LOAD COMPENSATION TECHNIQUES USING MULTI-TAPPED POTENTIOMETERS

by

K. C. GARNER
Load Compensation Techniques
Using Multi-tapped Potentiometers

- by -

K. C. Garner, B.Sc.(Eng.),
A.M.I.E.E., A.F.R.Ae.S.

SUMMARY

The theory is developed for loading error compensation techniques utilising multi-tapped potentiometers. Two methods of compensation are derived, and their relative advantages discussed. Design formulae are given for calculating appropriate values of the shunt resistors to be used in association with the tapped potentiometers. Numerical examples of each method are given.
## CONTENTS

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>Summary</td>
<td></td>
</tr>
<tr>
<td>List of Symbols</td>
<td></td>
</tr>
<tr>
<td>1. Introduction</td>
<td>1</td>
</tr>
<tr>
<td>2. Method A: Self compensated potentiometer</td>
<td>1</td>
</tr>
<tr>
<td>3. Method B: Ganged pair circuit</td>
<td>5</td>
</tr>
<tr>
<td>4. References</td>
<td>9</td>
</tr>
<tr>
<td>Appendix 1: Special Nonlinear</td>
<td></td>
</tr>
<tr>
<td>Compensating function for Method A</td>
<td>10</td>
</tr>
<tr>
<td>Appendix 2: Application for other than the Designed Load</td>
<td>12</td>
</tr>
<tr>
<td>Appendix 3: Method A: Example Design</td>
<td>13</td>
</tr>
<tr>
<td>Appendix 4: Method B: Example Design</td>
<td>16</td>
</tr>
</tbody>
</table>
LIST OF SYMBOLS

- $f(1)$: value of $f(K)$ when $K = 1$
- $f(K)$: a nonlinear function of potentiometer resistance with $K$
- $K$: Potentiometer wiper displacement
- $n = \frac{1}{m}$: segment length ratio; i.e. for a ten segment potentiometer $n = 0.1$
- $r$: unloaded and unshunted total resistance of the potentiometer winding in the general case of Method B
- $R$: unloaded and unshunted total resistance of a potentiometer winding
- $R_i$: shunt resistor value across segment 1 of a tapped potentiometer
- $R_L$: minimum input resistance of the loaded potentiometer circuit used in Method A
- $R_j$: shunt resistor value across segment $j$ of a tapped potentiometer
- $R_{m}$: shunt resistor value across segment $m$ of a tapped potentiometer
- $R_o$: total resistance of the special nonlinear potentiometer determined in Appendix 1
- $V_i$: voltage at the input of the potentiometer circuit
- $V_o$: voltage at the output of the potentiometer circuit
- $\left(\frac{V_o}{V_i}\right)_j$: voltage developed at the wiper output of the loaded potentiometer circuit of Method A when the wiper is at the $j$th tapping point
- $\alpha_j = \sum_{j=1}^{j} (y_1 + y_2 + \ldots + y_j)$
- $\alpha_m = \sum_{m=1}^{m} (y_1 + y_2 + \ldots + y_m)$
- $\gamma_j = \frac{R_i}{nR + R_j}$
- $\lambda_{K} = \frac{f(K)}{f(1)}$
- $\lambda_{L} = \frac{R_L}{f(1)}$
1. Introduction

The error in the relationship between voltage output and shaft position introduced by having a finite resistive load at the output of an otherwise linear potentiometer is a common difficulty in the design of servomechanisms and analogue computers. The use of tapped potentiometers with shunt resistors between tappings can provide a suitable means of compensating for this error and two methods are discussed, each with its special merits.

The first method, using only one tapped potentiometer, applies where an accurate full scale linear relation between the potentiometer angular setting and attenuation is required, and where the resistive load at the output is known and constant. This method is particularly suitable for helical potentiometers with geared dials so that the scale calibration is always valid in the specified loaded condition. Applications with loads less than that specified are described in Appendix 2.

The second method utilizes an accurately wound, linear potentiometer, ganged with a tapped potentiometer having specially chosen shunt resistors to provide the compensation for the load. This circuit enables a linear relationship to be obtained whatever the value of the load, although for any finite fixed load there is a fixed insertion attenuation. This latter effect may be turned to advantage when the potentiometer assembly is used as a position feedback pick-off in a servomechanism, since the load may be varied to provide a stiffness control.

2. Method A: Self compensated potentiometer

Using an accurate linear tapped potentiometer, it is necessary to calculate the values for the shunt resistors to be inserted between taps using the following specified data:-

(a) the minimum input resistance to be presented to the source
(b) the load resistance

It is of course necessary that item (a) should be rather less than (b).

Figure 1 shows the basic circuit configuration with a multi- tapped potentiometer having an unloaded and unshunted resistance $R$. The wiper is loaded by the resistance $R_w$. There are $m$ segments, and it is assumed that across each segment, between adjacent tapping points, there is a resistor $R_j$, where $j$ denotes that the resistor lies across the $j$th segment.
Such a network has been analysed in some detail in a previous article\(^{(1)}\) where it was shown that it could be used to generate a wide range of nonlinear functions. In this case a very special nonlinear function is required in the unloaded condition, so that with the load connected the final result is linear within the specified tolerance. This special nonlinear function is derived in Appendix 1 and is shown to be

\[
f(K) = \frac{R_o}{2} \left[ 1 - \frac{R_L}{K R_o} + \sqrt{\left(\frac{R_L}{K R_o} - 1\right)^2 + 4 \frac{R_L}{R_o}} \right]
\]

where \(K\) is the wiper displacement and \(R_o\) is the total resistance of the shunted potentiometer which gives the desired nonlinear law.

The problem is intractable in this form however, since \(R_o\) is not known until the design is complete. Nevertheless equation 1 indicates the feasibility of the method, and to overcome the above dilemma the following analysis has been developed.

Returning to Fig. 1, consider the input-output voltage ratios for the wiper set at each tapping point, with the load in circuit.

For the wiper at the junction of the first and second segments,

\[
\left(\frac{V_o}{V_i}\right) = \frac{\left(\frac{n R R_1}{n R + R_1}\right) R_L \left(\frac{n R R_1}{n R + R_1} + R_L\right)}{n R \left[\frac{R_1}{n R + R_1} + \frac{R_2}{n R + R_2} + \cdots + \frac{R_m}{n R + R_m}\right] + \left(\frac{n R R_1}{n R + R_1}\right) R_L \left(\frac{n R R_1}{n R + R_1} + R_L\right)}
\]

\[
= \frac{R_L \gamma_1}{(n R \gamma_1 + R_L) (\gamma_2 + \gamma_3 + \cdots + \gamma_m) + R_L \gamma_1}
\]

where \(\gamma_j = \frac{R_j}{n R + R_j}\)
Similarly
\[
\left( \frac{V_o}{V_1} \right)_{1-2} = \frac{R_L (\gamma_1 + \gamma_2)}{nR(\gamma_1 + \gamma_2) + R_L (\gamma_3 + \gamma_4 + \ldots + \gamma_m) + R_L (\gamma_1 + \gamma_2)}
\]

\[
\left( \frac{V_o}{V_1} \right)_{1-3} = \frac{R_L (\gamma_1 + \gamma_2 + \gamma_3)}{nR(\gamma_1 + \gamma_2 + \gamma_3) + R_L (\gamma_4 + \gamma_5 + \ldots + \gamma_m) + R_L (\gamma_1 + \gamma_2 + \gamma_3)}
\]

and by induction
\[
\left( \frac{V_o}{V_1} \right)_{1-j} = \frac{R_L (\gamma_1 + \gamma_2 + \ldots + \gamma_j)}{nR(\gamma_1 + \gamma_2 + \ldots + \gamma_j) + R_L (\gamma_{j+1} + \gamma_{j+2} + \ldots + \gamma_m) + R_L (\gamma_1 + \gamma_2 + \ldots + \gamma_j)}
\]

or
\[
\left( \frac{V_o}{V_1} \right)_{1-j} = \frac{1}{\left( \frac{nR}{R_L} + \frac{1}{\gamma_j} \right) (\gamma_m - \gamma_j) + 1}
\]

where
\[
\gamma_j = (\gamma_1 + \gamma_2 + \ldots + \gamma_j)
\]

and
\[
\gamma_m = (\gamma_1 + \gamma_2 + \ldots + \gamma_m)
\]

This is the general formula for the voltage ratio at any tapping point in the presence of the load \( R_L \). Thus if these voltages are arranged to lie along a linear law, then at least at these positions of the wiper, the resulting network performs as a linear potentiometer, as shown in Fig. 2. However some permissible error will have been prescribed, and since it is known that the loading effect will cause each segment of the generated curve to droop, a more accurate design will be achieved if these voltage ratios are chosen to lie high by an amount equal to the positive tolerance, as indicated in Fig. 3. This method of optimizing the design of tapped-potentiometer function generators is discussed fully in Ref. 1.
Assuming that the potentiometer resistance \( R \), and the load resistance \( R_L \) are specified, together with \( n \), the segment length ratio, it is now possible to write \((m-1)\) equations, where \( \gamma_i \) to \( \gamma_m \) are the unknowns. One more equation is required to make the solution possible. This is obtained by considering the minimum resistance, \( R_i \), allowed by the source power requirements. This value will occur with the wiper at the top of the potentiometer in the loaded condition.

Hence

\[
R_i = \frac{n R R_L}{n R \alpha_m + R} \quad (11)
\]

or

\[
\alpha_m = \frac{R_L R_i}{n R (R_L - R_i)} \quad (12)
\]

where \( \alpha_m \) will be a constant for any given specification. Substituting this value of \( \alpha_m \) in equation 8 yields

\[
\left( \frac{V_o}{V_i} \right)_j = \frac{1}{\left( \frac{n R}{R_L} + \frac{1}{\alpha_j} \right) \left[ \frac{R_L R_i}{n R (R_L - R_i)} - \alpha_j \right]^+ 1} \quad (13)
\]

Solving this for \( \alpha_j \) gives

\[
\alpha_j = \sqrt{\left[ \frac{V_o}{V_i} \right]_j \left( \frac{R_i}{R_L - R_i} \right) - 1} + \sqrt{\left[ \frac{V_o}{V_i} \right]_j \left( \frac{R_i}{R_L - R_i} \right) - 1}^2 \left[ \frac{V_o}{V_i} \right]_j \left( \frac{R_i}{R_L - R_i} \right) + 4 \left( \frac{V_o}{V_i} \right)_j \left( \frac{n R}{R_L} \right) \quad (14)
\]

Also it is known that

\[
\alpha_j - \alpha_{j-1} = \gamma_j \quad (15)
\]
Thus if all $\alpha$ terms are computed, all $\gamma$ terms can be derived. Therefore, since

$$y_j = \frac{R_j}{(nR + R_j)}$$

then

$$R_j = \frac{nRy_j}{1 - y_j}$$

Hence

$$R_{x_j} = \frac{nR(a_j - a_{j-1})}{(1 - a_j + a_{j-1})}$$

which is the general formula required in order to calculate the shunt resistances for specified values of $R_L$, $R_i$ and $R$.

3. **Method B: Ganged pair circuit**

The second method provides linear operation regardless of the choice of $R_x$. Assume the network shown in Fig. 4, where the tapped potentiometer has a nonlinear resistance characteristic $f(K)$, and the linear potentiometer is gauged to it.

The voltage ratio is then

$$\frac{V_o}{V_i} = \frac{KR_L}{R_L + KR(1 - K) + f(K)}$$

where $K$ is the wiper displacement.

Thus in order to generate the desired linear law

$$\frac{V_o}{V_i} = K$$

from this circuit, it would be necessary that

$$f(K) = -KR(1 - K)$$

which is clearly impossible with passive components.
However \( f(K) \) has a maximum negative resistance of \( \frac{R}{4} \) at \( K = \frac{1}{2} \).
Thus if it is arbitrarily chosen that

\[
 f(K) = \frac{R}{4} - K R(1 - K) \tag{22}
\]

the network is passive realizable. Any other value of the positive resistance greater than the chosen one of \( \frac{R}{4} \) is admissible but as shown below does not produce such an efficient design. The overall performance of the ganged pair of potentiometers is now given by

\[
 \frac{V_o}{V_i} = K \left[ \frac{R_L}{R_L + \frac{R}{4}} \right] \tag{23}
\]

where the factor \( R_L / (R_L + \frac{R}{4}) \) is a constant for any fixed load.

Therefore the desired linear law is modified only by a constant multiplying factor and the result is shown graphically in Fig. 5., i.e. the load resistance merely changes the slope but does not affect the linearity.

This configuration, unlike that of method A is not well suited for analogue computer applications, since for every change of \( R_L \), (usually the input resistor for the following amplifier), a new scale factor would have to be calculated. However the circuit is very suitable as a servo feedback potentiometer since the constant factor only affects the loop stiffness, and indeed can be used as the feedback gain control directly by varying \( R_L \), without affecting the position linearity in any way.

The calculation for the resistor values is simple, and derived as follows.

Referring to Fig. 6, where there are m equal segments of fractional length \( n = \frac{1}{m} \) where m is an even number. Only m/2 segments need be considered as the law \( f(K) \) is symmetrical. The potentiometer total resistance is \( r \).

For \( K = 0 \)

\[
 f(K)_o = \frac{R}{4} = n r \gamma_{m/2} + \gamma \left( \frac{m}{2} - 1 \right)^{-1} + \ldots + \gamma_2 + \gamma_1 \tag{24}
\]

where

\[
 \gamma_j = \frac{r_j}{nr + r_j} \tag{25}
\]
For \( K = n \)

\[
f(K)_n = \frac{R}{4} - nR(1-n) = nr(y_{m/2} + y_{m/2-1} + \ldots + y_2 + y_1)
\]

For \( K = 2n \)

\[
f(K)_{2n} = \frac{R}{4} - 2nR(1-2n) = nr(y_{m/2} + y_{m/2-1} + \ldots + y_4 + y_3)
\]

For \( K = 3n \)

\[
f(K)_{3n} = \frac{R}{4} - 3nR(1-3n) = nr(y_{m/2} + y_{m/2-1} + \ldots + y_5 + y_4)
\]

etc.,
or

\[
(y_{m/2} + \ldots + y_2 + y_1) = \frac{R}{4} - nR(1-n)
\]

\[
(y_{m/2} + \ldots + y_3 + y_2) = \frac{R}{4} - nR(1-n)
\]

\[
(y_{m/2} + \ldots + y_4 + y_3) = \frac{R}{4} - 2nR(1-2n)
\]

\[
(y_{m/2} + \ldots + y_5 + y_4) = \frac{R}{4} - 3nR(1-3n)
\]

etc.

Solving for \( y_1, y_2, \) etc.

\[
y_1 = \frac{R}{4} - \left[ \frac{R}{4} - nR(1-n) \right] = \frac{R}{r} (1-n)
\]

\[
y_2 = \frac{R}{r} (1-3n)
\]

\[
y_3 = \frac{R}{r} (1-5n)
\]

\[
y_4 = \frac{R}{r} (1-7n)
\]
and by induction

\[ \gamma_j = \frac{R}{r} \left[ 1 - (2j - 1)n \right] \]  

(37)

and from equation 25

\[ R_j = \frac{n r \gamma_j}{1 - \gamma_j} \]  

(38)

which from equation 37 gives

\[ R_j = \frac{n R \left[ 1 - (2j - 1)n \right]}{1 - \frac{R}{r} \left[ 1 - (2j - 1)n \right]} \]  

(39)

which is the general formula for the \( j \)th shunt resistor which will provide the desired compensation.

A design simplification is to make both potentiometers of equal resistance, i.e. \( r = R \), so that for this special case

\[ R_j = \frac{R \left[ 1 - (2j - 1)n \right]}{(2j - 1)} \]  

(40)

If unequal valued potentiometers are used it is, of course, essential that the tapped potentiometer must have a resistance \( \geq \frac{R}{4} \) when \( K = 0 \) and \( K = 1 \). The condition for this is that the pair of segments having the maximum incremental resistance can be achieved in the absence of shunt resistors across these segments.

In the general case considered the maximum slope occurs across segments 1 and \( m \), and it is necessary to consider the value of \( r \) which yields an infinite value for \( R_j \) in equation 39. This will be the minimum permissible value of \( r \) for a specified number of segments, and linear potentiometer resistance \( R \).

Thus for \( R_j \to \infty \)

\[ \frac{n R (1 - n)}{1 - \frac{R}{r} (1 - n)} \to \infty \]  

(42)

or

\[ 1 - \frac{R}{r} (1 - n) = 0 \]  

(43)

Hence the minimum value of \( r \) is given by

\[ r = R(1 - n) \]  

(44)
4. References

APPENDIX 1

SPECIAL NONLINEAR COMPENSATING FUNCTION FOR

METHOD A

It was assumed that it would be possible to construct a special nonlinear potentiometer which when loaded would yield an overall linear law. The purpose of this appendix is to derive the resistance law of this special potentiometer and to see if it could be constructed using only passive components. The result obtained in this appendix fully justifies this assumption. Thus what is to be determined is this nonlinear law \( f(K) \), that the potentiometer must possess initially, so that when loaded the overall result will be linear.

Using the nomenclature given in Fig. 7 it can be written that

\[
K = \frac{V_o}{V_i} = \frac{\frac{R_L f(K)}{R_L + f(K)}}{f(1) - f(K) + \frac{R_L f(K)}{R_L + f(K)}}
\]

\[
= \frac{R_L f(K)}{[f(1) - f(K)][R_L + f(K)] + R_L f(K)}
\]

(A1.1)

Let

\[
\frac{f(K)}{f(1)} = \lambda K \quad \text{and} \quad \frac{R_L}{f(1)} = \lambda L
\]

so that

\[
K = \frac{\lambda L \lambda K}{\lambda L + \lambda K - \lambda^2 K}
\]

(A1.4)

or

\[
\lambda^2 K + \left(\frac{\lambda L}{K} - 1\right)\lambda K - \lambda L = 0
\]

(A1.5)

Hence

\[
\lambda_K = \frac{1}{2} \left[ 1 - \frac{\lambda L}{K} \pm \sqrt{\left(\frac{\lambda L}{K} - 1\right)^2 + 4 \lambda L} \right]
\]

(A1.6)

which is positive for practical values of \( \lambda L \).
Now $f(1)$ is the total resistance of the nonlinear potentiometer. Let this be equal to $R_o$ so that

$$
\lambda_L = \frac{R_L}{R_o} \quad \text{and} \quad \lambda_K = \frac{f(K)}{R_o}
$$

(A1.7)

Substituting these terms in equation A1.6 and re-arranging gives

$$
f(K) = \frac{R_o}{2} \left[ 1 - \frac{R_L}{K R_o} + \sqrt{\left( \frac{R_L}{K R_o} - 1 \right)^2 + 4 \frac{R_L}{R_o}} \right]
$$

(A1.8)

which is the desired nonlinear function. This function may be obtained using a shaped former technique etc., or by using a tapped potentiometer as described in this note.
APPENDIX 2

METHOD A: APPLICATION FOR OTHER THAN THE DESIGNED LOAD

Particularly in analogue computer applications, the load will be changed from time to time as the problem set-up is renewed. This is a typical consideration with servomultipliers, and dial-set coefficient potentiometers. However this presents no particular obstacle if either of the following techniques are employed.

The less attractive method is to provide pre-wired sets of compensating shunt resistors which when connected suit the load present. This would necessitate carrying a large stock of such networks, although connection could easily be arranged using standard plug-in cards. However, it is yet another task in an already tedious patching procedure.

A second and far better solution suggested by J.E. Fisher, is to design the compensated potentiometer for the lowest likely load. Thus whenever the computer set-up is changed, the potentiometer output can readily be shunted with a variable resistance until the correct loading is achieved. This is readily adjusted by setting the compensated potentiometer wiper to say, mid-scale, so that it should indicate half the applied voltage. If this wiper voltage is then compared with a reference voltage of appropriate value, using a circuit similar to that shown in Fig. 8, then a null indication on the detecting instrument shows that the variable resistance has been set to the correct value. This procedure can readily be mechanized for a large installation having several servomultipliers.
APPENDIX 3

METHOD A: EXAMPLE DESIGN

In order to illustrate the simplicity of the method a numerical case is included as an example. The following typical specification will be assumed:

- Maximum load on source to be 50K
- Fixed load on potentiometer to be 100K
- Stock size of potentiometer to be 300K
- Number of segments between equally spaced taps to be 10

Figure 9 shows the circuit where the resistors $R_1$ to $R_{10}$ are to be determined. This is carried out by computing the values for $\alpha_1$ to $\alpha_{10}$ and substituting these values in equation 14.

We know that

$$\frac{R_1}{R_L - R_1} = \frac{50}{100 - 50} = 1.0$$

and

$$\frac{n R}{R_L} = \frac{0.1 \times 300}{100} = 0.3$$

which remain constant throughout the calculation.

Also we write down the voltage ratios:

<table>
<thead>
<tr>
<th>$\alpha$</th>
<th>$(\frac{V_o}{V_{ij}})$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.1 $^*$</td>
</tr>
<tr>
<td>2</td>
<td>0.2</td>
</tr>
<tr>
<td>3</td>
<td>0.3</td>
</tr>
<tr>
<td>4</td>
<td>0.4</td>
</tr>
<tr>
<td>5</td>
<td>0.5</td>
</tr>
<tr>
<td>6</td>
<td>0.6</td>
</tr>
<tr>
<td>7</td>
<td>0.7</td>
</tr>
<tr>
<td>8</td>
<td>0.8</td>
</tr>
<tr>
<td>9</td>
<td>0.9</td>
</tr>
<tr>
<td>10</td>
<td>1.0</td>
</tr>
</tbody>
</table>

* In this example the droop error optimizing procedure has been omitted for simplicity. e.g. for a 0.1% tolerance $\frac{V_o}{V_i}$ would be 0.101 instead of 0.1 as used here.
Hence
\[ a_1 = \frac{\left[(0.1)(1) - 1\right] + \sqrt{\left[(0.1)(1) - 1\right]^2 + 4(0.1)^2}}{2(0.1)(0.3)} = 0.366 \]

and similarly
\[ a_2 = 0.781 \]
\[ a_3 = 1.233 \]
\[ a_4 = 1.667 \]
\[ a_5 = 2.050 \]
\[ a_6 = 2.402 \]
\[ a_7 = 2.694 \]
\[ a_8 = 2.942 \]
\[ a_9 = 3.153 \]
\[ a_{10} = 3.333 \]

Substituting these values in equation 18 gives
\[ R_1 = 17.3 \, \text{K\,\Omega} \]
\[ R_2 = 25.7 \, \text{K\,\Omega} \]
\[ R_3 = 24.2 \, \text{K\,\Omega} \]
\[ R_4 = 23.0 \, \text{K\,\Omega} \]
\[ R_5 = 19.4 \, \text{K\,\Omega} \]
\[ R_6 = 15.6 \, \text{K\,\Omega} \]
\[ R_7 = 12.4 \, \text{K\,\Omega} \]
\[ R_8 = 9.90 \, \text{K\,\Omega} \]
\[ R_9 = 8.02 \, \text{K\,\Omega} \]
\[ R_{10} = 6.58 \, \text{K\,\Omega} \]

which will be found to be the desired compensating resistances.
Component Tolerances

A complete analysis to include the effect of the many component tolerances is possible but very tedious, and since the basic calculation is relatively rapid, it is easy enough to test one or two trial designs using component values at each end of their tolerance limits. It is however, worth noting the significant factors influencing the final accuracy. The nominal value of the manufactured potentiometer resistance is usually stated with a small production tolerance. Before commencing the above calculation it is worthwhile measuring the actual value, if only one unit is to be constructed. The same applies to the load resistance, which also has to be known to a fair degree of accuracy. Similarly, small errors will arise if the tapping points are not precisely positioned.

The better the linearity of the unloaded, unshunted potentiometer, the better the final result will be, although the effect of shunting sections of a potentiometer is to make it less sensitive to linearity errors. Lastly, while not always necessary, it is preferable to use high-stability 1% resistors to make up the shunt resistances. This is nearly always possible taking by suitable combinations of standard preferred values. If it is desired to compute the entire performance curve for the completed design reference should be made to an earlier article by the author(1), where a complete analysis of such a network is discussed.
APPENDIX 4

METHOD B: EXAMPLE DESIGN

Figure 10 shows a final circuit where $R_1$ to $R_{10}$ are to be determined to give the desired law. As a simplifying factor it is assumed that both the linear, and the tapped potentiometers have the same overall resistance $R$. Since the law is symmetrical it is apparent that $R_1 = R_{10}$, $R_2 = R_9$, etc. Thus it is only necessary to consider $R_1$ to $R_5$. By inspection $n = 0.1$.

Since the potentiometers are ganged we must consider the resistance of the tapped potentiometer at the tapping points at $K = 0.1, 0.2, 0.3, 0.4$ and $0.5$, and make the resistance at each setting conform with the law

$$f(K) = \frac{R}{4} - KR(1 - K)$$

Using the relation (equation 40)

$$R_j = \frac{R\left[1 - (2j - 1)n\right]}{2j - 1}$$

$\begin{align*}
R_1 &= 0.90R = R_{10} \\
R_2 &= 0.23R = R_9 \\
R_3 &= 0.10R = R_8 \\
R_4 &= 0.043R = R_7 \\
R_5 &= 0.011R = R_6
\end{align*}$

and by substituting a value for $R$ the shunt resistor values are known.
FIG. 1. GENERAL CIRCUIT FOR METHOD A.

FIG. 2. METHOD A COMPENSATION IGNORING TOLERANCE
FIG. 3  METHOD A COMPENSATION
UTILIZING TOLERANCE

FIG. 4  BASIC CIRCUIT FOR METHOD B
FIG. 5. METHOD B PERFORMANCE CURVES.

FIG. 6. GENERAL CIRCUIT FOR METHOD B.
FIG. 7. GENERALIZED NONLINEAR FUNCTION POTENTIOMETER.

FIG. 8. METHOD A TRIMMING CIRCUIT FOR USE WITH OTHER THAN DESIGN-SPECIFIED LOAD