A TURBULENCE PROBE UTILIZING AERODYNAMIC LIFT

by

T. E. Siddon

JUNE 1965

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A TURBULENCE PROBE UTILIZING AERODYNAMIC LIFT*

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* Patent applied for.
ACKNOWLEDGEMENT

The concept of the possibility of an aerofoil instrument came from Dr. H. S. Ribner who has collaborated with me in the development of a practical turbulence probe. His keen interest and support throughout the undertaking is gratefully acknowledged.

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SUMMARY

The concept of a new-type of anemometer for measuring the cross-stream or "$v"-component of turbulent velocity is presented.

Basic theory concerning the response characteristics of the so-called "aerofoil probe" is discussed.

Steps in the development of a satisfactory prototype of the "aerofoil probe" are outlined. Various problems concerning resonant frequencies, accelerometric effects, low-frequency response fall-off, and the effect of finite aerofoil size were encountered, and these are described in detail.

A method of dynamic calibration, employing a square-wave turbulence simulator is outlined.

Various experimental data, accumulated from measurements in a low-speed turbulent air jet are presented. These include velocity measurements, frequency spectra, autocorrelations, and two point space-time correlations. Comparisons with hot-wire data are made wherever possible.

Conclusions are drawn as to feasibility of using the aerofoil probe in preference to hot-wire techniques for turbulence measurements. Limitations of both techniques are compared.
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### NOTATION

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<tr>
<td>$C_E$</td>
<td>effective electrical capacitance of transducer</td>
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<td>$C_L$</td>
<td>lift coefficient</td>
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<td>$C_{L_C}$</td>
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<td>$C_m$</td>
<td>mechanical compliance of transducer</td>
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<td>$C(k)$</td>
<td>Theodorsen's function</td>
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<td>$D$</td>
<td>nozzle diameter of turbulent jet</td>
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<td>$D_{db}$</td>
<td>decibels $(20 \log \frac{e_o}{e_{ref}})$</td>
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<td>$E$</td>
<td>Young's modulus</td>
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<tr>
<td>$I(x)$</td>
<td>cantilever moment of inertia at position $x$</td>
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<td>$J_{o}(k), J_{1}(k)$</td>
<td>Bessel functions of the first kind</td>
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<td>$K$</td>
<td>calibration constant for aerofoil probe $(\frac{e}{UV})$</td>
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<tr>
<td>$L$</td>
<td>lift force</td>
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<tr>
<td>$M_m$</td>
<td>effective mechanical mass of transducer</td>
</tr>
<tr>
<td>$N$</td>
<td>transducing ratio of piezoelectric element</td>
</tr>
<tr>
<td>$R$</td>
<td>input resistance of amplifier; nozzle radius of turbulent jet $(\frac{D}{2})$</td>
</tr>
<tr>
<td>$R(\tau)$</td>
<td>autocorrelation coefficient $\frac{e_1(t)e_1(t+\tau)}{\sigma e_1^2}$</td>
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<td>$R(\xi, \tau)$</td>
<td>two-point space-time correlation coefficient $\frac{e_1(x,t)e_2(x+\xi, t+\tau)}{e_1'e_2'}$</td>
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<td>$S$</td>
<td>lifting area of aerofoil</td>
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<tr>
<td>$U$, $\bar{U}$</td>
<td>component of mean velocity parallel to plane of aerofoil</td>
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<td>$U_c$</td>
<td>convection velocity in turbulent flow</td>
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<td>exit velocity of turbulent jet</td>
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<td>$X(x)$</td>
<td>shape function of deformed beam</td>
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<td>$X_c$</td>
<td>capacitive reactance of transducer $(\frac{-i}{\omega C_E})$</td>
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<td>$c$</td>
<td>maximum stream-wise dimension of aerofoil</td>
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<td>$\text{cps}$</td>
<td>cycles per second</td>
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<td>$e_{o}$</td>
<td>transducer output voltage</td>
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<td>$f$</td>
<td>frequency - cycles per second</td>
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<td>$\text{fps}$</td>
<td>feet per second</td>
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<td>$k$</td>
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<td>$l$</td>
<td>effective beam length</td>
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<tr>
<td>$m(x)$</td>
<td>mass per unit length of beam</td>
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<tr>
<td>$\text{mv}$</td>
<td>millivolts</td>
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<tr>
<td>$\text{pf}$, $\mu\text{f}$</td>
<td>picofarad $(10^{-12} \text{ farad})$</td>
</tr>
<tr>
<td>$\text{rms}$</td>
<td>root mean square</td>
</tr>
<tr>
<td>$u$</td>
<td>stream-wise component of turbulent fluctuation (prime denotes rms value)</td>
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<tr>
<td>$v$</td>
<td>transverse component of turbulence - tangential in case of axially symmetric jet. (Prime denotes rms value.)</td>
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<td>$x$</td>
<td>coordinate measured along beam; streamwise coordinate in jet</td>
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<tr>
<td>$y$</td>
<td>radial coordinate in jet</td>
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angle of attack

low frequency response depletion due to impedance mismatch

stream-wise wavelength of turbulence

stream-wise probe separation

density

time delay introduced in correlation

Sears function (Eq. 7)

spectrum function

angular frequency - radians/second

ohms

Overbar denotes time average
I. INTRODUCTION

To date, the accepted method of measuring instantaneously the transverse or "v"-component of turbulent velocity employs the familiar crossed-wire probe in conjunction with dual channel hot-wire anemometer circuitry. Alternatively, for investigations where only root-mean-square values are required, a single slant wire can be used by rotating through 180 degrees between measurements. Both of these methods require expensive equipment and involve several inherent limitations. For two-point correlation work in particular, instantaneous values of "v" are required, which introduces the need for two crossed-wire probes and the accompanying four channels of electronics.

In an effort to circumvent the complexity of the crossed-wire method, a new and relatively simple type of probe has been conceived and developed at UTIAS (Ref. 1). The only associated electronic equipment required with the probe consists primarily of an inexpensive audio-frequency amplifier.

The probe itself basically comprises a small aerofoil and a force transducer that yields a voltage varying as the instantaneous value of "v". More specifically, the aerofoil—of circular or rectangular planform—experiences a randomly varying lifting force, due to turbulent fluctuations in the flow. The aerofoil is attached to a tapered cantilever beam in which is imbedded a piezoelectric transducing element. For turbulence intensity values up to approximately 30%, the piezoelectric element produces an output voltage directly proportional to "v".

Development of the so-called "aerofoil probe" was chiefly motivated by a desire to make two-point space correlation measurements for the "v"-component in the UTIAS low-speed free air jet facility (Ref. 2). Consequently, this facility was used during the various stages of development in order to evaluate the response characteristics of the probe in turbulent flow.

II. BASIC THEORY

The basic mechanism underlying response of the aerofoil probe to turbulence is illustrated in Fig. 1. We consider flow incident on the aerofoil with velocity \( V \), at some angle of attack \( \beta \). In turbulent flow \( V \) and \( \beta \) both vary in a random fashion. It is assumed that at an any instant of time we can apply the approximation of quasi-steady linear aerofoil theory, provided that the frequency is not too high:

\[
L = \frac{1}{2} \rho V^2 S \left[ \frac{dC_L}{d\alpha} \right] \cdot \alpha
\]  

(1)
To a consistent approximation (i.e. for low intensity turbulence— \( \mathcal{L} \) small), \( \mathcal{L} \) may be replaced by \( (v/U) \) and \( V \) by \( U \) to give:

\[
L \simeq \frac{1}{2} \rho U S \left[ \frac{dC_L}{d\mathcal{L}} \right] v
\]

Now we know from simple aerodynamic theory that up to a certain angle of attack \( \mathcal{L}_{\text{stall}} \), the lift-curve slope \( C_L \) is essentially constant. Furthermore, the response of standard transducing devices is normally linear over a wide range.

Thus, in the range that the above limitations are met, or more exactly in the range that the overall probe response is linear in the lift \( L \), the result is a voltage proportional to "\( v \)":

\[
e \simeq K U v \text{ or } \frac{e}{U^2} \simeq K \left( \frac{v}{U} \right)
\]

It is important to note that the probe calibration is dependent upon the local mean stream velocity \( U \).

III. PROBE DEVELOPMENT

Various considerations were necessary in the development of a satisfactory probe design, the chief of these having to do with the frequency response characteristics of the overall system.

3.1 Cantilever Design for Maximum Resonant Frequency

Several cantilever shapes and configurations were analyzed in an effort to arrive at a beam design with the highest possible fundamental mode of resonance. In these analyses, use was made of the energy method of Rayleigh-Ritz (Ref. 3) which gives:

\[
\omega_1^2 = \frac{\int_0^l EI(x) \left( \frac{d^2X_1}{dx^2} \right)^2 dx}{\int_0^l m(x)X_1^2 dx}
\]

Here \( \omega_1 \) is the fundamental or first harmonic natural frequency for the beam and \( X_1(x) \) is the particular shape function for this mode of oscillation. Following Timoshenko, \( X_1(x) \) was approximated by the first term of the infinite polynomial series.
\[ X_1(x) = a_1 \left(1 - \frac{x}{l}\right)^2 + a_2 \left(1 - \frac{x}{l}\right)^2 \cdot \frac{x}{l} + \ldots \]

An approximation to the fundamental frequency for a particular beam geometry was determined by substituting \( I(x) \), \( m(x) \) and \( X_1(x) \) into (4). \( I(x) \) and \( m(x) \) are respectively the moment of inertia variation, and mass per unit length variation along the beam.) The result in general varied according to:

\[
f_1 = \frac{\omega_1}{2\pi} \sim \sqrt{\frac{EI_0}{m l^4}} \tag{5}
\]

\( I_0 \) is merely some reference moment of inertia and indicates the effect of cross-sectional shape on \( f_1 \). Various sectional shapes were considered in the analysis. The choice of beam material is also seen to influence resonant characteristics. Materials such as steel and aluminum have a high elastic modulus to density ratio, and were therefore selected to maximize \( f_1 \). The beam length \( l \) was kept short in relation to cross-sectional dimensions at the base.

In all cases the cantilever consisted of a tapered section terminated by a short beam of uniform cross-section which supported the aerofoil. The structure was analyzed in sections and it was found that in most cases the uniform outer section dominantly influenced the harmonic properties.

The minimum permissible resonant frequency was established from a knowledge of the spectral distribution of turbulent energy in the UTIAS free air jet. A typical cumulative energy spectrum for turbulence in this jet indicates relatively little energy content for frequencies above 5,000 cycles per second. (Figure 2.)

Of the various beam geometries which were analyzed, three in particular seemed to afford a fundamental resonant frequency well above 5,000 cps. During the stages of development, probes incorporating beams of cruciform cross-section, wedge shape, and conical shape (Fig. 3) were all fabricated and their performance was evaluated in the turbulent jet.

3.2 Transducer Considerations

Feasibility studies were made of various transducing methods. It was concluded that a piezoelectric type of force transducer would best satisfy the particular requirement.
In order to achieve a maximum natural frequency for the probe, it was apparent that the transducing device should have a high mechanical strength and be mounted in such a way so as to have minimum effect on the cantilever stiffness.

Various piezoelectric materials such as quartz, Rochelle salt, and PZT-4* ceramic were utilized in early models of the probe. Because of its superior mechanical properties, high permittivity, availability, and relatively workable nature, the PZT ceramic was selected as the most desirable material and has been incorporated in the ultimate probe design. The higher permittivity of ceramic piezoelectrics is of considerable importance since the low frequency limit of the transducer is determined by its self-impedance, which is essentially capacitive in nature. This matter meets with further discussion in 3.3.2.

Custom-shaping of the piezoelectric elements (by the manufacturer) was found to be extremely expensive. Since it was desired to experiment with various shapes and sizes, a decision was made to attempt our own fabrication. This scheme proved quite successful. The shapes were rough-cut with a diamond saw from readily available PZT-4 ceramic transducer discs. (Discs were 1/8" thick-electroded on both faces.) Final dimensions were obtained by a lapping-on-glass process. In several cases the inter-electrode spacing was reduced. Silver-conducting paint provided new electrodes in these instances and was also employed to attach aluminum-foil leads. An epoxy-base adhesive was used for mounting the finished transducer.

As an alternative to the piezoelectric type of transducer an investigation into the application of strain gauge techniques was carried out. This would have the advantage of affording flat frequency response right down to D.C. Calculations revealed however that for a beam of the desired stiffness, maximum available mechanical strains were of the order of \(10^{-5}\) in./in. This is well below the ultimate sensitivity of conventional strain-gauge systems. In confirmation of the calculation, no detectible output was obtained from a strain gauge mounted on one of the probes. In order to obtain measurable strains, the stiffness of the system would have to be reduced to such an extent that mechanical resonance would become a serious problem.

It should be pointed out, however, that our endeavors with strain-gauge transducers were very limited. Possibly for some particular applications of the aerofoil probe (e.g. low frequency-large scale turbulence) this type of transducer would be both feasible and preferable. (A new type of semiconductor strain-gauge** might be quite useful in such a case.)

* Trade-Name of Clevite Corporation, Bedford, Ohio.

3.3 Frequency Response as a Design Criterion

Throughout the stages of probe development, frequency response characteristics were used as the prime design criterion. Frequency spectra obtained at a fixed position in the turbulent air jet (with various models of the aerofoil probe) were compared with that obtained from a crossed hot-wire probe placed at the same point. Figure 4 shows a few of many response curves which were so obtained. The curves are plotted in Decibels relative to the 1,000 cps point. It is noted that response of the latest probe design agrees almost exactly with that of the crossed-wire probe over the major spectral range. Curves obtained with earlier models exhibit certain anomalies which are discussed in the following paragraphs.

3.3.1 High Frequency Resonant Response

The nature of this particular anomaly follows from the discussion of 3.1. Some of the earlier cantilever designs (e.g. the cruciform beam) exhibited strong resonant response at the high frequency end of the spectrum (i.e. above 5,000 cps). By externally damping the outer end of the beam in these cases, some reduction in the resonant peak was realized, but it was far from adequate. These early beams were machined from mild steel. The tapered sections of later models (i.e. the wedge and conical beams) were aluminum. The combination of more efficient geometry and much lower mass give rise to a marked increase in resonant frequency (see Eq. 5) and a decrease of the resonant peak to permissible levels. (The beam length \( l \) was approximately the same for all models*.)

Decrease of the resonant peak is attributed to a greater damping effectiveness of the newer designs. This results from an increased volume per unit length, despite inherently poorer internal damping of aluminum as compared with steel.

3.3.2 Low-Frequency "Fall-Off"

Considerable difficulty was also encountered with the low frequency end of the response curve where a "falling-off" of response with decreasing frequency was seen to occur (Figs. 4 and 5). Apparently this phenomenon is characteristic of piezoelectric transducing devices and has been reported elsewhere (Ref. 4).

*It was apparent that the ratio \( l/d \) (\( d \) is the depth of the beam at its base) should be reasonably large in order that the probe would have minimum downstream influence on the flow. However, since the natural frequency varies inversely as \( l^2 \), \( l/d \) is restricted to small values. As a compromise we chose \( l/d \approx 2 \) for all designs - this proved quite satisfactory.
A simple electrical analogy to the complete transducer exemplifies the situation (Fig. 6). Similar analogies have been given by various authors (Refs. 5 and 6). The piezoelectric device can be represented by a force/voltage transformer of turns ratio \( N \). The electrical capacitance \( C_E \) of the transducer acts in series with the transformer output. At low frequencies the capacitive impedance \( X_c \) increases very sharply. At frequencies where this impedance becomes comparable with or greater than the input impedance of the output amplifier, a marked depletion of the output voltage \( e_o \) occurs. If we assume the amplifier impedance to be a pure resistance, then we can say:

\[
\frac{e_o (f)}{e} \approx \sqrt{\frac{R}{R^2 + \frac{1}{(2\pi f C_E)^2}}} = \left[ 1 + \frac{1}{(2\pi f C_E R)^2} \right]^{-\frac{1}{2}}
\]

(6)

Obviously, for large \( fC_ER \) there will be negligible depletion of voltage output. However at low frequencies the product \( C_ER \) becomes very significant.

In general, a change in geometry which brings about an increase in the capacitance of a piezoelectric element has an inverse effect on the transducing ratio \( N \) (Volts/unit force). Consequently, in order to obtain reasonably measurable outputs, the capacitance had to be kept fairly small and in most cases was of the order of 140 pF. Naturally occurring piezoelectrics such as quartz and Rochelle salt have a much lower permittivity than the ceramic materials, thus making it more difficult to obtain good low-frequency response. Experimentally a more pronounced fall-off was noted with a Rochelle salt transducing element.

The other means of reducing the low-frequency depletion is obviously to increase the input resistance \( R \) (or net impedance) of the output instrument. Various high impedance devices such as cathode followers and electrometer or charge amplifiers are available for this purpose. In our case it was found that an electrometer amplifier gave satisfactory response to 20 cps, as depicted in Fig. 5. An approximation to the low frequency depletion \( \Delta e_o (f) \) as given by Fig. 6 is also plotted on Fig. 5, for \( R = 10^7 \Omega \). It is seen to agree quite closely with the experimentally determined "fall-off" for the same \( R \). The deviation is likely due to neglect of cable impedance and amplifier input capacitance in Ref. 6.
3.3.3 Effect of Aerofoil Size and Shape

A noticeable response decay was found to occur for frequencies higher than 1,000 cps. (See Fig. 4.) This progressive falling of response grows with frequency until the condition of resonance sets in, and constitutes a departure from the assumed quasi-steady linear aerofoil theory of Sec. 2.

The effect was originally attributed in part to averaging of the higher frequency turbulence fluctuations over the area of the sensing aerofoil. However a closer look at the non-steady response characteristics of finite lifting surfaces rules out any significant averaging for the frequency range of interest. The curves given in Fig. 7 appear to be relevant to the situation. Curve A is the well-known Sears function (Ref. 7) for the lift response of a 2-dimensional aerofoil to a sinusoidal gust:

\[ \phi(k) = \left[ \frac{J_0(k) - iJ_1(k)}{k} \right] C(k) + iJ_1(k) \]  

(7)

\[ \phi(k) = \frac{J_0(k) - iJ_1(k)}{k} \] \[ C(k) \] is the Theodorsen function*. It does not appear that an exact solution exists for the case of sinusoidal gust response of an aerofoil of finite aspect ratio. However, Biot and Boehnlein (Ref. 8) have given what amount to approximate values of \( C(k) \) for wings of finite aspect ratio ranging from 1.0 to \( \infty \). Although these values were determined on the basis of simplifying assumptions, it is suggested that they should be sufficiently accurate for reduced frequency \( k \) less than 5. By using the modified values of \( C(k) \) in Eqs. (7) above, for \( R \approx 1.25 \) curve B was obtained.

Kochin (Ref. 10) has presented an approximate solution for a wing of circular planform, describing pitching and plunging motions. For small values of \( k \) and \( |\mathcal{C}| \), Kochin's solution for the pitching mode should give a reasonable approximation to the sinusoidal gust response of a circular wing. Curve C depicts the lift variation with \( k \), as given by Kochin.

Of the three curves, B probably best describes the averaging effect which occurs when the turbulence wavelength becomes comparable with the streamwise dimension of the lifting surface. (According to the Taylor "frozen" convection hypothesis, typical wavelengths in the turbulence are related to frequencies observed by a stationary observer through the relation \( \lambda f = U \).) We note that for

*See pages 213-215 of Ref. 9.
k > 2.0, the lift response gradually falls off. But at the flow velocity at which the actual probe response was determined (see curve D), and based upon the maximum chord C of the aerofoil, the reduced frequency k = 2 corresponds to roughly 10,000 cps. In contrast we note that the actual lift response of the aerofoil probe begins to fall markedly from k ≈ 0.2, or 1000 cps.

The deviation between the theoretical curve B and the experimental curve D is thought to arise in part from interaction of the sting and probe body with the wake shed by the aerofoil. This interaction must greatly modify the wake geometry and hence the contribution to net lift arising from wake induction. Furthermore, curve B involves some uncertainty in the theory even for an aerofoil without this downstream interaction: the theory of Biot and Boehnlein does not account for spanwise lift variation, and moreover the physical extent to which Kutta's condition is satisfied for large k is not known.

To avoid the fall off in frequency response it appears desirable to employ a lift sensor of the smallest possible physical dimensions. Conversely it is advantageous to maximize the lifting area in order to obtain sufficient output voltage. As a compromise it was found that discs (AR ≈ 1.25) of .075 inch diameter adequately satisfied both requirements. Rectangular planforms of comparable area and low aspect ratio (AR ≈ 2) were also used successfully.

3.3.4 Accelerometric Effects

In effect, the cantilever - piezoelectric pickup combination is a miniature accelerometer. Any vibration of the probe support tends to accelerate correspondingly the mass of the cantilever beam. There results an "inertial loading" of the beam which is detected (as an unwanted signal) by the piezoelectric element.

Some difficulty was encountered with small "bumps" appearing on the frequency spectrum due to this effect. Suffice it to say that this spurious signal was minimized to an undetectable level by:

i) Keeping the mass of the probe head (i.e. the cantilever and shroud) as low as possible. Thus the probe head was fabricated almost entirely from aluminum.

ii) Making the probe supporting sting as light and stiff as possible. (One-half inch diameter thin-wall steel tubing served this purpose.)

iii) Ensuring that the probe head and supporting sting were rigidly clamped to a massive base.
3.4 **The Final Probe Design**

The final probe configuration decided upon is depicted by a detail drawing (Fig. 8.) Figures 9a and 9b show a photograph of the actual probe and a typical oscilloscope trace obtained in the heavily sheared (mixing) region of the turbulent jet, respectively.

When used in conjunction with an a.c. amplifier of reasonably high input impedance \(4 \times 10^8 \Omega\), this latest design affords a frequency response which is flat within roughly 3 Decibels from 20 to 10,000 cps.

For turbulence intensity \((v'/U)\) of 15\%, a typical signal to noise ratio of 40 was observed, the root mean-square signal level being of the order of 3 mv, and stream velocity \(U \approx 95\) fps. Thus the measurement of very low intensity turbulence (1 or 2\%) is entirely feasible, provided that the mean flow speed is high enough. (The probe response goes as the product of \(U\) and \(v'\) - see Eq. 3.)

**IV. Probe Calibration**

Due to the difficulty of obtaining consistently accurate measurements of the "\(v'\)"-component with a crossed hot-wire probe, it was decided to employ an independent method for calibrating the aerofoil probe.

The approach consisted primarily of placing the aerofoil at some angle of attack in a jet flow which was chopped by rotating a segmented disc at the jet exit. A quasi-square wave output signal was obtained from the probe as depicted in Fig. 10b. The chopping frequency was kept constant at 180 cps, which is well above the low-frequency cut-off of the probe. (It was this low-frequency limitation, as discussed in 3.3.2, which made static calibration impossible.) The calibration mechanism is shown in Fig. 10a.

By varying the angle of attack, an almost linear variation in peak voltage of the square wave was observed. A typical calibration curve obtained by this method is given in Fig. 11. The turbulence level in the chopped jet was negligibly small, so the jet exit velocity \(V\) readily separates into components \(v\) and \(U\), normal and parallel to the aerofoil respectively. (See inset sketch on Fig. 11.) The probe output voltage \(e\) is seen to be essentially linear in the product \((Uv)\) for simulated turbulence intensities of up to 30\%, as was predicted by Eq. (3).

The value of peak jet velocity \(V\) was determined independently by both hot wire measurement and total head measurement.
(A knowledge of the waveform shape was required in order to determine $V_{\text{max}}$ from the mean dynamic head.) Values obtained by the two methods agreed within less than 5%.

It is felt that the velocities $v$ and $U$ as defined in Fig. 11 are equivalent to the same components in turbulent flow. (See Fig. 1.) Some question may arise as to the validity of this approach to calibration, because we are not simulating a true turbulent flow field. However the resulting calibration curve led to a very satisfactory measurement of $v$ in turbulent shear flow, as will be discussed in the following section.

V. EXPERIMENTAL MEASUREMENTS

5.1 Velocity Measurements

Root mean square values of $v$ were determined at various positions in the turbulent jet utilizing a calibration curve of the type just discussed.

Figure 12a depicts the radial variation of mean flow velocity near the end of the so-called "mixing region" of the jet (i.e. at $x/D = 4.5$). The corresponding turbulence intensity $(v'/U)$ is plotted as 12b. In comparison, the distribution of the stream-wise or $u$-component intensity is also shown. We note that $v$ is only about 75% of $u$ in this region, which is in agreement with the findings of Bradshaw (Ref. 11).

The data is replotted with the form $v/U_0$ in Fig. 13. Comparison is made with similar profiles obtained by Bradshaw at a station slightly further upstream (i.e. at $x/D = 4.0$). Agreement is excellent, with all four curves peaking at the radial position $y/R = 1$. The centerline turbulence appears to be somewhat weaker for the Bradshaw data - this being due to the fact that the laminar potential core of the jet normally extends to $x/D \approx 4.2$. The centerline turbulence intensity rapidly grows for downstream positions greater than this.

Measurements made at other points in the mixing region also agreed quite well with Bradshaw's published data.

Further evidence for the accuracy of probe calibration derives from measurements made far downstream, in the region of "fully developed" flow (i.e. $x/D > 10$). Corrsin (Ref. 12) gives results which indicate a region of relatively isotropic turbulence near the jet axis, at a station 20 diameters downstream. His Fig. 24 indicates virtual equality of the $u$ and $v$ components, the intensities being in the order of 27%. Measurements carried out in our jet at the same position, with
the aerofoil probe and a hot-wire probe, yielded values of u and v which differ by less than 1%. The turbulence intensity was approximately 28%, which is surprisingly close to that found by Corrsin.

On the basis of the foregoing comparisons, it was concluded that the probe calibration was accurate to within 5%.

5.2 Frequency Spectra

The absolute spectrum function $\phi(\omega)$ is plotted for the u and v components as Fig. 14. This function is related to the mean-square value of u and v by:

$$\overline{v^2} = \int_{0}^{\infty} \phi_2(\omega) \, d\omega$$

Integrated areas of the "v"-component spectrum determined both with the aerofoil probe and crossed hot-wire probe were in good agreement.

5.3 Autocorrelation

The UTIAS correlation computer is described in detail elsewhere (Ref. 13). With it we were able to determine the autocorrelation coefficient $R(\tau)$. This coefficient is essentially a Fourier cosine transform of the one-dimensional spectrum function:

$$R(\tau) = \frac{1}{\sqrt{2}} \int_{0}^{\infty} \phi_2(\omega) \cos \omega \tau \, d\omega$$

Autocorrelation functions obtained from the aerofoil probe signal and that of the crossed wire are compared in Fig. 15. Again the agreement is excellent except for a slight deviation for small values of time delay $\tau$. This corresponds to high frequency discrepancy in $\phi(\omega)$ and arises because of a slight resonance in the frequency response of the aerofoil probe.
5.4 Two-Point Space-Time Correlations

By correlating the signals from two probes separated in the flow by distance $\xi$, again with time delay $\tau$ introduced between the signals, the space-time correlation coefficient $R(\xi, \tau)$ can be evaluated. For various values of $\xi$ and $\tau$, a family of curves results such as that depicted in Fig. 16. These particular data were obtained for separations $\xi$ in the stream-wise direction, centered at $x/D = 4.5$ with $y/R = 0.8$.

An envelope drawn tangent to the series of curves describes what is known as the autocorrelation in the moving or "convected" frame of reference (Refs. 14 and 15). This function depicts the rate of decay of the turbulent structure with distance for an observer moving with the pattern.

From this type of data we are able to extract interesting parameters characteristic of the turbulence, such as space and time-scales, typical fluctuation frequencies, and the eddy convection speed.

Figure 17 depicts the determination of convection speed from the data of Fig. 16. For the particular station studied, it was found that $U_C = 87.4$ fps.

By similarly correlating the $u$-component signals obtained with two hot-wire probes, a comparable set of data resulted, which gave $U_C = 88.5$ fps. We would expect the convection speed to be invariant, regardless of which turbulence component were used to determine it. Within 1% our results bear this supposition out.

The local mean flow velocity at the same radial position is somewhat higher ($\approx 94$ fps). For radial stations inside $y/R = 1$, this is the expected case, as reported by Davis, Fisher and Barrat (Ref. 16).

For small separations it was found that vorticity shed by the upstream probe significantly altered the turbulence pattern detected by the downstream probe. The net result was a decrease in peak correlation for small $\xi$ as depicted by the curves for $\xi = 0.4$ and 0.8 in Fig. 16. Consequently, it would be rather difficult to obtain much information about the curvature of the moving frame autocorrelation function near the apex with aerofoil type probes, unless they could be produced on a much smaller physical scale.

VI. CONCLUDING REMARKS

Although this new type of turbulence anemometer is as yet only in the early stages of exploitation, it would appear to have advantages for turbulence measurements in a number of circumstances. Unlike the more complicated crossed-wire technique, the aerofoil probe provides
a single channel, direct measurement of $v$. A quick check on the axisymmetry of a flow is afforded by merely rotating the probe axially.

It is a well-known fact that in certain parts of some wind tunnels, the cross-stream component of turbulence greatly overrides that in the stream-wise direction. However, due to the difficulty to date of measuring $v$, single wire measurements of $u$ have been relied upon to define the turbulence level. The aerofoil probe would be of considerable utility here since the turbulence level based on $u$ is unconservatively low.

The major shortcoming of the probe arises from the fact that mean flow velocity $U$ must be measured by some independent means (i.e. a pitot tube) in order to determine the absolute value of $v$. However, by incorporating a total head detector and the appropriate analogue circuitry, direct values of $v$ could probably be obtained at any point in a flow. This would be particularly useful in a flow of non-constant $U$ (i.e. a shear flow).

The obvious simplicity of the device points to ease of manufacture at an economical cost. Superior durability and temporal stability should make possible many measurements which were hitherto difficult with conventional crossed-wires (i.e. high velocities, dirty flows, hydrodynamic turbulence).
REFERENCES


FIGURE 1 MECHANISM OF RESPONSE OF AEROFOIL PROBE TO "v" - COMPONENT OF TURBULENCE.
FIGURE 2  CUMULATIVE ENERGY DISTRIBUTION IN THE TURBULENT JET

\[ \frac{1}{\Omega} \int_0^f f \phi(f) df \]

\( \frac{f}{\text{Cycles/Second}} \)

\( X_D = 4.5 \)
\( Y_R = 1.0 \)
CRUCIFORM    WEDGE    CONICAL

FIGURE 3 CANTILEVER GEOMETRIES WHICH WERE EMPLOYED
FIGURE 4 FREQUENCY RESPONSE COMPARISONS IN THE TURBULENT JET
FIGURE 5 EFFECT OF AMPLIFIER INPUT CHARACTERISTICS ON LOW FREQUENCY RESPONSE

\[ \Delta_{\text{ref}} = 20 \log \left[ 1 + \frac{1}{(2\pi fRC)^2} \right]^{-\frac{1}{2}} \quad R=10^7 \Omega \]
FIGURE 6 ELECTRO-MECHANICAL ANALOGY TO A PIEZOELECTRIC TRANSDUCER
A - SEARS' FUNCTION $\phi(k)$ 2-DIM.
B - MODIFIED $\phi(k)$ FOR $\lambda = 1.25$
(FROM BIOT-BOEHNLEIN)
C - KOCHIN SOL'N FOR PITCHING CIRCULAR AEROFOIL
D - LIFT RESPONSE OF AEROFOIL PROBE

$\frac{C_l}{2\pi}$ vs $k = \frac{\omega c}{2U}$

$\approx 1000$ cps $\approx 10,000$ cps

$\lambda = c$

FIGURE 7 NON-STEADY RESPONSE OF LIFTING SURFACES
FOIL LEAD

EPOXY

21 GA. HYPODERMIC TUBING

PIEZOELECTRIC ELEMENT (ELECTRODE TOP & BOTTOM)

SECTION OF #10 SEWING NEEDLE

ALUMINUM or STEEL

SCALE = 5 X FULL SIZE

FIGURE 8 ASSEMBLY DRAWING OF THE FINAL PROBE DESIGN
FIGURE 9a PHOTOGRAPH OF THE PROBE WITH SHROUD IN PLACE

FIGURE 9b TYPICAL OUTPUT TRACE IN TURBULENT FLOW
FIGURE 10a PROBE CALIBRATING MECHANISM

FIGURE 10b OUTPUT TRACE FROM SQUARE-WAVE CALIBRATOR
FIGURE 11  TYPICAL CALIBRATION CURVE FOR THE AEROFOIL PROBE

\[ e/U^2 \approx K(v/U) \quad [\text{for } v/U < 0.3] \]

\[ K = 2.88 \times 10^{-6} \text{ Volt sec}^2/\text{ft}^2 \]
FIGURE 12 VELOCITY VARIATION ACROSS THE TURBULENT JET
FIGURE 13 TURBULENCE INTENSITY PROFILES
FIGURE 14  COMPARISON OF TURBULENCE SPECTRA

- $u$-COMPONENT / HOT WIRE
- $v$-COMPONENT / CROSSED HOT WIRES
- $v$-COMPONENT / AEROFOIL PROBE

$x/D = 4.5 \quad y/R = 1.0$

$U_o = 138$ fps
FIGURE 15 AUTOCORRELATION OF THE v-COMPONENT SIGNAL

- CROSSED WIRE PROBE
- AEROFOIL PROBE

$\frac{\alpha}{D} = 4.5 \quad \frac{y}{R} = 1.0$

$U_o = 138 \text{ fps}$
ZONE OF WAKE INTERFERENCE

$U_o = 138$ fps  \hspace{1cm} \frac{x}{D} = 4.5

$\bar{U} = 94.0$ fps  \hspace{1cm} \frac{y}{R} = 0.8

$U_c = 87.4$ fps

AUTOCORRELATION IN THE MOVING FRAME OF REFERENCE

FIG. 16  SPACE-TIME CORRELATION OF TRANSVERSE VELOCITY FLUCTUATIONS WITH DOWNSTREAM SEPARATION.
FIGURE 17 DETERMINATION OF EDDY CONVECTION VELOCITY $U_c$

The equation on the graph shows:

$$\frac{\xi}{U_0} = \frac{U_c}{U_0} = 0.633$$