ON THE RESPONSE OF PRESSURE MEASURING
INSTRUMENTATION IN UNSTEADY FLOW

(An investigation of errors induced by probe-flow interaction)

by

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SUMMARY

If a fast responding pressure probe of classical "static" probe geometry is placed in an unsteady flow it will not register the true instantaneous pressure (i.e. that which would have occurred in the absence of the probe). Interaction between the probe and the unsteady velocity field gives rise to an error between the measured pressure $P_m(t)$ and the true pressure $P_t(t)$. This error is not always small; it can in some circumstances be larger than the difference between $P_t(t)$ and the ambient pressure.

A major objective of the present work has been to explore the possibility of correcting $P_m(t)$ instantaneously, through use of an error-compensating probe. As preliminary steps the fundamental mechanisms of interaction error were examined and a general empirical error function was postulated. A probe configuration was adopted for which the error reduces to the simple form $BPV_n^2(t)$; $V_n(t)$ is the instantaneous resultant of the orthogonal velocity components $V(t)$ and $W(t)$ normal to the probe axis; $B$ is a coefficient which typically takes values ranging from $-\frac{1}{2}$ to $-\frac{1}{3}$. A probe was developed to measure simultaneously the unsteady components of $P_m(t)$ and $V_n(t)$ over a wide frequency range. Output signals from the probe were processed by analogue means to correct out the error term $BPV_n^2(t)$, providing an improved estimate of the true unsteady pressure.

A series of experiments were conducted in a number of contrived unsteady flows. Major experiments involved a periodic "rotating inclined nozzle flow" and some typical turbulent flows. The prime objectives were to substantiate the assumed form of the error function, to evaluate the error coefficient $B$, and to attempt the measurement of the corrected unsteady pressure by the error-compensating scheme. These goals were realized with reasonable success for a variety of circumstances. The error-compensation scheme was found to be effective, particularly for the specialized rotating inclined nozzle flow. However the measurements in turbulent flows revealed that the correction to root-mean-square pressure fluctuation level was small, generally amounting to less than 20%.
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ERRATA

Page vi - Definition of $\tilde{V}$, $\tilde{W}$, $\tilde{V}_n$ should read:
"steady (d.c.) part of $V$, $W$, or $V_n$"

Page 5 - Line 6, definition should read $R_v = \frac{V_n d}{v} < 1$

Page 13 - Second paragraph, line 6; 80 KHz should be 10 KHz

Fig. 14 - Sign change in definitions of $e'_p$ and $e_o$:

$$\frac{-45p_m}{Bq}; \quad \frac{+45p_c}{Bq}$$

- Insert capacitor before $e_o$ to take out d.c. part of signal (due to $B_p v_n^2$).

Fig. 13a - $U$ should be $U_o$

Fig. 28 - $U_o$ should be $\tilde{U}_o$
Symbols of limited use are defined where used.

A \quad \text{axial flow error coefficient}

B \quad \text{cross flow error coefficient}

b, h \quad \text{cross-sectional dimensions of rectangular channel}

D \quad \text{diameter of round jet}

d \quad \text{diameter of pressure probe}

e_p, e_v, e_w \quad \text{output voltages of pressure and velocity sensors}

F \quad \text{flatness factor of } v_n \left( \frac{v_n^4}{v_n^2} \right)

f \quad \text{frequency - Hertz (Hz)}

G_p, G_v, G_w \quad \text{gains of voltage amplifiers in analogue network}

L_v, L_w \quad \text{orthogonal components of side force acting on lifting body sensor}

L_x, L_y, L_z \quad \text{integral length scales based on u-component fluctuation (e.g., } L_y = 1/\overline{u^2} \int u(y)u(y-\eta)d\eta)\text{)

P_m(t) \quad \text{instantaneous pressure measured by a fast responding 'static' pressure probe in an unsteady flow}

P_t(t) \quad \text{instantaneous 'true' pressure which would have occurred in the absence of a probe}

\overline{P}_m, \overline{P}_t \quad \text{steady (d.c.) part of } P_m(t) \text{ or } P_t(t)

F(\theta) \quad \text{circumferential distribution of pressure about a cylindrical probe}

p_m(t), p_t(t) \quad \text{unsteady (a.c.) part of } P_m(t) \text{ or } P_t(t)

p_c(t) \quad \text{corrected unsteady pressure as measured with the error-compensating probe (} p_c = p_t \text{ if error compensation is precise) }

q \quad \text{dynamic head of mean flow (usually } \frac{1}{2} \rho \overline{U^2})

R_v \quad \text{Reynolds number } \frac{\overline{V}_n d}{\nu}

R_{ab}(\tau) \quad \text{normalized cross-correlation of any two time dependent variables } a \text{ and } b \left( \frac{a(t)}{a'b'}b(t-\tau) \right)

r \quad \text{radial coordinate in round jet}

v
\( S_p, S_v, S_w \)  
Sensitivities of pressure and velocity sensors (millivolts per millimeter of \( H_2O \))

\( S \)  
Strouhal frequency \( \frac{f d}{\bar{U}} \) \((\sim d/\lambda \text{ when } \bar{U} \sim \bar{U}_c)\)

\( t \)  
Time

\( U(t) \)  
Instantaneous component of velocity parallel to probe axis  
(alternatively used as x-components in Sections 11 and 12)

\( \bar{U} \)  
Steady (d.c.) part of \( U(t) \)

\( \bar{U}_c \)  
Mean convection speed of a turbulent disturbance

\( U_0 \)  
Nozzle velocity in Sections 10 and 12; \( \bar{U}_0 \) is centerline value of \( \bar{U} \) in Section 11

\( u \)  
Unsteady (a.c.) part of \( U(t) \)

\( V(t), W(t) \)  
Instantaneous orthogonal components of velocity perpendicular to the probe axis

\( V_n(t) \)  
Resultant of \( V(t), W(t) \); \( (V_n)^2 = v^2 + w^2 \)

\( \bar{V}, \bar{W}, \bar{V}_n \)  
Steady (d.c.) part of \( V, W \), or \( \bar{V}_n \)

\( v, w, v_n \)  
Unsteady (a.c.) part of \( V, W \), or \( V_n \); \( (v_n)^2 = v^2 + w^2 \)

\( x, y, z \)  
Coordinates defined in Sections 11 and 12

\( \alpha \)  
Angle of attack as defined in Figure 18 or Figure 24

\( \theta_0 \)  
Characteristic angle of inclined nozzle

\( \lambda \)  
Used in a general sense to represent the effective spatial "scale" of a pattern of turbulence; alternatively used as the wavelength of a particular spectral component

\( \rho \)  
Density

\( \sigma \)  
Parameter defined in Section 12

\( \tau \)  
Time delay between \( a \) and \( b \) in \( R_{ab}(\tau) \)

\( \tau^* \)  
Non-dimensional time delay \( \bar{U}_r/d \)

\( \Phi_u(S) \)  
Non-dimensional spectral density of \( u \); \( \int_0^\infty \Phi_u(S)dS = \frac{u^2}{\overline{u^2}} \)

\( \Phi_p(S) \)  
Non-dimensional spectral density of \( p \); \( \int_0^\infty \Phi_p(S)dS = \frac{p^2}{\overline{p^2}} \)
\text{mV} \quad \text{millivolts}

\text{mm} \quad \text{millimeters}

Overbars denote time averages

Primes denote root-mean-square values
1. INTRODUCTION

The problem is stated with the aid of Figure 1. When a pressure probe of classical static probe geometry is placed in an unsteady flow field the probe will register an "apparent" instantaneous pressure \( P_m(t) \). This measured pressure will not in general equal the true pressure \( P_t(t) \) which would have occurred in the absence of the probe. An error is induced by interaction between the probe and unsteady velocity field adjacent to it. In the present work the undisturbed velocity field will be represented by an instantaneous velocity component \( U(t) \), parallel with the direction of the probe axis, and two orthogonal components \( V(t) \) and \( W(t) \), perpendicular to \( U(t) \). These latter transverse components are usually regarded as small compared with \( U \). In subsequent discussion, \( V \) and \( W \) are often replaced with their instantaneous vector sum, denoted \( V_n(t) \), for convenience.

The difference between \( P_m \) and \( P_t \) is the interaction error which, in its most general form, can be expressed as an integral over the distorted velocity field local to the probe (strictly speaking, a non-steady boundary value solution of the Navier-Stokes equations). To make the problem tractable, the present investigation starts with an empirical approximation to the error. This approximation is dependent on the instantaneous local properties of the velocity field. When subjected to a number of restrictions it reduces to a simple functional form. Our prime objective has been to devise a system which will correct for the pressure error by evaluating the "error function" and subtracting it from the measured pressure \( P_m(t) \) instantaneously. To lend confidence to such a scheme it was necessary to examine the validity of the assumed error function for a number of practical circumstances.

It is important at this point to distinguish between the steady (d.c.) and unsteady (a.c.) parts of \( P(t) \). By the usual definition:

\[
P(t) = \bar{P} + p(t)
\]

In unsteady flow both \( \bar{P} \) and \( p(t) \) will be measured incorrectly, although the errors are interrelated. We have been mainly concerned with devising an improved means of measuring \( p(t) \). The technique is easily applied to \( \bar{P} \) as well. It should be noted that our investigation is concerned exclusively with pressure in the free stream (i.e., as distinct from wall pressure).

The paper is organized under three sections. The present Part A reviews fundamental aspects of the problem, culminating with the empirical error equations reported in Section 4. These equations are central to the subsequent work. Part B describes the development of a miniature probe and associated analogue instrumentation for cancelling the interaction error instantaneously. Part C outlines a number of experiments which were undertaken to investigate the error, to calibrate the error cancelling system, and to attempt the evaluation of a corrected \( p(t) \) in various unsteady flows.

The investigation is motivated by a well-documented requirement for some fresh insights into the problem [e.g., 1,2,3]. The problem is controversial, as was demonstrated in 1963 at a Round Table Specialists Discussion on the future of noise research [1]. A question was posed by the moderator, Professor
E. J. Richards:

"Is there any chance of measuring pressure accurately?"  
(i.e. fluctuating pressure in turbulent shear flows.)

The negative view was taken by Dr. Ffowcs Williams who responded:

"If you stop the flow at any one point, I doubt if it is possible, even in principle to measure the pressure".

Surely this statement is questionable. If the spatial scale of the turbulent eddies is large compared with probe dimensions it should be feasible to compensate for the interaction error. This philosophy is reflected in the reply of Dr. H. S. Ribner to the same question:

"This is a problem one has with probes in general. Anything you put in to make a measurement disturbs what it is you are measuring. However, it is possible to calibrate away the disturbance in many cases. For example, in the case of a probe in supersonic flow, the probe creates a bow wave that completely destroys what it is you wanted to measure. Nevertheless, you can determine what would have been there if the probe had not been put there. I think this is still true in the case of a static pressure probe; that it is in principle possible to determine the pressure that would have been there in terms of the reading that the probe gives when you put it there. True, you disturb the pressure, but you can recover the information that appears to have been thrown away".

Improved techniques for pressure measurement would benefit several aspects of unsteady flow research. For example, pressure-velocity correlations $p\nu$ appear in one term of the turbulence energy balance equation. Because of the uncertainty inherent in measurements of $p\nu$, it has not generally been possible to "close" the energy equation (in differential form). The usual approach is to measure all the other terms with hot-wire probes and then to infer the unknown pressure-transport term by subtraction [e.g. 4,5].

2. HISTORICAL SURVEY

In spite of the need for reliable pressure measurement techniques in unsteady flows, the problem has been explored only in a limited sense. This is reflected in the relative shortness of the list of references. In 1936, Goldstein [6] proposed that the steady pressure $P_m$ as measured with a classical static probe in a turbulent stream would be in error according to a relation of the form:

\* While a closed energy balance would be of interest it is a major undertaking and beyond the scope of the present investigation.
The coefficient \( + \frac{1}{4} \) is an approximation based on assumed isotropy of the turbulence. In the more general case Goldstein implicitly speculates that the coefficient (which we shall subsequently denote as \( B \)) lies between 0 and \( + \frac{1}{4} \).

Fage [7] reported experiments in turbulent pipe flow which supported equation (2) as an expression of the static pressure error. He evaluated \( B \) as about \( + \frac{1}{4} \), in agreement with Goldstein's prediction. However there is reason to question this result. The calculation of \( B \) is based on cross-stream profiles of \( v^2 \) and \( w^2 \). Fage determined \( v^2 \) and \( w^2 \) by an indirect method, of doubtful reliability. His profiles differ significantly from corresponding hot-wire measurements made by Laufer [4, 24]. The value of \( B \) is quite sensitive to this difference. For example by re-working Fage's calculations for the case of flow in a round duct, but using Laufer's profiles of \( v^2 \) and \( w^2 \), it is possible to deduce that \( B \approx - \frac{1}{4} \), not \( + \frac{1}{4} \).

The work of Barat [8] and Toomre [9] provides a more complete description of the cross-flow error. Both point out that the Goldstein correction is valid only when the turbulent eddies are small compared with the probe diameter. In more general circumstances the error coefficient \( B \) may be positive or negative, depending on the ratio of a typical eddy size or scale \( \lambda \) to the probe diameter \( d \). When \( \lambda \) is very much larger than \( d \) the probe may be regarded as located in a locally uniform, but fluctuating flow. The situation at any instant in time is similar to that of steady flow about an inclined (i.e. yawed) probe. The error equation then has the same form as (2), but \( B \) is negative (see Section 3).

Toomre placed theoretically estimated limits on \( B \) in terms of the scale/probe-size parameter \( \lambda/d \). His findings can be summarized as follows:

\[
\frac{P_m - P_t}{\rho} \approx B \rho \frac{v_n^2}{n} 
\]

where,

\[- \frac{1}{2} \leq B \leq + \frac{1}{2} \]

Large scale limit \hspace{1cm} Small scale limit

\[\lambda \gg d\] \hspace{1cm} \[\lambda \ll d\]

Toomre implied that the large scale limit is more likely to be encountered in practical situations. In other words, for the usual flows like jets, wakes, and grid turbulence the dominant (energy bearing) eddies will be somewhat larger than a typical probe diameter. Hence the widely accepted Goldstein correction is misleading. For turbulent boundary layers, of course, this argument is less creditable.

** \( v_n \) denotes the instantaneous resultant of \( v \) and \( w \), which are the two unsteady (a.c.) components of turbulent velocity normal to the probe axis \( (v_n^2 = v^2 + w^2) \). Equation (2) assumes that the probe is aligned with the mean flow. In other words, \( \vec{v} = \vec{w} = 0 \).
Bradshaw and Goodman [10] were able to confirm experimentally that both the sign and magnitude of $B$ are dependent on $\lambda/d$. Their Figure 2 indicates an asymptotic approach to a large scale limit. It is strange, however, that the apparent limiting magnitude of $B$ obtained from their data is considerably smaller than Toomre's predicted $\frac{1}{2}$. Using $v_n^2/U^2 = 0.112$, their data gives $B \simeq -0.07$, for $\lambda \gg d$.

Attempts to investigate the error in unsteady pressure $p(t)$ are relatively non-existent (where it is understood that we mean free-stream pressure). Corcos [3] has given a brief qualitative account of the difficulties involved, emphasizing the possible influence of cross-flow Reynolds number on the error. Exploratory experiments with a novel type of pressure probe were made by Kistler [11]. Strasburg [12] and Kobashi [13] have used equations for the unsteady error which are similar to the equation for static pressure error (2). For example, Strasburg proposed that:

$$p_m(t) - p_0(t) \simeq \frac{1}{80}(v_n^2 - v_n^2)$$

Strasburg adopted the coefficient $\frac{1}{80}$ on the strength of the Goldstein-Fage result. However, in view of Toomre's findings it would seem more preferable to use the variable coefficient $B$, which will depend on the ratio $\lambda/d$. For many practical situations, $B$ will be negative. In fact, equation (4) is probably inappropriate in the case of positive $B$ (see section 3).

Strasburg showed that for isotropic turbulence the mean-square error pressure $(p_m - p_0)^2$ is of the same order of magnitude as the true mean square pressure $p_t^2$ predicted by Batchelor [14]. This is significant in that it indicates the degree to which the cross-flow error might obscure the true unsteady pressure in an unsheared turbulent flow.

While others have made reference to the error in $p(t)$, the usual practice seems to be to assume $p_m(t) \simeq p_t(t)$ [13,15]. This approximation is of questionable accuracy although it provides a crude estimate in shear flow turbulence, where the mean-square error is typically 25% of $p_m^2$ [12].

3. FUNDAMENTAL ERROR MECHANISMS

It is useful to review some aspects of the flow about a static pressure probe. The flow will be regarded as quasi-steady. The result is a highly over-simplified description of the fundamental error mechanisms.

Axisymmetric flow about a static pressure probe - Historically, attention has been focused on the errors in $p(t)$ arising from fluctuating cross-velocity $V_n(t)$. However, it is important to recall that a probe immersed in pure axisymmetric flow (whether it be steady or unsteady, but with $V_n = 0$), will not always indicate the true pressure. An error may arise due to the combined effects of nose curvature and downstream stem blockage (Figure 2a). This error has long been defined by aerodynamicists in terms of the pressure coefficient of a particular probe. In our notation:

$$\bar{p}_m - \bar{p}_t \simeq A \rho U^2$$

*Batchelor's result can be put in the form: $\bar{p}_t^2 \simeq [0.3 \nu v_n^2]^2$
This U-dependent error may have both steady and unsteady parts just as does the cross-flow error. Good probe design involves proper profiling to minimize A.

Cross-flow about an infinite cylinder - At the large-scale limit ($\lambda >> d$), a crude interpretation of the cross-flow error is made by considering pure cross-flow about an infinite cylinder (Figure 2b). When the Reynolds number is very low ($R_v = Vn/d/\nu < 1$), the Stokes solution gives a surface pressure distribution around the cylinder of the form:

$$P(\theta) - P_\infty = \frac{1}{\frac{1}{2} \rho V_n^2} \frac{1}{R_v} \cos \theta$$

On the other hand a somewhat different distribution results from the inviscid (potential flow) solution:

$$P(\theta) - P_\infty = \frac{1}{\frac{1}{2} \rho V_n^2} - 1 - 4 \sin^2 \theta$$

For Reynolds numbers appropriate to the pressure measurement problem ($10 < R_v < 1000$)*, the distribution $P(\theta)$ typically falls somewhere between the viscous and inviscid solutions, as shown in the figure (e.g. see Apelt's numerical solution of the Navier-Stokes equations for $R_v = 40$ [16]. If $P(\theta)$ is averaged around the circumference of the cylinder, the average pressure (which we have denoted $P_m$) will equal the undisturbed free-stream pressure $P_\infty$ only at the viscous limit ($R_v \rightarrow 0$). For finite Reynolds number $P_m$ is generally lower than $P_\infty$. The difference $(P_m - P_\infty)$ can be regarded as a cross flow error, and is approximated reasonably well by:

$$P_m - P_\infty \approx B \frac{\rho V_n^2}{2}$$

The potential flow solution gives a value $B = -\frac{1}{2}$. This is equivalent to Toomre's large scale limit. However, from numerical and experimental evaluations of $P(\theta)$ such as those presented in [16] and [17], values of $B$ in the range $-1/5$ to $-1/3$ are found, with evidence of a moderate increase with Reynolds number for $10 < R_v < 1000$.

The two-dimensional cross-flow model is only weakly related to the more complicated case of a probe in an unsteady flow. The structure of the wake formed in the "dead-water" region of the probe must be strongly influenced by a large axial component of velocity ($U$) and by irregular variation of the azimuthal direction of $V_n$. Nevertheless, in a typical unsteady situation with $U >> V_n$, $P(\theta)$ was found to behave much as depicted in Figure 2b. More particularly, this was noted during experiments with the rotating inclined nozzle, to be discussed in Section 10.

Inclined probe in steady flow - A slightly more plausible model, appropriate to the large-scale limit, is that of a static pressure probe yawed to a steady mean flow (Figure 3c). It is well known that a yawed probe will underestimate the pressure. For small angles of yaw ($\alpha$) the error is described by equation (6) if we put $V_n = U \sin \alpha$ [e.g. see reference 7]. The error arises from a pressure distribution $P(\theta)$ similar to that for pure cross-flow. As

* This Reynolds number range is typical for the usual probe sizes and for cross-velocities normally found in unsteady air flows.
will be seen in Section 7, typical values of \( B \) range from \(-\frac{1}{4}\) to \(-\frac{1}{2}\), with some dependence on probe geometry.

Goldstein's stagnation mechanism - When the scale of unsteadiness is much smaller than a probe diameter the cross-flow error may be induced in a manner originally described by Goldstein [6]. Figure 3d depicts the situation. In any one eddy the velocity normal to the probe surface stagnates, producing a local pressure higher than the undisturbed pressure would have been. An ideal pressure probe registers a circumferential average of the local surface pressure. Hence, the steady part of the cross-flow error is expressed as in Equation (2), where the coefficient may take values as large as \( \pm \frac{1}{2} \). We have already implied that the Goldstein type of error is unimportant in cases where the energy-bearing eddies are larger than \( d \). Indeed if we were unable to make a probe small enough to meet this condition, it would not be possible to measure the local unsteady pressure accurately. Both \( p_\text{t}(t) \) and \( v_n(t) \) would be uncorrelated around the probe circumference. Hence, spatial resolution would be lost. A simple error compensation scheme would not work.

Effect of pressure holes - So far, only those errors associated with quasi-steady flow about an ideal probe (i.e. one which averages over \( p(\theta) \) uniformly) have been considered. A conventional static pressure probe usually employs a finite number of sensing holes, equally spaced about a circumference. An accurate continuous average over \( p(\theta) \) is effected only if the number of these holes is very large or, equivalently, if they are replaced by a circumferential slit. When the number is small (1-6), the average will be weighted by the azimuthal direction of \( V_n \). If the holes are too large in diameter an error dependent on their size and shape may become important [18]. Furthermore, if there is a significant amount of internal flow (from hole to hole within the probe), the surface pressure distribution will be influenced unduly. These secondary effects must be considered when designing a practical error-compensating probe (Section 5.).

4. EMPIRICAL ERROR EQUATIONS

Strictly speaking, the interaction error \( p_\text{e}(t) - p_\text{t}(t) \) is defined by an integral over the distorted velocity field and the surface of the probe (e.g. see Toomre's analysis for the error in steady pressure [9]). To evaluate this integral for a general unsteady situation would be virtually impossible. Nevertheless, it is possible to construct an empirical approximation if we pre-suppose that the properties of the undisturbed velocity field are more or less uniform over distances which are much larger than the probe diameter \( (\lambda >> d) \). In what follows, a low speed incompressible flow is assumed.

Using an approach similar to that of Strasburg [12], we postulate that the error depends primarily on the local instantaneous flow velocity within \( \lambda \), and we expand \( p_\text{m} - p_\text{t} \) in a power series of \( U(t) \), \( V(t) \), and \( W(t) \):

\[
P_\text{m}(t) - P_\text{t}(t) = a_1 U + a_2 U^2 + a_3 V + a_4 V^2 + a_5 W + a_6 W^2 + a_7 UV + a_8 UW + a_9 VW + \ldots
\]
Other higher order terms and derivatives are presumed to be of lesser significance. Viscosity dependent errors, which may be important at low cross-flow Reynolds number (i.e. \( R_v \approx 10 \) or less), have been neglected here. Additionally, one might conceive of inertial errors of the form \( a_{10} \partial v / \partial t \), associated with the acceleration of fluid around the probe. In Appendix A it is argued that such terms are probably small compared with those involving \( U^2 \), \( V^2 \) and \( W^2 \).

Symmetry considerations* allow us to put \( a_1, a_3, a_5, a_7, a_9 \) and \( a_9 \) equal to zero. For dimensional consistency, coefficients \( a_2, a_4 \) and \( a_6 \) must have units of fluid density \( \rho \). In fact, by comparison with equations (5) and (6) it seems reasonable to put \( a_2 = A \rho \) and \( a_4 = a_6 = B \rho \). With these changes, equation (7) can be re-written:

\[
\tilde{p}_m(t) - \bar{p}_t(t) \approx A \rho \left( \bar{U}^2 + \bar{u}^2 \right) + B \rho \left( \bar{V}^2_n + \bar{v}^2_n \right)
\]

(8)

The variables are now broken into steady and unsteady parts:

\[
P(t) = \bar{p} + p(t) \]
\[
U(t) = \bar{U} + u(t) \]
\[
V(t) = \bar{V} + v(t) \]
\[
W(t) = \bar{W} + w(t) \]

On substitution of these variables, equation (8) can be manipulated to give separate expressions for the error in steady (static) pressure and the error in unsteady (fluctuating) pressure:

**STEADY ERROR**

\[
\bar{p}_m - \bar{p}_t \approx A \rho (\bar{U}^2 + \bar{u}^2) + B \rho (\bar{V}^2_n + \bar{v}^2_n)
\]

(9)

**UNSTEADY ERROR**

\[
p_m(t) - p_t(t) \approx A \rho (2 \bar{u} \bar{U} + u^2 - \bar{u}^2) + B \rho (2 \bar{v} \bar{V} + 2 \bar{w} \bar{W} + v^2 - \bar{v}^2_n)
\]

(10)

(Here, for convenience, we have used the identities: \( \bar{V}^2_n = \bar{V}^2 + \bar{W}^2 \), \( v^2_n = v^2 + w^2 \)).

Equations (9) and (10) are merely more general forms of the Goldstein-Toomre-Strasburg expressions. It must be stressed that they are based on the assumption of quasi-steady flow, the acceleration terms being neglected.

* A conventional cylindrical probe cannot distinguish between positive and negative \( U, V \), or \( W \) (e.g. the error must have the same sign for +\( V \) or -\( V \)).
Nonetheless for unsteady flows where the energy bearing spectral content is concentrated at low frequencies, such that $\lambda/d$ is relatively large* these equations should provide an adequate description of the error. Certainly the coefficients $A$ and $B$ will not be totally independent of $\lambda/d$ and Reynolds number over the entire frequency range. However, they can be approximated as constants when the conditions are right.

Our earlier experience with the aerofoil turbulence probe [21] confirmed that quasi-steady assumptions remain valid for frequencies well in excess of 1000 Hz, provided that the probe is made small enough. While the aerofoil probe is a velocity sensor, there is no reason to discount the validity of similar assumptions in the case of a pressure probe.

If a probe can be devised for which $A/B < 0.002$, allowing the neglect of $U$-dependent errors, the unsteady error equation (10) becomes:

$$p_m(t) - p_t(t) \sim B \rho \left(2 \frac{\bar{V}}{v} + 2 \frac{\bar{W}}{w} + \frac{v^2}{v^2} + \frac{w^2}{w^2} - \frac{\bar{V}^2}{v^2} - \frac{\bar{W}^2}{w^2}\right)$$

When the probe is aligned with the mean flow ($\bar{V} = \bar{W} = 0$):

$$p_m(t) - p_t(t) \sim B \rho (v^2 + w^2 - \frac{\bar{V}^2}{v^2} - \frac{\bar{W}^2}{w^2})$$

The possibility exists of compensating for the interaction error. If a probe were contrived to measure $P_m, v, w$ simultaneously a more accurate estimate of the true pressure $p_t(t)$ could be obtained by instantaneous analogue solution of (11a). Such an error-cancelling system has been developed. A detailed description is given in the following section.

**PART B - INSTRUMENTATION DEVELOPMENT**

5. **DESIGN OF AN ERROR-COMPENSATING PRESSURE PROBE**

5.1 **General Criteria**

To be effective the proposed error compensating probe should meet with a number of constraints, of which several have already been mentioned. A more detailed summary is given now.

The probe must measure $P_m, v, w$ simultaneously at a near-common point in the flow. The pressure and velocity components must be transduced into electrical signals instantaneously. To ensure adequate spatial resolution the transducers must occupy a volume much smaller than a characteristic correlation volume of the turbulence ($d \ll \lambda$). The transducers must have frequency response compatible with the dominant spectral content of $P_m, v, w$ (typically 10 Hz to 10 KHz), and sensitivity sufficient to give a good signal to noise ratio ($> 30 \text{ dB}$). They should be insensitive to undesirable inputs such as vibration or temperature variation. The three measurement channels

*If the spatial pattern of the unsteadiness is "frozen" a single spectral component (frequency $f$) moving past the probe with a mean convection speed $\bar{U}_c$ has a characteristic scale or wavelength $\lambda = \bar{U}_c/f$. 
should be reasonably well phase-matched over the frequency range of interest.

The probe shape should be carefully tailored to ensure the condition

\[ A \ll B \]

Also the cross-flow 'error should be invariant with the azimuthal
direction of \( V_n \). This suggests the use of a circumferential pressure sensing
slit, rather than a finite number of holes.

With these restrictions in mind a number of alternative probe layouts
and transducing schemes were considered. A configuration as depicted in Figure
4 was finally adopted as the most promising. Components and features of the
probe are described in the following sub-sections.

5.2 Miniature Pressure Transducer

It was decided that a probe of 1/8 inch nominal diameter would satisfy
the condition \( \lambda \gg d \) for a range of turbulent flows. It was also felt feasible,
although difficult, to construct an error-compensating probe on this scale. The
first requirement then was for a pressure sensor which could be housed within
such a probe.

One method which has been exploited in the past [12,13] is to use a
probe-tube extension coupled to a conventional microphone. However acoustic
resonance effects restrict the usefulness of such a device to frequencies below
1000 Hz, for a typical coupling tube length (\( \sim 3 \) inches). Furthermore, the
associated acoustical phase shift is significant at much lower frequencies.

Another possible scheme would employ a cylindrical piezoelectric trans­
ducer as the pressure sensor. Past experience has shown though that such devices
are difficult to design for vibration insensitivity [3] and that they lack
adequate signal/noise ratio when used in low speed turbulent flows.

A third alternative is a miniature condenser microphone which could be
housed within the probe, in close proximity to the pressure sensing slit. The
use of a very thin, low-mass diaphragm would ensure good pressure sensitivity
over a wide frequency range while providing a high degree of vibration insensi­
tivity. The last approach was judged the most promising. There being no such
device commercially available at the time, it was decided to construct our own.
During a summer stay at the LTV Research Center, Western Division*, a miniature
condenser microphone was successfully developed [19]. The transducer has a
nominal .100 inch diameter, utilizing a tensioned stainless-steel diaphragm of
.0001 inch thickness (see Figure 5). Electrical interconnection was facilitated
by building the sensor directly onto the end of a Microdot Lepra-Con Coaxial
connector. When coupled to a \( \frac{1}{3} \) inch Bruel and Kjaer cathode follower, sensiti­
vity lies between -85 and -105 db (re 1 volt/\( \mu \) bar), with an equivalent SPL
noise level of 90 db. Frequency response of the free transducer (not as mounted
in the probe housing) is typically flat from 20 Hz to 80 KHz. Characteristics
as mounted in the p-v-w probe will be discussed at a later stage.

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5.3 Two-Component Cross-Velocity Sensor

One means of measuring the velocity components \( v \) and \( w \) simultaneously is to use an array of inclined hot wire elements (effectively, two cross-wire probes). However, the complexity of such an arrangement together with the necessary associated circuitry make it rather unattractive.

**Lifting body probe** - Our earlier success with the aerofoil probe device [20, 21, 22] suggested an alternate approach. By making a simple extension to the aerofoil probe concept, the so-called lifting body probe was evolved. In principle, the idea is to measure two orthogonal components of the unsteady lift (or side-force) acting on the axi-symmetric nose piece of the pressure probe (Figure 4). To the extent that quasi-steady slender body aerodynamics are applicable, the instantaneous lift force will be proportional to the effective angle of attack \( \alpha \). When the turbulence intensity is not too high \( \alpha \) can be approximated by \( v_n/U \). Hence the orthogonal components of lift will be:

\[
L_v \sim q (v/U) ; \quad L_w \sim q (w/U) \tag{12}
\]

Here, \( q \) is the dynamic head of the mean flow (\( \sim \frac{1}{2} \rho U^2 \) to the same order of approximation).

If a device is provided to linearly transduce \( L_v \) and \( L_w \) into voltages, the probe outputs will be:

\[
e_v \sim S_v q \frac{v}{U} ; \quad e_w \sim S_w q \frac{w}{U} \tag{13}
\]

The proportionality constants \( S_v, S_w \) are the sensitivities of the respective channels.

**Force transducer** - The development of a suitable transducing device proved to be a major problem of design and fabrication. In the final version a piezoelectric I-beam supports the lift-sensing nose piece. The I-beam consists of four piezoelectric plate elements, arranged to respond to lift-induced orthogonal bending moments. The elements are connected in pairs so that voltages developed by the appropriate component of transverse force (\( L_v \) or \( L_w \)) add, while those arising from spurious axial forces cancel out. The arrangement is illustrated in Figure 6.

The piezoelectric elements were cut from Bimorph strips of the type found in some phonograph cartridges. They were assembled with non-conductive epoxy, the electrical connections being made with conductive paint and fine magnet wire. During assembly it was important to maintain precise alignment to ensure that the channels were \( 90^\circ \) separated and to minimize cross-coupling. The difficulty of the task becomes apparent when it is realized that the finished beam is nominally .060 inches high by .120 inches long.

**Mounting the nose-piece** - To provide a wide frequency response range it was necessary to maximize the fundamental resonant frequency of the beam/nose-piece combination. This dictated a fairly rigid assembly and a very light nose-piece. (An equally important reason for a light nose-piece was to minimize unwanted inertial loading of the transducer beam, arising from stem vibration).

The pressure sensing slit was formed by leaving a gap between the base
of the nose piece (i.e. the v-w sensor) and the supporting probe stem (Figure 6). This ensured a fairly localized measurement of p, v, and w. The base attachment of the transducer beam was therefore complicated by the need to provide passageways connecting the slit with the pressure transducer (which was housed behind the I-beam). Nevertheless a four-edge epoxy mounting as shown in Figure 6 provided adequate stiffness.

To achieve lightness the nose-piece was made of balsa wood - machined on a jeweller's lathe. It was then encapsulated in a thin epoxy coating and re-machined to produce a smooth axisymmetric surface. During mounting (with epoxy) on the outer end of the I-beam, it was aligned very carefully, particular care being paid to details of the pressure-sensing slit formed at its base. To minimize any effect which the slit might have on the local flow the gap was kept very small (≈ .008 inches). It was found however that when the gap was made too small ( < .004 inches) the response of the pressure sensor became attenuated at the higher frequencies (probably due to enhanced influence of viscosity in the gap). In the final assembled state the lifting-body sensor has a fundamental resonant frequency of 12-15 KHz, the resonance being very well damped.

Preamplifiers - The piezoelectric force transducers are inherently low output, high impedance devices. Therefore to minimize noise pickup and to ensure good low frequency response they were coupled directly to simple unity gain preamplifier units housed in the probe supporting sting. The circuit is basically a high impedance source follower, utilizing a field effect transistor at the input (see Figure 7). With the 100 MΩ gate resistor as shown, low-end response of the v and w sensors is typically 3 db down at 9 Hz. More complete calibrated characteristics are given in the next sub-section.

5.4 The complete p-v-w probe

Many features of the p, v, and w sensors have already been discussed with reference to Figure 6. Close-up photographs of the probe and various related devices are shown in Figure 8.

Minimizing A - Because it was necessary to place the pressure sensing slit near the nose of the probe, the nose curvature error (Figure 2a) posed a potential problem. To offset it (i.e. to ensure that A « B) a geometric trick was employed; the cross-sectional area of the probe was increased slightly at a point approximately one diameter downstream of the pressure-sensing slit. This causes an opposing effect somewhat like the stem influence of Figure 2. The shape and position of the area transition were juggled until A was made negligibly small (discussion in Section 7). This scheme will only work well, of course, when λ is substantially larger than d - a premise which has already been accepted.

Mounting the probe - An early prototype version of the probe was mounted on a wooden support fairing which housed preamplifiers for the sensors (Figure 9a). This arrangement was not very satisfactory because the fairing was too close to the measurement point and thereby had considerable adverse effect on both the axial and cross-flow pressure errors (see discussion in Section 7). The difficulty was overcome by mounting the probe at the end of a co-axial supporting sting. The sting consists of a Bruel and Kjaer Type 2615 cathode follower (preamplifier for the pressure transducer) with the shell of a B. & K.
input adaptor* coupled behind it. This shell houses the two source follower preamplifiers for the velocity sensors. The entire unit is axi-symmetric for some 80 probe diameters downstream of the nose. Figure 9a depicts the configuration together with the earlier prototype.

The probe-sting combination is supported by a perpendicular stem which in turn is fixed to a vibration-isolated traversing mechanism (Figure 9b). The probe can be traversed along three axes. In addition, the angle of attack can be varied without sensor translation.

Calibration of the pressure sensor - A Bruel and Kjaer pistonphone (Type 4220) was used to determine the sensitivity ($S_p$) of the pressure sensor. The probe tip was inserted into the pistonphone cavity through an O-ring seal. Sinusoidal piston displacement at 250 Hz produced a root-mean-square pressure of 3.2 millimeters of water. ($S_p$ was recorded in units of mv/mm H$_2$O for convenience). After some initial difficulty it was discovered that the pressure sensitivity was quite dependent on ambient temperature. Therefore $S_p$ was evaluated for a wide range of temperature - the result is shown in Figure 10.

When the thermal correction had been reliably established it was possible to calibrate the pressure sensor to 5% accuracy. At room temperature, $S_p$ was about .5 mv/mm H$_2$O. The electronic noise level, in the bandwidth 10 Hz - 10 KHz, was typically 35 microvolts.

Frequency response was deduced by a comparison calibration. The p-v-w probe and a standard reference microphone (B. & K. Type 4134) were mounted in an earphone driven coupler cavity (supplied with the B. & K. probe tube kit No. UA 0040). Frequency of the earphone source was varied. By comparing the output of the probe with that of the reference microphone (of known frequency response) the response of the pressure sensor was determined. Figure 11 depicts the result. The passageways connecting the pressure sensing slit with the pressure transducer form a potential resonant cavity. Loose cotton plugs were inserted during assembly in an attempt to provide some viscous damping (Figure 6). Nevertheless a strong resonance occurred at 8500 Hz as evidenced in Figure 11. A simple tuned L - C rejection filter proved to be quite effective in eliminating the resonance. The final compensated frequency response was virtually flat between 3 db end points at 9 Hz and 7 KHz. The comparison calibration yielded a value of $S_p$ which agreed closely with the pistonphone result.

Calibration of the velocity sensors - The rotating inclined nozzle (described more completely in Section 10, see Figure 18) was convenient for calibrating the v and w sensors. Basically, flow through the nozzle is caused to "swirl", the velocity vector being inclined at a small angle $\theta_0$ to the axis of nozzle rotation. For calibration, the probe was placed so that its axis coincided with the axis of rotation. Consequently the v-sensor experienced a sinusoidally varying angle of attack $\theta = \theta_0 \sin 2\pi ft$. The output voltage $e_v$ was correspondingly sinusoidal and proved to be very pure (i.e. free from noise and harmonics). Because the w-sensor was orthogonally displaced it generated an output $e_w$, 90° out of phase with $e_v$. The sensitivities $S_{v,w}$ were readily deduced from Equation 13 (e.g. by replacing $v/U$ with $\theta_0 \sin 2\pi ft$). By varying

* This adaptor, B. & K. part no. JJ 2614 is conveniently supplied with each cathode follower.
the dynamic head of the nozzle flow \((q)\), linear calibration curves resulted as depicted in Figure 12. The influence of ambient temperature was checked and found to be small over the normal operating range \((800° - 100°F)\). \(S_v\) and \(S_w\) were determined with 5\% accuracy. For most of the experiments to be described, the signal to noise ratio was of order 100, for a bandwidth of 10 Hz - 10 KHz.

Frequency response was deduced by a comparison technique. At a common point in a typical turbulent flow the energy spectrum of a velocity fluctuation \((w)\) was measured both with the lifting body probe and with a crossed hot-wire probe. (The crossed-wire probe was operated with a linearized constant temperature hot-wire anemometer giving frequency response which was constant from D.C. to 80 KHz.). The measurements were made in the shear layer of a 4 inch diameter round jet. At the probe location the integral length scale \(L_x\) of the turbulent eddies was nominally 2\(\frac{1}{2}\) inches (20 probe diameters). The resulting spectra as measured with either channel of the lifting body probe agree very well with that given by the reference crossed-wire probe (see Figure 13). The data are plotted against Strouhal frequency \(f_d/\bar{U}\) which can be regarded as a measure of the probe diameter to wavelength ratio \(d/\lambda\). When \(f_d/\bar{U}\) exceeds \(\frac{1}{4}\), frequency response of the \(v-w\) sensors begins to fall (a consequence of the finite size of the nose-piece). The data indicates nevertheless that for turbulent flows of comparable eddy size, the frequency response will be quite uniform \((\pm 1\text{db})\) over the frequency range 20 Hz to 10 KHz.

Inter-channel calibration - It was necessary to verify that the \(v\) and \(w\) sensors were orthogonally displaced. The angular separation was therefore checked by a cross-correlation technique which utilized the rotating nozzle flow. The method is described in Appendix B. Angular separation was found to be within 5\% of 90°.

Consideration was also given to the problem of inter-channel phase matching. A certain amount of phase shift was unavoidable for a particular channel. Nevertheless it was possible to minimize the relative phase difference between channels for much of the active frequency range.

At the low frequency end (below 150 Hz), the response of the \(p, v\) and \(w\) sensors rolls off in a manner characteristic of R-C coupled transducer/pre-amplifier combinations. This amplitude roll off is accompanied by a leading phase shift which grows with decreasing frequency. At the 3 db down frequency the phase lead amounts to 45\(^\circ\). By careful electronic tailoring, the low end response of the three sensors was matched so as to have a common 3 db down point at 9 Hz. Thus the interchannel phase difference was minimized to less than 5\(^\circ\) for frequency above 9 Hz. (The phase response was checked by a correlation technique similar to that described in Appendix B, with the aid of the rotating nozzle flow. Frequency was variable from 3 Hz to 150 Hz).

At the high frequency extreme (frequencies > 500 Hz), the phase problem becomes rather more difficult to handle. Various factors cause the pressure signal \(e_p\) to lag behind \(e_v\) and \(e_w\). A substantial lag arises because a turbulent disturbance takes a finite time to convect from the effective aerodynamic center of the \(v-w\) sensors to the pressure sensing slit. This time delay may be expressed as \(\Delta t \simeq s/\bar{U}\) where \(s\) is the separation of aerodynamic center and pressure slit. For the present probe in a flow with convection speed \(\bar{U}_C\sim 100\text{ fps}\) the equivalent phase lag amounts to about 45\(^\circ\) at 1000 Hz. A lesser lag is
associated with acoustic propagation of the pressure disturbance from sensing slit to pressure transducer, within the probe (about 5° at 1000 Hz). Other sources of phase shift are likely to be insignificant below 1000 Hz and may be self-compensating to some extent. For example the phase response of the pressure transducer will progressively lag as the frequency of diaphragm resonance is approached; however, the output of the velocity sensors will lag in a similar fashion, thereby reducing the inter-channel shift.

Turbulent flow experiments were conducted in which the pressure and velocity signals were cross-correlated (Section 11). These experiments indicated that \(e_p\) lagged \(e_v\) and \(e_w\) by approximately .140 millisecond. This corresponds to 50° at 1000 Hz in agreement with the rough prediction made earlier. For applications of the error compensating probe to be described, the major spectral energy was concentrated somewhat below 1000 Hz. Hence no attempt was made to eliminate the lag. However in a more sophisticated version of the system the velocity signals could be delayed by electronic means - this would improve the accuracy of error compensation at the higher frequencies.

6. ANALOGUE SYSTEM FOR CANCELLATION OF CROSS-FLOW ERROR

The probe signals \(e_p\), \(e_v\), and \(e_w\) (as measured at the preamplifier outputs) were fed into three voltage amplifiers of gains \(G_p\), \(G_v\), and \(G_w\). The signals were amplified to levels appropriate for the subsequent analogue processing (gains being continuously variable from 0.1 to 1000).

An electrical analogue of the error equation 11a was programmed on a Philbrick Model RP Operational Manifold. The network was typically set up as depicted in Figure 14. The \(v\) and \(w\) signals were squared with two quadrant squaring circuits utilizing Philbrick Transconductors (Model PSQ-N) and Operational Amplifiers (EP 85). The sum of squares was then subtracted from the pressure signal (taking proper account of phase inversions, channel sensitivities, and the error coefficient B) to arrive at a signal corresponding to the corrected pressure \(p_c(t)\). The overall system had constant amplitude response and negligible phase shift for the entire useful frequency range of the probe.

PART C - EXPERIMENTS AND DISCUSSION

The first experiments to be described were conducted at early and intermediate stages of the investigation. They lent some new insights which aided in the development of a practical error-compensating pressure probe. Later sections deal with several applications of the \(p-v-w\) probe in unsteady flows. The objectives were to substantiate Equation 11 as an expression of the cross-flow error and to test the feasibility of the error-cancelling system.

7. STEADY FLOW EXPERIMENTS WITH STATIC PRESSURE PROBES

In view of the assumed quasi-steady nature of unsteady errors at the large scale limit it was decided that a re-examination of the fundamental steady flow errors would be worthwhile. A series of static pressure measurements were undertaken, the objective being to determine the error coefficients \(A\) and \(B\) for a variety of probe configurations. The error equation (9) can be written in the form:
\[
\frac{P_m - P_t}{2q} \approx A \cos^2 \alpha + B \sin^2 \alpha
\] (14)

This applies when the probe is placed at a small angle of attack \(\alpha\) in a steady flow (\(u^2 = v_n^2 = 0\)) of dynamic head \(q\). By fitting equation 14 to experimental plots of \(P_m - P_t\) versus \(\alpha\), coefficients \(A\) and \(B\) can be obtained.

The measurements were made in the exit plane flow of an open throat wind tunnel (8" x 12" section), for values of \(q\) ranging from 15 to 130 millimeters of water. A sensitive pressure transducer (PACE P7D) was used to monitor the error pressure \(P_m - P_t\) because of its small magnitude (generally less than 5% of \(q\)). Data was obtained with three basic probe types:

1) Standard static pressure probe - A probe of the standard classical configuration (Figure 1) was used for the first set of measurements. "A" was very small (< .001) for this probe. Thus it indicated the true pressure \(P_t\) when placed at zero angle of attack. The diameter \(d\) was .120 inches. Six pressure sensing holes were located around a circumferential generator at a point \(8\frac{1}{2}\) diameters from the nose.

This standard probe yielded data shown as Curve a on Figure 15. Equation 14 was fitted to the experimental points by putting \(A = 0\), \(B \approx - .55\). \(B\) was found to be relatively independent of the tunnel \(q\). Its magnitude was slightly larger than the potential theory value for pure cross-flow (-\(\frac{1}{2}\)).

ii) "Dummy" of prototype probe - A geometric copy was made of the prototype p-v-w probe (as mounted on the wooden support fairing - Figure 9a). This "dummy" probe was equipped to monitor only the static pressure \(P_m\) using a five-hole sensor. The variation of \(P_m - P_t\) with \(\alpha\) was obtained as for the standard probe - Curve b of Figure 15 depicts the result where \(\alpha\) has been taken in the plane of the support fairing. The most noticeable features are the pronounced asymmetry of the curve and the unacceptably large value of \(A\). This sizeable axial flow error arose primarily because the pressure sensing holes were not far enough removed from the probe support fairing. Flow stagnation at the leading edge of the fairing caused the upstream static pressure to be modified (stem influence in Figure 2a). The cross flow error varied with the azimuthal orientation of \(\alpha\) because the fairing interference was asymmetric relative to the probe axis.

iii) "Dummy" of final version probe - In view of the inadequacies of the prototype probe it became apparent that a new configuration would be required. Hence the final version probe (Figure 9a) was evolved. This version employs an axi-symmetric afterbody which extends a considerable distance downstream. The perpendicular supporting stem is located 80 probe diameters from the sensors, thereby ensuring minimum upstream influence.

Once again a dummy of the complete probe arrangement was constructed and the static error \(P_m - P_t\) was determined. On the initial attempt (Curve c - Figure 15), \(A\) was found to be negative. This arose primarily because of the nose curvature effect. In order to reduce the magnitude of \(A\) a shaping trick was resorted to, as mentioned in Section 6. The cross-sectional area of the probe was increased slightly but abruptly at a point about one diameter downstream of the pressure sensing slit. This area increase tended to retard the flow in the vicinity of the slit, thereby counteracting the accelerating influence.

15
of nose curvature. By adjusting the position of the area transition it was possible to make \( A \) very small. With the final profile shape, Curve d (Figure 15) was obtained; \( A \) was minimized to less than .001.

Surprisingly, \( B \) turned out much smaller than for the earlier probe types (Curve d was fitted by putting \( B \approx -0.23 \)). This difference is probably a consequence of geometric dis-similarity of the three configurations. It is quite likely that the area transition had some influence on the value of \( B \) as well as \( A \). On the other hand, the multiple-hole type pressure sensors of the earlier probes may have resulted in improperly weighted values of \( B \).

8. ROTATING ELLIPSE EXPERIMENT

The steady flow measurements provided estimates of \( A \) and \( B \) and enabled the probe shape to be refined. However, they did not help to establish the validity of the error equations (9) and (10) for cases of unsteady flow. To do this it was necessary to contrive a number of specialized unsteady flows, the properties of which could be predicted by analysis. The first of these calibration flows was the so-called rotating ellipse flow. While it did not fulfill its intended purpose the experiment proved to be interesting and will be described now.

A cylinder of elliptic cross-section was rotated at high speed about its axis. Steady flow was superimposed parallel to the axis, the cylinder being fitted with a suitably rounded nose at the upstream end. The elliptic shape induced a non-uniform pressure and velocity pattern which rotated with the cylinder, appearing unsteady to a stationary observer. This disturbed field was treated as two-dimensional in any rotational plane sufficiently downstream of the nose (the possibility of coupling with the steady axial velocity \( U \) being neglected). On the assumptions of potential theory, the appropriate potential function takes the form [23]:

\[
\phi = \frac{\omega}{4} (a + b)^2 e^{-2\xi \sin 2\eta}
\]  
(15)

Here \( \omega \) is the angular rate of rotation. \( a \) and \( b \) are the semi-major and semi-minor axes of the elliptic cross-section. The focal distance is \( c = \sqrt{a^2 - b^2} \). \( \xi \) and \( \eta \) are elliptical co-ordinates, being related to the polar co-ordinates \( r \) and \( \theta \) by:

\[
\begin{align*}
    r \cos \theta &= c \cosh \xi \cos \eta \\
    r \sin \theta &= c \sinh \xi \sin \eta
\end{align*}
\]

A sketch of the streamline pattern relative to the cylinder is shown in Figure 16.

Applying the Bernoulli equation for unsteady flow, the fluctuating pressure at a radius \( r \) is:

\[
P_t(t) - P_\infty = -\frac{1}{2} \rho \left[ v_\theta^2 + v_r^2 + 2\omega r v_\theta \right]
\]  
(16)

Velocity components \( v_r \) and \( v_\theta \) are derived from the potential function.
The properties $p_t$, $v_r$ and $v_\theta$ vary periodically, but not sinusoidally. Their waveforms are related in a complex manner to the radius parameter $r/c$ and the ellipse eccentricity $c/a$.

At the outset it was felt that the error equation (11a) could be effectively investigated by the rotating ellipse method since the "true" unsteady properties could be calculated. A spinning apparatus was built and two elliptic cylinders of different eccentricities were fabricated. Some measurements of $P_m(t)$, $v_\theta$ and $v_r$ were made in the frequency range 20 Hz to 200 Hz. The waveforms of $v_\theta$ and $v_r$ corresponded fairly well with theory. Also, rather surprisingly, the measured pressure signal $P_m(t)$ proved to be almost identical with the predicted (true) pressure $P_k(t)$. Figure 16, for example, shows a typical comparison. It became evident that for this particular unsteady flow the probe error was a very small fraction of $P_m(t)$. An examination of the right-hand-side of equation (16) reveals the reason. The term $\frac{1}{2} \rho(v_\theta^2 + v_r^2)$ is of the same order of magnitude as the cross-flow error (assuming that $|B| < \frac{1}{3}$). By contrast, the centrifugal term $\rho v_r v_\theta$ was substantially larger than $\frac{1}{2} \rho(v_\theta^2 + v_r^2)$ for the range of experimental conditions which were attainable. For example, with a rather oblate ellipse ($a = 1.4''$, $b = .875''$), $2w_r$ exceeded $v_\theta$ and $v_r$ by a factor of 50, at $r/c = 2$. This result is independent of frequency because $v_\theta$ and $v_r$ are proportional to $\omega$. In consequence of $P_m(t) - P_\infty$ being much larger than the cross-flow error, the evaluation of $B$ was impossible. The rotating ellipse flow exemplifies the fact that pressure probe errors can sometimes be negligible.

9. AXIAL VELOCITY FLUCTUATION EXPERIMENT

As described in Section 5, special steps were taken to make the pressure and velocity sensors insensitive to axial velocity fluctuations. The effectiveness of these measures was checked with the aid of a pulsating unidirectional flow. The flow was generated by making a minor modification to an existing air jet facility, as illustrated in Figure 17. A butterfly valve was inserted upstream of the settling chamber and allowed to auto-rotate, thereby chopping the inlet flow. At the exit nozzle the velocity was modulated in a smooth, almost sinusoidal manner for a range of frequencies in the vicinity of 100 Hz. (Frequency was varied by changing the butterfly size and the flow speed). The modulated velocity $U(t) = \bar{U} + u(t)$ was monitored with a hot wire anemometer; a typical trace is shown in Figure 17. The root-mean-square fluctuation $u'$ was typically $7\%$ of $\bar{U}$, which was about 100 fps.

The $p-v-w$ probe was placed on the centerline of the nozzle and carefully aligned with the flow direction, the sensors being located slightly upstream of the exit plane. In this region the flow was parallel ($V = W = 0$), with a very low natural turbulence level. The applicable reduced form of the unsteady error equation (10) is:

$$P_m(t) - P_t(t) \approx A \rho \left( 2 \bar{u} \bar{U} + u'^2 - u'^2 \right)$$  \hspace{1cm} (17)

To check the negligibility of $A$ for this specialized case of unsteady flow it was necessary to evaluate the error $P_m - P_t$. In terms of root-mean-square...
quantities, equation (17) can be written:

\[
\frac{(P_m - P_t)}{q} \approx 4 A \frac{u'}{U} \tag{18}
\]

Here, higher order terms have been neglected because \( u'/U \) was small for the case at hand.

The probe registered an apparent unsteady pressure \( P_m(t) \) which was periodic but rather erratic in waveform (as distinct from the velocity waveform). On the assumption of quasi-steady flow, the pressure disturbance should be one-dimensional within the nozzle (i.e. constant pressure on planes perpendicular to the nozzle axis). Thus it was assumed that a pinhole microphone flush mounted at the inner surface of the nozzle would respond to the "true" pressure \( P_t(t) \). The sensitivity of this reference microphone was matched to that of the pressure probe. An analogue system was provided for subtracting the electrical outputs of the two devices. By this means, signals corresponding to \( P_m, P_t \) and \( P_m - P_t \) were obtained as depicted in Figure 17. Inserting the appropriate r.m.s. quantities into equation 18 gave \( A \approx .006 \). This value is somewhat larger than the steady flow result \( A < .001 \) which was achieved by tailoring the probe shape. However the electronic noise levels and signals arising from real turbulence in the nozzle were of the same order of magnitude as the voltage corresponding to \( P_m - P_t \). Thus the .006 value is probably overestimated. It was concluded that pressure errors arising from unsteady axial flow could be neglected relative to cross-flow errors.

With the probe subjected to the pulsating flow the outputs of the velocity sensors were also monitored. Signals of about 0.2 millivolt were noted, reflecting a small sensitivity to the unsteady axial velocity. In turbulence however, the velocity sensors typically generate 5 - 10 millivolts in response to cross-velocity fluctuations. The spurious u-component response was thus shown to be negligible.

10. EXPERIMENTS WITH THE ROTATING INCLINED NOZZLE

The rotating inclined nozzle flow provided one of the most useful methods for investigating the cross-flow error. The basic configuration is illustrated in Figure 18a. The nozzle consists of a journal which is caused to rotate in a bearing housing. A cylindrical passageway is bored through the journal, its axis being inclined to an angle \( \theta_0 \) to the axis of rotation. When air-flow is passed through the nozzle it follows essentially the inclination of the passageway. Rotation causes this inclined flow to "swirl", the velocity vector \( \vec{U}_0 \) tracing out an imaginary cone of apex angle \( 2 \theta_0 \).* If a probe is inserted into the nozzle it will be subjected to sinusoidal components of cross-velocity and pressure. For example, Figure 18b shows typical velocity and pressure signals which were obtained with the p-v-w probe.

The actual nozzle apparatus is depicted in Figure 19. To minimize friction the journal was pneumatically supported by an air-bearing. Rotation

* This quasi-steady interpretation is only valid if the period of rotation is long compared with the time taken for a fluid particle to pass through the nozzle.
was provided with an air-driven turbine, frequencies up to 150 Hz being attainable. Because of its lack of axi-symmetry the nozzle was dynamically unbalanced which led to vibration problems at the higher frequencies. An adequate reduction in the unbalanced mass was achieved by making the nozzle in two parts; a thin outer cylindrical shell (steel), and a wooden insert through which the nozzle passage was bored. Nozzles with inclination angles of 2.0, 5.0, and 7.5 degrees were made. The journal diameter was 1-3/4 inches and the nozzle diameter 1-1/16 inches. The passageway was located so that its axis crossed the axis of rotation at a point one-half inch upstream of the exit plane (total passage length was 2 1/2 inches).

Flow was provided by mounting the nozzle in a wind tunnel test section as shown in Figure 19a. It was necessary to verify that the flow within the nozzle was smooth (non-turbulent) and followed the inclined passageway in a parallel manner. An investigation with a hot-wire anemometer indicated that these requirements were met over the operational frequency range (5-150 Hz), provided that the dynamic head was not too small. (When q went below 75 millimeters of H2O the flow became somewhat erratic, probably as a result of separation in the nozzle).

Investigation of the cross-flow error - The probe was positioned in the nozzle at a small angle of attack \( \alpha \) to the axis of rotation (as shown in Figure 18a). The sensors were carefully centered on the point where the nozzle axis and the axis of rotation intersect. This being a point of symmetry as far as the pressure is concerned, the true unsteady pressure \( p(t) \) is necessarily zero there. In addition to the steady angle of attack \( \alpha \), the probe experienced a rotating angle of attack \( \theta o \). The rotation of \( \theta o \) at frequency \( f \) induced unsteady cross-velocity components \( v \) and \( w \). These are given as follows (when \( \alpha \) and \( \theta o \) are regarded as small angles):

\[
\begin{align*}
v(t) & \simeq U \sin \theta o \sin 2\pi ft \\
w(t) & \simeq U \sin \theta o \cos 2\pi ft
\end{align*}
\]  

(19a)

Adopting the convention that \( V(t) = \tilde{V} + v(t) \) is the component of cross-velocity in the plane of \( \alpha \), the steady part of \( V(t) \) is:

\[
\tilde{V} = U_o \sin \alpha
\]  

(19b)

\( \tilde{V}, v, \) and \( w \) are substituted into the equation for unsteady cross-flow error (equation 11). Noting from (19a) that \( v^2 + w^2 = U_o^2 \sin^2 \theta o = \tilde{v}^2 + \tilde{w}^2 \), recalling that \( p(t) \) must be zero, and putting \( \tilde{W} = 0 \), (11) reduces to:

\[
p_m(t) \simeq 2 B \rho \tilde{V} v(t)
\]  

\[\simeq 2 B \rho U_o^2 \sin \theta o \cos \theta o \sin 2\pi ft
\]  

(20)

In terms of the root-mean-square measured pressure \( p'_m \), equation (20) can be written as follows (when \( B \) is negative):
Thus, if equation (11) is a valid expression of the cross-flow error the rotating nozzle device should induce a sinusoidal error pressure $p_m(t)$, in phase with $v(t)^*$, and with amplitude proportional to $\sin\alpha$. This is precisely what was found. We refer again to Figure 18b. The top trace $p_m(t)$ was measured at $\alpha = -15^\circ$. It is in phase with the velocity component $v(t)$. The other velocity component $w(t)$ leads by $90^\circ$. It is noteworthy that these signals are unfiltered.

The error coefficient was determined by plotting $p_m'$ against $\sin\alpha$. Figure 20 shows data which was obtained for three different values of $q$. The linear relation (Equation 21) is seen to hold reasonably well for $\sin\alpha$ less than 0.3; at larger values the various small angle approximations fail and linearity is lost. Fortunately, in typical turbulence (e.g. $v'/\bar{U} \approx 0.1$, crest factor of 3) the instantaneous apparent angles of attack are generally less than $20^\circ$ so that the cross-flow error would be expected to fall within the linear range.

For small values of $q$ a curious curvature appeared in the "linear" portion of the plotted data (especially notable for $q = 75$ mm H$_2$O, Figure 20). It is conceivable that this non-linearity could arise from the expected tendency for $B$ to increase with cross-flow Reynolds number. However any such increase should be moderate for $R_v > 10$ (see Section 3). Based on the steady component of cross-velocity at $\sin\alpha = 0.1$, $R_v$ was of order 1000 and varied by less than a factor of 2 for the three values of $q$. It is unlikely that such a small change in $R_v$ would effect the cross-flow error to the noted degree. It seems more probable that the non-linear behaviour is a spurious effect originating with the nozzle. As was mentioned earlier, the nozzle flow took on an erratic nature when $q$ was less than 75 mm H$_2$O. The apparent pressure $p_m(t)$ may have been influenced adversely, in a manner which is not understood, during the onset of this flow irregularity. In any event, similar evaluations of the cross-flow error made in turbulent flow, at equally small values of $q$, did not show the same non-linearity with $\sin\alpha$. These findings will be reported in Section 11.

Three different nozzle angles ($\theta_0$) were used to obtain the data shown in Figure 21. The three sets of results have a more or less common slope (for the linear range). It is notable that the range of linearity increased as $\theta_0$ was reduced. This is reasonable because the break-down of linear behaviour should depend on the maximum net angle of attack $(\alpha + \theta_0)$, not on $\alpha$ alone.

On applying equation 21 to the data (Figure 20 or 21), a value $B \approx -0.46$ was found. This estimate is accurate to $\pm 10\%$.

Several other facets of the cross-flow error were explored with the rotating nozzle facility. For instance, the p-v-w probe was placed at a fixed angle of attack $\alpha$ and rotated on its axis to check that $B$ was more or less invariant with azimuthal orientation. In another experiment, $B$ was evaluated over the frequency range of the nozzle (5 Hz to 150 Hz) and was found to be

* Actually, the in-phase condition applies only when $v$ (or $\sin\alpha$) is negative, because $B$ was found to be negative. When $\sin\alpha$ is positive, $p_m$ and $v$ are $180^\circ$ out of phase.
independent of $f$. This observation lent support for our neglect of the frequency dependent inertial errors which are discussed in Appendix A.

Cancellation of the cross-flow error - The effectiveness of the error-compensating scheme was demonstrated with the rotating nozzle flow. Electrical signals equivalent to $p_m(t)$, $v(t)$ and $w(t)$ were fed into the analogue network (Figure 14). To $v(t)$ was added a d.c. voltage, representing the steady cross velocity $\bar{V} = US\sin\alpha$. The instantaneous cross-flow error $E[p_m(v + \bar{V})^2 + w^2]$ was computed; ideally the unsteady part of this should reduce to $2E[p_m \bar{V} v(t)]$, and when subtracted from $p_m(t)$, should leave a remainder of zero (see equation 20). The upper two traces in Figure 22 depict $p_m(t)$ and the cross-flow error. Upon subtraction the bottom signal was obtained, corresponding to the corrected pressure. This signal is virtually zero, thus establishing the effectiveness of the error cancellation system (at least for the rotating nozzle flow).

The pin-hole probe experiment - One slightly different experiment was carried out with the rotating nozzle flow. A probe was constructed with the pressure sensing slit replaced by a single pin-hole. The probe was inserted into the nozzle and aligned with the axis of rotation. Thus it experienced a constant angle of attack $\theta_0$ rotating with the nozzle frequency as depicted in Figure 23. At any instant, provided that the frequency is low, the situation might be compared with that of an infinite cylinder subjected to steady cross-velocity $V_n = U\sin\theta_0$. Recalling the discussion of Section 3, the circumferential pressure distribution could be expected to look somewhat as sketched in Figure 23. With the actual probe, if $V_n$ rotates slowly enough, a similar pressure distribution should be "sampled" by the pin-hole sensor. In other words, the output unsteady waveform should have the shape of $p(\theta)$, provided that the pin hole is much smaller than the probe diameter.

The actual measured waveform shown in Figure 23 looked surprisingly similar to the expected distribution. Very little change was noted over the range of nozzle frequency. Assuming that the peak pressure was the "stagnation" pressure $\frac{1}{2} \rho V_n^2$ (relative to the true static pressure), it was possible to evaluate the error coefficient $B$ as $\sim 0.4$. Although accuracy was lacking, the result agreed quite well with that obtained by the earlier measurements.

11. MEASUREMENTS IN TURBULENT CHANNEL FLOW

A number of experiments were undertaken in three representative classes of turbulent flow: channel flow turbulence, jet turbulence, and large scale grid turbulence. The channel flow experiments are the subject of the present section.

The measurements were made in a rectangular duct of 8" by 12" cross-section. The probe was positioned at the test section location as depicted in Figure 24. The cross-sectional dimensions were constant for a distance of 20 feet upstream, at which point a rather coarse grid (1\frac{1}{8} inch diameter bars) was inserted to help initiate turbulence. Flow was provided by mounting the duct in the closed return circuit of an acoustically quieted** wind tunnel, driven by a 10 H.P. centrifugal blower. The dynamic head ($q$) on the test section center-

* For this ideal to be achieved, the $v$ and $w$ sensors must be orthogonally displaced and the analogue solution of equation (11) must be precise (i.e, with no appreciable phase shift, non-linearity, etc).

** To minimize spurious pressure fluctuations of acoustical origin (for example, fan noise), the entire tunnel, including the test duct, was lined with acoustical tile of \(\frac{1}{2}\) inch thickness.
line was variable up to a maximum of about 100 mm of H₂O (\( \bar{U} \approx 135 \, f/s \)).

**Preliminary measurements** - Before undertaking the investigation of pressure probe response, some of the basic properties of the flow were determined. For the measurements to be described, \( q \) was 96 mm of H₂O at the centerline. The Reynolds number based on the equivalent circular pipe diameter (\( \sqrt{4 \, \frac{bh}{\pi}} \)) and the centerline velocity \( \bar{U}_o \) was 700,000.

Distributions of \( q \) and mean velocity \( \bar{U} \) across the test section were obtained by Pitot probe traverse. Horizontal profiles (variable \( y, z = 0 \)) are plotted in Figure 25, where the centerline values \( q_c \) and \( \bar{U}_c \) have been used to normalize the ordinate scale. The half-breadth of the duct \((b/2)\) is the normalizing factor for \( y \).

The \( v \) and \( w \) sensors and a true r.m.s. voltmeter were employed to measure distributions of turbulence level (\( v'/\bar{U}, w'/\bar{U} \)). The resulting profiles are shown in Figure 26. Corresponding measurements made with a crossed hot-wire probe were in close agreement. Turbulence level at the center-line was about .028.

Energy spectra of the turbulence components were determined over the frequency range 20 Hz to 10 KHz by passing the probe signals through a Bruel and Kjaer Type 2107 spectrum analyzer. A filter selectivity of 25 db (\( \sim 1/3 \) octave bandwidth) was utilized. The resulting spectra of \( u, v \) and \( w \) as measured at the duct centerline are presented in Figure 27. (A hot-wire probe was used to obtain the \( u \)-component data. The turbulence level \( u'/\bar{U} \) was .033). The spectra are plotted in the normalized form \( \Phi(S) \) versus \( S \), where \( S \) is the Strouhal frequency \( \frac{fd}{\bar{U}_0} \). \( \Phi(S) \) is defined such that its integral over \( S \) equals the square of the turbulence level:

\[
\int_0^\infty \Phi_v(S) \, dS = \frac{v^2}{\bar{U}_0^2}
\]

Figure 27 reveals that a substantial amount of the turbulence energy is concentrated at frequencies below \( S = 0.1 \). For example at \( S = 0.1 \), \( \Phi_u \) is 20 db down from its maximum value, while \( \Phi_v \) and \( \Phi_w \) are at least 10 db down. If the approximation \( \bar{U}_0 \approx \bar{U}_c \) is made, \( S \) can be regarded as the ratio of probe diameter to wavelength (e.g. see Section 4). It is apparent that \( \lambda/d \) is fairly large over a considerable portion of the spectrum.

Autocorrelations of the \( u, v, \) and \( w \) components were computed at the centerline location. For this purpose a time delay signal correlator was employed (P.A.R. Model 100). Figure 28 gives the correlation curves \( R(\hat{\tau}) \), plotted against the non-dimensional time delay parameter \( \hat{\tau} = \bar{U}_0 \tau/d \) and normalized to unity at \( \hat{\tau} = 0 \). The most notable feature is the much longer time scale of \( R_{uu} \) as compared with \( R_{vv} \). This is consistent with the higher concentration of spectral density at very low frequencies, for \( \Phi_u \) (recalling that \( \Phi(S) \) and \( R(\hat{\tau}) \) are essentially Fourier transforms of one another).

If the turbulence is "frozen" and convects with the mean velocity \( \bar{U} \), it is a consequence of the Taylor hypothesis that: [e.g. see Reference 2].

\[
\int_0^\infty R_{uu}(\hat{\tau}) \, d\hat{\tau} = L_x/d
\]
Here, \( L \) is the integral length scale in the streamwise (x) direction. Thus, by integrating \( \mathcal{R}_{uu} \), the estimate \( \frac{L_x}{d} \approx 24 \) was obtained. Laufer [24] has evaluated the length scales \( L_x, L_y, \) and \( L_z \) for turbulent flow in a two-dimensional channel. His results indicate that the cross-stream scales \( L_Y = 0.1 \) to \( 0.5 \) of \( L_x \), and that they are relatively invariant with position across the channel (except in the near-wall region). Furthermore, \( L_Y \) and \( L_z \) do not vary appreciably with Reynolds number in the core region \( (y < b/4) \). These findings are probably equally applicable to the present situation, allowing us to conclude that \( \frac{L_Y}{d}, \frac{L_z}{d} \approx 10 \).

Method for determination of \( B \) - A novel means was devised for evaluating the cross-flow error coefficient \( B \). The p-v-w sensors were located on the duct centerline \( (y = z = 0) \), the probe being oriented at a small angle of attack \( \alpha \) in the horizontal \( (x - y) \) plane, as shown in Figure 24a. With this arrangement the error equation (11) takes the form:

\[
p_m(t) = p_t(t) + B\rho (2v_0 \sin \alpha + v^2 + w^2 - \overline{v^2} - \overline{w^2})
\]

Here, as with the corresponding rotating nozzle experiment, we have set \( \bar{v} = U \sin \alpha \). Because the angle of attack in the vertical \( (x - z) \) plane is zero, \( \bar{w} = 0 \). Strictly speaking, in accordance with our original definition, \( v \) is a turbulence velocity component perpendicular to the probe axis. However in Figure 24a we have taken \( v \) to be the component in the y-direction. Actually, these two definitions are approximately equivalent when \( \alpha \) is small. On multiplying (22) through by \( v \) and time averaging, the correlation \( \overline{p_mv} \) is formed:

\[
\overline{p_m v} = \overline{p_t v} + B\rho (2\overline{v^2} v_0 \sin \alpha + \overline{v^3} + \overline{vw^2})
\]

At the centerline of the duct the true pressure-velocity correlation \( \overline{p_v} \) must be zero. (This results from symmetry considerations similar to those used to show that \( \overline{pv} = 0 \) for isotropic turbulence). Also, because the probability density functions of \( v \) and \( w \) are normally distributed on the duct centerline it can likewise be shown that the odd-moment correlations \( v^3 \) and \( vw^2 \) must be zero. With these simplifications, equation (23) reduces to:

\[
\overline{p_m v} (\alpha) = 2B\rho \overline{v^2} v_0 \sin \alpha
\]

Thus, if equation (11) is a valid expression of the cross-flow error, \( \overline{p_m v} (\alpha)/\overline{v^2} \) should vary linearly with \( \sin \alpha \). Both \( \overline{p_m v} \) and \( \overline{v^2} \) can be measured with the p-v-w probe, for a range of \( \alpha \), making the evaluation of \( B \) quite straightforward.

The P.A.R. Model 100 signal correlator was used to compute the correlation function \( p_m(\alpha,t) v(\alpha,t-\tau) \), for discrete values of \( \alpha \). A typical family of correlation curves is shown in Figure 29. If \( p_m \) and \( v \) could have been monitored at a common point, the curves would peak at \( \tau = 0 \), the peak value being the desired correlation \( p_m v(\alpha) \). In actual fact the maximum correlation occurs for a small positive \( \tau (\approx 0.14 \text{ milliseconds}) \), owing to the slight streamwise separation of the pressure and velocity sensors. As was noted in Section 5.4, the lagging of \( p_m(t) \) behind \( v(t) \) may be regarded as a phase shift...
which increases with frequency.

When \( \alpha \) is large enough we might expect the right hand side of equation (22) to be dominated by the term \( 2Bp_U v S \alpha \). In this event, \( P_m(t) \) should be roughly proportional to \( v(t) \). Inset in Figure 29 is a photograph of two simultaneous oscilloscope traces corresponding to \( P_m(t) \) and \( v(t) \), for \( \alpha = -20^\circ \). As expected, there is a pronounced degree of similarity between the two signals.

The correlations \( P_m v(\alpha) \) were normalized in the form \( \frac{P_m v}{\sqrt{2p_U^2 U}} \) and plotted against \( S \alpha \). Figure 30 gives the result for two values of \( q_0 \). The expected linear relationship (Equation 24) is seen to hold quite well for \( S \alpha \) less than 0.3. In fact the data bears a striking resemblance to results of the corresponding rotating inclined nozzle experiment (Figures 20 and 21). On applying equation (24) to the linear portion of the curve a value \( B \approx 0.31 \) was found. This is somewhat smaller in magnitude than the value \( B \approx 0.46 \) obtained from the rotating nozzle experiment, suggesting that finite scale effects probably play a more important role here. Roughly speaking, the value of \( B \) as determined by the present method can be regarded as a weighted average over a spectrum of values of \( B(d/\Lambda) \). The weighting factor is essentially the spectral density \( \Phi_v(S) \), where we use \( S \) and \( d/\Lambda \) interchangeably. Thus for very small \( d/\Lambda \), \( B(d/\Lambda) \) may be closer to -0.46 than -0.31. As \( d/\Lambda \) becomes large (~1 or higher), \( B \) will tend to zero or may even become positive (see Sections 2 and 3). In the present case, \( \Phi_v(S) \) is a decreasing function of \( S \) for \( S > .01 \) (Figure 27). Hence the effective (average) \( B \) is strongly weighted in favour of values associated with small \( d/\Lambda \).

Measurement of fluctuating pressure - The distribution of root-mean-square pressure \( P'_m \) was obtained by orienting the probe parallel to the mean flow and traversing it across the duct. The error compensating system was employed to evaluate corresponding profiles of the corrected pressure \( P'_c \) (via equation 11a), for two values of \( B \). In view of the findings described in the previous paragraph, it seemed most appropriate to use \( B \approx -0.31 \). However the rotating nozzle value, \( B \approx -0.46 \) was also tried. The results are given in Figure 31, where the pressures are normalized by the local value of \( q \). It is apparent that the correction did not modify the pressure levels very significantly in the core region. For the \( B \approx -0.46 \) case (where the error has probably been overcorrected), \( P'_c/q \) exceeds \( P'/q \) by only 10% at the centerline position. By comparison, \( Bp v_n^2/q \) amounts to about 30% of \( P'_m/q \). The effect of the correction is much more pronounced near the wall, because the turbulence levels are higher.

Some difficulty was encountered with noise propagating down the channel. This noise was of such low frequency that the acoustic lining was ineffective in attenuating it. Spectrum analysis revealed that the noise contributed significantly to \( P_m(t) \) at frequencies below 100 Hz. The spectrum of \( P_m(t) \) at the centerline is given by the lower curve in Figure 32. The non-dimensional spectral density \( \Phi_p(S) \) has been chosen such that its integral over Stouhal frequency \( S \) equals the square of the pressure level \( (p^2/q^2) \). Originally the undesirable acoustic noise appeared as a distinct "bump" on the spectrum, rising to a level some 8 db above the peak of the present curve in the frequency range below 50 Hz. By making minor modifications to the tunnel it was possible to reduce the noise to the point where most of the "bump" was eliminated from the spectrum. Nevertheless, the extent to which the remaining noise contributes to \( P'_m \) can only be guessed at. If the shaded region were due
to noise, for instance, $p_m'$ would be overestimated by about 20%.

The upper curve in Figure 32 is the spectrum of the corrected pressure as measured with the error compensating system, using $B \approx -0.46$. For most of the frequency range below 1000 Hz the correction appears to be a constant 20% (.8 db) of the spectrum level. Above 1000 Hz the two spectra begin to deviate. This behaviour probably arises from a combination of two effects. First, the use of $B \approx -0.46$ results in a degree of overcorrection which becomes more and more pronounced as $S$ approaches unity. Second, the phase shift between pressure and velocity sensors becomes large enough at frequencies above 1000 Hz to seriously affect the accuracy of error correction. Nevertheless, at 1000 Hz the pressure spectrum is more than 20 db down from its peak value; thus the correction to $p_m'$ is not likely to be influenced by minor spectral discrepancies at higher frequencies.

12. MEASUREMENTS IN A ROUND TURBULENT JET

A round four inch diameter nozzle was coupled to the wind tunnel (via a suitable contraction section) to produce a free turbulent jet. For the measurements to be described, the dynamic head at the exit plane was about 140 mm H2O, corresponding to an exit velocity $U_o$ of 160 feet per second. Investigations were carried out at the downstream stations $x/D = 4$ and $x/D = 6$, $D$ being the nozzle diameter.

Preliminary measurements - The radial distribution of local dynamic head at the two downstream stations is given in Figure 33a. The corresponding profiles of turbulence level are shown in Figure 33b. For the jet flow, $v$ denotes the radial component of unsteady velocity and $w$ denotes the tangential component. The turbulence signals became noticeably intermittent in the vicinity of $r/D = 0.7$; for larger values of $r$ the accuracy of the $v$ and $w$ sensors is open to question. (The small angle approximations on which the aerofoil probe principle is based break down when the turbulence level is too high).

Figure 34 gives the spectral density of $v$ and $w$ for the positions $x/D = 4$, $r/D = 0.5$ and $x/D = 6$, $r/D = 0$. As in the previous section, the data has been plotted in the form $\Phi(S)$ versus $S$. The abscissa can again be regarded as a measure of the ratio $d/\lambda$. The condition $\lambda >> d$ appears to be met over a substantial portion of the spectrum in each case.

For $x/D < 6$ the integral length scale is given approximately by $L \approx 1.3x$, being relatively independent of $r$ [25]. Thus the ratio $L_x/d$ is about 17.5 at $x/D = 4$ and 26 at $x/D = 6$ (although there is evidence that the latter value drops by about 1/3, near the centerline [26]). The transverse integral scale $L_r$ is about 1/3 of $L_x$.

Evaluation of $B$ - Equation (24) was arrived at as a consequence of symmetry conditions ($p_{mv} = p_{m'v} = 0$) on the centerline of the turbulent channel flow. These same conditions apply on the centerline of a jet, downstream of the potential core ($x/D > 4$). Thus equation (24) is appropriate for a pressure probe inclined at an angle $\alpha$ on the axis of the jet. From measurements of $p_{mv}$ and $v^2$, for a range of $\alpha$, the error coefficient $B$ can be determined. Such an evaluation was carried out at $x/D = 6$, $r/D = 0$ - Figure 35 gives the resulting plot of $p_{mv}$ versus $\sin \alpha$. The slope of the linear part of the curve yielded a value $B \approx -0.35$. The onset of non-linearity occurs for
slightly smaller values of Sinα than in the case of the channel flow experiment (Figure 30). As was mentioned in Section 10, the breakdown of linearity depends not on α alone, but on the net instantaneous angle of attack (which may be roughly expressed as α + v'/U, in turbulence). Thus, because v'/U was considerably larger for the present experiment, we would expect a reduced linear range (in α).

Approximate estimates of B can be obtained for off-axis positions as well. Even though p_v, v^2, and vw^2 are not zero in general, they are essentially independent of α. Thus, according to equation (23), p_v should still vary as Sinα. A measurement was made in the mixing layer (x/D = 4, r/D = 1/2) which yielded more or less the same value for B as was found on the centerline (B ~ - 0.35).

Pressure measurements - Radial distributions of the uncorrected pressure p^m and the corresponding corrected pressure p_c were evaluated at the two stations x/D = 4 and x/D = 6. In computing p_c(t) with the error-compensating system, B was taken as - 0.35. For convenience, the probe was aligned geometrically with the jet axis. Because the radial component of steady velocity V is non-zero in a spreading jet (except as r = 0), a term 2BpVv was thereby unaccounted for in the error compensation. Nevertheless for x/D values between 3 and 6, measurements of V indicate that it is generally very small, reaching a maximum of less than 0.15U_0 at r = D/2 [15]. By contrast, v' and w' were of order 0.15U_0, at r = D/2. Thus the neglect of 2BpVv relative to the compensated error Bp (v^2 + w^2) was felt to be justified.

Profiles of the root-mean-square pressures p_m' and p_c', normalized by the nozzle dynamic head q_o, are given in Figures 36 and 37 for x/d = 6 and x/d = 4 respectively. The inset photographs show typical waveforms of p(t) and the error function. An interesting feature of the data is the apparent change from a negative correction (p_c' < p_m') for small values of r/D, to a positive correction for r/D > 0.5 or so. This is distinct from the result obtained in the turbulent channel flow, where p_c' was consistently larger than p_m'. A possible explanation lies in the relationship between p_m^2 and p_t^2. On rearranging, squaring, and time averaging equation (11a), one obtains:

\[ \overline{p_t^2} \simeq \overline{p_m^2} - 2Bp \overline{p_m v_n^2} + (F - 1)(Bp \overline{v_n^2})^2 \]  (25)

Here, F is the ratio v_n^4/\overline{v_n^2}, usually termed the flatness factor. Equation (25) can be re-written as:

\[ \overline{p_t^2} - \overline{p_m^2} \simeq (1 - \sigma)(F - 1)(Bp \overline{v_n^2})^2 \]  (26)

\[ \sigma = \frac{2\overline{p_m v_n^2}}{(F-1)Bp \overline{v_n^2}} \]

The difference between \overline{p_t^2} and \overline{p_m^2} is seen to be dependent on the quantities \sigma and F, both of which will vary with position in the flow.
The flatness factor $F$ is inherently positive and generally takes values larger than unity*. However, $\sigma$ may be positive or negative, depending on the sign of the correlation $\rho_m \rho_n$. The mean-square error may likewise be positive or negative. In the special case where $\sigma = +1$, $P_t^2$ will equal $P_m^2$ (in other words, an uncorrected probe will measure the correct root-mean-square pressure).

With the aid of the signal correlator, the quantities $\sigma$ and $F$ were evaluated for the two positions $x/D = 4, r/D = \frac{1}{2}$ and $x/D = 6, r/D = 0$. (Typical correlation functions, normalized by $P_m'$ and $\nu_0^2$, are shown in Figure 38). Thus $P_t'$ was calculated from $P_m'$, using equation 26. The results are summarized below:

<table>
<thead>
<tr>
<th></th>
<th>$x/D = 4, r/D = \frac{1}{2}$</th>
<th>$x/D = 6, r/D = 0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$F$</td>
<td>2.0</td>
<td>2.0</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>+ 0.95</td>
<td>+ 2.74</td>
</tr>
<tr>
<td>$P_m'/q_o$</td>
<td>0.0510</td>
<td>0.0302</td>
</tr>
<tr>
<td>$P_t'/q_o$</td>
<td>0.0516</td>
<td>0.0267</td>
</tr>
<tr>
<td>$P_c'/q_o$</td>
<td>0.0530</td>
<td>0.0272</td>
</tr>
</tbody>
</table>

The calculated values of $P_t'/q_o$ are included on Figures 36 and 37. For the $x/D = 6$ case, there is good agreement with the corresponding corrected measurement ($P_c'/q_o$), thereby providing a check on the internal consistency of the error compensating system. For the $x/D = 4$ case, the values of $P_m'/q_o$ and $P_t'/q_o$ are more or less equal. This typifies a situation where the correction is very small ($\sigma \approx 1$). The fact that $P_c'/q_o$ is slightly larger probably reflects a tendency for the error-compensating system to overcorrect when the turbulence level is too high (i.e., such that the linear approximations on which the system is based degenerate). Thus, for radial positions greater than $r/D \approx \frac{1}{2}$, the reliability of the $P_c'/q_o$ data cannot be attested to.

The spectra of $P_m(t)$ and $P_t(t)$ are compared in Figure 39, for $x/D = 6, r/D = 0$. As with the equivalent channel flow result, the correction is a more or less constant fraction of $\Phi_p$ for frequencies below 1000 Hz. The unusual behaviour at higher frequencies is probably due to phase-shift and over-correction effects as described in Section 11.

* For a situation where the velocity components $v$ and $w$ have a Gaussian probability distribution, and where the turbulence is isotropic, $F$ equals 3. In other circumstances however, $F$ may be substantially different than 3. For example, in the shear layer of a jet it is known that the flatness factor of an individual velocity component ($u, v$ or $w$) may vary from 2 to 9 or more, depending on the radial position [27].

27
13. MEASUREMENTS IN LARGE-SCALE GRID TURBULENCE

The final experiment was carried out in a field of large-scale, homogeneous, and approximately isotropic turbulence. A facility for generating such turbulence has been developed at UTIAS in conjunction with another research program; a detailed description of the facility and the resulting flow field has been given by Surry [28]. Basically, the turbulence is produced in the diffuser section (5° expansion) of the UTIAS low speed wind tunnel by placing a coarse grid at the upstream end. For the present measurements the grid consisted of crossed rectangular bars (1.75" x 2.75"), spaced on twelve inch centers. The experiment was conducted at a point ten feet downstream of the grid, where the cross-sectional dimensions are about 5' x 5'. The p-v-w probe was mounted near the center of the section of a 1½" round bar spanning the tunnel. (To minimize spurious unsteady pressures arising from flow interaction with the bar, the sensors were positioned about 10 inches ahead of, and 5 inches laterally displaced from the bar).

Hot wire measurements made at the test position by Surry [28] gave \( u'/\bar{U} \approx 0.10 \), the \( v \) and \( w \) turbulence levels being 10 to 20 per cent smaller. The longitudinal and transverse integral scales were evaluated as 4\( \frac{1}{2} \) inches and 2\( \frac{1}{2} \) inches respectively (\( L_x/d \approx 38, L_y/d \approx 19 \)). Flow properties were found to be reasonably uniform over the test section.

The present experiment involved making a single point measurement of the quantities \( v', w', v_n^2, v_n^4, p_{m}', p_{m}v_n^2, \) and \( p_{c}' \). Because of the inconvenient probe mounting arrangement a re-evaluation of \( B \) was not attempted. In the computation of \( p_{c}' \), \( B \) was taken as -0.35. The following results were obtained for a dynamic head of 26.4 mm H₂O at the test section (\( \bar{U} \approx 70 \) f/s):

\[
\frac{v'}{\bar{U}} = 0.11 \quad \frac{w'}{\bar{U}} = 0.10
\]

\[
\frac{v_n^4}{v_n^2} \equiv \bar{r} = 1.80
\]

\[
\frac{p_{m}v_n^2}{p_{m}'v_n^2} = +0.16
\]

\[
\frac{p_{m}'}{q} \approx 0.0187 \quad \frac{p_{c}'}{q} \approx 0.0207
\]

With \( \sigma = +0.65 \), equation 26 was used to calculate \( p_{t}' \) from \( p_{m}' \), giving:

\[
\frac{p_{t}'}{q} \approx 0.0204
\]
As in the earlier experiments, the difference between \( p_m' \) and \( p_c' \) is small, amounting to about 10%. The calculated value of \( p_t' \) is quite close to \( p_c' \), indicating that the error-compensating system performed in a proper manner.

Energy spectra of the horizontal velocity component \( v \), and of the pressures \( p_m \) and \( p_c \) are given in Figure 40. All three spectra exhibit a prominent peak at a Strouhal frequency of about 0.02. This corresponds with the shedding frequency of the 1/2 inch probe supporting bar; the peaks probably arise from a combination of aerodynamic and vibration effects induced by the wake of the bar. Another discrepancy in the form of a slight "bump" appears at the low frequency end of the pressure spectra. As with the duct flow experiment, this reflects the presence of spurious acoustic noise in the wind tunnel. In view of these spectral irregularities the r.m.s. measurements are probably overestimated. In fact, if our flow has been exactly isotropic and homogeneous we would have expected a lower pressure level than we observed. (Batchelor's prediction \( p_t' = 0.34 (\rho u^2) \) gives \( p_t'/q \approx 0.012 \) [14]).

Because the mean square error \( (p_m - p_t')^2 \) is of the same order of magnitude as \( p_t'^2 \) in isotropic turbulence [12], it has generally been concluded that an uncorrected pressure probe will give a very erroneous reading. This is supported in the present experiment, where \( (p_m - p_t') \) was almost as large as \( p_m' \). The correction modified the waveform of \( p_m(t) \) considerably - note the simultaneous oscilloscope traces inset on Figure 40. It is perhaps fortuitous that in spite of their instantaneous disparity, the r.m.s. values of \( p_m \) and \( p_c \) (or the calculated \( p_t' \)) were more or less equal.

14. CONCLUDING DISCUSSION

The present study has been handicapped by an absence of standards with which to make comparison; there are no documented measurements of the "true" fluctuating pressure \( p_t \), free of probe error. For this reason some of the experiments, particularly those involving turbulence *, are inconclusive. Nevertheless, when taken collectively the experimental results lend self-consistent support to our interpretation of the problem.

The empirical error equations, especially the cross-flow error equation (11), were found to be appropriate in a number of circumstances. Measured values of the error coefficient \( B \) are summarized in Table I, along with results from other sources. Our values lie between -0.3 and -0.5, suggesting that the large scale condition was met fairly well. A gradual approach to the large scale limit, with increasing \( \lambda/d \), is indicated. (In the manner of Bradshaw and Goodman [10], values of \( \lambda/d \) cited in Table I are based on the Strouhal frequency \( S \) at which the spectral density \( \Phi_v(S) \) per octave is a maximum - i.e., \( \Phi_v(S) \sim 1/S \)).

The effectiveness of the error compensating system was particularly evident in the case of the rotating nozzle experiment. The measurements in

* To predict root-mean-square pressures theoretically for realistic cases of turbulent flow is a formidable task. Idealized cases such as those treated by Batchelor [14] and Kraichman [29] are difficult to simulate in the laboratory.
turbulent flows revealed that the correction to root-mean-square pressure level was small, generally amounting to less than 20%. It was shown that the correction can be positive or negative; in fact in specific instances an uncorrected probe will indicate the correct r.m.s. pressure. The waveforms of corrected and uncorrected pressure may differ significantly in some cases, even when the r.m.s. levels are nearly the same. This suggests that for measurements where instantaneous values are important (such as cross-correlations) the necessity for error compensation is more stringent.

The error-compensation system would benefit from a few refinements. The error cancellation is inaccurate at high frequencies because of excessive phase shift arising from spatial separation of the velocity and pressure sensors; a partial solution could be effected by delaying the velocity signals (electronically) by an appropriate time increment Δt. The error compensation breaks down in regions of high turbulence level (>30% or so), because of non-linearity in the pressure error and in the response of the velocity sensors; performance might be improved by tailoring the analogue circuitry to take account of these non-linearities. As the frequency parameter S approaches unity the large scale assumptions are no longer valid and we cannot in principle correct for the error; to avoid spurious overcorrection, the correction term kρνh^2 - νh^2 should be suppressed at the higher frequencies, possibly by using an appropriately tailored low-pass filter. With these additional features (or even in its present form) the error compensating probe could be a useful aid to basic research in unsteady flows.

A large effort has been expended in developing the correction scheme. As a partial result of this effort we now know that in many practical circumstances, where one is only interested in r.m.s. pressures, an uncorrected probe may be sufficiently accurate. This was not known a priori; the present research helped to establish such a conclusion.
<table>
<thead>
<tr>
<th>Reference</th>
<th>Author(s)</th>
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<tbody>
<tr>
<td>1.</td>
<td>Ginoux, J. J. (Editor)</td>
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<td>Hinze, J. Q.</td>
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<td>Corcos, G. M.</td>
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<td>4.</td>
<td>Laufer, J.</td>
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<td>5.</td>
<td>Sami, S.</td>
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<td>6.</td>
<td>Goldstein, S.</td>
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<td>7.</td>
<td>Fage, A.</td>
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<td>8.</td>
<td>Barat, M.</td>
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<td>9.</td>
<td>Toomre, A.</td>
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<td>10.</td>
<td>Bradshaw, P.</td>
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<td>11.</td>
<td>Kistler, A.</td>
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<td>12.</td>
<td>Strasburg, M.</td>
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<td>13.</td>
<td>Kobashi, Y.</td>
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<td>14.</td>
<td>Batchelor, G. K.</td>
</tr>
<tr>
<td>15.</td>
<td>Sami, S.</td>
</tr>
</tbody>
</table>

**REFERENCES**


APPENDIX A: PRESSURE ERROR ARISING FROM UNSTEADY CROSS-FLOW

Some insight into the problem of a probe in an unsteady cross-flow can be obtained by use of an idealized flow model that ignores the viscosity. Consider a cylindrical pressure probe of diameter \( d \), subjected to a uniform unsteady cross-velocity \( V_n(t) \). Any coupling effect of the axial velocity \( U(t) \) is neglected. Assuming the flow to be irrotational, the appropriate potential function is: [23]

\[ \phi = V_n \left( r + \frac{d^2}{4r} \right) \cos \theta \]

The unsteady form of the Bernoulli equation gives:

\[ P(r, \theta, t) - P_t(t) = \frac{1}{2} \rho \left( V_n^2 - V_r^2 - V_\theta^2 \right) + \rho \frac{\partial \phi}{\partial t} \]

where,

\[ V_r = -\frac{\partial \phi}{\partial r}, \quad V_\theta = -\frac{1}{r} \frac{\partial \phi}{\partial \theta} \]

\( P_t(t) \) is the pressure which would have occurred at \( r = 0 \) in the absence of the probe (i.e., the 'true' pressure). At the surface of the probe \( (r = d/2) \), the pressure equation becomes:

\[ P(\theta, t) - P_t(t) = \frac{1}{2} \rho V_n^2 \left( 1 - 4 \sin^2 \theta \right) + \rho \dot{V}_n d \cos \theta \]

The first term on the right hand side is recognized as the pressure distribution for steady potential flow. The second term arises from unsteadiness. For a pressure probe which registers the exact circumferential average of \( P(\theta, t) \), (e.g., by means of a circumferential slit) the error will be:

\[ P_m(t) - P_t(t) = -\frac{1}{2} \rho V_n^2 \quad (i.e., B = -\frac{1}{2}) \]

In this ideal situation the part of the pressure distribution associated with the acceleration term \( \dot{V}_n \) does not contribute to the error.

A real probe will not take an exact average over \( P(\theta) \); therefore an additional error proportional to \( \dot{V}_n \) may arise:

\[ P_m(t) - P_t(t) = -\frac{1}{2} \rho V_n^2 + K \rho \dot{V}_n d \]

Roughly speaking, the coefficient \( K (\leq 1) \) represents the fractional inaccuracy of the average over \( P(\theta) \). If we regard \( V_n \) as sinusoidal, the \( \dot{V}_n \) error becomes increasingly important with frequency \( (\dot{V}_n \sim \omega V_n) \). Nevertheless, for a 1/8 inch diameter probe with an averaging inaccuracy of 5% and \( V_n \approx 10 \) feet per second, \( K \rho \dot{V}_n d \) is less than 5% of the \( V_n^2 \) error at 100 Hz.
Ideally, the axes of the $v$ and $w$ sensors should be orthogonally displaced; in practice the angular separation may differ from $90^\circ$ by some small angle $\epsilon$. To evaluate $\epsilon$, the p-$v$-$w$ probe was positioned in the rotating inclined nozzle flow with $\alpha = 0$ (Figure 18a). Thus the probe was subjected to constant angle of attack $\theta_0$ (and cross-velocity $U_0 \sin \theta_0$), rotating with frequency $f$:

$$
\begin{align*}
& \text{The } v \text{-channel sensor generated an output voltage:} \\
& e_v = S_v q \sin \theta_0 \cos 2\pi ft \\
& \text{The output of the } w \text{-channel sensor lagged } e_v \text{ by an angle } \pi/2 + \epsilon: \\
& e_w = S_w q \sin \theta_0 \cos (2 \pi ft - \pi/2 - \epsilon)
\end{align*}
$$

In principle, $\epsilon$ could be deduced by comparing simultaneous traces of $e_v$ and $e_w$ on an oscilloscope screen. However, a correlation technique is more accurate. For convenience we will set $S_v q \sin \theta_0 = S_w q \sin \theta_0 = \sqrt{2}$.

Then the autocorrelation of $e_v$ is:

$$
\frac{e_v(t)e_v(t + \tau)}{= \cos 2\pi ft}
$$

The cross-correlation of $e_v$ and $e_w$ is:

$$
\frac{e_v(t)e_w(t + \tau)}{= \sin (2 \pi f \tau - \epsilon)}
$$

These correlation functions have the following form:
Curves of the above type were obtained with the aid of the P.A.R. Model 100 Signal Correlator. It was found for the final p-v-w probe that the v and w sensors were orthogonal to within 5°.

A simple extension of this method was used to determine the electronic phase shift between the p, v and w signals for frequencies in the range of the rotating nozzle.
<table>
<thead>
<tr>
<th>SOURCE</th>
<th>B</th>
<th>( \lambda/d )</th>
<th>( L_x/d )</th>
</tr>
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<tbody>
<tr>
<td>Goldstein prediction [6]</td>
<td>+ ( \frac{1}{4} )</td>
<td>( \ll 1 )</td>
<td>---</td>
</tr>
<tr>
<td>Fage experiment [7]</td>
<td>+ ( \frac{1}{4} )</td>
<td>---</td>
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</tr>
<tr>
<td>Toomre prediction [9]</td>
<td>+ ( \frac{1}{2} )</td>
<td>( \ll 1 )</td>
<td>( \ll 1 )</td>
</tr>
<tr>
<td></td>
<td>- ( \frac{1}{2} )</td>
<td>( \gg 1 )</td>
<td>( \gg 1 )</td>
</tr>
<tr>
<td>Bradshaw and Goodman [10]</td>
<td>- 0.07</td>
<td>( \gg 1 )</td>
<td>---</td>
</tr>
<tr>
<td>Cross flow about a circular cylinder (Section 3)</td>
<td>- ( \frac{1}{4} )</td>
<td>( \infty )</td>
<td>---</td>
</tr>
<tr>
<td>Inclined static probe (Section 7)</td>
<td>- ( \frac{1}{4} ) → - ( \frac{1}{2} )</td>
<td>( \infty )</td>
<td>---</td>
</tr>
<tr>
<td>Rotating nozzle experiment (Section 10)</td>
<td>- 0.46</td>
<td>( \frac{U_0}{f d} &gt; 150 )</td>
<td>---</td>
</tr>
<tr>
<td>Channel flow experiment (Section 11)</td>
<td>- 0.31</td>
<td>40</td>
<td>24</td>
</tr>
<tr>
<td>Turbulent jet experiment (Section 12)</td>
<td>- 0.35</td>
<td>75</td>
<td>26</td>
</tr>
</tbody>
</table>

TABLE I. VALUES OF CROSS-FLOW ERROR COEFFICIENT B
$P(t) = \bar{P} + p(t)$

**FIGURE 1** CONVENTIONAL STATIC PRESSURE PROBE IN UNSTEADY FLOW
a) AXISYMMETRIC FLOW

At the static pressure holes:

\[ P_m - P_t \approx A \rho U^2 \]

b) CROSS FLOW

For \( 10 < R_v < 1000 \)

\[ P_m - P_t \approx \frac{1}{2 \pi} \int_0^{2\pi} (P_e - P_\infty) \, d\Theta \approx B \rho V_n^2 \]

\[ B \approx -\frac{1}{4} \text{ for } 10 < R_v < 1000 \]

FIGURE 2  FUNDAMENTAL ERROR MECHANISMS
c) INCLINED PROBE IN STEADY FLOW

\[ U_0 \]
\[ U \]
\[ V_n \]
\[ \alpha \]
\[ P_m(\alpha) - P_t \approx B \varepsilon (U_0 \sin \alpha)^2 \]
\[ \sim -1/4 \]

---

d) GOLDSTEIN STAGNATION MECHANISM

PROBE X-SECTION

'EDDY' (\( \lambda \ll d \))

In any one eddy velocity normal to the probe stagnates, giving:

\[ P(\Theta, t) - P_t \approx B \varepsilon v^2(\Theta, t) \]

On averaging over \( \Theta \) and time:

\[ P_m - P_t \approx B \rho v_n^2 \]
\[ \sim +1/4 \]

FIGURE 3 ERROR MECHANISMS (continued)
P (t) at Circumferential Slit Communicates to Microphone

Balsa Cross-Force Sensor (Cross-Force is ~ to vn)

\[ v_n^2 = v^2 + w^2 \]

Two Component Force Transducer (gives \( e_v, e_w \sim v, w \))

Miniature Condenser Microphone (gives \( e_p \))

\[ p_c(t) \approx K_1 \left[ e_p - K_2(e_v^2 + e_w^2) \right] \]

FIGURE 4 ERROR-COMPENSATING PRESSURE PROBE
Diaphragm - .0001" Stainless Steel / Pretensioned and Mounted with EC-1099 Cement

Conducting Epoxy

Backplate

'Microdot' Lepra/Con Miniature Connector - Series 142 (Tightening nut removed)

Diaphragm Mounting Collar

FIGURE 5 MINIATURE CONDENSER MICROPHONE
FIGURE 6 ASSEMBLY DETAILS OF THE p-v-w PROBE
FIGURE 7  SOURCE FOLLOWER PREAMPLIFIER FOR PIEZOELECTRIC TRANSDUCERS
a) The p-v-w Probe

b) Associated Devices

FIGURE 8 CLOSE-UP DETAIL OF THE PROBES
a) Probe-Preamplifier Combinations

b) Probe Mounted on Traversing Mechanism

FIGURE 9 MOUNTING ARRANGEMENTS
FIGURE 10  SENSITIVITY OF THE PRESSURE SENSOR
(Variation with Temperature)
FIGURE 11 - FREQUENCY RESPONSE OF PRESSURE SENSOR
FIGURE 12 - CALIBRATION OF VELOCITY SENSORS WITH ROTATING NOZZLE FLOW
FIGURE 13 COMPARISON OF TURBULENCE SPECTRA AS MEASURED WITH LIFTING BODY PROBE AND WITH X-WIRE PROBE IN A ROUND JET \((x/D = 4.5, r/D = 0.5)\)
FIGURE 14 ANALOGUE SYSTEM FOR CANCELLATION OF CROSS-FLOW ERROR
FIGURE 15 PRESSURE PROBE ERRORS IN STEADY FLOW
FIGURE 16 PRESSURE FIELD OF A ROTATING ELLIPTIC CYLINDER. COMPARISON OF EXPERIMENT AND CALCULATION
Rotating Butterfly Valve Modulates Flow

4" Diameter Nozzle

Reference Microphone Measures $p_t(t)$

Probe Measures $p_m(t)$

$U(t)$

$\bar{U} + u(t)$

$\text{VELOCITY}$

$\text{PRESSURE}$

$\left( p_m - p_t \right) \approx 2A \bar{U} u$ gives $A \approx 0.06$

FIGURE 17 AXIAL VELOCITY FLUCTUATION EXPERIMENT
Figure 18a  Schematic of Rotating Inclined Nozzle Flow

Figure 18b  Pressure and Velocity Signals of $p$-$v$-$w$ Probe in Nozzle Flow. Pressure Signal is Spurious Effect of Cross-Flow.
Figure 20: Determination of error coefficient $B$ with rotating inclined nozzle.

The graph depicts the relationship between $\frac{p'}{m}$ and $q \sin \theta$. The equations for different flow rates $q$ are:

- $q = 240 \text{ mm H}_2\text{O}$
- $q = 150 \text{ mm H}_2\text{O}$
- $q = 75 \text{ mm H}_2\text{O}$

The equation for $B$ is:

$$B = \frac{-p'}{2\sqrt{2}q \sin \theta \sin \alpha} = -0.46$$

Additionally, $R_v \approx 85 \sqrt{q}$. 

The graph shows the data points for each flow rate, with lines indicating the trend.
FIGURE 21  VARIATION OF CROSS FLOW ERROR WITH NOZZLE ANGLE

\[ \frac{p'_m}{q \sin \theta_o} \]

- $\theta_o = 7.5^\circ$
- $\theta_o = 5.0^\circ$
- $\theta_o = 2.0^\circ$

$B \approx -0.46$
FIGURE 22 CANCELLATION OF CROSS-FLOW ERROR IN ROTATING INCLINED NOZZLE FLOW
FIGURE 24a - TERMINOLOGY FOR TURBULENT CHANNEL FLOW EXPERIMENTS

FIGURE 24b - PROBE MOUNTED AT TEST SECTION IN 8" X 12" CHANNEL
FIGURE 25  PROFILES OF DYNAMIC HEAD $q$ AND MEAN VELOCITY $\bar{U}$ IN TURBULENT CHANNEL FLOW
FIGURE 26  PROFILES OF $v'$ AND $w'$ IN TURBULENT CHANNEL FLOW
FIGURE 27 SPECTRAL DENSITY OF TURBULENCE COMPONENTS u, v, AND w ON CHANNEL CENTERLINE
FIGURE 28 AUTOCORRELATION OF TURBULENCE COMPONENTS u, v, AND w ON DUCT CENTERLINE
FIGURE 29 CORRELATION OF $p_m$ AND $v$ IN TURBULENT CHANNEL FLOW (PROBE PLACED AT ANGLE OF ATTACK $\alpha$)
\[
\frac{p_m v}{(1/2) e v^2 \bar{U}_o}
\]

- \(q = 94 \text{ mm H}_2\text{O}\)
- \(q = 59 \text{ mm H}_2\text{O}\)

**FIGURE 30** DETERMINATION OF ERROR COEFFICIENT B IN TURBULENT CHANNEL FLOW

\[
B \approx \frac{p_m v}{2 e v^2 \bar{U}_o \sin \alpha} = -0.31
\]
FIGURE 31  PROFILES OF ROOT-MEAN-SQUARE PRESSURE $p'$ IN TURBULENT CHANNEL FLOW
FIGURE 32 SPECTRAL DENSITY OF FLUCTUATING PRESSURE AT DUCT CENTERLINE
FIGURE 33a RADIAL PROFILES OF DYNAMIC HEAD IN TURBULENT JET

FIGURE 33b RADIAL PROFILES OF TURBULENCE LEVEL
FIGURE 34 SPECTRAL DENSITY OF v AND w IN TURBULENT JET
\[ B \approx \frac{\frac{p_m v}{(1/2) \rho v^2 U}}{4 \rho v^2 U \sin \alpha} \approx -0.35 \]

**FIGURE 35** EVALUATION OF B ON CENTERLINE OF TURBULENT JET (\(q_o = 145\text{mm H}_2\text{O}, \ x/D = 6\))
FIGURE 36 PROFILES OF ROOT-MEAN-SQUARE PRESSURE $p'$ IN 4" ROUND JET ($x/D = 6$)
\[ p_m' / q_o \]
\[ p_c' / q_o \] (B \approx -0.35)
\[ p_t' / q_o \] (Calculated)

FIGURE 37 PROFILES OF ROOT-MEAN-SQUARE PRESSURE IN 4" ROUND JET (x/D = 4)
FIGURE 38 CORRELATIONS OF $p$ AND $v_n^2$ IN TURBULENT JET ($x/D = 6$, $r/D = 0$)
FIGURE 39 SPECTRAL DENSITY OF FLUCTUATING PRESSURE IN TURBULENT JET \((x/D = 6, r/D = 1/2)\)
Shedding frequency of probe support bar

\[ S = \frac{fd}{U} \]

**FIGURE 40 MEASUREMENTS IN GRID TURBULENCE**
ON THE RESPONSE OF PRESSURE MEASURING INSTRUMENTATION IN UNSTEADY FLOW
(An Investigation of Errors Induced by Probe-Flow Interaction)

Thomas E. Siddon

ABSTRACT
If a fast responding pressure probe of classical "static" probe geometry is placed in an unsteady flow it will not register the true instantaneous pressure interaction between the probe and the unsteady velocity field gives rise to an error between the measured pressure $P_m(t)$ and the true pressure $P_t(t)$. This error is not always small; it can in some circumstances be larger than the difference between $P_t(t)$ and the ambient pressure. A major objective of the present work has been to explore the possibility of correcting $P_m(t)$ instantaneously, through use of an error-compensating probe. A probe configuration was adopted for which the error reduces to the simple form $P_m(t) \approx B_pV_n^2(t)$, where $V_n(t)$ is the instantaneous velocity normal to the probe axis. The probe was devised to measure simultaneously the unsteady components of $P_m(t)$ and $V_n(t)$ over a wide frequency range. Output signals were processed by analogue means to correct out the error term providing an improved estimate of the true unsteady pressure. A series of experiments were conducted in a number of contrived unsteady flows. Major experiments involved a periodic "rotating inclined nozzle flow" and some typical turbulent flows. The error-compensating scheme was found to be effective, particularly for the specialized rotating inclined nozzle flow. However the measurements in turbulent flows revealed that the correction to root-mean-square pressure fluctuation level was small, generally amounting to less than 20%.
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On the Response of Pressure Measuring Instrumentation in Unsteady Flow

T. E. Siddon

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II. UTIAS Report No. 136

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