AN INVESTIGATION INTO CERTAIN ASPECTS OF THE DESCRIBING
FUNCTION OF A HUMAN OPERATOR CONTROLLING A SYSTEM OF ONE
DEGREE OF FREEDOM

by

M. Gordon-Smith

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SUMMARY

An experimental investigation has been carried out of the "remnant" portion of the mathematical model of the human operator. This model consists of the combination of a quasi-linear describing function and a remnant term. A single-axis tracking task, with random forcing functions and a compensatory display, was used to investigate the effect of the type of manipulator and the bandwidth of the forcing function on the remnant. Pressure and free-moving manipulators were employed with rate-control vehicle dynamics and filtered-white-noise forcing functions similar in spectral shape to those used in previous work. Data is presented which show the effects of the manipulator and of the forcing function on the describing function, on the performance measures of the system and on the power spectrum and the amplitude probability distribution of the remnant.
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NOTATION

That part of the precision model of the human operator which does not contain the transport delay, see eq. (B,12).

Impulsive response of A(s).

Threshold value of the indifference threshold nonlinearity of the human operator.

Break frequencies of the components of the transfer function fitted to $\Phi_{n_{e}e_{e}}$, see eq. (4.4.3,1).

Forcing function from Refs. 28 and 38, see section (4.1).

Time function of the output of the human operator, the units depend on the manipulator used.

Component of c(t) due to e(t).

Component of c(t) due to i(t).

Component of c(t) due to e(t), obtained using $Y_{PF}(j\omega)$ or $Y_{PM}(j\omega)$ respectively.

Component of c(t) due to e(t) obtained using an impulsive response time function of the same length as e(t), or truncated to $T_{m}'$, respectively.

As in $c_{e_{T}}(t)$ except that the initial 12 seconds of the time record are set to zero.

Coefficients in the impulsive response of the human operator $\chi_{P}(t)$, see eq. (B,14).

Intermediate coefficients in the calculation of $\chi_{P}(t)$.

Time function of the error signal displayed to the human operator.

Expected number of samples in a given class.

Component of e(t) due to the remnant.

Effective limb applied force.

Force applied to the manipulator.

F-statistic.

Muscle actuation elements.
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
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<tr>
<td>$G_E$</td>
<td>Sensing and computational characteristics, including all neural conduction and computation time delays up to the muscle actuation elements.</td>
</tr>
<tr>
<td>$G_{F1}$</td>
<td>Command feedforward element, representing synchronous tracking.</td>
</tr>
<tr>
<td>$G_{L}/G_M$</td>
<td>Transfer function relating effective limb applied force to the force actually applied to the manipulator.</td>
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<tr>
<td>$G_M$</td>
<td>Dynamics of the manipulator.</td>
</tr>
<tr>
<td>$H(s)$</td>
<td>Transfer function of general system element.</td>
</tr>
<tr>
<td>$H(j\omega)$</td>
<td>Frequency response of $H(s)$.</td>
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<tr>
<td>$H_1(j\omega)$</td>
<td>Closed-loop describing function relating $c(t)$ to $i(t)$.</td>
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<tr>
<td>$H_2(j\omega)$</td>
<td>Closed-loop describing function relating $e(t)$ to $i(t)$.</td>
</tr>
<tr>
<td>$h(t)$</td>
<td>Impulsive response of general system element.</td>
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<tr>
<td>$h_1(t)$</td>
<td>Impulsive response of $H_1(j\omega)$.</td>
</tr>
<tr>
<td>$i(t)$</td>
<td>Time function of the system input or forcing function.</td>
</tr>
<tr>
<td>$j$</td>
<td>Complex number, $\sqrt{-1}$.</td>
</tr>
<tr>
<td>$K$</td>
<td>Gain term used in calculation of the impulsive response of the human operator.</td>
</tr>
<tr>
<td>$K_c$</td>
<td>Gain of the transfer function of the controlled vehicle.</td>
</tr>
<tr>
<td>$K_{eq}$</td>
<td>Equivalent gain describing function of a nonlinear element or system.</td>
</tr>
<tr>
<td>$K_p$</td>
<td>Gain term of the describing function of the human operator.</td>
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<tr>
<td>$K_T$</td>
<td>Gain of the indifference threshold describing function.</td>
</tr>
<tr>
<td>$k$</td>
<td>Number of intervals in the amplitude probability distribution.</td>
</tr>
<tr>
<td>$m$</td>
<td>Integer.</td>
</tr>
<tr>
<td>$N,n$</td>
<td>Integers.</td>
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<tr>
<td>$n(t)$</td>
<td>Component of $c(t)$ due to the remnant.</td>
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<tr>
<td>$n_a$</td>
<td>Real part of the root of the second order component in $A(s)$ due to the neuromuscular system.</td>
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<tr>
<td>$n_c(t)$</td>
<td>Time function of the remnant considered as being injected at the operator's output, also known as &quot;the injected output remnant&quot;.</td>
</tr>
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</table>
\( n_e(t) \)  
Time function of the remnant considered as being injected at the visual input to the operator, also known as "the injected observation remnant".

\( o(t) \)  
Time function of the output of the closed-loop system.

\( o_i \)  
Observed number of samples in a given class.

\( p(x_l) \)  
Probability that \( x(t) \) will have an amplitude in the interval \( x_l < x < x_l + \Delta x \)

\( R \)  
Gain factor in the forcing-function filter.

\( R_{xy}(\tau) \)  
Cross-correlation function between \( x(t) \) and \( y(t) \), defined in eq. (C,4). When \( x(t) = y(t) \), this is known as the auto-correlation function.

\( R2.2 \)  
Forcing function from Ref. 28, see section (4.1).

\( s \)  
Laplace variable.

\( s_x \)  
Sample standard deviation of general variable \( x \), defined as

\[
 s_x^2 = \frac{1}{N-1} \sum_{i=1}^{N} (x_i - \bar{x})^2 \]

\( t \)  
Time, seconds.

\( T \)  
Time; also the time constant of the first order component of the forcing function filter.

\( T_1, T_L \)  
Lag and lead time constants of the precision model of the human operator.

\( T_K, T_K' \)  
Lag and lead time constants of the neuromuscular system in the precision model of the human operator.

\( T_{N_1} \)  
Lag time constant of the first order component of the neuromuscular system.

\( T_m, T_m' \)  
Maximum length of a time record, seconds.  
Maximum length of the truncated impulsive response of the human operator.

\( T' \)  
\( T \) modified to take into account the influence of the width of the spectral window, defined in Appendix C.

\( u, v \)  
Variables.

\( W(\omega) \)  
General weighting function, a function of frequency.

\( W_0(\omega) \)  
"Box-car" frequency weighting function, or spectral window, defined in eq. (B,7).

\( W_2(\omega) \)  
"Hanning" frequency weighting function or spectral window, defined in Ref. 7.
\( X(\omega), Y(\omega) \) Fourier transforms of \( x(t), y(t) \) respectively, defined in eqs. (C.14) and (C.15).

\( x(t), y(t) \) General variables, functions of time.

\( Y_c(j\omega) \) Transfer function of the controlled vehicle.

\( Y_p(j\omega) \) Describing function of the human operator.

\( Y_p(j\omega) \) Values of the precision model of the human operator obtained by fitting to the measured describing function.

\( Y_p(j\omega) \) Measured values of the describing function of the human operator.

\( Y_pY_c(j\omega) \) Open-loop describing function, the product of \( Y_p(j\omega) \) and \( Y_c(j\omega) \).

\( y_p(t) \) Impulsive response of the human operator.

\( \alpha \) Level of significance.

\( \alpha, \beta \) Zeros of \( A(s) \).

\( \alpha_c, \alpha'_c \) Nerve signals from the brain to the alpha motor neuron. Primed values are for the antagonist muscles.

\( \beta(t) \) General response window, a function of time, the inverse Fourier transform of \( W(\omega) \).

\( \beta_0(t) \) Response window resulting from the "box-car" frequency weighting function, \( W_0(\omega) \).

\( \beta_2(t) \) Response window resulting from the "Hanning" weighting function \( W_2(\omega) \).

\( \gamma \) General command signal to the position feedback loop elements in the neuromuscular system.

\( \gamma_c \) Nerve signal from the brain to the gamma motor neuron.

\( \Delta t \) Time interval.

\( \Delta x \) Amplitude interval.

\( \Delta \tau \) Time delay interval in the discrete correlation function.

\( \Delta \omega \) Frequency interval.

\( \delta(t-\tau) \) Dirac delta at \( t = \tau \).

\( \zeta_N \) Damping term of the second order component of the neuromuscular system model.

\( \zeta_F, \zeta_s \) Damping terms of the primary and secondary second order components respectively of the forcing-function filter.
\[ \theta_1, \theta_2 \] General neuromuscular command signals.

\[ \lambda_1, \lambda_2, \lambda_3 \] Roots of \( A(s) \).

\[ \mu_x \] Mean of the general time function \( x(t) \), defined as

\[
\mu_x = \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} x(t) \, dt
\]

Degrees of freedom.

\[ \nu_1, \nu_2 \] Correlation coefficient, defined in eq. (2.5,1).

\[ \rho(\omega) \] Relative remnant, defined in eq. (2.7.2,1).

\[ \rho_a^2 \] Real part of the Laplace variable, \( s = \sigma + j\omega \), or general standard deviation, depending on context.

\[ \sigma_x \] Standard deviation of the general time function \( x(t) \), defined as

\[
\sigma_x^2 = \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} x(t)^2 \, dt, \text{ or as in eq. (C,5)}.
\]

\[ \tau \] Time delay in the correlation function, or the time displacement in the convolution integral, depending on context.

\[ \tau_d \] Transport time delay in the describing function of the human operator, representing the summation of all the neural conduction, processing, sensing, and muscle activation time delays.

\[ \tau_m \] Maximum time delay in the correlation function, or time displacement in the convolution integral.

\[ \phi_{xy}(\omega) \] Cross-spectral density function between general variables \( x(t) \) and \( y(t) \), defined in eq. (C,2). If \( x(t) = y(t) \), this is known as the power-spectral density.

\[ \phi_{xy}(\omega, \tau_m, \Delta \tau) \] Cross-spectral density function for finite, discrete time records, defined in eq. (C,6).

\[ \phi_m \] Phase margin, defined in section (2.7.3).

\[ \chi^2 \] Chi-Square statistic calculated from the amplitude probability distribution of the general variable \( x(t) \), defined in eq. (2.9,2).

\[ \chi^2_{(k-3), \alpha} \] Chi-Square statistic calculated from samples of a true normal distribution with \( k-3 \) degrees of freedom and at \((1-\alpha)\) confidence level.
Frequency, radians/second; or the imaginary part of the Laplace variable \( s = \sigma + j\omega \), depending on context.

Imaginary part of the root of the second order component in \( A(s) \) due to the neuromuscular system.

Undamped natural frequency of the second order component of the neuromuscular system model.

Break-frequency of the first order component of the forcing-function filter.

Cross-over frequency, the frequency at which the open-loop describing function, \( Y_p Y_c(j\omega) \), passes through unity.

Cross-over frequency when \( \omega_i << \omega_c \).

Nominal bandwidth of the forcing-function spectrum.

Equivalent nominal bandwidth of the forcing function, defined in eq. (3.2.1,2).

Maximum frequency at which the describing function of the human operator is calculated.

Undamped natural frequencies of the primary and secondary second order components respectively of the forcing-function filter.

Undamped natural frequencies of the second order components of the fourth-order analogue pilot model.

Expected value of the general variable \( x \), defined as

\[
E[x] = \int_{-\infty}^{\infty} x p(x) \, dx
\]

Decibels, defined in eq. (4.2.1).

Root mean square.

Magnitude or amplitude ratio of the frequency response \( G(j\omega) \).

Phase angle of \( G(j\omega) \).

Sample mean of \( x(t) \), or Laplace transform, depending on context.

Sample mean square of \( x(t) \).

Dirac delta comb, defined in eq. (C,7).

Convolution, or when used as a superscript, the complex conjugate.

Inches.

Inverse Laplace transform.

Millivolts.
The human operator is an important and useful element of many control systems, but certain aspects of his performance are still relatively ill-defined. The advantages and disadvantages of retaining the human operator as an active element in a control loop, rather than as a simple monitor, have been under considerable discussion, an example of which is Ref. 32. In a situation where man can perform a useful role as an active system element, the problem arises that the inclusion of the human operator in the analytical design of the man-machine system requires a mathematical description of his behaviour.

The physical situation is shown in block diagram form in Fig. 1. The task of the operator is to minimize the system error signal, that being the difference between the forcing function of the system and the actual state of the system. This is usually presented to the operator in a visual form on a display which, if it indicates only the system error, is called a compensatory display. If the system forcing function has a random appearing nature and occurs only in a single axis, then the operator's task is known as single-axis, random input tracking. The response of the operator to the visual input is made by means of a manipulator of some form, and is further operated on by the dynamics of the controlled system (or vehicle) to produce the system output. The difficulty of the task can be altered by changing the controlled system dynamics, or by making the input more difficult to track. The performance of the operator is dependent on the restraints imposed on his response by the dynamics of the actuating-limb/ manipulator combination.

As a control element the human is recognized as being in general non-linear, adaptive, and time-varying; but under conditions which minimize the major nonlinearities, the human operator can be adequately represented by the combination of a best-fit linear model, or describing function, and what is known as the remnant. The remnant term contains all of the operator's response which is not described by the linear model. This form of model is shown in block diagram fashion in Fig. 2. It should be stressed that this type of model is purely functional and not structural. In other words, it attempts to describe the behaviour of the operator by relating his input and output, rather than to model the fine details of the psychological and physiological processes that occur between the visual input and the manipulator output. There has been some correlation established between the model and the physiology, but this is far from complete. An excellent classified bibliography of the field can be found in Ref. 43.

This form of representation has gained wide acceptance and the characteristics of the describing function have been well documented, [e.g. Refs. 26 and 27]. There have been several applications to the design of complex man-machine systems [Refs. 14 and 20], and to studies of aircraft handling qualities. Associated with the describing function are the "Parameter Adjustment Rules" which have been developed from large quantities of experimental data to predict the form of model required in a particular control situation.

There is developing interest in attempting to complete the description of the human operator's performance by the addition of a model of the remnant. The lack of a reliable model of the remnant has always been recognized, but the difficulty and expense of obtaining the data has prevented much
forcing function and the gain of the display can be adjusted such that the
error signal being presented to the operator is reasonably large, and he can
be assumed to be well motivated, the indifference threshold can also be
reduced to a unity gain. The predicted form of the model for these experi­
ments can then be simplified to the following:

\[ Y_p(j\omega) = K_p e^{-j\omega \tau} \left( \frac{T_K j\omega + 1}{T_K j\omega + 1} \right) \frac{\omega^2_N}{(T_N j\omega + 1)[(j\omega)^2 + 2\zeta_N \omega_N (j\omega) + \omega_N^2]} \]  

(2.1,1)

One of the principal reasons for the choice of rate control is the
above simplification, which allows the changes in the describing function
due to the manipulator to be easily isolated.

2.2 Remnant Sources

The two alternative source locations for the additive remnant signal
are shown in Fig.2. The remnant may be treated as a signal injected at the
operator's output as an "injected output remnant", \( n(t) \), or as a signal in­
jected at the operator's displayed input as an "injected observation remnant"
\( n(t) \). The representation of the remnant as a signal injected into the loop
by the operator is a mathematically convenient concept, but it is not meant
to imply that the sole source of the remnant is injected noise. There are
other possible sources and these are discussed below.

The remnant is accepted as coming from several sources of varying
importance. It is possible to infer the sources and their relative signifi­
cance from the characteristics of the operator's response. The four major
sources are:

1) Sampling Behaviour: psycho-physiological data,[Ref. 3,21 ],
suggests that the model of the human operator should include
some form of periodic sampling process.

2) Nonlinear Behaviour: The indifference and physiological
thresholds,limits on operator output amplitude and rate,
and bimodal pulsing responses are included in this source.
Also included are any nonlinear input/output characteristics
of the sensing, processing or actuation elements.

3) Noise Addition: This may occur at any or all of the sensing,
computational or actuation elements of the human operator.
This may be considered as true random noise, produced by
random fluctuations in the physiological processes.

4) Time-varying Behaviour: Since the parameters of the describing
function are defined as averages over the measurement period,
any short term variations in their values will contribute to
this source.

A review of the very limited data on the remnant published prior to,
and shortly after the initiation of this study [Refs. 12, 19, 26, 27], showed that the form of the describing function was not strongly amplitude dependent, and that with the proper choice of experimental conditions, the effects of thresholds and output amplitude and rate limiting could be minimized. In addition, the power spectrum of the operator's output did not show any clear evidence of spectral peaks associated with sampling behaviour, periodicities or major nonlinearities.

Further, the power spectrum of the remnant was shown to be a relatively smooth function of frequency and a major source of the remnant was considered to be time-variability of the parameters of the describing function, [Refs. 26, 40]. Reference 26 also indicated that the power spectrum of the injected observation remnant was dependent on both the forcing function bandwidth and the gain and order of the dynamics of the controlled element. The variability between subjects however, was found to be of the same order as that due to different controlled elements. The majority of the remnant data was obtained using system forcing functions which were made up of sums of sine waves. There was no data available on the effect of the manipulator on the remnant, nor were there any results available for tracking tasks using forcing functions with continuous spectra, except for those of Refs. 12 and 19, which were either of very low bandwidth or of uncertain quality due to the measurement techniques employed.

Only one attempt was made [Ref. 13] to extract the time record of the remnant and to measure the amplitude probability distribution. However, this was done for a very limited number of runs and for position-control* vehicle dynamics with a free-moving type of manipulator. To extract the remnant time record the operator was modelled using a set of parallel orthonormal filters together with a time delay, with the gains adjusted until the difference between the outputs of the model and of the operator was a minimum in a least squares sense. The time record of the injected output remnant was obtained as the difference between the outputs of the model and the operator. The results showed that the normality of the remnant signal was dependent on the bandwidth of the forcing function. The limited number of runs used did not allow much confidence in this result. Reference 26 reported measurements of the amplitude distribution of the error signal and of the operator's output. There were indications that for forcing functions of very wide bandwidth, or for high order controlled-vehicle dynamics, the operator tended to develop bi-modal pulsing, or limit-to-limit positioning of the manipulator. It was concluded that the occurrence of the behaviour was associated with non-Gaussian remnant time records. This nonlinear behaviour was interpreted as an attempt to generate the leads necessary to maintain system stability by overcoming the phase lags introduced by the transport delay and by the dynamics of the limb/manipulator combination. Hence, the type of manipulator employed could be expected to have some influence on this form of behaviour and to produce differences in the power spectrum and amplitude distribution of the remnant.

Much later during the course of this program more data on remnant power spectra became available [Refs. 20, 22, 25, 30] and they will be discussed in the appropriate sections.

* Constant input produces a constant output.
2.3 Neuromuscular System Dynamics

In order to be able to predict the effect of the restraints on the operator's performance resulting from the dynamics of the limb/manipulator combination, one needs a model of the neuromuscular system. With such a model it is then possible to determine the influence of various types of manipulator nonlinearities, such as hysteresis and backlash, on the describing function.

At the beginning of this study there was no information available on the effect of the manipulator on the describing function, but some performance studies had been carried out [Refs. 2, 10, 16, 31]. These studies showed that pilots preferred to use a manipulator which approached the rigid pressure type. But the very reliable describing function data of Ref.26 were of sufficiently wide bandwidth to show that several additional terms were required to explain adequately both the very low frequency and high frequency results. These additional terms were attributed to the neuromuscular system and consisted in the very low frequency lead/lag term and the third order high frequency term. These terms are shown grouped into the "neuromuscular system dynamics" in the model in Appendix A. An input-adaptive characteristic of the neuromuscular system terms was noted in that both the very low frequency phase lags and the high frequency phase lags were dependent on the forcing function bandwidth. It was concluded that some of the elements of the neuromuscular system model were under the control of the operator. Some attempts have been made to measure the neuromuscular system describing function directly from the muscle nerve signals [Ref.14]; however the very poor signal-to-noise ratio made these results of doubtful use.

A very simple functional block diagram of the neuromuscular system appears in Fig.3. The neuromuscular command feed-forward block is necessary to explain the ability of the operator to produce outputs with minimal time delays, for instance, during precognitive behaviour, as occurs for the synchronous tracking of a single sine wave forcing function. The changes to be expected in the organization of the neuromuscular system, depending on the type of manipulator being employed, are also shown. For the free-moving manipulator, the position feedback loop is of primary interest while for the pressure manipulator the force feedback loop dominates. By minimizing the inertia of the free-moving manipulator it is possible to cause the operator to develop the two extreme forms of neuromuscular system organization, one based on almost total reliance on position feedback, the other based on almost total reliance on force feedback. These two types of manipulator then should produce the largest changes in both the describing function and the remnant.

A review of the available physiological data on the neuromuscular system [Ref.42] showed that in fact, the system does have a closed-loop form and consists of the actuating muscles together with a variety of position and force feedback loops. Each limb is equipped with pairs of opposing muscles and movement of the limb is achieved by the co-ordinated increase in tension in one set, the "agonists", and the decrease in tension in the opposing set, the "antagonists". A muscle is made up of a very large number of individual muscle fibres which contract in response to nerve impulses from the alpha motor neuron nerve cells. Each alpha motor neuron innervates a group of muscle fibres and the tension level developed in the muscle is determined by the number of alpha motor neurons that are being stimulated to "fire" by the various centres in the brain and their actual firing rates.

Figure 4 shows a more detailed functional block diagram based on
physiological examination of the neuromuscular system. A typical agonist-antagonist muscle pair is depicted with the network of position and force feedback loops shown in a simplified form. The primary input to the neuromuscular system is the alpha motor neuron command signal, $\alpha$, which is transmitted through the spinal cord from the brain where it is produced as the result of the computation in the higher centres (the cortex). The alpha motor neuron also accepts inputs from the position and force receptors in the muscle and inputs from lower brain centres, such as the cerebellum, associated with co-ordination. The inner force and position feedback loops function so as to ensure that the muscle responds correctly to $\alpha$ and also take part in reflex activity. The outer feedback loops to the processing and co-ordinating centres of the brain supply additional feedback information which will allow the modification of the command signal.

The principal receptors for the force developed in the muscle and the limb position are; 1) the Golgi Tendon Organs, which supply force feedback information and which are situated in series with the muscle in the tendon that connects the muscle to the limb, and 2) the Muscle Spindle Receptors, which are basically stretch receptors and respond to muscle length. The spindle receptor acts in parallel with the muscle and supplies position feedback. Additional information on the position of the limb is available from specialized joint position receptors. The muscle spindle receptor is a very highly developed sense organ and is supported in the muscle by its own contractile fibres, known as the "intrafusal fibres", which are innervated by the gamma motor neuron which in turn receives its stimulation from the brain in the form of the gamma motor neuron command signal, $\gamma$. The primary output from the spindle receptor is the signal from the "I" nerve ending. As shown in Fig. 4 this output is a function of the length of the muscle due to the actual position of the limb and also the stretch in the receptor due to the tension in the supporting intrafusal fibres, which is produced as a result of the summation of the output of the gamma motor neuron and the average level of tension in the muscle brought about by the output of the alpha motor neuron.

The forces generated in the intrasual fibres by the output from the gamma motor neuron cause changes in the feedback signals supplied to the alpha motor neuron and hence in the tension level in that particular muscle. Thus it can be seen that the force developed at the limb is a function of two separate inputs from the brain and several feedback signals.

Under the normal conditions of manual tracking the actuating limb makes relatively small excursions in force or position from the mean output. A model has been suggested for this case [Ref.24] in which the $\alpha$ signals serve to maintain the average tension level in the agonist-antagonist muscle pair while the $\gamma$ signal causes the required forces to be developed through the complex feedback pathways. It should be pointed out that neither the position nor the force feedback loops will be totally inactive during operation of the manipulators to be used in this study. The inherent inertia of the limb will result in the development of some force feedback with the free-moving manipulator, while the elasticity of the muscle will cause changes in its length under tension when using the pressure manipulator, causing in turn a certain amount of output from the spindle receptor.

Remnant sources exist in the neuromuscular system. The muscle and feedback elements do not have linear characteristics and noise may be generated anywhere in the system. In addition, the parameters of the neuromuscular
system may well be time-varying. A true noise source known as "tremor" exists, but it occurs at frequencies which are higher than those of interest in this study.

Shortly after the initiation of this study information on the effects of the manipulator on the describing function became available [Refs. 23, 28], and recently Ref. 24 was published, containing a very detailed review of the physiological data and its correlation with the neuromuscular system model. The pertinent results will be discussed in the appropriate sections, but an interesting point of immediate relevance is that some of the parameters of the neuromuscular system were found to be functions of the average level of muscle tension, and since it is unreasonable to assume that the operator maintains the same tension level throughout the period of the tracking task, this is an obvious source of remnant.

2.4 Identification of the Describing Function:

Figure 2 shows the now familiar block diagram of the single-axis, closed-loop, compensatory display tracking task in which the human operator is an active element. The total output of the operator, \( c(t) \), may be considered as the sum of two parts: 1) that part, \( c_1(t) \), due to a linear operation on the system forcing function, and 2) that part, \( n(t) \), which is not described by the linear operation on the input, known as the closed-loop remnant. Thus we have:

\[
c(t) = c_1(t) + n(t)
\]  

(2.4,1)

where \( c_1(t) \) is obtained by the convolution of the closed-loop impulsive response \( h_1(t) \) with the system forcing function \( i(t) \), such that

\[
c_1(t) = \int_{-\infty}^{\infty} h_1(\tau) i(t - \tau) \, d\tau
\]  

(2.4,2)

where

\[
h_1(\tau) = 0 \quad \text{for} \quad \tau < 0
\]

and where \( h_1(\tau) \) is defined in terms of the inverse Fourier transform of the closed-loop relationship in the frequency domain between \( c_1(t) \) and \( i(t) \).

\[
h_1(\tau) = \frac{1}{2\pi} \int_{-\infty}^{\infty} H_1(j\omega) e^{j\omega\tau} \, d\omega
\]  

(2.4,3)

and

\[
H_1(j\omega) = \frac{Y_p(j\omega)}{1 + Y_p Y_c(j\omega)}
\]  

(2.4,4)

The identification of the describing function employs the technique of determining the linear model, \( h_1(t) \), which minimizes the mean square of the closed-loop remnant, \( n(t)^2 \). Thus the describing function is the best-fit linear model, in the mean square sense, which minimizes that part of the output of the operator which is not linearly correlated with the system forcing function. The formal minimization procedure has been adequately described in the literature and need not be repeated here. The procedure
may be performed in either the time domain [Refs. 27, 39], or the frequency domain [Ref. 33], with equivalent results. In the time domain, a Wiener-Hopf equation is developed whose solution, without regard for the physical realizability of the system derived [Refs. 29, 39], after Fourier transforming, yields the expression

\[ H_1(j\omega) = \frac{\Phi_{ic}(\omega)}{\Phi_{ii}(\omega)} \]  

(2.4,5)

and in addition the remnant and the system forcing function are found to be linearly uncorrelated, i.e.

\[ \Phi_{in}(\omega) = 0 \]  

(2.4,6)

From linear system analysis and equation (2.4,6) we can obtain the following relationship

\[ H_2(j\omega) = \frac{1}{1 + Y_p Y_c(j\omega)} = \frac{\Phi_{ie}(\omega)}{\Phi_{ii}(\omega)} \]  

(2.4,7)

and the combination of equations (2.4,5) and (2.4,7) yields the expression for the describing function, \( Y_p(j\omega) \), as the ratio of two cross spectra:

\[ Y_p(j\omega) = \frac{\Phi_{ic}(\omega)}{\Phi_{ie}(\omega)} \]  

(2.4,8)

It is recognized that this particular expression need not necessarily yield a physically realizable \( Y_p(j\omega) \) even though the underlying process which produced the two cross spectra exists physically. The definition of physical realizability in the frequency domain is that the describing function has no poles in the right half-plane, and in the time domain that the impulsive response is zero for negative time. The more complicated expression, the equivalent of eq. (2.4,8), which is obtained as the solution of the Wiener-Hopf equation with the constraint of physical realizability, is discussed with reference to the identification of the human operator describing function in Ref. 39. A test for physical realizability of the describing function can only be performed if the describing function is known as a continuous function of frequency over an infinite bandwidth. However, the describing functions obtained in this study are available only at a discrete set of frequencies for a limited bandwidth, and experience has shown that a linear, physically realizable describing function model that is a smooth function of frequency, can be made to pass through the points of the measured describing function, within the limits of the variability of the data.

The considerable body of experimental describing functions in the literature has not revealed any evidence that the minimization without the constraint of physical realizability produces a result that can not be fitted adequately with a smooth physically realizable function of frequency. Although this is not a conclusive result, for the purposes of this study the identification of \( Y_p(j\omega) \) using equation (2.4,8) will be considered adequate.
2.5 Correlation Coefficient:

The correlation coefficient, $\rho(\omega)$, is a measure of the linearity of the closed-loop system. Since the controlled vehicle is a linear system, with transfer function $Y(j\omega)$, then $\rho(\omega)$ is a measure of the linearity of the human operator alone. The correlation coefficient is defined in terms of its square as:

$$\rho^2(\omega) = \frac{|\Phi_{ic}|^2}{\Phi_{ii}\Phi_{cc}}$$  \hspace{1cm} (2.5,1)

This form of definition of $\rho(\omega)$ is used since it is directly related to the correlation coefficient used in statistical analysis. If the human operator behaves in a nearly linear fashion then $\rho(\omega)$ will have values close to unity and the remnant will be small, while low values of $\rho(\omega)$ indicate more non-linear performance and the corresponding remnant will be large. However, low values of $\rho(\omega)$ in general yield no information on the source of the remnant. Under the particular conditions of a forcing function made up of a sum of sine waves, low values of $\rho(\omega)$ at the frequencies of the sinusoidal components indicate time variations in the parameters of the describing function [Ref.26]. For the continuous spectrum forcing functions used in this study, time variation and noise are indistinguishable.

From linear system analysis, since $Y(j\omega)$ is a linear element, we can obtain the following expressions, (the dependence on frequency being understood):

$$\Phi_{io} = Y_c \cdot \Phi_{ic}$$ \hspace{1cm} (2.5,2)

and

$$\Phi_{oo} = |Y_c|^2 \Phi_{cc}$$ \hspace{1cm} (2.5,3)

The substitution of equations (2.5,2) and (2.5,3) into equation (2.5,1) yields an alternative definition for $\rho$:

$$\rho^2 = \frac{|\Phi_{io}|^2}{\Phi_{ii}\Phi_{oo}}$$ \hspace{1cm} (2.5,4)

This second definition allows improved estimates of $\rho$ at those frequencies for which $c(t)$ has very small signal levels while $o(t)$ has large signal levels. This is particularly useful at low frequencies, where the transfer function of the rate-control vehicle dynamics [$Y(j\omega) = K_0/j\omega$] has a very large amplitude ratio and the signal levels of the operator's output, $c(t)$, are very small, while those of the system output, $o(t)$, are of the same order as $i(t)$. The combination of the two expressions, eqs. (2.5,1) and (2.5,4), will yield the best estimate of $\rho$ over the whole frequency range. Good identification of $\rho$ is important since it is used to determine the power spectrum of the closed-loop remnant, $\Phi_{nn}$, as is shown below.

2.6 Power Spectral Density of the Remnant:

We can obtain the power spectrum of the closed-loop remnant at the

* It is clear from the definition that $\rho(\omega)$ may be thought of as the non-dimensional narrow-band covariance of $i(t)$ and $c(t)$.  

9
output of the operator, $\Phi_{nn}$, from the frequency domain equivalent of eq. (2.4,1). The power spectrum of the operator's output, $\Phi_{cc}$, can be treated as the sum of two linearly uncorrelated power spectra, one, $\Phi_{ci}c_i$, a linear operation of the closed-loop on the forcing function, and the other the closed-loop remnant. We have:

$$\Phi_{cc} = \Phi_{ci}c_i + \Phi_{nn} \tag{2.6,1}$$

where

$$\Phi_{ci}c_i = \left| \frac{Y_p}{1 + Y_pY_c} \right|^2 \Phi_{ii} \tag{2.6,2}$$

It is often more convenient to consider the slightly different expression for $\Phi_{nn}$ which is derived below:

From eq. (2.4,5)\[ \Phi_{ic} = H^\dagger \Phi_{ii} \tag{2.6,3} \]

Therefore, from eq. (2.5,1)

$$\rho^2 = \frac{|H^\dagger|^2 \Phi_{ii}}{\Phi_{cc}} \tag{2.6,4}$$

Hence

$$|H^\dagger|^2 \Phi_{ii} = \Phi_{cc} \cdot \rho^2 \tag{2.6,5}$$

Now, since

$$\Phi_{ci}c_i = |H^\dagger|^2 \Phi_{ii} \tag{2.6,6}$$

We can write

$$\Phi_{ci}c_i = \rho^2 \Phi_{cc} \tag{2.6,7}$$

Substitution of equation (2.6,7) into equation (2.6,1) yields:

$$\Phi_{nn} = (1 - \rho^2) \Phi_{cc} \tag{2.6,8}$$

As discussed earlier, the closed-loop remnant referred to the operator's output, $\Phi_{nn}$, can be represented as being the result of the injection into the loop of an additional signal. Two locations for the injection point of this signal may be considered, 1) at the output from the operator, the "injected output remnant", $n_c(t)$, or 2) at the input to the operator, the "injected observation remnant", $n_e(t)$. The corresponding power spectra of these signals, $\Phi_{nn}n_c$ and $\Phi_{nn}n_e$, are obtained directly from the closed-loop remnant, using linear system theory, as:
\[ \phi_{n_c n_c} = |1 + \frac{Y}{p_c} Y_p^2 \phi_{nn} \] (2.6,9)

\[ \phi_{n_e e} = \left| \frac{1 + \frac{Y}{p_c}}{Y_p} \right|^2 \phi_{nn} \] (2.6,10)

i.e.,

\[ \phi_{n_c n_c} = \left| \frac{Y}{p} \right|^2 \phi_{n_e e} \] (2.6,11)

2.7 Performance Measures:

The overall efficiency of the closed-loop system as a tracking device can be determined from various performance measures. They serve as convenient means of gross comparison between systems. We shall consider four performance measures, 1) score, 2) relative remnant, 3) cross-over frequency, and 4) phase margin.

2.7.1 Score:

Since the tracking task requires the operator to minimize the system error signal, \( e(t) \), an index of the operator's efficiency in this task is the normalized mean square error or "score". This quantity is defined as:

\[ \text{SCORE} = \left[ \frac{e^2}{i^2} \right] \times 100 \] (2.7.1.1)

If the operator were to make no response at all to the input, and maintain his output as identically zero, then the score would be 100. Conversely, if he were to act in an optimum manner then the score would be almost zero. A totally unsuitable response could well yield scores higher than 100.

2.7.2 Relative Remnant:

The relative remnant, \( \rho_a^2 \), can be considered as a gross, overall index of the linearity of the operator. As such it is useful as one means of comparing control techniques and performance. Values of \( \rho_a^2 \) near unity indicate high levels of linearity. The relative remnant is defined in terms of the total amount of power in the output from the operator that is correlated with the system forcing function, relative to the total power in the operator's output:

\[ \rho_a^2 = 1 - \frac{\phi_{cc}^2}{\phi_{nn}^2} \] (2.7.2.1)

This can also be written as:

\[ \rho_a^2 = \frac{1}{2\pi} \int_{-\infty}^{\infty} (\phi_{cc} - \phi_{nn}) \, dw \] (2.7.2.2)
2.7.3 Cross-over Frequency and Phase Margin:

The cross-over frequency, \( \omega_c \), is the frequency at which the amplitude ratio of the open-loop describing function, \( |Y_{pY_c}| \), goes through unity, and is a measure of the bandwidth of the closed-loop system. For "good" system performance the cross-over frequency should be greater than the bandwidth of the forcing function.

The phase margin, \( \phi_m \), is defined as the difference between the phase of the open-loop describing function, \( \angle Y_pY_c \), and \(-180^\circ\) at the cross-over frequency. The phase margin is a measure of the stability of the closed-loop system and also the level of gain being employed. Low values of \( \phi_m \) indicate that the open-loop describing function, \( Y_pY_c(j\omega) \), is operating with a high level of gain and the closed-loop system is near instability. A "good" linear system will have an open-loop describing function that has the highest level of gain possible without instability. The addition of a remnant signal into the closed loop of a system operating at a high gain level could cause instability and the highest level of gain that could be achieved without instability would then be reduced.

The behaviour of both the cross-over frequency and the phase margin of the open-loop describing function of the human operator, as a function of the control situation, is predicted by the parameter adjustment rules of Appendix A.

2.8 Remnant Time Record:

The extraction of the time record of the remnant is of major importance to this study. The most convenient form of the remnant to obtain is that referred to the operator's output, \( n_c(t) \). This remnant signal is isolated from the operator's total output by subtracting from the total output that part due to the linear operation of the describing function on the error signal.

Referring to Fig.2, it can be seen that the operator's output can be treated as the sum of two parts, 1) the result of the linear operation of the describing function, \( Y(p(j\omega)) \), on the error signal, which yields \( c_e(t) \), and 2) the remnant signal, \( n_c(t) \). Thus we have:

\[
c(t) = c_e(t) + n_c(t) \tag{2.8,1}
\]

i.e.,

\[
n_c(t) = c(t) - c_e(t) \tag{2.8,2}
\]

where the time record of the linear output, \( c(t) \), is obtained by convoluting the error signal, \( e(t) \), with the impulsive response of the human operator, \( y_p(\tau) \). This impulsive response is calculated by performing an inverse Fourier transform on the describing function, \( Y_p(j\omega) \), [Ref. 8, 15]. That is:

\[
c_e(t) = \int_{-\infty}^{\infty} y_p(\tau) e(t - \tau) \, d\tau \tag{2.8,3}
\]
where
\[ Y_p(\tau) = \frac{1}{2\pi} \int_{-\infty}^{\infty} Y_p(\omega) e^{j\omega \tau} d\omega \]  
(2.8,4)

and \( Y_p(\tau) = 0 \) for \( \tau < 0 \).

The derivation of this particular expression and the problems that were encountered when attempting to apply it in practice are discussed in Appendix B.

2.9 Amplitude Probability Distribution:

An estimate of the amplitude probability distribution for an ergodic process may be obtained from a finite time record of the process, by determining the relative length of time that the amplitude of the signal remains within specified amplitude intervals. For a variable \( x(t) \), the value of the amplitude probability density for amplitude \( x_1 \), \( p(x_1) \), is defined as the probability that \( x(t) \) will lie within the amplitude interval \((x_1, x_1 + \Delta x)\), divided by the width of the interval, \( \Delta x \):

\[ p(x_1) = \frac{1}{\Delta x} \Pr. [x_1 < x \leq (x_1 + \Delta x)] \]  
(2.9,1)

To estimate the amplitude probability density distributions of finite time records it is necessary to obtain samples from the time records that can be considered to be independent.

The time interval between samples to ensure near independence can be estimated from the auto-correlation function of the time record of interest. Independence of the samples will be obtained by choosing a time interval equal to, or greater than, the time delay at which the auto-correlation falls to zero [Ref. 5]. However, a good estimate of the minimum interval that may be used can be derived from the fact that the time records used in this study have power spectra of finite bandwidth. It has been shown [Ref. 4,5] that for a time record \( x(t) \) with a power spectrum \( \Phi_{xx}(\omega) \), where \( \Phi_{xx}(\omega) = 0 \) for \( |\omega| > B \) rad/sec, \( x(t) \) need only be sampled at intervals of \( \pi/B \) seconds to completely define the time record in the absence of noise. Any sampling period greater than this may be used, the larger the better, provided that a sufficiently large number of samples are obtained from the finite time record to allow reasonable confidence in the distribution.

The amplitude interval limits may be chosen such that the intervals are of equal or unequal width. A convenient procedure is to derive the interval limits from the values of the mean and variance calculated from the sampled points, such that the intervals span equal areas under the normal probability density curve. Thus, if an infinite number of samples were available from a true normal distribution, there would be an equal number of samples in each interval. A Chi-Squared test of fit to the normal distribution may then be performed using the following expression [Ref. 6,13]:

\[ \chi^2 = \sum_{i=1}^{K} \frac{(o_i - e_i)^2}{e_i} \]  
(2.9,2)
where \( k \) is the number of amplitude intervals, and \( o_i \) and \( e_i \) are the observed and the expected number of samples in the \( i \)th interval respectively. There are \((k-3)\) degrees of freedom for this test and the confidence level is \((1-\alpha)\). That is, the \( \chi^2 \) statistic calculated from samples obtained from a true normal distribution would be expected to exceed the value of \( \chi^2_{(k-3),\alpha} \) about \( (\alpha)\% \) of the time.

### III EXPERIMENTAL CONSIDERATIONS

#### 3.1 Physical Layout and Equipment:

The facility used in this study consisted of a modified CF-100 fixed-base flight simulator cockpit coupled with an EAI TR-48 analogue computer. A general view of the facility can be seen in Fig. 5. The analogue computer performed the following functions:

1. The mechanization of the control loop elements external to the operator.
2. The calculation of the mean-square values of the error and forcing function from which the score could be obtained.
3. The adjustment of the mean and mean-square values of the forcing function.
4. The generation of suitably conditioned signals for the data recording equipment. The circuit diagram of the system appears in Fig. 6.

The system forcing functions were pre-recorded on an analogue tape recorder, a Sony TC-500A. This was an amplitude modulation machine but a suitable modulator-demodulator combination allowed the equipment to operate at the low frequencies of the forcing functions. The four time records of interest: forcing function, error, operator output and system output, were recorded in digital form on a Kennedy D8370 incremental tape deck after passing through an EECO ZA37050 analogue-to-digital convertor. The time records were sampled simultaneously and recorded sequentially. The sampling rate of the A-to-D convertor was controlled by an external clock pulse. The sensitivity of this equipment, based on an 11 bit word and a maximum input voltage of 10 volts, was 4.89 mv/bit.

The simulator cockpit supplied an isolated environment for the subjects during the experimental runs, with controlled ambient light level and air-conditioning. Distracting sounds from the surrounding laboratory were masked by simulated jet engine noise. A standard service lap and shoulder harness was worn to provide restraint. The display was a 4.5 inch diameter CRT mounted above the instrument panel as shown in Fig. 7, and the conventional control stick was replaced by the manipulators used in the study. Communication with the subject in the cockpit was possible through the intercom system, and his performance could be monitored on an oscilloscope during the run. A sketch of the interior of the cockpit, giving dimensional details, is given in Fig. 8.

The single-axis, compensatory display presented the error signal to the subject as the vertical displacement of a moving horizontal line from fixed reference marks in the centre of the display. The display characteristic was linear with a gain of 1.0 inch/volt, and the maximum displacement was \( \pm 2.0 \) inches. The display-control relationship was that of an artificial horizon, namely, a pull back on the manipulator caused a downward movement of
the line which represented the horizon. This type of relationship was chosen originally because of the number of subjects who had previous flying experience. It was considered that it would be easier to train the naive subjects to track in this manner than to retrain the others. Subsequently all but one of the experienced subjects were unable to participate in the program.

The two manipulators, free-moving and pressure, used in this project, were centre sticks. The zero output position of the free-moving manipulator coincided with the position of the rigid pressure manipulator, such that approximately the same muscle groups were used with both. The hand grips of both manipulators were of the same diameter, 1.5 inches, and the vertical position of the hand on the grip was determined by a horizontal plate. The free-moving manipulator, which was extremely light, (6.0 ozs., C.G. 12 ins. from pivot axis), and frictionless with no damping or spring-restraint, was mounted at floor level and pivoted to move in the fore and aft direction only. The total travel of the free-moving manipulator was about 12 inches. Infinite resolution output was obtained by use of a conductive plastic potentiometer.

The pressure manipulator consisted of a double cantilever, strain-gauged force transducer, with four active gauges in the bridge, wired such that an output was produced only for fore-and-aft direct loading. The application of a torque about any horizontal axis produced no output, while the vertical axis was isolated from torques by constructing the outer shell of the hand grip such that it was free to rotate while still transmitting the loads to the force transducer. The input/output characteristics of both manipulators, as shown in Fig. 9, were linear within the ranges of interest with no dead-band or hysteresis. Figure 9 also gives a view of the two manipulators while Fig.10 is a sketch of the force transducer of the pressure manipulator. The circuitry associated with the two manipulators appears in Fig.6.

The gain, \( K_c \), in the transfer function of the controlled vehicle \( Y_c(j\omega) = \frac{K_c}{j\omega} \), was chosen such that all subjects felt comfortable with either manipulator, and excessive position or force outputs were not required in the course of tracking the random inputs. Following the convention of Ref.26, the gain is presented in terms of the steady-state values achieved by the output on the display in response to a step input to the manipulator. For rate-control the gain is given in terms of inches per second on the display per unit step input (of deflection or force) to the manipulator. The gains chosen were:

- Free-moving \( K_c = 0.632 \text{ ins/sec (display) / degree (stick)} \)
- Pressure \( K_c = 1.14 \text{ ins/sec (display) / lb. (stick)} \)

The sign convention used in this study can best be visualized by considering the system to represent an aircraft in pitch. In terms of positive values we have: 1) a positive input to the system is a command to pitch up, 2) this will produce initially a positive error signal indicating that the aircraft is nose down, i.e., the horizon line will be above the reference marks, 3) the operator makes a positive response by pulling back on the manipulator, and 4) this results in a positive system response, i.e. a pitch up, which causes a downward movement of the horizon line. Of course, with the compensatory display, the subject is tracking the error signal and not the forcing function directly, and so the display-control relationship is
apparently incompatible, in that a backward movement of the manipulator results in a downward movement of the line rather than an upward movement. This often caused control reversals in the early stages of training of naive subjects, or occasionally even of highly trained subjects under conditions of stress.

3.2 **Forcing Functions**:

3.2.1 **UTIAS**:

The type of forcing-function power spectrum chosen for this study was similar in nominal shape and bandwidth to the augmented rectangular spectrum of Ref. 26 but with a continuous spectrum instead of a line spectrum. The basic shape was that of a main low-frequency portion which contained over 90% of the total power of the signal, and a low amplitude shelf extending out to higher frequencies. Since the describing function could only be identified over that frequency range for which the forcing function existed, the purpose of the high-frequency shelf was to extend the bandwidth over which the describing function could be identified by supplying additional power at higher frequencies. This additional power was at a sufficiently low level that the low frequency performance of the operator was assumed to be unaffected by its presence.

The "UTIAS" forcing functions were obtained from the filtered output of a Gaussian noise generator. Three grades of task difficulty were used, being determined by the cut-off frequency, \( \omega_i \), of the main low-frequency section, with \( \omega_i = 1.5, 2.5 \) and 4.0 rad/sec. The forcing functions are identified in this report by these values of cut-off frequency. The general frequency response of the filter is given below and the values of the parameters required to obtain the different spectra appear in Table I.

\[
G(j\omega) = \left[ \frac{\omega_p^2}{(j\omega)^2 + 2\zeta_p \omega_p (j\omega) + \omega_p^2} \right]^2 \cdot \frac{1}{1 + \frac{1}{\frac{\omega_{s}^{2}}{(j\omega)^2 + 2\zeta_s \omega_s (j\omega) + \omega_s^2}}} \tag{3.2.1,1}
\]

where \( T = 1/\omega_{FO} \).

The cut-off slope of the main low-frequency section was 100 db/decade and that of the high-frequency shelf was 80 db/decade, with the shelf being attenuated by 20 db. The comparisons between the power transfer functions of the filters and the measured power spectra are shown in Fig. 11, while the measured forcing function spectra are compared in Fig. 12.

To check the assumption that the high-frequency shelf did not affect the low-frequency performance, an additional forcing function was made up with \( \omega_i = 2.5 \), but with no high-frequency shelf. This forcing function is identified as the \( \omega_i = 2.5^* \) forcing function and appears in both Fig's. 11 and 12.

The concept that the cut-off frequency of the forcing function directly defines the bandwidth of the power spectrum is only valid for rectangular spectra. For power spectra of other shapes it is possible to define an
equivalent rectangular bandwidth $\omega_{le}$, [Ref.26] as:

$$\omega_{le} = \left( \int_{-\infty}^{\infty} \Phi_{ii} (\omega) \, d\omega \right)^2 \int_{-\infty}^{\infty} [\Phi_{ii} (\omega)]^2 \, d\omega \tag{3.2.1,2}$$

The values of $\omega_{le}$ for each forcing-function spectrum are given in Table I.

3.2.2 STI:

During the experimental program it was found necessary to use a set of forcing functions that were identical to those of Ref.26. These "STI" forcing functions consisted of sums of ten sine waves of fixed amplitude but random phases, at the same frequencies as those used in the reference, with the high-frequency shelf attenuated by 20 db. The line spectra associated with these forcing functions are given in Fig.12, which shows the frequencies of the sine wave components and their relative magnitudes. However, the absolute values of the points are not necessarily correct.

All the forcing functions were pre-recorded in segments slightly longer than three minutes, and the analogue computer potentiometer settings were determined such that when the runs were actually used in the experiment, they would all have the same nominal zero mean and 0.5 inch r.m.s. levels, i.e., $\mu_i = 0$ ins. and $\sigma_i = 0.5$ ins. Amplitude probability distributions were obtained for all forcing functions and the goodness of fit to the normal distribution was tested using the Chi-Squared test. The forcing functions were found to be normal at the 95% confidence level without exception. A sufficient number of runs were pre-recorded for each forcing-function spectrum such that the subjects were not presented with the same runs too often during the prolonged training program. Six runs with each forcing function were reserved for the data collection such that the subjects would be tracking completely fresh inputs while the time records were being recorded.

3.3 Subjects and Training Program:

The experimental program was conducted with eight optically normal, right-handed subjects, six males and two females. All were volunteers from amongst the students and technical staff of the Institute and only one had previous flying experience. Table II gives the pertinent subject data, [Ref.35], including biographical details and measurements of visual acuity.

Apart from the initial basic instructions which were given to each subject concerning the operation of the system, the display-control relationship and the rate-control vehicle dynamics, each subject was allowed to develop his own control technique under the single restriction that the whole arm be used to operate the manipulator rather than just the wrist. The capability of monitoring the subjects' performance during a run proved useful in that instances of poor performance due to lack of concentration or fatigue could be detected and corrective action taken. In particular, it was observed during the training program that certain subjects tended to develop an abnormal control technique, involving high-frequency large amplitude outputs. An attempt was made to coach these subjects towards a more reasonable type of response but with only limited success. When it became apparent that these subjects actively preferred to track in this fashion, the coaching was
The training program was quite prolonged and the subjects were brought
to as high, and as consistent a level of performance as possible. At the
beginning of the program there were conflicting opinions amongst other researchers
as to the amount of tracking experience required to ensure consistent performance,
with the estimates ranging from 10 to 100 hours. The subjects were eventually
trained until they had accumulated in the region of 20 hours of experience.
About 400 runs per subject were required and, because of the limited availability
of both the subjects and the equipment, this took a considerable period of time.
Statistical testing of the scores at this stage showed that the subjects had
reached stable levels of performance.

The training program was split into four major phases: 1) Initial
training with the free-moving manipulator; during this period both subjects and
experimenters gained experience with the equipment, and this is described in
Ref.35, 2) Training with the free-moving manipulator and the forcing functions
to be used in the study, 3) Training with the pressure manipulator and all forcing
functions and 4) training with all the combinations of manipulator and forcing
function presented in a random order. The transfer of training from the free­
moving to the pressure manipulator was found to be extremely high. The hours
of experience accumulated by each subject in each phase of the training program
are given in Table II together with the duration of each phase in weeks.

The subjects supplied their own motivation in that they were all aware
of the objectives of the program and all were interested in its success. However,
an attempt was made to standardize the motivation by paying the subjects a small
sum ($1.50) for each data run recorded. In addition, the performance scores
were posted in the laboratory to foster a competitive spirit amongst the subjects.

3.4 Experimental Design:

The experimental variables to be investigated were manipulator type
and forcing function bandwidth; there were two manipulators, four forcing-function
spectra, and eight subjects. This yielded a three-way (2 x 4 x 8) factorial
design. Four data runs were recorded for each subject at each manipulator/
forcing-function combination (referred to as an experimental condition), and
this resulted in 32 runs per condition. Each subject tracked the same four in­
puts, each input being a different time record, for each forcing-function
spectrum with both manipulators, and thus was presented with the identical forcing­
function time record only twice. This particular design allowed between-manipu­
lator, between-forcing-function, and between-subject comparisons, and a form
of within-subject comparison. A true within-subject comparison would employ an
identical time record for each run, but this would raise the possibility of an
additional learning effect.

To obtain a statistically valid experimental design, i.e., one with­
out order effects etc., the experimental conditions were presented to each
subject in a random order, with a different order for each subject. The
randomization was implemented by assigning a number to each condition and using
a table of random numbers to determine the order of presentation. An additional
order effect was possible if the four runs of each forcing function were to be
presented to each subject in the same sequence. This effect was avoided by
arbitrarily separating the subjects into two groups and reversing the order of
the presentation of the forcing-function runs between the groups.
3.5 Data Analysis:

3.5.1 Choice of Parameters for Spectral Density Measurements:

Fundamental to the identification of the describing function and all the subsequent analysis is the measurement of the required power and cross-spectral densities of the time records of interest in the closed-loop system. In practice, infinite time records are unobtainable, and so the spectral densities can only be estimated. The procedure for obtaining the spectral estimates and the errors introduced by the measurement conditions of this study are discussed in Appendix C.

The restrictions on the choice of the critical parameters: sampling rate, \( \frac{1}{\Delta t} \), and the maximum delay in the correlation function, \( \tau_m \), are discussed in the appendix. The values chosen are given below:

1) Sampling Rate: Since the cut-off frequency of the forcing-function high-frequency shelf was at 15 rad/sec, it was felt that no signals in the closed-loop system would have appreciable power at frequencies much higher than 18 rad/sec. This was verified by the fact that the describing functions identified for the linear analogue models, discussed below, showed large increases in variability beyond 18 rad/sec. A compromise between the data storage limitations and the suggested sampling rate of ten times the maximum frequency of interest resulted in the choice of sampling rate as 25 samples/sec, i.e., \( \Delta t = 0.04 \) seconds.

2) Maximum Correlation Time Delay: During the investigation of the identification of linear analogue models, a range of values of \( \tau_m \) was tried. The main effects appeared to be on the very-low-frequency values of the correlation coefficient, while the describing function variations were found to be inconclusive. An increase in the value of \( \tau_m \) was found to decrease the values of \( \rho \) at very low frequency, while no significant increase in the variability of the describing function values could be noted. As is shown below, the problem of the low values of \( \rho \) at very low frequencies could be circumvented. Taking into consideration the restrictions of computation time and method, the desired frequency resolution of the power spectra, and the expected variability of the estimates, the final value chosen for \( \tau_m \) was \( \tau_m = 12.56 \) seconds. This value of \( \tau_m \) resulted in the variance of the power-spectral estimates, when using the Hanning \( \text{m} \) spectral window, of about \( \sigma/\mu = 0.24 \) for \( \omega > 1.3 \pi/\tau_m \). Thus, power and cross-spectral estimates could be calculated with a frequency resolution of 0.25 rad/sec from DC to 17.75 rad/sec. The possible presence of a spectral peak in the zero-frequency density estimate due to incorrect removal of the DC level in the signal made the zero-frequency estimate of doubtful use. In addition, as discussed in Appendix C, the use of the Hanning spectral window allowed only every second estimate to be considered independent. Therefore, the actual frequency points used in the describing functions and power-spectra have a frequency resolution of 0.5 rad/sec and are available from 0.25 rad/sec up to 17.75 rad/sec.

3.5.2 Evaluation of the Describing Function Identification:

Before attempting to identify the unknown system element, (the human operator), it was necessary to investigate the accuracy with which known system elements could be identified by the experimental and computational procedures used. This was done by replacing the operator in the loop by a variety of linear and
nonlinear analogue models and carrying out the complete identification process. The rest of the system was left unchanged, and the forcing functions used were the same ones later presented to the human subjects.

1) Pre-emphasis: Pre-emphasis is a technique whereby the signal-to-noise ratio of recorded data can be improved at high frequencies, where the signal levels are usually low, by passing the signal through a filter which increases the high-frequency power before recording.

A first order lag with transfer function: \( G(j\omega) = \frac{1}{1 + 0.2j\omega} \) was identified with and without simple first order lead/lag pre-emphasis circuits operating on the error signal and the model output. It was found that adequate identification was possible without this form of pre-emphasis and so it was not employed.

There is an alternative form of pre-emphasis, known as "system pre-emphasis" which is described in Ref.26. In this form, the controlled-vehicle dynamics are used to improve the low-frequency identification. As discussed in section (2.5), when rate control is used, the high gain of the transfer function \( Y(j\omega) \) at low frequencies results in the signal levels in the operator's output being very low and consequently the identification of \( Y(j\omega) \) and \( \rho \) may be in error. The appropriate expressions using the system output \( o(t) \) instead of \( c(t) \) are as follows:

\[
Y_p = \frac{\Phi_{io}}{\Phi_{ie}} \cdot \frac{1}{Y_c} \quad (3.5.2,2)
\]

\[
\rho = \frac{|\Phi_{io}|^2}{\Phi_{ii} \cdot \Phi_{oo}} \quad (3.5.2,3)
\]

and

\[
\Phi_{cc} = \Phi_{oo} \left| \frac{1}{Y_c} \right|^2 \quad (3.5.2,4)
\]

The effectiveness of system pre-emphasis was investigated with a fourth-order linear model. It was found that only the correlation coefficient, \( \rho \), benefitted substantially from the added complexity of this technique, and in the final analysis only this particular quantity was corrected in this manner.

2) Linear Analogue Models: To simulate the signal levels to be expected from the human operator, three linear models having approximately the correct amplitude and phase characteristics were identified: 1) A transport delay of 0.18 seconds, which was obtained using the head separation on an analogue tape recorder with a suitable tape speed, 2) A second order system with an additional source of random noise, uncorrelated with the forcing function, injected at the output of the model to simulate \( n(t) \), and 3) A fourth order system. Since the identifications of all three linear analogue models were excellent only the results for the fourth-order model will be discussed in detail.

The transfer function of the system was:
The r.m.s. levels of the forcing functions were 0.5 inches. A representative set of results for the fourth-order system is given in Fig.13. It can be seen that the agreement between the theoretical transfer function and the measured quantities is excellent and the variability is extremely small. The results shown are for the mean of four runs, and the bars indicate ±1.0σ. In particular, the values of the correlation coefficient are very close to unity.

3) Nonlinear Analogue Models: To further examine the capabilities of the experimental and computational procedures, a number of nonlinear analogue models were identified. The nonlinearities were of the amplitude dependent type, and the models used were: a) Absquare, b) Threshold, c) Saturation.

The input/output characteristics are sketched in Fig.14 with \( e(t) \) and \( c(t) \) being the input and output of the nonlinearity respectively. The models were implemented using the squaring circuits and diodes available on the analogue computer. Again, since the results for all three models were equally good only the saturation nonlinearity will be discussed. The actual characteristics were:

\[
c(t) = 5.36 \ e(t) \text{ ins. for } |e(t)| < 0.154 \text{ ins.} \\
= 0.825 \text{ ins. for } |e(t)| \geq 0.154 \text{ ins.}
\]

The comparison between the measured values of the describing function and the theoretical value of the equivalent gain, \( K_{eq} \), which was calculated using a method given in Ref.18, is shown in Fig.15. The agreement is good and the variability is quite low, the results again being for the mean of four runs. Since the saturation was an amplitude dependent nonlinearity, the phase lags were zero. The values of the correlation coefficient could not be predicted and are not presented.

The results of the linear and nonlinear analogue model identifications have shown that the data analysis system that was developed for this study was extremely powerful. It was capable of identifying linear elements over a frequency range of 0.25 rad/sec to at least 17.75 rad/sec with an accuracy of better than 0.2 db and 2 degrees. The correlation coefficients calculated for the linear elements were at least 0.995 at low frequencies. Nonlinear elements of the amplitude dependent type, were identified with an accuracy of about 4 db and 15 degrees. The low variability of the describing functions was very gratifying. It implied that the variability of the ratio of the two cross-spectra used to identify the describing function was apparently less than the expected variability of the individual spectra.

3.5.3 Evaluation of the Determination of the Remnant Time Records:

The remnant time record to be extracted was that of the remnant considered as being injected at the operator's output, \( n_c(t) \). Appendix B shows
that the use of the direct Fourier transform of the measured describing function as the impulsive response of the human operator introduced the problems of aliasing and the response window which were analogous to the problems of aliasing and the spectral window encountered in the estimation of the power and cross-spectral densities. The solution adopted was to curve-fit the measured describing function data with the "precision model" and perform the inverse transform on the fitted describing function.

However, further difficulties arose with this particular solution of the response-window problem, and they were 1) the effect of the extrapolation of the describing function up to infinite frequency, and 2) the definition of the remnant.

Examination of the measured describing functions and the power-spectra of the operator's output, $\Phi_{cc}$, indicated that the operator was capable of producing output beyond 18 rad/sec. The non-zero values of the correlation coefficient at 18 rad/sec showed that the output was still linearly correlated, to a certain extent, with the forcing function. However, the finite time records of this study made it impossible for the cross-spectrum, $\Phi_{de}$, and hence the correlation coefficient, to become identically zero. The fitting of the describing function with the precision model extrapolated the describing function to infinite frequency and assumed that the measured describing function behaved at high frequency in the same manner as the third-order term associated with the neuromuscular system. From physical considerations this did not appear to be an unreasonable assumption, but this extrapolation could cause the time record of the remnant to be in error.

The original concept of the remnant was that it constituted that part of the operator's output which was not linearly correlated with the forcing function within the frequency range for which the forcing function existed, in this case for $\omega < 18$ rad/sec, and all of the operator's output at higher frequencies. To obtain the correct amplitude probability distribution of the remnant it was necessary to extract the true remnant time record, that is, that which included the output of the operator above 18 rad/sec. If the error signal contained power above 18 rad/sec due to the remnant, then the operation of the infinite-bandwidth fitted-describing-function on the error signal would result in more of the operator's output being attributed to linear action than was correct. Fortunately, when the power-spectra of the forcing function and error were compared it was found that the bandwidth of the error signal was in fact about 18 rad/sec. This is shown for the case of $\omega_1 = 2.5$ and both manipulators in Fig.16. This confirmed that all of the output of the operator at frequencies higher than 18 rad/sec would be identified as part of the remnant.

The curve-fitting of the describing function, using the precision model, was performed with the help of a digital computer and a plotting attachment, and the goodness of fit was determined visually. Considerable time and effort was expended to develop a "Steepest Descent" gradient method for least squares fitting, but very little progress was made. The main difficulty was that the scatter of the individual data points at high frequencies made it extremely difficult for the automatic technique to converge to a good fit over the whole frequency range. Attempts were made to improve the convergence by heavily weighting the errors at lower frequencies but again with little success. It was necessary to fit both amplitude ratio and phase angle simultaneously, and the compromises required were best achieved visually.
The parameters of the fitted describing function will be discussed in a later section, but comments on the influence of two of them are in order here. It was found that the term in the impulsive response corresponding to the $T_K$ term in the describing function was a very slowly decaying exponential, while the cases of very low damping for the second-order term of the neuromuscular system resulted in a very slowly decaying oscillation. Under both these conditions the impulsive response still had non-zero values after a considerable length of time, and in order to reduce the computation time of the convolution integral it was necessary to truncate the impulsive response after about 12 seconds.

As a result of the possible errors introduced into the remnant by the curve-fitting procedure and the truncation of the impulsive response, a series of tests were performed to evaluate the effectiveness of this technique using actual human operator data.

1) Accuracy of Fit: A measure of the accuracy of the curve-fitting was obtained from the mean square of the linear response of the operator to the measured error signal, $c^2_e$, by comparing its value when derived using the measured describing function with that resulting from the use of the fitted model. The expressions used were as follows:

$$\overline{c^2_e}_M = \frac{1}{2\pi} \int_{-\infty}^{\infty} |Y_{PM}|^2 \Phi_{ee} d\omega$$

and

$$\overline{c^2_e}_F = \frac{1}{2\pi} \int_{-\infty}^{\infty} |Y_{PF}|^2 \Phi_{ee} d\omega$$

where $Y_{PM}$ and $Y_{PF}$ were the measured and the fitted describing functions respectively. The results of this test appear in Table III. A number of representative describing functions for various levels of neuromuscular system damping were considered. It can be seen that the agreement between the two methods of calculation of $c^2_e$ is better than 5%, except for the cases where large scatter in the high frequency data points made the fit rather approximate.

2) Effect of Impulsive Response Truncation: The effect of the truncation of the impulsive response was investigated by comparing the values of $c^2_e$ resulting from the convolution of the error signal with the impulsive response whose length was either equal to the run length, $T_m$, or was truncated to $T_m$, i.e.,

$$\overline{c^2_e}_L = \frac{1}{T_m} \int_0^{T_m} c_e(t)^2 dt$$

and

$$\overline{c^2_e}_T = \frac{1}{T_m} \int_0^{T_m} c_e(t)^2 dt$$

where $c_e(t)$ is the impulsive response.
where
\[ c_{eL} (t) = \int_{0}^{T_m} \mathcal{F} (\tau) e(t-\tau) \, d\tau \] (3.5.3,5)
and
\[ c_{eT} (t) = \int_{0}^{T_m'} \mathcal{F} (\tau) e(t-\tau) \, d\tau \] (3.5.3,6)

The actual values used were: \( T_m = 165 \) seconds and \( T_m' = 12.0 \) seconds.

The results are given in Table III for the same set of runs as in the first test. The agreement between the two values of \( c_{e}^{2} \) is excellent, differences being less than 2% in all cases. In addition, a further value of \( c_{e}^{2} \) was calculated for these same runs, based on a truncation of \( c_{e}(t) \). To avoid initial transients in \( c_{e}(t) \) due to the fixed starting point \( (t = 0) \) of the convolution integral, the first point of \( c_{e}(t) \) was calculated at \( t = 12.0 \) seconds. The value of \( c_{eT}^{2} \) obtained from this time is identified as \( c_{eT}^{2} \), where
\[ c_{eT} (t) = c_{eT} (t) \quad \text{for } t \geq 12.0 \text{ seconds} \] (3.5.3,7)
\[ = 0 \quad \text{for } t < 12.0 \text{ seconds} \]

and the integration in eq. (3.5.3,4) to yield \( c_{eT}^{2} \) was performed with the lower limit of \( t = 0 \) being replaced with \( t = 12.0 \) seconds, and using a total record length of 153 seconds.

Here again the agreement with \( c_{eP}^{2} \) is better than 5%, except where the scatter in the measured describing functions is large. The time record \( c_{eT}'(t) \), involving the truncation of both \( \mathcal{F}(t) \) and \( c_{eT}^{2}(t) \) was the one used to isolate the remnant time record from the output of the operator.

In Fig. 17 two representative measured describing functions for the two extremes of the damping term of the neuromuscular system are shown, with the corresponding fitted models, the resulting impulsive responses and the time records. Included in the time records are those of the remnant, \( n(t) \). The scatter in the measured values of the describing functions can be seen and the oscillatory nature of the impulsive response for low damping of the neuromuscular system is quite marked. The time records clearly show the difference between low and high values of damping, particularly in the operator's output and the remnant.

3) Remnant Power-Spectra: The third and last test of the technique compared the power-spectra of the remnant, \( n_{e}(t) \), obtained from the correlation coefficient and \( \Phi_{cc} \), with that obtained from the time record of \( n_{e}(t) \), i.e.,
\[ \Phi_{n_{e}n_{e}} = \left( 1 + \frac{Y_{P-M}}{Y_{M}} \frac{Y_{c}}{Y_{P-M}} \right)^{2} (1-\rho^{2}) \Phi_{cc} \] (3.5.3,8)
\[ \phi_n \phi_n = \left( \frac{1}{\phi_n} \right)^2 \phi_n \phi_n \]  

(3.5.3,9)

The results are shown in Fig.18, where it can be seen that the agreement between the two power-spectra is excellent. In fact, this comparison between these power-spectra was made for every single run and the two shown here indicate the largest errors that were found. Since the length of the time record of \( n_c(t) \) was 12 seconds less than the other measured time signals because of the truncation already discussed, the variability of the estimates of the power-spectrum, \( \Phi_n \phi_n \phi_n \), would tend to be slightly larger. The excellent agreement between the two power-spectra is all the more impressive when the number of steps in the computation required to obtain \( n_c(t) \) is taken into account.

The results of the three tests which have just been described show that the technique used to extract the time record of the remnant was extremely effective, and allowed considerable confidence in the measured values of its amplitude distribution.

3.6 Procedure:

Each training or data-recording session consisted of three, three minute runs, separated by two minute rest periods. The choice of a three minute run was dictated by the possibility of operator fatigue. For the data recording, the first run of the three was used as a warm-up and in addition, the first 12 seconds of the following two runs were not recorded, in order to allow any starting transients to die out. At the end of each run the performance score was calculated and given to the subject. During the program subjects often found it more convenient to remain at the facility for two or three consecutive sessions with a reasonable rest period between them. On these occasions it was often the case that the randomization of the experimental conditions resulted in the manipulator being changed between sessions. However, it was found during the training program that only the single warm-up run was required to transfer to the new manipulator.

The collection of data took a considerable time, (approximately two months), and it was necessary to check that the performance levels of the subjects had not changed markedly over the period. This check was performed by comparing the scores for two identical sets of experimental conditions (including the actual forcing-function run), one from before the data recording period and the other after it. The scores were compared using an analysis of variance with the two sets of scores being treated as replications of the experiment, and the results are given in Table IV. The method of testing the variance followed that of Ref.6 in that the triple and double interaction terms were compared with the experimental error term initially, and where possible the values of the mean square and the degrees of freedom were pooled with the residual term before testing the main effects. As shown in Table IV one double interaction term and two main effects, the conditions and the subjects, were found to be significant at the 99% level, but the result of primary interest is that the replication main effect was not significant. This implied that the subjects, as a group, showed no significant improvement in performance during the data-recording period. In addition, no significant improvement in performance was found on the part of any individual subject.

The initial analysis of the scores and the describing functions
showed effects that could only be attributed to the type of forcing-function spectra that were being employed. Consequently, the STI forcing functions, described in section (3.2.2), were made up and further score data were obtained after adequate training. Describing functions could not be derived for these inputs because the techniques of data analysis were unsuitable for the line spectra.

IV RESULTS AND DISCUSSION

In the following sections the experimental results are presented and discussed. Unless stated otherwise the values are given in the form of grand averages over the eight subjects. The describing functions of the human operator and the power-spectra of his output and the remnant are shown in units of inches on the display. Thus, the units of the amplitude ratio are inches/inch, while those of the spectra of the remnant and the operator output are (inches on the display)^2/radian/second. This is done simply by multiplying the quantity by K^2 or K^4 as appropriate. The units of the phase angle of the describing function are degrees.

The amplitude ratios and power-spectra are plotted in decibels. The definitions used throughout this report are as follows:

\[
\text{Decibels} = 20 \log_{10} (\text{Amplitude Ratio})
\]

\[
\text{Decibels} = 10 \log_{10} (\text{Power Quantity})
\]

(4,1)

with 0 db = 1.0 ins^2/rad/sec.

To avoid overcrowding of the points at high frequencies in the figures, the frequency interval has been doubled above 9 rad/sec. The negative phase angle of the describing function is often referred to as a positive "phase lag", such that a reduction in phase lag corresponds to a reduction in the absolute magnitude of the phase angle.

4.1 Scores:

The performance scores are presented in Fig. 19 and show the influence of both the manipulator and the bandwidth of the forcing function. The data are plotted as normalized mean-square error for the pressure manipulator against the free-moving manipulator. It can be seen that the former produces lower scores and hence better tracking performance for all forcing functions. A decrease in forcing-function bandwidth, \(\omega_1\), causes a decrease in score except for \(\omega_1 = 1.5\) and the pressure manipulator. An examination of the power-spectrum for this input shows that the slight increase in the contribution of the high-frequency shelf to the total power of the signal is sufficient to make the shelf appear to dominate the forcing function. Subjectively this produces an apparently more demanding tracking task than that of the \(\omega_1 = 2.5\) input.

The effect of the high-frequency shelf on the score is considerable. A comparison of the scores for \(\omega_1 = 2.5\), with and without the shelf, shows that its removal causes a large reduction in score and a consequent improvement in performance. This is contrary to the assumption made at the initiation of this study which was based on the results given in Ref. 26.
As mentioned in section (3.2.2), the assumption made in that reference was that the high-frequency shelf would make no difference to the low-frequency performance. Data was presented to support this, being the comparison of describing functions for a pure-gain controlled vehicle, \[ Y(j\omega) = K \], and forcing functions made up of sums of sine-waves, with and without the high-frequency shelf. Although a large number of sine-wave components were used, the shelf extended only to about 10 rad/sec. The current score results show that when the forcing functions are filtered Gaussian noise, this assumption does not hold.

The high scores obtained in this study for the filtered-noise forcing functions cast doubt on the performance levels of the subjects and so scores for the STI forcing functions were measured. These are presented in Table V where they are compared with the data for the identical forcing functions of Ref.38. It can be seen that the comparison is excellent, considering that a spring-restrained manipulator will yield slightly lower scores than a free-moving one [Ref.28]. This good agreement indicated that the subjects were in fact producing valid results and that their performance levels were adequate.

It was felt that the amplitude of the shelf components was the main factor in the larger scores. Supporting evidence for this was found in Refs. 28 and 38, for the "B5" forcing function. The "B5" forcing function had \( \omega_i = 2.5 \), \( \sigma_i = 1.0 \) ins. and a shelf attenuation of only 10 db. This forcing function produced much higher scores than the normal \( \omega_i = 2.5 \) forcing function and these are shown in Table V. In fact, the scores for the "B5" input were very similar to those for the current \( \omega_i = 4.0 \) forcing function.

Also shown in Table V, for comparison, are the scores from Ref.28 for the pressure and free-moving manipulators and the "R2.2" forcing function. This particular input was made up of sums of ten sine waves and had a rectangular spectrum, with \( \omega_i = 14.03 \) rad/sec and \( \sigma_i = 1.0 \) ins. The scores for this forcing function were very large and the pressure manipulator had poorer performance than the free-moving one.

4.2 Describing Functions:

The describing functions of the human operator, with the total variability, are presented in Fig. 20 and show the individual results for both manipulators and for \( \omega_i = 1.5, 2.5, 4.0 \) and 2.5*. The following sections will discuss these results in detail.

4.2.1 Variability Due to the Subjects:

Before discussing the describing functions as a whole, it is necessary to investigate the variability introduced into the data by the subjects. In addition to the total variability shown in Fig. 20, Figs. 21 and 22 present the between-subject and within-subject variability respectively. The between-subject variability is shown for the free-moving manipulator and a single run with the forcing function \( \omega_i = 2.5 \). The within-subject variability is given, for a representative subject, for the four runs with \( \omega_i = 2.5 \) and the free-moving manipulator. The error bars indicate the ± 1.0 \( \sigma \) levels. A single-sided error bar on the amplitude ratio implies that the variance of the data is larger than the mean.
and the \((\mu - \sigma)\) level is a negative quantity for which logarithms are not defined.

All forms of the describing function variability show the same basic pattern. The variability is small at medium frequencies and increases towards low and high frequencies. The relatively larger variability of the lowest frequency point, at \(\omega = 0.25\) rad/sec, is not necessarily due to the subjects alone, since the identification of the linear analogue pilot models showed that this point would have larger variability. The increase in size of the error bars towards each end of the frequency range is consistent with other published data [Ref. 26]. In our case, the large variability at the high-frequency end is due in part to the presence of a peak in the describing function, whose centre frequency varies from subject to subject.

In general the variability is smaller for the pressure manipulator, and from the results it can be seen that the within-subject variability tends to be smaller than the between-subject variability, showing very consistent performance by each subject. Although the pattern of the variability is the same for each forcing function, the magnitude tends to decrease slightly with an increase in forcing function bandwidth. This implies that as the forcing functions become more demanding, the behaviour of the subjects tends to become more constrained and the variability between them is reduced.

When the relative inexperience of the subjects and the difficulty of the tracking tasks are taken into account, the low variability is very satisfying, and is further evidence that the subjects have achieved high and consistent levels of performance.

4.2.2 Effect of the High-Frequency Shelf:

The effect of the high-frequency shelf on the open-loop describing function, \(Y_pY_c(j\omega)\), is presented in Fig. 23, for the free-moving manipulator and \(\omega_i = 2.5\). The presentation of this result and the following describing functions in the open-loop form is more meaningful from the overall system point of view, since the behaviour of the open-loop describing function has a direct bearing on the bandwidth and the stability of the closed-loop system.

When the high-frequency shelf is removed, the phase angle shows virtually no change while the amplitude ratio increases such that the cross-over frequency is increased by about 2 rad/sec. Since the frequency-dependent parameters of the describing function are apparently unchanged, the describing function differences can be modelled by a simple increase in gain. This gain increase causes lower mean-square errors and hence lower scores, and this was shown in Fig. 19. The corresponding comparison of Ref. 26 showed no change in the describing function due to the high-frequency shelf. The disagreement between the present effects of the high-frequency shelf and those reported in Ref. 26 is attributed to the fact that this study used an input with a continuous spectrum rather than a line spectrum. It is clear that the use of the more realistic forcing function with a continuous power-spectrum leads to quite different conclusions from those arrived at by using random appearing inputs made up of sums of sinusoids.

The decrease in the cross-over frequency due to the high-frequency shelf is called "Cross-over Frequency Regression", and this phenomenon is discussed more fully in a later section.
### 4.2.3 Effect of the Forcing-Function Bandwidth:

The effects on the describing function of the input bandwidth, $\omega_i$, are presented in Fig. 24 for both manipulators. In general terms, it can be seen that the amplitude ratio has a slope of about 20 db/decade in the region of cross-over, while at low and high frequencies the changes attributed to the neuromuscular system are apparent. The behaviour of the high-frequency amplitude ratio and phase angle is very similar to that of an under-damped second order system.

With an increase in forcing-function bandwidth the amplitude ratio shows an increase in gain which is relatively constant with frequency. The difference is large between the $\omega_i = 1.5$ and 2.5 cases but for $\omega_i = 4.0$ the gain of the amplitude ratio is only slightly larger than that for $\omega_i = 2.5$ except at low frequencies, where the gain tends to be lower for $\omega_i \geq 4.0$. The high-frequency neuromuscular peak is clearly visible for the free-moving manipulator. The natural frequency remains relatively constant but the damping decreases with an increase in forcing-function bandwidth.

The influence of $\omega_i$ on the phase angle is also marked. The phase lags decrease with an increase in bandwidth at low and medium frequencies, but at high frequencies the sudden change in phase angle corresponding to the amplitude-ratio peak is the dominant factor. Both manipulators show the same trends in the describing function but the differences tend to be smaller for the pressure manipulator.

The describing functions from Ref. 26, for STI forcing functions with $\omega_i = 1.5$, 2.5 and 4.0 and a spring-restrained manipulator (side stick), are shown in Fig. 25. Also shown are the "B5" and "R2.2" results from Ref. 28. Although these are not direct comparisons due to the manipulator and forcing function differences, the trends in the data are very similar to those presented here. The absolute values of the phase angles agree quite well, but those of the amplitude ratio do not, particularly in the region of cross-over. In addition, the amplitude ratio shows only relatively small change with forcing function in the referenced data. The differences between the amplitude ratios indicate that all the current describing functions are regressed. The regression of the cross-over frequency implies that the apparent bandwidths of the forcing functions of this study are much higher than the nominal bandwidths and this increase is mainly due to the presence of the high-frequency shelf.

### 4.2.4 Effect of the Manipulator:

The influence of the manipulator on the open-loop describing function for $\omega_i = 1.5$, 2.5 and 4.0 is presented in Fig. 26. The effect of the manipulator on the amplitude ratio is mainly confined to high frequencies, where the change to the pressure manipulator moves the neuromuscular peak to higher frequencies. The natural frequency of the neuromuscular peak for the free-moving manipulator is around 15 rad/sec while for the pressure manipulator it is in the region of 18 to 20 rad/sec.

The effect of the manipulator on the phase angle is much more pronounced. The change to the pressure manipulator produces large reductions in the phase lags, and they extend over a much wider frequency range than the differences in the amplitude ratio. The phase angle differences at low frequencies are small, except for $\omega_i = 4.0$, where the pressure manipulator
has larger phase lags. The increase in forcing-function bandwidth does not affect these trends; however, the high-frequency differences in the amplitude ratios are accentuated by an increase in input bandwidth.

The describing functions from Ref. 28 are shown in Fig. 27. These results are for pressure and free-moving side-stick manipulators and the "B5" and "R2.2" forcing functions. The side-stick manipulators will yield natural frequencies of the limb-manipulator combination which are higher than those used in this study. Again, no direct comparisons are possible, but the amplitude ratio shows a lack of dependence on the manipulator which is similar to the present result. The neuromuscular peak, if it exists, is not apparent in the referenced results due to insufficient bandwidth. However, the influence of the manipulator on the phase angle does not agree with the current results in that the effects are mainly restricted to the lower frequencies. This disagreement could be explained by the presence of the neuromuscular peak in the present data. The reduction in phase lags produced by the shift of the peak to higher frequencies with the change to the pressure manipulator would appear to dominate the phase angle over most of the frequency range.

The variability of the phase angle results shown in Ref. 28 contrasts sharply with the generally smooth data of this study. This allows considerably greater confidence in the present results and indicates that the measurement and analysis techniques that have been developed for this study are, in general, more precise than those of the referenced work. The large variability of the results of Ref. 28 may well be hiding high-frequency effects due to the manipulator which are similar to those in the present findings.

It is interesting to note that the describing functions in Ref. 25, although not shown here, are the only published results of sufficiently wide bandwidth and for a sufficiently demanding tracking task to show clear indication, in the amplitude ratio, of the high-frequency neuromuscular peak found in these experiments. This peak was also assumed to be present in some of the results in Ref. 26, but its presence was inferred from a single data point. This is adequate evidence that the neuromuscular peak in the present results is not an artifact associated with the particular conditions of this study.

The changes in the describing function due to the manipulator may be related to the physiology of the neuromuscular system. In general terms, with reference to Fig. 4, it is clear that the pressure manipulator will cause a considerably higher stiffness and considerably reduced inertia of the limb-manipulator combination and this will contribute to the fact that the natural frequency of the system with this manipulator is higher than that with the free-moving one. Reference 24 goes into the subject in far greater detail than is possible here. One important point that is brought out is that the use of the rigid pressure manipulator results in a much higher level of average tension in the agonist-antagonist muscle pair. The changes in the parameters of the neuromuscular system model due to this increased tension are such as to increase the natural frequency of the limb-manipulator combination. It appears that this increase in the average level of muscle tension may be brought about partly by a change in the alpha motor neuron command signal and partly by a change in the mean value of the gamma motor neuron command signal, which in turn affects the feedback signals from the muscle spindle receptors to the alpha motor neuron. The alpha motor neuron command signal itself is assumed to play no part in the "small perturbation" responses associated with random tracking.
4.3 System Performance Measures:

The scores have already been discussed in section (4.1). The other three performance measures: relative remnant, $\rho_a^2$, cross-over frequency, $\omega_c$, and phase margin, $\phi_m$, are presented in Table VI, showing the effects of the manipulator and forcing function.

4.3.1 Relative Remnant:

As defined in eq. (2.7.2,1), the relative remnant is a measure of the "overall" linearity of the operator. It can be seen in Table VI that an increase in $\omega_i$ produces an increase in $\rho_a^2$, while the pressure manipulator gives lower values than the free-moving one. This implies that as the forcing function becomes more difficult the operator tends to behave in a slightly more linear fashion, and that the pressure manipulator produces less linear performance. Also shown in Table VI are some averaged values of $\rho_a^2$ taken from Ref.26 for a spring-restrained side-stick manipulator and STI forcing functions. The agreement is reasonable and the trend of increase in $\rho_a^2$ with increase in $\omega_i$ is the same. However, the trend due to the manipulator which is shown in the present data does not agree with that of Ref.25, where the pressure manipulator was shown to give a higher value of $\rho_a^2$ than the free-moving manipulator.

Under the present experimental conditions the values of $\rho_a^2$ indicate that the relative amount of power in the output of the operator that is due to a linear operation on the system input is from 60-70% for the free-moving manipulator, and from 50-60% for the pressure manipulator.

4.3.2 Cross-over Frequency and Phase Margin:

The values of the cross-over frequency and phase margin, given in Table VI, serve to summarize the behaviour of the describing function at medium frequencies. As discussed earlier, the present describing functions are regressed. However, the cross-over frequency, $\omega_c$, is still slightly dependent on the forcing-function bandwidth, $\omega_i$, in that $\omega_c$ increases with an increase in $\omega_i$. The phase margin shows no clear trend with $\omega_i$. The non-regressed data of Ref.26 show a slightly greater dependence of $\omega_c$ on $\omega_i$, while $\phi_m$ is strongly dependent on $\omega_i$. The present results for $\omega_i = 2.5$ and 4.0 give better agreement in absolute values and trends in $\omega_c$ and $\phi_m$ than that for $\omega_i = 1.5$. Apparently the $\omega_i = 1.5$ forcing function causes more regression than the higher bandwidth inputs. The $\omega_i = 2.5$* forcing function (no high-frequency shelf) yields values of $\omega_c$ and $\phi_m$ which agree quite well with those for the STI forcing functions.

Table VI also contains results from Ref.26 for both free-moving and pressure manipulators and the "B5" and "R2.2" STI forcing functions. The values of $\omega_c$ and $\phi_m$ for both manipulators and the regressive "R2.2" input are quite similar to the current ones. This implies that when regression occurs the type of describing function that results, for the same controlled system, is relatively independent of the particulars of the forcing function that produced the regression. The scores obtained for the "R2.2" input (Table V) indicate that they produced much poorer performance than any of the inputs in the present study.
4.4 Power-Spectral Density of the Remnant:

The various forms of the power-spectral density of the remnant, together with the total variability due to the subjects, are presented in Figs. 28 to 30. In Fig.28, the relative magnitudes of $\Phi_{nn}$ and $\Phi$ are compared for $\omega_i = 2.5$ and both manipulators. The correlation coefficient, $\rho$, and the closed-loop output remnant, $\Phi_{nn}$, are shown in Fig.29, plotted for the individual combinations of manipulator and forcing function, and the corresponding plots are given in Fig.30 for the power-spectra of the observation remnant, $\Phi_{ne ne}$, and the output remnant, $\Phi_{nc ne}$.

4.4.1 Variability due to the Subjects:

The total variability due to the subjects is shown in Figs. 29 and 30 and includes both the between- and the within-subject variability. The error bars are for the $\pm 1.0\sigma$ levels.

The basic pattern of the variability of the correlation coefficient is somewhat similar to that of the describing function, in that it is quite small over the low and medium frequencies and increases towards high frequencies. The use of system pre-emphasis [section (3.5.2)] ensures that the identification of $\rho$ at low frequencies is good, and consequently the slight increase in variance at low frequencies is due almost entirely to the subjects. It can be seen that the variance of $\rho$ is very small within the bandwidth of the forcing function, increasing sharply at higher frequencies.

The variability of the power-spectra of the remnant shows a similar trend of low variance within the forcing-function bandwidth and in this case, only a slight increase at higher frequencies. Of particular interest are the relatively small values of the variance of $\Phi_{ne ne}$, when compared with those of $\Phi_{nc ne}$, in the region of the neuromuscular peak in the describing function. When the variability of the describing function and the correlation coefficient at these frequencies is taken into account, this small variance is quite remarkable.

4.4.2 Effect of the Forcing-Function Bandwidth:

Figures 31 and 32 show the influence of the forcing-function bandwidth on the correlation coefficient and the three forms of the power-spectrum of the remnant for both manipulators. The values of the correlation coefficient are close to unity up to the beginning of the high-frequency shelf at which point the value drops to around 0.8 and in the region of the neuromuscular peak there is a further reduction. At very low frequencies the value of $\rho$ is reduced for the $\omega_i = 4.0$ input. It can be seen that $\rho$ is strongly dependent on the forcing function at medium frequencies and that the operator maintains very linear performance within the forcing-function bandwidth.

The effect of the forcing-function bandwidth on $\Phi_{nn}$ is confined to the medium and high frequencies where an increase in $\omega_i$ results in an increase in $\Phi_{nn}$. At low frequencies, within the forcing-function bandwidth, $\Phi_{nn}$ is independent of $\omega_i$. However, the influence of $\omega_i$ on $\Phi_{nc ne}$ shown in Fig.32, is quite marked over the whole frequency range, while the behaviour of $\Phi_{ne ne}$ with $\omega_i$ is also different. The input bandwidth apparently has very little influence.
on $\Phi_{n,n}$ over the whole frequency range, except for the case of $\omega_1 = 4.0$ and the free-moving manipulator, for which condition the low frequency values of $\rho$ are lower. Of particular note is the large peak in $\Phi_{n_e n_e}$ at high frequencies, in the region of the under-damped second order term of the neuromuscular system model, which is not visible in $\Phi_{n_e n_e}$. The manipulator has little effect on these trends with the exception that the pressure manipulator tends to reduce the influence of $\omega_1$.

There are no published results with which direct comparisons may be made. The little data that is available is presented in Fig. 33, taken from Refs. 22 and 26. Both sets were obtained using STI inputs. In Ref. 22 $\Phi_{n_e n_e}$ is normalized with respect to mean-square error, $\sigma_e^2$, while in Ref. 26 the results are given in the form of $\Phi_{n_e n_e}/\sigma_i^2$. The values of $\Phi_{n_e n_e}$ for both manipulators which were obtained in this study are presented in both normalized forms in Fig. 33 for comparison. In addition, Ref. 1 showed one remnant spectrum, $\Phi_{n_c n_e}$, which was quite similar in general shape to those in Figs. 30 and 32.

The agreement in $\Phi_{n_e n_e}/\sigma_i^2$ is quite good for $\omega_1 = 1.5$, but the referenced results show considerably larger differences due to $\omega_1$ within the input bandwidth. At high frequencies both normalized spectra coalesce equally well with $\omega_1$, but the values given in the referenced work are about 5-10 db lower. This difference can be attributed to the presence of the neuromuscular peak. On the other hand, the agreement with $\Phi_{n_e n_e}/\sigma_i^2$ of Ref. 22 is not quite so good. The general shape of the curves and the lack of dependence on $\omega_1$ are similar, but the referenced results are about 10 db lower within the forcing-function bandwidth. This disagreement is difficult to explain in view of the relatively favorable comparison for $\Phi_{n_e n_e}/\sigma_i^2$ over the same frequency range, and especially since the techniques employed here to isolate the time record of the remnant have been shown to be both accurate and precise. Since the STI forcing functions of the type used in Ref. 22 would be expected to produce lower scores and hence lower values of $\sigma_e^2$ (for the same $\sigma_i^2$) than those of this work, it would appear that the non-normalized spectra of $\Phi_{n_e n_e}$ which were measured in that experiment were of considerably smaller magnitude than the ones obtained here.

The comparison of the present results with those of the references has shown that the basic trends in $\Phi_{n_e n_e}$ (due to $\omega_1$) are relatively independent of the particulars of the forcing function. However, since the experimental conditions of the two referenced investigations were more or less the same, the discrepancies that were found in the absolute magnitudes of $\Phi_{n_e n_e}$ (particularly at low to medium frequencies) must be attributed to the differences in measurement techniques; and in addition, lack of agreement with the results of this study may be caused by a combination of the measurement technique and the type of forcing function.

4.4.3 Effect of the Manipulator:

The influence of the manipulator on the correlation coefficient and the closed-loop output remnant is shown in Fig. 35, and the two remnant spectra, $\Phi_{n_c n_e}$ and $\Phi_{n_e n_e}$, appear in Fig. 36. Within the forcing-function bandwidth the effect of the manipulator on $\rho$ is negligible. At higher frequencies the pressure manipulator produces lower values. An increase in the $\omega_1$ tends to
reduce the effects of the manipulator at high frequencies, but slight differences
do appear at low frequencies.

The effect of the manipulator on $\Phi_{nn}$ is small at low frequencies, but
is more pronounced at higher frequencies. In general, the pressure manipulator
produces lower values of $\Phi_{nn}$ at low frequencies and higher values at high
frequencies. The influence of the manipulator on $\Phi_{nn}$ and $\Phi_{ne}$ is somewhat
similar; the pressure manipulator yields lower values at low frequencies, but
at higher frequencies the peak due to the neuromuscular system tends to domi-
nate $\Phi_{nn}$. However, as mentioned before, this peak is not apparent in $\Phi_{ne}$.
Again, an increase in $\omega_i$ tends to increase the low frequency differences and
to reduce those at higher frequencies.

The high-frequency peak in $\Phi_{nn}$ indicates that, when presented in this
form, the remnant receives an appreciable contribution from the neuromuscular
system. On the other hand, this remnant spectrum can be considered as being
due to the operation of the describing function on $\Phi_{ne}$ which appears to be
relatively independent of the neuromuscular system.

Again, there are no results with which direct comparisons may be made. However, some very recently published work [Ref.20], does show the effect of
the manipulator on the closed-loop observation remnant, i.e., $\Phi_{ne}$, where
$\Phi_{ne} = |Y|^{2} \Phi_{nn}$. The study was performed for acceleration-control
vehicle dynamics, STI forcing functions and for pressure and spring-restrained
manipulators. The pressure manipulator was found to yield lower values of
$\Phi_{ne}$ at low frequencies and higher values at high frequencies. This is the
same trend as is shown here.

The above results, which have described the influence of the forcing
function and the manipulator on the remnant, have clearly shown that the
conclusions that may be drawn are very dependent on the form of presentation
of the power-spectrum of the remnant. The form of the remnant spectrum that
coalesces best, over the whole frequency range, for the various conditions
of forcing function and manipulator, is that of the injected observation rem-
nant, $\Phi_{ne}$. When presented in this form, the remnant is only slightly de-
pendent on the forcing function and the manipulator.

The power-spectrum of the remnant could be considered as being the
result of passing "white noise" through a filter. The spectra plotted in
Fig.33 could, as a first approximation, be fitted using a filter whose trans-
fer function was that of a first order lag, with break frequency around
10 rad/sec, but the fit at low frequency would be rather poor. Since the
identification of the linear analogue pilot models indicated that the
measurement of $\rho^2$ was extremely accurate at low frequencies, then the values
of $\Phi_{nn}$ at these frequencies must be considered as correct. A better fit
could be obtained by using a filter with a frequency response, $G(j\omega)$, of
the form:

$$
G(j\omega) = \frac{a_2 a_3 (a_2 + a_3)}{a_2^2 + a_2 a_3} \left( \frac{a_1 + j\omega}{a_2^2 + j\omega(a_2 + j\omega)} \right)^{1/2} \quad (4.4.3.1)
$$
with $a_1 = 0.8 \text{ rad/sec}$; $a_2 = 0.1 \text{ rad/sec}$; $a_3 = 10.0 \text{ rad/sec}$ and $\sigma = 0.3 \text{ ins}$.

The power transfer function for this filter is shown in Fig. 34, fitted to $\Phi_{n_p n_e}/\sigma^2$ for both manipulators. The fit is reasonable for both sets of results.

Until more measurements of as high quality as those of this study become available, showing in particular the effects of the dynamics of the controlled vehicle, there is little point in attempting to model the power-spectrum of the remnant any more closely than this.

The general agreement between the present conclusions about the remnant power-spectra and those of Refs. 22 and 26, and the agreement in turn between the results from Ref. 26 and those for the single-sine-wave tracking experiments reported in Ref. 30, serve to bolster the impression that the remnant is due to a relatively stable (i.e. invariant) source within the human operator.

4.5 Fitted Describing-Function Parameters:

As discussed in section (3.5.3), an intermediate step in the isolation of the time record of the remnant was the curve-fitting of the complete set of 192 describing functions. The average values of the parameters used in this fitting process are presented in Table VII. The differences in the parameters due to the influence of the input bandwidth and the manipulator are quite evident, since these parameters simply reflect the corresponding changes in the describing functions themselves. The fitted parameters will not be discussed in detail, but it is of interest to note a few points.

Although the precision model was used to fit the data, in which the transport delay term, $\tau_d$, accounted for only the neural conduction and processing time delays, a difference in value of $\tau_d$, depending on the manipulator, was required to fit the data adequately. This difference amounted to about 0.06 seconds and was independent of the forcing function. In addition, the values of $\tau_d$ required for the pressure manipulator were at the lower limit of the delay based on physiological considerations [Ref. 26]. But then, even this limit was only about half that given for simple reaction times in Ref. 37. (For one particular subject, the values of $\tau_d$ required for the pressure manipulator and $\omega = 1.5$ were an incredibly low 0.01 seconds). This indicates that the subjects were able to develop a control technique which reduced the phase lags introduced by the neural conduction time delays etc., to very small values.

Further, it can be seen that the adaptive dynamics terms, $T_x$ and $T_q$, were required to fit the data. The difference between $T_x$ and $T_q$ increases with an increase in forcing function bandwidth, but the dependence on the manipulator is not clear. The effect of $\omega$ on the high frequency terms of the neuromuscular system is relatively small, but the free-moving manipulator results in much lower damping and lower natural frequencies for the second order neuromuscular term. Two representative describing functions and the fitted models have already been presented in Fig. 17. The two examples cover the extremes in the value of the neuromuscular system damping term, and the fitted parameters for these two cases are given in Table VII.

4.6 Time Records and Amplitude Probability Distributions:

The technique used to extract the remnant time record, $n_c(t)$, is
described in Appendix B, and it has been shown to be extremely powerful. The time record will contain all of the remnant within the frequency range for which the forcing function can be said to exist, and all of the operator's output at higher frequencies than this.

Two representative sets of system time records are shown in Fig. 17 for the cases of low and high damping of the neuromuscular system. All the time records are expressed in units of inches on the display and the following records are plotted: i(t), e(t), c(t), o(t), c(t), and n(t). This is the first time that such remnant time records have been published.

The system output can be seen to follow the forcing function quite well with a slight time delay. The differences between the levels of the neuromuscular system damping are most clearly shown in n(t). The low damping case gives a very oscillatory form of time record, while the high damping case results in a time record that looks much more like a random signal. Also shown in Fig. 17 are the values of the score and the Chi-Squared statistics for these particular time records. As will be discussed later, a most interesting point is that both of the remnant signals are normally distributed while the operator's output need not be.

Having obtained the time records of n(t), the amplitude probability distributions were estimated for all the signals in the loop, including the remnant, for all the experimental runs. Independent samples were obtained from each time record by sampling with a period of 0.8 seconds. Sixteen amplitude intervals were chosen, of unequal widths, as discussed in section (2.9), such that a normal distribution would have approximately the same number of points in each interval. There were a sufficient number of samples such that there would be about 10 samples per interval for a normal distribution.

The probability distributions are presented in the form of bar histograms, that is, as relative frequency plotted against non-dimensional amplitude. The heights of the bars indicate the relative number of samples in each interval, i.e.

\[
\text{Relative Frequency} = \frac{\text{Number of Samples in Interval}}{\text{Total Number of Samples}}
\]

The histograms for the individual combinations of forcing function and manipulator appear in Fig.37. The horizontal dotted line on each histogram represents the normal distribution. To obtain the probability density, the relative frequency is simply divided by the width of the interval, bearing in mind that the two extreme intervals are of infinite width. Chi-Squared tests of fit to the normal distribution were performed on the probability distributions for each set of time records and the averaged values of the Chi-Squared statistic are given in Table VIII.

The most striking aspects of the probability distributions and the values of \( \chi^2 \) are that the time records of the remnant are normally distributed without exception, while at the same time the output of the operator is non-normal. The histograms for the operator's output show a distinctly bi-modal appearance. The only case for which the output of the operator was found to be normally distributed was that for the free-moving manipulator and \( \omega_i = 1.5 \). It can be seen that the forcing function bandwidth has little effect on the probability distribution, and the pressure manipulator tends to accentuate
the non-normality of the output of the operator. The error signal is normally distributed, which means that the actual visual input to the operator is a Gaussian signal.

There is an apparent anomaly in these results, in that the two normally distributed signals, \( c(t) \) and \( h(t) \), have been summed at the operator's output to produce a non-normal \( c(t)^C \). This can be resolved by considering that the two signals are correlated, since \( c(t) \) contains remnant which has circulated around the loop. The probability distribution resulting from the sum of two correlated signals is not predictable. The theory which states that the summation of normally distributed signals will produce another normal variable applies only to uncorrelated signals [Ref.11].

The fact that the operator produces a non-normal output in response to a normally-distributed input indicates that his behaviour is nonlinear to a certain extent. The measured values of the relative remnant, \( c^2 \), have shown that the major part of the output of the operator is due to a linear response to the input, and thus this nonlinear behaviour should be considered as being superimposed on the linear output. A possible inference from the probability distributions is that the nonlinear behaviour consists of a pulsing form, or of periodic oscillations, which occur at a sufficiently high frequency that the attenuation of the rate-control vehicle dynamics is sufficient to restore normality to the system output. The amplitude of the pulsing is not at the limits of the manipulator, but is at a relatively consistent level of 1.0 - 1.5 \( \sigma \). It is now necessary to investigate the power-spectra of the operator's output for evidence of a spectral peak that could be associated with the source of nonlinear behaviour suggested by the probability and remnant data.

4.7 Power-Spectral Density of the Operator's Output:

The power-spectral density of the operator's output, \( \Phi_{cc} \), is presented in Fig.38 with the total subject variability. The results are for both manipulators and the three forcing functions. The spectra are directly compared in Fig.39. The variability is small except at high frequencies where some evidence of a spectral peak is present for \( \omega_4 = 1.5 \) and 2.5 for both manipulators. Since the variance is large, and the peaks occur at the same frequencies as the peaks in the describing functions, they could be attributed simply to the operation of the describing function on the error spectrum. However, the relatively low values of the correlation coefficient and the peak in \( \Phi_{nn} \) at the same frequencies, and the strong bi-modality of the operator's output, all serve to indicate that nonlinear behaviour is occurring.

A further important point is that the present results show that the operator is capable of producing high levels of output well beyond 18 rad/sec and attempts should be made to extend the measurement bandwidth to higher frequencies.

4.8 Behavioural Differences Between Subjects:

In order to determine what differences in behaviour between the subjects were being lost in the averaging process, all the individual spectra of the operator's output and the correlation coefficients were re-examined. It was found that the subjects could be divided into two groups, and that by coincidence there were four subjects in each group. Group A did not have a peak in the output spectrum within the measurement frequency range while
group B did, with a correspondingly lower value of correlation coefficient. Based on this grouping of the subjects, the rest of the describing function, remnant and probability distribution data were reviewed.

The grouped amplitude distributions are not shown but the values of $\chi^2$ appear in Table IX. The results show that both groups produced operator-output time records that were non-normal, except for $\omega_i = 1.5$, where group A tended to have normal outputs. Hence the difference between the groups was more a matter of degree than one of a fundamental mode of behaviour. Of particular interest are the grouped describing functions, shown in Fig. 40 for both manipulators and $\omega_i = 2.5$. The values of the fitted parameters, given in Table X, show that the main difference between the groups is in the damping terms of the second order component of the neuromuscular system and in the transport delay. The natural frequency of the second order term is unchanged, but group B has considerably lower damping than group A. Group B also has lower time delays and hence smaller phase lags. It can be seen that the change in the neuromuscular damping term has little effect on the amplitude ratio outside of the immediate vicinity of the peak, while the phase angle differences are visible over most of the frequency range. The low damping of group B produces much smaller phase lags. The differences between the groups are relatively independent of the forcing function and the manipulator.

The grouped performance measures of Table XI, show that the cross-over frequency is generally unchanged between the groups, while group B has larger phase margins, smaller relative remnants and lower scores than group A. The differences are less pronounced for the free-moving manipulator. The relative remnants and scores show that group B developed more nonlinear behaviour, but in doing so, managed to produce lower scores and hence better tracking performance. This improvement in score can be attributed to the lower phase lags that group B could achieve through the more marked nonlinear behaviour.

Figures 41 and 42 show the grouped spectra of the operator's output, $\Phi_{cc}$, and the remnant spectra, $\Phi_{nc}$ and $\Phi_{ne}$, for both manipulators and for $\omega_i = 2.5$. The differences between the groups are quite large and are relatively independent of the manipulator. It is interesting to note that the differences in $\Phi_{cc}$ between the groups are negligible at low frequencies, within the input bandwidth, but at higher frequencies, up to the frequency at which the peak occurs, group B produces lower values. Beyond this frequency group B has larger values of $\Phi_{cc}$. The total mean-square output of group B, $\sigma_c^2$, is larger, as is shown in Table XI, indicating that this group expends more energy during tracking by using larger manipulator outputs. The grouped remnant spectra of Fig. 42 reveal quite clearly the advantages of presenting these results in the form $\Phi_{nc}$ and $\Phi_{ne}$ exactly when plotted in this way, while large differences between the groups appear in $\Phi_{nc}$ at the natural frequency of the neuromuscular system.

4.9 Statistical Comparisons:

To place the differences between the various experimental conditions and the groups of subjects on a more quantitative basis, statistical tests were performed on a wide range of comparisons. The describing functions and power spectra which were involved included:

$$|Y_p|, \Delta Y_p, \rho, \Phi_{nn}, \Phi_{nc}, \Phi_{ne}, \Phi_{ee}, \Phi_{cc}, \text{and } \Phi_{oo}.$$
Before the statistical comparisons could be carried out it was necessary to check that the data to be tested were normally distributed. Since the analysis method was such that each frequency point could be treated as being independent of its neighbours, it was possible to perform the tests of normality and the comparisons of means on each frequency point individually.

The normality of the distributions was investigated for the following quantities, \(|\gamma|, \angle\gamma, \phi,\) and \(\Phi_{ne}\), for every frequency point for \(\omega = 4.0\) and both manipulators, and for selected frequency points and both manipulators for \(\omega = 1.5\) and \(2.5\). The method of testing the distribution of a particular variable was to calculate the cumulative probability distribution and compare it graphically with the normal distribution. A \(\chi^2\) test of fit could not be employed with any degree of accuracy because of the small number of samples. The cumulative probability distributions were plotted on normal probability paper in the form of standard scores, \((x - \bar{x})/ s\), where \(x\) is the value of the variable, \(\bar{x}\) is the sample mean, and \(s\) is the sample standard deviation. If a significant number of points lay \(x\) outside the confidence limits, which were also plotted for that particular sample size, then the data was considered to be non-normal at that confidence level. A representative set of results is shown in Fig. 43 with the 95% confidence limits.

It was found that the grand averages were normally distributed for all the quantities tested, for the three forcing functions and both manipulators, at low and medium frequencies. At high frequencies, where the neuromuscular-system differences occurred, the distributions tended to be non-normal. However, when the groups were examined, the high frequency distributions were again normal. For this reason, the mean values were compared finally only for the grouped results.

The testing of the differences between the means was performed using the Aspin and Welch form of the "t-test" since the variances of the distributions could not be assumed to be the same [Ref. 6]. Of the large number of variables that were tested only \(|\gamma|, \angle\gamma,\) and \(\Phi_{ne}\) are of particular interest, and these are presented for the various comparisons in Table XII. The results are given in the form of a blank, 1 or 2 to show that the differences between the means were non-significant, of possible significance (95% level), or significant (99% level), respectively.

The three comparisons investigated were, 1) Between the groups of subjects, 2) Between manipulators, and 3) Between forcing functions. No important new information was gained from the results of the statistical tests, particularly for the describing function differences, for which the tests tended simply to confirm the conclusions which had already been reached from visual examination. However, for the remnant spectrum, \(\Phi_{ne}\), some of the differences which had been previously ignored proved to be significant. These differences occurred for the group B results when comparing manipulators and for both groups when comparing forcing functions. The small variance of \(\Phi_{ne}\) produced these significant differences when visual examination showed that the magnitudes of the differences were in themselves relatively small. From the overall system point of view, these differences in \(\Phi_{ne}\), although statistically significant, are sufficiently small, and extend over small enough frequency ranges that they may be ignored.
A review of the results of the statistical comparisons shows that most of the differences which exist in the describing functions are significant when comparisons are made between subject groups, manipulators and forcing functions. On the other hand, the differences in the observation remnant spectra may be treated, on the whole, as being insignificant.

4.10 Cross-over Frequency Regression:

Inadvertently, this study has gathered a considerable amount of data on cross-over frequency regression. Regression is a phenomenon which occurs when the operator is presented with a forcing function which he feels is too difficult to track and still maintain stability, and he makes a radical change in his tracking technique, which appears as a reduction in gain in the describing function. In the present experiments its presence is most clearly shown in the results for the effect of the high-frequency shelf. The forcing function without the shelf allows the operator to develop performance levels which compare very well with published results of other experiments for forcing functions of the same nominal bandwidth, while the inclusion of the shelf results in both regression and nonlinear behaviour. The present results have shown regression to occur for both manipulators and for the three forcing functions with the shelf, for which the cross-over frequencies of the describing functions were in fact less than the input bandwidths.

Cross-over frequency regression deserves considerable further study, particularly since its onset cannot be properly predicted. Parameter adjustment rule "5-c" states that regression will occur when \( \omega_i \) approaches \( 0.8 \omega_c \), where \( \omega_c \) is the value of \( \omega_c \) when \( \omega_i \ll \omega_c \). This criterion is based on the use of rectangular-spectra forcing functions. For the non-rectangular spectra of this study the equivalent rectangular bandwidth, \( \omega_{ie} \), should be used instead. However, the present results have shown that, for continuous-spectrum forcing functions, \( \omega_{ie} \) is of very little use in predicting regression. This is quite obvious for the case of the inclusion of the high-frequency shelf to the \( \omega_i = 2.5 \) forcing function, which caused only a very small change in \( \omega_{ie} \), (see Table I), but which resulted in regression. Since the forcing functions used in this study were of the same nominal shape and bandwidth as the STI forcing functions of Ref. 26, it is apparent that the regression is due to the amplitude of the high-frequency shelf components. Thus it is necessary to find the minimum attenuation of the high-frequency shelf of a continuous-spectrum forcing function which will allow the identification of the describing function up to at least 18 rad/sec without causing regression. This is quite an important matter since it is often more convenient to develop data reduction methods based on filtered-noise forcing functions than on sums of sine waves. However, the large attenuation of the shelf components and the attendant low signal levels may require the use of more sophisticated recording techniques, including pre-emphasis.

Physically, regression is an attempt by the operator to improve the stability of the closed-loop system at the expense of the bandwidth, by reducing the gain of the open-loop describing function. The results presented here have indicated that regression is accompanied by nonlinear high-frequency pulsing behaviour, which allows an even further reduction in phase lags and consequently larger phase margins and increased stability. The reduction in bandwidth results in larger tracking errors and poorer performance scores.

For the single reliable case of regression in the literature, from Ref. 28,
shown in Fig.25, the cross-over frequency and phase margin agree very well with our results, as shown in Table VI. This implies that the characteristics of a regressed system are relatively independent of the type of forcing function that is being employed. In addition, the good agreement for the phase margins suggests that pulsing behaviour may have occurred in the case of Ref. 28 as well, since the low phase lags could be achieved by such behaviour. Unfortunately, the bandwidth of the describing functions is too low to identify the high-frequency peak that should be present. Further evidence for the generally invariant nature of regressed systems is the low variability of the describing functions of both the present experiments and the referenced data.

It is important to note that the "regressed" operator is still a useful system element. This study has shown that his linearity is not degraded and that the magnitude of the remnant is similar to that for non-regressed operators. From the overall system point of view, the only major changes that will be observed, when regression occurs, are that the tracking errors will be larger than they would be for the non-regressed case, and that the operator may employ nonlinear behaviour in the form of high-frequency pulsing, provided that the dynamics of the limb/manipulator combination allow it to develop.

The nonlinear pulsing behaviour could be described physically as being due to a source within the operator of random amplitude, high-frequency pulses or oscillations. Its occurrence is usually associated with acceleration-control vehicle dynamics, but apparently it can occur also for rate-control. The time records of the operator's output show that the oscillations are approximately sinusoidal in form. The amplitudes of the pulses are sufficiently well correlated with the forcing function that a large amount of this oscillatory output can be modelled by the second order component of the neuromuscular-system dynamics. The remainder is attributed to the remnant. The more pronounced the pulsing behaviour, the lower is the damping of the neuromuscular system. The examination of the results for the two groups of subjects showed that the principal effect of the pulsing behaviour was the reduction of phase lags over a wide frequency range, and a consequent improvement in performance. This leads to the conclusion that the operator makes use of the dynamics of the limb/manipulator combination to improve system performance. Since this behaviour coincided with regression, one could also conclude that the operator, having regressed in order to maintain stability, still tries to achieve a reasonable performance level and attempts to do so by considerably reducing the damping of the neuromuscular system. However, if the dynamics of the manipulator are such that the operator is unable to develop this behaviour without considerable effort, then the tracking performance may be severely degraded.

4.11 Status of the Information Available on the Remnant:

An exhaustive review of the present status of information about the remnant is beyond the scope of this study, but a brief summary of the main points is possible. The combination of the results of these experiments and the relatively small quantity of reliable remnant measurements that have been published, some of it very recently, allows the following summary:

1) The remnant power spectrum tends to coalesce best, and to be least dependent on other parameters, when it is referred to the input of the operator as the "injected observation remnant". In this case the variability of the spectrum due to the subjects is quite small.
2) In general, the absolute values of the remnant spectrum and the trends due to the forcing-function bandwidth have been shown to be independent of the type of forcing-function spectrum. The injected observation remnant is relatively independent of the forcing-function bandwidth and amplitude, when normalized with respect to the mean square input. This spectrum is also independent of the type of manipulator, and of the gains of the manipulator and the display. The gain of the controlled system has a slight effect, while the system order is of major importance.

3) The spectrum of the observation remnant is almost unaffected by the presence of pulsing behaviour and cross-over frequency regression. The time record of the remnant is normally distributed, irrespective of the controlled vehicle dynamics, the forcing function, the manipulator and nonlinear behaviour by the operator.

4) The remnant can then be considered to be due to time-variability, true noise injection and aperiodic sampling. Time-variability seems to be the major source. However, the form of presentation of the remnant spectrum will affect the conclusion as to the major physiological source. For instance, if the injected output remnant is considered, then the neuromuscular system appears to have a large effect, while on the other hand, the injected observation remnant shows little dependence on the state of the neuromuscular system. This leads to the impression that the main source of the remnant is time-variability in the parameters of the observational and computational processes "upstream" of the neuromuscular system. On the whole, the experimental results suggest that the remnant is produced by a relatively invariant source within the operator.

5) There are still not enough high-quality measurements available to permit the formation of a definitive model of the remnant. If necessary for system design purposes, and only as a first approximation, the injected observation remnant can be fitted by a power transfer function with the form of a first order lag or the more complex function developed in this study. These models ignore the differences due to the controlled vehicle dynamics. One promising recent model [Ref.22] treats the remnant as being due solely to noise from the physiological processes internal to the operator. In that approach the remnant is related to that quantity in the error signal in which the operator is assumed to be most interested. For a pure-gain controlled vehicle this is error magnitude; for acceleration control it is error rate; for rate control it is a combination of error and error rate. When correctly normalized, the spectrum of the injected observation remnant coalesces very well for the pure-gain and the acceleration controlled vehicles, while the remnant for the rate control case can be predicted reasonably well.

V SUMMARY AND CONCLUSIONS

Each individual section above has contained considerable detailed discussion, so this section will serve only to summarize the major points in the results and draw conclusions.

The objective of this study has been to add to the data base on the remnant portion of the mathematical model of the human operator by investigating the effects on the remnant of the bandwidth of the forcing function and the type of manipulator, for a single-axis tracking task. The mathematical model of the human operator comprises a describing function and a
remnant. The remnant is defined as that part of the output of the human operator which is not linearly correlated with the forcing function. The experiment involved the use of a compensatory display with rate-control vehicle dynamics and forcing functions of filtered Gaussian noise.

Eight subjects took part in the program, after accumulating about 20 hours of tracking experience each during the training phase. Pressure and free-moving manipulators were employed, and the forcing functions were similar in basic shape and bandwidth to the so-called STI forcing functions, which consist of a sum of several sine waves. Data was collected on the effects of the forcing function and the manipulator on the describing function, the performance measures of the system and on the power spectrum of the remnant. Time records of the remnant were isolated from the operator's output, and the effects of the forcing function and the manipulator on the amplitude probability distributions of all the signals in the loop, including the remnant, were investigated.

The results obtained during the course of this study have been shown to be, on the whole, both more accurate and more precise than those available in the literature. This is a consequence of the techniques developed for data measurement and reduction, of the experimental design, and of the training of the relatively large group of subjects.

The scores and describing functions have shown effects that can only be attributed to the type of forcing function employed. The low variability of the describing functions, and the agreement with previous measurements of scores for the STI forcing functions, allow considerable confidence in the results as a whole. The changes in the describing function due to the input bandwidth and the manipulator have been shown to be in general agreement with the few results that have been published. However, the measurement bandwidth is still insufficiently wide to completely determine the describing function at high frequencies.

Although the inputs used in this study were of the same nominal shape and bandwidth as those of the sum-of-sine-waves forcing functions in the literature, nearly all the describing functions obtained in the current study have been regressed. The principal cause has been the presence of the high-frequency shelf. The inputs made up of sums-of-sine-waves, while appearing random in nature, are obviously subjectively much easier to track than the more realistic inputs obtained from the filtering of random noise. The onset of cross-over frequency regression cannot be predicted for filtered-noise inputs, which have the high-frequency shelf, when using the criteria developed for sums-of-sine-waves forcing functions. Considerably more work is required in this area.

In conjunction with the regression, the describing functions have shown the presence of a high-frequency peak that can be modelled by the second order component of the neuromuscular system. The amplitude distributions and the spectra of the operator's output show that this peak is the result of nonlinear behaviour in the form of pulsing, which is sufficiently well correlated with the forcing function that some of its effect appears in the describing function, the rest being explained by a peak in the remnant spectrum at the same frequency.

Useful insights have been gained into the mechanics of cross-over fre-
quency regression. The process has been shown to be the result of a simple reduction in the gain of the describing function, the frequency-dependent parameters remaining unchanged. The variability of the regressed describing functions is very low and the values agree well with another case of regression in the literature. Regression is apparently independent of the type of forcing function which causes it, for the same controlled-vehicle dynamics. The pulsing behaviour which accompanies the regression is clearly a technique to reduce the phase lags, and this is shown by the better scores obtained when the pulsing behaviour is most pronounced.

The influence of the manipulator and of the forcing function on the power spectrum of the injected observation remnant is small, and the spectrum is relatively flat at medium frequencies for the rate-control vehicle dynamics, with an increase in magnitude at low frequencies and falling off sharply at high frequencies. The variability due to the subjects in the remnant spectrum is quite small, and this is an important result. It is excellent evidence for the description of the remnant as being due to a relatively stable, or invariant, source within the human operator.

The results of this study have brought to light a very striking characteristic of the remnant concerning its amplitude probability distribution. For the first time, it has been possible to obtain reliable estimates of the time record of the remnant and it has been found that the remnant is normally distributed, irrespective of the forcing function, the manipulator and the presence of regression and nonlinear behaviour. The only major source of variation in the remnant that remains to be investigated in depth is that due to the dynamics of the controlled vehicle.

Having reviewed the main points of the results we can now draw the following conclusions:

1) The data collection and analysis techniques employed in this study have allowed excellent identification of the describing function of the human operator and the isolation of the time record of the remnant.

2) By means of a carefully designed experiment and an extensive training program, it has been possible to obtain very good results of low variability from subjects who were initially inexperienced in manual tracking.

3) The type of forcing function used has been shown to be of major importance to the describing function. The results for the more realistic filtered-noise inputs and those made up of sum-of-sine-waves cannot be directly compared.

4) However, the power spectrum of the remnant, when it is considered as an independent signal injected at the visual input to the operator, has been found to be unaffected by the type or bandwidth of the forcing function, or the type of manipulator.

5) The time record of the remnant is normally distributed, irrespective of the forcing function, the manipulator and nonlinear behaviour by the operator.
6) When the forcing function presents a sufficiently demanding tracking task to the operator, he will develop pulsing behaviour as well as cross-over frequency regression in order to improve the level of performance, even for rate-control vehicle dynamics.

7) The regressed operator is still a useful system element, since the presence of regression has no influence on the linearity of the operator within the bandwidth of the forcing function.

8) The results have shown that the operator is capable of producing output beyond 18 rad/sec. It is apparent also that further measurements of the remnant are required before an adequate model can be developed. These additional studies should yield data that is of low variability and wide bandwidth, investigating in particular the effects of the dynamics of the controlled vehicle on the remnant.

9) Another fruitful area of research is that of the shape of the power spectrum of the forcing function when filtered Gaussian noise is used. It is necessary to determine the shape of the spectrum that will allow the identification of the describing function of the human operator over a wide bandwidth without causing cross-over frequency regression.
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APPENDIX A: The Precision Model of the Human Operator and the Parameter Adjustment Rules

The most general and complex form of the describing function model of the human operator was first presented in Ref. 26. This model is of course a functional model; it attempts to describe only the relationship between the input and output of the human operator and not the fine internal structural details. By examination of extensive data it has been possible to correlate various terms of the model with the known physiology, but still in functional terms. The form of the model is given below:

\[ Y_p(j\omega) = K_p K_T \left[ \frac{a_T}{\sigma_T} \right] e^{-j\omega \tau_d} \left( \frac{T_L j\omega + 1}{T_I j\omega + 1} \right) \left[ \frac{T_K j\omega + 1}{T_K' j\omega + 1} \right] \left( \frac{1}{T_N j\omega + 1} \right) \left( \frac{\omega_N^2}{(j\omega)^2 + 2\xi_N \omega_N (j\omega) + \omega_N^2} \right) \]

where \( K_p \) = Gain.
\( \tau_d \) = Reaction Time Delay.
\( K_T \left[ \frac{a_T}{\sigma_T} \right] \) = Indifference Threshold Describing Function.
\( \left( \frac{T_L j\omega + 1}{T_I j\omega + 1} \right) \) = Adaptive Equalization Characteristics.

The parameter adjustment rules are the second part of the analytical model and indicate the form that the model will take in a particular control situation. The adjustment rules have been confirmed and validated by a considerable body of experimental measurements and are set out below, taken from Ref. 26:

1) Stability: The human operator adapts the form of the equalizing characteristics to achieve stable control, i.e. the closed-loop system of which he is an element will produce a bounded output for a bounded input.

2) Form Selection - Low Frequency: The human adapts the form of his equalizing characteristics to achieve good low-frequency closed-loop system response to the forcing function. A low-frequency lag, \( T_L \), is generated when both of the following conditions apply:

   a. The lag would improve the system low-frequency characteristics.

   b. The controlled element characteristics are such that the introduction of the low-frequency lag will not result in destabilizing effects at higher frequencies which cannot be overcome by a single first order lead, \( T_L \), of somewhat indefinite but modest size.
3) Form Selection - Lead: After good low-frequency characteristics are assured, within the above conditions, lead is generated when the controlled element characteristics, together with the reaction time delay, are such that a lead term would be essential to retain or improve high-frequency system stability.

4) Parameter Adjustment: After adaptation of the equalizing form, the describing function parameters are adjusted so that:

   a. Closed-loop low-frequency performance, in operating on the forcing function, is optimum in some sense analogous to that of minimum mean-square tracking error.

   b. System phase margin, $\phi$, is directly proportional to $\omega$, the forcing-function bandwidth, for values of $\omega$ less than about 2.0 rad/sec. The strong effect of forcing-function bandwidth on the phase margin is associated with the variation of the neuromuscular system parameters with the same task variable.

   c. Equalization time constants, $T_L$ and $T_I$, when form selection requires $1/T_L$ or $1/T_I < \omega_c$, will be adjusted such that low-frequency response will be essentially insensitive to slight changes in $T_L$ or $T_I (\omega_i << \omega_c)$.

5) Invariance Properties of $\omega_c$:

   a. $\omega_c - K_c$ Independence: After initial adjustment, changes in controlled element, $K_c$, are offset by changes in operator gain, $K_o$, i.e. system cross-over frequency, $\omega_c$, is invariant with $K_c$.

   b. $\omega_c - \omega_i$ Independence: System cross-over frequency depends only slightly on forcing-function bandwidth for $\omega_i < 0.8 \omega_c$ ($\omega_c$ is that value of $\omega_c$ adopted for $\omega_i << \omega_c$).

   c. $\omega_c$ Regression: When $\omega_i$ nears or becomes greater than $0.8 \omega_c$, the cross-over frequency regresses to values much lower than $\omega_c$.

The considerable body of experimental results has shown that the human operator tends to adapt the form of the describing function model such that it looks very much like the one that would have been chosen for a "black box" to control the same system. In particular, the properties of a "good" feedback control system have been described as follows:

1) The ability to provide the specified input-output response characteristics. This means that the cross-over frequency, which determines to a large extent the bandwidth of the system, should be larger than the forcing function bandwidth, such that, within the forcing-function bandwidth, the amplitude ratio of the open-loop describing function is large.
2) The ability to suppress unwanted inputs and disturbances. This means that above $\omega_c$, the gain of the closed-loop system should be low so as to attenuate any inputs at higher frequencies.

3) Reduce the effects of time variations of elements in the loop.

One of the simplest "good" linear systems is one in which the open-loop describing function is approximately a single integrator. The human operator tends to adjust the parameters of his describing function such that the open-loop describing function, $Y_pY_C(j\omega)$, has precisely this form in the region of cross-over. At low and high frequencies, however, the neuromuscular system terms are dominant. Thus, for rate-control vehicle dynamics, the application of the parameter adjustment rules will result in the human-operator describing function having the form, at medium frequencies, of a pure gain together with the transport delay:

$$Y_p(j\omega) = K_p e^{-j\omega T_d} \quad (A,2)$$
APPENDIX B: The Impulsive Response of the Human Operator

It has been shown [Refs. 8, 15, 29] that the response, h(t), of a linear system with transfer function H(s), to an impulse δ(t), may be obtained from the inverse Laplace transform of the system transfer function. This response is known as the Impulsive Response, or Impulsive Admittance:

\[ h(t) = \frac{1}{2\pi} \int_{\gamma-j\infty}^{\gamma+j\infty} H(s) e^{st} \, ds \]  

(B,1)

If the system is stable, that is, all the poles of H(s) lie in the left half of the s-plane, then the Laplace variable, s = \( \sigma + j\omega \), can be replaced with s = j\omega. The transfer function then becomes the frequency response, \( H(j\omega) \), by replacing s with j\omega, and the inverse Laplace transform becomes the inverse Fourier transform:

\[ h(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} H(j\omega) e^{j\omega t} d\omega \]  

(B,2)

Now, if the frequency response is multiplied by some real function of frequency, \( W(\omega) \), and the inverse transform performed, we have:

\[ y(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} H(j\omega) W(\omega) e^{j\omega t} d\omega \]  

(B,3)

which may be shown to become:

\[ y(t) = \int_{-\infty}^{\infty} \beta(u) h(t-u) \, du \]  

(B,4)

If \( H(j\omega) \) is physically realizable, then

\[ h(t) = 0 \quad \text{for} \quad t < 0 \]

i.e.

\[ y(t) = \beta(t) \ast h(t) \]  

(B,5)

where

\[ \beta(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} W(\omega) e^{j\omega t} d\omega \]  

(B,6)

and the \( \ast \) denotes convolution.

Thus it can be seen that the inverse Fourier transform of the product of two functions of frequency is equivalent to the convolution of the individual transformed functions in the time domain. If the weighting function \( W(\omega) \) is an even function of frequency, and has a "box-car" shape, such that:
then the inverse transform of $W_o(\omega)$ yields:

$$\beta_o(t) = \frac{\omega_m}{\pi} \frac{\sin(\omega_m t)}{\omega_m t}$$

The product of the frequency response $H(j\omega)$ with the "box-car" weighting function, $W(\omega)$, exactly represents the frequency truncation of $H(j\omega)$ due to bandwidth limitations of the digital analysis in the present study, and the very best estimate of the impulse response that can be made under these conditions is that involving the convolution of the true impulse response with some "response window", $\beta(t)$.

The response window, $\beta(t)$, can be seen to be analogous to the spectral window involved in the estimation of the power spectral density discussed in Ref. 7 and Appendix C, with frequency being replaced by time as the independent variable.

Now, if the frequency response is also known only as a discrete function of frequency, that is, it is only known for a finite number of points with a frequency interval of $\Delta \omega$ then the additional problem of "aliasing" is involved, which results in the response window becoming the aliased response window (a continuous function of time), with the time interval between the main lobes proportional to $\Delta \omega$ and the shape that of the window $\beta(t)$, given in Fig. 44. Of particular interest in this study is the time averaging of the true impulse response caused by the response window. As shown in Ref. 7, the narrowest window is that resulting from the box-car weighting function, $W(\omega)$, and its equivalent width is $B = \pi/\omega_m$ seconds. The parameters chosen for the power and cross-spectral estimation, and the use of the Hanning window, result in a frequency interval of 0.5 rad/sec between estimates of the describing function $Y(j\omega)$, and the truncation in frequency takes place at 18 rad/sec. The resulting aliased response window then has an equivalent width of $B = 0.175$ seconds, and a recurrence interval of $A = 12.56$ seconds.

The impulsive response data of Refs. 36 and 40, although for different experimental conditions, show that the details of interest are contained in the first 0.8 seconds of the time function. Specifically, the important characteristics of the impulsive response that would suffer most from the smoothing effect of the response window are those of the transport delay and the terms associated with the high frequency response of the human operator. Even the narrowest window, $\beta(t)$, would have the effect of modifying these terms, and any time record obtained using this estimate of the impulsive response would be in error. The best possible estimate that could be obtained of the output of a linear model of the human operator would then be, for $e(t)$ and $c_e(t)$ the input and output of the model respectively:

$$c_e(t) = e(t) * \gamma_p(t) * \beta_o(t)$$

(B.9)
Since the accuracy of the estimate of the time record of the remnant was of major importance in this study, this particular approach was abandoned.

An alternative procedure was then developed, based on replacing the measured describing function with a physically realizable, linear, stable model and performing the inverse Fourier transform on this model. By this means the bandwidth of the describing function is artificially extended to infinity and the resultant response window becomes a delta function, such that all of the details of interest in the impulsive response can be identified. This process involves curve-fitting the describing function with some function of frequency such that the finite discrete function can be replaced with the continuous function of frequency of infinite bandwidth. The inverse transform now has a closed form solution.

A model of sufficient complexity, and which has been shown to give excellent agreement with actual measured describing functions is the precision model of Ref.26 described in Appendix A. For convenience it is repeated here, using the Laplace variable (s):

\[ Y_p(s) = K_p e^{-\frac{\tau_d}{T_p}} \left( \frac{T_L s+1}{T_I s+1} \right) \left( \frac{T_K s+1}{T_K s+1} \right) \left( \frac{1}{T_N s+1} \right) \frac{\omega_N^2}{(s^2+2\zeta_N \omega_N s+\omega_N^2)} \]  \hspace{1cm} (B,10)

In actual practice it proved simpler to perform the inverse transform as a Laplace transform by splitting \( Y_p(s) \) into partial fractions and using a table of transforms to find the terms of the solution [Ref.15]. If we let \( A(s) \) be that part of \( Y_p(s) \) which does not contain the time delay, \( \tau_d \), then eq. (B,10) can be rewritten as:

\[ Y_p(s) = e^{-\frac{\tau_d}{T_p}} A(s) \]  \hspace{1cm} (B,11)

where \( A(s) = K_p \left( \frac{T_L s+1}{T_I s+1} \right) \left( \frac{T_K s+1}{T_K s+1} \right) \left( \frac{1}{T_N s+1} \right) \frac{\omega_N^2}{(s^2+2\zeta_N \omega_N s+\omega_N^2)} \) \hspace{1cm} (B,12)

and hence

\[ y_p(t) = \mathcal{L}^{-1} \left[ e^{-\tau_d s} A(s) \right] = A(t-\tau_d) \]  \hspace{1cm} (B,13)

The inverse Laplace transform of \( A(s) \) is:

\[ A(t) = c_1 e^{\lambda_1 t} + c_2 e^{\lambda_2 t} + c_3 e^{\lambda_3 t} - c_4 e_a (c_5 \cos\omega_a t + c_6 \sin\omega_a t) \]  \hspace{1cm} (B,14)

where

\[ \lambda_1 = -1/T_I ; \lambda_2 = -1/T_K ; \lambda_3 = -1/T_N ; \]

\[ n_a = -\xi_N \omega_N ; \omega_a = \omega_N \sqrt{1-\xi_N^2} \]  \hspace{1cm} (B,15)
and where the coefficients $c_1, c_2, c_3, c_4, c_5, c_6$ are functions of $K_p, T_I, T_L$ etc.

Replacing $t$ by $(t-\tau_d)$ yields the complete impulsive response incorporating the transport delay. Since the model is physically realizable, the value of $\psi_p(t)$ will be zero for negative time. The presence of the time delay alters this criterion slightly to

$$\psi_p(t) = \Omega(t-\tau_d) \text{ for } t \geq \tau_d$$
$$= 0 \text{ for } t < \tau_d \quad (B,16)$$

For completeness the expressions for the coefficients $c_1$ to $c_6$ are written out below. Since the values of the coefficients were computed numerically, no attempt has been made to simplify the expressions and they are given in their basic forms. In addition to the roots $\lambda_1, \lambda_2, \lambda_3, n_a, \omega_a$ which were defined above, in eq. (B,15), we require the following substitutions for the zeros, $\alpha, \beta, \gamma, \omega_a$ and the constant term, $K$:

$$\alpha = -1/T_L; \quad \beta = -1/T_K \quad (B,17)$$

and

$$K = \frac{K_p \omega^2 N}{T_I T_K'} \frac{T_L}{T_K} \frac{T}{T_{N_1}} \quad (B,18)$$

We can then write:

$$c_1 = \frac{K(\lambda_1-\alpha)(\lambda_1-\beta)}{(\lambda_1-\lambda_2)(\lambda_1-\lambda_3)[(\lambda_1-n_a)^2 + \omega_a^2]} \quad (B,19)$$

$$c_2 = \frac{-K(\lambda_2-\alpha)(\lambda_2-\beta)}{(\lambda_1-\lambda_2)(\lambda_2-\lambda_3)[(\lambda_2-n_a)^2 + \omega_a^2]} \quad (B,20)$$

$$c_3 = \frac{K(\lambda_3-\alpha)(\lambda_3-\beta)}{(\lambda_1-\lambda_3)(\lambda_2-\lambda_3)[(\lambda_3-n_a)^2 + \omega_a^2]} \quad (B,21)$$

$$c_4 = \frac{K}{\omega_a (d_3^2 + d_4^2)}; \quad c_5 = (d_2 d_3 - d_1 d_4); \quad c_6 = (d_1 d_3 + d_2 d_4) \quad (B,22)$$

where

$$d_1 = (n_a-\alpha)(n_a-\beta) - \omega_a^2; \quad d_2 = \omega_a(2n_a-\alpha-\beta)$$

$$d_3 = (\lambda_3-n_a)[(\lambda_1-n_a)(\lambda_2-n_a) - \omega_a^2] + \omega_a^2(2n_a-\lambda_1-\lambda_2) \quad (B,23)$$

$$d_4 = \omega_a[(\lambda_3-n_a)(2n_a-\lambda_1-\lambda_2)-(\lambda_1-n_a)(\lambda_2-n_a) + \omega_a^2]$$

$\quad (B,24)$
APPENDIX C: The Estimation of Power and Cross-Spectral Densities

The power and cross spectral densities required in this study may be defined as the Fourier transforms of the appropriate auto and cross correlation functions:

\[ \Phi_{xx}(\omega) = \int_{-\infty}^{\infty} R_{xx}(\tau) e^{-j\omega \tau} \, d\tau \quad (C,1) \]

and

\[ \Phi_{xy}(\omega) = \int_{-\infty}^{\infty} R_{xy}(\tau) e^{-j\omega \tau} \, d\tau \quad (C,2) \]

for \(-\infty < \omega < \infty\)

where \(x(t)\) and \(y(t)\) are the time records and \(R_{xx}(\tau)\) and \(R_{xy}(\tau)\) are the corresponding correlation functions defined below:

\[ R_{xx}(\tau) = \lim_{T \to \infty} \frac{1}{\Delta T} \int_{-T}^{T} x(t) x(t+\tau) \, dt \quad (C,3) \]

and

\[ R_{xy}(\tau) = \lim_{T \to \infty} \frac{1}{\Delta T} \int_{-T}^{T} x(t) y(t+\tau) \, dt \quad (C,4) \]

and where

\[ \sigma^2_x = R_{xx}(0) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \Phi_{xx}(\omega) \, d\omega \quad (C,5) \]

These forms of definitions involving time averages are only appropriate for ergodic processes. An ergodic process is one for which the time average over a single infinite time record, produced by some process, is equivalent to the ensemble average, at one given time, over an infinite number of individual time records from the same process.

However, since the time records in this study are obtained by sampling at a predetermined rate and are of finite length, two types of errors are introduced into the estimation of the underlying spectra: 1) "Aliasing", the result of the discrete sampling of the time record and 2) The "Spectral Window", a consequence of the finite record length. These phenomena have been fully discussed in the literature, [Ref.7] and have been reviewed with relevance to the estimation of the describing function of the human operator in a companion study [Ref.33]. For completeness the techniques employed will be very briefly summarized here.

The time records are available only in finite, discrete form, as \(x(t)\) and \(y(t)\), where \(t = n\Delta t\), for \(n = 0,1,2,...,m-1\), and the record length is \(T_m\), where \(T_m = (m-1)\Delta t\). The sampling rate is \(1/\Delta t\). The integrals to form the correlation functions are replaced by summations, such that the correlations are known only for discrete values of \(\tau\), i.e. \(\tau = n\Delta t\), for \(-T_m < \tau < T_m\). The integrals of the Fourier transformations of the correlation functions \(m\) (yielding the spectral estimates) also become summations. These estimates of the spectra
are written as $\Phi_{xx}(\omega,\tau,\Delta \tau)$, and $\Phi_{yy}(\omega,\tau,\Delta \tau)$ to show that they are calculated from finite, discrete time records. We are primarily interested in the mean values and variances of these spectral estimates.

It can be shown that the Fourier transform of the finite, discrete correlation function can be written as the Fourier transform of the continuous correlation function, $R_{xy}(\tau)$, multiplied by a finite Dirac delta "comb", $\nabla_m(\tau,\Delta \tau)$, and the expected value of this estimate is given as:

$$E[\Phi_{xx}(\omega,\tau_m,\Delta \tau)] = \int_{-\infty}^{\infty} [\nabla_m(\tau,\Delta \tau) R_{xx}(\tau)] e^{-j\omega \tau} \, d\tau$$  \hspace{1cm} (C,6)

where

$$\nabla_m(\tau,\Delta \tau) = \frac{\Delta \tau}{2} \left[ \delta(\tau + \tau_m) + \delta(\tau - \tau_m) \right] + \Delta \tau \sum_{q=-m+1}^{m-1} \delta(\tau - q\Delta \tau)$$  \hspace{1cm} (C,7)

and $\tau_m = (m-1)\Delta \tau$.

The transform of the product of the two time functions can be rewritten as the convolution in the frequency domain of the transforms of the individual time functions, and the expected values of the spectral estimates are now:

$$E[\Phi_{xx}(\omega,\tau_m,\Delta \tau)] = W(\omega) \ast \Phi_{xx}(\omega)$$  \hspace{1cm} (C,8)

where

$$W(\omega) = \int_{-\infty}^{\infty} \nabla_m(\tau,\Delta \tau) e^{-j\omega \tau} \, d\tau$$  \hspace{1cm} (C,9)

is a weighting function called the "Aliased Spectral Window" which, for a box-car shaped function will have the shape of $W(\omega)$ shown in Fig.44. It will be noted that the width of the main lobe is determined by the maximum correlation delay, $\tau_m$, and the window is repeated every $2\pi/\Delta \tau$ rad/sec. This aliased window is a continuous function of frequency and exists for infinite frequency. The width of the main lobe and the contributions of the secondary side lobes, when convoluted with the true spectrum, produce a smoothing effect, and if the true spectrum has power over a wider bandwidth than $\pm \pi/\Delta \tau$ rad/sec then the adjacent windows will cause this power to be added to that at the centre frequency of the primary window, resulting in aliasing.

There are several types of spectral window. As discussed, a simple box-car truncation of the correlation function results in the $W(\omega)$ window of Fig.44. The large side lobes of the spectral window are a distinct disadvantage and another window, the "Hanning Window"[Ref.7] is often employed because of its much smaller side lobes. This window is shown in Fig.44 as $W_\omega(\omega)$. However, this window has a wider central lobe, with an equivalent width of $1.3\pi/\tau_m$ seconds. In order to obtain spectral estimates which are independent of each other it is then necessary to use only every second spectral estimate calculated with the Hanning Window, and the frequency resolution will then be $2\pi/\tau_m$ rad/sec.

The derivation of the expression for the variance of the estimates of the smoothed power spectral density is more complicated, and requires the
assumption that the true spectrum does not vary much within the width of the central lobe of the window. An approximate expression is given below for use with the Hanning window:

\[
\text{Variance } \left( \Phi_{XX}(\omega, \tau, \Delta \tau) \right) = 0.75 \frac{\tau_m}{T'} \text{ for } \omega \geq \frac{1.3 \pi}{\tau_m} \quad (C,10)
\]

increasing to \(1.5 \frac{\tau_m}{T'}\) for \(\omega \ll \frac{1.3 \pi}{\tau_m}\).

where \(T' = T_m - 0.3 \tau_m\).

The variance of the estimates of the smoothed cross spectral density has no simple expression and it can only be estimated from the experimental data for the particular measurement situation.

It can be seen that to obtain the least variance in the estimates we require as small a ratio of \(\tau_m/T'\) as possible, while for the best frequency resolution i.e. the narrowest spectral window, \(\tau_m\) should be as large as possible. Since \(T_m\) is usually restricted to some maximum value, a compromise between these conflicting constraints is required. Experience has shown that the ratio \(\tau_m/T'\) should not exceed 0.1. Further, for least aliasing effects the sampling rate should be as high as possible, at least twice the highest frequency at which power can be expected to exist in the spectrum to be measured. For signals containing noise it has been suggested that the sampling rate be maintained at about ten times the highest frequency of interest [Refs. 4,5]. An upper limit on the sampling rate is set by data storage limitations.

The correlation functions discussed above can be calculated directly from the definitions using the summation of the lagged products. However, when large time delays are employed, this can be very time-consuming, even on a high speed digital computer. An algorithm has been developed which allows considerable time and storage savings when combined with the Fast Fourier transform [Refs. 9, 17, 34]. The basic equations are given below, defined for finite continuous time records. The extension to finite discrete time records is trivial, provided that the time records are considered to be periodic, with period \(T_m\).

From eq. (C,4) we write the definition of the cross-correlation between two time records \(x(t)\) and \(y(t)\) as:

\[
R_{XY}(\tau) = \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} x(t) y(t+\tau) dt \quad (C,11)
\]

Since we are dealing with finite time records of length \(T_m\), such that \(x(t) = x(t)\) for \(|t| \leq T_m/2\) and is zero elsewhere, we can replace the limiting process in the above integral by one in which the end points are set to infinity, i.e.

\[
R_{XY}(\tau) = \frac{1}{T_m} \int_{-\infty}^{\infty} x(t) y(t+\tau) dt \quad (C,12)
\]

The substitution of the inverse Fourier transform of \(y(t+\tau)\) into this equation and the interchange of the order of integration, allows us to write:
\[ R_{xy}(\tau) = \frac{1}{2\pi T} \int_{-\infty}^{\infty} X^*(\omega) Y(\omega) e^{j\omega \tau} \, d\omega \quad (C,13) \]

where
\[ X^*(\omega) = \int_{-\infty}^{\infty} x(u) e^{j\omega u} \, du \quad (C,14) \]
and
\[ Y(\omega) = \int_{-\infty}^{\infty} y(v) e^{-j\omega v} \, dv \quad (C,15) \]

The time savings arise from the use of the Fast Fourier transform for the individual transforms in eqs. (C,14) and (C,15). The Fourier transforms of the correlation functions to obtain the spectral estimates are usually performed directly since the number of operations required is reasonably small.

A more detailed discussion of the algorithm and the Fast Fourier transform, as applied to the calculation of the power and cross-spectral estimates of this study, is given in Ref. 33. The actual computer program listings presented in that reference are basically similar to those used here.
## TABLE I

PARAMETERS OF FORCING FUNCTION FILTERS

<table>
<thead>
<tr>
<th>$\omega_i$</th>
<th>$\omega_{ie}$</th>
<th>$\omega_p$</th>
<th>$\xi_p$</th>
<th>$\omega_f0$</th>
<th>$\omega_s$</th>
<th>$\xi_s$</th>
<th>$R$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.5</td>
<td>1.56</td>
<td>1.5</td>
<td>0.75</td>
<td>0.75</td>
<td>15.0</td>
<td>0.75</td>
<td>0.1</td>
</tr>
<tr>
<td>2.5</td>
<td>2.47</td>
<td>2.5</td>
<td>0.75</td>
<td>1.50</td>
<td>15.0</td>
<td>0.75</td>
<td>0.1</td>
</tr>
<tr>
<td>4.0</td>
<td>3.77</td>
<td>4.0</td>
<td>0.70</td>
<td>2.0</td>
<td>15.0</td>
<td>0.75</td>
<td>0.1</td>
</tr>
<tr>
<td>2.5*</td>
<td>2.26</td>
<td>3.0</td>
<td>0.70</td>
<td>1.65</td>
<td>—</td>
<td>—</td>
<td>0</td>
</tr>
<tr>
<td>SUBJECT</td>
<td>AGE</td>
<td>SEX</td>
<td>VISUAL ACUITY (BOTH EYES)</td>
<td>GLASSES WORN</td>
<td>TRAINING PROGRAM (PER PHASE)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>---------</td>
<td>-----</td>
<td>-----</td>
<td>---------------------------</td>
<td>--------------</td>
<td>-----------------------------</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>20 ft.</td>
<td>14 in.</td>
<td>HOURS</td>
<td>WEEKS</td>
<td></td>
</tr>
<tr>
<td>BB</td>
<td>20</td>
<td>F</td>
<td>20/20</td>
<td>14/17.5</td>
<td>NO</td>
<td>9.6</td>
<td>6.4</td>
</tr>
<tr>
<td>WB</td>
<td>31</td>
<td>M</td>
<td>20/20</td>
<td>14/14</td>
<td>NO</td>
<td>9.0</td>
<td>3.8</td>
</tr>
<tr>
<td>JB</td>
<td>31</td>
<td>M</td>
<td>20/20</td>
<td>14/14</td>
<td>YES</td>
<td>12.4</td>
<td>3.6</td>
</tr>
<tr>
<td>BM*</td>
<td>42</td>
<td>M</td>
<td>20/20</td>
<td>14/21</td>
<td>NO</td>
<td>10.9</td>
<td>3.1</td>
</tr>
<tr>
<td>LR</td>
<td>24</td>
<td>M</td>
<td>20/15</td>
<td>14/14</td>
<td>NO</td>
<td>13.1</td>
<td>3.2</td>
</tr>
<tr>
<td>DS</td>
<td>25</td>
<td>M</td>
<td>20/20</td>
<td>14/14</td>
<td>NO</td>
<td>8.1</td>
<td>3.0</td>
</tr>
<tr>
<td>JS**</td>
<td>24</td>
<td>F</td>
<td>20/20</td>
<td>14/14</td>
<td>NO</td>
<td>9.2</td>
<td>2.3</td>
</tr>
<tr>
<td>NU</td>
<td>32</td>
<td>M</td>
<td>20/20</td>
<td>14/14</td>
<td>NO</td>
<td>8.5</td>
<td>3.3</td>
</tr>
</tbody>
</table>

NOTE:  
* 300 hours single engine light aircraft, all other subjects had no previous flying or tracking experience other than automobile driving.

** Four months absence between training phases 2 and 3.
TABLE III

MEAN SQUARE DATA USED TO TEST THE ACCURACY OF THE DESCRIBING FUNCTION CURVE-FITTING AND THE EXTRACTION OF THE REMNANT, $\xi_N$ A PARAMETER.

<table>
<thead>
<tr>
<th>$\xi_N$</th>
<th>$\overline{c_e^2}_M$</th>
<th>$\overline{c_e^2}_F$</th>
<th>$\overline{c_e^2}_L$</th>
<th>$\overline{c_e^2}_T$</th>
<th>$\overline{c_e^2}_T'$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.01</td>
<td>16.80</td>
<td>11.0</td>
<td>12.91</td>
<td>12.48</td>
<td>12.40</td>
</tr>
<tr>
<td>0.02</td>
<td>1.59</td>
<td>1.44</td>
<td>1.45</td>
<td>1.47</td>
<td>1.50</td>
</tr>
<tr>
<td>0.05</td>
<td>2.73</td>
<td>2.09</td>
<td>2.10</td>
<td>2.11</td>
<td>2.12</td>
</tr>
<tr>
<td>0.08</td>
<td>1.42</td>
<td>1.36</td>
<td>1.43</td>
<td>1.44</td>
<td>1.43</td>
</tr>
<tr>
<td>0.1</td>
<td>0.85</td>
<td>0.81</td>
<td>0.83</td>
<td>0.83</td>
<td>0.81</td>
</tr>
<tr>
<td>0.2</td>
<td>1.80</td>
<td>1.98</td>
<td>2.22</td>
<td>2.14</td>
<td>2.19</td>
</tr>
<tr>
<td>0.34</td>
<td>0.61</td>
<td>0.58</td>
<td>0.62</td>
<td>0.62</td>
<td>0.64</td>
</tr>
</tbody>
</table>
### TABLE IV

**ANALYSIS OF VARIANCE TABLE TO TEST THE STABILITY OF SUBJECT PERFORMANCE LEVELS**

<table>
<thead>
<tr>
<th>EFFECT NUMBER</th>
<th>SOURCE OF VARIATION</th>
<th>DEGREES OF FREEDOM</th>
<th>SUM OF SQUARES</th>
<th>MEAN SQUARE</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>REPLICATION(R)&lt;sup&gt;b&lt;/sup&gt;</td>
<td>1</td>
<td>94.6</td>
<td>94.6</td>
</tr>
<tr>
<td>2</td>
<td>SUBJECT (S)</td>
<td>7</td>
<td>8730.8</td>
<td>1247.3</td>
</tr>
<tr>
<td>3</td>
<td>CONDITION (C)&lt;sup&gt;c&lt;/sup&gt;</td>
<td>6</td>
<td>61934.9</td>
<td>10322.5</td>
</tr>
<tr>
<td>4</td>
<td>R x S</td>
<td>7</td>
<td>255.4</td>
<td>36.5</td>
</tr>
<tr>
<td>5</td>
<td>R x C</td>
<td>6</td>
<td>277.3</td>
<td>46.2</td>
</tr>
<tr>
<td>6</td>
<td>S x C</td>
<td>42</td>
<td>4763.2</td>
<td>113.4</td>
</tr>
<tr>
<td>7</td>
<td>R x S x C</td>
<td>42</td>
<td>986.8</td>
<td>23.5</td>
</tr>
<tr>
<td>8</td>
<td>RESIDUAL (EXPERIMENTAL ERROR)</td>
<td>224</td>
<td>4134.2</td>
<td>18.5</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>TEST</th>
<th>RATIO OF EFFECT NOS.</th>
<th>&lt;sup&gt;v&lt;/sup&gt;&lt;sub&gt;1&lt;/sub&gt;</th>
<th>&lt;sup&gt;v&lt;/sup&gt;&lt;sub&gt;2&lt;/sub&gt;</th>
<th>F&lt;sub&gt;&lt;sup&gt;v&lt;/sup&gt;&lt;sub&gt;1&lt;/sub&gt;,&lt;sup&gt;v&lt;/sup&gt;&lt;sub&gt;2&lt;/sub&gt;&lt;/sub&gt; 0.01</th>
<th>RATIO OF MEAN SQUARES</th>
</tr>
</thead>
<tbody>
<tr>
<td>TRIPLE INTERACTION</td>
<td>7/8</td>
<td>42</td>
<td>224</td>
<td>1.5970</td>
<td>1.2703</td>
</tr>
<tr>
<td></td>
<td>6/(7+8)</td>
<td>42</td>
<td>266</td>
<td>1.5815</td>
<td>2.700**&lt;sup&gt;a&lt;/sup&gt;</td>
</tr>
<tr>
<td>DOUBLE INTERACTION</td>
<td>5/(7+8)</td>
<td>6</td>
<td>266</td>
<td>2.8715</td>
<td>1.100</td>
</tr>
<tr>
<td></td>
<td>4/(7+8)</td>
<td>7</td>
<td>266</td>
<td>2.7081</td>
<td>0.8690</td>
</tr>
<tr>
<td>MAIN EFFECTS&lt;sup&gt;e&lt;/sup&gt;</td>
<td>3/(4+5+7+8)</td>
<td>6</td>
<td>279</td>
<td>2.8683</td>
<td>82.7787**</td>
</tr>
<tr>
<td></td>
<td>2/6</td>
<td>77</td>
<td>42</td>
<td>3.0078</td>
<td>10.9991**</td>
</tr>
<tr>
<td></td>
<td>1/6</td>
<td>1</td>
<td>42</td>
<td>7.1521</td>
<td>0.8342</td>
</tr>
</tbody>
</table>

**NOTE:**
- a) **: Significant at 99%
- b) Replication: Scores obtained before or after main data collection.
- c) Condition: Manipulator/forcing-function combination, (Pressure/ω<sub>1</sub> = 2.5* excluded).
- d) Ratio of Effect Numbers: Shows actual test performed: summation of effect numbers in denominator indicates which mean squares and degrees of freedom were pooled.
- e) Significance of (SxS) double interaction requires modification of testing procedure for the main effects, (see Ref.6).
### Table V

**Averaged Normalized Mean Square Errors for STI Forcing Functions and Rate Control Vehicle Dynamics**

<table>
<thead>
<tr>
<th>$\omega_i$</th>
<th>1.5</th>
<th>2.5</th>
<th>4.0</th>
<th>&quot;B5&quot;</th>
<th>&quot;R2.2&quot;</th>
</tr>
</thead>
<tbody>
<tr>
<td>Present Results (Free-moving)</td>
<td>7.1</td>
<td>12.3</td>
<td>29.8</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Ref. 38 (Spring-restrained)</td>
<td>5.7</td>
<td>8.2</td>
<td>23.5</td>
<td>52.0</td>
<td>-</td>
</tr>
<tr>
<td>Ref. 28 (Free-moving)</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>56.2</td>
<td>149</td>
</tr>
<tr>
<td>Pressure</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>53.2</td>
<td>214</td>
</tr>
</tbody>
</table>
### TABLE VI
AVERAGED SYSTEM PERFORMANCE MEASURES
PRESENT RESULTS

<table>
<thead>
<tr>
<th>MANIPULATOR</th>
<th>FREE-MOVING</th>
<th>PRESSURE</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\omega_i$</td>
<td>1.5 2.5 4.0</td>
<td>1.5 2.5 4.0 2.5*</td>
</tr>
<tr>
<td>$\omega_{ie}$</td>
<td>1.56 2.47 3.77 2.26</td>
<td>1.56 2.47 3.77 2.26</td>
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<tr>
<td>$\rho_a^2$</td>
<td>0.64 0.66 0.71 -</td>
<td>0.51 0.57 0.63 -</td>
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<tr>
<td>$\omega_c$ rad/sec</td>
<td>2.4 3.3 3.7 5.1</td>
<td>2.7 3.5 3.9 5.7</td>
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<tr>
<td>$\phi_m$ deg</td>
<td>54 45 52 28</td>
<td>60 57 59 27</td>
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**NOTE:** 2.5* - No Shelf

### REFERENCED DATA

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<td>1.5 2.5 4.0</td>
<td>1.5 2.5 4.0</td>
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<td>$\omega_c$ rad/sec</td>
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<td>- - -</td>
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<tr>
<td>$\phi_m$ deg</td>
<td>- - -</td>
<td>&quot;B5&quot; &quot;R2.2&quot;**</td>
<td>&quot;B5&quot; &quot;R2.2&quot;**</td>
</tr>
</tbody>
</table>

Ref.28

| $\omega_i$  | "B5" "R2.2"** | "B5" "R2.2"** | "B5" "R2.2"** |
| $\omega_c$ rad/sec | 4.4 2.4 | 5.2 3.2 | 6.6 3.8 |
| $\phi_m$ deg | 32 56 | 36 72 | 12 46 |

** Cross-over frequencies for these forcing functions are regressed.
TABLE VII

AVERAGED PARAMETERS OF THE FITTED DESCRIBING
FUNCTION MODEL, (INCLUDING TWO INDIVIDUAL RUNS)

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<th>PRESSURE</th>
<th>RUN NUMBER</th>
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<td>1.5 2.5 4.0</td>
<td>4066** 4079**</td>
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<tr>
<td>$K_{in/in}$</td>
<td>39.3 32.9 24.4</td>
<td>47.7 47.8 34.1</td>
<td>44.0 19.5</td>
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<td>$\tau_d$ sec</td>
<td>0.122 0.119 0.119</td>
<td>0.064 0.064 0.065</td>
<td>0.065 0.150</td>
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<td>$1/T_L$</td>
<td>0.62 0.34 0.31</td>
<td>0.37 0.57 0.40</td>
<td>0.40 0.40</td>
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<td>$1/T_I$</td>
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<tr>
<td>$1/T_K$</td>
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<td>0.40 0.15</td>
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<tr>
<td>$1/T_{K'}$</td>
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<td>0.01 0.01 0.01</td>
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<td>$1/T_{N_1}$</td>
<td>9.2 10.1 12.3</td>
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<td>14.0 16.0</td>
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<td>0.15 0.13 0.16</td>
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<td>18.3 18.0 18.1</td>
<td>19.0 13.5</td>
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** NOTE: **

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TABLE VIII

AVERAGED MEAN SQUARE AND CHI-SQUARED DATA

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<td>$\sigma_c$</td>
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<td>$\sigma_o$</td>
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<td>$\sigma_{nc}$</td>
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<td>1.24</td>
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<td>$\chi_e^2$</td>
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<td>$\chi_c^2$</td>
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<td>$\chi_o^2$</td>
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<td>$\chi_{nc}^2$</td>
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<td>13.9</td>
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</tbody>
</table>

NOTE: $\sigma_i = 0.50$ ins.

$\chi_{13,0.05} = 22.4$ and $\chi_{13,0.01} = 27.7$

* - Possible significance (95%)

** - Significant (99%)
### TABLE IX

**GROUPED AVERAGED MEAN SQUARE DATA AND CHI-SQUARED DATA**

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<td>2.47</td>
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<td>0.80</td>
<td>1.24</td>
<td>1.04</td>
<td>1.18</td>
<td>1.28</td>
<td>1.48</td>
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<td>15.9</td>
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**NOTE:**

' - Possible significance (95%)

* - Significant (99%)
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<td>42.2</td>
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<tr>
<td>( K_p ) in/in</td>
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<td>0.111</td>
<td>0.126</td>
<td>0.111</td>
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<td>0.121</td>
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<td>0.058</td>
<td>0.068</td>
<td>0.062</td>
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<td>( \tau_d ) sec</td>
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<td>0.65</td>
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<td>1.02</td>
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<td>1.08</td>
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<td>( 1/T_L )</td>
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<td>0.92</td>
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<td>1.02</td>
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<td>0.94</td>
<td>0.70</td>
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<td>( 1/T_{N1} )</td>
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TABLE XI
GROUPED MEAN PERFORMANCE MEASURES
AND MEAN SQUARE OPERATOR OUTPUT

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<td>( \phi_m^o )</td>
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<tr>
<td>( \sigma_c ) deg.or lb.</td>
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### TABLE XII-I

**STATISTICAL COMPARISONS OF MEANS**

| VARIABLE | $| \frac{\mathbf{y}}{p}$ | $\frac{\mathbf{y}}{p}$ | $\frac{\varphi_{n_e}}{n_e}$ | SIGNIFICANCE | NONE | POSSIBLE (95%) | DEFINITE (99%) |
|----------|----------------|----------------|----------------|-----------------|------|----------------|----------------|
| SYMBOL   | G             | P             | N             |                 |      |                |                |

#### A) BETWEEN MANIPULATORS

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B. Run 4079. Subject LR. $\omega_1 = 3.0$ rad/sec. Free-Moving Manipulator.
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A. Run 4066. Score = 33.4 \( \chi^2 = 29.6 \) \( \chi_{nc}^2 = 6.3 \)

B. Run 4079. Score = 66.4 \( \chi^2 = 10.2 \) \( \chi_{nc}^2 = 9.2 \)
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A. FREE-MOVING MANIPULATOR.

B. PRESSURE MANIPULATOR.
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\[ \omega_i = 2.5 \text{ RAD/SEC.} \]
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Figure 30.1
Average of 32 runs with ±1.0° error bars.

A. Free-moving manipulator.
B. Pressure manipulator.
Figure 30-2: Power Spectra of injected output remnant and injected observation remnant.

\[ \omega = 2.5 \text{ rad/sec} \]

\[ \omega = 4.0 \text{ rad/sec} \]

A. Free-Moving Manipulator.

B. Pressure Manipulator.
FIGURE 30-3. POWER-SPECTRA OF INJECTED OUTPUT REMNANT AND INJECTED OBSERVATION REMNANT. AVERAGE OF 32 RUNS WITH ±1.00 ERROR BARS. BOTH MANIPULATORS. \( \omega_i = 4.0 \) RAD/SEC.
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\( \omega_1 = 1.5 \) RAD/SEC.
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a) CLOSED-LOOP OUTPUT REMNANT

b) CORRELATION COEFFICIENT

- MANIPULATOR
- FREE-MOVING
- PRESSURE
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FIGURE 37-1.
AMplitude PROBABILITY DISTRIBUTIONS IN THE FORM OF HISTOGRAMS.
AVERAGE
\( \bar{\omega}_m = 1.5 \) RAD/SEC.

A. FREE-MOVING MANIPULATOR.

B. PRESSURE MANIPULATOR.
FIGURE 37-2. AMPLITUDE PROBABILITY DISTRIBUTIONS IN THE FORM OF HISTOGRAMS.

A. FREE-MOVING MANIPULATOR.

B. PRESSURE MANIPULATOR.
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A. Free-moving manipulator.

B. Pressure manipulator.
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FIGURE 38-3. POWER-SPECTRA OF OPERATOR OUTPUT. AVERAGE OF 32 RUNS WITH ±1.0σ ERROR BARS. BOTH MANIPULATORS. $\omega = 4.0 \text{ RAD/SEC}$. 

**a) FREE-MOVING MANIPULATOR**

**b) PRESSURE MANIPULATOR**
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\[ \omega_i = 2.5 \text{ RAD/SEC}. \]
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a) FREE-MOVING MANIPULATOR

b) PRESSURE MANIPULATOR
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FIGURE 44. ALIASED SPECTRAL OR RESPONSE WINDOWS.
An investigation into certain aspects of the describing function of a human operator controlling a system of one degree of freedom.

Gordon-Smith, M.

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1. Human Pilot Dynamics
2. Manipulators
3. Human Engineering
4. Vehicle Handling Qualities
5. Flight Simulation

An experimental investigation has been carried out of the "remnant" portion of the mathematical model of the human operator. This model consists of the combination of a quasi-linear describing function and a remnant term. A single-axis tracking task, with random forcing functions and a compensatory display, was used to investigate the effect of the type of manipulator and the bandwidth of the forcing function on the remnant. Pressure and free-moving manipulator were employed with rate-control vehicle dynamics and filtered-white-noise forcing functions similar in spectral shape to those used in previous work. Data is presented which show the effects of the manipulator and of the forcing function on the describing function, on the performance measures of the system and on the power spectrum and the amplitude probability distribution of the remnant.

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