<td>height of C.G. above ground</td>
</tr>
<tr>
<td>$\dot{h}$</td>
<td>descent rate</td>
</tr>
<tr>
<td>$h$</td>
<td>aircraft angular momentum about mass centre</td>
</tr>
<tr>
<td>$h_d$</td>
<td>decision height</td>
</tr>
<tr>
<td>$I$</td>
<td>identity matrix</td>
</tr>
<tr>
<td>$(\hat{B}, \hat{B}, \hat{B})$</td>
<td>unit vectors along $x_B, y_B, z_B$</td>
</tr>
<tr>
<td>$(\hat{E}, \hat{E}, \hat{E})$</td>
<td>unit vectors along $x_E, y_E, z_E$</td>
</tr>
</tbody>
</table>
\((\hat{S}, \hat{S}, \hat{S})\)
unit vectors along \(x_s, y_s, z_s\)

\(J\)
cost function or weighted performance measure

\(K_{1/3}, K_{2/3}\)
modified Bessel functions

\(K_1, K_2\)
feedback gain matrices

\(K_d\)
glide path deviation proportional feedback gain

\(K_{\bar{d}}\)
glide path deviation integral feedback gain

\(K_{\dot{d}}\)
glide path deviation rate feedback gain

\(K_E\)
elevator-actuator time constant

\(K_H\)
height deviation proportional feedback gain

\(K_{\bar{H}}\)
height deviation integral feedback gain

\(K_{\dot{H}}\)
vertical speed deviation proportional feedback gain

\(K_{n_z}\)
load factor proportional feedback gain

\(K_T\)
throttle actuator time constant or averaging time correction constant

\(K_u\)
airspeed deviation proportional feedback gain

\(K_{\bar{u}}\)
airspeed deviation integral feedback gain

\(K_{\dot{u}}\)
airspeed deviation rate feedback gain

\(K_\theta\)
pitch angle deviation proportional feedback gain

\(K_{\dot{\theta}}\)
pitch angle rate deviation feedback gain

\(L\)
turbulence integral scale

\(M\)
aerodynamic moment about the C.G.

\(m\)
aircraft mass

\(N\)
intensity matrix of white noise

\(\Delta n_z\)
normal load factor perturbation

\(P\)
block diagonal transformation matrix

\(P_f(x)\)
overall hard landing probability function

\(p_f(x)\)
local hard landing probability function
Q \quad \text{weighting matrix for quadratic state vector}

q_e \quad \text{dynamic pressure } \left( \frac{1}{2} \rho \mathbf{V}_e^2 \right)

q(\hat{\mathbf{h}}|\mathbf{w}) \quad \text{conditional hard landing probability function}

R \quad \text{weighting matrix for control signal vector, or state vector}

r \quad \text{range of searching in RCDS technique}

\mathbf{r} \quad \text{position vector}

S \quad \text{set of events}

s \quad \text{Laplace operator}

T \quad \text{diag} \{\tau_L T_E\}, \text{ time constant matrix, or transformation matrix of rotating}

T_t \quad \text{engine time constant}

(u \, v \, w) \quad \text{components of } \mathbf{V} \text{ in } \mathbf{F}_B

(u^E \, v^E \, w^E) \quad \text{components of } \mathbf{V}^E \text{ in } \mathbf{F}_B

\mathbf{U}(t) \quad \text{total turbulent wind vector}

u^* \quad \text{friction velocity}

\Delta u_c \quad \text{airspeed command signal}

\mathbf{V} \quad \text{velocity vector of vehicle M.C. relative to local air mass (airspeed vector)}

V_e \quad \text{airspeed at the reference trajectory}

\mathbf{V}^E \quad \text{velocity vector of vehicle M.C. relative to } \mathbf{F}_E

V_s \quad \text{stalling speed}

\mathbf{W} \quad \text{wind vector}

\mathbf{\dot{W}} \quad \text{wind gradient vector}

W_e \quad \text{wind speed at the reference trajectory}

W_m \quad \text{mean wind speed}

(x_E^E, y_E^E, z_E^E) \quad \text{spatial coordinates in } \mathbf{F}_E

(X \, Y \, Z) \quad \text{components of aerodynamic force in } \mathbf{F}_S
$\Delta \mathbf{x}$  
vector of state variables

$z_0$  
terrain roughness

$\Delta z$  
intermediate variable in solution for $\Delta \mathbf{x}$

$\alpha$  
angle of attack of the aircraft's zero lift line

$\alpha_f$  
angle of attack of the aircraft's fuselage reference line

$\alpha_0$  
angle between fuselage reference line and zero lift line

$\alpha_x$  
angle of attack of $\mathbf{\dot{i}}_5$

$\gamma$  
the angle $\mathbf{v}$ makes with the horizontal

$\gamma_G$  
glide slope angle with respect to the horizontal

$\Gamma$  
gamma function

$\delta(t-a)$  
impulse applied at $t = a$

$\Delta \delta_E$  
elevator deflection

$\Delta \delta_T$  
throttle setting measured as a fraction of full throttle

$\Delta \delta_C$  
command signal matrix

$\epsilon$  
error signal matrix

$\eta$  
terrain roughness parameter

$\mu$  
deviation due to mean shear

$\kappa$  
averaging time correction factor

$\sigma$  
standard deviation

$\Sigma(t)$  
covariance matrix

$\tau$  
time constant, or time shift $\tau = t_2 - t_1$

$\phi(\omega)$  
spectral function

$\omega_g$  
 equivalent angular velocity vector due to turbulence

$(\omega_x,\omega_y,\omega_z)$  
angular velocity of aircraft relative to $\mathbf{F}^E$

$\Omega$  
wave number

$\Omega(t,\tau)$  
impulsive response function matrix
Mathematical Symbols

\[ \dot{x} \] time rate of change as seen from inertial frame

\[ \ddot{x} \] time rate of change as seen from non-inertial frame, or relative derivatives

\[ A^H \] Hermitian transpose of matrix \( X \)

\[ X^* \] complex conjugate of matrix \( X \)

\[ \exp(A) \] matrix exponential, \( e^{A} = 1 + A + \frac{1}{2!} A^2 + \ldots + \frac{A^K}{K!} + \ldots \)

\[ \langle X(t) \rangle \] ensemble average of \( X(t) \)

\[ F_E \] reference frame defined as a vector \( (iE_jE_kE) \)

\[ F_E \times X \] vector \( \dot{X} \) expressed in reference frame \( F_E \), \( (x_1E^+ + x_2E^+ + x_3E^+) \)

\[ F_E \times X \] vector \( \dot{X} \) decomposed in reference frame \( F_E \), \( (x_1, x_2, x_3) \)

\[ F_E^{-1}X \] skew-symmetric matrix for vector \( \dot{X} \), expressed in frame \( F_E \)

\[ \hat{X} \] augmented matrix of the \( X \)

\[ \hat{X} \sim \hat{X} \] Fourier pair matrices

\[ \hat{X} \] Laplace transform of matrix \( X \)

Reference Frames

\( F_E: (x_E, y_E, z_E) \) Earth-fixed frame - \( \dot{0}_E \) points in direction of the central line of a runway

\( F_G: (x_G, y_G, z_G) \) glide-slope reference frame

\( F_B: (x_B, y_B, z_B) \) vehicle-fixed frame

\( F_S: (x_S, y_S, z_S) \) stability reference frame (vehicle-fixed)

\( F_W: (x_W, y_W, z_W) \) wind axes reference frame

Notation for aerodynamic coefficients and derivatives is standard, as for example in [47].
Superscripts, Subscripts and Abbreviations

B  body-fixed reference frame
CAT.II  category II landing
C  commanded
CCT  constrained correlation technique
E  Earth-fixed reference frame or elevator
e  linearization reference value
f  final or overall
FAA  Federal Aviation Administration (U.S.A.)
FRL  fuselage reference line
g  gust
G  glide slope
GE  ground effect
H  hybrid
ICAO  International Civil Aviation Organization
ILS  instrument landing system
\( \lambda \)  limit
n  nominal
o  initial
PSD  power spectral density
QSLFM  quasi-steady linear field model for aerodynamics
RCDS  random choice direct search technique
S  stability-axes reference frame
SFT  shaping filter technique
T  thrust or transpose
TD  touchdown
UTIAS  University of Toronto, Institute for Aerospace Studies
1.1 A Brief Review

Recently many serious fatal accidents/incidents on landing and take-off, in which wind shear and turbulence are cited as the contributing factors, have been officially reported [1, 2 and 3]. For example, turbulence has been recognized as major contributing factors in 183 accidents in the U.S.A. during eleven years from 1964 to 1975. Those catastrophic events have sparked a renewed interest in the analysis of the problems concerning the response of aircraft to winds.

In the earliest days of the era of powered flight, as reviewed by Etkin [4], the Wright brothers were faced with the side gust problem and successfully dealt with it. The wind problem was recognized as a very high priority item by NACA when it was originally established. In 1915 Wilson [5] carried out the analysis of aircraft response to discrete gusts and pointed out an important conclusion in the necessity of taking into account the gust gradients. This point is still valid today. Much of the work in the period preceding World War II focused on the gust loads placed on aircraft structures, as reported by Taylor [6]. Little work had been done on the study of lower level atmospheric boundary layer wind effect. After World War II researches into the low altitude effects of variable winds continued to be relatively uncommon, much of the focus having turned to the development of spectral and correlation techniques for studying responses to continuous turbulence [4 and 6]. The pioneer in the investigation of wind shear effects on flight is well known to be B. Etkin [7, 8]. In [8], Etkin analysed the problem of the effects of low altitude linear spatial wind gradients on glide and climb. In the early sixties, Grosso [9] and Zbrozek
also considered wind gradient effects, but at high altitude. Until the mid-sixties low altitude wind encounters were generally not considered to be hazardous. Some investigators concluded that the low level wind shear can be overcome without problems, as Browne discussed in 1961 [11]. Burnham [12] suggested that the wind shear was of concern only to light-aircraft pilots during take-off. Such views of the hazards of low altitude winds began to change in the later sixties and seventies. The reason for this change may probably attribute to the growing number of jet aircraft. This brought an increased awareness of the differences between the response characteristics of jet aircraft and propeller aircraft. Some of these characteristics, such as the large value of engine time lag, would tend to increase the risks associated with low level variable wind encounters. The fast developing computer science may also be a very important factor. It allows investigators to do much work in the time domain such that they could look with deeper insight into wind shear problems, particularly in the time-variant, nonstationary, inhomogeneous problems. As stated at the beginning of this chapter, this growing awareness was spurred by a number of low altitude aircraft accidents; e.g., the Eastern 66, Boeing 727 which crashed on approach, in a thunderstorm wind environment, at Kennedy International Airport on June 24, 1975 [2], and the DC-9 crash while landing in a rainstorm at Philadelphia International Airport on June 23, 1976, injuring 86 of the 106 persons aboard. In retrospect, Shrager [13] has concluded that low altitude variable winds may have been contributing factors in a number of accidents where winds had previously not been considered to be a factor. Table I lists 25 accidents in which wind shear was a probable cause.
Such catastrophic events have stimulated an explosion of investigations on many challenging topics of landing problems. These include work on wind modelling technique, wind shear detecting and warning technique, downburst modelling, digital simulation, design of 'wind-proof' aircraft controllers, control logic for flying through severe winds in flare-out or aborted-landing and methods for predicting the response of aircraft to winds. A sampling of the literature on the above aspects is given in [14 to 37].

1.2 Objectives and Scope of the Present Study

The scope of this research was restricted to the study of an analytical and computational technique which is intended to apply to evaluating aircraft system response to hazardous variable wind conditions including both wind shear and turbulence encountered on the landing approach-flare process. In the past, the response problem was separated into different branches:

(1) deterministic method - usually applied to wind shear problems;
(2) statistical method - usually applied to turbulence problems.

Considering the fact that the severe wind shear is a very rare event, i.e., the probability of encountering extreme shear which causes accidents is very small, it is not unreasonable to think of a given deterministic wind shear profile as a single realization of mean wind gradient plus turbulence. In this way the severe wind shear problem, the deterministic wind, could be put into a statistically analytical framework. This immediately brings out a merit that one may obtain insight into both magnitude and probability of interest from calculated results of the response to winds. This statistical method would be very useful to the aircraft design and safety analysis.
To establish a suitable analytical framework model, there are three items which must be specified:

1. the aircraft system model which includes the airframe dynamics and the automatic control system;

2. the wind model which includes a reasonable mean wind profile model and the turbulence model in the earth's boundary layer;

3. the pertinent method for predicting the system response to winds which should include an algorithm to analyse the probability of interest.

All of these three aspects will be dealt with in this study; however, as an engineering model no attempt will be made to view the turbulent wind from the mechanism viewpoint but rather to make use of the information available in a logical manner in order to accommodate the need for reasonably describing the atmospheric boundary layer winds. The emphasis will be laid on (3).

There are a number of experimental and analytical techniques that can be applied to predict the rms of the state variables. The problem of estimating the response of an aircraft to turbulence in the planetary boundary layer is somewhat more complicated than in high altitude because of the turbulence characteristics being no longer isotropic nor homogeneous. These factors point out the need to perform a time-domain analysis, which includes the effects of the vehicle's transient response and the nonstationary nature of the input. Past studies based on power spectral density techniques [38, 39] have often either omitted some of these effects or employed approximations to them. Considerable efforts have been expended on methods for the calculation of the response to turbulence in landing and a method for the terminal analysis is presented in [18].
Unfortunately, it still used the power spectral density method at decision height. This implies that the results from that technique only represent the response of an aircraft in horizontal flight over the ground by 30m. It cannot be considered a suitable method for the landing case, though it was claimed so. References [33, 27 and 21] introduced a method using modern system theory to find the covariance matrix of the state variables at an arbitrary moment during landing in a nonisotropic turbulence field. That is, undoubtedly, an elegant analysis method. Because it uses a shaping filter through which the noise is filtered to a colored noise to represent a real Gauss-Markov process, the approximation clearly exists. References [27, 33] employed the point approximation which ignored the size of an aircraft. It is questionable when applied to a large size aircraft. Although [21] used a five-point model to account for the gust gradient, it only dealt with the lateral case. Etkin et al, in [40 and 41] proposed a "Flight Correlation Technique" and showed an example calculation. However, it only applied to a stick-fixed STOL aircraft with a 15° glide slope was based on the point approximation assumption. The aircraft used in that work was a bare-frame, i.e., the problem was confined to an open-loop system.

Only a small amount of work has been reported on the failure probability analysis for landing in turbulence. In the present study, the basis is the application of "two-point space/time correlations" among the turbulence velocity components as calculated along the landing trajectory. The gust gradient is considered in the correlation function which is calculated based on Etkin's four-point model. The solution of covariance matrix is found using modern system theory. The nonhomogeneous, nonisotropic turbulence and the system's nonstationary properties are considered by "freezing" the aircraft system dynamics in a series of successive piecewise small intervals.
and by the "stratified assumption" that in each layer corresponding to each interval the turbulence flow field is homogeneous, isotropic and Gaussian. To be more realistic, the modified von Kármán model is used [42]. The airplane is equipped with an autoland system; thus we will deal with a closed-loop problem. The Weibull wind speed distribution is used in the present study, which is a more realistic model used in the field of meteorology recently. Thus, a hard landing probability analysis can be made to check the relative importance of the terrain feature, the Weibull distribution parameters, the operation limitation values and the autopilot control logic.
CHAPTER II
STATE EQUATIONS OF THE AIRCRAFT SYSTEM

The differential equations for describing flight through the atmosphere are presented in numerous literatures [43, 44 and 45]. The equations are derived using a Newton-Euler development method. In this report, the development of the equations of motion will mainly follow Etkin's work [43].

2.1 Basic Assumptions

Considering our goal is to deal with a specified scenario that the problems relate to the CTOL aircraft subjected to atmospheric disturbances during approach and flare, several assumptions are made as follows:

(1) Flat-Earth and the Earth-fixed reference frame is considered inertial.
(2) The gravitational field is constant, as well as the air density, temperature and pressure.
(3) Neglect the elastic properties of the aircraft in question.
(4) The mass of the aircraft is constant.
(5) The aircraft has a vertical symmetry plane.
(6) The effects of internal and external rotors, articulation of the controls and fuel sloshing are all negligible to the gross rigid body motion of the aircraft.

Some of the overall simplifying assumptions can be relaxed with a modest increase in complexity, if required.

2.2 Definition of the Reference Frames

As any vector can be expressed in any reference frame, a specified reference frame for the study of the landing of an aircraft is required. The following notation convention is defined:
This defines a reference frame system \( E \) with three orthogonal components \( \mathbf{i}^E \), \( \mathbf{j}^E \) and \( \mathbf{k}^E \) as a vector \( \mathbf{F}_E \). Therefore, we define

\[
\mathbf{F}_E = (\mathbf{i}^E, \mathbf{j}^E, \mathbf{k}^E) \quad (2.1)
\]

as the vector \( \mathbf{a} \) expressed in the reference frame \( \mathbf{F}_E \), and

\[
\mathbf{F}_E^T \mathbf{a} = (a_x^E, a_y^E, a_z^E) \mathbf{a}^E \quad (2.2a)
\]

as the vector \( \mathbf{a} \) decomposed in \( \mathbf{F}_E \). In this investigation, the following frames are employed;

\( \mathbf{F}_E \) - Earth-fixed reference frame - \( O_x^E \) points in the direction of the central line of a runway.

\( \mathbf{F}_G \) - Glide-slope reference frame. It is defined to have an origin at the runway intercept point of the glidepath and x-axis coincides with the glide slope.

\( \mathbf{F}_B \) - Body-fixed reference frame whose origin is at the mass centre of the vehicle and whose x-axis is nominally downwards in the plane of symmetry and the y-axis orientation follows from the right-hand rule.

\( \mathbf{F}_S \) - Stability axes reference frame (body-fixed) whose origin is also

* The author is indebted to Dr. P. C. Hughes for the use of vectrix notation which was introduced in the course "Spacecraft Attitude Dynamics" at UTIAS.
at the mass centre and its x-axis coincides with the airspeed vector in the equilibrium flight condition. The z-axis is defined as being nominally downwards in the plane of symmetry and the y-axis orientation follows from the right-hand rule.

\( \mathcal{F}_W \) - Wind axes reference frame. The origin of it is at the mass centre of the aircraft and its x-axis is aligned with the instantaneous airspeed vector \( \mathbf{v} \) and its y-axis orientation follows from the right-hand rule. This frame moves wrt \( \mathcal{F}_S \).

It is noticed that \( \mathcal{F}_E \), according to the basic assumption (1) in the previous section, is an inertia reference frame. Figure 1 shows the reference frames defined above.

2.3 Transformation Matrices, Position, Velocities and Angular Velocities

Transformations are required for the rotational relationships between reference frames. There are a number of methods which may be adopted to represent these rotations, including Euler angles, direction cosines, Euler variables and so forth. The traditional Euler angle method will be employed throughout this study.

2.3.1 Transformation Matrices

In the following, use is made of the notation conventions similar to [43]. Let \( T_{BA} \) denote a transformation matrix relating the components of a vector \( \mathbf{v} \) expressed in \( \mathcal{F}_A(\mathbf{v}^A) \) to the components of the same vector in \( \mathcal{F}_B(\mathbf{v}^B) \), where there is a certain rotating angle between those two frames, then

\[
\mathbf{v}^B = T_{BA} \mathbf{v}^A
\]

(2.3)
The following definition, geometric relationships, and matrices will be used in the derivation of the equations of motion.

(1) The transformation matrix relating $E_F$ and $S_F$:

$$
I_{SE} = \begin{bmatrix}
\cos\theta\cos\phi & \cos\theta\sin\phi & -\sin\theta \\
\sin\phi\sin\theta\cos\psi & \sin\phi\sin\theta\sin\psi & \sin\phi\cos\theta \\
\cos\phi\sin\theta\cos\psi & \cos\phi\sin\theta\sin\psi & \cos\phi\cos\theta
\end{bmatrix} = [t_{ij}] \quad (2.4)
$$

where $\phi$, $\theta$, and $\phi$ are the Euler angles defined by Etkin [43].

(2) The transformation matrix relating $E_E$ and $B_F$:

$$
I_{BE} = \begin{bmatrix}
t_{BE}
\end{bmatrix} \quad (2.5)
$$

where the elements $t_{BE}$ are identical to the elements $t_{ij}$ with $\phi$, $\theta$, $\phi$ replaced by $\phi_B$, $\theta_B$, and $\phi_B$. $\phi_B$, $\theta_B$, and $\phi_B$ are defined analogously to $\phi$, $\theta$, and $\phi$.

(3) The transformation matrix relating $S_S$ and $B_F$:

$$
I_{BS} = \begin{bmatrix}
\cos\alpha_{fe} & 0 & -\sin\alpha_{fe} \\
0 & 1 & 0 \\
\sin\alpha_{fe} & 0 & \cos\alpha_{fe}
\end{bmatrix} \quad (2.6)
$$
where $\alpha_{fe}$ is the angle of attack (measured from FRL) in the equilibrium condition.

(4) The transformation relating $F_W$ and $F_B$:

$$I_{BW} = \begin{bmatrix} \cos \alpha_f & 0 & -\sin \alpha_f \\ 0 & 1 & 0 \\ \sin \alpha_f & 0 & \cos \alpha_f \end{bmatrix} \quad (2.7)$$

where $\alpha_f$ is defined similar to $\alpha_{fe}$ but relating to the projection of the instantaneous airspeed vector onto the symmetry plane, as shown in Fig. 2.

(5) The transformation matrix relating $F_W$ and $F_S$:

$$I_{SW} = \begin{bmatrix} \cos \beta \cos \alpha_x & -\sin \beta \cos \alpha_x & -\sin \alpha_x \\ \sin \beta & \cos \beta & 0 \\ \cos \beta \sin \alpha_x & -\sin \beta \sin \alpha_x & \cos \alpha_x \end{bmatrix} = [t_{SWij}] \quad (2.8)$$

where $\beta$ and $\alpha_x$ are the aerodynamic angles which will be explained later. Note that the rotation order is $(-\beta, \alpha_x, 0)$.

(6) The transformation matrix relating $F_E$ and $F_G$:

$$I_{GE} = \begin{bmatrix} \cos \gamma_G & 0 & \sin \gamma_G \\ 0 & 1 & 0 \\ \sin \gamma_G & 0 & \cos \gamma_G \end{bmatrix} = [t_{GEij}] \quad (2.9)$$
where \( \gamma_G \) is the glide slope.

2.3.2 Velocities, Angles and Angular Velocities

For the needs of future applications, we now described the linear and angular velocities using the defined reference frames and then find the relationship between aerodynamic angles and velocity components.

1. The angular velocity of the aircraft with respect to \( \mathbf{E} \) written as components in \( \mathbf{S} \) and \( \mathbf{B} \):

\[
F_{\omega}^T = (p, q, r)^T \tag{2.10}
\]

\[
F_{\omega}^B = (p_B, q_B, r_B)^T \tag{2.11}
\]

2. The skew-symmetric matrix for the angular vector product in \( \mathbf{S} \) and \( \mathbf{B} \) are:

\[
F_{\omega}^S = \begin{bmatrix}
0 & -r & q \\
 r & 0 & -p \\
 q & p & 0
\end{bmatrix} \tag{2.12}
\]

\[
F_{\omega}^B = \begin{bmatrix}
0 & -r_B & q_B \\
 r_B & 0 & -p_B \\
 q_B & p_B & 0
\end{bmatrix} \tag{2.13}
\]

3. The airspeed of the aircraft written as components in \( \mathbf{S} \), \( \mathbf{B} \) and \( \mathbf{W} \):

\[
F_{\omega}^S = (u, v, w)^T \tag{2.14}
\]
\[
\begin{align*}
\mathbf{F}_B^T &= (u_B, v_B, w_B)^T \\
\mathbf{F}_W^T &= (V, 0, 0)^T
\end{align*}
\] (2.15) (2.16)

(4) A vector's derivative expressed in \( F_B \):

As shown in [43], the components of \( \dot{\mathbf{x}} \) in \( F_B \) can be written as

\[
\mathbf{F}_B^T \dot{\mathbf{x}} = \mathbf{T}_{BE} \mathbf{F}_E^T \dot{\mathbf{x}}
\] (2.17)

or

\[
\mathbf{F}_B^T \dot{\mathbf{x}} = \mathbf{F}_B^T \dot{\mathbf{x}} + \mathbf{F}_B^V \mathbf{F}_B^T \dot{\mathbf{x}}
\] (2.18)

where \( \mathbf{F}_B^T \dot{\mathbf{x}} \) means the time derivative of vector \( \mathbf{x} \) wrt the body axes frame expressed in \( F_B \), or called the relative derivative.

(5) The ground speed of the aircraft with respect to \( F_E \) written as components in \( F_S \), \( F_B \) and \( F_E \):

\[
\mathbf{F}_S^T v_E = (u^E, v^E, w^E)^T
\] (2.19)

\[
\mathbf{F}_B^T v_E = (u_B^E, v_B^E, w_B^E)^T
\] (2.20)

\[
\mathbf{F}_E^T v_E = (x_E^E, y_E^E, z_E^E)^T
\] (2.21)
The aerodynamic angles (as shown in Fig. 3):

\[ \alpha = \alpha_f + \alpha_o \]  
(2.22)

\[ \alpha_x = \alpha_f - \alpha_{fe} \]  
(2.23)

\[ \alpha_x = \arctan \left( \frac{w}{u} \right) \]  
(2.24)

\[ \alpha_f = \arctan \left( \frac{w_B}{u_B} \right) \]  
(2.25)

\[ \beta = \arctan \frac{v}{v_{xz}} = \arcsin \frac{v}{V} \]  
(2.26)

or

\[ \beta = \arctan \frac{v_B}{v_{xz}} = \arcsin \frac{v_B}{V} \]  
(2.27)

where

\[ v_{xz} = \sqrt{u^2 + w^2} = \sqrt{u_B^2 + w_B^2} \]  
(2.28)

\[ V = \sqrt{u^2 + v^2 + w^2} = \sqrt{u_B^2 + v_B^2 + w_B^2} \]  
(2.29)

Here, \( \alpha \) is the angle of attack with respect to the zero life line (zzl); \( \alpha_o \) is the angle between the fuselage reference line and the zero lift line; \( \alpha_f \) is the angle of attack wrt the \( x_B \) axis; \( \alpha_{fe} \) is the equilibrium \( \alpha_f \); \( \alpha_x \) is the angle of attack wrt the \( x_s \) axis, \( \beta \) is the sideslip angle; \( V \) is the total airspeed magnitude; and \( v_{xz} \) is the magnitude of the component in the
symmetry plane.

(7) The geometric relationships (see Fig. 3):

\[ \theta = \theta_B - \alpha_{fe} \] \hspace{1cm} (2.30)

\[ u = V \cos \beta \cos \alpha_x \] \hspace{1cm} (2.31)

\[ v = V \sin \beta \] \hspace{1cm} (2.32)

\[ w = V \cos \beta \sin \alpha_x \] \hspace{1cm} (2.33)

\[ u_B = V \cos \beta \cos \alpha_f \] \hspace{1cm} (2.34)

\[ v_B = V \] \hspace{1cm} (2.35)

\[ w_B = V \cos \beta \sin \alpha_f \] \hspace{1cm} (2.36)

(8) The wind speed wrt \( F_E \) written as components in \( F_E \):

\[ F_{EW}^T = (w_1, w_2, w_3)^T \] \hspace{1cm} (2.37)

(9) The gravity acceleration written as components in \( F_E \):

\[ F_{EG}^T = (0, 0, g)^T \] \hspace{1cm} (2.38)
(10) The aerodynamic forces and moments written as components in \( \mathbf{F}_S \) and \( \mathbf{F}_B \):

\[
\mathbf{F}_{SA}^T = (X, Y, Z)^T
\]  
\[ \text{(2.39)} \]

\[
\mathbf{F}_{BA}^T = (X_B, Y_B, Z_B)^T
\]  
\[ \text{(2.40)} \]

From the definition of \( \mathbf{F}_S \) and \( \mathbf{F}_B \), it obviously follows that \( Y = Y_B \).

Similarly,

\[
\mathbf{F}_{SM}^T = (L, M, N)^T
\]  
\[ \text{(2.41)} \]

\[
\mathbf{F}_{BM}^T = (L_B, M_B, N_B)^T
\]  
\[ \text{(2.42)} \]

Also

\[
M = M_B
\]

(11) The inertial matrix of the aircraft wrt its mass centre and expressed in \( \mathbf{F}_B \) and \( \mathbf{F}_S \):

\[
\mathbf{F}_{IB}^T = \begin{bmatrix}
I_{xx} & -I_{xy} & -I_{xz} \\
-I_{yx} & I_{yy} & -I_{yz} \\
-I_{zx} & -I_{zy} & I_{zz}
\end{bmatrix}
\]  
\[ \text{(2.43)} \]

From the assumption of (5) and the definition of \( \mathbf{F}_B \), it is evident that \( I_{xy} = I_{yx} = I_{yz} = 0 \). Also, the assumption of (5) implies that \( I_{zx} = \ldots \)
In the frame $\mathcal{F}_S$ we have

$$F_S^T \mathcal{I} = \begin{bmatrix} I_{xx} & 0 & -I_{xz} \\ 0 & I_{yy} & 0 \\ -I_{xz} & 0 & I_{zz} \end{bmatrix} \quad (2.44)$$

Here, the symmetric properties have been taken into account. As shown in [43], the inertia matrices $\mathcal{I}_B$ and $\mathcal{I}_S$ are related through a rotational transformation, i.e.,

$$F_S^T \mathcal{I}_B = \mathcal{I}_S \mathcal{B}_I^T \mathcal{I}_B \quad (2.45)$$

(Note that $\mathcal{I}_S = \mathcal{I}_B^T$.)

In scalar form, (2.43) can be expanded as

\begin{align*}
I_{xx} &= I_{xx_B} \cos^2 \alpha_{fe} - 2I_{xz_B} \cos \alpha_{fe} \sin \alpha_{fe} + I_{zz_B} \sin^2 \alpha_{fe} \\
I_{yy} &= I_{yy_B} \\
I_{zz} &= I_{xx_B} \sin^2 \alpha_{fe} + 2I_{xz_B} \cos \alpha_{fe} \sin \alpha_{fe} + I_{zz_B} \cos^2 \alpha_{fe} \\
I_{xz} &= (I_{xx_B} - I_{zz_B}) \cos \alpha_{fe} \sin \alpha_{fe} + I_{xz_B} \cos^2 \alpha_{fe} - I_{xz_B} \sin^2 \alpha_{fe} \quad (2.49)
\end{align*}
2.4 Development of the General Equations of Motion

Using the Newton-Euler Angle Method, we start with the fundamental equations [43]:

\[ \dot{p} = F \]  
\[ \dot{h} = M \]  

(2.50)  

(2.51)

\( \dot{p} \) is the momentum vector of the body, \( \dot{h} \) is the angular momentum of the body about its mass centre, \( \dot{F} \) is the external force vector acting on the mass centre of the body and \( \dot{M} \) is the external moment vector about the mass centre. \( \dot{p} \) may be written

\[ \dot{p} = m \dot{v}^E \]  

(2.52)

where \( \dot{v}^E \) is the velocity vector of the aircraft wrt \( E \) and \( m \) is the mass of the aircraft. An expression for \( \dot{h} \) follows from the relationship

\[ \dot{h} = \int_{(M)} [\dot{r} \times \dot{r}] \, dm \]  

(2.53)

or

\[ \dot{h} = \int_{(M)} [\dot{r} \times \dot{r} + \dot{r} \times (\omega_B \times r)] \, dm \]  

(2.54)

where \( \dot{r} \) is the position vector of an element mass \( dm \) of the body (see Fig. 4); \( \omega_B \) is the angular velocity of the body wrt \( E \); 'o' means a vector change rate wrt \( B \) and 'r' - wrt \( E \). (See [45], which contains a full discussion of vector differentiation.) Equation (2.54) may be written in
matrix notation as

\[ \mathbf{F}_B^T \mathbf{h} = \int (\mathbf{M}) \left[ \mathbf{F}_B^x \mathbf{r} \mathbf{F}_B^T \mathbf{r} - \mathbf{F}_B^x \mathbf{r} \mathbf{F}_B^T \mathbf{\omega} \right] \mathrm{dm} \quad (2.55) \]

for a rigid body

\[ \mathbf{F}_B^T \mathbf{r} = 0 \quad (2.56) \]

Since \( \mathbf{\omega}_B \) is a constant wrt the integration in (2.55), it follows that

\[ \mathbf{F}_B^T \mathbf{h} = \mathbf{F}_B^T \mathbf{I} \mathbf{F}_B^T \mathbf{\omega} \quad (2.57) \]

where

\[ \mathbf{F}_B^T \mathbf{I} = - \int (\mathbf{M}) \mathbf{F}_B^x \mathbf{r} \mathbf{F}_B^x \mathbf{r} \mathrm{dm} \quad (2.58) \]

The externally applied force \( \mathbf{F} \) is made up of an aerodynamic component \( \mathbf{A} \) and a gravitational component \( \mathbf{mg} \) such that

\[ \mathbf{F} = \mathbf{A} + \mathbf{mg} \quad (2.59) \]

The externally applied moment \( \mathbf{M} \) arises entirely from aerodynamic effects.

The position of the aircraft in \( \mathbf{\xi}_E \) can be found by integrating the components of the aircraft ground speed vector \( \mathbf{v}_E \) in \( \mathbf{\xi}_E \) where
\[ \dot{y}^E = \dot{y} + \dot{w} \]  \hspace{1cm} (2.60)

where \( \dot{y} \) represents the aircraft airspeed vector and \( \dot{w} \) is the wind speed vector (commonly measured in \( \xi_E \)).

Substituting (2.59) and (2.60) into (2.50), it becomes

\[ m(\dot{y} + \dot{w}) = \ddot{A} + mg \]  \hspace{1cm} (2.61)

and

\[ \dot{h} = \dot{M} \]  \hspace{1cm} (2.62)

For the purpose of consistency with the previous work [35, 36 and 46], in the later development the body-fixed reference frame \( \xi_B \) will be used.

By employing (2.18) to represent the components of \( \dot{y} \) in \( \xi_B \) and (2.17) to represent \( \dot{w} \). Eq. (2.61) can then be written in \( \xi_B \) components as

\[ m(F_B \dot{V} + F_B \dot{X} \omega F_B \dot{V} + T_{BE} F_E \dot{W}) = F_B \ddot{A} + mT_{BE} F_E \dot{Q} \]  \hspace{1cm} (2.63)

and (2.62) can be written as

\[ F_B \dot{h} + F_B \dot{X} \omega F_B \dot{h} = F_B \dot{M} \]  \hspace{1cm} (2.64)
Substituting (2.57) into the above expression obtains

\[ \mathbf{F}_B \mathbf{I} + \mathbf{F}_B^T \omega + \mathbf{F}_B^x \omega \mathbf{F}_B \mathbf{I} + \mathbf{F}_B^T \omega = \mathbf{F}_B^T \mathbf{M} \]  

(2.65)

Writing out (2.63) and (2.65) in scale form yields:

For the force equations:

\[ m(u_B + q_B w_B - r_B v_B + t_{BE11} W_1 + t_{BE12} W_2 + t_{BE13} W_3) = X_B + mg t_{BE13} \]  

(2.66)

\[ m(v_B + r_B u_B - p_B w_B + t_{BE21} W_1 + t_{BE22} W_2 + t_{BE23} W_3) = Y_B + mg t_{BE23} \]  

(2.67)

\[ m(w_B + p_B v_B - q_B u_B + t_{BE31} W_1 + t_{BE32} W_2 + t_{BE33} W_3) = Z_B + mg t_{BE33} \]  

(2.68)

For the moment equations:

\[ I_{xx} p_B - I_{xz} (r_B + p_B q_B) + (I_{zz} - I_{yy}) q_B r_B = L_B \]  

(2.69)

\[ I_{yy} q_B - I_{xz} (r_B - p_B) + (I_{xx} - I_{zz}) r_B p_B = M_B \]  

(2.70)

\[ I_{zz} r_B - I_{xz} (p_B + r_B q_B) + (I_{yy} - I_{xx}) p_B q_B = N_B \]  

(2.71)
Kinematic equations are also required for the linear and rotational position of the aircraft. The linear position equations follow from (2.60) in matrix form.

\[
\begin{align*}
F_B^T V^E &= F_B^T V + F_B^T W \
\end{align*}
\]

or

\[
\begin{align*}
F_E^T V^E &= T_{BE}^T F_B^T V + F_E^T W
\end{align*}
\]

An additional set of kinematical relationships exists which is required in order to solve the above equations. This set is derived in [43] and gives the Euler angle rates as

\[
\begin{align*}
\dot{\phi}_B &= p_B + q_B \sin \phi_B \tan \theta_B + r_B \cos \phi_B \tan \theta_B \\
\dot{\theta}_B &= q_B \cos \phi_B - r_B \sin \phi_B \\
\dot{\psi}_B &= q_B \sin \phi_B \sec \theta_B + r_B \cos \phi_B \sec \theta_B
\end{align*}
\]

The linear position equations are:

\[
\begin{align*}
X_E &= t_{BE11} u_B + t_{BE21} v_B + t_{BE31} w_B + W_1 \\
Y_E &= t_{BE12} u_B + t_{BE22} v_B + t_{BE32} w_B + W_2
\end{align*}
\]
The important variable of $d$ - the normal glidepath deviation is derived from Fig. 1. It follows that

$$
\begin{bmatrix}
\ddot{x}_G \\
\ddot{y}_G \\
\ddot{z}_G
\end{bmatrix} = I_{GE} \begin{bmatrix}
\ddot{x}_E \\
\ddot{y}_E \\
\ddot{z}_E
\end{bmatrix}
$$

(2.80)

where $I_{GE}$ is given by (2.9). Noting that

$$
d = -z_G
$$

(2.81)

and substituting (2.77) to (2.79) into (2.80) yields the differential equation for $d$ i.e.,

$$
\dot{d} = \sin\gamma_G \dot{x}_E + \cos\gamma_G \dot{h}
$$

(2.82)

or

$$
\dot{d} = (t_{BE_{11}} \sin\gamma_G - t_{BE_{13}} \cos\gamma_G)u_B + (t_{BE_{21}} \sin\gamma_G - t_{BE_{23}} \cos\gamma_G)v_B + (t_{BE_{31}} \sin\gamma_G - t_{BE_{33}} \cos\gamma_G)w_B + \sin\gamma_G w_1 - \cos\gamma_G w_3
$$

(2.83)
Here \( h \) is the height of the aircraft above the ground and is given by

\[
h = -z_E
\]  

(2.84)

### 2.5 Linearization of the Equations

#### 2.5.1 General Theory of Small Perturbation Technique

Linearization of the general equations of motion is common practice. The linearization of the differential equations (2.66) to (2.71) and (2.74) to (2.79), (2.83) is more conveniently considered if these equations are written in a general vector form:

\[
\dot{x}(t) = \bar{F}(\bar{x}, \bar{\delta}, \bar{w}, \bar{\dot{w}}, \bar{g}, t)
\]  

(2.85)

where the dependence of \( \bar{F} \) on control inputs \( \bar{\delta} \) (incorporated within the aerodynamic forces and moments), wind (turbulence and mean wind shear) inputs \( \bar{w} \) and wind gradient inputs \( \bar{\dot{w}} \) as well as the ground effect \( \bar{g} \) is explicitly noted. In writing (2.85) in this form it has been tacitly assumed that the state derivatives \( \dot{x} \) are separable from \( \bar{F} \). This is not particularly restrictive for aircraft dynamic models.

A trajectory of the system (2.85) is any vector function of time \( \bar{x}_e(t) \) which satisfies (2.85) for given \( \bar{\delta}_e(t), \bar{w}_e(t), \bar{\dot{w}}_e(t) \) and \( \bar{g}_e(t) \), i.e.,

\[
\dot{\bar{x}}_e(t) = \bar{F}(\bar{x}_e, \bar{\delta}_e, \bar{w}_e, \bar{\dot{w}}_e, \bar{g}_e, t)
\]  

(2.86)
Mathematically speaking, any of these trajectories are candidates for a reference trajectory about which linearization can take place. Thus, (2.85) may be written

\[
\dot{x}_e + \Delta \ddot{x} = F(x_e + \Delta x_1, \delta_e + \Delta \delta, \dot{w}_e + \Delta \dot{w}, \ddot{w}_e + \Delta \ddot{w}, s_e + \Delta s, t) \tag{2.87}
\]

where the 'Δ' symbol refers to a perturbation quantity and the subscript 'e' refers to a reference trajectory. In this study we only consider the cases where the perturbation solutions are small. The constraint of 'smallness' is imposed to yield a linear system for analysis, and the usual confidence in linear flight-dynamic solutions leads to the expectation that they will be of practical utility for realistic levels of disturbance. The perturbation equations of (2.87) can then be written as, to first order:

\[
\dot{x}_e + \Delta \ddot{x} = F(x_e, \delta_e, \dot{w}_e, \ddot{w}_e, s_e, t) + A_1(t) \Delta x + A_2(t) \Delta \delta \\
+ A_3(t) \Delta \dot{w} + A_4(t) \Delta \ddot{w} + A_5(t) \Delta s \tag{2.88}
\]

where

\[
A_1(t) = \begin{bmatrix}
\frac{\partial F_1}{\partial x_1} & \cdots & \frac{\partial F_1}{\partial x_n} \\
\cdots & \cdots & \cdots \\
\frac{\partial F_m}{\partial x_1} & \cdots & \frac{\partial F_m}{\partial x_n}
\end{bmatrix}
\]
where $F_i$ and $x_i$, etc., are the $i^{th}$ elements of $F$ and $X$ respectively. In terms of (2.88), after eliminating the reference solution, the perturbation equation can be written as

$$
\dot{\Delta x}(t) = A_1(t)\Delta X(t) + A_2(t)\Delta \delta(t) + A_3(t)\Delta \chi(t) + A_4(t)\Delta \dot{\delta}(t) + A_5(t)\ddot{\delta}(t) 
$$

(2.90)

2.5.2 Selection of Reference Trajectory

We have defined the linearized flight dynamics model which is, in general, a set of time-varying linear differential equations. As stated in the previous section, any special solution that satisfied the original nonlinear equations could be the reference trajectory. In the approach situation the simplest way to select as reference an equilibrium flight
condition corresponding to constant airspeed \((V_e)\) landing approach along a rectilinear glide slope of angle \(\gamma_E\) in the presence of a constant headwind \((W_e)\). Perturbations in the aircraft's state vector away from the reference equilibrium then arise from initial conditions on the equations, wind shear effects, turbulence, control inputs and other external perturbations. State variables are then written as \(X = X_e + \Delta X\) where \(X_e\) is the reference equilibrium value and \(\Delta X\) represents a small perturbation away from \(X_e\).

In the flare-out phase, the situation is more complicated. The complexity comes from: (1) the flare maneuver results in a curved flight path; (2) the ground effect becomes relatively significant as the aircraft approaches the ground; and (3) the variation of the properties of the mean wind shear and turbulence encountered (as a function of height or time) is not negligible. An exponential nominal flare path is suitable to be the reference trajectory. This implies that the system will be treated as a time-varying one. In order to apply the linear system theory, which commonly deals with the constant linear system, the changing properties in flying through the lower atmosphere level can be partially accounted for by dividing the planetary boundary layer into layers and assuming fixed but different homogeneous and isotropic properties for each layer. In other words, the system matrices in (2.90), i.e., \(A_i(t)\), are assumed piecewise constant for a small enough time interval \(\Delta t\), in which the reference trajectory chosen for the approach case will be applied by adjusting the equilibrium conditions \(V_e\) and \(\gamma_E\) for each layer.

For the assumed reference equilibrium the linearized variables are specified by the following (where \(V_e\) is the reference airspeed):

\[
(u, v, w) = (V_e + \Delta u, \Delta v, \Delta w)
\]  

(2.91)


\[(p, q, r) = (\Delta p, \Delta q, \Delta r)\]  
\[(W_1, W_2, W_3) = (W_{1e} + \Delta W_1, W_{2e} + \Delta W_2, \Delta W_3)\]  
\[(x_E, y_E, z_E) = (x_{Ee} + \Delta x_E, \Delta y_E, z_{Ee} + \Delta z_E)\]  
\[(X, Y, Z) = (X_e + \Delta X, \Delta Y, Z_e + \Delta Z)\]  
\[(L, M, N) = (\Delta L, \Delta M, \Delta N)\]  
\[(\psi, \theta, \phi) = (\psi_e + \Delta \psi, \theta_e + \Delta \theta, \Delta \phi)\]  
\[
\vec{\delta}^T = (\delta_E, \delta_T, \delta_A, \delta_R) = (\delta_{Ee} + \Delta \delta_E, \Delta \delta_T + \Delta \delta_{Te}, \Delta \delta_A, \Delta \delta_R) \]  
\[
\vec{Q}^T = (C_{xGE}, C_{zGE}, C_{mGE}) = (C_{xGEe} + \Delta C_{xGE}, C_{zGEe} + \Delta C_{zGE}, C_{mGEe} + \Delta C_{mGE}) \]  

2.5.3 Linearization

As a consequence of the above section, the kinematics of the reference equilibrium state are characterized by

\[p_e = 0\]  
\[q_e = 0\]
\[ r_e = 0 \quad (2.102) \]
\[ u_e = V_e \quad (2.103) \]
\[ v_e = 0 \quad (2.104) \]
\[ w_e = 0 \quad (2.105) \]

\( \psi_e \) is chosen such that the lateral drift due to \( W_{2e} \) is zero (i.e., \( \dot{Y}_{Ee} = 0 \)).

From the reference equilibrium geometry, as seen in Fig. 5, in general, it may be shown that

\[ V_e^E = [(V_e \cos \gamma_e \cos \psi_e + W_{1e})^2 + (V_e \cos \gamma_e \sin \psi_e + W_{2e})^2 \]
\[ + (V_e \sin \gamma_e)^2]^{1/2} \quad (2.106) \]

\[ \theta_e = \gamma_e = -\arcsin \left( \frac{V_{eE} \sin \gamma_e}{V_e} \right) \quad (2.107) \]

\[ \phi_e = 0 \quad (2.108) \]

\[ \phi_e = \arcsin \left[ \frac{-W_{2e}}{(\cos \gamma_e \cdot V_e)} \right] \quad (2.109) \]

\[ x_{Ee} = (V_e \cos \gamma_e \cos \phi_e + W_{1e})t + x_{E0} \quad (2.110) \]
\[ z_{ee} = -V_e \sin \gamma_e \cdot t \]  
(2.111)

\[ y_{ee} = 0 \]  
(2.112)

\[ x_{ee} = V_e \cos \gamma_e \cos \phi_e + W_1 e \]  
(2.113)

\[ y_{ee} = 0 \]  
(2.114)

\[ z_{ee} = -V_e \sin \gamma_e \]  
(2.115)

\[ \dot{\phi}_e = 0 \]  
(2.116)

\[ \dot{\theta}_e = 0 \]  
(2.117)

\[ \dot{\psi}_e = 0 \]  
(2.118)

The external forces and moments in the reference equilibrium are:

\[ X_e = mg \sin \theta_e \]  
(2.119)

\[ Y_e = 0 \]  
(2.120)

\[ Z_e = mg \cos \theta_e \]  
(2.121)

\[ L_e = 0 \]  
(2.122)
Equations (2.119), (2.121) and (2.123) can be expanded and put into non-dimensional form as follows:

\[ C_T e \cos(\alpha_e + \varepsilon_T) - C_D e - C_w e \sin \theta_e = 0 \]  
\[ C_T e \sin(\alpha_e + \varepsilon_T) + C_L e - C_w e \cos \theta_e = 0 \]  
\[ \sum C_m = 0 \]

Here the subscript 'e' denotes a value at the reference equilibrium state; \( \alpha_f \) is the angle that the \( x_S \) makes with respect to the FRL; \( \varepsilon_T \) is the angle that the thrust line makes with respect to the FRL; \( C_T \) is the total thrust coefficient; \( C_D \) is the drag coefficient; \( C_w \) is the weight coefficient; and \( \sum C_m \) is the sum of the moment coefficients of the aircraft with respect to \( E \) written as components in \( x_S \) and \( (x_E, y_E, z_E) \) are the inertial position coordinates, with the altitude \( h \), where

\[ h = -z_E \]

Substituting (2.91) to (2.99) into (2.4) and (2.66) to (2.71) and (2.74) to (2.79), assuming the small angle approximation is made and only the first order terms in the perturbation quantities are retained, then the following set of equations results:
Then the linearized perturbation equations become:

\[ m(\ddot{u} + \dot{\Omega}_1 \cos \theta_e - \dot{\Omega}_3 \sin \theta_e) = \Delta X - mg \Delta \theta \cos \theta_e \]  
(2.130)

\[ m(\ddot{v} + \dot{\Omega}_2 V_e + \dot{\Omega}_2 W_2) = \Delta Y + mg \dot{\phi} \cos \theta_e \]  
(2.131)

\[ m(\ddot{w} - \dot{\Omega}_1 V_e + \dot{\Omega}_1 \dot{\Omega}_3 \sin \theta_e + \dot{\Omega}_3 \cos \theta_e) = \Delta Z - mg \dot{\theta} \sin \theta_e \]  
(2.132)

\[ I_{xx} \ddot{\phi} - I_{xz} \ddot{r} = \Delta L \]  
(2.133)

\[ I_{yy} \ddot{q} = \Delta M \]  
(2.134)

\[ I_{zz} \ddot{r} - I_{xz} \ddot{\phi} = \Delta N \]  
(2.135)

\[ \Delta \ddot{X}_E = -V_e \dot{\theta} \sin \theta_e + \dot{u} \cos \theta_e + \ddot{w} \sin \theta_e + \dot{\Omega}_1 \]  
(2.136)

\[ \Delta \ddot{Y}_E = V_e \dot{\phi} \cos \theta_e + \dot{v} + \dot{\Omega}_2 \]  
(2.137)

\[ \Delta \ddot{Z}_E = -V_e \dot{\theta} \cos \theta_e - \dot{u} \sin \theta_e + \ddot{w} \cos \theta_e + \dot{\Omega}_3 \]  
(2.138)

\[ \Delta \dot{d} = \sin \gamma_e \Delta \dot{x}_E + \cos \theta_e \Delta \dot{h} \]  
(2.139)
\[ \Delta \phi = \Delta \rho + \Delta r \tan \theta_e \]  
(2.140)

\[ \Delta \dot{\theta} = \Delta q \]  
(2.141)

\[ \Delta \dot{\phi} = \Delta r \sec \theta_e \]  
(2.142)

2.5.4 Forces and Moments Acting on the Aircraft

(1) Quasi-Steady Linear-Field Model (QSLFM) Consideration

The aerodynamic derivative technique has been traditionally employed to model the aerodynamic forces and moments in rigid body dynamic simulation. It was first proposed by G. H. Bryan in the early 1900's and essentially consists of a Taylor series expansion of the aerodynamic forces and moments about an aerodynamic reference equilibrium state. In this expansion Bryan proposed that only linear terms be kept. Furthermore, the expansions are frequently assumed to include only terms in the Taylor series which involve the translational and/or rotational velocities except \( \dot{w}(\dot{\alpha}_f) \) and \( \dot{v}(\dot{\beta}) \) derivatives as these can be shown to arise from strictly quasisteady considerations due to the downwash and sidewash and this is to account for the transport lag in the wash flow at the tail and this is one of the most important unsteady aerodynamic effects. The quasisteady aerodynamics assumption is adequate for most problems dominated by rigid body modes - such as may occur in the study of guidance, control and handling qualities - but may not be suitable for problems involving significant participation of structural degrees of freedom [4].

In the landing approach-flare phase, according to the basic assumptions stated in (2.1), the air density, viscosity, Mach number and temperature
effects will not be considered. Therefore we may express the aerodynamic forces and moments function in a general formula:

\[ A = f(x, \dot{x}, \delta, W, \dot{W}, \omega, \varrho, t) \]  

\[ M = f(x, \dot{x}, \delta, W, \dot{W}, \omega_g, \varrho, t) \]  

where

\[ x^T = (u, v, w, p, q, \gamma, x_E, y_E, z_E, \psi, \theta, \phi) \]

\[ \delta^T = (\delta_E, \delta_A, \delta_R, \delta_T) \]

Here, the thrust being included is implied.

\[ \omega^T = (p_g, q_g, \gamma_g) \]  

\[ \varrho^T = (C_{x_E}, C_{z_E}, C_{m_E}) \]

In the past, little literature included terms due to \( \dot{W} \) and \( \omega_g \) as well as \( \varrho \) into the aerodynamic forces and moments. Only recently Etkin [4] has made a fully complete description of these forces and moments due to turbulent flow using the Uniform-Gust Approximation or the Linear-Field Approximation. In the present study the ground effect is included during the flare-out phase. Under the quasisteady assumption the augment \( t \) in (2.143) and (2.144) can be
omitted. With the Bryan expansion method, now we write the aerodynamic forces and moments in general form:

\[ \mathbf{A} = \mathbf{A}_e + \Delta \mathbf{A} \quad (2.145) \]

\[ \mathbf{M} = \mathbf{M}_e + \Delta \mathbf{M} \quad (2.146) \]

where, according to (2.143) and (2.144),

\[ \Delta \mathbf{A} = \left( \frac{\partial \mathbf{A}}{\partial \mathbf{x}} \right)_e \mathbf{x} + \left( \frac{\partial \mathbf{A}}{\partial \mathbf{\dot{x}}} \right)_e \mathbf{\dot{x}} + \left( \frac{\partial \mathbf{A}}{\partial \mathbf{\ddot{x}}} \right)_e \mathbf{\ddot{x}} + \left( \frac{\partial \mathbf{A}}{\partial \mathbf{\omega}} \right)_e \mathbf{\omega} + \left( \frac{\partial \mathbf{A}}{\partial \mathbf{q}} \right)_e \mathbf{q} \quad (2.147) \]

\[ \Delta \mathbf{M} = \left( \frac{\partial \mathbf{M}}{\partial \mathbf{x}} \right)_e \mathbf{x} + \left( \frac{\partial \mathbf{M}}{\partial \mathbf{\dot{x}}} \right)_e \mathbf{\dot{x}} + \left( \frac{\partial \mathbf{M}}{\partial \mathbf{\ddot{x}}} \right)_e \mathbf{\ddot{x}} + \left( \frac{\partial \mathbf{M}}{\partial \mathbf{\omega}} \right)_e \mathbf{\omega} + \left( \frac{\partial \mathbf{M}}{\partial \mathbf{q}} \right)_e \mathbf{q} \quad (2.148) \]

Here \( \frac{\partial \mathbf{A}}{\partial \mathbf{x}}, \frac{\partial \mathbf{A}}{\partial \mathbf{\dot{x}}}, \ldots \), and \( \frac{\partial \mathbf{M}}{\partial \mathbf{x}}, \frac{\partial \mathbf{M}}{\partial \mathbf{\dot{x}}}, \ldots \), are the aerodynamic influence coefficient matrices. For the quasisteady linear-field model only \( \mathbf{\dot{w}} \) and \( \mathbf{\dot{\gamma}} \) will be considered instead of the whole vector \( \mathbf{x} \). Assuming that the ground effect is the function of the state variables, i.e., \( \mathbf{q} = f(\mathbf{x}) \), actually \( \mathbf{q} = f(z_E) \), then
Once \( q = f(x) \) is available, \( (\frac{\partial A}{\partial x})^* \) and \( (\frac{\partial M}{\partial x})^* \) can then be found and lumped into the term \( (\frac{\partial A}{\partial x})_e^* \Delta x \), i.e., the \( z_E \) derivative as pointed out in [43]. Thus

\[
(\frac{\partial A}{\partial x})_e^* \Delta x + (\frac{\partial A}{\partial x})^*_e \Delta x = [(\frac{\partial A}{\partial x})_e + (\frac{\partial A}{\partial x})^*_e] \Delta x
\]

This implies that the ground effect can be taken into account by modifying the original aerodynamic influence coefficient matrices.

We also notice that by the physical meaning, the effects of the wind speeds (including mean wind and turbulence) and its gradients on the aerodynamic forces and moments are identical to the vehicle's speeds and angular velocities. There are two approaches to treat the effects due to winds as shown in [4, 43 and 27]. These treatments are actually identical. The difference arises from the reference frame chosen in the derivation of the equations of motion. In [27] the state variable vector includes the air speeds which are expressed in a body-fixed frame; the wind effects are explicitly included except those due to gust gradients. Etkin's derivations in [4 and 43] are slightly different where the state vector includes the ground speed. The effects of the winds and their gradients are considered by adding extra terms which represent forces and moments due to winds. By the Linear-Field Approximation the effective angular velocity concept is
introduced, e.g., \( p_g \), \( q_g \), etc., where \( p_g = \frac{\partial w_g}{\partial y} \), \( q_g = -\left(\frac{\partial w_g}{\partial x}\right) \). These terms are lacking in [27]. From the physical meaning point of view, Etkin's approach is more preferable. However, to be consistent with the previous work (computer codes) a modified approach of [27] is used in the present investigation; i.e., the gust gradient term will be included in the state equations by modifying the aerodynamic influence coefficient matrix relating to the aerodynamic moments \( M \).

(2) Description of Aerodynamic Forces and Moments

(i) Basic Aerodynamic Forces and Moments

Based on the Bryan expansion assumption discussed in the foregoing paragraph we write the general form in detail:

\[
\Delta X = X_u \Delta u + X_w \Delta w + X_v \Delta v + X_p \Delta p + X_q \Delta q + X_r \Delta r + X_\delta \Delta \delta_r + X_\delta \Delta \delta_w
\]

\[
+ X_\delta \Delta \delta_e + X_\delta \Delta \delta_t + X_\delta \Delta \delta_a + X_\delta \Delta \delta_r
\]

(2.153)

Similarly we can write out \( \Delta Z \) and \( \Delta M \).

The small perturbation assumption allows us to divide the general equations of motion into two decoupled sets, the "longitudinal and lateral perturbation equations of motion". Correspondingly we write the aerodynamics into two sets:

Longitudinal Aerodynamics

\[
\Delta X = X_u \Delta u + X_w \Delta w + X_v \Delta v + X_q \Delta q + X_\delta \Delta \delta_r + X_\delta \Delta \delta_t
\]

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\[
\Delta Z = Z_u \Delta u + Z_w \Delta w + Z_q \Delta q + Z_\delta \Delta \delta_E + Z_\delta \Delta \delta_T \quad (2.154)
\]

\[
\Delta M = M_u \Delta u + M_w \Delta w + M_q \Delta q + M_\delta \Delta \delta_E + M_\delta \Delta \delta_T
\]

\[
\Delta Y = Y_v \Delta v + Y_\delta \Delta \delta + Y_p \Delta p + Y_r \Delta r + Y_\xi A \Delta \delta A + Y_\xi R \Delta \delta R
\]

\[
\Delta L = L_v \Delta v + L_\delta \Delta \delta + L_p \Delta p + L_r \Delta r + L_\xi A \Delta \delta A + L_\xi R \Delta \delta R \quad (2.155)
\]

\[
\Delta N = N_v \Delta v + N_\delta \Delta \delta + N_p \Delta p + N_r \Delta r + N_\xi A \Delta \delta A + N_\xi R \Delta \delta R
\]

References [43 and 47] have developed the expressions of the stability derivatives including in the above force and moment specifications. Here, for the purpose of the present study, the longitudinal derivatives are presented in Appendix I.

(ii) Aerodynamic Forces and Moments due to Turbulence and Gust Gradients

The atmospheric disturbance input vector can be expressed as

\[
W^T = (u_g \quad v_g \quad w_g \quad p_g \quad q_g \quad r_{1g} \quad r_{2g}) \quad (2.156)
\]

To account for the effect of the turbulence speed to the aerodynamic forces is a very straightforward matter. Figure 6 gives those forces and moments acting on an aircraft in turbulence. Care must be taken when applying to
the approach-flare case because the turbulence scales become comparable in size to the airplane. Thus the Uniform-Gust Approximation (or called point approximation), which uses the turbulence velocity at mass centre to represent an overall uniform speed distribution, seems unreasonable. The works of [21, 43 and 48], introducing more complex aircraft representations to account for non-uniform gust fields, are analogous in that they account for spatial gust distributions by approximating the aircraft as several points (4 or 5), and assuming a linear gust variation between those points. In this work Etkin's 4-point model is followed which uses 4 planar points to represent an aircraft flying in the turbulence field, as shown in Fig. 7. Those points are chosen so that:

(i) point '0': at the mass centre of the aircraft,

(ii) points '1' and '2': positions 0.85(b/2) along the span of the right and left wings respectively. This was suggested by [21] to best represent an elliptical lift distribution.

Gust properties are calculated (or measured) at these points and are assumed to vary linearly between them, i.e., the Linear-Field Approximation.

In this investigation all turbulence speeds are taken to be the weighted average at the four points. Thus the input speeds are then:

\[ U_g = \frac{4}{k} \sum_{i=1}^{4} k_i U_i \]

\[ V_g = \frac{4}{k} \sum_{i=1}^{4} k_i V_i \]

\[ W_g = \frac{4}{k} \sum_{i=1}^{4} k_i W_i \]
where \( k_i \) are the weighting factors depending on the importance of the specified point contributing to the related aerodynamics. In this research, \( k_0 = k_1 = k_2 = 1.0, k_3 = \frac{S_t}{S_w} = \eta = 0.3 \) are selected. Here \( \frac{S_t}{S_w} \) is the ratio of the tail area to that of the wing.

The equivalent angular velocities due to the gust gradients are developed as follows:

\[
\frac{\omega_g}{\omega_c} = \left( \frac{dW}{d\mathbf{r}_B} \right) = \begin{bmatrix}
\frac{\partial u_g}{\partial x_B} & \frac{\partial u_g}{\partial y_B} & \frac{\partial u_g}{\partial z_B} \\
\frac{\partial v_g}{\partial x_B} & \frac{\partial v_g}{\partial y_B} & \frac{\partial v_g}{\partial z_B} \\
\frac{\partial w_g}{\partial x_B} & \frac{\partial w_g}{\partial y_B} & \frac{\partial w_g}{\partial z_B}
\end{bmatrix}
\]

where

\[
\mathbf{r}_B = (x_B, y_B, z_B)^T
\]

In order to simplify the calculation, the relative importance of various gusts on the response is considered. Since the four point model assumed that all points lie in a plane, the vertical gradients are eliminated: \( \frac{\partial u_g}{\partial z_B}, \frac{\partial v_g}{\partial z_B} \) and \( \frac{\partial w_g}{\partial z_B} \) vanish. In addition, the gradient \( \frac{\partial u_g}{\partial x_B} \) is assumed to have negligible effect on the aircraft's longitudinal response. The resultant gust contributions considered for longitudinal case are then:

\[
W = \begin{bmatrix}
u_g \\
w_g \\
\frac{\partial w_g}{\partial x_B}
\end{bmatrix}
\]
For the purpose of completeness the meaning of the terms contained in (2.160) is denoted later. This will be performed as in [4, 21 and 43] noting that they are equivalent to aircraft rotations in the opposite sense. As such $\partial w_g / \partial y, -(\partial w_g / \partial x)$ are assumed equivalent to the pitch and roll rates. Pursuant to convention of [4], we denote:

\[ p_g = \frac{\partial w_g}{\partial y} = \frac{(W_1 - W_2)}{b'} \]  
(2.162)

\[ q_g = \frac{-\partial w_g}{\partial x} = \frac{(W_3 - W_0)}{\lambda_t} \]  
(2.163)

\[ r_1 = \frac{-\partial u_g}{\partial y} = \frac{(U_2 - U_1)}{b'} \]  
(2.164)

\[ r_2 = \frac{\partial v_g}{\partial x} = \frac{(V_0 - V_3)}{\lambda_F} \]  
(2.165)

$u_g, v_g$ and $w_g$ are shown in (2.157) to (2.159).\[ u = (p - p_g) \] etc...

(iii) Aerodynamic Forces and Moments due to Ground Effect

The influence of ground proximity during the landing flare was modelled by extra terms $\Delta x_{GE}, \Delta y_{GE}$ and $\Delta M_{GE}$ as shown in the previous work [46]. In that work the aerodynamic properties were taken as functions of wheel height above the runway level and the angle of attack $\alpha_f$. After examining the effect of angle of attack on the influence of ground effect during flare it was found that by using an average value $\alpha_{av}$ in the flare phase, the ground effect can be reasonably taken into account without significant error
Thus, the augment \( \alpha \) can be removed from the expressions for calculating the extra forces and moments due to ground effect. Changes due to ground effect relative to unperturbed nominal automatic flare were modelled as derivatives with respect to height \( \Delta h \), where \( \Delta h \) is the deviation from nominal flare:

\[
\Delta C_{X_{GE}} = C_{X_h}(t) \cdot \Delta h(t) \quad (2.166)
\]

\[
\Delta C_{Z_{GE}} = C_{Z_h}(t) \cdot \Delta h(t) \quad (2.167)
\]

\[
\Delta C_{M_{GE}} = C_{M_h}(t) \cdot \Delta h(t) \quad (2.168)
\]

The coefficients \( C_{X_h}, C_{Z_h} \) and \( C_{M_h} \) are derived in this study as shown in Appendix II which gives an outline of the linearization relative to the nominal, unperturbed landing flare. Notice that the derivatives \( C_{X_h}, C_{Z_h} \) and \( C_{M_h} \) are functions of time as they are only supposed to be present in ground proximity. In the modelling process they will be set to zero until flare initiation.

2.5.5 Actuator Dynamics

The dynamic response of the control surface deflection, engine thrust due to changes in required thrust and control amount is modelled as a first order dynamic system with the differential equation:

\[
\Delta \delta = I^{-1} \Delta \delta_c - I^{-1} \Delta \delta_c
\]

(2.169)
where

\[ I^{-1} = \begin{bmatrix} \tau_T^{-1} & 0 \\ 0 & \tau_E^{-1} \end{bmatrix} \]  

(2.170)

for the longitudinal motions. \( \tau_T \) and \( \tau_E \) are the time constant of the first order engine response and elevator response respectively. \( \tau_E \) is statistically chosen to be 0.15 sec. The engine response to throttle cannot be dealt with as easily, even though the actual throttle command to the engine can be assumed to reach the engine instantly. This is a consequence of engine acceleration characteristics, and may lead to significant lags between commanded thrust and actual thrust in particular, for pure jet and turbofan engines. For large turbofan engines the spool-up time from an ideal setting to maximum thrust may be greater than 10 sec. Such lags are too large to be neglected, and contribute to the hazards of wind encounters [49 and 50]. In the present study \( \tau_T = 2 \sim 4.0 \) sec is used.

2.5.6 Linearized Longitudinal Equations of Motion

The following equations taken from (2.130) to (2.142) represent the longitudinal motion perturbations of the aircraft. To accommodate the effects of gust gradients the elements for \( \Delta q \) (pitch rate) need to be modified by adding the corresponding terms concerned with \( q \). The matrix form of the state equations is written as

\[ A_1 \Delta \dot{x} = A_2 \Delta x + B \Delta \delta + D_1 \Delta W + D_2 \Delta \dot{W} \]  

(2.171)

where, for the basic airframe dynamics, the state vector \( \Delta x \) is six-dimensional, i.e.,
\[ \Delta x = (\Delta u, \Delta w, \Delta q, \Delta \theta, \Delta x_E, \Delta h)^T \]  \hfill (2.172)

The control and wind vectors are

\[ \Delta \delta = (\Delta \delta_T, \Delta \delta_E)^T \]  \hfill (2.173)

\[ \Delta W = (u_g, w_g, q_g)^T \]  \hfill (2.174)

The matrices in (2.171) can be expressed as

\[
A_1 = \begin{bmatrix}
m & 0 & 0 & 0 & 0 & 0 \\
0 & m - Z \frac{Z}{Z} & 0 & 0 & 0 & 0 \\
0 & -M_w & I_y & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 \\
\end{bmatrix}
\]  \hfill (2.175)

\[
A_2 = \begin{bmatrix}
x_u & x_w & 0 & -mg \cos \theta_e & 0 & x_{GE} \\
y_u & y_w & mV_e + z_q & -mg \sin \theta_e & 0 & y_{GE} \\
M_u & M_w & M_q & 0 & 0 & M_{GE} \\
0 & 0 & 1 & 0 & 0 & 0 \\
\cos \theta_e & \sin \theta_e & 0 & -V_e \sin \theta_e & 0 & 0 \\
\sin \theta_e & -\cos \theta_e & 0 & V_e \cos \theta_e & 0 & 0 \\
\end{bmatrix}
\]  \hfill (2.176)
Noting that in the study of the glide-slope tracking problem (landing approach phase), \( \Delta d \) - the dispersion from the glide-slope beam - should be included in the state variables, and then the corresponding terms should be shown in the relating column of \( A_2 \) by using the relationship

\[
\Delta \dot{d} = \sin \gamma_G \Delta \dot{x}_E + \cos \gamma_G \Delta \dot{h}
\]
It should be mentioned that the above derivation of the equations of motion is a general form which can be used for both CTOL and STOL aircraft. For the CTOL case, since $\theta_e$ is always small, we may write $\cos\theta_e = 1$ and $\sin\theta_e = \theta_e$, $\cos\gamma_G = 1$ and $\sin\gamma_G = \gamma_G$ in all the equations.

2.5.7 Augmented System State Equations

When the aircraft is controlled by an autopilot, either in the 'glide-slope tracking' mode or in the 'autoflare' mode, the autopilot equation has the form of a static output feedback control low, i.e.,

$$\Delta\delta_c = K_1\varepsilon + K_2\dot{\varepsilon}$$

(2.180)

where $K_1$ and $K_2$ are gain matrices and

$$\varepsilon = \Delta y_C - \Delta y$$

(2.181)

Here $\Delta y$ is the observed output, $\Delta y_C$ is the commanded output. The observation equation is

$$\Delta y = C \Delta x$$

(2.182)

In the case of $C = I$, (2.180) becomes

$$\Delta\delta_c = (K_1\Delta x + K_2\dot{\Delta x}) - (K_1\Delta x + K_2\dot{x})$$

(2.183)

The gain matrices $K_1$ and $K_2$ contain, in the appropriate positions, the feedback gains such as $K_0$, $K_0^*$, $K_u$, $K_u^*$, etc. The $K$ matrices are time-varying because in the two phases of approach and flare the autopilot modes...
are different. The autopilot gains are then a function of time.

Substituting (2.183) into (2.169) obtains

\[
\Delta \dot{\delta} = -I^{-1}K_1 \Delta x_c + I^{-1}K_2 \Delta \dot{x} - I^{-1}K_1 \Delta x - I^{-1}K_2 \Delta \dot{x} - I^{-1}\Delta \delta
\]  

(2.184a)

or

\[
I^{-1}K_2 \Delta \dot{x} + \Delta \dot{\delta} = -I^{-1}K_1 \Delta x - I^{-1}\Delta \delta + I^{-1}K_2 \Delta \dot{x} + I^{-1}K_1 \Delta x_c
\]  

(2.184b)

Combining (2.184b) and (2.171) leads to the augmented matrix form for the closed feedback system equations:

\[
\begin{bmatrix}
A_1 & 0 \\
I^{-1}K_2 & I
\end{bmatrix}
\begin{bmatrix}
\Delta \dot{x} \\
\Delta \dot{\delta}
\end{bmatrix}
= 
\begin{bmatrix}
A_2 & B \\
-I^{-1}K_1 & -I^{-1}
\end{bmatrix}
\begin{bmatrix}
\Delta x \\
\Delta \delta
\end{bmatrix}
+ 
\begin{bmatrix}
I_1 \\
0
\end{bmatrix}
\Delta w
\]

(2.185)

Equation (2.185) can be written in a compact form as

\[
\hat{A}_1 \Delta \dot{x} = \hat{A}_2 \Delta \dot{x} + \hat{D}_1 \Delta w + \hat{D}_2 \Delta \dot{w} + \hat{C}_1 \Delta x + \hat{C}_2 \Delta \dot{x}_c
\]  

(2.186)

or

\[
\Delta \ddot{x} = \hat{A}_3 \Delta x + \hat{A}_4 \Delta w + \hat{A}_5 \Delta \dot{w} + \hat{A}_6 \Delta x_c + \hat{A}_7 \Delta \dot{x}_c
\]  

(2.187)

where

\[
(\hat{A}_3, \hat{A}_4, \hat{A}_5, \hat{A}_6, \hat{A}_7) = A^{-1}(\hat{A}_2, \hat{D}_1, \hat{D}_2, \hat{C}_1, \hat{C}_2)
\]  

(2.188)
and

\[ \hat{A}_1 = \begin{bmatrix} A_1 & 0 \\ I^{-1}K_2 & I \end{bmatrix} \]  

(2.189)

\[ \hat{A}_2 = \begin{bmatrix} A_2 & B \\ -I^{-1}K_1 & -I^{-1} \end{bmatrix} \]  

(2.190)

\[ \hat{D}_1 = \begin{bmatrix} D_1 \\ 0 \end{bmatrix} \]  

(2.191)

\[ \hat{D}_2 = \begin{bmatrix} D_2 \\ 0 \end{bmatrix} \]  

(2.192)

\[ \hat{C}_1 = \begin{bmatrix} 0 \\ I^{-1}K_1 \end{bmatrix} \]  

(2.193)

\[ \hat{C}_2 = \begin{bmatrix} 0 \\ I^{-1}K_2 \end{bmatrix} \]  

(2.194)

\[ \hat{X} = (\Delta u \ \Delta w \ \Delta q \ \Delta \theta \ \Delta x_E \ \Delta h \ \Delta \delta_T \ \Delta \delta_E)^T \]  

(2.195)

With Eqs. (2.186) or (2.187), it is convenient to employ a digital computer for calculating time response or making other analyses in the time domain.
3.1 Introduction

One of the more difficult tasks facing an autopilot designer is the synthesis of feedback logic which will maintain the desired attitude, position and descent rate at touchdown in the presence of wind shear or turbulence. In order to carry out an effective automatic landing, approach and flare, accurate information giving the aircraft's attitude and position, etc., is required. In the past decades, accurate and reliable navigation systems have been developed to provide this information. Such systems include inertially augmented ILS, the Microwave Landing System (MLS), and various attitude reference system. System accuracy and reliability have been greatly improved in recent years. Evidently, touchdown errors in automatic landing are mainly caused by wind shear and turbulence. In contrast with the navigation errors, there is little hope of reducing these disturbances. Therefore, finding an effective control logic for the autopilot to reduce the dispersion due to winds is essential. In particular, as landing speeds are reduced and the variations of the properties of the winds become important, as in the flare-out phase, the effects of aerodynamic disturbances appear more pronounced. The purpose of designing autopilots in this study is only to provide a reasonable test base for the investigation of a wind model and the relating algorithm of predicting the system response to winds in the statistic sense. The approach in designing the autopilot for glide-slope tracking essentially follows the one suggested in [51] and the resulting autopilots were presented in the previous work in detail [52]. Also, an autoflare system study was made in [46]. We will not attempt to review all the autopilot system design procedures herein. However, to improve the Fixed-Path Flare Algorithm, the Random Choice Direct Search
Method is introduced to optimize the control gain set which leads to much better performance during flare-out. A brief discussion on the autoland system follows.

3.2 Glide Slope Tracking Mode

The glideslope tracking autopilot takes the aircraft following a rectilinear ground referenced approach trajectory; this is essentially a regulator problem, provided that the reference trajectory is defined appropriately. In particular, the glideslope deviation is completely defined by \( d \). The autoland system uses the Instrument Landing System (ILS) for guidance along the localizer and glideslope path. It is assumed that the ground facilities transmit ideal signals. It will also be assumed that the aircraft has onboard equipment which will convert the angular deviations and the distance measuring equipment measurements into linear position coordinates. In other words, all noise from the beram and avionic devices are neglected. The classical approach to control system design was used in which the gains are determined through root locus techniques and specified gain and phase margin criteria. The modern control theory approach was partially applied in this study, i.e., the use of state equations (matrix form) to determine all the feedback gains which satisfy the classical criteria was made in the digital computer implementation though some trial and error procedures were still needed to account for the interaction between the multiple inputs and outputs of the system. Both auto-throttle and auto-elevator control were synthesized. The control laws are:

\[
\Delta \delta = K_u \Delta u + K_u \dot{\Delta} u + K_u \int \Delta u \, dt \tag{3.1}
\]

\[
\Delta \delta_{\text{EE}} = K_\theta \Delta \theta + K_\dot{\theta} \Delta \dot{\theta} + K_d \Delta d + K_{\dot{d}} \Delta \dot{d} + K_{d} \int \Delta d \, dt \tag{3.2}
\]
The block diagram is shown in Fig. 8, in which a washout filter is included in the attitude feedback loop to account for the wind shear or the need for aborted landing. A set of typical gains is shown as follows:

\[
\begin{align*}
K_u &= 0.06 & K_\theta &= -2.6 \\
K_u' &= 0.02 & K_d &= -0.018 \\
K_u'' &= 0.00287 & K_d' &= -0.045 \\
K_\theta &= -2.0 & K_d'' &= -0.00035
\end{align*}
\]

The closed-loop performances for a step input are given in Fig. 9. For a B-747 class jet transport flying through a severe wind shear field the flight path and the characteristics are shown in Fig. 10 and Table II.

3.3 Autoflare - Classical Design

A fixed-path flare law was used in the previous work [46]. This approach involves commanding the aircraft to fly a fixed flare trajectory that is explicitly defined as a function of runway distance. Flares of this type, we called path-in-space or h(x) flare laws, have many advantages. The flight path is unchanged for variations in approach speed. The explicitly defined path may be altered independently of the gains used to achieve better response. Conversely the effects of feedback gains can be studied without changing the flare path. When an estimate of aircraft position is available, such as can be provided by MLS or the other ground facilities, the flare is initiated at a preselected value of x and the path may be made a continuous extension of the glide path at the height of commencing the
flare. This approach would enable the flare law to reduce the effect of glideslope tracking errors at flare initiation on touchdown location. Finally, commands for $h(x)$, $\dot{h}(x)$ and $\ddot{h}(x)$ or $n_z(x)$ can be developed to provide close tracking of the desired flight path.

Reference [46] proposed a 'path-in-space' trajectory, which is designed for use with a nominal 3° glidslope:

$$H(x) = (H_o - H_\infty)e^{-\frac{x}{\tau_h V_{av}}} + H_\infty$$  \hfill (3.3)

where

- $x$ = distance
- $V_{av}$ = average speed during flare
- $H_o$ = height starting flare

in (3.3), $\tau_h$ and $H_\infty$ are chosen to satisfy the boundary conditions:

- $\dot{H}(0) = \dot{H}_{glide\ path}$ at flare initiation
- $H_o = 15.24\text{m}$ wheel height
- $\dot{H}_f = \dot{H}_{design\ value}$ at touchdown

The flare control law is as follows:

$$\delta E_c = K_H (h - H) + K_{\dot{h}} (\dot{h} - \dot{H}) + K_{\ddot{h}} (\ddot{h} - \ddot{H}) + K_\theta (\theta - \theta_e) + K_{\ddot{\theta}} + K_{n_z} \Delta n_z$$  \hfill (3.4)

or written as
\[
\delta E_c = K_H \Delta H + K_\dot{H} \Delta \dot{H} + K_\theta \Delta \dot{\theta} + K_\dot{\theta} \Delta \dot{\theta} + K_n \Delta n_z \quad (3.5)
\]

where \( H, \dot{H} \) are the control signals which are formed according to (3.3); \( h, \dot{h}, \text{ etc.} \), are the observed signals. The signal \( K_n \Delta n_z \) was for reducing the sink rate at touchdown in the presence of wind shear.

During flare, the throttle was assumed to be two types of control logic: open-loop algorithm, the throttle is mono-retarding such that the airspeed is linearly reduced from 1.3 \( V_s \) to 1.2 \( V_s \). The second is closed-loop algorithm with control law:

\[
\Delta \delta_\epsilon = K_u \Delta u + K_\dot{u} \Delta \dot{u} + K_\int \Delta u \, dt \quad (3.6)
\]

where

\[
\Delta u = u - u_c, \quad u_c = 1.3 \, V_s - 0.1 \, \frac{V_s}{\Delta h} \quad \frac{|h-h_0|}{h_0}
\]

The system block diagram is shown in Fig. 11. A set of response plots for the nominal case is given in Fig. 12. The selected gains are as follows:

\[
K_\theta = -2.0 \quad K_n = -0.085
\]

\[
K_\dot{\theta} = -3.5 \quad K_u = 0.006
\]

\[
K_H = -0.01 \quad K_\dot{u} = 0.02
\]

\[
K_n = -0.75
\]
It is noted that the above control logic resulting from the classical theory was unfavourable for the tracking performance, therefore the state variable deviations at touchdown in the presence of severe wind shear were too large to be accepted. It was recommended in [46] that to reduce the range deviation it would be effective to include a range-dependent feedback signal in the control loop. It is true that [53] had developed such a system in which a signal relating to the design range was included. The testing results showed that the longitudinal dispersion (1σ) was 28.0m. This deems a satisfactory result, but at the expense of increasing the complexity of the control system.

3.4 Autoflare - Optimal Design

The landing maneuver (abort and flare) requires simultaneous use of elevator and throttle (multi-inputs). Classical control design methods are not well adapted for such multi-input problems, whereas optimal control design methods are. In this section we will use the latter technique to design the control logic for an autoflare system. The optimization technique mostly uses a performance index which includes appropriately weighted aircraft state variables and control values. The performance index is then optimized, subject to the constraints of the aircraft dynamics. After solving a set of Riccati differential equations, a set of optimal feedback gains will be found. In general the gain matrix is time-varying but we may approximately use the steady values of the gain matrix which is solved by using a set of algebra Riccati equations. It is demanded that all the state variables must be fed back into the control system. In addition, the routine optimal method is based on the linear system theory and the constraints are also restricted. However, these limitations are avoided by using a new method, called 'the Random Choice Direct Search' (RCDS) optimization technique. This method was originally proposed by Rein Luus in
Liden Pan has recently developed and modified the method which makes it more effective [55]. Here, Pan’s modified RCDS technique will be adapted to the synthesis of autoflare system with some extensions.

3.4.1 A Brief Description of the Theory of the Random Choice Direct Search (RCDS) Technique

We formulate a nonlinear program as follows:

Maximize (or minimize)

\[ F = \phi(x_1, x_2, \ldots, x_n) \]  \hspace{1cm} (3.7)

subject to the constraints

\[ g_i(x_1, x_2, \ldots, x_n) < 0 \quad i = 1, 2, \ldots, m \]  \hspace{1cm} (3.8)

\[ h_j(x_1, x_2, \ldots, x_n) > 0 \quad j = 1, 2, \ldots, r \]  \hspace{1cm} (3.9)

\[ q_k(x_1, x_2, \ldots, x_n) = 0 \quad k = 1, 2, \ldots, s \]  \hspace{1cm} (3.10)

In order to have a meaningful problem, we must restrict \( s < n \) but no restrictions are placed on the number of inequalities \( m \) or \( r \). In most of the problems, each equality constraint can be readily removed by solving each equation in terms of a different variable seriatim. Thus without any loss of generality we may remove the equality constraint. The problem states that choosing \( x_i \) (\( i = 1, 2, \ldots, n \)) which will minimize (maximize) the function \( F \) under the conditions (3.8) and (3.9). To do this, the Direct Search Using Random Numbers Combined with Interval Reduction Algorithm is employed. The procedure of this algorithm is as follows:
(1) Take initial values for $x_1, x_2, \ldots, x_n$ denoted by $x_1^{0(o)}, x_2^{0(o)}, \ldots, x_n^{0(o)}$ and an initial range for each variable denoted by $r_1^{(0)}, r_2^{(0)}, \ldots, r_n^{(0)}$. Set the iteration index $j$ to 1.

(2) Read in a sufficient number of random numbers, say 2000, which are in between -0.5 and +0.5. Denote these by $y_{ki}$. If dividing these numbers into 100 groups, we then have a two-dimensional array $y(100, 20)$.

(3) Take $p \times n$ random numbers from $y(100, 20)$ and assign these to $x_1, x_2, \ldots, x_n$ so that we may have $p$ sets of values, each calculated by

$$x_i^{(j)} = x_i^{0(j-1)} + y_{ki} r_i^{(j-1)}, \quad i = 1, \ldots, n, \quad k = 1, \ldots, p \quad (3.11)$$

(4) Test Eqs. (3.8) and (3.9) and calculate a value of $F$ with each admissible set.

(5) Find the set which maximizes $F$ given by Eq. (3.7). Write out the maximum value of $F$ and the corresponding $x_i^{0(j)}, \quad i = 1, 2, \ldots, n$. Increment $j$ by 1 to $j+1$.

(6) If the number of iterations has reached the maximum allowed, end the problem. For instance, we may choose 200 to be the maximum number of iterations.

(7) Reduce the range by:

$$r_i^{(j)} = \phi \times r_i^{(j-1)} \quad (3.12)$$

Reference [54] was defining $\phi = 1 - \varepsilon$, where $\varepsilon = 0.05 > 0$ and $\varepsilon$ therefore $\phi$ is constant. Reference [55] improved the algorithm to
speed the search procedure by varying $\phi$ with $i$ and it is found that the following search procedure is favourable:

\[
\begin{align*}
\phi &= 1.0 & i &= 1 \\
&= 0.95 & 1 < i < 5 \\
&= 0.76 & 5 < i < 11 \\
&= 0.475 & i > 12
\end{align*}
\]

It is required that if after the present iteration one cannot obtain an optimal result, then the range must be kept constant unless there occur more than three sequential iterations which fail to find optimal results.

(8) Go to step (2) and continue.

In general, approximately 20 iterations will reach a satisfactory result.

3.4.2 Determination of the Cost Function for Autoflare System Design

To carry out the RCDS method, the function $F$, $g_i$ and $h_j$ shown in (3.7) to (3.9) are required to be specified. Obviously, the autoflare system is satisfactory only if certain prescribed performance requirements and constraints are satisfied. In general, these conditions are often described in terms of desired response performance, desired control signals and the limits on those values. In this work, the constraints are considered to be the prescribed flare-out path which is an ideal exponential fixed in space trajectory as shown in (3.3), the desired values at touchdown, e.g.,

\[0 > \dot{h}_{TD} > -0.61 \text{ m/s} \quad (3.13)\]
the in-flight performance limitations, e.g.,

\[ \alpha_f(t) < 0.8 \alpha_s \tag{3.15} \]

\[-25^\circ < \delta_E(t) < 25^\circ \tag{3.16} \]

where \( \alpha_s \) - stall angle of attack. The path tracking requirement can be converted by an inequality as

\[ 0 < \psi - \psi_d < \varepsilon \tag{3.17} \]

where \( \varepsilon \) is the allowance error matrix; ideally zero.

With the constraints at a specified time, e.g., (3.13) and (3.14), an impulse function is feasible to describe. For example,

\[ \theta^L_{lim} < \theta(t) \delta(t - t_{TD}) < \theta^H_{lim} \tag{3.15} \]

\[ 0 > \dot{h}(t) \delta(t - t_{TD}) > 0.61 \tag{3.16} \]

where the superscripts 'L' and 'H' denote the low and high limits; \( t_{TD} \) is the time to touchdown from the flare beginning; \( \delta(t - t_{TD}) \) is the impulse function.

A significant step in the optimal control design is the formulation of a mathematical error index. To a large extent, it determines the nature of the resulting optimum control, i.e., the resulting control may be linear,
nonlinear, stationary, or time-varying depending upon the form of the error index. Reference [56] made a complete discussion on the selection of a suitable cost function (or error index). For the flare problem, a weighted performance index would be suitable. The control engineer is interested not so much in an instantaneous value of the error measure as he is in the cumulative effect of this instantaneous measure throughout an interval of time. Thus, the error index $e(t)$ is often expressed as the time integral of the error measure over a suitable future interval $t$ to $T$, throughout which the system performance is of interest. That is:

$$e(t) = \int_{t}^{T} e_m(\sigma) d\sigma$$  \hspace{1cm} (3.17)

where $\sigma$ is a dummy time value; $e_m(\cdot)$ is the measured error. $T$ and $t$ are real time. Here we choose the lower limit to be 0 and $T$ - the ideal value of time to bring the aircraft from the height where the flare begins to the touchdown.

In general, the formulation for the weighted performance measure is

$$J = \min_{\delta} \int_{0}^{T} (x^T Q x + 2D^T \delta + \delta^T R \delta) dt$$ \hspace{1cm} (3.18)

where

- $Q$: $n \times n$  \hspace{1cm} $n$ - system's order
- $D$: $n \times 1$
- $E$: $1 \times 1$  \hspace{1cm} constant matrix, can be arbitrary
- $R$: $m \times m$  \hspace{1cm} $m$ - control dimension
Q and R are symmetric positive. To expand the above matrices explicitly, the flight path tracking criteria is adopted by applying (3.3). The first term in the right-hand side is identical to $-\dot{\hat{H}}$, thus we rewrite (3.3) as

$$H(t) = (H_0 - H_\infty)e^{-\frac{t}{\tau_h}} + H_\infty = -\tau_h \dot{\hat{H}}(t) + H_\infty$$

or

$$\tau_h \dot{\hat{H}}(t) + H(t) - H_\infty = 0 \quad (3.19)$$

Assuming that the state vector is expressed as

$$\dot{x}^T = (u, w, \dot{\phi}, \phi, \Delta x_E, \Delta h, \Delta \dot{h}) \quad (3.20)$$

then the weighting matrices are

$$Q = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & k \tau_h^2 \\ 0 & 0 & 0 & 0 & 0 & k \tau_h \\ 0 & 0 & 0 & 0 & k \tau_h & k \end{bmatrix} \quad (3.21)$$

$$D = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ -kH_\infty \tau_h \\ -kH_\infty \end{bmatrix} \quad (3.22)$$
\[ R = \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix} \]  

\[ E = (H_\infty^2) \]  

Equation (3.18) can be written in scalar form:

\[
J = \min_{\delta} \int_0^T \left\{ k(\tau_h \ddot{H} + H - H_\infty)^2 + \lambda_1 \delta_E^2 + \lambda_2 \delta_T^2 \right\} dt
\]  

This is the case of constant weighting of altitude error and elevator deflection. The value of \( k \) is determined by a consideration of the relative importance of altitude errors and elevator deflection. The initial height error is assumed 5m. The autoflare system will attempt to correct this error by calling for a large positive (or negative, depending on the sign of \( \Delta h_0 \)) elevator deflection. The largest permissible elevator deflection is assumed to be 25° or 0.4363 (rad). For this case, altitude errors and elevator deflection are assumed of equal importance. Therefore, the first two terms in (3.25) should contribute equally to the value of cost \( J \). Thus, the value \( k \) may be calculated by equating these two terms and substituting the above specified altitude error and elevator deflection values, i.e.:

\[
k(5^2) = (0.4363)^2
\]

where

\[
\lambda_1 = \lambda_2 = 1 \text{ is assumed}
\]

then

\[
k = 0.0076143
\]
3.4.3 Optimization Procedure

Having obtained the cost function $J$ and specified the weighting matrices, a suitable optimization algorithm is required. It is, of course, necessary to select the proper weighting constants $k, \lambda_i$ by the trial and error method during the optimization process. In [56] and [57], the autoflare system is designed by the Parameter Expansion Method (PEM) which is based on the principles of Dynamic Programming. In this study, a more straightforward, more simple method is used - the RCDS algorithm. The reasons for choosing this approach are:

1. to make the control system simple;
2. to accommodate some nonlinearities;
3. to compare with the results using the classical theory.

Hence, the same flare law is employed as the one used in [46]. Consequently the design task becomes to choose the proper gain matrices $K_1$ and $K_2$ to minimize the cost function $J$, in (3.25). In order to obtain the random number set $y(100, 20)$ which is with zero mean and between $-0.5$ and $+0.5$, two approaches have been adopted:

1. random number table provided by Dr. R. Luus' group at the University of Toronto;
2. using the Monte Carlo method. The theory and the corresponding computer code are presented in Appendix II.

The entire optimization procedure is seen in Appendix III, the main program.

The resulting optimal gain set is as follows:

$K_\theta = -1.3 \quad K_H^- = -3.5$
$K_\phi = -36.0 \quad K_U = 0.06$
$K_H = -0.19 \quad K_U^* = 0.02$
here the \( K_U \) and \( K_D \) values obtained in Sec. 3.3 were used. To account for the effect of wind shear which requires a relatively tight inner loop to ensure the acceptable phugoid mode, a set of suboptimal gain sets was selected, which determines the cost function to be a little off the optimal value.

Under the feedback gain sets \( K_1 \) and \( K_2' \), a nominal flare path is illustrated in Fig. 13. It shows a very good tracking performance. Compared to the gain sets \( K_1 \) and \( K_2 \) determined by classical approach (in Sec. 3.3), it is evident that the pitch rate feedback gain \( K_\theta \), \( K_H \), and \( K_H' \) are enhanced over ten times.

In many references, the actual flare trajectories were showed a significant deviation from the desired profile defined by (3.3). This was explained as the great delay in the flight path tracking, in particular, for the large size jet transport. The classical design approach was aimed to obtain a control logic which satisfied a set of commonly used criteria in the frequency domain. Sometimes the resulting autopilot is not satisfactory in time domain. In [46], an additional signal \( K_{nz} \) was introduced to improve the performance at touchdown. It didn't achieve an acceptable performance under very strong wind shear conditions. However, the resulting autoflare system in this study showed a very good wind-proof performance. It may be attributed to the high gains \( K_\theta \), \( K_H \) and \( K_H' \) which increased the pitch damping a great deal and then eliminated the time delay significantly in whole flare-out phase. The performance of these two kind of controllers in the linear wind shear conditions are given below:
<table>
<thead>
<tr>
<th>h (m/s)</th>
<th>Classical Design</th>
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<td>TD</td>
<td>Sink rate at touchdown</td>
<td>Sink rate at touchdown</td>
</tr>
<tr>
<td>Shear Conditions</td>
<td>horizontal: $\Gamma_1 = 0.24 \text{ s}^{-1}$</td>
<td>$-2.36$</td>
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<td></td>
<td>vertical: $\Gamma_3 = 0.20 \text{ s}^{-1}$</td>
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CHAPTER IV
MODELLING THE WINDS NEAR THE GROUND

4.1 Introduction

Turbulence is a chaotic motion of the air in which the velocity vector is a random function of space and time which cannot be predicted exactly. Therefore a statistical description is necessary. The main ingredients are probability, correlation and spectrum functions. The real turbulence is generally nonstationary, non-Gaussian and anisotropic. Evidently, a complete description would not be tractable without simplifying assumptions. One appealing simplification is to approximate the turbulence velocity distributions as normal or Gaussian. Then the turbulent wind is characterized by the mean and correlation functions.

Now consider the turbulence field as defined by Fig. 14. Assume that the two arbitrary points $p_0$ and $p$, shown in Fig. 14, separated by a vector $r = r_x \mathbf{i} + r_y \mathbf{j} + r_z \mathbf{k}$ in a homogeneous three-dimensional turbulence field as shown above. The flow is moving with mean velocity $\mathbf{W}_m$ along the x-axis of a fixed reference frame of which $p_0$ is the origin. $p_1$, $p_2$ and $p_3$ are the projections of $p$ on the coordinate axes. The turbulence motion at any point in the field is denoted by $\mathbf{U}(t) = u(t)\mathbf{i} + v(t)\mathbf{j} + w(t)\mathbf{k}$. Thus the total velocity in the x-direction at $p_0$ is

$$U_0(t) = W_m(t_1, T) + W_0(t)$$

(4.1)

where $W_m(t_1, T)$ is the mean velocity and is defined by

$$W_m(t_1, T) = \frac{1}{2T} \int_{t_1-T}^{t_1+T} U_0(t) dt$$

(4.2)
and is the same at all points in the field (homogeneity). The mean of the fluctuating component \( u_o(t) \) as defined in (4.1), for \( T \to \infty \), is zero. 

\( W_m(t_1, T) \) is a random function of \( t_1 \) for finite \( T \). If their statistical properties are independent of time in the limit at \( T \to \infty \), such winds are said to be stationary. For stationary flow \( W_m(t_1, T) \) is independent of \( t_1 \) and the mean velocity is given by

\[
W_m = \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} U_0(t) dt
\]

Thus

\[
U_0(t) = W_m + u_o(t)
\]

The correlation function is defined as

\[
C(r, \tau) = \langle C_{ij}(r_x, r_y, r_z, \tau) \rangle \langle u^T(t)u(t + \tau) \rangle
\]

(4.5a)

\[
C_{ij} = \langle u_i(r_1, t_1)u_j(r_2, t_2) \rangle \quad i, j = 1, 3, 3
\]

(4.5b)

where

\[
u = (u v w)^T = (u_1 u_2 u_3)^T
\]

The correlation with zero space and time separation is the covariance.

There is strong evidence that the turbulent velocity distribution in the atmosphere is not Gaussian [58, 59, 60 and 48]. Reeves [61] proposed a technique for the non-Gaussian simulation of atmospheric turbulence. References [39 and 62] developed, based upon Reeves' method, a technique that permits inclusion of strong 'patchiness' by selecting the fourth moment
of the distribution. However, the non-Gaussian behaviour of the wind is most apparent at higher altitudes and is related to the intermittent nature of the turbulence. It is known that a Gaussian input to a linear system yields a Gaussian output, the output probability function is not in general known if the input is non-Gaussian. A very important exception is that a sum of Gaussian inputs yields a sum of Gaussian outputs, the statistics of which can therefore be calculated. The linear analysis plays a very important role in predicting response to turbulence. With a linear system model, if we are given the necessary input spectrum functions and if the inputs are Gaussian, then everything we need to know about the response is available. For reasons of practicality and the aircraft flying in the lower boundary layer, the linear system theory and Gaussian distribution assumption are employed.

4.2 Simplifying Assumptions

Considerable progress on the investigation into the fluid mechanics and meteorology fields has been achieved over the years. This provides the opportunity to make several simplifying assumptions in order to reduce the tremendous amount of information required in dealing with the problems relating to turbulence.

In general, the stationary, frozen, homogeneous, isotropic and Gaussian assumptions are widely employed in the study of turbulence. However, when dealing with the problem in the atmospheric planetary boundary layer the situation is different from that of the high altitude. The mean wind in the lower layer varies with height and time - magnitude and direction. Strictly speaking, the turbulence properties are no longer isotropic, nor homogeneous; therefore a further simplification is also needed. Nevertheless the general simplifying assumptions are listed in the sequent context.
4.2.1 Stationary

The mean value vector is time-invariant and the correlation function must depend on the time separation only. For the landing problem with time scales less than a few minutes this assumption is approximately valid [63 and 43].

4.2.2 Homogeneity

This is essentially the spatial equivalent to being stationary in the time domain. It is generally assumed that the flow in the air is homogeneous in all spatial directions. This is approximately true for clear or free air turbulence. However, in the lower level, the mean wind and the correlations vary considerably with height. For relatively uniform terrain there is little variation horizontally. Therefore the assumption of horizontal homogeneity is appropriate. For some cases the horizontal homogeneity may not be valid. Urban airports are frequently surrounded by residential and industrial areas whose characteristic roughness is considerably greater than the flat terrain of the airport. At airports whose runways end near a lake or river the thermal activities between the pavement and the water body may cause horizontal inhomogeneity. Also, the wakes of large buildings and sometimes mountains can cause larger than usual turbulence fluctuations [64]. It is noted that this assumption is not valid for flight through a 'small' patch or turbulence. None of these effects will be considered in the present investigation.

4.2.3 Taylor's Hypothesis (Frozen Field)

Taylor's hypothesis states that temporal changes in the velocity field are negligible compared with the apparent temporal changes 'seen' by the aircraft as it flies through spatial gradients. This implies that in flight at speed V on the x-axis, in which the perceived rate of change is
the partial time derivative $\frac{\partial}{\partial t}$ can be neglected. This is known as the 'frozen field' approximation. Taylor's assumption reduces the correlations and spectra from a four- to a three-dimensional function, i.e.,

$$C_{ij}(r) = \langle u_i(r_1)u_j(r_2) \rangle \quad i, j = 2, 3, 3 \quad (4.7)$$

and

$$\theta_{ij}(\Omega) = \left( \frac{1}{2\pi} \right)^3 \int \int \int C_{ij}(r)e^{-i\Omega \cdot r} dr_1 dr_2 dr_3 \quad (4.8)$$

Etkin [4] analysed the valid condition of frozen assumption. It was found to be $V/W > 1/3$, where $V$ and $W$ are the ground speed of the aircraft and mean wind speed respectively. This implies that the frozen turbulence assumption fails only for "nearly convected" flight. All upwind flight and hovering flight have $V/W > 1$, and for these the assumption is acceptable. Since the relative speeds at which the assumption becomes doubtful are so small, only lighter-than-air and VTOL aircraft would possibly be affected.

4.2.4 Isotropy

This assumption implies that all properties are independent on directions of coordinate axes. A study of the scalar invariants of a tensor function [65, 58 and 66] yields the result that for a homogeneous vector field,

$$C_{ij}(\xi) = \sigma^2 \{(f - g) \frac{\xi_i \xi_j}{\xi^2} + \delta_{ij}g\} \quad (4.9)$$
\[ g(\xi) = f(\xi) + \frac{1}{2} \xi \frac{df}{d\xi} \]

where \( \sigma^2 = \) the variance; \( \sigma_{ij}^2 = C_{ij}(\omega) \)

\( f = f(\xi) \) is a normalized scalar correlation function

\[ \delta_{ij} = \begin{cases} 1 & \text{if } i=j \\ 0 & \text{if } i \neq j \end{cases} \] - Kroneker delta

\( \xi = |\xi| \)

\( \xi \) = the separation vector

Figure 15 shows the longitudinal and transverse correlation functions \( f(\xi) \) and \( g(\xi) \). In the light of this assumption the correlation as well as the spectrum function are all independent on coordinates.

There are two main isotropic models: the Dryden and the von Kármán models. The former is a simpler analytical form whereas the latter is a better fit to experimental data.

In the free atmosphere, isotropy is a fairly reasonable assumption, but this proves less reliable with decreasing altitude. Lappe et al [67] concluded from their observation (tower and aircraft measurement) at 91m height in unstable air that Taylor's hypothesis is roughly equally well satisfied regardless of what direction the aircraft flies wrt the wind. This indicates that the turbulence under these conditions is more or less horizontally isotropic. Gunter et al [68] tested isotropy for hundreds of hours of aircraft data taken at 76m and 229m by comparing experimental ratios of vertical-to-lateral and longitudinal-to-lateral component spectra with the corresponding ratios obtained from the isotropic von Kármán spectral equations. Their conclusion is that for most stability cases and for all combinations of height and surface conditions, the turbulence is
totally isotropic. This conclusion is enhanced by $\gamma_{uv}^2$ and $\gamma_{vw}^2$ coherence measurements, which were always less than 0.15 for all frequencies, and would of course be zero for isotropic turbulence. It is to be noted that the $\gamma_{uw}^2$ coherences, which one would expect to be larger than the other two in a shear flow, were not presented. It is evident from their data that there is a distinct reduced frequency $\Omega = 3 \times 10^{-3}$ cpf above which there is a very significant decrease in the amount of departure of data from the isotropic case. This suggests a 'local isotropy' region in which the turbulence is significantly closer to true isotropy than at lower frequencies.

The concept of local isotropy was first put forth by Kolmogorov. It postulates that in the so-called inertial subrange region of the energy spectrum, the turbulence is isotropic. In this region turbulent energy is neither produced nor dissipated - merely passed through from the large anisotropic eddies to smaller ones by inertial forces. This implies that in this subrange the effects of viscosity and the anisotropic production are negligible.

Measurements taken from meteorological towers [63, 70 and 69] indicate that in the surface layer, the variance and scale of the vertical gust component are somewhat smaller than the variance and scale of the horizontal components. Also, the correlation of the vertical component and the component along the wind is not zero due to the shear of the mean wind. Panofsky [71] reports that at 2m the scales of the horizontal components are elongated in the direction of the mean wind.

It was found from Kilmogorov's theory that the structure function (defined as the variance of the incremental velocity, $E[\left(\left|u(\xi+r)-u(r)\right|^2\right)]$) is proportional to $\xi^{2/3}$. This means that for separations in the inertial
subrange, the isotropic correlation function $f$ is given by

$$f(\xi) = 1 - \frac{\left(K \epsilon \xi \right)^{2/3}}{2a^2}$$  \hspace{1cm} (4.10)

where $\epsilon$ is the average viscous dissipation, and $K$ is a universal constant ($K \equiv 2$) as seen in [66].

For the correlation function, von Kármán suggests

$$f(\zeta) = \frac{2^{2/3}}{\Gamma(1/3)} \zeta^{1/3} K_{1/3}(\zeta)$$  \hspace{1cm} (4.11)

$$g(\zeta) = \frac{2^{2/3}}{\Gamma(1/3)} \zeta^{1/3} \left[K_{1/3}(\zeta) - \frac{1}{2} \Gamma K_{2/3}(\zeta)\right]$$  \hspace{1cm} (4.12)

where

$$\zeta = \xi / (aL)$$

and $\Gamma$, $L$ are gamma and modified Bessel functions of the second kind respectively, and $a = 1.339$. $L \equiv \int_0^\infty f(\zeta) d\zeta$, the integral scale.

The plots for $f(\zeta)$ and $g(\zeta)$ are presented in Fig. 15. It shows that the function of $f(\zeta)$ has the properties that for small $\zeta$ it is of the form of Eq. (4.10) and is everywhere positive and uniformly decreasing to zero for large $\zeta$. The function of $g(\zeta)$ has different properties; it decays more rapidly than $f(\zeta)$ and is negative when $\zeta$ is above a certain value, and tends to zero for large $\zeta$. Gault and Gunter [72] report that the von Kármán expression, Eqs. (4.11), (4.12), provide a good fit to their data.

To summarize, it can generally be said that turbulence in the planetary layer is not isotropic in the true sense. However, for some wave number range a region of local isotropy does exist. Horizontal isotropy, in which
the turbulence is independent of rotations of the coordinate system about the vertical axis, is also a reasonable assumption. Hence, a suitable model to fit the vertical anisotropic properties is required. Etkin and Reid et al at UTIAS [40, 41, 42] have presented a correlation model for turbulence along the glide slope - a modified von Kármán model based on the results from an 1.12×1.68m multiple-jet wind tunnel measurements using the stationary-probe method. This model will be adopted throughout this investigation.

4.2.5 Stratified

The planetary boundary layer can be divided into layers and assuming fixed but different homogeneous and isotropic properties for each layer. Furthermore, assume that the parameters of the turbulence in the various layers are independent of each other. Figure 16 shows the stratified model of the boundary layer.

4.3 Mathematical Model of the Planetary Boundary Layer

4.3.1 Introduction

As a summary to the preceding description of flow characteristics in the planetary layer, a mathematical model is suggested based on certain simplifying assumptions described in Sections 4.2.1 to 4.2.5, and the existing results contributed by abundant researchers.

It may well be pertinent to overview the turbulence characteristics in the earth's boundary layer before presenting the model.

At a considerable distance above the earth's surface wind is created, in the first instance, by differential heating of the atmosphere producing pressure gradients which are subsequently modified by the rotation of the earth. As with the flow of any fluid over a surface, a boundary layer is formed over the earth's surface, which is called the atmospheric boundary layer, in which the wind speed decreases from a maximum value at the top of
the layer to zero at the earth's surface. This reduction in velocity is due to both the friction drag of the surface and the drag of all bodies protruding into the air flow (mountains, trees and buildings). These retarding forces are transmitted through the layer by shear forces (Reynolds stresses). The process of momentum exchange between layers is the mechanism leading to the generation, and decay, of eddies, which is termed turbulence. The resulting mixing of the air produces, along all three orthogonal axes, fluctuations in wind speed, commonly called gust, which vary in size in both time and space. The above description indicates that the mechanisms producing turbulence in the lower level atmosphere are different from that in the top of the boundary layer, i.e., clear air turbulence which is mainly induced by thermal buoyancy, energy transition, etc. As far as the turbulence encountering or speed distribution probability is concerned, little information is found of the lower layer situation. Some references, such as [18], employed the turbulence model (exceeding probability distribution) for the clear air case to approximate low layer turbulence, such a surrogate would, of course, result in errors. In this study, a different approach is employed in which the turbulent wind properties are fixed by a selected mean wind speed at 10m (standard height for the meteorology observation) and the roughness of the terrain characterized by a roughness parameter $z_0$. The mean wind speed distribution is characterized by the Weibull distribution instead of the Gaussian distribution.

4.3.2 Mean Wind

(a) Profile

In the earth's boundary layer, it is well-verified that the Prandtl logarithmic law for the mean velocity in a neutral atmosphere is valid. The law for the mean velocity in a neutral atmosphere is valid. The law can be
obtained from von Kármán's similarity hypothesis (or from Prandtl's mixing length theory [16]). According to the dimensional analysis, the nondimensional mean wind shear is

\[ \frac{h}{u_*} \frac{dw}{dh} = f_1 \left( \frac{h}{L*} \right) \]  \hspace{1cm} (4.13)

where \( f_1 \) is a "universal" function which may be determined experimentally. Similar results for the variance and scale parameters are obtained:

\[ \frac{\sigma}{u_*} = f_2 \left( \frac{h}{L*} \right) \]  \hspace{1cm} (4.14)

\[ \frac{L}{h} = f_3 \left( \frac{h}{L*} \right) \]  \hspace{1cm} (4.15)

The friction velocity \( u_* \) is related to the surface friction stress by

\[ \tau_0 = \rho u_*^2 \]  \hspace{1cm} (4.16)

In the case of strong winds, the mechanically induced turbulence due to the mean shear predominates over the effect of buoyancy and the ratio of scales \( h/L_* \to 0 \). The functions \( f_1, f_2 \) and \( f_3 \) then tend to be constants. The neutral atmospheric stability will be assumed as it gives rise to the strongest turbulent fluctuations, and will present the most difficulty for the aircraft's landing. Therefore, the mean wind speed can be integrated to yield
\[
\frac{W_m}{u_*} = \frac{1}{K} \ln \frac{h-d}{z_0}
\]  

(4.17)

where

- \(W_m\) = mean wind speed at height \(h\)
- \(u_*\) = friction velocity
- \(K\) = von Kármán's constant = 0.4
- \(z_0\) = surface roughness length
- \(d\) = displacement of the zero plane

Above the surface layer, Coriolis forces increase, surface roughness effects decrease, and the logarithmic law begins to depart from the empirical data. However, treating the atmospheric boundary layer as a whole, it was found that a simple form of power law provides a satisfactory engineering model, i.e.,

\[
\frac{W}{W_{ref}} = \left( \frac{h}{h_{ref}} \right)^\eta
\]  

(4.18)

The power-law index \(\eta\) is a function of roughness height, varying from about 0.1 for a smooth terrain to about 0.40 for urban centres, as seen in Fig. 17 and Table III.

(2) **Direction**

In the free atmosphere, the mean wind is the gradient wind and it moves along the isobars. However, in the lower level, the presence of shear stresses in combination with the Coriolis forces causes a systematic deflection of the mean wind. The deflection is away from the isobars in the direction of decreasing pressure gradient, and is such that in the Northern Hemisphere the wind direction rotates clockwise with increasing height, as shown in Fig. 18. This change of direction needs to be considered as part of the wind shear problem. However, Harris [73] suggests that except for
very tall structures or those with special features making them extremely sensitive to wind direction, it should be reasonable to ignore the change of wind direction with height in strong winds over all types of terrain. Thus, in this research, the mean wind vector does not change direction in the boundary layer.

(3) **Weibull Wind Speed Distribution**

In recent years much effort has been made to construct an adequate statistical model for describing the wind speed frequency distribution. Most attention has been focused on the Weibull function, since this gives a good fit to the experimental data, as indicated by Hennessey [74]. Wentink [75] in his study of Alaskan wind power potential compared the Weibull model with other distributions such as the frequency distribution of Planck, the Rayleigh distribution and the Gamma distribution. Justus et al [76] applied the Weibull and log-normal distribution to wind speed data from more than a hundred stations of the U.S.A. National Climatic Center, and concluded that the Weibull distribution rendered the best fit. Stevens et al [77] presented an algorithm to estimate the parameters of the Weibull wind speed distribution. Originally, the Weibull distribution is a two-parameter function. However, the two parameter Weibull distribution function was reported too deformed or heterogeneous in the low or zero wind speed distribution. Takle and Brown [78] proposed a hybrid density function which is given for describing wind speed distributions having nonzero probability of "calm". Compared with other methods, the results showed the improved Weibull distribution to be a good method with less than 5% error of actual value.

Although worldwide meteorologists have adopted this method to cope with mean wind speed distribution for years, the aviation circle seems to still prefer the traditional means which usually employ the clear air
turbulence exceedance probability information to deal with the landing (or take-off) problems [18]. In this report, the attempt is made to adopt the Weibull distribution to describe the mean wind model in the planetary boundary layer. Incidentally, it is found that in [79] the histogram of wind speed distribution for 16 world-wide locations coincides with the Weibull distribution curve very well, as illustrated in Fig. 19 (of this report). This supports the reasons for applying the Weibull distribution in this study. Now turn to the mathematical description of the Weibull distribution.

With the two-parameter function, the probability density function is given by

$$f_W(w) = \frac{k}{c} \left( \frac{w}{c} \right)^{k-1} \exp \left[ -\left( \frac{w}{c} \right)^k \right]$$

and the cumulative distribution function by

$$F_W(w) = 1 - \exp \left[ -\left( \frac{w}{c} \right)^k \right]$$

(4.19)

(4.20)

where $w$ is the wind speed, $k$ - a shape parameter and $c$ - a scale parameter. The mean of the distribution is equal to

$$\bar{w} = c \Gamma \left( \frac{1}{k} + 1 \right)$$

(4.21)

where $\Gamma$ is the Gamma function. Defining $x = w/\bar{w}$ then (4.19) and (4.20) can be rewritten as
and

\[ f^W(x) = 1 - \exp\left[-\Gamma^k\left(1 + \frac{1}{k}\right)x^k\right] \]  \hspace{1cm} (4.23)

The variance of the distribution is equal to

\[ \sigma^2 = c^2\left\{\Gamma\left(1 + \frac{2}{k}\right) - \Gamma^2\left(a + \frac{1}{k}\right)\right\} \]  \hspace{1cm} (4.24)

The hybrid density function, which is a three parameter distribution function, is defined as

\[ f^H(w) = F_0\delta(w) + (1 - F_0)f^W(w) \]  \hspace{1cm} (4.25)

for all real \( w \), where \( F_0 \) is the probability of observing zero wind speed and \( \delta(w) \) the Dirac delta function. The corresponding cumulative distribution function is

\[ F^H(w) = \begin{cases} F_0 + (1 - F_0)f^W(w) & w > 0 \\ 0 & \text{otherwise} \end{cases} \]  \hspace{1cm} (4.26)

The mean and variance of this distribution are then, respectively,

\[ \bar{w}^H = c\left(1 - F_0\right)\Gamma\left(1 + \frac{1}{k}\right) \]  \hspace{1cm} (4.27)

\[ (\sigma^2)^H = c^2\left(1 - F_0\right)\Gamma\left(1 + \frac{2}{k}\right) - (\bar{w}^H)^2 \]  \hspace{1cm} (4.28)
(4) Determination of Parameters for the Weibull Wind Speed Distribution

To apply the Weibull distribution, several things must be specified.

(i) Mean wind speed profile

Using the power form and defining the airport terrain characteristics as well as the reference height, one can then get the wind profile desired. In general, the reference height is often chosen to be 10m.

Figure 20 gives a typical Weibull distribution curve.

(ii) Wind distribution parameters

A set of Weibull parameters representing 144 locations across the country of Canada was provided by Atmospheric Environment of Canada. Each location has a specified parameter set \( \{c, k, F_0\} \). Treating the Weibull distribution as a whole, we take the average over all 144 locations for each parameter and then a nationwide parameter set \( \{C^*, k^*, F_0^*\} \) is obtained, which is

\[
C^* = 4.7933 \text{ m/s} \\
k^* = 1.6539 \hspace{2cm} (4.29) \\
F_0^* = 0.1062 \\
\text{or } \{4.7933, 1.6539, 0.1062\}
\]

Note that the obtained set corresponds to one hour mean.

(iii) Averaging time correction

In dealing with the landing problem the annual hourly mean wind Weibull distribution parameter set has to be transferred to a reasonable average time value. Reference [80] gives an explanation to a reasonable average time as illustrated in Fig. 21. It shows that as the average time reducing the variance and the mean value are diminished. It is also evident that shorter average time will lead to increase the probability of occurring high speed. Reference [80] provides an algorithm to correct the effect due to the change of average time, which is characterized by a corrector constant
\[ K_T = 1.0 - 2.14 \left( \frac{\sigma_u}{w} \right)^{1/33} \log_{10} \left( \frac{T_{av}}{2700} \right) = 1.0 + 1.3979 \left( \frac{\sigma_u}{w} \right)^{1/33} \]

as shown in Fig. 22. Obviously, \( K_T \) is dependent on the roughness because \( (\sigma_u/w) \) is. Also, if \( T_{av} > 2700 \) (sec), i.e., 45 min, \( K_T = 1.0 \).

We define

\[
\kappa = \frac{K_T}{K^*(3600)} = \frac{K_T(T_{av})}{K_T(T_{av})}
\]

as the averaging time correction factor, where \( K^*(3600) \) corresponds to 1h average time and is 1.0. \( K_T(T_{av}) \) is a correction coefficient of the desired average time.

Examining the effect of \( k \) and \( c \) on the distribution probability, it is found that as \( k \) or \( c \) is reducing, the mean value is diminishing while the probability of encountering high wind speed is increasing, as shown in Fig. 23.

Thus, it is suggested that the Weibull parameter set multiplied by the average time correction factor \( \kappa \) leads to a new parameter set which represents the new probability distribution function corresponding to a desired average time. The average time relates to the aircraft's response characteristics, in particular, the phugoid mode. The principle of selecting average time is to guarantee the aircraft subject to the turbulence with small wave number (or large scale). It is likely that a typical runway length for the jet transport may be \( \sim3000\)m. The aircraft is expected to experience one or two times distance of the runway, i.e., 3000
~ 6000m. The time span would be approximately equal to ten times the phugoid period. For example, in the present study the phugoid period in the landing approach phase is about 20.0 sec. Taking this value one then obtains the average time of 200 sec which corresponds to travelling ~3000m. Finally, the ten minutes-average value is selected.

At this moment it is pertinent to comment on the world-wide Weibull wind distribution. In the overall certification sense, a world-wide wind model is required. Because of the lack of such meteorological data, an approximation is made. As Canada is very large, numerous typical weather phenomena can be found within the country. Therefore, it is not unreasonable to consider the nationwide Weibull wind distribution to be of world-wide without loss generality.

Secondly, the used Weibull distribution parameters in this study are based on the available results observed in the Canada nationwide 144 airports which correspond to 24 hours duration in each day. But airplanes seldom operate during the middle of the night - when winds are zero or light. Few flights occur between 11 p.m. and 7 a.m. in most places. The present Weibull distribution parameters, however, include a zero wind distribution parameter $F_0$ which of course covers all the events happening in whole day. Obviously, this statistics would over-estimate the calm distribution probability in normal airplane operation duration; i.e., diminishing the nonzero wind distribution probability $(1 - F_0)$. Therefore, the hard landing probability based on the present Weibull distribution parameter would be lower than that only happening in day time. Since we do not have such statistic data only for the duration of airplane operations in each day, we approximately use the Weibull parameters in this investigation which may affect the numerical results but would not change the general conclusions. It was tried that with setting $F_0 = 0$ which should entirely
eliminate the night-time calm wind probability distribution to the Weibull parameters, the resulting overall hard landing probabilities only had a few percent differences from those obtained by using the existing Weibull parameters. It is recommended that further work on how to modify the measured 24 hour Weibull parameters to obtain a hard landing probability more representative of actual operating conditions is needed.

4.3.3 Turbulence Model

(1) Intensity

Similarity theory suggests that, for a neutral atmosphere, \( \sigma_i \) (for \( i \) represents \( u, v \) or \( w \)) is proportional to \( u_\ast \), i.e., the quantity

\[
F_i = 0.4 \sigma_i / u_\ast = (\sigma_i / w) \ln(h/z_0)
\]  

(4.32)

is a constant [81].

In practice the shear stress is not invariant with height so that the right-hand side of (4.32) is not truly constant. For heights up to about 300m empirical correlations of world-wide sources of data for a near-neutral atmosphere, mainly strong winds, show that \( F_u \) is a function of \( h \) and \( z_0 \) and that \( F_v \) and \( F_w \) are functions of \( h \). These relationships are presented by the equations: (See Ref. [81])

\[
F_u = \left[ 0.867 + 0.556 \log_{10} h - 0.246(\log_{10} h)^2 \right] \lambda
\]  

(4.33)

where

\[
\lambda = 1.0 \quad \text{for} \quad z_0 < 0.02 \text{m}
\]

or

\[
\lambda = 0.76 / z_0^{0.07} \quad \text{for} \quad 0.02 \text{m} < z_0 < 1.0 \text{m}
\]  

(4.34)
or
\[ \lambda = 0.76 \quad \text{for } z_0 > 1.0m \]

\[ F_{vg} = 0.655 + 0.201 \log_{10} h - 0.095(\log_{10} h)^2 \quad (4.35) \]

and
\[ F_{wg} = 0.381 + 0.172 \log_{10} h - 0.062(\log_{10} h)^2 \quad (4.36) \]

The data used to derive the correlations cover a wide variety of terrains ranging from measurements over water to town centres and forests. This representation is deemed to be more reliable than those used in [16], etc. The ESDU turbulence intensity plots are illustrated in Fig. 24.

(2) Scale

For the planetary layer as a whole, we have to consider more than the two integral scales that characterize isotropic turbulence. With the definition:

\[ L_{ij} = \frac{1}{\langle u_i^2 \rangle} \int_{0}^{\infty} C_{ij}(\xi, 0) d\xi \quad (4.37) \]

where \( C_{ij}(\xi, 0) \) is the correlation function; the integration is on the \( \xi_j \) axis. For \( i \), use \([u, v, w]\); for \( j \), use \([x, y, z]\); therefore, there are 9 different scales:

**Longitudinal scales**

\[ L_{u} = \frac{1}{\sigma_u^2} \int_{0}^{\infty} C_{uu}(x) dx \]
\[ L_y^u = \frac{1}{\sigma_{w_g}^2} \int_0^\infty C_{u_g u_g}(y)dy \]  

Transverse scales

\[ L_x^w = \frac{1}{\sigma_{w_g}^2} \int_0^\infty C_{w_g w_g}(x)dx \]

\[ L_x^v = \frac{1}{\sigma_{v_g}^2} \int_0^\infty C_{v_g v_g}(x)dx \]

etc. ESDU gives expressions for scales:

\[ L_x^u = 25h^{0.35}/z_0^{0.063}(m) \]

\[ L_x^v = 5.1h^{0.48}/z_0^{0.086}(m) \] (4.39)

\[ L_x^w = 0.35h(m) \]

\[ L_y^w = L_x^w \]

Figure 25 shows how the most important scales vary with height.

(3) Power Spectra

The von Kármán equations are suggested as the model for the velocity component power spectra. In order to take the anisotropy into account,
values of integral scale and variance must be used, as specified, in each of the one-dimensional spectra.

\[ \phi_{uu}(\Omega) = \sigma^2 u_g \left( \frac{4L_u^x}{[1.0 + 70.7(L_u^x\Omega)^2]^{5/6}} \right) \]  \hspace{1cm} (4.40)

\[ \phi_{vv}(\Omega) = \sigma^2 v_g \left( \frac{4L_v^x}{\left[1 + 70.7(2L_v^x\Omega)^2\right]^{11/6}} \right) \]  \hspace{1cm} (4.41)

\[ \phi_{ww}(\Omega) = \sigma^2 w_g \left( \frac{4L_w^x}{\left[1 + 70.7(2L_w^x\Omega)^2\right]^{11/6}} \right) \]  \hspace{1cm} (4.42)

The cross-spectra \( \phi_{uv}(\Omega) \) and \( \phi_{vw}(\Omega) \) are assumed to be zero.

\[ \phi_{uw}(\Omega) = -Y_0 \left[ \frac{\phi_{uu}(\Omega)\phi_{ww}(\Omega)}{1 + 0.395(L_u^x\Omega)} \right]^{1/2} \]  \hspace{1cm} (4.43)

However, it will be neglected in this study. A typical turbulence spectrum function is shown in Fig. 26.

(4) Correlation Function

The correlation matrix is defined as

\[ C = \langle W W^T \rangle \]  \hspace{1cm} (4.44)

where

\[ W = (u_g \ v_g \ w_g \ p_g \ q_g \ r_1 \ r_2)^T \]
Altogether, there are 49 elements contained in $C$. In this study, only the longitudinal case will be considered and according to the basic assumption we ignore the cross-correlations without much accuracy loss because of its negligible effects, as pointed out in [27]. Thus, for the longitudinal correlation matrix we have

$$\frac{1}{\sigma_{ij}}$$

$$\begin{bmatrix} C_{uu} & C_{uw} & C_{uq} \\ C_{wu} & C_{ww} & C_{wq} \\ C_{qu} & C_{qw} & C_{qq} \end{bmatrix} \approx \begin{bmatrix} \sim C_{uu} & 0 & 0 \\ 0 & \sim C_{ww} & 0 \\ 0 & 0 & \sim C_{qq} \end{bmatrix}$$

(4.45)

where '$\sim$' means the normalized correlation.

Now we carry out the calculation of elements in (4.45). Recalling Section 4.2.4, we note that the isotropy assumption brings a significant convenience in the evaluation of correlation functions of the turbulence speed components because, in this case, it is only a function of distance between two points in question regardless of its direction. In the present investigation, the constrained correlation functions between two fixed points on the landing approach-flare trajectory are considered for the needs of applications later. As stated in the preceding sections, the turbulence features in the atmospheric boundary layer are no longer of isotropy. However, for some cases, the local isotropy assumption could be valid. Reid [42] suggested an algorithm based on experimental data in the UTIAS wind tunnel to develop a modified von Kármán model which is suitable for the application of landing problems. Here, the similar procedure is followed
with some extension. Reference [42] used the data obtained from the wind tunnel to fit the nondimensionalized correlation functions \( \tilde{C} \) which is obtained by dividing by the product of the rms of the turbulence velocities as measured at the two locations fixed on the glide slope. Figures 27 and 28 demonstrate the comparisons between wind tunnel correlation measurements and the von Kármán model. It indicates the acceptable agreements of the modified von Kármán model concept. Because of the lack of experimental data for the aircraft used in this study, the theoretical modified von Kármán model is then employed (see Chapter VII for details). Also, Etkin's four point model is used. The von Kármán model (based on homogeneous, isotropic, Gaussian and frozen flow) has the form

\[
\tilde{C}_{ij}(\xi_1, \xi_2, \xi_3) = \left[ f(\zeta) - g(\zeta) \right](\xi_i \xi_j / \xi^2) + g(\zeta) \delta_{ij} \tag{4.46}
\]

where \( \xi^2 = \xi_1^2 + \xi_2^2 + \xi_3^2 \) is the length of the separation vector. Under the local isotropy assumption, \( \xi \) can be expressed on any reference frame. There are two possible choices to select in reference frame: either \( ^{\text{G}}T_{\text{E}} \) glide path frame of \( ^{\text{E}}T_{\text{E}} \) Earth-fixed frame. The following relationship can be used in the transformation from one to the other:

\[
^{\text{G}}T_{\text{E}} = ^{\text{T}}T_{\text{E}}^{-1} \tag{4.47}
\]

where

\[
^{\text{T}}T_{\text{E}} = \begin{bmatrix}
\cos \gamma_G & \sin \gamma_G \\
0 & 1 \\
-sin \gamma_G & 0 \cos \gamma_G
\end{bmatrix}
\]
For example, we can express $\xi$ on $\xi_E$ then

$$\xi_1 = (h_2 - h_1)/\tan\gamma_E - W_m(t_2 - t_1)$$

$$\xi_2 = b' \text{ (lateral separation)}$$

$$\xi_3 = h_2 - h_1$$

where $W_m$ is the mean wind speed in the interval $\Delta t = t_2 - t_1$, $h_2$ and $h_1$ are the height above the Earth's surface corresponding to the two points fixed on the glide slope. $f(\xi)$ and $g(\xi)$ are already given in (4.11) and (4.12).

Appendix V gives the calculation formulation for $\sim C_{uu}$, $\sim C_{ww}$ and $\sim C_{qq}$ based on the four-point model in detail.
5.1 Introduction

A method to be chosen for calculating an aircraft's response to turbulence depends upon the interest of the task as well as the property of the system and the characteristics of the random inputs, i.e., whether the aircraft system (with or without autopilot) being treated as a linear or nonlinear one, whether the turbulence being described as a purely Gaussian random process (white noise), or a general Gaussian-Markow random process, when applying as the input into the system. Both time domain and frequency domain analyses are frequently being used. Also, linear or nonlinear analyses are used for different scenarios. The property of the input, generally speaking, dominates the analyses method. It is well known that a Gaussian input to a linear system yields a Gaussian output; the output probability function is not in general known if the input is non-Gaussian. We shall also see that linear analysis plays a very important part in calculating response to turbulence. With a linear model, everything needed to know about the response is available with reasonable computing effort if we know the necessary input spectrum functions and if the inputs are Gaussian. The response statistics for a non-Gaussian input or for a nonlinear system can usually be carried out only by direct simulation in the time domain, using a very large number of runs, or very long runs, to collect the statistics of rare events of which the distributions of certain events (or probability density) is lying on the tail of the probability distribution curve. Such an example is given in [82]. In a meaningful sense, the calculation of the response of a system to one set of specified random process realization is less useful unless for a check run. The time average of the response due to turbulence is also meaningless except for an
ergodic random process of which the ensemble and the time average are identical. Considering the convenience and economy of computer time, the linear analysis concerned with stationary ergodic processes is more attractive, and this shall be emphasized throughout this study. To obtain the statistic properties of a random process, the conventional or classical method - power spectral density method (PSD) which is carried on the frequency domain - is commonly used [83 and 43]. The main limitation of the PSD technique is in that its results only reflect the steady state characteristics. With the growing interest on landing problems, the time domain analysis is more and more attractive and challenging to aviation engineers. A number of investigators have proposed several techniques to handle the linear system statistic properties due to random atmospheric input, which can be categorized to be frequency domain (FD), frequency-time domain (FTD) and time domain (TD), as summarized in the sequent review.  

5.2 Review and Comparisons of Existing Methods  

In the first place, the response of stable linear-invariant systems and the response in the steady state are concerned. Then following are the linear time-dependent systems with the output of the covariance matrix. Assume the system subjected to turbulence can be represented by a mathematical model given by a set of state equations in matrix form, as already derived in the foregoing Chapter II. Here we rewrite it for convenience in a general form:

$$\dot{R} = A_1 R + B_1 W + B_2 \dot{W}$$  \hspace{1cm} (5.1a)$$

where $R$ - the state vector; $W$ - turbulence input matrix; $\dot{W}$ - gust gradient input matrix; $A_1$, $B_1$ and $B_2$ - system matrix. Here the autopilot is
As will be shown later, (5.1a) can be transformed into a canonical form by the 'Augment-Substituting' technique [41].

\[
\dot{\mathbf{R}} = A \mathbf{R} + B \mathbf{W}
\]

(5.1b)

The turbulence is assumed to be the Gauss-Markov random process, although this would not be necessary for some more general method. The ultimate goal of random process analysis is frequently to obtain probability of failure, lifetime, return period, signal-to-noise ratio, ride comfort, fatigue damage, etc. Under the assumed input property being Gaussian, all the above results can be deduced for any particular response variable \( r_i \) from the two quantities \( \mu_{r_i} \) and \( \sigma_{r_i} \) or \( \langle r_i^2 \rangle \), sometimes \( \langle \dot{r}_i^2 \rangle \) is needed, but it is easily achieved by using Taylor's hypothesis and using the same procedure as for \( \langle r_i^2 \rangle \), or adding \( \dot{r}_i \) to the system as an additional state variable. The mean value \( \mu_{r_i} \) is also not difficult to be estimated in the time domain by inputting a deterministic mean wind shear. Thus, the goal of a response calculation is simply to find the covariance \( \sigma_{r_i} \). More generally, one might have reason to obtain the whole covariance matrix \( C_{rr}(\tau) \) or spectrum matrix \( \Phi_{rr}(\omega) \) from which the above mean-square values can be deduced. Since \( C_{rr} \) and \( \Phi_{rr} \) are a Fourier integral pair, a knowledge of one implies the other.

This section outlines the existing techniques for calculating the mean-square value of state vector due to turbulence.
5.2.1 Power Spectral Density Technique (PSD)

This method is carried on the frequency domain. A FD problem is frequently characterized by the transfer function in Laplace or Fourier form, i.e., \( G(\omega) \) or \( G(s) \). The spectra of the response \( \mathbf{r}(t) \) and the input \( \mathbf{w}(t) \) are related by [43]

\[
\Phi_{rr}(\omega) = G(i\omega) \Phi_{ww}(\omega) G^H(i\omega) \quad (5.2)
\]

The desired mean-squares are then given by the diagonal elements of

\[
\sigma_r^2 = \frac{1}{\Phi_{rr}(\omega)} <\mathbf{r}^\top \mathbf{r}> = \int_0^\infty \Phi_{rr}(\omega) d\omega \quad (5.3)
\]

where \( G^H(i\omega) \) is the Hermitian transpose matrix, \( \Phi_{ww}(i\omega) \) is the turbulence power spectra given in (4.40) and (4.42).

This technique has two outstanding features:

(i) The simplification of computation, and

(ii) The availability of power spectral density models for homogeneous, isotropic turbulence.

The shortcomings of this method are:

(i) When the system has many lightly-damped modes, i.e., having sharp peaks in \( |G| \), the accurate evaluation of (5.3) may be with some difficulties.

(ii) Valid only for a linear set of motion equations.

(iii) The turbulent boundary layer is assumed to be homogeneous, isotropic and frozen in whole.
(iv) It does not take into account the nonstationary nature of the turbulence inputs to the aircraft.
(v) It can only calculate the steady state response value, does not include the transient behaviour of the aircraft.

5.2.2 Impulse Response Technique

Actually, this method is the version of PSD method in the time domain. The matrix of impulse response, $Q(t)$ is the Fourier transform of $G(i\omega)$, and yields the response by convolution with the input. In terms of $Q$, the response covariance matrix can be written as

$$C_{rr}(\tau) \triangleq \langle r(t + \tau) r^T(t) \rangle = \int_0^\infty \int_0^\infty Q(\alpha)C_{ww}(\beta - \alpha + \tau)Q^T(\beta) d\alpha d\beta \quad (5.4)$$

The desired mean-square values are given by setting $\tau$ to equal zero, i.e., $\sigma_r = C_{rr}(0)$.

The merits and limitations are the same as the previous method.

5.2.3 Direct Simulation of the Input

A direct method of finding the responses is of course to simulate the input random process by actually generating (or even measuring) the required random functions (by analog or digital - Monte Carlo method) and using them as inputs to either a computer simulation or a physical realization of the system. This procedure would be facilitated by using the shaping filter technique [21, 84].

This method can be used for both linear and nonlinear systems, and also it can simulate the non-Gaussian turbulence properties [61, 62], therefore the results would be more realistic. The limitations are the considerable
computer time consumption because very large number or very long time runs are needed to collect the very rare events.

5.2.4 Etkin Equivalent Deterministic Input Technique

This concept was first proposed by Etkin in 1961 [85]. Then, Etkin et al [86] have extended and developed it. This method can be used for calculating the response of a linear-invariant system to a set of random-process inputs in the time domain. It is an attractive approach to evaluate the responses of an aircraft to turbulence. Initially, it was tried to solve the problem investigated in this study. However, we eventually found that it was not suitable for the case involving time varying properties during landing approach - flare phase.

5.2.5 Dispersion Covariance Matrix Propagation Technique

This method uses basic principles of modern system theory. It can calculate the covariance matrix and allows the computation of variances or rms values of aircraft state variables in the case where system dynamics and statistical properties of disturbing signals are a function of time. The details of this technique are reserved for the next section.

5.3 Constrained Correlation Technique

A general state equation form is given in (5.1a), which includes the derivative term of the external disturbances. It is different from the canonical form. In the special case, one may use the approach proposed in [27 and 41] to evaluate the covariance matrix by eliminating the \( \dot{W} \) term, i.e., define

\[
R = Z + B_2W
\]  

(5.5)

Substituting (5.5) into (5.1a) yields
\[
\ddot{Z} + B \dot{W} = A(Z + B_2 W) + B_1 W + B_2 \dot{W}
\]  \hspace{1cm} (5.6)

or

\[
\dot{A} = A \dot{Z} + B \dot{W}
\]  \hspace{1cm} (5.7)

where

\[
B A = A B_2 + B_1
\]  \hspace{1cm} (5.8)

and the system matrices \(A, B_1, B_2\) are assumed constant. Equation (5.7) is a canonical state equation of linear system for state vector \(Z\). After obtaining \(Z\), one can then solve (5.5) for \(\dot{R}\) and readily find the covariance matrix of \(\dot{Z}(t)\), where

\[
\dot{Z}(t) = \langle R(t) R^T(t) \rangle_{\sigma_{rr}}(o).
\]

It is noted that this approach will be valid only if the initial winds (or turbulence intensity) are all assumed to be zero.

In this study, a different approach was employed to evaluate the covariance matrix for a system subject to Gauss-Markov process disturbances. Two category methods were developed according to whether the governing state equation includes derivative terms of the external disturbances or not.

5.3.1 Without Derivative Term Case

The canonical form of the state equation is

\[
\ddot{Z}(t) = A(t) \dot{Z}(t) + B(t) W(t)
\]  \hspace{1cm} (5.9)

where \(A(t)\) and \(B(t)\) are the time-dependent system matrices, \(W(t)\) is the Gauss-Markov random process. The solution of (5.9) is (see [88] and [89]):

\[
\dot{R}(t) = \phi(t, t_0) R(o) + \int_{t_0}^{t} \phi(t, \tau) B(\tau) W(\tau) d\tau
\]  \hspace{1cm} (5.10)
where the transition matrix can be found, in general:

$$\dot{\Phi}(t, t_0) = A \Phi(t, t_0)$$ (5.11)

One can solve (5.11) for $\Phi(t, t_0)$ by numerical calculation.

Let us define the covariance matrix as

$$\Sigma(t) \triangleq \langle R R^T \rangle + C_{rr}(t_0)$$ (5.12)

Differentiating (5.12) we obtain

$$\dot{\Sigma}(t) = \langle \dot{R} R^T \rangle + \langle R \dot{R}^T \rangle$$ (5.13)

From (5.9), it yields

$$\dot{R}^T = R A^T + W B^T$$ (5.14)

$R^T$ is readily obtained from (5.10):

$$R^T = R(o) \phi^T + \int_{t_0}^{t} W^T B^T \phi^T d\tau$$ (5.15)

Substituting (5.9), (5.14) and (5.15) into (5.13) yields:

$$\dot{\Sigma}(t) = A \Sigma + \Sigma A^T + B \langle W R^T \rangle + \langle R W^T \rangle B^T$$
\[ \sum(t) = A(t)\sum(t) + \sum(t)A^T(t) + \int_{t_0}^{t} B(\tau) <W^T(t)\Phi(t + \tau) B^T\sum(t, \tau) d\tau \]

\[ + \int_{t_0}^{t} \Phi(t, \tau)B(\tau) <W(t + \tau) W(t)B^T(t) d\tau \] \hspace{1cm} (5.17)

This expression is similar to that shown in \([90 \text{ and } 27]\). For the case of a linear time-invariant system (5.17) becomes

\[ \sum(t) = A \sum(t) + \sum(t)A^T + D(t) + D^T(t) \] \hspace{1cm} (5.18)

where

\[ D(t) = \int_{t_0}^{t} B <W(t)W^T(t + \tau)B^T\sum(t + \tau) d\tau \]

\[ = \int_{t_0}^{t} B C_{WW}(t, \tau)B^T\sum(t + \tau) d\tau \] \hspace{1cm} (5.19)

where
\( \phi(t) = e^{At} \) which can be calculated using approaches shown in Appendix VI. As long as the matrix \( \mathbf{D}(t) \) is specified, (5.18) can then be solved for \( \Sigma(t) \).

Here \( \mathbf{D}(t) \) is a weighted integral of the autocorrelation of the turbulence and we may call it the driving (or forcing) matrix. To determine \( \mathbf{D}(t) \), one must first find \( C_{WW}(t, \tau) \).

The turbulence autocorrelation \( C_{WW}(t, \tau) \) may be found in several ways depending on the method of describing the random process. Summarily, we may use two approaches to solve the covariance matrix:

1. Constrained Correlation Technique (CCT).
2. Shaping Filter Technique (SFT).

The 'CCT' method is a kind of direct method which directly puts the turbulence correlation function \( C_{WW}(t, \tau) \) into the governing equation (5.18) or (5.19). \( C_{WW}(t, \tau) \) may be obtained by an experimental approach in the wind tunnel or in flight [27, 41]. When flying in the earth's boundary layer at other than level flight, the turbulence properties encountered by the aircraft (in landing or take-off) change with time (or height) due to the effect of nonhomogeneity and anisotropy. Different landing conditions (such as glide slope, speed, flare path profile, etc.) correspond to a different correlation function; in other words, the turbulence correlation function is constrained by the flight path. Since the 'frozen' assumption is used, the correlation function only relates to the distance along the landing trajectory (actually only the vertical distance change is important). \( C_{WW}(t, \tau) \) is sometimes called the "flight path correlation matrix". In terms of flight path, we may define

\[
C_{WW}(t, \tau) = \langle W(t) W^T(t + \tau) \rangle \quad (5.20)
\]
where \( W(t) \) represents the turbulence input to the aircraft at time \( t \). These flight path correlations can be measured by using two fixed velocity probes positioned at pairs of points along the specified glide slope \([40]\). Assuming ergodicity it then follows that

\[
C_{\text{ww}}(t, \tau) = C_{\text{ww}}(t_1, t_2) = \langle W(t_1) W^T(t_2) \rangle \tag{5.21}
\]

where \( W(t) \) represents the time history of the turbulence as measured at the location on the glide slope (given by \( r_i \)) which is reached by the aircraft at time \( t_1 \) (see Fig. 29). This constrained correlation concept can also be applied to the theoretical calculation of turbulence correlations as shown in the present report.

Note that the 'CCT' method defines a Gauss-Markov process or coloured noise input for determining the covariance matrix \( \Sigma(t) \). The system state vector does not include the turbulence vector \( W \); thereafter the state equation does not have to be augmented to account for \( W \). Because the flight path correlation function comes from real measurements (even using the modified von Kármán turbulence model, less simplifying assumptions are made), the results would be more reliable than using another method.

With the SFT method, it is based on the fact that a Gauss-Markov random process can always be represented by the state vector of a continuous linear dynamic system forced by a Gassian purely random process where the initial state vector is Gaussian. The augmented system (original plus shaping filter) then has a white noise input and a state vector that includes the
original state variables as well as additional ones from the shaping filter, as seen in Fig. 30. With a white-noise input $n(t)$, there results a formula for the covariance matrix $\sum$ similar to (5.18) but simpler. In this case, we have

$$C_{\text{ww}}(t, \tau) = \langle n(t)n^T(t + \tau) \rangle = N\delta(t - \tau)$$  \hspace{1cm} (5.23)$$

where $N$ is the intensity matrix. Thus

$$D(t) = \int_{t_0}^{t} B(t') C_{\text{ww}}(t', \tau) B^T(t', \tau) d\tau$$

$$= \begin{cases} B(t)N(t)B^T(t)\Phi(t, \tau), & \tau > t_0 \\ 0, & \tau < t_0 \end{cases}$$ \hspace{1cm} (5.24)$$

This method simplifies the calculation considerably. It demands the proper filters. For doing this a suitable method for finding the filter matrices is required. One may use the spectra density matrix factorization technique similar to that shown in [87]. References [21 and 86] give the approximate or exact method.

5.3.2 With Derivative Term Case

Recalling (5.1a) we have

$$\dot{\mathbf{R}} = A\mathbf{R} + B_1\mathbf{W} + B_2\dot{W} \hspace{1cm} (5.1a)$$

It was found that in this case the approach to evaluate the covariance matrix, presented in the previous paragraph, is not valid, although the
governing equation can be transferred to a canonical form without derivative term. This is because the equivalent impulsive function, therefore the equivalent transition matrix for the system does not exist.

Therefore, a new method is developed in the present study and presented herein.

We start with solving (5.1a) for $\mathbf{R}$ in the light of system theory.

$$
\mathbf{R}(t) = e^{At} \mathbf{R}(0) + \int_0^t e^{A(t-v)} \mathbf{B}_1 \mathbf{W}(v) dv + \int_0^t e^{A(t-v)} \mathbf{B}_2 \mathbf{W}(v) dv
$$

(5.34)

The third term in (5.34) can be solved by parts

$$
\int_0^t e^{A(t-v)} \mathbf{B}_2 \mathbf{W}(v) dv = e^{A(t-v)} \mathbf{B}_2 \mathbf{W}(0) \bigg|_{v=0}^{v=t} - \int_0^t (-A)e^{A(t-v)} \mathbf{B}_2 \mathbf{W}(v) dv
$$

$$
= \mathbf{B}_2 \mathbf{W}(t) - e^{At} \mathbf{B}_2 \mathbf{W}(0) + e^{At} \int_0^t e^{-Av} \mathbf{B}_2 \mathbf{W}(v) dv
$$

since $Ae^{-Av} = e^{-Av} A$, we have

$$
\int_0^t e^{A(t-v)} \mathbf{B}_2 \mathbf{W}(v) dv = \mathbf{B}_2 \mathbf{W}(t) - e^{At} \mathbf{B}_2 \mathbf{W}(0) + e^{At} \int_0^t e^{-Av} \mathbf{B}_2 \mathbf{W}(v) dv
$$

(5.35)

Assuming $\mathbf{R}(0) = 0$, we finally find

$$
\mathbf{R}(t) = \mathbf{B}_2 \mathbf{W}(t) - e^{At} \mathbf{B}_2 \mathbf{W}(0) + \int_0^t e^{A(t-v)} \mathbf{B}_\sim \mathbf{W}(v) dv
$$

(5.36)

where $\mathbf{B}_\sim = \mathbf{B}_1 + A \mathbf{B}_2$
Based on (5.36) we now derive the covariance matrix expression of $R(t)$.

Replacing 't' in (5.36) with 's' and transposing the resulting version of (5.36) turns out

$$R^T(s) = W^T(s)B_2^T - W^T(o)B_2^T e^{A^Ts} + \int_0^s W^T(v)\tilde{W}^T(s-v)dv \quad (5.37)$$

Post-multiplying (5.36) by (5.37) and taking the average, it follows

$$<R(t)R^T(s)> = B_2^T <W(t) W^T(s)> - e^{At}B_2^T <W(o) W^T(s)> + \int_0^t e^{A(t-v)}\tilde{B}(v) <W(v) W^T(s)>dv$$

defining $C_{rr}(t,s) \triangleq <R(t)R^T(s)>$ and $C_{wr}(t,s) \triangleq <W(t) R^T(s)>$

then (5.38) becomes

$$C_{rr}(t,s) = B_2^T C_{wr}(t,s) - e^{At}B_2^T C_{wr}(o,s) + \int_0^t e^{A(t-v)}B C_{wr}(v,s)dv \quad (5.39)$$

Premultiplying (5.37) by $W(t)$ and taking the average, it yields

$$<W(t)R^T(s)> = <W(t) W^T(s)>B_2^T - <W(t) W^T(o)>B_2^T e^{A^Ts}$$

$$+ \int_0^s <W(t) W^T(v)>\tilde{B} e^{A^T(s-v)}dv$$

or

$$C_{wr}(t,s) = C_{ww}(t,s)B_2^T - C_{ww}(t,o)B_2^T e^{A^Ts}$$
where $\mathbf{C}_{ww}(t,s) \triangleq \langle \mathbf{W}(t)\mathbf{W}^T(s) \rangle$ is the constrained correlation matrix, as mentioned in section 5.3.1.

To get $\mathbf{C}_{wr}(o,s)$ including in (5.39), just simply set 't' in (5.40) to o, i.e.

$$\mathbf{C}_{wr}(o,s) = \mathbf{C}_{ww}(o,s)\mathbf{B}_2^T - \mathbf{C}_{ww}(o,o)\mathbf{B}_2^T \mathbf{A}_T + \int_0^s \mathbf{C}_{ww}(o,v)\mathbf{B}_2^T \mathbf{A}_T (s-v) \, dv$$

(5.41)

once the constrained correlation matrix $\mathbf{C}_{ww}(t,s)$ along a landing path is given, the covariance matrix $\sum_r(t)\mathbf{C}_{rr}(t,t)$ can then be essentially obtained by (5.39), (5.40) and (5.41).

A detailed derivation for the evaluation of covariance propagation matrix $\sum_r(t)$ is presented in Appendix VII.

It should be born in mind that the above theory is regarded to the linear time-independent system. In some circumstances, if the linear time-independent assumption for a system can't be satisfied, the direct integration technique may be used, which is developed and given in Appendix VIII.
CHAPTER VI
6.0 PROBABILITY OF "HARD" LANDING

6.1 Introduction

Automatic control of landing can be considered to be aimed at reducing the effects of external or internal disturbances acting on the controlled system (airframe plus autopilot). A number of sources of disturbances are of a random nature and cause random errors relative to a desired state. It has already been seen that the present study is mainly concerned with random disturbances due to atmospheric turbulence, but it is evident that the calculation method presented herein is also readily applied to effects of electronic and mechanical random noise.

The final goal for autolanding is to perform a safe landing which is characterized by the desired state at touchdown. A requirement associated with safe landing is the decision height window (for Cat. II it is a window at 30m height. Table IV gives different specifications for several scenarios). The problem for the analyst and designer is to predict, for a given situation, the probabilities associated with the decision to make a landing or abort and with unsafe touchdown (hard landing or failure). The state variables with which we would be concerned in discussing catastrophe at decision height or touchdown include the position vector \( \mathbf{r}^E \), the velocity vector \( \mathbf{v}^E \) and the attitude, nine scalar variables in all. The angular velocity is probably unimportant. In practice, it may well be good enough to define the window or touchdown state in terms of a severely restricted set of variables; for example, \( h \) (or \( z \)), \( y \), \( V^E \) and \( \dot{h} \).

Of the factors that contribute to dispersion at the desired location we can usefully separate out three (apart from control system errors):
(i) errors in initial condition;
(ii) the mean wind shear;
(iii) turbulence.

Figure 31 illustrates the above three contributions to state errors at a target position (may be a plane at the decision height or at touchdown). R is the point at which the reference state associated with correct initial conditions and zero turbulence lies on the target position. The reference trajectory accounts for the mean wind profile, which may include cross-wind, rotation of wind direction with height and effects of terrain. Curve a in Fig. 31 represents the dispersion around R associated with initial errors that may occur with an assigned probability $P_1$. A given initial error in position and/or velocity and/or attitude yields a single point in the target plane. The continuous curve a derives from a continuous set of initial errors $\sum_{i=1}^{N} (t_0)(i = 1, 2, \ldots, N)$. Curve b in Fig. 31 shows the additional dispersion at probability level $P_2$ associated with turbulence and a particular set of initial errors. We assume that the errors resulting from the initial conditions are independent of those produced by the turbulence, and hence that the joint probability is the product of the separate probabilities. For a linear system the dispersions from the two sources can be linearly superposed, therefore only one curve like b need be obtained (for zero initial error). This approximation may well be the case for an airplane controlled by a very effective autopilot - and combined with every point of a to yield the envelope c. The overall probability is then $P = P_1P_2$. We will only deal with the probability due to turbulence in the next section.

One of the key aspects in the safety analysis is a definition of the landing-approach-flare outcomes. This was accomplished in Table V (from [18] with some modification). Figure 32 shows an approach-flare outcome.
tree for the longitudinal case which indicates how aircraft dispersions can lead to the various approach-flare outcomes listed in Table V.

6.2 Consideration of the Critical Outcome

In practice, more than one of the basic outcomes listed in Table V may occur during an approach-flare. For example, a hard landing and running off the runway could both occur on the same landing. However, for such a case one might still consider this to be a single accident because it is not surprising to find that an aircraft ran off the runway after a particularly hard landing (where the undercarriage failed). Thus the 'critical' outcome can be considered to be a hard landing (in this case) even though another basic outcome also actually occurred. On the other hand, some combinations of basic outcomes deserve joint consideration if they occur on the same approach-flare. Reference [18] made a detailed analysis of this possibility. We can separate the outcomes into longitudinal and lateral and only the former will be considered in this study. Also we will not consider the joint outcome case. It is decided that the hard landing is considered to be the critical outcome. If necessary, the aborted-landing may be used for comparison purposes.

It is pertinent, at this point, to cite some materials of accident and incident statistics for several years in the U.S.A., as shown in Appendix IX, which indicates the relative likelihood of the various outcomes. From this statistical information we could learn which outcomes were most important and which ones were of least importance. It is felt that choosing hard landing as a criteria for the autoland case is reasonable, though the statistics showed the undershoot is prior to hard landing. This is because the results were for all kinds of landings - including both a manual and automatic operation - and it is believed that the probability of occurring short landing is less than that of hard landing in the autoland.
6.3 Calculating Missed Approach Probabilities

Each of the probabilities of interest can be represented by an area under a probability density distribution curve. For Gaussian distributions, the probability density function is

\[ p(x) = \frac{1}{\sqrt{2\pi} \sigma_x} \exp \left\{ - \frac{(x - \mu_x)^2}{2\sigma_x^2} \right\} \]  \hspace{1cm} (6.1)

and the cumulative distribution function [the area under \( p(x) \)] is represented by

\[ F(x) = \frac{1}{\sqrt{2\pi} \sigma_x} \int_{-\infty}^{x} \exp \left\{ - \frac{(\tau - \mu_x)^2}{2\sigma_x^2} \right\} d\tau \]  \hspace{1cm} (6.2)

where \( \sigma_x \) is the standard deviation or variance of \( x \) and is exactly the rms value \( \sigma_x = \sqrt{\sum_i} \); \( \mu_x \) is the mean deviation.

To determine the probability of interest, only three parameters are required:

(i) the mean value of the pertinent variable;
(ii) the rms deviation from the mean;
(iii) the critical limits that define the accident.

The example in Fig. 33 shows graphically how these quantities affect the probability of a landing interest event. The cumulative probability \( F(x) \) can be looked up in a table or it can be computed by (6.2) using a digital computer.

For the hard landing we may find (assumed Gaussian distribution)
\[ p_{h\lambda} = F \left( \frac{\mu_{h_{TD}} - \dot{h}_{\lambda}}{\sigma_{h_{TD}}} \right) \]  

(6.3)

where \( \mu_{h_{TD}} \) is the descent rate at touchdown due to mean shear; \( \dot{h}_{\lambda} \) is the maximum allowable descent rate at touchdown due to turbulence. For the other distribution, one can also obtain the interest probability level by integrating to find the area under a specified probability function curve.

The probability of a successful approach - within the Cat. II window is obtained as follows:

The Cat. II window is defined by FAA, as given in Fig. 34. This window defined two sets of limitations relating to longitudinal and lateral respectively as

**Longitudinal**

\[ |\Delta U| < U_{\lambda} \quad U_{\lambda} = 2.58 \text{ m/s} \]
\[ |\Delta d| < d \quad d_{\lambda} = 3.66 \text{ m/s} \]
\[ |\Delta h| < \Delta h_{\lambda} \quad |\Delta h_{\lambda}| = 1.7 \text{ m/s} \]

where \( U_{\lambda} \) is the limit of the airspeed dispersion at \( h_d \);
\( \dot{h}_{\lambda} \) is the limit of descending rate at \( h_d \);
\( d_{\lambda} \) is the limit of the glideslope dispersion at \( h_d \);
\( h_d \) is the decision height, 30m for Cat. II.

**Lateral**

\[ |\Delta y| < y_{\lambda} \quad y_{\lambda} = 21.95 \text{ m} \]

where \( y_{\lambda} \) is the limit of lateral position error at \( h_d \).

Note that the constraint of \( \Delta h_{\lambda} \) added to the longitudinal window was suggested in [35]. It is not contained in the original FAA Cat. II window.

Assuming the probability distribution for outside the window is
Gaussian, let us derive the outcome at \( h_d \). The probability of being outside the lateral limits of the Cat. II window is given by the area under the tails of the lateral displacement distribution that falls outside the window limits. Thus,

\[
P_{\text{out-lat}} = F \left( \frac{y_l + \mu_y}{\sigma_y} \right) + F \left( \frac{y_l - \mu_y}{\sigma_y} \right)
\]  

(6.4)

The probability of being outside the longitudinal window is a little more complicated in that it involves three events: altitude, airspeed and descent rate deviations. The probability theory gives the formula for calculating the overall probability of an event of a set as (see [93]):

\[
P(S) = \sum_{i=1}^{n} P(A_i) - \sum_{i<j=2}^{n} P(A_iA_j) + \sum_{i<j<k=3}^{n} P(A_iA_jA_k) + \ldots
\]

\[
+ (-1)^{n-1} P(A_1A_2 \ldots A_n)^*
\]  

(6.5)

where

\[
S \triangleq \bigcup_{i=1}^{n} A_i \triangleq (A_1 \cup A_2 \cup \ldots \cup A_n)
\]

In the present case we define

\[
E \triangleq (A_1 \cup A_2 \cup A_3)
\]

* An example of the meaning is given:

\[
\sum_{i<j<k=3}^{4} P(A_iA_jA_k) = P(A_1A_2A_3) + P(A_1A_2A_4) + P(A_1A_3A_4) + P(A_2A_3A_4)
\]

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as the aircraft being inside the window, where $A_1$, $A_2$ and $A_3$ are defined as the three events of altitude, airspeed and descent rate deviations respectively. Furthermore, it is assumed that $A_1$, $A_2$ and $A_3$ are independent of each other; i.e., $P(A_iA_j) = P(A_i)P(A_j)$. Thus we have

$$P(E) = P(A_1) + P(A_2) + P(A_3) - P(A_1A_2) - P(A_1A_3) - P(A_2A_3) + P(A_1A_2A_3)$$

(6.6)

Explicitly,

$$P_{out-long} = F\left(\frac{d\ell + \mu_d}{\sigma_d}\right) + F\left(\frac{d\ell - \mu_d}{\sigma_d}\right) + F\left(\frac{u_\ell + \mu_u}{\sigma_u}\right) + F\left(\frac{u_\ell - \mu_u}{\sigma_u}\right)$$

$$+ F\left(\frac{\dot{\ell} + \mu_{\dot{\ell}}}{\sigma_{\dot{\ell}}}\right) + F\left(\frac{\dot{\ell} - \mu_{\dot{\ell}}}{\sigma_{\dot{\ell}}}\right) - \left[ F\left(\frac{d\ell + \mu_d}{\sigma_d}\right) + F\left(\frac{u_\ell + \mu_u}{\sigma_u}\right) + F\left(\frac{u_\ell - \mu_u}{\sigma_u}\right) \right]$$

$$+ F\left(\frac{d\ell - \mu_d}{\sigma_d}\right) \left[ F\left(\frac{u_\ell + \mu_u}{\sigma_u}\right) + F\left(\frac{u_\ell - \mu_u}{\sigma_u}\right) \right] - \left[ F\left(\frac{d\ell + \mu_d}{\sigma_d}\right) + F\left(\frac{d\ell - \mu_d}{\sigma_d}\right) \right]$$

$$\left[ F\left(\frac{\dot{\ell} + \mu_{\dot{\ell}}}{\sigma_{\dot{\ell}}}\right) + F\left(\frac{\dot{\ell} - \mu_{\dot{\ell}}}{\sigma_{\dot{\ell}}}\right) \right]$$

$$+ F\left(\frac{u_\ell + \mu_u}{\sigma_u}\right) + F\left(\frac{u_\ell - \mu_u}{\sigma_u}\right) \left[ F\left(\frac{\dot{\ell} + \mu_{\dot{\ell}}}{\sigma_{\dot{\ell}}}\right) + F\left(\frac{\dot{\ell} - \mu_{\dot{\ell}}}{\sigma_{\dot{\ell}}}\right) \right] - \left[ F\left(\frac{\dot{\ell} + \mu_{\dot{\ell}}}{\sigma_{\dot{\ell}}}\right) + F\left(\frac{\dot{\ell} - \mu_{\dot{\ell}}}{\sigma_{\dot{\ell}}}\right) \right]$$

$$\left[ F\left(\frac{u_\ell + \mu_u}{\sigma_u}\right) + F\left(\frac{u_\ell - \mu_u}{\sigma_u}\right) \right]$$

$$\left[ F\left(\frac{\dot{\ell} + \mu_{\dot{\ell}}}{\sigma_{\dot{\ell}}}\right) + F\left(\frac{\dot{\ell} - \mu_{\dot{\ell}}}{\sigma_{\dot{\ell}}}\right) \right]$$
The combined probability of a successful Cat. II landing approach is to combine the above results, assuming that longitudinal and lateral deviations are independent,

\[ P_{MA} = P_{out-long} + P_{out-lat} - P_{out-long} \cdot P_{out-lat} \]  \hspace{1cm} (6.8)

and

\[ P_{\text{successful approach}} = 1 - P_{\text{MA}} = (P_{SA-long})(P_{SA-lat}) \]  \hspace{1cm} (6.9)

where

- \( P_{MA} \) is the probability of missing the Cat. II window;
- \( P_{SA} \) is the probability of successful approach (within the Cat. II window).

6.4 Calculating the Hard Landing Probability of Touchdown

As presented in the previous chapter, the covariance matrix during landing is determined by the driving function which is related to the external disturbances as well as the system's characteristics. Using the proposed model, the atmospheric disturbances in the planetary boundary layer are fixed by a specified mean wind speed at the reference height, 10m. For a particular case corresponding to a given mean wind speed, the obtained probability of interest is conditional probability and bears no relation to the actual landing approach-flare hard landing probability except when the
wind conditions are assumed. Therefore, a further sophistication will be introduced here to account for the distribution of wind levels. That is, the 'overall failure probability' is defined which will be more meaningful in terms of long-time nationwide or worldwide averages. This overall probability is preferred over the conditional probability as a performance index for comparison of systems, effects of operation environment, effects of certification criteria, etc., because it makes possible a quantitative assessment of relative system merit. The system comparisons should be made on the basis of overall probabilities in order to make differences between systems or different comparison conditions assessable in meaningful terms. However, the above argument does not necessarily mean that the conditional probability is of no use. Actually, it is significant in the aircraft design when the probability of exceeding a certain level of certification criteria, such as certain load values under a specific wind speed is used as a criterion.

Let each landing be characterized by touchdown sink rate $\dot{h}$ and by a reference wind speed $w$. The joint distribution function of ($\dot{h}$ and $w$) is denoted by

$$ p(\dot{h}, w) \quad (6.10) $$

The distribution functions of $\dot{h}$ and $w$ are, respectively,

$$ P_\dot{h}(\dot{h}) = \int_0^{w_L} p(\dot{h}, w)dw \quad (6.11) $$

where $w_L$ is the limiting reference wind speed, corresponding to 'no flight operation'. For each $w$ we can calculate $p(\dot{h}, w)$, then for each $w$, the hard
landing probability is

\[ p_f(w) = \int_{\hat{h}_l}^{\infty} p(\hat{h}, w) d\hat{h} = \text{probability} \{ \hat{h} > \hat{h}_l \} \quad (6.12) \]

where \( \hat{h}_l \) is the limiting touchdown rate, corresponding to an acceptable rate. Therefore, the overall probability of hard landing is calculated by

\[ p_f = \int_0^{W_f} p_f(w) dw = \int_0^{\hat{h}_l} \int_{\hat{h}_l}^{\infty} p(\hat{h}, w) d\hat{h} \ dw \quad (6.13) \]

To obtain the joint distribution of (\( \hat{h} \) and \( w \)), we need to find the individual probability.

Let the solution for a given \( w \) yield a probability for \( \hat{h} \) denoted by \( q(\hat{h}|w) \) such that

\[ \int_{-\infty}^{\infty} q(\hat{h}|w) d\hat{h} = 1.0 \quad (6.14) \]

Since the fraction of all flights in range \( dw \) is \( p_w(w) dw \), which is given by the Weibull distribution, then it follows that, according to the Conditional Probability Theorem,

\[ p(\hat{h}, w) = p_w(w)q(\hat{h}|w) \quad (6.15) \]

With (6.15) and (6.13) it is now a simple matter to compute the overall hard landing probability at touchdown. At this moment, it is pertinent to
comment on the term "hard landing" used in this report. It means that the sink rate at touchdown exceeds the defined value \( \dot{h}_L \). According to the values of \( \dot{h}_L \) which define the exceeding probability, "hard landing" may be classified into uncomfortable landing (|\( \dot{h}_L \)| value is small), hard landing (|\( \dot{h}_L \)| is moderate), and a landing that includes damage or crash (|\( \dot{h}_L \)| very high). We define |\( \dot{h}_L \)| = 3.82 m/s as hard landing in the later analysis.
CHAPTER VII

EXAMPLE CALCULATIONS AND DISCUSSIONS

As an example to demonstrate the proposed algorithm, the simulation of a B-747 type jet transport landing was carried out using a digital computer. The results are presented herein, and discussions are also made.

7.1 An Overview of the Computation Procedure

The last chapter showed that the quantities required to compute the desired probabilities are the means and rms deviations from the means for certain of the state variables. The linearity assumption of the system allows one to calculate those two values separately: the mean values of the state variables are computed from deterministic inputs, and the rms values or covariance matrix from zero-mean input - Gauss-Markov process - as indicated in Fig. 36. All the calculation cases are defined in Table VI.

7.2 Aircraft Dynamics and Autoland Systems

The aircraft dynamics used in this study are a large jet transport which has four jet-engines, in the approach configuration at a reference speed of 73.06 m/sec. Table VII gives the characteristics of this airplane.

The covariance matrix of aircraft deviations is computed during the approach and flare-out phase, starting at an altitude of about 50m down to touchdown. The reference glideslope is taken to be 3° resulting in a nominal descent rate of 3.82 m/sec.

The autoland system which includes glideslope tracking mode and autoflare mode is given in Chapter III. The former is seen in Fig. 8 and is characterized by Eqs. (3.1) and (3.2). The latter is illustrated in Fig. 11 and the gain matrices $K_1$ and $K_2$ resulting from RCDS technique are employed to the autoflare system. The control law for flare is represented by Eqs. (3.5) and (3.6), however the gains of $K_{U_1}$, $K_{\dot{H}}$, and $K_{n z}$ are assumed zero for
convenience. In the numerical example, the elevator and throttle actuator dynamics were included by selecting the suitable time constant matrix,

$$I = \begin{bmatrix} 2.0 & 0 \\ 0 & 0.15 \end{bmatrix}$$

All the ILS observation noise is neglected. This assumption was verified to be reasonable [18]. In the glideslope tracking mode, as seen in (3.2), the deviations $d$ (≡ $Δh$) relative to the nominal ILS glide path are used. The ILS installation, however, yields angular displacements as the observed variable. Thus, the effective gain $K_d$ (also $K_d^*$, $\bar{K}_d$), for example, is expressed in degrees with decreasing range to touchdown. In practice, this effective change in gain is compensated for by a range-to-go dependent function.

For the purpose of simplifying the calculation, we choose the same control law for both approach and flare phases with different gain values. This means that the approaching model is only tracking airspeed (via auto-throttle), attitude and height. It may affect the final range dispersion, but little affect the most interested value of touchdown sink rate.

7.3 Atmospheric Disturbances (Turbulence and Mean Wind)

The reference wind profile is assumed to be the power form and three terrain parameters were chosen as $η = 0.16$, 0.28, 0.40 representing quite different surface roughness. For each category terrain, wind speeds at the reference height 10m were assumed varying from 1.5 m/s to 20.0 m/s. The Canada nationwise Weibull distribution parameters were used to approximately represent the worldwide distribution; they are {C, K, $F_o$} = {4.7933, 1.6539, 0.1062} corresponding to annual hourly mean [94]. In the present study a
ten-minutes average time was used. Therefore the correction for average
time was made by the theory proposed in Chapter IV. The correction factor
wrt three terrains are then

\[
\kappa = \begin{cases} 
0.8559 & (\eta = 0.16) \\
0.7701 & (\eta = 0.28) \\
0.7021 & (\eta = 0.40) 
\end{cases} \quad i = 1, 2, 3
\]

and consequently three corrected Weibull distribution parameter sets were
obtained by \( \{C, K, F_0\}_i^* = \kappa_i \{C, K, F_0\} \) \( i = 1, 2, 3 \) where \( \{\cdot\}_i^* \) = corrected
set relating to the i-th terrain parameter.

The changes of integral scale lengths and intensities of the horizontal
and vertical turbulence with altitude were modelled according to Chapter IV,
4.3.3, as seen in Fig's 24 and 25. To account for the gust gradients and to
obtain a more reasonable average velocity distribution of the turbulence,
the four-point model is used. The geometrical parameters are chosen as
\( b' = 0.85b = 50.696m \) (effective span); \( l_t = 32.43m \) (effective tail arm);
\( \eta = 0.30 \) (the tail velocity weighting factor which reflects the relative
importance of the velocity at tail to the whole velocity distribution). The
glide path reference frame \( \vec{F}_G \) was used. Details are given in Appendix V.

7.4 Ground Effect

The linearized ground effect influence coefficients are seen in
Appendix II, which was only used for the calculation of the covariance
matrix in flare. In calculating the response of the airplane to mean wind
shear, the ground effect was considered to be an external input which does
not lump with the state vector, instead as an independent vector inputting
into the rhs of the state equations, as used in [46].
7.5 The Unperturbed Automatic Landing

The nominal automatic approach-flare path or state vector are those presented in [46]. The glidepath is a rectilinear making 3° with the horizontal, of which the reference airspeed is 73.06 m/s. The flare-out starts at the wheel height of 15m above the runway. It takes 8.5 sec to reach the ground. Assuming the CG is at a distance of 3m above the wheels in vertical plane, in terms of the CG height the aircraft actually begins the flare at 18m, then stops at 3m. The speed is assigned to be monodecreasing as approaching the ground by about 10%V_s. The nominal touchdown sink rate is of 0.61 m/s. In the diagram of Fig. 36 the unperturbed automatic landing trajectory is presented. The autopilot modes are changed at the moment when the flare begins.

7.6 Response to Mean Wind Shear

Three cases corresponding to different terrains were conducted for each reference wind speed. The mean wind profiles with different thickness are shown in Fig. 37. It is clear that the wind shear increases as height is reduced. A typical disturbed flight path by mean wind shear is given in Fig. 38. Table VIII presents the resulting dispersions of descent rate at touchdown, i.e., the \( \mu_h \). Results show the linear tendency between \( \mu_h \) and \( W_{ref} \) within the range 0 to 10 m/s. The nonlinearity, however, increases as the mean wind exceeds that range. It is also indicated that the effect of the terrain parameter on \( \mu_h \) is significant. The airplane landing through a very rough, wavy area will be subject to a harder landing compared to that of landing on flat terrain, though the reference wind speed is the same.

The computer codes for calculating the means are given in Appendix X.
7.7 Response to Turbulence

The covariance matrix during the landing approach-flare was calculated using the **Constrained Correlation Technique** as described in Chapter V, 5.3.2 and in Appendix VII. The detailed numerical calculation is outlined herein. The principal base for carrying out this calculation are Eqs. (5.39) through (5.41) and the core for obtaining the covariance matrix is in finding the constrained correlation matrix $C_{ww}(t, \tau)$ and the system transition matrix $e^{-At}$.

7.7.1 Reference Flight Path

In evaluating $\Sigma(t)$, a reference flight trajectory is needed; actually, turbulence is always associated with mean wind. Thus the reference flight path should be the actual trajectory subjected to mean wind. In order to simplify the calculation we take the unperturbed nominal landing approach-flare path as the reference trajectory. In fact, this implies that the 'perturbation on a perturbation' is neglected. This assumption would not produce significant errors because ILS maintains the perturbed flight path very close to the desired predescribed trajectory except for the extreme wind speed.

7.7.2 Constrained Correlation Matrix

To calculate $C_{ww}(t, \tau)$, the von Kármán model, as shown in Chapter IV, Eqs. (4.42) and (4.44) were utilized. In these equations, the modified (or hyperbolic) Bessel function $K_{1/3}(\zeta)$ and $K_{2/3}(\zeta)$ are included. An approximation to those special functions is given in Appendix XI, and the associated computer program is included. Figure 39 illustrates that these functions (order 1/3 and 2/3) vary with argument. These functions can be precalculated then stored off-line then referred to when using. Alternatively they can be calculated on-line.
The calculation of the constrained correlation matrix is based on a modified von Kármán model [42] which reads

\[ C_{ww}(t, \tau) = \sigma_a^2 \sigma_\beta^2 C_{ww}(t, t_\alpha, t_\beta; L_\beta) \] (7.1)

where \( \tau = t_\beta - t_\alpha (\beta > \alpha) \), \( t_\beta \) and \( t_\alpha \) are the times corresponding to the aircraft reaching those points of \( \alpha \) and \( \beta \) fixed on the reference flight path. \( \sigma_\alpha \) and \( \sigma_\beta \) are the turbulence intensities at point \( \alpha \) and \( \beta \) respectively; \( \sigma_\alpha \sigma_\beta = \sigma_\alpha \sigma_\beta \omega_\alpha \omega_\beta \), where \( \sigma \) is relative intensity. \( C_{ww}(\cdot) \) denotes the von Kármán correlation formula \( C(\xi) \). \( L_\beta \) denotes single value of \( L \) to be used for a reasonable fit to the experimental data.

Reid [42] suggested that if

1. using a scale \( L_\alpha^X \) corresponding to the lower probe location and taking \( L_\alpha^X \) varying for \( C_{uu} \) fit and \( L_\alpha^X = 59.13 \text{m} \) (constant) for \( C_{ww} \) fit;
2. in making nondimensionalization of \( C_{ww} \) to produce \( C_{ww}^* \), \( C_{ww}^* \) being divided by the product of the rms of turbulence velocities measured at the two probe locations;

these two conditions are satisfied, then the calculated von Kármán correlation curves are best fit.

Here we adopt the above suggestions. Also we use the relation of \( L_\beta = 2L_\omega^X \) for calculating \( C_{qq} \) and \( C_{ww} \). Note that when carrying out the integration to yield the covariance matrix, the integration domain is shown in Fig. 40. The larger the \( t_\beta \) the more computer time is required.

The Reid's model was verified to be correct only at the height above 100' and for STOL aircraft. However, it would not be unreasonable that that modified von Kármán atmospheric turbulence model accounting for the nonisotropy property in the vertical plane can be extended to below 100'
without significant errors. Also, for shallower glide slopes, such as the 3° used herein, the approximation suggested by Reid should be even better.

### 7.7.3 Covariance Matrix

As stated in Chapter V, the problem for computing the covariance matrix is discretized in time, that is, the desired covariance can be calculated by successively computing the responses of a stepwise changing system, the properties of which are set constant between successive steps. For the transition matrix it can be considered time-invariant as the aircraft flying above the height where flare begins, because we have chosen the 3° rectilinear glideslope with constant airspeed as the reference trajectory, while during flare the system dynamics and disturbance are all time-varying.

The method shown in Appendix VII had been employed to solve the covariance matrix in this research. Checking runs were carried out to compare the results by using another method. The results obtained by the present method were compared with those obtained by the power-spectral-density method for horizontal flight in homogeneous isotropic turbulence. It showed a substantial agreement. Appendix XII gives the computer codes for calculating the covariance matrix.

The calculation cases are shown in Table VII. The results are shown in Fig. 41 and Table IX and X. Cases 1 to 7 corresponding to $\eta = 0.16$ are considered to be the base line for the purpose of making comparisons. The data listed in Table IX can be used directly to compare the effects of the three terrain conditions.

We see that the rms of sink rate at touchdown for the case $\eta = 0.16$ are all acceptable even with reference wind level as high as 15 ~ 20 m/s. As $\eta$ increases to 0.28, the touchdown sink rate increases, in particular, at very high wind level ($w > 10$ m/s) the rms corresponding to the same mean winds is increased by double. The higher the $\eta$ value, the larger the increment.
This preliminarily reveals that the terrain parameter plays a very significant role in causing hard landing in wind environment.

Table X and Figure 41 demonstrate the propagation of the covariance matrix of the state vector and the sink rate. It shows that all the state variables are increasing as the airplane approaching to the ground prior to the flare. As starting flare at 18m, the dispersions are all beginning to reduce except the range erro. This is due to the autoflare system tracking a specified exponential flight path very well. In the present study, owing to using a not real glide slope tracking autopilot for the approach control system and using no range-erro feedback in the autoflare system, the range dispersion was monotonously increasing since landing began. Even so, the total range dispersion at touchdown in the effect of the maximum likelihood wins speed was less than a half length of the tested airplane shown in Fig. 41 (c) which gives a comparison to the airplane scheme in nearly the same scale as $\Delta x$.

It is would be necessary to make a comment concerning the sink rate dispersion time history shown in Fig. 41(b). As may be seen, the initial value of $h$ is surprisingly nonzero which is quite different from all the other state variable dispersions. The reason is in that $h$ was not included in the state vector but was treated as an observed variable (as an output of the system). $h$, in the presence of winds, is not only related to state variables but also to the external disturbances; in particular, it includes vertical wind. In the present study, the nonzero wind (turbulence) initial conditions were taken into consideration so that the nonzero $h$ in the initial time occurred. No matter what initial $h$ value could be, however, the sink rate would be always reduced during the descent due to the control of the automatic landing system.
7.8 Probability of Hard Landing

Having obtained the mean values due to mean shear and the covariance
matrix due to turbulence, the hard landing probability then can be
calculated by means of Eqs. (6.13) and (6.15). We are mainly concerned with
the touchdown sink rate which is taken as a critical outcome at touchdown in
the symmetric plane. Figures 42 to 48 illustrate the overall hard landing
probability varying with reference wind speed or some limited values. Table
XI contains those data plotted on the figures.

Figure 42 shows that the cumulative overall hard landing probability
against the reference wind speed and shows the effect of roughness of the
landing area. Figures 43 through 45 show how the average time $T_{av}$, the
terrain parameter $n$, the limit of sink rate $S_{lim}$ affect the overall hard
landing probabilities. The sensitivities of the allowable maximum reference
wind $w_{x}$ for safe flight, which corresponds to the overall hard landing
probability less or equal to $10^{-5}$ to the terrain feature and load limit are
also given in Figures 46 and 47. The criterion $10^{-5}$ means that if an
airplane makes an average 4 landings per day for 365 days per year, i.e.,
1460 landings per year, then a probability of $10^{-5}$ represents one hard
landing every 68 years.

Before making further analysis the hard landing probability, we need to
have a discussion concerning $\Delta h_{TD}$. As stated in Chapter V that the RMS
values obtained using the approach proposed in this study are measured from
the nominal flight path. So are the hard landing probabilities. Actually,
for the perturbated flight path, $\Delta h_{TD}$ corresponds to the point at which
the actual flight path penetrates through a plane which is intersected with the
nominal flight path, as seen in Fig. 29. The true hard landing for any
actual flight path realization takes place at the point where it touches the
ground. To obtain the ensemble average of the true hard landing for all the realizations one has to execute a great number of testing runs which would be of too complicated and too expensive to be achieved in practice. A realistic approach seems to use the method suggested in this study to calculate the hard landing probability on the target plan instead of on the horizontal plane. Because we are dealing with the automatic landing problem, the deviation of the state vector on the two planes would not be too large. Hence, this approximation is reasonable for the purpose of engineering applications.

In any case, the sensitivity of $p_f$ to variations in $\eta$, $w_x$ and $h_x$ would be expected to be correctly shown by the present method.

7.8.1 Overview of the Overall Hard Landing Probability

Figures 42(a) through 42(c) illustrate how the overall hard landing probability is growing up as the allowable maximum operation wind $w_x$ is increasing. Fig. 42(a) is the base line results - the large size CTOL airplane landed on a flat terrain airport. The criteria of hard landing was specified to be 3.823 m/s which is equal to the descent rate in the nominal glide slope. This limit sink rate value may not be considered as the limit of the structure strength rather an uncomfortable touchdown. If the maximum allowable load limit would be taken into consideration, the failure probability caused by winds should be much less than that shown in Fig. 42(a). Figures 42(b) and (c) are prepared for checking how the terrain parameter $\eta$ affects the overall hard landing probability of the same aircraft. Examining Fig. 42(a), it is showed that the probability increases steeply in the region of small reference winds, e.g., less than 4 m/s. As the reference wind increasing, the probability curve flattens gradually. As long as the mean wind exceeds 10 m/s, the overall probability becomes
insensitive to wind speed. For instance, as the mean wind speed increases from 2 m/s to 4 m/s, the probability increases about 10 decades; in contrast, as the mean wind speed from 6 m/s to 12 m/s, the probability increases only by 5 decades. Nevertheless, the strong winds always reduce the landing safety significantly. Viewing Figures 42(b) and 42(c), we see the similar tendency of the probability varying with wind speed. However, the values of the probability corresponding to the same wind speed are increased by 1 - 2 decades due to the effect of roughness on the landing area. We will discuss the results in detail in the consequent text.

7.8.2 Effect of the Averaging Time in Wind Statistics

As seen in Fig. 43, the averaging time, which affects the Weibull distribution parameters, as discussed in Chapter IV, does not affect the hard landing probability very much. For example, in the baseline case the difference in \( p_f \) corresponding to one hour mean and 20 minute mean is only about 35%. The operation limit \( W_\infty \) is almost no different. The results show that the hard landing probability increases as the averaging time increases (from \( 5.6 \times 10^{-5} \) to \( 7.2 \times 10^{-5} \) as \( T_{av} \) from 10 min. to 60 min.). This is expected because shortening the averaging time means that the truncated frequency will be lower; consequently, a part of the energy of low frequency turbulent flow is ignored.

It is recommended that one may not have to make an averaging time correction in the safety analysis, just simply using the 1 hr statistic values of the Weibull distribution parameters.

7.8.3 Sensitivity of Hard Landing Probability to Terrain Feature

In order to make a comparison, we pick up the probability values corresponding to the reference wind being 5 m/s, 10 m/s, 15 m/s and 20 m/s
respectively, and keep the parameters constant except those for comparison. The calculation results are given in Fig. 44. It shows that as \( \eta \) increases from 0.16 to 0.40, when \( W = 10 \text{ m/s} \sim 20 \text{ m/s} \), the probability of hard landing increases by about two decades; while the wind speed becomes small, e.g., 5 m/s, the corresponding increment is over 7 decades. This implies that an airplane landing over a city or terrain with an equivalent roughness of \( \eta = 0.40 \) would be much more likely to destroy its undercarriages than if landing over a smooth surface airport. This is due to the airplane encountering different atmospheric boundary layer structures. Over the high roughness area the mean shear is severe, and the turbulence intensities are large as well. From Table VIII we see that in the case of \( W = 10 \text{ m/s} \), as \( \eta \) increases from 0.16 to 0.40, the dispersion of \( \hat{h} \) at touchdown due to mean shear only changes a little: from -1.009 to -1.06 m/s. This implies that the turbulence plays a very important part in causing accident/incident. This seems to confirm the assertion that a given strong wind shear is a single realization of mean wind gradient plus turbulence. It is also revealed that a pilot should be very careful to handle landing when he changes his landing airport from a smooth surface to a new area near a city, because in this case the hard landing probability is very sensitive to the terrain feature.

In Figure 44, the shaded line is assumed the safety margin \( p_f < 10^{-5} \). We see that for a given airport, the operational maximum allowable wind is fixed. On the flat airport, much higher wind would be allowable than that on a very rough site.

### 7.8.4 Sensitivity of Hard Landing Probability to Allowable Maximum Sink Rate

Figure 45, which corresponds to the baseline case and only varies the sink rate limit values, shows that as \( |\hat{h}_x| \) changes by about twice, from
2.0 m/s to 4.0 m/s, the hard landing probability is drastically reduced by about 3 decades! It also shows that the relationship between $p_f$ and $|\dot{h}|$ is a hyperbola-like curve. The U.S. airworthiness standards [95] give a limit descent velocity of 10 fps (3.05 m/s) at the design landing weight for the specification of ground loads. This limit, for the airplane used in this study, corresponds to an overall hard landing probability of $1.0 \times 10^{-4}$ for $W_{ref} = 10$ m/s. This means that if the airplane is allowed to fly in the wind environment up to 10 m/s at 10m height then it might have a possibility of 1:10000 per landing to encounter hard landing (destroy the landing gear or a very uncomfortable landing for passengers). However, if the structure engineer exerts every effort such that the aircraft can be subject to a higher descent rate, say increasing about 15%, i.e., 3.56 m/s, then the hard landing probability will reduce to $1.9 \times 10^{-5}$, almost 10 times. This great benefit is only at the little expense of strengthening the structure or improving the quality of the materials. It is also obvious that the more great benefit could be achieved with the increased allowable maximum sink rate over 3.8 m/s due to the gradient of the curve becomes much steeper in the high value region of $\dot{h}$.

7.8.5 Sensitivity of Flight Operation Limit to $\eta$ and $\dot{h}_l$

Here, the value of $W_\perp$ is defined as a flight operation limitation which corresponds to a hard landing probability of $10^{-5}$, if exceeding $W_\perp$ than no flights are supposed to be allowed. This limitation is different from airport to airport, from airplane to airplane. The sensitivity of $W_\perp$ to the terrain feature, to the maximum allowable descent rate is demonstrated in Figures 46 and 47. Figure 46 indicates that the terrain feature $\eta$ affects $W_\perp$ very much, as the landing site shifts from a nominal airport to a rough area, the $W_\perp$ value would be reduced by 3.5 times. For a flat airport, the
flight operation limit is about 11 m/s. It is shown from Fig. 47 that increasing $|\dot{h}_x|$ is the most promising way in improving the landing safety. As seen in this figure, when $|\dot{h}_x|$ increases from 2.4 m/s to 5.0 m/s, $W_\dot{h}$ is increased about 3.3 times.

Figures 46 and 47 can be employed as a boundary diagram to determine or check the operation limit for a given aircraft and a specified airport to satisfy certain safety requirements. Also, it may serve as an airplane design or aircraft operation aid.

7.9 Sensitivity of the Aircraft Response to Feedback Gains of the Auto-
land System

Cases 45 and 46 were run to check the sensitivity of the response to turbulence to the automatic control system's feedback gains.

In a sense, varying particular gain value may not be useful in assessing the whole control system. Therefore, these two cases were designed to test two extreme conditions. Case 45 was for the situation when all the feedback gains of the automatic landing system were enlarged by double. Case 46 was for the opposite situation - halving the feedback gains. The tested results are presented in Table IX. Comparing those outcomes with the normal case we find that as the gain values increasing by double, the $\sigma_u$ doesn't change much, but $\sigma_h$ and $\sigma_{\dot{h}}$ have been reduced by about 20% ~ 30% respectively. In general, the changes of the RMS value of the state vector are minor in the case of increasing feedback gain values. In contrast, halving the gain values deteriorates the control quality significantly; the RMS values increase by about ten times! (except $\sigma_u$ and $\sigma_x$). For example, the touchdown sink rate was 2.989 m/s (in baseline case $\sigma_h = 0.325$ m/s). These results imply that the aircraft with a control system which has very high feedback gains is favourable in automatic landing.
through the turbulent boundary layer.

7.10 Discussion on Simplifying the Computation of the Covariance Matrix

For the case that the system with non-white noise inputs, in general, the calculation of covariance matrix in time domain is time-consuming. The algorithm for solving the covariance matrix proposed in this study is based on the constrained turbulence correlation matrix. In solving the response problem, we actually deal with a kind of convolution problem in double integration form in which the upper limits are the time measured from the landing begins. The total integrating steps are

$$S = \sum_{i=1}^{N} i^2 = \frac{N(N + 1)(2N + 1)}{6} \quad (7.2)$$

where $N = \frac{T}{\Delta t}$

- $T$ - total landing time
- $\Delta t$ - the interval of the integration.

We see that $S \propto N^3$. Reducing $N$ will lead to a substantial diminution of $S$, in particular, $N$ is large. Because the correlation transients drop off rapidly during first few seconds, also, the airplane responses to turbulence are quite fast in the short period mode, the interval $\Delta t$ should be chosen such that a reasonable accuracy can be achieved. In general, $\Delta t < 0.25$ sec. For a landing lasting 1 min. such an interval leads to quite a long integration execution (if equal interval is used in the calculation). Actually after the first few seconds the correlation function will be died out, even the aircraft's dynamics is still active, the following integration only has little contribution to the final results. Therefore, to reduce the calculation time, we need to cut-off the upper limit; or alternatively, one may use variable interval in the integration, which might cause some
inconvenience in the computation. In this study, a cut-off upper limit algorithm (equal interval) is used. It is based on the rapid convergence property of the correlation time history as observed in Figs. 20(a) and 20(b) in Ref. [4]. That is clear that as the argument $\frac{\xi}{\alpha L}$ becomes greater than 3 the nondimensional correlation function drops off close to zero. Assuming we choose $K_c = \frac{\xi}{\alpha L}$ as the integration upper limit cut-off criteria, i.e.,

$$\xi = 1.339 K_c L$$  \hspace{1cm} (7.3)

where $L$ = the turbulence scale, $L_u^X$ or $L_w^X$,

$\xi$ = the distance between two points along the flight path.

Equivalently, the integration upper limit is from $t_0$ to

$$t_u = t_0 + 1.339 K_c \frac{L}{V_e}$$  \hspace{1cm} (7.4)

where $t_0$ is the lower limit.

(7.3) implies that we will cut-off all the higher frequency waves which have wave lengths of $1.339 K_c$ times of the turbulence integration scale. It is expected that the ignored high frequency energy does not influence the flight path very much, which mainly depends on the low frequency energy. In addition, this cut-off wave length should be consistent with the use of guassistatic aerodynamics in this study. It shows in Ref. [4] that a reasonable estimate of the valid frequency region (longitudinal) is given by

$$\Omega_{10} = \frac{0.1}{C}$$  \hspace{1cm} (7.5)
In the present case \( \zeta = 8.324 \text{ m} \), it follows

\[ \omega_{10} = 0.01201 \text{ (1/m)} \] - wave number

which corresponds to

\[ \lambda = \frac{2\pi}{\omega_{10}} = 523.263 \text{ m} \]

In this investigation, because we use the modified von Kármán model which takes \( L_u^x = 59 \text{ m} \) (constant), with this value we obtain the \( \kappa_c = 8.8672 \). In practice, we only need to choose \( \kappa_c = 3 \sim 4 \) which corresponds much lower frequency region. This ideal has been used in this study and it showed a very beneficial merit: as \( \kappa_c = 3.5 \) being chosen, the computer CPU time for one full run was reduced to about 1/3, from 39 min./per run to 12 min./per run. The second approach to simplify the computation of the covariance is in that we observed that the RMS values of the state variables and the sink rate all showed a substantial linear relationship against the reference wind speed as seen in Fig. 48. For \( \dot{h} \), we may find an approximate linear equation by fitting the computation results which is given as follows:

\[ \sigma_{\dot{h}} = 0.09688 \ We \text{ (m/s)} \]  \( (7.6) \)

Similarly, we may find the corresponding expressions for the other state variables, but with different slopes. Unfortunately, it is impossible to obtain a similar linear relationship between RMS or sink rate and the terrain parameter \( \eta \) because of the severe nonlinearity. Nor is the relationship between the mean deviation values of the state vector and the
mean wind speed either. However, in the light of (7.6) combining with the "upper limit cut-off algorithm", we shorten the computing time a great deal. We may only need to compute one case of $W = 1 \text{ m/s}$ to obtain the covariance matrix and the RMS of sink rate. Then we easily deduce all the other results for any reference wind speed up to 20 m/s, with at least 95% confidence and at more than six times reduction of the computation time. In addition, we used the Simpson method in integrations and use the hyperbolic interpolation method in finding the values not available.
8.1 Summary and Conclusions

In this report the general equations of motion were derived and linearized. The forces and moments due to winds and their gradients were included in the state equations, which were based on Etkin's four-point model. The ground effect was also considered during the flare, based on the approximation of linearization wrt the unperturbed flight path.

The autoland system was reviewed, which was mainly based on the author's previous work, however a new technique of RCDS was applied to carry out the optimal autoflare system design.

The wind model of the planetary boundary layer for the study of landing problem was proposed based on the results available. The outstanding features of this model were in that the modified von Kármán turbulence model and the Weibull wind speed distribution model were integrated in the planetary boundary layer wind model which provides the possibility to evaluate the probability of interest. A set of nationwide Weibull parameters based on 144 locations spread across Canada was deduced. To correct the effect of the average time of the wind observation on the Weibull distribution, an approximate method was suggested.

A number of currently used techniques for predicting the aircraft response to random process inputs were reviewed. By using the modern system theory, a method for calculating the covariance matrix was developed, which dealt with the state equation that includes gust gradient terms and with nonzero initial wind conditions. Several techniques for solving the system transition matrix $e^{-At}$ were also summarized. A stepwise local stationary method was derived and used for solving the covariance matrix during the
airplane landing through the non-isotropic, nonstationary atmospheric boundary layer.

A method for predicting the overall probability of interest was developed. A numerical example for autolanding a large jet transport was presented and a sensitivity analysis was conducted which showed the effects of some parameters on the hard landing probability. Based on the results given in this research the following conclusions can be drawn:

(1) The suggested wind model of the turbulent planetary boundary layer is acceptable and feasible. Once the terrain and reference wind are available then the characteristics of winds (including mean wind shear and turbulence) are determined. Using this model one may study the problems of flying through the Earth's boundary layer. The statistical analysis then becomes possible.

(2) The constrained correlation technique for dealing with the response of a system subject to the Gauss-Markov process inputs is more reliable and realistic because less simplifying assumptions were used; it is particularly useful when the system's random inputs come from real life measurements - in flight or in the laboratory. It is also possible to reflect on the nonstationary, nonisotropy and nonhomogeneity properties and give the transient response.

(3) The sensitivity analysis showed that

(A) The response to mean wind shear and turbulence increases with increased terrain roughness, or equivalently with increased gradient and boundary layer thickness. However, the effects due to turbulence are much more profound than the mean shear. The hard landing probability of landing over a very rough terrain (e.g., $\eta = 0.40$) in a 10 m/s reference wind is as high as two decades compared to landing over a smooth surface airport. In a
relatively low wind speed condition, the difference will be even more higher.

(B) The maximum allowable sink rate at touchdown affects the hard landing probability significantly. Doubling the limit value will change the failure probability by more than 3 decades. Thus, the improvement on the structure is the most effective way to increase the safety in landing.

(C) The automatic system has considerable effects on the safety of landing. Doubling the whole feedback gain values will favourably reduce the RMS values by about 25%, however, halving the whole control system gains will deteriorate the performance significantly, in particular, the RMS of sink rate at touchdown increases by about ten times.

(D) The probability analysis framework based on both the Weibull distribution (for mean wind shear) and the Gaussian distribution (for turbulence) is considered reasonable, in particular, for very strong wind in which the deviations due to turbulence are quite large, falling within $3\sigma$ where the Gaussian distribution is valid.

The above conclusion is based on the results obtained with a particular type of aircraft, a specified autoland system, a particular flight condition, CG position, weight, etc., and a particular landing approach. It is predicted that all changes of these conditions would affect the numerical results. However it is felt that the general trends presented in this study are valid and should serve as a useful guide in choosing airport sites, in flight safety analysis, in making the certification criteria and in aircraft design (autopilot synthesis and structure design).
8.2 Recommendations

The following recommendations for future investigation are outlined which are based on the need to extend the work undertaken to cover more general, more complete situations than what has been done herein:

(1) Investigate other types of aircraft such as STOL/VTOL using the same algorithm.

(2) Study the effect of different automatic control law on the hard landing probability.

(3) Study the case in which the other critical outcomes such as the short landing probability will be included into the overall failure probability.

(4) Investigate the lateral landing performance which will also contribute to the overall failure probability.

(5) Check the sensitivity of the overall hard landing probability to the major aircraft parameters, e.g., C.G position, landing weight, etc.

(6) Further improve the computation technique in calculating the constrained correlation matrix and in solving the covariance matrix to reduce the computation time required.
REFERENCES


57. Watanabe, Akira and Horikawa, Yuso, "Simulation Study on Flare Control System by Optimization Theory" (Japanese), Japan, 1977.


<table>
<thead>
<tr>
<th>No.</th>
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Source: FAA-RD-77-169, p. 16
Table II
Aircraft Characteristics (With or Without Autopilot)

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<td>$\xi_{ph}$</td>
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sp: short period
ph: phugoid mode
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<th>$z_0 (m)$</th>
<th>Description of area within several kilometres upwind of site</th>
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<tr>
<td>$10^{-2}$</td>
<td>Centres of large towns, cities</td>
</tr>
<tr>
<td></td>
<td>Centres of small towns</td>
</tr>
<tr>
<td></td>
<td>Outskirts of towns</td>
</tr>
<tr>
<td></td>
<td>Many trees, hedges, few buildings</td>
</tr>
<tr>
<td>$10^{-1}$</td>
<td>Many hedges</td>
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<tr>
<td></td>
<td>Few trees, summer time</td>
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<tr>
<td></td>
<td>Isolated trees</td>
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<tr>
<td></td>
<td>Uncut grass</td>
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<tr>
<td>$10^{0}$</td>
<td>Few trees, winter time</td>
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<tr>
<td></td>
<td>Cut grass ($\approx 3$ cm)</td>
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<td>Natural snow surface (farmland)</td>
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<td>Large expanses of water</td>
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<tr>
<td>$10^{3}$</td>
<td>Calm open sea</td>
</tr>
<tr>
<td></td>
<td>Snow-covered flat or rolling ground</td>
</tr>
<tr>
<td>$10^{4}$</td>
<td>Ice, mud flats</td>
</tr>
</tbody>
</table>

[From ESDU 75001].
Table IV
Low-Visibility-Landing ILS Categories

<table>
<thead>
<tr>
<th>Category</th>
<th>Runway Visual Range (RVR) (ft)</th>
<th>Decision Height (DH) (ft)</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>2,400 (730 m)</td>
<td>200 (60 m)</td>
</tr>
<tr>
<td>II-A</td>
<td>1,600 (490 m)</td>
<td>150 (45 m)</td>
</tr>
<tr>
<td>II-B</td>
<td>1,200 (365 m)</td>
<td>100¹ (30 m)</td>
</tr>
<tr>
<td>III-A</td>
<td>700 (215 m)</td>
<td></td>
</tr>
<tr>
<td>III-B</td>
<td>150 (45 m)</td>
<td></td>
</tr>
<tr>
<td>III-C</td>
<td>Zero</td>
<td></td>
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</table>

¹ Sometimes called CAT II.
Table V  Considerations of Approach-Flare Outcomes

<table>
<thead>
<tr>
<th>Basic Outcome</th>
<th>Associated Performance Measures</th>
<th>Performance Metrics</th>
<th>Critical Limits (To achieve outcome)</th>
<th>Outcome probabilities (These are functions of performance metrics and critical limits)</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>Successful landing</td>
<td>Dispersions at decision height and/or reference position and at touchdown</td>
<td>$\mu_h, \sigma_h, \mu_y, \sigma_y$</td>
<td>Airplane must be within successful landing 'window'</td>
<td>$P_{OK}$</td>
<td></td>
</tr>
<tr>
<td>Successful missed approach</td>
<td>Dispersions at decision height</td>
<td>$\mu_h, \sigma_h, \mu_y, \sigma_y$</td>
<td>Airplane must be outside of successful approach window but within successful &quot;go-around&quot; window</td>
<td>$P_{MA}$</td>
<td></td>
</tr>
<tr>
<td>Short landing</td>
<td>Longitudinal touchdown location</td>
<td>$\mu_{XTD}, \sigma_{XTD}$</td>
<td>$X_{TD} &lt; X_{TDMIN}$</td>
<td>$P_{SL}$</td>
<td></td>
</tr>
<tr>
<td>Hard landing</td>
<td>Sink rate at touchdown</td>
<td>$\mu_{HTD}, \sigma_{HTD}$</td>
<td>$</td>
<td>h_{TD}</td>
<td>&gt;</td>
</tr>
<tr>
<td>Overrun runway during rollout</td>
<td>Airspeed and altitude errors at reference position</td>
<td>$\mu_E, \sigma_E$</td>
<td>$K_1\Delta h_R + K_2\Delta u_R &gt; E_{MAX}$</td>
<td>$P_{OR}$</td>
<td></td>
</tr>
<tr>
<td>Land off side of runway</td>
<td>Lateral touchdown location</td>
<td>$\mu_{YTD}, \sigma_{YTD}$</td>
<td>$</td>
<td>\gamma_{TD}</td>
<td>&gt; \gamma_{TDMAX}$</td>
</tr>
<tr>
<td>Drag a wing tip or engine pod during landing</td>
<td>Bank angle at touchdown</td>
<td>$\mu_{\phi TD}, \sigma_{\phi TD}$</td>
<td>$</td>
<td>\phi_{TD}</td>
<td>&gt; \phi_{TDMAX}$</td>
</tr>
<tr>
<td>Land with excessive misalignment angle (putting side loads on landing gear)</td>
<td>Side velocity at touchdown</td>
<td>$\mu_{vTD}, \sigma_{vTD}$</td>
<td>$</td>
<td>v_{TD}</td>
<td>&gt; v_{TDMAX}$</td>
</tr>
<tr>
<td>Run off side of runway during rollout</td>
<td>Lateral displacement deviations during rollout</td>
<td>$\mu_{\gamma RO}, \sigma_{\gamma RO}$</td>
<td>$</td>
<td>\gamma_{RO}</td>
<td>&gt; \gamma_{ROMAX}$</td>
</tr>
</tbody>
</table>

*From [18]*
Table VI
Physical and Aerodynamic Characteristics of the Aircraft Used in this Study

(A constant air density of 1.2442 Kg-m\(^{-3}\) was used for all calculations.)

The aircraft is a four-engined CTOL large jet transport. Reference 35 gives its longitudinal specifications as follows (landing approach):

\[
\begin{align*}
\text{m} &= 260885.1 \text{ Kg-sec}^2/\text{m} \\
\text{b} &= 59.6424 \text{m} \\
\text{I}_y &= 4.4656 \times 10^6 \text{ Kg-sec}^2/\text{m} \\
\text{S} &= 510.953 \text{ m}^2 \\
\overline{c} &= 8.324 \text{m} \\
\varepsilon_T &= 2.5^\circ \\
L_{th} &= 3.048 \text{m}
\end{align*}
\]

From the information given in Ref. 35 the following aerodynamic data have been estimated.

**Flight Conditions**

**Landing Configuration:**

| Height:  | sea level | \(\alpha_{fe} = 5.57^\circ\) |
| Gear:    | down     | \(V_e = 1.3 \times V_S = 73.0581 \text{ m/s}\) |
| Glide Slope: | \(\delta_F = 30^\circ\) |
|          | \(\delta_{Ee} = -6.3^\circ\) |

\[
\begin{align*}
C_Le &= 1.479 \\
C_D &= 0.2115 \\
C_L &= 5.67/\text{rad} \\
C_D &= 1.13/\text{rad} \\
C_{\alpha} &= -1.45/\text{rad} \\
C_{m\alpha} &= -3.3/\text{rad} \\
C_{Lq} &= 5.65/\text{rad} \\
C_{mq} &= -21.4/\text{rad} \\
C_{L\delta} &= 0.336/\text{rad} \\
C_{m\delta} &= -1.40/\text{rad}
\end{align*}
\]

**Power Plant Characteristics**

- Maximum Thrust: \(T_{\text{max}} = 90718.6 \text{ kg}\)
- Average Engine Constant: \(T_t = 4.0 \text{ sec}\)
<table>
<thead>
<tr>
<th>No.</th>
<th>Outcome</th>
<th>$\eta$</th>
<th>$W$</th>
<th>$h(h_{\infty})$</th>
<th>$\frac{K_{1,2}}{x}$</th>
<th>STOP-height</th>
<th>Note</th>
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<tbody>
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<td>1</td>
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<td>TD</td>
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<td>22</td>
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<td>(0.61)</td>
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<td>Baseline</td>
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<td>Baseline</td>
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### Table VII (contd)
#### Calculation Case Definitions

<table>
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<tr>
<th>No.</th>
<th>Outcome</th>
<th>$\eta$</th>
<th>$W_{m}$</th>
<th>$n_{2i}$</th>
<th>$K_1$, $K_2$</th>
<th>STOP-height</th>
<th>Note</th>
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<tr>
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<td>30(m)</td>
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<td>0.16</td>
<td>NOM**</td>
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<td>0.16</td>
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<td>0.16</td>
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<td>0.16</td>
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<tr>
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<td>probability</td>
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<td>0-10m/s</td>
<td>3.823</td>
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<td>TD</td>
<td>$T_{av}$=3600sec</td>
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<tr>
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<td>0-10m/s</td>
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<td>57</td>
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<td>NOM</td>
<td>TD</td>
<td>$T_{av}$=3600sec</td>
</tr>
</tbody>
</table>

- **NOM** nominal case
- **TD** touchdown with the ground
- **CTOL** climb to landing

A set of feedback gain was selected for the tested STOL aircraft which was used in Ref. 27 as a testing base without automatic control system. The gain set was $S_K = \{ K_u, K_\theta, K_\beta, K_H, K_A \} = \{ 0.03, 0.01, 1.0, 1.0, 0.009, 0.009 \}$.

### Table VIII
#### Deviations of Sink Rate at Touchdown due to Mean Wind Shear ($\mu$)

<table>
<thead>
<tr>
<th>$W$(m/s)</th>
<th>Case</th>
<th>$n$=0.16</th>
<th>$n$=0.28</th>
<th>$n$=0.40</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\Delta n_{TD}$ (m/s)</td>
<td>$\Delta n_{TD}$ (m/s)</td>
<td>$\Delta n_{TD}$ (m/s)</td>
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Table IX
Dispersions of the State Variables due to Turbulence (Baseline*)

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<th>$W_{10}$ (m/s)</th>
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<th>$\sigma_\alpha$ (rad)</th>
<th>$\sigma_q$ (rad/s)</th>
<th>$\sigma_\theta$ (rad)</th>
<th>$\sigma_x$ (m)</th>
<th>$\sigma_h$ (m)</th>
<th>$\sigma_{\delta T}$ (m/s)</th>
<th>$\sigma_{\delta E}$ (rad)</th>
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<td>0.007154</td>
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* $h_N = -0.61$ m/s; $h_T = 3.823$ m/s; $\eta = 0.16$
Table X  Covariance Matrix $\Sigma(t) = [R R^T]$ Propagation

### $h = 49.0m$

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Note: The values are given in scientific notation, where $D$ denotes the power of 10.
Table X- continued.

h=16.0m

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### Table XI  Overall Probabilities of Hard Landing

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<td>0.2482109D-03</td>
<td>0.1964250D-01</td>
<td>0.1265236D-02</td>
</tr>
<tr>
<td>1.95000000+02</td>
<td>0.2503605D-03</td>
<td>0.1964250D-01</td>
<td>0.1275663D-02</td>
</tr>
<tr>
<td>2.00000000+02</td>
<td>0.2567346D-03</td>
<td>0.1964250D-01</td>
<td>0.1275663D-02</td>
</tr>
</tbody>
</table>
Fig. 1  Typical Aircraft Descent Through the Planetary Boundary Layer.
Fig. 2
Equilibrium Flight Condition Forces and Geometry, and
Definition of $F_B$ and $F_S$. 

HORIZONTAL REFERENCE LINE
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\[ \mathbf{d} \mathbf{L} = (\mathbf{r} \times \dot{\mathbf{r}}) \, \mathbf{dm} \]

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\((r_e, \theta_e < 0; W_{1e}, W_{2e} < 0)\)
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Fig. 9  Time Responses for a Step $d_c$ Input
(Continued)
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STABILITY CATEGORIES:
A—very unstable
B—moderately unstable
C—slightly unstable
D—neutral
E—slightly stable
F—stable to very stable
W—Weibull distribution
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\[ p_{OK} = (1-p_1)(1-p_2)(1-p_3)(1-p_4) \]
\[ p_{OR} = p_4(1-p_1)(1-p_2)(1-p_3) \]
\[ p_{HL} = p_3(1-p_1)(1-p_2) \]
\[ p_{SL} = p_2(1-p_1) \]
\[ p_{MA} = p_1(1-p_5) \]
\[ p_{FA} = p_5p_1 \]
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Fig. 38(d)  Response to Mean Shear (Time History) - Concluded.
Fig. 39(a) Modified Bessel Function $K_{1/3}(x)$.

Fig. 39(b) Modified Bessel Function $K_{2/3}(x)$. 
Fig. 40: The Integration Domain.

\[ 0 \leq t_2 \leq t_{TD} \]
\[ 0 \leq t_1 \leq t_{TD} \]
\[ t_2 \geq t_1 \]
Fig. 41(a) Response to Turbulence ($\Delta u$ and $\Delta \alpha$).

($h_0=50.24$ m/s; $w_0=3.83$ m/s; $\eta=0.16$)
Fig. 41(b)  Response to Turbulence (Δθ and Δh)

(h₀=50.24 m/s; w₀=3.83 m/s; η=0.16)
Fig. 41(c)  Response to Turbulence ($\Delta h$ and $\Delta x_E$).

($h_0=50.24$ m/s; $w_0=3.83$ m/s; $n=0.16$)
Fig. 41(d) Response to Turbulence ($\Delta \sigma_E$ and $\Delta \sigma_T$).

$$(h_0 = 50.24 \text{ m/s}; \omega_0 = 3.83 \text{ m/s}; \theta = 0.16)$$
Fig. 42(a) The Cumulative Overall Hard Landing Probability

( η=0.16; h=3.823 m/s )
Fig. 42(b) The Cumulative Overall Hard Landing Probability

( $\eta=0.28; \ h=3.823 \ m/s$ )
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(\(W=10\) M/S; -x-10 MIN. -*--60 MIN.)
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Fig. 45  Sensitivity to the Limited Sink Rate at Touchdown.
Fig. 46  Sensitivity of $W_\phi$ to $\eta$ ($P_f = 10^{-5}$).

$h = 3.82 \text{ m/s}; P_f(h, w) = 10^{-5}$.
Fig. 47  Sensitivity of $W_\lambda$ to $\dot{h}_\lambda$

$C_n=0.16; P_{f}(h,w)=10^{-5}$
Fig. 48    RMS Value of $\frac{\sigma}{h}$ Against Reference Wind Speed (Baseline).
LONGITUDINAL STABILITY AXES DIMENSIONAL DERIVATIVES

The dimensional aerodynamic derivatives in Eqs. (2.154) and (2.155) may be written in terms of their nondimensional counterparts as follows (see also Etkin [47]):

\[
X_u = \rho V_e S C_L \tan \theta_e + \frac{1}{2} \rho S V_e C_x u
\]

\[
X_w = \frac{1}{2} \rho V_e S C_x a
\]

\[
X_q = t^* q_e S C_x q
\]

\[
Z_u = -\rho V_e S C_L^* + \frac{1}{2} \rho V_e S C_z u
\]

\[
Z_w = \frac{1}{2} \rho V_e S C_z \alpha
\]

\[
Z_p = \frac{1}{2} t^* \rho V_e S C_z^* \alpha
\]

\[
Z_q = t_q^* S C_z q
\]

\[
M_u = \frac{1}{2} \rho V_e S C \bar{c} m_u
\]
where

\[ q_e = \frac{1}{2} \rho V_e^2 \]

\[ C_{L^*} = C_{W_e} \cos \theta_e \]
\[ t^* = \frac{\bar{c}}{(2V_e)} \]

\[ \delta\alpha = \alpha_{fe} + \epsilon_T \]

\[ C_{x_{CT}} = -C_D^{*CT} \text{ or } C_{x_{CT}} = -[C_{D_{CT}} - \cos(\delta\alpha)] \]

\[ C_{x_u} = -2C_{x_{CT}} C_T e \]

\[ C_{z_u} = 2[C_{L_{CT}} + \sin(\delta\alpha)] C_T e \text{ or } C_{z_u} = 2C_{L_{CT}} C_T e \]

\[ C_{m_u} = -2C_{m_{CT}} C_T e \]

\[ C_{x_\alpha} = C_{L^*} - C_{D_\alpha} \]

\[ C_{x_q} = -C_{D_q} \]

\[ C_{z_\alpha} = -(C_{L_\alpha} + C_{D_{e^*}}) \]

\[ C_{z_{\alpha^*}} = -C_{L_{\alpha}} \]

\[ C_{z_{q^*}} = -C_{L_q} \]

\[ C_{x_{\delta E}} = C_D \delta_E \]
\[ C_{T_{T_{T}}} = C_{T_{T_{T}}}(V_e) \]

\[ C_{z_{T_{E}}} = -C_{L_{T_{E}}} \]
In Ref. [46] the ground effect was taken into account by adding extra perturbation terms into the state equations. These terms are given by the following functions of wheel height \( h \) implicitly:

\[
X_{GE} = C_{D_{GE}} q e^S
\]
\[
Z_{GE} = C_{Z_{GE}} q e^S
\]
\[
M_{GE} = C_{m_{GE}} q e^S \bar{c}
\]

where

\[
C_{D_{GE}} = k_A^A (2.308\alpha^3 - 0.979\alpha^2 - 0.1769\alpha - 0.0384)
\]
\[
C_{Z_{GE}} = -C_{L_{GE}} = -k_B^B (0.24) \cos[8.036(\alpha - 0.00526)]
\]
\[
C_{m_{GE}} = k_B^B (2.736\alpha^2 - 0.621\alpha - 0.115)
\]

\[
k_A^A = 0.0482 \times 10^{-6} h^3 - 0.0997 \times 10^{-4} h^2 - 0.4515 \times 10^{-2} h + 1.0
\]
\[
k_B^B = 0.1073 \times 10^{-6} h^3 - 0.4587 \times 10^{-4} h^2 + 0.8556 \times 10^{-3} h + 1.0
\]
Note that the angle of attack during the flare will not change drastically so that we may reasonably assume that taking an average value of $\alpha_{av}$ to replace the variable $\alpha$ in the above expressions will not result in much loss of accuracy.

If $h(t)$ is considered as a time-history consisting of a part $h_n(t)$ due to nominal flare and a part $\Delta h(t)$ caused by winds, i.e., $h(t) = h_n(t) + \Delta h(t)$, then we have

$$K_{GE}^A = f_1(h_n + \Delta h)$$

$$K_{GE}^B = f_2(h_n + \Delta h)$$

and the deviations of $K_{GE}^A$ and $K_{GE}^B$ wrt the nominal path can be expressed by

$$\Delta K_{GE}^A = f_1(h_n + \Delta h) - f_1(h_n) \quad (A2-9)$$

$$\Delta K_{GE}^B = f_2(h_n + \Delta h) - f_2(h_n) \quad (A2-10)$$

Substituting (A2-7) and (A2-8) into (A2-9) and (A2-10), after ignoring terms higher than second order, we obtain

$$\Delta K_{GE}^A = (0.1446 \times 10^{-6} h_n^2 - 0.1994 \times 10^{-4} h_n - 0.4515 \times 10^{-2}) \Delta h$$

$$\Delta K_{GE}^B = (0.3219 \times 10^{-6} h_n^2 - 0.9174 \times 10^{-4} h_n + 0.8556 \times 10^{-3}) \Delta h$$

then
\[ \frac{\partial \Delta \kappa^A}{\partial \Delta h} = 0.1446 \times 10^{-6} y_n^2 - 0.1994 \times 10^{-4} h_n - 0.4515 \times 10^{-2} \quad (A2-11) \]

and

\[ \frac{\partial \Delta \kappa^B}{\partial \Delta h} = 0.3219 \times 10^{-6} y_n^2 - 0.9174 \times 10^{-4} h_n + 0.8556 \times 10^{-3} \quad (A2-12) \]

We define

\[ \Delta \chi_{GE} = \Delta \chi_{GE} = \frac{\partial C_{D GE}}{\partial h} q_e S \triangleq C_{\chi h} q_e S \quad (A2-13) \]

\[ \Delta Z_{GE} = \Delta Z_{GE} = \frac{\partial C_{Z GE}}{\partial h} q_e S \triangleq C_{Z h} q_e S \quad (A2-14) \]

\[ \Delta M_{GE} = \Delta M_{GE} = \frac{\partial C_{M GE}}{\partial h} q_e S \triangleq C_{M h} q_e S \quad (A2-15) \]

Using (A2-4) to (A2-8) obtains

\[ C_{\chi h} = \frac{\partial \Delta \kappa^A}{\partial \Delta h} (2.308 \alpha^3_{av} - 0.979 \alpha^2_{av} - 0.1769 \alpha_{av} - 0.0380) \quad (A2-16) \]

\[ C_{Z h} = -\frac{\partial \Delta \kappa^B}{\partial \Delta h} (0.24) \cos[8.036(\alpha_{av} - 0.0526)] \quad (A2-17) \]

\[ C_{M h} = \frac{\partial \Delta \kappa^B}{\partial \Delta h} (2.736 \alpha^2_{av} - 0.621 \alpha_{av} - 0.115) \quad (A2-18) \]
where $\frac{\Delta K_{GE}^A}{\Delta h}$ and $\frac{\Delta K_{GE}^B}{\Delta h}$ are given in (A2-11) and (A2-12) respectively.

These derivatives are used in the state equations as derived in this report (see Chapter II).
APPENDIX III

RANDOM NUMBER GENERATOR (SEE [96])

A random number table [i.e., the array Y(101,20)] is given in the attached table.

The alternative approach to the generation of a random signal is based on the formula \( X_{n+1} = aX_n + b \mod m \). This technique is called the mixed congruential method which provides a simple and fast computation procedure. The constants \( a \) and \( b \) are selected to provide speed of computation and good statistical properties. Chambers [97] and Hamming [98] found that if \( a \) and \( b \) are selected as

\[
\begin{align*}
    a &= 4\cdot N + L \\
    b &= \text{odd} \\
    N &= 1, 2, 3, 4...
\end{align*}
\]

or

\[
\begin{align*}
    a &= b \cdot N + 3 \\
    b &= 0
\end{align*}
\]

then a maximum period can be achieved. In this investigation, the latter rule is used.

The computer program and an example calculation are shown as follows: Subroutine RANDU is combined in subroutine GAUSS, which is included in subroutine INPUT, to generate a random number array. The subroutine MEAN and SIGMA are used to check the mean value and the standard deviation of the generated random number series. The value 'a' is selected to be 65539 or \( N = 8192 \). Five runs were carried out and the results are shown in the following table.
<table>
<thead>
<tr>
<th>No. of run</th>
<th>Mean Value</th>
<th>Standard Deviation</th>
<th>Seed Number</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-0.00529</td>
<td>0.16497</td>
<td>1864286221</td>
</tr>
<tr>
<td>2</td>
<td>0.00227</td>
<td>0.16573</td>
<td>250707981</td>
</tr>
<tr>
<td>3</td>
<td>-0.00466</td>
<td>0.16571</td>
<td>2126790669</td>
</tr>
<tr>
<td>4</td>
<td>-0.00898</td>
<td>0.16460</td>
<td>1050083341</td>
</tr>
<tr>
<td>5</td>
<td>-0.00175</td>
<td>0.16568</td>
<td>1315553293</td>
</tr>
</tbody>
</table>

**Symbol List**

- **AVR**: mean value
- **N**: dimension number
- **NM**: control number for selecting different method
- **DT**: time increment
- **IX**: seed number
- **XR**: real part of the array \( \{x_i\} \)
- **XI**: imaginary part of the array \( \{x_i\} \)
- **SIG**: desired standard deviation
- **XB**: desired mean value
- **XDP**: normalized Gaussian random number
COMPUTER CODES

C RANDOM NUMBER GENERATOR

J=20
MJ=2020
M=101
DT=18.
N=2020
SIG=1.0
AVR=0.0
DO 444 II=1, N
   XR(II)=0.0
   IV=65549
   WRITE(6,222) AVR, SIG, IX
   CALL RANDNO(M, J, MJ, N, DT, IX, AVR, SIG, XI, Y)
   WRITE(6,222) AVR, SIG, IX
   WRITE(6,777) Y
   0077 FORMAT(1X, 20(F5.2, 1X))
   STOP
END

SUBROUTINE INPUTCN, N, MJ, DT, IX, XI, XR, XI, Y
C FILE NAME: RANDOM
DIMENSION XR(MJ), XI(MJ)
NM=1
CALL INPUT(N, NM, DT, IX, XR, XI)
CALL MEAN(N, XR, AVR)
CALL SIGMA(N, XR, AVR, SIG)
DO 555 JI=1, J
   DO 555 II=1, M
      V(JI,II)=XR((JI-1)*M+II)
   END
END

SUBROUTINE MEAN(N, NR, AVR)
DIMENSION XR(N)
SM=0.0
DO 111 I=1, N
   SM=SM+XR(I)
111 CONTINUE
AVR=SM/N
RETURN
END

SUBROUTINE SIGMA(N, XR, AVR, SIG)
DIMENSION XR(N)
SG=0.0
DO 222 I=1, N
   XX=XR(I)-AVR
   XX=XX
222 CONTINUE
SIG=SG+XX
SIGM2=SG*(N-1)
SIG=SGRT(SIGM2)
RETURN
END
APPENDIX IV

COMPUTER PROGRAM FOR THE "RANDOM CHOICE DIRECT SEARCH" METHOD

This is only a main program which is similar to the one in [46]. All the symbols were given there. The additional symbols are listed as follows:

ABC region reduction factor
FISEC the cost function
REG(I) array for regions of the gains to be searched
XX(I) array for gains to be selected, in the present step
XP(L) array for gains having been obtained in the last step
Y(M,N) array for random numbers
COMPUTER CODES

THIS PROGRAM IS FOR THE SIMULATION OF LANDING APPROACH, LANDING ABORT, LANDING FLARE
CASE=1 -----LANDING ABORT
CASE=2 -----LANDING APPROACH PLUS ABORT
CASE=3 -----LANDING FLARE
FV=DESCENT AIRSPEED FACTOR; PT=THRUST FACTOR; ADGE=WHITH GROUND EFFECT; CMSW=DNZE SWITCHING; MODE=CONTROL MODEL IN CASE 1 OR 2;

IMPLICIT REAL*B (A-H, O-Z)
REAL*B

DIMENSION FF(8, 2), C5(8, 2), A10(8, 2)
DIMENSION NO(8)

DIMENSION A1(B, 8), A2(B, 8), A1INV(8, B), B1(B, 8), B2(8, 2), B3(8, 2),

K1(2, 8), K2(2, 8), AA(8, 8), C1(2, 2), C2(2, 2), C3(2, 2), D1(8, B),

DIMENSION D1(B, 8), A1D2(B, 8), D1D(8, B), A1D2IN(8, B), WK(B, 8B), A3(B, 8), A4(B, 2),

A5(B, 2), A6(B, 2), IIR(B, 8), IIC(B, 8), COEFS(4), WWK(B, 8B)

DIMENSION IMK(B)

DIMENSION X(4), DFOX(4)

DIMENSION DX(8), UA(8), UB(8), DXD(8), A7(8, 8), A8(8, 8)

DIMENSION WVE(3)

DIMENSION Y(101, 20), XX(4), XP(4), X5(4), X7(4), REQ(4)

DIMENSION ZI(B, 8), ZN, W(8)

COMMON/NUMB/L, I1K

COMMON/TTGF/TTT1, TTT2

COMMON/ALN/VVE, AEROD, GACM

COMMON/WDDT/W3DT, W3DT, WVEE, WVEE3

COMMON/OUT/DZNC, DZ, DETC, DTC

COMMON/GE/GE, SY, CASE, ADGE

COMMON/ZFOF/ZOF, ZOF, ZOF, TAUF, TAUF, DELTE, GEFL

COMMON/WINDS/DDDUW, DDDUW, DVEE, CM, DMOVE, MNW

COMMON/CA/A3, A4, A5, A6, A7, A8, A1INV, A10, A12IN

COMMON/WD/WDD, W3DD, HII, HHD, W3DD2, WIND, WINN, XII

COMMON/CAB/KT, KE, K1, A1FAE, CCLLA, CCLAD, XXA, Q

COMMON/CLA/DDMU, DDDW, DDU, DDAFF, DDTHC, DDHHC, DDRC, DDT, DDO, RDGA

COMMON/CC/B, KK, CCDDO, APSTT, CCWE, THETE, CCLLA, CCLLA, ALFAO, LTH, C, CCMO,

COMMON/GEN2/DNZCQ, XGQ, ZGQ, DMQE

COMMON/STVE/DD, DDD, DZQ, AKD, DAF, DGC, DTHC, DDD, DNZC, DETTE,

COMMON/GSSM/W3K, KTH, KTH, KAL, KALF, KU, KU, KUT, GLIDP, GLIDD,

/KH, KHD, ENHAN, AKNZ, AKINTH, AKS, AKUB, AIRSPE, DHD, VUST, AKC, TDL,

/HHDF, HD

COMMON/FLAR/TSS, HOST, VVEE, DH, HHCD, ENTCH, ENTQ, HHK, HHDC, VVEE, SINGQ

/COMMON/GEDE/XGQ, ZGQ, DMQE, CCLLA, CCLAD, CCMQ, CCLAD, CCMQ, CCLAD, CCMQ,

/COMMON/EA: XGQ, DMQE, CCLLA, CCLAD, CCMQ, CCLAD, CCMQ,

/EXTERNAL F

EXTERNAL CALC

CALL RANTAB(Y)

READ(5, 1)IDO

READ(5, 1)DX0, DX1, DX2, DX30, DX40, DX50, DX60, DX70, DX80

READ(5, 3164)CM, OK1, OK2, XXA, CASE, MODE, ITIMR, ITIMR

READ(5, 1)KU, KUD, KALF, KTH, KALFD, KTHD, KH, KHD

READ(5, 2)IWDD, IWDD2, IWDD

READ(5, 8009)IWDD, IWNN, WINL, WINL

READ(5, 8009)AKS, AKINTH, ENHAN, AKNZ

NARY1=803

NARY2=60

NARY3=10

IF (CASE. EQ. 1) NOPT=601

IF (CASE. EQ. 2) NOPT=2500

IF (CASE. EQ. 3) NOPT=801

IF (CASE. EQ. 1) DX60=30.0

IF (CASE. EQ. 2) DX60=152.4

IF (CASE. EQ. 3) DX60=18.24

DDWU=6.5

AIRSPE=1.0

HMCC=50.0

HHDF=0.0

VERT=0.0

GAMG=0.05236

PT=1.0

FTR=1.0

FV=1.3

ADGE=1.0

TDL=20.0

DUC=5.62

DAFC=0.1115

GEFL=0.0

DDUC=0.00

DDDF=0.0

DDSF=0.00

DDTHC=0.00

DDHHC=0.00

DRC=0.00

DTC=0.00

DDUC=0.00

HD=30.0

TT=4.0

IF (CASE. EQ. 2) TT=2.0

AKUB=0.00

KUT=-0.02

KUTD=0.04

IV-2
L=2

IV-3
TANΘE = SINΘE / COSΘE
COSΘQ = DCOS(QAMGG)
SING = DSIN(QAMGG)
TANG = SING / COSΘQ

C THIS COMPLETES THE CALCULATION OF THE REFERENCE EQUILIBRIUM CONDITIONS

C
X(1) = 1.0
X(2) = 0.1
X(3) = 1.0
X(4) = 0.1
CALL NLS(4, X, F, DFX, 1, D-8)
CCTE = X(1)
ALFAFE = X(2)
CCLLE = X(3)
DELTE = X(4)
DALFD = ALFAFE * 180 / PI
CCDDE = CCDDE + KK * CCLLE ** 2
DETTEE = CCTE * OE * SY / TM
WRITE(6, 2222) ALFAO, GAME, THETE, SIE, VVEEE
WRITE(6, 105) CCLLE, CCDDE, CCTTE, DALFD, DELTE
105 FORMAT (IHO, 5X, 'CCLLE=', IPE14.6, 5X, 'CCDDE=', IPE14.6, 5X, 'CCTTE=', /1PE14.6, 5X, 'DALFD=', 1PE14.5/1HO, 'DELTE=', 1PE14.5)

CCTAD = -5.65
CCMAD = -3.3
CCM = -21.4
CCXDEE = 0.0
CCZDEE = -356
CCCLLT = 0.0
CCDDCT = 0.0
COSTE = DCOS (THETE)
SINT = DSIN (THETE)
TANTE = SINT / COSTE
DA = ALFAFE + APSST
SIND = DSIN (DA)
COSDA = DCOS (DA)
CCLLS = CCLLE + SIND * CCTTE
CCLLCT = CCLLLT + SINDA
CCDDS = CCDDE - COSDA * CCTTE
CCDDCT = CCDDCT - COSDA
TS = C / (2.0 * VVE)
CCMCT = LTH / C
CCXCT = CCDCT
CCXU = -2.0 * CCTXCT * CCTTE
CCZU = 2.0 * CCLLCT * CCTTE
CMU = -2.0 * CCMMC * CCTTE
CCXG = 0.0
CCZG = -CCLLA - CCDDE
CCX = CCLLS - CCDDA

FTR FOR ESTIMATING THE ENGINE FAIL EFFECTS ON PERFORMANCE

CCDTDT = TM / OE / SY * FTR
XU = RH0 * VVE * SY * CCLLS * TANTE + 0.5 * RH0 * SY * VVE * CCXU
XH = 0.5 * RH0 * VVE * SY * CCXA
XG = TS * GE * SY * CCXG
XZ = RH0 * VVE * SY * CCLLS + 0.5 * RH0 * VVE * SY * CCZU
ZU = 0.5 * RH0 * VVE * SY * CCZA
ZWD = 0.5 * TS * RH0 * VVE * SY * CCZAD
ZQ = TS * GE * SY * CCZG
MU = 0.5 * RH0 * VVE * SY * CCMA
MU0 = 0.5 * TS * RH0 * VVE * SY * CCMAD
MQ = TS * GE * SY * CCMG
XDEE = GE * SY * CCXDEE
XTTT = GE * SY * CCTXCT * CCTDT
ZDEE = GE * SY * CCDEE
ZDDE = GE * SY * CCLLCT * CCTDT
MDEE = GE * SY * CCMDEE
MDT = GE * SY * CCMCCT * CCTDT
WRITE(6, 500)
WRITE(6, 501) CCLLS, CCLLCT, CCDDE, CCDDE
WRITE(6, 502) CCMMC, CCTDT, CCXCT, CCXU
WRITE(6, 503) CCZU, CCMM, CCXA, CCZA
WRITE(6, 504) CCTDT, CCZAD, CCZG, CCMA
WRITE(6, 505) CCMMAD, CCMM, CCZDEE, CCMMDE
WRITE(6, 506)
WRITE(6, 507) XU, XW, XG, ZU
WRITE(6, 508) ZW, ZG, MU
WRITE(6, 509) MM, MMG, MJ, XDEE
WRITE(6, 510) XDTT, ZDEE, ZDTT, MDEE
WRITE(6, 511) MDTT, DDW, DDWG, DDTHE

C TO DEFINE MATRICES

DO 31 I = 1, N
DO 31 J = 1, N
AI(I,J) = 0.0
31 CONTINUE
AI(1,1) = MASS
AI(2,1) = MASS-ZWD
AI(3,2) = -WWD
AI(3,3) = ΙΙΙΥΥ
A2(1,1) = XU
A2(1,2) = XW
A2(1,3) = XG
A2(1,4) = -MASS * 0 * COSTE
A2(2,1) = ZU
A2(2,2) = ZW
A2(2,3) = MASS * VVE * ZG
A2(2,4) = -MASS * O * SINTI
A2(3,1) = MU

C IV-4
A2(3.2) = Mw
A2(3.3) = MG
A2(4.3) = 1.0
A2(5.1) = COSTE
A2(5.2) = SINTE
A2(5.4) = VVE*SINTE
A2(6.1) = SINTE
A2(6.2) = COSTE
A2(6.4) = VVE*COSTE
DO 44 I = 1, N
DO 44 J = 1, L
B1(I, J) = 0.0
B2(I, J) = 0.0
B3(I, J) = 0.0
B1(I, 1) = XDTT
B1(2.1) = ZDTT
B1(3.1) = MDTT
B1(1.1) = XDEE
B1(2.2) = ZDEE
B1(3.2) = MDEE
B1(6.2) = KE
B3(1.1) = MASS*COSTE
B3(1.2) = MASS*SINTE
B3(2.1) = MASS*SINTE
B3(2.2) = MASS*COSTE
B2(5.1) = L
B2(6.2) = 1.0
K1(1.7) = 1.0
K1(2.1) = KU
K1(2.2) = KM
K1(2.4) = KTH
K1(2.6) = KHH
K2(2.1) = KUD
K2(2.2) = KUD
K2(2.4) = KTHD
K2(2.6) = KHD
A2(1.7) = B1(1.1)
A2(1.8) = B1(1.2)
A2(2.7) = B1(2.1)
A2(2.8) = B1(2.2)
A2(3.7) = B1(3.1)
A2(3.8) = B1(3.2)
A1(8.1) = KUD*KE
A1(8.2) = KE*KWD
A1(8.4) = KE*KTHD
A1(8.6) = KE*KHD
A1(8.8) = 1.0
A2(7.7) = KT
A2(8.1) = KE*KU
A2(8.2) = KE*KW
A2(8.4) = KE*KTH
A2(8.8) = KE
IF(ITIMR.NE.1) GO TO 1221
DO 55 I = 1, L
DO 55 J = 1, N
K1(I, J) = 0.0
K2(I, J) = 0.0
DO 9060 I = 1, N
DO 9060 J = 1, L
9060 B1(I, J) = 0.0
1221 CONTINUE
C
C TO CALCULATE TRANSIENT DISTURBANCE FORCES DUE TO FLAPS AND GEAR
C
C IDOT=4
AKCD=0.95
CCLDF=2.307
CCDOG=0.04988*AKCD
CCDEF=GE*SY*CCLDF
ZOF=ZDEF*0.17452
XOF=GE*SY*CCDOF
XOG=GE*SY*CCDEF
CALL LINV2F(A1, N, N, A1IN, IDOT, WK, IER)
WRITE(6, 88) IER
WRITE(6, 988) IDOT
CALL VMULFF(A1INV, A2, N, N, N, N, AA, N, IER)
WRITE(6, 881) IER
CALL VMULFF(A1INV, B1, N, N, L, N, C1, N, IER)
WRITE(6, 882) IER
CALL VMULFF(A1INV, B2, N, N, L, N, C2, N, IER)
WRITE(6, 883) IER
CALL VMULFF(A1INV, B3, N, N, L, N, C3, N, IER)
WRITE(6, 884) IER
CALL VMULFF(Cl, K1, N, L, N, L, D1, N, IER)
WRITE(6, 885) IER
CALL VMULFF(C1, K2, N, L, N, L, D2, N, IER)
WRITE(6, 886) IER
DO 99 I = 1, N
DO 99 J = 1, N
AD1(I, J) = 0.0
99 AID2(I, J) = 0.0
DO 100 I = 1, N
DO 100 J = 1, N
D11(I, J) = D11(I, J)
100 AID2(I, J) = AA(I, J) + D11(I, J)
DO 1188 I = 1, N
DO 1188 J = 1, N
1188 AID2(I, J) = D2(I, J)
DO 1199 I = 1, N
1199 AID2(I, I) = 1.0 + D2(I, I)
IDOT=4
CALL LINV2F(AID2, N, N, AID2IN, IDOT, WK, IER)
WRITE(6, 887) IER
CALL VMULFF(AID2IN, AD1, N, N, N, N, A3, N, IER)
WRITE(6, 888) IER
CALL VMULFF(AID2IN, C1, N, N, L, N, A4, N, IER)
WRITE(6, 889) IER
CALL VMULFF(AID2IN=1, N.L.N.N.A5, N, IER)
WRITE(6, 890) IER
CALL VMULFF(AID2IN=C3.N.N.L.N.A6, N, IER)
WRITE(6, 891) IER
CALL VMULFF(AID2IN=D1.N.N.N.N.A7, N, IER)
WRITE(6, 892) IER
C
C
GERATIONS OF EIGENVALUES
C
IJOB=2
CALL EI0RF(A3.N.N. IJOB, WW.ZI.N.WMK, IER)
WRITE(6, 3302)
WRITE(6, 3303)(WW(I), I=1, N)
WRITE(6, 3304)IER, WW(K)
500 FORMAT(1HI, 'THE STABILITY DERIVATIVES ARE')
501 FORMAT(IHO, 'CCLLS=', FB. 4, 5X, 'CCLST=', FB. 4, 5X, 'CCDTS=', FB. 4, 5X, 'CCDDT=', FB. 4)
502 FORMAT(IHO, 'CMCT=', FB. 4, 5X, 'CCTDT=', FB. 4, 5X, 'CCXT=', FB. 4, 5X, 'CCXU=', FB. 4)
503 FORMAT(IHO, 'CUU=', FB. 4, 5X, 'CMU=', FB. 4, 5X, 'CXA=', FB. 4, 5X, 'CXU=', FB. 4)
504 FORMAT(IHO, 'CCDT=', FB. 4, 5X, 'CZAD=', FB. 4, 5X, 'CCZ=', FB. 4, 5X, 'CCMA=', FB. 4)
505 FORMAT(IHO, 'CCMAD=', FB. 4, 5X, 'CMQ=', FB. 4, 5X, 'CCM=', FB. 4, 5X, 'CCMDEE=', FB. 4)
506 FORMAT(1HI, 'THE DERIVATIVES OF FORCES AND MOMENT')
88 FORMAT(IHO, 'ERROR FOR A1INV', I6)
888 FORMAT(IHO, 'IDT=', I9)
881 FORMAT(IHO, 'ERROR FOR A2', I9)
882 FORMAT(IHO, 'ERROR FOR A1', I9)
883 FORMAT(IHO, 'ERROR FOR A2', I9)
884 FORMAT(IHO, 'ERROR FOR A3', I9)
885 FORMAT(IHO, 'ERROR FOR D1', I9)
886 FORMAT(IHO, 'ERROR FOR D2', I9)
887 FORMAT(IHO, 'ERROR FOR AID2IN', 15)
888 FORMAT(IHO, 'ERROR FOR A3', I9)
889 FORMAT(IHO, 'ERROR FOR A4', I9)
890 FORMAT(IHO, 'ERROR FOR A5', I9)
891 FORMAT(IHO, 'ERROR FOR A6', I9)
3302 FORMAT(IHO, 'DIAGNOSIS')
3303 FORMAT(IHO, 'POSTA', 19)
3006 FORMAT(IHO, 'ERROR FOR A7', I9)
692 FORMAT(IHO, 'ERROR FOR A7', I9)

WRITE(6, 247)(XX(I), I=1, N)
247 FORMAT(1HO, 'KTH', FB. 10, 5, 'KTHD', FB. 10, 5, 'KHD', FB. 10, 5, 'KH', FB. 10, 5)
DO 81 I=1, NN
XSI(I)=XP(I)
XT(I)=XS(I)
REG(I)=-2.0*XP(I)
81 CONTINUE
TESTI=1.E10
DO 100 JC=1, 2
KK1=KK(I)+1
IF(KK1 LE 20) DO 94 K=1, 50
IF(JC GT 10) DO 311 I=1, NN
IF(JC LE 4) GO TO 19
ABC(I)=0.76
GO TO 19
311 ABC(I)=7.5
19 CONTINUE
DO 18 K=1, NN
18 XX(LO)=XX(LO)+Y(K, LG+KK1)*REG(LO)
KTH=XX(1)
KTHD=XX(2)
KHD=XX(3)
KH=XX(4)
WRITE(6, 9709) KTH, KTHD, KHD, KH
9709 FORMAT(1X, 3HKID, FB. 10, 5, 3X, 4HKTID, FB. 10, 5, 3X, 3HKTID, FB. 10, 5, 3X, 2HKID, FB. 10, 5)
T=0.000
AINTD=0.0
AINTU=0.0
ALFAMS=0.0
ALTIE=0.0
AKC=0.00
AKD=0.00
AINDEX=0.0
DDAW=0.0
DNZC=0.00
DNZ=0.0
DAF=0.0
DETDD=0.0
DD=0.0
DDD=0.0
DDD=0.0
DO 196  I=1,4 
XN(1)=0.000 
196 
YN(1)=0.000 
IF (11KK.GE.2) GO TO 1983 
DO 22356  I=1.5 
22356  NO(I)=0 
DO 8681 I=1.NARY1 
8681  SXX(I,J)=0.000 
DO 197 I=1.NARY1 
DO 197 J=1.NARY3 
DO 197 XPLOT(I,J)=0.000 
197  CONTINUE 
DO 23457 CONTINUE 
CHD=VVEEE*SINGG 
HHDW=DX(6)+CHD 
DTeDD=(DX(6)+DX(8))*57.3 
DAF=DX(2)/VVE*57.3+DALFD 
GAQQ=TANGG 
23457 CONTINUE 
XN(1)=0.000 
YN(1)=DHDD 
XN(2)=DX(1)+GAGG 
YN(2)=0.000 
XI=(VVEEE*COSGG*T+OXXO)+DX(4) 
HI=OHHO 
00TTO=CCTTE*QE*SY/90718.6 
THRUaTM=(OX(7)+DETTEEI 
GAMAP=OX(4)-OALFO 
OoOWU=ODWU 
OoOWW=ODWW 
DO 0""= oDDWU 
DDOWV=OOoWW 
OoOWU=DODWU-WWEE 
VVAA=DSERT((VVE+DX(1))*2+DX(2)*DX(2)) 
VVEE=DSERT((VVAA*DCOS(GAMAP)+DDRWH)**2+(VVAA*DSIN(GAMAP)+DDRWH)**2) 
/##2) 
GAMC=57.3+GAMAP 
ALTIE=(ALFAE-GAMGG)*57.3 
GAMMT=DX(4)+57.3+ALTIE 
DNZ=-((DX(2)-2430T-DX(3)*VVEE-XXA+DX(3))/G*DCOS(ALFAE) 
ALFAMS=DALFD+(DX(4)-DX(6))/VVE*57.3 
CALL CONTR(CASE.MODE.T.DX.UB.OXo.CALC) 
WRITE(6,15) DX(1).1. DAF. HHDW.GAMMT.XII.HII.DTDD.THRU.HHC 
ISPA=12 
ISK=ISPA*(11KK-1) 
DH=2 5D-2 
DH=0.1 
FISEC=0.0 
10 CALL RUNKU(N.DH.T.DX.UA.UB.DX.DCALC) 
CC=0.0 
LK=LK+1 
JK=JK+1 
HHD=0.0 
AM1=0.4363
IV-8
30 CONTINUE
94 CONTINUE
WRITE(6,79) JC, NDD, FM1, Z1
79 FORMAT(1X, 215, 5F20.8)
IF(XP(1).EQ.XS(1)) GO TO 98
Z1=Z1*ABC
DO 97 I=1, NN
XT(I)=XP(I)
XP(I)=XS(I)
REQ(I)=REQ(I)*ABC
97 CONTINUE
N5=0
GO TO 1000
98 CONTINUE
N5=N5+1
N6=N5/3
N7=N6*3
IF(N5.NE.N7) GO TO 1000
DO 399 I=1, NN
399 REQ(I)=REQ(I)*ABC
Z1=Z1*ABC
1000 CONTINUE
22559 CONTINUE
STOP
END
For a large airplane, it is considered to be more realistic that for $u_g$ and $v_g$ we take them to be the average at the four points. By choosing points 0, 1, 2, 3 as shown in the above sketch, sweepback is neglected, and
there is some loss in fidelity as a consequence.

The inputs for the longitudinal case are then:

\[ u_g = (k_0u_0 + k_1u_1 + k_2u_2 + k_3u_3)/(k_0 + k_1 + k_2 + k_3) \]  \hspace{1cm} (A5-1)

\[ w_g = (k_0w_0 + k_1w_1 + k_2w_2 + k_3w_3)/(k_0 + k_1 + k_2 + k_3) \]  \hspace{1cm} (A5-2)

\[ w'_g = (k_0w'_0 + k_1w'_1)/(k_0 + k_1 + k_2) \]  \hspace{1cm} (A5-3)

\[ q_g = (w_3 - w'_g)/\lambda_t \]  \hspace{1cm} (A5-4)

The correlations are calculated with the aid of (4.9):

\[ \tilde{c}_{uu}(\tau) = \langle u_g u_g' \rangle = \langle (k_0u_0 + k_1u_1 + k_2u_2 + k_3u_3)(k_0u_0' + k_1u_1' + k_2u_2' + k_3u_3') \rangle \]

\[ = [k_0^2\langle u_0u_0' \rangle + k_1^2\langle u_1u_1' \rangle + k_2^2\langle u_2u_2' \rangle + k_3^2\langle u_3u_3' \rangle + k_0k_1\langle u_0u_1' \rangle + k_0k_2\langle u_0u_2' \rangle + k_0k_3\langle u_0u_3' \rangle + k_1k_2\langle u_1u_2' \rangle + k_1k_3\langle u_1u_3' \rangle + k_2k_3\langle u_2u_3' \rangle + k_1k_0\langle u_1u_0' \rangle + k_2k_0\langle u_2u_0' \rangle + k_2k_1\langle u_2u_1' \rangle + k_3k_0\langle u_3u_0' \rangle + k_3k_1\langle u_3u_1' \rangle + k_3k_2\langle u_3u_2' \rangle ]/k \]  \hspace{1cm} (A5-5)

\[ \tilde{c}_{ww}(\tau) = \langle w_g w_g' \rangle = \langle (k_0w_0 + k_1w_1 + k_2w_2 + k_3w_3)(k_0w_0' + k_1w_1' + k_2w_2' + k_3w_3') \rangle/k \]

\[ = [k_0^2\langle w_0w_0' \rangle + k_1^2\langle w_1w_1' \rangle + k_2^2\langle w_2w_2' \rangle + k_3^2\langle w_3w_3' \rangle + k_0k_1\langle w_0w_1' \rangle \]
\[+ k_0 k_2 \langle w_0 w_2' \rangle + k_0 k_3 \langle w_0 w_3' \rangle + k_1 k_2 \langle w_1 w_2' \rangle + k_1 k_3 \langle w_1 w_3' \rangle + k_2 k_3 \langle w_2 w_3' \rangle + k_1 k_0 \langle w_1 w_0' \rangle + k_2 k_0 \langle w_2 w_0' \rangle + k_2 k_1 \langle w_2 w_1' \rangle + k_3 k_0 \langle w_3 w_1' \rangle + k_3 k_1 \langle w_3 w_1' \rangle + k_3 k_2 \langle w_3 w_2' \rangle \]

\[= k \frac{k_0 + k_1 + k_2 + k_3}{k_0 + k_1 + k_2 + k_3} \]

For convenience, we recall the von Karman model:

\[\zeta_{qq}(\tau) = \langle w_3 - \frac{k_0 w_0 + k_1 w_1 + k_2 w_2}{k_0 + k_1 + k_2} \rangle (w_3 - \frac{k_0 w_0 + k_1 w_1 + k_2 w_2}{k_0 + k_1 + k_2}) \]

\[= \frac{1}{g_{\xi}} \left[ (3w_3 - w_0 - w_1 - w_2) (3w_3 - w_0 - w_1 - w_2) \right] \]

\[= \frac{1}{g_{\xi}} \left[ 9 \langle w_3 w_3' \rangle + \langle w_0 w_0' \rangle + \langle w_1 w_1' \rangle + \langle w_2 w_2' \rangle - 3 \langle w_3 w_0' \rangle - 3 \langle w_3 w_1' \rangle - 3 \langle w_3 w_2' \rangle - 3 \langle w_3 w_3' \rangle + \langle w_0 w_1' \rangle + \langle w_0 w_2' \rangle - 3 \langle w_0 w_0' \rangle + \langle w_0 w_1' \rangle + \langle w_0 w_2' \rangle \right] \]

\[= \frac{1}{g_{\xi}} \left[ 9 \langle w_3 w_3' \rangle + \langle w_0 w_0' \rangle + \langle w_1 w_1' \rangle + \langle w_2 w_2' \rangle - 3 \langle w_3 w_0' \rangle - 3 \langle w_3 w_1' \rangle - 3 \langle w_3 w_2' \rangle - 3 \langle w_3 w_3' \rangle + \langle w_0 w_1' \rangle + \langle w_0 w_2' \rangle - 3 \langle w_0 w_0' \rangle + \langle w_0 w_1' \rangle + \langle w_0 w_2' \rangle \right] \]

where \(k_0 = k_1 = k_2 = 1\) is assumed.

For convenience, we recall the von Karman model:

\[\zeta_{ij}(\xi) = [f(\xi) - g(\xi)] \frac{\xi_{ij}}{\xi^2} + g(\xi) \delta_{ij} \]

where

\[\delta_{ij} = \begin{cases} 0 & (i \neq j) \\ 1 & (i = j) \end{cases} \quad (i, j = 1, 2, 3)\]

V.3
and \((u + 1, v + 2, w + 3)\) is assigned.

\[
\zeta = \frac{a}{aL}
\]

\[
\xi^2 = \xi_1^2 + \xi_2^2 + \xi_3^2
\]

\(a = 1.229\)

In the sketch, it is readily recognized that

\[
\begin{align*}
\tau_1 &= V^E - wt \cos \gamma_G \quad (V^E \text{ - ground speed}) \\
\tau &= V \tau \\n\end{align*}
\]

or

\[
\begin{align*}
\tau_2 &= \sqrt{\tau_1^2 + (\omega/2)^2} \\
\tau_3 &= \sqrt{\tau_1^2 + \omega^2} \\
\end{align*}
\]

\[
\begin{align*}
\tau_4 &= |\tau_1 + \lambda_t| \\
\tau_5 &= |\tau_1 - \lambda_t| \\
\end{align*}
\]

\[
\xi_3 = wt \sin \gamma_G \quad \text{or} \quad \xi_3 = h(\alpha) - h(\beta)
\]

In the above expressions, for example, \(u_0, u_1\) are values of \(u_g\) at points 0 and 1 at time \(t\), and \(u'_0, u'_1\) are values at the same points at the later time \((t + \tau)\).

We get for homogeneous isotropic turbulence:

\[
<u_0'u_0'> = \tilde{C}_{11}(r_1, 0, \xi_3) = [f(\xi_3) - g(\xi_3) \Delta \tilde{C}_{u0}(\tau)]\frac{r_1^2}{r_{13}^2} + g(\xi_3) \Delta \tilde{C}_{u0}(\tau)
\]

V.4
where

\[ r_{13} = \sqrt{r_1^2 + \xi_3^2} \quad \text{and} \quad \zeta_{13} = r_{13}/aL \]

Similarly,

\[ \langle u_0 u_2^* \rangle = \langle u_1 u_0^* \rangle = \langle u_2 u_0^* \rangle = \tilde{C}_{u1}(\tau) \]

\[ \langle u_0 u_3^* \rangle = \tilde{C}_{11}(r_5, 0, \xi_3) = \left[ f(\zeta_5^3) - g(\zeta_5^3) \right] \frac{r_5^2}{r_{53}^2} \]

\[ + g(\zeta_5^3) \triangleq \tilde{C}_{u3}(\tau) \]

\[ \langle u_0 u_1^* \rangle = \tilde{C}_{11}(r_1, \frac{b'}{2}, \xi_3) = \left[ f(\zeta') - g(\zeta') \right] \frac{r_1^2}{r_{12}^2} + g(\zeta') \triangleq \tilde{C}_{u3}(\tau) \]

where

\[ r_{53}^2 = \sqrt{r_5^2 + \xi_3^2} \quad r_1^2 = \sqrt{r_1^2 + \xi_3^2 + (b'/2)^2} \]

\[ \zeta_5^3 = r_{53}/aL, \quad \zeta' = r'/aL \]

\[ \langle u_1 u_2^* \rangle = \tilde{C}_{11}(r_1, b', \xi_3) = \left[ f(\zeta) - g(\zeta) \right] \frac{r_1^2}{r^2} + g(\zeta) \triangleq \tilde{C}_{u2}(\tau) \]

where

\[ r = \sqrt{r_1^2 + b'^2 + \xi_3^2} \quad \text{and} \quad \zeta = r/aL \]

Similarly,

\[ \langle u_2 u_1^* \rangle = \tilde{C}_{u2}(\tau) \]

\[ \langle u_1 u_3^* \rangle = \tilde{C}_{11}(r_5, \frac{b'}{2}, \xi_3) = \left[ f(\zeta_5^3) - g(\zeta_5^3) \right] \frac{r_5^2}{r_{53}^2} \]

\[ + g(\zeta_5^3) \triangleq \tilde{C}_{u4}(\tau) \]
where

\[ r_{53} = \sqrt{r_5^2 + \frac{b_1^2}{4} + \xi_3^2} \quad \text{and} \quad \zeta_{53} = r_{53}/aL \]

Similarly,

\[ u_2 u_3 = \tilde{c}_{u4}(\tau) \]
\[ u_3 u_1 = \tilde{c}_{11}(r_r, \frac{b}{2}, \xi_3) = \left[ f(\zeta_{43}) - g(\zeta_{43}) \right] \frac{r_4^2}{r_{43}^2} + g(\zeta_{43}) \]

where

\[ r_{43} = \sqrt{r_4^2 + \frac{b_2^2}{4} + \xi_3^2} \quad \text{and} \quad \zeta_{43} = r_{43}/aL \]

Similarly,

\[ u_3 u_2 = \tilde{c}_{u5}(\tau) \]
\[ u_3 u_0 = \tilde{c}_{11}(r_4, 0, \xi_3) = \left[ f(\zeta_{43}) - g(\zeta_{43}) \right] \frac{r_4^2}{r_{43}^2} + g(\zeta_{43}) = \tilde{c}_{u6}(\tau) \]

where

\[ r_{43} = \sqrt{r_4^2 + \xi_3^2} \quad \text{and} \quad \zeta_{43} = r_{43}/aL \]

Using the assumptions \( k_0 = k_1 = k_2 = 1.0, k_3 = \eta \) and \( K^2 = (3 + \eta)^2 \), (A5-5) becomes

\[ \tilde{c}_{uu}(\tau) = \frac{1}{(3+\eta)^2} \left[ (3 + \eta^2)\tilde{c}_{u0}(\tau) + 4\tilde{c}_{u1}(\tau) + 2\tilde{c}_{u2}(\tau) + \eta\tilde{c}_{u3}(\tau) \right. \]
\[ + 2\eta\tilde{c}_{u4}(\tau) + \eta\tilde{c}_{u6}(\tau) + 2\eta\tilde{c}_{u5}(\tau) \]

(A5-8)
Using the similar procedure, we may find

\[ \tilde{c}_{22}(r_1, 0, \xi_3) = [f(\xi_{13}) - g(\xi_{13})] \frac{\xi_{13}^2}{r_{13}^2} + g(\xi_{13}) \triangle \tilde{c}_{w0}(\tau) \]

\[ \tilde{c}_{22}(r_1, b', \xi_3) = [f(\xi') - g(\xi')] \frac{\xi_{13}^2}{r_{13}^2} + g(\xi') \triangle \tilde{c}_{w1}(\tau) \]

\[ \tilde{c}_{22}(r_1, b', \xi_3) = [f(\xi) - g(\xi)] \frac{\xi_{13}^2}{r_{13}^2} + g(\xi) \triangle \tilde{c}_{w2}(\tau) \]

\[ \tilde{c}_{22}(r_5, 0, \xi_3) = Lf(\xi_{53}) - g(\xi_{53}) \frac{\xi_{53}^2}{r_{53}^2} + g(\xi_{53}) \triangle \tilde{c}_{w3}(\tau) \]

\[ \tilde{c}_{22}(r_5, b', \xi_3) = [f(\xi_{53}) - g(\xi_{53})] \frac{\xi_{53}^2}{r_{53}^2} + g(\xi_{53}) \triangle \tilde{c}_{w4}(\tau) \]

\[ \tilde{c}_{22}(r_4, b', \xi_3) = Lf(\xi_{43}) - g(\xi_{43}) \frac{\xi_{43}^2}{r_{43}^2} + g(\xi_{43}) \triangle \tilde{c}_{w5}(\tau) \]

\[ \tilde{c}_{22}(r_4, 0, \xi_3) = [f(\xi_{43}) - g(\xi_{43})] \frac{\xi_{43}^2}{r_{43}^2} + g(\xi_{43}) \triangle \tilde{c}_{w6}(\tau) \]

Thus, (A5-6) and (A5-7) become

\[ \tilde{c}_{ww}(\tau) = \frac{1}{(3+\eta)^2} \left[ (3 + \eta^2)\tilde{c}_{w0}(\tau) + 4\tilde{c}_{w1}(\tau) + 2\tilde{c}_{w2}(\tau) + \eta\tilde{c}_{w3}(\tau) \right] \]
Equations (A5-8) through (A5-10) are used in this study. Note that the integration scales are taken to be

\[ L^x_u \text{ for } \tilde{c}_{uu}(\tau) \]
\[ L = \left\{ \begin{array}{ll}
L^x_u & \text{for } \tilde{c}_{uu}(\tau) \\
2L^x_w & \text{for } \tilde{c}_{ww}(\tau) \text{ and } \tilde{c}_{qq}(\tau)
\end{array} \right. \]
APPENDIX VI

METHODS FOR CALCULATING $e^{At}$

(1) Series Development Method

By definition (see [91]),

$$e^A = I + A + \frac{1}{2!} A^2 + \frac{1}{3!} A^3 + \ldots + \frac{1}{i!} A^i + \ldots$$

In practice, $i$ must be finite, say $i < n$. Thus there is a truncated error due to selecting the finite value of $n$. In order to obtain the desired accuracy, $n$ should be chosen such that the error is satisfied. In general, the number of terms needed to calculate in (A6-1) depends on the property of $A$.

(2) Laplace Transfer Method

$$e^{At} = L^{-1}[(S - A)^{-1}]$$

This method is only suitable for the simple case where $A$ is a low order matrix.

(3) Block Diagonal Transformation Method

Let $P$ be the block diagonal transformation; $\lambda_i$ ($i = 1, 2, \ldots, n$) the
eigenvalues of the system matrix $A_j$ and $V_i$ ($i = 1, 2, \ldots, n$), the corresponding eigenvectors. Then we transfer $A$ to a diagonal matrix:

$$p^{-1}Ap \Delta A = \begin{bmatrix} \Lambda_1 & 0 \\ 0 & \Lambda_2 & \ddots \\ & & \ddots & 0 \end{bmatrix}$$  \hspace{1cm} (A6-3)

where

$$\Lambda_i = \begin{bmatrix} \sigma_i \\ -\omega_i \\ \omega_i & \sigma_i \end{bmatrix}$$ for a real eigenvalue

$$\Lambda_i = \begin{bmatrix} \sigma_i & -\omega_i \\ \omega_i & \sigma_i \end{bmatrix}$$ for a pair of complex conjugate eigenvalues

Here

$$\lambda_i = \sigma_i + j\omega_i, \hspace{1cm} \chi_i = \sigma_i - j\omega_i$$

The similarity transformation matrix $p$ is constructed as follows: Let $p = [p_1 p_2 \ldots p_i \ldots] (i = 1, 2, \ldots, n)$. $p_i$ are the columns of $p$ matrix, where

$$p_i = V_i$$ for a real eigenvalue, and

$$p_i = \text{Re}\{V_i\} + \text{Im}\{V_i\} \hspace{1cm} \text{for a pair of complex conjugate eigenvalues}$$

By means of this transformation, the system state transition matrix $e^{At}$ becomes

$$e^{At} = pe^{At} p^{-1} = pe^{(P^{-1} A P)t} p^{-1}$$ \hspace{1cm} (A6-5)

where

VI.2
and

\[
e^{At} = \begin{bmatrix}
\lambda_1 t \\
e^{\lambda_1 t} & \lambda_2 t & 0 \\
0 & e^{\lambda_2 t} & \ddots \\
& & & e^{\lambda_N t}
\end{bmatrix}
\]  \hspace{1cm} (A6-6)

(4) Sylvester Expansion Technique

This method uses the Sylvester Expansion Theorem to determine the state transition matrix [92]. That is:

\[
e^{At} = F_1 e^{\lambda_1 t} + F_2 e^{\lambda_2 t} + \cdots + F_N e^{\lambda_N t}
\]  \hspace{1cm} (A6-8)

where

\[
F_i \text{ are the column matrices in } F
\]

\[
F = (I \ A \ A^2 \ \cdots \ \ A^{N-1})(V^T)^{-1}
\]  \hspace{1cm} (A6-9)

and

\[
V \Delta \begin{bmatrix}
1 & 1 & 1 & \cdots & 1 \\
\lambda_1 & \lambda_2 & \lambda_3 & \cdots & \lambda_N \\
\vdots & \vdots & \vdots & & \vdots \\
\lambda_1 & \lambda_2 & \lambda_3 & \cdots & \lambda_N
\end{bmatrix}
\]
may be called the "Vandermonde matrix". Here $\lambda_1, \lambda_2 \ldots \lambda_N$ are the eigenvalues of the system matrix A.

Equation (A6-8) can be proven by the Hamilton Theorem.

In this study, the Block Diagonal Transformation Method and the Sylvester Expansion Method are used and the computer program codes are given here.

```fortran
SUBROUTINE EXPAT(A.T.EIGR.EIGL.EXPAS.INNOW)
C AT FIRST, INNOW=0--PRINT OUT THE TRANSITION MATRIX
C
IMPLICIT REAL*8(A-H,O-Z)
DIMENSION A(B,B),EIGR(B),EIGL(B),STAT(B,B),SPS(B,B),
EXPAS(B,B),EX(B,B),COMPLEX*16 CA(B,B),CAL(B,B),CA2(B,B),TCA(B,B),DENOM(B),
CEC(B,B)
COMMON/TRANS/EIGR,EIGL
1000 FORMAT(HO,5X,'THE ELEMENTS OF THE STATE MATRIX')
1001 FORMAT(HO,5X,'THE MATRICES COEFFICIENT OF
1002 FORMAT(HO,5X,'THE MATRICES COEFFICIENT OF
1003 FORMAT(HO,5X,'THE MATRICES COEFFICIENT OF
1004 FORMAT(HO,5X,'THE MATRICES COEFFICIENT OF
1005 FORMAT(HO,5X,'THE MATRICES COEFFICIENT OF
1006 FORMAT(HO,5X,'THE MATRICES COEFFICIENT OF
1007 FORMAT(HO,5X,'THE MATRICES COEFFICIENT OF
1008 FORMAT(HO,5X,'THE MATRICES COEFFICIENT OF
1009 FORMAT(HO,5X,'THE MATRICES COEFFICIENT OF
1010 FORMAT:H0,5X,'THE MATRICES COEFFICIENT OF
1011 FORMAT:H0,5X,'THE MATRICES COEFFICIENT OF
1012 FORMAT:H0,5X,'THE MATRICES COEFFICIENT OF
1013 FORMAT:H0,5X,'THE MATRICES COEFFICIENT OF
1014 FORMAT:H0,5X,'THE MATRICES COEFFICIENT OF
1015 FORMAT:H0,5X,'THE MATRICES COEFFICIENT OF
1016 FORMAT:H0,5X,'THE MATRICES COEFFICIENT OF
1017 FORMAT:H0,5X,'THE MATRICES COEFFICIENT OF
1018 FORMAT:H0,5X,'THE MATRICES COEFFICIENT OF
1019 FORMAT:H0,5X,'THE MATRICES COEFFICIENT OF
1020 FORMAT:H0,5X,'THE MATRICES COEFFICIENT OF
1021 FORMAT:H0,5X,'THE MATRICES COEFFICIENT OF
1022 FORMAT:H0,5X,'THE MATRICES COEFFICIENT OF
1023 FORMAT:H0,5X,'THE MATRICES COEFFICIENT OF
1024 FORMAT:H0,5X,'THE MATRICES COEFFICIENT OF
1025 FORMAT:H0,5X,'THE MATRICES COEFFICIENT OF
1026 FORMAT:H0,5X,'THE MATRICES COEFFICIENT OF
1027 FORMAT:H0,5X,'THE MATRICES COEFFICIENT OF
1028 FORMAT:H0,5X,'THE MATRICES COEFFICIENT OF
1029 FORMAT:H0,5X,'THE MATRICES COEFFICIENT OF
1030 FORMAT:H0,5X,'THE MATRICES COEFFICIENT OF
END

SUBROUTINE EXPAS(INOW)
IMPLICIT REAL*8(A-H,O-Z)
COMPLEX*16 LAMAAAD(B),LUAAD(B),LUAADI(B,B),CDEP
COMPLEX*16 STA(B,B),STI(B,B),SPS(B,B)
DIMENSION EXPAS(B,B)
COMMON/EXP/LAMAAAD,LUAAD,LUAADI
DO 1 I=1,B
ZBIG(I,1)=+T0.000,0.000
TCAK,L=I(0,0.000,0.000)
DO 3 LK=1,N
DO 5 LK=1,N
DO 2 L=1,B
CEXPS(I,J)=(0.000,0.000)
DO 3 LK=1,N
2 CEPSB(I,J)=CEPSB(I,J)+LUAADI(I,K)*ZBIG(K,1)
DO 4 I=1,B
DO 4 J=1,B
4 EXPAS(I,J)=CEPSB(I,J)
RETURN
END

VI.4
APPENDIX VII

EVALUATION OF COVARIANCE MATRIX $\Sigma_r(t)$

In order to calculate expressions (5-39) through (5.41) in the text on a digital computer, we now further discuss the detailed procedures. We begin with the basic equations:

\[
\Sigma(t) = B_2 C_{wr}(t, t) - e^{A t} B_2 C_{wr}(0, t) + \int_0^t e^{A(t-\nu)} B C_{wr}(\nu, t) d\nu \quad (A7-1)
\]

\[
C_{wr}(t, s) = C_{ww}(t, s) B_2^T - C_{ww}(t, 0) B_2^T e^{A^T t} + \int_0^s C_{ww}(t, \sigma) B_2^T e^{A^T(t-\sigma)} d\sigma \quad (A7-2)
\]

\[
C_{wr}(0, t) = C_{ww}(0, t) B_2^T - C_{ww}(0, 0) B_2^T e^{A^T t} + \int_0^t C_{ww}(0, \sigma) B_2^T e^{A^T(t-\sigma)} d\sigma \quad (A7-3)
\]

After changing the notation of arguments in (A7-2), we have

\[
C_{wr}(v, t) = C_{ww}(v, t) B_2^T - C_{ww}(0, v) B_2^T e^{A^T t} + \int_0^t C_{ww}(\mu, \sigma) B_2^T e^{A^T(t-\sigma)} d\sigma \quad (A7-4)
\]

It is very important to remember that in the landing case the $C_{ww}(v, \sigma)$ is not only a function of $(\sigma-v)$, but also the function of $v$. Also, $e^{A \gamma}$, $B$ may not be constant because the system matrices $A$, $B_1$ and $B_2$ may change with time. Therefore, under the piecewise-time-independent hypothesis we may calculate $e^{A \gamma}$ backwards step-by-step from $\gamma = 0$ (i.e., $t = t$) to $\gamma = t$ (i.e., $t = 0$) in advance, where $\gamma = t-\sigma$ or $\gamma = t-v$. Then store it in an

VII.1
array for further use.

In integrating (A7-1) through (A7-4) we also may use an approximation that uses $\bar{A}(v)$ and $\bar{B}(v)$ as the average values for calculating $e^{A(t-v)}$ and $\bar{B}$ in each interval. This treatment is good enough because the transients of the aircraft dynamics die out with time and the system matrices $A$ and $B$, etc., only have a few changes during landing.

Now let

$$\beta = \sigma - v \quad \alpha = t - v$$

Hence we have $v = t - \alpha$, $dv = -d\alpha$ and $d\beta = d\sigma$. Denoting

$$C_{ww}(v, \sigma) = C_{ww}(\beta)$$

where the superscript $(v)$ means that the correlation matrix $C_{ww}(\beta)$ is measured starting from $t = v$.

Since

$$t - \sigma = t - v + v - \sigma = \alpha - \beta$$

we have

$$e^{A^T(t-\sigma)} = e^{A^T(\alpha - \beta)}$$

Replacing the notations of $v$ and $\sigma$ in (A7-1) through (A7-4) with $\alpha$ and $\beta$ and noting the lower and upper limits, (A7-4) becomes

$$C_{wr}(\alpha) = C_{ww}^{(v)}(\alpha)B_2^T - C_{ww}^{(0)}(0, t-\alpha)B_2^T e^{A^T t} + \int_0^\alpha C_{ww}^{(v)}(\beta)B_2^T e^{A^T(\alpha - \beta)} d\beta (A7-5)$$

Similarly, (A7-1) becomes

$$\Sigma_r(t) = B_2 C_{wr}(t, t) - e^{A^T t} B_2 C_{wr}(0, t) + \int_0^t e^{A^\alpha} B C_{wr}(\alpha) d\alpha \quad (A7-6)$$

VII.2
By definition,

$$C_{wr}(0, t) = \frac{C^{(0)}(\alpha=t)}{C_{wr}}$$

$$C_{wr}(t, t) = \frac{C(t)(\alpha=0)}{C_{wr}} = \frac{C(t)(0)B_{2}^{T}}{C_{ww}} - \frac{C^{T}(0)(0)B_{2}^{T}}{C_{ww}} e^{A^{T}t}$$

$$+ \int_{0}^{t} \frac{C(t)(\beta)B_{2}^{T} e^{-A^{T}t}}{C_{ww}} d\beta$$

To compute

$$I = \int_{0}^{t} \int_{0}^{\alpha} e^{A\alpha} B_{2}^{T} C_{ww}(\beta) e^{A^{T}(\alpha-\beta)} d\beta d\alpha$$

We should look at the original form of this double integration expression in terms of \(v\) and \(\sigma\). It is readily seen from (A7-4) and (A7-1) that

$$I = \int_{0}^{t} \int_{0}^{\alpha} e^{A(t-v)} B_{2}^{T} C_{ww}(v, \sigma) e^{A^{T}(\sigma-\sigma)} d\sigma dv$$

This is the same as that shown in Eq. (B-2) of Ref. [27] except the notations being slightly different \((v \sim t_{1}, \sigma \sim t_{2})\). Following the approach proposed in [27], we take advantage of the symmetry condition

$$C_{ww}(v, \sigma) = C_{ww}^{T}(\sigma, v)$$

Noting the definitions

$$\alpha = t-\mu, \quad \beta = \sigma-\mu$$

VII.3
we finally obtain

\[ I = \int_0^t \int_0^\alpha (D + D^T) d\beta \ d\alpha \]  

(A7-9)

where

\[ D = e^{A\alpha} \tilde{B} C_{\tilde{W}}(\nu) \tilde{B}^T e^{A^T(\alpha-\beta)} \]  

(A7-10)
APPENDIX VIII

EVALUATING THE COVARIANCE MATRIX BY DIRECT INTEGRATION METHOD

We begin with the state equation

\[ \dot{\mathbf{R}}(t) = A \mathbf{R}(t) + B_1 \mathbf{W}(t) + B_2 \dot{\mathbf{W}}(t) \]  \hspace{1cm} (A8-1)

Let

\[ s = t + \tau \]

Postmultiplying (A8-1) by \( \mathbf{R}^T(s) \) it follows that

\[ \dot{\mathbf{R}}(t)\mathbf{R}^T(s) = A \mathbf{R}(t)\mathbf{R}^T(s) + B_1 \mathbf{W}(t)\mathbf{R}^T(s) + B_2 \dot{\mathbf{W}}(t)\mathbf{R}^T(s) \] \hspace{1cm} (A8-2)

If the random process is stationary, there exists the following property:

\[ \frac{d}{dt} \mathbf{R}(t)\mathbf{R}^T(s) = \frac{d\mathbf{R}(t)}{ds} \mathbf{R}^T(s) \quad \text{or} \quad \frac{d}{ds} \mathbf{R}(t)\mathbf{R}^T(s) = \mathbf{R}(t) \frac{d\mathbf{R}^T(s)}{ds} \]

Taking the ensemble of (A8-2) and defining

\[ C_{R}(t, s) \triangleq \mathbf{R}(t)\mathbf{R}^T(s) \]

\[ C_{W}(t, s) \triangleq \mathbf{W}(t)\mathbf{R}^T(s) \]
which are the constrained auto- or cross-correlations by a flight path. Therefore, (A8-2) turns to

\[
\frac{d}{dt} C_{rr}(t, s) = A C_{rr}(t, s) + B_1 C_{wr}(t, s) + B_2 \frac{d}{dt} C_{wr}(t, s) \quad (A8-3)
\]

The initial conditions are

\[
C_{rr}(0, s), \quad C_{wr}(0, s) \quad \text{and} \quad C_{wr}(0, s)
\]

In (A8-3), the argument is 't'; 's' should be specified, say s*. Replacing 't' in (A8-1) by 's' and transposing (A8-1) yields

\[
\dot{R}^T(s) = R^T(s)A^T + W^T(s)B_1^T + W^T(s)B_2^T \quad (A8-4)
\]

Premultiplying (A8-4) by \(W(t)\) and taking the average, we get

\[
\frac{d}{ds} C_{wr}(t, s) = C_{wr}(t, s)A^T + C_{ww}(t, s)B_1^T + \frac{d}{ds} C_{ww}(t, s)B_2^T \quad (A8-5a)
\]

or

\[
\frac{d}{ds} C_{wr}^T(t, s) = A C_{wr}^T(t, s) + B_1 C_{ww}^T(t, s) + B_2 \frac{d}{ds} C_{ww}^T(t, s) \quad (A8-5b)
\]

The initial conditions are:

\[
C_{wr}(t, 0), \quad C_{ww}(t, 0) \quad \text{and} \quad C_{ww}(t, 0)
\]

In (A8-5), the argument is 's'; 't' should be specified, say t*. In order to solve (A8-3), we need to find its initial conditions, i.e.,
In doing so, let 't' in (A-5a) be 0. It follows that

$$\frac{d}{ds} C_{\text{wr}}(0, s) = C_{\text{wr}}(0, s) A^T + C_{\text{ww}}(0, s) B_1^T + \frac{d}{ds} C_{\text{ww}}(0, s) B_2^T$$

(A8-6)

The initial conditions of (A8-6) are

$$C_{\text{wr}}(0,0), \quad C_{\text{ww}}(0,0) \quad \text{and} \quad C_{\text{ww}}(0,0)$$

In the case of studying the take-off or landing problems, we may take

$$C_{\text{wr}}(0,0) = 0$$

Here $C_{\text{ww}}(0,0)$ and $C_{\text{ww}}(0,0)$ can be calculated or measured. Integrating (A8-6) from $s = 0$ to $s = s^*$, we may solve (A8-6) for $C_{\text{wr}}(0, s^*)$ and $C_{\text{wr}}(0, s^*)$.

To find $C_{\text{rr}}(0, s^*)$, premultiplying (A8-4) by $R(0)$ and taking the average, it follows that

$$\frac{d}{ds} C_{\text{rr}}(0, s^*) = C_{\text{rr}}(0, s^*) A^T + C_{\text{rw}}(0, s^*) B_1^T + C_{\text{rw}}(0, s^*) B_2^T$$

(A8-7)

The initial conditions are

$$C_{\text{rr}}(0,0), \quad C_{\text{rw}}(0,0) \quad \text{and} \quad C_{\text{rw}}(0,0)$$
As long as the above values are given, we are able to solve (A8-7) for \( C_{rr}(0, s^*) \) which is used in solving (A8-3) as the initial condition.

Thus, using (A8-6), (A8-7), (A8-5a) and (A8-3) one may obtain the covariance matrix

\[
\Sigma_r(t^*) \triangleq C_{rr}(t^*, t^*)
\]

It is clear that this approach can be used for a nonlinear and nonstationary system.
APPENDIX IX

ACCIDENT/INCIDENT STATISTICS

IX.A Number of Air Carrier Aircraft Operations at FAA Facilities

<table>
<thead>
<tr>
<th>Year</th>
<th>1964</th>
<th>1965</th>
<th>1966</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>7,447,434</td>
<td>7,819,144</td>
<td>8,206,322</td>
</tr>
</tbody>
</table>

IX.B Number of Air Carrier Instrument Approaches

<table>
<thead>
<tr>
<th>Year</th>
<th>1964</th>
<th>1965</th>
<th>1966</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>564,195</td>
<td>620,645</td>
<td>664,435</td>
</tr>
</tbody>
</table>

IX.C Overall Summary of Terminal Accidents

<table>
<thead>
<tr>
<th>Type of Operation</th>
<th>Occurrence Probability $\times 10^7$</th>
<th>Number of Accidents 1964-1966</th>
</tr>
</thead>
<tbody>
<tr>
<td>Takeoff</td>
<td>9.372 (Takeoff Acc/Takeoff)</td>
<td>11</td>
</tr>
<tr>
<td>Landing (Total)</td>
<td>48.57 (Landing Acc/Landing)</td>
<td>57</td>
</tr>
<tr>
<td>Terminal</td>
<td>28.97 (Accidents/Operation)</td>
<td>68</td>
</tr>
</tbody>
</table>

1 Follow [18].
# IX.D

## TYPES OF TERMINAL AREA INCIDENTS AND THEIR STATISTICS*

<table>
<thead>
<tr>
<th>TYPE OF INCIDENT</th>
<th>PHASE OF FLIGHT</th>
<th>OCCURRENCE PROBABILITY $\times 10^7$</th>
<th>NUMBER IN YEARS</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>(TYPE INC./TAKEOFF)</td>
<td>1960-1966</td>
</tr>
<tr>
<td>Premature lift-off, stall, low airspeed</td>
<td>Roll, initial climb</td>
<td>2.556</td>
<td>3</td>
</tr>
<tr>
<td>Ground loop, aborted takeoff</td>
<td>Roll, aborted takeoff</td>
<td>16.189</td>
<td>19</td>
</tr>
<tr>
<td>Deviation from prescribed course</td>
<td>Initial climb</td>
<td>1.704</td>
<td>2</td>
</tr>
<tr>
<td>Evasive maneuver</td>
<td>Initial climb, roll</td>
<td>5.112</td>
<td>6</td>
</tr>
<tr>
<td>Dragged wing tip, swerve</td>
<td>Roll, aborted takeoff</td>
<td>11.077</td>
<td>13</td>
</tr>
<tr>
<td>Ground system failure</td>
<td>Roll, initial climb</td>
<td>0.852</td>
<td>1</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>TYPE OF INCIDENT</th>
<th>PHASE OF FLIGHT</th>
<th>OCCURRENCE PROBABILITY $\times 10^7$</th>
<th>NUMBER IN YEARS</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>(TYPE INC./LANDING)</td>
<td>1960-1966</td>
</tr>
<tr>
<td>Inadvertent gear retraction/failure to extend</td>
<td>Final approach, roll</td>
<td>1.704</td>
<td>2</td>
</tr>
<tr>
<td>Overshoot, ground loop</td>
<td>Roll, level off/touchdown</td>
<td>17.041</td>
<td>20</td>
</tr>
<tr>
<td>Undershoot</td>
<td>Final approach</td>
<td>15.357</td>
<td>18</td>
</tr>
<tr>
<td>Hard landing</td>
<td>Level off/touchdown</td>
<td>15.357</td>
<td>18</td>
</tr>
<tr>
<td>Line up, swerve off runway</td>
<td>Final approach, level off/touchdown</td>
<td>49.419</td>
<td>58</td>
</tr>
<tr>
<td>Dragged wing tip</td>
<td>Level off/touchdown, roll</td>
<td>14.485</td>
<td>17</td>
</tr>
<tr>
<td>Ground facility/system unsafe, failed</td>
<td>Initial approach, final approach, level off/touchdown, roll</td>
<td>7.668</td>
<td>9</td>
</tr>
<tr>
<td>Evasive maneuver</td>
<td>Initial approach, final approach</td>
<td>1.704</td>
<td>2</td>
</tr>
</tbody>
</table>

*These data have been summarized from public records made available through the Flight Standards Division of the FAA. The data have been culled from the public records and classified according to the ground rules discussed in the 10 Oct. 1968 Quarterly Progress Report. In particular, incidents involving equipment failures only are generally excluded from consideration.

---

IX-2
IX.E

TYPES OF TERMINAL AREA ACCIDENTS AND THEIR STATISTICS

<table>
<thead>
<tr>
<th>TYPE</th>
<th>SUBPHASE</th>
<th>OCCURRENCE PROBABILITY X 10^7</th>
</tr>
</thead>
<tbody>
<tr>
<td>TYPES OF TAKEOFF ACCIDENTS</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Premature lift-off — stall — low airspeed</td>
<td>Roll, initial climb</td>
<td>5.112</td>
</tr>
<tr>
<td>Ground loop — aborted takeoff</td>
<td>Roll, aborted takeoff</td>
<td>3.408</td>
</tr>
<tr>
<td>Deviation from prescribed course</td>
<td>Initial climb</td>
<td>0.852</td>
</tr>
<tr>
<td>Evasive maneuver</td>
<td>Initial climb</td>
<td>0</td>
</tr>
<tr>
<td>Dragged wingtip</td>
<td>Roll, aborted takeoff</td>
<td>0</td>
</tr>
<tr>
<td>TYPES OF LANDING ACCIDENTS</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Inadvertent gear retraction/failure to extend</td>
<td>Final approach, roll</td>
<td>5.112</td>
</tr>
<tr>
<td>Overshoot — ground loop</td>
<td>Roll, level off/touchdown</td>
<td>12.78/16.32</td>
</tr>
<tr>
<td>Hydroplaning factor</td>
<td>Roll</td>
<td>(4/2)</td>
</tr>
<tr>
<td>Undershoot</td>
<td>Final approach (IFR or VFR)</td>
<td>16.19/27.03</td>
</tr>
<tr>
<td>Hard landing</td>
<td>Level off/touchdown</td>
<td>10.22</td>
</tr>
<tr>
<td>Line-up — swerve off runway</td>
<td>Final app., level off/touchdown</td>
<td>1.704/5.40</td>
</tr>
<tr>
<td>Dragged wingtip</td>
<td>Level off/touchdown, roll</td>
<td>0.852</td>
</tr>
<tr>
<td>Ground facility unsafe or failed</td>
<td>Initial approach (IFR or VFR), final approach, level off/touchdown, roll</td>
<td>1.704/0</td>
</tr>
<tr>
<td>Evasive maneuver</td>
<td>Initial approach (IFR or VFR), final approach (IFR)</td>
<td>0/0</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>TYPE</th>
<th>SUBPHASE</th>
<th>OCCURRENCE PROBABILITY X 10^7</th>
</tr>
</thead>
<tbody>
<tr>
<td>TYPES OF TAKEOFF ACCIDENTS</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Premature lift-off — stall — low airspeed</td>
<td>Roll, initial climb</td>
<td>5.112</td>
</tr>
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<tr>
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<td>Initial climb</td>
<td>0</td>
</tr>
<tr>
<td>Dragged wingtip</td>
<td>Roll, aborted takeoff</td>
<td>0</td>
</tr>
<tr>
<td>TYPES OF LANDING ACCIDENTS</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Inadvertent gear retraction/failure to extend</td>
<td>Final approach, roll</td>
<td>5.112</td>
</tr>
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<td>Overshoot — ground loop</td>
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</tr>
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<td>Hydroplaning factor</td>
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<td>(4/2)</td>
</tr>
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<td>1.704/0</td>
</tr>
<tr>
<td>Evasive maneuver</td>
<td>Initial approach (IFR or VFR), final approach (IFR)</td>
<td>0/0</td>
</tr>
</tbody>
</table>


* Includes U. S. Air Carrier accidents occurring in the 50 states, possessions, and protectorates.

* When two numbers appear, the first number is the total number of accidents of given type for visual and instrument flight; the second number is for those accidents which arise from instrument flight.
### IX.F

**TERMINAL AREA ACCIDENTS AND INCIDENTS BY TYPE OF AIRCRAFT FOR 1964-1966**

<table>
<thead>
<tr>
<th>TYPE AIRCRAFT</th>
<th>ACCIDENTS</th>
<th>INCIDENTS</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>TAKEOFF</td>
<td>LANDING</td>
</tr>
<tr>
<td>B-707</td>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>B-702</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>B-701</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>DC-3</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>DC-7</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>DC-6</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>CV-580</td>
<td>3</td>
<td>1</td>
</tr>
<tr>
<td>CV-600</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>CV-620</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>H-199</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>C-109 (184)</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Vickers (V-810, 810, 755, 700)</td>
<td>1</td>
<td>P</td>
</tr>
<tr>
<td>A-11</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Argosy AM-650</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>F-167</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>C-41</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>Beech 18*</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Grumman G-73</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Grumman G-44</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>WAC 7-11</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

*Does not meet gross weight ground rule.

**IX-4**
APPENDIX X

CALCULATION OF MODIFIED BESSEL'S FUNCTION

The modified (or hyperbolic) Bessel differential equation is

$$t^2 \frac{d^2 u}{dt^2} + t \frac{du}{dt} (t^2 + n^2) u = 0$$  \hspace{1cm} (A7-1)

The solution to this equation can be expressed as follows:

$$K_n(t) = \frac{1}{2} \pi \frac{I_{-n}(t) - I_n(t)}{\sin(n \pi)}$$  \hspace{1cm} (A7-2)

where \( n \) is not an integer. In the case of \( n \) being an integer, the limit then must be taken.

\( I_n(t) \) is defined by

$$I_n(t) = \sum_{k=0}^{\infty} \frac{1}{k! \Gamma(n + k + 1)} \left( \frac{t}{2} \right)^{n+2k}$$  \hspace{1cm} (A7-3)

The following computer codes are for the solution of \( K_n(t) \). In the program, \( x \) is an argument. \( P \) is the order of the modified Bessel function.

The example calculations for \( K_{1/3}(x) \) and \( K_{2/3}(x) \) are given in Fig. 39.
Appendix XI

COMPUTER PROGRAM FOR SOLVING MEAN VALUE (μ)

RESPONSE TO MEAN SHEAR SEPT. 10, 1983

This program is for the simulation of landing approach, landing abort, landing flare.

CASE 1 ——- LANDING ABORT
CASE 2 ——- LANDING APPROACH PLUS ABORT
CASE 3 ——- LANDING FLARE

FY — DESCENT AIRSPEED FACTOR, PT = THRUST-FACTOR, ADGE = WHITH GROUND EFFECT, CMS = DNZC SWITCHING, MODE = CONTROL MODEL IN CASE 1 OR 2, WIND = WIND MODEL

IMPLICIT REAL*S (A-H.O-Z)
IKU.KTH.KUD.KUB . KTHB.KTHD.MDEE.MDTT.KAE.KFW.KW. KWD.KALF . ~LFD.
REAL*S KDT.KOD
REAL*S KUT. KUDT
REAL*S .
AWK (1'2)
REAL*4 GAOG
REAL*4 SXX(1'00,4S).TVEC(S)
REAL*4
XPLOT(1~.4S).XN(4).YN(4).ANOLE
DI MENS ION
FF(B.2).C~(B.2).Al0(B.2)
DIMENSION IWK(S)
DIMENSION X(4).DFDX(4)
DIMENSION DX(S).UA(B).UB(B).DXD(B).A7(B.B).AB(B.B)
DIMENSION WSWE(3)
COMPLEX*16 ZI(B.B).ZN. WW(B)
COMMON/NUMB/L. I1K
COMMON/TTGF/TTT1. TTT2
COMMON/ALN/VVE. Aerod. GAMC
COMMON/HADD/WDT. WBDT. MMEE. WHEE3
COMMON/OUT/DZNG.DZN. DETC. DTTC
COMMON/DE/GE. SY. CASE. ADGE
COMMON/ZGF/GZP. KFG. XFG. TAF. TAUF. TAUH. DELTE. GEFL
COMMON/WINDS/DDDMU. DDDMM. DVEE. CM. DME. DME
COMMON/HADD/WCH. WD3D. HII. HI0. W3DD2. WINK. WINK. XII
COMMON/A7B/KT. KE. GKI. ALFAFE. CCLG. CCLLD. XA. 6
COMMON/CA/DDLW. DDDW. DUDC. DDLC. DTHC. DDDC. DDC. DDTT. DDDO. TDGA
COMMON/CB/KK. CCDDE. APSTT. CCME. THETE. CCLL. CCLL. ALFAO. LTH. C. CCMO.
/CAMA. CCMDEE. CCLLDE
COMMON/GNZ/DNZCD
COMMON/SVE/DD. DDD. DNZI. AKD. DALFG. QUDC. DTHC. DDDO. DNZC. DETEE.
/THCFC. CH. HDDD. DTDTO

COMMON/GAN/CMSW. KTH. KTHD. KALF. KALFD. KU. KUD. KUT. KUDT. QLIDP. QLIDD. /KH. KHD. ENHAN. AKNZ. AKINTH. AKS. AKUB. AIRSPE. DHDO. VVST. AKC. TDL. /HHDF. HD. SWTO. XXDD. HHDD. DDOD
COMMON/FLAR/TSS. HOST. VVEEE. DH. HHCD. ENCH. ENTOU. HHC. HHDC. VVEE. SING . /COSDD
COMMON/GEOM/XXOE. ZGE. DMMGE. CCLLGE. CCDDGE. CCMMGE. CCLLGE. CCDDGE. /CMMGE. ZGGE. XXG. DMMGE
COMMON/MINSH/WRRD. HHSTA. AAFF
EXTERNAL F
EXTERNAL CALC
READ(5,1)I0
READ(5,11)KU.KUD.KALF.KTH.KALFD.KTHD.KH.KHD
READ(5,11)DFX0. DX20. DX30. DX40. DX50. DX60. DX70. DX80
READ(5,1014)CM. GKI. GKI2. XA. CASE. MODE. IWIN. ITIMR
READ(5,667)W3DD. W3DD2. DWW
READ(5,8009)DWD. DWEE. WINH. WINL
READ(5,8009)AKS. AKINTH. ENHAN. AKNZ
DO 22559 INT=1. I00
READ(5,2)WRRD. HHSTA. AAFF. IPR1=5
IPRT=5
CASE 2
NARY1=1500
NARY2=48
NARY3=48
NOPT=NARY1
AIRSPE=1
HHC=30.0
DDDO=18.24
HDDF=0.0
VERT=0
CMSW=1.0
GAMSO=0.05236
PT=1.0
FTR=1.0
FV=1.3
ADGE=1.0
TDL=20.0
DUAC=6.62
DALFG=0.1115
GEFL=0.0
DDDO=0.00
DDDO=0.00
DDALFC=0.00
DDTCH=0.00
DDHRC=0.00
DDRC=0.00
DDTC=0.00
DDUC=0.00
HD=18.29
XXOD=0.00
HHDD=0.00
SWFLR=0.0
TT=4.0
IF(CASE.EQ.2)TT=2.0
C CALCULATION OF REFERENCE EQUILIBRIUM VALUES

C

CCDA=1.13
CCLLA=5.67
CCMDEE=-1.40
CCMA=-1.45
CCLLDE=0.356
DELTO=-0.109948
CCDDDEO=0.263
CCLDEO=1.76
ALFAO=0.148342
ALFADO=CCDDDEO-CCLLDE=DELTO/CCLLA

C THIS COMPLETES THE CALCULATION OF THE REFERENCE EQUILIBRIUM CONDITIONS

C

XI-2
CCTTE=X(1)
ALFAFE=X(2)
CCCLLE=X(3)
DELTE=X(4)
DALFD=ALFAFE*180./PI
CCDDE=CCDDE+XX+CCCLLE+2
DETTEE=CCTTE*QE*SY/TM
GO TO 595
WRITE(6,2222)ALFAF,GAME,THETE,SIE,VVEEE
2222 FORMAT(1HO,5X,'ALFA='','F10.4,3X,'GAME='','F10.4,3X,'THETE='','F10.4,
/3X,'SIE='','F10.4,3X,'VVEEE='','F10.4)
WRITE(6,105)CCCLLE,CCDDE,CCTTE,DALFD,DELTE
105 FORMAT(1HO,5X,'CCCLLE='','IPE14,5X,'CCDDE='','IPE14,5X,'CCTTE='',
/IPE14,5X,'DALFD='','IPE14,5X,'DELTE='','IPE14,5X)
599 CONTINUE
CCZAD=6.7
CCZG=-5.65
CCMAD=-3.3
CCMG=-21.4
CCXDEE=0.0
CCZDEE=0.356
CCCLCT=0.0D0
CCDDCT=0.0
COSTE=DCOS(THETE)
TANTE=DSIN(THETE)
DELTE=DSIN/DCOS
DA=ALFAFE+APSTT
SINDA=DSIN(DA)
COSDA=DCOS(DA)
CCCLLS=CCCLLE+SINDA*CCTTE
CCCLCT=CCCLCT+SINDA
CCDDE=CCDDE-COSDA*CCTTE
CCDDCT=CCDDCT-COSDA
TS=C/(2.0*VVE)
CCMCT=LT/CC
CCXTC=CCDDCT
CCXU=-2.0*CCXCT*CCTTE
CCZU=2.0CCCLCT*CCTTE
CCMU=-2.0CCCMCT*CCTTE
CCXG=-0.0
CCZA=-CCLLA-CCDDE
CCXA=-CCLLS-CCDDA

FTR FOR ESTIMATING THE ENDING FAIL EFFECTS ON PERFORMANCE

CCTTDT=TM/GE/SY=FTR
XU=RHO*VVE*SY*CCCLLS*TANTE+0.5*RHO*SY*VVE*CCXU
XW=0.5RHO*VVE*SY*CCXA
XG=TS*GE*SY*CCXG
ZU=RHO*VVE*SY*CCCLLS+0.5*RHO*SY*VVE*CCZU
ZW=0.5*RHO*VVE*SY*CCZA
ZW=0.5TS*RHO*VVE*SY*CCZAD
ZG=TS*GE*SY*CCZG

MU=0.5*RHO*VVE*SY*CCMU
MM=0.5*RHO*VVE*SY*CCMA
MD=0.5*TS*RHO*VVE*SY*CCMAD
MG=TS*GE*SY*CCMG
XDEE=GE*SY*CCXDEE
XDTT=GE*SY*CCXCT*CCTTDT
ZDEE=GE*SY*CCZDEE
ZDTT=GE*SY*CCCLCL*CCTTDT
MDEE=GE*SY*CCCMDEE
MDTT=GE*SY*CCCMCT*CCTTDT

TO DEFINE MATRICES

DO 31 I=1,N
DO 31 J=1,N
A1(I,J)=0.0
31 A1(I,J)=0.0
DO 32 I=4,N
A1(I,1)=1.0
32 CONTINUE
A1(1,1)=MASS
A1(2,2)=MASS-ZWD
A1(3,2)=MWD
A1(3,3)=IYY
A2(1,1)=XU
A2(1,2)=XW
A2(2,1)=ZU
A2(2,2)=ZW
A2(4,1)=MASS*O-COSTE
A2(4,2)=MASS*VVE*SINTE
A2(3,1)=MU
A2(3,2)=MW
A2(3,3)=MQ
A2(4,1)=MASS*VVE+ZG
A2(4,2)=MASS*O*SINTE
A2(5,1)=COSTE
A2(5,2)=SINTE
A2(5,3)=SINTE
A2(5,4)=VVE*SINTE
A2(6,1)=SINTE
A2(6,2)=COSTE
A2(6,3)=COSTE
A2(6,4)=VVE*COSTE
DO 44 I=1,N
DO 44 J=1,L
B1(I,J)=0.0
44 B1(I,J)=0.0
B3(I,J)=0.0
B1(I,1)=XDTT
B2(I,1)=ZDTT
B1(I,3)=MDTT
B1(I,2)=XDEE
B1(I,4)=ZDEE
B1(3,2)=MDEE
B1(8,2)=KE
CALL VMLUFF(A1INV, B1, N, N, L, N, N, C1, N, IER)
CALL VMLUFF(A1INV, B2, N, N, L, N, N, C2, N, IER)
CALL VMLUFF(A1INV, B3, N, N, L, N, N, C3, N, IER)
CALL VMLUFF(C1, K1, N, L, N, N, L, D1, N, IER)
CALL VMLUFF(C1, K2, N, L, N, N, L, D2, N, IER)
DO 99 I=1, N
DO 99 J=1, N
AD1(I, J)=0.0
99
AD2(I, J)=0.0
DO 100 I=1, N
DO 100 J=1, N
D1(I, J)=-D1(I, J)
100
AD1(I, J)=AA(I, J)+D1(I, J)
DO 1188 I=1, N
DO 1188 J=1, N
1188 AD2(I, J)=D2(I, J)
DO 1199 I=1, N
1199 AD2(I, J)=AD2(I, J)+D2(I, J)
IDGT=4
CALL LINV2F(AID2, N, N, AID2IN, IDGT, WK, IER)
CALL VMLUFF(AID2IN, AD1, N, N, N, N, AD2, N, IER)
CALL VMLUFF(AID2IN, C1, N, L, N, N, A4, N, IER)
CALL VMLUFF(AID2IN, C2, N, L, N, N, A5, N, IER)
CALL VMLUFF(AID2IN, C3, N, L, N, N, A6, N, IER)
CALL VMLUFF(AID2IN, D1, N, N, N, N, A7, N, IER)
CALL VMLUFF(AID2IN, D2, N, N, N, N, A8, N, IER)

CALCULATION OF EIGENVALUES

IJOB=2
CALL EIGRF(A3, N, N, IJOB, WK, ZI, N, WWK, IER)
IF(ITIMR . NE. 1) GO TO 22559
T=0.000
AINSTD=0.0
AINTD=0.0
ALFAMS=0.0
ALTIE=0.0
AKC=0.000
AKD=0.000
AINDEX=0.0
DDDT=0.0
DNZC=0.000
DNZ=0.0
DAF=0.0
DETDD=0.0
DD=0.0
DDD=0.0
DDWD=0.0
DDWD=0.0
EN gon=0.0
ENTG=0.0
GAMMT=0.0
GAMAP=0.000
GAMC=0.0

TO CALCULATE TRANSIENT DISTURBANCE FORCES DUE TO FLAPS AND GEAR

IDGT=4
AKCD=0.95
CCLLDF=2.307
CCDDO=0.0498*AKCD
CCDDF=0.0498*(1.0-AKCD)
ZDEF=GE*SY*CCLLDF
ZGFF=ZDEF*0.17452
XQF=GE*SY*CCDDF
XQG=GE*SY*CCDDO
CALL LINV2F(A1, N, N, A1INV, IDGT, WK, IER)
CALL VMLUFF(A1INV, A2, N, N, N, N, AA, N, IER)
~

LK-LK+l
.JK-,N.+ 1
HHIfl-o. 0
AI'1\--O . 43ó3
AP1A-o . 4363
DI'1AX-AI'1A-OELTE
OI'1IN-AP1I-OELTE
IF(OX(8"OE . OMAX)OX(8)-DMAX
IF(DX(S"LE . DMIN)DX(S)=DP1IN
TTI'1AX-l.O-DETTEE
TTI'1IN--DETTEE
IF(DX(7). GE . TTMAX)DX(7)-TTMAX
IF(DX(7"LE . TTMIN)DX(7)-TTMIN
AlTIE-(ALFAFE-0AP100)*S7 . 3
HHOD-OXD( ó) +CHO
OAF-OX(2)/VVE*57. 3+DALFO
DETDD-(DELTE+DX(S) )*57 . 3
THRlJ-TI'1*(DX (7'.1 +DETTEE)
XII-(VVEEE*COSGG*T+DXXO)+DX(5)
HII=(-VVEEE*SINQG*T+DHHO)+DX(ó)
IF(SWFLR.GT.O . O) GO TO 11011
IF(HiI. LE . IB. 29) SWFLR"I . 0
IF(SWFLR . EO.l)XXOO=XII
IF(SWFLR.EO.l)HHOO=HII
IF(SWFLR. EO . l)CASE=3. 0
011 CONTINUE
HHO-DXbO
IF(DDDWW. NE . O. O)VERT=I . 0
IF(VERT. EO.l) GO TO 99999
IF(CASE.EO. 3.ANO. VERT. EO. 1. 0) HHD-HHO+DOWWIDAB5(W3D02)
'999 CONTINUE
CALL WIND5(T,DX,DXO,5INOQ,VVEEE)
DDDWV-DDDWW
DDDloH-DDDWU
DDDWU-DDDWU-WWEE
VVAA-OSORT«VVE+DX(1»**2+DX(2)*DX(2»
VVEE_D5ORT«VVAA*OCOS(GAP1AP)+ODDWH)**2+(VVAA*OSIN(GAP1AP)+ODDWV)
/**2)
OAMAP-OX(4)-DX(2)/VVE-0AP1GG
GAMC-57 . 3*0AP1AP
GAMl'1TsOX(4)*57 . 3 +ALTIE
DNZ_-(DXD(2)-W30T-DX(3)*VVEE-XXA*DXD(3»/G*DCOS(ALFAFE)
ALFAP1S-DALFD+(DX(4)-(DXO(ó»/VVEE)*S7.3
IF(DXD(b). GE. CHD)AKC"'I . 0
IF (DXD(6) QE. CHD)AKD=1. 0
CALL CONTR(CASE,MOOE,T.OX. DXD. OTTC.DETC)
IF(CASE.EO. 3. OR . HD . EO. 0.0) QO TO 1109

1949

1957

1966
1109

1010

7794
INDEX AND SWITCH TIME CALCULATION
IF(SWT1 . GT. 0.0)00 TO 1949
IF(DABS(HHDD) . LE. 5 . 0D-02)5WT1-l . 0
IF(SWTl . EO.1.0HTT1=T
IF(SWTl . EG.llHI'1IN-HII

15

)CT

-6

IF(SWTI . EO . 1. 0) DHMIN"'30. 0-....,IN
IF (SWT 1. EO . 1. 0 )5IGMAl= (Hl 1 -t+fCC )*( HII-HHCC ) 130 . 0/30 . 0*0 . 5
CONTINUE
IF(SWT2 . 0T . O. Ol GO TO 1957
IF (H-mo . OT . O. O. AND . DAB5(HI I-HHCC) . LE 1. OD-01lSWT2-1. 0
IF(SWT2 . EG . 1. 0)TTT2-T
IF(SWT2 EG . 1.0)SIOMA2=DX(1)*DX(I)/VVE/VVE
CONTINUE
IF(SWT3 . GT . 0. 0) GO TO 19óó
IF(DABS(DX (4)-0 . 2443)*57. 3 . LE . 1. OD-02)5WT3-1. 0
IF(SWT3 . EG. 1. 0ITTT3=T
IF (SWT3 . EO . 1. 0 ISIGMA3=DX( 1) *DX (11 /VVE/VVE
CONTINUE
AINDEX=5IGMA1+5IGMA2+5IQMA3
CONTINUE
5XX(JK. (1+1510 ) ..T
5XX(JK. (2+1510 )=VVAA
5XX(JK. (3+1510 I=VVEE
5XX(JK. (4+15IU I=GAMMT
5XX(JK. (5+1510 )=DAF
5XX(JK. (ó+1510 )=DETDD
5XX(JK. (7+1511..1 )"HHC
SXX(JK. (8+ISK) )'*lHOC
SXX(JK. (9+151'.) )=DOOWU+WWEE
SXX(JK. (10+15K»-DDDWW
SXX(JK. (11+15K»-THRU
SXX(JK. <12+ISK»-HHOD
XPLOT(JK. (1+IISJ) )"XII
XPLOT (JK. (2+11 SJ) ) =HII
IF (lJ(1 IPRT*IPRT . EO . LK . AND . HIJ . LE . 50. 0)WRITE(6. 15>T. DX( 1). DAF.
IHHOD.GAMMT. XII.HII.DETDO.THRU.OAP1C
IFCT. GE 150. 0) GO' TO 1010
IF(HII . LE . 0 50) GO TO 1010
IF(LK . LE . NOPTlGO TO 10
XXTO=XII
OHDTD:HHDD
NPRI=JK
NO( IIKK)=JK
IF(NO( IIKK) . GE . NOPT) NO(IIKK)-NOPT
TTOo=T
VVEETD=WEE
AT ITUD=GAMMT
WRITE(ó. 1011)XXTO, DHDTD.TTDO.VVEETD.ATITUD,NO(IIKK)
WRlTE(ó.l7>
DO 7794 JK=ó90,NPRI. IPRT
WR ITE (ó. 7Ó54)SXX( JK. (1 +ISK». SXX (JK. (9+ISK». SXX( JK. (10+151'.»,
/5XX(JK. (3+ISK) ),SXX(JK. (2+151'.) ).SXX(JK. (&+ISK) 1.5XX(.JK. (7+ISKI)
CONTINUE
WRITE(ó.1985)
WRITE(ó. 1984)KU,KUD,AKUB.AKNZ.AKINTH.KTH.KTHD.ENHAN,AKS.KH,KHD.
IHHDF
WRITE(ó.9999)TTTO.TTT1.TTT2.TTT3
WRITE(6.7009)AINDEX . DHMIN
FORMAT(lX. F5. 2.1X. 9E12 . 4)


CONTINUE
CALL VMULFF(A1INV, GEO, N. N. 1, N. N, Q. 1, N. IER)
CALL VMULFF(A1D2IN, GEO, N. N. 1, N. N, Q. 2, N. IER)
DO 444 I=1, N
AGE(I)=0. 000
CONTINUE
DO 850 I=1, L
DR(I)=0. 0
DO 128 I=1, N
A4R(I)=0. 0
DO 128 J=1, L
A4R(I)=A4R(I)+A4(I, J)*DR(J)
CONTINUE
DO 851 I=1, N
DXCD(I)=0. 0
CONTINUE
DO 129 I=1, N
DXCD(6)=DDHMC
DO 129 I=1, N
A4B(I)=0. 0
DO 129 J=1, N
ABC(I)=ABC(I)+A8(I, J)*DXCD(J)
CONTINUE
DO 849 I=1, L
DW(I)=0. 0
CONTINUE
DO 849 I=1, N
DW(I)=0. 0
CONTINUE
DO 123 I=1, N
A3X(I)=0. 0
DO 123 J=1, N
A3X(I)=A3X(I)+A3(I, J)*DX(J)
CONTINUE
DO 124 I=1, N
A7C(I)=0. 0
DO 124 J=1, N
A7C(I)=A7C(I)+A7(I, J)*DX(J)
CONTINUE
DO 125 I=1, N
A5W(I)=0. 0
DO 125 J=1, L
A5W(I)=A5W(I)+A5(I, J)*DW(J)
CONTINUE
DO 126 I=1, N
A6WD(I)=0. 0
DO 126 J=1, L
A6WD(I)=A6WD(I)+A6(I, J)*DW(D)
CONTINUE
DO 127 I=1, N
DXD(I)=0. 0
CONTINUE
CALL VMULFF(A1INV, GEO, N. N. 1, N. N, Q. 1, N. IER)
CALL VMULFF(A1D2IN, GEO, N. N. 1, N. N, Q. 2, N. IER)
DO 444 I=1, N
AGE(I)=0. 000
CONTINUE
DO 850 I=1, L
DR(I)=0. 0
DR(2)=DDRC
DO 128 I=1, N
A4R(I)=0. 0
DO 128 J=1, L
A4R(I)=A4R(I)+A4(I, J)*DR(J)
CONTINUE
DO 851 I=1, N
DXCD(I)=0. 0
CONTINUE
DO 129 I=1, N
DXCD(6)=DDHMC
DO 129 I=1, N
A4B(I)=0. 0
DO 129 J=1, N
ABC(I)=ABC(I)+A8(I, J)*DXCD(J)
CONTINUE
DO 849 I=1, L
DW(I)=0. 0
CONTINUE
DO 849 I=1, N
DW(I)=0. 0
CONTINUE
DO 123 I=1, N
A3X(I)=0. 0
DO 123 J=1, N
A3X(I)=A3X(I)+A3(I, J)*DX(J)
CONTINUE
DO 124 I=1, N
A7C(I)=0. 0
DO 124 J=1, N
A7C(I)=A7C(I)+A7(I, J)*DX(J)
CONTINUE
DO 125 I=1, N
A5W(I)=0. 0
DO 125 J=1, L
A5W(I)=A5W(I)+A5(I, J)*DW(J)
CONTINUE
DO 126 I=1, N
A6WD(I)=0. 0
DO 126 J=1, L
A6WD(I)=A6WD(I)+A6(I, J)*DW(D)
CONTINUE
DO 127 I=1, N
DXD(I)=0. 0
CONTINUE
CALL VMULFF(A1INV, GEO, N. N. 1, N. N, Q. 1, N. IER)
CALL VMULFF(A1D2IN, GEO, N. N. 1, N. N, Q. 2, N. IER)
DO 444 I=1, N
AGE(I)=0. 000
CONTINUE
DO 850 I=1, L
DR(I)=0. 0
DR(2)=DDRC
DO 128 I=1, N
A4R(I)=0. 0
DO 128 J=1, L
A4R(I)=A4R(I)+A4(I, J)*DR(J)
CONTINUE
DO 851 I=1, N
DXCD(I)=0. 0
CONTINUE
DO 129 I=1, N
DXCD(6)=DDHMC
DO 129 I=1, N
A4B(I)=0. 0
DO 129 J=1, N
ABC(I)=ABC(I)+A8(I, J)*DXCD(J)
CONTINUE
DO 849 I=1, L
DW(I)=0. 0
CONTINUE
DO 849 I=1, N
DW(I)=0. 0
CONTINUE
DO 123 I=1, N
A3X(I)=0. 0
DO 123 J=1, N
A3X(I)=A3X(I)+A3(I, J)*DX(J)
CONTINUE
DO 124 I=1, N
A7C(I)=0. 0
DO 124 J=1, N
A7C(I)=A7C(I)+A7(I, J)*DX(J)
CONTINUE
DO 125 I=1, N
A5W(I)=0. 0
DO 125 J=1, L
A5W(I)=A5W(I)+A5(I, J)*DW(J)
CONTINUE
DO 126 I=1, N
A6WD(I)=0. 0
DO 126 J=1, L
A6WD(I)=A6WD(I)+A6(I, J)*DW(D)
CONTINUE
DO 127 I=1, N
DXD(I)=0. 0
CONTINUE
CALL VMULFF(A1INV, GEO, N. N. 1, N. N, Q. 1, N. IER)
CALL VMULFF(A1D2IN, GEO, N. N. 1, N. N, Q. 2, N. IER)
DO 444 I=1, N
AGE(I)=0. 000
CONTINUE
DO 850 I=1, L
DR(I)=0. 0
DR(2)=DDRC
DO 128 I=1, N
A4R(I)=0. 0
DO 128 J=1, L
A4R(I)=A4R(I)+A4(I, J)*DR(J)
CONTINUE
DO 851 I=1, N
DXCD(I)=0. 0
CONTINUE
DO 129 I=1, N
DXCD(6)=DDHMC
DO 129 I=1, N
A4B(I)=0. 0
DO 129 J=1, N
ABC(I)=ABC(I)+A8(I, J)*DXCD(J)
CONTINUE
DO 849 I=1, L
DW(I)=0. 0
CONTINUE
DO 849 I=1, N
DW(I)=0. 0
CONTINUE
DO 123 I=1, N
A3X(I)=0. 0
DO 123 J=1, N
A3X(I)=A3X(I)+A3(I, J)*DX(J)
CONTINUE
DO 124 I=1, N
A7C(I)=0. 0
DO 124 J=1, N
A7C(I)=A7C(I)+A7(I, J)*DX(J)
CONTINUE
DO 125 I=1, N
A5W(I)=0. 0
DO 125 J=1, L
A5W(I)=A5W(I)+A5(I, J)*DW(J)
CONTINUE
DO 126 I=1, N
A6WD(I)=0. 0
DO 126 J=1, L
A6WD(I)=A6WD(I)+A6(I, J)*DW(D)
CONTINUE
DO 127 I=1, N
DXD(I)=0. 0
CONTINUE
CALL VMULFF(A1INV, GEO, N. N. 1, N. N, Q. 1, N. IER)
CALL VMULFF(A1D2IN, GEO, N. N. 1, N. N, Q. 2, N. IER)
DO 444 I=1, N
AGE(I)=0. 000
CONTINUE
DO 850 I=1, L
DR(I)=0. 0
DR(2)=DDRC
DO 128 I=1, N
A4R(I)=0. 0
DO 128 J=1, L
A4R(I)=A4R(I)+A4(I, J)*DR(J)
CONTINUE
DO 851 I=1, N
DXCD(I)=0. 0
CONTINUE
DO 129 I=1, N
DXCD(6)=DDHMC
DO 129 I=1, N
A4B(I)=0. 0
DO 129 J=1, N
ABC(I)=ABC(I)+A8(I, J)*DXCD(J)
CONTINUE
DO 849 I=1, L
DW(I)=0. 0
CONTINUE
DO 849 I=1, N
DW(I)=0. 0
CONTINUE
DO 123 I=1, N
A3X(I)=0. 0
DO 123 J=1, N
A3X(I)=A3X(I)+A3(I, J)*DX(J)
CONTINUE
DO 124 I=1, N
A7C(I)=0. 0
DO 124 J=1, N
A7C(I)=A7C(I)+A7(I, J)*DX(J)
CONTINUE
DO 125 I=1, N
A5W(I)=0. 0
DO 125 J=1, L
A5W(I)=A5W(I)+A5(I, J)*DW(J)
CONTINUE
DO 126 I=1, N
A6WD(I)=0. 0
DO 126 J=1, L
A6WD(I)=A6WD(I)+A6(I, J)*DW(D)
CONTINUE
10 R=E(F(X))
IF(KK. GE. 100) GO TO 20
GO TO 22
20 WRITE(6,21)
FORMAT('1X,' 'ITERATION TIMES ==100')
RETURN
22 CONTINUE
IF(R. LT. EPS) RETURN
SUM=0. 000
DO 7 I=1, N
W=X(I)
IF(W. 1. 2. 1)
2 DXI=1 D-6
GOTO3
1 DXI=(1. D-6)*W
3 X(I)=X(I)+DXI

XI-8
DFDX(I) = (F(X(I)) - R)/DXI
SUM = DDFX(I) / (SUM+2)

7 X(I) = W
IF (SUM.LE. 1E-6) SUM = 1E-6
RLD = R/SUM
DO 8 I = 1, N
8 X(I) = X(I) - RLD * DDFX(I)

2 KK = KK + 1
GO TO 10

END

SUBROUTINE RUNKUCN(H, X, V, YA, YB, F, FCT)
IMPLICIT REAL*8 (A-H, O-Z)
DIMENSION Y(N), YA(N), YB(N), F(N), EC5)

'DATA E/:Z • . 500. :Z.1. 000 • .

DO 11 I = 1, N
VA(I) = V(I)
VB(I) = V(I)
DO 10 I = 1, N
10 CALL FCT(N, X, YB, F)
X = XA + E(J) * H

51 Y(I) = Y(I) + E(J) * H * F(I)
RETURN

END

SUBROUTINE WINDICT(DX, DXD, SINO, VVEE, VVEE)
** LINEAR WIND MODEL

****** COMPILE ON NOV. 5, 1981
IMPLICIT REAL*8 (A-H, O-Z)
DIMENSION DX(8), DXD(8)
COMMON/WINDS/DDDUU, DDDWW, DVEE, CM, DNVE, DNWE
COMMON/WINDS/ODWDU, ODWWU, OODUC, ODALF, OOTHC, OOHHC, DOOOG, DTOD

IF (W30D.EQ.0.0) GO TO 2
HEND = HHO - (DABS(WINHL) + DABS(WINLL)) / DABS(WIDD)

IF (HII . GE. HHO) DDDMU = WINHL
IF (HII . LT. HEND) DDDMU = WINLL

WID = WIDD

IF (HII . GE. HHO) OR (HII . LT. HEND) WID = 0.0

IF (HII . LE. HHO) AND (HII . LT. HEND) DDDMU = WINLL * (HII-HEND) * WID
WID = WID + DXI(6) - VVEE * SINGG
CONTINUE
RETURN

END

SUBROUTINE CONTR(CASE, MODE, T, DX, DXD, DTTC, DETC)
IMPLICIT REAL*8 (A-H, O-Z)
REAL*8 SX, TX, W, T
COMMON/STVE/DD, DOD, DNZ, AKD, AKINTH, AKS, AKUB, AKSP, AK, AKD, T

IF (CASE . EQ. 1) GO TO 3467
IF (CASE . EQ. 3) GO TO 5467
IF (CASE . EQ. 5) GO TO 9467
CONTINUE

GO TO 11111
/*(DX(1)-GDUC)-KTHD*AKD*DX(3)-AKD*KUD*DXD(1)
GO TO 6467
J2222 CONTINUE
DETC=2.6*(DX(4)-THCA)+2.6*DX(3)-KUT*AKD*(DX(1)-GDUC)*AIRSPE
/-KUDT*AKD*DXD(1)**AIRSPE
GO TO 6467
J3333 CONTINUE
IWASS=1.0-1.0*DEXP(T/TDL)
DETC=2.6*IWASS*(DX(4)+DX(3))-KALF*(DX(2)/VVE-DALFG)-KALFD*
/DXD(2)/VVE-KUT*AKD*(DX(1)-GDUC)*AIRSPE-KUDT*AKD*DXD(1)**AIRSPE
GO TO 6467
J4444 CONTINUE
THCAF=DALFG+DXD(6)/VVEE
IF (DX(4) .Q. 0.35) THCAF=0.0
DETC=2.6*(DX(4)-THCAF)+2.6*DX(3)-KALF*(DX(2)/VVE-DALFG)-KALFD*
/DXD(2)/VVE-KUT*AKD*(DX(1)-GDUC)*AIRSPE-KUDT*AKD*DXD(1)**AIRSPE
GO TO 6467
J467 CONTINUE
XIX-XIX=XXOO
HII=HII
TSS=DDHO//-(VVE=SIGG-HHDF)
TSSS=AKS*TSS
HOST=TTSS+HHDF
HHA=DDQO-HOST
HHC=HHA/DEXP(XIX/TSS/VVE)+HOST
HNDH=HHA/DEXP(XIX/TSSS/VVE)+HOST
HHDC=HHC/TSS
THCOM=(HHC+SIGG)/VVE+ENHAN
ENTGH=ENTGH+AKINTH*DH*(HII-HHC)
THCOM=0.0
ENTGH=0.0
DETC=-KH*(HII-HHC)-KH*(HNDH-HHDC)-KTH*(DX(4)-THCOM)-KTHD*DX(3)-
/ENTGH-ANZ*INZ
VVCV=VVE-0.25*VYST/DDHO//-(DDHO-HHC)
ENTGU=ENTGU+AKUB*DH*(DX(1)+VVE-VVCC)
DTTC=-KUD*(DX(1)+VVE-VVCC)-KUD*DXD(1)-ENTGU
J5467 CONTINUE
RETURN
END
SUBROUTINE WIND4(T,DX,DXD,SINGG,VVEE)
IMPLICIT REAL*8(A-H,O-Z)
DIMENSION DX(8),DXD(8)
COMMON/WDDT/WIDD,W3DD,HHII,HHIO,W3DD2,W3NH,W3NL
COMMON/WINDS/DDDUW,DDNW,DVEE,CM.DNVE,DMWE
COMMON/CAL/DDDUW,DDNW,DDAC,DDALFC,DDTHC,DDHC,DDRC,DDTC,DX60,TDGA
DDNW=0.0 DDW=0.0 W3DD=0.0 W3D0=0.0 USTA=0.5 ZD0=0 CARM=0.4
W3DD=USTA/CARM/(-1.0+HII/ZD0)
W3DD=0.0 DDW=DDUW DDNW=DDNW
W3DD=DDNW W3D=DDNW DDNW=DDNW
W3DD=DDNW RETURN
END
SUBROUTINE WIND5(T,DX,DXD,SINGO,VVEE)
IMPLICIT REAL*8(A-H,O-Z)
DIMENSION DX(8),DXD(8)
COMMON/WDDT/WIDD,W3DD,HHII,HHIO,W3DD2,W3NH,W3NL
COMMON/WINDS/DDDUW,DDNW,DVEE,CM.DNVE,DMWE
COMMON/CAL/DDDUW,DDNW,DDAC,DDALFC,DDTHC,DDHC,DDRC,DDTC,DX60,TDGA
COMMON/MINSH/DDDUW,HHSTA,AAFF
DDNW=0.0 DDW=0.0 W3D0=0.0 W3DD=0.0 DDNW=DDNW
RETURN
END
SUBROUTINE WIND3(T, DX, DXD, SIGNW, VVEEE)
IMPLICIT REAL*(A-H,O-Z)
DIMENSION DX(8), DXD(8)
COMMON/WWDDT/WIDT, WWEE, WWEE3
COMMON/WWOD/W1DO, W30D, W3002, WINHL, WINLL
COMMON/WNOS/ODOWU, ODDWW, OVEE, CM, DNOE, DNOE
COMMON/CAL/DDWU, DDWW, DDUC, DDALFC, DDTHC, DDMHC, DDC, DDTC, DX60, TDOA
W3DDWU=0.0
W3DT=0.0
DDWU=0.0
DDDDWU=0.0
W1DT=0.0
W3DT=0.0
IF(HII.LT.60.96)W1DD=0.0454
IF(HII.GE.60.96.AND.HII.LT.148.13)W1DD=0.292
IF(HII.GE.148.13.AND.HII.LT.167.34)W1DD=0.167
IF(HII.GE.167.34.AND.HII.LT.194.77)W1DD=0.157
IF(HII.GE.194.77.AND.HII.LT.205.44)W1DD=0.189
IF(HII.GE.205.44.AND.HII.LT.230.73)W1DD=0.186
IF(HII.GE.230.73)W1DD=0.0
IF(HII.LT.40.23)W3DD=0.0
IF(HII.GE.40.23.AND.HII.LT.84.73)W3DD=0.0447
IF(HII.GE.84.73.AND.HII.LT.120.70)W3DD=0.0122
IF(HII.GE.120.70.AND.HII.LT.137.46)W3DD=0.02
IF(HII.GE.137.46.AND.HII.LT.163.37)W3DD=0.0348
IF(HII.GE.163.37.AND.HII.LT.177.70)W3DD=0.0412
IF(HII.GE.177.70.AND.HII.LT.194.77)W3DD=0.0343
IF(HII.GE.194.77.AND.HII.LT.247.65)W3DD=0.101
IF(HII.GE.247.65.AND.HII.LT.482.50)W3DD=0.00467
IF(HII.GE.482.50)W3DD=0.0
IF(HII.LT.60.96)HS1=0.0
IF(HII.GE.60.96.AND.HII.LT.84.73)HS1=0.36
IF(HII.GE.84.73.AND.HII.LT.148.13)HS1=0.13
IF(HII.GE.148.13.AND.HII.LT.148.13)HS1=0.36
IF(HII.GE.148.13.AND.HII.LT.167.34)HS1=0.24
IF(HII.GE.167.34.AND.HII.LT.167.34)HS1=0.13
IF(HII.GE.167.34.AND.HII.LT.194.77)HS1=0.45
IF(HII.GE.194.77.AND.HII.LT.194.77)HS1=0.34
IF(HII.GE.194.77.AND.HII.LT.205.44)HS1=0.15
IF(HII.GE.205.44.AND.HII.LT.230.73)HS1=0.12
IF(HII.GE.230.73)HS1=0.44
IF(HII.LT.40.23)HS3=0.0
IF(HII.GE.40.23.AND.HII.LT.60.96)HS3=0.0
IF(HII.GE.60.96.AND.HII.LT.84.73)HS3=0.0
IF(HII.GE.84.73.AND.HII.LT.120.70)HS3=0.93
IF(HII.GE.120.70.AND.HII.LT.120.70)HS3=0.96
IF(HII.GE.120.70.AND.HII.LT.120.70)HS3=0.96
IF(HII.GE.120.70.AND.HII.LT.120.70)HS3=0.96
DDWU=-(DDWU+W1DD*(HII-HS1))
DDDDWU=-(DDDDWU+W3DD*(HII-HS3))
W1DT=-(W1DT*(DDX60)-VVEEE*SINQQ)
W3DT=-(W3DT*(DX60)-VVEEE*SINQQ)
RETURN
END
SUBROUTINE WIND2(T, DX, DXD, SINGG, VVEEE)
C
C WIND PROFILE OF A THUNDERSTORM 1976(FDP)
C
IMPLICIT REAL*8(A-H,O-Z)
DIMENSION DX(8), DXD(8)
COMMON/WWDI/WIDT, W30T, WVEE, WVEE3
COMMON/WWDI/WIDD, W3DD, HII, HHO, W3DD2, WINNL, WINLL, XII
COMMON/WINDS/DDDWU, DDDWW, DVEE, CM, DNVE, DNWE
COMMON/CAL/DDDWU, DDWW, DDUC, DDALFC, DDTHC, DDHHC, DDRC, DDTC, DX60, TDGA
DDDW=0.000
WIDT=0.000
W3DT=0.000

IF(HII . GE . 0.0 AND HII . LT . 12.4) WIDD=-0.7194
IF(HII . GE . 12.40 AND HII . LT . 48.21) WIDD=-0.47486
IF(HII . GE . 48.20 AND HII . LT . 128.0) WIDD=-0.05589
IF(HII . GE . 128.0 AND HII . LT . 152.4) WIDD=0.06148
IF(HII . GE . 152.4) WIDD=0.0

IF(HII . GE . 0.0 AND HII . LT . 51.82) W3DD=-0.1164
IF(HII . GE . 51.82 AND HII . LT . 60.96) W3DD=0.000
IF(HII . GE . 60.96 AND HII . LT . 91.44) W3DD=-0.1776
IF(HII . GE . 91.44 AND HII . LT . 104.96) W3DD=-0.04591
IF(HII . GE . 104.94)W3DD=0.000

IF(HII . GE . 0.0 AND HII . LT . 12.4) DDDWJ=6.5
IF(HII . GE . 0.0 AND HII . LT . 51.82) DDDWJ=0.000
IF(HII . GE . 51.82 AND HII . LT . 60.96) DDDWJ=-2.42
IF(HII . GE . 60.96 AND HII . LT . 91.44) DDDWJ=-19.42
IF(HII . GE . 91.44 AND HII . LT . 104.96) DDDWJ=-19.42
IF(HII . GE . 104.94) DDDWJ=-20.27

DDDI=WIDD*(HII-HSI)
DDDDWW=DDWW+W3DD*(HII-HS3)
WIDT=WIDT+(DXD(6)-VVEEE*SINGG)
W3DT=W3DD+(DX(6)-VVEEE*SINGG)
RETURN
END

This program is the same as the one in [46] except the subroutine for wind inputs which is replaced by the power law mean wind profile denoted by WINDS in this study.
Appendix XII

Computer Program for Solving Covariance Matrix (σ)

This program computes the response of the aircraft to constrained turbulence correlation inputs. The system equations and corresponding matrices are similar to those described in Appendix VII of [46]. The notation convention and the parameters to be specified follow [46]. The flow chart to evaluate the covariance matrix Σ(t) is shown in the attached sketch. The subroutines used for this program are listed as follows:

1. EQUCON and NLS for the computation of equilibrium values called by NLS and defines the nonlinear equations
2. F(X) used for the interpolation
3. INTER
4. LINV2F an IMSL subroutine for finding the inverse of a real matrix
5. VMULFF an IMSL subroutine for the calculation of the product of two real matrices (MATML - the same function)
6. EIGRF an IMSL subroutine for the computation of eigenvalues and eigenvectors of a linear system
7. MINVCD invert a complex matrix
8. MATSUB substraction between two real matrices called by MINVCD, which finds the largest element of a complex matrix
9. SUBMCD calculates the time history of a state variable vector
10. RUNKU

XII-1
FLOW CHART TO EVALUATE THE COVARIANCE MATRIX $\Sigma(t)$

- Reference path
- Mean wind $W_m(t_i)$
- Turbulence $\sigma_1(t_i), L_1(t_i)$
- Constrained correlation matrix
- Control law $K_1(t_i), K_2(t_i)$
- System matrices $A(t_i), B_1(t_i), B_2(t_i)$
- Transition matrix $A(t_i)_{t_i}$
- Covariance matrix equations
- Covariance matrix $\Sigma(t_i)$

XII-2
11. XJN(x,p) a function for calculating a modified Bessel function

12. GG(x) and FF(x) functions for calculating the longitudinal and traverse correlations, which are called by JN(x,p)

13. CCRR calculates the correlation matrix using four-point model
called by CCRR

14. RELMT this subroutine is for calculating the integrand called by CWRAF

called by CCRR

15. GRND this subroutine is for calculating the system transition matrix $e^{At}$ using
Block Diagonal Transformation method
calculated by EXPAT

16. EXPAT calculates $e^{At}$ using Sylvester Expansion technique
to make the eigenvalues of the system matrix in order
that the complex eigenvalues will be brought into
the subroutine EXPAT2 in pair with the root with the negative imaginary part first

calculated by EXPAT

17. EXPAT2
calculates $e^{At}$ using Sylvester Expansion technique

to make the eigenvalues of the system matrix in order
that the complex eigenvalues will be brought into
the subroutine EXPAT2 in pair with the root with the negative imaginary part first

calculated by EXPAT

18. ORDER this subroutine is for calculating the covariance matrix $\Sigma(t)$

19. CWRAF this subroutine is for calculating the covariance matrix $\Sigma(t)$
COMPUTER CODES

THIS PROGRAM CALCULATES THE COVARIANCE MATRIX OF THE STATE VARIABLES IN LANDING THROUGH THE PLANETARY BOUNDARY LAYER USING THE CONSTRAINED CORRELATION FUNCTION TECHNIQUE.

REVISED IN JULY TO SEPTEMBER 1984, UTAAG, CANADA, S. ZHU

IMPLICIT REAL*8(A-H,O-Z)

REAL*4 SXX(200,60), ANGLE

REAL*8 LUX, LWX, KK, MU, MW, MD, MG, MASS, IIYY, KT, KE, LTH, KU,

IU, KUD, KTH, KD, KH, KDD, MDD, MDT, HH, NDEX

DIMENSION A1(8,8), A2(8,8), AINV(8,8), B2(8,3), B3(8,3),

I1A(8,8), C2(8,3), C3(8,3), WK(100), IIR(8), IIC(8), X(4), DFX(4),

DWM3(3), RNSQ(8), HHH2(1,1), HHH3(1,1), HCM(1,8), HCH(1,8), HCT(8,1)

DIMENSION EXPAST(8,8), EXT(8,8), EXPASS(8,8), HH(200), XSTAT(8,1),

IVVE(200), SING(200), COS(200), SQRD(8,8), RNSQ(8),

DIMENSION SINOQ(200), RBBDD(8,3,200), RBBDD(8,3,200), IVVEEE(200),

1, COB(8,200), HHII(200), REAM(8,200), S5O(8,8), SS11(8,8),

DIMENSION RTI(8,8), ROI(8,8), SIGM(8,8), SS22(8,8), SSS(8,8),

1, TEMPS(8,8), SOURCE(8,8), EATST(8,8), ROR(8,8),

ZROOR(8,8), RCR(8,8)

DIMENSION TADD(8,8), WWK(180), D1B(8,3), SIGSIQ(8,8), BBD(8,3),

COMPLEX(16) LAMAA(8), UVAAD(8,8), UUAAD(8,8), WW(100), ZI(8,8), DET

COMMON/STAT/IVVEE, HHII, SING(8), COS(8), REAM, RBBDD, RBBDD

COMMON/CA/LUX, LWX, AT, BP, HSTAT

COMMON/CB/KK, CDCDO, APST, CCM, THETE, CCLA, CCLLO, ALFA,

ILTH, CCMO, CCA, CCMDO, CCLDE

COMMON/PASS/LAMAA, UAAAD, UAAAD

COMMON/SIOXZ/2D, RND, ICAS, MW, NDEX

EXTERNAL F, FF, G0, XJN, FCNJ

READ(5,*) NPD

DO 11111 IIK=1,NPD

WRITE(6,269)

WRITE(6,*) 'CALCULATING THE COVARIANCE MATRIX'

WRITE(6,269)

NPT5=1

READ(5,*) WW, DT, ICAS, IPRINT, NPLT, HSTP, HSTAT, IGAIA, ITEST

READ(5,*) AUT1, AUT1D, AUT4, AUT4D, AUT6, AUT6D, GAMSQO2

WRITE(6,*) 'AUT1=', AUT1, ' AUT1D=', AUT1D, ' AUT4=', AUT4, ' AUT4D=

1 AUT6, ' AUT6D=', AUT6D, ' AUT7=', AUT7, 'GAMSQO2=',

1GAMSQO2

IF (ICAS.EQ.15) NDEX=0.16

IF (ICAS.EQ.25) NDEX=0.28

IF (ICAS.EQ.35) NDEX=0.40

IF (ICAS.EQ.15) ZD=0.01

IF (ICAS.EQ.25) ZD=0.04

IF (ICAS.EQ.35) ZD=1.50

IF (ICAS.EQ.15) RND=1.0

IF (ICAS.EQ.25) RND=0.8103

IF (ICAS.EQ.35) RND=0.76

IST=0

WRITE(5,*) '------------------ CASE== ICAS ------------------'

WRITE(6,*) '-------------------------- END --------------------------'

111 CONTINUE
CALCULATION OF REFERENCE EQUILIBRIUM VALUES

CCDDA=1.13
CCLLA=5.67
CCMDEE=-1.40
CCMA=1.45
CCLLDE=0.356
CCZAD=-6.7
CCZOM=-5.65
CCZAD=-3.3
CCMG=-21.4
CCXDEE=0.0
CCGDEE=-0.356
DDT=-0.109948
CCDDDE=0.263
CCLLDE=1.76
ALFAO=1.143421
ALFAO=CCCLLO=CCCLLE+ALFAO-CCLLDE=DDT)/CCLLA
CCMN=0.0369343
CCLLLE=CCDDA/CCLLAS/CCLLLE/2.0
CCLLO=CCLLA*ALFAO
CCDDD=CCDDDE-KK*CCLLDE=CCLLE
GEO=RHO*VWE+VVE/2.0

CCWEE=MASS*G/(GE*SY)
CALL GUCON(VVE, GAMES, WMSE, VVEE, GAME, THETE, SIE)
COSGE=DCOS(GAME)
SINE=DSIN(GAME)
TANGE=SINE/COSGE
COSTE=DCOS(THETE)
SINTE=DSIN(THETE)
TANTE=SINTE/COSTE
IF (IST EQ. 1) VSPD1=VVEEEDSIN(GAMES)
VSPD=VVEEDSIN(GAMES)
IIST=IST+1
IF (IST EQ. 2) GO TO 7666
IF (ND GT 1) GO TO 91
IF (HSTP. GT. 18. 24) GO TO 92
GO TO 93
92
MFL=MM
GO TO 91
93
MFL=(HSTAT-HSTP)/VSPD1/DT+1
MFL=4.765013*DLOG(HSTP/18.24)/DT
MM=NFL+MFL-1
91
MM=MM+1
VVEE=(ND)=VVEE
SINE=DSIN
COSN=DSIN
IF (ND EQ. 1 OR. ND EQ. (MM1-NFL)) GO TO 3
GO TO 4
3
WRITE(6,2)KU.KUO.KTH.KTHO.KHD.KH
FORMAT(('KU-6. FI0. 5,2X, 'KUO'-6. FI0. 5,2X, 'KTH'-6. FI0. 5,2X, 'KTHO'-6. FI0. 5,2X, 'KHD'-6. FI0. 5,2X, 'KH'-6. FI0. 5,2X)
WRITE(6,22)-'CONTINUE'
22
THE EQUILIBRIUM VALUES
WRITE(6,2222)ALFAO, GAME, THETE, SIE, VVEE
4545
FORMAT(1HO, 'THE EQUILIBRİUM VALUES')
WRITE(6,2222)ALFAO, GAME, THETE, SIE, VVEE
2222
32 CONTINUE
A1 (7, 1) = KE*KU
A1 (8, 4) = KE*KH
A1 (8, 6) = KE*KD
A2 (1, 1) = XU
A2 (1, 2) = XW
A2 (1, 3) = XG
A2 (1, 4) = MASS*0*COSTE
A2 (1, 6) = XH
A2 (1, 7) = XDIT
A2 (1, 8) = XEDE
A2 (2, 1) = ZU
A2 (2, 2) = ZH
A2 (2, 3) = MASS*VVE*ZG
A2 (2, 4) = MASS*0*SINTE
A2 (2, 6) = ZH
A2 (2, 7) = ZDIT
A2 (2, 8) = ZEDE
A2 (3, 1) = MU
A2 (3, 2) = MW
A2 (3, 3) = MG
A2 (3, 7) = MDIT
A2 (3, 8) = MDEE
A2 (4, 3) = 1.0
A2 (5, 1) = COSTE
A2 (5, 2) = SINTE
A2 (5, 4) = VVE*SINTE
A2 (6, 1) = SINTE
A2 (6, 2) = COSTE
A2 (6, 3) = MU
A2 (6, 4) = VVE*COSTE
A2 (7, 1) = KT*KU
A2 (7, 7) = KT
A2 (8, 4) = KE*KH
A2 (8, 6) = KE*KD
A2 (8, 8) = KE
DO 44 I1 = 1, N
DO 44 J1 = 1, L
B2 (1, 1) = 0.0
B2 (1, 3) = 0.0
B2 (2, 3) = -ZG
B2 (3, 3) = M0
B2 (5, 1) = 1.0
B2 (6, 2) = -1.0
B3 (1, 1) = MASS*COSTE
B3 (1, 2) = MASS*SINTE
B3 (2, 1) = MASS*SINTE
B3 (2, 2) = MASS*COSTE
IDG = 0
CALL LINV2F(A1, N, N, A1INV, IDGT, WK, IER)
CALL VMULFF(A1INV, A2, N, N, N, N, AA, N, IER)
CALL VMULFF(A1INV, B2, N, N, L, N, N, C2, N, IER)
CALL VMULFF(A1INV, B3, N, N, L, N, N, C3, N, IER)
"OUTCRTI.N.N"
WRITE(6, #, 'SIGM')
CALL MOUT(SIGM, N, N)
CONTINUE

IK=IK+1
IA=MM1-IK
IF (IK. EQ. MM1) GO TO 5052
GO TO 5053

5052 CONTINUE

DO 5056 I=1, N
DO 5056 J=1, N

SIGSIG(I, J)=SIGM(I, J)+RTI(I, J)-ROI(I, J)-ROR(I, J)+ROQR(I, J)

5056 SIGRD(I, J)=SIGM(I, J)+RTI(I, J)+ROR(I, J)

IF (HSTP . LE . 18.24) VVE=73.04
IF (HSTP . LT . 18.24) VVE=67.3846
DO 1199 I=1, N
DO 1199 J=1, N
HC(I,J)=0.00
HC(I,1)=SINGE
HC(I,2)=COSGE
HC(I,4)=VVE+COSGE

HCT(I,J)=HC(I, J)

1199 CONTINUE

DO 309 I=1, N
DO 309 J=1, N
IF (I . EQ. J) RMSQ(I)=DSQRT(DABS(SIGSIG(I, J)))
IF (I . EQ. J) RMSQ(I)=DSQRT(DABS(SIGRD(I, J)))

309 XSTAT(I,1)=RMSQ(I)

CALL MATML(I, N, HC, SIGSIG, HCM)
CALL MATML(I, N, HC, HCT, HDH)
CALL MATML(I, N, HC, XSTAT, HH22)

RMSG(3)=DSQRT(DABS(HDH(1,1)+WIND3*WIND3-2.0+WIND3+HH22(1,1)))
RMSG(2)=RMSG(2)/VVE
RMSG(2)=RMSG(2)/VVE

IF (HSTP . LT . 18.24) TIME=(HSTAT-HSTP)/VSPD

DO 1200 I=1, N
ISK=NPLT(I, Ikk-1)+I
SXX(Ikk, ISK)=RMSG(I)
SXX(Ikk, ISK+1)= TIME

1200 CONTINUE

WRITE(6, #) 'RMS'
WRITE(6, 505)


WRITE(6, 533) (RMSG(I), I=1, N)

333 FORMAT(1X, B10.8)
WRITE(6, 269)
WRITE(6, #) 'SIGSIG'
WRITE(6, 535) (SIGSIG(I, J), J=1, N)
WRITE(6, 269)
SUBROUTINE CWRAF(DT,NL,IK,MM,IA,SOURCE,RTI,ROI,ROR,RCOR,RCR,
1 WIND3)
IMPLICIT REAL*(A-H,O-Z)
REAL*8 LUX,LWX,LT,NDEX,LUXK,LWXK
DIMENSION SOURCE(8,8),ROK(8,8),SSll(8,8),TEMP3(8,8),SS22(8,8),
1 VVVEEE(200),SINGQ(200),COSQGQ(200),QROD(8,8),SSK(8,8),
2 HHII(200),RCOR(8,8),RCR(8,8),RB22(8,3,200),RBDD(8,3,200),
3 REAGM(8,8,200),RTI(8,8),ROI(8,8),TEMP3(3,8),SSDD(8,8),GQ(8,8),
4 DIMENSION ROX(8,8),CWX(3,3),QROD(8,8),RCOX(8,8),RCG(8,8),
5 IEAT(8,8),EAM(8,8),EAMT(8,8),ECX(8,8),EEE(8,8),B2E(8,8),
6 2CBA(3,8),BCA(8,8),BB(8,3),BZ(8,3),B2T(3,8),RICOX(3,8),
7 3RCB(8,8),RRB(R8,8),TEMP3(8,8),CBT(3,8),EABK(3,8),EACK(3,8),
8 COMMON/CA/LUX,LWX,LT,NDEX,LUXK,LWXK
COMMON/STAT/VVVEEE,HHII,SINGQ,COSQGQ,REAGM,RBDD,RB22
COMMON/SIQMQ/SIQMG,SIQMG,SIQMG,SIQMG
COMMON/SIQMG/STAT,INDEX,LUXK,LWXK
COMMON/DRL/R1,AKS3
COMMON/MODEL/LCAS,HKKK,ICNT
11=1
ICNT=1
L=3
JK=1
IF (IK.EQ.1 .AND. JK.EQ.1) GO TO 25
GO TO 26
25 DO 27 I=1,NL
DO 27 J=1,NL
SSll(I,J)=0.000
SS22(I,J)=0.000
SOURCE(I,J)=0.000
TEMP3(I,J)=0.000
ROI(I,J)=0.000
RTI(I,J)=0.000
27 CONTINUE
READ(*,*) LCAS,CUT
WRITE(6,*) 'M V K','LCAS,' CUTOFF=',CUT
SINGQ=0.5*(SINGQ(1)+SINGQ(MM))
HSTAT=HHII(1)
VAV=0.5*(VVVEEE(1)+VVVEEE(MM))
HHTD=HHII(MM)
VVK=VVVEEE(MM)
RK=VAV*MM**DT*WO/(10.0**INDEX)*VAV*SINGQ*(INDEX+1.0)**((HSTAT-VAV*
*SINGQ/MM)**DT)**(INDEX+1.0)**HSTAT)**((INDEX+1.0))
CALL SIGLX1(ZO,ANDA,HHTD,SIGNAT,SIGNAT,LUXK,LWXK)
IF (LCAS.EQ.1 .AND. HHTD .GE. 30.5) LWX=59.13
IF (LCAS.EQ.1 .AND. HHTD .LT. 30.5) LWX=1.938689*HHTD
WNEK=0.70605*(HHTD-0.01)**0.15*WO
SIGUK=SIGMU*WK
SIGUK=SIGMU*WK
26 CONTINUE
HHD=HHII(1)
HKKK=HHD
CALL SIGLX1(ZO,ANDA,HHD,QMU,QMUD,LUK,LUX)
IF (LCAS.EQ.1 .AND. HHD .GE. 30.5) LWX=59.13
IF (LCAS.EQ.1 .AND. HHD .LT. 30.5) LWX=1.938689*HHD
WNEO=WNEO*(HHD/O.0)**INDEX
SIGQM=SIGMU*WNEO
SIGQM=SIGMU*WNEO
IF (IK.EQ.1 .AND. JK.EQ.1) RRO=1D-8
VVOO=VVVEEE(IK)
COSQG=COSQGQ(IK)
RROO=RROO+VVOO*(COSQG-QMWD)*DT
AKS3=HSTAT-HHD
RI=RROO
IF (IK.EQ.1) R1=ID-8
IF (IK.EQ.1) AKS3=ID-8
SIMGW=SIQWO
CONTINUE
SIMGW=SIQWO
CALL GRND(JK,IK,MA,MM,NI,L,ORD1,CW)
ICNT=ICNT+1
DO 30 I=1,NL
DO 30 J=1,NL
30 EAT(I,J)=REAGM(I,J,IA)
IF (IK.NE.1) GO TO 32
DO 31 I=1,NL
DO 31 J=1,L
BB(I,J)=RBDD(I,J,IK)
B2(I,J)=RB22(I,J,IK)
31 B2T(I,J)=RB22(I,J,IK)
DO 33 I=1,NL
DO 33 J=1,NL
33 EAM(I,J)=REAGM(I,J,MM)
EAMT(I,J)=REAGM(I,J,MM)
CALL MATML(NI,NL,1,B2T,EMT,TEMP3)
CALL MATML(L,NL,NL,CW,TEMP3,RICX)
CALL MATML(NL,NL,B2,RICX,RIC0)
CALL MATML(NL,NL,NL,EAM,RIC0,RCR)
CONTINUE
CALL MATML(L,NL,NL,B2T,EMT,B2E)
CALL MATML(NL,NL,NL,CW,B2E,BC)
CALL MATML(NI,NL,B2,BC,BC)
CALL MATML(NL,NL,NL,EAM,BC,ECE)
DO 33 I=1,NL
DO 33 J=1,NL
EEE(I,J)=ECE(I,J)+ECE(I,J)
ROI(I,J)=ROI(I,J)+EEE(I,J)+TEMP3(I,J)**DT/2.0
TEMP3(I,J)=EEE(I,J)
33 IF (IK.EQ.MM) GO TO 34
GO TO 35
34 CALL MATML(L,NL,NL,CW,B2T,CBT)
CALL MATML(NL,NL,NL,B2T,BCB)
CALL MATML(NL,NL,NL,EAM,BCB,ROR)
DO 36 I=1,NL
DO 36 J=1,NI
36 ROR(I,J)=ROR(I,J)+ROR(I,J)
CONTINUE
R1=RK-RROO
SUBROUTINE HATSUB(L,M,A,B,C)
IMPLICIT REAL*8(A-H,0-Z)
DIMENSION A(L,M),B(L,M),C(L,M)
DO I=1,L
DO J=1,M
C(I,J)=A(I,J)+B(I,J)
RETURN
END

SUBROUTINE HATML(L,M,N,A,B,C)
IMPLICIT REAL*8(A-H,0-Z)
DIMENSION A(L,M),B(L,N),C(L,N)
DO I=1,L
DO J=1,N
DO K=1,M
C(I,J,K)=C(I,J,K)+A(I,J)*B(J,K)
CONTINUE
RETURN
END

SUBROUTINE SUBMCDCA,IA,JA,MA,NA,IR,IC,I
IMPLICIT REAL*8(A-H,0-Z)
COMPLEX*16 ACIA(I,J)
DIMENSION IR(MA),IC(NA)
I=0.000
J=0.000
TEST=0.000
DO 5 K=1,MA
IF(IR(K).NE.0) GO TO 5
DO 4 L=1,NA
IF(IC(L).NE.0) GO TO 4
X=CDABS(A(K,L))
IF(X.LT.TEST) GO TO 4
I=K
J=L
TEST=X
CONTINUE
RETURN
END

IF(IR(I).EQ.2.AND.RI.GE.RLIM)GO TO 28
IF(JK.EQ.IA)GO TO 21
CONTINUE
J=JK+1
CONTINUE
GO TO 10
CONTINUE
RETURN
END

IF(IR(I).EQ.2)GO TO 28
IF(JK.EQ.IA)GO TO 21
CONTINUE
J=JK+1
CONTINUE
GO TO 10
CONTINUE
RETURN
END
SUBROUTINE MINVCO(A, IA, MA, OETA, IR, IC)
IMPLICIT REAL*8(A-H, O-Z)
COMPLEX*16 A(IA, IA), PIV, OETA, TEMP, PIV1
DIMENSION IR(MA), IC(MA)
DO 1 I=1, MA
IR(I)=0
IC(I)=0
OETA=(1.0D0, 0.0D0)
S=0.0D0
R=MA
CALL SUBMCO(A, IA, IA, MA, MA, IR, IC, I, J)
PIV=A(I, J)
DETA=PIV*OETA
Y=CDABS(PIV)
IF(Y.EQ.0) GO TO 17
IR(I)=J
IC(J)=I
PIV=(1.0D0, 0.0D0)/PIV
A(I, J)=PIV
DO 5 K=1, MA
IF(K.NE. J) A(I, K)=A(I, K)*PIV
DO 9 K=1, MA
IF(K.EQ. I) GO TO 9
PIV1=A(K, J)
DO 8 L=1, MA
IF(L.NE. J) A(K, L)=A(K, L)-PIV1*A(I, L)
CONTINUE
DO 11 K=1, MA
IF(K.NE. I) A(K, J)=PIV*A(K, J)
S=S+1.0D0
IF(S.LT. R) GO TO 2
DO 16 I=1, MA
K=IC(I)
M=IR(I)
IF(K.EQ. I) GO TO 16
DETA=DETA
DO 14 L=1, MA
TEMP=A(K, L)
A(K, L)=TEMP
A(L, M)=A(L, M)
A(L, I)=A(L, I)
A(L, I)=TEMP
IC(M)=K
IR(K)=M
CONTINUE
RETURN
100 CONTINUE
TF=TF*K
IF(K.EQ.0) TF=1.0D0
IF(X.EQ.0) DO X=1, D-9
TF=DEXP((DP(L)+2.0D0)*DLOG(X/2.0D0))
TG=DGAMMA(1.0D0+DP(L)+K)
XINC=TP/TF/TG
T=DABS(XINC)
IF(T.LT.TOL) GO TO 101
XJ(L)=XJ(L)*XINC
K=K+1
IF(K.GE.30) GO TO 101
GO TO 100
101 CONTINUE
110 CONTINUE
XJN=(XJ(2)-XJ(1))*3.14159/2.0D0
XJN=XJN/DSCIN(P*3.14159)
RETURN
END

DOUBLE PRECISION FUNCTION XJN(X, P)
IMPLICIT REAL*8(A-H, O-Z)
DIMENSION DP(2), XJ(2)
IF(X.GE.10.0) X=10.0
TOL=1.0D0
DP(1)=P
DP(2)=-1.0D0
DO 110 L=1, 2
TF=1.0D0
K=0
XJ(L)=0.0D0
CONTINUE
TF=TF*K
IF(K.EQ.0) TF=1.0D0
IF(X.EQ.0) DO X=1, D-9
TF=DEXP((DP(L)+2.0D0)*DLOG(X/2.0D0))
TG=DGAMMA(1.0D0+DP(L)+K)
XINC=TP/TF/TG
T=DABS(XINC)
IF(T.LT.TOL) GO TO 110
XJ(L)=XJ(L)*XINC
K=K+1
IF(K.GE.30) GO TO 110
GO TO 100
110 CONTINUE
110 CONTINUE
XJN=(XJ(2)-XJ(1))*3.14159/2.0D0
XJN=XJN/DSCIN(P*3.14159)
RETURN
END

DOUBLE PRECISION FUNCTION XJN(X, P)
IMPLICIT REAL*8(A-H, O-Z)
DIMENSION DP(2), XJ(2)
IF(X.GE.10.0) X=10.0
TOL=1.0D0
DP(1)=P
DP(2)=-1.0D0
DO 110 L=1, 2
TF=1.0D0
K=0
XJ(L)=0.0D0
CONTINUE
TF=TF*K
IF(K.EQ.0) TF=1.0D0
IF(X.EQ.0) DO X=1, D-9
TF=DEXP((DP(L)+2.0D0)*DLOG(X/2.0D0))
TG=DGAMMA(1.0D0+DP(L)+K)
XINC=TP/TF/TG
T=DABS(XINC)
IF(T.LT.TOL) GO TO 110
XJ(L)=XJ(L)*XINC
K=K+1
IF(K.GE.30) GO TO 110
GO TO 100
110 CONTINUE
110 CONTINUE
XJN=(XJ(2)-XJ(1))*3.14159/2.0D0
XJN=XJN/DSCIN(P*3.14159)
RETURN
END

FUNCTION Q0(X)
IMPLICIT REAL*8(A-H, O-Z)
EXTERNAL FF, XJN
Q0=FF(X)-0.296279*X**1.3333*X,JN(X, 0.33333)
RETURN
END

FUNCTION FF(X)
IMPLICIT REAL*8(A-H, O-Z)
EXTERNAL XJN
FF=0.592557*X**0.33333*XJN(X, 0.33333)
RETURN
END
SUBROUTINE CCRR(RRDATA)
IMPLICIT REAL*(A-H.O-Z)
REAL*LUX, LXW, L2WX, LT, LT2
DIMENSION VVVEE(200), SINGGO(200), RRDATA(3,3),
1CDOSGOG(200), HII(200), REAGM(B, B, 200), RBB22(B, B, 200),
2LBBD(B, B, 200)
COMMON/DIST/R12, R22, R32, AK53
COMMON/RWLL/I, RC01, RC11, RC21, RC31, RC51, RC61, QGR13,
1QGRP, QGRP3, QGR, QGRP5, QGRP43, QGRP43
COMMON/STAT/VVVEE, HII1, SINGGO, CDOSGOG, REAGM, RBBD2, RBB22
COMMON/CA/L, T.LUX, L2W, LT, BP, HSTAT
COMMON/SIN/SINU, SINU2, SINO, SIMU, SIMU2
COMMON/DRI/R11, R32
COMMON/DEL/LE/LR1, ICNT
EXTERNAL F, FF, X
IF(LWX . LE . 1.5) LWX = 1.5
IF(LUX . LE . 1.5) LUX = 1.5
L2WX = 2.4*LWX
IF(LCAS . LE . 1) L2WX = LWX
LT2 = LT + LT
SIG1 = S1M1 + SIG1U1
SIG1M2 = SIG1M + SIG1M2
AT2 = AT + AT
DO 555 I = 1, 3
DO 555 J = 1, 3
RRDATA(I, J) = 0.000
CALL RELMT(LUX, LT, AT, BP)
RUO = RC01 + R12 + QGR13
RU1 = RC11 + R12 + QGRP
RU2 = RC21 + R12 + QGR
RU3 = RC31 + R12 + QGRP3
RU4 = RC41 + R12 + QGRP5
RU5 = RC51 + R12 + QGRP43
RU6 = RC61 + R12 + QGRP43
RRDATA(1, 1) = (-3.0*AT2)*RUO + 4.0*RU1 + 2.0*RU2 + AT*RU3 +
12.0*AT*RU4 + AT*RU6 + 2.0*AT*RU5)/(3.0*AT)/ (3.0*AT)
SIG1M2 = SIG1M + SIG1M2
CALL RELMT(L2WX, LT, AT, BP)
RWO = RC01 + AKS32 + QGR13
RW1 = RC11 + AKS32 + QGRP
RW2 = RC21 + AKS32 + QGR
RW3 = RC31 + AKS32 + QGRP3
RW4 = RC41 + AKS32 + QGRP5
RW5 = RC51 + AKS32 + QGRP43
RW6 = RC61 + AKS32 + QGRP43
RRDATA(2, 1) = (-3.0*AT2)*RWO + 4.0*RW1 + 2.0*RW2 + AT*RW3 +
12.0*AT*RW4 + AT*RW6 + 2.0*AT*RW5)/(3.0*AT)/ (3.0*AT)
SIG1M2 = SIG1M + SIG1M2
IF(LT2 . LE . 0.000) GO TO 121
RRDATA(3, 1) = (12.0*RWO + 4.0*RW1 + 2.0*RW2 + 3.0*RW3 - 6.0*O
1RW4 - 6.0*RW5 - 3.0*RW6)/9.0/LT2*SIG1M2
GO TO 122
RRDATA(3, 3) = 0.000
CONTINUE
RETURN
END

FUNCTION F(X)
IMPLICIT REAL*(A-H.O-Z)
REAL*LUX, LTH
DIMENSION X(4)
COMMON/CA/CCCG, CCDD, APSTT, CCWME, THETE, CCLLA, CCLLO, ALFAO,
1LTH, C, CCMD, CCMA, CCMDDE, CCLLDE
Y1 = X(1) + DCOS(APSTT + X(2)) - CCDDO - AKS32 + X(3) - CCWME + DSIN(THETE)
Y2 = X(1) + DCOS(APSTT + X(2)) + X(3) - CCWME + DCOS(THETE)
Y3 = 3.0*L4*AT2 - X(3)*CCLLO + X(4)*CCLLDE
Y4 = CCMA + X(1)*LTH + CCMA*X(2) + CCMMDDE*X(4)
F = Y1*Y1 + Y2*Y2 + Y3*Y3 + Y4*Y4
RETURN
END

SUBROUTINE SIGLX(ZO, RNOA, HI1, Sigmua, Sigmw, LUX, LWX)
IMPLICIT REAL*(A-H.O-Z)
REAL*LUX, LTH
IF(HI1 . LE . 1D-6) WRITE(6, *) '***** HI1=0. PAUSE RNDA= ', RNDA
IF(HI1 . LE . 121.92) Sigmua = (304.8*HI1*0.00065923
IF(HI1 . LE . 121.92) Sigmua = 1.0/DLOG(HI1/1D0)
Sigmua = 0.52*Sigmua
LUX = 11.0417*DSQRT(HI1)
LWX = 0.12192*HI1
RETURN
END

SUBROUTINE NLS(N, X, F, OFDX, EPS)
IMPLICIT REAL*(A-H.O-Z)
DIMENSION X(N), OFDX(N)
R = F(X)
IF(R . LE . EPS) RETURN
SUM = 0.000
DO 7 I = 1, N
W = X(I)
IF(W . LE . 1.2) 1, 2, 1
DX = 1.0
DO 10 I = 1, N
X(I) = W + DX
10 CONTINUE
IF(DX <= 1.0 - 6.0) W
X(I) = W + DX
DFDX = F(X) - R/DFX
SUM = DFX + SUM
X(I) = W
RLD = R/SUM
DO 8 I = 1, N
X(I) = X(I) - RLD*DFDX(I)
8 CONTINUE
GO TO 10
END

XII-12
SUBROUTINE RELMT(LX, LT, AT, BP)
IMPLICIT REAL*8(A-H, O-Z)
REAL*8 LX, LT
DIMENSION VVVEEE(200), SINGGG(200), RBBDD(8, 3, 200)
COMMON/DIST/R1, R42, R52, A3K32
COMMON/RRLL/RC01, RC11, RC21, RC31, RC41, RC51, RC61, QGR13
COMMON/STAT/VVVEEE, HII, SINGGG, COSGGG, REAGM, RBBDD, RBB22
EXTERNAL OO, FF
COMMON/DR1/R1, A3K32
ALX=1. 339* LX
R12=R1*R1
R22=R2*R2
R3=R3*R3
R4=R4*R4
R5=R5*R5
A3K32=A3K32*A3K32
R13=DSQRT(R12+A3K32)
R13L=R13/ALX
QGR13=QGR13/R13L
RC01=(FF(R13L)-QGR13)/R13L
RP=DSQRT(R12+A3K32+0. 25*BP*BP)
RP2=RP*RP
RP3=RP3*RP3
RP53=RP53/RP3
RP=RP53/GQ(3, 4)
QC01=(FF(R13L)-QGR13)/R13L
RC11=(FF(RPL)-QGRP)/RP2
RP3=DSQRT(R52+A3K32)
RP32=RP3*RP3
RP33=RP33/RPL
QGRP3=QGRP3/RP33
RC31=(FF(RP33L)-QGRP3)/RP33
RR=DSQRT(R12+BP*BP+A3K32)
RR2=RR*RR
RRL=RRL/RPL
QGRP=QGRP/RPL
RC21=(FF(RPL)-QGRP)/RR2
R53=DSQRT(R52+A3K32+0. 25*BP*BP)
R53L=R53/RPL
GC03=GC03/RPL
RC31=(FF(R3L)-QGR31)/R532
R43=DSQRT(R42+A3K32+0. 25*BP*BP)
R432=R43*R43
R43L=R43L/RPL
GGR3=GGR3/RPL
RC51=(FF(R43L)-QGR43)/R432
R432=RP32*RP32
RP432=RP43*RP43
RP43L=RP43L/RPL
GGRP43=GGRP43/RP43L
RC61=FF(RP43L)-QGRP43)/RP432
RETURN
END

SUBROUTINE MOUT(A, M, N)
IMPLICIT REAL*8(A-H, O-Z)
DIMENSION A(B, B)
WRITE(6, *)
DO 1 I=1, M
WRITE(6, 200)(A(I,J), J=1, N)
1 CONTINUE
200 FORMAT(1HX, 8015, 5)
WRITE(6, *)
END

SUBROUTINE OUT38(A, L, N)
IMPLICIT REAL*8(A-H, O-Z)
DIMENSION A(3, B)
WRITE(6, *)
DO 1 I=1, L
WRITE(6, 200) (A(X; J), J=1, N)
1 CONTINUE
200 FORMAT(IHX, 8015, 5)
WRITE(6, *)
END

SUBROUTINE SIGLX1(ZO, RNDA, HI11, SIGMUU, SIGMWW, LUX, LWX)
IMPLICIT REAL*8(A-H, O-Z)
REAL*8 LUX, LWX
IF (HI11 LE 10-8) WRITE(6, *) '****** HI11=O, PAUSE'
DLG=DLG0(10*HI11)
SIGM=X=RNDA/DLGQ(HI11/ZO)*0. 867+0. 556*DLG+10. 246*DLG**2
SIGMWW=1. 0/DLGQ(HI11/ZO)*0. 381+0. 172*DLGQ+0. 062+1DLGQ+DLG**2
LUX=25 *HI11**0. 35/(Z0**0. 063)
LWX=0. 33*HI11
RETURN
END
SUBROUTINE EXPAT(N, A, T, EIQ, EIQR, EXPAS, IKNOW)
A = NN; EIQ = EIQR WITH ORDER OF THE NEGATIVE EIQR
AT FIRST: IKNOW = 0 -- PRINT OUT THE TRANSITION MATRIX

IMPLICIT REAL*8(A-H, O-Z)
DIMENSION A(N, N), EIQ(N), EIQR(N), CHI(N, N), SPS(N, N), EXPS(N, N), EXPAS(N, N), DENOM(N, N), ICEIQ(N), ICEQR(N)
COMPLEX*16 CA(N, N), CA1(N, N), CA2(N, N), TCA(N, N)

1000 FORMAT(1HO, 5X, 'THE ELEMENTS OF THE STATE IMATRIX')
1001 FORMAT(1HO, 5X, 'THE MATRIX COEFFICIENT OF EXP', 1IP9.3, 1P013.6, 'T')
1002 FORMAT(1IP9.3, 1P013.6, 'T')
1003 FORMAT(1HO, 5X, 'THE MATRIX COEFFICIENT OF EXP', 1IP9.3, 1P013.6, 'T')
1004 FORMAT(1HO, 5X, 'THE MATRIX COEFFICIENT OF EXP', 1IP9.3, 1P013.6, 'T')
1005 FORMAT(1HO, 45('H'))
DO 916 K=1, N
DO 916 J=1, N
916 EX(I,J)=0.000
IF(IKNOW .NE. 0) GO TO 800
WRITE(6,1000)
DO 300 K=1, N
CEIQ(K)=DCMPLX(EIQ(K), EIQR(K))
DO 10 L=1, N
CA(K,L)=DCMPLX(A(K,L), 0.000)
10 IF(IKNOW .NE. 0) GO TO 700
WRITE(6,1000)
700 DO 15 K=1, N
IF(J-1) 100, 500, 200
200 IF(J-1) 110, 110, 150
200 IF(J-1) 100, 100, 150
300 IF(J-1) 110, 110, 150
300 IF(J-1) 110, 110, 150
400 IF(J-1) 110, 110, 150
400 IF(J-1) 110, 110, 150
110 DO 5 K=1, N
DO 5 L=1, N
5 CA1(K,L)=CA(K,L)
DO 20 K=1, N
CA1(K,K)=CA1(K,K)-CEIQ(J)
DO 20 L=1, N
20 CA1(K,L)=CA1(K,L)/DENOM(J)
GO TO 500
500 DO 40 K=1, N
40 DO 40 L=1, N
40 CA2(K,L)=CA(K,L)
DO 25 K=1, N
CA2(K,K)=CA2(K,K)-CEIQ(J)
DO 25 L=1, N
25 CA2(K,L)=CA2(K,L)/DENOM(J)
DO 30 K=1, N
DO 30 L=1, N
30 CONTINUE

WRITE(6,1000) EIQ(I), EIQR(I)
DO 55 K=1, N
DO 55 L=1, N
SPS(K,L)=REAL(CA1(K,L))*2.0
DO 56 K=1, N
DO 56 L=1, N
CHI(I,K,L)=SPS(K,L)
56 CONTINUE
IF(IKNOW .NE. 0) GO TO 600
DO 90 K=1, N
90 WRITE(6,1000) EIQ(I), EIQR(I)
DO 80 K=1, N
DO 80 L=1, N
SPS(K,L)=DIMAG(CA1(K,L))*2.0
DO 81 K=1, N
DO 81 L=1, N
CHI(I,K,L)=SPS(K,L)
56 CONTINUE
IF(IKNOW .NE. 0) GO TO 600
DO 83 K=1, N
83 WRITE(6,1000) EIQ(I)
DO 50 K=1, N
50 WRITE(6,1000) EIQ(I)
DO 80 K=1, N
DO 80 L=1, N
SPS(K,L)=REAL(CA1(K,L))
DO 61 K=1, N
DO 61 L=1, N
CHI(I,K,L)=SPS(K,L)
61 CONTINUE
IF(IKNOW .NE. 0) GO TO 600
WRITE(6,1000) EIQ(I)
DO 83 K=1, N
83 WRITE(6,1000) EIQ(I)
DO 50 K=1, N
50 WRITE(6,1000) EIQ(I)
DO 80 K=1, N
DO 80 L=1, N
SPS(K,L)=REAL(CA1(K,L))
DO 61 K=1, N
DO 61 L=1, N
CHI(I,K,L)=SPS(K,L)
61 CONTINUE
IF(IKNOW .NE. 0) GO TO 600
WRITE(6,1000) EIQ(I)
DO 83 K=1, N
83 WRITE(6,1000) EIQ(I)
DO 50 K=1, N
50 WRITE(6,1000) EIQ(I)
DO 80 K=1, N
DO 80 L=1, N
SPS(K,L)=REAL(CA1(K,L))
DO 61 K=1, N
DO 61 L=1, N
CHI(I,K,L)=SPS(K,L)
61 CONTINUE
IF(IKNOW .NE. 0) GO TO 600
WRITE(6,1000) EIQ(I)
DO 83 K=1, N
83 WRITE(6,1000) EIQ(I)
DO 50 K=1, N
50 WRITE(6,1000) EIQ(I)
DO 80 K=1, N
DO 80 L=1, N
SPS(K,L)=REAL(CA1(K,L))
DO 61 K=1, N
DO 61 L=1, N
CHI(I,K,L)=SPS(K,L)
61 CONTINUE
IF(IKNOW .NE. 0) GO TO 600
WRITE(6,1000) EIQ(I)
DO 83 K=1, N
83 WRITE(6,1000) EIQ(I)
DO 50 K=1, N
50 WRITE(6,1000) EIQ(I)
DO 80 K=1, N
DO 80 L=1, N
SPS(K,L)=REAL(CA1(K,L))
DO 61 K=1, N
DO 61 L=1, N
CHI(I,K,L)=SPS(K,L)
61 CONTINUE
IF(IKNOW .NE. 0) GO TO 600
WRITE(6,1000) EIQ(I)
DO 83 K=1, N
83 WRITE(6,1000) EIQ(I)
DO 50 K=1, N
50 WRITE(6,1000) EIQ(I)
DO 80 K=1, N
DO 80 L=1, N
SPS(K,L)=REAL(CA1(K,L))
DO 61 K=1, N
DO 61 L=1, N
CHI(I,K,L)=SPS(K,L)
61 CONTINUE
IF(IKNOW .NE. 0) GO TO 600
WRITE(6,1000) EIQ(I)
DO 83 K=1, N
83 WRITE(6,1000) EIQ(I)
DO 50 K=1, N
50 WRITE(6,1000) EIQ(I)
DO 80 K=1, N
DO 80 L=1, N
SPS(K,L)=REAL(CA1(K,L))
DO 61 K=1, N
DO 61 L=1, N
CHI(I,K,L)=SPS(K,L)
61 CONTINUE
IF(IKNOW .NE. 0) GO TO 600
WRITE(6,1000) EIQ(I)
DO 83 K=1, N
83 WRITE(6,1000) EIQ(I)
DO 50 K=1, N
50 WRITE(6,1000) EIQ(I)
DO 80 K=1, N
DO 80 L=1, N
SPS(K,L)=REAL(CA1(K,L))
DO 61 K=1, N
DO 61 L=1, N
CHI(I,K,L)=SPS(K,L)
61 CONTINUE
IF(IKNOW .NE. 0) GO TO 600
WRITE(6,1000) EIQ(I)
DO 83 K=1, N
83 WRITE(6,1000) EIQ(I)
DO 50 K=1, N
50 WRITE(6,1000) EIQ(I)
DO 80 K=1, N
DO 80 L=1, N
SPS(K,L)=REAL(CA1(K,L))
DO 61 K=1, N
DO 61 L=1, N
CHI(I,K,L)=SPS(K,L)
61 CONTINUE
IF(IKNOW .NE. 0) GO TO 600
WRITE(6,1000) EIQ(I)
DO 83 K=1, N
83 WRITE(6,1000) EIQ(I)
SUBROUTINE EXPAT(T,EXPAS)
IMPLICIT REAL*8(A-H.O-Z)
COMPLEX*16 LAMAAD(8),UUAAAD(8.8),UUAADI(8.8),CDEXP
COMPLEX*16 ZSIG(8),CEXPAS(8.8)
DIMENSION EXPAS(8.8)
COMMON/PASS/LAMAAD,UUAAD,UUAADI
DO 1 I=1,8
   ZSIG(I)=LAMAAD(I)*T
1   ZSIG(I)=CDEXP(ZSIG(I))
DO 2 I=1,8
   DO 2 J=1,8
      CEXPAS(I,J)=(0. 000, 0. 000)
   2
   DO 2 K=1,8
      CEXPAS(I,J)=CEXPAS(I,J)+UUAAD(I,K)*ZSIG(K)*UUAADI(K,J)
   3
   DO 4 J=1,8
   4
   EXPAS(I,J)=CEXPAS(I,J)
RETURN
END

SUBROUTINE INTER(X,Y,XP,YP,M)
IMPLICIT REAL*8(A-H.O-Z)
DIMENSION X(M),Y(M)
M1=M-1
DO 33 J=2, M1
   IF(XP-X(J)) GT 22.22.33
   33 CONTINUE
   B=(Y(J)-Y(J-1))/(X(J)-X(J-1))
   C=((Y(J+1)-Y(J))/(X(J+1)-X(J))-B)/(X(J+1)-X(J-1))
   YP=Y(J-1)+B*(XP-X(J-1))+C*(XP-X(J-1))*(XP-X(J))
RETURN
END

SUBROUTINE EGUCON(VVE,QAMGQ,WWSE,VVEEE,GAME,THETE,SIE)
IMPLICIT REAL*8(A-H.O-Z)
DIMENSION WWSE(3)
ARG1=ASIN(-WWSE(2)/VVE)
VVEVV=VVE*DCOS(ARG1)
ARG2=DCOS(QAMGQ)
VVEEE=WWSE(1)*ARG2+DSQRT(WWSE(1)*WWSE(1)-(ARG2*ARG2-11. 000)*VVEVV*VVEVV)
GAME=ASIN(VVEEE*DSIN(QAMGQ)/VVEVV)
THETE=ASIN(-DSIN(GAME)*DSQRT(VVE*VVE-WWSE(2)*WWSE(2)))/VVE
SIE=ASIN(-WWSE(2)/(DCOS(THETE)*VVE))
RETURN
END

XII-15
In this report a statistical approach to automatic landing in turbulent planetary boundary layer is presented which is valuable for use in aircraft design and the analysis of terminal operation safety. The effects of gust gradients on the motion were considered. The wind is modeled as a multi-dimensional random process, characterized by the mean wind shear and turbulence. In the planetary boundary layer the mean wind is adequately described by a power law and by Weibull wind speed distribution. The turbulence is assumed locally isotropic. The modified von Karman model adequately represents correlations along an approach-flare trajectory. The forces and moments are considered to depend linearly on uniform gust and gust gradient components which are obtained by Etkin's four-point method which utilizes the air velocity at 4 points on the aircraft. As a test base, an autopilot of a jet transport for the approach and flare was designed. The "Random Choice Direct Search" technique was employed to find a set of optimal feedback gains for the autoland system. A method to calculate the covariance matrix, as a function of time, of a linear system perturbed by the atmospheric turbulence, which is described as a Gauss-Markov process, is presented. Having found the perturbations of the state variables due to mean wind shear and turbulence, a hard landing probability analysis was carried out. The primary purpose of the proposed method is to establish a structure containing the system elements, disturbances and the certification limits in an analytical framework. With it, the relative effects of changes in the various system elements, mean wind and turbulence parameters and operational limitations on precision of control and available margins of safety can be estimated. A numerical example calculation for a large jet transport with autoland was conducted to demonstrate the proposed analysis method. In the case of a piloted aircraft with the automatic system model replaced by a human pilot model, this analysis approach is still valid.

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