SCHLIEREN DIAGNOSTICS
OF A HYPersonic GAS TARGET NEUTRON GENERATOR

by

A. A. Haasz and J. H. Lever

February, 1980

UTIAS Report No. 242
CN ISSN 0082-5255
SCHLIEREN DIAGNOSTICS
OF A HYPERSONIC GAS TARGET NEUTRON GENERATOR

by

A. A. Haasz and J. H. Lever

Submitted October, 1979
Acknowledgements

The funding for this research project was provided by the Natural Sciences and Engineering Research Council of Canada under Grant No. A9188.
Abstract

The gasdynamic behaviour of a planar model of the Los Alamos geometry hypersonic Gas Target Neutron Generator (GTNG) was investigated using Schlieren flow visualization photographs, static and total pressure and spill flow measurements. The model consisted of two symmetrical expansion nozzles with 220 μm throats producing a combined flow of about Mach 4 in the GTNG channel. Stagnation pressures of 100-800 kPa were used. Two basic flow configurations, spill line closed and spill line open, were studied in order to gain insight into the complex boundary layer development near the nozzle exit planes. Both flow configurations are discussed qualitatively, making use of the pressure measurements and theoretical analysis.
## CONTENTS

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>Acknowledgements</td>
<td>ii</td>
</tr>
<tr>
<td>Abstract</td>
<td>iii</td>
</tr>
<tr>
<td>Contents</td>
<td>iv</td>
</tr>
<tr>
<td>Notation</td>
<td>v</td>
</tr>
<tr>
<td>1. INTRODUCTION</td>
<td>1</td>
</tr>
<tr>
<td>2. SCHLIEREN THEORY AND TECHNIQUE</td>
<td></td>
</tr>
<tr>
<td>2.1 Principle of Operation</td>
<td>1</td>
</tr>
<tr>
<td>2.2 Typical Configurations</td>
<td>2</td>
</tr>
<tr>
<td>2.3 Operational Parameters Associated with Schlieren Systems</td>
<td>3</td>
</tr>
<tr>
<td>2.4 Physical Optic Considerations</td>
<td>6</td>
</tr>
<tr>
<td>3. EXPERIMENTAL APPARATUS</td>
<td>7</td>
</tr>
<tr>
<td>3.1 Test Section and Pumping Facilities</td>
<td>7</td>
</tr>
<tr>
<td>3.2 Schlieren System</td>
<td>8</td>
</tr>
<tr>
<td>3.3 Pressure Measurement Equipment</td>
<td>9</td>
</tr>
<tr>
<td>4. EXPERIMENTAL RESULTS</td>
<td>9</td>
</tr>
<tr>
<td>4.1 Schlieren Photographs</td>
<td>10</td>
</tr>
<tr>
<td>4.2 Pressure Measurements</td>
<td>11</td>
</tr>
<tr>
<td>4.3 Spill Flow Measurements</td>
<td>11</td>
</tr>
<tr>
<td>5. FLOW FIELD ANALYSIS</td>
<td>12</td>
</tr>
<tr>
<td>5.1 Boundary Layer Development in the 10° Nozzles</td>
<td>12</td>
</tr>
<tr>
<td>5.2 Spill-Closed Flow Field Analysis</td>
<td>14</td>
</tr>
<tr>
<td>5.3 Spill-Open Flow Field Analysis</td>
<td>15</td>
</tr>
<tr>
<td>6. CONCLUSIONS</td>
<td>17</td>
</tr>
<tr>
<td>REFERENCES</td>
<td>19</td>
</tr>
<tr>
<td>APPENDIX A: THE EFFECT OF OPTICAL ABERRATIONS IN THE DESIGN OF SCHLIEREN SYSTEMS</td>
<td></td>
</tr>
<tr>
<td>APPENDIX B: DETERMINATION OF THE 10° NOZZLE BOUNDARY LAYER PROFILES</td>
<td></td>
</tr>
<tr>
<td>TABLES</td>
<td></td>
</tr>
<tr>
<td>FIGURES</td>
<td></td>
</tr>
<tr>
<td>PHOTOGRAPHS</td>
<td></td>
</tr>
<tr>
<td>Symbol</td>
<td>Description</td>
</tr>
<tr>
<td>--------</td>
<td>-------------</td>
</tr>
<tr>
<td>a</td>
<td>Speed of sound</td>
</tr>
<tr>
<td>A</td>
<td>Nozzle area</td>
</tr>
<tr>
<td>b</td>
<td>Light source dimension measured parallel to knife edge</td>
</tr>
<tr>
<td>c</td>
<td>Speed of light</td>
</tr>
<tr>
<td>f</td>
<td>Focal length of mirror or lens</td>
</tr>
<tr>
<td>G</td>
<td>Boundary layer function</td>
</tr>
<tr>
<td>h</td>
<td>Light source dimension measured perpendicular to knife edge</td>
</tr>
<tr>
<td>I</td>
<td>Schlieren system screen illumination</td>
</tr>
<tr>
<td>( \dot{m}_{\text{tot}} )</td>
<td>Total mass flow rate in the gas target</td>
</tr>
<tr>
<td>M</td>
<td>Mach number</td>
</tr>
<tr>
<td>N</td>
<td>Index in power law function for mean velocity distribution in a boundary layer</td>
</tr>
<tr>
<td>P</td>
<td>Pressure</td>
</tr>
<tr>
<td>R</td>
<td>Specific gas constant</td>
</tr>
<tr>
<td>T</td>
<td>Temperature</td>
</tr>
<tr>
<td>u</td>
<td>Flow velocity</td>
</tr>
<tr>
<td>x, y, z</td>
<td>Distances in perpendicular directions; defined where used</td>
</tr>
<tr>
<td>( \beta )</td>
<td>Angular deflection of a light ray produced by a density gradient perpendicular to its direction of travel</td>
</tr>
<tr>
<td>( \gamma )</td>
<td>Specific heat ratio</td>
</tr>
<tr>
<td>( \delta )</td>
<td>Boundary layer disturbance distance</td>
</tr>
<tr>
<td>( \delta^* )</td>
<td>Boundary layer displacement thickness</td>
</tr>
<tr>
<td>( \eta )</td>
<td>Index of refraction of light</td>
</tr>
<tr>
<td>( \theta )</td>
<td>Boundary layer momentum thickness</td>
</tr>
</tbody>
</table>
\( k \)  
Gladstone-Dale constant.

\( \mu \)  
Absolute viscosity.

\( \nu \)  
Prandtl-Meyer angle.

\( \rho \)  
Density.

\( \tau \)  
Shear stress.

\( \phi \)  
Mach wave angle.

**Subscripts**

\( 0 \)  
Stagnation conditions.

\( e \)  
Free-stream conditions.

\( s \)  
Physical light source dimensions.

\( k \)  
Conditions with knife edge in position.

\( NC \)  
Nozzle contour wall.

\( SW \)  
Side wall of nozzle.

\( r \)  
Reference conditions.

**Superscripts**

\( * \)  
Nozzle throat conditions.

\( \prime \)  
Light source image dimensions.

\( - \)  
Free-stream conditions.
1. **INTRODUCTION**

Researchers in the fusion energy field have expressed considerable interest in the development of a Gas Target Neutron Generator (GTNG) facility to study materials damage resulting from a high energy neutron environment. The 14 MeV neutron spectrum typical of an operational fusion reactor can be generated by the impacting of a tritium ion \((^3H^+)^\) beam into a deuterium \((^2H)_2\) gas target. The capability of dissipating the 300 kW of heat deposited by the tritium beam is the major advantage of the gas target approach. \((1,2)\)

This study investigates the cold-flow (no ion beam) characteristics of a planar model of the hypersonic GTNG proposed by the Los Alamos Scientific Laboratory. The Schlieren photographs, pressure measurements and associated analysis presented in this report provide a starting point towards an understanding of the more complex gasdynamics of the beam heated GTNG.

Directly related to the subject of this thesis, Johnston \((3)\) conducted a flow visualization study of a geometrically similar, though not identical, planar gas target. Also, de Leeuw et al \((4)\) conducted spill flow measurements on a hypersonic gas target nearly identical to the one used in this study. Comparisons with the results presented in these two papers will be made.

2. **SCHLIEREN THEORY AND TECHNIQUE**

2.1 **Principle of Operation**

Schlieren methods can be applied to the study of any phenomenon which produces density variations in a gaseous medium. This is because the index of refraction, \(\eta\), of a gas is primarily a function of its density, \(\rho\) \((5)\). We can write with sufficient accuracy:

\[
\eta - 1 = K\rho \tag{1}
\]

where \(K\) is called the Gladstone-Dale constant, and is constant for a particular gas and wavelength of light \((6)\). For nitrogen at 0°C, 101.3 kPa the index of refraction of 5893 Å light is 1.000297.

The speed of light, \(C\), is determined by the index of refraction such that

\[
C = \frac{C_0}{\eta} \tag{2}
\]

where \(C_0\) is the speed of light in vacuo.

Consider a plane wavefront entering a region where the index of refraction varies perpendicular to the direction of travel, as in Figure 1. The rays passing through regions of higher \(\eta\) will be retarded with respect to the remaining rays. This will result in a turning of the wavefront and the emerging rays will be deflected. It can be shown \((5,6)\) that for light passing through a test region of inhomogenous refractive index, the total angle through which the rays will be deflected, \(\beta\), is given by
\[ \beta = \int \frac{1}{\eta} \frac{\partial \eta}{\partial y} \, dz \]  

(3)

where the integration is along the path travelled. For \( \eta = 1 \), as with nitrogen, the above expression can be reduced to

\[ \beta = \int \frac{\partial \eta}{\partial y} \, dz \]  

(4)

By inserting the density dependance of \( \eta \) from (Eq. 1), we get

\[ \beta \approx K \int \frac{\partial \rho}{\partial y} \, dz \]  

(5)

From Figure 1, we see that the rays are bent towards the region of higher refractive index. Thus rays passing through an inhomogenous gaseous medium will be deflected in the direction of increasing density. The purpose of a Schlieren system is to detect such deflections.

2.2 Typical Configurations

Application of the Schlieren method to the study of gaseous flows is generally attributed to Topler (7). In this method, undisturbed light passing through the test section is brought to focus by a field element (a mirror or a lens). At the focus is placed a knife edge stop, adjusted so that some of the undisturbed rays are blocked off. In this way, the image of the test section becomes darkened uniformly. Rays which pass through optical inhomogeneities will be deflected, as described earlier, and will be brought to focus slightly above or below the knife edge, depending on the direction of the deflection. Thus, the images of such inhomogenous regions will appear either darker or lighter than their surroundings.

The Schlieren visualization technique has been extensively applied to the study of supersonic gas flows (8-11), acoustic wave studies (12-14), boundary layer flows (15,16) and other areas (17,18). Three of the most common arrangements will be discussed here.

Figure 2 illustrates a typical double pass Schlieren system. Light from the source, located near the mirror's center of curvature, passes through the test section, reflects off the mirror and passes again through the test section as it is focused onto the knife edge. When studying phenomenon with small density gradients, such as convection flows, the angular deflections of the light rays are small enough that the rays pass essentially through the same region twice. The system sensitivity is effectively doubled in this case. However, the image of a stronger density gradient, such as shock waves, will be blurred slightly since larger deflections are produced. Also, the double pass of the rays is equivalent to a single pass through two spatially separated test sections. The screen cannot be placed at the conjugate focus of both test sections so that some lateral displacement of the image of a disturbance with respect to the homogenous background must occur (5). This is a shadowgraph effect and cannot be eliminated.

The two-lens system shown in Figure 3 eliminates some of the shortcomings of the double pass system. The light source, generally a slit or a pinhole, lies at the focus of lens \( L_1 \). A parallel beam of light is thus produced between the
two lenses in which the test section can be placed. Lens L2 focuses the beam onto a knife edge, which is used to uniformly darken the image of the working section cast onto the screen. Since the light rays are parallel between L1 and L2, the response of the system to an optical disturbance is independent of the position of that disturbance. Furthermore, the screen can be placed at the conjugate focus of the test section through L2 so that all the rays passing through a given point in the test section are brought to focus at a distinct point on the screen. This means that while a region may produce a local deflection of the parallel beam, leading to a lightening or darkening of its image, that image will not be spatially displaced on the screen. A complete analysis of the operation of the double lens Schlieren system is presented later.

The major drawbacks of this configuration arise from the use of lenses as field elements. These lenses need to be of very high quality and corrected for aberrations. Such lenses are necessarily more expensive and difficult to obtain than concave mirrors of the same aperture; this leads to the most popular Schlieren layout for general use, viz., the twin mirror Z style system, shown in Figure 4.

The twin mirror system operates similarly to the double lens system, except that the light source and knife edge must be located off the central axis if the entire working field is to remain unobscured. Large diameter, front surface concave mirrors are much less expensive than the equivalent lens elements, and are inherently achromatic. Unfortunately, unless rare off-axis focal point mirrors are used, aberrations will be introduced at both the knife-edge and screen planes. While these aberrations can be minimized by reducing the offset angle and increasing the focal length of the mirrors, they cannot be eliminated (see Appendix A for a more complete discussion of the effect of optical aberrations on the design of a Schlieren system). Furthermore, increasing the focal length of the second field element leads to diffraction problems arising from the edges of planar test sections used for internal flow field studies. In fact, the diffraction pattern produced by the edges of the planar GTNG model used in this study was sufficiently severe that a very short focal length second element was required to minimize its intrusion on the Schlieren image of the flow field. For this reason, an unusual layout consisting of a long focal length mirror as the first field element and a short focal length lens as the second was eventually chosen.

2.3 Operational Parameters Associated with Schlieren Systems

(1) Sensitivity

The sensitivity of a Schlieren system is its ability to detect the small light ray deflections produced by density gradients in the flow field. The expression for the sensitivity of a two element, single pass, parallel beam layout will now be derived.

Recall that the angular deflection of a light ray due to a density gradient normal to its path is

\[ \beta \approx K \int \frac{\partial \rho}{\partial y} \, dz \]  

(5)
and is in the direction of increasing density (see Figure 5). In general, the ray exiting the test section will undergo an additional deflection as a result of the difference in refractive index between the test region and the ambient. When the working fluid is air or nitrogen however, \( \eta \approx 1 \), and this additional deflection can be neglected.

It can be shown (5,6) that rays deflected through an angle \( \beta \) will pass above the knife edge (according to the directional conventions used in Figure 5) by an amount

\[
\Delta h = f_2^2 \beta
\]

Since the screen is placed at the conjugate focus of the test section through \( L_2 \), all rays, regardless of their direction, originating at point 0 will be brought to focus at 0'. This means 0' will be slightly lighter, but not displaced, with respect to its surroundings.

It is important to note here that the source image size at the knife edge \( b' \times h' \) is related to the physical light source dimensions, \( b_s \times h_s \), through the relationship

\[
\frac{b'}{b_s} = \frac{h'}{h_s} = \frac{f_2}{f_1}
\]

(7)

The knife edge is used to cut off all but height \( h_k \) of the undisturbed source image. This darkens the screen uniformly to an intensity

\[
I_k = \frac{h_k}{h'} I'
\]

(8)

where \( I' \) is the initial screen illumination.

A region of uniform density gradient in the test section will deflect the rays passing through it so that they form an image of the light source at the knife-edge plane which is displaced by the amount \( \Delta h \). The change in illumination, \( \Delta I \) caused by the disturbance can be written as

\[
\Delta I = \frac{\Delta h}{h'} I' = \frac{\Delta h}{h_k} I_k
\]

(9)

The relative change in illumination, or contrast, is defined as

\[
\text{Contrast} = \frac{\Delta I}{I_k} = \frac{\Delta h}{h_k}
\]

Substitution of (Eq. 6) yields

\[
\text{Contrast} = \frac{f_2^2 \beta}{h_k}
\]

(10)

The sensitivity of a Schlieren system is defined as the rate of change of contrast with respect to angular deflection \( \beta \). That is

\[
\text{Sensitivity} = \frac{d(\text{Contrast})}{d\beta}
\]
Insertion of (Eq. 10) produces the expression for sensitivity generally found in the literature, namely

\[
\text{Sensitivity} = \frac{f_2}{h_k}
\]  \hspace{1cm} (11)

Left in this form, the sensitivity expression indicates that for high sensitivity, \( f_2 \) should be as large as possible and \( h_k \) should be made small. However, these conditions cannot always be met for the following reasons:

a. The intrusion of the diffraction pattern produced by the edges of a planar test section will be more severe for a long focal length second element. The reason for this is that although the angular spread of the diffraction pattern is determined by the wavelength of the light (19), the further the screen is placed from the test section, the greater the lateral spread of the pattern will be. The minimum test section to screen distance is \( 2f_2 \). Hence \( f_2 \) should be made as small as possible in Schlieren systems designed to study planar internal flow fields.

b. For the working range of the system to be equal in both directions, \( h_k \) is set to \( \frac{1}{2}h' \) (see discussion on Range below).

c. As the knife edge is moved in to cut off more of the source image, geometric optics ceases to correctly predict the screen illumination and the wave nature of the light must be taken into account.

For equal working range in both directions of source image displacement, the knife edge is set at the optical axis, so that

\[
h_k = \frac{1}{2}h' \hspace{1cm} (12)
\]

From (Eq. 7) we find that

\[
h' = h_s \times \frac{f_2}{f_1}
\]  \hspace{1cm} (13)

Then by substitution of (Eq. 12) and (Eq. 13) into (Eq. 11), the new sensitivity expression becomes (20)

\[
\text{Sensitivity} = \frac{2f_1}{h_s}
\]  \hspace{1cm} (14)

Equation (14) indicates that for high sensitivity a long focal length first element should be used and that the source dimension perpendicular to the knife edge should be as small as possible. The Schlieren sensitivity under this mode of operation is independent of \( f_2 \), which means that a short focal length second element can be utilized to reduce the edge diffraction effect discussed earlier. Although (Eq. 14) was derived for rectangular sources, it is approximately valid for circular sources with the knife edge set at half cut-off.

(2) Working Range

The working range of a Schlieren system is the limiting angular deflection of a light ray which will produce a change in screen illumination. It corresponds to the maximum density gradient uniquely detectable by the apparatus.
From Figure 6, it is seen that a lateral displacement of the source image by $h_k$ towards the knife edge would completely blacken the screen. This condition is called saturation and any further deflections in the same direction would cause no change in the screen illumination. A lateral displacement of $h_k$ corresponds to an angular deflection of $h_k/f_2$ at the test section, which is thus the working range of the apparatus towards the knife edge. Similarly, the working range away from the knife edge is $(h'-h_k)/f_2$. Generally, it is desirable to have the range equal in both directions by setting $h_k = \frac{1}{2}h'$. This means that the maximum density gradient producing a change in screen illumination will do so in either direction. Actually, it can be shown that the working range of a Schlieren system is equal to the inverse of its sensitivity. Care must then be taken to avoid saturation of highly sensitive configurations, otherwise some flow field detail may be lost.

(3) **Light Intensity**

For any given light source, the background screen illumination will be determined by the choice of the first field element and the knife edge position. A compromise between sensitivity and light intensity must be made for both choices. The speed, or light gathering ability of the first element is inversely proportional to the square of its $f$/number (21), where

$$f/\text{number} = \frac{\text{effective focal length}}{\text{minimum clear aperture}}$$

Once the diameter of the parallel beam test region is chosen, the light gathered by the first element can be increased only by reducing its effective focal length. The system sensitivity will drop as a result. Generally, first element with an $f$/number between 6 and 10 is considered a good compromise (6,22).

In addition to creating equal working ranges in both directions, placing the knife edge at the optical axis (i.e., $h_k = \frac{1}{2}h'$) is also recommended (22) as satisfactory trade off between sensitivity and light intensity.

### 2.4 Physical Optic Considerations

The above discussion of Schlieren operational parameters is based solely on geometric optics. Under certain conditions, the wave nature of light must be taken into account. A detailed treatment of the physical optics of Schlieren systems can be found in References 22 to 24. A summary of the results and conclusions of these reports is presented here.

The undisturbed background illumination of the screen has light bands outlining the aperture of the test section, as depicted in Figure 7. This corresponds to the edge diffraction pattern discussed earlier. The division between two regions of dissimilar refractive indices will not appear distinct in the Schlieren image. Rather, the intensity distribution will be non-uniform and asymmetric, and will vary continuously, as shown in Figure 8a. Similarly, a rectangular refraction region will not have finite boundaries (see Figure 8b). Hence the illumination patterns of two closely spaced Schlieren objects will overlap one another, and will tend to limit the spatial resolution of the system.

The above irregularities in screen illumination become more severe as the
knife edge is used to cut off more of the light source image. For general use, the optimum position of the knife edge is at the optical axis. To improve sensitivity, a smaller light source, rather than more cut-off, should be used. High sensitivity Schlieren systems work well qualitatively, detecting the presence of small density gradients in the test section. However, due to the complicated superposition of the illumination patterns of a detailed flow field, such systems should not be used to gather quantitative information.

3. EXPERIMENTAL APPARATUS

3.1 Test Section and Pumping Facilities

The test section under study is a planar model of the hypersonic GTNG proposed by the Los Alamos Scientific Laboratory. A perspective sketch describing the basic configuration of this model is shown in Figure 9, with details of the half-nozzle assemblies given in Figure 10. The test section generally operates as follows: the flow field is set up by supplying nitrogen gas at a known pressure in the stagnation chambers. From there, the gas passes through the throats and expands into the two 10° nozzles. Most of the gas turns towards the main channel and is pumped away, however, if the spill line is open, some of the gas will turn the corner at the exit of the nozzle into the beam line.

The two half-nozzle assemblies are identical, and are symmetrically assembled in the test section so that the gap between them defines the flow channel. The throat width, which can be adjusted, was set to 0.22 mm for both half-nozzles. Four reinforced rubber gaskets were used to seal the two stagnation chambers. These gaskets were cut to conform to the contour of the flow channel, and were carefully bonded to the metal half-nozzle surfaces using a premium quality contact cement. The thickness of the flow channel, including the rubber gaskets, were measured as 10.7 mm.

Plate glass windows, 13 mm thick, were lightly clamped to either side of the assembled model (see Figure 11), to facilitate flow visualization experiments. The glass-to-rubber interface was sealed with vacuum wax. The entire test section, when assembled, could withstand chamber pressures of up to 1 MPa with no appreciable leakage. In addition, the leakage rate from the atmosphere into the test section under vacuum was measured to be less than $10^{-3}$ times the lowest total mass flow studied.

The test section was connected between two separate pumping facilities. The main flow (see Figures 9 and 11) was pumped away by a 60 l/s Kinney mechanical pump, while the spill flow was dumped into a 20,000 l vacuum chamber, which was in turn pumped by two 205 l/s Kinney pumps. This arrangement permitted two distinct flow configurations to be studied:

(1) spill line closed, main flow pumped away, and

(2) spill line open, both main and spill flows pumped away.

The spill channel represents the location of the incoming tritium ion beam in a full scale GTNG, and is therefore a critical area of interest. Differences in the flow fields under the above two modes of operation are helpful in determining the effect of the nozzle boundary layers on the spill flow, as will be discussed later.
3.2 Schlieren System

The layout of the Schlieren system used for the flow visualization experiments is shown in Figure 12, with the relevant overall dimensions noted. More detailed illustrations of the arrangement of the optical components are given in Figure 13.

Since the density in the flow field under study varies between $10^{-1}$ and $10^{-2}$ times atmospheric density, the Schlieren system was designed to have a fairly high sensitivity. The important features of the system components (as numbered in Figure 12) are described below:

(1) The light source was a 150W Xenon arc lamp encased in a glass envelope. The luminous flux is given by the manufacturer as 2200 lumens from an arc $0.5 \text{ mm} \times 1.5 \text{ mm}$. The lamp was enclosed in an aluminum housing with a glass port, and was fan cooled. The cooling air was exhausted outside the building, to safeguard against possible ozone contamination in the laboratory.

(2) An $f/2.8$, 76 mm focal length condensing lens was used to produce an image of the arc onto a pinhole. The pinhole and arc were placed 152 mm on either side of the lens in order to produce a full size arc image.

(3) A pinhole of $0.38 \text{ mm} \pm 0.01 \text{ mm}$ diameter, was used to produce the effective Schlieren light source. Owing to the arc lamp's high intensity, this small source diameter still allowed enough light to enter the system that visual, as well as photographic observations could be made. Since the arc intensity was uniform over a region several times larger than the pinhole, a very uniform beam was obtained (see Photo 1).

(4) Lens $L_1$ was a high quality, achromatic doublet which focused the cone of light emerging from the pinhole onto the parabolic mirror. The effective focal length of the first Schlieren field element, which consisted of $L_1$ and the parabolic mirror, was 1.14 m.

(5) A front surface, plane mirror folded the diverging light beam onto the parabolic mirror. This plane mirror was placed close to the axis of the parallel beam so that the effective off-axis angle of the light source was less than 20°.

(6) The parabolic mirror was an $f/8.75$, 3.56 m focal length front surface mirror.

(7) The test section, described previously contained the planar hypersonic flow field between two plate glass windows. The quality of the glass was sufficiently high that the windows introduced no noticeable distortion into the system.

(8) The second Schlieren field element was an $f/2.4$, 178 mm focal length Aero-Ektar anastigmatic lens.

(9) The knife edge was the movable half of an adjustable slit. It was found to be slightly more convenient to use than a razor blade and yielded identical Schlieren photographs.
A Polaroid plate film holder was custom mounted behind an adjustable exposure shutter. The film plane was located 356 mm from L2, as was the test section, producing a full size image of the test section onto the film. To record the Schlieren images, Polaroid Type 55 Positive/Negative film was used.

The entire parallel beam section of the Schlieren system was enclosed in a box to prevent the detection of atmospheric convection currents. The use of the 178 mm focal length lens was necessary to reduce the intrusion of the model edge diffraction pattern into the Schlieren image. In fact, this diffraction pattern was so severe that if the screen were placed at a distance at 2 m from the test section, the model edges could not be distinctly identified.

The full aperture setting of lens L2 was required so that the entire flow field could be examined in each photograph. Unfortunately, this introduced some spherical aberration at the knife-edge plane, which in turn gave rise to a non-uniform background illumination on the screen (see Photo 2). When the knife edge orientation was perpendicular to the centerline of the test model's flow channel, the so-called 90° direction, irregular background illumination was evident in the Schlieren photographs (i.e., the "no-flow" photograph, Photo 3). However, when the knife edge was oriented parallel to the flow channel centerline, the 0° position, the effective aperture was much narrower, and the Schlieren photographs did not exhibit significant background irregularities (see Photo 4). Fortunately, the 0° knife edge orientation yielded the most detail about the nature of the flow field, and it was felt that the spherical aberration introduced by the 178 mm lens was an acceptable price to pay for the necessary reduction in the model edge diffraction pattern.

The remainder of the Schlieren system layout was experimentally optimized to produce clear, detailed photographs of the flow field. The resulting system sensitivity, calculated using Equation (14), was $6 \times 10^5$ (percentage illumination change per radian deflection).

### 3.3 Pressure Measurement Equipment

For static pressure measurements, one glass window was replaced by an aluminum plate containing a series of 0.50 mm diameter holes. (See Figure 14 for the location of the static taps relative to the model flow channel.) These taps covered only one side of the flow channel, since the flow fields of interest appeared to be symmetric in the Schlieren photographs.

A 2 mm diameter pitot tube was used to measure total pressures at various points along the channel centerline. These total pressures corresponded to locations just downstream of the pitot tube bow shock (see Photo 5).

The lines leading from the static taps and the pitot tube were manifol ded into two sets of Wallace & Tiernan dial pressure gauges which were previously calibrated against an MKS capacitance manometer. The entire gauge/manifold/lines/test section assembly was lead tested to ensure the accuracy of the results.
4. EXPERIMENTAL RESULTS

4.1 Schlieren Photographs

At each stagnation pressure, for each of the two flow conditions (spill open; spill closed), two Schlieren photographs were taken; one with the knife edge parallel to the axis of the channel (0° position) and one perpendicular to it (90° position). In addition, "no-flow" photographs were taken at both knife edge orientations to record the image background illumination. Table 1 identifies the photographs according to the above flow and knife edge conditions.

When the knife edge was set at the 0° position, density gradients perpendicular to the channel axis were detected by the Schlieren system. Dark regions in these photographs indicate increasing density towards side A of the channel (see Figure 15a). Conversely, light regions indicate increasing density towards side B.

When the knife edge was rotated to the 90° position (see Figure 15b), density gradients parallel to the channel axis were detected. In this case, dark regions indicate increasing density in the downstream direction for photos up to 500kPa. For 90° photos at 600kPa or greater, the knife edge was rotated through 180° as this seemed to produce better image contrast. Hence the direction of the density gradient for a dark region in these photographs was in the upstream direction. Note that the no-flow photo for this knife edge orientation is shown in Photo 3.

Some general comments can now be made concerning the nature of the flow fields based on qualitative assessment of the Schlieren photographs. Firstly, it must be said that all flow configurations were steady. No instabilities were ever detected visually or with high speed (1/500 sec.) photography. Next, it can be seen from the photographs that the flow field density gradients lie mainly in the direction perpendicular to the channel axis. This is especially true for the spill-open case. In fact, density gradients parallel to the axis are so small for the spill-open cases that only flows of 300kPa or more stagnation pressure show any appreciable detail in 90° photographs. The pitot tube bow shock in these photos indicates that the Schlieren system would be able to detect strong axial density gradients in the flow, if any existed.

The existence of dark and light bands along the walls of the two 10° nozzles indicates the presence of boundary layers. The intensity of these bands, points out that the density generally increases across the boundary layers towards the center of the nozzle.

Additionally, the major difference between spill-open and spill-closed flow fields can be seen at the exist of each 10° nozzle: the spill-open flow can turn around the corner while the spill-closed flow at this point creates a viscous mixing layer between the expanding supersonic flow exiting the nozzle and the near stagnant gas trapped in the spill line. Because of this difference, the spill-open supersonic flow turns downstream through an oblique shock, while the spill-closed flow turns more gradually in a manner analogous to the separated wake behind a supersonic body. Much more will be said about this interesting region later.

Finally, an anomaly exists in the spill-closed flow fields at stagnation pressures of 100kPa and 200kPa. Depending upon whether these stagnation
pressures are approached from above or below, asymmetric flow fields can be set up. No such asymmetries exist at higher pressures. Also, all spill-open flow fields are symmetric. Small disparities in the exact shape of the two nozzles, resulting in non-symmetric boundary layer development, could produce asymmetric flow fields. It then appears that as the boundary layers get thinner with increasing stagnation pressure, or with the spill channel open, the small disparities become insignificant and the flows become symmetric.

To help confirm the symmetry of the higher pressure flow fields, Schlieren photographs were taken with the knife edge rotated through 180° from the positions used in Table 1 (see Photos 15A & B, 16A & B). These photographs show that the flow fields are indeed symmetric above 300kPa.

4.2 Pressure Measurements

Four sets of static and total pressure measurements were taken at the following flow conditions: 300kPa and 600kPa spill-closed; and, 300kPa and 600kPa spill-open. The purpose of these pressure measurements was to provide empirical input into the analysis of the flow fields, as well as to help confirm the validity of the Schlieren photographs. With this in mind, different sets of static pressure tap locations were used for the spill-closed and spill-open configurations. Tables 2 and 3 contain the results of these experiments.

It is important to note here that each of the four walls defining the flow channel will be covered by a fully developed compressible boundary layer. A nearly inviscid supersonic free-stream will flow along the center of the channel (25). In the analysis presented in Section 5, the above static pressures, which are measured along the wall of the test section, are assumed to be the values which exist in the free stream directly above the corresponding tap locations. An order of magnitude analysis (25-27) confirms the validity of this assumption by showing that \( \frac{\partial P}{\partial Y} \approx 0 \) across both laminar and turbulent compressible boundary layers.

Since the total pressure measurements were made in the free stream directly above static tap holes, the Rayleigh pitot formula can be used to determine local Mach numbers (see Reference 28). Mach numbers thus calculated are also shown in Tables 2 and 3.

During the pressure experiments, the stagnation temperature of the nitrogen gas was measured in one of the stagnation chambers. It was found to be steady at \( 21.0 \pm 0.02^\circ C \). The gas temperature in the other stagnation tank was assumed to be the same, since both tanks were interconnected.

4.3 Spill Flow Measurements

The mass flow rate into the spill lines was determined by allowing the spill to flow into the 20,000 l vacuum chamber for a known length of time, and then measuring the chamber pressure rise. The spill rates were assumed to be constant over the time interval of the experiments, since the increases in the chamber pressure were small (typically 50Pa). The results are shown below:

- \( P_0 = 300kPa \), Spill Rate: 57.4 mg/s
- \( P_0 = 600kPa \), Spill Rate: 112 mg/s
The no-flow leakage rate of gas into the test section was measured and found to be negligible (less than 1 mg/s).

The mass flow rate into the main channel was measured by inverting the test section and repeating the above procedure. The following results were obtained:

\[
P_0 = 300\text{kPa}, \text{ Main Flow Rate: } 3.14 \text{ g/s}
\]
\[
P_0 = 600\text{kPa}, \text{ Main Flow Rate: } 6.27 \text{ g/s}
\]

For these experiments, the spill flow was also pumped away; and again the chamber pressure rise was small so that the main flow rate was assumed to be constant over the measurement interval.

The mass flow through a choked nozzle can be calculated from the isentropic one-dimensional flow equations. In reduced form, the expression for the total mass flow through both model nozzles (kg/s) is shown in Reference 4 to be

\[
\dot{m}_{tot} = 0.07948 A^* P_0 \sqrt{T_0}
\]

where \(A^*\) is the throat area of one nozzle (m\(^2\)), \(P_0\) is the stagnation pressure (Pa) and (\(T_0\)) is the stagnation temperature (K). Thus, the predicted total mass flow through the test section is 3.27 g/s at \(P_0 = 300\text{kPa}\), and 6.55 g/s at \(P_0 = 600\text{kPa}\). These figures agree (within 3%) with the combined main and spill flow mass rates measured.

The above spill rates are 1.8% of the total measured mass flow, for both stagnation pressure cases. This figure agrees well with the cold-flow spill measurements reported in (4) for an identical hypersonic GTNG model.

5. FLOW FIELD ANALYSIS

5.1 Boundary Layer Development in the 10° Nozzles

The presence of boundary layers along the walls of the 10° nozzles was first indicated by the 0° Schlieren photographs. The light and dark bands which line the nozzles represent regions of increasing density towards the respective nozzle centerlines. These bands would not exist in an ideal one-dimensional supersonic expansion.

The static pressures measured along the 10° nozzle are shown in Table 4. The static pressures which would be expected in the nozzle if no boundary layers were present are shown in the last column of Table 4. These calculated pressures are based on the ratio of the geometric nozzle area at each tap location to the area of the nozzle throat. From these area ratios, Mach numbers and pressure ratios, \(P/P_0\), can be determined from the one-dimensional isentropic flow equations. Multiplication of the pressure ratios by the appropriate stagnation pressure yields the desired inviscid values of static pressure.

Inspection of the static pressures in Table 4 leads to the following general conclusions:
1. The actual flow expands somewhat slower than an inviscid flow along the same 10° nozzle. This slower rate of expansion can be attributed to the growth of the boundary layers along the nozzle walls, that is, the effective area ratio seen by the inviscid core flow is reduced due to the presence of the viscous shear flow along the walls.

2. The rate of expansion of the core flow is approximately the same regardless of whether the spill line is open or closed. This means that the pressure in the spill channel has very little influence on the nozzle boundary layer development, even though the effect of that pressure can be propagated upstream through the subsonic portion of the boundary layer. Thus there is no need to discriminate between the spill-open and spill-closed configurations in the analysis of the flow in the 10° nozzles.

3. The rate of expansion of the core flow is greater at $P_0 = 600$ kPa than at $P_0 = 300$ kPa. This agrees with supersonic viscous flow theory which predicts that the boundary layer thickness is an inverse function of free-stream total pressure (see Appendix B).

Figure 16 illustrates the nature of the boundary layer development in the 10° nozzles. There are two measures of the boundary layer thickness which are significant in supersonic nozzle analysis: the "disturbance distance", $\delta$ and the "displacement thickness", $\delta^*$. The disturbance distance is defined as the distance into the flow field (measured normal to the wall) that the presence of the viscous shear layer can be detected. For convenience, $\delta$ is generally taken to be the location for which the boundary layer velocity, $u(y)$, equals 99% of the free-stream velocity, $\bar{u}$, (see Figure 17).

As far as the inviscid core flow is concerned, the influence of the boundary layer on the effective nozzle geometry is determined by the displacement thickness (25-27). Fundamentally, $\delta^*$, can be written as

$$\delta^* = \int_0^\delta \left( 1 - \frac{\rho(y)}{\rho_\infty} \right) dy$$

(16)

where $\rho_\infty$, $\rho$ are the density in the boundary layer and the free stream, respectively. Physically, the mass flow retardation due to the presence of the boundary layer is equivalent to an inward shift of the wall by $\delta^*$.

To determine the boundary layer profile in a nozzle, expressions for $\rho$ and $u$ must be obtained as functions of both $x$, the distance downstream of the nozzle throat, and $y$, the distance normal to the wall. The general approach to the problem is to assume a mean velocity distribution, valid along the length of the boundary layer, as a power law shape function:

$$\frac{u}{\bar{u}} = \left( \frac{y}{\delta} \right)^{1/N}$$

A choice of $N=7$, common in turbulent incompressible theory, is said to yield results in good agreement with experiments (25). The density ratio, $\rho/\rho_\infty$ can be determined from an energy balance, which then yields an expression for $T/\bar{T}$ (the local temperature ratio), since the static pressure is assumed to be constant across the boundary layer.
In Appendix B, the displacement thickness profiles are calculated numerically for both the nozzle contour and the side wall boundary layers using the method described in Reference 25. By assuming that the core flow expands isentropically along a nozzle whose effective area ratio is modified by the calculated displacement thickness profiles, this numerical viscous theory provides an estimate of the actual flow conditions.

A comparison of the free-stream Mach numbers at each tap location is shown in Table 5. The Mach numbers based on the measured static pressures, (columns (a)) were determined from the isentropic one-dimensional flow equations upon substitution of the measured pressure ratios, $P/P_0$. This approach is valid provided the boundary layers on opposite walls do not touch, as is the case here, and enables an experimental check to be made on the two theoretical predictions. The Mach numbers in column (b) of Table 5 are determined from a one-dimensional expansion of an inviscid gas from the throat to the appropriate nozzle location, with no allowances made for the boundary layer growth along the walls. This analysis is identical to the one used previously to generate the inviscid values of the static pressures shown in Table 4, and again overestimates the true rate of expansion in the nozzle. Lastly, the Mach numbers in columns (c) of Table 5 are calculated from the theoretical effective area ratio seen by the core flow, which involves a correction in the location of the nozzle walls accounted for by boundary layer displacement thickness profiles as generated in Appendix B. Notice the good agreement between the Mach numbers based on the viscous analysis and those experimentally determined. It is reasonable to assume that the predicted boundary layer profiles are close to those actually existing in the nozzles. Thus the core flow conditions at the 10° nozzle exit planes can be estimated from viscous theory (see Table 6) and used in the subsequent analyses of the main channel spill-closed and spill-open flow fields.

It was stated in Section 4.1 that boundary layer density gradients along the nozzle walls were evident on the Schlieren photographs. This is because viscous shear dissipates energy across the entire width of the boundary layer, which leads to increasing static temperature towards the walls (16,26) and, since the static pressure is constant across the boundary layer, to increasing density towards the free-stream. This result appears to be consistent with the density gradients observed in the 0° Schlieren photographs.

5.2 Spill-Closed Flow Field Analysis

When the spill channel is closed, the flow exiting the 10° nozzles behaves very similarly to the separated flow behind a supersonic body. The analysis of this so called "near wake" region was pioneered by Chapman et. al. (29) for the ideal case of negligible boundary layer thickness at the separation point, and can be treated theoretically. Figure 18 shows the basic flow field generated for this ideal case. The uniform upstream flow separates from the body at point $S$ and reattaches at $R$. Because the flow is assumed to be steady, the same streamline which separates also reattaches. The region so trapped is called the dead air space. Due to viscous shear, a low speed circulation is set up in the dead air space, and the velocity increases away from this region across the viscous mixing layer, towards the free-stream. Some expansion occurs in the free-stream around the separation point corner and the characteristic expansion waves are produced. Also, the reattachment process occurs nearly isentropically so that a series of compression waves are produced which turn the flow parallel to the body once again. As a result, the static pressure
along the reattachment zone rises, (see Figure 19).

Figure 20 qualitatively sketches the spill-closed flow field as seen in the Schlieren photographs and identifies the important details. The near wake analogy is fairly complete for that portion of the core flow exiting the nozzle which can feel the influence of the corner. The nozzle flow separates from the corner and reattaches along the channel centerline (the equivalent of a solid boundary since the model is symmetric). The closed spill channel thus becomes the dead air region. The first expansion wave from the nozzle corner propagates at an angle $\phi$, given by:

$$\phi = \sin^{-1} \frac{1}{M_e}$$

where $M_e$ in this case is the exit plane free-stream Mach number given in Table 6. Thus, for the 300 kPa stagnation pressure case, $M_e = 3.74$ and $\phi = 15.5^\circ$. The mixing layer between the dead air and the expanded free-stream is clearly visible in the $0^\circ$ Schlieren photographs, which also show increasing density across the mixing layer towards the free-stream. Since energy is dissipated in this viscous shear layer as it is in a boundary layer, the direction of the transverse density gradient here is indeed towards the free-stream. Finally, the compression waves associated with the reattachment process are clearly visible in the Schlieren photographs, and are indicated in the sketch in Figure 20. Additionally, a rise in centerline static pressure as predicted by the near wake theory was measured both in the current investigation (static taps 10-13) and by Johnston (3) in the corresponding region of his spill-closed flow field.

Further analysis of the separated flow region becomes complicated by the fact that the boundary layers at the exit of the $10^\circ$ nozzles must be taken into account. Such analysis is beyond the scope of the present study. However, the interested reader is referred to Reference 30 for an extension of the near wake problem to include finite boundary layers at the separation point.

The $10^\circ$ nozzle core flow which does not feel the influence of the exit corner expands downstream as though the nozzle continued. At static tap 6, assuming further isentropic expansion of this portion of the free stream, the Mach number based on the measured pressure ratio of $4.53 \times 10^{-3}$ (again for $P_0 = 300$ kPa case) is $M_e = 4.28$. The flow will continue to expand in this way until the influence of the $16^\circ$ expansion corner is encountered. It is interesting to note here that the expansion fan around the $16^\circ$ corner is preceded by a weak oblique shock (identified in Figure 20 and seen most clearly as a light line preceding the dark expansion fan in the $0^\circ$ Schlieren photographs 13B and 14B). The occurrence of such a shock is a result of boundary layer separation from the channel walls (15). The crossing of the weak shock/expansion fan combination with the mixing layer and the reattachment compression waves can also be clearly seen in the Schlieren photographs.

Very little quantitative analysis can be carried out on the spill-closed flow field due to the complex nature of the viscous effects.

5.3 **Spill-Open Flow Field Analysis**

When the spill line is open, the subsonic portion of the boundary layer nearest the spill channel is siphoned off. The supersonic boundary layer and
free stream will then expand around the exit corner of the nozzle. Since there still exists a velocity gradient across the supersonic boundary layer, viscous losses will continue to occur. This makes it very difficult to determine the angle of turning of the flow near the nozzle corner. However, the gas close to the nozzle centerline should be sufficiently distant from the boundary layers that it will expand isentropically.

The core flow streamlines which pass above static tap 6 originate near the nozzle center at the exit plane. It is possible to determine the oblique shock angle required to turn these streamlines parallel to the main channel axis. Far from the nozzle corner, the expansion waves will appear to be originating from the corner over a range of angles given by isentropic theory. That is, the first influence of the corner will propagate at an angle of \( \phi \), given by:

\[
\phi = \sin^{-1} \left( \frac{1}{M_e} \right)
\]

Again for the \( P_0 = 300 \text{ kPa} \) case, \( M_0 = 3.74 \) and \( \phi = 15.5^\circ \). Figure 21 shows that it is possible for this first expansion wave to intersect the central streamlines before tap 6 is encountered. Thus some turning of the core flow will occur. The measured static pressure, \( P_6 = 1.55 \text{ kPa} \), yields a local Mach number of \( M_6 = 4.18 \), under the assumption of isentropic expansion of the free-stream. The difference in Prandtl-Meyer angles between the exit plane (\( \phi_e = 62.2^\circ \)) and tap 6 (\( \phi_6 = 68.1^\circ \)) is \( \Delta \phi = 5.9^\circ \). Without some knowledge of the boundary layer profile in the main channel and details of the non-centered expansion fan produced through the turning of the supersonic boundary layer around the exit corner, it is not possible to determine the breakdown of the change in Mach number between directional expansion (due to influence of exit corner) and radial expansion (due to effective area ratio changes before influence of exit corner is encountered). Instead, two limiting cases will now be considered.

(a) Assume that the \( 14.5^\circ \) streamline passes directly above tap 6 without encountering the corner expansion waves; the increase in the Mach number being entirely due to the further radial expansion of the core flow. Expanding radially at the same rate to the approximate location of the oblique shock (as obtained from the Schlieren photos), the \( 14.5^\circ \) streamline will be approximately Mach 4.30. The angle of the oblique shock required to turn this streamline parallel to the main channel axis (from Chart 2, Reference 28) is \( \theta = 26.7^\circ \). Thus the oblique shock will make an angle of \( 12.2^\circ \) with the nozzle axis (see Figure 21a) and will yield an after shock Mach number of 3.2.

(b) The second limiting case is where no further expansion of the core flow occurs beyond the nozzle exit plane until the corner expansion waves are encountered. The \( 14^\circ \) streamline will pass very near tap 6 as it turns through the \( \Delta \phi = 5.9^\circ \) required to achieve Mach 4.18. Assuming that the flow continues to turn at approximately the same rate until the shock is encountered, an additional \( \Delta \phi \) of 5.9° would yield a before shock Mach number of 4.70. The oblique shock angle required to turn the flow parallel to the axis in this case is \( 37.5^\circ \) (see Figure 21b), so that the resulting after shock Mach number is 2.30, and the shock angle with respect to the channel axis is 11.7°.

The true shock angle, as measured from the Schlieren photographs, is approximately \( 10^\circ \) so that either analysis yields about the same degree of
accuracy. The analysis does show however that the more the core flow turns, the lower will be the aftershock Mach number, and the shock angle with respect to the axis. By extension then, the core flow streamlines near the nozzle corner, which turn the most, will tend to recompress through stronger oblique shocks, at reduced angles to the channel axis, and will result in lower after-shock Mach numbers than streamlines originating further from the nozzle corner. All after-shock streamlines will be parallel to the nozzle axis, and the corresponding Mach numbers will increase with distance from the axis (see Figure 22). This results in a region of inviscid shear flow between the center-line and the oblique shock, where although shear forces can be neglected, adjacent layers of gas have different flow velocities.

Since the before-shock conditions (Mach number, density, etc.) are nearly constant along the shock, the density recovery will be greatest at highest after-shock Mach number. Thus the above analysis confirms two flow details seen in the Schlieren photographs: (1) the oblique shock curves gently away from the nozzle axis, and (2) the parallel flow after the oblique shock increases in density away from the nozzle axis.

Figure 23 is used to qualitatively identify the spill-open flow details seen in the Schlieren photographs. The weak shock-expansion fan near the 16° corner is shown again, as it is also visible in the spill-open Schlieren photos 13D and 14D.

Due to the viscous boundary layer effects, quantitative analysis becomes increasingly complex and would yield little additional useful information. An analytical prediction of the spill rate, for instance, would have to take into account the fact that the spill line suction could be felt throughout the subsonic boundary layers on all four channel walls, while at the same time the viscous shear force between adjacent fluid layers would tend to resist this suction. Such a calculation is beyond the scope of this report.

6. CONCLUSIONS

(1) A sensitive Schlieren system was constructed utilizing the fact that sensitivity depends upon the focal length of the first field element and the source size (i.e., Sensitivity = 2f2/h3). All optical aberrations except spherical aberration were found to be insignificant in this Schlieren system design. The main purpose of the unusual mirror/lens arrangement, to minimize model edge diffraction effects, was achieved.

(2) Schlieren photographs of the hypersonic GTNG flow fields showed that they are steady, but that there are marked differences between the spill-closed and spill-open configurations. All spill-open and the higher pressure (Po ≥ 300 kPa) spill-closed flow fields are symmetric. However, the 100 and 200 kPa spill-closed flows were seen to have two stable asymmetric configurations. These asymmetries were attributed to small discrepancies in the exact shape of the two 10° nozzles.

(3) Static and total pressure measurements were made to help analyze the flow in the model. The static pressures measured in the 10° nozzle indicated a reduction in the effective nozzle area as a result of boundary layer development along the walls.
(4) The mass flow rate through the spill channel was measured and was found to be in agreement with previously published results (3,4) (i.e., approximately 1.8% of the total mass flow, for both 300 and 600 kPa cases).

(5) The flow conditions at the exit of the 10° nozzles were determined using a numerical technique to calculate the boundary layer displacement thickness profiles. Starting with these exit plane conditions, the spill-closed and spill-open flow fields were analyzed qualitatively. The spill-closed flow was found to resemble the near wake region behind a supersonic body, with a viscous mixing layer separating the dead air trapped in the spill channel from the inviscid core flow. The spill-open flow was found to partially expand around the nozzle exit corner, then recompress through an oblique shock to turn parallel to the main channel axis. The flow fields were not analyzed in more detail due to the complexity of the boundary layer/core flow interactions.
REFERENCES

1. Colombant, D.
   High Intensity 14MeV Neutron Source.

2. Emigh, C. R., Brolley, J. E., Cline, M. C.
   A Proposal for a 14MeV Intense Neutron Source at Los Alamos, New Mexico.

3. Johnston, S. C.
   Gasdynamic Measurements for the LASL Intense Neutron Source.

4. de Leeuw, J. H., Haasz, A. A., Stangeby, P. C., Youle, I. S.
   Experimental Simulation of a Hypersonic Gas Target Neutron Generator.

5. Eckert, E. R. G., Goldstein, R. J.
   Measurements in Heat Transfer.

6. Holder, D. W., North, R. J.
   Schlieren Methods.
   National Physical Laboratory Notes on Applied Science No. 31.

7. Topler, A.
   Vol. 127, 1866 and other sources listed in Reference 5.

8. Glass, I. I.
   The Design of a Wave Interaction Tube.
   UTIA Report No. 6, University of Toronto, 1950.

9. Soloukhin, R. I.
   Schlieren Method of Measuring a Density Jump in a Shock Wave.
   Fizika Goreniya i Vzryva.

10. Hendrix, R. E., Dugger, P. H.
    Hyperbalistic Range of the von Karman Gas Dynamics Facility.
    Photographic Applications in Science, Technology and Medicine,
<table>
<thead>
<tr>
<th></th>
<th>Author(s)</th>
<th>Title</th>
<th>Source/Details</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Author(s)</td>
<td>Title and Details</td>
<td></td>
</tr>
<tr>
<td>---</td>
<td>-----------</td>
<td>-------------------</td>
<td></td>
</tr>
</tbody>
</table>
APPENDIX A: THE EFFECT OF OPTICAL ABERRATIONS ON THE DESIGN OF SCHLIEREN SYSTEMS

It is necessary to consider the effect of optical aberrations on the quality of the images produced at the two planes of focus of a Schlieren system: distortion of the light source image at the knife-edge plane limits the system's sensitivity; while aberrations seen at the screen affect the spatial resolution of the Schlieren image produced (23).

The aberrations of importance in Schlieren system design are coma, astigmatism, and spherical and chromatic aberration. Each of these are briefly discussed below.

(a) Coma

Coma is a lateral smearing out of the image of an off-axis beam of light. The operation of concave mirrors inclined at an angle to the optical axis of the Schlieren system introduces coma at both planes of focus. While the coma at the screen can only be reduced through the use of long focal length mirrors, the coma at the knife edge plane can be eliminated entirely by utilizing two equal focal length mirrors set at equal, but opposite, off axis angles. Hence, the reason for the Z-style layout used for most twin mirror systems.

No coma is introduced into the Schlieren system through the use of lenses, since these are aligned parallel to the optical axis.

(b) Astigmatism

An optical element exhibiting astigmatism will focus an off-axis object point into two perpendicular image lines, called the tangential and sagittal foci.

With the off-set twin mirror arrangement, two light source images are produced at the knife-edge plane. However, this situation does not affect the sensitivity, provided a vertical source slit is used in conjunction with a vertical knife edge placed at the tangential focus, and a horizontal slit used with a horizontal knife edge located at the sagittal focus (23).

Again, this aberration is not exhibited by a lens used on axis, so that no astigmatism would be seen at the knife-edge plane of a dual lens Schlieren system. Unfortunately, all layouts introduce astigmatism at the screen since test objects in the working field are necessarily distributed over a range of off-axis positions. This effect is essentially independent of the choice of element focal length.

(c) Spherical Aberration

Spherical aberration can be defined as the variation of focal length with aperture (21). Figure A1 illustrates the effect of spherical aberration at the knife-edge plane on the background screen illumination. The exact illumination pattern will vary with knife-edge position, but it will not be possible to uniformly darken the screen.

Spherical aberration can be eliminated in the knife-edge plane by employ-
ing two parabolic mirrors. Spherical aberration will nevertheless be present at the screen plane regardless of the choice of field elements, although its effects can be reduced by placing both test section and screen at distances $2f_2$ on either side of the second element (21,23).

(d) **Chromatic Aberration**

The variation of focal length with light frequency occurs only with lenses; mirrors are inherently achromatic. Most high quality lens systems are color corrected for two widely separated bands (i.e., yellow and blue). However, it is still possible for the light source images formed at the knife-edge plane by two other colors to be axially separated sufficiently that the screen will be divided into two color regions (see Figure A2).

Because the focal length of a lens is a function of frequency, so too the magnification of the Schlieren image on the screen is color dependant. Generally, the longitudinal chromatic aberration shown in Figure A2 is more significant.

Chromatic aberrations in lens systems can be eliminated through the use of monochromatic light sources (i.e., laser). However, this aberration is much less noticeable in photographs made from panchromatic film than it is in visual observations.
FIG. A1 BACKGROUND SCREEN ILLUMINATION UNDER THE EFFECT OF SPHERICAL ABERRATION AT THE KNIFE-EDGE PLANE. THE KNIFE EDGE IS SET AT THE OPTICAL AXIS.

FIG. A2 CHROMATIC ABERRATION RESULTING IN A TWO-COLOUR SCREEN.
APPENDIX B: DETERMINATION OF THE 10° NOZZLE BOUNDARY LAYER PROFILES

The boundary layer growth along a wall of a supersonic expanding nozzle differs from that along a thin flat plate primarily due to the presence of the axial pressure gradient in the core flow. This "favourable" pressure gradient produces boundary layers which are slightly thicker than those predicted by flat plate theory.

The boundary layers which develop in supersonic nozzles are usually turbulent and are generally assumed to begin at the nozzle throat [25, 26]. That the boundary layer thickness in the throat can be neglected in the current investigation is confirmed by the mass flow measurements (Section 4.3). The presence of significant boundary layer development in the nozzle throats could only reduce the total mass flow, whereas it was measured to be approximately 2-3% higher than inviscid theory predictions.

As was stated previously, the flow field in a real supersonic nozzle consists of a primarily inviscid core (free stream) flow surrounded by layers of viscous flow at the channel walls. Since $\partial P/\partial y = 0$ across the boundary layer, the axial pressure gradient existing in the nozzle can be obtained from the one-dimensional expansion of the core flow. Also, provided the boundary layers are thin, the uncorrected pressure field obtained from the inviscid expansion of the gas along the 10° nozzle will be sufficiently close to the true pressure distribution that it may be used as input to boundary layer thickness calculations.

Following the assumptions made in Reference 25 (adiabatic, constant temperature walls; Prandtl number unity) the expression for the displacement thickness along the nozzle contour, $\delta^*_N$, can be written as

$$\delta^*_N = \frac{1}{G} \int_0^x \frac{\delta^*}{\theta} \left( \frac{\tau_o}{-2\rho u^2} \right) G dx \quad (B1)$$

and the displacement thickness along the side wall, $\delta^*_SW$, as

$$\delta^*_SW = \frac{1}{G} \frac{A^*}{A} \int_0^x \frac{\delta^*}{\theta} \left( \frac{\tau_o}{-2\rho u^2} \right) G \left( \frac{A}{A^*} \right) dx \quad (B2)$$

For a 1/7 power law turbulent boundary layer velocity distribution, the local skin friction coefficient, $\tau_o/\rho u^2$, can be determined from the empirical formula

$$\frac{\tau_o}{\rho u^2} = 0.0131 \left( \frac{\dot{u}}{\rho u x} \right)^{1/7} \quad (B3)$$

and the shape functions $G$, $A/A^*$, and $\delta^*/\theta$ can be expressed in terms of the free stream Mach number, $M$. $G(M)$ and $\delta^*/\theta$ are plotted as functions of $M$ in [25] over the range of interest ($0 \leq M \leq 4$).

To illustrate the dependence of the displacement thickness on the flow stagnation pressure, (B3) can be broken down as follows:
a) The free-stream absolute viscosity $\tilde{\mu} = \mu_r \left( \tilde{\mu} / \mu_r \right)$
- from \[28\] $\tilde{\mu} / \mu_r = \left( \frac{T}{T_r} \right)^{0.76}$ \quad $167 \leq T \leq 500$ K
- if the reference is chosen as the stagnation conditions, $T_r = T_o$ and $\mu_r = \mu_o = 1.82 \times 10^{-5}$ Ns/m$^2$ at $T_o = 294$ K
- then
  $$\tilde{\mu} = 1.82 \times 10^{-5} \left( \frac{T}{T_o} \right)^{0.76} \quad \text{(B4)}$$

b) The free-stream density $\tilde{\rho} = \rho_o \left( \tilde{\rho} / \rho_o \right)$
- at $294$ K, 101.3 kPa $\rho_r = 1.20$ kg/m$^3$
- then $\rho_o = \rho_r \left( P_o / P_r \right)$ for $T_r = T_o = 294$ K
- thus
  $$\tilde{\rho} = 1.20 \left( \frac{P_o}{101.3} \right) \left( \frac{\tilde{\rho}}{\rho_o} \right) \text{kg/m}^3 \quad \text{(B5)}$$
  for $P_o$ in kPa

c) The free-stream velocity $\tilde{u} = a^* \left( \tilde{u} / a^* \right)$
- $a^*$, the speed of sound at the nozzle throat is given by
  $$a^* = \sqrt{\gamma RT^*} = \sqrt{\gamma R T_o x T^*/T_o}, \quad \text{but } T^*/T_o = 0.8333$$
  $$= 0.913 a_o$$
  where $a_o$ is the speed of sound in the stagnation tank, 344 m/s at $21^\circ C$ (294 K)
- thus
  $$\tilde{u} = 314 \left( \frac{\tilde{u}}{a^*} \right) \text{m/s} \quad \text{(B6)}$$

Substitution of Equations B4, B5, B6 into B3 yields
$$\frac{\tau_o}{\rho u^2} = 1.18 \times 10^{-3} \left[ \left( \frac{T}{T_o} \right)^{0.76} \left( \frac{P_o}{101.3} \right) \left( \frac{\tilde{\rho}}{\rho_o} \right) \left( \frac{\tilde{u}}{a^*} \right) \right]^{1/7} \quad \text{(B7)}$$
It is important to note that the ratios $\frac{T}{T_0}$, $\frac{\rho}{\rho_0}$, $\frac{u}{a^*}$ are all functions of the free-stream Mach number, $\bar{M}$. Notice that at a given position in the nozzle, for a given Mach number, $\frac{T_0}{\rho_0 u^2} \propto (1/P_0)^{\frac{1}{7}}$. It follows that the boundary layer profiles given by (B1) and (B2) will also be inverse functions of the stagnation pressure.

Casting of the skin friction coefficient into the form of (B7) helps in the numerical formulation of the displacement thickness equations. By assuming the inviscid flow conditions as inputs, the initial Mach number distribution in the nozzle will be determined by the one-dimensional isentropic equations. Thus, for each stagnation pressure, the displacement thickness profiles can be numerically generated as functions of the axial distance from the nozzle throat.

Table B1 shows the breakdown of the parameters needed to solve (B1) and (B2) numerically at each position along the nozzle. The integration becomes summation if these parameters are held constant over each respective nozzle increment, and the displacement thickness equations can be written approximately as

$$
\frac{\delta_{NC}}{\delta_{SW}} \approx \frac{1}{G(\bar{M}_1)} \sum_{j=1}^{i} \left\{ \frac{\delta^*_{j}}{\bar{M}_j} \frac{T_0}{\rho_0 u^2} (\bar{M}_j, x_1, P_0) G(\bar{M}_j) \Delta x_j \right\} 
$$

(B8)

and

$$
\frac{\delta_{NC}}{\delta_{SW}} \approx \frac{1}{G(\bar{M}_1)} \frac{\bar{\rho}/\bar{\rho}^*}{A/\bar{A}^*(x_1)} \sum_{j=1}^{i} \left\{ \frac{\delta^*_{j}}{\bar{M}_j} \frac{T_0}{\rho_0 u^2} (\bar{M}_j, x_1, P_0) G(\bar{M}_j) \frac{A}{\bar{A}^*} (\bar{M}_j) \Delta x_j \right\} 
$$

(B9)

To determine the displacement thickness at the three static tap locations and the nozzle exit, a linear interpolation between adjacent values of $x_1$ is sufficiently accurate. These results are shown in Table B2 along with the effective area ratio at each of the four positions.

The isentropic flow equations can now be used to calculate the nozzle Mach numbers based on this viscous theory analysis. The results are shown in the main text, Table 5. Finally, since $\delta^*/\delta$ is plotted in [25] as a function of $\bar{M}$, the disturbance distances can also be determined.
TABLE B1

Numerical parameters for the calculation of the boundary layer displacement thickness profiles.

<table>
<thead>
<tr>
<th>i</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x_i$ (mm)</td>
<td>0</td>
<td>0.64</td>
<td>1.27</td>
<td>1.19</td>
<td>3.18</td>
<td>4.45</td>
<td>6.35</td>
<td>9.53</td>
<td>12.7</td>
<td>15.9</td>
</tr>
<tr>
<td>$x_i$ (mm)</td>
<td>0.64</td>
<td>0.64</td>
<td>0.64</td>
<td>1.27</td>
<td>1.91</td>
<td>1.91</td>
<td>3.18</td>
<td>3.18</td>
<td>3.18</td>
<td>3.18</td>
</tr>
<tr>
<td>$A/A^*$</td>
<td>1</td>
<td>1.51</td>
<td>2.03</td>
<td>2.54</td>
<td>3.57</td>
<td>4.59</td>
<td>6.13</td>
<td>8.70</td>
<td>11.3</td>
<td>13.8</td>
</tr>
<tr>
<td>$\tilde{M}$</td>
<td>1</td>
<td>1.86</td>
<td>2.21</td>
<td>2.46</td>
<td>2.82</td>
<td>3.08</td>
<td>3.39</td>
<td>3.77</td>
<td>4.05</td>
<td>4.29</td>
</tr>
<tr>
<td>$T/T_o$</td>
<td>.591</td>
<td>.506</td>
<td>.452</td>
<td>.386</td>
<td>.345</td>
<td>.303</td>
<td>.260</td>
<td>.234</td>
<td>.214</td>
<td></td>
</tr>
<tr>
<td>$\rho/\rho_o$</td>
<td>2.69</td>
<td>1.82</td>
<td>1.38</td>
<td>.926</td>
<td>.700</td>
<td>.506</td>
<td>.346</td>
<td>.264</td>
<td>.211</td>
<td></td>
</tr>
<tr>
<td>$\bar{u}/a^*$</td>
<td>1.57</td>
<td>1.72</td>
<td>1.81</td>
<td>1.92</td>
<td>1.98</td>
<td>2.05</td>
<td>2.11</td>
<td>2.14</td>
<td>2.17</td>
<td></td>
</tr>
<tr>
<td>$G(M)$</td>
<td>1.67</td>
<td>1.50</td>
<td>1.34</td>
<td>1.07</td>
<td>0.88</td>
<td>0.70</td>
<td>0.53</td>
<td>0.43</td>
<td>0.35</td>
<td></td>
</tr>
<tr>
<td>$\bar{5}\times/\theta (\bar{M})$</td>
<td>2.8</td>
<td>2.2</td>
<td>4.1</td>
<td>4.9</td>
<td>5.6</td>
<td>6.5</td>
<td>7.6</td>
<td>8.6</td>
<td>9.4</td>
<td></td>
</tr>
</tbody>
</table>
| $\tau_o\sqrt{\rho u^2(M,x_i)}$ | \begin{tabular}{c}
$P_o = 300$ KPa \\
$P_o = 600$ KPa
\end{tabular} | \begin{tabular}{c}
3.10  \hfill 2.88 \hfill 2.77 \hfill 2.66 \hfill 2.50 \hfill 2.53 \hfill 2.47 \hfill 2.43 \hfill 2.41 \\
2.81  \hfill 2.61 \hfill 2.51 \hfill 2.41 \hfill 2.35 \hfill 2.30 \hfill 2.24 \hfill 2.20 \hfill 2.18
\end{tabular} |
| $5^{\times}NC$ | \begin{tabular}{c}
$P_o = 300$ KPa \\
$P_o = 600$ KPa
\end{tabular} | \begin{tabular}{c}
0.005  \hfill .012  \hfill .021  \hfill .043  \hfill .080  \hfill .131  \hfill .233  \hfill .355  \hfill .507 \\
0.005  \hfill .011  \hfill .019  \hfill .039  \hfill .072  \hfill .119  \hfill .211  \hfill .321  \hfill .459
\end{tabular} |
| $5^{\times}SW$ | \begin{tabular}{c}
$P_o = 300$ KPa \\
$P_o = 600$ KPa
\end{tabular} | \begin{tabular}{c}
0.005  \hfill .010  \hfill .017  \hfill .032  \hfill .057  \hfill .085  \hfill .139  \hfill .199  \hfill .272 \\
0.005  \hfill .009  \hfill .015  \hfill .029  \hfill .052  \hfill .077  \hfill .126  \hfill .180  \hfill .246
\end{tabular} |


**TABLE B2**

Boundary layer displacement thickness and effective area ratio at the three static tap positions and the nozzle exit plane.

<table>
<thead>
<tr>
<th>Location</th>
<th>Axial Distance (mm)</th>
<th>Stagnation Pressure (kPa)</th>
<th>Displacement</th>
<th>Thickness</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>$\delta_{NC}$ (mm)</td>
<td>$\delta_{SW}$ (mm)</td>
</tr>
<tr>
<td>Tap 1</td>
<td>8.26</td>
<td>300</td>
<td>0.20</td>
<td>0.12</td>
</tr>
<tr>
<td></td>
<td></td>
<td>600</td>
<td>0.18</td>
<td>0.11</td>
</tr>
<tr>
<td>Tap 2</td>
<td>11.4</td>
<td>300</td>
<td>0.30</td>
<td>0.17</td>
</tr>
<tr>
<td></td>
<td></td>
<td>600</td>
<td>0.28</td>
<td>0.16</td>
</tr>
<tr>
<td>Tap 3</td>
<td>14.6</td>
<td>300</td>
<td>0.44</td>
<td>0.24</td>
</tr>
<tr>
<td></td>
<td></td>
<td>600</td>
<td>0.40</td>
<td>0.22</td>
</tr>
<tr>
<td>Exit Plane</td>
<td>15.6</td>
<td>300</td>
<td>0.49</td>
<td>0.27</td>
</tr>
<tr>
<td></td>
<td></td>
<td>600</td>
<td>0.45</td>
<td>0.24</td>
</tr>
</tbody>
</table>
TABLE 1
Identification of the Schlieren Photographs

<table>
<thead>
<tr>
<th>Stagnation Pressure (kPa)</th>
<th>Spill Line Closed</th>
<th>Spill Line Open</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Knife Edge Orientation</td>
<td>Knife Edge Orientation</td>
</tr>
<tr>
<td></td>
<td>0°</td>
<td>90°</td>
</tr>
<tr>
<td>No-flow</td>
<td>6A</td>
<td>6B</td>
</tr>
<tr>
<td>100</td>
<td>7A, 7B</td>
<td>7C</td>
</tr>
<tr>
<td>200</td>
<td>8A, 8B</td>
<td>8C</td>
</tr>
<tr>
<td>300</td>
<td>9A</td>
<td>9B</td>
</tr>
<tr>
<td>400</td>
<td>10A</td>
<td>10B</td>
</tr>
<tr>
<td>500</td>
<td>11A</td>
<td>11B</td>
</tr>
<tr>
<td>600</td>
<td>12A</td>
<td>12B</td>
</tr>
<tr>
<td>700</td>
<td>13A</td>
<td>13B</td>
</tr>
<tr>
<td>800</td>
<td>14A</td>
<td>14B</td>
</tr>
</tbody>
</table>
### TABLE 2

Spill-Closed Static Pressures, Total Pressures, and Mach Numbers

#### 300 kPa Stagnation Pressure

<table>
<thead>
<tr>
<th>Tap No.</th>
<th>Static Pressure (kPa)</th>
<th>Total Pressure (kPa)</th>
<th>Mach Number</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4.63</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>3.29</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>2.91</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>1.37</td>
<td></td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>1.66</td>
<td></td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>1.27</td>
<td></td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>1.37</td>
<td></td>
<td></td>
</tr>
<tr>
<td>11</td>
<td>1.69</td>
<td></td>
<td></td>
</tr>
<tr>
<td>12</td>
<td>2.03</td>
<td></td>
<td></td>
</tr>
<tr>
<td>13</td>
<td>2.45</td>
<td>5.60</td>
<td>1.16</td>
</tr>
<tr>
<td>14</td>
<td>1.47</td>
<td>5.40</td>
<td>2.48</td>
</tr>
<tr>
<td>15</td>
<td>2.53</td>
<td>19.86</td>
<td>2.40</td>
</tr>
</tbody>
</table>

#### 600 kPa Stagnation Pressure

<table>
<thead>
<tr>
<th>Tap No.</th>
<th>Static Pressure (kPa)</th>
<th>Total Pressure (kPa)</th>
<th>Mach Number</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>9.17</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>5.65</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>4.83</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>2.16</td>
<td></td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>2.46</td>
<td></td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>1.72</td>
<td></td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>2.41</td>
<td></td>
<td></td>
</tr>
<tr>
<td>11</td>
<td>3.73</td>
<td></td>
<td></td>
</tr>
<tr>
<td>12</td>
<td>5.37</td>
<td></td>
<td></td>
</tr>
<tr>
<td>13</td>
<td>6.43</td>
<td>16.00</td>
<td>1.23</td>
</tr>
<tr>
<td>14</td>
<td>2.67</td>
<td>24.53</td>
<td>2.60</td>
</tr>
<tr>
<td>15</td>
<td>6.20</td>
<td>30.40</td>
<td>1.85</td>
</tr>
<tr>
<td>Tap No.</td>
<td>Static Pressure (kPa)</td>
<td>Total Pressure (kPa)</td>
<td>Mach Number</td>
</tr>
<tr>
<td>---------</td>
<td>-----------------------</td>
<td>----------------------</td>
<td>-------------</td>
</tr>
<tr>
<td>1</td>
<td>4.63</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>3.33</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>2.83</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>0.03</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>0.03</td>
<td></td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>1.55</td>
<td></td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>1.28</td>
<td></td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>0.03</td>
<td></td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>0.03</td>
<td></td>
<td></td>
</tr>
<tr>
<td>13</td>
<td>2.03</td>
<td>12.27</td>
<td>2.08</td>
</tr>
<tr>
<td>14</td>
<td>0.93</td>
<td>6.53</td>
<td>2.26</td>
</tr>
<tr>
<td>15</td>
<td>2.47</td>
<td>13.47</td>
<td>1.97</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Tap No.</th>
<th>Static Pressure (kPa)</th>
<th>Total Pressure (kPa)</th>
<th>Mach Number</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>9.17</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>5.64</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>4.84</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>0.03</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>0.03</td>
<td></td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>2.95</td>
<td></td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>3.11</td>
<td></td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>0.03</td>
<td></td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>0.03</td>
<td></td>
<td></td>
</tr>
<tr>
<td>13</td>
<td>5.89</td>
<td>20.93</td>
<td>1.54</td>
</tr>
<tr>
<td>14</td>
<td>2.07</td>
<td>14.53</td>
<td>2.26</td>
</tr>
<tr>
<td>15</td>
<td>4.20</td>
<td>27.33</td>
<td>2.16</td>
</tr>
</tbody>
</table>
### TABLE 4

Measured and calculated (based on one-dimensional inviscid theory) static pressures.

<table>
<thead>
<tr>
<th>Tap No.</th>
<th>Stagnation Pressure (kPa)</th>
<th>Measured Static Pressure (kPa)</th>
<th>Calculated Static Pressure (kPa) (Inviscid Theory)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Spill Closed</td>
<td>Spill Open</td>
</tr>
<tr>
<td>1</td>
<td>300</td>
<td>4.63</td>
<td>4.63</td>
</tr>
<tr>
<td>2</td>
<td>300</td>
<td>3.29</td>
<td>3.33</td>
</tr>
<tr>
<td>3</td>
<td>300</td>
<td>2.91</td>
<td>2.83</td>
</tr>
<tr>
<td>1</td>
<td>600</td>
<td>9.17</td>
<td>9.17</td>
</tr>
<tr>
<td>2</td>
<td>600</td>
<td>5.65</td>
<td>5.64</td>
</tr>
<tr>
<td>3</td>
<td>600</td>
<td>4.83</td>
<td>4.84</td>
</tr>
</tbody>
</table>

### TABLE 5

Nozzle Mach numbers at each static tap location as calculated from (a) measured static pressures, (b) inviscid theory, and (c) viscous theory.

<table>
<thead>
<tr>
<th>Tap No.</th>
<th>(a) Measured static pressures</th>
<th>(b) Inviscid theory</th>
<th>(c) Viscous theory</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$P_o = 300$ kPa</td>
<td>$P_o = 600$ kPa</td>
<td>$P_o = 300$ kPa</td>
</tr>
<tr>
<td>1</td>
<td>3.39</td>
<td>3.39</td>
<td>3.62</td>
</tr>
<tr>
<td>2</td>
<td>3.62</td>
<td>3.74</td>
<td>3.93</td>
</tr>
<tr>
<td>3</td>
<td>3.72</td>
<td>3.85</td>
<td>4.18</td>
</tr>
</tbody>
</table>
### TABLE 6

Summary of the flow conditions at the exit plane of the 10° nozzles

<table>
<thead>
<tr>
<th>Stagnation Pressure</th>
<th>Mach Number</th>
<th>Boundary Layer Displacement (mm)</th>
<th>Boundary Layer Thickness (mm)</th>
<th>Boundary Layer Disturbance Distance (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>( \delta_{\text{NC}} )</td>
<td>( \delta_{\text{SW}} )</td>
<td>( \delta_{\text{NC}} )</td>
</tr>
<tr>
<td>300</td>
<td>3.74</td>
<td>0.49</td>
<td>0.27</td>
<td>2.16</td>
</tr>
<tr>
<td>600</td>
<td>3.79</td>
<td>0.45</td>
<td>0.24</td>
<td>1.10</td>
</tr>
</tbody>
</table>

Note: Subscripts NC and SW refer respectively to "nozzle contour" and "side wall" boundary layers, as shown in Figure 16.
FIG. 1 THE BENDING OF LIGHT RAYS PASSING THROUGH AN INHOMOGENEOUS MEDIUM.

FIG. 2 THE DOUBLE PASS SCHLIEREN SYSTEM.
FIG. 3 THE DOUBLE LENS, SINGLE PASS, SCHLIEREN SYSTEM.

FIG. 4 THE TWIN MIRROR, Z STYLE SCHLIEREN SYSTEM.
FIG. 5 SINGLE PASS, PARALLEL BEAM LAYOUT USED FOR SENSITIVITY CALCULATION.

FIG. 6 VIEW OF THE LIGHT SOURCE IMAGE AT THE KNIFE-EDGE PLANE.
FIG. 7 BACKGROUND SCREEN ILLUMINATION AS PREDICTED BY GEOMETRIC AND PHYSICAL OPTICS. THE KNIFE EDGE IS SET AT THE OPTICAL AXIS.

FIG. 8a SCHLIEREN SYSTEM RESPONSE TO A STEP CHANGE IN REFRACTIVE INDEX.

FIG. 8b SCHLIEREN SYSTEM RESPONSE TO A RECTANGULAR REFRACTION REGION.
FIG. 9 GENERAL LAYOUT OF THE HYPERSONIC GAS TARGET MODEL.
FIG. 10 DETAILS OF THE HALF-NOZZLE ASSEMBLIES.
FIG. 11 ASSEMBLED TEST SECTION, SHOWN HERE WITH STATIC PRESSURE PLATE IN POSITION.
FIG. 12a GENERAL LAYOUT OF SCHLIEREN SYSTEM; PLAN VIEW.

LEGEND:

1. ARC LAMP AND HOUSING
2. CONDENSING LENS CL
3. PINHOLE
4. LENS L1
5. PLANE MIRROR
6. PARABOLIC MIRROR
7. TEST SECTION
8. LENS L2
9. KNIFE EDGE
10. SCREEN OR FILM PLANE

[OPTICAL BENCH OR MOUNTS]
FIG. 12b EXPERIMENTAL APPARATUS, SHOWING SCHLIEREN SYSTEM, TEST SECTION, AND 60 l/s KINNEY PUMP.
FIG. 12c  GAS TARGET TEST SECTION IN POSITION.
NOTE THAT THE SPILL LINE IS NOT CONNECTED.
FIG. 13 DETAILS OF THE SCHLIEREN SYSTEM'S OPTICAL COMPONENTS.
FIG. 14 LOCATION OF STATIC PRESSURE TAPS.

FIG. 15a DIRECTION OF FLOW FIELD DENSITY GRADIENTS FOR 0° KNIFE EDGE.

FIG. 15b DIRECTION OF FLOW FIELD DENSITY GRADIENTS FOR 90° KNIFE EDGE FOR $P_o < 600$ kPa. (FOR $P_o \geq 600$ kPa DIRECTION OF GRADIENT IS REVERSED.)
THROAT

PARAMETERS

\[ A^* = 0.22 \times 10.7 = 2.35 \text{ mm}^2 \]

\[ W(x) = 0.22 + 0.175x \text{ mm} \]

\[ \frac{[A/A^*]_{\text{geometric}}}{A^*} = \frac{W(x)}{10.7} + 0.795x \]

or

\[ \frac{[A/A^*]_{\text{effective}}}{A^*} = \frac{[W(x) - \delta_{\text{NC}}]}{10.7 - \delta_{\text{SW}}} \]

FIG. 16 BOUNDARY LAYER DEVELOPMENT ON 10° NOZZLE WALLS.

FIG. 17 A TYPICAL BOUNDARY LAYER VELOCITY PROFILE.
FIG. 18 THE SIMPLIFIED SEPARATED FLOW PATTERN IN THE NEAR WAKE.

FIG. 19 STATIC PRESSURE RISE THROUGH THE REATTACHMENT ZONE.
Identification of spill-closed flow details.
(Numerical results are for $P_0 = 300$ kPa.)
FIG. 21 CALCULATION OF THE SPILL-OPEN OBLIQUE SHOCK WAVE ANGLE.

(a) ASSUMING NO EXIT CORNER EXPANSION WAVES ARE ENCOUNTERED.

(b) ASSUMING NO FURTHER RADIAL EXPANSION OF THE CORE FLOW.
**BEFORE SHOCK CONDITIONS:** \( M_1 > M_2, \quad \rho_2 > \rho_1 \)

**AFTER SHOCK CONDITIONS:** \( M'_2 > M'_1, \quad \rho_2 > \rho_1 \)

**FIG. 22** VARIATION IN THE AFTER-SHOCK MACH NUMBER WITH DISTANCE FROM THE CHANNEL AXIS.
FIG. 23 IDENTIFICATION OF SPILL-OPEN FLOW DETAILS.
(NUMERICAL RESULTS ARE FOR \( P_0 = 300 \text{ kPa} \).)
Photo 1. The uniform intensity parallel beam.

Photo 2. The non-uniform screen background illumination resulting from spherical aberration at the knife-edge plane.
Photo 3. No-Flow schlieren photograph, 90° knife edge.

Photo 4. No-Flow schlieren photograph, 0° knife edge.

Photo 5. Bow shock in front of pitot tube.
Photo 6A. No-Flow, spill closed, 0° knife edge.

See Photo 4.

Photo 6B. No-Flow, spill closed, 90° knife edge.

Photo 6C. No-Flow, spill open, 0° knife edge.

Photo 6D. No-Flow, spill open, 90° knife edge.
Photo 7A. $P_0 = 100$ KPa, spill closed, $0^\circ$ knife edge.

Photo 7B. $P_0 = 100$ KPa, spill closed, $0^\circ$ knife edge.

Photo 7C. $P_0 = 100$ KPa, spill closed, $90^\circ$ knife edge.

Photo 7D. $P_0 = 100$ KPa, spill open, $0^\circ$ knife edge.
Photo 8A. $P_0 = 200$ KPa, spill closed, $0^\circ$ knife edge.

Photo 8B. $P_0 = 200$ KPa, spill closed, $0^\circ$ knife edge.

Photo 8C. $P_0 = 200$ KPa, spill closed, $90^\circ$ knife edge.

Photo 8D. $P_0 = 200$ KPa, spill open, $0^\circ$ knife edge.
Photo 9A. $P_0 = 300$ KPa, spill closed, $0^\circ$ knife edge.

Photo 9B. $P_0 = 300$ KPa, spill closed, $90^\circ$ knife edge.

Photo 9C. $P_0 = 300$ KPa, spill open, $0^\circ$ knife edge.

Photo 9D. $P_0 = 300$ KPa, spill open, $90^\circ$ knife edge.
Photo 10A. $P_0 = 400$ KPa, spill closed, $0^\circ$ knife edge.

Photo 10B. $P_0 = 400$ KPa, spill closed, $90^\circ$ knife edge.

Photo 10C. $P_0 = 400$ KPa, spill open, $0^\circ$ knife edge.

Photo 10D. $P_0 = 400$ KPa, spill open, $90^\circ$ knife edge.
Photo 11A. \( P_0 = 500 \text{ KPa}, \text{ spill closed, 0° knife edge.} \)

Photo 11B. \( P_0 = 500 \text{ KPa}, \text{ spill closed, 90° knife edge.} \)

Photo 11C. \( P_0 = 500 \text{ KPa}, \text{ spill open, 0° knife edge.} \)

Photo 11D. \( P_0 = 500 \text{ KPa}, \text{ spill open, 90° knife edge.} \)
Photo 12A. \( P_0 = 600 \text{ KPa} \), spill closed, \( 0^\circ \) knife edge.

Photo 12B. \( P_0 = 600 \text{ KPa} \), spill closed, \( 90^\circ \) knife edge.

Photo 12C. \( P_0 = 600 \text{ KPa} \), spill open, \( 0^\circ \) knife edge.

Photo 12D. \( P_0 = 600 \text{ KPa} \), spill open, \( 90^\circ \) knife edge.
Photo 13A. $P_0 = 700$ KPa, spill closed, $0^\circ$ knife edge.

Photo 13B. $P_0 = 700$ KPa, spill closed, $90^\circ$ knife edge.

Photo 13C. $P_0 = 700$ KPa, spill open, $0^\circ$ knife edge.

Photo 13D. $P_0 = 700$ KPa, spill open, $90^\circ$ knife edge.
Photo 14A. $P_0 = 800$ KPa, spill closed, $0^\circ$ knife edge.

Photo 14B. $P_0 = 800$ KPa, spill closed, $90^\circ$ knife edge.

Photo 14C. $P_0 = 800$ KPa, spill open, $0^\circ$ knife edge.

Photo 14D. $P_0 = 800$ KPa, spill open, $90^\circ$ knife edge.
Photo 15A. $P_0 = 300$ KPa, spill closed, $180^\circ$ knife edge.

Photo 15B. $P_0 = 300$ KPa, spill open, $180^\circ$ knife edge.

Photo 16A. $P_0 = 500$ KPa, spill closed, $270^\circ$ knife edge.

Photo 16B. $P_0 = 500$ KPa, spill open, $270^\circ$ knife edge.
The gas dynamic behaviour of a planar model of the Los Alamos geometry hypersonic Gas Target Neutron Generator (GTNG) was investigated using Schlieren flow visualization photographs, static and total pressure and spill flow measurements. The model consisted of two symmetrical expansion nozzles with 220 μm throats producing a combined flow of about Mach 4 in the GTNG channel. Stagnation pressures of 100-800 kPa were used. Two basic flow configurations, spill line closed and spill line open, were studied in order to gain insight into the complex boundary layer development near the nozzle exit planes. Both flow configurations are discussed qualitatively, making use of the pressure measurements and theoretical analysis.