A REVIEW OF AERODYNAMIC NOISE

BY

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The financial assistance received from the Defence Research Board of Canada is gratefully acknowledged.
A detailed review is presented of the theoretical and experimental advances that have been made in the study of aerodynamic noise and in devising means for its suppression. Since many workers in the fields of aerodynamics and aerophysics may be unfamiliar with acoustic principles, the necessary background of laws and ideas from the field of acoustics is included. The theories for noise caused by subsonic disturbances, which may include turbulence fields in overchoked jets, (Lighthill, Proudman) and for those noise sources peculiar to overchoked jets (Lighthill, Powell, Ribner) are considered. Experimental results are quoted complete with numerous graphs, notes on correlation of data for model and engine jets and a comparison with theory. The results of attempts at noise suppression are discussed, noting both untried and extensively tested suggestions. A list of references is included.

Those phases of aerodynamic noise research concerning which there is disagreement or in which there is confusion because of insufficient theoretical and experimental work, or which appear to have been neglected, are thus brought to attention.

It is hoped that this review will prove useful in the initiation of research programs in the field of aerodynamic noise and its control.
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(2)

\( \lambda \) Wave length of sound

\( Q \) Mass per unit volume per unit time

\( \epsilon \) Arbitrarily small increment in the space co-ordinate

\( U \) A velocity typical of the flow

\( \ell \) A length typical of the flow

\( x \) Magnitude of the vector \( \mathbf{x} \)

\( \tau(\mathbf{z}) \) The component of mean shear in the direction of shear at the origin (Eq. 43)

\( \gamma \) Kinematic viscosity of atmosphere

\( Re \) Reynolds number

\( M \) Mach number

\( St \) Strouhal number (Section 6.1.5)

\( \delta_{ij} \) Unit diagonal tensor

\( \Theta \) Angle of sound direction with jet axis (direction of flow motion)

\( d \) Jet pipe or jet diameter at flow exit

\( \rho_0 \) Atmosphere density

\( \rho'_0 \) The density appropriate to the mean temperature at the fluctuating fluid

\( \tau \) The magnitude, \( |\mathbf{x} - \mathbf{y}| \), of the distance from source to observer

\( M_c \) Mach number of quadrupole convection and of reference frame translation

\( s \) Cell length or distance between two successive shock waves

\( \eta_d \) Number of cycles of the stream disturbance created in a given time interval

\( \eta \) Number of wave lengths between the orifice and the sound source

\( h \) Distance between the effective sound source and the orifice
N  Integer
q  Rate of amplification of the flow disturbances
\( \eta_s \)  The factor in the gain criterion associated with the actual sound production by the disturbances interacting with the cellular shock wave pattern (Eq. 61)
\( \eta_t \)  The factor in the gain criterion associated with transmission of sound to the orifice and its directionality (Eq. 61)
C  A constant
\( \alpha \)  Sign of proportionality
\( \approx \)  Sign of approximation
\( \rightarrow \)  Approaching
\( \eta_d \)  The factor in the gain criterion associated with the initiation of the flow disturbances and the pressure ratio across the jet boundary at the orifice (Eq. 61)
\( P_a \)  Atmospheric (static) pressure
\( P_c' \)  Critical stagnation pressure in jet flow reservoir (corresponding to sonic exit velocity)
\( P' \)  Stagnation pressure in jet flow reservoir
\( P_N \)  Static pressure in jet flow at nozzle exit
O  Instantaneous local power output per unit mass for isotropic turbulence
\( f^* \)  Double correlation function
L  Length scale of turbulence
S  Space correlation of the second time derivatives of \( (\nabla_x^2 - \nabla_y^2) \) at points \( \vec{y} \) and \( \vec{y'} \)
\( \Theta_{\text{max}} \)  Angle of maximum intensity peak with jet axis
K  Acoustic power coefficient (Eq. 59)
\( f_p \) Frequency of the intensity peaks

\( \text{St}_p \) Peak Strouhal number

\( \eta_N \) Efficiency of noise production (Eq. 58)

\( \alpha \) Expansion angle of overchoked jets (see Fig. 5)

\( \phi \) Angle of spread of a jet measured from the jet axis (see Section 7.1.1)

\( P'_N \) Stagnation pressure in jet flow at nozzle exit

\( P^* \) Jet pressure ratio = \((P'_N - P_a) \cdot P_a^{-1}\)

**Subscripts**

i, j, k, equal to 1, 2, 3

**Superscripts**

- average values
II INTRODUCTION

The high noise levels characteristic of powerful aircraft engines and propellers are a source of annoyance to all in their vicinity and may be the cause of actual injury to those who must work in close proximity to aircraft. The problem of aeroplane noise has become progressively more important with the introduction of piston engines of ever increasing power with propellers achieving supersonic tip speeds.

A new urgency was attached to noise suppression with the abrupt increase in power provided by jet engines, especially when such engines were considered as the propulsive units for civil aircraft. Civil airports, for practical use, must be close to centres of population; and as a result are usually surrounded by or close to expanding suburbs of metropolitan areas.

Each new jet engine features another increase in thrust and in noise, with the result that extensive research is now under way, particularly in the U.K. and the United States, with the aim of noise reduction. Significant advances have been made.

This review results from a study of the published research in the subject to date, and was undertaken to provide a background for a research program at the Institute of Aerophysics. It is essentially a literature survey, and is not meant to be a critical review of research in this field.

III THE NOISE PROBLEM

Those most affected by noise from jet engines are:

(a) the turbojet manufacturers (testing engines)

(b) military users (ground crews, carrier crews, and flying personnel), and

(c) civil aviation (ground crews, flying personnel, passengers, and the public near airports).

In cases (a) and (b) the problem is to avoid possible physical damage, especially to the ears, besides the impairment of communication by speech. In case (c) it is more the annoyance caused by the noise of normal airport activities. In all cases there are two main avenues of attack:
(a) the reduction of the overall noise level (in decibels), and

(b) the reduction of the noise nuisance.

The noise nuisance means the offensiveness of the noise, i.e. the adverse psychological effects of the various noises on those subjected to them. These effects should be studied in a joint effort with psychologists. Especially noteworthy are the effects of ultrasonic noise and that of a very low frequency of about ten cycles per second known as "belly shaking", which is most unpleasant. The noise of frequencies within the range of human speech has to be reduced in all cases to decrease their highly annoying disturbance of conversation.

The noise nuisance is difficult to define since:

(a) it is different with different people and the circumstances in which it is heard may not be comparable,

(b) noise coming from all around the observer is more annoying than an even louder noise from an easily localized source,

(c) noise of a distinct frequency is more annoying than noise of a more indiscriminate nature, and

(d) the preparedness for noise has an important bearing on its nuisance value (e.g. the sonic bang gives no warning).

There is still another and most serious aspect of jet noise, i.e. its effect on adjacent aeroplane structures. The pressure fluctuations radiating from the jet flow, especially from an over-choke d jet, cause vibrations and possible fatigue failures when the jet passes in close proximity to the aeroplane structure. Research work in this field must also be given high priority.

IV THE CONCEPT OF "AERODYNAMIC NOISE"

"Aerodynamic noise" is noise generated as a by-product of an unsteady (turbulent) airflow, or noise produced without participation of solid boundaries. Its frequency spectrum reaches from the sub-audible to the ultrasonic, encompassing a wide range of frequencies of comparable intensity. Therefore, it is called "noise" as opposed to "sound", which is usually considered to be composed of a few discrete frequencies.

Aerodynamic noise is produced by air jets, boundary layers, vortices and wakes, edge tones and related phenomena. Propeller noise does not belong in this category as it is noise created by solid bodies in motion.
As stated above, aerodynamic noise comprises ultrasonic noise as well. It cannot be "heard", but its effect on human beings or animals can be detrimental, and requires investigation by medical research workers.

V SOME BASIC ACOUSTIC CONCEPTS

5.1 Sound Intensity and Sound Pressure

Sound intensity $I$ in a specified direction at a point, is the average rate of sound energy transmitted in the specified direction through a unit area normal to this direction at the point. In general

$$I = \frac{1}{T} \int_{0}^{T} P u \cos \gamma \, dt$$

where $T$ is the period or a time much longer than a period, $P$ is the instantaneous sound perturbation pressure at the point, $u$ is the instantaneous particle velocity, and $\gamma$ is the angle between the instantaneous particle velocity and the specified direction. In a gas of density $\rho_0$, for a plane or spherical wave having a velocity of propagation $a_0$, the average sound intensity over a cycle at a point, $x$, is:

$$I(x) = \frac{P^2}{2 \rho_0 a_0}$$

(1)

From Eq. (1) it follows that

$$I(x) \propto P^2(x)$$

(2)

and it should be noted that the root mean square (rms) value of the perturbation pressure is referred to in acoustics as "sound pressure".

5.2 Sound Level and Sound Loudness

It is standard practice in acoustic measurements to relate the sound intensity $I$ and the sound pressure $P$ to reference values $I_0$ and $P_0$ which correspond approximately to the intensity and rms pressure fluctuation of the faintest audible sound at a frequency of 1000 cps. The noise level (also sound or intensity level) is measured in decibels (db).

$$\text{Intensity level (db)} = 10 \log_{10} \frac{I}{I_0}$$

$$\text{Pressure level (db)} = 20 \log_{10} \frac{P}{P_0}$$
These two levels are equal for pressure and particle velocity in phase as in the case of a plane wave. This phase relationship, of course, does not always exist. Numerically

\[ I_0 = 10^{-16} \text{ watts per sq. cm.} \]

\[ P_0 = 2 \cdot 10^{-4} \text{ dynes per sq. cm.} \]

The noise loudness (also sound loudness) is a subjective concept, and is measured in terms of the response of the "average" human ear. The loudness, measured in phons, of a tone of frequency \( f \) equals the intensity \( (I_s) \) in \( \text{db} \) of a tone of frequency 1000 cps which sounds equivalently loud to the human ear (see Fig. 1).

\[ \text{loudness level (phons)} = 10 \log_{10} \frac{I_s}{I_0} \]

From Fig. 1 it can be seen that a reading in decibels must be numerically greater than a reading in phons at frequencies below and above approximately 600 and 6000 cps respectively. The phon scale gives the loudness in close agreement with the characteristics of the human ear.

Normally, noise levels are quoted in decibels relative to \( P_0 \) or \( I_0 \). Both values correspond only approximately to the threshold of hearing (a rather difficult value to define, as it can only be an average value), and \( 2 \times 10^{-4} \) dynes per square cm. is about 0.2 decibels less than \( 10^{-16} \) watts per square cm.

5.3 Sound Level and the Human Ear

The values which represent the threshold of hearing have been discussed in Section 5.2. The human ear is capable of tolerating without discomfort an intensity up to about \( 10^{12} \) times the lowest audible intensity or

\[ 10 \log \frac{10^{12} \cdot 10^{-16}}{10^{-16}} = 120 \text{ db} \]

Measurements of sound waves show that the maximum pressure variations in the loudest sounds which our ear can tolerate are of the order of 280 dynes per square cm. above and below the atmospheric pressure of about \( 10^6 \) dynes per square cm.

The following noise table indicates decibel values which we encounter in daily life.
### Table I - Noise Levels (db) Due to Various Sources

<table>
<thead>
<tr>
<th>Description of Noise</th>
<th>Noise Level (db)</th>
<th>Noise Effect on Humans</th>
</tr>
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<tbody>
<tr>
<td>Rustle of leaves</td>
<td>10</td>
<td>None</td>
</tr>
<tr>
<td>Average whisper</td>
<td>20</td>
<td></td>
</tr>
<tr>
<td>Living room, quiet office</td>
<td>40</td>
<td>Interference with speech</td>
</tr>
<tr>
<td>Typical office</td>
<td>60</td>
<td></td>
</tr>
<tr>
<td>Ordinary conversation</td>
<td>65</td>
<td></td>
</tr>
<tr>
<td>Heavy traffic</td>
<td>80 and over</td>
<td>Discomfort</td>
</tr>
<tr>
<td>Passing train</td>
<td>90</td>
<td>The highest noise level in which one can work for normal periods</td>
</tr>
<tr>
<td>Pneumatic drill</td>
<td>90-100</td>
<td>Discomfort</td>
</tr>
<tr>
<td>A Derwent jet engine on ground 1/2 mile away</td>
<td>80-90</td>
<td>Actual pain</td>
</tr>
<tr>
<td></td>
<td>85</td>
<td>Permanent damage to the ear</td>
</tr>
<tr>
<td>Riveter</td>
<td>95</td>
<td></td>
</tr>
<tr>
<td>Comet take-off (4 ghost engines)</td>
<td>110</td>
<td></td>
</tr>
<tr>
<td></td>
<td>120</td>
<td></td>
</tr>
<tr>
<td></td>
<td>140</td>
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<tr>
<td></td>
<td>160</td>
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</tbody>
</table>

The average power developed as sound waves by a person speaking in a normal conversational tone is about $10^{-5}$ watts. Thus, one million persons would only develop a power of 10 watts if speaking at the same time. On the other hand, imagine an auditorium in the shape of a sphere which is 25 meters in radius, in which the sound intensity at the surface is to be $10^{-5}$ watts per cm². Here the acoustic power output of a loudspeaker at the centre of the auditorium would have to be 250 watts.

### 5.4 Sound Energy Attenuation

A modern jet engine produces noise energy at a rate of the order of 100 HP, which if radiated uniformly in all directions is still painful at a distance of about 100 feet, and in spite of attenuation with distance would still interfere with conversation at a distance of 3000 ft.
With actual jet engines, the noise is not radiated uniformly but has peaks in certain directions, hence the effect can be even worse in these directions.

The following measurements may illustrate the effect of distance on jet noise. The distance is given ahead of various aeroplanes on a five degree take-off path, at which their noise levels still interfere with speech. According to Table I, those noise levels are at about 80 or above.

- A Dakota would produce 80 db up to 2.5 miles ahead.
- A Stratocruiser would produce 80 db up to 4 miles ahead.
- A 6 engine jet airliner with reheat would produce 80 db up to 12 miles ahead.
- A 6 engine airliner with supersonic propellers would produce 80 db up to 16 miles ahead.

The atmosphere affects the transmission of sound energy by:

(a) absorption (due to an acoustic resistance $= \rho \cdot a_o$),
(b) dissipation caused by turbulence,
(c) refraction caused by wind,
(d) temperature and humidity,
(e) density gradients,
(f) translational effects of wind, and
(g) reflections from clouds or the stratosphere.

The atmospheric attenuation of sound energy by absorption - that is the conversion of sound energy (orderly mass motion) into heat (random molecular motion) - amount to:

- 1 db per 1000 ft. in the frequency range of 37.5 to 75 cps.
- ≈30 db per 1000 ft. in the frequency range of 4800 to 9600 cps.

Noise with a "flat" spectrum (called "white" noise, the sound intensity of which is the same in each consecutive octave) at 100 ft. distance from the sound source is no longer flat at 1000 feet, where the high frequencies are much reduced in intensity. The rate of absorption of sound energy by gases at low pressures is known to increase with the square of the frequency, if $f < 50,000$ cps. When the sound vibrations are of a frequency at which molecules can absorb energy internally, we get "supercritical" absorption (Ref. 1). Supercritical absorption may yield absorption
values which may be up to several hundred times higher than those predicted by the \( f^2 \) law.

As an illustration of the effect of wind on sound energy attenuation, a very light breeze of only 10 knots would cause a reduction of 4 db per 1000 feet in the frequency range of 37.5 to 75 cps. and of \( \approx 18 \) db per 1000 feet in the frequency range of 4800 to 9600 cps. in the downstream direction. In the upstream direction the loudness level drops by \( \approx 20 \) phons compared with the downstream values.

5.5 Some Useful Shortcuts

Below are quoted some relationships, which are useful to remember.

(a) An increase in loudness level by 9 phons makes a sound appear twice as loud to our ear.

(b) White noise with a sound intensity of 100 db in each consecutive octave throughout the audible range would yield a total noise level of 109 db or a loudness level of 125 phons.

(c) If the distance between a noise source and an observer is doubled, the noise intensity, due to the inverse square law is reduced to a quarter or the noise level is reduced by 6 db.

(d) If the intensity at any point is doubled, the noise level is increased by 3 db. If it is tenfold or hundredfold of the original intensity, the noise level is increased by 10 or 20 db respectively.

(e) The displacement amplitude in the loudest tolerable sounds of a frequency \( f = 1000 \) cps. is only about \( 10^{-3} \) cm., and in the faintest sounds about \( 10^{-6} \) cm. (for yellow light \( \lambda = 6 \times 10^{-5} \) cm., and the diameter of a molecule \( d = 10^{-8} \) cm. for comparison).

VI GENERAL THEORY

Lighthill's paper "On Sound Generated Aerodynamically" (Ref. 2 and 3) provides the general theory of the production of aerodynamic noise by subsonic jets. Since its publication (Ref. 2) in 1952, this paper has become a standard reference for sound or noise created without the aid of vibrating or moving solids, as aerodynamic noise is defined.
Proudman (Ref. 4) has applied Lighthill's theory to the case of isotropic homogeneous (uniform throughout) turbulence.

No theory for choked and overchoked jet noise comparable to that of Lighthill for subsonic jets is at present available. Lighthill contributed a paper (Ref. 5) in which he shows how turbulence creates sound when it traverses a shock wave. Ribner (Refs. 6 and 78) worked on the same problem. Lighthill's theory is used by Powell (Ref. 7) as a basis to explain the production of noise in overchoked jets.

6.1 Lighthill's Theory for Subsonic Jet Noise

6.1.1 Introductory Note

The momentum equation for a continuous medium subject to external forces $\mathbf{F}_i$ per unit volume is in Euler's form

$$\rho \frac{\partial \mathbf{v}_i}{\partial t} + \mathbf{v}_j \frac{\partial \rho}{\partial x_j} = \mathbf{F}_i - \frac{\partial}{\partial x_j} p_{ij}$$  \hspace{1cm} (4)

Here $p_{ij}$ is the compressive stress tensor representing the force in the $x_i$ direction acting on a portion of fluid, per unit surface area with inward normal in the $x_j$ direction.

For air, assuming the stress components are independent of the rate of change of density (a Stokesian gas),

$$p_{ij} = p \delta_{ij} + \mu \left\{ - \frac{\partial v_i}{\partial x_j} - \frac{\partial v_j}{\partial x_i} + \frac{2}{3} \left( \frac{\partial v_k}{\partial x_k} \right) \delta_{ij} \right\}$$  \hspace{1cm} (5)

Inserting in the left side of Eq. (4) the equation of continuity

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x_j} (\rho \mathbf{v}_j) = 0$$  \hspace{1cm} (6)

multiplied by $\mathbf{v}_i$, one obtains

$$\rho \frac{\partial \mathbf{v}_i}{\partial t} + \mathbf{v}_j \frac{\partial \rho}{\partial t} + \mathbf{v}_j \frac{\partial (\rho \mathbf{v}_j)}{\partial x_j} + (\rho \mathbf{v}_j) \frac{\partial \mathbf{v}_i}{\partial x_j} + \frac{\partial}{\partial x_j} (p_{ij}) = \mathbf{F}_i$$  \hspace{1cm} (7)
or

\[ \frac{\partial}{\partial t} (\rho v_i) + \frac{\partial}{\partial x_j} \left[ (\rho v_i v_j) + p_{ij} \right] = F_i \]  \hspace{1cm} (8)

which is the Reynolds form of the momentum equation for a medium subject to external forces \( F_i \).

From Eq. (8) one obtains the appropriate equation for sound propagation in a field subject to external forces, by noting that for the case of infinitesimal disturbances the viscous effects and the second order terms in the velocities will be negligible. Eq. (8) then becomes

\[ \frac{\partial}{\partial t} (\rho v_i) + \alpha_o^2 \frac{\partial \rho}{\partial x_i} = F_i \]  \hspace{1cm} (9)

where \( \frac{\partial \rho}{\partial x_i} = \frac{\partial \rho}{\partial \theta} \frac{\partial \theta}{\partial x_i} \) has been replaced by \( \alpha_o^2 \frac{\partial \theta}{\partial x_i} \).

In case of no external forces or \( F_i = 0 \), Eq. (8) becomes

\[ \frac{\partial}{\partial t} (\rho v_i) + \frac{\partial}{\partial x_j} \left[ (\rho v_i v_j) + p_{ij} \right] = 0 \]  \hspace{1cm} (10)

which represents the exact equation of momentum in an arbitrary continuous medium under no external forces. Similarly Eq. (9) becomes

\[ \frac{\partial}{\partial t} (\rho v_i) + \alpha_o^2 \frac{\partial \rho}{\partial x_i} = 0 \]  \hspace{1cm} (11)

which is an approximate equation of momentum governing the propagation of sound in a uniform medium, without sources of matter or external forces.

6.1.2 Derivation of Equivalent Source Distribution

A fluctuating fluid flow of limited extent (e.g. an air jet) is considered embedded in an infinite volume (e.g. the atmosphere).

The exact momentum equation for such an arbitrary continuous medium under no external forces is as shown in the previous...
section given by

$$\frac{\partial}{\partial t}(\rho \nu_i) + \frac{\partial}{\partial x_j}(\rho \nu_i \nu_j + p_{ij}) = 0$$  \hspace{1cm}(10)$$

which may be rewritten as

$$\frac{\partial}{\partial t}(\rho \nu_i) + a_0^2 \frac{\partial \rho}{\partial x_i} = - \frac{\partial}{\partial x_i} T_{ij}$$  \hspace{1cm}(12)$$

with

$$T_{ij} = \rho \nu_i \nu_j + p_{ij} - a_0^2 \rho \delta_{ij}$$  \hspace{1cm}(13)$$

Eq. (12) has the form of Eq. (9), which governs the propagation of sound in a continuous medium at rest due to external forces \( F_i \). Interpreting Eq. (12) in this manner one considers the tensor \( T_{ij} \) as an externally applied fluctuating stress and the applied force \( F_i \), per unit volume equals its inwards flux

$$- \frac{\partial}{\partial x_i} T_{ij}.$$ 

The tensor \( T_{ij} \) may be neglected outside the fluctuating flow (jet flow) where the velocities \( \nu_i \) and \( \nu_j \) are the infinitesimal local velocities of sound motion, and both viscous stresses and heat conduction effects are negligible (see Eqs. (5) and (13)). Thus outside the jet flow the density satisfies the usual equations of sound propagation in a uniform medium at rest, free from external stresses and without sources of matter, as given by Eq. (6) and Eq. (11).

Using Eq. (6) for substitution in Eq. (9) results in the relation

$$\frac{\partial^2}{\partial t^2} \rho - a_0^2 \nabla^2 \rho = - \frac{\partial F_i}{\partial x_i}$$  \hspace{1cm}(14)$$

One now notes that these equations governing the medium in the case of applied forces bear a certain resemblance to those describing the fundamental case of a medium free from external stresses but containing in some limited space a continuous distribution of fluctuating sources of additional matter such that a mass \( Q(\vec{x}, t) \) per unit volume per unit time is introduced at \( \vec{x} \) at time \( t \). The equation of continuity in this case is

$$\frac{\partial}{\partial t} \rho + \frac{\partial}{\partial x_i} (\rho \nu_i) = Q$$  \hspace{1cm}(15)$$
which taken together with the momentum equation (11) for infinitesimal disturbances yields

\[
\frac{\partial^2 \rho}{\partial t^2} + \alpha_0 \nabla \cdot \rho = \frac{\partial Q}{\partial t}
\]  

(16)

The density field for such a source distribution in an unbounded medium is given by the volume integral over all space,

\[
\rho(x, t) - \rho_0 = \frac{1}{4\pi \alpha_0^3} \int \frac{Q(y, t - \frac{|x - y|}{\alpha_0})}{|x - y|} \, dy_1 \, dy_2 \, dy_3
\]  

(17)

the "retarded potential" of electromagnetic theory. The essential factor governing sound production is seen (from Eq. (17)) to be the time rate change of the rate of mass introduction \(\frac{\partial Q}{\partial t}\) and for the present purpose it is denoted as the "source strength" per unit volume.

Comparing the continuity-momentum relationships of Eq. (14) and (16) for the cases of applied forces and a source distribution respectively one sees that

\[
\frac{\partial Q}{\partial t} \quad \text{corresponds to} \quad -\frac{\partial F_i}{\partial x_i}
\]

such that the fluctuating force field is equivalent in its effect on density to a source distribution whose strength per unit volume \(\frac{\partial Q}{\partial t}\) equals the inwards force flux \(-\frac{\partial F_i}{\partial x_i}\). Hence in the case of applied stresses, \(T_{ij}\), in which the equivalent force field \(F_i\) is given by \(-\frac{\partial T_{ij}}{\partial x_i}\),

the effect on density is seen to be the same as for a source distribution of strength per unit volume

\[
\frac{\partial^2}{\partial x_i \partial x_j} T_{ij}
\]

6.1.3 Deduction of the Quadrupole Field

In the estimation of the acoustic power output of the above equivalent source field, one must take due note of the fact that the equivalent source strength is a space derivative and not a simple source. The effect of the double differentiation in space is shown as follows.

In the equivalent source distribution of strength \(-\frac{\partial F_i}{\partial x_i}\)

the term \(-\frac{\partial F_i}{\partial x_i}\) is equivalent in the limit \(\varepsilon \rightarrow 0\) to the source distributions \(\varepsilon^{-1} F_i(x_1, x_2, x_3)\) and \(-\varepsilon^{-1} F_i(x_1 + \varepsilon, x_2, x_3)\), such that any value \(\varepsilon^{-1} F_i(x_1, x_2, x_3)\) occurs with positive sign at \((x_1, x_2, x_3)\) and negative sign at \((x_1, -\varepsilon, x_2, x_3)\). These two in the limit form a
dipole of strength $\mathbf{F}_i$ with axis in the positive $x_i$ direction. For the general vector $\mathbf{F}_i$ it is seen that a choice of coordinates such that the $x_i$ axis is in the direction of $\mathbf{F}_i$, yields by the above argument the fact that the corresponding dipole is of strength equal to the magnitude of $\mathbf{F}_i$ and has its axis in the direction of the vector $\mathbf{F}_i$. Hence the force field $\mathbf{F}_i$ per unit volume emits sound as a volume distribution of dipoles with the vector strength per unit volume, $\mathbf{F}_i$.

In the present case, however, $\mathbf{F}_i$ is itself the space derivative $\frac{\partial F_i}{\partial x_i}$, thus the term $\frac{\partial F_i}{\partial x_i}$ for example being equivalent in the limit $(\epsilon \to 0)$ to the dipole fields $\epsilon^{-1} \mathbf{T}_{i1}(x_1, x_2, x_3)$ and $-\epsilon^{-1} \mathbf{T}_{i1}(x_1+\epsilon, x_2, x_3)$ such that $\epsilon^{-1} \mathbf{T}_{i1}(x_1, x_2, x_3)$ occurs with positive sign at $(x_1, x_2, x_3)$ and negative sign at $(x_1-\epsilon, x_2, x_3)$. This pair of dipoles in the limit yields a quadrupole with strength equal to the magnitude of the vector $\mathbf{T}_{i1}$ and axes in the directions of $\mathbf{T}_{i1}$ and $\mathbf{x}_i$. Again generalizing the argument as done for $\mathbf{F}_i$, one sees that an applied fluctuating stress field $\mathbf{T}_{ij}$ per unit volume emits sound as a volume distribution of quadrupoles of strength per unit volume $\mathbf{T}_{ij}$. Since there is no indication that $\mathbf{T}_{ij}$ is in turn a space derivative it is concluded that the sound field may be treated as the effect of this quadrupole distribution.

6.1.4 Density Field

In Lighthill's consideration of the density field the equivalent source distribution $-\frac{\partial \rho_i}{\partial x_i}$ replaces $(\frac{\partial Q}{\partial t}) |\mathbf{x} - \mathbf{y}|^{-1}$ in the integrand for the simple source (see Eq. (17)) by

$$\lim_{\epsilon \to 0} \epsilon^{-1} \sum_{i=1}^{3} \left\{ \left( \frac{\mathbf{F}_i}{|\mathbf{x} - \mathbf{y}|} x_i \right) \left( \frac{\mathbf{F}_i}{|\mathbf{x} - \mathbf{y}|} x_i + \epsilon \right) \right\}$$

such that the R.H.S. of Eq. (17) becomes

$$-\frac{1}{4\pi a_0^2} \frac{\partial}{\partial x_i} \int \frac{\mathbf{F}_i}{|\mathbf{x} - \mathbf{y}|} \left( \mathbf{y}, t - \frac{|\mathbf{x} - \mathbf{y}|}{a_0} \right) d\mathbf{y}_1 d\mathbf{y}_2 d\mathbf{y}_3$$

Replacing $\mathbf{F}_i$ by $-\frac{\partial \mathbf{T}_{ij}}{\partial y_j}$ and repeating the argument yields

$$\rho(\mathbf{x}, t) - \rho_0 = \frac{1}{4\pi a_0^2} \frac{\partial^2}{\partial x_i \partial x_j} \int \mathbf{T}_{ij}(\mathbf{y}, t + \frac{|\mathbf{x} - \mathbf{y}|}{a_0}) d\mathbf{y}_1 d\mathbf{y}_2 d\mathbf{y}_3$$

One obtains the simpler form

$$\rho(\mathbf{x}, t) - \rho_0 \approx \frac{1}{4\pi a_0^2} \int (\mathbf{x}_1 - \mathbf{y}_1)(\mathbf{x}_2 - \mathbf{y}_2) \frac{1}{a_0^2} \frac{\partial^2}{\partial t^2} \mathbf{T}_{ij}(\mathbf{y}, t + \frac{|\mathbf{x} - \mathbf{y}|}{a_0}) d\mathbf{y}_1 d\mathbf{y}_2 d\mathbf{y}_3$$
for points at a distance by retaining only the differentiations of $T_{ij}$ and neglecting those terms involving differentiation of $|x - y|^{-1}$ which vary as the inverse second and third powers of the distance from the fluctuating flow field. Eq. (19) is considered to be valid for $x$ at a distance from the flow field large compared with $\lambda$, where $\lambda$ is a typical wavelength since fluctuations of a function are less than those of its time derivative by a factor of the order $2\pi/\lambda$. (A fluctuating quantity $q = Be^{-i2\pi ft}$ having the time derivative $-i2\pi f q$).

A further simplification can be introduced for distances large relative to the dimensions of the fluctuating flow (the "far" field) such that $(x_i - y_i)$ in Eq. (19) may be replaced by $x_i$ provided the origin is taken within the flow. Thus without neglecting any terms of order $|x|^{-1}$ Eq. (19) becomes

$$
\rho(x, t) - \rho_0 \approx \frac{1}{4\pi a_0^2} \frac{x_i x_j}{x^3} \int \frac{2}{a_0^2} T_{ij}(\gamma, t - \frac{|x - y|}{a_0}) dy_1 dy_2 dy_3
$$

Equations (19) and (20), depending principally on the second time derivative of the equivalent applied stress $T_{ij}$ are the basic result of Lighthill's paper. Note that Eq. (20) is not applicable to the so-called "near" field consisting of the space external to the fluctuating jet flow at a distance not large relative to its dimensions. The more exact forms of Eq. (19) or even Eq. (18) are needed in this case.

6.1.5 Dimensional Analysis

The dependence of the density variations in Eq. (19) on $\rho_0, a_0, \gamma$, a typical velocity and length of the flow ($U$ and $\ell$) is inferred for geometrically similar mechanisms by noting that $T_{ij}$ is proportional to $\rho_0 U^2$ (see Eq. (13)). In general, a typical flow frequency $\ell, \gamma$ varies as $U/\ell$ (from the relative constancy of the Strouhal number $St = \frac{f \ell}{U}$ of the order of $U$). Hence, the fluctuations in $\frac{\partial^2 T_{ij}}{\partial t^2}$ are proportional to $(\rho U^2)(U^2/\ell^4)$. Density variations $(\rho - \rho_0)$ at a given directed distance $x$ are then proportional (see Eq. (19)) to

$$
\frac{1}{a_0^2} \frac{1}{x} \frac{1}{a_0^2 (\ell)} \rho_0 U^2 \ell^3
$$
The intensity of sound is

$$\rho_o^3 \rho_o^{-1} \left[ (\rho - \rho)^2 \right]$$

(22)

where the term in square brackets is the mean square fluctuation of $\rho$ at the point considered. From Eq. (19) one sees that $(\rho - \rho_o)$ is a time derivative and hence has a zero mean value in time such that the mean density is $\rho_o$ and the mean square fluctuation is the mean value at the point of $(\rho - \rho_o)$ squared. Dimensionally the intensity is thus proportional to

$$\rho_o^3 U^4 a_o^{-4} x^{-1}$$

(23)

The total acoustic power output obtained by integrating the intensity over the surface of a sphere of radius large compared with the dimensions of the flow field is then proportional to

$$\rho_o U a_o^{-5} l^2$$

(24)

The proportionalities referred to above are not exact and hence expectations should be conservative regarding these dimensional formulae.

6.1.6 Frame of Reference in Translation

One considers a coordinate system in translation with uniform velocity $a_o \frac{\partial}{\partial \tau}$ and $M_c < 1$ and coinciding with the previously considered fixed axes at time $t$. The axes then move a distance $M_c |\dot{x} - \dot{y}|$ while sound travels from $y$ to $\tilde{y}$ such that $\tilde{y}$ in the argument of $T_{ij}$ is replaced by $\tilde{y} + \dot{y} + M_c |\dot{x} - \dot{y}|$ (see Fig. 2). The density field of Eq. (18) then becomes

$$\rho(x, t) = \frac{1}{4\pi a_o^2} \sum_{i,j} \int_{T_{ij}} \left( T_{ij, t} \right) \frac{d\eta_i}{a_o} \frac{d\eta_j}{a_o} \frac{d\eta_3}{a_o}$$

(25)

At points at a distance from the jet flow large compared with $\frac{\lambda}{2\pi}$ this equation simplifies, by applying the differentiation to $T_{ij}$ only, (as in the derivation of Eq. (19)) to

$$\rho(x, t) - \rho_o \approx \frac{1}{4\pi a_o^2} \int_{|x-y|^2 = |x-y|^2} \frac{1}{|x-y|^3} \frac{d\eta_i}{a_o} \frac{d\eta_j}{a_o} \frac{d\eta_3}{a_o}$$

(26)
As previously stated (see Eq. (22)) the intensity field is given by
\( I(x) \approx \frac{1}{16 \pi^2 \rho_0 a_0^5} \int \frac{(x_i - y_i)(x_j - y_j)(x_k - y_k)(x_l - y_l)}{(x - \bar{y})^3} \left( \frac{1}{x - \bar{z}} - M_c \cdot (x - \bar{z}) \right)^3 \)

\[
\frac{\partial^2}{\partial t^2} \overline{t_i}(\bar{r}, t - \frac{|\bar{x} - \bar{y}|}{a_0}) \frac{\partial^2}{\partial t^2} T_{ikl}(\bar{r}, t - \frac{|\bar{x} - \bar{y}|}{a_0}) d\eta_i d\eta_k d\xi_i d\xi_k
\]

In integrating this mean square over a large sphere to obtain power output, the \( y_i \) and \( z_i \) may be neglected relative to the \( x_i \) and the integral

\[
\int \frac{x_i x_j x_k x_l}{(x - M_c \cdot x)^6} dS
\]

must be evaluated. With the \( x_i \) axis chosen in the direction of \( \vec{M}_c \), (28) vanishes by symmetry unless the suffixes \( i, j, k, l \) are equal in pairs. The power output for the case \( M_c = 0 \) (coordinate system at rest) is then multiplied by

\[
a) \frac{1}{(1 - M_c^2)^3} \quad b) \frac{1 + 5M_c^2}{(1 - M_c^2)^4} \quad \text{or} \quad c) \frac{1 + 10M_c^2 + 5M_c^4}{(1 - M_c^2)^5}
\]

according as neither, one or both the pairs of suffixes agree with the direction of \( \vec{M}_c \). These cases correspond to quadrupoles with neither, one or both their axes in the direction of motion.

At distances large compared with the maximum separation of points \( \bar{y} \) and \( \bar{z} \) in the flow for which there is a significant covariance, both \( (x_i - y_i) \) and \( (x_i - z_i) \) in Eq. (27) may be replaced by \( |x - \bar{y}| \cos \theta \) or \( |x - \bar{y}| \sin \theta \) depending on whether \( i \) is 1 or not (assuming one of the lateral quadrupole axes is coincident with the direction of \( \vec{M}_c \)).

In the special case with source velocity \( a_o \cdot M_c \) in the direction \( x \), and \( \cos \theta = \frac{x_i - y_i}{x} \) assumed to be constant for all positions of the sources in the fluctuating flow relative to an observer at \( \bar{x} \), the intensity may be written in the form

\[
I(x, \theta) \approx \frac{(\cos^2 \theta K_i + 2 \cos \theta \sin \theta K_a + \sin^2 \theta K_d)^2}{16 \pi^2 \rho_0 a_o^5 x^2 (1 - M_c \cdot \cos \theta)^6}
\]
where $K_1, K_2, K_3$ are functions of $M_c$ and are independent of $\theta$ (see Ref. 8).

The effect of translation on frequency is to cause the band of frequencies considered in $T_f(\hat{x}, t)$ to be responsible for radiation to a distant point $x$ of a band of frequencies obtained by multiplying those of the original band by the factor

$$\left\{ 1 - \frac{M_c \cdot (x - \hat{x})}{|x - \hat{x}|} \right\}^{-1}$$

The frequency is increased for sound emitted forwards and decreased for backwards emission. In the special case considered above for the derivation of Eq. (30), the sound intensity $I_f(\hat{x})$ measured in a small band around the frequency $f$ by an observer at $\hat{x}$, allowing for Doppler effect is

$$I_f(\hat{x}, \theta) \propto \frac{x^2 (\cos^2 \theta K_4 + 2 \cos \theta \sin \theta K_5 + \sin^2 \theta K_6)}{\rho_0 \cdot a_0^5 x^2 (1 - M_c \cos \theta)^2}$$

(31)

where $K_4, K_5, K_6$ are functions of $M_c$ and $f$, independent of $\theta$ (see Ref. 8).

6.2 Proudman's Theory of Isotropic Turbulence

Proudman, considering an isotropic region of turbulence with stationary boundaries embedded in an infinite expanse of compressible fluid, derived the rate of conversion of the kinetic energy of turbulence into acoustic energy. Assuming the turbulent medium to be incompressible reduces the number of parameters on which mean values are dependent to three (by Heisenberg's hypothesis). Only eddies not dissipating energy by viscosity are considered to contribute to noise generation at large Reynolds numbers, their contribution being independent of Reynolds number. These considerations, dimensional analysis and Lighthill's result of Eq. (24) yield

$$\theta = \beta \cdot \varepsilon \cdot M_c^5$$

(32)

for the acoustic power output per unit mass, $\theta$. Here $\beta$ is a constant to be determined and $\varepsilon$, the mean rate of dissipation of energy per unit mass is given by

$$\varepsilon = - \frac{3}{2} \frac{d\nu^2}{dt}$$

(33)
where \( \overline{V^2} \) is the mean square of one of the three velocity components and the Mach number is defined as

\[
M^* = \frac{\overline{V^2}}{a_0}
\]

It is further assumed that the mean temperature of the turbulence approximates that of the surrounding fluid, that the turbulence is adiabatic, that the Reynolds number is large, the Mach number small, and that the space correlation, \( S_{xx} \), of the second time derivatives of \( \overline{V_x^2 - V_x^2} \) at points \( \overrightarrow{y} \) and \( \overrightarrow{y'} \) (where \( V_x \) is the velocity component in the \( x \) direction) is zero for points further separated than some distance \( \ell \), and the time correlation for these quantities is unity at a point for a time interval less than \( \ell/a_0 \) (hence the smallest wavelength of significant acoustic energy must be much larger than the largest eddies generating noise.)

Lighthill's equation for the density (Eq. (20)), under the assumptions there noted, reduces in this case to

\[
\rho(x', t) = \rho_0 \frac{1}{4\pi a_0^4} \int \left[ \frac{\partial^2}{\partial t^2} V_x^2 \right]_{t = \frac{x}{a_0}} d\gamma_1 d\gamma_2 d\gamma_3
\]

since \( T_{ij} \) is simplified to \( \rho_0 V_x \gamma_i \) by the assumption of the present article. The equation for the intensity (Eq. (32)) then becomes

\[
I(x', t) = \frac{\rho_0}{16 \pi^2 a_0^5 x^2} \int \left[ \frac{\partial^2}{\partial t^2} (V_x^2 - \overline{V_x^2}) \right]_{t = \frac{x}{a_0}} d\gamma_1 d\gamma_2 d\gamma_3 d\gamma_3'
\]

or

\[
I(x', t) = \frac{\rho_0}{16 \pi^2 a_0^5 x^2} \int [d(\gamma_1' - \gamma_1) d(\gamma_2' - \gamma_2) d(\gamma_3' - \gamma_3)] d\gamma_1 d\gamma_2 d\gamma_3
\]

being independent of the direction of \( \overrightarrow{x} \) for this case of isotropy such that the intensity field is non-directional like that for a simple source.

The local instantaneous power output per unit mass is then

\[
\frac{\rho_0}{4\pi a_0^5} \int S d(\gamma_1' - \gamma_1) d(\gamma_2' - \gamma_2) d(\gamma_3' - \gamma_3)
\]
and $\beta$ is determined by evaluation of this integral.

The correlation $S$ being of fourth order in the velocity and its first two time derivatives, the assumption is made that these three quantities at two points in space have a normal joint probability distribution allowing expression of $S$ in terms of second order correlations. These in turn, by the assumption of incompressibility, are elements of solenoidal tensors (tensors for which the divergence is zero) and may be expressed in terms of three double correlation functions. The terms involving two of these in the expression for $S$ may be expressed in terms of the third $f^*$ by use of the dynamical equations and normality hypothesis such that $\theta$ is a function involving only $\overline{\nu^2}$, $f^*$ and its derivatives with respect to $|\nabla' - \nabla|$ and $t$. $\beta$ is then given by a sum of integrals involving $f^*$ and its derivatives with respect to $x = \frac{1}{2} \nabla'-\nabla' \cdot L^{-1}$ (where $L = (\overline{\nu^2})^{\frac{1}{2}} \epsilon^{-1}$ is the length scale of the turbulence) in which the terms depending on the instantaneous forms of the correlation functions occur separately from those depending on their time rates of change, allowing separate calculation of the contributions of instantaneous and decay terms. Using Heisenberg's form of the correlation function $f^*(x)$ (which has strong experimental support for large Reynolds numbers) $\beta$ was found to be 37.7. Considering the greatest reasonable variation in the shape of $f^*(x)$ by using $f^*(x) = e^{-\frac{\pi x^2}{4}}$ yielded $\beta = 13.2$. The contribution of the decay terms in each case was roughly 1%.

In the case of steady anisotropic turbulence fields, maintained by an external energy source (becoming now the rate of energy supply) any variation in power output is expected to be explained by the form of the correlation functions, since decay alone is not an important feature in the isotropic case. It has been shown, however, that $\beta$ is relatively insensitive to changes in the correlation function and hence for most types of turbulence it is expected that $\beta$ will be in the range of 10 to 100.

6.3 Lighthill's Extension of Subsonic Noise Theory

6.3.1 Turbulence Eddies Con vected at Non-Negligible Mach Number

In Ref. 3 Lighthill considers an extension of the theory of aerodynamic noise to the case of non-negligible fluctuation Mach number obtaining the expression

$$i(x) \approx \frac{x_1 x_2 x_3 x_4}{16 r_0 a_0^2 (x - M_c \cdot x)^6} \left( \frac{\partial^2 T_i(x, t)}{\partial t^2} - T_i M_c(x, t) \right) dx_1 dx_2 dx_3$$

(36)
for the intensity at \( \vec{x} \) per unit volume of turbulence at the origin. This expression is derived from the general intensity equation of Ref. 2 (see Section 6.1) given as

\[
I(\vec{x}) \propto \frac{1}{16 \, x^2 \, \rho_0 \, a_o^5} \int \int \frac{(x_i - y_i)(x_j - y_j)(x_k - z_k)(x_l - z_l)}{|x - y|^3 \cdot |x - z|^3} \ (37)
\]

by letting \( \vec{y} = 0 \), dropping the integration with respect to \( \vec{y} \) and neglecting differences between \((\vec{x} - \vec{y})\) and \((\vec{x} - \vec{z})\) on the assumption that \( \vec{x} \) is far from the origin compared with an average eddy size. The average eddy volume has a diameter \( l \) such that the correlation in Eq. (37) is negligible for points separated by a distance exceeding \( l \). The difference in the retarded times of the two values of \( \tau_i \) is made negligible by basing the analysis in a frame of reference, moving at the local eddy convection velocity \( a_o \, M_c \), which is such that the eddies alter slowest when viewed from this frame. The time scale of the turbulent fluctuations is then of the order of an eddy size divided by a typical departure of the velocity from its mean. This ratio is in general large relative to \( l \, a_o^{-1} \) since the Mach number of the velocity fluctuations is almost always small. The difference between the retarded times (being less than \( l \, a_o^{-1} \)) is then negligible compared with the times significant in the turbulent fluctuations. The results of Ref. 2 concerning a moving frame of reference are then applied, multiplying the intensity field of an element \( d\gamma_i \, d\gamma_2 \, d\gamma_3 \) by the factor

\[
\left[ 1 - \frac{\vec{M}_c \cdot (\vec{x} - \vec{y})}{|\vec{x} - \vec{y}|} \right]^{-6} \ (38)
\]

While the eddy convection velocity \( a_o \, M_c \) need not be uniform as in Ref. 2, but varies over the field, the result may still be used provided \( M_c \) shows small variation in a distance of order \( l \), since two points radiate sound independently if separated by a greater distance.

6.3.2 The Importance of the Shear in Aerodynamic Noise

The amplifying effect of large mean shear is seen by considering the time rate of change of momentum flux (the chief factor governing intensity) in the form
\[
\frac{\partial}{\partial t}(p v_i v_j) = p_{ij} \frac{\partial v_i}{\partial x_j} + p_{kl} \frac{\partial v_i}{\partial x_l} - \frac{\partial}{\partial x_k} \left[ (p v_i v_j + p_{ij}) + p_{jk} v_j \right] \tag{39}
\]

(Using the equations of momentum and continuity to express time derivatives as space derivatives). The last term being a pure space derivative represents an octupole field and hence may be dropped as a negligibly weak radiation source. The viscous contribution to the stresses may also be ignored for Reynolds numbers of the order found in fully developed turbulence, yielding

\[
\frac{\partial}{\partial t}(p v_i v_j) = p \left( \frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i} \right) = p \, e_{ij} \tag{40}
\]

i.e. the product of the pressure \( p \) (measured from atmospheric as zero) and the rate of strain (\( e_{ij} \) being the rate of strain tensor).

### 6.3.3 Note on Eddy Size

An intermediate size of eddy is shown (Ref. 3) to be of most importance in sound generation by replacing \( \frac{\partial}{\partial t} T_{ij} \), in Proudman's theory (Ref. 4) for isotropic turbulence, by \( p \, e_{ij} \). The power output per unit volume is then

\[
\theta^* = \frac{1}{\pi p_0 a_o^5} \int \frac{\partial}{\partial t} \left( p \frac{\partial v_i}{\partial x_j} \right) \, d\mathbf{z} \tag{41}
\]

\[
= 38 \, p_0 (\nabla v_i)^4 \, L^{-1} \, a_o^{-5} \propto 38 \, \varepsilon (\nabla v_i)^5 \, p_0 a_o^{-5}
\]

as shown by Eq. (32), where \( L = (\nabla v_i)^3 \, \varepsilon^{-1} \) and \( \varepsilon \) is the mean rate of energy dissipation per unit mass. Proudman evaluated the integral

\[
\int \left( \frac{\partial p}{\partial t} \right)_{x^j = 0} \, d\mathbf{z} \propto 1.6 \, p_0^2 \, (\nabla v_i)^3 \, L \tag{42}
\]

The order of magnitude of the fluctuations in velocity gradient contributing most to \( p e_{ij} \) is inferred from the square root of the ratio of Eq. (41) to (42) to be \( 8 \, (\nabla v_i)^2 \, L^{-1} \). Hence the effective velocity gradients in sound production are roughly 8 times those typical of the largest energy bearing eddies.
6.3.4 Aerodynamic Noise in the Presence of Large Shear

In turbulence with large mean shear a single term (e.g. $p \vec{e}_{12}$) predominates, hence the quadrupoles are aligned to radiate maximum sound at $45^\circ$ to the direction of motion. The sound is amplified if the mean shear $\vec{e}_{12}$ exceeds $30(\vec{v}_i)^2 L^{-1}$ (using the facts that the effective fluctuating velocity gradients in isotropic turbulence are $\approx 8(\vec{v}_i)^2 L^{-1}$ and the sound radiated by one lateral quadrupole orientation is $\frac{1}{15}$ that by all orientations in equal strength).

The intensity field per unit volume of turbulence at the origin is

$$i(x) \sim \frac{x^2}{(x - M_e x_1)^6} \cdot \frac{\tau(0)}{4 \pi^2 \rho_o a_o^5} \cdot V$$

obtained from Eq. (36) by assuming that $\frac{\partial}{\partial t} T_{ij}$ is predominantly $p \vec{e}_{12} \propto p \frac{\partial \vec{v}_i}{\partial x_2}$, that the direction of eddy convection is parallel to the only significant mean velocity $\vec{v}_i$ and that only the components $\vec{e}_{12}, \vec{e}_{21}$ of $\vec{e}_{k\ell}(x)$ need be considered (as the covariance is significant only when $x$ is close to the origin at which time the mean shears should be similarly oriented at $x$ and the origin), $\tau(x) \propto \vec{e}_{12}(x)$ is the component of mean shear in the direction of shear at the origin. $V$ is the average eddy volume, defined as

$$V = \int \frac{\tau(x)}{\tau(0)} \frac{\frac{\partial}{\partial t} p(0,t) \cdot \frac{\partial}{\partial t} p(x,t)}{\left(\frac{\partial}{\partial t} p(0,t)\right)^2} \cdot dx_1 dx_2 dx_3$$

The power output per unit volume under these conditions is

$$\frac{1 + 5 M_e^2}{(1 - M_e^2)^5} \cdot \frac{\tau^2(0) \left(\frac{\partial p}{\partial t}\right)^2}{15 \pi \rho_o a_o^5} \cdot V$$

by integrating Eq. (43) over a large sphere centred at $x = 0$. 
The total power output of a thin shear layer is

\[
\frac{1 + 5 M_c^2}{(1 - M_c^2)^4} \int \tau(\vec{y}) \tau(\vec{z}) \frac{\partial p(\vec{y}, t)}{\partial t} \frac{\partial p(\vec{z}, t)}{\partial t} \, dx \, dy_1 \, dy_2 \, dy_3 \, dz_1 \, dz_2 \, dz_3
\]

(46)

Assuming the covariance varies much more slowly than \( \tau(\vec{y}) \) or \( \tau(\vec{z}) \) as \( \vec{y}, \vec{z} \) cross the layer, integration yields the power output per unit area of turbulent shear layer.

\[
\rho' = \frac{1 + 5 M_c^2}{(1 - M_c^2)^4} (\delta v)^2 \left( \frac{\partial p}{\partial t} \right)^2 A \frac{1}{15 \pi \rho_o a_o^5}
\]

(47)

where \( \delta v \) is the velocity change across the shear layer and

\[
A = \int \left[ \frac{\partial}{\partial t} p(\vec{y}, t) \right]^2 \, dA_x
\]

(48)

is the average eddy area at a point \( \vec{y} \) with \( dA_x \) an element of area of the shear layer in \( \vec{z} \) space.

The result of Eq. (47) applies also to the annular shear layer of a jet if area in Eq. (48) is taken to mean projected area on the tangent plane at \( \vec{y} \). In addition, the combination of lateral quadrupoles in the \((x_1, x_2)\) and \((x_1, x_3)\) planes yields a directional distribution proportional to

\[
\frac{x_1^2 (x_2^2 + x_3^2)}{(x - M_c x_1)^6} = \frac{\sin^2 \Theta \cos^2 \Theta}{x^2 (1 - M_c \cos \Theta)^6}
\]

(49)

The frequencies in shear flow turbulence are expected to be of the order \((2\pi)^{-1} \cdot \) times the mean shear because of the term \( v_2 \frac{dV_1}{dx_2} \) in the acceleration \( \frac{Dv_1}{Dt} \).

6.3.5 Aerodynamic Noise Arising From a Difference of the Mean Sonic Velocity in the Turbulent Region From That in the Surrounding Medium.

Lighthill also shows that pressure fluctuations are in
general of small importance in generating aerodynamic noise through their direct sonic field, by considering the main term in \( \tau_{ij} \) other than momentum flux. This is the source field due to that part of the turbulent pressure fluctuations which is not balanced by accompanying local density fluctuations in the free acoustic vibrations of the atmosphere, having a source strength per unit volume of

\[
\left[ 1 - \left( \frac{\alpha_0}{\bar{a}} \right)^2 \right] \cdot \frac{1}{\alpha_0^4} \frac{\partial^2 p}{\partial t^2} \tag{50}
\]

where \( \bar{a} \) is the mean velocity of sound at the point and pressure fluctuations are considered to be nearly adiabatic. Viscous contributions to \( p_{ij} \) are neglected.

The power output of the above source field per unit volume of turbulence at the origin is shown by the previous arguments to be

\[
\frac{1}{4\pi \rho_0 a_0^5} \left[ 1 - \left( \frac{\alpha_0}{\bar{a}(x)} \right)^2 \right] \left( 1 - \left( \frac{\alpha_0}{\bar{a}(z)} \right)^2 \right) \frac{\partial^2}{\partial t^2} p(0,t) \cdot \frac{\partial^2}{\partial t^2} p(x,t) \, dx_1 \, dx_2 \, dx_3 \tag{51}
\]

omitting quadrupole convection.

The amplification due to pressure fluctuations can only be important if this term is at least comparable to the power output due to momentum flux.

For isotropic turbulence with \( \bar{a} \gg \alpha \) (as in a heated jet) Eq. (51) differs from Eq. (42) by a single time differentiation. The same integral with no time differentiation is obtained from Batchelor's results (Ref. 24) as \( \frac{2}{\rho_0} (\bar{v}_1^2)^2 \cdot L^{-3} \) showing the dominant frequency for pressure fluctuations to be of the order of \( (\bar{v}_1^2)^2 \cdot L^{-1} \) and Eq. (51) to be of the order of

\[
\frac{\rho_0 (\bar{v}_1^2)^4 \cdot L^{-1}}{4\pi \alpha_0^5}
\]

Hence this term is negligible relative to the momentum flux contribution (see Eq. (41)). This result is considered applicable to the case of the jet.

In heavily sheared turbulence (e.g. the mixing region of a jet) the power output of momentum flux (which in general takes the form of Eq. (51) with \( \frac{2}{\rho_0} \frac{\partial^2 p}{\partial L^2} \) replaced by \( \frac{\partial p \cdot \bar{e}_{ij}}{\partial t} \) and hence exceeds Eq. (51) if \( \partial x^{-1} \bar{e}_{ij} \) exceeds the dominant frequencies of pressure fluctuation) is reduced by 4/15 by consideration of a single lateral quadrupole field instead of a source field (comparing Eq. (46))
and (51). The relative importance of
\[
\left( \frac{\partial^2 p}{\partial t^2} \right) \quad \text{to} \quad \left( \bar{e}_{ij} \frac{\partial p}{\partial t} \right)
\]
is greatest in the high frequency bands where a possible amplification
of \(10 \log (1 + 15.4) = 6 \text{db}\) is thus an upper limit.

In heavy gases having \(\bar{a} < a_o\), greater amplification at high
frequency in sheared turbulence is expected; the theoretical ratio of
the intensity field to the above case with \(\bar{a} \gg a_o\) being 18 to 1 for Freon
as an example.

6.4 Some Physical Implications of the Theory

The following sections may help one to a better understanding
of Lighthill's theory, and aid in its comparison later with experimental
results.

6.4.1 The Equivalent Stress Tensor, \(T_{ij}\)

In the region outside the fluctuating jet flow (the atmosphere) \(T_{ij}\)
may be neglected as explained in Section 6.4.2. In the region of the
fluctuating flow itself, the assumption that the viscosity term in \(p_{ij}\)
(see Eq. (5)) may be neglected permits \(T_{ij}\) to reduce to \(\sigma v_i v_j\). If,
in addition, the flow system is adiabatic (as it may be assumed, e.g. in a
cold jet) the further simplified form

\[
T_{ij} \approx \sigma_0 v_i v_j
\]

may be used. The error involved in this form is of the order of \(M^{k2}\)
since the relative changes in density are of the order of \(M^{k2}\)
and the ratios of fluctuations in pressure to those in density depart from
\(\sigma_0^2\) by a proportional error of the order of \(M^{k2}\). The error is
small, therefore, for low fluctuation Mach numbers.

In a "cold" jet flow one expects only the first term \(\sigma v_i v_j\)
in \(T_{ij}\) representing momentum flux to be important (Ref. 3). The
sound field will therefore, be large when there are large variations in
\(v_i\) and \(v_j\) as, e.g. in regions of high velocity shear and high
turbulence. \(\sigma v_i v_j\) can fluctuate most widely when the fluctuations
of \(\sigma v_i\) are amplified by a heavy mean value of \(v_j\). In other words,
turbulence of given intensity can generate more sound in the presence
of a large mean shear.
In a jet flow of non-uniform temperature or fluid composition (e.g., a heated jet) the speed of sound varies widely in the turbulent flow from that in the surrounding atmosphere, and the pressure terms in $T_{ij}$ (see Eq. (13)), $\rho v_i v_j$, may become important. In this case, the pressure fluctuations in the turbulence are only partly balanced by $\alpha_o^2$ times the density fluctuations and the remainder contributes to $T_{ij}$.

Lighthill assumes this contribution to $T_{ij}$ is still small compared with that resulting from momentum flux $\rho v_i v_j$ except in the case of sheared turbulence in heavy gases (Section 6.3.5).

### 6.4.2 Directionality

Lighthill's theory considered in previous sections deals with the general quadrupole field in the case of a frame of reference at rest, and extends it to the case of a frame of reference in translation. He shows that an analysis of the equivalent stress tensor $T_{ij}$ into a pressure and a single pure shearing stress effects an analysis of the quadrupole into three equal mutually orthogonal longitudinal quadrupoles, each of strength

$$T = \frac{1}{3} T_{ii} = \frac{1}{3} \rho v_i^2 + p - \alpha_o^2 \rho$$

and one lateral quadrupole. The sum of these three longitudinal quadrupoles is equivalent to a simple source of strength

$$\alpha_o^2 \frac{\partial^2 T}{\partial t^2} = \frac{1}{3} \left( \frac{1}{\alpha_o^2} \frac{\partial T_{ii}}{\partial t^2} \right)$$

at least as far as their sound radiation field outside the stress field is concerned.

#### (a) Stationary Field

The sound radiation fields of a stationary lateral quadrupole and a simple source are shown in Fig. 3. It shows that the lateral quadrupole radiation field has intensity maxima at angles of $\theta = 45^\circ$ and $135^\circ$ to the jet axis and shows no radiation of sound energy at $\theta = 90^\circ$. The simple source radiation is uniform in all directions.

#### (b) Moving Field

The density fields for the general quadrupole field at
rest and in motion are given by Eqs. (19) and (26) respectively for the same simplifying assumptions. A comparison of these two equations shows that Eq. (26) is nothing else than Eq. (19) multiplied by a factor \((1-M_c \cos \Theta)^{-3}\) due to the motion of the quadrupole field.

Applying this factor to the radiation field equations of the stationary lateral quadrupole and the simple source, we get the changes in the radiation fields due to translation as shown in Fig. 3 for a Mach number of \(M_c = 0.9\). It clearly indicates the heavy increase in emission of sound energy forwards, which is much greater than the reduction in rearwards emission. Fig. 4 shows the rapid increase of the sound intensity maxima in the downstream direction and their shifting towards smaller angles \(\Theta\) with increasing Mach number of translation. One also notes a distinct decrease in upstream noise levels.

One of the weaknesses of Lighthill's theory is that it does not allow for refraction of noise. The turbulence of a jet may cause severe refraction of noise passing through it.

6.4.3 Dimensional Reasoning

When the typical flow velocity \(U\) and length \(l\) are chosen to be the jet flow exit velocity \(U_e\) and the nozzle diameter \(d\), the total power output (Eq. (24)) is seen to be roughly proportional to

\[ \rho_o U^8 a_o^{-5} d^2 \]  

Thus theory predicts that the sound intensity increases roughly as the eighth power of the jet exit velocity \(U\), and the square of the jet diameter.

The sound pressure \(P\) being proportional to the density fluctuations is roughly proportional (see Eq. (21)) to \(U^4\) and \(d\).

It may be noted here that in the dimensional expression \(\rho_o U^2\) for \(T_{ij}\), the \(\rho_o\) is a reference density appropriate to the fluctuating flow field and hence in the case of heated jets or jets of a gas different from the surrounding medium (atmosphere) is not the "atmospheric" density \(\rho_o\) existing outside the jet. In such cases the relationship is clarified by using \(\rho'_o U^2\) for \(T_{ij}\) such that the sound pressure \(P\) is seen to be proportional to the mean density in the jet

\[ \rho'_o U^4 a_o^{-4} d x^{-1} \]
In a jet flow of non-uniform temperature or fluid composition (e.g., a heated jet) the speed of sound varies widely in the turbulent flow from that in the surrounding atmosphere, and the pressure terms in $T_{ij}$ (see Eq. (13)), $P_{ij} - a_o^2 \delta_{ij} \varphi$, may become important. In this case, the pressure fluctuations in the turbulence are only partly balanced by $a_o^2 \times$ the density fluctuations and the remainder contributes to $T_{ij}$.

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and one lateral quadrupole. The sum of these three longitudinal quadrupoles is equivalent to a simple source of strength

$$a_o^2 \frac{\partial^2 T_{ii}}{\partial t^2} = \frac{1}{3} \left( \frac{1}{a_o^2} \frac{\partial^2 T_{ii}}{\partial t^2} \right)$$

at least as far as their sound radiation field outside the stress field is concerned.

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\[ \rho_o' U^4 a_o^{-4} d x^{-1} \]  

(56)
and the intensity is proportional to the mean density squared

$$\rho_o^{-1} \left( \rho_o U^8 a_o^{-5} d^2 \right)^{-1/2}$$

Finally, using dimensional reasoning an efficiency of noise production can be defined as the ratio of the total acoustic power output to the supply of jet energy

$$\eta_N \approx K \frac{\rho_o^5}{\rho_0^5 (U/a_o)^5}$$

This relationship indicates that turbulence at low Mach numbers is a quite exceptionally inefficient producer of sound (for $M = 0.2$, $\eta_N \approx 0.3\%$; for $M = 1$, $\eta_N \approx 0.01\%$). The factor $K$ is called the "acoustic power coefficient" and defined as

$$K = \frac{\text{acoustic power (measured)}}{\rho_o U^8 a_o^{-5} d^2}$$

when the density of the jet flow is equal to that of the atmosphere, $\rho_o$. If the jet flow density $\rho'$ differs from $\rho_o$ we get

$$K = \frac{\text{acoustic power (measured)}}{\rho_o^{-1} \rho_o^{12} U^8 a_o^{-5} d^2}$$

6.4.4 Mach Number, Reynolds Number, and Strouhal Number

The density variations in the sound radiation field (at distances from the turbulent flow large compared with $\sqrt{a_d}$) are proportional to $M^4$ (Eq. (21)). Density changes in the flow itself are of the order of $\rho' M^2$. This difference indicates that sound radiation is a Mach number effect, because of its origin from a quadrupole source, $T_{ij}$, which is of strength proportional to $U^2$. 
An increase in $Re$ is expected to increase $K$ (Ref. 2) since the energy of the turbulence is borne principally by frequencies such that the Strouhal number $(St = \frac{f d}{U})$ is less than one, and these frequencies grow gradually with $Re$. Counter to this, the corresponding eddy sizes (and hence range of $|\hat{Y} - \hat{Z}|$ for which the covariance in Eq. (37) is not negligible) are smaller.

From previous experience a constant value of unity may be considered for $St$ such that the predominant flow frequency would satisfy the relationship $f \propto \frac{U}{d}$. The frequency has a lower limit of $\approx 0.2 \frac{U}{d}$ (imposed by the scale of the jet system) while its high values are damped by viscous action. Hence the Strouhal number has a corresponding lower limit and grows slowly with increasing Reynolds number. The Strouhal number would prove a useful parameter in an analysis of the dependence of $K$ on frequency at various $Re$ values. The variations in shape of the frequency spectra might give information as to which aspects of the turbulent flow contribute most to the noise.

6.5 Theory for the Choked and Overchoked Jet

So far there does not exist a quantitative theory for the choked and overchoked jet comparable to that of Lighthill for the subsonic jet flow. All knowledge available is based on empirical results.

In the choked jet flow we have again a field of moving turbulence (eddies) as in the subsonic jet. In addition, a standing shock wave pattern is formed which grows in strength the more the jet flow becomes overchoked. Experimental evidence shows a marked change in the nature of the noise produced when the pressure ratio at the nozzle exit is increased beyond the critical (i.e., that at which the jet exit velocity first becomes sonic or the jet becomes "choke"). The conclusion was drawn that an eddy-shock wave interaction may be responsible for this change in the nature of jet noise.

Lighthill (Ref. 5) has put the general case of the noise produced by the interaction of turbulence with shock waves on a firm mathematical basis. He derived formulae for the sound energy scattered when a sound wave passes through a turbulent fluid flow or when a unit of turbulence (eddy) passes through a shock wave. Ribner (Refs. 6 and 78) examined the same problem theoretically. Powell (Ref. 7) used the theory of noise generated by eddy-shock wave interaction to explain the sound production in choked jets as follows:

If a moving eddy crosses a standing shock wave sound is generated according to theory. This will happen at each shock of the standing shock wave pattern the eddy is traversing. The generated
sound field will be a complex one being composed of the radiation from several sources located at the shocks. The phases of these sources are of course related, their relationship depending on the ratio of cell length \( s \) (see Fig. 5) to disturbance wavelength \( \lambda \) and velocity \( a_o \cdot M_2 \). Since the velocity of the disturbance can be measured by means of a newly developed device (see Ref. 7) the resultant directional properties of the total sound field can be calculated.

The above explanation does not account for the very powerful noise of a completely different character which is often described as a "whistle" or "screech" due to its almost discrete frequencies. It is usually of increased intensity for, and is characteristic of, certain ranges of pressure ratios above the critical one.

In analogy with the classical edge tone phenomenon (see Ref. 9 and 10) a "back reaction mechanism" has been suggested and Powell proposed a qualitative analysis for this phenomenon.

The back reaction mechanism can be explained as follows. The sound waves generated by the eddy shock wave interactions propagate pressure fluctuations (sound waves) back (upstream) to the orifice which cause the pressure ratio of the jet \( \frac{P_t}{P_n} \) at the nozzle edge to fluctuate. Since the expansion angle \( \alpha \) of the jet flow (see Fig. 5) is a function of the pressure ratio, the angle \( \alpha \) fluctuates too. As the sound waves arriving at opposite sides of the orifice are not in phase (see edge tone phenomenon, Refs. 9 and 10) both sides of the jet will be deflected in like directions, giving the flow a sinuosity. If the frequency of these oscillations is of the right order, they will become amplified and new eddies will be generated, which in turn produce additional sound waves when traversing the shock waves. Thus, the whole process is perpetuated.

Powell (Ref. 7) shows that in order to maintain this resonating mechanism certain conditions of phase and gain must be fulfilled as expressed by the phase equation

\[
\frac{\Theta}{\dot{\varphi}} = \frac{N + C}{\dot{\varphi}} = \int \frac{dh}{M \cdot a_o} + \frac{h - n \cdot \lambda}{a_o} \quad (60)
\]

and by the gain criterion

\[
q \cdot \eta_s \cdot \eta_t \cdot \eta_d \geq 1 \quad (61)
\]
The phase equation represents the time it takes a disturbance to travel from the orifice to the effective source plus the time it takes a sound wave of similar phase to that just generated to reach the orifice. Rough rules for the wavelength of the sound and its frequency are

\[ \lambda = 3 \left[ \frac{P'}{P_a} - \frac{P_c'}{P_a} \right]^{\frac{1}{2}} \cdot d \]  

(62)

and

\[ \ell = \frac{1}{3 \left[ \frac{P'}{P_a} - \frac{P_c'}{P_a} \right]^{\frac{1}{2}}} \cdot \frac{a_0}{d} \]  

(63)

for the 3-dimensional or axially symmetric case. Besides, Powell states

\[ \frac{s}{d} = 1.2 \left[ \frac{P'}{P_a} - \frac{P_c'}{P_a} \right]^{\frac{1}{2}} \]  

(64)

and \( \frac{\lambda}{s} \) is nearly constant at a value of \( \frac{\lambda}{s} = 2.5 \). There is little yet known about the factors of the gain criterion. Powell asserts that the task of solving Eqs. (60 and 61) is "very formidable". They serve only for qualitative estimates.

The noise emanating from a choked jet is of a very complex nature. In addition to that generated in the way discussed above there is also that produced by the general turbulence interacting with the shock wave pattern. Further, there is the noise generated by turbulent mixing commencing with the break-up of the laminar boundary layer near the orifice and developing into the subsonic eddy flow far downstream, just as in the subsonic jet. Although discrete notes (screech) may be predominant, it is not the only noise present in an overchoked jet.

In experiments with model jets it has been found that as the pressure ratio of the jet is continuously increased above choking values the growth of the noise is characterized by a series of states in which discrete notes are predominant, interspersed with states in which the spectrum is relatively flat. Test data concerning this phenomenon are not available for full scale jets.

VII EXPERIMENTAL INVESTIGATION OF JET NOISE

The greatest limitation, considering the theory put forward to date, arises from lack of quantitative or even qualitative knowledge of the aerodynamics of jet flows in general. A detailed experimental study of both engine and model jets, heated and cold, their flow structures, aerodynamic noise sources, their sound intensity and radiation fields is essential to a complete understanding of the jet noise problem.
7.1 The Structure of Jets

7.1.1 Subsonic Jets

Figure 6 shows the turbulent mixing of a subsonic jet (M = 0.7). The flow is laminar only for a very short length adjacent to the nozzle orifice, then turns into fine grain turbulence which increases in scale with downstream distance from the orifice. The eddies increase in size, and finally penetrate to the centre of the jet at about 5d from the nozzle exit. These large eddies travel substantially with the speed of the jet flow as was found by tests (in particular by Powell, Ref. 7).

It was further found that the extent of the noise producing jet flow in the downstream direction varies with frequency. For low frequencies it is about 30d or even up to 50d for very low frequencies (f < 100 cps).

The angle of spread of the jet boundaries, \( \phi \), has been found to be between 7.5° and 10° for Mach numbers of the order of 0.7.

It may be noted that the possible existence of toroidal or ring vortices was traced by one investigator (Ref. 12, Fig. 1) in subsonic jets.

Figure 7 shows a schematic diagram of a subsonic jet, its structure and the location of its aerodynamic noise sources (as discussed in Section 7.2.1).

7.1.2 Choked and Overchoked Jets

When the pressure ratio, \( \frac{P_j}{P_o} \), of the jet reaches its critical value the jet velocity at the nozzle exit becomes sonic (choked jet). A further increase in pressure ratio causes the jet flow to become overchoked. This means that the flow expands around the nozzle corner and accelerates to become supersonic as it passes through the expansion fan (see Fig. 5). The jet spreading angle \( \alpha \) increases with increasing \( \frac{P_j}{P_o} \). The expansion wave is reflected from the opposite jet boundary as a compression wave which now causes a jet contraction. The compression lines coalesce in the section N' - N' creating here a flow state which is almost the same again as that at the orifice section N - N. If there were no turbulent mixing developing from the jet boundary, this flow pattern characteristic of the overchoked jet would repeat itself until the pressure ratio was asymptotically decreased by heat losses to values less than critical with distance from the orifice.
Figure 8 shows a schlieren photograph of an overchoked jet. The regions of expansion (dark), mixed expansion and compression and the region of compression (light) are very distinct. The static pressure in the dark field of expansion is decreased below atmospheric pressure. A conical shock is clearly visible in the (light) compression region which is formed by the running together of the compression lines (Fig. 5). This conical shock is strongest at the jet boundary.

In three-dimensional jets as many as 12 cells and in two-dimensional jets up to 5 cells (section between N - N and N' - N') could be photographed. It seems therefore that the subsonic jet is more subject to turbulent mixing than the overchoked jet, when the distances are compared at which the turbulent mixing region penetrates to the centre line of the jet flow (Ref. 7). Other tests with a 1" model jet, just choked, (Ref. 11, Fig. 36) show the mixing region reaching the jet centre line as early as 4.5*d.

There exists a good deal of experimental evidence to show that the rate of spread of the turbulent mixing region in an overchoked jet is much smaller than that of the subsonic jet (Ref. 13). Lighthill’s explanation is that the intensity of the turbulence in the mixing region decreases continuously with increasing flow Mach number owing to the amount of flow energy lost in the form of sound radiation. Measurements (Ref. 14) indicate that turbulence introduced artificially into a jet flow by two 90° bends decreases with increasing Mach number.

The angle of spread \( \phi \) of a supersonic jet has been found to exceed that for subsonic jets by approximately 5° (Ref. 7).

7.2 Location of the Aerodynamic Noise Sources

7.2.1 Subsonic Jets

In a subsonic jet, the turbulent mixing region is the main producer of aerodynamic noise. The high frequency sound sources have been found to be located in the severely sheared mixing region very close (within \( 1\times d \)) to the orifice.

The low frequency (\( f < 3200 \) cps) noise emanates from sources farther downstream (up to \( 40\times d \)) where the fine grain turbulence from near the orifice has developed into large scale eddies which travel with a speed of between 0.34 and 0.75 of that of the jet flow itself. The extent of the source field radiating high frequency noise at the orifice is very small compared with that of the low frequency noise.

The sound intensity level (\( d_b \)) from the low frequency noise sources is much higher than that from the high frequency sources.
If (as explained in Section 7.4) a whistle or discrete note arises from a subsonic jet, it is due to discrete toroidal (ring) vortices emanating from the laminar flow immediately aft of the nozzle.

A way to prove the location of the low frequency sound sources is as follows. A microphone is moved along a path very close to the jet boundary at a small angle $\theta$ to the jet axis from far downstream (high $\frac{Ma}{\sqrt{\alpha}}$ values) towards the jet nozzle. The intensity level increases continuously up to a certain distance after which it decreases for the lower frequency bands (20 to 3200 cps). This variation signifies the motion of the microphone past the sources of low frequency sound which are thus shown to be far downstream.

Further illustrations of the location of sound sources by near field measurements are given in Section 7.3.

### 7.2.2 Choked and Overchoked Jets

The sources of aerodynamic noise in an overchoked jet include those due to turbulence - shock wave interaction in addition to those in the turbulent mixing region.

The former are located where the turbulence (eddies) interacts with each shock of the standing shock wave pattern when passing through it. Under certain conditions (see Section 6.5) a resonance system may be set up in the jet flow which can intensify the noise radiated from these stationary sources to become a predominant "hiss" or "screech" which is a characteristic of overchoked jets only.

In finding the location of aerodynamic noise sources in a jet flow, sound wave photographs (see Ref. 15 and 16) and near field noise measurements are used. It was from sound wave photographs of a high pressure ratio jet that another group of sound sources was discovered due to small eddies in the fine grain turbulent mixing region near to the orifice. These small eddies, moving with sonic or supersonic velocities give rise to weak shock waves or impulse sound waves. Further noise is generated when they traverse the main shock waves. All this noise is of very high frequency and is supposed to be represented by the sound waves of Fig. 15 and 16 in Ref. 17 between $\theta = 45^\circ$ and $60^\circ$. Other photos of jets have shown sound waves which are out of phase and which originate from the region at the end of the cellular pattern where of course the flow disturbances are greatest.

In Ref. 11 the jet spread angle for a 1" model jet is given as about $6.5^\circ$. Further, the extent of the sound sources in the downstream direction is given for the same 1" model jet as a function
of frequency in the following table.

<table>
<thead>
<tr>
<th>Frequency Band (cps)</th>
<th>Extent of Sound Sources</th>
</tr>
</thead>
<tbody>
<tr>
<td>6400-12,800</td>
<td>Jet nozzle to 10 jet diameters (d) downstream</td>
</tr>
<tr>
<td>3200-6,400</td>
<td>14</td>
</tr>
<tr>
<td>1600-3,200</td>
<td>16</td>
</tr>
<tr>
<td>800-1,600</td>
<td>18</td>
</tr>
<tr>
<td>400-800</td>
<td>24</td>
</tr>
<tr>
<td>37.5-75</td>
<td>From 25d downstream to 30d downstream</td>
</tr>
</tbody>
</table>

The total extent of the field of noise sources was from the jet nozzle to 30d downstream. The strength of the sources reaches a maximum near the downstream end of their distribution along the jet axis.

7.3 Noise Fields and Directionality

If sound intensity levels measured around a jet at various distances from the nozzle exit and at different angles θ are plotted in a polar diagram and the equal intensity contours are drawn, a complete picture of the sound field is obtained. It can be taken for the whole frequency band of the jet or for bands of small width only. An important factor, however, is the distance from the jet nozzle at which the intensity levels are measured, whether in the so-called "near" or "far" fields.

Near and far field measurements stress quite different properties. Very close to the jet one can find the intensity and frequency of the sound leaving any particular point on the jet boundary but no indication of any preferred direction in which that sound is proceeding. Far field measurements allow one to find the intensity levels due to the entire source field, the frequencies and the directions given preference in its radiation, i.e. its "directionality", but not the location of the noise source.

In between these two fields is a complicated transitional region where measurements are difficult to interpret, as points so located will be always nearer to one part of the source field than to another due to the extent of the sound field (up to 40d). Hence the intensity of the nearer sources are over-emphasized.

In practice the far field is at a distance λ at which the plotted equal intensity contours show the true directionality of the field, or at which the whole source field can be considered as small (or even as a single source) compared with λ. Therefore λ is taken not less than 10 times the extent of the source field (30 to 40·d) or up to 400 jet diameters (Ref. 11 recommends 500d) for low frequencies.
Measurements taken too close to the jet boundary (near field) may be affected by gusts which would distort or mask the measured sound field. So far most of the observation distances in tests were much smaller than 300d and such measurements can therefore only be used to indicate the general trend with respect to directionality.

Measurements taken at 500d distant from an actual engine jet of $d = 1.5$ ft, would mean $r = 750$ feet. This seems to be a rather great distance, at which the magnitudes of the sound pressure fluctuations may be noticeably influenced by atmospheric and terrain attenuation (Section 5.4). Besides a greater sound pressure reduction than that predicted by the inverse square law \( \frac{P'}{P} = \frac{r^2}{r_0^2} \), the frequency spectrum would be changed, as the sound pressure reduction with distance is a function of frequency.

7.3.1 Subsonic Jets

Sound fields for a subsonic jet ($\frac{V}{c} = 1.5$) are illustrated in Fig. 10. Figure 10b for a frequency band of 6400-12,800 cps shows a distinct directionality of the high frequency noise at an angle $40^\circ < \theta < 45^\circ$ and confirms what was said above about directionality and far field measurements. Figure 10a for the lower frequency band of 800-1600 cps is taken at the same distances as Fig. 10b, but this distance is not sufficient to place one in the far field for this lower frequency band. In spite of this imperfection an apparent directionality can be observed at $\theta \approx 30^\circ$. (As used here, the term "directionality" means a sound intensity peak at some angle $\theta$.)

In both figures there are small peaks at $\theta \approx 100^\circ$. There is definitely sound radiated at $\theta = 90^\circ$, and apparently at angles up to and exceeding $150^\circ$. The sound intensity of the 800-1600 cps peak at $\theta \approx 30^\circ$ is obviously higher than the 6400-12,800 cps peak at $\theta = 45^\circ$.

All subsonic jet investigations confirm the fact that directional maxima of low frequency noise are at much smaller angles $\theta$ than those of high frequency noise. Besides, this angle $\theta$ decreases if the frequency is reduced or the flow Mach number, $M$, is increased. $\theta_{\text{max.}}$ may often become too small for measurements to be unaffected by jet gusts.

Figure 11 shows the directionality of the total noise of various model air jets for the conditions stated. All curves display the same expected characteristic, i.e. that the intensity level decreases with increasing $\theta$. The shapes show a considerable scatter which may be due to the different $\frac{A_d}{A}$ values at which the measurements were taken, as well as to the instruments and techniques used and to the initial turbulence in the jets. Westley and Lilley's results for $\frac{\mu}{d} = 108$ show the position of the maximum intensity peak at $\theta = 30^\circ$ for the total noise, and as they are taken farthest from the jet they are the most reliable with respect to directionality.
7.3.2 Choked and Overchoked Jets

Sound fields for an overchoked jet \( \left( \frac{P^*}{P_{0x}} = 2.0 \right) \) are shown in Fig. 12. For the low frequency band of 800-1600 cps a very distinct noise maximum is located at \( \Theta = 30^0 \) which shifts to about \( 40^0 \) in the 6400-12,800 cps band. There is again a small peak at \( \Theta = 100^0 \) in the low frequency band which develops into an intensity maximum for the high frequency band. At angles \( \Theta > 140^0 \) the equal intensity contours in Fig. 12b at least indicate the existence of another peak.

Both figures again provide sufficient evidence that there is sound radiated at an angle of \( \Theta = 90^0 \).

Another interesting feature of overchoked jets, shown by experiments with models, is that with increasing pressure ratio (or \( M \)) the noise levels do not increase uniformly. They increase through a series of states characterized by predominant frequencies which change abruptly as the next member of the series is attained, these states being separated by states of harsh noise. Test data concerning this phenomenon are not available for full scale jets.

The total noise as recorded in Fig. 13 is poorly defined in these regions. This phenomenon is due to the previously discussed resonance system (see Section 6.5). It causes a definite fundamental frequency peak in the upstream direction close to the jet axis and a second peak of exactly double the upstream peak frequency at about \( 90^0 \). Both peaks are of much higher intensity level than those of subsonic jets. Figure 16 of Ref. 17 shows the very intense sound radiations in the upstream direction. The intensity peak at \( \approx 90^0 \) was found to be quite a narrow beam.

Tests with 1" diameter axially symmetric model jets (see Ref. 7) have proved another very high frequency peak to exist at an angle \( \Theta \approx 60^0 \). This frequency (\( f = 40 \) to 50 kcps) is well above the audio range and was found for even smaller jet diameters (1/4") to be as high as 80 to 100 kcps.

7.4 Factors Governing Jet Noise

7.4.1 Turbulence and Shear

Turbulence and shear in a jet flow are intimately related to each other, and both are of equal importance with respect to their contribution to the production of aerodynamic noise. This is established by theory as well as by experiments.

Lighthill (Ref. 3) states, that turbulence of given intensity \( (\varphi v_i) \) can generate more sound in the presence of a large mean shear \( (\frac{\partial v_i}{\partial x_i}) \) or \( \varphi v_i \bar{v}_i \) can fluctuate most widely if the fluctuations of \( \varphi v_i \) are amplified by a large gradient in \( \bar{v}_i \) (Sec. 6.3.2, Eq. 40).
High shear means in general a high velocity gradient at the jet boundary. The velocity gradient in turn is dependent on the velocity in the jet boundary. Therefore, as experiments have proved (see Fig. 24), a jet with an almost rectangular exit velocity profile (as a result of high initial turbulence) is noisier than a jet with an exit velocity profile of the "pipe flow" type, if both jets have the same maximum velocity and are of equal thrust (which means a larger diameter in the latter case). The high frequency noise which mainly originates from shear forces near the nozzle exit is particularly greater in the case of a rectangular velocity profile.

If the velocity profile is changed from the rectangular to the pipe flow shape, keeping the jet diameter d and the jet thrust constant, the jet becomes noisier as the increased velocity near the centre has a greater effect than that of the shear reduction at the jet edges.

Figure 14 illustrates the effect of initial turbulence on jet noise (sound pressure). Measurements were taken at \( \frac{d}{a} = 16 \) from a 3" model jet for both low and high turbulence (introduced artificially by two 90° pipe bends upstream of the nozzle exit) over the same range of jet exit velocities. The high turbulence jet gives a drastic increase in overall sound pressure. Corresponding values of an actual turbojet lie between the model jet curves. Another interesting feature of high turbulence jets is that they show less directionality than those of low turbulence if compared at a constant speed of 1000 feet per second (see Fig. 15 of Ref. 14, but note that these measurements were taken at an \( \frac{d}{a} \) of only 16).

Experimental evidence leaves no doubt that even in subsonic jets there is a close relationship between the frequencies of maximum turbulence in the jet flow and maximum sound just outside the jet boundary as shown in Fig. 15. To measure the disturbance frequencies a new method was developed by Powell (Ref. 7). It is based on standard schlieren equipment, where instead of the photographic plate a pinhole (0.009" dia.) and a photomultiplier tube are used. In this way the time fluctuations of a component of the mean density gradient in the flow can be obtained. Using two pinholes close together, the disturbance speed can be measured. This method revealed that the most powerful intensity lobe of overchoked jets in the upstream direction (\( \approx 180^\circ \)) close to the axis is of exactly the eddy frequency in the jet flow; and the lobe at an angle of just over 90° is of exactly double the eddy frequency. It proved further that these powerful intensity lobes are precisely related to the eddy frequency and sound pattern in the pressure ratio regions which give discrete notes (resonance system). In the cases of no discrete notes, no consistency between the eddy and sound frequency curves could be traced on the oscilloscope screen. In Fig. 16 the frequency of the flow disturbance is plotted against the jet pressure and in Fig. 25 the ratio of the fundamental radiated wavelength
and shock wave pattern cell length is plotted against \( \left( \frac{P'}{P_0} - \frac{P_0'}{P_0} \right)^{1/2} \),
the square root of the excess pressure ratio above that for choking. Figure 17 illustrates the same measurements with a three-dimensional jet. Note the correspondence between irregularities in frequency and in total noise level at particular jet pressures in Fig. 17a and Fig. 13 independent of the difference in nozzle exit diameter.

Reference 7 gives a numerical value for the disturbance speed in a three-dimensional overchoked model jet as about 0.7 times the speed of sound in the jet. Further, it points to evidence which suggests that in some cases the speed of the flow disturbances increases somewhat as they pass downstream. As a result of jet flow spreading and the loss of flow energy a decrease in eddy speed would have been expected.

Little is known so far about turbulence levels in jet flows. The turbulence level is defined as the ratio of the root mean square velocity fluctuation to that of the mean velocity at the same point. Measurements, using a 0.75" jet nozzle and introducing high turbulence artificially by two 90° bends upstream of the nozzle exit show a degree of turbulence of \( \approx 9.5% \) at 400 feet per second which reduces to \( \approx 6.6% \) at 600 feet per second, a rather unexpected result (Ref. 7). Low turbulence jets for comparison have a turbulence of only \( \approx 1% \). These measured turbulence levels of model jets are higher than those of wind tunnels, but they are in quantitative agreement with results quoted in Ref. 18.

More evidence on the importance of initial turbulence in aerodynamic noise production follows from observations of the Fairey Aviation Co., Ltd., with pressure jets as used on the tips of helicopter rotors. Very high turbulence levels occur in pressure jets due to flame stabilization devices. Measurements indicated noise levels were about 15 db higher for a jet pressure of 10 lb./in.² (subsonic) than the average values of model jets (see Fig. 13) at the same pressure. With increasing jet pressure the sound level rose less rapidly to about the same value as in Fig. 13 for a jet pressure of 30 lb/in². It can be assumed that the high subsonic noise levels must be due to the unusually high degree of turbulence which loses importance as soon as choking of the jet becomes effective.

Research workers agree on the importance of turbulence, but they do not agree on the amount of turbulence which is still present in a jet flow when turbulence is introduced anywhere upstream of the nozzle exit.

The intimate relation between turbulence and shear in the jet flow is especially obvious in testing noise suppression devices as will be seen in Section 10.

7.4.2 Jet Velocity and Noise

The most influential parameter in the production of aerodynamic noise was found to be the jet flow velocity.
From test results plotted in Fig. 18 for various actual jets and a model jet, it follows that the sound pressure increases as the fourth power of the jet velocity, \( U \), or correspondingly (see Eq. 3) that the sound power output varies as the eighth power of \( U \). The separation of the data into individual curves is due to different nozzle diameters, accounting for about 2 db of the total spread of 7 decibels. The remaining 5 db may be due to differences in measuring techniques or (what is still more likely) in initial turbulence levels of the various jets (see Ref. 14).

Other tests (Ref. 14) using 3" subsonic model jets of both high and low turbulence, Fig. 14, indicate that the sound pressure is proportional to \( U \) to the power 2.1 to 3.0 for the high and approximately 3.0 to 3.7 for the low turbulence jets. Note that measurements were taken at \( \theta = 90^\circ \), where no eddy convection effects occur (as \( (1 - M_c \cos \theta )^{-6} = 1 \)). The lower power of \( U \) for the high turbulence level jets can be explained by those tests (see Ref. 7 and Section 7.4) which revealed a decrease in turbulence with increasing jet velocity. It was reported in Ref. 14 that, as the jet velocity increases, the maximum amplitudes of sound intensity occur at higher frequencies, and that this effect was only detected with high turbulence jets. It may be therefore a function of the jet turbulence level, but this phenomenon needs further investigation.

When the jet pressure was increased above its critical value in model tests and the total jet noise level was plotted as a function of jet pressure as shown in Fig. 13, the noise curve steepened its slope but rose in a stepwise fashion having short segments of relatively small slope. The jumps in noise level correspond to resonance levels as explained in Section 6.5. Steeper slopes indicate proportionality to higher powers of \( U \). The total acoustic power radiated was found to vary as \( U^{12} \) to \( U^{15} \) for overchoked model jets and as \( U^{10} \) to \( U^{18} \) for full scale (hot) overchoked jets. Powers of \( U \) as high as 29 are claimed to have been found in cases of very strong discrete notes due to resonance.

Finally, it may be mentioned that a 13% reduction in the jet velocity achieves a 5 db noise reduction in the case of a subsonic jet \( (I(x) \propto U^8) \) and an even greater noise reduction for overchoked jets in which case the intensity is proportional to higher powers of \( U \).

7.4.3 Jet Diameter ("d")

Model jet tests (Ref. 14) indicate that the most intense sound of a jet has a wavelength of about 3 to 4 d and originates at between 5 to 10d downstream from the orifice. They further proved for jets of diameters varying from 0.75" to 12" that the total noise level is proportional to \( d^2 \).
Measurements at a given jet velocity of the noise level in a fixed direction (θ = const.) are found to be independent of the jet diameter (within the experimental accuracy) when they are always taken at the same number of diameters distant from the orifice. This is in accord with the dimensional analysis of Section 6.4.3, where Eq. 56 shows the sound pressure to be dependent on the ratio $d/\lambda$.

7.4.4 Mach Number, Reynolds Number, and Strouhal Number

Experiments using model jets of 1/4" to 1-1/2" diameter (Ref. 20) have indicated that the Mach and Reynolds numbers of the flow influence the acoustic power coefficient $\kappa$ only very slightly if at all.

If the Strouhal number is expressed in terms of the peak intensity frequency, $f_p$, these peak Strouhal numbers $St_p = \frac{f_p \cdot d}{U}$ found by some investigators can be compared. From Ref. 21, peak Strouhal numbers vary between 0.3 and 0.6, from Ref. 14 they are fairly constant at about 0.5 as $U$ is varied. Values from Refs. 11 and 22 are hard to define due to the very flat peaks of the sound spectra. All the curves bracket the frequency corresponding to a Strouhal number of 0.5 which may be considered as an average experimental value, deduced from the following test results.

<table>
<thead>
<tr>
<th>Ref. No.</th>
<th>M</th>
<th>St</th>
<th>$\theta_{\text{max}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>14</td>
<td>0.8</td>
<td>&gt; 1.5</td>
<td>40°</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.4 - 0.8</td>
<td>30°</td>
</tr>
<tr>
<td></td>
<td></td>
<td>&lt; 0.2</td>
<td>30° $&gt;\theta_{\text{max}}$ &gt; 15°</td>
</tr>
<tr>
<td>11</td>
<td>1.0</td>
<td>&gt; 0.7</td>
<td>40°</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.3 - 0.7</td>
<td>30°</td>
</tr>
<tr>
<td></td>
<td></td>
<td>&lt; 0.2</td>
<td>30° $&gt;\theta_{\text{max}}$ &gt; 15°</td>
</tr>
<tr>
<td>22</td>
<td>higher</td>
<td>&gt; 0.6</td>
<td>40°</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.5</td>
<td>35°</td>
</tr>
<tr>
<td></td>
<td></td>
<td>&lt; 0.3</td>
<td>20°</td>
</tr>
</tbody>
</table>

7.4.5 Frequency

If the peak Strouhal number (see previous section) is taken to have its experimentally found average value of $St_p = 0.5$, one has the peak frequency

$$f_p = \frac{U}{2d}$$
Further experimental evidence (Refs. 21 and 11) indicates that $\frac{dP}{U}$ is not directly proportional to $U$. Considering Ref. 14 as well, the experimental evidence indicates slower increase of $\frac{dP}{U}$ with $U$ than that corresponding to a direct proportionality. This point needs further investigation.

Figure 19 presents the frequency as a function of the jet noise level with the locality at which the measurements were taken as a parameter. It proves again that the low frequency noise arises some distance downstream and (what is more important) that at any point close to the jet, the spectrum is in fact sharply peaked while the frequency of this peak systematically falls as the distance downstream from the orifice increases.

In Ref. 7 the following results from 2-dimensional overchoked model jets are quoted. The frequency and sound intensity vary with jet pressure along a smooth curve having no points of inflection, Fig. 16. That is not the case for 3-dimensional (round) jets, where the curves showing variation of frequency and intensity with jet pressure suffer changes in sign of the second derivative corresponding to regions of instability and sudden transition (Fig. 17). Between the stages a and b, the transition is very confused, between b and c it is quite sudden and between c and d, a hysteresis is present.

Figure 20 obtained from an overchoked 2-dimensional model jet shows the frequency relationship between the powerful upstream intensity peak ($\theta \approx 180^\circ$) and that at about $90^\circ$. The $\theta \approx 90^\circ$ peak is of precisely twice the frequency of the upstream intensity peak. The downstream peak (at $\theta \approx 30^\circ$) was found to be of lower intensity but the frequency is that of the upstream peak. Thus the frequency in either the upstream or the downstream intensity peaks is that of the fundamental disturbance.

Figure 21 shows the change of the velocity exponent (theoretically 8) with frequency as obtained from the model jet experiments of Ref. 22. One sees that for low frequencies the power factor of higher than 8 falls off to a minimum of $\approx 6.5$ and then rises with increasing frequency. So far it is not possible to say what this means since the reasons can be manifold (e.g. effects of $Re_c$, acoustic source distribution along the jet axis and changes in jet velocity distributions).

In Ref. 11 the index $b$ (from $I \sim \left(\frac{P^3 - P_0}{P_0}\right)^{b(\theta)}$) increases with frequency as shown in Fig. 22, but varies as well with angle $\theta$, suggesting a power law varying with $\theta$ in accordance with the distribution of the acoustic source field along the jet axis.

Other measurements have indicated that the frequency of
the sound radiated is dependent upon the jet pressure ratio \( \left( \frac{P' - P_a}{P_a} \right) \). Using model test results, Powell in Ref. 7 derived the following equation for the frequency of a 2-dimensional choked cold jet

\[
\ell = \frac{1}{5} \frac{a_0}{d} \left( \frac{P' - P_a}{P_a} \right)^{-\frac{1}{3}}
\]

and for the overchoked axially symmetric jet

\[
\ell = \frac{1}{3} \frac{a_0}{d} \left( \frac{P' - P_a}{P_a} \right)^{-\frac{1}{3}}
\]

Another important relationship is that of frequency and jet diameter. The larger the jet, the lower the frequency of the noise intensity peak (Ref. 14).

It is known from general sound fields (see Fig. 10 and 12) that different frequency bands have different directional characteristics. The higher the frequency, \( \ell_p \), of the noise intensity peak, the greater its angle \( \Theta \) as shown in Fig. 23. The following values are quoted in Ref. 17 where

\[
\ell_p = \frac{a_0}{\lambda_p}
\]

is replaced by its corresponding wavelength, \( \lambda_p \)

\[
\frac{\lambda_p}{d} > 6 \quad \Theta = 15^\circ
\]

\[
= 2 \quad \Theta = 30^\circ
\]

\[
= 1/4 \quad \Theta = 45^\circ
\]

These results (from Ref. 14) are obtained from near field measurements and therefore the angles \( \Theta \) for the low frequencies are probably too small. The trend at least is obvious.

Subsonic and overchoked jets possess characteristic noise spectra (octave noise level plotted against octave mid-band frequency) which are readily distinguished (see Fig. 10 and 12). Figure 24 from Ref. 23 shows the noise spectra of subsonic jets. They extend over many octaves and have a single flat maximum which becomes flatter as the angle \( \Theta \) at which the intensity peak occurs decreases. An average value for the width of the noise frequency band of subsonic jets is given in Ref. 11 and 14 as about seven octaves. The larger the peak angle \( \Theta \),
the more pronounced is the peak in the frequency curve.

The terms "high frequency" and "low frequency" have been used a number of times in the foregoing pages. These terms are now given a more precise meaning. From the test results of Refs. 11, 14, and 22 Lighthill (Ref. 3) uses the Strouhal number to define high frequency as

\[ f > 0.7 \frac{U}{d} \]

(with a directional intensity maximum at slightly less than \( \theta = 45^\circ \))

and low frequency as

\[ f \leq 0.3 \frac{U}{d} \]

(with a peak angle of \( \theta \geq 20^\circ \)). High and low frequencies mean in this sense "sensibly above or below the peak frequency".

7.4.6 Jet Flow Density

Model jet tests using air (\( \rho = 0.00279 \text{ at } 68^\circ \text{F.} \)), helium (\( \rho = 0.00046 \)) and Freon 12 (\( \rho = 0.1130 \)), using a 0.75" nozzle and taking measurements at \( \theta = 90^\circ \) and \( \Theta/d = 16 \) were undertaken (Ref. 14) to investigate the effect of flow density on sound intensity or sound pressure. When the flow exit Mach number range of all three gases was kept constant the results presented in Fig. 26 were obtained showing sound pressure curves of approximately equal slopes. When a comparison was made at equal jet exit velocities the medium of highest density (Freon 12) produced the highest sound pressure. It follows that the sound pressure ratios for the various gases are approximately the same as their respective density ratios. Thus the sound pressure appears to vary directly as the stream density for a given exit velocity. If the sound pressure values of Fig. 26 are multiplied by their respective air-gas density ratios, a single straight line is obtained.

Reference 26, referring to the wide variation found in total noise diagrams, suggests the possibility of a relative change of high and low frequency noise with jet flow density.

7.4.7 Cell Length

The length of the cell adjacent to the orifice is defined as "cell length" \( s \) (see Fig. 5) due to the fact that its length is easiest to measure. The more significant cell, however, is that at the downstream end of the cellular pattern where the predominant noise sources are located. Experimental evidence indicates that for higher pressure ratios the cell length increases in the downstream direction.
Relating $s$ to a characteristic dimension of the nozzle, the ratio $\frac{s}{d}$ was found (Ref. 15) for the 2-dimensional case of an overchoked jet to be:

$$\frac{s}{d} = 1.89 \left( \frac{p_1'}{p_a} - \frac{p_2'}{p_a} \right)^{\frac{1}{3}}$$

where $d$ is the smaller dimension of the rectangular exit slot. For the 3-dimensional case

$$\frac{s}{d} = 1.2 \left( \frac{p_1'}{p_a} - \frac{p_2'}{p_a} \right)^{\frac{1}{3}}$$

Another useful relationship is that between the predominant wavelength of sound, $\lambda$, and $s$. From Fig. 25 it follows that for the 2-dimensional case $\frac{\lambda}{s} = 2.5$ while from Fig. 17 for the axially symmetric case the value of 2.5 does not hold satisfactorily in the higher jet pressure range. It is seen, that a decrease in $s$ (by as much as 20%) for the high pressure ratios would adjust the $\frac{\lambda}{s}$ value to the approximately constant value of 2.5 again (Ref. 7).

### 7.4.8 Acoustic Power Coefficient

The Acoustic Power Coefficient is of importance in the equation for the efficiency of aerodynamic noise production (Eq. 58). It is defined as

$$K = \frac{\text{acoustic power (measured)}}{\rho_0 \cdot U^5 \cdot a_0^{-5} \cdot d^2}$$

Direct measurements of $K$ up to this date are known only from Ref. 21. Here the acoustic power coefficient is given as lying between 0.6 $\times$ 10^{-4} and 1.2 $\times$ 10^{-4} as $U$ varies by a factor of 2.5 and $d$ by a factor of 2. Reference 21 suggests a Reynolds number effect, but Lighthill thinks there is little evidence for it. For higher Mach numbers $K$ was found to be close to 0.9 $\times$ 10^{-4}.

Other investigators (Ref. 14) measure only intensity at various positions round the jet at $U = 1000$ ft./sec. and for $d = 0.75''$ to 12''. A very approximate integration, estimating the upstream intensity values, results in an average value of $K = 0.5 \times 10^{-4}$. For $d = 0.75''$ it was thus found to be only $K = 0.3 \times 10^{-4}$.

In Ref. 22 the measurements of sound intensity round a large sphere enclosing a jet with $d = 1''$ are integrated. The acoustic power coefficient is found to be
From all the above references, Lighthill (Ref. 3) deduces

\[ K = 0.6 \times 10^{-4} \]

as a typical value for \( 0.3 \leq M \leq 1 \) (subsonic case). Knowing \( K \) allows one to evaluate the acoustic efficiency of aerodynamic noise production (see Eq. 58).

7.4.9 Re-heated Jets (After-burning)

The jet velocity is the predominant factor in the production of aerodynamic noise and unfortunately re-heating results in increasing it.

By burning additional fuel in the jet pipe, the jet temperature \( T \) is raised and the jet velocity \( U \) is increased \( \propto T^{1/2} \). If the size of the jet engine with after-burner is now decreased to deliver the same thrust as without reheat (nozzle diameter remaining constant), the jet noise would be increased in proportion to \( U^8 \) or \( T^4 \). The upper limit of this process is given by the stoichiometric temperature. The maximum in noise increase with this type of jet engine would be up to about 18.5 db. due to the increase of jet velocity alone. Increasing the nozzle diameter to get the original mass flow would add another 2.5 db. \((I(x) \propto d^{1/2})\). Thus, re-heat applied to jet engines of to-day's type would increase noise by 21 db. as a maximum. It would bring modern jet engines as e.g. the Avon up to 160 db.

The effect of re-heat on jet noise can be expressed in another way. If an increase in thrust is obtained by building a larger engine, jet noise is increased proportional to the thrust. If the same increase in thrust is obtained by applying re-heat, the noise increases as thrust to the ninth power.

In general, normal jet engines with after-burner are not overchoked at sea level. As the after-burning process essentially raises the jet temperature, both the jet velocity and the sound velocity in the jet are raised. The jet flow Mach number remains approximately constant and the very much noisier re-heated jet is in most cases subsonic. Unfortunately (as will be shown later) subsonic or nearly sonic jets are a more difficult problem for noise reduction than overchoked jets.

\[ K = 0.4 \times 10^{-4} \quad M = 0.8 \quad \text{for the 36" long jet pipe, and} \]
\[ K = 0.8 \times 10^{-4} \quad M = 0.3 \quad \text{for the 54" long jet pipe.} \]
Another aspect of re-heat application is the increased liability to structural fatigue failures due to vibration caused by the much stronger jet pressure fluctuations. Fortunately, the very large increases in thrust with re-heat (theoretically up to 70%) have not yet been realized due to obstructions introduced into the jet pipe in the form of devices for the stabilization of combustion (these obstructions reducing jet velocity) and the unavailability of heat resisting materials required to withstand the temperatures produced by complete combustion. These obstructions on the other hand increase the turbulence which together with the extension of the combustion zones even into the free jet adds to the noise level of re-heated jets.

Figure 33 shows the results of tests on a Derwent engine without and with re-heat. It is very interesting to note from these tests that the increase of noise due to re-heat is no greater than the values which follow from the increase of the jet velocity. As there is undoubtedly a considerable increase in turbulence in the jet stream, this observation is hard to explain. It can be imagined that there is probably not much increase in turbulence in the boundary layer in the jet pipe. It is perhaps only the boundary layer turbulence which affects the noise production (Ref. 25). More test results are needed before this question can be answered satisfactorily.

VIII MODEL AND FULL SCALE CORRELATION

If model tests are undertaken instead of full scale testing, with the intention of applying model test results to the full scale case, the relationship between the phenomena in each of these cases must be known.

In the cases of model and full scale jets the principal differences are obviously scale, temperature and initial turbulence.

Most testing done to date has been on model jets of 1" diameter (but tests with 1/4" to 12" jet diameter are also available) and a temperature of about 20°C. Actual engine jets tested varied in diameter between 16"and20", and were at temperatures up to 650°C (without re-heat). More data on full scale jet engines tested is given below from Ref. 19 and 25.

<table>
<thead>
<tr>
<th>Engine Type</th>
<th>Exit Temp. °C</th>
<th>Exit Veloc. ft./sec.</th>
<th>d inch</th>
<th>Thrust lb.</th>
<th>rpm</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nene</td>
<td>604±3/4%</td>
<td>1620±1/2%</td>
<td>18.9</td>
<td></td>
<td></td>
</tr>
<tr>
<td>? (take-off)</td>
<td>600</td>
<td>1800</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Derwent</td>
<td>460</td>
<td>1400</td>
<td>16</td>
<td>13500</td>
<td></td>
</tr>
<tr>
<td>? (sea level)</td>
<td>690</td>
<td>1400</td>
<td>16</td>
<td>5000</td>
<td>7950</td>
</tr>
<tr>
<td>Avon</td>
<td>440</td>
<td>1750</td>
<td>19.1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>?</td>
<td>445</td>
<td>1730</td>
<td>21.0</td>
<td>9000</td>
<td></td>
</tr>
</tbody>
</table>
With respect to the various jet engines tested it should be mentioned that the sound field is in general independent of the type of engine, as one would expect if the noise pattern is entirely due to the interaction of the high velocity jet with the surrounding atmosphere. The data from model jet tests substantially confirm the following results of the theory:

a) sound intensity $I(\vec{x}) \propto U^2$; $P(\vec{x}) \propto U^4$

b) sound intensity $I(\vec{x}) \propto d^2$; $P(\vec{x}) \propto d$

c) sound intensity $I(\vec{x}) \propto \frac{1}{\kappa^2}$; $P(\vec{x}) \propto \frac{1}{\kappa}$

d) sound intensity $I(\vec{x}) \propto \rho'^2$; $P(\vec{x}) \propto \rho'$

e) sound frequency $\nu \propto \frac{1}{d}$

In addition, the test results yield the approximate relation

$$\frac{\lambda_p}{d} \approx 2.5$$

From relationships b) and c) it follows, assuming identical jet velocities and angles $\theta$, that the overall sound pressures for various sizes of cold jets are approximately equal at equal values of the non-dimensional distance parameter $\frac{\lambda_p}{d}$, thus compensating for scale effect. If under this condition the results of a cold and a hot jet are to be compared, the effect of jet temperature and initial turbulence comes in, as discussed in subsequent sections.

The Reynolds number, $Re$, (based upon the nozzle diameter and jet exit velocity) of a 1" model jet (cold) at choking is about 0.75 x 10^6 compared with 3 x 10^6 of an actual (engine) jet of 20" diameter. This relatively small increase in $Re$ is due mainly to the decrease caused by high temperature.

### 8.1 Noise Frequency Spectrum

Figure 27 (from Ref. 15) shows the spectrum of a 2" diameter cold model jet extrapolated to the full scale jet exit velocity of 1640 ft./sec. and shifted 3 octaves to the left to account for the difference in jet diameter. The full scale results from Ref. 28 (diameter $\approx 20"$) were reduced to the octave noise level at $\frac{\lambda_p}{d} = 24$.

Both jet curves in Fig. 27 are of similar character, i.e. their radiated noise is of similar frequency structure. (The "hollow" in the full scale jet curve is not typical). Their near coincidence indicates that the rules (see Section 8) holding for model (cold) jet results can be applied to actual (hot) jets to give useful results.
In Ref. 14 the frequency spectra of a 12" model and a 15" actual engine jet at \( \gamma = 16 \) and \( \theta = 90^\circ \) show good qualitative agreement and confirm the results stated above.

8.2 Jet Noise Intensity

Figure 28 presents more evidence that the rules derived empirically from model jets hold for an engine jet as well. The majority of jet engine test points lie somewhat above the extrapolated line from cold jet measurements. This discrepancy may be due to varying initial turbulence, temperature effects and probably also to the shift of the intensity peaks, with increasing jet velocity, from smaller angles to angles of approximately \( 30^\circ \).

The total noise energy of various American jet engines is plotted against the kinetic energy of the jets in Fig. 29, taken from Ref. 29. This figure suggests that the total noise energy is proportional to the kinetic jet energy or to \( U^3 \) roughly up to jet energies of \( \approx 10^6 \) watts. It thus shows an unexplained discrepancy in comparison with the body of results for which the variation is as \( U \) to the 6th to 8th power.

8.3 Jet Noise Directionality

With engine jets the total noise intensity peaks seem to occur at an angle \( \theta \approx 30^\circ \) (see Ref. 14, 28 and 30). The maximum intensity of a given frequency band shifts to higher angles as the frequency is increased. This rule applies in the same way for the model jet. In Fig. 30, each point represents a band of frequencies, usually of about one octave, taken from engine and model test results. There is a considerable scatter in frequency of the intensity peaks which may appear at a given angle \( \theta \). One sees from Figs. 28 and 30 a remarkable resemblance between the noise generated in the mixing region of engine and model jets. Thus the important assumption that it is the aerodynamic noise from subsonic jets which is the predominant noise appears to be confirmed, at least when the engine is operating at high power. (At lower powers, the compressor noise may predominate, Refs. 14 and 30).

In Fig. 31 the directional properties of turbo-jet engine noise and those of air and helium jets are plotted for comparison. The helium jet has a higher flow and sound velocity and a lower density, the air jet a lower flow and sound velocity and a higher density than the turbo-jet. In Fig. 31 the turbo-jet curve lies between the two others and it may be concluded that the significant parameters for the noise of model jets (air, helium) apply as well for hot or turbo-jets.

8.4 Jet Flow Temperature

The most obvious difference between engine and model jets is the jet temperature, but experimental results suggest that it has only a small effect. To accept the indication as fact one must have much more
complete experimental evidence. In particular, it must be established that:

a) the rate of jet mixing is the same for a hot or cold jet surrounded by cold air,

b) if the main jet flow Mach number is near unity, the Mach number variation in the region of boundary mixing is unaffected by jet temperature, and

c) in overchoked jets where the turbulence - shock wave interaction makes the main contribution to the noise level, no appreciable difference is caused by a change in the shock wave pattern near the jet boundary due to a change in jet temperature.

As sound pressure is proportional to density and density inversely proportional to temperature, the sound pressure should be reduced with increasing temperature. Reference 18 refers to tests which proved that a decrease in jet density (by heating) results in a more rapid spread of the jet, which might affect the noise level produced. Rolls Royce made model tests with $1/4''$ subsonic jets at $0^\circ$ and $250^\circ$C, constant Mach number, and measuring at a distance of 48·d on a line parallel to the jet axis. They found an increase in total sound level, Fig. 32, of about 10 db due to the increased jet flow velocity in the hot jet, but not an appreciable one due to temperature. It must be kept in mind, however, that in practical cases the temperatures of hot jets are nearly three times as high.

Reviewing the experimental evidence it seems that the total effects of heating a jet do balance each other fairly well, since model (cold) and heated jet noise characteristics agree too well to suggest any major temperature influence.

8.5 Jet Flow Velocity

Flow velocities in engine jets have been much higher than those in model jets. This fact can be considered as partially responsible for the difference in directional patterns of full scale and model jets (see Fig. 31).

Eddies in the boundary mixing region may be forced to move at supersonic speeds relative to the atmosphere in fast jet flows. They have been observed to produce shock waves (Ref. 17) which in turn would produce additional noise. This mechanism is not to be confused with the "back-reaction" mechanism (Section 6.5).
8.6 Initial Jet Flow Turbulence

There is no doubt that the state of the jet flow as it leaves the nozzle is an important factor in aerodynamic noise production and this state depends partly on the initial turbulence of the flow, i.e. the degree of turbulence in the flow at the nozzle exit.

In general, engine jets are more turbulent than model jets, due mainly to turbulence introduced in the compressor, combustion chambers and turbine. It is not agreed upon by research workers, how much of this turbulence is still left in the jet flow at the nozzle exit and this initial turbulence is the important factor.

Tests in Ref. 14, presented in Fig. 14, show the effect of initial turbulence on sound pressure of a model jet. The corresponding curve for an actual turbojet lies between the model jet curves, but closer to that for low turbulence. This is reasonable as the sound pressure of the turbojet is increased due to a higher value of initial turbulence than that of the low turbulence model jet. On the other hand, the temperature effect in reducing the jet density decreases the sound pressure. The shear may have also been altered radically in introducing the increased turbulence artificially by means of two 90° bends upstream of the nozzle. As there is no record of a numerical value for the initial turbulence nor of the velocity profile existing at the nozzle exit, these results cannot be properly correlated. Thus no final conclusion can be drawn as to the variation in noise intensity (or sound pressure) with initial turbulence and shear.

8.7 Combustion Effects

The combustion noise, passing along in the jet and being refracted and reflected outside the jet pipe into the atmosphere certainly adds to the aerodynamic noise of an engine jet. Its relative magnitude is unknown as yet. Tests at the National Gas Turbine Establishment (Ref. 25) indicated that pressure fluctuations in a normal fully developed jet engine due to the combustion system were such as to make little contribution to the aerodynamic jet noise, but research workers do not completely agree with this statement. More work must be done on noisy combustion, and a distinction must be made between combustion in and after the flame tubes or combustion even in the open atmosphere as is encountered with re-heated engines, ram jets and rockets.

From knowledge presently available it may be concluded that combustion within the combustion chamber does not seem to be an effect of the first order of magnitude. Things may change if "rough burning" or unsteady burning occurs. Some sort of thermo-acoustic resonance seems then to be involved, which results in a fluctuating mean jet velocity at the nozzle exit, giving rise to additional noise. This sort of noise can apparently be distinguished from aerodynamic jet noise as
was observed on American jet engines (Refs. 27, 31, and 32). The pulse jet was found to be a classical example of this phenomenon.

Still, there are unexplained differences between various jet engines (see Fig. 29, which indicates $I \propto U^3$) and between engine and model test results (see Fig. 28). Further research is necessary to resolve these discrepancies.

8.8. Final Remarks

It should be mentioned here that in some cases, the 3-dimensional model jet had to be replaced by a 2-dimensional one, especially for the study of overchoked jet phenomena. The features of both jets were found to be identical except for frequency (see Section 7.6.5).

Considering the effect of scale in comparing actual jets and model jets, it was found that for a given angle $\Theta$ the significant parameter is the non-dimensional ratio $\sqrt{\lambda}$. At equal values of $\sqrt{\lambda}$ approximately equal overall sound pressures were measured for different jet sizes, whereas at a fixed point, measurements of the total sound pressure would vary due to jet diameter and density, velocity, Mach number, initial turbulence, etc.

In view of the good agreement between full scale and model jet test data it may be deduced that the principal source of jet engine noise is the external aerodynamic noise.

IX A THEORY - TEST RESULT COMPARISON

In a theory - test result comparison, no more than an order of magnitude agreement can be expected in the subsonic flow case. In the case of the choked and overchoked jet, no theory in the sense of Lighthill's theory is available, only a qualitative explanation of the experimental evidence.

9.1 Jet Flow Velocity

Lighthill derived $I(\lambda) \propto U^3$ for the subsonic case and his theory suggests that the power of the velocity may increase for choked and overchoked jets. The velocity $U$ is defined as a "typical" velocity in the turbulent flow. In general, the nozzle exit velocity is chosen for $U$ but with respect to the importance of the large scale eddies in the downstream (nearly isotropic) turbulence in aerodynamic noise production, the eddy convection velocity has to be taken into account as well, especially at higher flow Mach numbers. In the absence of the convection effect the $I(\lambda) \propto U^3$ was found
to be at fault at least for the higher subsonic Mach numbers.

That this is so follows from all experiments, where the total sound intensity measurements were carried out at $\theta = 90^\circ$. At this angle the convection effect is absent as the convection factor 

$$(1 - M_c \cdot \cos 90^\circ) = 1.$$ 

The powers of the velocity are smaller for higher than for lower flow Mach numbers. Since the convection factor for $\theta < 90^\circ$ increases more rapidly for higher than for lower flow Mach numbers, such a result is in agreement with the theory. Total sound intensity measurements at $\theta = 30^\circ$ verify the $U^8$ law almost perfectly for model and engine jets. There are even test results available from jet engines running well above the critical pressure ratio (Ref. 19) which also verify the $U^8$ law (see Fig. 18).

The $I(\kappa) \propto U^8$ law can be violated, especially in the subsonic range, by initial turbulence which may be produced in the combustion system. Pressure jets with high turbulence due to flame stabilization devices in their combustion chambers are the most drastic examples of increased velocity exponents in the overchoked jet region. Subsonic jets with high turbulence levels show velocity exponents as low as 5. Lighthill suggests that turbulence in a high velocity flow may lose sufficient energy by sound radiation from layers of heavy shear to reduce the general level of turbulence in the jet (Ref. 3), thus causing a decrease in the velocity exponent for higher subsonic jet velocities. This is supported by test results (e.g. Ref. 14), see Fig. 14.

9.2 Eddy Convection Mach Number

The importance of the velocity of the turbulence (eddy) in the jet flow with respect to the $U^8$ law was pointed out in the previous section. Measurements of this velocity (Ref. 7) indicate that it increased in the downstream direction as much as twice within a $2d$ distance. This means that the turbulence in the downstream jet flow is moving much faster than that close to the orifice.

If the velocity of the turbulence is known, the measured sound field can be theoretically predicted with good agreement. In Ref. 22, an attempt is made to determine the convection Mach number of the high frequency sound sources from the measured noise radiation field. The noise field is simulated by superposition of a lateral and a longitudinal sound field. The sum of both fields is then modified by multiplying by the convection factor \( (1 - M_c \cdot \cos \theta)^{-6} \) to get the best fit with the measured sound field. In this way, $M_c$ is found to be $0.4 \cdot M$. This result agrees well with observations in Ref. 33.

From the convection factor \( (1 - M_c \cdot \cos \theta)^{-6} \) it follows that an eddy convected at a known $M_c$ would produce theoretically infinite sound (in practice a maximum in noise intensity) at an angle $\theta = \sec^{-1}(M_c)$. 
No experimental evidence is available in this connection.

9.3 Jet Diameter

Theory predicts $\tilde{I}(\vec{x}) \propto d^3$.

Ref. 25 compares total noise measurements by different research workers using various jet engines and made under different conditions. This comparison (see Fig. 44) was inconclusive.

Measurements on a Derwent jet engine of various diameters with and without reheat (Ref. 25) and different jet velocities indicate a very good agreement with theory, if measurements are corrected by the $\tilde{I}(\vec{x}) \propto U^6 \cdot d^3$ law as shown in Fig. 33. Still, the changes in jet diameter are not big enough to allow a final conclusion.

Tests with model air jets of diameters varying from 0.75" to 12" show good agreement with the $\tilde{I}(\vec{x}) \propto d^3$ law (Ref. 14).

9.4 Noise Field and Directionality

From the theory it follows that the noise sources can be considered as a field of lateral and longitudinal quadrupoles. The lateral quadrupoles are predominant in regions of high velocity shear whereas the longitudinal ones predominate in the region of more nearly isotropic turbulence and low shear. High velocity shear occurs in the region immediately adjacent to the orifice and the isotropic turbulence predominates farther downstream. Both lateral and longitudinal quadrupoles are moving but their velocity is different from that of the jet flow. The sound radiation fields both for a lateral quadrupole and a simple source are shown in Figs. 3 and 4 for the cases of emission while stationary and while in translation. From theory it follows further that noise created in a heavily sheared flow is of a high frequency of the order of magnitude $f = \frac{c_s}{2 \pi x}$ (see Section 6.3).

In principle, test results confirm the above points of Lighthill's theory. It is difficult to determine the location of the noise sources in the jet flow directly. Plots of contours of equal intensity were used for a given frequency band (Refs. 11 and 22). These are not too helpful because measurements in the near field may contain sound other than that radiated directly from the sources in question; while far field measurements are masked owing to the larger extent of the radiating source field, especially at low frequency. A better approach with a directional type of microphone was undertaken in Ref. 14 with the result that "the higher frequency noise components emanate from the region immediately outside the jet pipe". In Ref. 22, another method, based on far field measurements of
contours of equal intensity is tried to determine the high frequency source location. Varying the flow Mach number from 0.3 to 1.0 at a frequency of 8500 cps, the source is found to vary from 1 to 9 jet diameters downstream of the orifice. (Note, that \( f = 8500 \) at \( M = 1 \) would result in \( St = 0.65 \), and therefore is barely high frequency, see Section 7.6.5). Applying this method to the results of Ref. 11 leads to similar results.

Passing to the directional noise distribution it is obvious that most of the total noise, at least in the subsonic range, is radiated at angles \( \Theta < 90^\circ \). A frequency analysis for a \( M = 1 \) jet proves this to be true except for the two highest frequency bands used (Ref. 11). The highest total noise intensity measured in the upstream direction (\( \approx 180^\circ \)) was found to be about 15 db smaller than that of the downstream total intensity peak. Ref. 22 obtained similar results at high subsonic Mach numbers with smaller differences at lower Mach numbers.

The downstream directional noise distribution is best checked with theory on the basis of high and low frequency noise. Test results with their high frequency peak at an angle \( \Theta \ (45^\circ > \Theta > 40^\circ) \) agree very well with the theoretical concept of a lateral quadrupole having maximum radiation at \( \Theta = 45^\circ \). If this lateral quadrupole is considered moving at a small \( M_c \) of e.g. 0.2, the results can be made to fit theory perfectly and there is no reason from experimental evidence to reject a small translational motion of the lateral quadrupole.

The sound field from the path of the jet further downstream with its directionality at \( \Theta \approx 30^\circ \) can be best explained by theory as a mixture of a strongly directional quadrupole radiation due to the interaction of the high turbulence with the low velocity shear and due to turbulence alone. The combination of their directional and omnidirectional radiation fields, when modified to allow for convection, would account for the directional intensity peak being at angles considerably less than \( 45^\circ \) and for the observed fact that this angle decreases further with increasing \( M_c \) or decreasing frequency.

Theory predicts that there is no sound radiation from a lateral quadrupole at \( \Theta = 90^\circ \). The observed noise levels are certainly not zero at \( \Theta = 90^\circ \) for both low and high frequencies. The low frequency radiation is due to the omnidirectional radiation characteristic of the equivalent simple source. The high frequency discrepancy may be explained by scattering effects, as half of the radiated sound energy (Ref. 17) is radiated into and through the jet and Lighthill's theory does not account for refraction. Another explanation is offered by Ref. 8, suggesting that a more complicated quadrupole field has to be taken, thereby taking account of an inflow at right angles to the jet in the region adjacent to the orifice by means of longitudinal quadrupoles with their radiation maxima at \( \Theta = 0 \) and \( \Theta = 90^\circ \).
Using the argument of refraction again, Powell in Ref. 17 tries to explain the high frequency intensity peaks at angles up to \( \theta = 80^0 \) for hot jet flows. The refraction effects can be calculated by geometrical acoustics for the lateral quadrupole as well as the non-directional equivalent simple source which taken into account together with the effect of a disturbance (eddy) velocity, shift the intensity peaks up to larger angles of \( \theta \). For the cold jet, Ref. 17 gives values up to \( \theta = 55^0 \) and for the hot jet up to \( \theta = 80^0 \). Lighthill in Ref. 3 suggests that refraction effects cannot be considered to be so pronounced as would follow from geometrical acoustics.

The theoretical explanation of a directional intensity peak observed at about \( \theta = 100^0 \) for overchoked jets is rather tentative. Lighthill explains it (Ref. 3) as being possibly associated with the normal or nearly normal shocks in the jet.

9.5 Near Field - Far Field

Tests on an actual jet engine verified that the sound intensity level varies in accordance with the theoretical inverse square law for far field measurements (\( \lambda_0 > 100 \)). Measurements as close as \( \lambda = 50 \) to the jet indicated intensity levels which were in general 1 - 2 db. above the theoretical values. This means that a considerable deviation from the inverse square law might be obtained for the very near field.

A theoretical consideration shows, however, that the measured values should be below the theoretical ones. Thus the test results of 1 - 2 db. above the theoretical values may be due to sound reflection from the aeroplane structure (Ref. 19).

9.6 Jet Temperature

Theory implies that noise is independent of jet temperature, and model test results support this theory. Full scale test results with temperature variations of 1450°C (Derwent) and 700°C (Avon) have not produced any deviation from a straightforward power law relationship between noise intensity and jet velocity \( I(\lambda) \propto U^8 \).

9.7 Noise Frequency

Lighthill (Ref. 2) originally derived the relationship \( f \propto U \cdot d^{-1} \) in his dimensional analysis for the frequency. Refs. 11 and 22 suggest that this is not the case. The experimentally found mean value for the peak frequency is \( f_p \approx \frac{U}{2d} \) where \( f_p \) was found not to be directly proportional to \( U \) but to a power of \( U \) smaller than unity. The relationship \( f \propto \frac{1}{d} \) was, however, confirmed.
There is still a great need for further investigation, particularly into the relationship between velocity and frequency as not all test results available agree with the above references (see Refs. 21 and 14). Reference 14 shows frequency spectra indicating direct proportionality to velocity using jets which issue directly from a reservoir through a converging nozzle.

The directional maxima for the higher frequency sound at high $\theta$ and for the lower frequency sound at lower angles $\theta$ are well verified by test results (see Section 9.4).

9.8 Jet Flow Density

From theory (Eqn. 57) it follows that the intensity varies as the square of the jet density, $\varphi'$; hence sound pressure is directly proportional to the density.

Test results (Ref. 14) have substantiated this prediction.

X NOISE SUPPRESSION DEVICES

Quite an effort has been made in the past two to three years to find methods by which the noise of actual jet engines can be reduced permanently or temporarily to lower values. So far, ad hoc methods have been more successful in proposing useful noise suppression devices than those derived from systematic research into jet noise.

10.1 Mechanism of Noise Reduction

The theoretical approach to, and the experimental investigations of, jet noise resulted in the following hints for the correct approach to methods of noise suppression.

10.1.1 Subsonic Jets

The most effective noise reduction results from a reduction in jet flow velocity due to $I(x) \propto U^g$.

Noise reduction is thought to be obtained from a reduction of velocity shear at the nozzle exit, but decreasing the velocity shear may mean increased turbulence. From Eqn. 40 it is seen that noise suppression devices are effective only if the decrease in the shear terms $\left( \frac{\partial v_1}{\partial x_1}, \frac{\partial v_1}{\partial x_3} \right)$ overbalances the accompanying increase in turbulence $\left( \frac{\partial v_2}{\partial x_1}, \frac{\partial v_3}{\partial x_1} \right)$ such that there is a net decrease in $\overline{T_{ij}}$ (which for the case of sheared turbulence in the jet boundary is $p \overline{e_{ij}}$ with the $p \overline{e_{23}}$ term neglected).
Another way of noise reduction is to decrease the initial flow turbulence; or by means of a drastic reshaping of the jet to modify the "rolling-up" process of the laminar boundary layer adjacent to the orifice (Fig. 7). It is this "rolling up" which is responsible for the downstream large scale turbulence that produces the bulk of the subsonic jet noise.

10.1.2 Choked and Overchoked Jets

All that was said above for the subsonic jet is appropriate for the choked and overchoked jet as well. Due to the special characteristics of overchoked jets, there are two more major factors which can contribute to an effective noise reduction.

(1) Reduce the shock strength

(2) Prevent a back reaction mechanism with the accompanying fluctuations in $\alpha$

The shock strength can be reduced by using a Laval nozzle instead of a plain nozzle. In the ideal case, a diffuser of appropriate expansion ratio should be adjustable to take care of changing jet pressure ratios.

Another means to reduce shock strength could be based on throttling away the excess static pressure at the nozzle exit.

Back reaction with the formation of discrete eddies can be prevented by:

(a) increasing the turbulence outside the nozzle exit, reducing the velocity shear, or

(b) preventing transmission of sound to the orifice

10.2 Methods of Noise Reduction

Overchoked jets are noisier than subsonic ones and fortunately noise suppression devices attain higher reduction with overchoked jets.

Extensive tests proved that noise reduction with model jets is more effective than that achieved when noise suppression devices are applied to engine jets. This is due to the fact that model jets are in general of higher noise frequency $f \propto \frac{1}{d}$ and less turbulent. As the high frequency noise sources are located near the nozzle orifice they can be affected more successfully by noise suppression devices than the low frequency sources far downstream.
Additional turbulence may, especially in the case of the overchoked model jet (see previous section) prevent the setting up of a back reaction mechanism which in engine jets, due to the much higher initial turbulence, may never exist.

When comparing the noise intensity or sound pressure levels of standard engine jets with those equipped with noise suppression devices, a common basis of comparison should be agreed upon. So far the following values are compared in publications (see Fig. 34):

(a) the difference between the value of the maximum intensity peak of the jet without a suppression device and that at the same angle $\theta$ with the suppression device applied,

(b) the difference between the maximum intensity peaks without and with the noise suppression device applied, and

(c) the difference between the total radiated noise powers with and without suppression devices which corresponds to the area under the curves in Fig. 34.

Case (b) would give the truest indication of the noise reduction achieved from a practical viewpoint while case (a) is certainly the easiest method to apply.

10.2.1 Lower Jet Flow Velocity

The use of lower jet velocities as the most effective means of noise reduction is only applicable in the design stage of new jet engines. It would mean larger and heavier engines; a penalty acceptable only to civil airliners where noise abatement is most desirable. One such jet engine, the "Conway", already exists and it is hoped that it will be tested soon. If these engines could be designed for 400°C. jet flow temperature (instead of 650°C), the corresponding decrease in jet velocity would account for another noise reduction of about 5 db.

There would be a great advance in this sense if new jet engines were designed to avoid choking under any operating condition.

10.2.2 Teeth (Fingers) or Notches

Figure 35 shows such noise suppression devices tested full scale in the high subsonic range on a Derwent jet engine. It proves the device listed first to be the most effective one (see also Fig. 36, curve 4) whereas the last one does not reduce but slightly increases the noise. Note the 8 db. reduction of the low frequency intensity peak which is nearly
balanced by a noise intensity increase of 6 db. at the high frequency peak. This 8 db. reduction is more than could be expected from model tests (4-5 db. only) in the subsonic range and it was achieved (Ref. 25) without a noticeable effect on engine performance (thrust or fuel consumption). Only a slight increase in jet pipe temperature was found.

More recent test results (Ref. 79) show a loss of thrust amounting to 2.5% and 6% for a 6 and 12 toothed nozzle respectively.

The failure of the notched device in the subsonic range is assumed to be due to too great an increase in turbulence which outweighs the decrease in velocity shear at the jet edges and its effect on noise reduction.

In Fig. 36 the effect of various tooth arrangements on noise intensity is shown as a function of jet pressure ratio. A 5 to 6 db. reduction is obtained for the subsonic jet and up to over 14 db. for the overchoked jet at a pressure ratio \( \frac{P\prime - P_0}{P_0} \) of about 1.8. In the overchoked jet it can be assumed that the appreciable noise reduction is due to eliminating the resonance mechanism.

The notched device which caused no noise reduction in the subsonic jet behaves as effectively as teeth in the overchoked jet. Applied to pressure jet nozzles of helicopter wing jets, noise reductions of over 15 db. could be obtained in the overchoked jet, but noise levels are increased considerably in the subsonic range as shown in Fig. 37. Fingers or toothed devices applied to pressure jets proved to be also effective in the subsonic range but the reductions in noise are much less than for the subsonic jet engine. The reason is probably due to the high intensity combustion chambers with their extremely high turbulence (as a result of combustion stabilisation devices). The teeth cannot therefore have much effect on the jet flow.

With respect to teeth or notches, some critical questions are still unanswered.

(a) The effect of teeth and notches is based on balanced changes in velocity shear and turbulence in the jet flow. If the initial flow turbulence is so important a factor in noise production as test results suggest, will the effect of teeth and notches differ from one type of engine to another?

(b) Teeth and notches cause a spreading of the jets. Will this bring additional parts of the aeroplane structure into the region of jet temperature and pressure fluctuations with resultant undesirable vibrations?
(c) A total noise level reduction may still leave the annoyance level of the noise unchanged if the reduction concerns only the high frequency noise (which can be more easily affected than the low frequency sources located further downstream). Tests (Ref. 11) indicate clearly that for some types of teeth the noise reduction is practically constant with frequency down to at least 200 cps. Which reduction devices are best considering this factor?

More full scale tests in which the initial turbulence, noise levels and thrust are measured have to be undertaken to answer these questions.

Testing teeth and notches on model jets yields results which, due to different initial conditions, cannot be applied directly to engine jets. It was found that noise reductions achieved on subsonic model jets were much lower (4-5 db.) than those obtained from jet engine testing, while the opposite is true for overchoked jets. Again the 6 toothed device proved to be the most efficient one, reducing noise by 5 db. subsonically and 12 - 14 db. in the overchoked case

\( \frac{P_n - P_a}{P_a} = 2.0 \).

10.2.3 Gauze Cylinders

Gauze cylinders used as an extension of the nozzle exit are a very effective means of reducing the shock strength, by bleeding away the excess static pressure at the orifice which otherwise would cause the angle \( \alpha \) to fluctuate (Refs. 11 and 15). The expansion makes the shock waves disappear (Refs. 8 and 26) and thereby reduces the possibility of eddy shock wave interaction. The noise reduction obtained of only 5 db. was disappointing. Further tests proved the maintenance of a high noise level was due to another noise source which was introduced by the gauze cylinder itself, i.e. by the air issuing from the gauze. Eliminating this effect by ducting this air away brought the total noise reduction up to 20 db. for the overchoked jet. There was practically no reduction in subsonic jets.

It was also found (Ref. 11) that with increasing excess pressure, the length of the gauze cylinder must be increased to allow for a complete expansion of the excess pressure down to atmospheric pressure.

10.2.4 Radial Swirl Vanes

Another means of increasing the turbulence in the mixing region adjacent to the nozzle exit is the use of radial vanes in the exit itself (Ref. 34). They alter the jet structure and tend to diffuse the shock wave pattern. The results achieved in noise reduction are shown in Fig. 38 and compared with those of notches. We see that vanes are more
effective than notches but notches are presently the most practical of all means proposed, and their influence on engine performance does not seem to be appreciable.

Figure 39 shows the effect on noise reduction of radial vanes of different depth. Reduction by as much as 20 db. is achieved at high jet pressure ratios. As no great difference in noise level was found when the camber angle of the swirl vanes was altered it may be concluded that the noise reduction mechanism is not in general connected to the amount of swirl but rather to the amount of turbulence and the reduction of velocity shear.

It may be mentioned that prewhirl of $5 - 10^\circ$ introduced in the contraction area of the nozzle had no appreciable effect below choking and even increased the noise level above choking.

10.2.5 By-Pass Air, Diffusers, and Ejectors

In certain jet engines where by-pass air is exhausted (Refs. 17 and 23) at slower than jet velocity by means of a ring duct around the jet, noise reductions were found. In principle this device amounts to changing the jet exit velocity profile to one with slower boundary flow (pipe flow velocity profile). Such a change is only effective with respect to noise reduction if the maximum jet flow velocity is not increased as a result of this change.

The effect of the turbulent pipe flow velocity profile on overchoked jet flow is a shift in the onset of the sudden noise level increase to higher jet pressure ratios (Ref. 23). A maximum reduction in noise levels of up to 10 db. was found.

In Fig. 45, another type of by-pass engine is shown (Ref. 79) which not only reduces the velocity of the by-pass air but also that of the primary air in the jet centre. The noise reductions achieved, as shown in Fig. 45, are considerable and are greater for the mixed than for the unmixed jet due to a somewhat more reduced jet temperature and therefore velocity.

Figure 40 shows the diffuser noise suppression device. The results as yet obtained are not satisfactory. Further tests using this basic idea should be made (Refs. 11 and 26). The largest noise reduction was experienced when the nozzle end projected into the diffuser ($h \rightarrow 0$). This reduction is considered to be due to the re-attachment of the flow in the diffuser due to the expansion of the overchoked flow at the nozzle exit.

Figure 46 shows an ejector configuration and the range investigated. No significant decrease in total radiated acoustic power was found (Ref. 79).
10.2.6 Noise Suppression Devices under Preliminary Test

Below some devices are mentioned which can be applied only under special circumstances or which have not been sufficiently tested for final conclusions to be drawn.

(a) Corrugated jet pipes

Model jet results are very promising. In the overchoked jet, 15 - 20 db. reductions were found, but subsonically there was none at all (Ref. 27). The advantage of this device is easy maintenance.

(b) Contracting or expanding nozzle edge

The former brought a slight decrease in noise level due to the reduction in mass flow for a given jet pressure ratio.

The latter increased the noise level considerably due to breakaway of the flow. This suggests that the reduction of shear by thickening the jet boundary layer may not result in a decrease in noise level; presumably because of the greatly increased periodic turbulence which more than outweighs the decrease in velocity shear due to the resultant spreading of the jet (Ref. 26).

(c) Sharp edged nozzles

It was found that the magnitude of the sound pressure just at the orifice is important with respect to the amplitude of the particle displacement. If these sound waves could be reflected (as in the case of a jet exhausting from a hole in a wall) the sound pressure would be practically be doubled, causing larger $\propto$ fluctuations, etc. and therefore a noise level increase of many decibels. Sharp edges at the nozzle exit should be used (Ref. 17).

(d) Interaction of two jets

Too little is known at present as to how the noise energy radiated will be affected by the mutual interference of two jets. So far it is merely a consideration (Ref. 26).

(e) Water injection

Tests (Ref. 35) have shown a definite noise reduction with a model jet of a bifuel rocket 4.5" in diameter. Two peripheral rings mounted 1" and 6" downstream of the nozzle exit injected water into the jet flow at a rate of 12 lb/sec (a quantity prohibitively large for application to jet engines in flight). The problem is however worth further study for possible use in test bed and ground running.
The mechanism of the noise reduction is unknown but it is of the same order as that obtained from teeth and notches. It would be interesting to study the water injection effect on a jet which is already fitted with a toothed device.

(f) **High initial turbulence**

High initial turbulence (due to combustion or the turbine) may be expected to work in the same way as notches. On the other hand, it is believed that high initial turbulence in subsonic jets increases the noise level considerably. More investigation is necessary in the overchoked case.

(g) **Screens and grids**

A screen or grid fixed at right angles to the flow at about 0.5 to 1 d downstream of the nozzle exit was found to be an effective means of noise suppression for both model and full scale subsonic jets (Refs. 43, 79), particularly in the low frequency bands. Figure 47 shows a typical polar sound field with and without such a screen.

Engine thrust was reduced by about 40% (Ref. 79) due to back pressure effects of the screen. This method of noise suppression may however be of considerable value in the ground running of jet engines.

(h) **Convergent and convergent-divergent (Laval) nozzle**

In Fig. 48, the sound power levels radiated by a jet issuing from a convergent and from a Laval nozzle are shown as a function of jet total to static pressure ratio. Both curves show a sharp increase of power level at a pressure ratio of about 2.1, but the curve for the Laval nozzle returns to the \( \sqrt{U} \) law curve again at a design pressure ratio of 3.0 (Ref. 79). It seems that a considerable reduction in noise power level can be achieved by using a Laval nozzle designed to give a shock free expansion of the jet.

10.2.7 **Final Remarks**

Fortunately experiments have shown that the greatest reductions in noise can be achieved for the overchoked jet. Values as high as 25 db have been obtained under ideal conditions whereas for the subsonic jet only up to about 5 db can be claimed with any ad hoc devices tested. It is clear that the ultimate solution is not yet in sight.
XI ASSOCIATED PROBLEMS

The problems briefly discussed below are associated with the subject of aerodynamic noise either as jets other than engine jets or as non-aerodynamic noise sources contributing to the jet noise (such as noise suppression devices).

11.1 Various Propulsion Jets

Various types of jets have been suggested for use on helicopter rotor blade tips for supplying the motive power for rotation.

(a) Pulse jets

Experiments suggest that as a most optimistic estimate the noise level of pulse jets would be about 105 db. at a 50 foot radius, a value much too high for civil aircraft operating in highly populated areas (Refs. 27 and 36).

(b) Ram jets

So far no noise level measurements are available. The low efficiency may bar their application in civil helicopters, but use by military fighters and helicopters appears probable.

(c) Pressure jets

These were already mentioned in Section 10.2.2. Air compressed to pressure ratios of 3 to 4 by a compressor is led to the combustion chambers at the wing tips. The combustion products are emitted at super critical nozzle pressures, resulting in heavily overchoked jets.

Noise level measurements are reported in Ref. 37. Summarizing the effects of noise suppression devices it can be said that the noise levels are still about 10 db. higher than permissible. Further reductions can be obtained by fundamental design changes, i.e., reduction of pressure ratios and increase in mass flow to keep the thrust constant.

(d) Rockets

Little is known about rocket jets except that they are very noisy. Figure 41 shows the overall noise intensity compared with that of other propulsion devices.
11.2 Non-Aerodynamic Noise Sources

These are of interest in the degree to which they contribute to the total jet noise to be reduced.

(a) Centrifugal compressors

There is a very shrill noise radiated in the upstream direction through the air intakes, which predominates over the jet noise at low and up to medium jet engine speeds. It is characteristic of centrifugal compressors and is noticed as a high frequency "screaming", especially when the engine is opened up or closed down. It was found by one investigator to be about 3/4 of the actual jet noise radiating from the intake and it decreases as engine speed approaches the design speed.

(b) Axial flow compressors

Engines with axial flow compressors do not differ in their noise pattern from that of centrifugal compressor engines at high engine speeds. Much less high frequency noise radiates from the intake at lower engine speeds, but this compressor whine may become of importance in by-pass engines with a high by-pass/primary air ratio. The whine may eventually outweigh the jet noise.

(c) Horn effect of intake duct

The installation of a forward facing intake and a fairly straight duct gives a well defined directional effect with the duct acting as a horn. This, of course, increases the radiation efficiency.

(d) Combustion noise

A further noise source may be uneven combustion in and aft of the flame tubes. The contribution of the turbulence introduced by the combustion process to the aerodynamic noise of the jet has been discussed in Section 8.7.

Concerning the noise of combustion and its direct contribution to the total jet noise, little can be said due to the lack of experimental results. One knows only that combustion should be completed in the flame tubes and avoided in the atmosphere. In this case combustion noise does not seem to be of first order importance.

11.3 Boundary Layer and Wake Noise

Previous studies of boundary layer noise have been mainly concerned with the noise of air flows in ducts. In the attempt to analyse the mechanism of the noise generation three different types of
radiators have been suggested as its components.

At the outer edge of the laminar sublayer, where large velocity shear combines with turbulence, a quadrupole distribution in accordance with Lighthill's theory is expected.

The interaction of a rigid plane surface with the fluctuating normal pressures was assumed by Lighthill to imply a dipole distribution and a theoretical treatment of the noise from turbulent boundary layers was presented (Lighthill, Ref. 39). Powell suggested that such a dipole distribution must be considered together with its image to yield a distribution of longitudinal quadrupoles with axes normal to the surface. Phillips (Ref. 44) agrees with the longitudinal quadrupole distribution for a plane surface and suggests that for curved surfaces one has in addition a normal dipole distribution. A further paper by Lighthill and Powell jointly is expected on this subject.

The third component, suggested by Phillips (Ref. 44), is a distribution of dipole radiators with axes lying in the surface. These are considered as the result of fluctuating shear stresses on the surface.

The separation of flow from surfaces introduces a new generator of noise, the turbulent wake. No appreciable analysis of wake noise as such has appeared. The wake may be considered as a "negative" jet, producing a concave velocity profile instead of the convex profile of a true jet (Fig. 9) when referred to the velocity of the aircraft. Hence the characteristic features of shear and turbulence are expected to be similar (although less intense) and the theory for subsonic jet noise (Lighthill, Refs. 2, 3) properly applied should yield useful results.

From suitable measurements available from air flows in ducts, it follows that:

(a) the acoustic efficiency \( \eta \) is about \( 10^{-4} \), a value which agrees with that obtained from other vortex producing flows (Ref. 25),

(b) the frequency at maximum intensity is given by the relation (Ref. 29)

\[
\frac{f_{max}}{f(Re)} = \frac{U}{d}
\]
(c) the noise intensity increases as $U^4$ to $U^6$ (Ref. 38).

Experiments suggest that in general the intensity of boundary layer noise (aerodynamic noise) increases as aerodynamic drag increases (i.e., aerodynamically clean aircraft will generate lower noise levels associated with boundary layers and wakes than those having areas of separated flow).

The problem of noise radiating from boundary layers, wakes and regions of separated flow is important not only in aeronautics (aeroplane surface and cabin noise), but in many other fields as well (duct flow noise, wind noise on cars, etc.). The need for research on this aspect of the subject is important for its application to those fields, as well as for its value in understanding the fundamental mechanism.

11.4 Ground Running of Jet Engines

There are three main cases:

(a) a jet aircraft, taxiing at comparatively low engine speeds,

(b) a jet aircraft making its pre-take off engine checks, and

(c) jet engine testing in a maintenance area whereby prolonged running is necessary for full power checks or test-bed running of jet engines at the factory.

Case (c) is a very important one and has to be considered with respect to noise reduction.

In these cases, in addition to noise suppression devices, acoustic screening can be applied. There are several methods which can be useful in one way or another:

(a) **Aeroplane orientation**

The aeroplane may be oriented in such a way that its jet engines radiate their most intense noise (at angles $\theta$ between $30^\circ$ to $45^\circ$) across the airfield and hence away from built up areas.

(b) **Screening by walls**

This method showed promising test results (Ref. 40). A wall of 40 foot height and 110 foot width was used and a Comet and Viking were tested. The main conclusions which can be drawn from the test work are the following:
(1) With aircraft close to the screen wall, facing it, up to 25 db. overall reduction was obtained in the forward direction for at least half a mile, and a similar reduction in the wall shadow in other directions for about a quarter of a mile.

(2) Trees and buildings are not very effective as noise suppressors unless the tests are made close to the buildings.

(3) Noise radiated against the wind falls off in intensity more quickly than that radiated in any other direction.

(4) Walls are more effective than pits or sunken enclosures.

(5) Attenuation is least for the low frequencies. Walls have to be very solid and of heavy material to be as successful in reducing low frequency as they are with high frequency noise.

(c) Ground Mufflers

Ground mufflers are already becoming standard equipment for ground running of jet engines (Ref. 41) in the U.S.A. Test results allow one to draw the following conclusions.

(1) A special muffler (Northrop) placed forward as well as aft of the engine reduced the total high frequency noise by as much as 30 db. at a distance of 100 feet. For the low frequency noise a 20 db. reduction was obtained. Using the aft muffler only, a 20 db. reduction was still achieved in the high frequency range.

(2) The disadvantage of these mufflers is that they must fit very accurately on the aircraft to achieve such noise reductions. Therefore they are very costly and can be used only for one aircraft type. Because of their expense and inconvenience they should not be considered as the final answer for this phase of the noise reduction problem.

(d) Screens and grids

A screen of rods normal to the jet at a downstream distance of about 0.6 d achieved a considerable noise reduction mainly due to velocity reduction, especially in the low frequency noise range (Ref. 79). For pre-flight ground running the reduction in engine performance due to back pressure does not matter. This device is very cheap.
11.5 Aerodynamic Noise at Altitudes

Noise at subsonic flight speeds is due to turbulence such as that occurring in jets, wakes, boundary layers, regions of separation, vortices and regions of shear flow. In all such noise sources the viscosity plays a predominant part. Without viscosity (non-viscous fluids) there would be no mechanism of creation for aerodynamic noise. Ordered pressure gradients (in contrast to the random fluctuations considered above) are in general too shallow to produce audible noise, in passing with sonic speed by an observer.

At supersonic speeds, the ordered pressure gradients, (e.g. in a shock wave) are sufficiently strong and steep to be quite audible in their passage with greater than sonic speeds by an observer even if the fluid were inviscid. These pressure changes are known as "sonic bangs" and in a shock wave the pressure varies almost discontinuously. For example, a pressure pulse of 1 lb./ft.\(^2\) represents a sound of about 130 db., although it means an aerodynamic pressure coefficient of about 0.001 \[ \frac{\Delta P}{\frac{1}{2} \rho \cdot U^2} \]
at the sonic velocity.

At high altitudes jet engines become severely overchoked, in general giving rise to very high noise levels if a simultaneous reduction in noise is not brought about by:

(a) a reduction in jet power (thrust),

(b) the reduced jet exit velocity relative to the surrounding air \( U_{\text{jet}} = U_{\text{jet, absolute}} - U_{\text{flight}} \), or

(c) less turbulent mixing which may reduce the lower frequency noise.

On the other hand, a high frequency noise intensity peak at about \( \Theta = 90^\circ \) is associated with heavily overchoked jets, which adds to the cabin noise level and adds another 4 - 5 db. to the level measured on the ground (in the case of jets which are subsonic at ground level and operate at altitudes at which they become overchoked.)

Theoretically cabin noise should decrease with altitude. Test with a Comet confirmed this, but measurements on a Canberra undertaken by Rolls Royce indicate that at constant flight speeds the cabin noise is independent of altitude (Ref. 25). This was not expected as cabin noise is primarily a function of viscosity which decreases with altitude. Perhaps the characteristic intensity peaks in the upstream direction \( (\Theta \approx 180^\circ) \) and at \( \Theta \approx 90^\circ \) can be held responsible.
In Fig. 43 the noise levels measured in a Comet cabin are shown as a function of noise frequency. The maximum noise level curve previously recommended by RAE for civil aircraft cabin noise is shown, along with a new cruising standard level, suggested by Rolls-Royce Ltd.

11.6 Jet Noise as a Vibration Source

Severely overchoked jets (at high altitudes) close to the aeroplane structure may cause serious vibrations. Very intense vibration levels have been measured in close proximity to the jet, even at subsonic jet speeds (Ref. 14).

It is important to realize that from the point of view of structural fatigue, the frequency is in general of more vital significance (due to the danger of resonance) than the amplitude. Thus if the jet noise spectrum covers the whole range the jet has to be moved further away from the structure or the structure has to be stiffened. Subsonically vibrations are assumed to be mainly energized by the low frequency eddy noise radiation, while overchoked jets add high frequency radiation from the turbulence – shock wave interactions. It is known that teeth and notches may appreciably reduce the high frequency vibrations but no adequate cure has been found for the low frequency vibrations so far.

In Fig. 42 the variation of over-all oscillating pressures with axial and radial distance from the 1" model jet orifice is shown. The clearances are indicated that are needed between the jet and the aeroplane structure depending upon the oscillating pressures that may be tolerated.

To avoid the occurrence of dangerous resonance in aeroplane structures, the natural frequency of all aircraft parts close to the jet must be checked. A change in the natural frequency or an outward movement of the jets should help to prevent excessive vibration. Noise suppression devices may be a help in the high frequency range, but unfortunately they cause a spreading of the jet which must be taken into account.

Further test work is needed to permit analysis of the predominant noise frequencies in the region between the jet stream and the structure of the aeroplane. These frequencies and the natural frequency of the structure must of course be as different as possible. More information on jet spreading is needed.

No severe physiological effects were found on personnel if given adequate ear protection when exposed to the vibrations of a GE 14.6 turbo-jet engine for 1 - 2 hours daily (Ref. 42).
11.7 **Edge Tones**

New theories for the classical problem of "sensitive" jets and edge tones (tones produced by blowing a jet of air against a rigid edge) have been put forward (Refs. 9 and 10) as by-products of the jet noise problem.

Noise from choked jets and edge tones relies essentially on a back reaction mechanism which in turn gives some foundation to the assumption that in an overchoked jet flow the shock waves replace the sharp edge (see Powell theory for choked and overchoked jets, Ref. 7 and Section 6.4).

Earlier experiments have shown that a jet flow pattern under certain conditions can be greatly modified if sound waves are allowed to impinge upon the jet in the immediate neighbourhood of the jet orifice, the principal condition being that the flow from the orifice must be laminar. Later it was found that even if the flow is turbulent, sufficiently intense sound waves influence the flow pattern. If, however, the flow is in a very turbulent state and subjected to a high intensity sound beam the edge tone phenomenon has been observed (Ref. 17).

**XII POSSIBLE RESEARCH PROJECTS**

In the case of a subsonic jet, future research aims at a comparison with Lighthill's theory. For the overchoked jet no quantitative theory now exists, the knowledge available being empirical. Thus there is a great need for theoretical research at this time in the field of overchoked jets.

The research programs proposed can be divided into three main groups:

1. Investigation into the fundamental nature of the jet flow,

2. Investigation into the way in which jet flow produces noise and the characteristics of such noise, and

3. Investigation into the problem of noise reduction.

A more detailed consideration of these groups follows.

1. A further detailed investigation into the structure of the jet flow is necessary, considering

   i. the rate of growth of the mixing region,
ii. the precise mechanism by which sound waves induce disturbances at the orifice of choked jets,

iii. the rate of growth of these disturbances while moving downstream through the mixing region.

iv. initial turbulence and its influence on the turbulence produced by shear,

v. the Mach number effect on the mixing process at the edges of a jet (jet spreading),

vi. further turbulence analysis either by hot wire techniques or by optical methods for both subsonic and overchoked jets,

vii. influence of initial turbulence on the exit velocity profile,

viii. the jet temperature effect with respect to jet density, velocity, spread and noise efficiency.

2. A broader investigation is required concerning the basic acoustic generators (including their location in the jet field) whose combined effects constitute the total noise field. Points for consideration are:

i. the production of sound by a disturbance (eddy) - shock wave interaction or interaction with a standing shock wave pattern, e.g.

   (a) shock wave - eddy interaction, using a shock tube and an eddy generator, or a
   (b) shock wave - turbulence interaction, using an open end shock tube to fire a shock wave into a turbulent stream issuing from a nozzle.

ii. the frequency of turbulence in jet flow and that of noise just outside the jet boundary,

iii. the correlation of jet noise with the structure of the jet using:

   (a) two identical microphones in the far field with measurements of cross correlation (which is a maximum when the line joining the microphones is perpendicular to the line joining the microphones and the mean centre of the source). A given frequency band could be selected and its source located.
(b) two identical microphones, one in the near, the other one in the far field. One might correlate readings taking account of the appropriate time delay corresponding to the distance between the microphones. In moving the near field microphone parallel to the jet axis, the maximum cross correlation would be found when the microphones and the source in question were collinear.

iv. the effect of the turbulence level in the jet on the noise radiated, determining:

(a) the relationship between the turbulence level in the jet and the intensity peak frequency (Lighthill's $f_p = \frac{\nu}{\lambda d}$)

(b) the relationship between the turbulence level and the velocity exponent (in $\overline{I(x)} \propto \nu^p$).

(c) the influence of initial turbulence on the jet noise intensity and frequency.

(d) the eddy convection Mach numbers in different jets, the characteristic frequencies of which cover a wide range.

v. the effect of jet density on noise frequency (high and low)

vi. boundary layer noise. A critical experiment is required to test the theory of dipole radiation associated with fluctuating shear stresses between the fluid and the solid boundary

vii. the effect of flight speeds on jet noise

viii. structural problems associated with the fluctuating pressure field near the jet

ix. comparison between a supersonic jet (Laval nozzle) and an overchoked jet (converging nozzle) with respect to noise intensity, directionality, etc.

3. More investigation is needed into the effects of:

(a) gauze cylinders, screens, and grids

(b) jet diffusers and by-pass air, and

(c) changes in jet velocity exit profile.
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Fig. 1 LOUDNESS LEVEL CONTOURS.

Fig. 2 FRAME OF REFERENCE IN TRANSLATION.
Fig. 3 RADIATION FIELDS OF A LATERAL QUADRUPOLE AND A SIMPLE SOURCE AT REST AND IN TRANSLATION TO THE RIGHT AT A MACH NUMBER OF $M_C = 0.9$. 
Fig. 4 CHANGE IN THE DIRECTIONAL INTENSITY DISTRIBUTION AS A RESULT OF TRANSLATION OF (a) A LATERAL QUADRUPOLE IN THE DIRECTION OF ONE OF ITS AXES, (b) A SOURCE AT A MACH NUMBER $M_c$. $\theta$ IS MEASURED FROM THE DIRECTION OF TRANSLATION.
Fig. 5 SCHEMATIZED OVERCHOKED JET FLOW. (Ref. 17)
E - EXPANSION REGION, C - COMPRESSION REGION,
EC - MIXED REGION.

Fig. 6 TURBULENT MIXING OF A SUBSONIC AIR JET. (Ref. 17)
Fig. 7 SCHEMATIC DIAGRAM OF A SUBSONIC JET. REGION A - HIGH VELOCITY SHEAR, FINE GRAIN TURBULENCE, HIGH FREQUENCY NOISE SOURCES. REGION B - LOW FREQUENCY NOISE SOURCES, NEARLY ISOTROPIC TURBULENCE, LARGE SCALE EDDIES.
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Fig. 9 UPPER HALF - MEAN VELOCITY AND AXIAL FLUCTUATION OF VELOCITY DISTRIBUTIONS.
LOWER HALF - CONSTANT INTENSITY CONTOURS OF OCTAVE ANALYSIS OF NEAR SOUND FIELD. NUMBERS ARE MEAN $\lambda \cdot d^{-1}$ VALUES, THOSE IN BRACKETS ARE NOISE LEVELS IN $\text{dB}$. (Ref. 17)
Cranfield subsonic noise spectrum. Frequency band 800-1600 c.p.s.


Fig. 10 SUBSONIC NOISE FIELDS.

Fig. 11 DIRECTIONALITY OF THE TOTAL NOISE OF AIR JETS. EXIT VELOCITY \( \approx 1000 \) ft./sec. DATA REDUCED TO \( \lambda \cdot d^{-1} = 24 \). (Ref. 17)
Cranfield noise spectrum (over-choked $P' = 2.0$).
Frequency band 800-1,600 c.p.s. ($P'$ is pressure ratio).

Noise spectrum (over-choked $P' = 2.0$).
Frequency band 6,400-12,800 c.p.s. ($P'$ is pressure ratio).

Fig. 12 NOISE FIELDS OF OVERCHOKED JETS.

Fig. 13 TOTAL NOISE LEVEL OF TWO INCH DIAMETER JET EXHAUSTING TO ATMOSPHERE. (Ref. 26)
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Fig. 17a FREQUENCY OF STREAM DISTURBANCE IN AXially SYMMETRIC CASE. (Ref. 7)
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Fig. 25  RELATION BETWEEN FUNDAMENTAL RADIATED WAVELENGTH AND CELL LENGTH, TWO DIMENSIONAL CASE. (Ref. 7)

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<th>Medium</th>
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<td>Helium</td>
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<tr>
<td>Air</td>
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Fig. 26  OVER-ALL SOUND PRESSURE AS A FUNCTION OF JET EXIT VELOCITY FOR THREE JET MEDIA. θ = 90°, r.d⁻¹ = 16 d = 0.75 INCHES. (Ref. 14)
Fig. 27  MODEL AND FULL SCALE SPECTRA FOR \( \Theta = 30^\circ \), REDUCED TO \( \kappa d^{-1} = 24 \), EXIT VELOCITY 1640 FT/SEC, DIAMETER 16 INCHES. (Ref. 17)

Fig. 28  TOTAL NOISE LEVEL AT \( \Theta = 30^\circ \) REDUCED TO \( \kappa d^{-1} = 24 \). (Ref. 17)
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Fig. 30  ANGLE AT WHICH NOISE OF GIVEN $\lambda/d$ REACHES MAXIMUM.  ○ ENGINE (Ref. 28).  ▲ ENGINE (Ref. 30)  ● MODEL (Ref. 14).  (FROM Ref. 17)
Fig. 31  COMPARISON OF OVER-ALL SOUND PRESSURE RADIATION PATTERNS FOR TURBOJET EXHAUST AND MODEL JETS. (Ref. 26)

Fig. 32  ROLLS-ROYCE NOISE TESTS ON 0.25 INCH JET. (Ref. 26)
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(Ref. 19)
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Fig. 36 EFFECT OF JET TEETH ON TOTAL NOISE VARIATION WITH JET VELOCITY. $\theta = 30^\circ$, $\alpha \cdot d^{-1} = 108$, $\gamma$ IS ANGLE OF TEETH TOWARDS JET AXIS. (Ref. 26)
Fig. 37  INFLUENCE OF NOTCHING JET ORIFICE.  (Ref. 26)

![Graph showing influence of notching jet orifice.](image)

Fig. 38  INCREASED NOISE LEVELS ARISING ABOVE CHOKING WHICH COMMENCES AT C. BROKEN LINES ARE REDUCED NOISE LEVELS OBTAINED BY NOTCHES (N) AND VANES (V).  (Ref. 15)

![Graph showing increased noise levels.](image)
Total noise levels at 90°, 4 ft. from jet exit. Addition of six cambered radial vanes of depth (i) 1/2 in., (ii) 1 in., (iii) 3/4 in. to nozzle "Bp" of 2 in. diam.

Fig. 39 VARIATION OF NOISE LEVEL WITH VANE DEPTH. (Ref. 26)

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<td>δ = 25°</td>
<td>D = 1 inch</td>
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<td></td>
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<td>δ = 50°</td>
<td>D = 1 inch</td>
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<td>δ = 50°</td>
<td>D = 1.125 inches</td>
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Fig. 40 NOISE REDUCTIONS EFFECTED BY DIFFUSERS OF VARIOUS ANGLES, δ, THROAT DIAMETERS, D, AND NOZZLE TO DIFFUSER GAPS, A, \( h \cdot d^{-1} = 108 \), θ = 30°. (Ref. 26)
Fig. 41 ESTIMATED OVER-ALL SOUND INTENSITIES AT $h = 30$ FT. AS A FUNCTION OF $\beta$ FOR 10,000 LB. THRUST. (Ref. 52)

Fig. 42 VARIATION OF OVER-ALL OSCILLATING PRESSURE WITH AXIAL AND RADIAL DISTANCE FROM MODEL JETS WITH CONSTANT EXIT VELOCITY. (Ref. 58)
Fig. 43  NOISE LEVEL MEASUREMENT ON COMET.  (Ref. 25)
Fig. 44  VARIATION OF NOISE WITH JET VELOCITY. REDUCED 
\( r \cdot d^{-1} = 45, \ \theta = 30^\circ \) . (Ref. 25)
Fig. 45 BY-PASS ENGINES AND NOISE REDUCTION ACHIEVED. (Ref. 79)
DIAMETER RATIOS $\frac{d_2}{d_1} = 1.2, 1.4$

SPACING RATIOS $.15 < \frac{s}{d_1} < 1.50$

Fig. 46 EJECTOR CONFIGURATIONS. (Ref. 79)
Fig. 47 EFFECT OF SCREEN ON ENGINE NOISE. (Ref. 79)
Fig. 48 EFFECT OF PRESSURE RATIO ON TOTAL POWER LEVEL. (Ref. 79)
A detailed review is presented of the theoretical and experimental advances that have been made in the study of aerodynamic noise and in devising means for its suppression. Since many workers in the fields of aerodynamics and aeronautics may be unfamiliar with acoustic principles, the necessary background of laws and ideas from the field of acoustics is included. The theories for noise caused by subsonic disturbances, which may include turbulence fields in overchoked jets, (Lighthill, Proudman) and for those noise sources peculiar to overchocked jets (Lighthill, Powell, Ribner) are considered. Experimental results are quoted complete with numerous graphs, notes on correlation of data for model and engine jets and a comparison with theory. The results of attempts at noise suppression are discussed, noting both untried and extensively tested suggestions. A list of references is included.

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