THE EFFECT OF HIGH INTENSITY TURBULENCE
ON THE AERODYNAMICS OF A RIGID CIRCULAR
CYLINDER AT SUBCRITICAL REYNOLDS NUMBER

by

D. Surry
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SUMMARY

The interaction of high intensity turbulence with the flow past a rigid circular cylinder has been studied experimentally at subcritical Reynolds Numbers. Grids were used to produce homogeneous turbulence fields with longitudinal scales ranging from 0.36D to 4.40D, and with longitudinal intensities greater than 10%. Power and cross-spectra of the turbulence components (the 'system input') have been measured in order to carefully define the turbulence characteristics. In particular, lateral coherences of the longitudinal component have been found to collapse well when plotted versus $\xi/\lambda$ (lateral separation/wavelength) as suggested by Davenport.

A model with which measurement of arbitrary two-point pressure correlations could be made was used in the response experiments. Subsequent integrations yielded the spectral properties of the unsteady drag and lift. Measurement of mean drag and Strouhal frequency indicate that to some extent even severe large-scale turbulence can be considered equivalent to an increase in the effective Reynolds Number. Vortex shedding is not disrupted drastically by severe turbulence, but is affected more by that at low frequency than at high. The unsteady lift response is still dominated by the Vortex shedding, whereas the unsteady drag is primarily a response to turbulence. The cross-spectra of the drag collapse well when plotted versus $\xi/\lambda$, and have been used, for one grid, to derive a 'describing function' for the drag 'response' to turbulence. This describing function is the central element needed for the calculation of structural response in the drag direction.
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NOTATION*  

A  
wind tunnel cross-sectional area  
b  
grid bar width or diameter  
b'  
grid bar depth  
\( C_d \)  
sectional mean drag coefficient, \( C_d = \bar{d} / qD \)  
\( C_p \)  
static pressure coefficient \( C_p = \frac{p - p_m}{q} \)  
\( C_{xy}(f) \)  
real or co-spectral density of \( x(t) \) and \( y(t) \)  
\( C_{ff}(\tau, \xi) \)  
coefficient form of force cross-correlation,  
\[ C_{ff}^2(\tau, \xi) = \frac{R_{ff}(\tau, \xi)}{q^2 D^2} \]  
D  
cylinder diameter  
\( d_T(t, y) \)  
total instantaneous sectional drag at \( y \)  
\( \bar{d}(y) \)  
mean sectional drag at \( y \), \( \bar{d}(y) = \bar{d}_m(t, y) \)  
d  
diameter of hot wire sensor  
\( d(t, y) \)  
fluctuating component of sectional drag at \( y \), \( d(t, y) = 0 \)  
E  
linearized voltage output of hot wire system  
e_o  
wave analyzer rms output voltage  
f  
frequency, Hz.  
f_c  
wave analyzer filter centre frequency  
f_o  
Shannon frequency, \( f_o = 1/2 \Delta \tau \)  
f_s  
frequency of Strouhal vortex shedding.;  
\( \Delta f \)  
frequency increment for which spectral estimates are calculated, \( \Delta f = 1/2 \tau_m \)  
\( \Delta f_e \)  
effective bandwidth of digital spectral analysis  
H  
heating ratio of hot wire sensor  
H(\( f \))  
frequency response function  

* This table is not complete in that some symbols are not included or are used in other definitions than listed here. In these cases, the symbols are locally defined in the text and hence should not lead to confusion.
\( K \)  
microphone calibration constant - see Appendix C.

\( K_u \)  
calibration constant of hot wire system for velocities in the u-direction.

\( L \)  
finite length of cylinder.

\( L_u(x) \)  
integral scale of \( u \) in the x-direction.

\( L_p(\theta) \)  
integral scale of surface pressure for constant \( \theta \).

\( \ell \)  
length of 'hot wire' sensor.

\( \ell(t, y) \)  
fluctuating sectional lift at \( y \), \( \ell(t, y) = 0 \).

\( M \)  
grid mesh = bar centre-line to centre-line distance.

\( P \)  
total instantaneous static pressure at a point.

\( P_T \)  
flow stagnation pressure.

\( P_\infty \)  
reference static pressure corresponding to undisturbed flow.

\( p \)  
fluctuating component of surface static pressure, \( \bar{p} = 0 \).

\( Q_{xy}(f) \)  
imaginary or quad-spectral density of \( x(t) \) and \( y(t) \).

\( q \)  
dynamic pressure of mean velocity, \( q = 1/2 \rho \bar{U}^2 \).

\( R_{gh} \)  
cross-correlation function of \( g(t) \) and \( h(t) \).

\( \text{Re} \)  
Reynolds Number.

\( r \)  
cylinder radius.

\( S \)  
grid solidity, \( S = \frac{b}{M}(2 - b/M) \).

\( St \)  
Strouhal Number, \( St = f_s D/\bar{U} \).

\( s \)  
experimental estimate of the standard deviation.

\( T \)  
absolute temperature; record length.

\( T' \)  
effective record length.

\( t \)  
time.

\( \Delta t \)  
sampling period.

\( U \)  
total x-component of velocity, \( U = \bar{U} + u(t) \).
mean x-component of velocity

fluctuating x-component of velocity, \( \bar{u}(t) = 0, \bar{u} = \sqrt{\frac{u^2}{T_x}} \)

voltage

fluctuating y-component of velocity, \( \bar{v}(t) = 0, \bar{v} = \sqrt{\frac{v^2}{T_y}} \)

fluctuating z-component of velocity, \( \bar{w}(t) = 0, \bar{w} = \sqrt{\frac{w^2}{T_z}} \)

downstream coordinates, see Figure 2-1

random variable

downstream coordinate, measured from entrance of diffuser

lateral coordinates, see Figure 2-1

random variable

vertical coordinates, see Figure 2-1

temperature coefficient of resistance of \( i \) at a reference temperature \( T_o \)

air density

air density at standard temperature and pressure

non-dimensional lateral separation, \( \eta = f \xi/\bar{U} \)

magnitude of lateral separation

wavelength

time delay

magnitude of maximum time delay

time delay increment

standard deviation

mean

true circumferential angle measured from front stagnation point

circumferential angle measured from horizontal

**SUBSCRIPTS**

f

force - lift or drag

r

reference conditions in wind tunnel test section
M model centre-line conditions
G grid position conditions

MATHEMATICAL NOTATION

rms root mean square
Δ equal by definition
~ approximately equal
Re real part of
Im imaginary part of
\bar{a} mean value of a
\text{rms} a rms value of a
I INTRODUCTION

In recent years, the desire for more realistic design criteria for both Earth-fixed structures and aircraft has focussed increasing attention upon the prediction of the response of such structures to turbulence in the wind. This problem can be considered as made up of three sub-problems. These are:

(i) to obtain an adequate description of the atmospheric environment;
(ii) to relate the atmospheric velocity (inputs) to pressures, forces, and moments on the body (outputs);
(iii) to derive the motion response of the body, knowing the forces acting upon it.

The last of these involves the application of well-known principles by well-developed techniques, and hence can be considered to be in a quite satisfactory state. The first two sub-problems are currently under intensive investigation in many quarters, and the methods and information needed are slowly becoming available.

Perhaps the most complex area of the atmospheric environment to define adequately for this problem is the ground boundary layer formed by wind blowing over the earth's surface. It extends to altitudes of the order of a thousand feet, and hence is shared both by buildings and by low-flying aircraft. It is typically a non-homogeneous region of strong vertical shear and can provide severe turbulence with intensities of the order of 20% or more. Furthermore, its properties depend strongly on such variables as atmospheric stability, type of terrain, altitude, etc. (e.g. Refs. 18, 29, 30).

The problem of relating the atmospheric turbulence inputs to pressures and forces on the body theoretically is, in most cases, unsolved. Theories of aircraft and structural response to turbulence (e.g. Refs. 19-23) usually are restricted in application by this inability at present to describe adequately general relations between the forces induced on the body and the turbulence inputs. The resulting linearizations and simplifications used to model these relations are successful in predicting the body response in some regimes - notably the response of aircraft to turbulence at high altitude (e.g. Ref. 24) or "line-like" structural response (Refs. 9,25,26) - where the input energy is concentrated at wavelengths large with respect to the characteristic body dimensions. However, for low altitude flight, and for structures which do not fulfill the "line-like" assumptions, the atmospheric boundary layer presents turbulence inputs with energy concentrated at wavelengths of the same order as the characteristic body dimensions. (The longitudinal scale, L, in the ground boundary layer is of the order of 200 to 2,000 feet). In this case, the simplifications in the usual assumptions of force-velocity relations become suspect.

In particular, for buildings, (which includes such problems as the rocket on the launch pad), the aerodynamics of the body response becomes extremely complex. Structures in non-turbulent flow would in any case be subjected to unsteady forces. These are caused by large wakes and vortex shedding phenomena, which are sensitive to the particular body geometry and its flexibility, as well as to the flow variables themselves. Hence the addition of turbulence produces a situation which in many cases yields only to direct experimental investigation.

Much work has already been carried out to determine the effects of winds
on buildings and structures (viz Refs. 31,32). However, much research remains to be done - in particular for the problems introduced by severe turbulence.

Much of the work done to date in this field is characterized by measurement of overall structural response in particular cases (e.g. Refs. 9,25,26, 33), either to test theories of response using simplified force-turbulence relations, or directly to measure the response of complex structures for design purposes. To date the research effort into the fundamental aerodynamics of even simple shapes exposed to turbulence has been sparse. Notable exceptions are the investigations of Wardlaw and Davenport (Ref.27) into the forces on rigid flat plates, and of Vickery (Ref.28) into those on a rigid square cross-sectioned cylinder.

In this report, the subject of investigation is a rigid circular cylinder exposed to homogeneous turbulence whose characteristics have been well defined. There has been no attempt to simulate the atmospheric boundary layer precisely, but rather the approach has been to provide a variety of controlled turbulence fields so as to study the effect of turbulence parameter changes. However, an awareness of the atmospheric problem is reflected in the choice of these parameters to be realistic atmospheric values, as far as the restriction of homogeneity allows.

The choice of a rigid circular cylinder for the model was dictated partially by the many similar real structures of this shape operating in the lower atmosphere (chimneys, rockets, cables, etc) and partially by the fundamental nature of the flow phenomena about it.

In particular, the aerodynamic information sought for the circular cylinder is that which is central to the design problem - i.e. a description of the cross-spectrum of unsteady forces acting upon the cylinder. With this information, and its relation to the unsteady flow field, the necessity of assuming simple analytical relations can be avoided. Then it is relatively straight-forward to obtain the unsteady structural response (see Ref.26).

1.1 Flow Properties around a Rigid Circular Cylinder

It is of interest to review briefly what is essentially the reference state for this experiment - i.e. the flow properties about such a cylinder in steady flow with a low turbulence level. (The flow is assumed to be incompressible and at normal temperature and pressure). Under such conditions, the cylinder aerodynamics is primarily dependent on the $Re$, the cylinder surface roughness, and the turbulence level in the airstream. At least to some degree all of these parameters can be related, in that the effects of both small turbulence level and surface roughness often produce effects similar to an increase in $Re$.

The reason for this is that both increase the turbulence present in the boundary layer on the cylinder. The state of the boundary layer is the prime factor in determining when the flow separates from the cylinder. For instance, in the $Re$ range of approximately $10^3$ to $2 \times 10^5$ the boundary layer is laminar over the entire front surface of the cylinder. The flow in the boundary layer is then subjected to both the high decelerating viscous force present in a laminar layer as well as the externally impressed pressure field due to the essentially inviscid flow outside the boundary layer. The latter results in an adverse pressure gradient which is felt by the boundary layer before the $90^\circ$ point (from the front stagnation point) and which causes local flow reversal next to the surface and hence separation. Above the critical $Re$ (about $4 \times 10^7$), the boundary layer naturally becomes turbu-
lent prior to meeting the adverse pressure gradient. The increased mixing in the
turbulent boundary layer increases the boundary layer energy and retards separation
significantly. This leads to a marked decrease in wake width and an accompanying
drop in the drag coefficient. The effect of stream turbulence or surface rough-
ness is then to produce a boundary layer state like that for a non-turbulent
higher Re, referred to here as the effective Re. The change in effective Re can
be seen most easily by noting the critical Re, at which the large change in
drag coefficient occurs.

Even under steady upstream conditions, however, the flow around the
cylinder is unsteady, since it is subjected to a strong flow-instability pheno-
menon over a wide range of Re. This instability consists of the alternate
shedding of vortices from the two sides of the cylinder at a regular frequency.
The frequency of shedding from one side is used to define the Strouhal Number,
$St = f_D/\bar{U}$. In addition, flow separation from the cylinder results in a turbulent
wake which also induces unsteady sectional forces on the cylinder.

The extent of the contribution of these two sources of unsteady forces
to the unsteady aerodynamics of the cylinder, and to its mean drag and shedding
frequency have been studied in detail over a wide range of Re. Much of the work
for the Re of interest in this experiment ($\sim 40,000$) has been reviewed by Keefe
(Ref.34). More recent work has generally dealt with extending similar investi-
gations to higher Re's in order to approach those represented by large cylindrical
structures in high winds (e.g. Refs. 35,36,37,38). For reference here, the beha-
vior of the Strouhal Number and the mean drag coefficient with Re is shown in
Figure 1-1 (adapted from Ref.37). It appears that at high Re the vortex shedding
phenomenon disappears over a wide range of Re ($4 \times 10^5$ to $1.5 \times 10^6$) but reappears
above this range. The Re of the present experiment is such that the cylinder's
properties are subcritical and do not change rapidly with Re.

Thus, for the reference state of the present experiment, the cylinder
is already subject to broad-band forces due to its wake, and strong periodic
sectional lift and drag forces occurring at $f_s$ and $2f_s$ respectively, due to vor-
tex shedding. The effect of these sectional forces on the integrated lift and
drag on a finite length of the cylinder will of course depend on the degree to
which they are laterally correlated. Since the major part of the force is origi-
nating from the vortex shedding, it is then essentially dependent on the length
of cylinder over which the vortex shedding occurs as a coherent sheet. Although
at small Re the shedding is coherent over a large length, for Re in the subcriti-
cal range, it has been found that this coherence length is of the order of two
to four diameters (Refs.39,40,41). This then normally results in little inte-
grated effect on a long rigid cylinder unless a coupling mechanism such as a
mechanical vibration of the cylinder or an acoustical reflection is available
to increase the coherency.

1.2 Experimental Concept

This section is intended to explain briefly the compromises and
restrictions which led to the particular attack taken on the problem, and to
provide a framework for the detailed discussions of the experimental method and
results to follow.

A variety of methods are available for producing turbulence in a wind
tunnel. A turbulent boundary layer builds naturally along a wall and its pro-
properties and rate of growth can be controlled by the addition of roughness or
screens. The mixing region of a jet also provides high intensity turbulence. However, it is difficult in both of these methods to develop a homogeneous intense turbulence region in a short streamwise distance. The method adopted here is the use of coarse square-mesh grids. These provide a crude but simple method of providing homogeneous turbulence whose characteristics are simply related to the physical grid characteristics (See Section 4). An alternative method which also appears suitable, but has only recently been developed, is that of the ejector driven wind tunnel (Ref.42).

The model design was required to allow investigation of the input/output relation between various types of turbulent fields and the resulting fluctuating aerodynamic loads. In particular, the interaction between the turbulence and the vortex-shedding phenomenon was to be investigated. Due to the physical limitations on model size, and the requirement of measuring force characteristics on a very small lateral segment to approximate sectional characteristics, the direct measurement of local forces proved impractical. Instead, the approach taken was to design the model to allow any two surface pressure points to be sampled simultaneously. In this way (as shown in Section 2.2) the statistical properties of the fluctuating sectional forces can be determined by integration. Furthermore, this method offered the advantage of directly giving the fundamental surface pressure information.

The force-response characteristics of the cylinder as a function of frequency were expected to be of a form which approached quasi-steady behaviour at low frequencies and zero at high frequencies. In terms of the parameter \( \frac{D}{\lambda} \), it is noted that the vortex shedding is expected at \( \frac{D}{\lambda} = 0.2 \). Hence it was desirable to try measure the unsteady aerodynamic response to a value of \( \frac{D}{\lambda} \) up to at least 1, and it would be ideal to approach 10 for this parameter. In actual fact a range from approximately 0.02 to 1 was realized.

Acceptable turbulence characteristics were then required to provide a reasonable energy density over the above range, and to provide turbulence intensities and relative scales \( \frac{L_u(x)}{D} \) of the same order as would be realistic in the atmospheric case. Hence, intensities of up to 20% accompanied by \( \frac{L_u(x)}{D} \geq 1 \) would be desirable. The values of \( \frac{L_u(x)}{D} \) attained here were of the order of five and lower, and the intensities ranged between 10 and 15%. Since a detailed modelling of atmospheric characteristics was not intended, these parameter ranges were considered acceptable.

The lack of experimental data and the inherent nonlinearities of this type of flow problem make it difficult to generalize as to the sensitivity of the cylinder's aerodynamic response to all of the parameters of the general atmospheric turbulence spectra (i.e. the complete tensor field). Nevertheless, it is of interest to note that Vickery (Ref.43) has shown a good comparison between grid turbulence and atmospheric turbulence over a wide range of \( \lambda/D \) for one-dimensional spectra.

II ANALYSIS METHODS

It is not intended here to derive the basic relations used, since they exist in common references, but rather to display them in common notation.

The basic techniques are those of random variable analysis (Ref.11). In this work it is assumed that the data analyzed are both stationary and ergodic. The prime statistical properties examined are the means, variances, and power
spectral distributions of the signals. The experimental techniques usually involved
derivation of the power spectra from direct measurement of the correlation function.
These functions are defined and related in Section 2.1.1. The power spectral informa-
tion has led naturally to the examination of frequency response functions to
describe the input/output aerodynamic relations involved. These are defined in
Section 2.1.2. The particular application of these methods in this experiment are
outlined in the latter parts of Section 2.

2.1.1 Correlations and Spectra

For any two random variables \( x(t) \) and \( y(t) \), the cross-correlation function
between \( x(t) \) at time \( t \) and \( y(t) \) at time \( t + \tau \) is defined as

\[
R_{xy}(\tau) = \lim_{T \to \infty} \frac{1}{T} \int_{0}^{T} x(t) y(t+\tau) \, dt
\] (2-1)

The corresponding physically-realizeable one-sided cross-power spectral
density of \( x(t) \) and \( y(t) \) is represented as a complex expression

\[
\Phi_{xy}(f) = C_{xy}(f) - i Q_{xy}(f)
\]

where it can be shown that \( C \) and \( Q \) are respectively even and odd functions of \( f \).
The cross-power spectral density and the cross-correlation functions are related
by the expressions

\[
\Phi_{xy}(f) = 2 \int_{0}^{\infty} R_{xy}(\tau) e^{-i 2\pi f \tau} \, d\tau
\] (2-2)

\[
R_{xy}(\tau) = \int_{0}^{\infty} [C_{xy}(f) \cos 2\pi f \tau + Q_{xy}(f) \sin 2\pi f \tau] \, df
\] (2-3)

These expressions reduce naturally to the auto-correlation and power
spectral density functions of a single time record which are, respectively

\[
R_{xx}(\tau) = \lim_{T \to \infty} \frac{1}{T} \int_{0}^{T} x(t) x(t+\tau) \, dt
\] (2-4)

and

\[
\Phi_{xx}(f) = 2 \int_{-\infty}^{\infty} R_{xx}(\tau) \cos 2\pi f \tau \, d\tau
\] (2-5)

\[
= 4 \int_{0}^{\infty} R_{xx}(\tau) \cos 2\pi f \tau \, d\tau
\]

\[
= \int_{0}^{\infty} \Phi_{xx}(f) \cos 2\pi f \tau \, df
\] (2-6)

2.1.2 Frequency Response Functions

For a physically-realizeable constant-parameter single-input linear
system which is subject to a stationary random input \( x(t) \), the frequency response function of the system \( H(f) \) can be determined from the relation

\[
H(f) = \frac{\phi_{xy}(f)}{\phi_{xx}(f)} \tag{2-7}
\]

where \( y(t) \) is the output of the system for \( x(t) \) as input, \( H(f) \) is a complex frequency response which can be represented in polar notation as

\[
H(f) = |H(f)| e^{-i\theta(f)} \tag{2-8}
\]

where \( |H(f)| \) is called the system gain factor and the associated phase angle \( \theta(f) \) is called the system phase lag. The determination of the system gain factor \( |H(f)| \) alone does not require cross-spectral information and can instead be determined from

\[
|H(f)|^2 = \frac{\phi_{Y_1Y_2}(f)}{\phi_{X_1X_2}(f)} \tag{2-9a}
\]

Furthermore, if two parallel identical linear systems are examined each with a frequency response function \( H(f) \), such that the two outputs are \( y_1(t) \) and \( y_2(t) \) and the two inputs are \( x_1(t) \) and \( x_2(t) \), then a form similar to equation 2-9a relates their respective cross-spectra, i.e.

\[
|H(f)|^2 = \frac{\phi_{Y_1Y_2}(f)}{\phi_{X_1X_2}(f)} \tag{2-9b}
\]

In this report, linear system approaches are applied to aerodynamic input/output relations which are at least to some extent non-linear. The resulting linear model may only be applicable over a limited range of inputs and hence is referred to in the text as a describing function.

2.2 Derivation of the Statistical Properties of Forces from those of Pressures

For a circular cylinder in a turbulent flow (see Fig.2.1), the total force acting upon a section at \( y \) can be resolved into a time varying lift, \( \ell(t,y) \) where \( \ell(t,y) = 0 \) and a total instantaneous drag, \( d_T(t,y) \) where

\[
d_T(t,y) = \bar{d}(y) + d(t,y)
\]

and \( \bar{d}(t,y) = 0 \)

Considering initially only the sectional drag, the time-varying component can be written in terms of the time-varying surface pressures at two stations as
\[ d(t,y) = \int_0^{2\pi} r \ p(t,y,\alpha) \cos \alpha \ d\alpha \] (2-10)

and

\[ d(t+\tau, y+\xi) = \int_0^{2\pi} r \ p(t+\tau, y+\xi,\beta) \cos \beta \ d\beta \]

Hence, to obtain the space-time correlation of drag between these two lateral stations, the two expressions above are multiplied together and averaged. Assuming the order of averaging and integration can be interchanged, then

\[ \overline{d(t,y) \ d(t+\tau, y+\xi)} = \int_0^{2\pi} \int_0^{2\pi} r^2 p(t, y, \alpha) p(t+\tau, y+\xi, \beta) \cos \alpha \cos \beta \ d\alpha \ d\beta \]

Then, if we further assume homogeneity and stationarity of the drag

\[ R_{dd}(\tau, \xi) = \int_0^{2\pi} \int_0^{2\pi} r^2 R_{pp}(\tau, \xi, \alpha, \beta) \cos \alpha \cos \beta \ d\alpha \ d\beta \] (2-11)

\[ R_{dd}(\tau, \xi) \] is the cross-correlation function of drag between two stations separated by \( \xi \). Due to homogeneity, it immediately follows that

\[ R_{dd}(\tau, \xi) = R_{dd}(-\tau, \xi) \]

Hence, the cross power spectral density of the drag will have no imaginary component and can be written as

\[ \phi_{dd}(f, \xi) = 4 \int_0^{\infty} R_{dd}(\tau, \xi) \cos 2\pi f \tau \ d\tau \] (2-12)

Note that this expression contains the sectional drag's autopower spectra as a special case for \( \xi = 0 \).

If the characteristics of the fluctuating load on a finite length \( L \) of the cylinder are required, then a similar approach to the above is adopted, i.e. the time-varying component of the total drag on the length \( L \) is

\[ D(t;L) = \int_0^L d(t,u) \ du \quad \text{where} \quad D(t;L) = 0 \]

Then

\[ R_{DD}(\tau;L) = \int_0^L \int_0^L R_{dd}(\tau, \xi) \ du \ dv \]

and

\[ \phi_{DD}(f;L) = 4 \int_0^{\infty} R_{DD}(\tau;L) \cos 2\pi f \tau \ d\tau \]

The above two expressions give the auto-correlation and the auto-power spectra of the total fluctuating drag on the finite length \( L \).
Parallel derivations for the fluctuating lift on a section and a finite length yield:

\[ R_{ll}(\tau,\xi) = \int_0^{2\pi} \int_0^{2\pi} r^2 R_{pp}(\tau,\xi,\alpha,\beta) \sin \alpha \sin \beta d\alpha d\beta \] (2-13)

\[ \phi_{ll}(\tau,\xi) = 4 \int_0^\infty R_{ll}(\tau,\xi) \cos 2\pi f \tau d\tau \] (2-14)

\[ R_{LL}(\tau,L) = \int_0^{L} \int_0^{L} R_{ll}(\tau,\xi) d\xi d\eta \]

\[ \phi_{LL}(\tau,L) = 4 \int_0^\infty R_{LL}(\tau,L) \cos 2\pi f \tau d\tau \]

The integration of equations 2-11 and 2-13 then requires definition of the time delayed pressure correlation between any two arbitrary circumferential angles. In practice this requires defining a complete matrix of pressure correlations for \( \alpha \) and \( \beta \). The digital integration technique used, and its accuracy are discussed in Appendix D.

### 2.3 Analytic Aerodynamic Transfer Function Relations

For later comparison with experimentally derived describing functions, it is of interest to consider the analytic form assumed in the cylindrical drag response study by Etkin (Ref. 26). In this theory, the sectional drag is assumed to obey a "strip theory," i.e.

\[ d_T(t,y) = F_1(y) \dot{U}(t) + F_2(y) \ddot{U}(t) \]

where

\[ F_1(y) = \frac{\rho}{2} C_d(y) D(y) \]

\[ F_2(y) = \rho k(y) D^2(y) \]

and

\[ \ddot{U}(t) = \frac{dU}{dt} \]

In the case considered here, the sectional drag coefficient \( C_d \), the additional mass coefficient \( k \), and the diameter \( D \) are assumed to have no \( y \)-dependence. Etkin then shows that on linearization, the turbulence induced fluctuating load is given by

\[ d(t) = 2 F_1 \dot{U} u(t) + F_2 \ddot{U}(t) \]

Thus,

\[ R_{dd}(\tau,\xi) = 4 F_1^2 \dot{U}^2 R_{uu}(\tau,\xi) + F_2^2 R_{\dddot{U}}(\tau,\xi) \]

\[ + 2 F_1 F_2 \dot{U} [R_{uu}(\tau,\xi) + R_{\dddot{U}}(\tau,\xi)] \]
On Fourier transforming the above equation, and making use of the relations
\[ \phi_{uu} = (2\pi f)^2 \phi_{uu} \]
\[ \phi_{uu} = i 2\pi f \phi_{uu} \]
\[ \phi_{uu} = -i 2\pi f \phi_{uu} \]
we obtain that
\[ \phi_{dd}(f,\xi) = [4 \rho^2 C_d^2 U^2 + 4\pi^2 f^2 C_a^2] \phi_{uu}(f,\xi) \]

Hence the expected value of \( |H_{du}(f)|^2 \) is
\[ \frac{\phi_{dd}(f,\xi)}{\phi_{uu}(f,\xi)} = \frac{\rho^2 D^2 C_d^2 U^2 + 4\pi^2 \rho^2 k^2 D^4 f^2}{4} \]  

(2-15)

Using the appropriate experimental \( C_d \) (Re), and the theoretical value of \( k = \pi/4 \), both Campbell (Ref.26) and Cooper (Ref.9) have shown that this approach to the aerodynamic transfer function gives good predictions of an elastic cylinder's base bending moment at low frequency. Cooper's results show that the experiment and theory start to diverge for values of \( \lambda/D < 36 \) although agreement is reasonable to \( \lambda/D \approx 10 \). (Based on a mean profile velocity).

A similar strip theory approach can be used for the lift response (neglecting the vortex excitation). In this case, the assumption is made that the turbulence intensities are small enough such that the cross-component of turbulence, \( w(t) \) causes an angle of attack change of the instantaneous velocity vector, but does not appreciably change its magnitude. In this case, the fluctuating lift is simply a component of the fluctuating drag, i.e.
\[ \ell(t) = d_{\ell}(t) \frac{w(t)}{U} \]

which, when linearized to first order terms gives
\[ \ell(t) \approx F_{\ell} \bar{U} w(t) \]

and then the expected value of \( |H_{\ell\ell}(f)|^2 \) is
\[ \frac{\phi_{\ell\ell}(f,\xi)}{\phi_{ww}(f,\xi)} = \frac{\rho^2}{4} D^2 C_d^2 U^2 \]  

(2-16)

which is just one-quarter of the drag response at low frequency.
III DESCRIPTION OF APPARATUS AND ITS VERIFICATION

3.1 Model Hardware

The model required for the experimental measurements was a rigid circular cylinder, which would allow measurement of any arbitrary surface pressure correlation $R_{pp}(\tau, \xi, \alpha, \beta)$. To ensure a minimum of cylinder movement, the cylinder was designed to pass through the tunnel without contact. An overall schematic of the set-up is shown in Figure 3-1.

The cylinder was constructed in two sections, each cantilevering from one of the external mounts and held together by an external compression load. Each cantilever was made up of a section of dummy cylinder (thick-walled steel tubing) and a screw-on module. Each module had a single surface hole (see Figure 2-1 and 3-2) which communicated the point surface pressure to the module interior. For static pressure measurements the modules were otherwise sealed except for a plastic tubing connection to a Betz manometer. For fluctuating pressure measurements the modules contained microphones (described in Section 4.4) as pressure transducers. The modules in various configurations are shown in Figures 3-2 and 3-3. Also shown are spacers which were used to vary the lateral displacement, $\xi$, of the measuring stations.

The end compression load applied to the cantilevers and spacers was such as to prevent any surface stresses on the two piece beam from becoming tensile. The two modules were made with a small male/female type fit to ensure proper alignment. Because of the cylindrical symmetry, and lack of any actual fastening between the two modules, rotation of both modules to independent arbitrary angles could be performed without disassembling the model. Furthermore, the fabrication of various lengths of spacers (Figure 3-2) allowed arbitrary lateral spacings to be obtained. The only compromise in this approach is that there is a minimum lateral spacing obtainable which was 0.16" or 0.129D in this case.

The complete cylinder model as seen looking downstream in the diffuser is shown in Figure 3-4. The surface finish of the cantilever parts of the model was that obtained by centrelessly grinding the steel tube to its final diameter of 1.24". The modules themselves were gold plated inside and gold and nickel plated outside to prevent corrosion. This improved still further the surface finish and hence the main sources of roughness were the holes themselves and the mating seams between modules and spacers. The holes were 1/16" in diameter and hence subtend angles of 5.78°. The mating seams produced surface discontinuities of the order of .002" or less. Figures 3-1 and 3-4 also indicate the type of end seals used. The end fairings produced a locally parallel duct, thus allowing the cylinder with its attached end plates to be rotated arbitrarily. The gap between the end plate and the inner wall of the recess was nominally 1/8", forming a simple labyrinth seal for air flowing behind the end plate. In addition, the recess was sealed from the external tunnel environment by a thin rubber tube connecting the wind tunnel to the external cylinder end mount. This rubber tube was the only physical connection between the tunnel and the cylinder mounting, other than through the floor to the end mount. The elaborate end seals were incorporated to eliminate the end effects observed in previous work by Keefe (Ref.34), who found that preventing appreciable wake leakage was vital to the fluctuating lift and drag measurements in low turbulence flow.

The external mounting of the cylinder (Figure 3-5) was such that each cantilever was held in a 12" long steel block. The block in turn was adjustable
to provide the four degrees of freedom required to align the cantilever properly before the end load was applied. One end mount then clamped its cantilever, while the other end mount provided the compression load using a single leaf spring (Figure 3-5). The angular orientation of each cantilever was determined by a ring which slipped over the outer end of each cantilever and was prevented from rotating with respect to the cylinder by means of a set screw riding in a longitudinal V-groove machined in the cantilever. The ring bore 2° scribe lines, and was read against a needle pointer attached to the end support block. The accuracy of angular position obtainable is approximately ± 0.25°.

The entire cylinder end mount assembly on each side rides on a mount designed to prevent any residual external vibrations in the floor from reaching the cylinder, and to prevent any appreciable cylinder/mount response due to aerodynamic inputs. Each vibration isolation mount consisted of a weighted steel box supported by four coil springs and foam rubber pads in the lift direction. It was restrained from motion in the drag direction by a leaf spring, and supported the compression end load on the cylinder by means of tension flexures connecting the sprung mass to the unsprung structure. The two unsprung structures were rigidly connected underneath the tunnel by steel I-beams. Each sprung mass was approximately 400 lbs., while the unsprung structure was of the order of 1000 lbs.

The natural frequencies of the sprung end supports were approximately 4 cps in the lift direction and 2 cps in the drag direction.

3.2 Working Section Calibration Equipment

To provide means for measuring flow characteristics along the proposed model centre-line, a dummy cylinder was built, consisting of three pieces of steel tube similar to that used for the cantilever parts of the model. The three pieces bolted together to provide a sliding rod which could be used to position probes mounted on it at arbitrary points across the working section. The same end mounts were used, but were moved to a second location 5.5" behind and 1" below the model. This allowed probes to traverse the exact model position. Longitudinal V-grooves were machined into the outer surface to provide an alignment reference for small aluminum blocks which could be slid spanwise (see Figure 3-6).

Investigation of flow field gradients requiring positions off the model were made using a secondary rod of adjustable length which was mounted directly to the diffuser walls. No effects of the higher vibration level could be detected in such measurements (using hot wire probes), although rod vibrations could be seen.

3.3 Calibration of Cylinder End Compression Loads

In order to calibrate the end compression load as a function of leaf spring deflection, a plastic spacer was fabricated, and strain gauged. The plastic was required to obtain easily measurable strains. The spacer was itself calibrated initially by directly loading it with weights. Typical resulting spring calibrations when it formed part of the model cylinder are shown in Figure 3-7.

During the experiment, the minimum compression load maintained was 100 lbs. which is roughly four times the minimum required.
3.4 Cylinder Motion

In order to determine the cylinder motion when subjected to turbulence, an accelerometer was clamped to one of the hot wire probe mounting blocks which was in turn placed at the centre of the dummy cylinder. The dummy cylinder was used for simplicity, since its mechanical properties were identical to those of the model across the working section, except for the absence of the microphone modules. The accelerometer was calibrated by noting the increment in voltage due to the 2g effective change in acceleration incurred by turning it over. The turbulence used for the test condition was the highest intensity and largest scale used in the later tests (Grid 4). Accelerations obtained at the mid-point of the dummy cylinder were tape-recorded for both the lift and drag direction, and were reduced to power spectra by the method of Appendix B.2.2. The displacement spectra were then derived from the acceleration spectra by using the relation

\[ \phi_{\eta} = \frac{1}{16\pi^4 f^4} \phi_{\eta \eta} \]

where \( \eta \) is a displacement in either the lift or drag direction.

Typical displacement spectra are shown in Figure 3-8. The accelerometer was specified as having a critically damped naturally frequency at 75 Hz. This was not corrected for in the spectral data. When observing the acceleration data on an oscilloscope, the drag response appeared almost purely sinusoidal at the first natural mode which occurs at 40 Hz. This corresponds to a calculated first mode frequency of 60 Hz, assuming perfectly rigid end mounts. The lift response, however, was a mix of a number of frequencies as can be seen. The maximum rms acceleration was obtained in the drag direction. The total rms displacement can only be obtained from this data by integrating \( \phi_{\eta \eta} \). However, the order of overall displacement of the cylinder can be obtained directly in the drag case by assuming that all of the displacement was occurring at 40 Hz. The total rms acceleration measured in this case was 0.49 g's = 19 ins/sec^2 which implies that the equivalent rms displacement at 40 Hz is 3 \times 10^{-4} ins. As a comparison, the thickness of a laminar boundary layer on a circular cylinder at \( \text{Re} = 40,000 \) is of the order of 5 \times 10^{-3} ins. (Ref.44). Hence, the cylinder can be considered rigid.

IV BASIC INSTRUMENTATION

Section IV describes the general capabilities of the instrumentation used. In some areas, specialized techniques have been developed. These cases are briefly mentioned below and are discussed in more detail in the Appendices.

4.1 Wind Tunnel

The wind tunnel available for the experiments is shown schematically in Figure 4-1. It is a closed circuit design with a maximum speed when empty, of approximately 200 fps. The test section has an octagonal cross-section and is 48" x 32" x 4 ft long. The tunnel's reference wind speed is taken as the equivalent test section speed, \( U_r \), and is derived from measuring the test section dynamic pressure, \( \frac{1}{2} \rho U_r^2 \). This is read directly from a Betz manometer connected to static taps at the entrance to the test section and in the stagnation chamber. This value of \( U_r \) agrees within 0.5% with that obtained directly in the test section using an NPL standard pitot-static probe.
4.2 Analog Computer

The laboratory's EAI 221R analog computer was used primarily as a central on-line data processor. In this role, transducer signals are fed into it through a trunkline network, and the computer then performs various real-time manipulations on the signals to produce required forms. The major uses of the analog computer in this experiment were:

1) linearization of hot wire signals (see Section 4.3)

2) retrieval of zero-time-delay correlation coefficients, and

3) removal of D.C. offsets, low and high pass filters.

A block diagram illustrating the production of zero-time-delay correlations is shown in Figure 4-2. For any two time varying input signals, it produces estimates of the two means, the two mean squares, and the mean product by integrating the relevant quantities over a fixed time. These five outputs were usually transferred to IBM cards so that the correlation coefficient could be calculated and plotted. Figure 4-2 also shows the frequency response of this circuit.

Since the fluctuating components of the signals were usually of most importance, D.C. removal was required prior to such operations as F.M. tape recording or zero-time correlation. However, the removal of low frequency information distorts the measured estimates, hence some care is required. The effects of truncation at low frequencies on rms measurements is discussed in Appendix E. The effect of a low frequency cutoff on measured correlation functions is discussed in Reference 45. In this experiment, the microphone outputs included a simple D.C. offset which was corrected in kind with no additional low frequency effects. Hot wire outputs, prior to correlating or recording, included a D.C. level corresponding to the mean flow velocity. A simple analog low pass filter was used to separate the mean so that it could be read on a digital voltmeter. The required high pass filter was then formed by differencing the original signal with the output of the low pass filter. A schematic of the circuitry, and its frequency response, is shown in Figure 4-3.

The 221R is a 100 volt system equipped with a variety of readout devices, including a digital voltmeter with a 10 mv. resolution. The overall accuracy available depends on the particular number and type of manipulations used. For the circuits used here, the errors introduced were primarily nonlinearities in the hot wire linearizations - which automatically show up in system calibrations - and additional electronic noise. The latter was usually the most significant electronic noise source. A single amplifier's output noise for grounded input is less than 1 mv. rms. The noise output of the complex linearizer circuit plus hot wire amplifier was measured as less than an equivalent turbulence level of .03%. The frequency response of the electronic components was typically D.C. to 40 khz. (3dB down).

4.3 Hot Wire Instrumentation

Both mean and fluctuating properties of the flow were investigated using hot wire techniques (Ref.1). Four of the hot wire amplifiers available were of a Kovasznay design, produced by Leslie Miller of John Hopkins University. These amplifiers were modified to provide an option of temperature compensation.
(see Appendix A.1) when in use in the closed circuit wind tunnel. A Disa model 55A01 was also available. The unlinearized hot wire signals were fed into the analog computer, where they could be linearized accurately and simply by circuits employing summers and X2 cards (fixed diode function type). The setup is discussed in more detail in Appendix A.3. The computer provided up to four simultaneous channels of linearization, including high pass filters for removing D.C. levels in preparation for recording.

The hot wire probes employed were of two basic conventional types - u-probes and X-probes (see Figure 3-6). Both used .00020 in. dia. tungsten wire with an active length of 0.135 ins. (length/diameter = 675). The u-probes were simply Disa 55A25 probes with the needles spread slightly apart to take the longer wire length. The wire in this case was welded directly to the needle supports, using a capacitance discharge technique such that the wire was perpendicular to the axis of the probe body. The X-probes were of a "home-built" variety, with each probe body having two pairs of needle supports. Each pair of needles supported a length of tungsten wire at 45° to the probe axis and at 90° to each other in two parallel planes. The two tungsten wires in this case were copper plated except for an active element of 0.135" in the centre and were soldered to the supports. The lateral distance between the planes of the two wires was 0.10".

The frequency response of the hot wire systems is discussed in Appendix A.2, including the effect of the finite hot-wire length on the measurement of three-dimensional turbulence.

The availability of four simultaneous and independently linearized hot wire channels allowed a variety of measurements to be taken including two-point longitudinal and lateral velocity component correlations. The flexibility of the computer linearization greatly simplified the cross-component measurements since it allowed matching of X-wires with different physical properties. Details of the techniques employed in measuring such typical quantities as $u(t)$, $w(t)$, $uw(t)$, $u_1u_2(t)$, $w_1w_2(t)$ are outlined in Appendix A.3.

4.4 Microphones

The instruments chosen to act as pressure transducers for the cylinder surface pressure measurements were Bruel & Kjaer type 4132 one-inch condenser microphones with type 2613 cathode followers. Both the microphones and cathode followers were modified to allow a useable pressure response between 1 Hz and 2000 Hz. The actual frequency response was affected by the cavity coupling the microphone diaphragm to the surface pressure sensing hole on the model (see Figure 3-2). Hence, the complete physical system was calibrated. The resulting normalized mean amplitude and phase response is shown in Figure 4-4. These measurements are described in Appendix C.

The sensitivity of the system at the cathode follower output is of the order of 2 volts/ph. The maximum linear rms pressure level (4% distortion) is 8.4 psf compared to experimental pressures which did not exceed 2 psf (rms). The inherent electronic noise level of the system detectable with a Random Noise Meter (including the latter's contribution) is equivalent to less than .0015 psf (rms). The microphones' residual sensitivity to such factors as atmospheric pressure, vibration, etc. are discussed in Reference 46, but are of negligible effect in this experiment. Of particular interest is perhaps the temperature sensitivity. However, over the normal wind tunnel temperature range of 68 - 122°F., Reference 46 shows approximately 0.5% change in microphone sensitivity, and hence this
effect was also neglected.

4.5 Random Noise Meters

Two Bruel & Kjaer type 2417 random noise meters were used for the majority of rms measurements of both velocities and pressures. These meters provide true rms readings for sine wave or Gaussian random noise inputs. Their accuracy is $\pm 1\%$ of full scale deflection. The frequency response is quoted by the manufacturer as $\pm 0.2\text{dB}$ from 2 Hz. to 20,000 Hz. Over the range of interest in this experiment (from DC - 2,000 Hz), experimental frequency response measurements showed the meters to be flat above 2 Hz. ($\pm 1\%$). The shape of the low frequency cut off is shown in Figure E-2. Its truncation effects on the data are discussed in Appendix E.

The meters provide variable time constants of 0.3 to 100 seconds. For statistical purposes, these correspond to effective record lengths from 0.6 to 200 secs. (Ref.2).

4.6 Analog Tape Recorder

All the analog tape recording was performed using an Ampex SP 300 four track AM/FM tape recorder. In this experiment, the FM mode was used for all data recording. The AM mode was used for recording timing pulses when required. The FM system has a usable frequency response from DC to 2,500 Hz at 15 inches per second. The upper frequency limit is proportionally reduced for the other available tape speeds of 1-7/8, 3-3/4 and 7-1/2 ips. Recording and playback at different speeds allowed frequency expansion and compression when required. Because the frequency response of the recorder was not flat over the experimental range, extensive calibrations were performed for each record/reproduce combination required. It was found that the fall-off with frequency could in all cases be reasonably approximated by a first order amplitude response. A typical experimental amplitude response and its first order fit are shown in Figure 4-5. The analytical fit was then used for data correction purposes. No phase shift calibrations were performed. Only in cross-spectral measurements between two data channels would this be important, and in this case only the difference in phase shifts of the two channels would be necessary. This was assumed small for the only applicable case (the coherence measurements, Sections 6.4 and 8.3).

Since the input gain control on the SP 300 was continuously variable, the actual gain for each record was obtained by recording a known amplitude 60 Hz signal (to avoid D.C. drift) and observing its output level when that record was being analyzed.

4.7 Spectral Analysis Systems

Three different spectral analysis techniques were employed during the experiment. The first was an analog filter type of approach. The other two both first obtained correlations, and then spectra by a Fourier transformation. A brief description of the physical systems is given below. Discussion of the implications of the digital sampling techniques inherent in the last two systems in Appendix B.2. The relative merits of the three systems are discussed in Appendix B.3, including comparisons of the spectral estimates of identical data obtained from all three systems.
4.7.1 Analog Wave Analysis

A Bruel & Kjaer Audio Frequency Analyzer Type 2107 was used for "on line" spectral analysis. It provides a choice of six different bandwidth filters, whose centre frequencies are continuously tunable from 20 - 20,000 hz. The filters have effective bandwidths ranging from approximately 8% to 33% of the centre frequency. The instrument was useful in applications where very few spectra were required and it also provided a simple means for verifying the accuracy and consistency of the other two more sophisticated systems. A brief description of the method of data reduction involved, and of its overall accuracy, is included as Appendix B.1.

Due to the wave analyzer's lower limiting frequency of 20 hz., data compression using the Ampex SP 300 was required to obtain "direct" spectral measurements below 20 hz. In this way, the lowest frequency analyzable was 2.5 hz.

4.7.2 Digital Data Reduction System

In order to streamline the collection of the large number and variety of spectral distributions required to describe the flow fields and cylinder surface pressures, a digital data reduction system was utilized. The method is shown schematically in Figure 4-6. First, the data of interest was recorded on the Ampex SP 300 in the F.M. mode (one, two, or three channels). Simultaneously, a series of timing pulses were recorded on another channel in the A.M. mode.

At a later date, the analog tape was replayed through some data conditioning electronics (see Appendix B.2.1) into an EECO model ZA 37050 Data Acquisition System. The latter provides a high speed digitizing mode whereby it accepts one channel of data at a time which it samples at one of three fixed rates - 400, 500 and 600 per sec. Only the highest of these rates was used, implying a theoretical upper limit of 300 hz which could be detected in the data (see Appendix B.2). However, using various analog tape recorder speed reductions on playback, the effective sampling frequency could be increased to 8 x 600 = 4,800 samples/sec, providing a theoretical upper limiting frequency of 2,400 hz. In practice, useable data could be obtained as high as 2,000 hz.

The controlling of the EECO system with tape recorded pulses introduced a variety of problems due to:

i) the requirement that the first recorded pulse be clean,

ii) occasional bad spots of tape which reduced the recorded pulse level below the EECO trigger level.

An electronic gate solved the first problem simply. However, the solution of the second was more complex and is displayed in Figure 4-6.

The EECO system's output was a digital tape written at 200 bits per inch, containing a series of records in ungapped format. To convert this to a Fortran compatible tape, a special machine language program for the IBM 7094 computer was obtained, which translated the "raw" tape into standard gapped format at 556 bpi. In the high speed EECO mode, no information was available as to whether the data was actually written correctly at the time of recording, hence the translation program accepted whatever it read on the raw tape without demanding correct parity. However, an indication was provided of the accuracy of the translation by writing out whether any read errors had been encountered (and hence accepted) during each
Finally, the translated tapes were used as the input to a Fortran program on an IBM 7094 computer which produced digital estimates for the auto- and/or cross-correlation functions and their respective spectra (Appendix B.2.2 and Reference 13). Generally, these estimates were obtained in punched card form and were later manipulated further (averaging, plotting, correcting, etc.) on an IBM 1130 computer.

The results analyzed using this system consist of everything presented except the two-point surface pressure correlations used in the derivation of the time-varying lift and drag results, or where an alternate analysis system is specifically noted.

4.7.3 Hybrid Data Reduction System

Late in the project, a new type of instrument became available which was capable of producing time-delayed correlations "on-line" and hence did not require the previously described time-consuming process of digitizing the time record itself. This Princeton Applied Research (PAR) model 100 or 101 correlator provides 100 estimates of the cross-correlation function of the two inputs. These 100 values are stored in analog fashion in capacitive memories. The memory can be scanned at various rates so as to allow oscilloscope displays or plotting. Furthermore, the scanning can be controlled by a digital output device. In the case used here, (see Figure 4-7), the correlator was interfaced to a CIMRON model 6840 Data Logging System and then connected to an IBM 526 Summary Punch. The CIMRON is essentially a Digital Multimeter which acts as an A/D Converter. Its output is then simply punched on cards by the Summary Punch. The correlator allows sampling rates from 10/sec. to $10^6$/sec. Any particular one of the 100 estimates of the correlation function is averaged using a first order filter (essentially the capacitive memory). Thus, the effective record length is simply twice the filter's time constant. The latter is easily variable by means of internal resistance changes but is normally left constant for a particular experiment. In the case examined here, the filter time constant was 20 sec., and hence the effective record length was 40 secs.

In practice, the signals to be correlated were connected through some signal conditioning electronics (see Appendix B.2.2) into the correlator. Then, to obtain recording of the positive part of an auto- or cross-correlation required waiting five filter time constants (100 secs.) - to allow the memory to come within 1% of its true value - and then required initiating the readout of the memory on to 8 punched cards ($\approx 45$ secs). This punched card output was Fourier transformed on an IBM 1130 to provide spectral estimates.

Due to the nature of the sampling used by the correlator, this transform is not entirely straightforward. It is discussed further in Appendix B.2.2.

V GENERATION OF TURBULENCE

As mentioned in Section I, the turbulent fields required were produced by placing uniform, biplanar grids in the air-flow. The turbulence behind grids has been studied extensively in the past. It has been found that immediately behind the grid, the flow is highly inhomogeneous with a strong memory of the particular grid geometry. The flow becomes more homogeneous and the turbulence in-
tensity decreases with downstream distance. Typically the flow approaches homogeneity between about five and ten mesh lengths, and approaches isotropy after about twenty mesh lengths. Most of the past interest has been concentrated on the fundamental properties of the quasi-isotropic turbulence, (e.g. Ref. 49). However, the region of interest here is that occurring when the flow first becomes homogeneous, since it offers intensities of the same order as atmospheric turbulence. The use of this region for model testing has been developed in this project (Refs. 47 and 48) and by others, notably Vickery (Ref. 43). Basic design data is available from the extensive tests in this region performed by Baines and Peterson (Ref. 50) and from extrapolations of data given by Batchelor and Townsend (Ref. 49).

5.1 Grid Design

The basic design objective for this project was to produce homogeneous turbulence of various scales up to the largest practical, together with turbulence intensities in the range from 10-20%. In general, these grid design requirements are essentially separable. Following Reference 49, a square-mesh grid can be thought of as having two characteristics which are the primary determinants of the turbulent flow behind the grid. The first is the drag per unit area, since it directly determines the amount of turbulent energy created by the grid. The second is some physical length, M', associated with the grid, since this determines the dimensions of the initial wakes. M' will be some function of the mesh M, the bar width b, and the bar depth b'. Hence, at some non-dimensional distance downstream x/M', the scale will be determined by M' and the intensity by the grid drag coefficient, C_G.

Batchelor and Townsend found that their results behind circular cross-sectioned grids for x/M > 20 could be accurately described by

$$\frac{\overline{u'^2}}{\overline{u'^2}} = \frac{\beta}{C_G} \left( \frac{x}{M'} - \frac{x_0}{M'} \right)^n$$

with n = 1, \( \beta = 106 \), M' = M, and the drag coefficient (for grids with circular cross-sectioned bars) given by

$$C_G = \frac{S}{(1 - b/M)^4}$$

Here b is the bar diameter, and S is the grid solidity, or ratio of closed to total area, S = b/M (2-b/M). The parameter x_0 is the extrapolated virtual origin of the data which in these experiments varied between 3 and 20M.

Baines and Peterson found that their data for biplane grids with square cross-sectioned elements could be described well by

$$\frac{\overline{u'^2}}{\overline{u'^2}} = 0.785 \left( \frac{x}{b} \right)^{10/7}$$

after an initial establishment region, which tended to be about 2M to 3M in each case. Equations 5-1 and 5-3 approach a similar form except for the value of the exponent n if x_0 is assumed small and if C_G = (b/M)^n. For n = 1, this latter fact is true for small solidity as seen from equation 5-2. Because of the collapse of the flow establishment length scaled in mesh lengths, it is suggested that a
good collapse of Baines and Peterson's data may also have occurred in the form of equation 5-1 if experimentally measured values of $C_G$ had been introduced.

Nevertheless, for this project, the data of the above two investigations adequately defines the intensity variation with downstream distance. Figure 5-1 shows the mean curves of experimental results as presented in Reference 50, and also shows the comparative form of equation 5-1.

Whereas the intensity varies rapidly with downstream distance, the turbulence scale increases slowly due to the dissipation of the high frequency turbulence components. For the range of interest here ($z/M = 5$ to $20$), the longitudinal scale is of the order of $0.4M$. Thus for a grid with three members in a square tunnel cross-section, having $M/2$ clearances with the wall, the expected scale will be of the order of $13\%$ of the tunnel width. This in fact appears to be a practical maximum as found both here and by Vickery.

The design process for a uniform duct using the above information is then very straightforward. The overall grid dimensions determine the scale, and the required intensity is obtained by using a suitable grid solidity and downstream distance. The grid solidity determines the grid drag coefficient, and hence the load to be supplied by the wind tunnel fan.

In our particular case, a further complication arose due to the small length of the test section which did not allow enough distance for flow establishment for the large turbulence scales required. This led to using the first diffuser as the test region (see Figure 4-1), and hence introduced some doubt as to the accurate application of the available grid data.

In order to obtain intensities as high as possible, the load characteristic of the wind tunnel was measured so that the grid drag could be maximized. In practice this meant the tunnel was operated just around stalling speed. The UTIAS wind tunnel's experimentally derived load characteristic is plotted in Figure 5-2. In this case $P$ is the pressure drop measured at a particular station in the diffuser due to a blockage introduced in the test section. It was found experimentally from wall pressure taps at several stations in the diffuser that the pressure gradient present in the diffuser was always within experimental error of that predicted by the Bernoulli equation, i.e. that the total pressure at some distance $x'$ measured downstream from the test section could be expressed reasonably well by

$$P_T(x') = P_T - \Delta P_L - \Delta P$$

(5-4)

where $\Delta P_L$ is a loss associated with the breather slot and the diffuser entrance up to the first measuring station, and $\Delta P$ is the pressure drop across the arbitrary blockage placed in the test section. Then equation 5-4 becomes (assuming continuity and uniform velocity profiles)

$$\frac{P(x') - P_T}{q_r} = \left[1 - \left(\frac{A_r}{A(x')}\right)^2\right] - \frac{\Delta P_L}{q_r} - \frac{\Delta P}{q_r}$$

(5-5)

The first two terms on the right side of 5-5 represents the empty tunnel diffuser pressure distribution, and hence the addition of a load simply displaces the whole curve by $\Delta P/q_r$. Experimental verification of this behaviour is shown
in Figure 5-3, which shows data typical of that used to define Figure 5-2.

The design speed through the test section for maximum turbulence intensity was then set at 120 fps, from Figure 5-2, thus setting a maximum allowable \( \Delta P/q_x \) for any grid at 0.65.

Data for the pressure drop across square-bar grids is given by Reference 50, and is reproduced here as Figure 5-4. Although the grids used here had bars of rectangular cross-section, this data proved adequate for the iterative technique which proved most simple due to the uncertain diffuser effects.

In summary, then, the design steps were as follows:

1. The grid mesh was taken as 2.5 times the required longitudinal scale (with an upper limit of 13% tunnel width).

2. The homogeneity requirement was normally set at 9 or 10 mesh lengths between the grid and the working section and hence fixed the physical location of the grid.

3. The maximum \( b/M \) of the grid was then determined from Figure 5-4, where

\[
\frac{\Delta P}{q(x')} = 0.65 \left[ \frac{A(x')}{A_r} \right]^2
\]

and where \( A(x') = 1 + 0.00497x' \) (where \( x' \) is in inches) for the UTIAS first diffuser.

4. Figure 5-1 then defines the approximate expected longitudinal intensity.

If this value of intensity is insufficient, then a compromising second design must be made in which either homogeneity or scale is sacrificed. The latter allows \( \Delta P/q(x') \) to be increased by moving the grid further down the diffuser and hence increasing \( [A(x')/A_r]^2 \).

It should be noted that Baines found that the flow behind grids of solidity greater than 0.5 becomes unstable. This was also indicated by some preliminary grid development work carried out by Kupcis at UTIAS (Ref.47).

5.2 Grid Development

When a design was finalized as above and built, it was generally found that it did not conform closely enough to design requirements in terms of intensity and maximum test section speed. This implies that either the tunnel was stalled or higher intensities were possible by increasing the grid solidity. In this case a perturbation calculation was performed, using the uniform duct data to predict the required solidity change to the experimental situation in order to modify the measured results to those desired.

The process was carried out as follows. Given an initial grid design aimed at maximizing intensity, of solidity \( s' \), that allows a test section speed of \( U_x \) which is near, but not exactly the 120 fps required, then Figure 5-2 immediately yields the change in grid drag required as
where $\delta(Ap/q_g)$ is simply the difference in $Ap/q_g$ at $U_0$ and at the design speed. Then a similar process yields the change in grid solidity required as

$$\delta(s) = \left[ \frac{d(Ap/q_g)}{ds} \right]_{s=s_0} \delta(Ap/q_g)$$

where the derivative is found from Figure 5-4. Finally the new solidity is implemented by increasing the bar width as required.

This procedure proved successful in one iteration in all cases attempted.

However, a further problem often remained, in that the flow at the working section was not as uniform as desired, or as predicted by uniform duct results at similar downstream locations. It became apparent, and has also been reported by others, that the flow is more sensitive than expected to the grid construction tolerances (which were quite loose in this case), especially for large solidities. Furthermore, the diffuser may accentuate this effect. This problem was solved to a great extent by minor empirical changes in the geometry of the individual grids (such as local changes in bar width or small modifications to the leading edges of particular bars in the grid). This process was somewhat time-consuming but effective as shown in Figure 5-5 for an early grid design.

In order to verify that the resulting flow was actually homogeneous to a good approximation, some statistical properties of the turbulence at various spanwise locations were measured. For instance, the sensitivity of the lateral correlations of the streamwise component has been investigated. Figure 5-6 shows the measured lateral correlations of $u$ at various spanwise positions accompanied by the mean flow distribution for an early version of grid 4. The results are identical within experimental error.

### 5.3 Final Grid Designs and Verification

The final outcome of this turbulence field development was four grids producing longitudinal scales between 0.45 ins. and 5.5 ins. ($\approx 9\%$ of local tunnel width) and intensities of 10% or greater at the intended model position. Table 1 summarizes the physical and flow properties associated with these grids, including the empty tunnel case. Figure 5-7 shows the grids mounted in the diffuser. In particular, they can be characterized according to their scale and intensity combination. Figure 5-8a shows longitudinal intensity vs. scale in order to indicate the range of parameters covered.

An indication of the degree of anisotropy present at the working section is given by Figure 5-8b, which shows mean values (over the test region) of the three turbulent velocity components for each turbulence case considered. Anisotropy is present in all cases, but is most severe for the highest intensity. Part of the anisotropy is inherent in the grid turbulence and part is due to the diffuser effect. However, it is not known to what degree each effect is present.

The flow at the model section was also investigated to determine the local gradients of the flow parameters there. As previously mentioned, the static pressure gradient in the diffuser is given reasonably well by the combination of
continuity and the incompressible Bernoulli equation as:

$$\frac{d \left( \frac{P-P_M}{q_M} \right)}{d(x/D)} = 2 \left( \frac{A_M}{M} \right)^3 \frac{d (A/A_M)}{d (x/D)}$$

$$= .00541$$

at the model section since $A/A_M = .438 (1 + .00616 x'/D)$. Similarly, the associated gradient of the mean velocity is given by continuity as

$$\frac{d \left( \frac{\bar{U}/U_M}{d(x/D)} \right)}{d(x/D)} = - \left( \frac{A_M}{A} \right)^2 \frac{d (A/A_M)}{d (x/D)}$$

$$= -.0027 \text{ at } x = x_M$$

Figure 5-9 shows the measured centre-line variation of u-intensity for an early version of grid 4. Also shown is the expected gradient from equation 5-1 ($n = 1, x_0 = 0$) as given by

$$\frac{d \left( \sigma/\sigma_M \right)}{d(x/D)} = - \frac{1}{2} \left( \frac{x}{x_M} \right)^{3/2} \frac{D}{x_M}$$

$$= - \frac{D}{2x_M} \text{ at } x = x_M$$

where $\sigma = \bar{u}/\bar{U}$

The mean velocity gradients are similar for all grids. The measured longitudinal intensity gradient is approximately that expected, indicating that the diffuser effect is not strong. It is similar for cases 3 and 4, but is worse in case 1 where equation 5-8 predicts 7.1% per diameter. This value should be larger than actually present, although it was not measured directly for this case.

As can be seen, all other stream-wise gradients are less than 1% per model diameter and are hence expected to have negligible effect on the fluctuating components of the lift and drag on the cylinder model.

The lateral gradients of the mean and fluctuating velocity components were also investigated both in the vertical ($z$) and spanwise ($y$) directions. The latter are included in the detailed grid properties for the cylinder centre-line presented in Section VI. The former were investigated briefly and found to be of the same order as the spanwise gradients and hence are not presented. An exception is the case of the empty tunnel, where the larger divergence angle of the floor and ceiling produce the vertical mean velocity profile shown in Figure 5-10. This profile was speed dependent and was associated with rapidly increasing turbulence intensities within a few inches of the $x=y=0$ plane. This should be noted in examining later results measured under empty tunnel conditions.

VI RESULTS AND DISCUSSION OF FLOW MEASUREMENTS AT THE MODEL SECTION

For each of the cases examined, the following measurements were taken
along the proposed centre-line of the model cylinder in order to specify the flow
characteristics.

1. Lateral profiles of  
   i) the mean velocity, $\bar{U}$
   ii) the turbulence intensities,
      $\frac{u}{U}, \frac{v}{U}, \frac{w}{U}$
   iii) the turbulence shear coefficients,
      $\frac{uv}{u v}, \frac{uw}{u w}$

2. Auto-correlations and power spectra of $u$, $v$, and $w$ at seven locations.

3. Zero time-delay lateral cross-correlations of $u$ and $w$.

4. Power and cross-spectra of $u$ for the eight lateral spacings at which
   two-point pressure correlations were to be obtained.

First, lateral profiles were measured over the entire tunnel width. Then,
preliminary lateral correlation measurements in the largest scale turbulence (grid
4) were used to define an influence region equal to the maximum separation intended
between the pressure modules (10D) plus twice the distance required for grid 4's
lateral u-correlation to become zero. Since this distance was found to be 10D, the
total influence region extended for 30D. The $Y$-location of the fixed pressure
module was then chosen so that this influence region location would encompass the
best compromise in homogeneous flow for all five cases. This resulted in the
$y = 0$ point being located at $Y = -6.2D$, and the influence region extending from
$y = -10D$ to $y = 20D$.

6.1 Lateral Profiles

The flow properties over the influence region are shown for the five
cases in Figures 6-1 to 6-5. Note that this region is conservative for the smaller
scale cases. It can be seen that over the 30D lateral distance the mean velocity
for each case is within $\pm 5\%$ of the median and that the turbulence component in-
tensities are similarly within $\pm 7\%$ except for the empty tunnel and case 2 which
was not used in response tests. All values were derived from hot wire anemometer
measurements. The intensities have not been corrected for any hot wire frequency
response effects (which are small - see Appendix A.3) or measuring instrumentation
truncation effects (see Appendix E).

The $uv$ and $uw$ measurements indicate the degree of shear remaining in the
flow. They were generally less than 0.2 except again for the empty tunnel.

A summary of the mean values of these properties used during other phases
of data reduction is given in Table 1, where the values have been weighted most
heavily over the region $0 \leq y/D \leq 10$. 
6.2 Power Spectra of Turbulence Components

Time records of \( u, v \) and \( w \) were recorded at seven lateral positions \( (y/D = 0, \pm 4.83, \pm 9.65, 14.5, 19.3) \) for each case and analyzed using the method of Appendix B.2.1. The digital spectral estimates were initially produced in the form \( \phi_{uu}'/S_{uu}'^2 \), where \( S_{uu}'^2 \) is the estimate of the variance from the digital records used, and \( \phi_{uu}' \) represents the uncorrected spectral estimates. The data was then corrected for filter and tape recorder frequency response, and multiplied by the ratio \( S_{uu}'^2/S_{uu}^2 \) where \( S_{uu}^2 \) is the estimate of the variance obtained directly in the experiment using the Bruel & Kjaer rms meters. The resulting data in final form \( \phi_{uu}'/S_{uu}'^2 \) is subject only to statistical variability inaccuracies, hot wire frequency response effects (see Appendix A.3) and to the effect of truncation errors in the measurement of \( S_{uu}^2 \). As seen from Appendix E, this latter error is generally less than 3%.

These normalized spectral estimates were averaged across the seven positions to form mean normalized spectral estimates which are shown in Figures 6-6 to 6-9 for grid cases 0, 1, 3 and 4. Such data is not available for case 2 since it was excluded from final testing, as explained in Section 7.2. These curves are shown for simplicity as continuous although they are made up of the best fit to the two spectral estimates resulting from the high and low frequency 'cuts' for each case. (Note that the data was analyzed separately for two frequency bandwidths - every 2Hz. for 0 to 250Hz. and every 20Hz. for 0 to 2000Hz. - for reasons detailed in Appendix B.2.1. These two analyses, one accurate at lower frequencies, the other at higher frequencies, are the two 'cuts' referred to). An example of the basic data is shown in Figure 6-10. This displays the high and low frequency cuts for the \( u \)-spectra of case 4, averaged over the seven locations, as two different plots, each accompanied by the overall final curve.

Furthermore, error bands are also shown. These bands are defined as

\[
\left[ \frac{\phi_{uu}}{S_{uu}^2} \right] \pm \left\{ \text{var} \left[ \frac{\phi_{uu}}{S_{uu}^2} \right] \right\}^{1/2}
\]

where the variance is calculated directly from the seven estimates available at each point. These error bands are typical for all cases except 0 which was subject to error bands of approximately twice the width. This coincided with the visual observations of the hot wire signal which was extremely intermittent - probably caused by an unstable boundary layer on the floor and ceiling of the diffuser, as indicated in Section 5.3.

As is expected in the results of Figures 6-6 to 6-9, the collapse of the \( w \) and \( v \) spectra show that the flow is at least axisymmetric, and further show the typical behaviour of such spectra which is to be biased towards higher frequencies than that of the \( u \)-distributions. The \( u \)-data can be fitted reasonably well by the von-Karman spectrum model (Ref.53) as shown typically in Figure 6-8. Also, shown is the Dryden model fit (see Ref.53 or Appendix E) which is not as good but has been used elsewhere in this report due to its simple form. Although not shown, the lateral von-Karman spectrum model shows a similar degree of fit using the measured lateral scale in the form indicated in Figure 6-8.

6.3 Correlation of Turbulence Components

In the course of the above data reduction, mean auto-correlations of each component were also obtained. These are shown for case 4 in Figure 6-11, plotted in a normalized form versus \( \bar{U}t \). These were integrated to provide estimates of the longitudinal and lateral scales, where
and are listed for all cases in Table 1. These integrations are sensitive to the tail of the correlation, which is controlled by the low frequency data. Since these low frequency components are subject to higher variability, the integrations were performed by overlaying the high and low frequency-cut estimates, hand-fitting a smooth curve, and integrating the areas directly using a planimeter. The correlations were not corrected for tape recorder and filter frequency responses. However, this only affects the higher frequencies which do not play a large part in determining the scales. For instance, the effect of the missing high frequencies in the low cuts of Figure 6-11 is primarily an early rounding off of the correlation near the origin. In all cases, the fitted curves were forced through \( R = 1.0 \).

Estimates for \( L_u(y) \) and \( L_w(y) \) were obtained by taking on-line zero time-delay correlations of \( u \) and \( w \) between two appropriate probes for different \( y \)- spacings. The zero-time correlations were formed through a direct 50 sec. integration of the mean products and the mean squares of the two signals (see Section 4.2). The results for all cases were again integrated using a planimeter and are also tabulated in Table 1. The experimental \( u \) and \( w \) lateral correlation points for case 4 are plotted in Figure 6-12, accompanied by the mean auto-correlation of \( w \) from Figure 6-11.

In homogeneous, isotropic turbulence, the three curves should be identical and \( L_u(x) = 2L_y(x) \) and \( L_v(x) = L_w(x) = L_u(y) = L_w(y) \). From Table 1, it can be seen that the trend in the data is for the longitudinal scale to be greater than twice the lateral scale, in fact the ratio varies between 1.86 and 4.10 (excluding case 0). These values are scattered but the trend agrees with Vickery's result of 3 in similar flow conditions.

6.4 Cross-Spectra of \( u \)

At the eight lateral spacings at which two-point pressure correlations on the model were to be examined, simultaneous time records of \( u(0) \) and \( u(y) \) were recorded for spectral analysis as described in Appendix B.2.1. Each of the eight sets provides the cross-spectra for that value of \( y = \xi \), and also estimates of the power spectra both at \( y = 0 \) and \( y = \xi \). As checks on the method, and to provide extremely reliable spectral data in the influence region, two further averaged \( u \)-spectra were formed from these eight sets of power spectral estimates.

One was the average of the eight replicates at \( y = 0 \). The other was the average of the eight values for the different non-zero \( y \)-spacings. These two \( u \)-spectral estimates are shown in Figure 6-13 accompanied by the estimate derived in Section 6.2 for case 4. This shows the reliability of this spectral data.

The prime reasons for obtaining the cross-spectral data at these eight
lateral spacings were, firstly, to allow examination of the pressure, drag, or lift describing function using equation (2-9b) and, secondly, to investigate further a collapsing form for the cross-spectra of turbulence suggested by Wardlaw and Davenport (Ref. 27).

Four of the eight cross-spectra for case 4 are presented in Figure 6-14, accompanied by the auto-spectra mean line. These plots show an increasing amount of variability as the probe spacing and frequency increase. Note that each of these plots represents only a single run, as against the previous averaged power spectra.

It is of particular interest to examine the cross-spectral results in terms of a coherence, $\gamma^2(f)$

$$
\gamma^2(f) = \frac{C_{u_1u_2}^2(f) + Q_{u_1u_2}^2(f)}{\phi_{u_1u_1}(f) \phi_{u_2u_2}(f)}
$$

(6-1)

where in homogeneous flow, this reduces to

$$
\gamma(f) = \frac{C_{u_1u_2}(f)}{\phi_{uu}(f)}
$$

(6-2)

It has been suggested by Davenport (Refs. 7 and 27) that for lateral cross spectra of $u$, $\gamma$ collapses when it is plotted versus $f \xi/\bar{U}$, and in fact that

$$
\gamma(\eta) = e^{-b\eta}
$$

(6-3)

where $\eta = f \xi/\bar{U}$ and $b$ is a constant between 5 and 8. This has further been verified by other investigators (e.g. Ref. 9).

Figure 6-15 shows the data obtained here for case 4, plotted as $\gamma$ vs. $\eta$ (using the complete form for $\gamma$ - equation (6-1)). Also shown in the lower part of the Figure is the fraction of total power represented in the imaginary component. For reference, the best fit curve found from the following has been superimposed. Note that the data represents 16 overlays resulting from high and low frequency cuts at eight lateral spacings. This plot emphasizes the lack of smoothing due to the short record lengths (single runs) involved. It becomes particularly bad at large $\eta$ where the signal to noise level of the system is becoming smaller, and the frequencies of interest are becoming lower and hence less statistically reliable (i.e. the record length includes fewer periods). This can be seen in both $\gamma$ and the fractional imaginary power. However, note that the latter curve is essentially symmetrical, indicating the scatter does not have any apparent non-zero trend.

Recognizing that the coherence data appears to be of exponential form in the region where its significance is not lost in noise, the data was replotted logarithmically. However, none of the experimental data could be fitted very well by equation (6-3). Instead it was found that the best fit was of the form

$$
\gamma(\eta) = e^{-(b\eta)^n}
$$

(6-4)
The best value of \( n \) appears to be 1.4 as shown by Figure 6-16 which shows the data of Figure 6-15 presented for \( n = 1, 1.3, 1.4, \) and 1.5. Figure 6-16 is plotted logarithmically such that equation 6-4 will appear as a straight line. The data best fits the straight line for \( n = 1.4 \). Note that the value of \( n \) does not affect the collapsing of the data, only the final curve shape.

Davenport refers to \( \gamma \) as a narrow band cross-correlation as which it can be interpreted quite easily.* In fact, two wave analyzers and a zero time-delay correlator allow direct measurements in the form of equation (6-2), with much better accuracy than that obtained here. In this form, however, it is also suggested that a value of \( n = 1 \) is physically unlikely, due to the finite slope of equation (6-3) at \( \eta = 0 \). Note that for any value of \( n > 1 \), equation (6-4) has a zero slope at the origin. Data for all four cases examined show behaviour similar to that shown in Figure 6-16. Furthermore, data presented in Reference 27 also appears to be fitted better by equation (6-4), as seen in Figure 6-17 where this data has been reproduced. However, the value of \( b \) required is different.

The value of \( b \) was determined for the present data by a final plot of the data in a form to make greatest use of the most reliable area of the data. Hence, Figure 6-18 shows the four cases replotted in the form \( [-\log_{10}(\gamma(\eta))]^{1/1.4} \) versus \( \eta \). The best fits are marked on each, except for case 1 where the average value of \( b \) from the other three cases is shown. All cases except 1 consist of overlays of high and low frequency cuts from the eight lateral spacings. For case 1, only the first four spacings are presented. It is believed that the low coherency values for this case are due to the low signal to noise ratios obtained for this case at low frequencies and the presented spacings (c.f. \( L_m(y) = 0.12D \)). To a good approximation, then, most of the data can be described by equation (6-4) with

\[
\begin{align*}
n &= 1.4 \pm 0.1 \\
b &= 6.4 \pm 0.5
\end{align*}
\]

Because of the scatter in the present data at high frequencies and large spacings, it is not possible to conclude that this is a valid collapse over the entire range of significant lateral cross-spectra. It is suggested that more precise measurements, especially for the high frequency components, would be of interest because of the powerful implications of this spectral model.

VII SURFACE PRESSURE MEASUREMENT-TECHNIQUES AND VERIFICATIONS

The surface pressure on the model was sampled at a point by the small hole connecting the surface to the interior of each of the modules described in Section 3.1. The physical details of the cavity coupling the microphone diaphragm

* It is of interest to note that the cross-spectra referred to in this report and used in the definition of \( \gamma(f) \) are really neither spectrum functions nor correlations in the rigorous sense. The correlation function from which \( \Phi_{uu}(f) = \phi_{uu}(f,\xi) \) was derived is actually the component of the general correlation tensor given by Reference 51 as \( R_{ll}(r_1, r_2, 0) \) where the subscript \( l \) denotes the x-direction, \( r_2 = \xi \), and \( r_1 \) is related to \( \tau \) through Taylor's hypothesis, \( r_1 = \bar{U}\tau \). Hence, the form \( \phi_{uu}(f,\xi) \), after a one-dimensional Fourier transform, is halfway between a correlation function and the two-dimensional cross-spectrum. It is then not entirely straightforward to understand the physical implications of the collapse of \( \gamma(\eta) \).
to the surface are described in Appendix C, accompanied by a description of the module calibrations. The resultant experimental frequency response curve for the microphone/module system is displayed as Figure 4-4. The modules were matched such that the average amplitude response described both to within $\pm 3\%$ for frequencies less than 850 Hz. Measurements cannot be reliably corrected above 800 Hz. because the resonance is amplitude dependent.

7.1 Experimental Validity

Several preliminary experiments were undertaken to ascertain the validity of some of the assumptions inherent in the method, and to investigate some anomalous behaviour observed for one grid. The latter was grid 2 where the lateral surface pressure correlations did not approach zero as the module separation became large. This investigation forms part of Section 7.1.4. The major assumptions of the method adopted here are as follows:

i) The flow is homogeneous across the primary influence region and flow non-homogeneities outside the primary influence region do not affect the pressure characteristics within it.

ii) Motion-induced pressures are negligible.

iii) Surface irregularities of the order present on the model are of little significance.

iv) Other extraneous inputs into the system are small. These are typically electronic noise, and acoustic pressures due to the noisy wind tunnel environment.

7.1.1 Flow Homogeneity

The homogeneity of the flow in the primary influence region has been established in Section VI. The effects of the flow non-homogeneities outside of the primary influence region are more difficult to ascertain. As indicated previously, the end-sealing problems of Keefe (Ref.34) were avoided by using elaborate end-seals. These prevented flow leakage to the outside of the tunnel and also simulated a normal end-wall effect by the use of a large circular end-plate (Diam. = 10.5D) attached directly to the model cylinder. However, there remains the fact that, due to the non-uniformity of the mean flow (including that due to the wall boundary layer), lateral static pressure gradients are induced on the cylinder. Such gradients are particularly important in the stagnant wake. The exact effect of such gradients present here is not known; however the similarity of the static pressure distributions obtained 9.78D apart (see Figs. 8-1 and 8-2) in all cases indicates this effect is small. Note also that the overall cylinder length between end plates is 50D, which should reduce any residual end effects.

7.1.2 Effects of Residual Cylinder Motion

The small residual motion of the model (see Section 3.4) has two effects. The first is the direct effect on the surface pressures due to the velocity of the cylinder surface with respect to a stationary coordinate system. This has been neglected, since it should be of the same order as the effect of an additional turbulence input with longitudinal and lateral spectra given by the cylinder's own two lateral velocity spectra in the x and z directions. In the worst case cited
in Section 3.4, this motion-induced rms velocity is of the order of $6.3 \times 10^{-3}$ fps, which is negligible even when compared with the empty tunnel turbulence which was of the order of 1 fps rms. As expected, the above was verified by the absence of any peaks in the cylinder pressure spectra at the modes natural frequency.

The second but indirect effect is that of the acceleration on the microphone characteristics. This is also negligible because of the low level ($\sim 0.05$ g's) and the fact that the direction of application is parallel to the diaphragm rather than normal to it.

7.1.3 Surface Irregularities

The basic surface finish of the cylinder was described in Section 3.1. However, there were occasional pock marks created due to handling, and slight surface level discontinuities where the two modules and spacers mated. These were of the order of 0.002". The 1/16" diameter hole itself subtends an angle of 5.78°, which is effectively a local flat spot with maximum depth of 0.008" for the cross-section through the centre of the hole. It is known that very small disturbances can create large differences in the surface pressure fluctuations on a circular cylinder in smooth flow at some Re (Ref. 38). However, due to the large impinging turbulence levels in this case, this effect was expected to disappear. To confirm this, discontinuities were added to the cylinder surface just upstream of one of the holes by means of adhesive tape layers while monitoring the rms pressures and two point correlations. The result is shown in Figure 7-1, and it is seen that it is safe to assume that surface irregularities of the order of those present were negligible in this experiment.

7.1.4 Extraneous Input

A schematic of the equipment used in the microphone/pressure measurement is shown in Figure 7-2. The electronic noise levels produced by the microphone/cathode-follower system were less than 0.15% of the typical rms signal level (2 to 3 volts), as measured directly on a Bruel & Kjaer Random Noise Meter. The noise level of the computer amplifier used in the system (with grounded input) was less than 0.1% of the typical rms signal level. Hence the latter determined the overall system electronic noise level.

However, the model under test was also subject to acoustical inputs, whose magnitudes were difficult to determine directly, but the lack of unexpected peaks in the surface pressure spectra, and lateral correlations of surface pressures which approached zero for large $\zeta$, both imply the lack of any acoustical field of significance compared to the turbulence induced pressures. Both approaches showed no evidence of significant acoustical inputs except for grid 2. For this case only, the lateral pressure correlations approached value of $\mathcal{R} \approx 0.2$ as $\zeta$ became large. This discovery prompted a number of experiments to determine the origin. Since the lateral velocity correlations were known to be well-behaved, it was concluded that the correlated pressure field had to be acoustical. However, a variety of sources were possible. The eventual explanation was that the combination of the wooden grid mounted directly onto a relatively flexible area of the wooden tunnel wall was forming a mechanical system with a very lightly damped natural frequency at about 40 hz. Since this frequency was near the peak of the wake energy being produced by the grid, the grid/wall response was substantial. The resulting wall motion produced the troublesome acoustic pressures. In fact, it was surmised that, since the acoustical wavelength was roughly four times the
tunnel width, conditions were also suitable for a resonance across the duct. Attempts were made to change the natural frequency of the system by wire bracing and by addition of mass to the grid. However, no substantial change could be accomplished. Since all other cures appeared to be too time-consuming, this grid was dropped from the experimental pressure-measurement program, although it appears likely that redesign of the grid and its mounting arrangement could have eliminated the problem.

During the course of this investigation, a number of other possible sources of strong acoustical fields were checked. The most interesting in its own right was that of the wake directly behind the grid. Since close proximity of cylinders is known to couple the vortex shedding phenomena, it was of interest to look at the behaviour of the individual bar wakes close to the grid. In the brief experiment conducted, no evidence of any preferred frequency was found in the bar wake at a distance of 0.55 mesh lengths downstream, or in the potential region just upstream of one of the grid members. This suggests that the square lattice is effective in completely disrupting the vortex shedding phenomenon.

7.2 Method for Measuring Static Surface Pressures

In order to use the modules to measure static surface pressures, an insert was made to seal the module off (as per Section 3.1 and Fig.3-2) except for a pressure nipple which was connected by means of Tygon tubing directly to a Betz manometer. The tunnel static pressure at the location of the model was obtained by manifolding 8 taps located circumferentially around the tunnel at the model section. The resulting pressure difference was recorded for each of the four grid cases considered, for the two lateral locations \( y = 0 \) and \( y = 9.78D \), and for circumferential angles from \( 0^\circ \) to \( 360^\circ \). The measurements were taken for \( 5^\circ \) increments except in the wake (\( 140^\circ - 220^\circ \)) where \( 1^\circ \) increments were used.

The fine traverse was particularly intended to provide an accurate indication of the error in angle of attack from the assumed horizontal flow. In all cases cited in this report, the angles are nominally referred to horizontal, except for those used in the lift and drag integrations where the experimental angles were corrected for the real flow angle. In the former case, this implies that the true circumferential angle equals the indicated angle minus an error given in Table 1.

For the measurements of the surface pressure distribution behind grid 1, some of the wall taps were directly in the wake of the grid mounting brackets. The resulting error in indicated working section static pressure was bypassed by forcing the \( C_p \) data at \( \theta' = 0^\circ \) to go through the mean stagnation pressure recorded with the other grids.

Although the value of \( C_p \) at the stagnation point would be expected to be dependent on the turbulence level in the free stream, as pointed out by Hinze (Ref.1) it is not clear what this relation is. This point warrants further research. The present results do not show any clear trend nor do they consistently agree with the assumption of total stagnation of one or all of the turbulence components. This may be due in part to the increased error in the absolute value of \( C_p \) introduced by working in the diffuser. This increased error is caused by the dependence of the results on the calibrated mean velocity ratio between the working section and the test section, and by the measurement of wall static pressures under conditions of high intensity turbulence. Hence, the method of setting the absolute values of the \( C_p \) data for grid 1 was judged to be satisfactory.

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VIII RESULTS AND DISCUSSION OF PRESSURE MEASUREMENTS ON THE CIRCULAR CYLINDER

Pressure measurements were taken for cases 0, 1, 3 and 4. They include static pressure distributions at two stations \((y/D = 0\) and \(9.80\)) and the following sets of fluctuating pressures.

A. For all four cases, for eight lateral spacings \((\xi/D = .129, .290, .443, .935, 1.58, 2.56, 4.64, \text{ and } 8.19)\) and for circumferential angles from \(0^\circ\) to \(180^\circ\) in \(30^\circ\) increments

i) rms pressure levels were obtained directly using B & K Random Noise Meters.

ii) time records were analog recorded for later spectral analysis as described in Appendix B.2.1. Specifically, power spectra were obtained at \(y = 0\) and \(y = \xi\) for each spacing and for each angle, and cross-power spectra were obtained between points at different spacing but at the same angle.

B. For case 4, and for the same eight lateral spacings, a complete matrix of two-point pressure correlations were obtained using the PAR/526 system (Appendix B.2.2) using an angle increment of \(15^\circ\) in order to perform the integration of equations 2-11 and 2-13. At the same time, an independent set of zero-time correlations were also obtained using the analog computer.

8.1 Static Pressure Distributions and Mean Drag Results

The static pressure distributions are shown in Figures 8-1 and 8-2 accompanied by the potential flow theoretical prediction of \(C = 1-4\sin^2\theta\). The average sectional drag coefficient (see Table 1) was obtained from this data, using

\[
Cd = \frac{1}{2} \int_0^{2\pi} \bar{C}_p(\theta') \cos \theta' d\theta'
\]

where \(\bar{C}_p(\theta')\) is the average \(C_p(\theta')\) recorded at the two stations examined in each case. Note that the correction of data necessary for case 1 has no effect on the value of \(C_d\), since the latter is independent of the reference pressure used. Due to the use of the diffuser as a test section, both the pressure distributions and \(C_d\) values are subject to error due to the static pressure gradient present. As discussed in Section 5.3, this gradient is of the order of \(0.54\%\) per diameter.

It can easily be shown that the pressure gradient results in an increment of drag coefficient in the raw results given by

\[
\Delta C_d = - \frac{\pi}{4} \frac{d \left( \frac{P-P_M}{\frac{\partial P}{\partial M}} \right)}{d (x/D)}
\]

assuming that the effect of pressure gradient can simply be superimposed. In this case, \(\Delta C_d = -0.0043\). This correction is hence negligible.

The static pressure distributions at the two stations examined agree
well for all cases except for that of the empty tunnel, which has already been shown to be quite inhomogeneous. Hence, all of the grid cases appear to have little indication of severe lateral pressure gradients which would compromise the results.

It is evident, however, that the four cases differ markedly. Although at approximately the same Re ($\approx 4 \times 10^4$), the pressure distributions and $C_d$ values are significantly different. In particular, the presence of turbulence has altered the pressure distributions to those corresponding to higher Re's. An indication of the effect of turbulence is given by

$$\frac{\bar{u}}{U} \left[ \frac{D}{L_u(y)} \right]^{0.2}$$

which was suggested by Taylor as the significant parameter to predict the onset of the critical flow regime for spheres. Bearman (Ref. 52) has shown that the critical Re's of circular cylinders can also be predicted by this parameter, at least for some ranges of turbulence intensity and scale. The value of the parameter has been calculated for the present results and is presented in Table 1 for each case accompanied by the base pressure coefficients and peak negative pressures obtained. It can be seen that, although none of the cases examined here have actually reached the critical Re (Bearman takes this where $C_d = 0.8$) the parameter does correctly order the results in terms of decreasing $C_d$, decreasing base pressure, and increasing magnitude of the negative peaks. With reference to Figure 1-1, this behaviour is consistent with equivalent Re's of the order of $10^5$ to $2 \times 10^5$. Furthermore, the St behaviour described later and recorded in Table 1 is also consistent with this equivalent Re range.

8.2 One-Point Fluctuating Surface Pressure Characteristics

Distributions of rms pressures for the four turbulence cases are shown in Figures 8-3 and 8-4. Each plot contains two sets of data. For $0^\circ$, $30^\circ$, $60^\circ$, etc., one is the average of the eight replicates at $y = 0$. The second set is the average of the eight measurements at different $y = \zeta$ for the same angle. The other measurements (at $15^\circ$, $45^\circ$, etc) are single measurements and hence are subject to more variability.

This data has not been corrected for module response characteristics. As pointed out in Appendix E, this is a good approximation for all cases except grid 1, where the energy is concentrated at higher frequencies and the resonance and truncation errors are more severe. A conservative view of the error in case 4, for example, is provided by the ratio of output rms to input rms pressure signal recorded across the 400 hz natural frequency filters used in the method of Appendix B.2.2. This ratio is plotted in Figure 8-4 and shows a maximum decrement of about 6%. Since the microphones are flat within 3db to 650 cps compared to the filters 3db point at 400 cps, it can be concluded that for the large scale turbulence field the measurements are not compromised by the module response. It is interesting to note that the minimum ratio across the filter occurs at $120^\circ$ and $180^\circ$, which is consistent with later spectral results, indicating that these points have the largest high frequency content.

The most noticeable effect of increasing turbulence is on the $0' = 0^\circ$ measurement, which directly reflects the free stream turbulence level. In fact, also plotted for reference is the rms pressure expected from the simple linearization $p(t) = p_0 U u(t)$.
This provides a good estimate of the measured value in all cases. The increasing turbulence also generally raises the level of pressure response as expected, although this is not a strong effect as the fluctuating pressures resulting from vortex shedding are already powerful, and the effect of turbulence is more of a levelling effect. The peak of the rms pressure is associated with the neighbourhood of the mean separation point. In particular, the higher effective Re for case 1 is reflected in the peak pressures occurring later and a reduction of wake pressure level.

The characteristics of the fluctuating surface pressures can be seen in more detail by examining the pressure spectra of Figures 8-5 to 8-11. These results are completely corrected for module response, and represent the mean curves obtained by averaging the eight replicates of the spectra derived for \( y = 0 \). Again the \( 0^\circ \) results show the reflection of the input turbulence characteristics. No evidence of vortex shedding was observable at this angle. It can usually be observed under smooth flow conditions, but the empty tunnel case here has sufficient turbulence to mask its presence. For the two angles in the attached flow regime (30° and 60°), the influence of the input turbulence appears to become less on the background pressure spectra as can be seen most explicitly for case 1. Of most interest is the effect of the turbulence on the vortex shedding. Comparing cases 0, 3 and 4, which are of similar scale, the effect of increasing intensity is to broaden and lower the pressure peak. However, the latter effect is small and may be due to the increase in effective Re noted earlier, since these three peaks order directly according to their mean drag coefficient. Case 4 also shows a significant change in St which is recorded in Table 1. Case 1, which has similar intensity to case 3 but a much reduced scale, does not show a broadening of the peak, but merely a shift in St. Furthermore it does not order according to its mean drag coefficient, suggesting that large scale turbulence tends to spread the vortex shedding, hence reducing the peak energy. However, none of the results indicate that the turbulence significantly interferes with the vortex shedding phenomena.

The results at 90° for the empty tunnel and for case 3 already show presence of the second harmonic of the vortex shedding, which can also be seen for 120°, 150° and in a dominant role as expected at 180°. However, it was not expected to find the third harmonic of \( f_s \) at 120° and 150° which is clearly visible for both cases 0 and 3. It is suggested that it is produced by the non-linearity of the pressure oscillation or possibly by a form of heterodyning between the two fundamental frequencies at \( f_s \) and 2\( f_s \). This third harmonic is not apparent at 180°, although a peak at \( f_s \) is still visible for cases 0 and 3. The distributions at 120° again show an increase in the high frequency spectral density compared to the spectra for other angles, which appears to be associated with the turbulence at separation. The pressure spectra at 180° reflect the differences in wake flow due to the change in effective Re as well as the levelling effect of the free stream turbulence. That is, cases 3 and 4 show peaks which are ordered according to both input turbulence levels, and mean drag coefficients. However, both cases are well above the spectral levels of cases 0 and 1. The latter has been reduced due to the increase in effective Re which was most marked for this case.

The effect of the turbulence on the fundamental vortex shedding is again seen with reference to the normalized auto-correlations of the surface
pressure shown in Figure 8-12 and 8-13 for some representative circumferential angles. These auto-correlations indicate the phase coherence of the vortex shedding, and show immediately that the strongest disruption is that of case 4, followed by case 3. In particular, it is again evident that turbulence scale is more effective in modifying vortex-shedding than is turbulence intensity. This behaviour suggests that high-frequency turbulence is not "seen" by the Strouhal phenomenon. In fact, it may be possible to associate the dispersion of the Strouhal peak as a linear phenomenon associated with the variance of the input turbulence below a certain frequency. Obviously the variance associated with input turbulence below, say, the fundamental vortex shedding frequency is much less for case 1 than for cases 3 or 4.

8.3 Two-Point Fluctuating Surface Pressure Characteristics

At increments of 30°, and for equal θ, zero time-delay pressure correlations were obtained for various lateral spacings. These are of the form shown for cases 0 and 4 in Figure 8-14. Integrating these curves give lateral pressure scales as

\[ L_p(\theta') = \int_0^\infty R_{p_1p_2}(0, \xi, \theta', \theta') d\xi \]

The distribution of \( L_p(\theta')/D \) around the cylinder is shown for all four cases in Figure 8-15. Again, the turbulence is seen to have a levelling effect for the large scales where the lateral turbulence scale is of the same order as the lateral correlation of the vortex shedding. Furthermore, the lateral pressure scales reflect the input turbulence scales as would be expected if a quasi-linear relation holds between the turbulence and the pressure response. Hence, even in the region dominated by Strouhal shedding (60°-120°), the overall lateral scales are reduced due to the effects of the input turbulence. Also shown on 8-15 are lateral pressure scales of vortex shedding obtained by Prendergast (Ref.39) in smooth flow, which agree well with the present results. Not shown are results obtained by El Baroudi (Ref.40), who measured the correlation of velocities near the surface of a cylinder and obtained results which are about 80% higher than the pressure results shown at 90°. Unfortunately, the presence of the turbulence does not allow the results of Figure 8-15 to be interpreted directly in terms of lateral coherence of the vortex shedding. Hence, the narrow band correlation technique of Section VI was used to investigate this lateral coherence further. A typical plot of \( \gamma \) vs \( \eta \) is shown in Figure 8-16 for case 4 for the angles 0° and 90°. Since the pressure response at 0° was expected to reflect the input turbulence, this data was plotted in the form to show a linear collapse similar to the u-inputs. However, the pressures at other than 0° do not show any sign of collapsing when plotted against \( \eta \). Nevertheless, as seen in Figure 8-16, the coherence at the Strouhal frequency is significantly higher than that for the background pressure data. Each set of data shows peaks at \( \eta_i \) where \( \eta_i = f_s \xi_1/D \), where the subscript \( i \) refers to one of the eight values of \( \xi \) used. Note that the envelope of the peaks determines the coherence behaviour of the vortex shedding alone. It can thus be removed from the effects of the background pressure response which clouds the interpretation of the total lateral pressure correlation. Again it was found that a good fit to the peaks is given by

\[ \gamma_s = e^{-(b/\xi_s)^{1.4}} \]

as indicated in Figure 8-17, where the data has been plotted for all four cases.
and where the data presented is restricted to just that surrounding the vortex shedding peaks. As can be seen, the fitted curve is a good approximation, even as far as the sixth and seventh peaks (2.56D and 4.65D respectively). Since each set of cross-spectral data was the result of only one set of measurements, rather than the multiple averages used for previous power spectra, it is felt that the lack of fit where it occurs is primarily the result of scatter in the data, rather than in the fundamental phenomenon, and that better results could be obtained simply by using twin analog filters at the Strouhal frequency in combination with a zero time-delay correlator. Nevertheless, the results obtained here in terms of \( b_s(\theta') \) are plotted in Figure 8-18, together with corresponding scales determined analytically from

\[
\frac{L_p(\theta')}{D} = \frac{1}{St} \int_{0}^{\infty} \gamma_s \, d\eta
\]

Note that for \( \theta' = 180^\circ \), \( \gamma_s \) is based on the peaks at \( 2f_s \) and hence in the above expression, \( St = 2f_s D/\bar{U} \) for this case.

When viewed with this narrow band approach, the vortex shedding coherence in turbulence as indicated by the behaviour of \( b_s \) is seen to be similar to that discussed previously in that case 1 has not been affected to the same degree as case 3, although both have similar intensities. For similar scales, case 4's larger intensity shows slightly more interference than that of case 3. In all cases, even when subjected to turbulence intensities of 15%, the vortex shedding still dominates the fluctuating surface pressures. Note that the apparent collapse of \( L_p(\theta')/D \) is due to the change in \( St \) in the different cases and hence \( \lambda_{sc}/D \), rather than a collapse of coherence.

### 8.4 Pressure-velocity Describing Functions

The lack of collapse of the pressure coherences for other than \( \theta^\circ \) indicates that the relationship between pressure and input velocity is not describable as a simple linear relation. This was expected, and it is not within the scope of the present work to investigate in detail this relation. However, because of the collapse of the pressure coherence at \( \theta^\circ \) and the good agreement between the measured and linearly predicted rms pressures at that point as seen in Section 8.2, describing functions were derived for the four cases at \( \theta^\circ \). As per Section 2, \( |H_p(f)|^2 \) is shown in Figure 8-19, non-dimensionalized by the simple linear prediction given by \( p^2 U^2 \). It is of interest to note that the prediction is generally too low at low frequency which might be expected since this approach neglects the effect of cross-components, acceleration effects, and the pressure field of the cylinder itself. However, it can be seen that the rapid drop-off in pressure response at high frequency is similar in all four cases and serves to indicate a frequency at which the stream turbulence has negligible effect on the flow field of the cylinder, at least near the stagnation point. This frequency is of the order of 160 Hz, i.e. at \( D/\lambda \approx 0.3 \). These results indicate that the correct non-dimensionalizing parameter for these results is \( fD/\bar{U} \) (not, for example, \( fL/\bar{U} \)). Strasberg, Reference 8, has shown good agreement between fluctuating total head measurements and that predicted using the simple theory to values of \( D/\lambda \) of 3 for ordinary stagnation tubes. Hence, it is felt that more investigation of the behaviour of different stagnation geometries would be of value, since the present results show a drop-off at an unexpectedly large value of \( \lambda/D \).
8.5 Fluctuating Drag and Lift Results

This data is available for case 4 only.

8.5.1 Basic Results

Equation 2-11 and 2-13 were integrated by the method outlined in Appendix D to yield the two functions $R_{dd}(T, S)$, $R_{ll}(T, S)$. Each integrated function consisted of 100 estimates along the $T$ axis (effectively at $(n + \frac{1}{2})\Delta T$; $n = 0, 1, \ldots, 99$) for each of the eight $S$ values examined. These functions are shown in Figures 8-2D and 8-2E in the form

$$C_{ff}^2(\tau, \xi) = \frac{R_{ff}(\tau, \xi)}{R_{ff}(\Delta \tau/2, \xi)}$$

where

$$C_{ff}^2(\tau, \xi) = \frac{R_{ff}(\tau, \xi)}{q_m^2 D^2}$$

and $\xi_1/D = 0.129$.

The integrated correlations show the separation of drag and lift very well. The damped oscillation in lift denotes a broad Strouhal peak, whereas the absence of any significant oscillation in drag implies that the second order fluctuating drag at twice the Strouhal frequency is submerged in the response to turbulence. It can also be seen that even at the maximum separation of 8.18D, the Strouhal frequency is clearly visible in the lift correlation. This indicates the sensitivity of the experimental method.

Several investigators have measured the rms fluctuating lift and drag for a circular cylinder in smooth flow. Keefe (Ref.34) found that $C_{ld}(0,0) = 0.044$ and $C_{ll}(0,0) = 0.455$ as measured on an instrumented section one diameter long at a similar mean flow Re. These values are compared to the present ones found in a turbulent flow in Figures 8-22 and 8-23. (Note that Keefe's results are plotted at $\xi/D = 0$ although they actually correspond to data somewhere between 0 and 1). The present results show the square root of the integrated zero-time delay correlation $C_{ff}(0, \xi)$ and similar values found from the general integral for $n = 0$, i.e., essentially $C_{ff}(\Delta \tau/2, \xi)$. It should be noted that these latter results are subject to the 400 Hz cut-off filters, whereas the zero time correlation results include the entire pressure signal, without compensation for microphone module response. However, as noted in Section 8.2, this is a small effect for this case, and the two methods agree very well. It is evident from Figures 8-22 and 8-23 that the fluctuating drag is roughly six times bigger in this turbulence field than in the smooth stream case examined by Keefe. Hence the input turbulence is of prime importance. This is not the case for the lift response, as the values obtained here are not significantly greater than those found for smooth flow. Hence, when combined with the previous observations of the strength of the vortex shedding even in this severe turbulence, it appears evident that vortex shedding is still the primary cause of fluctuating lift.

The data of Figures 8-22 and 8-23 can be integrated to yield lateral
force scales according to

\[ \frac{L_f}{D} = \int_0^\infty \frac{c_{ff}^2(0, \xi)}{c_{ff}^2(0,0)} \, d\left(\frac{\xi}{D}\right) \]

This process yields

\[ \frac{L_d}{D} = 2.80 \quad \frac{L_f}{D} = 2.32 \]

which compare well with \( L_u(y)/D = 2.41 \) and the order of the lateral pressure scales of Figure 8-15 respectively.

Figure 8-24 shows the behaviour of the components zero time-delay pressure correlations used in producing \( c_{ff}(0, \xi) \). This indicates the complex form of the general two point pressure correlations. Note that the points plotted do not represent the maximum correlation between the two points except when \( \alpha = \beta \). Otherwise the maximum occurs at some finite time delay.

On Fourier transforming the correlations, spectral estimates were obtained of the form in Figure 8-25 for two values of \( \xi \). These spectral estimates are corrected for all gains and frequency responses of the system, in contrast to the correlations which are not corrected for frequency responses. Figure 8-25 shows the frequency intervals at which the spectral estimates were obtained and the decrease in reliability of the data with increasing frequency and lateral spacing. The behaviour at high frequency is typical of such transforms as shown in Appendix B.3, which also shows the scatter to be around a reliable mean. Generally, the best fitted curve is taken as reliable up to the vicinity of the first negative spectral estimate. Figure 8-25 also shows an anomaly in the drag spectrum at the Strouhal frequency in the form of a significant dip in an area where the data is taken as highly reliable. It is considered to be an anomaly since

1) it does not occur in any other drag cross-spectrum

2) there is no physical mechanism by which the vortex shedding can modify the drag response at the lift frequency. If at all, it would be expected at twice the lift frequency

No conclusive explanation of this anomaly can be given now, and hence it has not been taken into account in any of the following discussions.

Overlays of the mean spectra derived from data such as shown in Figure 8-25 are displayed for the drag and lift for all 8 spacings in Figures 8-26 and 8-27. As indicated previously, the lift response is dominated by the Strouhal shedding, although the peak is broadened over that occurring in steady flow. As expected, the drag cross-spectra diminish rapidly with \( \xi \) and the curves indicate that the spectral density at high frequency falls off more rapidly than at low frequency. This suggested an investigation of the lateral coherence of these forces as detailed below.
8.5.2 Coherences and Describing Functions

Initially, the method of equation 2-9b was applied to the drag data in order to see if a common describing function could be defined. The results are shown in Figure 8-28. They do not collapse as predicted by the simple theory of Section 2.3. However, the theoretical model (equation 2-15) does agree reasonably well with the results at low frequency, using the experimental value of $C_d$ and the theoretical value of $k = \pi/4$.

Since Figures 8-26 and 8-28 suggest a better collapse when plotted as a function of $\eta$, the drag data was re-examined, using the "narrow-band correlation" technique. Figure 8-29 shows that $(\phi_{d_1 d_0}/\phi_{d_1 d_0})$ collapses well when plotted against $\eta'_i$, where $d_i$ refers to the drag at $y = y_i$ and $d_0$ refers to the drag at some $y_i = \hat{y}_i$. Note that if an exponential relation of the form

$$\frac{\phi_{d_1 d_0}}{\phi_{d_0 d_0}} = e^{-\eta_i^n}$$

is assumed, where $\eta_i = \frac{\eta_i}{\hat{y}_i}$, then

$$\frac{\phi_{d_1 d_0}}{\phi_{d_1 d_0}} = e^{-\eta_i'^n}$$

where

$$\eta_i' = (\eta_i^n - \bar{\eta}_i^n)^{1/n}$$

Assuming $n = 1.4$, the data was plotted logarithmically to determine $c$. The best fit using $c = 7.9$ is also shown on Figure 8-29, accompanied for comparison by a "fit" using $c = 8$ and $n = 1$, and by the fitted velocity relation from Section 6.4.

The fact that the decay rates of the exponential functions are different for the drag and the u-component of velocity suggest that a velocity-drag relation can be described by the form

$$\frac{\phi_{d_1 d_0}}{\phi_{u_1 u_0}} = |H(f)|^2 e^{-(d_{1.4})^{1.4}}$$

(8.1)

where $d = [c^{1.4} - b^{1.4}]^{1/1.4}$

and

$$|H(f)|^2 = \frac{\phi_{d_1 d_0}}{\phi_{u_1 u_0}}$$

Estimates for $|H(f)|^2$ were produced from equation 8-1 directly using
d = [7.9^{1.4} - 6.4^{1.4}]^{1/1.4} = 3.0 and are shown on Figure 8-30. There is considerable scatter in these results; however, the mean behaviour has a consistent trend.

Although the relation 8-1 is a good approximation to the data obtained here, more work is required to determine whether its form is generally applicable. Since the exponential is essentially a fitting term to a non-linearity, the value of d may be a function of Re and turbulence characteristics.

It should be noted that the expression 8.1 fulfills the aim discussed in Section I of providing a definitive form for the drag cross-spectra in terms of the velocity cross-spectra, and hence allows more reliable structural responses to be calculated in the drag direction. In this regard it is probable that the relation |H(f)|^2 should actually be regarded as a function of the non-dimensional parameter fD/U when applied to cylinders of different dimensions.

Since no w-cross-spectra were measured, the only data available to test the simple model of equation 2-16 was to form \( \phi_{ll} / \phi_{ww} \) where \( \xi_1 = 0.129D \). This plot is shown in Figure 8-32 accompanied by the simple theory of equation 2-16.

This data does not lead to a simple fitting form, although the theory is of the correct order below the Strouhal number, and the region of the Strouhal peak can be considered by using the experimental data directly, in conjunction with the coherence form near the Strouhal number as given by equation 8-2. Generally, the most critical response is not in the lift direction, but in the drag direction because the latter is superimposed on a large steady state drag.

**IX CONCLUSIONS**

The main conclusions arrived at in this work are as follows:

1) Lateral coherence data for the longitudinal turbulence component obtained for three of the four cases examined support Davenport's observation that narrow band cross-correlations collapse when plotted as functions of \( \eta = f \xi / \bar{U} \). However, the results obtained here indicate that the data is fitted better by the form...
than by that given by Davenport. The fourth case concentrated the turbulent energy at high frequencies, and resulted in low signal levels at wavelengths associated with the \( \xi \) values examined. The results indicated coherences less than expected.

2) Mean drag coefficients and vortex shedding frequencies in turbulence are consistent with an effective increase in \( \text{Re} \) which is determined by both intensity and scale. The values of \( C_d \) measured are ordered correctly when using the Taylor parameter

\[
\frac{\bar{u}}{\bar{U}} \left[ \frac{D}{L_u(y)} \right]^{0.2}
\]

although it is not likely that this parameter is definitive for all aspects of the turbulence interaction.

3) Intense turbulence does not interfere drastically with the vortex shedding phenomenon on a rigid circular cylinder.

4) The effect of turbulence on the shedding phenomenon arises primarily from its low frequency spectral content, and serves mainly to reduce the phase coherence of the shedding (i.e. broaden the Strouhal peak). The peak energy and lateral coherence is only slightly reduced by severe turbulence whose lateral scale is approximately 2.4D.

5) Under low turbulence conditions, surface pressure spectra in the range \( \theta = 90^\circ \) to \( 150^\circ \) show small peaks at \( 3f_s \).

6) The spectra of drag and lift (\( \bar{u}/\bar{U} = .147, L_u(y)/D = 2.41 \)) show that in homogeneous turbulence the fluctuating lift is still dominated by the broadened Strouhal peak, while the unsteady drag is primarily due to streamwise turbulence.

7) The data obtained here show that the drag cross-spectrum can be deduced from the velocity cross-spectrum using the relation

\[
\frac{\phi_{d,u}}{\phi_{u,u}} = |H(f)|^2 \frac{-(d\eta_i)^{1.4}}{\bar{u}_0}
\]

where \( d \) was found to be 3.4. It is not known whether such a form is generally useful for other turbulent fields.
REFERENCES


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* Height and Width of empty octagonal section.
APPENDIX A: HOT WIRE INSTRUMENTATION

A.1 Temperature Compensation

Due to the use of a closed circuit wind tunnel, continuous running led to a gradually increasing flow temperature. Typically at the beginning of a run the ambient temperature was about 75°F. The upper limit, set by tunnel motor requirements (135°F.), was normally reached at an indicated stagnation chamber temperature from 120 to 125°F. The time required to reach this upper limit varied with tunnel loading and ambient conditions but was of the order of two hours. This range of 50°F. introduced severe errors in the uncorrected hot wire measurements, and thus the following method was developed to obtain a high degree of temperature compensation.

The temperature compensation principle utilizes the assumption that the hot wire calibration in the 'constant temperature' technique is primarily determined by the temperature difference maintained between the hot wire sensor and the flow. Hence, if this temperature difference is maintained by a system sensitive only to the long term trend of ambient temperature change, the primary effects of the temperature change on the probe characteristics should be eliminated.

Hinze (Ref.1) gives a working form of Kramer's relation as:

$$I^2 \frac{R_w(T_w)}{R_w(T_w)-R_w(T_g)} = A + B\sqrt{U}$$

where

$$A = \frac{0.42\mu k(T_f)\ell}{[P_r]^{1/5}} \left[ \frac{\rho d}{\mu} \right]^{1/2}$$

$$B = \frac{0.57\mu k(T_f)\ell}{[P_r]^{1/3}} \left[ \frac{\rho d}{\mu} \right]^{1/2}$$

and

$$T_f = (T_w + T_g)/2. \quad \text{In these equations, } I = \text{current through the probe, } R_w = \text{resistance of the probe wire, } k = \text{heat conductivity of the gas, } \alpha = \text{first order temperature coefficient of electrical resistivity, } P_r = \text{Prandtl Number, } \mu = \text{viscosity of the gas, } T_w = \text{temperature of the heated wire, and } T_g = \text{temperature of the ambient gas.}$$

Or, in terms of $V = IR_w$ and $R_w(T_w) - R_w(T_g) = \alpha R_w(T_o)[T_w - T_g]$,

$$V^2 = \alpha R_w(T_o)R_w(T_w)[A + B\sqrt{U}] [T_w - T_g]$$

This indicates the first order effect of the temperature difference; however, as pointed out in Reference 1, the parameter also shows temperature dependence through the heat conductivity $k(T_f)$. The grouping of terms in $P_r$ and $k(\rho/\mu)^{1/2}$ virtually eliminates the effect of the temperature sensitivity of their individual components. Also, a system which acts to keep $(T_w - T_g)$ constant will have to increase $R_w(T_w)$. This is a small effect and is compensated for by empirical adjustment of the final system.

Referring to the sketch in Figure A-1, the system used was to make
another arm (3) of the bridge sensitive to the long term flow temperature changes, and to provide it with a variable effective temperature coefficient of resistance so that it could be matched with that of the hot wire sensor. In practice, arm 3 was made up on a non-inductive coil of nickel wire (whose temperature coefficient of resistance is greater than that of tungsten) and two trimming potentiometers. The nickel resistance wire was placed inside the wind tunnel at the model section. The analysis of the system in order to predict settings for $R_a$ and $R_b$ as a function of sensor resistance is outlined below.

For bridge balance, with a heated wire

$R_1R_3(T_g) = R_2R_w(T_w)$  \hspace{1cm} (A-1)

where

$R_3(T_g) = R_a + \frac{R_bR_w(T_w)}{R_N(T_g)}$  \hspace{1cm} (A-2)

Assuming a reference temperature of $T_0$, to the first order, the tungsten hot wire resistance at any wire temperature $T_w$ is

$R_w(T_w) = R_w(T_0) [1 + \alpha_w(T_w - T_0)]$

Now, for a rise in ambient temperature of $\Delta T$, we require the wire temperature to rise by $\Delta T$ and hence we require an increase in $R_w$ of

$\Delta R_w = R_w(T_0) \alpha_w \Delta T$

Hence for continued bridge balance

$\Delta R_3 = \frac{R_2}{R_1} \Delta R_w = \frac{R_2}{R_1} R_w(T_0) \alpha_w \Delta T$

But $\Delta R_3$ can also be written in terms of the effective temperature coefficient of arm 3, $\alpha_E$, and $\Delta T$ as

$\Delta R_3 = R_3(T_0) \alpha_E \Delta T$

Thus

$\alpha_E = \frac{R_2}{R_1} \cdot \frac{R_w(T_0)}{R_3} \cdot \alpha_w$

Note that if the reference condition is taken as the initial ambient temperature, and that the initial heating ratio, $H = R_w(T_w)/R_w(T_0)$ where $(R_2/R_1) \cdot R_w(T_w)/R_3(T_0)=1$, then

$\alpha_E = \frac{\alpha_w}{H}$  \hspace{1cm} (A-3)

$\alpha_E$ can be found in terms of the parameters of arm 3 as follows: The resistance $R_N$ at some temperature $T$ can be related to its initial value by

$R_N(T) = R_N(T_0) [1 + \alpha_N(T-T_0)]$
Hence

\[ R_3(T) = R_a + \frac{R_b R_n(T)}{[R_b + R_n(T)]} \cdot \frac{[1 + \alpha_n(T-T_0)]}{[1 + \frac{R_n(T)}{R_b + R_n(T)} \cdot \alpha_n(T-T_0)]} \]

\[ \approx R_3(T_0) \cdot [1 + \alpha_B(T-T_0)] \]

where

\[ R_3(T_0) = R_a + \frac{R_b \cdot R_n(T_0)}{[R_b + R_n(T_0)]} \]

and

\[ \alpha_B = \frac{R_b^2 \cdot R_n(T_0)}{[R_b + R_n(T_0)]^2 \cdot R_3(T_0)} \cdot \alpha_n \]

Hence equation (A-3) becomes

\[ \frac{R_b^2 \cdot R_n}{(R_b + R_n)^2} = \frac{R_3 \cdot \alpha_w}{H \cdot \alpha_n} \triangleq K \]  

(A-4)

where all quantities are evaluated at \( T_0 \).

For example, the hot wire amplifiers as initially used had \( R_a = 50 \) ohms, \( H \) was required to be in the range 1.5 to 2.3, and the ratio \( \alpha_W/\alpha_n \) was typically of the order of 1.3. (Temperature coefficients of the tungsten and nickel wire used were experimentally measured as \( 0.00187/\text{°F} \) and \( 0.00242/\text{°F} \) at \( 70^\circ\text{F} \)). These values of the parameters require a design range of \( K \) between approximately 10 and 30.

For \( R_3 = 50 \) ohms the case of no temperature compensation \( (R_b = 0) \) requires \( R_a \) to be trimmable up to 50 ohms. The most suitable value of \( R_n(T_0) \) to provide values of \( K \) between 10 and 30 was found from combining equations A-2 and A-4 to yield

\[ R_n \cdot K = (R_3 - R_a)^2 \]  

(A-5)

Equation A-5 is plotted in Figure A-1 for \( R_3 = 50 \) as \( K \) vs. \( R_a \) with \( R_n \) as a parameter. The final choice of \( R_n = 75 \) ohms obviously gives a good compromise of \( K \) range and low sensitivity to setting accuracy of \( R_a \). The upper limit required for \( R_b \) can be found from equation A-2 as a function of \( R_a \) in the form

\[ R_b = \frac{R_3 (R_n - R_a)}{R_n - R_3 + R_a} \]

In the limit, for \( R_a = 0 \), \( R_b = 150 \) ohms for the parameters mentioned above, but for sensible \( K \) values, the upper limit of 100 ohms used was more than adequate.
The final system can be utilized in a number of different ways. For the initial work in this project, probes were made from .00013 inches diameter tungsten wire with a nominal cold resistance of $R_w(T_0) = 13.5$ ohms. The bridge circuit was then such that for $R_a(T_a) = 50$ ohms the probe's hot resistance at $70^\circ$F. flow temperatures was $27$ ohms ($H=2$). The exact value of $H$ would vary from probe to probe, depending on the exact cold resistance. In this case, we can write

$$K = \frac{R_3(T_0)}{R_1} \cdot \frac{\alpha_w}{\alpha_i} = \frac{R_3(T_0)}{R_w(T_w)} \cdot \frac{\alpha_w}{\alpha_i} \cdot R_w(T_0)$$

$$= h \cdot R_w(T_0)$$  \hspace{1cm} (A-6)

where

$$h = \frac{R_2}{R_1} \cdot \frac{\alpha_w}{\alpha_i}$$  \hspace{1cm} (A-7)

The procedure was then to measure $R_w(T_0)$ which gave $K$ directly. Then $R_a$ is determined from equation (A-5) as

$$R_a = R_3 - (R_3 K)^{\frac{1}{2}} = 50 - (75K)$$  \hspace{1cm} (A-8)

Once $R_3$ was set to this value, $R_1$ would be set such that the total arm resistance $R_3 = 50$ ohms. The probe would then be calibrated and the settings would be left untouched until a complete recalibration was done.

Due to the relatively low power available from the Miller hot wire amplifiers, the probes could not always be driven to an overheating ratio of 2 without overloading the amplifiers. More power is required for higher test speeds and for wires with larger physical dimensions. Hence, to obtain the optimum performance from the system, it was first necessary to run an operating probe at the highest speed expected in a test (usually the expected mean speed plus 2.5 times the expected turbulence standard deviation). The value of $R_3(T_0)$ would then be initially set to provide $H = 2$. If overloading occurred, then $R_3$ was reduced until the overloading condition just disappeared. Temperature compensation could then be accomplished using the same value of $h$ to derive $K$, but using the reduced value of $R_3$ in equation (A-8). In particular, for the probes used through the majority of this work (.00020" dia., 13.5 ohms), $R_3$ was reduced to 42 ohms, making the nominal value of $H = 1.68$.

The value of $h$ was initially calculated directly. However, as expected, initial tests showed that the temperature compensation was not perfect and thus $h$ had to be varied empirically. Under-compensation requires an increase in $h$. The initially calculated value of $h$ was about 20% low.

The effectiveness of the temperature compensation can be seen in Figure A-2, which shows compensated and uncompensated linearized calibrations. The uncompensated drift is of the order of 0.5% per $\circ$F.
A.2 Techniques for Measuring Turbulence Quantities

A.2.1 Linearization and Longitudinal Component Measurements

As mentioned in Section 4.3, the prime hot wire instrumentation was made up of four channels of unlinearized hot wire amplifiers, which were linearized using analog computer components. The linearization law used was that of King or Kramer, in the form (Ref.1)

\[ E = K_u U = \left[ \frac{v^2 - a}{b} \right]^2 \]

This was adopted for simplicity rather than the more general relation

\[ v^2 = a' + b' U^n \]

where the best value of \( n \) is still in some dispute but appears to be closer to 0.45 (Ref.4) than King's 0.5. In the range of interest in this experiment (10 < \( U < 100 \) fps), it is interesting to note in Figure A-3 that data which actually follows a \( U^{0.45} \) law can be accurately fitted by a \( U^{0.5} \) law. However, all of the experimental calibrations do consistently show some curvature, suggesting that the most accurate value of \( n \) is less than 0.5.

A typical channel of linearization is shown in Figure A-4. The procedure for its use in a U-calibration is as follows: After setting the bridge parameters and temperature compensation network as described in Appendix A.1, then:

i) Run the probe in the wind tunnel at the maximum expected speed, \( U_{\text{max}} \);

ii) Set \( k_1 \) such that the magnitude of the first stage output = 100 volts = 100 \( k_1^2 V_{\text{max}}^2 \);

iii) Reduce the tunnel speed in steps and plot 100\( k_1^2 V^2 \) versus \( \sqrt{U} \);

iv) From this plot, extrapolate a best linear fit to obtain 100 \( k_1^2 V_o^2 \) as required for the linearizer. Note that this will not be the same as the circuit output at zero velocity;

v) Set \( a = 100 k_1^2 V_o^2 \);

vi) Set \[ \frac{n}{10} = \frac{100 k_1^2 V_{\text{max}}^2}{1000 k_1^2 (V_{\text{max}}^2 - V_o^2)} = \frac{10}{(100-100 k_1^2 V_o^2)} \text{ for this case.} \]

This then provides a linearized output \( E = K_u U \) where

\[ K_u = \frac{100}{U_{\text{max}}} \text{ volts/fps} \]

Usually, as a check, and to ensure accuracy, a final direct calibration was
done of \( E \) vs \( U \). A typical first stage and overall calibration is given in Figure A-5, showing the typical residual curvature caused by assuming \( n = 0.5 \).

A number of the calibrations performed during the experiments were re-examined in order to define a value of \( n \) for future improvement of the linearization circuitry. The results were fitted by a least squares technique for various values of \( n \), until a minimum residual least squares total was found. For 4 runs, the values of \( n \) were 0.41 to 0.43. These are somewhat lower than the 0.45 expected but are similar to those found by Portfors (Ref.6) using a similar technique. He reported \( n = 0.42 \) for \( 10 < U < 30 \) fps and \( n = 0.38 \) for \( 20 < U < 60 \) fps.

Calibrations were very repeatable, except for residual temperature drifts that were normally less than 2%. However, a program of wind tunnel cleanliness had to be maintained to prevent drift occurring from dirt accumulation on the wires. A particularly extreme example of the effect of dirt accumulation can be seen in the photograph of a wire (Fig.A-6) exposed to flow at about 50 fps for 30 minutes when welding smoke was present in the laboratory. Also shown is a picture of a wire exposed during an extended period of running (3 to 4 hours), which had shown very little calibration drift. However, some accumulant is present - which appeared to be tiny oil droplets. A similar wire state was observed by Collis (Ref.2).

### A.2.2 X-wire Technique - Cross-component Measurements

The use of the analog linearizer simplified greatly the cross-component measurements using X-wires. Because each channel could be independently linearized, it was not required that they be exactly physically matched. Figure A-7 shows the notation used. Note that in this section, \( v \) is the cross-component being measured but that the discussion applies equally to \( w \) through a 90° rotation of the probe axis.

Assuming the wire responds only to the magnitude of the normal component of the velocity (Ref.1), then the linearized output of wire 1 is

\[
E_1 = k_1 \left\{ \left[ (\bar{U} + u)\sin\phi_1 + v\cos\phi_1 \right]^2 + w^2 \right\}^{1/2} \\
\approx k_1 \left[ (\bar{U} + u)\sin\phi_1 + v\cos\phi_1 \right] = E_1 + e_1 \tag{A-9}
\]

Similarly,

\[
E_2 = k_2 \left[ (\bar{U} + u)\sin\phi_2 - v\cos\phi_2 \right] = E_2 + e_2 \tag{A-10}
\]

Hence

\[
e_1 - e_2 = u \left( k_1 \sin\phi_1 - k_2 \sin\phi_2 \right) + v \left( k_1 \cos\phi_1 + k_2 \cos\phi_2 \right)
\]

Then, the experimental procedure is such that for a carefully defined probe axis/mean velocity vector alignment, each channel is calibrated vs. \( \bar{U} \) and the gains are set such that \( K_{u_1} = K_{u_2} \). Since \( K_{u_1} = k_1 \sin\phi_1 \) in this case, then we are left with

\[
e_1 - e_2 = K_v U_2
\]
The final calibration to obtain $K_v$ was done directly in steady flow by yawing the probe through an angle $\theta$ to obtain an effective mean cross-component = $\bar{U}\sin\theta$ and a mean longitudinal component = $\bar{U}\cos\theta$. Then $K_v$ was found from a plot of $(E_1 - E_2)$ vs. $\bar{U}\sin\theta$. A typical calibration is shown in Figure A-8 for a number of mean speeds, showing independence of $\bar{U}$ to a good approximation.

There are a number of assumptions in this X-wire technique which bear further discussion. Obviously, in all the hot wire analysis so far, spatial coherence of the velocity fluctuations is assumed. This is discussed further in Section A.3. Beyond this fundamental flow assumption, there may be errors introduced by:

i) intermittently exceeding the probe's velocity or angle limitations

ii) misalignment of the probe axis during re-installation for final measurements.

iii) residual u-sensitivity, i.e., $K_{u_1}$ and $K_{u_2}$ not exactly equal

iv) errors in the original normal component law

i) If the angle $\theta$ is defined as that between $\bar{U}$ and $\bar{V}$, then, neglecting the effect of $w$,

$$\tan \theta = \frac{v}{\bar{U} + v}$$

For the present experiment, a conservative case can be examined by assuming $u = v = w = 0.15\bar{U}$. If the turbulence components are also taken as Gaussian and independent, they each exceed 0.3 $\bar{U}$ in magnitude about 5% of the time. The value of $\theta_{\text{max}}$, as defined by

$$\tan \theta_{\text{max}} = \frac{0.30 \bar{U}}{\bar{U} - 0.30\bar{U}} = 3/7$$

$$\theta_{\text{max}} = 23.4^\circ$$

can then be taken as a representative design value. In actual fact the probability of exceeding any angle $\theta$ whose tangent is $\beta$ is (see Fig. A-9)

$$P_r \{ \theta > \theta_{\text{max}} \} = 1 - \frac{1}{\pi \phi^2} \int_0^\infty du \int_0^\infty \exp\left(\frac{u^2}{2\sigma^2} - \frac{v^2}{2\sigma^2}\right) dv$$

$$= 1 - \frac{1}{\sqrt{2\pi}} \int_0^\infty \exp\left(-\frac{u^2}{2\sigma^2}\right) \cdot \text{erf}\left(\frac{\beta (\bar{U} + u)}{\sqrt{2}\sigma}\right) du$$

A simpler approach can be adopted by noting that
\[ P_r \left\{ \theta > \theta_{\text{max}} \right\} < P_r \left\{ \sqrt{u^2+v^2} < R \right\} \]

where

\[ R = \frac{\bar{U} \beta \cos \theta}{\sqrt{1 + \beta^2}} \]

and

\[ P_r \left\{ \sqrt{u^2+v^2} < R \right\} = 1 - \int_c \frac{1}{2\pi \sigma^2} \cdot \exp \left( -\frac{u^2}{2\sigma^2} - \frac{v^2}{2\sigma^2} \right) .du.dv \]

\[ = 1 - \int_0^R \frac{1}{2\pi \sigma^2} \cdot \exp \left( -\frac{r^2}{2\sigma^2} \right) .2\pi r .dr \]

\[ = \exp \left( -\frac{R^2}{2\sigma^2} \right) \]

Hence, for \( \sigma/\bar{U} = 0.15 \), \( \beta = 3/7 \)

\[ P_r \left\{ \theta > \theta_{\text{max}} \right\} < 3.4\% \]

The nominal wire angles \( \phi \) were \( 45^\circ \). This provided a good compromise between sensitivity, which requires the angles \( \phi_1 \) and \( \phi_2 \) to be small, and the fact that obedience to the normal component law is only good down to angles of about \( 15^\circ \) (Ref.3). The results of the above analysis show that in isotropic turbulence of 15\% intensity, the relative angle between the velocity vector and either wire is less than \( 21.6^\circ \) less than 3.4\% of the time. Hence the effect of occasionally exceeding the probe's angle limitation is not significant in this experiment.

The normal velocity component relative to a \( 45^\circ \) wire can be shown to be (again neglecting \( w \))

\[ U_n = \frac{1}{\sqrt{2}} (\bar{U} + u + v) \]

As before, reasonable design points are \( u=v=2\bar{U} = 0.30\bar{U} \) since

\[ P_r \left\{ u + v > 4\sigma \right\} < P_r \left\{ \sqrt{u^2+v^2} > 2\sqrt{2}\sigma \right\} \quad \text{(see Fig.A-9)} \]

\[ < e^{-4.0} = 0.0183 \]

In 15\% isotropic turbulence then the normal component will be expected to exceed \( (1.6/\sqrt{2})\bar{U} = 1.13 \bar{U} \) less than 1.83\% of the time. Thus, the bridge parameters were set such that one of the X-wires operated normally to the flow would not over­load at 1.13 times the intended mean test speed. This ensured negligible error due to the velocity exceeding the probe system limits.

ii) Since the probes are calibrated directly, the probes will thereafter measure the cross-component relative to the originally calibrated alignment axis. During
calibration, this alignment axis is not necessarily the probe axis, but is simply the mean velocity vector's orientation in a probe-fixed reference frame during the process of equalizing $k_1 \sin \phi_1$ and $k_2 \sin \phi_2$. Thus, if a misalignment error $\Delta \theta$ is introduced when the probe is placed in a measuring location, then

$$E_1 - E_2 \approx K_v [v + \bar{u} \Delta \theta + u \Delta \theta]$$

The mean value $K_v \bar{u} \Delta \theta$ will indicate the misalignment if the local mean velocity vector has no inherent mean cross-flow. This D.C. component is easily removed, but the error term $u \Delta \theta$ contributes to the mean square value, i.e. the indicated cross-component

$$v' = v + u \Delta \theta$$

and

$$v'^2 = v^2 + u^2 \Delta \theta^2 + 2uv \Delta \theta$$

Thus, for isotropic turbulence ($uv = 0$) the effect of misalignment on the indicated rms is

$$v' \approx v (1 + \frac{\Delta \theta^2}{2})$$

or an error of about $0.02\%$ per degree. However, if a large degree of shear exists, then in the limit as $uv = u'\bar{v}$ then

$$v' \approx v (1 + \Delta \theta)$$

or an error of about $2\%$ per degree.

iii) A similar behaviour is displayed due to residual u-sensitivity caused by a slight error in the initial channel equalization. If

$$k_1 \sin \phi_1 - k_2 \sin \phi_2 \neq 0$$

$$= \mu K_v$$

then the above argument holds exactly for $\Delta \theta$ replaced by $\mu$. Hence an error of $0.02 K_v$ in equalization of channels would result in negligible error in cross-component measurement for no shear, but up to $2\%$ error for a high degree of shear.

iv) The original normal component law is known to be inaccurate as the velocity vector becomes more nearly parallel with the hot wire. Hinze (Ref.1) suggests a better law should show a parallel component dependence for the effective velocity as

$$U_E = U [\sin^2 \phi + A^2 \cos^2 \phi]^{\frac{1}{2}}$$

(Hinze suggests values of $A$ between 0.1 and 0.3. Champagne (Ref.5) has made a detailed investigation of inclined wire behaviour and has found $A$ to decrease with increasing $L/d$ ratio of the wire used. For the value used here ($L/d = 675$), $A$ was found to be less than 0.04 for a wire at $Re \approx 10$. Since the typical wire $Re$ in this work is approximately 5, it is in the same general flow behaviour region (which roughly extends between $Re$'s of 0.1 to 44) and hence we can assume negligible tangential component dependence. It may be noted, however, that any
A small tangential component effect can be linearized to the form

\[ U_E = (\bar{U} + u)S_1(\phi)\sin\phi + vS_2(\phi)\cos\phi \]  

(A-12)

where \( S_1(\phi) \) and \( S_2(\phi) \) are the results of the linearization of equation A-11 and are detailed in Reference 5. Hence, these first order effects introduce no additional error due to the direct calibration technique involved, but only mean that channel equalization demands

\[ k_1S_1(\phi_1)\sin\phi_1 = k_2S_1(\phi_2)\sin\phi_2 \]

and that the analytical expression for

\[ K_v = k_1S_2(\phi_1)\cos\phi_1 + k_2S_2(\phi_2)\cos\phi_2 \]

which is then found by a direct yaw calibration.

A.2.3  X-Wire Technique - Shear Measurements

The following method allows measurement of the instantaneous values of the shear terms \( uv \) or \( uw \). Although it was used in this work and is included here for completeness, its development is due to Mr. T. R. Nettleton, who is conducting a parallel project involving the response of wings to shear turbulence.

It has previously been shown how the two channels of the X-wire output are combined to provide a \( v \) response. It is now further shown that the same outputs can be combined to provide a \( u \) response. Using equation (A-9) and (A-10), we obtain

\[ \mu_1E_1 + \mu_2E_2 = (\mu_1k_1\sin\phi_1 + \mu_2k_2\sin\phi_2)(\bar{U} + u) \]

\[ + (\mu_1k_1\cos\phi_1 - \mu_2k_2\cos\phi_2)\nu \]

where \( \mu_1 \) and \( \mu_2 \) are arbitrary constant gains chosen such that

\[ \mu_1k_1\cos\phi_1 = \mu_2k_2\cos\phi_2 \]
\[ \mu_1/\mu_2 = k_2\cos\phi_2/k_1\cos\phi_1 \]

Since the initial channel equalization requires that

\[ k_1\sin\phi_1 = k_2\sin\phi_2 \]

we then have

\[ \mu_1/\mu_2 = \tan\phi_1/\tan\phi_2 \]  

(A-13)

Since only the ratio is important, either \( \mu_1 \) or \( \mu_2 \) can be defined as 1.0. In practice, the gains were applied with potentiometers so that they would be equal to, or less than, 1.0. The ratio \( (\tan\phi_1/\tan\phi_2) \) requires definition of the two wire
angles \( \phi_1 \) and \( \phi_2 \). Since they were nominally equal, a perfect probe would result in \( \mu_1 = \mu_2 = 1 \). To determine the wire angles, it was noted that during the yaw calibration to determine \( K_y \), the two wire outputs can be written (for a yaw angle \( \theta \), and mean calibrating speed \( \bar{U} \))

\[
E_1(\theta) = k_1 \bar{U} \sin(\phi_1 + \theta) \\
E_2(\theta) = k_2 \bar{U} \sin(\phi_2 - \theta)
\]

Hence,

\[
\frac{E_1(\theta)}{E_1(0)} = \frac{\sin(\phi_1 + \theta)}{\sin(\phi_1)}
\]

And

\[
\frac{E_2(\theta)}{E_2(0)} = \frac{\sin(\phi_2 - \theta)}{\sin(\phi_2)}
\]

Figure A-10 shows a plot of the function

\[
\frac{E(\theta)}{E(0)} = \frac{\sin(\phi + \theta)}{\sin(\phi)}
\]

as a function of \( \phi \) with \( \theta \) as a parameter. The calibration procedure was then simply to record the individual wire outputs \( E_1 \) and \( E_2 \) during the \( K_y \) calibration, and plot the above ratios on the appropriate yaw angle curve. A complete yaw calibration at every 5° between \( \pm 30° \) would then yield 12 independent estimates for each of \( \phi_1 \) and \( \phi_2 \) and the two best vertical straight lines would then define \( \phi_1 \) and \( \phi_2 \). The scatter in the estimates was typically \( \pm 1° \), although it was found that the most consistent results were obtained if the probe was subjected to about 100 fps. for a half hour prior to calibration. This allowed for an initial deformation of a newly strung wire and hence change in its effective angle.

Application of the ratio \( \mu_1/\mu_2 \) as found above then gave

\[
\mu_1 E_1 + \mu_2 E_2 = K_\bar{U} (\bar{U} + u)
\]

where

\[
K_\bar{U} = \mu_1 k_1 \sin(\phi_1) + \mu_2 k_2 \sin(\phi_2)
\]

and \( K_\bar{U} \) was again found by direct calibration.

Finally, an output proportional to the instantaneous value of \( uv \) is given by

\[
(u_1 e_1 + u_2 e_2) (e_1 - e_2) = K_\bar{U} uv
\]

If desired, an overall check on the system can then be accomplished by noting that when yawed at an angle \( \theta \) in steady flow, the system output should be \( uv = \bar{U} \sin 2\theta \). Calibrations of this type at various values of \( \bar{U} \) show good agreement.

Errors introduced into this method can be treated similarly to those discussed in Section A.2.2. In particular, the assumption of a strict normal component law has simplified the procedure considerably. As noted before the tan-
gential component correction is very small for this case, and this is verified to some degree by the repeatability of the direct uv calibration with different values of $\bar{U}$, and the agreement among the estimates of wire angles obtained at different flow yaw angles. Note that for shorter wires where an equation of the form (A-12) must be assumed, the above approach would still be applicable. However, equations (A-13) and (A-14) would be modified to read

$$\mu_1 = \frac{\tan \phi_1 S_1(\phi_1)}{\tan \phi_2 S_1(\phi_2)}, \quad \mu_2 = \frac{S_2(\phi_2)}{S_2(\phi_1)}$$

and

$$\frac{B(\theta)}{E(\theta)} = \frac{\cos \theta \sin \phi + [S_2(\phi)/S_1(\phi)] \sin \theta \cos \phi}{\sin \phi}$$

(A-15)

and the application of equation (A-15) to obtain the wire angles would require an initial estimate of $S_2(\phi)/S_1(\phi)$.

The main sources of error are those due to probe axis misalignment, and residual sensitivities of $u$ to $v$ and $v$ to $u$ due to inaccuracies in calibration set up, wire drift, etc. Since they can be treated as equivalents as noted before, consider a misalignment angle of $\Delta \theta$. Then, priming the indicated values,

$$v' = v + \bar{U} \Delta \theta + u \Delta \theta$$
$$u' = u = v \Delta \theta$$

Hence

$$u'v' = uv + u \bar{U} \Delta \theta + (u^2 - v^2) \Delta \theta$$

Thus, although some inaccuracy may be present in the instantaneous value of the indicated shear terms, as long as $u^2 \approx v^2$, then $u'v' \approx uv$. Otherwise

$$\frac{u'v'}{uv} = \frac{uv}{uv} + \Delta \theta \left( \frac{v^2 - 1}{v} \right)$$

where $\frac{u^2}{v} = \frac{v^2}{v}$. For a typical shear coefficient of 0.5 and $v = 2$, the error introduced is about 2.5% per degree.

A.3 Frequency Response Considerations

The frequency response of a hot wire system can be viewed as made up of two separate parts:-

i) the response of the system to a coherent, two dimensional sinusoidal gust

ii) the attenuation of the real three-dimensional turbulence due to the finite length of the hot wire

The basic system response of a constant temperature hot wire anemometer has been treated widely - both as to the thermal lag effects of the wire and to the electronic characteristics of the anemometer networks. Due to the relatively
low frequencies (0-2000 Hz.) considered in this experiment, the upper frequency limit of this aspect of the hot wire sets was not investigated directly. Instead a comparative check between the Miller amplifiers and the Disa hot wire anemometer was carried out and showed good agreement within the experimental range. The analog computer linearizer was included in this test but was not restrictive as components are typically flat from DC to above 15kHz.

The second aspect of the anemometer's response arises because of the three dimensional nature of the turbulence. The wire then responds to the integrated average velocity fluctuation along the wire if a locally two-dimensional cooling relation is assumed. That is, for a wire, the effective turbulent cooling velocity is

\[ u_E(t) = \frac{1}{l} \int_0^l u(y,t) \, dy \]

then we can write

\[ R_{uE uE}^{(\tau)} = \frac{1}{l^2} \int_0^l \int_0^l \alpha u_\alpha(t) u_\beta(t+\tau) \, d\alpha d\beta \]

If homogeneity can be assumed so that

\[ R_{u_\alpha u_\beta}^{(\tau)} = R_{u_\alpha u_\beta}^{(\xi)} \]

where \( \xi = |\beta - \alpha| \) then simple geometrical interpretation of the above integral yields

\[ R_{uE uE}^{(\tau)} = \frac{1}{l^2} \int_0^l (l-\xi) R_{u_\alpha u_\beta}^{(\xi)} \, d\xi \] (A-16)

Hence, the error in the mean square measurements can be found as given in Reference 1:

\[ u_E = R_{uE uE}^{(0)} = u \left[ \frac{1}{l^2} \int_0^l (l-\xi) R_{uu}^{(\xi)} \, d\xi \right] \]

Reference 1 discusses the form of these corrections in detail. They are usually small because the correction is an attenuation of the higher frequency components which usually contain only a small fraction of the energy. For this experiment, they are considered negligible.

However, it is difficult to infer from these relations the effect of the finite wire length on the spectral density measurements at high frequency. A complete theoretical discussion of the effect of finite wire length if given in Reference 10. However, its application requires solution of an integral equation involving the true energy spectrum function. A simpler approach based on recent empirical evidence is given below.

Recently, Davenport (Ref. 7 and 27) has shown that the form of the lateral correlation for any particular frequency of turbulence appears to be a collapsible function of \( f \xi/\bar{U} \). This has been verified elsewhere, including the present work in homogeneous turbulence, and Cooper (Ref. 9) in shear turbulence. This "narrow band" correlation function is actually simply the square root of the coherence,

A13
which is given by

\[ \gamma^2 = \frac{c^2}{\phi_{u^2}(f)\phi_{u^2}(f)} \]

for homogeneous turbulence. Davenport found that to a good approximation his data was described by

\[ \gamma = e^{-\frac{b\xi}{\lambda}} \]  

(A-17)

where \( b \) is a constant whose value lies between 5 and 8. Hence, Fourier transforming each side of equation (A-16), we obtain

\[ \phi_{E^2}(f) = \frac{1}{\ell^2} \int_0^\ell (\xi - y) \phi_{u^2}(\xi, f) \, dy \]

Since, for homogeneity,

\[ \phi_{u^2}(f) = \phi_{u^2}(f) \quad \text{and} \quad \phi_{u^2}(f) = C_{u^2} \]

then

\[ \phi_{E^2}(f) = \frac{1}{\ell^2} \int_0^\ell (\ell - \xi) \, d\xi \]

For the empirical form of \( \xi \), this equation can be integrated to yield

\[ \phi_{E^2}(f) = \phi_{u^2}(f) \left[ \frac{2}{\xi} - \frac{2}{\xi^2} \left( 1 - e^{-\xi} \right) \right] \]  

(A-18)

where

\[ \xi = \frac{bfl}{U} \]

This function is plotted as Figure A-11. For typical experimental parameters used here (\( U = 55 \) fps, \( \ell = 0.135" \)) and a representative \( b \) value of 7, this theory shows the measured spectrum to be approximately 20% low at 500 cps and 53% low at 2,000 cps.

The corrections indicated by the above approach are considerably higher than those assumed in the past. Since the only assumption in the above is that of the collapse of the coherence function when plotted versus \( f\xi/U \), the main element of doubt is to the applicability of this result over all spectra and including the highest frequency components. Although some differences in the constant \( b \) have been reported, they are not marked, and there is no evidence that this empirical result is not true. There is an important need for verifying the regime of applicability of Davenport's approach.

Some experimental evidence exists as to the effect of finite wire length on measured spectral results, but the data is somewhat contradictory. Portfors (Ref.6) made measurements of the same turbulence with three wire lengths for two
different turbulence regimes. In both cases his results show less effect than that predicted by Uberoi and Kovazcnay's approach (Ref.10). Furthermore, the results show different effects for the same frequency in the two different cases. This latter result questions the validity of equation (A-17). However, a similar but brief experiment by Teunissen of UTIAS, using the Disa 55A01 hot wire anemometer with two different probe lengths for one turbulence case showed a difference in spectral results agreeing well with that predicted by the above approach. In both experimental cases, however, the wire parameters were necessarily changed and hence the basic system response to a coherent gust was altered. This should not be severe at the frequencies involved. However, it is obvious that clarification of the situation is required both by exploring the applicability of the Davenport result and by hot wire measurements which include system response measurements to a coherent input. Because of the weight of hot wire investigations which have previously neglected a length correction, and because of the lack of concrete experimental verification, the above corrections were not made to the data presented herein.
APPENDIX B: SPECTRAL ANALYSIS TECHNIQUES

Three methods of spectral analysis were used during the course of the experiments. Their application and a comparison of their relative merits are discussed below.

B.1 Analog Spectral Analysis

The Bruel & Kjaer 2107 Wave analyzer used was of a constant percentage bandwidth variety. That is

\[ g(f/f_c) = \frac{e_o^2(f)}{e_o^2(f_c)} \]

is a function only of \( f/f_c \) where

\[ f_c = \text{the frequency at which the filter is centered} \]

and

\[ e_o^2(f) = \text{the mean square output for a sinusoidal input of frequency } f. \]

Hence when a wide band time varying signal \( R(t) \) is applied to the filter input, where

\[ R^2 = \int_0^\infty \phi_{RR}(f) \, df \]

the output \( r(t) \) has a mean square given by

\[ \overline{r^2(f_c)} = \int_0^\infty \phi_{RR}(f) \, g \left( \frac{f}{f_c} \right) \, df \]  \hspace{1cm} (B-1)

Now, \( g \left( \frac{f}{f_c} \right) \) is usually nearly symmetric about \( f/f_c = 1 \), and falls rapidly to zero on either side. Hence, if \( \phi_{RR}(f) \) is constant, or is anti-symmetric about its value at \( f = f_c \) in the vicinity of \( f/f_c = 1 \), then to a good approximation

\[ \overline{r^2(f_c)} \approx \phi_{RR}(f_c) \int_0^\infty g(f/f_c) \, df = \phi_{RR}(f_c) G f_c \]  \hspace{1cm} (B-2)

where

\[ G = \int_0^\infty g(f/f_c) \, d(f/f_c) \]

is the experimentally measured effective bandwidth of the wave analyzer. Thus, the estimate for the spectral density function of \( R(t) \) at \( f_c \) is then given by

\[ \hat{\phi}_{RR}(f_c) = \frac{\overline{r^2(f_c)}}{G f_c} \]

In practice, whether analyzing a real time or pre-recorded signal, the wave analyzer outputs were measured using a Bruel & Kjaer 2417 Random Noise Meter. This allowed a time constant between 0.3 and 100 seconds to be used. Bendat and Piersol (Ref.11) give the statistical reliability of such spectral measurements as
\[
\frac{\sigma^2(\phi)}{\mu^2(\phi)} = \frac{\text{var}\{\hat{\Phi}_{RR}(f_c)\}}{\left[\text{avg}_{RR}(f_c)^2\right]} \approx \frac{1}{\Delta f_c T}
\]

where \(\Phi_{RR}(f_c)\) is the true spectral density at \(f_c\) and \(\Delta f_c\) is the effective bandwidth of the filter and is here taken equal to \(G f_c\).

Hence for the worst case at frequencies of 20 Hz using an 8\% bandwidth, \(\sigma(\phi)/\mu(\phi) = 0.056\) for a 100 sec. time constant. \((T = 200)\). This statistical reliability of approximately 5\% was typical of all measurements as the time constant was normally reduced for higher frequency measurements and was consistent with the agreement obtained in repeatability tests.

Systematic inaccuracies were introduced in the spectral estimates (assuming a perfect input) due to

i) the assumptions used to derive equation (B-2) from (B-1)

ii) experimental determination of \(G\)

iii) power response of the wave analyzer over the measurement range

iv) power response of the random noise meters used to measure the output

The error involved in (i) is dependent upon the spectral shape involved. For the typical spectra analyzed in this report errors are largest near peaks in the spectra, or where the spectra are falling off rapidly according to a power law. In the latter case, the comparisons of Section B.3 show that this method tends to overestimate the spectral density slightly as might be expected. For the limiting case of peaks where the data contains a true spike, as used in Section B.3 to compare the spectral analysis methods, then the spectral estimates in the neighborhood of the spike frequency form an image of the analysis window, i.e., of the filter's frequency response. In the case of real vortex peaks as encountered in the surface pressure spectra, the analysis window tends to broaden and lower the peak. Most of these latter cases were handled using the digital methods discussed in later sections, which have narrower analysis windows. However, comparison between their results and those obtained using the analog wave analyzer show little difference, indicating that the ratio of analysis window width to real peak width is small enough not to introduce appreciable distortion.

The experimental determination of \(G\) was carried out at several frequencies over the range of interest with resulting values of \(G\) falling within a range of \(\pm 1\%\).

The power response of the wave analyzer was measured as the variation of \(e_0^2(f)\) with \(f\) for a constant amplitude sine wave input. The power response varied within \(\pm 10\%\) over the range 20-2,000 Hz. It was approximated by a set of linear corrections which allowed maximum errors of \(\pm 4\%\).

The power response of the noise meters was determined for sine wave inputs as a function of both input amplitude and frequency by comparing the meter outputs to the output of an analogue squaring and averaging circuit. The results show the frequency response to be essentially constant for the range 2-2,000 Hz. The power response as a function of input amplitude is accurate within \(\pm 3\%\) maximum over the normally used amplitude range (upper two-thirds of the scale). The
meters are designed to give true rms outputs for sine wave or Gaussian inputs. The statistical characteristics of the inputs were considered closely enough to Gaussian or Gaussian plus sinusoid that this was not considered a source of error.

Total maximum systematic errors in analog spectral estimates are then of the order of \( \pm 8\% \), and hence the probable deterministic error in any particular spectral estimate is of the order of half this, or \( \pm 4\% \).

B.2 Digital Analysis Techniques

The two techniques described below are both essentially digital techniques, although the second incorporated a hybrid instrument to form 100 analogue correlation estimates which are then digitally sampled.

B.2.1 Direct Data Sampling Technique - EECO System

This section refers to the digital data reduction system described in section 3.7.2 utilizing the EECO A/D system. The theoretical considerations underlying the production of spectral estimates from equi-spaced data samples have been considered in detail by Blackman and Tukey (Ref.12) and have been summarized as applied to the system used at UTIAS by Reid (Ref.13). The method used in this experiment is essentially the same as that used by Reid, except that the data was sampled at speeds consistent with this experiment's requirements. This required the use of the high speed mode in the EECO system, which could only handle one input at a time. Hence, simultaneous data samples from two records for cross-correlation purposes required consecutive digitizing of tape-recorded data with pre-recorded timing pulses. However, once digitized, the spectral estimates were obtained using the same subroutines described by Reid. Thus, only the idiosyncrasies of the present system will be described, making use where necessary of the results given by Reid.

There are two major effects of digitally sampling time records at intervals \( \Delta t \) in order to produce digital estimates of correlation functions and spectra (both power and cross).

Due to inevitable physical limitations, the correlation function must be truncated. This results in the spectral estimates being viewed through a window which averages the spectrum over a finite band - a situation similar in concept to that of the analog analyzer's window as described by the function \( g(f/f_c) \), except that here the window is the same at all frequencies - i.e. it could be described by \( g'(f-f_c) \). The spectral window can be modified to a certain extent by changing the manner in which the correlation function is truncated. (Truncation here is thought of as multiplying the infinitely extending correlation function by a second function which is zero for time delays \( \tau \) greater than some \( \tau_m \)). Typically the characteristic shapes of digital windows are similar to the \( \sin x/x \) function. It has been found that the damped oscillatory side bands can be reduced to less than 3\% by incorporation of a truncation function due to Hanning (see Ref.12). It was used for all the digital estimates presented here. The effective bandwidth of the main lobe, \( \Delta f_e \), is given by Tukey as

\[
\Delta f_e = \frac{1.30}{\tau_m}
\]

The second major effect is that of aliasing. It arises due to the
inability of a particular sampling rate $f_s$ to define frequencies higher than the Shannon limit $f_o = f_s/2$.

Aliasing leads to the fact that the spectral estimate developed at some frequency $f$ in the analyzable range less than $f_s$ is actually a sum of the power existing in the input signal at frequencies of $0, 2nf + f$ where $n = 0, 1, 2, \ldots$. This is often viewed geometrically as an accordion type of folding of the spectrum at frequency multiples of $f_o$.

The combination of truncation and aliasing merely involves having a spectral window centered at each of the aliasing frequencies.

The experimental design of sampling rates and record lengths requires compromises due to the above two effects, and the statistical variability inherent in spectral estimates. The latter is given by Reference 12 as

$$\frac{\sigma^2(\hat{\phi})}{\mu^2(\hat{\phi})} = \frac{3}{4} \frac{T_m}{T'}$$

for $f > 3T_m/4$ increasing to a maximum of $1.5 \frac{T_m}{T'}$ at $f = 0$ where $T' = T - 0.3T_m$ for the Hanning window, and where here $\hat{\phi}$ denotes a spectral estimate.

Hence, for the case at hand, the relations are

$$\frac{\sigma(\hat{\phi})}{\mu(\hat{\phi})} = \sqrt{\frac{3}{4} \frac{T_m}{T'}}$$

$$\Delta f_e = \frac{1.3}{T_m}$$

$$f_o = \frac{1}{2\Delta \tau} \quad (\Delta t = \Delta \tau \text{ in this work})$$

$$S = f_s T$$

where $S$ is the total number of samples and $T$ is the record length in seconds.

Note that spectral estimates are usually derived at frequency increments of $\Delta f = 1/2T_m$ and hence adjacent points are not independent. However, in this work all points developed were plotted except those beyond the aliasing limit discussed shortly.

The above expressions can be combined to give

$$\frac{\sigma(\hat{\phi})}{\mu(\hat{\phi})} = \left[ \frac{0.976}{\frac{\Delta f_e}{f_o} \cdot \frac{S}{2} - 0.39} \right]^{1/2}$$

The fast fourier transforms technique employed in the programming (see Ref.13) and core storage restrictions of the IBM 7094 computer led to a choice of $S = 4,096$. The above equation is plotted in Figure B-1 for this value of $S$. The EECO system restricted the sampling frequency to 600 per sec. and hence $f_o$ to 300 hz. However, data compression using the SP300 tape recorder allowed
effective Shannon limits of eight times this to be obtained.

In the spectra dealt with here where the energy is typically falling off as $f^{-2}$ for frequencies above 250 hz. the major aliasing problem is that due to the first alias - i.e.

$$\phi_{\text{meas.}}(f) \approx \phi(f) + \phi(2f_0 - f)$$

For a $1/f^2$ spectrum, the first alias contributes 10% to the measured power at $f/2$ and increases to double the power at $f_0$.

This problem was avoided by incorporating in the system a sixth order Butterworth filter through which the data was replayed prior to digitizing. The filter's circuit and its theoretical and experimental amplitude response are shown in Figure B-2. This reduces the aliasing problem to be less than 2% in power at $f/f_0 = 0.833$. The experimental power response was used to correct the data, which was found to be reliable up to this value of $f/f_0$. Note that since all data was passed through the same filter, the phase change was identical for all data and hence cross-power spectral densities were unaffected.

In order to keep the statistical reliability within reason and to maintain a small $\Delta f_e$, the data was analyzed in two separate frequency bands. Both pressure and velocity measurements were first subjected to a low frequency, high resolution "cut" and then a high frequency, low resolution cut. The parameters used are shown below

<table>
<thead>
<tr>
<th>Case</th>
<th>$f_0$ (hz.)</th>
<th>$f_{\text{max}}$ (hz.)</th>
<th>$\Delta f$ (hz.)</th>
<th>$\Delta f_e$ (hz.)</th>
<th>$\sigma(\phi) / \mu(\phi)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Velocities and Pressures - low cut</td>
<td>300</td>
<td>250</td>
<td>2.0</td>
<td>5.2</td>
<td>0.167 / 0.084</td>
</tr>
<tr>
<td>2. Pressures - high cut</td>
<td>1200</td>
<td>1000</td>
<td>7.6923</td>
<td>20.0</td>
<td>0.173 / 0.087</td>
</tr>
<tr>
<td>3. Velocities - high cut</td>
<td>2400</td>
<td>2000</td>
<td>20.0</td>
<td>52.0</td>
<td>0.150 / 0.075</td>
</tr>
</tbody>
</table>

($f_{\text{max}}$ is that frequency to which the data is considered acceptable).

To reduce further the statistical variability, four sets of data were digitized for each frequency cut. The assumption was then made that the sets were far enough apart in time so that they constituted four statistically independent sets. In this case, for $x_i(t_i)$ being the $i$th set from the same record, and the sum

$$X(t) = \sum_{i=1}^{4} x_i(t_i)$$

then

$$R_{xx} = \sum_{i=1}^{4} R_{x_i x_i}$$

In this way, the sum of the four data cuts can be analyzed, which
effectively multiplies the record length by four, reduces the statistical variability by a factor of two, while only requiring additional core storage for one temporary record. The time allowed between data sets in terms of the number of periods associated with the spectral estimate at \( f = \Delta f \) was greater than 20 in all cases. In order to verify the method, the spectrum derived from \( R_{xx} \) was compared with that derived from \( \sum_{i=1}^{n} R_{x_i x_i} \) where the latter was created by individually obtaining each \( R_{x_i x_i} \). A typical result is shown in Figure B-3 where both frequency cuts are shown for the assumed and the true method of analysis. No significant difference is evident.

B.2.2 Hybrid Technique - PAR/526 System

The Princeton Applied Research Models 100 and 101 correlators produce 100 estimates of the cross-correlation function of any two inputs. The 100 estimates are essentially equi-spaced in the \( \tau \)-range from zero to \( \tau_m \) at intervals of \( (n+\frac{1}{2})\Delta \tau \); \( n = 0, 1, 2, ..., 99 \). The physical method of developing these estimates is detailed in Reference 14, but for the purposes here, it can be viewed as follows.

Of the two inputs \( A(t) \) and \( B(t) \), \( B(t) \) is connected directly to one input of a bank of 100 two-input analog multipliers. The second input, \( A(t) \), is sampled and the sampled values are shifted along the bank of multipliers every \( \Delta \tau \) seconds - i.e. whenever a new sample is taken and sent to the first multiplier. Hence channel \( n \)'s output \( (n = 0, 1, ..., 99) \) is always \( A(t) B(t+\Delta \tau) \) where \( t_0 \) is the time at which the \( A \) sample is taken, and \( n\Delta \tau \leq \tau \leq (n+\frac{1}{2})\Delta \tau \). If the output of multiplier \( n \) is referred to as \( r_n(t) \), then it can be seen that for stationary inputs, the average multiplier output

\[
R_n'' = \lim_{T \to \infty} \frac{1}{T} \int_{0}^{T} r_n(t) \, dt = \frac{1}{\Delta \tau} \int_{0}^{\Delta \tau} R(n\Delta \tau + \tau) \, d\tau \quad (B-3)
\]

where \( R(\tau) \) is the true correlation function of \( A \) and \( B \).

In practice, each multiplier output is fed to a capacitive memory (first order RC filter) which provides an estimate of \( R_n'' \). After sufficient time has elapsed to allow a steady state output to be reached (typically 5 times the RC time constant for 1% accuracy), the variability of the correlation estimates and the resulting digitally transformed spectra can be treated as in B.2.1, where the record length is given by two times the RC time constant (Ref.11). In this experiment, this equivalent record length was 40 secs.

The result of each channel's averaging as per equation B-3 is effectively that of obtaining the spectral estimates through a low pass filter. This can be shown for power spectra as follows, following Blackman and Tukey (Ref.12).

If we define a function

\[
R'(\tau) = \frac{1}{\Delta \tau} \int_{0}^{\Delta \tau} R(\tau + x) \, dx
\]

then the ideal correlator's output (no noise, \( RC = \infty \)) is given by

\[
R_n'' = R'(\tau) \delta (\tau - n\Delta \tau) D_0 (\tau)
\]
where $\delta$ is the dirac delta function, and $D_0(\tau)$ is a box-car truncation function, $D_0(\tau) = 1$ for $-\tau_m < \tau < \tau_m$ and $D_0(\tau) = 0$ for $\tau > \tau_m$.

Note that since $R(\tau)$ is symmetrical, $R'(\tau)$ is symmetrical about $\tau = -\Delta \tau / 2$ and hence $R''$ can be defined for negative values of $n$ through the relation $R'' = R''_m$. Then, following Blackmann and Tukey, we can write that the two-sided aliased power spectral estimates

$$\tilde{P}_a(f) = \int_{-\infty}^{\infty} [\nabla_m (\tau; \Delta \tau) R'(\tau)] e^{-i\omega \tau} \, d\tau$$

where

$$\nabla_m (\tau; \Delta \tau) = \frac{\Delta \tau}{2} \delta(\tau + m \Delta \tau) + \Delta \tau \sum_{q = -m+1}^{m-1} \delta(\tau - q \Delta \tau)$$

and as shown by Tukey,

$$\tilde{P}_a(f) = Q_0(f) * A(f; 1/\Delta \tau) * P'(f) \tag{B-4}$$

where

$$A(f; 1/\Delta \tau) = \sum_{q = -\infty}^{\infty} \delta(f - q \Delta \tau)$$

and the $*$ indicates convolution, which is commutative. $P'(f)$ is the true power spectrum associated with $R'(\tau)$. However, we require an expression relating $P'(f)$ to the true power spectrum $P(f)$ associated with the true, non-averaged correlation function $R(\tau)$.

Noting that

$$R'(\tau) = \frac{1}{\Delta \tau} \int_{-\infty}^{\infty} R(y) H(\tau - y) \, dy$$

where $H(u)$ is everywhere zero except for $-\Delta \tau < u < 0$ where $H(u) = 1$, and making use of the convolution theorem, then

$$P'(f) = S_2(f) P(f)$$

where

$$S_2(f) = \frac{1}{\Delta \tau} \int_{-\infty}^{\infty} H(u) e^{-i\omega u} \, du$$
It can be readily shown that
\[ S_2(f) = \frac{\sin X}{X} e^{iX} \]

where
\[ X = \pi f \Delta \tau \]

Then we finally obtain from equation B-2 that
\[ \tilde{P}_a(f) = Q_o(t) \ast \left[ \sum_{q=\infty}^\infty P'(f - q/\Delta \tau) \right] \]

\[ = Q_o(f) \ast \left[ \sum_{q=-\infty}^\infty P(f - 2qf_o) S_2(f - 2qf_o) \right] \quad (B-5) \]

introducing the Shannon frequency \( f_o = 1/2\Delta \tau \).

Under conditions where the aliases can be neglected, then the power spectral estimates, \( P(f) \) are simply
\[ \tilde{P}(f) = Q_o(f) \ast \left\{ \frac{\sin X}{X} e^{iX} P(f) \right\} \quad (B-6) \]

This indicates that the filtering action of the correlator is given by \( \sin X/X \ e^{iX} \) and the resulting filtered spectrum is viewed through the zeroth spectral window due to the truncation function \( D_0(\tau) \). In practice, the resulting raw estimates as given above were manipulated such that the final estimates were those seen through a Hanning filter window as in Section B.2.1, changing \( Q_o(f) \) to \( Q_2(f) \) (see Blackman and Tukey, Ref.12, for plots of these windows). Note that the above expression is equally applicable to one-sided or two-sided spectra.

The order of the aliasing problem can be determined from considering the relative attenuation at \( f-f_o \) to that at the first alias. The first alias for a one-sided spectrum occurs at \( 2f_o-f \). It is equivalent to the combined effect of \( q = 1 \) for \(+|f|\) and \( q = -1 \) for \(-|f|\) in the two sided spectrum of equation (B-5).

The relative attenuation is simply
\[
\sin \left[ \frac{\pi (2f_o-f)}{2f_o} \right] \quad \frac{\pi f}{2f_o} = \frac{f/f_o}{2-f/f_o}
\]

Thus, at \( f/f_o = 0.5 \), an input spectrum which is flat over the frequency range \( f < 2f_o \), and zero beyond would have a first alias of 33.3\%. Naturally, real spectra usually fall off rapidly with frequency, and hence add another attenuation factor. However, assuming an \( f^{-2} \) relation for the spectra, it was still found necessary to use fourth order filters in the input data to avoid aliasing of greater than 2.5\% at \( f/f_o = 0.8 \).
These filters are also of the Butterworth type but are of fourth order and have a natural frequency of 400 hz. In this case, careful attention was paid to matching the characteristics of the filters so as to avoid differences in phase shift. The schematics and analytical and experimental amplitude responses are shown in Figure B-4. Phase differences were measured using the technique of having both signals on a single oscilloscope beam. This method could detect phase shifts of about ± 1°. Under these conditions no phase differences could be detected up to 400 hz, hence removing any need for such correction to cross-spectral results.

The \( e^{iX} \) term in equation (B-4) results from the basic assymetry of \( R'(\tau) \). Two main alternatives are possible for compensating for this term. Firstly, power spectra could be obtained directly by the method previously followed. For example, raw one-sided power spectral estimates can be derived using the formulation

\[
\hat{\Phi}_{xx}(k\Delta f) = 4\Delta f \left( \frac{F_{xx}(0)}{2} + \sum_{n=1}^{99} F_{xx}(n\Delta f) \cos \left( \frac{n\eta k}{100} \right) + \frac{F_{xx}(100)}{2} \cos[nk] \right)
\]

where \( F_{xx} \) is a symmetrical function and is defined in terms of the experimental measurements as:

\[
F_{xx}(0) = R''
\]

\[
F_{xx}(n) = \frac{R'' + R''}{2} = \frac{R'' + R''}{2}
\]

\[
F_{xx}(100) = \frac{R'' + R''}{2} \approx R''
\]

Then the expected values of \( \hat{\Phi}_{xx}(k\Delta f) \) are given by the real part of \( \tilde{P}(f) \) as:

\[
\text{Re}[\tilde{P}(f)] = Q_o(f) * \left[ \frac{\sin X}{X} \cdot \cos X \cdot P(f) \right]
\]

and the results can be corrected accordingly. Note that the cosX term drives the estimates to zero at the Shannon limit. This method can be modified to include use of the imaginary component, which does not approach zero, and can be made more exact by using only 99 points in the Fourier transform, rather than 100, so that the approximation for \( F_{xx}(100) \) is not required.

The second method, which is that used in the results for this report, essentially shifts the discrete transform so as to compensate for the fact that the correlation estimates are more closely associated with the \( (n + \frac{1}{2})\Delta \tau \) points due to the averaging over the interval.

This can be shown more explicitly by noting that from equation (B-6) (neglecting the spectral window \( Q_o(f) \)), the one-sided true power spectrum \( P(f) \) can be written

\[
P(f) = \frac{X}{\sin X} e^{-iX} \tilde{P}(f)
\]
\[ \frac{2X}{\sin X} \int_{-\infty}^{\infty} \left[ \nabla_m(\tau;\Delta \tau) R'(\tau) \right] e^{-i\omega \tau} \, d\tau \]
\[ = \frac{2X}{\sin X} \int_{-\infty}^{\infty} \left[ \nabla_m(\tau;\Delta \tau) R'(\tau) \right] e^{-i \frac{\pi}{f_0} \left( q + \frac{1}{2} \right)} \, d\tau \quad (B-7) \]

using the fact that the presence of the \( \nabla_m \) function allows replacement of \( e^{-i\omega \tau} \) by \( e^{-2\pi fq \Delta \tau} \).

The final digital computational formulae for transforming the 100 correlation estimates follow directly from equation (B-7), requiring only the additional assumption that
\[ R_{100}'' \cos \left( \frac{\pi f}{f_0} \left( 100\frac{1}{2} \right) \right) \approx R_{99}'' \cos \left( \frac{\pi f}{f_0} \left( 99\frac{1}{2} \right) \right) \]
since \( R_{100}'' \) is not available experimentally. (Although an \( m=99 \) point transform would avoid this assumption). The expressions used are given below, incorporating the \( \Delta \tau/2 \) shift in the transform and using the usual \( \Delta f = 1/2\tau \_m = f_\circ/100 \). The raw one-sided spectral estimates are:
\[ \hat{\phi}_{xx}(k\Delta f) = 4 \Delta \tau \sum_{n=0}^{99} R_{xx}(n\Delta \tau) \cos \left( \frac{\pi k}{100} (n + \frac{1}{2}) \right) \]
\[ \text{Re} \hat{\phi}_{xy}(k\Delta f) = 4 \Delta \tau \sum_{n=0}^{99} \left[ \frac{R_{xy}(n\Delta \tau) + R_{yx}(n\Delta \tau)}{2} \right] \cos \left( \frac{\pi k}{100} (n+\frac{1}{2}) \right) \]
\[ \text{Im} \hat{\phi}_{xy}(k\Delta f) = 4 \Delta \tau \sum_{n=0}^{99} \left[ \frac{R_{xy}(n\Delta \tau) - R_{yx}(n\Delta \tau)}{2} \right] \sin \left( \frac{\pi k}{100} (n+\frac{1}{2}) \right) \]

Modifications to the estimates to provide the Hanning window and introduction of the \( \sin(X)/X \) correction are applied similarly in all three of the above cases and gives the final estimate of the input spectra as
\[ \phi(k\Delta f) = \frac{\pi k/200}{\sin(\pi k/200)} \left( 0.25 \hat{\phi}[(k-1)\Delta f] + 0.5 \hat{\phi}[k\Delta f] \right) + 0.25 \hat{\phi}[(k+1)\Delta f] \quad (B-8) \]
\[ \phi(99\Delta f) = \frac{\pi k/200}{\sin(\pi k/200)} \left( 0.5 \hat{\phi}[(k-1)\Delta f] + 0.5 \hat{\phi}[k\Delta f] \right) \]
It is interesting to note that although the correction for the $e^{ix}$ term in this manner should reduce the frequency response of the correlator simply to $\sin x/x$, the practical application of the cosine transform shifted by $\frac{1}{2}$ interval inevitably gives $\hat{X}_{yy}(f_0) = 0$ since for $f_0$, $k = 100$.

In fact, from direct computation, it was found that the manner in which the estimates approach zero at $f_o$ depends strongly on the spectrum shape. Since the $e^{ix}$ term results solely from the $\frac{1}{2}$ interval shift ($\sin x/x$ originates from averaging over each $\Delta T$), the method adopted was to choose the auto correlation/power spectrum pair given by

$$R(\tau) = \frac{1}{1 + (2\pi f_0/A)^2}$$

$$\phi(f) = \frac{A}{f_o} e^{-Af/f_o}$$

These functions allow simple variation of the drop-off rate of the spectrum since

$$\frac{d\log \phi}{d\log f} = -A \quad \text{at} \quad f = f_o$$

Values of $R(\tau)$ were analytically determined for values of $\tau = (n + \frac{1}{2}) \Delta T$; $n = 0, 1, \ldots, 99$. These were then used to determine spectral estimates as per equations B-8. The ratio of estimate to analytic spectral values are plotted in Figure B-5. Also shown are similar ratios originating from the analytic pair

$$R(\tau) = e^{-B\tau}$$

$$\phi(f) = \frac{4B}{B^2 + (2\pi)^2}$$

In this case the curve shown is an average for values of $B$ between 5 and 500, noting that in all cases

$$\frac{d\log \phi}{d\log f} = -2 \quad \text{as} \quad f \to \infty$$

Hence, it appears that empirically the previous digital method gives better results as the spectra drop off more sharply across the Shannon limit. Although aliasing is not considered to be a strong factor in this behaviour (since the spectral estimates are always too low), the use of the de-aliasing technique incorporating high order filters is seen to be advantageous. In this experiment, the combination of the fourth order filters, and the natural data fall off resulted in inputs to the correlator with spectral behaviour $d\log \phi/d\log f < -8$. Hence, the tendency to zero at $f_o$ is of negligible effect for the $f/f_o < 0.8$ range considered for the experimental results.

The expected statistical variability of the results can be considered as in B.2.1 using the effective record length of 40 secs. This is illustrated below for some typical $\tau_m$ values.
The main application of this analysis technique was to obtain the set of two-point pressure correlations ($\tau_m = 0.1$ sec.) used in the integration leading to the lift and drag correlations. The statistical variability of the latter is thus expected to be very low.

### B.3 Comparison of Spectral Analysis Techniques

The analytical consideration of the methods has been covered in the previous sections. In their practical application, other desiderata become apparent - primarily those of elapsed time, simplicity, and direct control of statistical variability. Essentially there are only two methods - analog and digital. While they are capable of giving equally reliable results, they are subject to slightly different compromises. The analog method has the advantage of giving the experimenter immediate and direct control over the statistical variability of each point since the output meter's fluctuations are visually controllable by adjusting the time constant. However, when taking large quantities of data, the analog method becomes intractable. The digital method's prime advantage is that of simplicity in repetitive measurements. However, the reduction of statistical variability is difficult without resorting to multiple runs in the method of B.2.1. or large time constants and hence increased experimental times in the method of B.2.2. Furthermore, the digital method's application is traditionally such as to produce estimates at equal frequency increments. For data of the nature of turbulence, this appears wasteful as against the analog analyser's constant percentage bandwidth. This situation leads inevitably to the "two-cut" approach used here. It is felt that more efficient digital methods should be developed which model the constant percentage bandwidth approach rather than a constant frequency increment. It might be expected that this would lead to deriving digital correlation estimates at intervals increasing (exponentially?) with $\tau$. This would match the data's inherent form of concentrating the high frequency information at small $\Delta \tau$'s.

In order to compare the experimental methods operationally spectral estimates of the same data were derived, using all three approaches. The input consisted of the output of a hot wire anemometer in one of the grid-produced turbulent fields.
The data was analyzed on-line directly, using the wave-analyzer, and was also tape-recorded. This tape-recorded data was re-analyzed using all three techniques. Basically, these final three comparisons are of the greatest importance. The initial on-line analysis provided insight into degradation of data due to recording, and a check on the entire method of gain calibrations and frequency response corrections. In all comparisons, the analog analysis is used as a reference, due to the long time constants and hence low variability of the spectral estimates. A comparison of the three methods of analysis is shown for this turbulence data in Figure B-6. Both digital methods incorporated a high and low frequency cut. The method of section B.2.1, using the sum of four records was used for the EECO-derived data, but otherwise each set of data was not averaged further. There is a slight gain error evident between the means of the two digital estimates and that found from wave analysis, which is most severe at high frequencies. This is partially due to inaccuracies in the correction for wave analyzer response, and partially due to the broad wave analyzer bandwidth at high frequency, which gives a spectral estimate associated with a frequency less than its centre frequency for a spectrum of this shape.

The variability in the two digital methods is due to three causes:

1) the basic statistical nature of stationary random data estimates given approximately by

\[ \frac{\sigma(f)}{\mu(f)} \approx \left( \frac{1}{\Delta f T} \right)^{1/2} \]

which was discussed in Section B.2.

2) the degree of non-stationarity present in the input due to long term trends of the wind tunnel performance. These are considered small in this case.

3) noise introduced by the instrumentation involved in forming the correlation estimates.

The last cause is relatively unimportant in the method of B.2.1, but is probably the major cause of the high frequency variability in the estimates using the PAR correlator.

This can be appreciated by examining the sensitivity of the Fourier transformation process. This was done on a computer using analytical correlation and spectrum functions. The results are not shown here. However, the work pointed out the sensitivity of the high frequency spectral estimates (high with respect to \( f \)) to small errors in the first few points of the correlation. In this case, the 1% correlator accuracy leads to significant high frequency errors. It can be seen from Figure B-6 that in both cases of digital data reduction, the variability appears to be about a reliable mean curve.

Due to the expectation of a peak in the pressure data induced by vortex shedding, further sets of data were analyzed as above, except that a known amplitude sine wave was added to the data using an operational amplifier. The sine wave's amplitude was set to approximate the same fraction of energy in the peak as that measured for some preliminary surface pressure spectra at \( \theta = 90^\circ (f_p \approx 120 \text{ hz}) \) and at \( \theta = 180^\circ \) (peak at \( f = 2f_s \)). The results for three cases investigated are similar. Hence, only the data for a 240 hz peak are shown in Figure B-7. The amplitudes of the peaks are not directly comparable, although the increased smearing with \( \Delta f_e \) can be seen. The prime concern was of the peak's effect on the surrounding
spectral estimates. It should essentially produce the analysis window centered at the spike. This can be seen, especially for the low frequency cut, but the effect is negligible within a few $\Delta f$ steps to either side of the peak. Since the case of a spike is an extreme test compared to the broadened experimental peaks actually obtained, it is felt that the experiment spectral estimates in these regions are extremely good.

The final test of the data reduction system applied only to the method of B.2.1. The same u-data was recorded simultaneously on both channels. Analysis of the two channels showed identical values within plotting accuracy ($< 2\%$). Furthermore, the real cross-spectra agreed to within the same tolerance, whereas the imaginary cross-spectra was of the order of $10^{-3}$ times the real part. Hence this verifies the consecutively digitized analysis method used.
APPENDIX C: MICROPHONE CALIBRATIONS

The two microphones used as pressure transducers in the experiment were supplied by Bruel & Kjaer. They and their associated cathode followers were matched in amplitude and phase response and were specially modified to provide a usable amplitude response to fluctuating pressures between 1 Hz and 9000 Hz (3db down). Their flat range (+2%) is approximately 10 Hz to 2000 Hz. However, when used to sense the cylinder surface pressures, the frequency response is altered by the small surface hole and internal cavity (see Figure 3-2). The dimensions of the cavity, which follows a similar design by MacGregor (Ref.15), are approximately 1/16" deep by 1" in diameter. The connecting hole to the surface is 1/16" in diameter by .025" long. These dimensions lead to a calculated Helmholtz resonance of approximately 1900 Hz. (Reference 16).

To insure accuracy over as wide a range of frequency as possible, a calibration box (Fig.C-1) was constructed, also similar to that used by MacGregor. The box provided a uniform pressure field for test purposes which was created in one of two ways. For frequencies above 10 Hz, usable pressure levels were produced by an acoustic horn driver (University 25 watt SA-HF) driven through a power amplifier by a laboratory oscillator. For an overlapping frequency range between 0.8 Hz and 50 Hz, an approximately sinusoidal volume fluctuation was introduced by driving a small piston with a D.C. electric motor (Fig.C-1). The piston was simply that from a 0.10 cu.in. displacement model aircraft engine, running in its own crank case, but with a new sleeve which mated directly into the calibration box.

The pressure field was measured simultaneously by:

1) a microphone buried in its module,

2) directly by the second microphone acting as a reference,

3) by a reluctance type pressure transducer (Pace Model P7D ± 0.1 psid).

This allowed the two microphone/module systems to be calibrated in turn relative to the two references. Absolute calibrations were obtained by direct calibration of the two references. In the case of the Pace transducer, a D.C. calibration was performed against a Betz manometer, and it could be inferred from the relative calibrations that its amplitude response remained flat to 50 cps (+1%). The reference microphone was calibrated directly at 250 cps using a Bruel and Kjaer pistomphone (+2%). This then provided two independent, absolute calibrations of each microphone/module system. These agreed within ±3% for each combination. It was also found that the matching of the two systems was within this tolerance so that the average normalized amplitude response curve shown in Figure 4-4 described both systems to within ±3% except near the resonant peak (above 850 cps) where ±10% is a more representative accuracy and where the systems become strongly non-linear.

It should be noted in reference to the amplitude response that the low frequency fall-off is a property of the cathode follower, rather than of the cartridge itself. (the cartridge also has a low frequency cut-off due to a pressure equalization hole across the diaphragm, but this cut-off is well below the cathode
follower cut-off). However, the high frequency resonance is entirely due to the coupling cavity characteristics. The discrepancy between the resonance shown (950 hz) and that predicted (1900 hz) is quite large. However, it was actually found that both cavities showed a double resonance. The first as shown is at about 950 hz. The second occurred at about 1600 hz. It is thought that the second peak is that calculated at 1900 hz, since the calculation did not consider the equivalent volume of the microphone itself (which reduces the predicted resonance by about 10%) or the small volume associated with clearance around the microphone cartridge. The remaining difference is reasonably assigned to manufacturing tolerances and approximations in the theory. The first resonance, however, remains unexplained. It is interesting to note that MacGregor shows results up to 900 hz. for similar geometry, which also indicate an early resonance. The results found here are not presented above 1000 hz. because of their strong non-linearity.

Phase response was also measured for both systems using a chopped oscilloscope trace of the module microphone and the reference microphone. Phase differences could be determined to within ±1°. Although each system produced phase lags with respect to the reference of up to 90° at 1000 hz, the phase angles were matched extremely well between the systems. Zero phase difference existed below 200 hz. and the phase angles agreed to within a mean error of ±5° up to 1000 hz. The mean phase lag between a microphone/module system and the reference microphone is also shown in Figure 4-4. This is not an absolute phase response, since Bruel and Kjaer show phase angles of about 5° for a standard 4132 microphone at 1000 hz.

The repeatability of the calibration was checked, especially following removal and replacement of the microphone from the modules between calibrations. All tests showed the calibrations to be repeatable to within the above tolerances.

The non-linearity of the microphone/module systems was negligible below 400 hz. This is illustrated in Figure C-2, where the indicated output of the microphone/module system is normalized to that found with an rms input of 0.40 psf. Since the typical total rms outputs recorded during the experiment never exceeded 2.0 psf, the effect of the non-linearities were considered negligible below 800 hz. justifying correction of the data using Figure 4-4, which is derived with rms inputs of approximately 0.40 psf.

The assumption of a uniform pressure in the calibration box was tested by moving the pressure sensing hole from its normally central location in the box to extremes of rotation and translation allowed by the geometry. For the frequency range up to 800 hz. maximum differences of 6% were recorded. However, mean errors were not greater than the tolerances previously indicated. Near the resonance peak (1000 hz), considerable sensitivity to position within the cavity was displayed with errors ranging up to 50%. It can thus be concluded that the measured amplitude and phase characteristics must be accepted with caution above 800 hz.

The experimental measurements on this circular cylinder were actually carried out using two different sets of matched microphones, since one of the first set failed during the experiment. Only the microphone cartridges were replaced, and, as noted above, this would not be expected to alter the frequency response. This was verified by a recalibration of the system using the second pair of cartridges. The second set of cartridges was used only in the two point pressure correlations leading to the drag and lift results. All other one- and two-point pressure measurements were made with the first set.
The absolute calibrations for the microphone systems are given below for 200 volts polarization:

<table>
<thead>
<tr>
<th>Module</th>
<th>Cartridge</th>
<th>Cathode Follower</th>
<th>K (psf) (volt)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>145779</td>
<td>146852</td>
<td>0.543</td>
</tr>
<tr>
<td>B</td>
<td>145712</td>
<td>146853</td>
<td>0.543</td>
</tr>
<tr>
<td>A</td>
<td>167386</td>
<td>146852</td>
<td>0.473</td>
</tr>
<tr>
<td>B</td>
<td>115283</td>
<td>146853</td>
<td>0.505</td>
</tr>
</tbody>
</table>

where the amplitude response is found by dividing K by the normalized function of Figure 4-4.
APPENDIX D: INTEGRATION OF EQUATIONS 2-11 AND 2-13

The integration of equations 2-11 and 2-13 was performed numerically using two straightforward integration rules. A two dimensional Simpson's rule (Ref. 17) was used to produce the main results, with the box, or histogram, rule carried out simultaneously to provide a check and insight into the sensitivity of the process to the integration method used. Hence, if we write

\[
R_{\tau \xi} = r^2 \int_{0}^{2\pi} \int_{0}^{2\pi} R_{\tau \xi}(\tau, \xi, \alpha, \beta) \cos \alpha \cos \beta d\alpha d\beta
\]

\[
R_{\tau \xi} = r^2 \int_{0}^{2\pi} \int_{0}^{2\pi} R_{\tau \xi}(\tau, \xi, \alpha, \beta) \sin \alpha \sin \beta d\alpha d\beta
\]

Then both can be represented by:

\[
R(\tau, \xi) = r^2 \int_{0}^{2\pi} \int_{0}^{2\pi} f_{\alpha \beta}(\tau, \xi, \alpha, \beta) d\alpha d\beta
\]

where \( f_{\alpha \beta}(\tau) \) is a convenient short form for the integrand in either of the upper two expressions. The \( \xi \)-dependence is neglected since the integration in each case involved a particular \( \xi \) and 100 fixed values of \( \tau \).

The experimental measurements took the form of \( R_{\tau \xi}(\tau, \xi, m\Delta \alpha, n\Delta \beta) \) where \( m, n = 0, 1, \ldots, 2N \) (\( N \) is an integer). In this case \( \Delta \alpha = \Delta \beta = \pi/N \), and the experimental measurements included both the front and rear stagnation points.

The box or histogram rule used was then simply

\[
R_{Bx}(\tau) = r^2 \left( \frac{\pi}{N} \right)^2 \sum_{m=0}^{2N-1} \sum_{n=0}^{2N-1} f_{m,n}(\tau) \tag{D-1}
\]

and the Simpson's rule was

\[
R_{Si}(\tau) = r^2 \left( \frac{\pi}{N} \right)^2 \sum_{m=0}^{2N} \sum_{n=0}^{2N} a_{m,n} f_{m,n}(\tau) \tag{D-2}
\]

where \( a_{m,n} \) is the matrix of Simpson's rule coefficients given in Reference 17 and is of the form

\[
a_{0,n} = 1, 4, 2, 4, 2, \ldots, 2, 4, 1
\]
\[
a_{1,n} = a_{3,n} = a_{5,n} = \ldots = 4a_{0,n}
\]
\[
a_{2,n} = a_{4,n} = a_{6,n} = \ldots = 2a_{0,n}
\]
\[
a_{2N,n} = a_{0,n}
\]
The number of two-point pressure correlations required experimentally was minimized by applying symmetry conditions and the assumption of homogeneity. Symmetry implies:

\[ R_{p_1p_j}(\tau, \xi, \alpha, \beta) = R_{p_1p_j}(\tau, \xi, 2\pi-\alpha, 2\pi-\beta). \]

Noting that

\[ R_{p_1p_j}(\tau, \xi, \alpha, \beta) = \frac{R_{1}(t, \alpha).p_j(t+\tau, \beta)}{p_i(t, \alpha).p_i(t+\tau, \beta)} \]

and

\[ R_{p_jp_1}(\tau, \xi, \alpha, \beta) = \frac{R_{2}(t, \alpha).p_j(t-\tau, \alpha)}{p_i(t, \beta).p_i(t-\tau, \alpha)} \]

then homogeneity implies

\[ R_{p_jp_1}(\tau, \xi, \alpha, \beta) = R_{p_i}(\tau, \xi, \alpha, \beta) \]

Hence,

\[ R_{p_jp_1}(\tau, \xi, \alpha, \beta) = R_{p_i}(\tau, \xi, -\alpha, \beta) \]

Further noting that the terms \(\sin \alpha. \sin \beta, \cos \alpha. \cos \beta\), and \(a_{m,n}\) also display homogeneity and symmetry, we can write the two conditions as

\[ f_{m,n}(\tau) = f_{2N-m,2N-n}(\tau) \]

and

\[ f_{m,n}(\tau) = f_{n,m-\tau} \]

The application of these two conditions (noting that 0 and 2N are identical points) gives the number of independent two-point pressure correlations required to completely define the integrals \((D-1)\) and \((D-2)\), as \(N^2+N+1\). The table below indicates this number as a function of the angular increment:

<table>
<thead>
<tr>
<th>(\Delta \alpha = \Delta \beta) (degrees)</th>
<th>No. of Independent (R_{pp})</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>1,296</td>
</tr>
<tr>
<td>10</td>
<td>343</td>
</tr>
<tr>
<td>15</td>
<td>157</td>
</tr>
<tr>
<td>20</td>
<td>91</td>
</tr>
<tr>
<td>30</td>
<td>43</td>
</tr>
<tr>
<td>45</td>
<td>21</td>
</tr>
<tr>
<td>60</td>
<td>13</td>
</tr>
<tr>
<td>90</td>
<td>7</td>
</tr>
</tbody>
</table>
Since the angular increment subtended by the pressure sensing hole is 5.8° (Fig.3-2), a conservative value of angle increment of 15° was chosen.

The application of the symmetry and homogeneity conditions is most easily accomplished with reference to a typical matrix of $f_{m,n}(\tau)$ as shown for $N = 4$ ($\Delta \alpha = 45°$) in Figure D-1. The result is a mapping onto the terms in the area outlined. Analytically, the sum of Equation (D-2) is reduced as follows, using $f'_{m,n}(\tau) = a_{m,n} f_{m,n}(\tau)$. From symmetry:

$$
\sum_{m=0}^{2N} \sum_{n=0}^{2N} f'_{m,n}(\tau) = f'_{N,N}(\tau) + 2 \sum_{m=0}^{N-1} f'_{m,m}(\tau) + \left\{ \sum_{m=0}^{N-1} f'_{m,2N-m}(\tau) + \sum_{m=0}^{2N} f'_{m,2N-m}(\tau) \right\}
$$

which can be simplified using homogeneity as

$$
\sum_{m=0}^{2N} \sum_{n=0}^{2N} f'_{m,n}(\tau) = f'_{N,N}(\tau) + 2 \sum_{m=0}^{N-1} f'_{m,m}(\tau) + 2 \left\{ \sum_{m=0}^{N-1} f'_{m,2N-m}(\tau) + \sum_{m=0}^{2N} f'_{m,2N-m}(\tau) \right\}
$$

Writing

$$
f''_{m,n}(\tau) = \frac{f'_{m,n}(\tau) + f'_{m,n}(-\tau)}{2},$$

noting that $f''_{m,m}(\tau) = f'_{m,m}(\tau)$, and using symmetry to equate $f'_{0,n}$ and $f'_{0,2N-m}$ we finally obtain

$$
\sum_{m=0}^{2N} \sum_{n=0}^{2N} f'_{m,n}(\tau) = f''_{N,N}(\tau) + 2 \left\{ \sum_{m=1}^{N-1} f''_{m,m}(\tau) + \sum_{m=1}^{2N} f''_{m,2N-m}(\tau) \right\} + 4 \left\{ f''_{0,0}(\tau) + f''_{0,N}(\tau) \right\} + \sum_{m=1}^{N-1} \sum_{n=m+1}^{N-1} f''_{m,n}(\tau) + 8 \sum_{n=1}^{N-1} f''_{0,n}(\tau)
$$

(D-3)
The result produces a sum symmetrical about the \( \tau = 0 \) axis as required by the homogeneity assumption. The corresponding expression for equation (D-1) is:

\[
\sum_{m=0}^{2N} \sum_{n=0}^{2N} f'_{m,n}(\tau) = f''_{0,0}(\tau) + f''_{2N,0}(\tau) + 2 \left\{ \sum_{m=1}^{N-1} f''_{m,m}(\tau) + \sum_{m=1}^{N-1} f''_{2N-m,m}(\tau) + f''_{0,N}(\tau) \right\}
\]

where here \( a_{m,n} = 1 \).

Equations (D-3) and (D-4) are those used as a basis for the integration subroutines. In practice, the computer program for \( \Delta \alpha = 15^\circ \) reads 157 sets \( (N = 12) \) each consisting of \( R_{P_1P_2}(\tau) \) for a particular \( \zeta, \alpha \) and \( \beta \). The values for \( -\tau \) and \( \tau \) are averaged to form \( f''_{m,n}(\tau)/a_{m,n} \) and then a simple set of logic tests determines the appropriate coefficient (including \( a_{m,n} \)) to apply before storing it in a running sum. A copy of the subroutine, including the sorting logic applicable for all even \( N \) values (if \( N \) is odd, the value of \( a_{m,n} \) for the \((N,N)\) point alters) is shown in Figure D-2. Integration for values of \( N \) less than 4 involved so few terms that the values of term coefficients were read into the computer as a predetermined array.

The results presented in Section 8.5 were all derived using the full \( N = 12 \) matrix for \( \Delta \alpha = 15^\circ \). As a test of the sensitivity of the integration to the incremental angle, \( \Delta \alpha \), the integration of some cases was repeated, using a larger \( \Delta \alpha \). In particular, the zero time-delay results, \( R_{P_1P_2}(0,\zeta,\alpha,\beta) \), obtained from the analog computer correlation circuit, were integrated for \( \Delta \alpha = 15^\circ, 30^\circ, 45^\circ, 60^\circ, \) and \( 90^\circ \). The results for four values of \( \zeta \) are shown in Figure D-3, normalized by the result for \( \Delta \alpha = 15^\circ \). Also shown at \( \Delta \alpha = 15^\circ \) is the difference between the box and Simpson's rule results as a percent of the latter. These results indicate that at least for zero time delay, the answers agree to within about \( \pm 10\% \) for \( \Delta \alpha \leq 45^\circ \).

For the minimum \( \zeta \) case, the entire \( R(\tau) \) integration was then carried out for \( \Delta \alpha = 30^\circ \) and \( 45^\circ \). The results were Fourier transformed and compared to those obtained from the \( \Delta \alpha = 15^\circ \) "reference" case. It was of interest to find that the anomaly discussed in Section 8.5 (a significant dip in the drag spectrum at the Strouhal frequency) recurred in both the \( 30^\circ \) and \( 45^\circ \) integrations. Otherwise the alteration in the drag and lift spectra were comparable, and only the lift spectra calculated using Simpson's rule are presented in Figure D-4. In all three graphs, the mean curve is that from the \( \Delta \alpha = 15^\circ \) case as used in the data reduction process of Section 8. Hence it can be seen that the spectra derived from a \( 30^\circ \) and \( 45^\circ \) integration do not differ appreciably in mean value, but do show high scatter at high frequency. This is probably a result of the reduced number of terms in the integration. However for future work, it is felt that this increased scatter does not outweigh the advantage gained of considerably reducing the experimental time required to obtain similar results (see previous table of number of \( R_{pp} \) required vs. \( \Delta \alpha \)).
APPENDIX E: ERRORS IN RMS MEASUREMENTS

The RMS measurement of fluctuating velocities and pressures were taken directly on Bruel and Kjaer 2417 Random Noise Meters. As such, they are subject to three classes of inaccuracy:

1) sensor calibration errors, and basic meter inaccuracy

2) statistical variability

3) errors due to spectral distortion

Sensor calibration errors are discussed in Appendices A and C. Meter error for rms measurements was found to be ± 1.5% maximum for sine wave inputs.

Statistical variability of the rms readings can be treated by considering a mean square value as simply a wide band spectral measurement. As such, the statistical variability of the rms reading is roughly $1/\sqrt{\Delta f e^*}$, where $T$ is twice the meter time constant and $\Delta f e^*$ is the effective bandwidth of the data. The latter is dependent upon the particular input spectrum but is obviously large enough in all cases so that a suitable meter time constant can be chosen to reduce the variability to a negligible level.

Errors due to spectral distortion originate in two ways. Firstly, the sensor output (the term "sensor" as used here includes the electronics up to the meter input) is usually a distortion of the true physical variable being examined. Secondly, the meter's frequency response acts to truncate this distorted spectrum at both the high and low frequency end, although in this case the former is not restrictive (20,000 Hz).

For velocities, spectral models of the longitudinal and lateral components show that the distribution of energy is dependent on the parameter $z = fL/\bar{U}$, where $L$ is the longitudinal turbulence scale. Hence for consideration of truncation effects it is useful to consider the Dryden model of the $u$ and $w$ spectra given by

$$\frac{f\phi_{uu}(f)}{\sigma^2 u^2} = \frac{4z}{1 + (2\pi z)^2}$$

$$\frac{f\phi_{ww}(f)}{\sigma^2 w^2} = \frac{2z[1 + 3(2\pi z)^2]}{[1 + (2\pi z)^2]^2}$$

where $\sigma^2 = \bar{u}^2 = \bar{v}^2 = \bar{w}^2$ and then define corresponding cumulative energy distribution functions of the form

$$F(f) = \int_0^f \frac{\phi(f)}{\sigma^2} \, df$$

which can be integrated to give
\[ F_{uu}(f) = \frac{2}{\pi} \tan^{-1}(2\pi f) \]
\[ F_{ww}(f) = \frac{2}{\pi} \left\{ \tan^{-1}(2\pi f) - \frac{1}{f} \sin[2\tan^{-1}(2\pi f)] \right\} \]

These two functions are shown plotted in Figure E-1. As mentioned, the rms meter has essentially no upper limit for this experiment. However, the range of reliable hot wire response was only verified to 2,000 Hz., and is also subject to the frequency response discussion of Appendix A-3. The meter's low frequency power response was measured directly for sine wave inputs and is shown for the two UTIAS samples in Figure E-2. Taking the effective low frequency power cut-off at 0.7 Hz, the expected low frequency truncation and the percent of energy expected above the 2,000 Hz. limit of this experiment's analysis is shown in the table below.

<table>
<thead>
<tr>
<th>CASE</th>
<th>0</th>
<th>1</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>% truncation error, below 0.7 Hz.</td>
<td>u, v, w</td>
<td>u, v, w</td>
<td>u, v, w</td>
<td>u, v, w</td>
</tr>
<tr>
<td>4.0</td>
<td>2.0</td>
<td>0.2</td>
<td>0.1</td>
<td>2.3</td>
</tr>
<tr>
<td>% energy expected above 2,000 Hz.</td>
<td>u, v, w</td>
<td>u, v, w</td>
<td>u, v, w</td>
<td>u, v, w</td>
</tr>
<tr>
<td>0.5</td>
<td>2.0</td>
<td>8.0</td>
<td>12.0</td>
<td>0.7</td>
</tr>
</tbody>
</table>

Hence, since the error in the measured energy is small, the error in the rms measurements is approximately one-half of the values in the top row, and is such that the measured values are too low.

For pressures, the situation is complicated by their spectral dependence on both the turbulence characteristics and angular location around the circumference. Furthermore, the power response of the microphone/cathode follower system (see amplitude response in Figure 4-4) produces a considerable distortion of the spectra at both the low and high frequency ends. The power response is down 50% at about 2 Hz. and up 50% at about 500 Hz. Hence here, the added distortion due to the meter response is unimportant. Two indications are available as to the order of the error introduced into the rms readings. Firstly, for case 4, for which the fluctuating lift and drag results were obtained, fourth-order filters were used to de-alias the data as described in Appendix B.2.2. Simultaneous rms measurements of the pressure data were made both before and after the signals were filtered. The ratio of rms filter output to rms filter input for the actual pressure signals are tabulated below as a function of \( \theta \).

<table>
<thead>
<tr>
<th>( \theta ) (degrees)</th>
<th>rms Filter output/rms Filter input</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.988</td>
</tr>
<tr>
<td>15</td>
<td>0.988</td>
</tr>
<tr>
<td>30</td>
<td>0.988</td>
</tr>
<tr>
<td>45</td>
<td>0.992</td>
</tr>
<tr>
<td>60</td>
<td>0.991</td>
</tr>
<tr>
<td>75</td>
<td>0.992</td>
</tr>
<tr>
<td>90</td>
<td>0.984</td>
</tr>
<tr>
<td>105</td>
<td>0.946</td>
</tr>
<tr>
<td>120</td>
<td>0.935</td>
</tr>
</tbody>
</table>
This indicates that, of the total power output of the sensor as measured by the rms meter, a minimum of 87% (0.935$^2$) is concentrated below the effective filter frequency limit of 400 hz. (50% down in power) where the sensor output is essentially flat. Hence, the mean square error introduced in $p^2$ by the sensor response above 400 hz. must be a small fraction of at most 13%, and the error in the rms measurements will be reduced by another factor of 2. The error must then be of the order of $2\%$ or less, and acts to make the measured value too high.

The turbulence spectra of cases 0 and 3 are similar in scale to the above, but the associated surface pressure spectra have more energy concentrated in the spectral peak. This suggests that the errors will likewise be small.

All three of these cases are subject to the truncation error at the low frequency end (below 2 hz). To assess this and to verify the above, the second approach was adopted of directly integrating the spectra for the worst expected cases of $\theta' = 0^\circ$ and $\theta' = 180^\circ$.

Since the true underlying spectra over the complete frequency range are not known, the results tabulated below were obtained by integrating the sensor output spectrum over particular frequency ranges so as to indicate the proportion of energy each range contributes to the measured variance.

<table>
<thead>
<tr>
<th>% energy in sensor output spectrum in frequency range</th>
<th>CASE</th>
<th>0</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\theta' = 0^\circ$</td>
<td></td>
<td>&lt;3</td>
<td>&lt;2</td>
<td>&lt;4</td>
</tr>
<tr>
<td>$\theta' = 180^\circ$</td>
<td></td>
<td>&lt;2</td>
<td>&lt;4</td>
<td>&lt;4</td>
</tr>
<tr>
<td>$\theta' = 0^\circ$</td>
<td></td>
<td>&lt;4</td>
<td>&lt;2</td>
<td>&lt;4</td>
</tr>
<tr>
<td>$\theta' = 180^\circ$</td>
<td></td>
<td>&lt;2</td>
<td>&lt;4</td>
<td>&lt;9</td>
</tr>
<tr>
<td>0-2 hz.</td>
<td></td>
<td>&lt;3</td>
<td>&lt;2</td>
<td>&lt;4</td>
</tr>
<tr>
<td>500-1000 hz.</td>
<td></td>
<td>&lt;1.5</td>
<td>&lt;8</td>
<td>&lt;2</td>
</tr>
<tr>
<td>above 1000 hz.</td>
<td></td>
<td>&lt;3</td>
<td>&lt;16</td>
<td>&lt;4</td>
</tr>
</tbody>
</table>

Because the microphone power response is down to 50% at 2 hz., the true power truncation at low frequency is of the order of twice the energy in the sensor output spectra, and hence, the low frequency truncation effect of rms readings is roughly the same percentages as those given in the first row of the above table, and acts to make the measured values too small.

The second row of the above table indicates that the percentage of the sensor output energy over the module resonant range is less than 8% in all cases. The sensor output overestimates the true energy content of the surface pressure again by roughly a factor of 2 (see microphone/module power response = square of amplitude response given in Fig. 4-4). Hence, the errors in variance measurements induced over this range are approximately one-half of the values in row two, and
the errors in the rms measurements are then roughly one-quarter of the values in row two. This effect is such as to make the measured values too large.

The sensor output energy above 1000 hz. is fairly small in all cases, although as expected it is highest in the wake. Since, as indicated in Section C, the sensor response has a second resonance at 1600 hz. and then falls off sharply, and since only a small fraction of the total surface pressure energy exists above 1000 hz., it is felt that the distortion in this range does not produce an appreciable error in the rms measurements.

The above considerations suggest that for these three cases, the major error in the rms pressures is due to low frequency truncation, which leads to experimental values underestimating the true values. This error appears to be of the order of 5% or less.

For the final turbulence input, case 1, the turbulence scale is considerably smaller and hence the energy is concentrated at higher frequency (see previous table for velocity component errors). The distribution of energy in the resulting sensor-distorted pressure spectra was examined as above and provided the following results.

<table>
<thead>
<tr>
<th>% energy in</th>
<th>$\theta' = 0^\circ$</th>
<th>$\theta' = 30^\circ$</th>
<th>$\theta' = 60^\circ$</th>
<th>$\theta' = 90^\circ$</th>
<th>$\theta' = 120^\circ$</th>
<th>$\theta' = 150^\circ$</th>
<th>$\theta' = 180^\circ$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0-2 hz.</td>
<td>&lt;0.5</td>
<td>&lt;0.5</td>
<td>&lt;0.5</td>
<td>&lt;1.0</td>
<td>&lt;0.3</td>
<td>&lt;0.3</td>
<td>&lt;1.0</td>
</tr>
<tr>
<td>500-1000 hz.</td>
<td>&lt;14</td>
<td>&lt;11</td>
<td>&lt;5</td>
<td>&lt;3</td>
<td>&lt;25</td>
<td>&lt;8</td>
<td>&lt;10</td>
</tr>
<tr>
<td>above 1000 hz.</td>
<td>&lt;15</td>
<td>&lt;16</td>
<td>&lt;14</td>
<td>&lt;2</td>
<td>&lt;23</td>
<td>&lt;7</td>
<td>&lt;14</td>
</tr>
</tbody>
</table>

In this case, as expected, the low frequency truncation is negligible. Overestimation of rms readings due to the resonance region between 500 and 1000 hz. is, as before, roughly one-quarter of the value in row two which is of the order of 2 or 3% except in the case of $120^\circ$. Here, the increase in high frequency content due to separation, as noted in the spectra of section VIII, causes a larger error of roughly 6 or 7% in the rms readings. For the region above 1000 hz, the overall effect is not completely clear but is likely to be small except again perhaps for $120^\circ$. The small fraction of energy above 1000 hz. even for this high frequency input case is indicative of the concentration of energy at the Strouhal peak and the lack of high frequency aerodynamic pressures as shown by the pressure-velocity describing functions of section VIII.

The above suggests that in this case, the major error in the rms pressures is an overestimation due to the resonance and is of the order of 5% except for $\theta' = 120^\circ$ where the error may increase to about 10 to 15%.
1.1 VARIATION OF STROUHAL NUMBER AND DRAG COEFFICIENT FOR A CIRCULAR CYLINDER WITH REYNOLDS NUMBER. (Adapted from Reference 37)
NOTE: \( X = x = 0 \) is located at grid plane

2.1 SKETCH OF CYLINDER IN AIRSTREAM TO INDICATE AXES, SIGN CONVENTIONS AND NOTATION.
3.1 SKETCH OF CYLINDER INSTALLATION IN WIND TUNNEL.
3.2 CROSS-SECTIONS OF THE PRESSURE SENSING MICROPHONE MOUNTED IN THE CYLINDRICAL MODULE.
3.3 PHOTOGRAPHS OF MICROPHONE, MODULE AND CYLINDRICAL SPACERS.
3.5 DETAILS OF CYLINDER END MOUNTING AND COMPRESSION DEVICE.
3.6 PROBES MOUNTED ON DUMMY CYLINDER
3.7 CALIBRATION OF THE CYLINDER END LOAD.
3.8 TYPICAL DISPLACEMENT SPECTRA FOR THE CYLINDER.
4.1 AERODYNAMIC OUTLINE OF THE WIND TUNNEL AND DIFFUSER INSTALLATION LOCATIONS.
NOTE: Control circuit (not shown) automatically limits operating time to T seconds.

4.2 CIRCUIT BLOCK DIAGRAM FOR ZERO-TIME DELAY CORRELATIONS AND ITS FREQUENCY RESPONSE.
\[ E_0 = \text{Low Pass Filter Output} \]
\[ e_0 = -(E_1 - E_0) \]

\[ e_0 = \text{High Pass Filter Output} \]

\[ E_0 = \text{Low Pass Filter Output} \]
\[ e_0 = \text{High Pass Filter Output} \]

\[ \frac{2\pi fT}{(1 + 4\pi^2 f^2 T^2)^{1/2}} \]

For \( k = 0.2, T = 5 \)

4.3 HIGH/LOW PASS ANALOG FILTER AND FREQUENCY RESPONSE
Fitted first order curve used for correction:

\[
\frac{1}{1 + 10^{-h_f}}
\]

4.5 Typical tape recorder frequency response.
(Channel 1, Record Speed = 7\(\frac{1}{2}\) ips, Reproduce Speed = 7\(\frac{1}{2}\) ips)
Electronic Gate
A.M.
SP300
Data Signals
(Zero Mean)
F.M.
F.M.
Analog Tape Recorder
Analog Tape
Pulse Generator

SP300
A.T.R.

Analog Tape

Pulse Trains

Butterworth
6th-order
filter, \(f_n = 250\text{Hz}\)

EECO
A/D
system

"raw" digital
tape

IBM 7094
machine language
program

Fortran tape

IBM 7094
Fortran analysis
Program

Correlations,
spectra, as
card output

IBM 1130
final data
corrections, etc.

output data,
calcomp plots

Oscillator:
Free-running Pulse
Rate = 1200/sec.
Syncs. with input
whenever present

Electronic
Gate opens
2.1 msecs
after first
pulse

Fortran
analysis
spectra, as
card output

EECO
A/D
system

external
pulse input
at \(2f_s\)

"raw" digital
tape

IBM TR-48 Analog Comp.

Data
Input

"raw" digital
tape

EAI TR-48 Analog Comp.

Pulse Shaper

Timing

4.6 DIGITAL DATA ACQUISITION SYSTEM.
DATA SIGNAL, A(t)

Matched Butterworth 4th order filters

\[ f_n = 400 \text{ hz} \]

DATA SIGNAL, B(t)

PAR Model 100 signal correlator

R(\tau) → CIMRON digital voltmeter

CARD OUTPUT OF CORRELATION FUNCTIONS

IBM 526 SUMMARY PUNCH

CARDS

IBM 1130 Fourier Transforms data manipulation

Output data, CALCOMP plots

4.7 HYBRID CORRELATOR DATA ACQUISITION SYSTEM.
\[ \frac{\bar{u}^2}{u^2} = \frac{8}{C_G} \left[ \frac{x - x_0}{M} \right] \]

where \( C_G \) has been chosen to force curves through \( x_b = 100, \bar{u}/\bar{u} = 0.042. \)

**Surry Data Point**

5.1 DECAY OF LONGITUDINAL INTENSITY WITH DISTANCE DOWNSTREAM (References 49, 50)
5.2 TUNNEL LOADING CHARACTERISTICS
5.3 LONGITUDINAL STATIC PRESSURE MEASUREMENTS FOR VARIOUS LOADS.
54 DRAG AND GRID GEOMETRY VS. SOLIDITY FOR SQUARE MESH GRIDS (Reference 50).
5.5 TAILORING OF MEAN VELOCITY PROFILE BEHIND A GRID.
5.6 STABILITY OF LATERAL CORRELATION WITH LOCATION ACROSS THE TEST REGION FOR AN EARLY GRID DESIGN (MEAN VELOCITY AND TURBULENCE INTENSITY PROFILES GIVEN).
5.7 FOUR FINAL GRID DESIGNS USED TO GENERATE TURBULENCE
Numbers Designate Grid (See Table 1)

D = Model Diameter

= 1.24 ins

5.8a COMPOSITE OF THE TURBULENCE INTENSITIES AND SCALES FOR THE FOUR EXPERIMENTAL GRIDS DOWNSTREAM AT THE MODEL TEST LOCATION
5.8b COMPOSITE OF THE THREE TURBULENCE COMPONENTS FOR ALL GRIDS
5.9 MEASURED AND EXPECTED DOWNSTREAM INTENSITY VARIATIONS.
|Z_{wall}| = 30.75 ins
\[ U_r \approx 100 \text{ fps} \]
\[ X = Y = 0 \]

5.10 VERTICAL MEAN VELOCITY PROFILE.
6.1 CHARACTERISTICS OF TURBULENCE ALONG MODEL TEST LOCATION FOR CASE 0 (empty tunnel).
6.2 CHARACTERISTICS OF TURBULENCE ALONG MODEL TEST LOCATION FOR
6.3 CHARACTERISTICS OF TURBULENCE ALONG MODEL TEST LOCATION FOR CASE 2.
6.4 CHARACTERISTICS OF TURBULENCE ALONG MODEL TEST LOCATION FOR CASE 2
6.5 CHARACTERISTICS OF TURBULENCE ALONG MODEL TEST LOCATION FOR CASE 4.
6.6 $u$, $v$, $w$ SPECTRA OF TURBULENCE AT MODEL CENTRE LINE FOR CASE 0.
6.7 $u$, $v$, $w$ SPECTRA OF TURBULENCE AT MODEL CENTRE LINE FOR CASE 1.
\[
\frac{\phi_{xx}}{s_x^2} = \frac{4L_1^2}{u} \left[ \frac{1}{1 + \left( \frac{1.339}{0} \right)^2} \right]^{5/6}
\]

\[
\frac{\phi_{vv}}{s_v^2} = \frac{4L_2^2}{u} \left[ 1 + \frac{8/3 \left( \frac{1.339}{0} \right)^2}{1 + \left( \frac{4\pi L_2^2}{0} \right)^2} \right]^{11/6}
\]

where \( L_1 \) = longitudinal integral scale
\( L_2 \) = lateral integral scale

6.8 \( u, v, w \) SPECTRA OF TURBULENCE AT MODEL CENTRE LINE FOR CASE 3.
6.9 $u, v, w$ SPECTRA OF TURBULENCE AT MODEL CENTRE LINE FOR CASE 4.
6.10 TYPICAL SPECTRAL DATA (u-spectrum for case 4) SHOWING HIGH AND LOW FREQUENCY CUTS WITH ERROR BANDS.
6.11 $u, v, w$ AUTO CORRELATIONS FOR CASE 4.
6.12 \( u, w \) LATERAL CORRELATIONS FOR CASE 4, INCLUDING \( w \)-AUTO CORRELATION.
LENS

average of seven spectral estimates uniformly spaced across the influence region

average of eight spectral estimates taken at $y = 0$

average of eight spectral estimates for $0 < y < 10.16$ at positions where cross-spectra were obtained

6.13 OVERLAYS OF 3 SPECTRAL ESTIMATES OF $u$ FOR CASE 4.
VARIATION OF u-CROSS SPECTRA WITH LATERAL SPACING, INCLUDING u-AUTO SPECTRUM FOR COMPARISON.

\[ \frac{\Phi_{u_1u_2}}{S_u} \]
6.15 TYPICAL NARROW BAND CROSS CORRELATION FOR LONGITUDINAL VELOCITY COMPONENT.
6.16 FITTING VARIOUS FORMS OF THE EXPONENTIAL FUNCTION TO THE NARROW BAND CROSS CORRELATION.
COMPARISON OF THE FIT OF TWO FORMS OF EXPONENTIAL FUNCTIONS TO DATA PUBLISHED BY DAVENPORT AND WARDLAW (REFERENCE 27, FIGURE 11)
6.18 NARROW BAND CROSS CORRELATIONS FOR FOUR GRIDS.
1.1 EFFECTS OF SURFACE DISCONTINUITIES IN CYLINDER ON RMS PRESSURE

- Normal Condition
- Minimum End Load for Structural Integrity
- Rubber Cement Sealing
- Spacer Seams
- Scotch Tape over Seams (.002" thick)
  - d + Circumferential Strip of Masking Tape between Measuring Stations (.003" thick)
  - d + 2 Masking Tape Strips as in e
- c + 10D Longitudinal Strip of Masking Tape covering region $0 < \theta < 90^\circ$

**Legend**
- Normalized RMS Pressure, $R_{p1p2}$
- $\theta = 45^\circ$, $\gamma/D = 0.443$
- $\theta = 45^\circ$, $\gamma/D = 0.40$
- $\theta = 45^\circ$, $\gamma/D = 0.2$

**Axes**
- Normalized RMS Pressure, $R_{p1p2}$
- $0 \leq R_{p1p2} \leq 10$

**Note:** The diagram includes various symbols and lines indicating different conditions and measurements related to the effects of surface discontinuities on cylinder RMS pressure.
7.2 SCHEMATIC OF MICROPHONE SIGNAL ANALYSIS SYSTEM.
8.1 MEAN PRESSURE DISTRIBUTIONS AROUND THE CIRCUMFERENCE OF THE CYLINDER CASES 0 AND 1.
8.2 MEAN PRESSURE DISTRIBUTIONS AROUND THE CIRCUMFERENCE OF THE CYLINDER, CASES 3 AND 4.
CASE 0

LEGEND

- Module A Single Datum Point
- Module B
- Module A Averaged Data
- Module B

CASE 1

8.3 RMS PRESSURE DISTRIBUTIONS AROUND THE CIRCUMFERENCE OF THE CYLINDER CASES 0 AND 1.
8.4 RMS PRESSURE DISTRIBUTIONS AROUND THE CIRCUMFERENCE OF THE CYLINDER CASES 3 AND 4.
8.5 PRESSURE SPECTRA FOR FOUR TURBULENCE INPUTS AT STAGNATION POINT.
8.6 PRESSURE SPECTRA FOR FOUR TURBULENCE INPUTS AT 30°.
8.7 PRESSURE SPECTRA FOR FOUR TURBULENCE INPUTS AT 60°.
8.8 PRESSURE SPECTRA FOR FOUR TURBULENCE INPUTS AT 90°.
8.9 PRESSURE SPECTRA FOR FOUR TURBULENCE INPUTS AT 120°.
8.10 PRESSURE SPECTRA FOR FOUR TURBULENCE INPUTS AT 150°.
8.11 PRESSURE SPECTRA FOR FOUR TURBULENCE INPUTS AT 180°.
8.12 PRESSURE AUTOCORRELATIONS AT VARIOUS CIRCUMFERENTIAL ANGLES
FOR CASES 0 and 1.
CASE 3

CASE 4

8.13 PRESSURE AUTOCORRELATIONS AT VARIOUS CIRCUMFERENTIAL ANGLES FOR CASES 3 and 4.
LATERAL PRESSURE CORRELATIONS AT VARIOUS CIRCUMFERENTIAL ANGLES FOR CASES 0 and 4.
8.15 NONDIMENSIONAL LATERAL PRESSURE SCALES VS CIRCUMFERENTIAL ANGLE FOR ALL TURBULENCE CASES.

Prendergast (Ref. 39),
R.N. = 4.7 x 10^4
8.16 TYPICAL NARROW BAND CROSS CORRELATIONS FOR PRESSURES 0° and 90°.
8.17 TYPICAL NARROW BAND PRESSURE CROSS CORRELATIONS IN THE NEIGHBOURHOOD OF THE STROUHAL PEAK.
8.18 PLOT OF $b_s$ AND ASSOCIATED LATERAL SCALES AS A FUNCTION OF CIRCUMFERENTIAL ANGLE FOR ALL TURBULENCE CASES.
8.19A PRESSURE - u - COMPONENT DESCRIBING FUNCTION AT 0° FOR CASES 0 AND 1.
8.19B PRESSURE - u - COMPONENT DESCRIBING FUNCTION AT 0° FOR CASES 3 AND 4.
8.20 NONDIMENSIONALIZED CROSS CORRELATIONS OF DRAG AT VARIOUS LATERAL SPACINGS
8.21 NONDIMENSIONALIZED CROSS CORRELATIONS OF LIFT AT VARIOUS LATERAL SPACINGS
8.22  NONDIMENSIONALISED SQUARE ROOT OF DRAG CROSS CORRELATION AT ZERO TIME DELAY AS FUNCTION OF LATERAL SPACING.
Figure 8-22

See Figure 8-22 for Legend

8.23 NONDIMENSIONALISED SQUARE ROOT OF LIFT CROSS CORRELATION AT ZERO-TIME DELAY AS FUNCTION OF LATERAL SPACING
8.24 LATERAL ZERO-TIME CROSS CORRELATIONS OF PRESSURE FOR VARIOUS ANGLES AROUND THE CYLINDER.
8.25 TYPICAL LIFT AND DRAG CROSS-SPECTRAL DATA AT TWO LATERAL SPACINGS.
8.26 DRAG CROSS SPECTRA FOR 8 LATERAL SPACINGS.
8.27 LIFT CROSS SPECTRA FOR 8 LATERAL SPACINGS.
$\frac{\phi_{d, d_0}}{\phi_{u, u_0}} = \frac{1}{p^{2D^2C_2^{2UY^2}}} \times (9.69)$

LEGEND

- $\varphi / D$
- 0.129
- 0.29
- 0.443
- 0.935
- 1.58
- 2.56
- 4.64
- 8.19

[ ] data at limit of reliability

Second Spectral Estimate is unreliable

8.28 SIMPLE DRAG-u-COMPONENT DESCRIBING FUNCTION FOR VARIOUS LATERAL SPACINGS.
See Figure 8-28 for Legend.

8.29 DRAG COHERENCE DATA.
See Figure 8-28 for Legend

8.30 MAGNITUDE OF THE FINAL DRAG-\(u\)-COMPONENT DESCRIBING FUNCTION.
\[
\frac{\phi_{1,1}}{\phi_{1,0}} \text{ for Legend, also:}
\]

- **Strouhal Peak Ratios**

See Figure 8-28

\[
\exp \left\{ - \left( 1.5 \frac{r \tau'}{U} \right)^{1.4} \right\}
\]

8.31 LIFT COHERENCE DATA FOR VARIOUS LATERAL SPACINGS.
\[
\frac{\phi_{11}(f, \zeta_1)}{\rho \frac{\pi^2 b^2 c_2 g^2}{d^2} \phi_{ww}(f)}
\]

8.32 MAGNITUDE OF THE LIFT-\(w\)-COMPONENT DESCRIBING FUNCTION.
A-I  HOT WIRE TEMPERATURE COMPENSATION CIRCUIT AND DESIGN CHART.
A-2 HOT WIRE CALIBRATIONS WITH AND WITHOUT TEMPERATURE COMPENSATION.

LEGEND
- INITIAL CALIBRATION, NEW PROBE, \( H = 1.65 \)
- RECALIBRATION AFTER ABOUT 1/2 HOUR'S RUNNING AT 35 fps
- FINAL RECALIBRATION

NO TEMPERATURE COMPENSATION
A.3 (VELOCITY)^0.45 vs (VELOCITY)^0.5 (See text).
A.4 TYPICAL CHANNEL OF HOT WIRE SIGNAL LINEARISATION.
A-5  TYPICAL CALIBRATIONS OF THE LINEARIZATION CIRCUIT.
A.6 EXAMPLES OF CONTAMINATED WIRES.
A.8 CALIBRATION OF X WIRE PROBES.
A.9 NOTATION FOR DISCUSSION OF PROBABILITY DISTRIBUTION OF VELOCITY VECTOR.
A.10 DESIGN CHART FOR DETERMINING X WIRE ANGLES.
A.11 SIMPLE THEORETICAL HOT WIRE RESPONSE TO 3-DIMENSIONAL TURBULENCE.
B.1 STATISTICAL VARIANCE FOR DIGITAL SPECTRAL ANALYSIS.
$R_1 = R_2 = R_3 = 10k\Omega$

$C_1 = .105\text{mF}$

$C_2 = .044\text{mF}$

$C_3 = .125\text{mF}$

$C_4 = .027\text{mF}$

$C_5 = .010\text{mF}$

$C_6 = .034\text{mF}$

B-2  CIRCUIT OF 6TH - ORDER FILTER, WITH THEORETICAL AND EXPERIMENTAL AMPLITUDE RESPONSE.
B-3 VERIFICATION OF DIGITAL SPECTRAL ANALYSIS METHOD.
\[ R_1 = R_2 = 10k\Omega \]
\[ C_1 = 0.0647\text{mF} \]
\[ C_2 = 0.025\text{mF} \]
\[ C_3 = 0.156\text{mF} \]
\[ C_4 = 0.0105\text{mF} \]

![Circuit Diagram](image)

**EXPERIMENTAL DATA USED FOR CORRECTION PURPOSES**

**ANALYTIC 4TH ORDER FILTER RESPONSE, 400 Hz NATURAL FREQUENCY**

- CHANNEL 1: EXPERIMENTAL DATA USED
- CHANNEL 2: FOR CORRECTION PURPOSES

**B-4 CIRCUIT OF 4TH-ORDER FILTER, WITH THEORETICAL AND EXPERIMENTAL AMPLITUDE RESPONSE.**
RATIO = \frac{\text{PAR estimate of spectrum from } R = 1 / \left(1 + \left(\frac{2\pi f_0^2}{A}\right)^2\right)}{f_{\text{max}} = f_0}

\text{True spectrum from } \phi = \frac{A}{f_0^2} \exp\left(-A \frac{f}{f_0}\right)

\text{NOTE: log-log slope of true spectrum at } f_0 = -A

\text{Ratio for } R = e^{-B_t}; \phi = \frac{4B}{B^2 + (2\pi f)^2} \text{ averaged for } B = 5, 50, 500 \text{ (increasing flatness)}

\text{Note: log-log slope as } f \rightarrow \infty = -2 \text{ for all these curves}

B.5 EFFECT OF SPECTRAL SHAPE ON SPECTRAL ESTIMATES DERIVED FROM PAR/526 SYSTEM.
LEGEND

- WAVE ANALYSIS OF TAPED DATA
- "ON-LINE" WAVE ANALYSIS

B-6A COMPARISON OF SPECTRAL ANALYSIS METHODS: WAVE ANALYSIS
Eeco System

--- Mean line from wave analysis off tape

Note: For clarity, only typical spectral estimates are shown at high frequency.
B-6C  COMPARISON OF SPECTRAL ANALYSIS METHODS: PAR/526 SYSTEM ANALYSIS
B-7A COMPARISON OF SPECTRAL ANALYSIS METHODS FOR DATA INCLUDING A SPIKE: EECO SYSTEM ANALYSIS.
Figure showing comparison of spectral analysis methods for data including spike:

- PAR/526 System
- High Cut, Δf = 20Hz, f₀ = 2000Hz
- Low Cut, Δf = 2Hz, f₀ = 500Hz

Legend: Mean line from wave analysis off tape, negative data points.

Note: For clarity, only typical spectral estimates are shown at high frequency.
C-1  VIEWS OF THE MICROPHONE CALIBRATION EQUIPMENT.
Equal by Homogeneity

Equal by Symmetry

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HISTOGRAM RULE

\[ \Delta \alpha = 15^\circ, \quad N = 4 \]

D.1 TYPICAL MATRIX OF COMPONENTS OF LIFT AND DRAG INTEGRATION.
BEGINNING OF INTEGRATION SYSTEMS

PROGRAMMER MUST DEFINE N=180/ANGLE INCREMENT, N= MUST BE EVEN

TWO DATA SETS (RXY AND RYX) HAVE BEEN PREVIOUSLY READ, CORRECTED AND

AVERAGED TO FORM F*X/AMN

MN=2*N

III=III+1
IF (I=J) 100,101,100
101 IF (I) 102..201+102
102 IF (J) 201..201+101
100 I=I+J
IF (J=MN) 103..201..103

103 IF (I) 202..104..+202
104 IF (J=MN) 204..204..204
200 DO 400 M=1,100
RDBX(M)=RDBX(M)+RTMP(M)
400 RDDSI(M)=RDDSI(M)+4.0*RTMP(M)
GOTO 2

201 JTEST=J/2/J/2
IF (JTEST=J) 401.402.401
401 DO 403 M=1,100
TEMPI=RTMP(M)*CO(I)*CO(J)*4.0
TEMP2=RTMP(M)*SI(I)*SI(J)*4.0
RRBX(M)=RRBX(M)+TEMP2
RRLSI(M)=RRLSI(M)+8.0*TEMP2
RDBX(M)=RDBX(M)+TEMPl
RRDSI(M)=RRDSI(M)+8.0*TEMPl
GOTO 2

204 JTEST=J/2/J/2
406 IF (JTEST=J) 406.406.406
406 DO 408 M=1,100
TEMPI=RTMP(M)*CO(I)*CO(J)*+4.0
TEMP2=RTMP(M)*SI(I)*SI(J)+4.0
RRBX(M)=RRBX(M)+TEMP2
RRLSI(M)=RRLSI(M)+8.0*TEMP2
RDBX(M)=RDBX(M)+TEMPl
RRDSI(M)=RRDSI(M)+8.0*TEMPl
GOTO 2

202 JTEST=J/2/J/2
407 DO 409 M=1,100
TEMPI=RTMP(M)*CO(I)*CO(J)*4.0
TEMP2=RTMP(M)*SI(I)*SI(J)*4.0
RRBX(M)=RRBX(M)+TEMP2
RRLSI(M)=RRLSI(M)+16.0*TEMP2
RDBX(M)=RDBX(M)+TEMPl
RRDSI(M)=RRDSI(M)+16.0*TEMPl
GOTO 2

NP=NN**2+NN+1

2000 READS TWO NEW DATA SETS -- ARRAYS OF RXY AND RYX

14 AA=AA/9.0
15 BB=BB/9.0
DO 15 M=1,100
RDBX(M)=AA*RDBX(M)
RLLBX(M)=AA*RLLBX(M)
RRDSI(M)=BB*RRDSI(M)
RLLSI(M)=BB*RLLSI(M)

D-2 INTEGRATION PORTION OF THE DIGITAL PROGRAM.
% Deviation = % deviation from value obtained from N = 12 integration

\[ \Delta_{ll} = 100 \left( \frac{C_{ll}^0(0, \zeta) - C_{ll}^*(0, \zeta)}{C_{ll}^0(0, \zeta)} \right) \]

\[ \Delta_{dd} = 100 \left( \frac{C_{dd}^0(0, \zeta) - C_{dd}^*(0, \zeta)}{C_{dd}^0(0, \zeta)} \right) \]

D.3 EFFECT OF ANGLE INCREMENT ON INTEGRATED ZERO TIME DELAY LIFT AND DRAG CROSS CORRELATION.
EFFECT OF ANGLE INCREMENT ON LIFT SPECTRA.
E.1 CUMULATIVE ENERGY DISTRIBUTIONS FOR DRYDEN SPECTRA.
E.2 LOW FREQUENCY POWER RESPONSE FOR BRUEL AND KJAER 2417 RMS METER.
The interaction of high intensity turbulence with the flow past a rigid circular cylinder has been studied experimentally at subcritical Reynolds Numbers. Grids were used to produce homogeneous turbulence fields with longitudinal scales ranging from 0.36D to 4.40D, and with longitudinal intensities greater than 10%. Power and cross-spectra of the turbulence components (the 'system input') have been measured in order to carefully define the turbulence characteristics. In particular, lateral coherences of the longitudinal component have been found to collapse well when plotted versus $\Lambda/\lambda$ (lateral separation/wavelength) as suggested by Davenport. A model with which measurement of arbitrary two-point pressure correlations could be made was used in the response experiments. Subsequent integrations yielded the spectral properties of the unsteady drag and lift. Measurement of mean drag and Strouhal frequency indicate that to some extent even severe large-scale turbulence can be considered equivalent to an increase in the effective Reynolds Number. Vortex shedding is not disrupted drastically by severe turbulence, but is affected more by that at low frequency than at high. The unsteady lift response is still dominated by the Vortex shedding, whereas the unsteady drag is primarily a response to turbulence. The cross-spectra of the drag collapse well when plotted versus $\Lambda/\lambda$, and have been used, for one grid, to derive a 'describing function' for the drag 'response' to turbulence. This describing function is the central element needed for the calculation of structural response in the drag direction.

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