DYNAMIC PROPERTIES OF BONE AND SOME
ENGINEERING CONSIDERATIONS IN THE DESIGN OF
INTERNAL ORTHOPAEDIC PROSTHESES

by

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Summary

The behaviour of human compact bone under compressive impact is investigated experimentally. A viscoelastic model is presented to give a satisfactory phenomenological description of bone over a wide range of strain rates. Storage fluids were found not to affect appreciably the elastic stiffness of human compact bone but caused a pronounced change of the viscous stiffness. A method of quantifying microscopic damage under impact is suggested and speculations are made on its relevance to spontaneous fractures. Some hypothetical composite materials are considered for possible use in internal orthopaedic prosthetic and fixative devices.
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Notation

$a_n$  Constant
$b_n$  Constant
$c$  Speed of sound, compliance
$c^*$  Complex compliance
$c_1, c_2$  Components of complex compliance
$E$  Elastic stiffness, modulus
$f_n$  A function
$g$  Acceleration due to gravity
$g_n$  A function
$G$  Relaxation modulus, shear modulus
$h$  Height
$H$  Relaxation spectrum
$J$  Creep compliance
$J_0$  Instantaneous compliance
$K$  Creep kernel
$L$  Retardation spectrum
$M$  Modulus
$M^*$  Complex modulus
$M_1, M_2$  Components of complex modulus
$P$  Linear differential operator
$P'$  Non-linear differential operator
$Q$  Linear differential operator
$Q'$  Non-linear differential operator
$R$  Relaxation kernel, resolvent
Greek Symbols

\( \delta \)  
Phase angle

\( \epsilon \)  
Strain

\( \epsilon_i \)  
Incident strain

\( \epsilon_o \)  
Strain in output bar

\( \epsilon_R \)  
Reflected strain

\( \dot{\epsilon} \)  
Strain rate

\( \epsilon_{ij} \)  
Strain tensor

\( \phi \)  
Aging function

\( \sigma \)  
Stress

\( \sigma_{ij} \)  
Stress tensor

\( t \)  
Time

\( T_{\sigma} \)  
Retardation time

\( \omega \)  
Circular frequency

\( \eta \)  
Damping coefficient

\( \eta_i \)  
i\textsuperscript{th} damping coefficient

Subscripts

Ave  
Average

c  
Composite

Equiv  
Equivalent

f  
Fiber

i  
i\textsuperscript{th} element

m  
Matrix

0  
Zero time
1. INTRODUCTION

1.1 General

The topic for this thesis forms part of the subject matter of the discipline of biomedical engineering.

It may be reasonable to attribute the enhanced interest in this field in recent times to a realization that compartmentalisation of science is inhibitive to development. The multi-disciplinary approach of this field of endeavor is all too often declared to be novel. However, this approach may be viewed as a reversion to the concept of a 'complete science of man', a concept successfully utilized for a synthesis of knowledge by Aristotle (384-327 B.C.), Leonardo da Vinci (1452-1519), Rene Descartes (1596-1628) and others.

The integrative approach of biomedical engineering has been used for different purposes and for solving different types of problems at various times in history. For example, Descartes (1)* described the mechanical action of muscles on bone, responsible for the movement of limbs. In his passions of the soul, Article XI, he wrote:

"...the sole cause of all the movements of the members is that certain muscles contract, and that those opposite to them elongate..."

The problems that need to be tackled today are quite different from the ones tackled by Descartes. There are three factors typical of the 20th century that have alerted the biomedical engineer. They are,

(i) high speed ground travel in a modern industrial society (which is almost a necessity) and space flight (2),
(ii) extension of human lifespan, uncovering essentially unexpected mechanical weaknesses of body parts. That is, the effective life of the whole is outlasting that of individual components (3). As an example of this, we find that a very large number of patients in the age range of 65-80 years suffer fracture of the neck of the femur.

and

(iii) the revolutionary developments in reconstructive surgery not unrelated to (i) and (ii) above.

The problems arising from the three factors mentioned above need, for their solution, information on 'man-machine interactions'. The term, 'man-machine interactions', is used here in a very broad sense to include tissue-implant (tissue = man, implant = machine) interactions, host-transplant interactions, patient - dialyser interactions and so forth. Tissue-implant interactions include bone-bone plate, bone-intramedullary nail, heart-artificial heart valve, heart-pacemaker and breast-mammary prosthesis interactions. These interactions may be mechanical, chemical, biochemical, electrical, psychological or a combination of them. For example, in the tissue-implant interactions one needs information on,

(a) the mechanical interaction between the two in terms of forces, displacements, stresses, strains, rate of loading, etc.

* Numbers in brackets refer to References given at the end.
(b) the electro-chemical interaction in terms of corrosion resistance of the implant if it is made of a metal

and,

(c) the biochemical interaction in terms of carcinogenicity and toxicology of the implant.

The psychological dimension of man-machine interactions is brought out most dramatically in the form of a psychosis called 'post-cardiotomy delirium' occurring, for example, in patients who have undergone open-heart surgery for replacement of the defective natural heart valves with artificial heart valves (4).

Various experimental and theoretical methods have been used in studies concerning man-machine interactions. The studies were concerned with (a) the whole body or person, (b) parts of the body or organs (e.g., whole bones, heart, kidney, etc), and (c) tissues or materials (e.g., muscle, cartilage, bone, etc). Experiments have been conducted in vivo and in vitro, on humans and other animals. However, most of the experiments have employed model systems.

Some of the problems in biomedical engineering that have received interest in the past fifty years are:

(i) kinetics of locomotion (as opposed to kinematics of locomotion) (5 to 13)*

(ii) dynamic response of tissues, organs or body parts (14 to 20)

(iii) whole body response under vibration, impact, and external pressure loads due to acoustic effects, blast waves and decompression (21)

(iv) load bearing capacity of bone grafts (22)

(v) electro-chemical and mechanical tissue-implant interactions (23 to 30)

(vi) influence of earth gravity, hypergravity, and simulated weightlessness on bone (31).

This thesis deals with the human body primarily at the level of tissues, and the particular tissue studied is bone.

1.2 The Research Program

1.2.1 Factors Influencing the Choice of the Problems for Study

(i) It was observed by Souter (32) that among patients with tibial fractures which united clinically within 4 months of injury, 50% were caused by industrial and road traffic accidents. Among patients with delayed union of the tibia, 79% of the fractures were caused by industrial and road traffic accidents. A typical example of dynamic loading of the skeletal bone structure was reported by Kirkup (33) who described the case of a male 20 years of age involved in a motor cycle-car collision.

* These references are not meant to compose an exhaustive listing
The probable sequence of events leading to the extrusion of the lower femoral shaft, as illustrated by Kirkup, is shown in Fig. 1.

Since the mechanical behaviour of bone under high velocity impact conditions (which are usually associated with industrial and road-traffic accidents) is not well understood, this study was partly undertaken to throw some light on this problem.

(ii) More often than not, practical considerations impose a delay between the death of the patient or animal whose bone is being tested and the actual testing of the bone. Bone is usually either frozen or stored in a preservative to minimize the effect of delaying the testing. Before choosing a method of storage, it is essential to evaluate the influence of the method of storage on the properties of interest.

A great volume of literature has been published on the influence of storage conditions on the quasi-static mechanical properties of compact bone (34 to 41). Swanson (35) has reviewed these results, some of which are briefly stated below:

(a) **Effects of Freezing:** Freezing does not significantly affect the Young's modulus, stress at fracture and energy absorbed to fracture when the specimens are loaded under quasi-static conditions.

(b) **Effects of Fixation in Formalin:** The experimental results on the influence of fixation in formalin on quasi-static mechanical properties are inconclusive.

(c) **Effects of Storage in Saline:** The quasi-static mechanical properties are not significantly affected by storage in saline.

Compact bone is known to be viscoelastic (37, 39). Piekarski (37) has given convincing arguments in support of the view that the fluids in bone contribute significantly to its viscous properties. One would therefore expect that the changes occurring in bone (if any) upon storage (in a fluid different from the fluid that it is exposed to in the body), would primarily be in its fluid contents. These changes could be due to ionic diffusion, chemical changes, or simply changes in the volume of fluid contained within the pores. If there are such changes in the fluid content of bone upon storage, it is reasonable to expect the viscoelastic properties to be more sensitive to the storage fluid than the quasi-static mechanical properties. Except for the work done by Tennyson et al. (42, 43) on bovine compact bone, the author is not aware of any other results reported on the effect of storage on the viscoelastic properties of compact bone. Hence, it was decided to investigate the influence of storage on the viscoelastic properties of human compact bone.

(iii) If the fluid in which the bone specimen is stored does affect its viscoelastic properties, then the duration of storage after death of the patient or animal (the Post-Mortem Age or PMA) should have a bearing on the viscoelastic properties. Hence, it was decided to investigate the effect of PMA on the viscoelastic properties of bone.

(iv) Experiments to determine the mechanical properties of compact bone can be classified as those done on (a) human bone in vivo

(b) " " in vitro

(c) animal " in vivo

and (d) " " in vitro
Almost all the available data belongs to categories (b), (c) and (d). The purpose of most experiments, however, was to determine, even if very approximately, the properties of human bone in vivo. There have been very few experiments performed on human bone in vivo due to the simple fact that such experiments must be non-destructive. If non-destructive tests on human bone in vivo can be performed reliably, they would be very useful not only in determining the basic mechanical properties, but also in the diagnosis or quantification of those pathological conditions of bone that influence its mechanical properties (e.g., Osteopetrosis, Osteoporosis, Avascular Necrosis, Osteomalacia, etc) and possibly in the determination of fracture healing. Some work has been reported on the non-destructive testing of compact bone (44 to 47). However, as there are no direct methods available at the present time for determining the high strain rate viscoelastic properties of human bone in vivo, indirect methods have to be used for this purpose. If the viscoelastic properties are sensitive to PMA, then the determination of the viscoelastic properties as a function of PMA would serve as an indirect method for estimating the viscoelastic properties of human bone in vivo. The in vivo properties can be estimated by extrapolating the experimental values to a PMA of zero. Clearly there is no way of guaranteeing that these extrapolations define the in vivo values, but it is plausible to expect that they provide the best estimate one can obtain until better methods are available.

(v) As noted earlier, the quasi-static properties of compact bone have been studied quite extensively. There are several excellent reviews available on this topic (34 to 36, and 41). In particular, stress relaxation properties of compact bone have been considered by Lugassy and Korostoff (48), Piekarski (37), and Sedlin (39). Creep and creep recovery properties of compact bone were also considered by Renton and Piekarski (49), Piekarski (37), Bursttein and Frankel (50), Sedlin (39), and Smith and Walmley (51, 52). However, dynamic properties of compact bone have not been evaluated, although forced vibration experiments have been performed by Laird and Kingsbury (53), Uezaki (54), and Smith and Keiper (47). Compact bone impacted with projectiles was also considered by Bird and Becker (55 to 57). McElhaney (2, 58) studied fresh bovine compact bone and embalmed human compact bone using strain rates of 0.001 sec\(^{-1}\) to 1,500 sec\(^{-1}\) and dynamic loading of whole bones was considered in Refs. 59 to 65.

It is almost analytic to say that a material has to be studied over a broad range of strain rates for a complete understanding of its mechanical properties. A single test can usually cover only about two to three orders of magnitude of strain rate. For example, creep and stress relaxation tests cover approximately the range of \(10^{-7}\) to \(10^{-5}\) sec\(^{-1}\). Quasi-static tests occur in the range of \(10^{-4}\) to \(10^{-1}\) sec\(^{-1}\) while dynamic tests using either fast acting hydraulic or pneumatic machines, or using forced vibration techniques, describe the range of \(10^{-1}\) sec\(^{-1}\) to \(10^{1}\) sec\(^{-1}\). Mechanical impact tests generally include the range of \(10^{1}\) to \(10^{3}\) sec\(^{-1}\) and hypervelocity impact is defined above \(10^{4}\) sec\(^{-1}\). The different tests relevant for the various strain rate regimes (see Lindhelm (66)) are shown in Fig. 2.

As noted earlier, the published literature on compact bone describes the whole physiologically relevant strain rate range from stress relaxation and creep to mechanical impact. There have, however, been very few attempts to obtain viscoelastic models for compact bone. Sedlin (39) proposed a viscoelastic model which qualitatively exhibits all the viscoelastic phenomena exhibited by compact bone. However, he did not attempt to evaluate the constants
for his model. Burstein et al. (50) propose the same model for compact bone as Sedlin. They too did not evaluate the constants for the model. To the author's knowledge, Piekarski (37) was the first to quantify a viscoelastic model for compact bone by representing the stress relaxation behaviour by a Maxwell model in which he evaluated the stiffness and viscosity of the elements. Similarly, he represented the creep behaviour of compact bone by a Kelvin (Voigt) model and again evaluated the constants for the model. A viscoelastic model is more than a mere qualitative mechanical analogy. It can be used for predictive purposes if the constants of the model are quantified. Thus, Piekarski's quantified viscoelastic models are more useful than the earlier qualitative, viscoelastic models.

However, it is highly desirable to obtain a single quantified viscoelastic model for compact bone governing its response over the entire relevant strain rate spectrum. To the author's knowledge, such a model has not been obtained so far.

The literature surveyed indicated that a considerable amount of information was available to obtain a fairly reliable quantified viscoelastic model for compact bone at the creep end of the spectrum. There was also information at discrete points in the intermediate range of strain rates. However insufficient reliable data was available to obtain a quantified viscoelastic model for compact bone at the impact end of the spectrum of strain rates. It was thus decided that if two quantified viscoelastic models can be defined at the two ends of the spectrum, it is possible to synthesize the two models to obtain a single quantified model representing the total spectrum of strain rates. The ability of the model thus obtained to predict the behaviour of compact bone at intermediate strain rates can be verified by comparing the model predictions with the experimental data already available at discrete points in this range.

Although McElhaney (2, 58) obtained some data on compact bone at the impact end of the spectrum, his data was not suitable for defining quantitatively a reliable viscoelastic impact model for the following reasons:

(a) The data was obtained at only two strain rates (300 sec\(^{-1}\) and 1500 sec\(^{-1}\)) in the impact range. Data at several strain rates in the impact range was required.

(b) The human bone specimens used to cover the entire strain rate range of 10\(^{-3}\) sec\(^{-1}\) to approximately 10\(^{4}\) sec\(^{-1}\) came from a single femur. Thus the number of specimens tested at each strain rate must have been small. A larger sample size is required to obtain reliable data, particularly in view of the large scatter inherent in bone properties.

(c) The human femur used was embalmed, dissecting room material and the post-mortem age was not reported. Data on fresh material is also required.

Based on the above considerations, it was decided to obtain data on fresh human compact bone to enable one to formulate a quantified viscoelastic model for the impact end of the spectrum.

(vi) A great majority of the data reported in the literature for compact bone was obtained from tests on samples taken from a femur and tested parallel to the long axis of the bone. The reasons for this are:
(a) A large fraction of the applied stresses (compressive and bending) acting on long bones in vivo act parallel to the long axis.

(b) A large number of the long bone fractures encountered in practice occurs either in the femur or in the tibia.

(c) It is easier to obtain specimens of sufficiently large size for testing purposes from the femur than any other long bone.

(d) For a given size of specimen, the femur provides a larger number of specimens per bone than other long bones.

Hence, it was decided to conduct the experiments on samples taken from the femur in compression and parallel to the longitudinal axis of the femur.

(vii) The fracture behaviour of compact bone has been considered by Pope, et al (67), Piekarski (37,68,73), Bird, et al (56,57), Gordon (69), Bonfield, et al (70,71) and Takezono, et al (72). Reviews of the fracture behaviour of compact bone may also be found in Refs. 34 and 35.

Pope et al, established the toughness and fracture energy of pre-cracked compact bone. They studied the mode of both transverse and longitudinal fractures.

Piekarski made both optical and scanning electron microscope (SEM) observations of compact bone fractures. He was able to explain these observations on the basis of the theory of composite materials. Besides the fracture behaviour, Piekarski also explained the strength characteristics of compact bone fairly successfully by means of the elementary theory of composite materials.

Bird et al, classified the condition of their specimens after impact testing as survival, micro cracking and shatter. They were the first to introduce a damage classification for impacted specimens.

Gordon employed 'work of fracture' calculations and estimated the critical crack length of living bone to be between 1.0 and 1.5 cms. This length is roughly comparable to the thickness of bones of moderate sized animals and humans. In these cases there is a good chance that after a mishap, a bone is only partially broken and may heal. Gordon commented that this is the equivalent of a crack being found by an inspector in an engineering structure. In both cases remedial action must be taken.

Bonfield, et al show bone to be notch-sensitive. They studied the temperature dependence of fracture in the range from -196°C to 900°C. They also make comparative remarks on the fracture behaviour of bone and ivory.

Takezono, et al, have reported Izod impact data on compact bone.

Bird, et al used a qualitative approach in classifying the microscopic and gross damage caused by impact. As a result, an attempt was made in the present study to quantify microscopic damage caused by impact.
A correlation was obtained between damage and the intensity of the impact as indicated by pulse amplitude, strain rate or particle velocity. This work thus represents an extension of the work done by Bird, et al. A useful outcome was the finding that even very low impact loads caused considerable microscopic damage, thus warning us against the use of a specimen for more than one test.

(viii) In a modern industrial society, high speed travel is so common (and is almost a necessity) that the designer of internal orthopaedic prostheses or implant devices is under an obligation to consider the response of the prosthesis or implant to impact. For those devices that are intended to reside inside the body permanently, the following design objective for impact loads may be useful:

The injury suffered in an accident by an individual with a prosthetic or fixative device inside the body, in the long term, should be no greater than the injury suffered by a normal individual under similar circumstances.

To achieve the above design objective it is necessary to make the load transfer between bone and the device as efficient as possible. Thus for efficient load transfer under impact conditions, it is necessary to make the rate-sensitive (viscoelastic) behaviour of bone and the device compatible. Consequently, a knowledge of the viscoelastic behaviour of bone is essential for the design of internal orthopaedic prosthetic and fixative devices.

It should also be noted that the lack of satisfactory quantified viscoelastic models for compact bone has been pointed out by Kraus (36) and Herrmann, et al (34). The need for such information in the design of implants or artificial substitutes was also stressed by Hirsch, et al (19).

1.2.2 Summary of Objectives for the Research Program

(a) Study the mechanical behaviour of compact bone from human femurs under high rates of compressive strain parallel to the longitudinal axis of the bone.

(b) Investigate the influence of storage fluids on the viscoelastic properties of human compact bone.

(c) Investigate the influence of duration of storage i.e., post-mortem age (PMA), on the viscoelastic properties.

(d) If the viscoelastic properties are sensitive to PMA, then estimate the properties at zero PMA.

(e) Attempt to develop a quantified viscoelastic model for compact bone to represent its behaviour over the entire relevant strain rate spectrum ($10^{-7}$ to $10^3$ sec$^{-1}$).

(f) Attempt quantification of the microscopic damage caused by impact, and correlate the microscopic damage with the intensity of impact as indicated by the pulse amplitude, strain rate or particle velocity.
Study the suitability of various biomedically acceptable materials for internal orthopaedic prosthetic or fixative devices, with their rate-dependent properties in mind.

2. THEORETICAL CONSIDERATIONS AND VISCOELASTICITY

2.1 Introduction

It is well known that many biological (e.g., muscle tissue and bone), geological (e.g., glacial soils) and synthetic materials (e.g., high polymers) exhibit "memory" of their past history of stress and strain. This "memory" property of materials is physically explained by relating it to three other properties of these materials; namely elasticity, viscosity (liquid friction) and plasticity (solid friction).

Materials (biological, geological and synthetic) undergo "aging", caused by either physico-chemical interactions with the environment, or non-equilibrium processes within the material itself, independent of the external loading history. Some examples of aging follow:

(a) Biological tissues undergo both "in vivo aging" and "post-mortem aging". The post-mortem aging of intervertebral disks was considered by Fitzgerald and Freeland (17).

(b) Glacial soils undergo "aging" due to seasonal temperature variations (74).

(c) High polymers undergo "aging" due to the fact that polymerisation and cross-linking take place over a period of time. Aging may also occur due to environmental degradation.

A rigorous phenomenological constitutive equation describing the stress-strain-time behaviour of a material should include both "memory" and "aging". For example, a description of the creep behaviour of some biological materials studied in vitro should include aging.

The complexity of phenomenological descriptions of materials depends to a large extent on the time-scale of the process under consideration. As an example, when creep of ice over geological periods of time is considered, the description would be visco-elasto-plastic. The phenomenological description of water is that of a solid elastic body when the time scale is of the order of 10^-11 secs.

In Sections 2.1.1 and 2.1.2, some simple and some not so simple, linear and non-linear phenomenological descriptions of materials are considered. In Section 2.2, a phenomenological model is proposed for compact bone.

2.1.1 Linear Phenomenological Descriptions

Several excellent references are available on this subject (74 to 81). The present treatment, largely based on Zaretskii (74), is oriented towards Volterra integral equations. The following presentation is in summary form and the interested reader is referred to Appendix A for details.
2.1.1.1 Aging Linear Hereditary (or "Memory") Materials

The Arutyunyan constitutive equation for aging linearly viscoelastic materials is expressed as (74):

\[ \varepsilon(t) = \frac{1}{E_0(t)} \left\{ \sigma(t) + \int_{t_1}^{t} K(t,\tau) \sigma(\tau) \, d\tau \right\} \quad (2.1) \]

where \( K(t,\tau) \) is the "creep kernel" and denotes the creep rate. \( t_1 \) is the age of the material at the instant of loading. Aging materials may be classified into coupled and decoupled types based on whether or not the aging process is affected by the loading history. Most metals used in internal orthopaedic applications undergo coupled aging while concrete and some soils can be approximated as decoupled aging materials. For decoupled aging, the creep kernel can be expressed in the following form:

\[ K(t,\tau) = \phi(\tau)K_1(t-\tau) \quad (2.2) \]

where, \( \phi(\tau) \) is the "aging function".

For coupled aging, the kernel is of the following form:

\[ K(t,\tau) = K(\sigma; t, \tau) \quad (2.2a) \]

2.1.1.2 Non-Aging Linear Memory Materials

Integral Representation of the Constitutive Equation:

The Boltzmann integral equation (derived in 1875) using a simple superposition principle is,

\[ \varepsilon(t) = \frac{1}{E_0} \left\{ \sigma(t) + \int_{0}^{t} K(t-\tau)\sigma(\tau) \, d\tau \right\} \quad (2.3) \]

Equation (2.3) can also be represented in the following equivalent form:

\[ \sigma(t) = E_0 \left\{ \varepsilon(t) - \int_{0}^{t} R(t-\tau) \varepsilon(\tau) \, d\tau \right\} \quad (2.4) \]

where \( R(t-\tau) \) is the "relaxation kernel" or the "resolvent of the creep kernel". The constitutive equations (Eqs. (2.1), (2.3) and (2.4)) presented above are Volterra integral equations of the second kind.

Differential Representation of the Constitutive Equation:

A linear phenomenological description of materials can also be given in a differential form (75):

\[ P_\sigma = Q\varepsilon \quad (2.5) \]
where, \( P \) and \( Q \) are linear differential operators (see Appendix A). \( \sigma \) and \( \varepsilon \) can be appropriately interpreted for both uniaxial and multiaxial loading (Appendix A).

**Mechanical Model Representation of the Constitutive Equation:**

A number of familiar viscoelastic models obtained by combinations in series, parallel and series-parallel of two basic elements are shown in Fig. 3.

**Summary:** Three different approaches to a phenomenological description of linearly viscoelastic materials were presented above. It is shown in Appendix A that the differential and mechanical model representations are derivable from the integral forms. Hence, a phenomenological description using Volterra integral equations of the second kind forms the most general of the approaches presented.

**2.1.1.3 Alternative Integral Representations for Linear Hereditary Materials**

An alternative form for Eq. (2.3) can be written as,

\[
\varepsilon(t) = \int_0^t J(t-\tau) \frac{d\sigma}{d\tau}(\tau) \, d\tau
\]  

(2.6)

Similarly, an alternative form for Eq. (2.4) is,

\[
\sigma(t) = \int_0^t G(t-\tau) \frac{d\varepsilon}{d\tau}(\tau) \, d\tau
\]  

(2.7)

where \( J(t) \) is the "creep compliance" and \( G(t) \) is the "relaxation modulus". It can be shown that

\[
K(t) = \frac{dJ}{dt}(t)
\]  

(2.8)

\[
R(t) = -\frac{dG}{dt}(t)
\]  

(2.9)

**2.1.1.4 Interrelations Between Creep, Stress Relaxation, and Vibration Tests**

As pointed out in Appendix A, for linearly viscoelastic materials, interrelations exist among the following viscoelastic functions: (a) creep compliance, (b) relaxation modulus, (c) retardation spectrum, (d) relaxation spectrum, (e) components of the complex compliance and (f) components of the complex modulus. Exact and approximate interrelations among these functions are given by Ferry (78).

**2.1.2 Non-Linear Phenomenological Descriptions**

In general, nonlinear descriptions are obtained by simply generalizing the corresponding linear descriptions. This is done by replacing, \( \sigma \) by a function of stress, \( f(\sigma) \), which is nonlinear, or \( \varepsilon \) by \( g(\varepsilon) \) which is nonlinear. A detailed consideration of the subject may be found in Appendix B. The presentation below
is a condensed version.

2.1.2.1 Integral Representations

**Aging Materials:**

Equation (2.1) is generalized as follows:

\[ \varepsilon(t) = \frac{1}{E(t)} \left\{ f[\sigma(t)] + \int_{\tau_1}^{t} K(t, \tau)f[\sigma(\tau)] \, d\tau \right\} \]  

(2.10)

**Non-Aging Materials:**

Equation (2.3) is generalized as follows:

\[ \varepsilon(t) = \frac{1}{E_0} \left\{ f[\sigma(t)] + \int_{0}^{t} K(t-\tau)f[\sigma(\tau)] \, d\tau \right\} \]  

(2.11)

Ward and Onat (82) gave a nonlinear phenomenological description of the following form:

\[ \varepsilon(t) = J_1(t-\tau) \frac{d\sigma}{d\tau} (\tau_1) \, d\tau_1 + \int_{-\infty}^{t} \int_{-\infty}^{t} J_2(t-\tau_1, t-\tau_2) \frac{d\sigma}{d\tau_1} (\tau_1) \frac{d\sigma}{d\tau_2} (\tau_2) \, d\tau_1 \, d\tau_2 \]

\[ + \int_{-\infty}^{t} \int_{-\infty}^{t} \cdots \int_{-\infty}^{t} J_N(t-\tau_1, \ldots, t-\tau_N) \frac{d\sigma}{d\tau_1} (\tau_1) \cdots \frac{d\sigma}{d\tau_N} (\tau_N) \, d\tau_1 \cdots d\tau_N \]

(2.12)

The first term here represents the independent contribution of the loading \( d\sigma(\tau_1) \) to the final elongation. The second term represents the joint contribution of \( d\sigma(\tau_1) \) and \( d\sigma(\tau_2) \) to the final elongation.

Another form of the integral representation may be given as follows (74):

\[ \varepsilon(t) = f_0 [\sigma(t)] + \int_{0}^{t} K(\sigma, t, \tau)f[\sigma(\tau)] \, d\tau \]

(2.13)

2.1.2.2 Mechanical Model Representations

The *Eyring Model* (83): This model is shown in Fig. 4. Eyring used a dashpot whose constitutive equation is:
\[ \dot{\varepsilon} = K \sinh(\alpha \sigma) \; ; \; \alpha = \text{const}, \; K = \text{const.} \quad (2.14) \]

Its relevance to relaxation processes in metals is discussed in Appendix B.

Mechanical Models Incorporating a Simple Second Degree Dashpot:

Some mechanical models of this type are shown in Fig. 4. These models are useful in describing those materials that exhibit higher viscous resistance at higher deformations. The author's experimental observations suggest that bone is such a material. Compressive impact tests on soils (84) indicate that for some soils, the attenuation of the impact strain (or stress) is greater at greater values of the impact strain (or stress). These models may also be found useful for describing such soils.

2.1.2.3 Differential Representations

Equation (2.5) may be generalized as follows:

\[ P' \sigma = Q' \varepsilon \quad (2.15) \]

where,

\[ P' = f_n(\sigma, \varepsilon) + f_{n-1}(\sigma, \varepsilon)D + f_{n-2}(\sigma, \varepsilon)D^2 + \ldots + D^n \quad (2.16) \]

\[ Q' = g_m(\sigma, \varepsilon) + g_{m-1}(\sigma, \varepsilon)D + \ldots + D^m \quad (2.17) \]

and

\[ D = \frac{d}{dt} \quad (2.18) \]

2.2 Proposed Phenomenological Descriptions for Compact Bone

It is well known that compact bone is inhomogeneous and non-isotropic (86). However, a comparison of the mechanical properties of single osteons with macroscopic samples of compact bone shows a close correspondence (87 to 89). Hence, the osteon may be considered as the basic mechanical unit of bone. In a transverse section of a human long bone, microscopic observation of the cortex reveals that roughly \(10^2\) osteons intersect a circular area of 0.25 inches diameter. This fact permits one to assume that bone is transversely homogeneous on a macroscopic level. It is also known that bone is transversely isotropic with respect to its quasi-static mechanical properties (86). The mechanical behaviour of a transversely isotropic and transversely homogeneous elastic solid at small strains is definable by five elastic constants (82). Thus, the elastic behaviour of compact bone can be described by the following constitutive equation: (the longitudinal axis or the direction parallel to the long axis of the bone is denoted by subscript '3', and the other two orthogonal axes lying in the transverse section of bone are denoted by subscripts '1' and '2'):

\[ [\varepsilon] = [S][\sigma] \quad (2.19) \]

where

\[ [\varepsilon] = \begin{bmatrix} \varepsilon_{11} \\ \varepsilon_{22} \\ \varepsilon_{33} \\ \varepsilon_{12} \\ \varepsilon_{23} \\ \varepsilon_{13} \end{bmatrix} = \text{strain vector} \quad (2.20) \]
\[
[s] = \begin{bmatrix}
S_{11} & S_{12} & S_{13} & 0 & 0 & 0 \\
S_{12} & S_{11} & S_{13} & 0 & 0 & 0 \\
S_{13} & S_{11} & S_{33} & 0 & 0 & 0 \\
0 & 0 & 0 & S_{44} & 0 & 0 \\
0 & 0 & 0 & 0 & S_{44} & 0 \\
0 & 0 & 0 & 0 & 0 & 2(S_{11} - S_{12})
\end{bmatrix} \tag{2.21}
\]

= compliance matrix

\[
[\sigma] = \begin{bmatrix}
\sigma_1 \\
\sigma_2 \\
\sigma_3 \\
0 \\
0 \\
0
\end{bmatrix} = \text{stress vector} \tag{2.22}
\]

The compliance coefficients \( S_{ij} \) are related to the familiar elastic moduli as follows (82):

Longitudinal Young's modulus = \( E_1 = 1/S_{33} \) \tag{2.23}

Transverse Young's modulus = \( E_T = 1/S_{11} \) \tag{2.24}

Shear modulus for torsion about the long axis, \( G = 1/S_{44} \) \tag{2.25}

It may be noted that \( S_{33} \) for compact bone is different in compression and tension (69,73). It is known that compact bone is viscoelastic or exhibits time-dependent behaviour (47 to 58). The author's own experimental observations suggest that its viscoelasticity under compressive impact can be characterized as nonlinear. Hence, a more complete characterization of compact bone would include nonlinear viscoelasticity in its constitutive equation. Borrowing some ideas from Biot (85), this may be done by making the compliance coefficients in the matrix of Eq. (2.21) time-dependent. Depending on the accuracy and nature of the available experimental data, any degree of complexity may be introduced into the time-dependent characterization of the compliance coefficients of Eq. (2.21), using the methods outlined in Sections 2.1.1 and 2.1.2.

It was observed that the experimental results reported in the literature up to the end of 1972, would permit characterization of compact bone as viscoelastic only in the longitudinal direction (subscript '3'). The relevant part of Eq. (2.19), in this direction is,

\[ \epsilon_3 = S_{13} (\sigma_1 + \sigma_2) + S_{33} \sigma_3 \] \tag{2.26}

Further, most of the tests were conducted under uniaxial stress. Thus, \( \sigma_1 + \sigma_2 = 0 \). Hence,
Using Eq. (2.23), and dropping subscripts one obtains

\[ \varepsilon(t) = \frac{1}{E(t)} \sigma(t) \]  

(2.28)

2.2.1 An Integral Representation for Compact Bone

If it is desired to characterize compact bone as an aging nonlinear viscoelastic material, Eq. (2.10) may be used. A comparison of Eqs. (2.10) and (2.28) yields,

\[ \frac{1}{E(t)} = \frac{1}{E_0(t)} \left\{ \int_{t_1}^{t} K(t,\tau) d\tau \right\} \]

(2.29)

A phenomenological description using Eq. (2.29) is mathematically sophisticated, but there are other considerations discussed in Section 2.3 which make it desirable to consider mechanical models for a description of the viscoelastic behavior of compact bone.

2.2.2 Mechanical Model Representation for Compact Bone

A most general mechanical model representation for compact bone is suggested in Fig. 5(a). It includes both viscoplasticity and viscoelasticity. Viscoplasticity is represented by a modified St. Venant body and a damper connected in parallel. The constitutive equation for the modified St. Venant body is shown in Fig. 6. Thus, the viscoplastic portion of the model includes work-hardening.

In the present thesis, the author is concerned only with the viscoelastic behavior of compact bone. For this case, the general model reduces to that shown in Fig. 5(b). Thus, the viscoelastic behavior is represented by 'n' Kelvin-Voigt elements connected in series. The first element is a Kelvin-Voigt element modified by the introduction of a simple second degree nonlinear damper in lieu of the Newtonian damper.

The mechanical behavior exhibited by this model includes:

(i) Instantaneous elasticity (by making the retardation time of one of the elements small in comparison to the quasi-static loading time)

(ii) Delayed elasticity
   (a) creep (elastic fore-effect or strain retardation)
   (b) creep recovery (elastic after-effect)
   (c) stress relaxation

The mechanical or stress-strain behavior of this model can be represented by the following system of equations (see Appendix C for a complete derivation):
\[
\varepsilon = \sigma \left( \frac{1}{E} + \sum_{i=2}^{n} \frac{1}{E_i} \right) - \left( \frac{\gamma_{\varepsilon}}{E} \dot{\varepsilon}_1 + \sum_{i=2}^{n} \frac{\tau_i}{E} \dot{\varepsilon}_i \right) \tag{2.30}
\]

where

\[
\varepsilon_1 = \sigma - \frac{\gamma_{\varepsilon}}{E} \dot{\varepsilon}_1 \tag{2.31}
\]

\[
\varepsilon_i = \frac{\sigma}{E_i} - \frac{\tau_i}{E_i} \dot{\varepsilon}_i, \quad i = 2 \text{ to } n \tag{2.32}
\]

and

\[
\tau_i = \frac{\eta_i}{E_i}, \quad i = 2 \text{ to } n \tag{2.33}
\]

2.3 Ultra-Phenomenological Considerations

(i) Compact bone is known to be a composite material (35,73). Piekarski (73) was fairly successful in explaining some of the quasi-static mechanical properties of compact bone in terms of the theory of composite materials. The theory of composite materials attempts to derive analytically the properties of the composite in terms of the properties of its constituents. It is desirable to correlate the properties of the constituents with the properties of the composite, even for types of loading other than quasi-static. The author suggests that a mechanical model representation of the viscoelastic properties (say, by the model of Fig. 5) is more likely to lead to an explanation of the properties of compact bone in terms of the properties of its constituents than an integral representation (say, of the form of Eq. (2.29)).

(ii) If the viscoelastic properties of compact bone can be represented fairly accurately by a mechanical model, it may become possible to identify certain pathological conditions in bone with a change in the properties of some specific elements of the model. For example, it may become possible to identify osteoporosis with an increase in the stiffness of some of the springs in Fig. 5(b) or with a change in the damping of some of the dampers or both. Similarly, it may be possible to identify osteomalacia with a reduction in the stiffness of some of the springs of Fig. 5(b) with possible changes also in the dashpots.

(iii) If certain pathological conditions of bone are identifiable with specific changes in the properties of some of the elements in Fig. 5(b), then it is possible to obtain clues to the method of treatment. Treatment may be aimed at bringing about appropriate specific changes in the constituents that would reverse the changes which brought about the pathological condition in the first place.

(iv) Experimental observations are subject to experimental error. Similarly, analytical representation of experimental data has curve-fitting errors associated with it. The curve-fitting errors can

* see Appendix D.
normally be minimized by an increase in the complexity of the analytical representation. It does not pay to increase the analytical complexity very much beyond the point where the curve-fitting error is of the same order of magnitude as the experimental error. The mechanical model in Fig. 5(b) is capable of giving a phenomenological description within experimental accuracy for manageable values of 'n'.

The above considerations were responsible for choosing the mechanical model in Fig. 5(b) for a phenomenological description of compact bone.

3. EXPERIMENTAL TECHNIQUE

The split-Hopkinson-bar method is used in this investigation to study the viscoelastic properties of compact bone under compressive impact.

Experimental studies of the rate sensitivity of materials under impact can be said to have begun with John Hopkinson (76). John Hopkinson published his results in 1872. His son Bertram Hopkinson continued the work of his father. The results of the Hopkinsons showed conclusively that the tensile strength of steel wires increases with increasing loading rate.

The theory of propagation of elastic waves through solid media was extensively developed before the year 1900 (90). Experimental investigations in the laboratory lagged behind theoretical developments for lack of suitable instrumentation. As late as 1947, investigators were struggling with very inefficient measuring devices. Davies (91), in a paper presented to the Royal Society of London in 1947, gave an excellent analysis of the Hopkinson pressure bar used for studying the impact response of materials. Davies pointed out the various sources of error in this method (dispersion of waves, errors due to imperfections in axial symmetry, non-uniformity of stress distribution, friction at the interfaces etc) and also discussed methods of minimizing these errors.

Developments in electrical resistance strain-gages with good dynamic response characteristics eliminated several of the problems faced by investigators in the earlier part of the century and made measurements of dynamic stresses and strains a matter of ordinary laboratory routine. The Hopkinson-bar apparatus has been used in recent years to study the impact properties of a wide variety of materials including fibre-reinforced composites, rocks, biological materials, metals and plastics.

In Section 3.1, the split-Hopkinson-bar apparatus is described. In Section 3.2, the theory of the apparatus is considered and some of the important equations are given. Subsequently, Section 3.3 presents a discussion of the method of analysis of the experimental data and Section 3.4 critically evaluates the assumptions associated with the experimental method.

3.1 Description of the Apparatus

The split-Hopkinson-bar apparatus used in this investigation is shown in Fig. 7. The design of this apparatus is based on the configuration described by Maiden and Green (92). This apparatus is designed to study the stress-strain behaviour of materials at high strain rates (corresponding to the impact regime shown in Fig. 2). The apparatus consists of two slender steel bars, known as the striker bar and the anvil bar, each 15 inches long and 1/4 inch in diameter. These bars are mounted on bearings which permit axial motion with negligible friction. The specimen is a circular cylinder 3/8 inch in length and 1/4 inch in diameter (with a L/D ratio of 1.5). The specimen is sandwiched between the
striker bar and the anvil bar after smearing a bit of silicone lubricant on the flat faces of the specimen. The lubricant minimizes radial friction between the specimen and the bars and also ensures contact between the specimen and bar surfaces, thereby permitting good wave transmission through the interface. Nitrogen gas from a high-pressure reservoir is used to drive a cylindrical projectile down the barrel of a gas gun. The projectile impacts against the striker bar thereby generating a one-dimensional step-function strain wave which propagates along the length of the bar. Electrical resistance foil gages are bonded to the surfaces of the striker and anvil bars very close to the specimen-bar interfaces. These gages measure the strains at the two specimen-bar interfaces. The strain signals from these gages are converted to voltage signals through an appropriate bridge circuit and displayed on a dual-trace oscilloscope. The camera attachment to the oscilloscope permits photographic recording of the strain-time history of the two gages during impact tests. A typical set of strain gage signals obtained in testing a bone specimen are shown in Fig. 8. In order to check the alignment of the striker and anvil bars and the strain-gage system, tests were conducted with the two bars in direct contact, without a specimen sandwiched between them. Typical results obtained are shown in Fig. 9. It is evident that the stress pulse is perfectly transmitted from the striker bar to the anvil bar, thereby confirming the checks of the system made by other means. It was necessary to calibrate the firing pressure of the gas gun against the maximum stress produced in the bone specimens in order to maintain good control over the stresses generated in the specimens during impact. Such a calibration curve is shown in Fig. 10. The oscilloscope trace for a grossly damaged specimen differs significantly from the trace of an intact specimen. This is shown in Fig. 11. Reports on the Hopkinson-bar from other users may be found in Refs. 93 to 99.

3.2 Theory of the Split-Hopkinson-Bar

In the test, a one-dimensional elastic strain pulse of magnitude $\varepsilon_s$ (see Fig. 8) is propagated along the input (striker) bar. The incident pulse ($\varepsilon_I$) is partly reflected ($\varepsilon_R$) and partly transmitted when it reaches the first (bar-specimen) interface, the fractions reflected and transmitted being determined by the relative acoustic impedance of the two materials. When the transmitted part of the pulse reaches the second (specimen-bar) interface, again part of the pulse ($\varepsilon_I$) is transmitted through the output (anvil) bar and part of the pulse is reflected. The duration of the pulse is made long compared to the time required for the wave to traverse the length of the specimen, so that a number of reflections occurs in the specimen within a fraction of the duration of the pulse. This permits the establishment of an almost quasi-static loading situation in the specimen. The strain-gage on the input-bar measures ($\varepsilon_I + \varepsilon_R$) and the strain-gage on the output bar measures $\varepsilon_o$. The apparatus is operated within the elastic region of the stress-strain curves for the input and output bars.

At time $\tau$, the displacements $u_1$ and $u_2$ at the first and second interfaces of the specimen are given by,

$$ u_1 = c \int_0^\tau (\varepsilon_I - \varepsilon_R) \, dt \quad (3.1) $$

$$ u_2 = c \int_0^\tau \varepsilon_o \, dt \quad (3.2) $$
where, \( c \) = one-dimensional elastic wave velocity through the bars.

The interface velocities \( v_1 \) and \( v_2 \) are given by

\[
v_1 = c (\varepsilon_I - \varepsilon_R)
\]

\[
v_2 = c \varepsilon_o
\]

If the initial length of the specimen is \( L \), the average strain and strain-rate in the specimen at time \( T \) can be evaluated from the following equations:

\[
\varepsilon = \frac{u_1 - u_2}{L} = \frac{c}{L} \int_0^T (\varepsilon_I - \varepsilon_R - \varepsilon_o) \, dt
\]

\[
\dot{\varepsilon} = \frac{v_1 - v_2}{L} = \frac{c}{L} (\varepsilon_I - \varepsilon_R - \varepsilon_o)
\]

The stresses \( \sigma_1 \) and \( \sigma_2 \) at the two interfaces can be calculated as,

\[
\sigma_1 = E (\varepsilon_I + \varepsilon_R)
\]

\[
\sigma_2 = E \varepsilon_o
\]

and the average stress \( (\bar{\sigma}) \) in the specimen is given by,

\[
\bar{\sigma} = \frac{\sigma_1 + \sigma_2}{2} = \frac{E}{2}(\varepsilon_I + \varepsilon_R + \varepsilon_o)
\]

3.3 Analysis of the Oscilloscope Traces

The oscilloscope traces, shown in Fig. 8, were digitized using a PDP-8 computer and a writing tablet. The digitized data was subsequently fed into a computer program, given in Appendix E. The program was written in Fortran IV for use on the IBM 1130 and was consequently longer than what would be required if it were written for use on a larger computer like the IBM 360.

The program takes the values of \( \varepsilon_I, (\varepsilon_I + \varepsilon_R) \) and \( \varepsilon_o \) as input data and makes use of Eqs. (3.5), (3.6) and (3.9) to evaluate \( \varepsilon, \dot{\varepsilon} \) and \( \bar{\sigma} \). It further evaluates \( \dot{\sigma} \) using a numerical differentiation procedure. At this stage, there is an output of values of \( \sigma, \varepsilon, \dot{\varepsilon} \) and \( \bar{\sigma} \) as a function of time. The second stage of the program performs a linear regression analysis of isochronal values of \( \sigma \) and \( \varepsilon \) to evaluate the modulus \( E \).

3.4 Assumptions in the Use of the Split-Hopkinson-Bar Method for Testing Bone

Bell (100) has criticized some of the assumptions of the Split-Hopkinson bar method. He confined his criticisms mainly to studies of large deformations.
in metals. It has been verified by an experiment (described in the next para-
graph) that Bell's criticisms do not apply to the results reported in this
study. Even for large deformations in metals, the validity of Bell's criti-
cisms is a matter of controversy (101).

It is assumed in the analysis of the Hopkinson bar experiment that the
stresses in the specimen are uniform, that the loading is quasi-static and that
effects of longitudinal inertia and radial motion (giving rise to contact sur-
face frictional forces) are negligible. To determine the conditions under which
these assumptions are valid, epoxy specimens (whose compressive modulus is of
the same order of magnitude as that of wet compact bone) containing an incapsulated
foil strain gage (1/8 inch in length) were tested in the split-Hopkinson bar
(102). Measurements from the incapsulated gage were compared with the analysis
of signals from the Hopkinson bar (Fig. 12). It was found that the above assump-
tions are valid if (a) the specimen L/D ratio is greater than unity, and (b)
the contact surfaces are coated with a lubricant. Hence, all the specimens used
in this study had a L/D ratio of 1.5 and the contact surfaces were carefully
lubricated before the test.

With respect to compact bone, it is assumed that the material is trans-
versely homogeneous and transversely isotropic (103). It has already been
shown in Section 2.2 that these two assumptions are valid at the macroscopic
level. Longitudinal waves propagate in a pure mode along the symmetry axis
of a transversely isotropic material (104). Hence, the assumption of a pure
longitudinal mode of wave propagation which is necessary for the Hopkinson bar
analysis is valid.

4. EXPERIMENTAL MATERIAL

The experimental material consisted of cortical specimens obtained from
human and bovine femurs. The reasons for using femoral specimens are:

(i) A great majority of the data reported in the literature for
compact bone was obtained from tests on samples taken from
the femur. To obtain a viscoelastic model capable of de-
scribing the behaviour of compact bone over a wide range
of strain-rates, experimental data from different types of
tests (creep, impact, vibration, stress relaxation, etc)
obtained by various researchers needed to be synthesized.
Such data was available only for femoral samples.

(ii) Femoral fractures form a significant fraction of those
clinically encountered. Hence a fundamental understanding
of the viscoelastic behaviour of femoral compact bone is
important.

(iii) It is easier to obtain specimens of sufficiently large size
for testing purposes from the femur than any other long bone.

(iv) For a given size of specimen, the femur gives a larger
number of specimens per bone than other long bones.

4.1 Human Specimens

Human specimens were obtained from patients who did not have any bone
diseases. Healthy femurs were removed from the body at autopsy and stored in a preservative until machining. The specimens were machined within 24 hours after removal from the body, and in two cases, within 2 hours after removal from the body. The machined specimens were placed in a storage fluid until testing. The testing was carried out at various intervals of time after the death of the patient (i.e., at various post-mortem ages or PMA), to evaluate the influence of this variable. The age and sex of the patients from whom femurs were obtained were noted to see if any significant trends could be attributed to these parameters. These specimens were obtained from the middle third of the shaft of the femur and no attempt was made to identify the specimens as belonging to the anterior, posterior, medial, or lateral quadrants. Evans and Lebow (105) found that there are no significant differences between the modulus of elasticity of the various quadrants, though the modulus varied from \(1.96 \times 10^6\) psi in the proximal third of the shaft to \(2.11 \times 10^6\) psi in the middle third of the shaft.

A total of 120 specimens from 8 femurs of 7 patients (4 male and 3 female) was tested at various PMA and strain-rates ranging from 50 sec\(^{-1}\) to 1000 sec\(^{-1}\). The statistics on the age and sex of the patients, cause of death and other test variables are listed in Table 1. Each specimen was tested only once and then prepared for microscopic examination.

4.2 Bovine Specimens

The initial work done on bovine femurs was reported in the paper by Tennyson, et al (42). These tests were conducted in a range of PMA from 1 to 240 days with strain-rates varying from 10 to 450 sec\(^{-1}\). A total of 43 specimens was tested and the specimens were obtained from femurs belonging to several animals. The study was exploratory at this stage and all the significant experimental variables were not known. The individual femurs were not identified and thus specimens could not be traced back to the femur of their origin. Each specimen was tested several times until complete fragmentation prevented further use. Later, a detailed analysis of experimental results and microscopic examination revealed that results obtained from specimens that have been subjected to previous compressive impact are unreliable (see Section 5.1.5).

In view of the above information, additional tests were conducted to obtain data free of the above mentioned errors. Two bovine femurs were obtained from an 8-year-old animal half an hour after its death. A total of 11 specimens was tested. The specimens were taken from the middle third of the shaft of the femur. Location of the specimen with respect to the various quadrants was ignored for reasons indicated in Section 4.1. The specimens were machined within two hours after the death of the animal and each specimen was tested only once.

4.3 Specimen Preparation

The method of specimen preparation was the same for human and bovine samples. The shaft of the femur was initially sectioned into proximal, middle and distal thirds using a band saw. The proximal and distal thirds were disposed of. The middle third was then cut into quadrants using the band saw. Subsequently, the quadrants were cut to suitable sizes with a jeweller's hack-saw. The jeweller's hack-saw has a very fine blade and causes negligible damage to the bone. These operations produce blocks of cortical
bone roughly the shape of a rectangular parallelepiped. These blocks were then turned on a lathe to produce circularly cylindrical specimens of 0.25" diameter and 0.375" length. The turning operation was carried out under a constant jet of physiological saline with the flat faces of the specimen given a very good polish to ensure proper contact between the faces of the specimen and the faces of the striker and anvil bars when the specimen is sandwiched between them. The sawing and turning operations were carried out with the minimum possible damage to the specimen. Microscopic examination of both longitudinal and transverse sections of a machined and an uncut specimen (obtained from bovine compact bone) revealed no differences attributable to machining.

All specimens were machined with their length parallel to the long axis of the bone. The reasons for this were discussed in Section 1.2.1.

4.4 Specimen Storage

(a) Human Specimens:

Two storage fluids were used for the human specimens to examine their influence on the compressive impact behaviour of compact bone.

(i) Storage Fluid No.1: 8.6 gms of NaCl, 0.3 gms of KCl, 0.33 gms of CaCl₂ in a litre of a neutral solution of 4% formaldehyde.

This fluid will be referred to as "formalin" for convenience of reference. The storage was at room temperature.

(ii) Storage Fluid No.2: This is the so-called "physiological saline", or "normal saline" containing 9 gms of NaCl per litre of distilled water. The storage was at room temperature.

(b) Bovine Specimens:

The bovine specimens were stored in normal saline at room temperature.

4.5 Test Conditions

Testing was completed within three minutes after removal from the storage fluid. Loss of fluid by evaporation in this period was considered negligible. All the tests were conducted at laboratory temperature and relative humidity which were in the range of 70°F - 76°F and 15% - 30%, respectively.

5. EXPERIMENTAL COMPRESSIVE IMPACT RESULTS

5.1 Human Specimens

Table 1 provides some statistics on the specimens used in these tests. The experimental results are shown in Figs. 13 to 22. The following sections discuss the method of data reduction and the results obtained.

5.1.1 Stress vs. Strain at Constant Strain Rate

The Hopkinson-bar analysis described in Section 3.2 is applicable only after quasi-static conditions are established in the specimen. Quasi-static conditions
are established after several reflections of the stress-wave occur within the specimen. It takes a time \( \tau_0 \) (approximately 20 \( \mu \)secs for the tests reported here) after the wave-front reaches the input-bar-specimen interface, for quasi-static conditions to exist. Therefore, Eq. (3.5) needs to be modified as follows:

\[
\varepsilon(\tau > \tau_0) = \varepsilon(\tau = \tau_0) + \frac{c}{f} \int_{\tau_0}^{\tau} (\varepsilon_I - \varepsilon_R - \varepsilon_o) \, dt
\]

(5.1)

This can be expressed as,

\[
\varepsilon(\tau > \tau_0) = \varepsilon(\tau = \tau_0) + \varepsilon'
\]

(5.2)

where,

\[
\varepsilon' = \frac{c}{f} \int_{\tau_0}^{\tau} (\varepsilon_I - \varepsilon_R - \varepsilon_o) \, dt
\]

(5.3)

The set of equations relevant for the Hopkinson bar analysis are Eqs. (5.2), (5.3), (3.6) and (3.9). All these equations are valid only for \( \tau \geq \tau_0 \).

The input to the computer program discussed in Section 3.3 consists of data points corresponding to \( \tau \geq \tau_0 \). The program computes \( \varepsilon' \) by numerical integration using Eq. (5.3). It then performs a linear regression between \( \sigma \) and \( \varepsilon' \) to evaluate \( \varepsilon(\tau = \tau_0) \). The program subsequently computes \( \varepsilon \) using Eq. (5.2). A graphical method of computing \( \varepsilon(\tau = \tau_0) \) is illustrated in the next paragraph to clarify the procedures utilized by the computer.

Graphs of \( \sigma \) vs. \( \varepsilon' \) at constant \( \dot{\varepsilon} \) are shown in Figs. 13.1, 13.2 and 13.3. This data comes from tests conducted at a PMA of 17 days on compact bone specimens from a 43 year old male stored in formalin. A straight line drawn through the experimental points (the regression line) intersects the X-axis at a point 'A'. If the origin of the \( \varepsilon' \)-scale is denoted by 'B', the distance 'AB' gives the value of \( \varepsilon(\tau = \tau_0) \). The \( \sigma \) vs. \( \varepsilon' \) graphs can also be converted to \( \sigma \) vs. \( \varepsilon \) graphs by simply shifting the origin from B to A. The \( \sigma \) vs. \( \varepsilon \) graphs thus obtained from Figs. 13.1, 13.2 and 13.3 are shown in Fig. 13.4. The computer program given in Appendix E is designed to perform these operations analytically and evaluate the slopes \( \dot{\varepsilon} \) of the \( \sigma \) vs. \( \varepsilon \) graphs at constant strain-rate \( \dot{\varepsilon} \).

5.1.2 Modulus \( \bar{E} \) vs. Strain Rate \( \dot{\varepsilon} \) at Constant PMA

Graphs of \( \bar{E} \) vs. \( \dot{\varepsilon} \) are shown in Figs. 14.1 and 14.2 for specimens from a 43 year old male at two different values of the post-mortem age (PMA = 17 days and PMA = 9 days). The experimental data is consistent with a linear relation between \( \bar{E} \) and \( \dot{\varepsilon} \) of the following form:

\[
\bar{E} = E + \eta \dot{\varepsilon}
\]

(5.4)

The graphs of Fig. 14.1 and 14.2 are superimposed in Fig. 14.3. It is seen that there is a small increase in \( E \) (from 0.75 x 10^6 psi to 0.85 x 10^6 psi) and a decrease in \( \eta \) (from 833 psi-sec to 583 psi-sec) as the PMA increased from 9 days to 17 days. This trend in the decrease of \( \eta \) with increase in PMA becomes
more pronounced at small values of PMA as will be seen later.

Figures 15.1, 15.2 and 15.3 show $\bar{E}$ vs. $\dot{\varepsilon}$ graphs for specimens from a 52-year-old female at a PMA of 13 days, 7 days and 1 day, respectively. It is seen that the rate-sensitivity (as denoted by $\eta$) is quite high when the specimens are fresh ($\text{PMA} = 1 \text{ day}$).

Figures 16.1, 16.2 and 16.3 show $\bar{E}$ vs. $\dot{\varepsilon}$ graphs for specimens from a 85-year-old female at a PMA of 21 days, 13 days and 6 days, respectively.

Figure 17 shows a $\bar{E}$ vs. $\dot{\varepsilon}$ graph for specimens from a 76-year-old male at a PMA of 52 days.

The graphs discussed thus far (shown in Figs. 13 to 17) are for specimens stored in "formalin". The graphs for specimens stored in saline are shown in Figs. 18 to 20.

Figures 18.1, 18.2 and 18.3 contain $\bar{E}$ vs. $\dot{\varepsilon}$ graphs for specimens from a 62-year-old female, stored in saline, and tested at a PMA of 19 days, 9 days and 1 day respectively. It is again noted that the rate-sensitivity (as denoted by $\eta$) decreases with increasing PMA. Thus, the changes in $\eta$ with changes in PMA show the same trend in both formalin and saline.

Figures 19.1, 19.2 and 19.3 show $\bar{E}$ vs. $\dot{\varepsilon}$ graphs for specimens from a 47-year-old male at a PMA of 21 days, 11 days and 5 days, respectively.

Figures 20.1, 20.2, 20.3, 20.4, and 20.5, provide $\bar{E}$ vs. $\dot{\varepsilon}$ graphs for specimens from a 52-year-old male at a PMA of 26 days, 26 days, 18 days, 16 days, and 8 days, respectively.

5.1.3 Stiffness $'E'$ and Damping Coefficient $'\eta'$ As a Function of PMA

It was seen from Figs. 13 to 20 that the experimental data on human compact bone specimens under compressive impact at various PMA (with storage in two different fluids) was consistent with a linear relationship of the form of Eq. (5.4) between the modulus $\bar{E}$ and the strain rate $\dot{\varepsilon}$. The stiffness $'E'$ and damping coefficient $'\eta'$, which are the constant terms in Eq. (5.4), are plotted as a function of PMA in Figs. 21 and 22, respectively.

5.1.3.1 Influence of Duration of Storage (PMA) on $E$

5.1.3.1.1 Storage in "Formalin"

It is seen from Fig. 21 that the value of $E$ lies in general between $0.5 \times 10^6$ psi and $1.2 \times 10^6$ psi. Though there appears to be a very small increase in $E$ with an increase in PMA, it is not considered significant. With the relatively small number of femurs tested, no significant trends are discernible with respect to age and sex.

5.1.3.1.2 Storage in Saline

Even the very small increase in $E$ with increasing PMA seen in formalin is not seen in saline. Here again, with the small number of femurs tested, no significant trends with respect to age or sex are evident.
5.1.3.2 Influence of Duration of Storage (PMA) on \( \eta \)

5.1.3.2.1 Storage in "Formalin"

It is seen from Fig. 22 that the value of the damping coefficient \( \eta \) decreased with increasing PMA for specimens (from the 43-year-old male, 85-year-old female and 52-year-old female) stored in formalin. Specimens from the 85-year-old female had a lower value of \( \eta \) than specimens from the 43-year-old male and 52-year-old female. The observations are consistent with the proposition "p" that the damping coefficients of specimens from the older donors are smaller than the damping coefficients of specimens from the middle-aged. The above proposition is not meant to be a categorical imperative, since our sample size is small. To say the least, the difference in the values of \( \eta \) between the 43-year-old male and 52-year-old female are very small. This small difference could be due to age, sex or error. Hence, no statement can be made on the influence of sex on \( \eta \) with the available data.

5.1.3.2.2 Storage in Saline

It is again seen from Fig. 22 that the value of \( \eta \) decreased with increasing PMA for specimens (from the 52-year-old male, 47-year-old male, and 62-year-old female) stored in saline. The values of \( \eta \) for the 62-year-old female are somewhat lower than the values for the 52-year-old male and the 47-year-old male. This observation is again consistent with proposition "p" of Section 5.1.3.2.1. The data is again insufficient to make a statement on the influence of sex on \( \eta \). The values of \( \eta \) for the 52-year-old male and the 47-year-old male are roughly the same.

5.1.3.3 Discussion

No significant trends were discernible in the values of \( E \) due to variations in the storage fluid, duration of storage (PMA), age and sex.

The value of \( \eta \) decreased with increasing PMA for all the femurs tested. The tests on samples from a 85-year-old female and a 62-year-old female were consistent with the proposition that the damping coefficients of specimens from the old are smaller than the damping coefficients of specimens from the middle-aged. It should, however, be noted that the death of the 85-year-old female was caused by a fibrosarcoma of the left lung, while the death of most of the other patients was due to heart disease. It is not known that the sarcomatous cells from the left lung of the 85-year-old female had metastasised in her femur. There were no gross indications of such a metastasis. There is a negligible but finite chance of such a metastasis having occurred. Thus it is conceivable that the low values of \( \eta \) for specimens from the 85-year-old female were at least partly due to bone pathology.

The data on \( \eta \) did not show any significant differences attributable to specific storage fluids. The data was insufficient to show any clear-cut influences of age and sex on \( \eta \).

5.1.4 Stiffness \( E \) and Damping Coefficient \( \eta \) at Zero PMA

5.1.4.1 \( E \) at Zero PMA

It is seen from Fig. 21 that the value of \( E \) is roughly independent of PMA.
The values of $E$ extrapolated to zero PMA lie in the range of $0.5 \times 10^6$ psi to $1.2 \times 10^6$ psi.

5.1.4.2 $\eta$ at Zero PMA

It is seen from Fig. 22 that $\eta$ is a decreasing function of PMA. When the values of $\eta$ are extrapolated to zero PMA, they lie in the range of 900 psi-sec to 1350 psi-sec.

In Section 1.2, it was pointed out that where non-destructive tests to determine a set of properties in vivo are not available, the method of extrapolation to zero PMA provided at least an estimate of the in vivo properties. It is not possible to prove that the extrapolated values do, in fact, represent the in vivo properties.

5.1.5 Influence of Repeated Use of Specimens on $E$ and $\eta$

It was emphasized several times that the results reported are for specimens that were tested only once. The reasons become obvious after looking at Figs. 23.1, 23.2 and 23.3.

The modulus $E$ is plotted against strain-rate $\varepsilon$ in Fig. 23.1 for specimens from a 43-year-old male. These specimens were not tested previously. Figure 23.2 shows a graph of $E$ vs. $\varepsilon$ for specimens from the same 43-year-old male. However, in this case, the results are for specimens that have been previously tested under compressive impact. In both Figs. 23.1 and 23.2, it is possible to fit a straight line through the data. The results of Figs. 23.1 and 23.2 are shown on a single graph in Fig. 23.3. It is seen that repeated use of specimens tends to increase the stiffness $E$ and reduce the damping coefficient $\eta$. One possible explanation is that compressive impact drives out a fraction of the fluids in compact bone in an irreversible manner. Another explanation may lie in the observation that even at relatively low compressive impact stresses, the specimen is damaged at a microscopic level.

5.2 Bovine Specimens

5.2.1 Modulus $E$ vs. Strain-Rate $\varepsilon$ at Constant PMA

The modulus $E$ is plotted against the strain-rate $\varepsilon$ for bovine compact bone (obtained from an animal 8 years old) in Figs. 24.1, 24.2 and 24.3 at a PMA of 3 days, 1 day and 4 hours, respectively. It is seen that for bovine compacta, a linear relationship between $E$ and $\varepsilon$ of the form of Eq. (5.4) is consistent with the experimental data.

5.2.2 Stiffness $'E'$ and Damping Coefficient $'\eta'$ as a Function of PMA

The variation of $E$ and $\eta$ with PMA is shown in Fig. 25. The stiffness $E$ for bovine compact bone increased from $0.825 \times 10^6$ psi at a PMA of 4 hours to $1.875 \times 10^6$ psi at a PMA of 3 days. When stored in saline at room temperature, the stiffness of bovine compact bone appears to be very sensitive to PMA particularly at small values of PMA. The increase in $E$ as the PMA increased from 4 hours to 1 day is larger than the increase in $E$ as PMA increased from 1 day to 3 days.

For the damping coefficient $\eta$, it was found that it decreased with
increasing PMA. The trend here is the same as in human compact bone.

5.2.3 Stiffness 'E' and Damping Coefficient 'D' at Zero PMA

The extrapolated value of E at zero PMA for compact bone from this animal is 0.65 x 10^6 psi. The extrapolated value of the damping coefficient at zero PMA is 1230 psi·sec. These values fall within the range observed for human compact bone.

5.3 Discussion of the Results

5.3.1 Storage in "Formalin"

The storage fluid that is referred to as "formalin" in this report is different from regular formalin in that it is a Ringer's solution prepared in formalin (Ringer's solution is a solution resembling the blood serum in its salt constituents. It contains 8.6 gms NaCl, 0.33 gms CaCl_2, and 0.3 gms KCl in a litre of distilled water).

(a) Some Related Results Reported in the Literature:

Sedlin (38,39) tested ten fresh human compact bone samples, 2 x 2 x 20 mm, in tension at a deformation rate of 2 mm/minute until a load of 8-9 Kg was reached. The specimens were unloaded and placed in 10% formalin solution for three weeks and then retested. It was noted that there was a 4.3% increase in the modulus of elasticity, but the change was considered not significant.

McElhaney (40) tested bovine compact bone specimens fresh within 48 hours of death and after at least 15 hours immersion in one of four embalming fluids containing ethyl alcohol and formalin. The deformation rate was 0.05 inches/min and the specimen size was 0.11 x 3/8 x 13/8 inches. It was found that the modulus of elasticity in compression was reduced by 6.2% due to embalming. It was also shown that the average difference between embalmed and unembalmed properties was much smaller than the natural variation from sample to sample.

Evans (35) reported in 1964 that embalmed bone gave a tensile stress at fracture 4% higher than unembalmed bone. Evans surveyed the results obtained by many different workers in 1957 and noted that the average tensile stress at fracture recorded on specimens of embalmed bone was usually considerably lower than for fresh bones. Tsuda (35) found that fixation in formalin produced no significant change in the fracture load, but a reduction of about 20% in the deflection at fracture was found in his three-point bending test on wet specimens.

It should be noted that all the results reported in the literature on the influence of fixation are for quasi-static loading rates (i.e., for static tests).

(b) Observations from the Present Study:

The influence of storage in "formalin" on compressive impact properties was considered. To the author's knowledge, this is the first time that such results are being reported.

For human compact bone, it is seen from Fig. 21 that there may be a
very small increase in stiffness \( E \) with an increase in PMA when specimens are stored in "formalin".

It is seen from Fig. 22 that the value of the damping coefficient for human compact bone decreases with increasing PMA when stored in "formalin".

5.3.2 Storage in Saline

(a) Some Related Results Reported in the Literature:

Gray (106) found that compressive fatigue life of bovine compact bone decreased significantly with increasing PMA when the specimens were stored in saline at room temperature. Gray did not find a significant change in fatigue life when specimens were stored in saline at 32\( ^\circ \)F as the PMA changed from 3 days to 7 days. These results have to be viewed cautiously (especially in view of the large scatter generally observed in fatigue tests) since the sample size at various PMA was as low as one or two specimens.

Yamada (106) reported that bone samples stored in saline in a refrigerator showed constant strength values when tested at different PMA.

Tsuda (35) used a three-point bending test on specimens stored for up to 30 days in saline. Both the load and deformations at fracture were reduced after about 10 days storage.

Ko (35) performed tensile tests on specimens stored in saline at room temperature for periods of up to one year. He found no significant change in the stress-strain curve, tensile stress at fracture or strain at fracture compared to fresh specimens.

(b) Observations from the Present Study:

The influence of storage in saline on compressive impact properties was considered. This is probably the first time that such results are being reported.

For human compact bone, it is seen from Fig. 21 that the stiffness \( E \) is relatively insensitive to PMA when specimens are stored in saline.

It is shown in Fig. 22 that the damping coefficient \( \eta \) of human compacta decreased with increasing PMA when specimens were stored in saline.

It can be noted from Fig. 25 that the stiffness \( E \) of bovine compact bone increased considerably with PMA for small values of PMA when specimens were stored in saline.

The damping coefficient \( \eta \) of bovine compact bone stored in saline decreased with increasing PMA as also shown in Fig. 25.

5.3.3 Comments

There was an inevitable delay between the death of the patient and the performance of an autopsy on the patient's body that was not under the control of the author. Thus, it was not possible to test human specimens at a PMA of less than 1 day. In the case of bovine specimens, it was possible to perform the
tests at a value of PMA as low as 4 hours. It was found in the case of bovine specimens that the value of \( E \) was very sensitive to PMA at low values of PMA. Thus, it is conceivable that human specimens possess a value of \( E \) less than that observed in the present study, at a PMA of less than 1 day.

6. QUANTIFIED VISCOELASTIC MODEL FOR COMPACT BONE

Viscoelastic models are useful not only for describing material behaviour qualitatively, but also for making quantitative predictions. It was pointed out in Section 1.2 that a quantified model for compact bone governing its response over the entire strain-rate spectrum has not been proposed so far. It is the aim of this section to develop such a model. In Section 2.2, two phenomenological descriptions of compact bone were considered. It was then pointed out that a mechanical model representation for compact bone was preferable due to the ultra-phenomenological considerations discussed in Section 2.3. In Section 5 experimental results were presented for human and bovine compact bone subjected to compressive impact. In Section 6.1 the mechanical model discussed in Section 2.2.2 for a phenomenological description of the viscoelastic behaviour of compact bone will be quantified by using the experimental results of Section 5 and some experimental data from Ref. 49. In Section 6.2, predictions of the model are compared with some experimental results.

6.1 Determination of the Constants of the Model

Model \( M_1 \) of Fig. 4 was discussed in Appendix B, Section B.3.2. It was indicated there that the constitutive equation of Model \( M_1 \) is:

\[
\sigma = E \varepsilon + \eta \varepsilon \dot{\varepsilon} \tag{6.1}
\]

By differentiating Eq. (6.1), with respect to \( \varepsilon \) at constant \( \dot{\varepsilon} \)

one obtains,

\[
\frac{d\sigma}{d\varepsilon} \bigg|_{\dot{\varepsilon} = \text{const.}} = E + \eta \dot{\varepsilon} \tag{6.2}
\]

Let,

\[
\frac{d\sigma}{d\varepsilon} \bigg|_{\varepsilon = \text{const.}} = \bar{E} \tag{6.3}
\]

From Eqs. (6.2) and (6.3),

\[
\bar{E} = E + \eta \dot{\varepsilon} \tag{6.4}
\]

It was shown in Sections 5.1.2 and 5.2.1, that the experimental results on human and bovine compact bone under compressive impact satisfy Eq. (6.4). Thus, the nonlinear viscoelastic model (\( M_1 \)) of Fig. 4 is suitable for a phenomenological description of the behaviour of compact bone under compressive impact.

Differentiation of Eq. (6.1) with respect to time yields,
\[ \dot{\sigma} = E \dot{\varepsilon} + \eta \varepsilon^2 + \eta \varepsilon \varepsilon \]  

(6.5)

For constant strain-rate data,

\[ \varepsilon = 0 \]  

(6.6)

and Eq. (6.5) reduces to,

\[ \dot{\sigma} = E \dot{\varepsilon} + \eta \varepsilon^2 \]  

(6.7)

Equation (6.7) may be re-written as,

\[ (\dot{\varepsilon} + E/2\eta)^2 = (1/\eta) (\dot{\sigma} + E^2/4\eta) \]  

(6.8)

which is an equation for a parabola. As a further check on the suitability of model M1 for describing compact bone response under compressive impact, the experimentally measured variations of stress-rate with strain-rate were plotted for a 52-year-old female and a 62-year-old female at a PMA of 1 day (see Fig. 26). Similarly, a graph of \( \dot{\sigma} \) vs. \( \varepsilon \) was plotted for a 43-year-old male and a 62-year-old female at a PMA of 9 days (see Fig. 27). Considering the large scatter that is normally observed in experimental data on bone, it is fair to say that Eq. (6.7) adequately fits the data of Figs. 26 and 27.

If one defines

\[ \dot{\varepsilon}_1 = \frac{\dot{\sigma}}{\varepsilon} \]  

(6.9)

then Eq. (6.7) can be re-written as,

\[ \dot{\varepsilon}_1 = \frac{\dot{\sigma}}{\varepsilon} = E + \eta \varepsilon \]  

(6.10)

Comparing Eqs. (6.4) and (6.10),

\[ \dot{\varepsilon}_1 = \dot{\varepsilon}_1 \]  

(6.11)

A graph of \( \dot{\varepsilon}_1 \) (Eq. (6.3)) against \( \dot{\varepsilon}_1 \) (Eq. (6.9)) is shown in Fig. 28. Again, there is reasonable agreement between Eq. (6.11) and the experimental data.

Although Model M1 is very satisfactory for a phenomenological description of impact behaviour (a short-characteristic-time phenomenon), it is not suitable for describing creep and stress relaxation behaviour (long-characteristic-time phenomena) of compact bone. It is, however, possible to add more elements to Model M1 of Fig. 4 such that its creep and stress relaxation characteristics are altered while the impact behaviour remains unchanged. The next section describes how this is done.
Renton and Pierkarski (49) have proposed a creep compliance function for compact bone based on their experiments on wet compact bovine material. This function is (49):

\[ J(t) = \left[ 4.5 - (0.15 e^{-t/91} + 0.23 e^{-t/3.4} + 0.20 e^{-t/3.7} ) \right] \times 10^{-11} \text{cm}^2/\text{dyne} \]  

Equation (6.12) is of the form,

\[ J(t) = J_0 U(t) \]  

where

\[ U(t) = \left[ 1 + \sum_{i=2}^{4} \left( \frac{J_i}{J_0} \right)(1 - e^{-t/\tau_i}) \right] \]  

Comparison of Eqs. (6.13) and (6.14) with Eq. (6.12) gives,

\[ \begin{align*}
\tau_2 &= 0.37 \text{ secs} \\
\tau_3 &= 3.4 \text{ secs} \\
\tau_4 &= 91 \text{ secs}
\end{align*} \]  

\[ \begin{align*}
\frac{J_2}{J_0} &= 0.051 \\
\frac{J_3}{J_0} &= 0.059 \\
\frac{J_4}{J_0} &= 0.038
\end{align*} \]  

The function \( U(t) \) of Eq. (6.14), represents a discrete retardation spectrum for compact bone which is shown in Fig. 29. Its physical significance can be understood by analogy with the explanation given in Appendix A (Section A.4) for a continuous spectrum. In the treatment to follow, it is assumed that the scatter in experimental creep results is due to scatter in \( J \) and that \( U(t) \) remains constant. Physically, this is equivalent to assuming that the shape of the retardation spectrum is an unchanging fundamental creep characteristic of compact bone. The assumption can also be interpreted to mean that all creep curves of compact bone are geometrically similar. This is, no doubt, a simplifying assumption. A more complex analysis is necessary to account for deviations from this assumption.

A "normalized creep" or "unit creep" model is defined here as a mechanical model whose creep compliance function is given by Eqs. (6.13), (6.14), (6.15), and (6.16), with \( J_0 = 1 \) unit.

Such a model is shown in Fig. 30. For \( J_0 = 1 \) unit,

\[ J(t) = U(t) \]  

(6.17)
Thus \( U(t) \) is the creep compliance function of the unit creep model, and hence may be called the "unit creep compliance". Besides the first term which is unity, there are six other terms on the right hand side of Eq. (6.14), which are given in Eqs. (6.15) and (6.16). The seven terms of the unit creep compliance of Eq. (6.14) are related to the four springs and three dashpots of Fig. 30 as follows:

\[
\frac{E_1}{E_0} = 1
\]

\[
\frac{E_i}{E_0} = \frac{1}{(J_{1i}/J_0)}, \text{ for } i = 2 \text{ to } 4
\]

\[
\frac{\eta_i}{E_0} = (E_i/E_0) \tau_i, \text{ for } i = 2 \text{ to } 4
\]

For the unit creep model, one has by definition

\[
E_0 = 1/J_0 = 1 \text{ unit}
\]

Substituting Eqs. (6.15) and (6.16) into Eqs. (6.19) and (6.20) one obtains,

\[
\begin{align*}
\frac{E_2}{E_0} &= 19.6 \\
\frac{E_3}{E_0} &= 17 \\
\frac{E_4}{E_0} &= 26.1
\end{align*}
\]

\[
\begin{align*}
\frac{\eta_2}{E_0} &= 7.24 \text{ secs} \\
\frac{\eta_3}{E_0} &= 58 \text{ secs} \\
\frac{\eta_4}{E_0} &= 2370 \text{ secs}
\end{align*}
\]

In Fig. 30, the spring \( E_1 = E_0 \) represents the instantaneous elasticity of the model. Thus for impact loading, where the rise times are far shorter than the shortest retardation time in Fig. 30 (i.e., for \( t_{\text{rise}} \ll 0.37 \text{ secs} \)), the model of Fig. 30 degenerates to a pure spring \( E_1 \). However, it was seen in the early parts of this section that model M1 of Fig. 4 gave a very satisfactory phenomenological description of the impact behaviour of compact bone. Hence, the spring \( E_1 \) of Fig. 30 is now replaced by Model M1 of Fig. 4, as shown in Fig. 31, to account for the impact behaviour of compact bone. If Fig. 31 is compared with Fig. 5b, it will be noticed that it is the same model as was proposed in Section 2.2.2 (the value of \( \eta \) in Fig. 5b is now given by \( n = 4 \)). The values of \( E \) and \( \eta \) in Fig. 31 may be obtained from Figs. 21 and 22 for human compact bone. It was noted in Section 5.1.4.1 that \( E \) at zero PMA had a range of values from \( 0.5 \times 10^6 \text{ psi} \) to \( 1.2 \times 10^6 \text{ psi} \). Similarly, it was noted in Section 5.1.4.2 that \( \eta \) at zero PMA had a range of values from 900 psi-sec to 1350 psi-sec. From these ranges of values for \( E \) and \( \eta \), average values may be calculated.
When the values of $E$ and $\eta$ from Eq. (6.24) are substituted into the model of Fig. 31, the model shown in Fig. 32 describing the impact and creep behaviour of human compact bone is obtained.

6.1.1 Reasons for Incorporating the Retardation Spectrum of Bovine Compact Bone into the Model for Human Compact Bone:

(a) A comparison of the mechanical properties of bovine compact bone with those of human compact bone under quasi-static loading (see Table 2), under vibration (47,53), and under impact (58) shows that the differences between the two are small compared to the scatter observed within each type of bone test data. It was also shown in Section 5.2.3 that the $E$ and $\eta$ values for bovine compact bone under compressive impact fall within the range of values observed for human compact bone.

(b) Histologically, bovine compact bone does not differ significantly from human compact bone.

(c) Historically, investigators have used bovine compact bone to model human compact bone with good justification.

(d) At the present time, the only quantitative information available on the compressive creep of compact bone is that of Renton and Piekarski (49) obtained from experiments on bovine compact bone. Experiments are being planned in the author's laboratory to obtain quantitative compressive creep data on human compact bone. Until such information becomes available, the only alternative is to use the data on bovine bone.

(e) If a method of obtaining a comprehensive phenomenological model for human compact bone describing its behaviour over the entire strain-rate spectrum is developed, then altering the model to account for small differences between the creep behaviour of bovine and human bone would be a simple matter.

6.1.2 Derivation of the Behaviour of the Viscoelastic Model (Fig. 32) on Physical Grounds

6.1.2.1 Impact Behaviour

Under this heading, the stress-strain behaviour of the viscoelastic model for compact bone, when subjected to stress or strain pulses with risetimes of the order of 20 μssecs, is considered. Let us consider the application of a step-function of stress to the model. The model shown in Fig. 32 consists of 4 Voigt elements in series. The first one is non-linear, but the other three are linear. It can be shown that for the $i^{th}$ Voigt element (78) ($i = 2$ to 4),

$$\frac{\varepsilon_i(t)}{\varepsilon_i(t = \infty)} = 1 - e^{-t/\tau_i}$$

(6.25)
where $\varepsilon_i(t)$ is the strain in the $i$th Voigt element at time $t$ and $\varepsilon_i(t = \infty)$ is the strain in the same element at infinite time. $\tau_i = \eta_i/E_i$ is the retardation time of the $i$th element. Equation (6.25) is tabulated in Table 3.

Using the retardation times shown in Eq. (6.15), and Table 3, it can be shown that,

$$\begin{align*}
\varepsilon_2 (3000 \mu\text{sec}) &< 1\% \text{ of } \varepsilon_2 (t = \infty) \\
\varepsilon_3 (3000 \mu\text{sec}) &< 1\% \text{ of } \varepsilon_3 (t = \infty) \\
\varepsilon_4 (3000 \mu\text{sec}) &< 1\% \text{ of } \varepsilon_4 (t = \infty)
\end{align*}$$

(6.26)

At $t = \infty$, the strain contributed by each Voigt element is inversely proportional to the stiffness of its spring. Therefore, the following relations hold:

$$\begin{align*}
\varepsilon_2 (t = \infty) &= 5.1\% \text{ of } \varepsilon_1 (t = \infty) \\
\varepsilon_3 (t = \infty) &= 5.9\% \text{ of } \varepsilon_1 (t = \infty) \\
\varepsilon_4 (t = \infty) &= 3.8\% \text{ of } \varepsilon_1 (t = \infty)
\end{align*}$$

(6.27)

At magnitudes of strain for which the proposed model is valid (i.e., in the elastic region of strain), the retardation time of the first element is in the neighbourhood of 1 $\mu\text{sec}$ as indicated in Fig. 32. Using this value and Eq. (6.25), one finds,

$$\varepsilon_1 (3000 \mu\text{sec}) \approx \varepsilon_1 (t = \infty)$$

(6.28)

Using Eqs. (6.26), (6.27) and (6.28) gives,

$$\sum_{i=2}^{4} \varepsilon_i (3000 \mu\text{sec}) < 0.15\% \text{ of } \varepsilon_1 (3000 \mu\text{sec})$$

(6.29)

Thus, for $t < 3000 \mu\text{sec}$, the combined strain in the Voigt elements 2, 3 and 4 is less than 0.15% of the strain in the first element. Hence, for impact loading where the times of interest are of the order of 20 $\mu\text{sec}$, the stress-strain behaviour of the model is entirely governed by the first element.

Thus for the case of impact loading, the model of Fig. 32 leads to the degenerate case shown in Fig. 33.

6.1.2.2 Creep Behaviour

The characteristic times of the creep process are long (of the order of seconds) compared to the characteristic times of impact (of the order of $\mu\text{sec}$).

It was shown in Eq. (6.28) that $\varepsilon_1 (3000 \mu\text{sec}) \approx \varepsilon_1 (t = \infty)$. Thus for
t > 3000 µsecs, the nonlinear damper of the first element is not active. Consequently for creep, the model of Fig. 32 leads to the degenerate case shown in Fig. 34.

6.2 Comparison of the Predictions of the Viscoelastic Model with Some Experimental Results

It is well-known that there is an enormous scatter in the values of the modulus reported by various investigators. Table 2 adequately demonstrates this fact. McElhaney (40) reported in 1964 a range of 2.8 x 10^6 psi to 5.1 x 10^6 psi for the quasi-static compressive modulus of embalmed bovine bone. The average quasi-static modulus of elasticity reported by the same author for fresh bovine bone in compression was 4.18 x 10^6 psi. McElhaney (58), in 1966, reported results showing that the compressive modulus changed from an average value of 2.7 x 10^6 psi at 0.001 sec^-1 to an average value of 4.8 x 10^6 psi at 300 sec^-1. Thus taking an extreme example, a change of six orders of magnitude in strain-rate produced an average change in the modulus of the same order of magnitude as the scatter. Hence it is possible in some cases for the scatter in results to mask the influence of strain-rate on the modulus. The situation becomes much worse when results from different authors need to be compared.

It appears, however, that notwithstanding the large scatter in the values of the modulus, the ratio of viscous modulus (viscous stiffness) to elastic modulus (elastic stiffness) shows a relatively small scatter.

In view of the above considerations, it was decided to use the viscoelastic model to make predictions of the ratio of viscous to elastic stiffness along the strain-rate spectrum and compare these predictions with the experimental results of several investigators. It is not meaningful to make comparisons of the results of several investigators with the model predictions (with respect to the modulus) due to the large scatter mentioned above. The results of this study lie at the lower boundary of the scatter band with respect to the modulus.

6.2.1 Predictions of the Ratio of Viscous to Elastic Stiffness or the Loss Tangent

6.2.1.1 The Impact Range

It was shown in Section 6.1.2.1 that the total model of Fig. 32 degenerates to the model of Fig. 33 for this range. Thus, the constitutive equation for this case can be represented by Eq. (6.4),

\[ \tilde{E} = E + \eta \dot{\varepsilon} \]  
(6.4)

The modulus is now made up of two terms. The first term is the elastic component and the second term is the viscous component. The ratio of the two terms is expressed as,

\[ \frac{\text{Viscous Stiffness}}{\text{Elastic Stiffness}} = \frac{\eta \dot{\varepsilon}}{E} \]  
(6.30)

Equation (6.30) is plotted in Fig. 35 using the values of E and \( \eta \) obtained in this investigation.
6.2.1.2 Predictions in the Vibration Range Using the Model for the Creep Range:

It was shown in Section 6.1.2.2 that the complete model of Fig. 32 reduces to the model of Fig. 34 for creep. It was indicated by Ferry (78) that the loss tangent, \( \tan \delta \) (ratio of viscous to elastic stiffness), can be determined in a simple fashion from the creep compliance if \( \tan \delta \ll 1 \). This condition is satisfied as will be demonstrated soon. Hence, the method indicated by Ferry was used to determine \( \tan \delta \) from the model of Fig. 34 for the creep range. The method is illustrated in Fig. 36. The creep compliance \( J(t) \) for the model of Fig. 34 is drawn as a function of time on a double-logarithmic plot. From the slope of this graph, \( \tan \delta \) is calculated as shown in Fig. 36. The value of \( \tan \delta \) so obtained is shown in Fig. 35. It is possible to calculate the loss tangent from the relaxation spectrum obtained from a stress relaxation experiment. The data of Lugassy and Korostoff (48) which gives rise to a flat relaxation spectrum was used to perform such a calculation (see Appendix F) and the result so obtained is shown in Fig. 35.

6.2.1.3 Experimental Results in the Vibration Range

The experimental data of Laird and Kingsbury (53), Smith and Keiper (47) and Uezaki (54) are shown in Fig. 35 for comparison with model predictions. It is seen that there is reasonable agreement.

6.2.2 Other Comparisons

6.2.2.1 Predictions

Using the model of Fig. 34, two further predictions can be made.

(i) Creep:

\[
\frac{\epsilon(t = \infty) - \epsilon(t = 0)}{\epsilon(t = 0)} \approx 15\%
\]

(ii) Stress Relaxation:

\[
\frac{E(t = \infty)}{E(t = 0)} \approx 86\%
\]

6.2.2.2 Experimental Data

(i) Creep:

From the data of Renton and Piekarski (49), one obtains,

\[
\frac{\epsilon(t = \infty) - \epsilon(t = 0)}{\epsilon(t = 0)} \approx 15\%
\]
(ii) Stress Relaxation:

From the data of Lugassy and Korostoff (48), one can calculate,

\[ \frac{E(t = \infty)}{E(t = 0)} \approx 84\% \]

6.2.2.3 Comments

It is seen that there is excellent agreement between predictions and the experimental results. The agreement with the data of Renton and Piekarski (49) is, of course, no surprise. The agreement with the data of Lugassy and Korostoff (48) is unexpectedly good.

7. STUDY OF MICROSCOPIC DAMAGE IN HUMAN COMPACT BONE UNDER COMPRESSIVE IMPACT

7.1 Introduction

Some fracture studies on compact bone have been reported by Pope, et al (67), Piekarski (37,68,73), Bonfield, et al (56,57) and Takezono, et al (72). Good reviews of the fracture behaviour of compact bone may be found in Refs. 34 and 35.

To the author's knowledge, Bird, et al (56,57) were the only ones to have studied the microscopic damage in compact bone under compressive impact. Their studies were on bovine femoral compact bone. Bird, et al classified the condition of their specimens after impact testing as survival, micro-cracking and shatter. This was the first attempt at a detailed damage analysis. Their analysis was qualitative however, and they did not make quantitative measurements of damage. Their study was undertaken to understand the mechanics of gun-shot wounds. Bullets generally strike bone either tangentially or radially and for this reason, longitudinal impact is not very important for gun-shot wounds.

In the present investigation, a quantitative damage analysis of human femoral compact bone under longitudinal compressive impact is made.

In Fig. 1, longitudinal compressive impact in a road traffic accident leading to the extrusion of the lower femoral shaft is shown schematically. Such damage may be referred to as gross damage. "Sub-threshold trauma" may be defined as trauma not leading to gross bone damage. Sub-threshold trauma would give no radiographic indications of damage to bone. It is possible, however, for sub-threshold impact trauma to lead to extensive microscopic bone damage. If a patient with extensive microscopic bone damage is permitted to subject the bone to fatigue stresses induced by daily activities before the micro-cracks are healed, they may propagate and lead to a "spontaneous fracture" (also referred to as "stress fracture", "insufficiency fracture", "fatigue fracture", or "march fracture"). Hence, it is important to know the level of microscopic damage caused under sub-threshold impact.

7.2 Experimental Data on Microscopic Damage

Some details of the method used in this investigation to quantify microscopic damage under sub-threshold compressive impact stresses are described in Appendix G. The Hopkinson bar apparatus used for this study is not designed to measure
particle velocities imparted to the specimen. The level of impact can, however, be measured with the Hopkinson bar set-up by the peak value of the stress pulse. The particle velocities equivalent to a given magnitude of the stress pulse can be calculated by an approximate equation (56) given below:

\[ v = \frac{\sigma}{\sqrt{E\rho}} \]  

(7.1)

where, 
\( v \) = particle velocity or velocity of impact 
\( \sigma \) = peak value of the stress pulse 
\( E \) = compressive modulus for compact bone 
\( \rho \) = density of compact bone

A plot of Eq. (7.1) is shown in Fig. 37. With an increase in the impact stress due to increasing impact velocities, the specimens are also subjected to increasing strain-rates. Experimentally measured values of strain-rate at various impact stresses are plotted in Fig. 38. Since the impact stress, impact velocity and strain-rate are related according to Figs. 37 and 38, specifying any one of them specifies the other two. Therefore, microscopic damage can be described as a function of any one of these parameters. In the graphs to be discussed later, microscopic damage is expressed as a function of the impact stress.

The tests performed for assessing microscopic damage roughly fall into three groups:

**Group A:** Low stress level tests: 6 to 10 Ks

**Group B:** Medium stress level tests: 12 to 20 Ks

**Group C:** High stress level tests: 23 to 30 Ks

In groups A and B, a visual examination of the specimens after testing revealed no damage. In group C, most of the specimens revealed visually observable damage upon careful examination. At the lower range of stress in group C, many specimens were still intact. At the higher range of stress in group C, the specimens shattered into many pieces by splitting longitudinally. For convenience, the stresses of groups A and B are called "sub-threshold impact stresses". Sub-threshold impact stresses do not lead to bone damage visible to the naked eye. Figure 39 shows some Hopkinson bar strain gage signals for groups A, B and C. Some typical photomicrographs of specimens from groups A, B and C are also shown in Figs. 40, 41 and 42, respectively. It was not at all uncommon to find in the specimens in group C long cracks formed by the joining of several micro-cracks running from one Haversian canal (see Appendix H) to another. The photomicrographs of a specimen which shattered at 29 Ks are shown in Fig. 43. There are strong indications that this specimen failed by the mechanism of cracks propagating from one Haversian canal to another. Micro-cracks were also observed in many group C specimens along the cement lines of the osteons. Thus, two mechanisms of fracture seem to be operating simultaneously: (i) crack propagation along the weak cement lines and (ii) crack propagation from one Haversian canal to another with the cracks freely crossing the material in-between.

The microscopic damage has been quantified in this study by measuring the "average micro-crack length" and the "micro-crack density" (see Appendix G). A plot of micro-crack density as a function of impact stress is shown in Fig. 44.
It is seen that for group A specimens, the micro-crack density increased with increasing impact stress. For group B specimens, the micro-crack density decreased with increasing impact stress. For group C specimens, there was relatively little change in the micro-crack density with increasing impact stress. The average micro-crack length is plotted in Fig. 45 as a function of the impact stress. It was found that the average length increased with increasing stress for all three groups. It appears, however, that the rate of increase of length with stress is higher for group C than for the other two groups. In an attempt to determine if there was any relationship between the microscopic damage parameters (average micro-crack length and micro-crack density) and the variables of PMA, age and sex, it was concluded that there were no significant trends attributable to these variables.

7.3 An Hypothesis

The differences in micro-crack density between groups A, B and C shown in Fig. 44 may be explained by the following hypothesis. In group A, most of the impact energy is used in initiating micro-cracks at several locations. Thus, an increase in impact stress for group A leads to an increase in crack density. In group B, the impact energy at the higher stresses is used to propagate the cracks initiated at lower stresses. Thus it is possible at this level for micro-cracks to join up and form long cracks. The process of joining up of micro-cracks reduces the number of micro-cracks per unit area and so the micro-crack density is lowered. In group C, it appears that most of the micro-cracks that can join with others before fracturing the specimen have already done so at a lower stress. Hence, a further reduction in micro-crack density by the process of joining up of micro-cracks is not possible any more. Thus, the micro-crack density reaches a "limiting value". A further increase in impact stress at this stage causes complete fracture of the specimen.

8. SOME CONSIDERATIONS IN THE DESIGN OF INTERNAL ORTHOPAEDIC PROSTHETIC OR FIXATIVE DEVICES

8.1 Introduction

Progress in materials science and surgical procedures are responsible for the increased use of operative techniques in the treatment of fractures. Both conservative (closed) and operative (open) techniques have a place in the reduction of fractures, but for a good number of fractures arguments can be presented both in favour of and against each of the approaches. Some of the arguments often presented against conservative methods are as follows:

(i) They cause an unnecessary delay in union.
(ii) In the case of femoral fractures, there may be residual knee stiffness.
(iii) There may be residual deformity due to improper alignment of the fractured ends.
(iv) Immobilization for extended periods occurs thereby increasing hospital costs and loss of useful time for the patient.
(v) There is a higher incidence of re-fracture in conservatively treated cases.

There are also disadvantages in the operative methods. For example, they damage blood vessels in the process of surgical intervention, possibly leading to avascular necrosis. For details on this subject the reader is referred to Burwell (107), Charnely (108) and Brodetti (109). Some techniques that attempt
to combine the advantages of both methods have also been developed (110). Once the operative method is decided upon as the preferred treatment, engineering considerations become very important. The preferred method of treatment depends to a considerable extent on the materials available to the surgeon.

Improvements in the design of prostheses, nails, plates and screws in the past two decades have led to a high success rate in open methods. Despite the high success rate, some of the basic mechanisms of bone healing are still incompletely understood. The role of compression and rigidity in fracture repair is still a fertile area for research (111 to 114). The Swiss AO group has recently reported on the use of an improved device for applying compression, known as the dynamic compression plate (26).

Several methods of joining fractured bone ends are in use. Where screws or other simple means are not sufficient to hold the broken ends together, there are two basic devices at the disposal of the surgeon; (i) plates and screws, and (ii) nails. Several of these devices have been tested for strength and rigidity, both singly and in combination (115). It was found that a 13 mm Kuntscher nail and two 7 cm Venables plates at 90° gave roughly the same rigidity in bending, while the plates were far superior in torsion. The poor performance of the nail was due to an inefficient method of joining the nail to the bone.

There are four basic methods of joining intramedullary nails and prostheses to bone: (a) impaction, (b) screws through the implant and both cortices, (c) porous sleeves (116, 117), and (d) bonding* with cement (27, 118 to 127). A survey of the literature left the author with the impression that bonding is the best among the methods mentioned.

Selection of proper materials is crucial for the success of operative methods in orthopaedic surgery. Some criteria for the selection of materials are presented in Section 8.2 with specific materials of interest in orthopaedic surgery discussed in Section 8.3. Composite materials have also come into prominence in the last two decades. The greatest asset of these materials is the possibility of tailoring their mechanical properties to suit the needs of the user. Some theoretical composites are considered in Section 8.4 for internal orthopaedic use.

8.2 Criteria for Materials Selection

Criteria used in the selection of materials for internal orthopaedic use can be broadly divided into: (a) biological criteria and (b) mechanical criteria.

8.2.1 Biological Criteria

(i) The material should be non-toxic
(ii) " " " non-carcinogenic
(iii) " " " non-thrombogenic

The reader is referred to Refs. 28, 29 and 128 to 130 for information on the toxicological, carcinogenic and thrombogenic aspects of implant materials.

* There is a restricted use of the term meaning adhesion by chemical bonding. Here the term is used in a more general sense to include mechanical interlocking of surfaces.
8.2.2 Mechanical Criteria

(i) Strength Criteria: These include requirements concerning static strength, fatigue strength, impact strength etc.

(ii) Stiffness Criteria: These include requirements pertaining to exten­sional, flexural and torsional stiffness of the material.

(iii) Miscellaneous Criteria: Under this heading, one may include machin­ability, abrasion resistance, frictional coefficient requirements, etc.

8.2.3 Biological and Mechanical Interactions

The biological environment of the body normally has a degrading influence on the mechanical properties of implants. The degradation products are quite often toxic to the body. Thus, there is an interaction between biological and mechanical requirements.

8.3 Some Materials of Interest in Orthopaedic Surgery

8.3.1 Metals and Alloys

Metals and alloys currently used by orthopaedic surgeons are shown in Table 4. Among these materials, stainless steel corrodes most rapidly. Titanium and titanium alloys are the best from the point of view of corrosion. The Co-Cr alloys, also known as vitallium, rank in the middle. For a detailed consideration of these metals and alloys, the reader is referred to Refs. 23 to 25, 29 and 131 to 134.

8.3.2 Plastics

Several plastics have been used in the human body. Some plastics that have found fairly extensive applications in orthopaedic surgery are polymethylmethacrylate (PMMA), polyethylene, and polytetrafluoroethylene (PTFE). Charnely (27, 118 and 121 to 124) has reported extensive use of PMMA. His long-term results indicate that PMMA is well tolerated by the body. PMMA was also used by the Judets (30) in hip prostheses. For details on the tissue tolerance of PMMA, polyethylene and PTFE, the reader is referred to Ref. 28. Some mechanical properties of these plastics are also given in Table 5.

8.4 Suggestions for Designing New Materials with Improved Properties

When two materials are joined together (e.g., bone and intramedullary nail), the stress concentrations that would inevitably arise at such joints generally would be a minimum when the rigidities of the two adherends are equal. The rigidity of a bone-plate depends both on its modulus of elasticity and its thickness. The rigidity of a bone-plate can be equalized to that of bone by either altering the modulus or the thickness. In the case of an intramedullary nail, its diameter is fixed by the diameter of the medullary cavity and hence the only way to alter its rigidity is by altering its modulus of elasticity. Thus it is more difficult to equalize the rigidity of an intramedullary nail with that of bone than it is to equalize the rigidity of a bone-plate with bone. Hence the present treatment is mainly concerned with nails though the results so obtained can also be used for plates. The radius of the femoral medullary cavity is approximately 0.25" and the thickness of the femoral cortex is approximately 0.375". These are approximately of the same order of magnitude. However the modulus of
metals like stainless steel and vitallium which are currently used for intramedullary nails is about 15 times as large as the modulus of cortical bone. The only way to equalize the rigidity of the nail with that of cortical bone is to find biologically compatible materials with a lower modulus. Among the metals and alloys currently in use, Ti-alloys have the lowest modulus and the highest strength. Thus from the strength and stiffness points of view, Ti-alloys are the most suitable. They are also the best from the point of view of corrosion. Although Ti-alloys are the best among the materials shown in Table 4, they have two main drawbacks: (i) their modulus is still too high compared to that of compact bone, (ii) they are not strain-rate sensitive in the elastic range as compared to compact bone which is strain-rate sensitive.

An attempt is made now to lower the modulus and impart strain-rate sensitivity by incorporating Ti-alloys as fibres in a composite. PMMA appeared to be the best choice for a matrix material due to its bio-compatibility and strain-rate sensitivity. The stress-strain curves of PMMA obtained by Maiden and Green (136) at various strain-rates are shown in Fig. 47. The properties of a hypothetical composite of PMMA reinforced with Ti-alloy fibres were calculated using the properties of PMMA from Fig. 47 and the properties of Ti-alloys from Table 4. The rule of mixtures (137) was used in the computations. Some computations of interest are shown in Tables 6, 7 and 8. The final results of the calculations are shown in Fig. 48 and Table 9. The whole range of volume fractions shown in Fig. 48 is not useful for our purposes. For a square array of fibres of circular cross-section, it can be shown that the maximum possible volume fraction of fibres \( V_f \) is \( \frac{1}{7} \times 100\% \). If it is stipulated, on the basis of currently used materials, that the UTS of the composite should be greater than 100,000 psi, then a lower limit for \( V_f \) is obtained. These two limits for \( V_f \) set the range of practical interest for the composite. This is shown in Fig. 48. The initial computations were done for a PMMA-matrix and Ti-alloy fibres because these materials are widely used in orthopaedicsurgery. Figure 48 shows that the composite has a modulus closer to that of compact bone than the Ti-alloy, but is still not close enough.

The mechanical properties of a number of other materials are shown in Table 10. This table was searched for those fibre materials that have a modulus close to that of compact bone, but at the same time possess high strengths. E glass and 970S glass were found to have interesting properties. Some fibre materials chosen for further consideration are shown in Table 11. A design criterion used in these composite calculations is that the composite should have a tensile strength greater than 100,000 psi. With this design objective, the values of \( V_f \) of interest for composites with various fibres are demarcated in Table 11. Computations were done for two more hypothetical composites. One composite considered was PMMA reinforced with 970S glass and the other was PMMA reinforced with E glass. The results are shown in Figs. 49 and 50. It is seen that these composites have a modulus closer to that of compact bone in their useful range of volume fractions than the composite considered earlier. Calculations were also made for composites using a high density polyethylene and PTFE as matrix materials. These are shown in Table 12. The results for the various hypothetical composites discussed so far are plotted in Fig. 51 for comparison with compact bone.

The theoretical considerations of this section suggest that among the hypothetical materials considered (see Fig. 51), the following would probably be the most suitable for intramedullary nails from the point of view of the dynamic compression modulus, although other mechanical properties must also be taken into account in the final choice;
9. CONCLUSIONS

(i) The mechanical behaviour of human femoral compact bone under compressive impact parallel to the long axis of the femur has been experimentally investigated using the Hopkinson bar apparatus. A non-linear model was found suitable for a phenomenological description of compact bone under compressive impact.

(ii) The influence of storage in saline and in formalin (Ringer's solution prepared in formalin) on the compressive impact properties was studied. It was found that the storage fluids had no significant effect on the elastic stiffness $E$ of human compact bone, but they had a pronounced effect on the damping coefficient $\eta$.

(iii) The damping coefficient was found to be a function of the post-mortem age (PMA). The damping coefficient $\eta$ at zero PMA was obtained by extrapolation of the experimental data. It was suggested that, in the absence of non-destructive test data in vivo, this provided the best estimate of the in vivo damping coefficient for human compact bone under compressive impact.

(iv) A quantified viscoelastic model for compact bone was developed. The model gave a satisfactory phenomenological description of compact bone in both the impact and creep ranges of strain-rate. The model was used for the prediction of the loss tangent in the vibration range of strain rates. Such predictions agreed well with available experimental results.

(v) The clinical significance of sub-threshold impact trauma was suggested. It is possible that such impact is responsible for some of the spontaneous fractures observed in practice. The microscopic damage caused by sub-threshold impact was quantified by two microscopic damage parameters. The parameters were (a) the micro-crack density, and (b) the average crack length. A hypothesis was proposed to explain the shape of the observed micro-crack density vs. impact stress curve.

(vi) A number of hypothetical composite materials were theoretically considered with the hope of improving the mechanical properties of currently used orthopaedic implant materials. The computations were done with intramedullary nails and prostheses in mind. The theoretical considerations suggest that among the hypothetical materials, the following would probably be the most suitable for intramedullary nails from the point of view of the dynamic compressive modulus:

(a) PMMA - E glass, $V_f = 30$
(b) PMMA - 970 S glass, $V_f = 20$

noting again that other mechanical properties of the composite must also be taken into account in the final design of the material system.
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APPENDIX A: LINEAR PHENOMENOLOGICAL DESCRIPTIONS

This subject has been extensively dealt with in the literature (74 to 81). In the present treatment, which is largely based on Zaretskii (74), Volterra integral equations are taken as the starting point

\[ \int_0^t K(t, \tau) \phi(\tau) \, d\tau = f(t) \quad \text{(A.1)} \]

\[ \phi(t) - \lambda \int_0^t K(t, \tau) \phi(\tau) \, d\tau = f(t) \quad \text{(A.2)} \]

Equations (A.1) and (A.2) are known as linear Volterra integral equations of the first and second kind respectively. \( K(t, \tau) \) is known as the "kernel" of the integral equation and satisfies the condition,

\[ K(t, \tau) = 0, \text{ if } t < \tau \quad \text{(A.3)} \]

\( \lambda \) is an arbitrary complex number.

A.1 Aging Linear Hereditary (Memory) Materials

The Arutyunyan constitutive equation of aging linear viscoelastic materials can be represented in the following integral form (74):

\[ \varepsilon(t) = \frac{1}{E_0(t)} \left\{ \sigma(t) + \int_{\tau_1}^t K(t, \tau) \sigma(\tau) \, d\tau \right\} \quad \text{(A.4)} \]

This is a Volterra integral equation of the second kind. \( K(t, \tau) \) is called the "creep kernel", and denotes the creep rate (in a creep test, \( K(t, \tau_1) = \dot{\varepsilon}(t, \tau_1)/\varepsilon_0 \)). \( \tau_1 \) is the "age" of the material at the instant of loading.

For conceptual convenience, the aging process can be classified into two types.

(i) "Coupled Aging": In this case there is interaction between the external loading history and the aging process. Examples of this are found in stress corrosion and corrosion fatigue. The coupled aging process is very important for internal orthopaedic prosthetic and fixative devices. Rigorous constitutive equations for materials in these applications should include coupled aging.

(ii) "Decoupled Aging": In this case the aging process takes place independent of the load history. The aging in concrete and some soils can be considered to be decoupled.

The creep kernel for decoupled aging materials can be expressed in the form,

\[ K(t, \tau) = \phi(\tau) K_1(t-\tau) \quad \text{(A.5)} \]
where $\phi(\tau)$ is the "aging function", characterizing the process of time variation of material properties.

For coupled aging materials, the kernel is of the following form:

$$K(t, \tau) = K(\sigma, t, \tau) \quad (A.5a)$$

The integral operator,

$$\tilde{K} \int_0^t K(t, \tau) \phi(\tau) \, d\tau \quad \text{(A.6)}$$

transforms every periodic function $\phi(t)$ with period $T$ into another periodic function with the same period $T$, if and only if

$$K(t, \tau) = K(t - \tau) \quad \text{(A.7)}$$

Thus, the integral in Eq. (A.6) is insensitive to the origin of the time scale if the creep kernel is of the form of Eq. (A.7). For aging materials a choice for the creep kernel of a form other than Eq. (A.7) renders the constitutive equation Eq. (A.4) sensitive to the origin of the time scale $\tau_1$.

A.2 Non-Aging Linear Memory Materials

Integral Representation: If the material is non-aging or if the time scale of the process under consideration is such as to make aging negligible in the period of interest, the aging function of the decoupled material becomes

$$\phi(\tau) = 1 \quad \text{(A.8)}$$

Then, the creep kernel of Eq. (A.5) reduces to Eq. (A.7) and the constitutive equation Eq. (A.4) becomes insensitive to $\tau_1$, the age of the material at the instant of loading. Under these conditions, Eq. (A.4) reduces to

$$\epsilon(t) = \frac{1}{E_0} \left\{ \sigma(t) + \int_0^t K(t-\tau) \sigma(\tau) \, d\tau \right\} \quad \text{(A.9)}$$

Equation (A.9) may be recognized as the Boltzmann integral equation which was derived in 1875 using a simple superposition principle.

The solution of Volterra integral equations of the second kind can be found by Picard's method of successive approximations. Using Picard's method the following result may be proved:

If the Euclidean norms of $f(t)$ and $K(t, \tau)$ are bounded, the solution of Eq. (A.2) can be represented in the form

$$\phi(t) = f(t) + \lambda \int_0^t R(t, \tau)f(\tau) \, d\tau \quad \text{(A.10)}$$

The function $R(t, \tau)$ is called the "Resolvent". From the above result, it
follows that the solution of the Boltzmann integral equation, Eq. (A.9), can be expressed in the form:

\[ \sigma(t) = E_o \left\{ \varepsilon(t) - \int_0^t R(t-\tau) \varepsilon(\tau) \, d\tau \right\} \]  

(A.11)

\( R(t-\tau) \) denotes the rate of stress relaxation (in a stress relaxation test, \( R(t) = -\dot{\sigma}(t)/\sigma_o \)) and is known as the "relaxation kernel" or the "resolvent of the creep kernel".

The linear constitutive equations discussed so far were Volterra integral equations of the second kind.

**Differential Representation:**

An alternative linear phenomenological description of materials can be given in the differential form (75):

\[ P \sigma = Q \varepsilon \]  

(A.12)

where, \( P \) and \( Q \) are linear differential operators.

\[ P = a_n + a_{n-1}D + a_{n-2}D^2 + \ldots + D^n \]  

(A.13)

\[ Q = b_m + b_{m-1}D + \ldots + D^m \]  

(A.14)

\[ D = d/dt \]  

(A.15)

And, if \( \sigma_{ij} \) and \( \varepsilon_{ij} \) are the stress and strain tensors, \( S_{ij} \) and \( e_{ij} \) are the deviatoric stress and strain tensors, then for triaxial deviatoric deformations,

\[ \sigma = S_{ij} = \sigma_{ij} - \frac{1}{3} \sigma_{kk} \delta_{ij} \]  

(A.16)

\[ \varepsilon = e_{ij} = \varepsilon_{ij} - \frac{1}{3} \varepsilon_{kk} \delta_{ij} \]  

(A.17)

\[ \delta_{ij} = 1, \text{ when } i = j \]  

(A.18)

\[ = 0, \text{ when } i \neq j \]

and for dilatational deformations,

\[ \sigma = \sigma_{ii} \]  

(A.19)

\[ \varepsilon = \varepsilon_{ii} \]  

(A.20)

Substitution of Eqs. (A.13), (A.14), and (A.15) into Eq. (A.12) yields

\[ \frac{d^n \sigma}{dt^n} + a_1 \frac{d^{n-1} \sigma}{dt^{n-1}} + \ldots + a_n \sigma = F(t) = \frac{d^m \varepsilon}{dt^m} + b_1 \frac{d^{m-1} \varepsilon}{dt^{m-1}} + \ldots + b_m \varepsilon \]  

(A.21)
Volterra showed that the solution of Eq. (A.21) can be reduced to the solution of the following equation.

$$\phi(t) - \int_{0}^{t} K(t-\tau)\phi(\tau)\,d\tau = f(t)$$  \hspace{1cm} (A.22)

where,

$$\phi(t) = \frac{d^n\sigma}{dt^n}$$  \hspace{1cm} (A.23)

$$K(t-\tau) = \sum_{\alpha=1}^{n} a_{\alpha} \frac{(t-\tau)^{\alpha-1}}{(\alpha-1)!}$$  \hspace{1cm} (A.24)

$$f(t) = F(t) - c_{n-1}a_n - (c_{n-1}t + c_{n-2})a_2 - \cdots$$

$$\cdots - (c_{n-1} \frac{d^{n-1}}{(n-1)!}) + \cdots + c_1t + c_0a_n$$  \hspace{1cm} (A.25)

$$C_i = \frac{d^{(i)}\sigma}{dt^i} \bigg|_{t=0}, \quad i = 0 \text{ to } (n-1)$$  \hspace{1cm} (A.26)

It can be seen that Eq. (A.22) is a special case of Eq. (A.2), where

$$\lambda = 1$$  \hspace{1cm} (A.27)

$$K(t,\tau) = K(t-\tau)$$  \hspace{1cm} (A.28)

Thus, the linear differential form of phenomenological description Eq. (A.22) is a special case of the Volterra integral equation of the second kind, Eq. (A.2).

**Mechanical Model Representation:**

A number of familiar viscoelastic models obtained by combinations in series, parallel, and series-parallel, of two basic elements (the Hooke element and the Newton element) are shown in Fig. 3. It can be shown that if arbitrary numbers of Hooke and Newton elements are combined in various ways, the resulting viscoelastic model would have a constitutive equation of the type of Eq. (A.21). The order of the linear differential equation for such a model would equal the number of viscous elements in the model. Thus, all mechanical models obtained by a combination of Hookean and Newtonian elements form special cases of the linear differential form of phenomenological description, which in turn is a special case of the phenomenological description using Volterra integral equations of the second kind. Thus, all possible mechanical model representations can be obtained from either Eq. (A.9) by an appropriate choice of the creep kernel or from Eq. (A.21) by an appropriate choice of the coefficients $a_i$ and $b_i$. 
Summary:

Three different approaches to a phenomenological description of linearly viscoelastic materials have been discussed.

(i) **Integral Representation:**

(a) By Volterra integral equations of the second kind
(b) Using Boltzmann superposition principle for linear heredity, but also giving rise to a Volterra integral equation of the second kind.

(ii) **Differential Representation**

(iii) **Mechanical Model Representation.**

It has been shown that a phenomenological description using Volterra integral equations of the second kind, forms the most general of the approaches presented.

A.3 **Alternative Integral Representations for Linear Hereditary Materials**

An alternative form for Eq. (A.9) may be expressed as:

$$\varepsilon(t) = \int_0^t J(t-\tau) \frac{d\sigma}{d\tau}(\tau) d\tau$$

Equation (A.9) can be obtained from Eq. (A.29) by integrating the right hand side of Eq. (A.29) by parts.

Similarly, an alternative form for Eq. (A.11) may be expressed as:

$$\sigma(t) = \int_0^t G(t-\tau) \frac{d\varepsilon}{d\tau}(\tau) d\tau$$

J(t-\tau) is the "creep compliance" and G(t-\tau) is the "relaxation modulus". The "creep compliance" is the time dependent strain per unit stress in a creep experiment. The "relaxation modulus" is the time-dependent stress per unit strain in a stress relaxation experiment.

The "creep kernel" in Eq. (A.9) and the "creep compliance" in Eq. (A.29) are related as follows:

$$K(t) = \frac{dJ}{dt}(t)$$

The "relaxation kernel" in Eq. (A.11) and the "relaxation modulus" in Eq. (A.30) are similarly related

$$R(t) = -\frac{dG}{dt}(t)$$
A.4 Some Definitions

(a) The Retardation Spectrum

Definition: The retardation spectrum $L$ is defined by,

$$J(t) = J_0 + \int_0^\infty L (1-e^{-t/\tau}) \, d\ln\tau$$

where, $J(t)$ is the "creep compliance" and $J_0$ is the instantaneous value of the creep compliance.

Physical Interpretation:

The hatched area shown in the figure represents the contribution to creep associated with retardation times which lie in the range between $\ln\tau$ and $\ln\tau + d(\ln\tau)$.

(b) The Relaxation Spectrum

Definition: The relaxation spectrum $H$ is defined by

$$G(t) = G_\infty + \int_0^\infty H e^{-t/\tau} \, d\ln\tau$$

where, $G(t)$ is the "relaxation modulus" and $G_\infty$ is the value of the modulus at infinite time or the "relaxed modulus".

Physical Interpretation:
The hatched area shown in the figure represents the contribution to the modulus associated with relaxation times which lie in the range between \( \ln \tau \) and \( \ln \tau + d(\ln \tau) \).

(c) **Parameters of Vibration Experiments**

In a harmonic vibration experiment conducted at a circular frequency \( \omega \), on a linear viscoelastic solid, let the amplitude of the stress be \( \sigma_o \), and the amplitude of the strain be \( \varepsilon_o \). Let the strain lag behind the stress by a phase angle of \( \delta \).

\[
\sigma(t) = \sigma_o \sin(\omega t + \delta) \\
\varepsilon(t) = \varepsilon_o \sin \omega t
\]

The modulus \( M \), and the compliance \( c \), are defined below

\[
M = \frac{\sigma}{\varepsilon_o} ; \quad c = \frac{\varepsilon}{\sigma_o} = \frac{1}{M}
\]

Further, the complex modulus \( M^* \), and the complex compliance \( c^* \) are defined as follows

\[
M^* = M_1 + i M_2 ; \quad c^* = c_1 + i c_2
\]

where,

\[
M_1 = M \cos \delta ; \quad c_1 = c \cos \delta \\
M_2 = M \sin \delta ; \quad c_2 = c \sin \delta
\]

Further, \( \sigma^* \), \( \varepsilon^* \), \( M^{**} \) and \( c^{**} \) are defined as follows:

\[
\sigma^* = \sigma_o (\cos \delta + i \sin \delta) ; \quad \varepsilon^* = \varepsilon_o (\cos \delta - i \sin \delta) \\
M^{**} = \frac{\sigma^*}{\varepsilon^*} ; \quad c^{**} = \frac{\varepsilon^*}{\sigma^*}
\]

With these definitions, it follows that

\[
M^* = \frac{\sigma^*}{\varepsilon_o} ; \quad c^* = \frac{\varepsilon^*}{\sigma_o} \\
c^* = \frac{1}{M^*} ; \quad c^{**} = \frac{1}{M^{**}}
\]

Thus, the vibration experiment described above can be characterized by any one of the following sets of quantities:

(i) \[ \{ M(\omega) , \delta(\omega) \} \]
(ii) \[ \{ c(\omega) , \delta(\omega) \} \]
(iii) \[ \{ M_1(\omega) , M_2(\omega) \} \] or \[ \{ M^*(\omega) \} \]
(iv) \[ \{ c_1(\omega) , c_2(\omega) \} \] or \[ \{ c^*(\omega) \} \]
The inter-relations between these four sets of quantities are tabulated in Table A.1.

The quantities M* and c* are known as the "complex modulus" and the "complex compliance" respectively. Alternatively, they are called the "dynamic modulus" and the "dynamic compliance" respectively.

M_1 and M_2 are known as the components of the complex modulus. c_1 and c_2 are known as the components of the complex compliance.

A.5 Inter-Relations Between Creep, Stress Relaxation, and Vibration

For linear viscoelastic materials, inter-relations exist among the following quantities:

(a) creep compliance, J(t)
(b) relaxation modulus, G(t)
(c) the retardation spectrum, L(t)
(d) the relaxation spectrum, H(t)
(e) components c_1(\omega), c_2(\omega) of the dynamic compliance
(f) components M_1(\omega), M_2(\omega) of the dynamic modulus.

Exact and approximate inter-relations among these various functions are given by Ferry (78).
APPENDIX B: NON-LINEAR PHENOMENOLOGICAL DESCRIPTIONS

B.1 Integral Representations

B.1.1 Aging Materials

The Arutunyan constitutive equation for linear hereditary materials, Eq. (A.4) can be generalized for this case as follows:

\[ \varepsilon(t) = \frac{1}{E_0(t)} \left\{ f\left[ \sigma(t) \right] + \int_{t_1}^{t} K(t,\tau)f\left[ \sigma(\tau) \right] d\tau \right\} \]  \hspace{1cm} (B.1)

B.1.2 Non-Aging Materials

The Boltzmann integral equations Eq.(A.9) can be generalized for this case as follows:

\[ \varepsilon(t) = \frac{1}{E_o} \left\{ f\left[ \sigma(t) \right] + \int_{0}^{t} K(t,\tau)f\left[ \sigma(\tau) \right] d\tau \right\} \]  \hspace{1cm} (B.2)

Alternatively, the Boltzmann equation, Eq. (A.29) can be generalized as follows:

\[ \varepsilon(t) = \int_{0}^{t} J(t,\tau) \frac{d}{d\tau} f\left[ \sigma(\tau) \right] d\tau \]  \hspace{1cm} (B.3)

Leaderman (82), used the following equation:

\[ \varepsilon(t) = \frac{1}{E_o} \left\{ \sigma(t) + \int_{-\infty}^{t} J(t,\tau) \frac{d}{d\tau} f\left[ \sigma(\tau) \right] d\tau \right\} \]  \hspace{1cm} (B.4)

Rozovskii (74) suggested a similar equation of state:

\[ \varepsilon(t) = \frac{1}{E_o} \left\{ \sigma(t) + \int_{0}^{t} K(t,\tau)f\left[ \sigma(\tau) \right] d\tau \right\} \]  \hspace{1cm} (B.5)

Ward and Onat (82), attempted to generalize the Boltzmann superposition principle in a formally acceptable mathematical manner.

\[ \varepsilon(t) = F \left[ \frac{d\sigma}{d\tau} (\tau) \right]_{\tau=-\infty}^{t} \]  \hspace{1cm} (B.6)

If the functional F is linear and continuous, Eq. (B.6) reduces to the Boltzmann equation of linear viscoelasticity. Frechet (82), has shown that where F is continuous and non-linear, the functional F can be represented to any desired degree of accuracy as follows:

B1
\[ \epsilon(t) = \int_{-\infty}^{t} J_1(t-\tau) \frac{d\sigma}{d\tau_1}(\tau_1) d\tau_1 + \int_{-\infty}^{t} \int_{-\infty}^{t} J_2(t-\tau_1, t-\tau_2) \frac{d\sigma}{d\tau_1}(\tau_1) \frac{d\sigma}{d\tau_2}(\tau_2) d\tau_1 d\tau_2 \]

\[ \text{and} \]

\[ \int_{-\infty}^{t} \int_{-\infty}^{t} J_N(t-\tau_1, \ldots, t-\tau_N) \frac{d\sigma}{d\tau_1}(\tau_1) \ldots \frac{d\sigma}{d\tau_N}(\tau_N) d\tau_1 \ldots d\tau_N \quad (B.7) \]

A physical interpretation of this mathematical representation is obtained by considering that the loading programme consists of a superposition of infinitesimal loading steps. The integrand of the first term in Eq. (B.7) can then be interpreted as representing the individual and independent contribution of the loading step \( d\sigma(\tau_1) \) to the final elongation. The integrand of the second term represents the joint contribution of the loading steps \( d\sigma(\tau_1) \) and \( d\sigma(\tau_2) \) to the final elongation.

When the stress-strain relations for \( t = 0 \) and \( t > 0 \) are different, and the creep curves are geometrically dissimilar (the kernel depends on the stress), the equation of state may be represented as (74).

\[ \epsilon(t) = \int_{0}^{t} [\sigma(t)] + \int_{0}^{t} K(\sigma; t, \tau)f[\sigma(\tau)] d\tau \quad (B.8) \]

B.2 Power Law Representations

In this case the phenomenological description is given by means of a power law relationship between stress, strain and time.

B.3 Mechanical Model Representations

B.3.1 The Eyring Model (83)

This model is shown in Fig. 4. Eyring used a dashpot whose constitutive equation has the form,

\[ \dot{\epsilon} = K \sinh(\alpha \sigma) \quad (B.9) \]

The dashpot is thus characterized by two constants \( K \) and \( \alpha \). This model was largely inspired by concepts that were used very successfully in explaining relaxation processes in metals (81). The relaxation of preferential distributions of atoms induced by stress in disordered solid solutions of cubic structure, and relaxation of ordered distributions of crystals (81) can, in particular, be explained in terms of the activation energy. This explanation gives rise to exponential viscosities. Eyring's model leads to exponential viscosities when \( \alpha \) is very large and Newtonian viscosity when \( \alpha \) is very small. Thus, Eyring's dashpot is a more general dashpot than either the exponential or the Newtonian.

The constitutive equation for Eyring's model becomes manageable, only under very special circumstances. When the strain rate is made a constant, \( \dot{\epsilon} = \sigma = \text{const.} \), the constitutive equation becomes,
\[
\frac{d\theta}{dt} = \beta - \sinh \phi
\]  
(B.10)

where,
\[
\begin{align*}
\beta &= \frac{\rho}{K} \\
\phi &= \omega \tau
\end{align*}
\]  
(B.11)

B.3.2 Mechanical Models Incorporating a Simple Second Degree Dashpot

The mechanical models discussed in this section incorporate a simple second degree dashpot whose constitutive equation is given by
\[
\sigma = \eta \varepsilon
\]  
(B.12)

where, \( \eta = \text{const.} \), is the damping coefficient.

For a Newtonian dashpot,
\[
\sigma = \eta' \dot{\varepsilon}
\]  
(B.13)

where, \( \eta' = \text{const.} \), is the damping coefficient.

In terms of the Newtonian dashpot, the damping coefficient for the second degree dashpot becomes (comparing Eqs. (B.12) and (B.13))
\[
\eta' = \eta \varepsilon
\]  
(B.14)

The viscous resistance to deformation increases linearly with strain for this second degree damper. From compressive impact tests on soils (84), it is known that the attenuation of the impact strain (or stress) is greater at greater values of the impact strain (or stress). The author's own observations on bone under compressive impact indicate that its viscous resistance increases with increasing strain. The expectation that such materials can be approximated by mechanical models incorporating the damper described by Eq. (B.12) led the author to consider the constitutive equations of such models.

Some of the models considered are shown in Fig. 4. Model M1 is a non-linear form of the Kelvin-Voigt model. Its constitutive equation is given by
\[
\sigma = E \varepsilon + \eta \varepsilon
\]  
(B.15)

Model M2 is a non-linear form of the Maxwell model. Its constitutive equation is
\[
\sigma = \eta \varepsilon - \frac{\eta}{E'} \frac{d}{dt} (\sigma \varepsilon) + \frac{\eta}{E'} \sigma \dot{\varepsilon}
\]  
(B.16)

Model M3 is a non-linear form of the standard linear solid. Its constitutive equation is
\( \sigma = E \varepsilon + \eta \left( 1 + \frac{E^2}{E'}^2 \right) \varepsilon + \frac{1}{E'} \sigma \dot{\varepsilon} - \frac{\eta}{E'} \left( 1 + \frac{E}{E'} \right) \frac{d}{dt} (\sigma \varepsilon) \) \hspace{1cm} (B.17)

**B.4 Differential Representations**

The phenomenological description of non-linear viscoelastic materials can also be given in the form of a differential equation:

\[ P' \sigma = Q' \varepsilon \] \hspace{1cm} (B.18)

where, \( P' \) and \( Q' \) are non-linear differential operators given by,

\[ P' = f_n(\sigma, \varepsilon) + f_{n-1}(\sigma, \varepsilon) D + f_{n-2}(\sigma, \varepsilon) D^2 + \ldots + D^n \] \hspace{1cm} (B.19)

\[ Q' = g_m(\sigma, \varepsilon) + g_{m-1}(\sigma, \varepsilon) D + \ldots + D^m \] \hspace{1cm} (B.20)

and

\[ D = \frac{d}{dt} \] \hspace{1cm} (B.21)
APPENDIX C: DERIVATION OF THE CONSTITUTIVE EQUATION FOR THE
MECHANICAL MODEL IN FIG. 5(b)

The stress acting on each of the Kelvin-Voigt elements is the same. So, we can write,
\[ \sigma_i = \sigma, \quad i = 1 \text{ to } n \]  
(C.1)

A subscript or superscript 'i' refers to the ith Kelvin-Voigt element. For the first element in the series, which has a simple non-linearity, we can write (see Eq. (B.15))
\[ \sigma_1 = E \epsilon_1 + \eta \epsilon_1 \dot{\epsilon}_1 \]  
(C.2.1)

For the ith element,
\[ \sigma_i = E_i \epsilon_i + \eta_i \epsilon_i \dot{\epsilon}_i, \quad i = 2 \text{ to } n \]  
(C.2.2)

Since the strain of the total system equals the sum of the strains of the individual elements, we can write
\[ \epsilon = \sum_{i=1}^{n} \epsilon_i \]  
(C.3)

Equations (C.2.1) and (C.2.2) can be rewritten as,
\[ \frac{\sigma_1}{E} = \epsilon_1 + \frac{\eta_1}{E} \dot{\epsilon}_1 \]  
(C.4.1)

\[ \frac{\sigma_i}{E_i} = \epsilon_i + \frac{\eta_i}{E_i} \dot{\epsilon}_i, \quad i = 2 \text{ to } n \]  
(C.4.2)

Define,
\[ \tau_i = \frac{\eta_i}{E_i} \]  
(C.5)

Substituting Eq. (C.5) into Eq. (C.4.2) and rearranging Eqs. (C.4.1) and (C.4.2),
\[ \epsilon_1 = \frac{\sigma_1}{E} - \frac{\eta_1}{E} \dot{\epsilon}_1 \]  
(C.6.1)

\[ \epsilon_i = \frac{\sigma_i}{E_i} - \frac{\tau_i}{E_i} \epsilon_i, \quad i = 2 \text{ to } n \]  
(C.6.2)

Substituting Eq. (C.1) in Eqs. (C.6.1) and (C.6.2),
\[ \epsilon_1 = \frac{\sigma}{E} - \frac{\eta_1}{E} \dot{\epsilon}_1 \]  
(C.7.1)
\[ \varepsilon_i = \frac{\sigma}{E_i} - \frac{\tau^i}{\tau^i_E} \dot{\varepsilon}_1 \], \quad i = 2 \text{ to } n \quad (C.7.2) \]

Combining Eq. (C.3) with Eqs. (C.7.1) and (C.7.2),

\[ \varepsilon = \sigma \left( \frac{1}{E} + \sum_{i=2}^{n} \frac{1}{E_i} \right) - \left( \frac{\eta \varepsilon_1}{E} \dot{\varepsilon}_1 + \sum_{i=2}^{n} \tau^i \dot{\varepsilon}_i \right) \quad (C.8) \]

Thus, the stress-strain behaviour of the model is given by the following system of equations:

\[ \varepsilon = \sigma \left( \frac{1}{E} + \sum_{i=2}^{n} \frac{1}{E_i} \right) - \left( \frac{\eta \varepsilon_1}{E} \dot{\varepsilon}_1 + \sum_{i=2}^{n} \tau^i \dot{\varepsilon}_i \right) \quad (C.8) \]

where,

\[ \varepsilon_1 = \frac{\sigma}{E} - \frac{\eta \varepsilon_1}{E} \dot{\varepsilon}_1 \quad (C.7.1) \]

\[ \varepsilon_i = \frac{\sigma}{E_i} - \frac{\tau^i}{\tau^i_E} \dot{\varepsilon}_1 \], \quad i = 2 \text{ to } n \quad (C.7.2) \]

and

\[ \frac{\tau^i}{\tau^i_E} = \frac{\eta_i}{E_i} \], \quad i = 2 \text{ to } n \quad (C.5) \]
APPENDIX D: MEANING OF "PHENOMENOLOGICAL DESCRIPTIONS" AND "ULTRA-PHENOMENOLOGICAL" CONSIDERATIONS

Phenomenology:

Phenomenology is a philosophical trend founded by the Austrian philosopher Edmund Husserl (1859-1938). Husserl's phenomenology is a methodological principle. A phenomenologist, in his observations, uses an absolutely unbiased approach and observes phenomena as they manifest themselves and only as they manifest themselves. This is accomplished by a method that Husserl called "epoche". The observer excludes from his mind not only any judgement of value about the phenomena but also any affirmation whatever concerning their cause and background. With this method, previously unnoticed structures of phenomena become apparent. Phenomenology was born as a reaction against the classical methods of observation which took note of only those data that fit the pre-conceived models of the observer.

"Phenomenological Descriptions" in Viscoelasticity:

A "phenomenological description" is a description of the phenomenon as it manifests itself and only as it manifests itself.

The phenomenological equations that have been described in Section 2 are used as empirical equations describing the observed relationship between stress, strain and time. At the present time, most of the viscoelastic equations of state used are purely phenomenological in character (74), that is, they have no significance beyond the fact that they represent vast amounts of experimental data by means of a concise (sometimes not so concise) mathematical expression. This empirical relationship between the mathematical equations and the experimental data is considered sufficient justification for the use of the equations.

"Ultra-Phenomenological" Considerations:

The disadvantage of purely phenomenological descriptions of experimental observations is that an unlimited number of variations and generalizations of mathematical equations are possible. Therefore, a successful description of properties is not conclusive and no single physical theory can be established on this basis.

Therefore, a two-stage procedure for scientific investigations is proposed.

Stage I: In the stage of experimental data-gathering, Husserl's phenomenological method is used.

Stage II: At the second stage, where the collected data has to be used for practical purposes, ultra-phenomenological considerations of the type discussed in Section 2.3 become important. At this stage, attempts are made to give physical explanations of the observed data.
APPENDIX E: Computer Program for the Analysis of Hopkinson-Bar Data

// FOR
*LIST SOURCE PROGRAM
*I0CS(CARD,TYPEWRITER,1132 PRINTER)
*ONE WORD INTEGERS

COMMON MX,MY
5 READ(2,1)KK
1 FORMAT(12)
 IF(KK)6,6,7
6 GO TO 5
7 IF(KK-2)2,2,3
2 CALL BM1(KK)
 WRITE(2,1)KK
 GO TO 5
3 CALL EXIT
 END

FEATURES SUPPORTED
ONE WORD INTEGERS
10CS

CORE REQUIREMENTS FOR
COMMON 2 VARIABLES 2 PROGRAM 56

END OF COMPILATION

// XEQ
1

*LOCAL, GDATA, ORDER, MINV, MULTR

// FOR
*LIST SOURCE PROGRAM
*ONE WORD INTEGERS

SUPROUTINE BM1(KK)
C DATA REDUCTION FOR THE HOPKINSON SPLIT BAR APPARATUS
C
C = SOUND SPEED IN STEEL (FT/SEC)
C DT = TIME INTERVAL (SEC = SEC/DIV*DIV/CM*CM)
C DVCM = MAGNIFICATION OF TRACE PHOTO (CM/DIV)
C DX = TRACE X-AXIS MEASUREMENT INTERVAL
C E = ELASTIC MODULUS OF STEEL (PSI)
C IRUN = RUN NUMBER
C ISPEC = SPECIMENT NUMBER
C N = NUMBER OF DATA POINTS
C FP = GUN FIRING PRESSURE (PSI)
C RHO = DENSITY OF STEEL (SLUGS/CU.FT)
C SF = TRACE PHOTO SCALE FACTORS (V/CM = V/DIV*DIV/CM)
C TDIV = SWEEP RATE (SEC/DIV)
C VDIV = VOLS/DIV
C YMAX = INCIDENT STRESS (CM)
C SL = SPECIMEN LENGTH (IN)
C DB = DIAMETER OF SPECIMEN (IN)
C WT = WEIGHT OF SPECIMEN (GM)
C DENS=DENSITY OF SPECIFICA (GM/CC)
C B1,B2 = BRIDGE CALIBRATION FACTORS (IN/IN/V)
C E1,E2 = BRIDGE POWER SUPPLY VOLTAGES
C Y1,Y2 = OUTPUT FROM EIGHT ANVIL BRIDGES (CM)
C Z1,Z2 = TRACE ZERO (CM)
DIMENSION SIG(50),SNR(50),EPS(50),Y1(50),Y2(50),Y3(50)
DIMENSION STRR(50)
COMMON MX,MY
MX=3
MY=2
F=30000000.
RHO=7.86*62.4/32.2
TDIV=O.00007
DX=O.05
READ(MY,1001)IRUN,E1,E2
B1=F/(F1*1.95)
B2=E/(E2*1.95)
C=SQRT(E*144./RHO)
Q=C*RHO/1728.
RA=E*B1
RB=E*B2
READ(MY,1001)IRUN,ISPEC,IPMA,FP,VDIV,DVC,N,SL,DT,Y1MAX,Z1,Z2
F=FORMAT (313,F4.0,F6.3,F4.1,I3,F6.3,3F6.2)
999 DO 3 I=1,N
READ(MY,1001)IRUN,Y1(I),Y2(I)
1001 FORMAT(I3,F6.2)
CONTINUE
SF=VDIV/DVC
DT=TDIV*(1./DVC)*DX
S=ABS(Y1MAX-Z1)*RA*SF*(.375/DB)*(375/DB)
SLA=SL
A=ABS(Y1(I)-Z1)*SF
B=ABS(Y2(I)-Z2)*SF
DENS=W/(SLA*(DB/2.)*(DB/2.)*3.1412*15.625)
SA=A*RA*(.375/DB)*(375/DB)
SB=B*RB*(.375/DB)*(375/DB)
SIG(1)=(SA+SB)/2.
VA=(S*2.-SA)/Q
VB=SR/Q
SNR(1)=(VA-VB)/SLA
EPS(1)=O.
DO 1 I=2,N
A=ABS(Y1(I)-Z1)*SF
B=ABS(Y2(I)-Z2)*SF
SA=A*RA*(375/DB)*(375/DB)
SB=B*RB*(375/DB)*(375/DB)
SIG(I)=(SA+SB)/2.
VA=(S*2.-SA)/Q
VB=SR/Q
SNR(I)=(VA-VB)/SLA
J=I-1
EPS(I)=EPS(J)+(SNR(I)+SNR(J))*DT/2.
SLA=SL-EPS(I)*SL
Y1(I)=SIG(I)/EPS(1)
Y2(I)=SNR(I)/EPS(1)
CONTINUE
CALL BONE1(DT,SIG,STRR,N,IER)
Y1(I)=0.
Y2(1) = 0.0
Y3(1) = 0.0
DO 5 I = 2, N
5 Y3(I) = STRR(I) / EPS(I)
WRITE(*,3000) IRUN, ISPEC, FP
1 F4.0, ' (PSI) ')
WRITE(*, 2001) DT
2001 FORMAT(1H0, 5X, 'TIME INTERVAL = ', F9.7, ' (SEC)', 2X, 'NUMBER OF DATA
1 POINTS = ', '12)
WRITE(*,2002) E, RHO
2002 FORMAT(1H0, 5X, 'E (STEEL) = ', 'F10.1, ' (PSI)', 2X, 'RHO (STEEL) = ',
1 F6.2, ' (SLUG/CU. FT) ')
WRITE(*, 2011) C
2011 FORMAT(1H0, 5X, 'C (STEEL) = ', 'E11.4, ' (FT/SEC) ')
WRITE(*, 2003) H1, B2
2003 FORMAT(1H0, 5X, 'BRIDGE FACTORS, B1, B2 = ', '2E12.4,
1 ' (IN/IN/V' )
WRITE(*, 2005) SL
2005 FORMAT(1H0, 5X, 'SPECIMEN LENGTH = ', 'E10.3, ' (IN) ')
WRITE(*, 2006) S
2006 FORMAT(1H0, 5X, 'INCIDENT STRESS=', 'E11.4, ' (PSI) ')
WRITE(*, 2020) DENS
2020 FORMAT(1H0, 5X, 'SPECIMEN DENSITY=', 'F6.3, ' GM/CC ')
WRITE(*, 2021) IPMA
2021 FORMAT(1H0, 5X, 'SPECIMEN POST MORTEM AGE=', 'I4, ' DAYS' ')
WRITE(*, 9) IER
WRITE(*, 1234) F1, E2
1234 FORMAT(1 ', 'E1=', 'F6.3, 3X,'E2=', 'F6.3)
9 FORMAT(1 ', 'IER=', 'I4)
WRITE(*, 2007) 1ER
2007 FORMAT(1H0, 1I ', 2X, ' STRESS', '2X, ' STRAIN', '2X,
1 ' STRESS RATE', ' STRAIN RATE', ' MODULUS', ' STRAIN RATE/
2 STRAIN', '4X, 'STRESS RATE/STRAIN')
WRITE(*, 2010) 1ER
2010 FORMAT(1H0, 5X, 'PSI) ', '2X, ' (IN/IN) ', '2X,
1 ' (PSI/SEC) ', '2X, ' (IN/IN/SEC)', '2X, ' (PSI)', ',
2 ' (PER SEC
11X, ' (PSI/SEC) ')
WRITE(*, 2008) (1, SIG(I), EPS(I), STKR(I), SNR(I), Y1(I), Y2(I), Y3(I),
1 I = 1, N)
2008 FORMAT(1H0, 1I ', 36E14.4, 7X, E14.4)
READ(*, 1002) IRUN
1002 FORMAT(12)
IF(KK=1) 909, 7, 8
7 WRITE(*, 1003) (Y1(I), Y2(I), Y3(I), Y3(I), Y3(I), Y3(I), Y3(I), Y3(I),
1 I = 1, N)
GO TO 909
8 WRITE(*, 1003) (SIG(I), EPS(I), STKR(I), SNR(I), Y1(I), Y2(I), Y3(I),
1 I = 1, N)
1003 FORMAT(6E12.4)
909 RETURN
END

E3
// FOR
*LIST SOURCE PROGRAM
*ONE WORD INTEGERS

SUBROUTINE BONE1 (H, Y, Z, NDIM, IER)

DIMENSION Y(50), Z(50)

IF (NDIM=3) I4, 1, 1

1 IF (H) 2, 5, 2

2 HH = 5 / H

NDIM2 = NDIM - 2

YY = Y(NDIM2)

R = Y(2) + Y(2)

B = HH * (R + R - Y(3) - Y(1) - Y(1) - Y(1))

DO 3 I = 3, NDIM

A = B

1 M 2 = I - 2

BB = HH * (Y(I) - Y(I-2))

3

Z(I-2) = A

IER = 0

NDIM1 = NDIM - 1

A = Y(NDIM1) + Y(NDIM1)

Z(NDIM1) = HH * (Y(NDIM1) + Y(NDIM1) + Y(NDIM) + Y(NDIM) - A - A + YY)

Z(NDIM1) = B

RETURN

4 IER = -1

RETURN

5 IER = 1

RETURN

END

FEATURES SUPPORTED
ONE WORD INTEGERS

CORE REQUIREMENTS FOR BONE1
COMMON U VARIABLES 16 PROGRAM 216

RELATIVE ENTRY POINT ADDRESS IS 0016 (HEX)

END OF COMPILATION

// FOR
*LIST SOURCE PROGRAM
*ONE WORD INTEGERS
*IOCS (CARD, TYPEWRITER, 1132 PRINTER)

COMMON MX, MY

CALL BM2

CALL EXIT

END
// FOR
*LIST SOURCE PROGRAM
*ONE WORD INTEGERS
SUBROUTINE RN2
DIMENSION SIGEX(50), EPS(50), STRR(50), SNR(50), L(50), X(150)
COMMON MX, MY
M = 3
READ(2, 12) MX, MY
12 READ(2, 12) MY
20 READ(MY, 21) IRUN
21 FORMAT(14)
WRITE(MX, 22) IRUN
22 FORMAT('1', 'IRUN=', 14)
READ(MY, 12) N
READ(MY, 12) K
READ(MY, 12) (L(I), I=1, K)
DO 1 I = 1, K
KI = K + I
KK = 2*K + I
READ(MY, 2) X(I), X(KI), X(KK)
1 CONTINUE
2 FORMAT(3E12.4)
READ(MY, 6) (SIGEX(I), EPS(I), STRR(I), SNR(I), I=1, N)
6 FORMAT(4E12.4)
17 READ(MY, 12) NNN
ITER = 0
3 CALL BONE2(X, K, M, E, YETA, TAU, NNN)
18 IF( NNN = 213, 14, 8
14 STIF1 = -1.0*YETA/TAU
STIF2 = 1.0 + (E*TAU)/YETA
STIF2 = E/STIF2
DASH = YETA/STIF2
WRITE(MX, 15) STIF1, STIF2, DASH
15 FORMAT(' ', 'STIF1=', E12.5, 'STIF2=', E12.5, 'DASH=', E12.5)
GO TO 16
13 TAU = 0.0
16 CALL BONE3(SIGEX, EPS, STRR, SNR, L, K, N, E, YETA, TAU, X, ERR2)
17 IF(ERR2 = 100.0) 8, 8, 7
7 ITER = ITER + 1
8 IF(ITER = 2) 9, 9, 10
9 GO TO 3
8 GO TO 20
10 WRITE(MX, 11)
11 FORMAT(' ', 'ITERATIONS EXCEEDED')
GO TO 20
23 RETURN
END
DO 115 J=1,K
L=ISAVE(J)
WRITE(MX,4)L,XBAR(L),STD(L),RY(J),B(J),SB(J),T(J)
WRITE(MX,5)
L=ISAVE(M)
WRITE(MX,4)L,XBAR(L),STD(L)
WRITE(MX,6)ANS(1),ANS(2),ANS(3)
WRITE(MX,7)
L=ANS(8)
WRITE(MX,8)K,ANS(4),ANS(6),ANS(10),L,ANS(7),ANS(9)
L=N-1
SUM=ANS(4)+ANS(7)
WRITE(MX,2)L,SUM
CONTINUE
E=ANS(1)
YETA=B(1)
TAU=B(2)
RETURN
END

// FOR
*LIST SOURCE PROGRAM
*ONE WORD INTEGERS
SURROUTINE RONE3(SIGEX,EPS,SNR,L,K,N1,E1,YETA1,TAU1,X,ERR2)
DIMENSION SIGEX(50),EPS(50),SNR(50),STRR(50),SIGC(50)
DIMENSION L(50)
DIMENSION X(1)
COMMON MX,M,Y
WRITE(MX,3)E1,YETA1,TAU1,N1
3 FORMAT('0',E = 'E12.5',2X,YETA = 'E12.5',2X,TAU = 'E12.5',2X,N = ',',12)
WRITE(MX,8)
8 FORMAT(' ',ERROR '13X,SNR/EPS '12X,'E *EPS',9X, 
YETA *SNR',5X,'TAU*STRR',7X,'MODULUS')
EEPSO=0.0
DO 15 I=1,K
KK=L(I)
15 EEPSO=EEPSO+SIGEX(KK)-E1*EPS(KK)-YETA1*SNR(KK)-TAU1*STRR(KK)
RN1=K
EEPS0=EEPSO/RN1
EPS0=EEPS0/E1
DO 18 I=1,N1
18 EPS(I)=EPS(I)+EPSO
ERR1=0.0
DO 10 I=1,N1
R1=E1*EPS(I)
R2=YETA1*SNR(I)

E7
R3 = TAU1 * STRR(1)
SIGC(1) = R1 + R2 + R3
ERROR = (1.0 - SIGC(1)/SIGEX(1)) * 100.0
ERR1 = ERR1 + ARS(SIGEX(1) - SIGC(1)) / X
YM0D = SIGEX(1) / EPS(1)
XXX = SNR(1) / EPS(1)
R1 = R1 * 100.0 / SIGEX(1)
R2 = R2 * 100.0 / SIGEX(1)
R3 = R3 * 100.0 / SIGEX(1)
WRITE (MX, 7) ERROR, XXX, R1, R2, R3, YM0D
10 CONTINUE
ERR2 = 0.0
DO 16 I = 1, K
KK = L(I)
16 ERR2 = ERR2 + ABS(SIGEX(1) - SIGC(1)) / K
WRITE (MX, 11) EPS0, EPS0, ERR1, ERR2
11 FORMAT (' ', 'EPSO=', E12.5, 4X, 'ERR1=', E12.5, 4X, 'ERR2=', E12.5)
DO 13 I = 1, K
KK = L(I)
X(I) = SIGEX(KK) / EPS(KK)
KI = K + I
X(KI) = SNR(KK) / EPS(KK)
KKI = 2 * K + I
X(KKI) = STRR(KK) / EPS(KK)
13 CONTINUE
RETURN
END

FEATURES SUPPORTED
ONE WORD INTEGERS

CORE REQUIREMENTS FOR BONE3
COMMON 2 VARIABLES 138 PROGRAM 624

RELATIVE ENTRY POINT ADDRESS IS 00FC (HEX)

END OF COMPILATION
APPENDIX F: CALCULATION OF THE LOSS TANGENT FROM STRESS RELAXATION DATA

The experimental data of Lugassy & Korostoff (48) is used here to calculate the loss tangent for stress relaxation. Define,

\[-b = \frac{d \left\{ \frac{\sigma(t)}{\sigma_0} \right\}}{d \log t} \quad \text{(F.1)}\]

where, \(\sigma(t)\) is the stress measured as a function of time 't' in a stress relaxation experiment. \(\sigma_0\) is the instantaneous stress in the same experiment.

In a stress relaxation experiment where the strain is held constant,

\[\frac{E(t)}{E_0} = \frac{\sigma(t)}{\sigma_0} \quad \text{(F.2)}\]

where, \(E(t)\) is the relaxation modulus and \(E_0\) is the instantaneous modulus.

From p.89 of Ref. 78,

\[- \frac{d E(t)}{d \ln t} \bigg|_{t=\tau} = H(\tau) \quad \text{(F.3)}\]

where, \(H(\tau)\) is the relaxation spectrum (see Appendix A, Section A.4). Equation (F.3) is known as Alfrey's Rule.

Using Eqs. (F.1, F.2 and F.3), we get

\[H(\tau) = \frac{b E_0}{2.303} \quad \text{(F.4)}\]

If \(b\) is constant, it is seen from Eq. (F.4) that the relaxation spectrum is "flat" \((H(\tau) = \text{constant})\). From p. 98 of Ref. 78,

\[M_2(\omega) = \left[ \frac{H}{B} \left(1 - |m|\right) \right] \tau = 1/\omega \quad \text{(F.5)}\]

where, \(M_2(\omega)\) is the imaginary part of the complex modulus (see Appendix A, Section A.4) as a function of the circular frequency '\(\omega\)', \(H\) is the relaxation spectrum, \(B\) is a function tabulated on p. 106 of Ref. 78, and 'm' is the negative logarithmic slope of \(H\). For a flat relaxation spectrum,

\[m = 0 \quad \text{and} \quad B = 0.637 \quad \text{(F.6)}\]
Substitution of (F.6) into (F.5) gives

\[ M_2(\omega) = \left[ \frac{H}{0.637} \right] \frac{1}{\tau} = \frac{1}{\omega} \] (F.7)

Substitution of Eq. (F.4) into Eq. (F.7) yields,

\[ M_2(\omega) = b E_o / (2.303 \times 0.637) = 0.68 b E_o \] (F.8)

From the data of Lugassy & Korostoff (48) for bovine femoral cortical bone,

\[ b = 0.026, \text{ for high strain amplitude-high strain rate tests} \]
\[ b = 0.033, \text{ for low strain-low low low} \]

In the following calculation, an intermediate value of \( b = 0.03 \) is used.

\[ b = 0.03 \] (F.9)

From p.99 of Ref. 78,

\[ M_1(\omega) = \left[ E(\tau) + H(\tau) \psi(m) \right] \frac{1}{\tau} = \frac{1}{\omega} \] (F.10)

where, \( M_1(\omega) \) is the real part of the complex modulus (see Appendix A, Section A.4), \( E(\tau) \) is the relaxation modulus, \( H(\tau) \) is the relaxation spectrum, \( \psi(m) \) is a function tabulated in Ref. 78, on p.106, and \( m \) is the negative logarithmic slope of \( H \). For a flat relaxation spectrum,

\[ m = 0 \]
\[ \psi(m) = 0.577 \] (F.11)

Using Eqs. (F.4), (F.10) and (F.11),

\[ M_1(\omega) = \left[ E(\tau) + 0.577 \frac{b E_o}{2.303} \right] \frac{1}{\tau} = \frac{1}{\omega} \] (F.12)

Substituting the value of 'b' from Eq. (F.9) into Eq. (F.12),

\[ M_1(\omega) \approx E(\tau) \bigg|_{\tau=1/\omega} \] (F.13)

The loss tangent, \( \tan \delta \), is defined as,

\[ \tan \delta = \frac{M_2(\omega)}{M_1(\omega)} \] (F.14)

Using, Eqs. (F.8), (F.9), (F.13) and (F.14),
\[ \tan \delta \approx 0.02 \frac{E_0}{E(\tau)} \bigg|_{\tau = 1/\omega} \]  

\[ \text{(F.15)} \]

From the data of Lugassy & Korostoff (48), the value of \( \frac{E(\tau)}{E_0} \) varies from 1 at \( \tau = 0 \) to 0.84 at \( \tau = 1000 \) minutes.

Substituting these values into Eq. (F.15),

\[ \tan \delta \approx 0.02 \text{ to } 0.017 \]  

\[ \text{(F.16)} \]
APPENDIX G: METHOD USED TO QUANTIFY MICROSCOPIC DAMAGE UNDER COMPRESSION IMPACT

Human femoral compact bone specimens 0.375" in length and 0.25" in diameter were subjected to compressive impact stresses in the Hopkinson bar apparatus. After impacting, each specimen was cut across its length into two unequal parts. The shorter piece was used for observing the cross-section of the specimen while the longer piece was used for observing the longitudinal section. The two pieces were placed in a non-adhering cylindrical plastic mold. A commercial resin mixed with its hardener was then poured into the mold. The resin was allowed to cure for 24 hours, and then the hardened cylindrical resin block with the specimen embedded in it is separated from the casting mold. The specimens embedded in plastic are conveniently polished under a constant stream of water using standard metallurgical or geological techniques (metallurgists use this technique to observe the microstructure of metals; geologists use the same technique to study the mineral composition and structure of rock samples).

The polished specimens are observed under a reflecting optical microscope at X100 magnification. The specimens were, on occasion, stained with eosin yellow or aniline blue to obtain a better contrast. Staining did not necessarily improve the quality of the observations. After making one set of observations on a polished section of a specimen, it was repolished to make a new set of observations on a different part of the specimen at a different depth. On each specimen, observations were made at four different sections to average out regional differences. For each specimen, the number of microcracks and the length of each microcrack were measured over an area of 0.0432 square inches. The "micro-crack density" for each specimen is defined as the number of microcracks per 0.0432 sq. inches of section observed. The "average microcrack length" is obtained by summing the lengths of all microcracks observed and dividing the sum by the number of microcracks. The results of these observations are plotted as graphs of microcrack density vs. impact stress and average microcrack length vs. impact stress. Though the greatest interest was in sub-threshold trauma, some observations were also made for stresses greater than the threshold values.
APPENDIX H: MICRO-STRUCTURE OF COMPACT BONE

There are several textbooks on the subject. Some of them are listed at the end of this appendix.

A brief description of the micro-structure of femoral compact bone is attempted here. A transverse section of human femoral compact bone is shown in Fig. H.1 at X100 magnification. It is seen that compact bone has a lamellar structure. The lamellae are arranged in concentric circles around longitudinal canals called the Haversian canals which anastomose freely with each other by oblique and transverse passages. Some of the passages which interconnect the Haversian canals and run transverse to them are called Volkmann's canals. This continuous network of canals contains blood vessels which nourish the bone cells called osteocytes. The osteocytes lie in depressions in the bone matrix called lacunae. Cytoplasmic processes of osteocytes extend into canaliculi which radiate from the lacunae. The canaliculi provide a path for the exchange of metabolites between the blood stream and the osteocytes. In Fig. H.1 the dead osteocytes were removed in the process of preparing the bone section for microscopic observation by metallurgical or geologic techniques (see Appendix G). The Haversian canal, the concentric lamellae surrounding the canal and the osteocytes within the lamellae constitute the fundamental structural unit of compact bone called an osteon or a Haversian system. Bone is continuously reconstructed during life, depending on the functional demands placed on it, according to a law called the Wolff's law. The intervals between osteons contain interstitial lamellae which are the remnants of old osteons destroyed in the process of reconstruction. Adjacent lamellar systems are bonded together by a cement line. The cementing substance is chiefly composed of acid mucopolysaccharides. Bone matrix is composed of 35% by weight of organic substance. The organic material is principally in the form of collagen fibres. The remaining 65% by weight is inorganic substance. The inorganic substance mainly consists of crystals of calcium phosphate arranged in the structure of hydroxy apatite. The collagenous fibres in each lamella are arranged helically with the helix angle varying from one lamella to the next. This varying helical angle enables adjacent lamellae to be distinguished.

The osteons are principally aligned parallel to the long axis of the bone. This makes bone appear very similar to fibre-reinforced plastics.

References:

(2) Ham, A. W. Histology, J. B. Lippincott Co, 1969.
Table 1: Statistics on Human Femurs Used in the Tests

<table>
<thead>
<tr>
<th>Age of patient at death (years)</th>
<th>Diagnosis</th>
<th>Sex</th>
<th>Storage* fluid</th>
<th>PMA (days)</th>
<th>No.of Specimens tested</th>
</tr>
</thead>
<tbody>
<tr>
<td>43</td>
<td>coronary heart disease</td>
<td>M</td>
<td>1</td>
<td>17</td>
<td>6</td>
</tr>
<tr>
<td>52</td>
<td>rheumatic heart disease (mitral valve replacement)</td>
<td>F</td>
<td>1</td>
<td>13</td>
<td>6</td>
</tr>
<tr>
<td>85</td>
<td>Fibrosarcoma of left lung</td>
<td>F</td>
<td>1</td>
<td>21</td>
<td>6</td>
</tr>
<tr>
<td>76</td>
<td>Bronchopneumonia</td>
<td>M</td>
<td>1</td>
<td>52</td>
<td>6</td>
</tr>
<tr>
<td>62</td>
<td>Chronic rheumatic heart disease</td>
<td>F</td>
<td>2</td>
<td>19</td>
<td>6</td>
</tr>
<tr>
<td>47</td>
<td>Acute myocardial infarction &amp; chronic rheumatic heart disease</td>
<td>M</td>
<td>2</td>
<td>21</td>
<td>6</td>
</tr>
<tr>
<td>52</td>
<td>Atherosclerotic heart disease</td>
<td>M</td>
<td>2</td>
<td>28</td>
<td>6</td>
</tr>
</tbody>
</table>

* Storage Fluid 1: 8.6 gms NaCl, 0.33 gms CaCl₂, 0.3 gms kcl in a litre of a neutral solution of 1% formaldehyde

Storage Fluid 2: Normal saline (9 gms of NaCl per litre of distilled water)
### Table 2: The Range of Values Reported in the Literature for the Quasi-Static Mechanical Properties of Compact Bone

<table>
<thead>
<tr>
<th>Property</th>
<th>Wet Human femoral cortex</th>
<th>Wet Human femoral cortex</th>
<th>Wet Bovine femoral cortex</th>
<th>Wet Bovine femoral cortex (demineralized)</th>
<th>Wet Bovine femoral cortex (deglycosylated)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Longitudinal:</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Tensile Modulus (x10^6 psi)</td>
<td>1.12</td>
<td>1.71</td>
<td>2.01</td>
<td>0.030</td>
<td></td>
</tr>
<tr>
<td></td>
<td>3.06</td>
<td>3.67</td>
<td>3.9</td>
<td></td>
<td></td>
</tr>
<tr>
<td>UTS, psi</td>
<td>6,990-18,840</td>
<td>8,740-31,500</td>
<td>9,200-22,300</td>
<td>2,400</td>
<td>983</td>
</tr>
<tr>
<td>Tensile yield stress (psi)</td>
<td>10,000</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Max. Tensile strain (μin/in)</td>
<td>5,600-25,500</td>
<td>2,900-11,300</td>
<td>3,000-9,000</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Compressive Modulus (x10^6 psi)</td>
<td>1.26-2.70</td>
<td>1.756-2.57</td>
<td>1.48-5.1</td>
<td>-</td>
<td>1.00-1.14</td>
</tr>
<tr>
<td>UCS, psi</td>
<td>16,000</td>
<td>21,050-31,000</td>
<td>12,800-36,800</td>
<td>-</td>
<td>4,300-5,580</td>
</tr>
<tr>
<td>Compressive yield stress psi</td>
<td></td>
<td></td>
<td></td>
<td>22,494</td>
<td></td>
</tr>
<tr>
<td>Max. compressive strain (μin/in)</td>
<td>11,000-26,500</td>
<td>3,000</td>
<td>28,900</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Shear Modulus (x 10^6 psi)</td>
<td></td>
<td></td>
<td></td>
<td>0.767</td>
<td></td>
</tr>
<tr>
<td>Shear strength (psi)</td>
<td>6,020-15,300</td>
<td>3,580-12,620</td>
<td>6,730</td>
<td>10,000</td>
<td></td>
</tr>
</tbody>
</table>
Table 3: Variation of Creep Strain with Time, for a Voigt Element, at Small Values of Time

<table>
<thead>
<tr>
<th>( \frac{t}{\tau_i} )</th>
<th>( \epsilon_i(t) )</th>
<th>( \epsilon_i(t = \infty) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.01</td>
<td>0.01</td>
<td></td>
</tr>
<tr>
<td>0.02</td>
<td>0.02</td>
<td></td>
</tr>
<tr>
<td>0.03</td>
<td>0.03</td>
<td></td>
</tr>
<tr>
<td>0.041</td>
<td>0.04</td>
<td></td>
</tr>
<tr>
<td>0.051</td>
<td>0.05</td>
<td></td>
</tr>
</tbody>
</table>

Table 4: Properties of Surgical Metals or Alloys (131)

<table>
<thead>
<tr>
<th>Material</th>
<th>Specific gravity</th>
<th>Young's Modulus, ( 10^6 ) lbf/in²</th>
<th>Yield or Proof stress*, ( 10^3 ) lbf/in²</th>
<th>Ultimate tensile strength, ( 10^3 ) lbf/in²</th>
<th>Elongation at fracture, %</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stainless steel</td>
<td>7.9</td>
<td>25-29</td>
<td>40-120</td>
<td>90-145</td>
<td>45-10</td>
</tr>
<tr>
<td>Co-Cr (east)</td>
<td>7.9</td>
<td>29</td>
<td>65</td>
<td>95</td>
<td>c.10</td>
</tr>
<tr>
<td>Co-Cr (wrought)</td>
<td>7.9</td>
<td>29</td>
<td>57</td>
<td>130</td>
<td>c.30</td>
</tr>
<tr>
<td>Titanium</td>
<td>c.4.5</td>
<td>15-18</td>
<td>31-67</td>
<td>63-110</td>
<td>27-15</td>
</tr>
<tr>
<td>Titanium alloys</td>
<td>c.4.5</td>
<td>15-17</td>
<td>128-152</td>
<td>139-180</td>
<td>10</td>
</tr>
</tbody>
</table>

* Yield stress (onset of plastic deformation) is often difficult to observe exactly, and therefore a proof stress is defined as the stress necessary to cause a specified amount of plastic deformation (0.1 per cent or 0.2 per cent for the figures quoted here.

† The figure obtained for elongation depends on the proportions of the test piece, and the figures given are therefore not necessarily directly comparable, although they do give an indication of relative ductility or brittleness.
<table>
<thead>
<tr>
<th>Property</th>
<th>Polyethylene (Low density)</th>
<th>Polyethylene (Med. density)</th>
<th>Polyethylene (High density)</th>
<th>Poly(methylmethacrylate)</th>
<th>Polyethylene (PFA)</th>
<th>Epoxies</th>
<th>Polyamides (nylon)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Modulus of elasticity (x10^6 psi)</td>
<td>0.014</td>
<td>0.035</td>
<td>0.085</td>
<td>0.35</td>
<td>0.033</td>
<td>&gt; 0.30</td>
<td>0.07</td>
</tr>
<tr>
<td>Tensile strength (psi)</td>
<td>1,500</td>
<td>1,600</td>
<td>2,500</td>
<td>6,000</td>
<td>1,500</td>
<td>&gt; 2,000</td>
<td>6,000</td>
</tr>
<tr>
<td>(psi)</td>
<td>2,400</td>
<td>2,500</td>
<td>5,000</td>
<td>10,000</td>
<td>3,000</td>
<td></td>
<td>12,000</td>
</tr>
<tr>
<td>Ultimate elongation (%)</td>
<td>400</td>
<td>200</td>
<td>10</td>
<td>2</td>
<td>120</td>
<td>2</td>
<td>60</td>
</tr>
<tr>
<td></td>
<td>700</td>
<td>500</td>
<td>300</td>
<td>10</td>
<td>350</td>
<td>6</td>
<td>300</td>
</tr>
<tr>
<td>Yield stress (psi)</td>
<td>1,100</td>
<td>1,500</td>
<td>2,400</td>
<td>N.A.</td>
<td>1,600</td>
<td>---</td>
<td>6,000</td>
</tr>
<tr>
<td>(psi)</td>
<td>1,700</td>
<td>2,600</td>
<td>5,000</td>
<td>2,000</td>
<td>12,000</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Yield strain (%)</td>
<td>20</td>
<td>10</td>
<td>5</td>
<td>N.A.</td>
<td>50</td>
<td>---</td>
<td>3.6</td>
</tr>
<tr>
<td></td>
<td>40</td>
<td>20</td>
<td>10</td>
<td>75</td>
<td></td>
<td></td>
<td>30</td>
</tr>
<tr>
<td>Rockwell hardness</td>
<td>---</td>
<td>---</td>
<td>R30</td>
<td>M34</td>
<td>J75</td>
<td>M75</td>
<td>R45</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>R50</td>
<td>M105</td>
<td>J95</td>
<td>M110</td>
<td>R118</td>
</tr>
<tr>
<td>Notched Izod impact strength (ft.lb/in)</td>
<td>N.A.</td>
<td>N.A.</td>
<td>1</td>
<td>0.4</td>
<td>2.5</td>
<td>0.16</td>
<td>0.6</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>10</td>
<td>0.6</td>
<td>4</td>
<td>0.7</td>
<td>&gt; 16</td>
</tr>
<tr>
<td>Sp. gravity</td>
<td>0.912</td>
<td>0.926</td>
<td>0.942</td>
<td>1.18</td>
<td>2.1</td>
<td>1.115</td>
<td>1.09</td>
</tr>
<tr>
<td></td>
<td>0.925</td>
<td>0.941</td>
<td>0.965</td>
<td>1.19</td>
<td>2.3</td>
<td></td>
<td>1.14</td>
</tr>
</tbody>
</table>
### Table 6: The Range of Yield Properties for Titanium and Its Alloys

<table>
<thead>
<tr>
<th>Material</th>
<th>Property</th>
<th>Modulus $= 15 \times 10^6$ psi</th>
<th>Modulus $= 18 \times 10^6$ psi</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ti</td>
<td>yield strength (psi)</td>
<td>31,000</td>
<td>67,000</td>
</tr>
<tr>
<td>Ti</td>
<td>yield strain (%)</td>
<td>0.207</td>
<td>0.446</td>
</tr>
<tr>
<td>Ti-alloys</td>
<td>yield strength (psi)</td>
<td>128,000</td>
<td>152,000</td>
</tr>
<tr>
<td>Ti-alloys</td>
<td>yield strain (%)</td>
<td>0.856</td>
<td>1.01</td>
</tr>
</tbody>
</table>

### Table 7: Tangent Modulus of PMMA at Various Stains and Strain Rates Calculated from the Data of Maiden and Green (136)

<table>
<thead>
<tr>
<th>Strain Rate, $\dot{\varepsilon}$</th>
<th>0.005 sec$^{-1}$</th>
<th>0.07 sec$^{-1}$</th>
<th>0.5 sec$^{-1}$</th>
<th>3 sec$^{-1}$</th>
<th>1210 sec$^{-1}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Strain $\varepsilon$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2%</td>
<td>0.5</td>
<td>0.55</td>
<td>0.65</td>
<td>0.75</td>
<td>0.97</td>
</tr>
<tr>
<td>3%</td>
<td>0.29</td>
<td>0.37</td>
<td>0.43</td>
<td>0.54</td>
<td>0.82</td>
</tr>
<tr>
<td>4%</td>
<td>0.22</td>
<td>0.28</td>
<td>0.34</td>
<td>0.43</td>
<td>0.69</td>
</tr>
<tr>
<td>5%</td>
<td>0.15</td>
<td>0.20</td>
<td>0.25</td>
<td>0.32</td>
<td>0.56</td>
</tr>
</tbody>
</table>

### Table 8: Critical Volume Fracti on of Fibers for a Hypothetical Composite (Compression at a Strain Rate of 0.005 sec$^{-1}$)

<table>
<thead>
<tr>
<th>Matrix-fiber</th>
<th>Yield stress of fiber, $E_f$ (psi)</th>
<th>Modulus of fiber, $E_f$ (x10$^6$ psi)</th>
<th>Yield strain of fiber, $\varepsilon_f$ (%)</th>
<th>Matrix ultimate strength, $\sigma_{um}$ (psi)</th>
<th>Stress at yield of fiber, $\sigma_{E}$ (psi)</th>
<th>Critical volume fraction of fibers, $V_f$ (%)</th>
<th>Minimum volume fraction of fibers, $V_{min}$ (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>PMMA-Ti</td>
<td>31,000</td>
<td>15</td>
<td>0.172</td>
<td>22,600</td>
<td>1,000</td>
<td>72</td>
<td>41</td>
</tr>
<tr>
<td></td>
<td>67,000</td>
<td>15</td>
<td>0.446</td>
<td>22,600</td>
<td>2,280</td>
<td>31.4</td>
<td>23.3</td>
</tr>
<tr>
<td></td>
<td>128,000</td>
<td>15</td>
<td>0.712</td>
<td>22,600</td>
<td>3,630</td>
<td>15.2</td>
<td>12.9</td>
</tr>
<tr>
<td>PMMA-Ti alloy</td>
<td>152,000</td>
<td>15</td>
<td>1.01</td>
<td>22,600</td>
<td>5,100</td>
<td>11.9</td>
<td>10.3</td>
</tr>
</tbody>
</table>
Table 9: Compressive Modulus of a Hypothetical Composite at Various Strain Rates

<table>
<thead>
<tr>
<th>$\dot{\varepsilon}$ (sec$^{-1}$)</th>
<th>log $\dot{\varepsilon}$</th>
<th>$E_f$ (x10$^6$ psi)</th>
<th>$E_m$ (x10$^6$ psi)</th>
<th>$V_f$(%)</th>
<th>40</th>
<th>50</th>
<th>60</th>
<th>70</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.005</td>
<td>-2.301</td>
<td>15</td>
<td>0.5</td>
<td>6.30</td>
<td>7.75</td>
<td>9.20</td>
<td>10.65</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>18</td>
<td>0.5</td>
<td>7.50</td>
<td>9.25</td>
<td>11.00</td>
<td>12.75</td>
<td></td>
</tr>
<tr>
<td>0.07</td>
<td>-1.155</td>
<td>15</td>
<td>0.55</td>
<td>6.33</td>
<td>7.78</td>
<td>9.22</td>
<td>10.66</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>18</td>
<td>0.55</td>
<td>7.53</td>
<td>9.28</td>
<td>11.02</td>
<td>12.76</td>
<td></td>
</tr>
<tr>
<td>0.5</td>
<td>-0.301</td>
<td>15</td>
<td>0.65</td>
<td>6.39</td>
<td>7.82</td>
<td>9.26</td>
<td>10.70</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>18</td>
<td>0.65</td>
<td>7.59</td>
<td>9.32</td>
<td>11.06</td>
<td>12.80</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>0.477</td>
<td>15</td>
<td>0.75</td>
<td>6.45</td>
<td>7.88</td>
<td>9.30</td>
<td>10.72</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>18</td>
<td>0.75</td>
<td>7.65</td>
<td>9.38</td>
<td>11.10</td>
<td>12.82</td>
<td></td>
</tr>
<tr>
<td>1210</td>
<td>3.828</td>
<td>15</td>
<td>0.97</td>
<td>6.58</td>
<td>7.98</td>
<td>9.39</td>
<td>10.79</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>18</td>
<td>0.97</td>
<td>7.78</td>
<td>9.48</td>
<td>11.19</td>
<td>12.89</td>
<td></td>
</tr>
</tbody>
</table>
Table 10: Mechanical Properties of Some Materials of Interest

<table>
<thead>
<tr>
<th>Material</th>
<th>Tensile strength (psi)</th>
<th>Modulus of elasticity in tension ($\times 10^3$ psi)</th>
<th>Tensile strength to modulus ratio expressed as a percentage</th>
</tr>
</thead>
<tbody>
<tr>
<td>PTFE or Polytetra-fluoroethylene*</td>
<td>3,000*</td>
<td>0.065*</td>
<td>4.6%*</td>
</tr>
<tr>
<td>High-Density Polyethylene*</td>
<td>5,000*</td>
<td>0.16*</td>
<td>3.12%*</td>
</tr>
<tr>
<td>Epoxy</td>
<td>&gt; 2,000</td>
<td>&gt; 0.30</td>
<td>~ 0.67%</td>
</tr>
<tr>
<td>PMMA or Polymethyl-methacrylate*</td>
<td>10,000*</td>
<td>0.50*</td>
<td>2.0%*</td>
</tr>
<tr>
<td>Compact bone*</td>
<td>13,000</td>
<td>2.0*</td>
<td>0.65%*</td>
</tr>
<tr>
<td>Low-density carbon</td>
<td>120,000</td>
<td>6.0</td>
<td>2.0%</td>
</tr>
<tr>
<td>Low-density graphite</td>
<td>90,000</td>
<td>6.0</td>
<td>1.5%</td>
</tr>
<tr>
<td>E Glass*</td>
<td>450,000*</td>
<td>10.5*</td>
<td>4.29%*</td>
</tr>
<tr>
<td>S Glass*</td>
<td>650,000*</td>
<td>12.3*</td>
<td>5.29%*</td>
</tr>
<tr>
<td>970 S Glass*</td>
<td>800,000 **</td>
<td>14.5*</td>
<td>5.5%**</td>
</tr>
<tr>
<td>Titanium alloys*</td>
<td>140,000-180,000*</td>
<td>15-18*</td>
<td>(1.0%)*</td>
</tr>
<tr>
<td>(160,000)</td>
<td>(16)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Titanium</td>
<td>63,000-110,000*</td>
<td>15-18*</td>
<td>(0.54%)</td>
</tr>
<tr>
<td>(87,000)</td>
<td>(16)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Stainless steel</td>
<td>90,000-145,000*</td>
<td>25-29*</td>
<td>(0.43%)</td>
</tr>
<tr>
<td>(117,000)</td>
<td>(27)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Vitallium-wrought</td>
<td>130,000</td>
<td>29</td>
<td>0.45%</td>
</tr>
<tr>
<td>Vitallium-cast</td>
<td>94,000</td>
<td>29</td>
<td>0.32%</td>
</tr>
<tr>
<td>Medium-density graphite</td>
<td>180,000-250,000*</td>
<td>25-40*</td>
<td>(0.65%)</td>
</tr>
<tr>
<td>(215,000)</td>
<td>(33)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Boron on tungsten</td>
<td>400,000</td>
<td>60</td>
<td>0.67%</td>
</tr>
<tr>
<td>High-density graphite</td>
<td>250,000-500,000*</td>
<td>40-100</td>
<td>(0.54%)</td>
</tr>
<tr>
<td>(375,000)</td>
<td>(70)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Table 11: Approximate Undirectional Composite Strength That Can be Expected from Various Fibres at Several Volume Fractions of Fiber

<table>
<thead>
<tr>
<th>Fiber</th>
<th>$\sigma^\prime$, psi</th>
<th>$\sigma_c = \sigma^\prime \cdot V_f$, psi</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>$V_f=$10%</td>
</tr>
<tr>
<td>Low density carbon</td>
<td>120,000</td>
<td>12,000</td>
</tr>
<tr>
<td>E Glass</td>
<td>450,000</td>
<td>45,000</td>
</tr>
<tr>
<td>970 S Glass</td>
<td>800,000</td>
<td>80,000</td>
</tr>
<tr>
<td>Ti-alloys</td>
<td>180,000</td>
<td>18,000</td>
</tr>
</tbody>
</table>

Table 12: Modulus of Some Hypothetical Composites Using Polyethylene and PTFE as Matrix Materials

<table>
<thead>
<tr>
<th>Fiber</th>
<th>$V_f=$20%</th>
<th>$V_f=$30%</th>
<th>$V_f=$40%</th>
<th>$V_f=$50%</th>
<th>$V_f=$60%</th>
<th>$V_f=$70%</th>
</tr>
</thead>
<tbody>
<tr>
<td>E Glass</td>
<td>3.028</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>2.90</td>
<td>-</td>
</tr>
<tr>
<td>970 S Glass</td>
<td>4.462</td>
<td>3.197</td>
<td>4.35</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Ti-Alloys</td>
<td>5.896</td>
<td>4.200</td>
<td>5.80</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>E Glass</td>
<td>7.330</td>
<td>5.250</td>
<td>7.25</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>970 S Glass</td>
<td>8.706</td>
<td>6.300</td>
<td>8.70</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Ti-Alloys</td>
<td>9.6</td>
<td>7.350</td>
<td>10.15</td>
<td>11.2</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>
Table A.1: Inter-Relations Between the Various Representations of Vibration Test Data

<table>
<thead>
<tr>
<th></th>
<th>(M, 5)</th>
<th>(C, 5)</th>
<th>(M₁, M₂)</th>
<th>(c₁, c₂)</th>
</tr>
</thead>
<tbody>
<tr>
<td>M =</td>
<td>-</td>
<td>1/C</td>
<td>(M₁² + M₂²)² / 2</td>
<td>(c₁² + c₂²)² / 2</td>
</tr>
<tr>
<td>5 =</td>
<td>5</td>
<td>5</td>
<td>tan⁻¹(M₂/M₁)</td>
<td>tan⁻¹(c₂/c₁)</td>
</tr>
<tr>
<td>C =</td>
<td>1/M</td>
<td>-</td>
<td>1/(M₁² + M₂²)² / 2</td>
<td>(c₁² + c₂²)² / 2</td>
</tr>
<tr>
<td>5 =</td>
<td>5</td>
<td>-</td>
<td>tan⁻¹(M₂/M₁)</td>
<td>tan⁻¹(c₂/c₁)</td>
</tr>
<tr>
<td>M₁ =</td>
<td>M cosθ</td>
<td>(1/C)cosθ</td>
<td>-</td>
<td>c₁/(c₁² + c₂²)</td>
</tr>
<tr>
<td>M₂ =</td>
<td>M sinθ</td>
<td>(1/C)sinθ</td>
<td>-</td>
<td>c₂/(c₁² + c₂²)</td>
</tr>
<tr>
<td>C₁ =</td>
<td>(1/M)cosθ</td>
<td>C cosθ</td>
<td>M₁/(M₁² + M₂²)</td>
<td>-</td>
</tr>
<tr>
<td>C₂ =</td>
<td>(1/M)sinθ</td>
<td>C sinθ</td>
<td>M₂/(M₁² + M₂²)</td>
<td>-</td>
</tr>
</tbody>
</table>
FIG. 1. PROBABLE SEQUENCE OF EVENTS LEADING TO EXTRUSION OF THE LOWER FEMORAL SHAFT [33]
<table>
<thead>
<tr>
<th>USUAL METHOD OF LOADING</th>
<th>DYNAMIC CONSIDERATIONS IN TESTING</th>
</tr>
</thead>
<tbody>
<tr>
<td>HYPER VELOCITY IMPACT</td>
<td>LIGHT GAS GUN OR EXPLOSIVE IMPACT</td>
</tr>
<tr>
<td>MECHANICAL IMPACT</td>
<td>SHOCK WAVE PROPAGATION</td>
</tr>
<tr>
<td>DYNAMIC IMPACT</td>
<td>MECHANICAL RESONANCE IN SPECIMEN AND MACHINE</td>
</tr>
<tr>
<td>&quot;STATIC&quot; HYDRAULIC</td>
<td>CONSTANT STRAIN RATE TEST</td>
</tr>
<tr>
<td>CREEP</td>
<td>STRAIN VS. TIME OR CREEP RATE RECORDED</td>
</tr>
</tbody>
</table>

FIG. 2. STRAIN RATE REGIMES [66]
Constitutive Equation: \( P\sigma = Q\varepsilon \), where \( P \) and \( Q \) are linear differential operators.

<table>
<thead>
<tr>
<th>Figure</th>
<th>Name of Model</th>
<th>Order of the linear differential constitutive equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a)</td>
<td>Hooke body</td>
<td>0</td>
</tr>
<tr>
<td>(b)</td>
<td>Newton body</td>
<td>1</td>
</tr>
<tr>
<td>(c)</td>
<td>Maxwell body</td>
<td>1</td>
</tr>
<tr>
<td>(d)</td>
<td>Kelvin body, Voigt body, or Kelvin-Voigt body</td>
<td>1</td>
</tr>
<tr>
<td>(e)</td>
<td>Standard linear body</td>
<td>1</td>
</tr>
<tr>
<td>(f)</td>
<td>Jeffreys body</td>
<td>2</td>
</tr>
<tr>
<td>(g)</td>
<td>Lesersich body</td>
<td>2</td>
</tr>
<tr>
<td>(h)</td>
<td>Burgers body</td>
<td>2</td>
</tr>
</tbody>
</table>

FIG. 3. SOME FAMILIAR MECHANICAL MODELS OF LINEAR VISCOELASTICITY.
The constitutive equation of the Non-Newtonian dashpot in the Eyring model is given by $\dot{\varepsilon} = K \sinh(\alpha \sigma)$. Thus, the dashpot is characterized by two constants.

All the models shown below use a dashpot with the constitutive equation $\sigma = \eta \dot{\varepsilon}$.

Constitutive equations:

(i) $\sigma = E\varepsilon + \eta \dot{\varepsilon}$

(ii) $\sigma = \eta \dot{\varepsilon} - \frac{n}{E'} \cdot \frac{d}{dt} (\sigma \varepsilon) + \frac{\eta}{E'n^2} \sigma \dot{\sigma}$

(iii) $\sigma = E\varepsilon + \eta (\frac{1}{1 + E'z}) \varepsilon \dot{\varepsilon} + \frac{\eta}{E'n^2} \sigma \dot{\sigma} \frac{\eta}{E'} (1 + \frac{E'}{E}) \frac{d}{dt} (\sigma \varepsilon)$

FIG. 4. SOME MECHANICAL MODELS OF NONLINEAR VISCOELASTICITY.
Plastic flow damper

A simple second degree damper

FIG. 5. MECHANICAL MODEL PROPOSED FOR COMPACT BONE
(b) Model for visco-elastic behavior

FIG. 5. MECHANICAL MODEL PROPOSED FOR COMPACT BONE
PLASTIC

\[ \sigma < \sigma_y, \quad \epsilon = 0 \]
\[ \sigma = \sigma_y, \quad \epsilon = \text{indeterminate} \]

(a) Rigid Plastic Body or St. Venant Body

ELASTO-PLASTIC

\[ \sigma < \sigma_y, \quad \epsilon = \frac{\sigma}{E} \]
\[ \sigma = \sigma_y, \quad \epsilon = \text{indeterminate} \]

(b) Perfectly Plastic Body or Elastic-Perfectly Plastic Body or Prandtl Body

\[ \sigma < \sigma_{y1}, \quad \epsilon = \frac{\sigma}{E} \]
\[ \sigma_{y1} < \sigma < \sigma_{y2}, \quad \epsilon = \epsilon_1 \]
\[ \sigma_{y2} < \sigma < \sigma_{y3}, \quad \epsilon = \epsilon_2 \]
\[ \sigma = \sigma_{y3}, \quad \epsilon = \text{indeterminate} \]

where, \[ \sigma_{y2} = \sigma_{y1} + \Delta \sigma_{y1} \]
\[ \sigma_{y3} = \sigma_{y2} + \Delta \sigma_{y2} \]
\[ \epsilon_2 = \epsilon_1 + \Delta \epsilon_1 \]

(c) Modified St. Venant Body

\[ \sigma < \sigma_y, \quad \epsilon = 0 \]
\[ \sigma = \sigma_y, \quad \epsilon = \text{indeterminate} \]

where, \[ \sigma_{y2} = \sigma_{y1} + \Delta \sigma_{y1} \]
\[ \sigma_{y3} = \sigma_{y2} + \Delta \sigma_{y2} \]
\[ \epsilon_2 = \epsilon_1 + \Delta \epsilon_1 \]

(d) Modified Prandtl Body

FIG. 6: SOME SIMPLE PLASTIC AND ELASTO-PLASTIC BODIES
FIG. 7 SPLIT-HOPKINSON-BAR APPARATUS

BAR APPARATUS SCHEMATIC

PROJECTILE

STRIKER BAR

SAMPLE

BRIDGE CIRCUITRY

DUAL TRACE OSCILLOSCOPE

STRAIN GAGES

ANVIL BAR

GUN AND BAR APPARATUS
FIG. 8 INTERPRETATION OF SIGNAL TRACES FROM THE SPLIT HOPKINSON BAR APPARATUS
FIG. 9 STRAIN GAUGE SIGNALS FROM INPUT-OUTPUT BARS WITH NO SPECIMEN

FIG. 10 THE EFFECT OF FIRING PRESSURE ON MAXIMUM INCIDENT COMPRESSIVE STRESS IN BONE FOR A 3 LB PROJECTILE
FIG. 11 DYNAMIC STRAIN - TIME PROFILES DETERMINED FROM SPLIT HOPKINSON BAR TESTS ON SPECIMENS WHICH (a) REMAINED INTACT (b) FRACTURED
FIG. 12 COMPARISON OF INTERNALLY & EXTERNALLY MEASURED STRAIN-TIME HISTORIES
**FIG. 13.1. STRESS VS STRAIN**

- Age: 43 years
- Sex: Male
- PMR: 17 days
- Preservative: Formalin

- $\varepsilon = \frac{9.3 \text{ ksi}}{0.010 \text{ in/in}} = 0.93 \times 10^6 \text{ psi}$
- $\varepsilon = \frac{9.3 \text{ ksi}}{0.010 \text{ in/in}} = 0.93 \times 10^6 \text{ psi}$

**FIG. 13.2 STRESS VS STRAIN**

- Age: 43 years
- Sex: Male
- PMR: 17 days
- Preservative: Formalin

- $\varepsilon = \frac{10.5 \text{ ksi}}{0.010 \text{ in/in}} = 1.05 \times 10^6 \text{ psi}$
FIG. 13.3. STRESS VS STRAIN

Age: 43 years
Sex: Male
PMI: 17 days
Preservative: Formalin

\[ \varepsilon = 778 \text{ in/in/sec} \]

\[ \varepsilon = 143 \text{ in/in/sec} \]

\[ \varepsilon = 304 \text{ in/in/sec} \]

FIG. 13.4. STRESS VS STRAIN
Age: 43 years
Sex: Male
PMA: 17 days
Preservative: Formalin

\[ E = E + \eta \dot{\varepsilon} \]

\[ E = 0.85 \times 10^6 \text{ psi} \]
\[ \eta = 583 \text{ psi-sec} \]

FIG. 14.1. \( \overline{E} \) vs \( \dot{\varepsilon} \)

Age: 43 years
Sex: Male
PMA: 9 days
Preservative: Formalin

\[ E = E + \eta \dot{\varepsilon} \]

\[ E = 0.75 \times 10^6 \text{ psi} \]
\[ \eta = 833 \text{ psi-sec} \]

FIG. 14.2. \( \overline{E} \) vs \( \dot{\varepsilon} \)
Age: 43 years
Sex: Male
Preservative: Formalin

\[ \bar{E} = E + \eta \]

PMA = 9 days; \( \bar{E} = 0.75 \times 10^6 + 833 \dot{\epsilon} \)

PMA = 17 days; \( \bar{E} = 0.85 \times 10^6 + 583 \dot{\epsilon} \)

FIG. 14.3, \( \bar{E} \) vs \( \dot{\epsilon} \)
FIG. 15.1 $\bar{F}$ VS STRAIN RATE

FIG. 15.2 $\bar{F}$ VS STRAIN RATE
Age: 52 years
Sex: Female
PMA: 1 day
Preservative: Formalin

\[ \bar{E} = E + \eta \dot{\varepsilon} \]

\[ E = 0.5 \times 10^6 \text{ psi} \]

\[ \eta = 1000 \text{ psi-sec} \]

FIG. 15.3. \( \bar{E} \) VS STRAIN RATE
FIG. 16.1. $E$ VS STRAIN RATE

$E = E + n\dot{\varepsilon}$
$E = 1.175 \times 10^6$ psi
$n = 250$ psi-sec

FIG. 16.2. $E$ VS STRAIN RATE

$E = E + n\dot{\varepsilon}$
$E = 1.0 \times 10^6$ psi
$n = 375$ psi-sec
Age: 85 years
Sex: Female
PMA: 6 days
Preservative: Formalin

\[ \bar{E} = E + n\dot{\varepsilon} \]
\[ E = 0.95 \times 10^6 \text{ psi} \]
\[ n = 616 \text{ psi-sec} \]

FIG. 16.3. \( \bar{E} \) VS STRAIN RATE
Age: 76 years
Sex: Male
PMA: 52 days
Preservative: Formalin

\[ \bar{E} = E + \eta \dot{\varepsilon} \]
\[ E = 1.1 \times 10^6 \text{ psi} \]
\[ \eta = 166 \text{ psi-sec} \]

FIG. 17. \( \bar{E} \) VS STRAIN RATE
Age: 62 years  
Sex: Female  
PMA: 19 days  
Preservative: Saline

\[ \bar{E} = E + \eta \dot{\varepsilon} \]

\[ E = 0.55 \times 10^6 \text{ psi} \]

\[ \eta = 375 \text{ psi-sec} \]

**FIG. 18.1.** $\bar{E}$ VS STRAIN RATE

Age: 62 years  
Sex: Female  
PMA: 9 days  
Preservative: Saline

\[ \bar{E} = E + \eta \dot{\varepsilon} \]

\[ E = 0.625 \times 10^6 \text{ psi} \]

\[ \eta = 625 \text{ psi-sec} \]

**FIG. 18.2.** $\bar{E}$ VS STRAIN RATE
Age: 62 years
Sex: Female
PMA: 1 day
Preservative: Saline

\[ E = E + \eta \varepsilon \]

\[ E = 0.575 \times 10^6 \text{ psi} \]
\[ \eta = 875 \text{ psi-sec} \]
Age: 47 years  
Sex: Male  
PMR: 21 days  
Preservative: Saline  

\[ \bar{E} = E + \eta \hat{\varepsilon} \]  
\[ E = 0.625 \times 10^6 \text{ psi} \]  
\[ \eta = 416 \text{ psi-sec} \]

FIG. 19.1. \( \bar{E} \) VS STRAIN RATE

Age: 47 years  
Sex: Male  
PMR: 11 days  
Preservative: Saline  

\[ \bar{E} = E + \eta \hat{\varepsilon} \]  
\[ E = 0.575 \times 10^6 \text{ psi} \]  
\[ \eta = 708 \text{ psi-sec} \]

FIG. 19.2. \( \bar{E} \) VS STRAIN RATE
Age: 47 years  
Sex: Male  
PMA: 5 days  
Preservative: Saline

\[ \bar{E} = E + \eta \dot{\varepsilon} \]
\[ E = 0.6 \times 10^6 \text{ psi} \]
\[ \eta = 916 \text{ psi-sec} \]

**FIG. 19.3.** \( \bar{E} \) VS STRAIN RATE
E (x 10^6 psi) 2.0
1.5
1.0
0.5
0

200 400 600 800 1000 \dot{\varepsilon} (in/in/sec)

\text{FIG. 20.1. } \bar{E} \text{ VS STRAIN RATE}

E (x 10^6 psi) 2.0
1.5
1.0
0.5
0

200 400 600 800 1000 \dot{\varepsilon} (in/in/sec)

\text{FIG. 20.2. } \bar{E} \text{ VS STRAIN RATE}

Age: 52 years
Sex: Male
PPA: 26 days
Preservative: Saline

\bar{E} = E + \eta \dot{\varepsilon}
E = 0.85 \times 10^6 \text{ psi}
\eta = 375 \text{ psi-sec}

Age: 52 years
Sex: Male
PPA: 26 days
Preservative: Saline

\bar{E} = E + \eta \dot{\varepsilon}
E = 0.875 \times 10^6 \text{ psi}
\eta = 416 \text{ psi-sec}
Age: 52 years
Sex: Male
PMA: 18 days
Preservative: Saline

\[ E = E + \eta \varepsilon \]

\[ E = 1.075 \times 10^6 \text{ psi} \]
\[ \eta = 500 \text{ psi-sec} \]

FIG. 20.3. \( E \) vs Strain Rate

Preservative: Saline
Age: 52 years
Sex: Male
PMA: 16 days

\[ E = E + \eta \varepsilon \]

\[ E = 0.925 \times 10^6 \text{ psi} \]
\[ \eta = 500 \text{ psi-sec} \]

FIG. 20.4. \( E \) vs Strain Rate
Age: 52 years
Sex: Male
PMA: 8 days
Preservative: Saline

\[ \bar{E} = E + \eta \dot{e} \]

\[ E = 0.925 \times 10^6 \text{ psi} \]
\[ \eta = 833 \text{ psi-sec} \]

FIG. 20.5. \( \bar{E} \) VS STRAIN RATE
FIG. 21. STIFFNESS AS A FUNCTION OF PMA

<table>
<thead>
<tr>
<th>SYMBOL</th>
<th>AGE (YEARS)</th>
<th>SEX</th>
<th>PRESERVATIVE</th>
<th>PMA (DAYS)</th>
</tr>
</thead>
<tbody>
<tr>
<td>○</td>
<td>43</td>
<td>M</td>
<td>Formalin</td>
<td>9,17</td>
</tr>
<tr>
<td>●</td>
<td>76</td>
<td>M</td>
<td>&quot;</td>
<td>52</td>
</tr>
<tr>
<td>○</td>
<td>85</td>
<td>F</td>
<td>&quot;</td>
<td>6,13,21</td>
</tr>
<tr>
<td>▽</td>
<td>52</td>
<td>F</td>
<td>&quot;</td>
<td>1,7,13</td>
</tr>
<tr>
<td>△</td>
<td>52</td>
<td>M</td>
<td>Saline</td>
<td>8,16,18,26,28</td>
</tr>
<tr>
<td>□</td>
<td>47</td>
<td>M</td>
<td>&quot;</td>
<td>5,11,21</td>
</tr>
<tr>
<td>■</td>
<td>62</td>
<td>F</td>
<td>&quot;</td>
<td>1,9,19</td>
</tr>
</tbody>
</table>

$E \times 10^6$ psi

PMA (days)
FIG. 22. DAMPING COEFFICIENT AS A FUNCTION OF PMA

<table>
<thead>
<tr>
<th>SYMBOL</th>
<th>AGE (YEARS)</th>
<th>SEX</th>
<th>PRESERVATIVE</th>
<th>PMA (days)</th>
</tr>
</thead>
<tbody>
<tr>
<td>◦</td>
<td>43</td>
<td>M</td>
<td>Formalin</td>
<td>9,17</td>
</tr>
<tr>
<td>•</td>
<td>76</td>
<td>M</td>
<td>&quot;</td>
<td>52</td>
</tr>
<tr>
<td>○</td>
<td>85</td>
<td>F</td>
<td>&quot;</td>
<td>6,13,21</td>
</tr>
<tr>
<td>▼</td>
<td>52</td>
<td>F</td>
<td>&quot;</td>
<td>1,7,13</td>
</tr>
<tr>
<td>▲</td>
<td>52</td>
<td>M</td>
<td>Saline</td>
<td>8,16,18,26,28</td>
</tr>
<tr>
<td>□</td>
<td>47</td>
<td>M</td>
<td>&quot;</td>
<td>5,11,21</td>
</tr>
<tr>
<td>■</td>
<td>62</td>
<td>F</td>
<td>&quot;</td>
<td>1,9,19</td>
</tr>
</tbody>
</table>
**FIG. 23.1. FRESH (PREVIOUSLY UNTTED) SPECIMENS**

\[ E = 0.75 \times 10^6 \text{ psi} \]
\[ \eta = 583 \text{ psi-sec} \]

**FIG. 23.2. PREVIOUSLY USED SPECIMENS**

\[ E = 1.1 \times 10^6 \text{ psi} \]
\[ \eta = 833 \text{ psi-sec} \]
\[ E = E + n \dot{\varepsilon} \]

Age: 43 years  
Sex: Male  
PMA: 9 days  
Preservative: Formalin

Previously used specimens
- \( E = 1.1 \times 10^6 \) psi
- \( n = 583 \) psi-sec

Fresh specimens
- \( E = 0.75 \times 10^6 \) psi
- \( n = 833 \) psi-sec

FIG. 23.3. \( E \) VS STRAIN RATE
Bovine Bone
Age: 8 years
PM: 3 days (72 hours)

\[ \bar{E} = E + n\dot{e} \]
\[ E = 1.875 \times 10^6 \text{ psi} \]
\[ n = 166 \text{ psi-sec} \]

**FIG. 24.1. \( \bar{E} \) VS STRAIN RATE**

Bovine Bone
Age: 8 years
PM: 1 day (24 hours)

\[ \bar{E} = E + n\dot{e} \]
\[ E = 1.4 \times 10^6 \text{ psi} \]
\[ n = 833 \text{ psi-sec} \]

**FIG. 24.2. \( \bar{E} \) VS STRAIN RATE**
Bovine bone
Age: 8 years
PMA: 4 hours

\[ \tilde{E} = E + \eta \dot{e} \]
\[ E = 0.825 \times 10^6 \text{ psi} \]
\[ \eta = 1083 \text{ psi-sec} \]

**FIG. 24.3.** \( \tilde{E} \) VS STRAIN RATE
Bovine Bone
Age: 8 years
Storage: Saline

FIG. 25. $E$ and $\eta$ AS A FUNCTION OF PMA
Theoretical curve: 
\[ \dot{\varepsilon} = E\varepsilon + \eta \dot{\varepsilon}^2 \]

\[ = 0.527 \times 10^6 \dot{\varepsilon} + 937 \dot{\varepsilon}^2 \]

FIG. 26. \( \dot{\varepsilon} \) vs \( \dot{\sigma} \) AT A PMA OF 1 DAY

Theoretical curve: 
\[ \dot{\varepsilon} = E\varepsilon + \eta \dot{\varepsilon}^2 \]

\[ = 0.687 \times 10^6 \dot{\varepsilon} + 730 \dot{\varepsilon}^2 \]

FIG. 27. \( \dot{\varepsilon} \) vs \( \dot{\sigma} \) AT A PMA OF 9 DAYS
<table>
<thead>
<tr>
<th>SYMBOL</th>
<th>AGE (YEARS)</th>
<th>SEX</th>
<th>PRESERVATIVE</th>
<th>PMA (DAYS)</th>
</tr>
</thead>
<tbody>
<tr>
<td>⊙</td>
<td>52</td>
<td>F</td>
<td>Formalin</td>
<td>7</td>
</tr>
<tr>
<td>▼</td>
<td>52</td>
<td>M</td>
<td>Saline</td>
<td>8</td>
</tr>
<tr>
<td>□</td>
<td>52</td>
<td>M</td>
<td>&quot;</td>
<td>18</td>
</tr>
<tr>
<td>○</td>
<td>47</td>
<td>M</td>
<td>&quot;</td>
<td>5</td>
</tr>
</tbody>
</table>

\[ \bar{E} = \frac{\sigma}{\epsilon}, \text{ (x10}^6 \text{ psi)} \]

FIG. 28. \( \bar{E} \) vs \( \bar{E}_1 \).
**FIG. 29.** DISCRETE RETARDATION SPECTRUM FOR COMPACT BONE
(UUSING DATA FROM RENTON AND PIEKARSKI [48])

**FIG. 30.** "NORMALISED CREEP" OR "UNIT CREEP" MODEL
FIG. 31. MODEL FOR COMPACT BONE ACCOUNTING FOR BOTH ITS IMPACT AND CREEP BEHAVIOR
FIG. 32. A QUANTIFIED VISCOELASTIC MODEL GIVING A
PHENOMENOLOGICAL DESCRIPTION OF HUMAN
COMPACT BONE

\[ E = 0.85 \times 10^6 \text{ psi} \]

\[ \eta' = \eta \epsilon = 1125 \epsilon \text{ psi-sec} ; \quad \tau \sim 1 \mu \text{sec} \]

\[ E_2 = 16.6 \times 10^6 \text{ psi} \]

\[ \eta_2 = 6.16 \times 10^6 \text{ psi-sec} ; \quad \tau_2 = 0.37 \text{ secs} \]

\[ E_3 = 14.5 \times 10^6 \text{ psi} \]

\[ \eta_3 = 49.5 \times 10^6 \text{ psi-sec} ; \quad \tau_3 = 3.4 \text{ secs} \]

\[ E_4 = 22.2 \times 10^6 \text{ psi} \]

\[ \eta_4 = 2020 \times 10^6 \text{ psi-sec} ; \quad \tau_4 = 91 \text{ secs} \]

Constitutive equation for the model:

\[
\epsilon = \sigma \left( \frac{1}{E} + \sum_{i=2}^{n} \frac{1}{E_i} \right) - \left( \frac{\eta \epsilon_1 \dot{\epsilon}_1}{E} + \sum_{i=2}^{n} \frac{i \epsilon_i}{\tau_i} \dot{\epsilon}_i \right)
\]

where,

\[ \epsilon_1 = \frac{\sigma}{E} - \frac{\eta \epsilon_1 \dot{\epsilon}_1}{E} \]

\[ \epsilon_i = \frac{\sigma}{E_i} - \frac{i \epsilon_i}{\tau_i} \dot{\epsilon}_i , \quad i = 2 \text{ to } n \]

and,

\[ \frac{i \epsilon_i}{\tau_i} = \frac{\eta_i}{E_i} , \quad i = 2 \text{ to } n \]
\[ E = 0.85 \times 10^6 \text{ psi} \]

\[ n_1 = n_1 \varepsilon = 1125 \varepsilon \text{ psi-sec}; \tau_1 = 1 \mu\text{sec} \]

**FIG. 33. DEGENERATE CASE OF THE MODEL FOR IMPACT LOADING**

\[ E = 0.85 \times 10^6 \text{ psi} \]

\[ E_2 = 16.6 \times 10^6 \text{ psi} \]

\[ n_2 = 6.16 \times 10^6 \text{ psi-sec}; \tau_2 = 0.37 \text{ secs} \]

\[ E_3 = 14.5 \times 10^6 \text{ psi} \]

\[ n_3 = 49.5 \times 10^6 \text{ psi-sec}; \tau_3 = 3.4 \text{ secs} \]

\[ E_4 = 22.2 \times 10^6 \text{ psi} \]

\[ n_4 = 2020 \times 10^6 \text{ psi-sec}; \tau_4 = 91 \text{ secs} \]

**FIG. 34. DEGENERATE CASE OF THE MODEL FOR CREEP**
Viscous stiffness
Elastic stiffness
or the loss tangent, tan $\delta$

Model proposed in this study:
\[
\frac{\eta_{\text{ave}}}{E_{\text{ave}}} = 1.32 \times 10^{-3} \times \varepsilon
\]

Experimental Impact Results from this study at zero PMA

FIG. 35. COMPARISON OF THE PREDICTIONS OF THE MODEL WITH SOME EXPERIMENTAL RESULTS
\[ m = \frac{d[\log J(t)]}{d[\log t]} \approx 0.021 \]

\[ \tan \delta = \frac{m \pi}{2}, \text{ for } \frac{m \pi}{2} \ll 1 \]

\[ \tan \delta = 0.021 \times \frac{\pi}{2} = 0.033 \]

FIG. 36. A LOG-LOG PLOT OF CREEP COMPLIANCE VS TIME
FIG. 37. IMPACT VELOCITY AS A FUNCTION OF IMPACT STRESS

FIG. 38. VARIATION OF STRAIN RATE WITH IMPACT STRESS
FIG. 39. HOPKINSON-BAR STRAIN-GAGE SIGNALS FOR THREE LEVELS OF IMPACT STRESS
(A) LOW (6 to 10 KSI)
(B) MEDIUM (12 to 20 KSI)
(C) HIGH (23 to 30 KSI)
FIG. 40. TYPICAL PHOTOMICROGRAPH FOR IMPACT STRESSES OF 6 to 10 KSI (X 100) LONG. SECT.

FIG. 41. TYPICAL PHOTOMICROGRAPH FOR IMPACT STRESSES OF 12 to 20 KSI (X 100) LONG. SECT.
FIG. 42. TYPICAL PHOTOMICROGRAPH FOR IMPACT STRESSES OF 23 to 30 KSI (X 100) LONG. SECT.

FIG. 43. IMPACT FAILURE OF SPECIMEN AT A STRESS OF 29 KSI. (CRACKS PROPAGATED FROM ONE HAVERSIAN CANAL TO ANOTHER) X 100 LONG. SECT.
"Micro-crack density"

or

$\text{n}_2$ of microcracks per 0.0432 sq. in. of long. sect.

Sub-threshold compressive impact

Visible damage

Impact Stress $\sigma$ (ksi)

FIG. 44. MICROCRACK DENSITY VS IMPACT STRESS
FIG. 45, AVERAGE MICROCRACK LENGTH VS IMPACT STRESS

SYMBOL | AGE (YEARS) | SEX | PMA (DAYS)
--- | --- | --- | ---
○ | 85 | F | 13
▼ | 85 | F | 21
○ | 52 | F | 1
□ | 43 | M | 17

"Average microcrack length" (inch)

Impact Stress \( \sigma \) (ksi)
FIG. 46. VELOCITY AT IMPACT AS A FUNCTION OF THE HEIGHT OF FREE FALL.
FIG. 47. COMPRESSION STRESS–STRAIN CURVES OF PMMA AT VARIOUS STRAIN RATES [136]
Fig. 48, A Hypothetical PMMA - Ti Alloy Composite
FIG. 49. A HYPOTHETICAL PMMA-970 S GLASS COMPOSITE

Vol. fraction of fibers, $V_f$ (%)

Region of interest

$\dot{\varepsilon} = 0.005 \text{ sec}^{-1}$

$\dot{\varepsilon} = 1210 \text{ sec}^{-1}$

FIG. 50. A HYPOTHETICAL PMMA - E GLASS COMPOSITE

Vol. fraction of fibers, $V_f$ (%)
Compressive Modulus

$12 \times 10^6 \text{ psi}$

FIG. 51. COMPARISON OF COMPACT BONE WITH SOME HYPOTHETICAL COMPOSITES

- High density polyethylene-Ti alloy or PTFE-Ti alloy, $V_f = 70\%$
- PTFE-970 S Glass, $V_f = 70\%$
- High density polyethylene-Ti alloy or PTFE-Ti alloy, $V_f = 60\%$
- PTFE-970 S Glass, $V_f = 60\%$
- High density polyethylene-E Glass, $V_f = 70\%$
  or PTFE-E Glass
- PMMA-Ti alloy, $V_f = 60\%$; $E_f = 10 \times 10^6 \text{ psi}$
- PMMA-Ti alloy, $V_f = 70\%$; $E_f = 15 \times 10^6 \text{ psi}$
- PMMA-970 S glass, $V_f = 60\%$
- PMMA-E Glass, $V_f = 70\%$

Upper boundary, from McElhaney [1966]

- PMMA-E Glass, $V_f = 30\%$
- PMMA-970 S Glass, $V_f = 20\%$
- PTFE-970 S Glass, $V_f = 20\%$
- Lower boundary, from the present investigation

Stress band for compact bone

Log($\varepsilon, \text{sec}^{-1}$)
FIG. H.1. TRANSVERSE SECTION OF HUMAN FEMORAL COMPACT BONE (X 100).
The behaviour of human compact bone under compressive impact is investigated experimentally. A viscoelastic model is presented to give a satisfactory phenomenological description of bone over a wide range of strain rates. Storage fluids were found not to affect appreciably the elastic stiffness of human compact bone but caused a pronounced change of the viscous stiffness. A method of quantifying microscopic damage under impact is suggested and speculations are made on its relevance to spontaneous fractures. Some hypothetical composite materials are considered for possible use in internal orthopedic prosthetic and fixative devices.