INFLUENCE OF STRUCTURAL FLEXIBILITY
ON A SINGLE-AXIS LINEAR ATTITUDE CONTROLLER

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by

Tarek Abdel-Rahman
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Abstract

The effect of structural flexibility on a linear attitude control system employing a reaction wheel is investigated. The parameters of a compensator in the form of (proportional + integral + derivative) feedback are chosen in an optimal way: to minimize the real part of right-most root of the system characteristic equation. This is done assuming the satellite to be rigid. Then the effects of flexibility are investigated through stability diagrams drawn by inspection of the real parts of the roots of the characteristic equation, showing stable and unstable regions. Root locus plots for different parameter values are also used to illustrate the effect of flexibility on the roots. Increasing the structural damping and designing the appendages as rigidly as possible are important means of insuring stability of the control system. Otherwise, the controller should be designed with structural flexibility explicitly included.

The flexibility investigations have been carried out for two general types of flexible vibration, namely 'rod-like' behaviour and 'membrane-like' behaviour. A method to counteract the destabilizing effect that may arise due to structural flexibility is also given.
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NOTATION

I  Moment of inertia of spacecraft about the axis (flexible + rigid)
    = \( I_f + I_r \)

\( I_f \)  The inertia associated with the flexible part of the structure about the control axis

\( I_r \)  The inertia associated with the rigid portion of the structure about the control axis

k  Proportional gain factor of the controller

\( k_D \)  Rate gain factor of the controller

\( k_I \)  Integrator gain factor of the controller

\( k_{II} \)  Double integrator gain factor of the controller

\( k_m \)  Motor gain factor

\( K_n \)  Appendage modal gains

\( Z_n \)  Modal damping ratios

\( \beta = \frac{I_f}{I} \)  Relative inertia of the appendages

\( \omega_m = \frac{1}{\tau_m} (\tau_m \text{ is the motor time constant}) \tau_m = 38.40 \text{ secs for CTS} \)

\( \omega_s = \frac{1}{\tau_s} (\tau_s \text{ is the sensor time constant}) \)

\( \Omega_n \)  Natural frequencies of appendage vibration
1. INTRODUCTION

The dynamical interaction between spacecraft attitude control systems and flexible appendages has received considerable attention because many appendages cannot reasonably be designed with sufficient rigidity to justify the optimistic assumption that the dynamic response to attitude control devices can be uncoupled from vehicle vibrations. When coupling is present, the result may vary from slight performance degradation to instability.

This study was motivated by the Communication Technology Satellite (CTS). Reference 1 gives an overall description of CTS. An artist's impression of the satellite appears in Fig. 1, and a conceptual diagram of the subsystems in Fig. 2. CTS is a high-power communication satellite and is scheduled for launch early in 1976. The power needs will be supplied by two arrays of solar cells which generate about 1.2 kilowatts. These arrays make CTS interesting from a control standpoint, since they contribute the predominant structural flexibility.

The pitch axis (parallel to the orbit normal) of the satellite coincides with the axis of the array. It is evident that twisting motions of the array will excite, and will be excited by, the pitch motion of the satellite and this fact has ramifications for the pitch attitude control system (Ref. 2). Assuming the ideal symmetry which is suggested by the overall spacecraft design (Fig. 1), pitch/twist oscillations are not coupled to any of the other modes of oscillation. The principal dynamical element in the control system is a nongimballed momentum reaction wheel whose axis coincides with the pitch axis. Thus roll and yaw are mutually coupled but not (ideally) to pitch. The twisting modes of the symmetrically deployed array may be further sub-classified into symmetric ones (in which both panels rotate equally one way, and the main body rotates the other) and skew-symmetric ones (in which the panels rotate equally in opposite directions, and the main body does not rotate). Only the former are directly of interest since only they involve the pitch attitude motion of the body, where the sensors and antenna are located. References 2-5 have treated this satellite in a relatively straightforward manner by modelling the array blanket as a simple membrane. A distinction is made between 'constrained' and 'unconstrained' modes of flexible vibration. The former concern the array only (with a fixed base); the latter one for the whole spacecraft. The constrained modes will be used here since they are inherently characterizations of the appendages alone (Ref. 2), and they are thus easier to deal with when we come to choosing 'general' appendage models. Motivated by the pitch attitude control system of CTS then, a general class of single-axis attitude control systems is shown in Fig. 3. The flexible appendage dynamics appears on the right-hand side of the figure.

2. CONTROL SYSTEM COMPONENTS

2.1 Reaction Wheel

The control actuator used in the control system shown in Fig. 3 is a reaction wheel. This device produces torques on the satellite by accelerating an inertia wheel (using the principal of conservation of angular momentum) and can be represented for most purposes as a linear system (Refs. 6,7). Its basic function is to counteract environmental disturbing torques which are cyclical.
FIG. 1: Artist's Impression of the Communications Technology Satellite (Courtesy Spar Aerospace Products Ltd.)
FIG. 2: Communications Technology Satellite as Deployed in Synchronous Orbit (from Ref. 1)
FIG. 3 BLOCK DIAGRAM REPRESENTATION OF SINGLE-AXIS CONTROL SYSTEM OF A FLEXIBLE VEHICLE EMPLOYING PROPORTIONAL + INTEGRAL + DERIVATIVE CONTROLLER
(i.e., have zero average). The action of secular components of disturbing torque leads to a buildup in wheel speed until a saturation is met. It then becomes necessary to 'unload' or 'dump' the excess angular momentum to prevent the speeds of the reaction wheel exceeding its limit. The most obvious means for unloading is through the use of a mass expulsion system. This system would be energized when the speed of the wheel exceeds a preset threshold. There are two principal methods of mechanizing such an operation:

(i) The appropriate jet is commanded ON and remains ON until the wheel speed falls below a lower threshold. The reduction of wheel speed is accomplished through the normal action of the wheel control loop, which treats the jet torque as a disturbing torque.

(ii) A change of control mode is initiated, the new mode controlling the attitude of the spacecraft purely by use of jet torque. Simultaneously, maximum deceleration torque is applied to the wheel. When the wheel speed falls below an appropriate threshold, the normal control mode is re-enabled.

Reaction wheel unloading may also be accomplished through the use of magnetic torquing to save fuel (Ref. 7). Recently proposals have been made (Ref. 9) for unloading systems which make use of knowledge or estimation of disturbing torque time variations in order to operate both economically and with minimum frequency.

2.2 Compensator

The output voltage of the compensator is assumed to be the combination of three signals. The first is proportional to the error and, remembering that this voltage goes into driving the wheel, its basic function is to improve the transient response. The second is proportional to the rate of change of error; its merit lies in that it is a measure of how fast the error signal is changing and thus corresponds to anticipation. The third is the integral type, and is provided to get a constant steady-state attitude error in response to a constant torque.

A rationale must now be found for the selection of the compensator parameters $k$, $k_T$ and $k_D$ (see Fig. 3). The criterion of minimizing the rightmost root of the characteristic equation of the control system will be applied (Ref. 10). The control system characteristics will then be obtained by using a Nyquist plot. The steady-state error in response to a constant disturbing torque is mentioned. Another possible choice for the compensator will also be discussed.

2.3 An Optimum Design Criterion

The criterion chosen for optimization is the transient response, a measure of which is the real part of the right-most system root. The time constant $T$ may be defined as follows: the distance from the imaginary axis to the right-most root is $a = 1/T$. The optimization process then reduces to choosing the system gains so as to minimize $T$ (i.e., to maximize $a$).
The attitude control system block diagram shown in Fig. 3 can be reduced to that shown in Fig. 4, assuming the satellite to be rigid. The relationship between the old and new constants is:

\[ k'_I = \frac{k \cdot k \cdot k}{s \cdot m} \]

\[ k'_s = \frac{k \cdot k}{s \cdot m} \]

\[ k'_D = k \cdot k \cdot k / I \]

(2.1)

\[ \Sigma = \omega_s + \omega_m \]

\[ \Pi = \omega_s \omega_m \]

The characteristic equation of this system can be written as follows:

\[ s^4 + \Sigma s^3 + (\Pi + k'_D)s^2 + k's + k'_I = 0 \]

(2.2)

Since \( k', k'_D \) and \( k'_I \) are free parameters, then to maximize the magnitude of the real part of the right-most root, the four roots have to be all located together at a distance

\[ a_I = \Sigma / 4 \]

(2.3)

from the imaginary axis. This occurs when the controller gains are selected as follows:

\[ k'_D = 6a^2_I - \Pi \]

\[ k' = 4a^3_I \]

(2.4)

\[ k'_I = a^4_I \]

For the values (typical of CTS)

\[ \omega_s = 0.5 \text{ sec}^{-1}; \quad \omega_m = 1/3840 \text{ sec}^{-1} \]

(2.5)
$$\frac{k'_I + k'_S + k'_D S^2}{S^2 (S^2 + \sum S + \pi)}$$
in particular, (2.3) and (2.4) give

\[ a_I = 0.125 \]
\[ k'_D = 9.37 \times 10^{-2} \]  
(2.6)
\[ k' = 7.82 \times 10^{-3} \]
\[ k'_I = 2.45 \times 10^{-4} \]

2.4 Control System Characteristics

Figure 5 shows a Nyquist plot for the open loop control function shown in Fig. 4,

\[ G(s) = \frac{k'_I + k's + k'_D s^2}{s^2(s^2 + 2s + \Pi)} \]
(2.7)

The characteristics of the control system for the gains given in (2.6) can be summarized as follows:

- Gain margin = 0.199 = 14.0 db
- Phase margin = 0.761 rad = 43.6°
- Phase crossover frequency \( (\omega_c) \) = 0.557 sec\(^{-1}\)
- Gain crossover frequency \( (\omega_g) \) = 0.181 sec\(^{-1}\)  
(2.8)
- Band width = 0.316 sec\(^{-1}\)
- Resonance peak = 1.61
- Resonant frequency = 0.113 sec\(^{-1}\)

Applying the final value theorem, the steady-state error in response to constant disturbance torque \( T_d \) is:

\[ \theta_{ss}(t) = \lim_{s \to 0} \frac{T_d}{s} \frac{k}{k \frac{1}{s} \frac{s^2}{s^2 + k_k m (k + k_k + k_D s^2)}} = \frac{T_d}{s} \]
(2.9)
2.5 Another Possible Compensator

This section is concerned with the discussion of the possibility of adding a double integrator to the controller. Thus the controller block diagram will be as shown in Fig. 6.

The characteristic equation will be, in this case,

\[ s^2(s^2 + \Sigma s + \Pi) + k'_D s^3 + k'_I s^2 + k'_I s + k'_{II} = 0 \]  \hspace{1cm} (2.10)

where,

\[ k'_{II} = \frac{k k_{m_{II}}}{m_{II}} \]  \hspace{1cm} (2.11)

Applying the same definition of optimality given in Section 2.3, the five roots of the polynomial can again be located altogether by choosing \( a_{II} = 1/T \) as follows:

\[ a_{II} = \frac{\Sigma}{5} \]

\[ k'_D = 10a_{II}^2 - \Pi \]

\[ k' = 10a_{II}^3 \]  \hspace{1cm} (2.12)

\[ k'_I = 5a_{II}^4 \]

\[ k'_{II} = a_{II}^5 \]

For the values of \( \Sigma \) and \( \Pi \) given in (2.6), the numerical values for (2.12) are, approximately,

\[ a_{II} = 0.100 \]

\[ k'_D = 0.100 \]

\[ k' = 10^{-2} \]  \hspace{1cm} (2.13)

\[ k'_I = 5.01 \times 10^{-4} \]

\[ k'_{II} = 10^{-5} \]
The steady-state attitude error in response to constant disturbance torque is zero. Thus the addition of a double integrator improves the steady-state response but deteriorates the transient response, since $a_{II} < a_i$.

3. INFLUENCE OF STRUCTURAL FLEXIBILITY ON ATTITUDE STABILITY

The effects of structural flexibility are now investigated by considering the interaction of the control system with a general flexible appendage. In an effort to limit the number of appendage parameters to as few as possible while at the same time retaining the greatest possible generality, the suggestions made in Ref. 2 will be adopted.

3.1 General Appendage Model

Firstly, concerning the natural frequencies of vibration ($\Omega_1, \Omega_2, \ldots$) it is noted (Ref. 2) that generally

$$n \leq \frac{\Omega}{\Omega_n} \leq n^2$$

(3.1)

The bounds in (3.1) are not always satisfied for all structures. However they seem reasonable general estimates of the magnitude of $\Omega_n$. Since $\Omega_n = n\Omega_1$ is satisfied by strings, membranes, etc., the lower bound in (3.1) will be called 'membrane-like' behaviour. Similarly, since $\Omega_n = n^2\Omega_1$ for a rod (at least for large $n$), the upper bound in (3.1) will be termed 'rod-like' behaviour.

Reference 2 goes on to suggest that the modal gains ($K_1, K_2, \ldots$; see Fig. 3) might reasonably be expected to be estimated by

$$K_n = \left( \frac{I_r}{I} \right) \frac{P_M}{n^2}$$

(3.2)

for membrane-like behaviour, where

$$P_M = \left\{ \sum_{n=1}^{\infty} \left( \frac{1}{n^2} \right) \right\}^{-1} = \frac{6}{\pi^2}$$

(3.3)

while, for rod-like behaviour

$$K_n = \left( \frac{I_r}{I} \right) \frac{P_R}{n^4}$$

(3.4)

where

$$P_R = \left\{ \sum_{n=1}^{\infty} \left( \frac{1}{n^4} \right) \right\}^{-1} = \frac{90}{\pi^4}$$

(3.5)
FIG. 6  PROPORTIONAL + DERIVATIVE + INTEGRAL + DOUBLE INTEGRAL
CONTROLLER BLOCK DIAGRAM

FIG. 7  THE REDUCED SINGLE-AXIS ATTITUDE CONTROL SYSTEM BLOCK DIAGRAM FOR
THE FLEXIBLE SATELLITE
In (3.2) and (3.4), \( I_f \) is the inertia of the flexible portions of the spacecraft, and \( I \) is the inertia of the entire spacecraft. Clearly,

\[
0 \leq \frac{I_f}{I} \leq 1
\]  

(3.6)

Finally, as to damping, in the absence of any better assumption, it will be assumed that

\[
Z_1 = Z_2 = Z_3 = \ldots = Z
\]  

(3.7)

The advantage of the model described in the preceding paragraph is that only three appendage parameters emerge (\( \Omega_1, I_f/I \) and \( Z \)) plus a decision on rod-like vs membrane-like behaviour.

3.2 Formulation of the Characteristic Polynomial

The block diagram shown in Fig. 3 can be reduced, for \( T_d(s) = 0 \), to that shown in Fig. 7, where

\[
G_f(s) = \frac{G_{fn}(s)}{G_{fd}(s)} \sum_{n=1}^{\infty} \left\{ \prod_{i=1, i \neq n}^{\infty} \left( s^2 + 2Z_i \Omega_i s + \Omega_i^2 \right) \right\}
\]

(3.8)

An approximate value for \( G_f(s) \) can be obtained by employing only \( N \) modes of constrained flexible vibrations. Thus

\[
G_f(s) = \frac{G_{fn}(s)}{G_{fd}(s)} \approx \sum_{n=1}^{N} \left\{ \prod_{i=1, i \neq n}^{N} \left( s^2 + 2Z_i \Omega_i s + \Omega_i^2 \right) \right\}
\]

(3.9)

3.3 Stability Diagrams

Two groups of stability diagrams have been generated both for rod-like behaviour and for membrane-like behaviour. In the first group, two flexible modes have been incorporated. These stability diagrams, prepared using \( I_f/I \) (= \( \beta \)) and \( \Omega_i/\omega_c \) as parameters, have been generated by a direct investigation of the real parts of the roots of the characteristic polynomial. The subroutine DPRBM was used which employs Bairstow's method for finding the roots of a polynomial.

Figures 8-10 show stability diagrams for \( Z = 0.001, .01 \) and \( 0.05 \), respectively, and rod-like behaviour. Figures 11-13 show stability diagrams for \( Z = 0.001, 0.01 \) and \( 0.05 \) and membrane-like behaviour.
In the second group four flexible modes have been incorporated. The resulting stability diagrams coincide exactly with those shown in Figs. 8-13. This implies that the third and fourth modes, at least under the current assumptions, do not contribute materially to the stability question. Figures 8-10 were also checked using the Routh-Hurwitz criteria. The resulting figures coincided exactly, as expected.

A second check, using Nyquist's stability criterion, was then carried out for \( Z = 0.001, 0.01, 0.05 \) and for both rod-like and membrane-like behaviour. The results for \( \beta = 0.1 \) are shown in Tables 1 and 2. It is clear that these results agree with those obtained with \( \beta = 0.1 \) in the stability diagrams. Of course they give much more information, gain and phase margin, for the stable cases. Figures 14 and 15 show sketches of the open-loop transfer function for a typical stable case, and a typical unstable case, respectively.

From the stability diagrams it is clear that the unstable region shrinks as the structural damping ratio \( Z \) increases until the unstable region disappears completely for a sufficiently high \( Z \) (perhaps as low as 0.05). Secondly, for a given structural damping ratio, the unstable region becomes more wider as \( \beta \) increases (i.e., as a greater fraction of the satellite is flexible).

Of interest is the possibility of a stable region in which \( \Omega_1 < \omega_c \), even for low \( Z \). In this region the satellite is so flexible that the controller can, in effect, control in a quasi-steady manner. The torques from the appendages on the main body change only very slowly.

3.4 Root Locus Plots

Deeper insight into the effect of flexibility on the transient response is now obtained by studying the effect on the roots of the characteristic equation of varying \( \Omega_1 \). The latter indicates the degree of flexibility of the appendages. Root loci for both rod-like and membrane-like behaviour employing both two and four modes are shown in Figs. 16-51. These figures show that the roots obtained by using two constrained modes agree very well with the corresponding roots obtained by using four modes. The largest deviation (still small) occurs in the higher frequency roots for membrane-like behaviour, very low structural damping ratio, and \( \beta = 0.9 \) (as in Figs. 38-39). The instability for a certain interval of \( \Omega_1 \) is seen to occur due to the crossing by three roots of the imaginary axis (in the case of four modes), and by two roots when two modes are used.

In each root locus the effect of flexibility (obtained by varying \( \Omega_1 \)) on the roots is displayed for specific values of the parameters \( Z \) and \( \beta \). The values of the structural damping ratio parameter \( Z \) have been chosen, as before, to be 0.001, 0.01, 0.05 to represent very low, low and moderate structural damping, respectively. The values for the parameter \( \beta \) (namely 0.1, 0.5 and 0.9) have been chosen to represent small, moderate and large appendages, respectively. A discussion of the significant trends in these root loci follows.

3.4.1 Effect of Structural Flexibility on Root Loci

For given \((Z, \beta)\), \( \Omega_1 \) may be considered as a measure of flexibility; the rigidity increases as \( \Omega_1 \) increases. If the appendages are designed with sufficient
FIG. 14 NYQUIST PLOT

\[ \text{(N=4, } \beta=0.1, \ zeta=0.010, \ \omega_1=0.04428, \ \text{'ROD-LIKE BEHAVIOUR'}) \]

FIG. 15 NYQUIST PLOT

\[ \text{(N=4, } \beta=0.1, \ zeta=0.001, \ \omega_1=0.04428, \ \text{'ROD-LIKE BEHAVIOUR'}) \]
Fig. 16 Root Locus (N=2; Beta=0.1; Zeta=0.001 'Rod-Like Behaviour')
(+ = Band Width)

Fig. 17 Root Locus (N=4; Beta=0.1; Zeta=0.001 'Rod-Like Behaviour')
(+ = Band Width)
FIG. 18 ROOT LOCUS ( N=2 , BETA=0.5 , ZETA=0.001 'ROD-LIKE BEHAVIOUR' )

( + = BAND WIDTH )

FIG. 19 ROOT LOCUS ( N=4 , BETA=0.5 , ZETA=0.001 'ROD-LIKE BEHAVIOUR' )

( + = BAND WIDTH )
FIG. 20 ROOT LOCUS (N=2; BETA=0.1; ZETA=0.001 'ROD-LIKE BEHAVIOUR')

( + = BAND WIDTH )

FIG. 21 ROOT LOCUS (N=4; BETA=0.9; ZETA=0.001 'ROD-LIKE BEHAVIOUR')

( + = BAND WIDTH )
FIG. 22 ROOT LOCUS (N=2, BETA=0.1, ZETA=0.010 'ROD-LIKE BEHAVIOUR')
(+ = BAND WIDTH)

FIG. 23 ROOT LOCUS (N=4, BETA=0.1, ZETA=0.010 'ROD-LIKE BEHAVIOUR')
(+ = BAND WIDTH)
FIG. 24 ROOT LOCUS (N=2, BETA=0.5, ZETA=0.010 'ROD-LIKE BEHAVIOUR')

(+= BAND WIDTH)

FIG. 25 ROOT LOCUS (N=4, BETA=0.5, ZETA=0.010 'ROD-LIKE BEHAVIOUR')

(+= BAND WIDTH)
FIG. 26 ROOT LOCUS \( N=2, \beta =0.9, \zeta =0.010 \), "ROD-LIKE BEHAVIOUR"

( + = BAND WIDTH )

FIG. 27 ROOT LOCUS \( N=4, \beta =0.9, \zeta =0.010 \), "ROD-LIKE BEHAVIOUR"

( + = BAND WIDTH )
FIG. 28 ROOT LOCUS ( N=2 , \( \beta = 0.1 \) , \( \zeta = 0.050 \) 'ROD-LIKE BEHAVIOUR')

( + = BAND WIDTH )

FIG. 29 ROOT LOCUS ( N=4 , \( \beta = 0.1 \) , \( \zeta = 0.050 \) 'ROD-LIKE BEHAVIOUR')

( + = BAND WIDTH )
FIG. 30 ROOT LOCUS (N=2, BETA=0.5, ZETA=0.050, 'ROD-LIKE BEHAVIOUR')

(+= BAND WIDTH)

FIG. 31 ROOT LOCUS (N=4, BETA=0.5, ZETA=0.050, 'ROD-LIKE BEHAVIOUR')

(+= BAND WIDTH)
FIG. 32 ROOT LOCUS (N=2, BETA=0.9, ZETA=0.050 'ROD-LIKE BEHAVIOUR')
(+ = BAND WIDTH)

FIG. 33 ROOT LOCUS (N=4, BETA=0.9, ZETA=0.050 'ROD-LIKE BEHAVIOUR')
(+ = BAND WIDTH)
FIG. 34 ROOT LOCUS ( N=2, BETA=0.1, ZETA=0.001 'MEMBRANE-LIKE BEHAVIOUR')

( + = BAND WIDTH )

FIG. 35 ROOT LOCUS ( N=4, BETA=0.1, ZETA=0.001 'MEMBRANE-LIKE BEHAVIOUR')

( + = BAND WIDTH )
FIG. 36 ROOT LOCUS ( N=2 • BETA=0.5 • ZETA=0.001 'MEMBRANE-LIKE BEHAVIOUR' )
( + = BAND WIDTH )

FIG. 37 ROOT LOCUS ( N=4 • BETA=0.5 • ZETA=0.001 'MEMBRANE-LIKE BEHAVIOUR' )
( + = BAND WIDTH )
FIG 38  ROOT LOCUS  ( N=2 ; BETA=0.9 ; ZETA=0.001 ; 'MEMBRANE-LIKE BEHAVIOUR')
( + = BAND WIDTH )

FIG 39  ROOT LOCUS  ( N=4 ; BETA=0.9 ; ZETA=0.001 ; 'MEMBRANE-LIKE BEHAVIOUR')
( + = BAND WIDTH )
FIG. 40 ROOT LOCUS (N=2; BETA=0.1; ZETA=0.010 'MEMBRANE-LIKE BEHAVIOUR')

( + = BAND WIDTH )

FIG. 41 ROOT LOCUS (N=4; BETA=0.1; ZETA=0.010 'MEMBRANE-LIKE BEHAVIOUR')

( + = BAND WIDTH )

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FIG. 42 ROOT LOCUS ( $N=2 \cdot \beta=0.5 \cdot \zeta=0.010$ 'MEMBRANE-LIKE BEHAVIOUR')

( + = BAND WIDTH )

FIG. 43 ROOT LOCUS ( $N=4 \cdot \beta=0.5 \cdot \zeta=0.010$ 'MEMBRANE-LIKE BEHAVIOUR')

( + = BAND WIDTH )
FIG. 44 ROOT LOCUS (N=2, BETA=0.19, ZETA=0.010 'MEMBRANE-LIKE BEHAVIOUR')
(+ = BAND WIDTH)

FIG. 45 ROOT LOCUS (N=4, BETA=0.19, ZETA=0.010 'MEMBRANE-LIKE BEHAVIOUR')
(+ = BAND WIDTH)
FIG. 46 ROOT LOCUS ( \( N=2 \), \( \beta=0.1 \), \( \zeta=0.050 \) 'MEMBRANE-LIKE BEHAVIOUR')
(+ = BAND WIDTH)

FIG. 47 ROOT LOCUS ( \( N=4 \), \( \beta=0.1 \), \( \zeta=0.050 \) 'MEMBRANE-LIKE BEHAVIOUR')
(+ = BAND WIDTH)
FIG. 48 ROOT LOCUS \( n=2 \), \( \beta=0.5 \), \( \zeta=0.050 \) 'MEMBRANE-LIKE BEHAVIOUR'

(• = BAND WIDTH)

FIG. 49 ROOT LOCUS \( n=4 \), \( \beta=0.5 \), \( \zeta=0.050 \) 'MEMBRANE-LIKE BEHAVIOUR'

(• = BAND WIDTH)
FIG. 50 ROOT LOCUS (N=2, BETA=0.9, ZETA=0.05, 'MEMBRANE-LIKE BEHAVIOUR')

(\(+=\) BAND WIDTH)

FIG. 51 ROOT LOCUS (N=4, BETA=0.9, ZETA=0.05, 'MEMBRANE-LIKE BEHAVIOUR')

(\(+=\) BAND WIDTH)
rigidity, the roots associated with the attitude control system (the lower frequency roots in this case) deviate very little from those obtained in Section 2.4 for the rigid satellite, and the roots associated with the flexible appendages deviate very little from the appendage modal characteristics. However, this deviation increases as $\Omega_1$ decreases.

An improvement in the damping ratio of the roots associated with flexibility can be noted when $\Omega_1$ becomes within the bandwidth of the control system. In other words, the attitude control helps to damp the appendage oscillations. As the appendages become even more flexible, the roots travel nearer to the imaginary axis, bringing about a marked decrease in stability margin, or even instability. Also it can be noticed that 'rod-like behaviour' tends to affect the control system roots more than 'membrane-like behaviour' does.

3.4.2 Effect of Structural Damping on Root Loci

Instability tends to occur for low structural damping ratio. For all the cases studied, increasing the structural damping reduces the instability region. In fact, for sufficiently high damping ratio ($Z = 0.05$), stability is assured.

3.4.3 Effect of Appendage Inertia on Root Loci

As the appendage inertia increases (relatively), the region of instability is increased. Also, for low $Z$, greater deterioration in the right-most root occurs.

4. ATTITUDE CONTROL SYSTEM IMPROVEMENT

Section 3 shows a great need to improve control system parameters in order to stabilize a spacecraft with a large, flexible appendage whose damping is low.

A straightforward approach is to vary the gains of the compensator through multiplication of each of the compensator parameters ($k_D^1$, $k'_1$, $k_I^1$) by an "improving gain factor", $g$. Thus the control system block diagram will be as shown in Fig. 52. Values of $k_D^1$, $k'_1$ and $k_I^1$ remain those calculated in Eq. (2.4) for the rigid satellite. An unstable case has been chosen to illustrate the effect on the roots of varying $g$. Figure 53 shows that as $g$ increases it stabilizes the (flexibility) roots that caused instability. The notation (+) shown in Fig. 53 determines the roots at $g = 4.4$. The Nyquist stability criterion requires $g > 4.319$ which agrees with the root locus shown. Further increases in $g$ above 4.319 improve the minimum damping ratio for the system until, when $g = 100$, the minimum damping ratio reaches the value of the structural damping ratio. No further improvement in the minimum damping ratio could be obtained by further increasing $g$. This further increase in $g$ is seen to deteriorate the damping ratio of the higher frequency root. It should also be noted that as $g$ grows greater than unity, the steady-state attitude error in response to a constant disturbing torque is reduced. It will be recalled that it is proportional to the inverse of the integrator gain.
FIG. 52 SINGLE-AXIS ATTITUDE CONTROL SYSTEM WITH IMPROVING GAIN FACTOR $g$
5. CONCLUDING REMARKS

Great care has to be taken when designing the control system of a flexible satellite, since even for a control system that is optimally designed considering the satellite to be rigid, instability may arise due to structural flexibility. This is especially true for very low structural damping ratios, and large, very flexible appendages.

Stability margins are improved markedly as the lowest natural frequency of the appendage goes higher than $\omega_c$ (the frequency at which the imaginary part of the open loop of the rigid satellite equals zero). It has also been shown that two modes of vibration approximate the appendage reasonably well, especially for 'rod-like' appendages.

Increasing the compensator gains improves the transient performance and also reduces the steady-state error but other control design considerations beyond the scope of this work would limit these gains.
REFERENCES

1. Franklin, C. A.
   Davison, E. H.

2. P. C. Hughes
   S. C. Garg

3. Hughes, P. C.

4. Hughes, P. C.

5. Hughes, P. C.


7. Frick, Martin A.
   "Attitude Stabilization of Satellite in Orbit", "Rotational Dynamics", AGARD-LS-45-71

8. Auclair, G. F.

9. Johnson, C. D.
    Skelton, R. E.

10. Hughes, P. C.
    "Optimized Reaction Wheel Attitude Control Systems", CRC Memo 6665-10-1 (NSTL), August 10, 1970.
### Table 1 - Rod-Like Behaviour Gain Margin (GM) and Phase Margin (PM) for $I_r/I = 0.1$

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### Table 2 - Membrane-Like Behaviour Gain Margin (GM) and Phase Margin (PM) for $I_r/I = 0.1$

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INFLUENCE OF STRUCTURAL FLEXIBILITY ON A SINGLE-AXIS LINEAR ATTITUDE CONTROLLER

Abdel-Rahman, Tarek

The effect of structural flexibility on a linear attitude control system employing a reaction wheel is investigated. The parameters of a compensator in the form of \((\text{proportional} + \text{integral} + \text{derivative})\) feedback are chosen in an optimal way: to minimize the real part of right-most root of the system characteristic equation. This is done assuming the satellite to be rigid. Then the effects of flexibility are investigated through stability diagrams drawn by inspection of the real parts of the roots of the characteristic equation, showing stable and unstable regions. Root locus plots for different parameter values are also used to illustrate the effect of flexibility on the roots.

Increasing the structural damping and designing the appendages as rigidly as possible are important means of insuring stability of the control system. Otherwise, the controller should be designed with structural flexibility explicitly included.

The flexibility investigations have been carried out for two general types of flexible vibration, namely 'rod-like' behaviour and 'membrane-like' behaviour. A method to counteract the destabilizing effect that may arise due to structural flexibility is also given.

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