Computation of Tortuosity of Two Dimensional Vessels

Abstract—Vessel tortuosity is an important parameter in determining the presence or the severity of various diseases that are diagnosed based on the pathology of vessels. An accurate quantification of vessel tortuosity would therefore be of great clinical significance. Usually, the branches of blood vessels are also captured, along with the major blood vessel, in the medical images, thereby posing a challenge to the computation of tortuosity. This necessitates some pre-processing of the images, to remove the branches of blood vessels, accomplished using contour tracing techniques, as proposed in this paper. In this paper, we propose a novel metric to compute tortuosity for two dimensional vessels, with brief indications of its implementation using image processing techniques, and provide evidence of its advantages over other methods.

Keywords—Branches, tortuosity, vessel segmentation, retinopathy of prematurity, retinal images

I. INTRODUCTION

Tortuosity is the quality or condition of being tortuous; twistedness or crookedness. Tortuosity of vessels is an important parameter that is useful to determine severity of diseases that are diagnosed based on the pathology of vessels. Retinopathy of Prematurity (ROP) is one such potentially eye blinding disease that is diagnosed based on observation of retinal vessels. Several methods have been proposed to identify it automatically based on analysis of retinal images. The methods segment the blood vessels in retina and analyze them to determine the severity of the disease.

Computation of tortuosity of retinal vessels from 2D images requires some pre-processing which includes extraction of the major blood vessel followed by obtaining the central skeleton. However, this skeleton also contains the branches of major vessel. It is essential to remove these branches from the major vessel before measuring tortuosity since they would degrade the estimate of it. However, this is a challenging task since there are no features distinguishing the vessel from its branches in the skeleton (one pixel thick). In this paper, we present a method to discard the branches on the major vessel using a contour tracing technique and obtain the coordinates of only the major vessel, thereby facilitating the evaluation of tortuosity.

Quite a few metrics have been proposed to quantify the tortuosity of blood vessels but only some of them are in accord with the clinical perception of tortuosity. Distance Metric (DM) [1], [2] and Mean Direction Angle Change (MDAC) [3] are commonly used measures of tortuosity. An algorithm to compute tortuosity considering the thickness of the vessel was proposed in [4]. A method based on ergodicity defect which involves the notion of how a signal samples space and explores the connection between ergodicity and vessel tortuosity was proposed in [5].

The Sum of Angles Metric (SOAM) proposed in [6], to determine the tortuosity for three dimensional vessels, measures two angles at the points chosen on the vessel: (i) an in-plane angle between the two vectors joining three consecutive points, (ii) a torsional angle between the two vectors obtained by calculating the cross-product of two successive vector pairs. Then, the total angle at each point is computed as the square-root of sum of squares of in-plane and torsional angles, measured in radians. The SOAM is then defined as the ratio of the sum of total angles at all points to the sum of the Euclidean distances between successive points. In the case of two dimensional vessels, since the cross-product of vectors is always out of the plane, the torsional angle is always zero. So, SOAM for 2D vessels accounts for only the in-plane angle and is similar to Mean Direction Angle Change. The algorithm proposed in [7] provides a clinical measurement of retinal tortuosity using changing curvature sign on the points manually chosen on the vessel.

In this paper, we propose a novel metric to quantify tortuosity effectively based on the measure of angles. Section II provides the background on two of the pre-existing tortuosity metrics against which we compare our method and a correlation coefficient used to compare their performance. Section III explains the proposed method to discard the vessel branches and the novel tortuosity metric. The performance of the metrics is evaluated and a comparison is made in Section IV. Conclusion of this paper is drawn in Section V.

II. BACKGROUND

A. Distance Metric (DM)

Distance metric, for a curve, is calculated as the ratio of length of the curve (L_c) to the length of line joining the endpoints of the curve (L_s). It assumes that a straight line is non-tortuous and tortuosity is a measure of deviation of the curve from the line joining the endpoints of the curve.

\[ \text{Tortuosity} = \frac{L_c}{L_s} \]
Figure 1. Center skeleton of the blood vessels (with branches)

B. Mean Direction Angle Change (MDAC)

This method [3], based on local variation in the direction of the vessel, calculates the mean of the angles between the lines joining successive pairs of sampled points on the vessel. For \( k \)th point on the vessel with coordinates \( d_k \), two vectors \( \vec{v}_{k+n} \) and \( \vec{v}_{k-n} \) are defined as:

\[
\vec{v}_{k+n} = d_{k+n} - d_k \\
\vec{v}_{k-n} = d_{k-n} - d_k
\]

The vessel direction at any \( k \)th point is obtained as:

\[
\theta_k = \cos^{-1}\left( \frac{\vec{v}_{k+n} \cdot \vec{v}_{k-n}}{|\vec{v}_{k+n}| |\vec{v}_{k-n}|} \right)
\]

Then, the mean direction angle change (MDAC) is defined as:

\[
MDAC = \frac{1}{N - 2n} \sum_{k=1}^{N-n} \theta_k
\]

where \( N \) is the number of vessel samples and \( n \) is the step size, a fixed parameter.

C. Spearman Rank Correlation Coefficient

The Spearman Rank correlation coefficient [8] is a measure of statistical dependence or monotonicity of the relationship between two variables. We use it to compare the performance of various tortuosity metrics. To compute the Spearman rank correlation coefficient between any two variables, initially, ranks are assigned to their raw data (in ascending or descending order of values). Then the correlation is computed as:

\[
\rho = 1 - \frac{6 \sum d_i^2}{n(n^2 - 1)}
\]

where \( d_i \) is the difference between the ranks of each observation on the two variables and \( n \) is the number of observations. The sign of \( \rho \) indicates the direction of association between the two variables i.e. positive when one increases with the other and negative when one decreases as the other increases. \( \rho \) varies from -1 to +1, where +1 signifies perfectly monotonically related variables.

III. PROPOSED METHODS

Retinal blood vessels, normally straight or mildly curved, become tortuous and take a convoluted path under some diseased conditions. Tortuosity is a salient indicator of presence or severity of some diseases. The tortuous nature of vessels is commonly measured, qualitatively, by human observers. Since there is no definite scale, there could be imprecision in grading the tortuosity of vessels between different observers and also with the same observer at different times. Hence there is a need for quantitative measure of tortuosity which can be used to produce consistent results. Removal of vessel branches is an important pre-processing operation in automated computation of tortuosity. So, we propose a method to remove branches using a contour tracing technique and then use a novel metric to quantify tortuosity.

A. Removal of vessel branches by contour tracing

First, we perform vessel segmentation on the images using multi-scale Hessian based approach and then, we obtain the central skeleton of the vessels, as shown in Figure 1, by performing center line extraction on the segmented vessels. Then, the vessel is traversed, beginning from the start point using Moore’s boundary tracing algorithm [10]. The usual stopping criterion in Moore’s algorithm is to terminate it on crossing the first pixel for the second time in the same direction. Given that we already know the end points of the vessel, we terminate it on encountering the specified end point for the first time.
As the curve is traversed from start point to end point in anti-clockwise direction, the pixels on the major vessel would only be traversed once. However, some of the branches will be traversed twice as shown in Figure 2. As we terminate the contour tracing operation when the end point is reached, the rest of the branches will be left untraversed as depicted in Figure 3. Hence, those points that are untraversed or traversed twice are discarded to eliminate the branches from major vessel. However, the tip of the branch will be traversed only once and will also be discarded by checking if the preceding and succeeding points are traversed twice. Thus the resultant contour will consist of points belonging to the major vessel only (Figure 4).

**B. Proposed Metric for computation of tortuosity**

We use the coordinates of the major vessel devoid of its branches to compute tortuosity. The points $P_i$ ($i = 1, 2, \ldots, N$) on the major vessel, where $N$ is less than or equal to the total number of points on it, will be used for our computation. Then, for each point $P_i$, three vectors $\mathbf{v}_1$, $\mathbf{v}_2$, $\mathbf{v}_3$ will be defined as follows:

$$
\mathbf{v}_1 = P_i - P_{i-1} \\
\mathbf{v}_2 = P_{i+1} - P_i \\
\mathbf{v}_3 = P_{i+2} - P_{i+1}
$$

Now, the angle subtended between the angular bisectors $\mathbf{a}_1$, $\mathbf{a}_2$ of the vector pairs $\mathbf{v}_1$, $\mathbf{v}_2$ and $\mathbf{v}_2$, $\mathbf{v}_3$ is measured. Since the resultant of two unit vectors bisects the angle between them, the vectors $\mathbf{a}_1$, $\mathbf{a}_2$ are obtained by:

$$
\mathbf{a}_1 = \frac{\mathbf{v}_1}{|\mathbf{v}_1|} + \frac{\mathbf{v}_2}{|\mathbf{v}_2|} \\
\mathbf{a}_2 = \frac{\mathbf{v}_2}{|\mathbf{v}_2|} + \frac{\mathbf{v}_3}{|\mathbf{v}_3|}
$$

The vectors $\mathbf{a}_1$, $\mathbf{a}_2$ computed using the above equations, are actually perpendicular to the ones depicted in Figure 5 and hence the angle would be the same as that between the actual vectors. The angle, $\Phi_i \in [0, \pi)$ at point $P_i$, subtended between the angular bisectors $\mathbf{a}_1$ and $\mathbf{a}_2$ is calculated as

$$
\Phi_i = \cos^{-1}\left(\frac{\mathbf{a}_1 \cdot \mathbf{a}_2}{|\mathbf{a}_1||\mathbf{a}_2|}\right)
$$

This angle exploits the tortuous nature of the curve as shown in Figure 5. It can be observed that, for the points with high curvature, $\Phi_i$ is large whereas for the points on the curve where the curve is without turns, almost straight, $\Phi_i$ is negligibly small as depicted in Figure 6. Further, $\Phi_i$ will be large for points both before and after the high curvature point.

As a measure of overall tortuosity of the curve, we calculate the sum of angles at all points, $P_i$. As the tortuosity of the curve increases, so does the angle. However, angle alone does not serve as a measure of tortuosity, since it does not distinguish a short tortuous curve from a long non-tortuous curve. In order to avoid this condition, we normalize it by dividing it by the sum of distance between consecutive points. Hence the tortuosity metric is defined as:

$$
\text{Tortuosity} = \frac{\sum \Phi_i}{\sum |P_i - P_{i+1}|}
$$

where $|P_i - P_{i+1}|$ is the Euclidean distance between the points $P_i$, $P_{i+1}$.
IV. PERFORMANCE EVALUATION

Table I. Comparison of Tortuosity Metrics

<table>
<thead>
<tr>
<th>S.No</th>
<th>Tortuosity Metric</th>
<th>Spearman rank correlation</th>
</tr>
</thead>
<tbody>
<tr>
<td>01.</td>
<td>Distance Metric (DM)</td>
<td>-0.3637</td>
</tr>
<tr>
<td>02.</td>
<td>Mean Direction Angle Change (MDAC)</td>
<td>0.8118</td>
</tr>
<tr>
<td>03.</td>
<td>Proposed method</td>
<td>0.8901</td>
</tr>
</tbody>
</table>

We evaluated the performance of our method by using the 30 curves, in increasing of order of tortuosity, as used in [7]. Each set of these curves comprise of several handpicked points along the centerline of respective vessels. For these curves, we compute tortuosity using different metrics and determine the rank of tortuosity for each curve. To compare different metrics, we calculate the Spearman rank correlation between the actual rank and the rank obtained from each metric (Table I). The closer the value of the Spearman rank correlation coefficient is to +1, the better is the performance of the metric.

It can be observed that the proposed metric outperforms both DM and MDAC. Distance Metric is totally distance dependent and hence it does not take into account the curvature. It is possible that a highly tortuous curve with long distance between its end points has a lower distance metric than a less tortuous curve with short distance between its end points. Hence, DM is not a reliable tortuosity metric.

The proposed method, where the angle between angular bisectors is significant only at bends and is negligible in the straight portions of the curve, gives a more accurate measure of tortuosity. Further, the higher value of it both before and after the points with high curvature, also accounts for the presence of a curve twice ($\Phi_i$ and $\Phi_{i+1}$, as depicted in Figure 5) during the calculation, unlike MDAC, where it is accounted just once. This highlights the change in curvature and hence performs better than MDAC. Also, dividing the total angle measure by the sum of Euclidean distances of successive points improves the tortuosity metric by making it independent of length of the curve.

V. CONCLUSION

We proposed a method to calculate tortuosity of two dimensional vessels. We used a technique based on contour tracing to remove branches on the main vessel, provided, the branches do not intersect each other, forming a loop. Angle between angular bisectors was used to define an efficient metric for calculation of tortuosity. Quantitative evaluation shows that the proposed method outperforms two of the metrics commonly used.

REFERENCES