HYPERVELOCITY LAUNCHERS

PART 2: COMPOUND LAUNCHERS - DRIVING TECHNIQUES

by

I. I. Glass
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SUMMARY

The critical survey of hypervelocity launchers and their research applications to reentry physics, hypervelocity impact, gasdynamics and aerodynamics, which was started in Part I (see UTIAS Review No. 22, 1962, and ARL Report No. 63-86) is continued in the present report. The various subsections consider the improvements that may be obtained over a simple launcher by using chambrage, combustion drivers, piston compressors, and electrical discharges. It is shown that although significant but modest improvements are obtained, these techniques do not appear to offer actual and startling increases in muzzle velocities. Consequently, novel methods will have to be employed in order to achieve the desirable current hyper-velocities of integral aerodynamic shapes at 50,000 ft/sec or greater. The application of implosion wave dynamics appears to offer such a possibility and it will be considered in a subsequent report.
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*Note: Subsections 3.6 and 3.7 will be considered in the forthcoming report entitled "Hypervelocity Launchers, Part 3: Compound Launchers - Explosive Drivers"

The remaining parts, as outlined in Part I, UTIAS Review No. 22, 1962, will be issued as they are completed.
### NOTATION

<table>
<thead>
<tr>
<th>Symbol</th>
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<tr>
<td>A</td>
<td>area</td>
</tr>
<tr>
<td>a</td>
<td>sound speed</td>
</tr>
<tr>
<td>b</td>
<td>covolume (Eq. 27)</td>
</tr>
<tr>
<td>C</td>
<td>see Eq. 71; constant (Eq. 73); capacitance (Eq. 84)</td>
</tr>
<tr>
<td>C_p</td>
<td>specific heat at constant pressure</td>
</tr>
<tr>
<td>C_v</td>
<td>specific heat at constant volume</td>
</tr>
<tr>
<td>D</td>
<td>detonation velocity</td>
</tr>
<tr>
<td>d</td>
<td>diameter</td>
</tr>
<tr>
<td>e</td>
<td>internal energy per unit mass</td>
</tr>
<tr>
<td>E</td>
<td>total internal energy (Eq. 72); voltage (Eq. 82)</td>
</tr>
<tr>
<td>G</td>
<td>mass of gas (Eq. 34)</td>
</tr>
<tr>
<td>g</td>
<td>$= f(M, \gamma)$, see Eq. 9a</td>
</tr>
<tr>
<td>h</td>
<td>distance (Eq. 34); enthalpy per unit mass (Eq. 52)</td>
</tr>
<tr>
<td>L</td>
<td>length (Eq. 77); inductance (Eq. 82)</td>
</tr>
<tr>
<td>M</td>
<td>Mach number ($u/a$)</td>
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<tr>
<td>m</td>
<td>mass</td>
</tr>
<tr>
<td>p</td>
<td>pressure</td>
</tr>
<tr>
<td>Q</td>
<td>heat of combustion; total electrical energy</td>
</tr>
<tr>
<td>R</td>
<td>universal gas constant</td>
</tr>
<tr>
<td>Q</td>
<td>gas constant per unit mass; resistance (Eq. 82)</td>
</tr>
<tr>
<td>T</td>
<td>temperature</td>
</tr>
<tr>
<td>t</td>
<td>time</td>
</tr>
<tr>
<td>(\dot{t})</td>
<td>nondimensional time (Eq. 1)</td>
</tr>
<tr>
<td>(\bar{t})</td>
<td>nondimensional time (Eq. 1)</td>
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</table>
u \quad \text{particle or projectile velocity}

\hat{u} \quad \text{nondimensional velocity (Eq. 1)}

\bar{u} \quad \text{nondimensional velocity (Eq. 1)}

\hat{u} \quad \text{escape speed (Eq. 1)}

V \quad \text{steady flow velocity; molar volume}

v \quad \text{specific volume (1/\rho )}

w \quad \text{wave speed}

x \quad \text{distance}

\hat{x} \quad \text{nondimensional distance (Eq. 1)}

\bar{x} \quad \text{nondimensional distance (Eq. 1)}

Z \quad \text{compressibility factor (Eq. 64)}

\text{Subscripts: 0, 1, 2, 3, 4, 5, n and * (state properties)}

E, e \quad \text{equilibrium}

F, f \quad \text{frozen}

f \quad \text{final}

i \quad \text{initial}

p \quad \text{piston}

r \quad \text{reference state}

s \quad \text{shock wave}

\alpha = \frac{\gamma + 1}{\gamma - 1} \quad \text{(Eq. 1)}

\beta = \frac{\gamma - 1}{2 \gamma} \quad \text{(Eq. 4a)}

\gamma \quad \text{specific heat ratio (C_p/C_v)}

\eta \quad \text{electrical energy transfer efficiency}

\rho \quad \text{density}

\sigma \quad \text{see Eq. 70}
3. DRIVERS FOR HYPERVELOCITY LAUNCHERS

It was noted previously that the function of a launcher is to convert available potential energy in chemical, mechanical, and electrical form into kinetic energy of a projectile accelerated to hypervelocity. In a light-gas gun this is accomplished by transferring the potential energy to the driver gas which expands and accelerates and maintains an ever decreasing pressure at the base of the projectile to accelerate it in turn. It was shown that the most efficient energy transfer would occur if the base pressure could ideally be kept constant at its initial value. Although in practice this can never be achieved, various methods have been developed in recent years in an attempt to approach this idealized concept in order to attain the largest hypervelocities possible.

The present section will consider some of the methods used in achieving this energy transfer, their limitations, and possible improvements.

3.1 Effects of Driver Pressure and Temperature

It is worthwhile at this stage to recapitulate some of the basic ideas obtained in Subsections 2.3.1 and 2.3.5 of Part I. The ideal internal ballistics equation was developed in Subsection 2.3.1, by using the simplifying assumptions of an inviscid gas, frictionless motion, zero counterpressure, and a long driver chamber in order to avoid a reduction in projectile base pressure from the reflected rarefaction wave. Under these assumptions it was shown that the path of the projectile in the non-dimensional time-distance plane is given by the following equations (Eqs. 57 to 59, Part I)

\[
\Delta x = \frac{A p_4}{m a_4^2} x, \quad \Delta t = \frac{A p_4}{m a_4} t, \quad \Delta u = \frac{u_3}{a_4}
\]

\[
\Delta x = \frac{4}{\gamma_4^2 - 1} \left[ 1 - \frac{\gamma_4 - 1}{2} \frac{\Delta u}{u} \right] - \frac{\gamma_4 + 1}{\gamma_4 - 1} \left( \frac{2}{\gamma_4 - 1} \left[ 1 - \frac{\gamma_4 - 1}{2} \frac{\Delta u}{u} \right] \right) + \frac{2}{\gamma_4 + 1}
\]

\[
\Delta t = \frac{2}{\gamma_4 + 1} \left[ 1 - \frac{\gamma_4 - 1}{2} \frac{\Delta u}{u} \right] - \frac{\gamma_4 + 1}{\gamma_4 - 1} \frac{2}{\gamma_4 + 1}
\]

(1a)
Alternately, (from Eqs. 53 and 55, Part I)

\[
\begin{align*}
\bar{x} &= \frac{A P_4}{m \hat{u}^2} \cdot x, \quad \bar{t} = \frac{A P_4}{m \hat{u}^2} \cdot t, \\
\bar{u} &= \frac{u_3}{\hat{u}}, \quad \hat{u} = \frac{2 a_4}{\gamma_4 - 1}.
\end{align*}
\]

\[
\bar{x} = \frac{1}{\lambda_4 (\lambda_4 - 1)} \left[ \frac{\lambda_4 \bar{u} - 1}{(1 - \bar{u}) \lambda_4} + 1 \right]
\]

\[
\bar{t} = \frac{1}{\lambda_4} \left[ \frac{1}{(1 - \bar{u}) \lambda_4} - 1 \right]
\]

It was pointed out that to obtain high projectile muzzle velocities \((u_3 \text{max at } x = L)\), it was necessary to have a driver gas with a small acoustic impedance, as expressed by (Eq. 13, Part I),

\[
\frac{\text{du}}{\text{dp}} = - \frac{1}{\rho a} \tag{2}
\]

For a perfect gas Eq. (2) becomes (Eq. 23, Part I)

\[
\frac{\text{du}}{\text{dp}} = - \sqrt{\frac{R}{p_4}} \left( \frac{T}{\gamma m_4} \right) \left( \frac{p_4}{p} \right)^{\frac{\gamma_4 + 1}{2 \gamma_4}} \tag{3}
\]

A survey of available physical gases shows that hydrogen and helium both possess very low values of the acoustic impedance, especially at high temperatures when dissociation and ionization occurs.

From an examination of the above equations (Sec. 2.3) it can be concluded that to obtain large hypervelocities high-pressure, high-temperature, light-gas drivers are required to accelerate light projectiles in long launcher barrels. However, practical considerations such as structural strength usually will limit the pressure \(p_4\) (and temperature \(T_4\)). Contamination arising from erosion of launcher and model materials will limit the driver gas temperature \(T_4\). Acceleration loads will usually determine the smallest mass of the projectile that can be used without it disintegrating. Finally, bore friction will limit the overall launcher length. Consequently, an optimization of all the relevant parameters subject to actual conditions is desirable for the attainment of the largest hypervelocity.

By neglecting viscous effects, heat transfer, bore friction and counterpressure the remaining requirements can be effectively illustrated (see also Subsection 2.3) by using particular examples as shown in Figs. 1 to 4, inclusive (Ref. 1). Figure 1 shows the motion of a projectile in the nondimensional \((\hat{x}, \hat{t})\)-plane under the following assumptions. The
kinematic (dynamic) solution is based on Eq. 3, Part I, assuming a constant base pressure, that is, the parabolic path given by $x = 1/2 t^2$. The unsteady solution is given by Eq. 1, above. The "characteristic solution" for a chambered gun (Subsection 3.2) must in general be computed in a step-by-step manner giving the piston path and the flow properties. It can be seen from Fig. 1 that the expansion characteristics (shed by the moving projectile) in the nonstationary rarefaction wave are reflected from the chambered plane ($x = 0$) as compression waves in order to satisfy the boundary conditions that higher pressure exists in the chamber, thus increasing the projectile base pressure and velocity. Whereas the transmitted portion of the rarefaction wave pulses move into the driving chamber and reflect from the chamber head as rarefaction pulses further decreasing the chamber pressure and temperature. The pulses then reflect and are transmitted as rarefaction waves at the chambered plane. The transmitted rarefaction pulses catch up with the projectile and lower the base pressure and velocity if the chamber length is not long enough to avoid this process before the projectile leaves the muzzle. Therefore, the flow can become quite complex for short driver chambers. The process is inefficient owing to a rapid reduction of projectile base pressure and a simultaneous decrease in projectile velocity.

It is seen that the dynamic solution (constant base pressure) gives the greatest muzzle velocity, as expected. A gun without chamberage (unsteady solution) has the lowest muzzle velocity and one with large chamberage (characteristic solution) provides intermediate values of the muzzle velocity for a given launcher length. At very small times the unsteady and the characteristic solutions approach the dynamic solution, since very little expansion has taken place and the base pressure is effectively constant at the value of the initial driver pressure. However, the characteristic solution is much closer to the kinematic solution.

Figures 2, 3, and 4 illustrate the effects of driver pressure and sound speed (temperature), projectile area and mass, and launcher length on the ideally attainable hypervelocity with a chambered driver (characteristic solution). The standard case was taken as having a driver sound speed $a_4 = 15,000$ ft/sec, driver pressure $p_4 = 50,000$ psi, projectile base area $A = 5 \times 10^{-2}$ in$^2$ and projectile mass $m = 5 \times 10^{-4}$ lb (227 mgm). Figures 2 and 3 show that excessive launcher lengths are required to attain a given hypervelocity with low driver pressures and sound speeds, a large projectile mass, and a small base area. Figure 4 is very similar to Fig. 19, Part 1, and is very effective in illustrating the combined effects of driver pressure and sound speed on a given light-gas gun. It is seen that it is not sufficient to increase just one of these parameters to achieve large hypervelocities. To accomplish this goal both the driver pressure and sound speed must be high.

In practice (Fig. 5), the driver sound speeds are limited by radiation losses and erosion products from the model, sabot, or launcher
inlet, which increase the driver gas molecular weight and heat capacity, thereby lowering the driver temperature or sound speed. It can be seen from Fig. 5 that the experience obtained at AEDC would indicate that for their guns an optimum sound speed in helium would be about 18,000 ft/sec over a driver pressure range of $60 \times 10^3 < p_4 < 350 \times 10^3$ psi for corresponding muzzle velocities of $15 \times 10^3 < u_4 < 22 \times 10^3$ ft/sec, using a gun with a 200 calibre launcher barrel, 1.0 calibre plastic projectile, no chambrage, and an infinite chamber length. Each muzzle velocity appears to have a minimum pressure. Increasing the pressure and sound speed together gives a reduced muzzle velocity, which is indeed very surprising. Further considerations are required to settle this question and it will be discussed in Section 5.

Some final remarks are in order concerning the method used by Bjork (Ref. 3) to analyze light-gas guns driven by dissociated hydrogen and high-temperature unionized helium (Subsection 2.3.5). Bjork made a very significant observation when he noted that an imperfect gas like dissociated hydrogen can be analyzed on the basis of a perfect gas if the correct equilibrium isentropic index ($\gamma_e$) appropriate to the initial driver temperature and pressure is used. Consequently, the ballistics equation, Eq. 1, can be employed without change. Otherwise, an actual curve or a polynomial would have to be fitted to a plot of pressure vs. velocity for an imperfect gas in equilibrium expansion in order to compute the muzzle velocity for a set of initial driver conditions. The foregoing is illustrated in the following set of curves.

It is assumed that a hydrogen driver is initially at a pressure $p_4 = 10^5$ psi (6800 atm, $14.4 \times 10^6$ psf), and temperature $T_4 = 6200^0K$. The driver gas is then expanded isentropically and considered as a perfect gas flow, equilibrium flow (instant adjustment of all degrees of freedom), and frozen flow (vibration and dissociation frozen at their initial values). The thermodynamic quantities and particle velocities for the three cases are then compared. Figure 6 shows the variation of temperature vs. pressure through the nonstationary expansion. It is seen that the equilibrium temperature is always greater than the other two expansions, since the recombination energy acts as a heat source keeping the flow temperature high. The perfect gas temperature is computed on the basis of $\gamma = 1.4$ throughout and the frozen flow has a constant isentropic index $\gamma = 1.44$ for an initial degree of dissociation $\alpha = 0.132$ and can be treated as a perfect gas. Since $\gamma > \gamma$, the frozen flow temperature curve lies below that of the perfect gas flow. The values of the three isentropic indices $\gamma$, $\gamma$ and $\gamma_e$ appear in Fig. 7. It is seen that over a substantial portion of the high-temperature range [for a given set of initial conditions, for example $(3000^0K \leq T \leq 6000^0K)$], $\gamma_e$ does not change too rapidly and in this regime $\gamma_e$ may be assumed for convenience to be a constant (in this particular case, $\gamma_e \sim 1.24$), as Bjork has done. It is seen that at temperatures below about $3000^0K$, this assumption becomes increasingly less valid and deviations from an exact analysis.
of an imperfect, equilibrium, nonstationary expansion will become increasingly greater as the pressure and temperature fall and $\gamma_e \rightarrow 1.40$, the perfect gas value.

Figure 8 shows the variation of density with pressure. As expected, the equilibrium density curve lies below the perfect and frozen cases owing to the fact that for a given pressure the temperature is larger for the equilibrium flow (see Fig. 6). Since the isentropic index for any gas (see Ref. 4) can be written as $\left(\frac{\partial \ln p}{\partial \ln \rho}\right)_s$, the slopes of these curves give the indices for the perfect, frozen, and equilibrium expansions as noted on the graph. It can be seen that because of the log log plot the variations in $\gamma_4$ at lower pressures are lost. Consequently, this is not a sensitive method of finding an isentropic index. It is worth noting that the same behaviour is exhibited by air at high temperatures (see Part I, Fig. 7) and therefore it can also be analysed on the basis of a constant $\gamma_e$ in shock-tube or launcher flows.

The effect of temperature also shows up in the plot of sound speed vs pressure (Fig. 9), where the equilibrium value is greater over most of the range. At the highest pressures and temperatures the combined effects of the compressibility factor ($Z = 1 + \alpha$) and the isentropic index ($\Gamma$) result in the frozen sound speed having the highest values in this range. The values of the equilibrium sound speed obtained by using a constant $\gamma_e = 1.23$, suggested by Bjork (Ref. 3), are also shown. It is seen that the agreement with the exact analysis is quite fair down to pressures of about $15 \times 10^3$ psf (see Fig. 8).

Figure 10 illustrates the variation of the inverse of the acoustic impedance $(1/\rho a)$ with pressure. An integration of these curves ($u = - \int dp/\rho a$, see Part I, Subsection 2.2) yields the particle velocity. It is seen that the smallest acoustic impedance is obtained from the equilibrium expansion and as a consequence the particle velocity is also the largest for this process as shown on Fig. 11. The velocity curve that is obtained by using Bjork's value of $\gamma_e = 1.23$ is also shown. It is seen that it will overestimate the particle velocity as the expansion pressure and temperature acquire low values when $\gamma_e$ is no longer a constant and approaches a value $\gamma_e \rightarrow 1.4$, as noted previously. However, it is seen that for practical purposes the assumption of a constant $\gamma_e$ for the entire expansion process is a very reasonable one indeed. It is also seen that should the expansion process remain frozen then a very considerable reduction in particle velocity would result. For such circumstances it appears that the particle velocity analysis can be based on a perfect gas without too much loss in accuracy (Fig. 9, Part I; Fig. 11, Part II) for guns and shock tubes.

Figure 12 shows the variation of the local flow Mach number with pressure. It is seen that during the early phase of the expansion the equilibrium flow Mach number is highest and later this trend is reversed and the frozen value is largest.
Figure 13 illustrates the variation of the flow velocity with sound speed. As expected (Eq. 18, Subsection 2.1, Part I), the perfect gas has a slope of -5, the frozen flow of -4.54, and the equilibrium-flow slope is no longer a constant, as discussed in Subsection 2.2, Part I.

The details of these calculations are summarized in Tables 1 to 3, inclusive, for the perfect, frozen and equilibrium flows. It is quite certain that none of these limiting flows will prevail in an actual expansion. What can be expected is an initially complex nonequilibrium flow very much like that encountered in a steady expansion around a corner (Ref. 5). During the nonstationary expansion the flow will start with near-frozen flow and will approach near-equilibrium flow after some time has elapsed depending on the collision processes that occur during the expansion. Such a study for high-pressure, high-temperature hydrogen and helium drivers would be very valuable for an understanding of actual unsteady expansions (Refs. 6 and 7). Even if the idealizations of inviscid flow (without catalytic wall effects) and perfect diaphragm rupture were made for the shock-tube case the results would still be very informative. In the case of a gun the analysis would be somewhat more complex owing to the effects of an accelerating projectile and chambrage on the characteristic lines in the flow. A similar set of results for the case of hydrogen at 12,000°K and 100,000 psi appears in Figs. 14 to 20 and Tables 4 to 6 inclusive. The previous remarks apply equally well for this case. However, at this temperature the low molecular weight of the nearly dissociated hydrogen (\(\alpha = 0.20\)) is very beneficial but the increase in the value of the isentropic index (\(Y_e = 1.57\)) tends to reduce this gain (Subsection 2.3.5).

In summary, it can be stated that the method suggested by Bjork of analyzing launcher performance on the basis of a constant isentropic index of the initial imperfect driver gas as the equilibrium expansion proceeds is a very reasonable one. The accuracy of the analysis will decrease as the projectile base pressure and temperature fall to lower values. (The same technique can be used to analyse shock-tube performance for an imperfect equilibrium expansion). It is also found that the frozen flow velocity will differ very little from the perfect gas flow velocity and the latter can be used for quick estimates of the frozen velocity. Neither of these two limiting velocities applies throughout an actual nonstationary expansion and there is therefore a definite need to investigate the nonequilibrium flow problem for a few cases. Such a study would provide a basis for assessing the importance of nonequilibrium effects on muzzle velocity, since the two extremes of frozen and equilibrium flow yield hypervelocities that differ by considerable amounts, especially when long barrels are used that result in low pressures and temperatures at the projectile base.
3.2 Chambrage and Chamber Length

The foregoing subsection dealt with simple launchers where the projectile (piston) was driven by a nonstationary expansion wave generated from the hot, high-pressure gas. The driver chamber was assumed infinite in length. That is, the chamber is considered to be sufficiently long such that the head of the rarefaction wave which is actually reflected from the closed end (breech) of the chamber was assumed not to overtake the projectile during its passage through the barrel. Also, the barrel cross-section was considered to be identical with that of the chamber. A shock tube or launcher is said to have "chambrage" (or is chambered) when the driving chamber has a larger cross-sectional area than the launcher barrel. The transition between chamber and barrel may be abrupt or gradual. The effect of chambrage, especially on guns, has been considered quite extensively over the past few years by Seigel (Refs. 8 to 12). In guns, the term chambrage is often taken to mean the ratio of chamber diameter to bore diameter. In the present report the area ratio will be used for generality.

3.2.1 Chambrage and Infinite Chamber Length

In the following it will be shown that chambrage improves the muzzle velocity of a projectile owing to a higher base pressure and flow velocity at any point along the barrel for a given set of initial driver conditions. This result may be illustrated for convenience by using a shock tube and then applying the same method to launchers.

If the high pressure chamber is considered as a flow reservoir, then from Eq. 14, Subsection 2.1, the pressure-velocity relation for a nonstationary Q-rarefaction wave is given by

\[ du = - \frac{dp}{\rho a} \]  
(4)

The equivalent relation for a steady flow expansion can be obtained from the one-dimensional mass and momentum equations (similar to Eqs. 25 and 26, Subsection 2.1) as

\[ dV = - \frac{dp}{\rho V} \]  
(5a)

Since Eq. 4 is essentially a momentum equation (Ref. 4) the equivalent steady flow form of the momentum relation is also used here for comparison purposes. However, in fact a steady flow expansion involves the energy equation, which shows how thermal energy is converted into directed energy. For an isentropic process the steady flow energy equation for a perfect or imperfect gas can be written as (see Eqs. 27 and 41, Part I)

\[ d(1/2V^2) = - \frac{dp}{\rho} \]  
(5b)
Consequently, it is the quantity \( d(1/2V^2) \) which has to be maximized in a steady flow for a given pressure drop \( dp \), and this occurs for a gas which has a large value of \( 1/\rho \). That is, the criterion for efficient unsteady expansion was shown to be large values of \( 1/\rho a \) vs \( p \) and for steady expansions large values \( 1/\rho \) vs \( p \) are required.

An examination of Figs. 7 and 8, Part I, and Figs. 8, 10, 16 and 18 of Sec. 3.1, shows that both \( 1/\rho \) and \( 1/\rho a \) are higher for imperfect gas flows in equilibrium rather than perfect or frozen flows. Therefore, both requirements for steady and unsteady expansions will usually be met in high-temperature equilibrium flows. However, since the flow will probably be in nonequilibrium during the initial projectile motion the full benefit of equilibrium expansion may not be realized. In addition, Seigel (Ref. 11) has shown that for high-pressure cold driver gases the effect of chambrage can be very beneficial since the steady-flow factor \( 1/\rho \) dominates, rather than the unsteady-flow factor \( 1/\rho a \), in this type of expansion, as a result of intermolecular forces (attractive and repulsive) coupled with chambrage.

A rough estimate of the efficiency of acceleration can be obtained from a comparison of Eqs. 4 and 5 (Ref. 11) which shows that \( dV/du = a/V = 1/M \). That is, for subsonic flows \( M < 1 \), a steady flow expansion will produce a greater particle velocity \( dV \) for a given pressure drop \( dp \), whereas, the converse is true for \( M > a \) or \( M > 1 \), for supersonic flows. Alternately, for an accurate comparison, the integrated equations for perfect unsteady flows

\[
\frac{p}{p_4} = (1 + \frac{\gamma - 1}{2} M) \quad (4a)
\]

and perfect steady flows

\[
\frac{p}{p_o} = (1 + \frac{\gamma - 1}{2} M^2) \quad (5c)
\]

can be compared for \( M \approx 1 \). (Note Eq. 5c is derived from the energy equation, Eq. 27, Part I, where \( p_o \) is the reservoir pressure equivalent to \( p_4 \).)

The equivalents to the above equations as well as two others (Eq. 9d and 3-26, Ref. 13a) are plotted in Fig. 21 in the \( (p, u) \)-plane. It is seen that a common solution exists (for \( M > 0 \)) and it is found by equating Eqs. 4a and 5c and solving for \( M \) to yield the so-called cross-over Mach Number \( M_x = 4/(3 - \gamma/4) \) or \( u_{3x} = 4 a_4/([\gamma/4] + 1) \). For all real gases \( 1 \leq \gamma/4 \leq 5/3 \), \( 2 \leq M_x \leq 3 \) and \( 2 \leq u_{3x} \leq 3/2 \).

It is worth noting the different driven-driven geometrical configurations used for current shock tube and launcher facilities that can be analysed by using the \( (p, u) \)-plane. Sketch A) shows the simplest shock tube and launcher configuration, while C) shows the usual modification resulting from chambrage (larger driver than driven section). Sketches B)
and D) illustrate geometries that are of interest to produce increased running time in shock tunnels by using a nozzle at the diaphragm station and for low density shock tube flows, respectively. The \((p, u)\)-plane shows at once that for \(u_3 < u_{3x}\) C), B), A) and D) are the most efficient, in this order, in converting thermal energy into directed energy of motion and for \(u_3 > u_{3x}\), the order is C), A), B), and D). Consequently, the most efficient driver configuration over the entire Mach number range is configuration C), the chambered shock tube or launcher (see additional examples in Refs. 13a to c). However, the geometries have their particular use. Therefore, the increased muzzle velocity in a chambered launcher (C) is a result of the more efficient energy conversion over the subsonic portion of the expansion as shown in Fig. 21. A quantitative expression for the increase in particle velocity, applying the assumptions of a perfect inviscid flow, with ideal diaphragm rupture can be derived as follows (Refs. 14a, 14b, and 4).

Consider Fig. 22a for the two cases shown, a) \(M_3 \leq 1\) and (b) \(M_3 \geq 1\). Case a) occurs when the shock-tube diaphragm pressure ratio \(p_4/p_1\) is sufficiently low. The result is that a steady expansion occurs through the chambered section of area ratio \(A_4/A_1\), from the uniform state (5) generated by the nonstationary rarefaction wave \(R_1\) to the uniform subsonic state (3). As \(p_{41}\) is increased from values giving \(M_3 < 1\), the pressure ratio \(p_{45}\) across \(R_1\) and Mach number \(M_5\) increase to critical values \(p_{4}/p_{5cr}\) and \(M_{5cr}\) such that sonic flow occurs in state (3) or \(M_e = M_3 = 1\). For steady, isentropic, one-dimensional flow \(p_{45cr}\) and \(M_{5cr}\) depend only on the contraction ratio \(A_4/A_1\) and \(\gamma_4\). If \(p_{4}/p_{1}\) is increased beyond the value for \(M_3 = M_e = 1\), \(p_{4}/p_{5}\), \(M_5\) and \(M_e\) are unaffected and remain at their respective values of \(p_{4}/p_{5cr}\), \(M_{5cr}\) and 1. Expansion from \(M = M_e = 1\) at station e-e, area \(A_1\), to \(M = M_3 > 1\) in state (3) occurs through the nonstationary rarefaction wave \(R_2\), case (b). The head of \(R_2\) always remains at station e-e(x = 0) regardless of the increase in \(p_4/p_1\). However, the tail of the wave \(R_2\) expands (moves more rapidly) with increasing \(p_4/p_1\), until it achieves a limiting speed \(u\) when \(p_4/p_1 \to \infty\) as \(p_1 \to 0\).

Assuming that the idealized wave system shown in Fig. 22a is quickly established after the diaphragm is ruptured (see Ref. 13c), then for case (b) where \(M_3 \geq 1\), \(M_5\) can be found from the one-dimensional steady flow relation

\[
\frac{A_4}{A^*} = \frac{A_4}{A_1} = \frac{1}{M_5} \left[ 2 + \left( \frac{\gamma_4 - 1}{\gamma_4} \right) \frac{M_5^2}{\left( \frac{\gamma_4}{2} + 1 \right)} \right] \tag{6}
\]

(where * indicates sonic conditions at \(x = 0\))

Note that when \(M_5 \to 0\), \(A_4/A_1 \to \infty\), that is, \(u_5 = 0\) and \(R_1\) does not exist. The pressure ratio of the rarefaction wave \(R_1\) which must accelerate the gas from the rest state (4) to \(M_5\) is given by the unsteady flow relation (see Eq. 21, Part I).

\[
\frac{p_4}{p_5} = \left( 1 + \frac{\gamma_4 - 1}{2} M_5 \right) \tag{1a}
\]
Further acceleration to reach sonic velocity in \( A_1 \), at \( x = 0 \), requires a pressure ratio \( p_5/p^* \) given by the steady flow relations

\[
\frac{p_4}{p_5} = (1 + \frac{\gamma_4 - 1}{2} M^2) \frac{\gamma_4}{\gamma_4 - 1}
\]

(5c)

and for \( M_5 = 1 \)

\[
\frac{p_4}{p^*} = \left( \frac{\gamma_4 + 1}{2} \right) \frac{\gamma_4}{\gamma_4 - 1}
\]

(5d)

or

\[
\frac{p_5}{p^*} = \left[ \frac{\gamma_4 + 1}{2 + (\gamma_4 - 1) M_5^2} \right] \frac{\gamma_4}{\gamma_4 - 1}
\]

(7)

where, \( p_4 \) is the conserved reservoir pressure. Finally, to reach \( M_3 > 1 \), \( R_2 \) is required to accelerate the flow. Imagine that the flow is accelerated to \( M = 1 \) by an nonstationary rarefaction wave, in which case

\[
\frac{p_4}{p^*} = (1 + \frac{\gamma_4 - 1}{2} M) \frac{2 \gamma_4}{\gamma_4 - 1} = \left( \frac{\gamma_4 + 1}{2} \right) \frac{2 \gamma_4}{\gamma_4 - 1}
\]

and then to

\[
M = M_3 \text{ or } \frac{p_4}{p_3} = (1 + \frac{\gamma_4 - 1}{2} M_3)
\]

giving the required pressure ratio

\[
\frac{p^*}{p_3} = \left[ \frac{2 + (\gamma_4 - 1) M_3}{\gamma_4 + 1} \right] \frac{2 \gamma_4}{\gamma_4 - 1}
\]

(8)

Consequently, when \( M_3 > 1 \) for a tube of known chambrage \( A_4/A_1 \), the overall pressure ratio across the system of unsteady and steady expansions is expressed by

\[
\frac{p_4}{p_3} = \frac{P_4}{P_5} \cdot \frac{p_5}{p^*} \cdot \frac{p^*}{p_3}
\]

(9)

If the appropriate quantities are substituted then it can be shown that

\[
g \cdot \frac{p_4}{p_3} = \left( 1 + \frac{\gamma_4 - 1}{2} M_3 \right) \frac{2 \gamma_4}{\gamma_4 - 1}
\]

(9a)

where, \( g = f(M_5, \gamma_4) \), and is plotted in Fig. 22b.
For the limiting case of \( A_4/A_1 \to \infty \), \( M_5 \to 0 \), \( P_{45} \to 1 \) (\( R_1 \) is absent), and \( g \) reduces to

\[
g = \left( \frac{\gamma_4 + 1}{2} \right) \frac{\gamma_4}{\gamma_4 - 1}, \text{ giving}
\]

\[
\frac{p_4}{p_3} = \left(\frac{2}{\gamma_4 + 1}\right) \frac{\gamma_4}{\gamma_4 - 1} \left[ 1 + \frac{\gamma_4 - 1}{2} \frac{M_3}{\gamma_4 - 1} \right] \tag{9b}
\]

or

\[
\frac{p_3}{p_4} = \left(\frac{\gamma_4 + 1}{2}\right) \frac{\gamma_4}{\gamma_4 - 1} \left[ 1 - \sqrt{\frac{2}{\gamma_4 + 1}} \frac{\gamma_4 - 1}{2} \cdot \frac{u_3}{a_4} \right] \frac{2\gamma_4}{\gamma_4 - 1} \tag{9c}
\]

or

\[
\frac{p_3}{p_4} = g \left[ 1 - g \frac{\gamma_4 - 1}{2} \frac{\gamma_4 - 1}{\gamma_4} \cdot \frac{u_3}{a_4} \right] \frac{2\gamma_4}{\gamma_4 - 1} \tag{9d}
\]

Equations (9c) and (9d) can be compared with their counterparts Eqs. (21) and (22), Part I, for a simple nonstationary rarefaction wave (rather than these resulting from a mixed steady and unsteady flow). From Eq. (9d), it is seen at once that when \( p_3 \to 0 \), \( u_3 \to \hat{u} \)

\[
\frac{\hat{u}}{a_4} = \sqrt{\frac{\gamma_4 + 1}{2}} \cdot \frac{2}{\gamma_4 - 1} \tag{9e}
\]

and the escape speed can be increased over its nonstationary value by a maximum factor of \( \sqrt{(\gamma_4 + 1)/2} \) as a result of the more efficient steady flow subsonic acceleration. This increase is most effective when \( \gamma_4 = 5/3 \) and ineffective when \( \gamma_4 \to 1 \). For area ratios \( 1 \leq A_4/A_1 \leq \infty \) the escape speed can be obtained from Eq. 9c (and Figs. 22b, c and d for \( \gamma = 1.4 \)) as

\[
\frac{\hat{u}}{a_4} = (g') \frac{\gamma_4 - 1}{2\gamma_4} \cdot \frac{2}{\gamma_4 - 1} \tag{9f}
\]
A plot of Eq. (9c) is shown in Fig. 23a for $\gamma_4 = 5/3$ and area ratios $A_4/A_1 \to \infty$ and $A_4/A_1 = 1$ (with $g = 1$, Part I, Eq. 21). The same information is replotted in Fig. 23b including the limiting case of $\gamma_4 = 1$. All other gases with $1 \leq \gamma_4 \leq 5/3$ will have values between these two curves. The very significant increase in particle velocity $u_3$ at the tail of the second expansion wave (or contact front velocity) for a given pressure at the wave tail (or contact surface) for infinite chambrage is quite apparent. In turn this means a stronger shock wave produced in a shock tube or a higher muzzle velocity produced in a gun as a result of the higher base pressures and higher expansion velocities compared with the unchambered case. It will be shown subsequently that an area ratio as low as $A_4/A_1 = 5$ nearly achieves most of the gain available from chambrage without going to higher values (see Fig. 22d).

It is worth noting that Seigel (Ref. 11) has shown that for the supersonic flow case when $M_3 > 1$, Eq. (9c) can be approximated (within 5%) by the relation

$$\frac{p_3}{p_4} = \left[ 1 - \frac{u_3 - \frac{a_4}{2} \left( 1 - \frac{A_1}{A_4} \right)}{\frac{2}{\gamma_4 - 1} a_4} \right]^{\frac{2 \gamma_4}{\gamma_4 - 1}} \quad (9g)$$

which simplifies when $A_4/A_1 \to \infty$ to (see also Eq. 33)

$$\frac{p_3}{p_4} = \left[ 1 - \frac{u_3 - \frac{a_4}{2}}{\frac{2}{\gamma_4 - 1} a_4} \right]^{\frac{2 \gamma_4}{\gamma_4 - 1}} \quad (9h)$$

The above are useful forms of the pressure-velocity relationship for chambered tubes in the range $1 \leq A_4/A_1 \leq \infty$.

The particle velocity $u$ may also be obtained in a manner similar to the pressure. For the first rarefaction wave $R_1$ (Subsection 2.1, Eq. 18)

$$\frac{a_5}{a_4} = \left[ 1 + \frac{\gamma_4 - 1}{2} M_5 \right]^{-1} \quad (10)$$
For a steady expansion from $A_4$ to $A_1$

$$a^* = \left( \frac{p^*}{p_5} \right) \frac{\gamma_4 - 1}{2 \gamma_4} = \left[ \frac{2 + (\gamma_4 - 1) M_5^2}{\gamma_4 + 1} \right]^{1/2} \quad (11)$$

For the second rarefaction wave

$$a^* = \left( \frac{p^*}{p_3} \right) \frac{\gamma_4 - 1}{2 \gamma_4} = \frac{2 + (\gamma_4 - 1) M_3}{\gamma_4 + 1} \quad (12)$$

Therefore, the particle (contact surface) velocity $u_3$ is found from

$$\frac{u_3}{a^*} = \frac{u_3}{a_3} \cdot \frac{a_3}{a^*} = \frac{M_3 (\gamma_4 + 1)}{2 + (\gamma_4 - 1) M_3} \quad (13)$$

or

$$\frac{u_3}{a_4} = \frac{u_3}{a^*} \cdot \frac{a^*}{a_5} \cdot \frac{a_5}{a_4} \quad (14)$$

In the limit when $A_4/A_1 \to 1$, $u_5 \to 0$ and

$$\frac{u_3}{a_4} = \sqrt{\frac{\gamma_4 + 1}{2}} \left[ \frac{M_3}{\gamma_4 - 1} \right] \quad (14a)$$

For the subsonic case (which is not of practical interest for launchers) when $M_3 < 1$, the pressure ratio across the first rarefaction wave $R_1$ is given by

$$\frac{p_4}{p_5} = \left[ 1 + \frac{\gamma_4 - 1}{2} M_5 \right]^{2 \gamma_4} \quad (15)$$

Since the second rarefaction wave $R_2$ does not exist the required pressure ratio $p_5/p_3$ can be obtained from steady flow relations using $M_5$ and $M_3$. Imagine a fictitious reservoir pressure $p_o$ from which $M_5$ and $M_3$ are obtained such that

$$\frac{p_5}{p_3} = \frac{p_5}{p_o} \cdot \frac{p_o}{p_3} = \left[ \frac{2 + (\gamma_4 - 1) M_3^2}{2 + (\gamma_4 - 1) M_5^2} \right]^{\gamma_4 - 1} \quad (16)$$
where, \( M_3 \) is found from the area ratio
\[
\frac{A_4}{A_1} = \frac{M_3}{M_5} \left[ \frac{2 + (\gamma_4 - 1) M_5^2}{2 + (\gamma_4 - 1) M_3^2} \right] \frac{\gamma_4 + 1}{2(\gamma_4 - 1)}
\]  
(17)

The overall pressure ratio \( \frac{p_4}{p_3} \) is obtained from
\[
\frac{p_4}{p_3} = \frac{p_4}{p_5} \cdot \frac{p_5}{p_3}
\]  
(18)

When \( \frac{A_4}{A_1} \to \infty \), \( M_5 \to 0 \) and only a steady flow expansion exists expressed by
\[
\frac{p_4}{p_3} = \left[ 1 + \frac{\gamma_4 - 1}{2} M_3^2 \right] \left( \begin{array}{c} \frac{\gamma_4}{\gamma_4 - 1} \\ \frac{\gamma_4}{\gamma_4 - 1} \end{array} \right)
\]  
(18a)

or
\[
\frac{p_3}{p_4} = \left[ 1 - \frac{\gamma_4 - 1}{2} \left( \frac{u_3}{a_4} \right)^2 \right] \left( \frac{\gamma_4}{\gamma_4 - 1} \right)
\]  
(18b)

or
\[
\frac{u_3}{a_4} = \sqrt{\frac{2}{\gamma_4 - 1} \left[ 1 - \left( \frac{p_3}{p_4} \right) \frac{\gamma_4}{\gamma_4 - 1} \right]}
\]  
(18c)

In a similar manner, the particle velocity \( u_3 \) can be determined. For the rarefaction wave \( R_1 \)
\[
\frac{u_5}{a_4} = \frac{u_5}{a_5} \cdot \frac{a_5}{a_4} = \frac{M_5}{1 + \frac{\gamma_4 - 1}{2} M_5}
\]  
(19)

From the one-dimensional, steady-flow, continuity equation
\[
\rho_3 u_3 A_1 = \rho_5 u_5 A_4
\]  
(20)
or
\[
\frac{u_3}{u_5} = \frac{\rho_5}{\rho_3} \cdot \frac{A_4}{A_1} = \frac{M_3}{M_5} \left[ \frac{2 + (\gamma_4 - 1) M_5^2}{2 + (\gamma_4 - 1) M_3^2} \right]^{1/2}
\] (21)

and
\[
\frac{u_3}{a_4} = \frac{u_3}{u_5} \cdot \frac{u_5}{a_4}
\] (22)

When \( \frac{A_4}{A_1} \to \infty, \ M_5 \to 0 \)

and
\[
\frac{u_3}{a_4} = \frac{M_3}{\left[ 1 + \frac{\gamma_4 - 1}{2} M_3^2 \right]^{1/2}}
\] (23)

It is worth noting that for the supersonic case (b) where \( M_3 > 1 \), the following limits are obtained from Eqs. 9a and 14a for the same flow Mach number (it is important to stress that these ratios are not as simple when compared on a particle velocity basis as can be seen from Eqs. (9c) and (9d)).

\[
\frac{(p_3/p_4)_{A_4/A_1}}{(p_3/p_4)_{A_4/A_1 = 1}} = g
\] (24)

In particular, when \( \frac{A_4}{A_1} \to \infty, \ g = \left( \frac{\gamma_4 + 1}{2} \right)^{\frac{\gamma_4}{\gamma_4 - 1}} \) (24a)

and for \( \gamma_4 = 1, \ 1.4, \ \text{and} \ 1.66 \) has limits of 1.65, 1.893 and 2.052, respectively.

Similarly
\[
\frac{(u_3)_{A_4/A_1}}{(u_3)_{A_4/A_1 = 1}} = \left( g \right) \frac{\gamma_4 - 1}{2 \cdot \gamma_4}
\] (25)

and when \( \frac{A_4}{A_1} \to \infty, \ \left( g \right) \frac{\gamma_4 - 1}{2 \cdot \gamma_4} = \sqrt{\frac{\gamma_4 + 1}{2}} \) (25a)
and has limits of 1, 1.095 and 1.155 for \( \gamma_4 = 1, 1.4 \) and 1.66, respectively (see Figs. 22b and c). Consequently, for all physical gases in the range \( 1 \leq \gamma_4 \leq 5/3 \) chambrage will increase the pressure behind the rarefaction wave (or launcher base pressure) and will in turn increase the particle velocity by a smaller amount. For any other area ratios the values of \( g \) can be found from Eq. 9 or Fig. 22b and the effect will decrease with decreasing area ratio. It is also seen that the effect of chambrage decreases for low values of \( \gamma_4 \). However, as can be seen from Eq. (9c)

\[
\left( \frac{\hat{u}_{A_4}/A_1}{A_4/A_1 = 1} \right) / \frac{\hat{u}_{A_4}/A_1}{A_4/A_1 = 1} = (g - 1) = \sqrt{\frac{\gamma_4 + 1}{2}} - 1.
\]

When \( \gamma_4 = 5/3, 7/5 \) and \( 5/4 \) say, \( \hat{u}_{A_4}/A_1 = 1 = 3a_4, 5a_4 \) and \( 8a_4 \), respectively, and the net gain achieved through infinite chambrage is \( 0.465 a_4, 0.475 a_4 \) and \( 0.48 a_4 \), respectively. That is, a gain of about \( 1/2 a_4 \) ft/sec (see Eq. 9b) over the usual range. In view that it is very difficult, using present methods, to get significant increases in velocity at hypervelocities over 30,000 ft/sec, this maximum gain of \( 1/2 a_4 \) is very significant, especially when \( a_4 \) is large, such as for hot dissociated or ionized gases.

From Eq. (9e) for a shock tube with infinite chambrage using air in the chamber and in the channel at room temperature, the escape speed \( \hat{u} \) as \( p_3 \to 0 \), has a value \( \hat{u} = \sqrt{\frac{\gamma_4 + 1}{2}} a_4 = 5.48 a_4 \). Consequently, the strongest shock that can be produced (Eq. 29, Part I), has a pressure ratio \( p_2/p_1 = 52.5 \) compared with \( 44(\gamma_4 = 1.4) \) for an unchambered shock tube. Alternately, from Eq. 34, Part I,

\[
M_s \sim \frac{\gamma_4 + 1}{2} \frac{a_4}{a_1} = \frac{\gamma_4 + 1}{2} \sqrt{\frac{\gamma_4 + 1}{2}} \frac{a_4}{a_1}
\]  

applies approximately to any driver gas combination. In the foregoing example, for an air-to-air combination \( M_s \sim 6.6 \) compared with 6.0 for the unchambered shock tube. The more exact limits are \( M_s = 6.75 \) compared with \( M_s = 6.16 \), which can be obtained from Eq. 29, Part I, based on the above value of \( \hat{u} \).

It is of interest to note that even moderate chambrage can produce very effective gains. For example, from Fig. 24 it can be seen that an area ratio \( A_4/A_1 = 25 (d_4/d_1 = 5) \) produces a rarefaction wave pressure-velocity relation that is nearly equal to that for infinite chambrage, and when \( A_4/A_1 = 2.25 (d_4/d_1 = 1.5) \) nearly fifty percent of the maximum gain is achieved. Consequently it is not necessary to go to extreme chambrage to derive most of its benefit (see Fig. 22d).

Seigel (Ref. 11) has also shown that in dense gases the inter-molecular forces (attractive and repulsive forces tend to decelerate and accelerate a dense gas, respectively, compared with a perfect gas) play an important role, owing to the manner in which these terms enter the acoustic impedance \( \rho a \) and the density \( \rho \). Consequently, the
attratice forces can actually decrease the performance of an unchambered shock tube or gun by increasing the value of $\rho a$ (Eq. 4), whereas, owing to the dominant role of the repulsive forces in the expression for the density $\rho$, in the steady expansion occurring in a chambered tube, they can bring about a marked improvement in the acceleration of the gas by decreasing the value of $\rho$ (Eq. 5b). For example, in the Abel-Noble gas model for a very dense gas, where the attractive force term $\alpha\rho^2$ is dropped from the van der Waal equation (Ref. 4) because it is small relative to the repulsive force term $b/\rho$, then the following relations apply (Ref. 11)

$$p \left( \frac{1}{\rho} - b \right) = RT$$

$$p \left( \frac{1}{\rho} - b \right) \frac{\gamma_4}{\gamma_4} = K(S)$$

$$\rho a = \left[ \frac{\gamma_4 P_4^2}{R T_4} \left( \frac{P}{p} \right) \right]^{1/2}$$

$$\rho = \left[ \left( \frac{p_4}{p} \right)^{1/\gamma_4} \frac{RT_4}{p_4} \right]^{\gamma_4 - 1}$$

$$u = \frac{A_4}{A_1} \cdot \frac{2}{\gamma_4 - 1} \sqrt{\gamma_4 P_4 (\gamma_4 - b)} = \frac{A_4}{A_1} \cdot \frac{2}{\gamma_4 - 1} \sqrt{\gamma_4 RT_4}$$

for $b/\gamma_4 \rightarrow 1$, where, $b$ is the covolume occupied by the molecules. (For hydrogen $b = 0.027$ litres/mole; at 10,000 atm and 25°C, $b/(\gamma_4 - b) \sim 8$).

It is seen from the above that $\rho a$ is unaffected by $b$, whereas $\rho$ decreases with increasing $b$ and therefore from Eq. 5b the steady expansion should be very much improved. Alternately, the escape speed becomes very large for $b/\gamma_4 \rightarrow 1$. Both of these points are illustrated in Figs. 25 and 26. Figure 25 shows that no gain in performance occurs in an unchambered tube no matter how dense the gas may be, whereas very large gains are possible in chambered tubes (as noted previously the value of $b/(\gamma_4 - b) = 8.09$ would apply to hydrogen at 10,000 atm and 25°C). Figure 26 shows the escape speed as a function of $b$ and $d_4/d_1$. Again when $A_4/A_1 = 1$, then there is no improvement in the escape speed, but marked increases (even infinite values) occur in a chambered tube with increasing covolume $b$. Equations and curves showing the relationship between pressure and velocity for various chamberage ratios for an Abel-Nobel gas can also be found in Ref. 11.
Although the above results are of interest they are limited to cold drivers. Despite the fact that the relative increases look impressive, they are small by comparison with the possible performance from a hot high-pressure driver. For this purpose the results shown in Table 3 for a tube with $A_4/A_1 = 1$, using a hydrogen driver at 6600 atm, $6200^\circ$K, $\gamma_e = 1.23$ and $\dot{u} = 17.3 \times 10^4$ ft/sec are also plotted on Fig. 25, but $u_3$ is normalized by the existing escape speed $\hat{u}$. It is seen that the hot driver lies close to the perfect gas (hydrogen) with $\gamma_4 = 1.4$, whose escape speed at $25^\circ$C is only about $0.21 \times 10^4$ ft/sec, that is, the hot gas has an escape speed about two orders of magnitude greater than the cold gas. Hence, even if the very dense gas $b/(v_4 - b) = 100$ does give a sixfold, say, increase in particle velocity, it is still small compared with what may be obtained from a hot, high-pressure, hydrogen driver.

It was noted previously (Fig. 1) that unlike a shock tube, where the contact surface (massless projectile) attains a uniform velocity instantly, an actual projectile has to accelerate to a given velocity. As long as the tube is unchambered in both cases, the isentropic expansion is similar (in a shock tube the rarefaction wave is centered at the origin and in a gun it is not - see Eq. 34) and the pressure-velocity relationships are identical (Fig. 27). However, in the case of a chambered tube the expansion characteristics are still centred in a shock tube, but in the case of a gun they undergo additional complex reflections as compression pulses and are transmitted into the chamber, in general, as expansion pulses at the chambrage plane and the pressure-velocity relations for the two processes are no longer the same as illustrated in Fig. 27. It is seen that for a given pressure ratio $p_3/p_4$ the flow velocity $u_3$ is larger in the shock tube case because of the instantaneous acceleration of the gas particles in this case. However, just as in the case of no chambrage, $A_4/A_1 = 1$, the two curves for the case of infinite chambrage $A_4/A_1 \to \infty$ tend to become identical when $u_3$ is large and $u_3 \to \hat{u}$ after an infinite distance and time.

This can be illustrated analytically as follows. It was already pointed out previously that because of the more complex characteristics diagram of a chambered gun the problem of the projectile motion must be solved graphically or numerically. However, it is possible to make the following simplifying approximations to obtain a flow model for an infinitely chambered gun. It is assumed that the projectile is driven by a steady expansion (in order to approximate the unsteady expansion-compression characteristics) until it has achieved sonic velocity and then by an unsteady expansion beyond that point. In such a model therefore no characteristics are generated or reflected at the chambered plane during the subsonic portion of the motion and for the supersonic motion the characteristics move downstream anyway.
When \( M_3 \leq 1 \), the equations of motion are described by

\[
\int u_3 \, d\tau = \frac{\mathcal{A}_4}{m} \cdot x
\]

or

\[
\frac{\mathcal{A}_4}{ma^2_a} \cdot x = \left[ 1 - \frac{\mathcal{Y}_4 - 1}{2} \left( \frac{u_3}{a_4} \right)^2 \right] - \frac{1}{\mathcal{Y}_4 - 1} - 1
\]

for \( \mathcal{Y}_4 = 1.4 \)

When the flow is sonic \( u_3/a_4 = \sqrt{2/(\mathcal{Y}_4 + 1)} \) or \( u_3/a_4 = 0.913 \)

\[
\Delta \frac{\mathcal{A}_4}{ma^2_a} \cdot x = \left( \frac{\mathcal{Y}_4 + 1}{2} \right) - 1
\]

or \( \Delta x = 0.57 \) for \( \mathcal{Y}_4 = 1.4 \)

Since \( \frac{u_3}{a_4/(\mathcal{Y}_4 - 1)} = \frac{u_3}{\hat{u}} = \hat{u} = 0.18, \quad \Delta x = \frac{\mathcal{A}_4}{x/\mu^2} \)

or \( \Delta x = 1/25, \Delta = 0.023 \) for \( \mathcal{Y}_4 = 1.4 \).

Similarly,

\[
\frac{\mathcal{A}_4}{ma^2_a} \cdot t = \frac{\mathcal{Y}_4 - 1}{2} \left[ \frac{u_3/a_4}{1 - \frac{\mathcal{Y}_4 - 1}{2} \left( \frac{u_3}{a_4} \right)^2} \right] - \frac{1}{\mathcal{Y}_4 - 1}
\]

When the flow is sonic \( u_3/a_4 = \sqrt{2/(\mathcal{Y}_4 + 1)} \)

and

\[
\Delta \frac{\mathcal{A}_4}{ma^2_a} \cdot t = \frac{\mathcal{Y}_4 - 1}{2} \left( \frac{\mathcal{Y}_4 + 1}{2} \right) \frac{3 - \mathcal{Y}_4}{2(\mathcal{Y}_4 - 1)}
\]

or \( \Delta t = 0.288 \) for \( \mathcal{Y}_4 = 1.4 \).

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For \( \gamma_4 = 1.4 \), \( t = \frac{A_p t}{m \hat{u}} = 1/5 \) t or \( t = 0.058 \). Consequently, the acceleration to sonic velocity (\( M_3 = 1 \)) consists of a fixed value of \( \bar{x} \) and \( \bar{t} \).

(It should be noted that these values are not entirely accurate owing to the initial assumption which neglected the reflected characteristics.)

For the portion of the motion beyond \( M_3 = 1 \), the following relations apply (using Eq. (9c) for \( p_3/p_4 \))

\[
\left( \frac{\gamma_4 + 1}{2} \right) \frac{1}{\gamma_4 - 1} \left( \frac{\gamma_4 - 1}{2} \right) \frac{A_p}{m \bar{a}_4^2} \cdot x = \frac{2}{\gamma_4 + 1} \left[ 1 - \sqrt{\frac{2}{\gamma_4 + 1}} \right] \frac{\gamma_4 - 1}{2} \frac{u_3}{a_4} \]

\[
- \left[ 1 - \sqrt{\frac{2}{\gamma_4 + 1}} \right] \frac{\gamma_4 - 1}{2} \frac{u_3}{a_4} - \left( \frac{\gamma_4 + 1}{2} \right) \frac{1}{\gamma_4 - 1} - 1 \quad (31a)
\]

Alternately

\[
x = \frac{A_p}{m \bar{a}_4^2} \cdot x = \frac{u_3}{a_4} \sqrt{\frac{2}{\gamma_4 + 1}} \frac{\gamma_4 - 1}{2} \left[ \frac{2}{\gamma_4 + 1} \right] + \left( \frac{\gamma_4 + 1}{2} \right) \frac{1}{\gamma_4 - 1} \quad (31b)
\]

The last two terms in both relations arise from the subsonic portion of the piston motion (\( M_3 \leq 1 \)) and are added to the supersonic path (\( M_3 \geq 1 \)).

(Equation 31a was developed independently of Eq. 31b, given in Ref. 43.)

Similarly

\[
\left( \frac{\gamma_4 + 1}{2} \right) \frac{3 \gamma_4 + 1}{2(\gamma_4 - 1)} \frac{A_p}{m \bar{a}_4} \cdot t = \left[ 1 - \sqrt{\frac{2}{\gamma_4 + 1}} \right] \frac{\gamma_4 - 1}{2} \frac{u_3}{a_4} \left( \frac{2}{\gamma_4 + 1} \right) \left( \frac{\gamma_4 + 1}{\gamma_4 - 1} \right) \]

\[
+ \frac{\gamma_4 - 1}{2} \left( \frac{\gamma_4 + 1}{2} \right) \left( \frac{\gamma_4 + 1}{2(\gamma_4 - 1)} \right) \quad (32a)
\]
or

$$t = \frac{A p_4}{m a_4} = \sqrt{\frac{2}{\gamma_4 + 1} \left( \frac{2}{\gamma_4 + 1} \right) \left[ \frac{\gamma_4 + 1}{2} - \frac{\gamma_4 - 1}{2} \frac{u_3}{a_4} \right] - 1}$$ \hspace{1cm} (32b)

$$+ \frac{\gamma_4 - 1}{2} \left( \frac{\gamma_4 + 1}{2} \right) \frac{3 - \gamma_4}{2 (\gamma_4 - 1)}$$

The last term in the above relations expresses the time taken to reach $M_3 = 1$, during the subsonic portion of the motion.

The above equations resemble those for the unchambered gun (Eq. 1) and have the correct limiting value of the escape velocity for the projectile of $\hat{u} = \sqrt{((\gamma_4 + 1)/2 \cdot 2 a_4/(\gamma_4 - 1)}$ when $x \to \infty$ or $t \to \infty$, (or for the shock tube $m = 0$ and $\hat{u}$ is obtained instantly). The motion is described with very fair accuracy despite the initial assumptions that were made regarding the projectile motion. It will be shown in the next paragraph that the steady expansion assumption slightly underestimates the distance $x$ covered by the projectile when $u_3/a_4 = \sqrt{2/((\gamma_4 + 1)}$ and the corresponding time $t$ when compared with the correct characteristics solution obtained in Refs. 8 and 9 and shown in Figs. 28 to 30, inclusive. This is to be expected since a lower driving pressure exists at the piston by neglecting the compression waves reflected from the chambrage plane. However, the difference is only significant at large values of $\bar{u}$ or $\bar{x}$, and for the case of $A_4/A_1 \to \infty$, the above equations can be used without recourse to the more time-consuming method of characteristics. These equations will be reconsidered in Subsection 3.4.

An actual portion of a characteristic diagram for $\gamma_4 = 1.4$ and a chambrage ratio $A_4/A_1 = 2.28$ ($d_4/d_1 = 1.511$) is shown in Fig. 28a and for $A_4/A_1 \to \infty$ in 28b. The details of the calculations appear in Tables 7 and 8, respectively. During early times only small gains appear for the case of $A_4/A_1 \to \infty$ compared with $A_4/A_1 = 2.28$. A comparison of the muzzle velocities that can be achieved with a perfect gas ($\gamma_4 = 1.4$) for chambrage area ratios of $A_4/A_1 = 1$, $A_4/A_1 = 2.28$, and $A_4/A_1 \to \infty$, can be seen in Figs. 29a and b. These figures reflect the pressure-velocity curve shown in Fig. 24, in that for a chambrage area ratio of $A_4/A_1 = 2.28$ about half of the effectiveness of infinite chambrage is already achieved, again confirming that one does not need to go to excessive area ratios to obtain sizeable benefits. It can also be seen that the gain in particle or muzzle velocity from chambrage becomes really effective only at the higher muzzle velocities or when $\bar{u} > 0.3$ to $0.4$ or $u_3 > 1.5 a_4$ to $2 a_4$. (When $A_4/A_1 \to \infty$ and $\bar{u} = 0.18$, sonic flow occurs at the projectile base, $\bar{x} = 0.024$. This is in good agreement with the value of $\bar{x} = 0.023$ obtained
from Eq. 29a.) Figure 30 shows the two extreme cases of \( \frac{A_4}{A_1} \to \infty \) and \( \frac{A_4}{A_1} = 1 \), at much greater muzzle velocities and distances. It is of interest to note that both curves maintain nearly a constant separation distance of about 0.09 \( \bar{u} \) for all \( x > 10 \) and reflects the statement that can be derived from Eq. 25 that the maximum velocity difference

\[
\left( \frac{\bar{u}}{\frac{A_4}{A_1}} - \frac{\bar{u}}{\frac{A_4}{A_1} = 1} \right) \left/ \frac{\bar{u}}{\frac{A_4}{A_1} = 1} \right. \right. = g \left( \frac{\gamma_4 - 1}{2 \gamma_4} - 1 \right)
\]

and for \( \frac{A_4}{A_1} \to \infty \) this value is 0.095 or \( g \left\vert \frac{A_4}{A_1} \to \infty \right. - 1 \sim 0.1 \)

or

\[
\left\{ \left( \frac{\bar{u}}{\frac{A_4}{A_1} \to \infty} - \frac{\bar{u}}{\frac{A_4}{A_1} = 1} \right) \right\} \sim 0.1 \frac{a_4}{a_{44}}
\]

That is, at best, for \( \gamma_4 = 1.4 \) chambrage will result in a 10% increase in ultimate velocity. Seigel (Ref. 10) points out that at the lower muzzle velocities a maximum percentage gain of about 28% is possible (\( \gamma_4 = 1.4 \)) owing to the nonlinearity of the two curves. However, it is not known explicitly whether such high increases in muzzle velocity have ever been attained as a result of chambrage under actual operating conditions.

It is worth noting from Fig. 22c that when \( \frac{d_4}{d_1} = 1.5 \), 50% of the infinite chambrage escape velocity is obtained, and from Figs. 29a and b it is seen that for \( \frac{d_4}{d_1} = 1.5 \) this percentage persists not for the escape velocity but for all velocities at any \( x \). Consequently, from Eq. 33, the identity \( \left( u_{\text{max}} \left| \frac{A_4}{A_1} = 1 \right. \right) \left/ \left( \frac{A_4}{A_1} \to \infty \right) \right. - \left( u_{\text{max}} \left| \frac{A_4}{A_1} = 1 \right. \right) \right. \) has been used as the ordinate in Fig. 22c. In view of the foregoing results, it is now possible to construct fairly accurate approximate curves (not analytic curves) for any chambered gun (with an effectively infinite chamber length) from the unchambered and infinite chambrage results. For example (Ref. 9), consider a gun with a chambrage area of \( \frac{A_4}{A_1} = 4 \) or \( \frac{d_4}{d_1} = 2 \), with a dimensionless barrel length \( \bar{x} = 0.015 \) for a driver gas with \( \gamma_4 = 1.4 \). From Fig. 22d or Eq. 33 find that the percentage increase in muzzle velocity for that chambrage over a gun with infinite chambrage is 70%.

From Fig. 29a, find \( \left( u_{\text{max}} \left| \frac{d_4}{d_1} = 1 \right. \right. = 0.126 \) and \( \left( u_{\text{max}} \left| \frac{d_4}{d_1} \to \infty \right. = 0.149 \). Therefore, from Eq. 33

\[
\frac{u_{\text{max}} \left| \frac{d_4}{d_1} = 2 \right. - 0.126}{0.149 - 0.126} = 0.70
\]
or \[
\bar{u}\left|_{\frac{d_4}{d_1}} = 2 = 0.142 \quad \text{or} \quad u_3 = 0.71 a_4
\]

Consequently, a new \(\bar{u}\) vs \(\bar{x}\) curve for \(\frac{d_4}{d_1} = 2\) or any other area ratio can be constructed once the two curves for \(\frac{A_4}{A_1} = 1\) (analytic) and \(\frac{A_4}{A_1} \to \infty\) (method of characteristics) (Fig. 30) have been drawn for a given \(\gamma_4\). Such a set of curves for \(\gamma_4 = 1.25\) appear in Fig. 31. The curve labelled \(\frac{d_4}{d_1} \to \infty\) was not constructed in a step-by-step manner as in Fig. 31. Instead, approximate relations were used over different ranges of the particle velocity (Ref. 8). Consequently, both the \(\frac{d_4}{d_1} \to \infty\) and the derived curve \(\frac{d_4}{d_1} = 1.5\) are labelled as "approximate".

### 3.2.2 Chambrage and Effectively Infinite Chamber Lengths

The foregoing analysis was based on a chamber length which was assumed to be infinite in length. In practice this requirement can be met for a given gun by making the driving chamber sufficiently long, such that the head of the rarefaction wave that strikes the end of the chamber and reflects does not overtake the base of the projectile while it is moving in the gun barrel, and thereby avoiding a reduction in the base pressure. Such a gun is said to have an effectively infinite chamber length.

It was noted in the previous subsection that when \(\frac{A_4}{A_1} \to \infty\) the first rarefaction wave \(R_1\) in the case of a shock tube disappears and in the case of a launcher the rarefaction pulses are reflected from the chambrage plane as compression pulses, but no transmitted rarefaction pulses are propagated into the chamber. Essentially, this means that the initial reservoir conditions are unchanged during the mass outflow through the chambrage plane during the short time the projectile is in the launcher barrel. (A very quick estimate can be made of this condition by comparing the mass of gas in the driving chamber (G) with the mass of the projectile (m). If the mass of driver gas is about five to tenfold greater, then this condition will usually be satisfied, as shown subsequently.)

The conditions that exist in a short chamber behind the projectile, when the head of the rarefaction wave (and other characteristics) reflected from the breech do catch up with the base of the projectile, is shown in Fig. 32a for a unchambered gun (Ref. 16). It is seen that four distinct regions exist in this case. Region I or state (4) is the rest state. Region II is the expansion wave region whose farthest extent ends at the overtake point \((x_1, t_1)\). It was shown by Stanyukovich (Ref. 17) that a projectile driven by the expansion of a gas in a chamber generates a centred rarefaction wave (Ref. 4) whose center is given by
\[ x_c = \frac{\mu^2}{Ap_4} \frac{1}{\gamma_4 \left( \frac{1}{\gamma_4} - 1 \right)} = \frac{2}{\gamma_4 + 1} \frac{x_4}{G/m} = \frac{2}{\gamma_4 + 1} \frac{ma_4^2}{p_4 A} = h \]

\[ t_c = -\frac{h}{a_4} \]  \hspace{1cm} (34)

where, the mass of driver gas \( G = Ax_4 \) \( \frac{\gamma_4 p_4}{a_4^2} \cdot Ax_4 \) or \( G/m = \gamma_4 \left( \frac{2}{\gamma_4 - 1} \right)^2 x_4 \) (see Eq. 43).

A chamber can be made to be effectively infinite in extent if the projectile path ends at \((x_1, t_1)\). That is, the base pressure of the projectile at this point is unaltered by the expansion characteristics reflected from the breech. If the barrel is made longer then it can be seen from Fig. 32a that the base pressure will be reduced by the reflected rarefaction wave pulses (characteristics) which overtake the projectile. For an effectively infinite chamber exact relations have been given in terms of the mass ratio \( G/m \) by Stanyukovich (Ref. 17) and in terms of \( h \) as follows (Refs. 16 and 18).

**Projectile path in \((x, t)\)-plane,**

\[ x = \hat{u}t + \gamma_4 h \left[ 1 - \left( 1 + \frac{a_4 t}{h} \right) \frac{2}{\gamma_4 + 1} \right] \]  \hspace{1cm} (35)

**Projectile velocity versus barrel length,**

\[ x = h \left[ 1 + (\gamma_4 \bar{u} - 1) (1 - \bar{u}) \gamma_4 \right] \]  \hspace{1cm} (36)

This relation is identical with Eq. 1b

**Projectile velocity versus time,**

\[ t = \frac{h}{a_4} \left[ \frac{1}{(1 - \bar{u}) \gamma_4} \right] - 1 \]  \hspace{1cm} (37)

This relation is equivalent to Eq. 1b.

**Projectile pressure versus time,**

\[ p_3 = p_4 \left[ 1 + \frac{a_4 t}{h} \right] \frac{2}{\gamma_4 + 1} \]  \hspace{1cm} (38)
Projectile acceleration versus time,

\[ \ddot{x} = \frac{a_4^2}{\gamma_4 x_4} \frac{G}{m} \left[ 1 + \frac{a_4^2}{h} \right] \frac{2}{\gamma_4 + 1} \]  

(39)

Along and across the characteristics generated by the projectile

\[ x = (u - a) t + h \propto \gamma_4 \bar{u} \]  

(40)

\[ \frac{2a}{\gamma_4 - 1} + u = \frac{2a_4}{\gamma_4 - 1} \]  

(41)

Using Eq. 41, Eq. 40 can be rewritten as

\[ (x - h) = (u - a) (t + h) \]  

(42)

That is, a projectile moving in an unchambered gun, owing to an isentropic expansion, will generate a rarefaction wave that is centred in the \((x, t) \)-plane at the point \((h, -h)\), as shown in Fig. 32a. In the case of a shock tube \(m = 0 = h\) and the centre is at \((0, 0)\). Stanyukovich (Ref. 17) has shown that this result also applies to a dense gas where the covolume \(b\) is significant. That is, \(p = A(s) \propto \gamma^2 / (1 + b \gamma) \) applies (see Eq. 27). Alternately, the perfect gas is a special case of the more general dense-gas driver.

Path of reflected rarefaction wave head,

\[ x_r = \frac{2}{\gamma_4 - 1} a_4 t + \gamma_4 h - \gamma_4 (x_4 + h) \left[ \frac{a_4^2 t + h}{x_4 + h} \right] \frac{3 - \gamma_4}{\gamma_4 + 1} \]  

(43)

for all \(t \geq x_4/a_4\)

Time when reflected wave head reaches projectile base,

\[ t_1 = \frac{x_4}{a_4} \left[ 2 + \frac{x_4}{h} \right] \]  

(44)

Position corresponding to Eq. (44),

\[ \frac{x_1}{x_4} = \frac{2}{\gamma_4 - 1} \left( 2 + \frac{x_4}{h} \right) + \gamma_4 \frac{h}{x_4} \left[ 1 - \left( \frac{x_4}{h} + 1 \right) \frac{4}{\gamma_4 + 1} \right] \]  

(45)

For a given gun barrel \(x_1\), gas mass \(G\), and projectile mass \(m\), Eq. 45 can be solved for \(x_4\) to given an effectively infinite launcher length.
Alternately, if the pressure $p_4$ and the sound speed $a_4$ are fixed, then $G = \gamma_4 p_4 A x_4 / a_4^2$ only depends on $x_4$ and Eq. 45 can be rewritten as

$$\frac{x_1}{h} = \frac{2\gamma_4 x_4}{\gamma_4 - 1} \left( \frac{x_4}{h} + a_4 \frac{1}{h} \left( 1 - \frac{x_4}{h} + 1 \right) \right)^4$$

and Equation (46) can then be solved for the required effectively infinite chamber length when the launcher length $x_1$ and the other quantities ($a_4$, $p_4$, $m$) are specified. (Of course, in Fig. 32, $x_4$ would have to be much longer to match the given barrel length $x_1 = L$.) To solve for the effects of a short chamber as shown in Fig. 32a, it is also necessary to solve for the penetration region (III) of effectively two equal strength expansion waves (the breech plane can be considered as an image plane when the rarefaction waves are equal, as in this case, see Refs. 20, 19 and 4), as well as the expansion region (IV) following the penetration. Usually this is done graphically or numerically but analytic solutions are given by Love and Pidduck (Ref. 21) and Parkinson (Ref. 16).

Parkinson considered a gun with the following parameters:

$L = 54$ ft, $A = 11.63 \text{ in}^2$ or $d = 3.85$ in, $m = 0.0349$ slugs (1-1/8 lb), $p_4 = 90,000$ psi, $a_4 = 10,000$ ft/sec, using helium as the driver gas ($\gamma_4 = 5/3$, $u_3 = 30,000$ ft/sec). From Eq. 34, $h = 2.5$ ft, and from Eq. 41, $x_4 = 8.39$ ft, the effectively infinite chamber length, or $x_1/x_4 = 6.44$.

From Eq. 36 it can be found that at $x_1 = L = 54$ ft, $u_3 = 15,630$ ft/sec and from Eq. 34, $G/m = 4.2$.

If the chamber is reduced to 7.5 ft, $G/m = 3.75$, then the muzzle velocity is only reduced to $u_3 = 15,550$ ft/sec. In this case, the head of the rarefaction wave overtakes the projectile base when it has travelled 78.7% of the barrel length. As noted in Ref. 17, for

$$G/m \geq \frac{2 \gamma_4}{\gamma_4 + 1} \left( 2 \frac{\gamma_4}{2} - 1 \right)$$

or in this case

$$G/m \geq 15/4 = 3.75$$

the waves reflected from the projectile never return to the breech. If the chamber is shortened to 1.944 ft, $G/m = 0.973 \sim 1$, then $u_3 = 10,870$ ft/sec, a reduction of 30.5%. In this case the reflected rarefaction wave head overtakes the projectile very quickly, after it has covered only 4.63% of the barrel length. The foregoing results from Ref. 16 are summarized in Figs. 32b, c and d. Figure 32b shows the rapid fall off in base pressure for the short chamber compared with the effectively infinite chamber as a function of time. Figure 32c is a plot of the projectile path and the rarefaction wave in the $(x, t)$-plane. It is seen that the head of the rarefaction
wave overtakes the projectile after it has travelled only 2.5 ft down the barrel. As expected, the trajectories of the projectile for the short chamber and effectively infinite chamber change from the point of overtake \((x_1, t_1)\). It takes the slower projectile 1.04 msec longer to pass through the 54 ft barrel as shown.

Figure 32d compares the velocity-position history for both chambers. It is seen that for the short chamber the velocity achieves a nearly asymptotic value of about 11,000 ft/sec, whereas the effectively infinite chamber has its maximum value of the muzzle velocity (about 16,000 ft/sec) for the given barrel length, and the velocity could be further increased in this case even if the barrel were lengthened and a rarefaction wave overtake did take place (this is to be expected, since \(x_1/x_4 \big|_\infty = 54/8.39 = 6.45\), whereas \(x_1/x_4 \big|_{1.944} = 54/1.944 = 27.8\)). The results illustrate very convincingly that short chambers or values of \(G/m\) say 75% lower than for the effectively infinite chamber length can drastically reduce the muzzle velocity of a gun.

Although the method used in Ref. 16 is very illustrative, it is in general tedious to compute for all values of \(\gamma_4\) for short chambers, and a graphical or numerical characteristics computation would be employed, especially with chambrage. This problem will be considered in the subsequent paragraphs.

For an unchambered gun, an alternate expression to Eq. 40 is given in Ref. 22 as

\[
\bar{x}_4 = \frac{1}{\lambda_4 (\lambda_4 - 1)} \left[ \left( 1 - \frac{\bar{u}_{x_1}}{\bar{u}_{x_1}} \right)^{-\frac{\lambda_4}{2}} - 1 \right]
\]

or

\[
x_4 = h \left( 1 - \frac{\bar{u}_{x_1}}{\bar{u}_{x_1}} \right)^{-\frac{\lambda_4}{2}} (1 - 1) \quad (48)
\]

where, \(\bar{u}_{x_1}\) is the velocity of the projectile as it leaves the muzzle at \(\bar{x} = \bar{x}_1\), and where the reflected pulse just overtakes it. The velocity can be found from Eq. 36 or Eq. 1b, for a given launcher barrel \(\bar{x}_1\) as

\[
\bar{x}_1 = \frac{1}{\lambda_4 (\lambda_4 - 1)} \left[ \left( \frac{\lambda_4}{\lambda_4 - 1} \left( 1 - \frac{\bar{u}_{x_1}}{\bar{u}_{x_1}} \right)^{-\frac{\lambda_4}{2}} \right) + 1 \right] \quad (1b)
\]

or

\[
x_1 = h \left[ \left( \frac{\lambda_4}{\lambda_4 - 1} \left( 1 - \frac{\bar{u}_{x_1}}{\bar{u}_{x_1}} \right)^{-\frac{\lambda_4}{2}} \right) + 1 \right] \quad (36)
\]
The results of such calculations for $\gamma_4 = 1.4$ for an unchambered gun ($d_4/d_1 = 1$) appear in Figs. 33 and 34. As expected from Eq. 48, the nondimensional chamber length, which is required for an effectively infinite chamber length operation, increases with projectile velocity (it would be infinite when the projectile attains the escape speed). This is illustrated in Fig. 33. Figure 34 shows a cross plot of the foregoing results by showing the ratio of barrel length to chamber length $x_1/x_4$ when the barrel length or chamber length is known for a given set of initial conditions. It is seen that as the launcher length $\bar{x}_1$ or $\bar{u}_{x_1}$ increases, the ratio $x_1/x_4$ increases, that is, as the muzzle velocity goes up ($\bar{x}_1$ or $\bar{u}_{x_1}$ can be increased by reducing $m$ or raising $p_4$ for a given $x_1$) the chamber length for a fixed barrel length must correspondingly be increased to avoid overtaking by the reflected rarefaction wave. (For example, at $\bar{x}_1 = 25 = 6$, $x_1/x_4 \approx 4$ for $A_4/A_1 = 1$ and $\bar{x}_1 \times 25 = 16$, $x_1/x_4 \approx 7$; owing to this nonlinear relation $x_4$ must increase by a factor of $12/7$ if $\bar{x}_1$ is increased by increasing $x_1$.) On the other hand, if $\bar{x}_1$ is increased by reducing $m$ or raising $p_4$ but keeping the barrel length $x_1$ fixed then $x_1/x_4$ is increased and this means that $x_4$ must decrease. That is, if the muzzle velocity is increased for a gun with a constant barrel by reducing the projectile mass or increasing the chamber pressure, then the chamber length for effectively infinite chambering goes down. This is reasonable since in this case the speed of sound in the chamber is not altered but the projectile is travelling faster so that a shorter chamber can be used before overtake can occur. This is analogous to the overtake problem in a shock tube (Ref. 4).

If the chambering is infinite then theoretically there should be no overtaking by the rarefaction wave head since it does not exist. However, to obtain a physical limit Seigel (Ref. 9) assumed that both the incident and the reflected wave heads will travel at the initial sound speed since conditions in the chamber are unaltered. Under this assumption the total time taken for the reflected head to reach the chamber plane is $2x_4/a_4$ (or $4 \bar{x}_4/(\gamma_4 - 1)$ in nondimensional coordinates). The characteristic equivalent to $P_I$ (Fig. 32a) is then found from the characteristics diagram used previously to determine the projectile path for the case of $A_4/A_1 \rightarrow \infty$ (Fig. 28b) or the reverse procedure can be used, that is, for any line $P$ find the corresponding $x_4$ by using the above time of $2x_4/a_4$. For chambering less than $A_4/A_1 \rightarrow \infty$, only the time $t_4 = x_4/a_4$ is known but now $P_I$ must be traced from the characteristic network. This has been done by Seigel (Ref. 9) and the results are shown in Figs. 33 and 34 for $A_4/A_1 \rightarrow \infty$ $A_4/A_1 = 2.25$ or $d_4/d_1 = 1.5$, for $\gamma_4 = 1.4$. It can be seen that the results for $d_4/d_1 = 1.5$ lie midway between those of $d_4/d_1 \rightarrow \infty$ and $d_4/d_1 = 1$, consistent with some of the previous results (Figs. 24 and 29).

From Fig. 33 it is seen that the chamber length required to achieve a given muzzle velocity $\bar{u}_{x_1}$ decreases with increasing chambering. For example, when $\bar{u}_{x_1} = 0.30$, $\bar{x}_4 = 0.045$ for $d_4/d_1 = 1.5$ and $\bar{x}_4 = 0.064$ for $d_4/d_1 = 1$, that is, a chamber 30% shorter for $d_4/d_1 = 1.5$. Figure 34 shows this point more effectively. It is readily seen that for a fixed barrel length
the ratio $x_1/x_4$ increases as $A_4/A_1$ or $d_4/d_1$ increases or the length of chamber decreases. Alternately, for a fixed chamber length $x_4$, the ratio $x_1/x_4$ increases with increasing $A_4/A_1$, or longer barrels can be used (higher muzzle velocities) for the same initial conditions. For example, consider a driver gas of the type shown in Table 3, that is, hydrogen at 100,000 psi and 6200°K with $\gamma_4 = 1.4$ (perfect gas) and $\bar{u} = 5 \times 1.97 \times 10^4 = 9.85 \times 10^4$ ft/sec. Assume that the muzzle velocity $u_3 = u_{x_1} = 30,000$ ft/sec or $\frac{u_3}{\bar{u}_{x_1}} = 0.305$. From Fig. 18, Part I, or Eq. 1, find $x_1 = 0.28$, for a single calibre projectile $x_1/d_1 = \frac{P_m}{P_4} \left( \frac{2}{\gamma_4 - 1} \right)^{1/2} \gamma_4 = 0.28 \times (1/5.21 \times 10^{-2}) \times 25 \times 1.4 = 188$ calibres, say a 200 calibre barrel. Then from Fig. 33, $x_4 = 0.066$ for $d_4/d_1 = 1$. That is, a chamber length of $188 \times 0.066/0.28 = 44.3$ or nearly 45 calibres would be required. For a single calibre projectile $G/m = (P_4/P_m) \left( \frac{A_4}{A_1} \right) (x_4/d_1) = 2.3$. For a chambrage diameter ratio $d_4/d_1 = 1.5$ this is reduced to 32 calibres or $G/m = 3.7$, that is, more driver gas is required for the chambered gun to avoid overtake. This is a very interesting result and stems mainly from the fact that the mass of driver gas goes up as the square of the driver diameter whereas it decreases more slowly as a result of the reduction of the chamber length due to chambrage. With the present method of calculation for $d_4/d_1 \to \infty$, the chamber length is further reduced to 24 calibres, about 1/8 of the gun barrel length.

The foregoing analysis due to Seigel was substantiated in Ref. 10. A series of tests was conducted on a uniform bore gun (0.520 in. dia.) and with chambrage (chamber diameters of 0.575, 0.750, 1.125 and 2.44 in. dia.). Gun barrels 16.75 in. and 30.5 in. long were used in the experiments. The chamber lengths were made considerably longer than required to give effectively infinite chamber lengths. A shear-type projectile weighing about 1 gm was used. The projectile was driven by 3000 psi air at room temperature. Corrections were made for friction (experimentally based, and assumed to be the same for all chambers with identical initial conditions and barrel lengths) and for the fact that the gun barrels were not evacuated thereby producing a counterpressure as a result of the formation of a normal shock wave (Part I, Subsection 2.3.2.).

The final results are shown in Figs. 35 and 36. The lines for zero chambrage and infinite chambrage on Fig. 35 were drawn from the theoretical results of Fig. 29. The other lines are a least-square fit to the experimental runs (6 to 8 runs for each average point that appears on the graph). Figure 36 shows the data plotted on a graph identical to that of Fig. 22c. It is seen that the chambrage diameter ratio $d_4/d_1 = 4.69$ or $A_4/A_1 = 22$ already achieves over 90% of the gain that is possible with infinite chambrage. The results may be taken as a very satisfactory substantiation of the theory of chambrage for guns whose initial conditions are well-known and stationary. This is usually not the case with conventional military guns where the combustion and motion of solid propellants makes the analysis considerably more complex (Ref. 23).
Recently, Seigel (Ref. 12) has also confirmed that the effect of using chambers shorter than the effectively infinite chamber length can produce a rapid deterioration in the muzzle velocity. If the mass ratio of propellant gas to projectile is written as

\[ \frac{G}{m} = \frac{\rho_4 A_4 x_4}{m} = \frac{A_4}{A_1} \cdot \frac{\gamma_4 A_1 p_4}{ma_4^2} = \frac{4}{A_1} \cdot \frac{A_4}{\gamma_4} \left[ \frac{2}{\gamma_4 - 1} \right]^2 \frac{A_1 p_4}{m u^2} x_4 \]

then \( G/m \to \infty \) means essentially that \( x_4 \to \infty \). However, in practice for an effectively infinite chamber \( x_4 \) will be finite and can be obtained from Fig. 33 as noted. When \( G/m < \infty \) then \( x_4 \) will be reduced and the muzzle velocity will start to decrease. This result is illustrated in Fig. 37. The curve labelled \( G/m \to \infty \) is effectively the \( A_4/A_1 \to \infty \) curve shown on Fig. 29. The other values of \( G/m \) are for \( x_4 \) less than the effectively infinite length. It is seen that for short chambers the muzzle velocity is greatly reduced and its maximum value does not increase with launcher length (as shown previously in Fig. 32d). The ratio \( G/m \) is physically very significant and shows that the mass of gas that is used to accelerate a projectile of a given mass must be about five (for an unchambered gun) to tenfold in magnitude (for a chambered gun) in order to achieve the muzzle velocity that is attainable with an effectively infinite chamber of a chambered gun. It is seen that the curve with \( G/m = 5 \) (half an order) lies quite close to the curve \( G/m \to \infty \), especially for the smaller barrel lengths or muzzle velocities.

Similar results for helium (\( \gamma = 5/3 \)) have also been calculated recently in Ref. 24 in some detail. Figure 38 shows the characteristic network in the \((x, t)\)-plane (it is worth noting that for \( \gamma = 5/3 \), \( t = 3t \) and \( \hat{x} = 9\hat{x} \)) for a chamber length \( \hat{x}_4 = 0.5 \) and a chamberage area ratio \( A_4/A_1 = 4 \). It is seen that the head of the rarefaction wave overtakes the projectile quite early in this case (\( \hat{x} \sim 1 \)). During the subsonic motion of the piston the rarefaction pulses generated by the motion will propagate towards the chamber plane where they will be transmitted into the chamber as rarefaction pulses (for all \( A_4/A_1 < \infty \)) and reflected into the barrel or launch tube as compression pulses. The pulse moving towards the breech will be reflected as a rarefaction pulse and later will be transmitted and reflected as a rarefaction pulse at the chamberage plane. The cumulative effect of transmitted compression and rarefaction pulses or characteristics will determine the projectile base pressure at a given time, as discussed previously. The net results are shown on Figs. 39 and 40. Figure 39 shows the \((\hat{x}, \hat{u})\)-relationships for the following conditions: \( A_4/A_1 \to \infty \), \( \hat{x}_4 \to \infty \); \( A_4/A_1 = 4 \), \( \hat{x}_4 \to \infty \) (\( G/m \to \infty \)); \( \hat{x}_4 = 0.5 \) (\( G/m = 3.33 \)), \( \hat{x}_4 = 0.25 \) (\( G/m = 1.66 \)), \( \hat{x}_4 = 0.125 \) (\( G/m = 0.83 \)); \( A_4/A_1 = 1 \), \( \hat{x}_4 \to \infty \), \( \hat{x}_4 = 2 \) (\( G/m = 3.33 \)),

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\( \hat{x}_4 = 1.0 \) (G/m = 1.66), \( \hat{x}_4 = 0.5 \) (G/m = .83). The following observations may be made. The two curves (Fig. 39a) \( A_4/A_1 \to \infty \) and \( A_4/A_1 = 1 \) for \( x_4 \to \infty \) approach a constant difference of 0.465 \( a_4 \sim 0.5 \) as \( \hat{u} \to 3 \), the escape velocity \( \hat{u} \), as given by Eq. 25. The curve \( A_4/A_1 = 1 \) for \( \hat{x}_4 \to \infty \) and \( \hat{x}_4 = 2 \) differ only at very high \( \hat{u} \), as expected. In other words, when G/m = 3.3 a practically infinite chamber length is achieved. The curve \( A_4/A_1 = 4 \) and \( \hat{x}_4 \to \infty \) has reached about 70% of the values between \( A_4/A_1 \to \infty \) and \( A_4/A_1 = 1 \) for \( x_4 \to \infty \) and \( \hat{x}_4 = 2 \), for velocity \( \hat{u} > 1.5 \). The same results for smaller velocities (barrel lengths) are shown in Fig. 39b, to a larger scale. The same remarks apply to the projectile paths in the launcher tube as shown in Figs. 40a and b. The detrimental effects of a short chamber (longest time to reach a given distance) are readily seen from these curves.

Some additional curves from Ref. 25, which are useful summary plots for launcher calculations, appear in Figs. 41 and 42. Figure 41 shows the \((\hat{u}, \hat{x})\)-relations for \( \gamma_4 = 5/3, 7/5 \) and \( 5/4 \). (The curve showing the gain in velocity of infinite chambrage to unchambered guns appears in error for large values of \( \hat{x} \) where the asymptotic value for \( \gamma_4 = 5/4, 7/5 \) and \( 5/3 \) are 1.061, 1.095 and 1.155, respectively. As indicated previously larger gains are apparently possible at the lower values of \( \hat{x} \) as noted, see Tables 7 and 8 and Fig. 30.) Figure 42 shows the chamber \((\hat{x}_4)\) and barrel \((x_1)\) lengths for effectively infinite chamber length operation for \( \gamma_4 = 7/5 \) and \( A_4/A_1 = 1 \), 2.25 and \( \infty \) and \( \gamma_4 = 5/3 \) for \( A_4/A_1 = 1 \), 4 and \( \infty \). If the chamber length is fixed then the barrel length can be calculated to provide the above condition.

In view of the foregoing, one should design a gun along the following lines perhaps. Decide the maximum mass of projectile to be used in the experiments and choose a driving gas mass about fivefold greater for a chambrage area ratio of about 2 to 4 and perhaps about tenfold greater for 4 to 9. Structural and model configurations will usually limit \( d_4 \) and \( d_1 \). The driver gas temperatures and pressures will limit the muzzle velocity \( u_3 \) and this will in turn limit the launcher barrel length \( x_1 \). With these quantities known, for a limited range, the chamber length can be made adequately long to suit this range.
In summary, it can be stated that although the maximum gain due to chambrage is only about 10 percent of the maximum muzzle velocity or $1/2 a_4$ (and perhaps actual gains of 10-15 percent at lower velocities), it is nevertheless very significant, especially at high muzzle velocities where additional gains become increasingly difficult to obtain even when all the initial conditions are optimized (pressure, temperature, projectile mass).

3.3 Combustion Heated Driver Gases

It was shown in Subsection 3.1 (Eqs. 2 and 3) that low values of the acoustic impedance $(\rho a)$ are required for an effective hypervelocity driver gas. It was noted that the light gases such as hydrogen or helium are very suitable for this purpose, especially when they are heated to high temperatures and pressures. It can be seen from Eq. 3 that when structural limitations set an upper limit on the driver pressure $p_4$, it is important to match (see Fig.19, Part 1) and keep the driver sound speed $a_4$ as high as possible in order to achieve hypervelocities.

One method of approaching this desired result is to burn stoichiometric mixtures of oxygen-hydrogen in the driver chamber and use the released energy to heat the remaining hydrogen or helium gas that is used as a diluent driver gas. Following the combustion process a number of chemical reactions take place such as the dissociation of the formed water vapor and other species. Consequently, a dissipation of some of the energy of combustion takes place followed by a drop in the final temperature of the driver gas. Nevertheless, significant improvement (70 - 80% for constant volume combustion) in sound speed over cold hydrogen can be readily achieved despite the increase in molecular weight. (A mixture of 1 mole of $O_2$ and 9 moles of $H_2$ will have a molecular weight of 5.)

In principle, this method of achieving efficient drivers for launchers, shock tubes and shock tunnels is quite feasible. (For very large hypervelocity guns (up to 14 in. dia., see Ref. 26a), it is perhaps the only practical method of heating the driver gas, otherwise, by using piston compression prohibitive chamber lengths would be required.) However, in practice this method requires the combustion of such mixtures at initial pressures of 10,000 - 15,000 psi and rather than deflagrating combustion, detonation combustion sets in with a resulting loss in repeatability of runs and an ever-present hazard of explosion. These detonations may arise from the effects of driver geometry, order of gas admission, insufficient time to permit proper mixing of the gaseous components, methods of ignition and other unknown causes perhaps (see Ref. 26b for some recent and worthwhile tests). As a result, many laboratories have abandoned this technique for producing efficient driver gases. Nevertheless, it is of importance to examine this process, for if the above problems can be solved it may become a very practical technique for heating driver gases for special applications. Some recent work at UTIAS has shown that this method of driving shock tubes is very good indeed, provided attention is paid to the composition of the driver gas, the use of impulsively heating tungsten ignition wires, and carefully controlled diaphragms (Refs. 13c and d).
In order to interpret some of the subsequent experimental results it may be useful to consider briefly some of the properties of combustion waves. A detonation wave, which is considered to consist of a shock wave and an attached combustion wave, propagates at supersonic velocity while a deflagration wave moves at subsonic velocity. Both waves are assumed to be thin (less than 1 mm at high pressure) and may be treated for simplicity as transition fronts. Consequently, an analysis similar to normal shock waves can be used (Part I, Eqs. 24 to 34). Consider the combustion wave in a one-dimensional flow (Fig. 43a) which is travelling into the unburned mixture in state (1) at rest, with a speed \(-w\), and induces a particle velocity \(u_2\). In stationary coordinates with respect to the moving wave, the equations of the motion can be written as (Ref. 27)

\[
\begin{align*}
\rho_1 V_1 &= f_2 \quad V_2 = m \\
p_1 + \rho_1 V_1^2 &= p_2 + \rho_2 V_2^2 \\
h_1 + \frac{1}{2} V_1^2 + Q &= h_2 + \frac{1}{2} V_2^2
\end{align*}
\]

(50)  
(51)  
(52)

Where, \(u_1\) and \(u_2\) are the absolute velocities of the unburned and burned gases respectively and \(V_1 = u_1 - w\) (since \(u_1 = 0\), \(V_1 = -w\)); \(V_2 = u_2 - w\); \(V_1\) and \(V_2\) are the relative velocities of the unburned and burned gases with respect to the flame front; \(V_1\) is also known as the burning velocity of the combustible mixture; \(Q\) = heat released at the wave per unit mass).

From Eqs. 50 and 51

\[
\frac{p_2 - p_1}{v_1 - v_2} = m^2, \quad V_1^2 = \frac{\rho_2}{\rho_1} \cdot \frac{p_2 - p_1}{\rho_2 - \rho_1}, \quad V_2^2 = \frac{\rho_1}{\rho_2} \cdot \frac{p_2 - p_1}{\rho_2 - \rho_1}
\]

(53)

where, \(v = 1/\rho\), specific volume.

From Eq. 53, it can be concluded that the numerator and the denominator must both have the same sign and as a consequence (Eq. 51) the relative velocity difference \((V_1 - V_2)\) must also have the same sign. Consequently, the existence of two possible waves are indicated. In stationary coordinates, one type, the detonation wave, produces an increase in the thermodynamic quantities (pressure, density, etc.) but a decrease in the dynamic quantity (relative velocity \(V_2\)). The other type, the deflagration wave, produces an increase in the relative velocity \(V_2\), and temperature \(T_2\) and a decrease in the pressure \(p_2\) and density \(\rho_2\), as shown in Fig. 43b. It is of interest to note that for an adiabatic process \((Q = 0)\) the normal shock wave corresponds to the detonation wave, whereas the deflagration wave has no counterpart (an expansion shock violates the condition that the entropy must increase, and therefore it does not exist).

Schematic diagrams of the structure of the steady and non-stationary portions of a detonation wave are shown in Figs. 43c (Ref. 36b) and 43d (Ref. 37c), and some relevant comments appear in the figures. The detailed structure of detonation waves with nonequilibrium effects is still not fully understood.
A substitution of Eq. 53 in Eq. 52 results in the so-called Hugoniot relations

\[ h_2 - h_1 = \frac{1}{2} (p_2 - p_1) (v_1 + v_2) + Q \]

or

\[ e_2 - e_1 = \frac{1}{2} (p_2 + p_1) (v_1 - v_2) + Q \]

where, \( h = e + pv \), the enthalpy.

Since \( h = h(p, v) \), Eq. (54) defines the loci of final states (2) that can be reached from initial states (1) through a combustion wave for different values of \( Q \). Such loci are illustrated in Fig. 43b, for \( Q = 0 \) for a normal shock wave, and \( Q > 0 \) for deflagration and detonation waves. The locus going through the initial state \((1, 1)\) is the usual one for a normal shock wave when \( Q = 0 \), which has a slope of \(-\frac{\partial}{\partial v_2}\) at that point. Only the upper branch has physical significance; the lower branch violates the second law of thermodynamics, as noted previously. The locus for \( Q > 0 \) lies to the right of the initial state \((1, 1)\) and has two branches, namely, the detonation branch AJB and the deflagration branch CKD. Tangents drawn from the initial state to the upper and lower branches result in the so-called Chapman-Jouguet detonation and deflagration, respectively. The hypothesis is made that \( M_2 = 1 \), at \( J \) and \( K \). This hypothesis is based on arguments that at this point the detonation wave is stable. It can be seen, for example, that the slope from the initial state \((1)\) to the final state \((2)\) at \( J \) has an isentropic slope given by

\[ \left. -\frac{\partial p_2}{\partial v_2} \right|_S = \frac{p_2 - p_1}{v_2 - v_1} \text{ or } a_2^2 = \frac{\rho_1}{\rho_2} \frac{p_2 - p_1}{\rho_2 - \rho_1} \]

Comparing, Eqs. 55 and 53, it is seen that at \( J, V_2 = a_2 \) or \( M_2 = 1 \) (the Chapman-Jouguet condition). The Chapman-Jouguet hypothesis has been substantiated by a number of experiments although the existence of stronger overdriven detonations have also been found. From considerations of transport phenomena weak detonations are ruled out as possible fronts.

It is worth noting that the condition \( M_2 = |V_2|/a_2 = 1 \) or \((w - u_2)/a_2 = 1\) in the case of a detonation wave is equivalent to stating that the absolute detonation velocity \( w_{\text{det}} = D = u_2 + a_2 \) (alternately from Eq. 50, \( D = \frac{\rho_2 a_2^2}{\rho_1} \)). Since nonequilibrium processes take place the question arises regarding the appropriate sound speed, (frozen composition and vibration or equilibrium) to be used in this case. For a uniformly moving detonation the equilibrium sound speed is used (Ref. 28a), since the frozen characteristics up to the position of the equilibrium wave head in a nonequilibrium expansion wave will decay (Ref. 5). It is seen that \( u_2 + a_2 \) is just the characteristic velocity of a \( P \)-type disturbance pulse and it cannot overtake the detonation front. Hence unlike a normal shock wave the detonation front is isolated from upstream influences and can therefore maintain stable properties (see Ref. 36b for a good review).
The deflagration states lie along CK where $M_2 < 1$. Those lying along KD are not possible as $M_2 > 1$, in violation of the second law of thermodynamics. It has been found that for normal deflagrations the final velocity $V_2$ is such that $p_2 \omega p_1$, at a point $n$, just below C on the deflagration branch. Nevertheless, Troshin (Ref. 29) points out that for turbulent combustion final velocities from $V_n$ to $V_{K} (M_2 = 1)$ are physically possible. The segment BC has no physical meaning since it yields imaginary values of $V_2$.

It should be noted that when a combustible mixture is ignited it usually starts to burn through a deflagration wave, which in time may or may not develop into a detonation wave. The time or distance required for the onset of detonation is known as the induction time or induction distance, respectively. The induction time can be very short (microsecond) or very large and depends on the chemical kinetics of the combustible mixture, the initial temperature and pressure, the energy and method of ignition and the geometry and character of the boundary layers of the combustion chamber. While the deflagration flame front travels, it behaves like a piston and it generates compression pulses that coalesce to form a shock front. This additional compression and heating changes the initial conditions in which the detonation wave forms. Consequently, when the detonation wave is finally generated it is much stronger than that based on the pre-ignition properties of the combustible gas. Analytical methods for predicting the induction time or distance are as yet not available. Further details are beyond the scope of this report and may be found in Refs. 17 (Chapter 7) and 27 to 36.

In using combustion heated driver gases, the combustion-wave dynamic processes are usually ignored and the final states after combustion are computed using the methods of chemical thermodynamics (Refs. 32 and 37). The combustible mixtures of interest are stoichiometric mixtures of oxygen and hydrogen diluted with helium or hydrogen. These can be expressed symbolically as

$$(2 + m)H_2 + O_2 + nHe$$

Where, the molar indices $m$ and $n$ take on values of zero or positive values. The final products are $H_2O$, $OH$, $H_2$, $H$, $O$ and He. It is seen that the use of oxygen gives rise to water vapor and other diatomic molecules that can dissociate. (As noted previously, the presence of oxygen and its products of course increases the molecular weight of the driver gas, a disadvantage that must be accepted.) It is possible to find the final composition and thermodynamic quantities by solving a set of nonlinear algebraic equations which employ the conservation of mass for each species, equilibrium of each dissociation reaction, conservation of energy, Dalton's law, and the equation of state for a constant volume reaction. The details of such calculations can be found in Refs. 37a to c and are illustrated in Figs. 44 to 50 inclusive.
Figure 44a shows the final pressure $p_f$, as a function of helium dilution in moles $n$ for various initial pressures $p_i$. It is seen that the maximum pressure occurs for the stoichiometric mixture and then it decreases with increasing dilution. The highest pressure ratio $p_f/p_i$ (Fig. 44b) occurs when $n \sim 0.5$. The asymptotic or zero dissociation limit (infinite pressure) is given by the uppermost curve, with the subscript A.

The final temperature curves $T_f$ are shown in Fig. 44c. It is seen that at low pressure and weak dilutions, when a high degree of dissociation can occur, the final temperature is lower since the combustion energy is expended in dissociating the molecules. However, at high pressures and weak dilutions this energy goes into increasing the translational temperature (thermal energy). At very strong dilutions the differences are small, as expected. The temperature ratios $T_f/T_i$ appear in Fig. 44d and exhibit characteristic shapes similar to Fig. 44c. The maximum values are attained at the stoichiometric conditions. If an excess of oxygen had been used (that is, less than 67 molar percent of hydrogen) lower values would have been obtained.

The final molecular weight $m_f$ of the products of combustion (Fig. 44e) shows that as the helium dilution increases the molecular weight drops and the effects of dissociation are small. However, for stoichiometric mixtures dissociation does lower the molecular weight, especially at a low initial pressure. The limiting value of $m_fA = 18$ can be seen.

The final frozen composition specific heat ratio $\gamma_f$ appears in Fig. 44f. It is seen that lowest values of $\gamma_f$ are attained on the no-dissociation curve $\gamma_fA$, where the heat of combustion goes into exciting fewer molecules in their translational, rotational and vibrational modes and approaches a limit of $7/6$ for the H$_2$O molecule at high temperature with full vibrational excitation (36 cal/mole).

Figure 44g shows the final frozen composition sound speed $a_f$. It is seen that the maximum sound speed locus varies from about $n = 7$ (70 percent helium dilution), for the no-dissociation case, to $n = 11$ (79 percent helium dilution) for $p_i = 1$ psi. However, the variation in $a_f$ is only about 50 ft/sec over the pressure range $1 \leq p_i \leq 10,000$ psi. The final to initial sound speed ratio $a_f/a_i$ appears in Fig. 44h. It can be seen that the maximum ratio occurs for one mole helium dilution.

Two other graphs which may be of some interest are the final frozen composition specific heat at constant pressure $C_{pf}$ and a typical species concentration such as water vapor, as shown in Figs. 45 and 46, respectively. Figure 45 shows the limiting value of $C_{pf}$ for the zero-dissociation case of 14 cal/mole for a stoichiometric mixture. For lower pressures and strong dilutions the values decrease. Figure 46 is typical of a number of graphs in Ref. 37b on concentration of species such as H$_2$, O$_2$, OH, H$_2$O, H, O and He. It is seen that at low pressures and weak dilutions considerable water vapor can be dissociated.
Similar results for hydrogen dilution appear in Figs. 47a to h, 48a and 48b. As expected, the limiting values for stoichiometric mixtures \((m = 0 \text{ or } n = 0)\) are identical. However, the diluted mixtures for diatomic hydrogen and monatomic helium are quite different. For example, similar results for final pressure temperature and molecular weight are obtained with a dilution of 6 moles of hydrogen compared with 12 moles of helium, that is, the zero dissociation case is approached for a lower dilution with hydrogen. The values of the specific heat ratios \(c_f\) are quite different (except at \(m = 0, n = 0\)), as expected. Like helium, the final sound speed \(a_f\) (Fig. 47g) appears to be approaching a maximum with a 7 mole dilution (but these values have not been computed for \(m > 6\)).

From Eqs. 1, 36, or 59, it is seen that the escape speed is a very significant index for hypervelocity launchers. A plot of the escape velocity \(u_F\) based on the values of the frozen sound speed and specific heat ratio (Figs. 44f, 44g, 47f, and 47g) appears in Fig. 49a. It is seen that the largest escape velocity occurs for the stoichiometric mixture at very high pressure (no dissociation, perfect gas). The maximum value of 22.3 km/sec may be compared with that of the diluent gases at 0°C, where \(u_{H_2} = 6.35 \text{ km/sec and } u_{He} = 2.89 \text{ km/sec}\).

The effects of dissociation on the escape speed are very marked. For helium the escape speed decreases monotonically with decreasing pressure and increasing dilution; for hydrogen the decrease is not as rapid and a locus of maxima appears to exist as shown by the dashed curve.

When Fig. 49a is viewed in conjunction with the slowly varying final pressure curves, Figs. 44a and 47a, it is apparent that the hydrogen dilution is best for hypervelocity drivers. However, it should be noted that the equilibrium specific heat ratio \(c_{fe} = \frac{\partial p}{\partial T} \mid_s\) should correctly have been used, but were not available.\(^*\) (It can be inferred from the results shown in Fig. 50g, for detonations, that significant differences will occur mainly at low initial pressures.)

Dunn (Ref. 37c) has recently computed the properties of detonating hydrogen-oxygen mixtures \((H_1 \equiv 0.6H_2 + O_2, H_2 \equiv 1.05 H_2 + O_2, H_3 \equiv 2H_2 + O_2, H_4 \equiv 2.34 H_2 + O_2, H_5 \equiv 2.55 H_2 + O_2, \text{ that is a molar percentage of hydrogen of } 20, 35, 66 2/3, 78 \text{ and } 85, \text{ respectively})\). Some results are shown in Figs. 50a to 50h, inclusive.

The equilibrium pressures for Chapman-Jouguet (C-J) detonations (following the shock front (von Neumann spike), induction zone and reaction zone) are shown in Fig. 50a. Like the constant-volume combustion the maximum pressures \(p_f\) are obtained for stoichiometric mixtures and decrease for oxygen or hydrogen dilutions. However, as can be seen from Fig. 50b and Table 9, below, the pressure ratio \(p_f/p_i\) for stoichiometric mixtures \((H_3)\) is nearly doubled for the detonation wave compared with the constant-volume combustion. However, the final temperature \(T_f\) (Fig. 50c) is only somewhat greater in the detonation case.

\(^*\) Note in final proof: The equilibrium results appear in Appendix 1 and in Fig. 49b. Higher equilibrium escape speeds are obtained, as expected.
TABLE 9

Comparison of Constant-Volume Combustion Properties with Those Obtained in a Chapman-Jouguet Detonation for a Stoichiometric Hydrogen-Oxygen Mixture

<table>
<thead>
<tr>
<th>Initial Pressure Atm (300°K)</th>
<th>Final Temperature, °K</th>
<th>Final Pressure, Atm</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Constant Volume Combustion</td>
<td>C-J Detonation</td>
</tr>
<tr>
<td>1</td>
<td>3503</td>
<td>3667</td>
</tr>
<tr>
<td>100</td>
<td>4366</td>
<td>4623</td>
</tr>
<tr>
<td>1000</td>
<td>4793</td>
<td>5103</td>
</tr>
</tbody>
</table>

(Ref. 37c)

It can be seen from Figs. 50a, b, and c that as the dilution with oxygen or hydrogen increases the final pressure and temperature decrements compared with the stoichiometric mixture, especially at high dilution (H1), since relatively little heat of combustion is available to raise these quantities.

The final molecular weight \( m_f \) and molecular weight ratio \( m_f/m_i \) appear in Figs. 50d and c. As may have been expected, negligible variations occur for strong dilutions (H1). The maximum variations occur for stoichiometric mixtures at low initial pressures when significant dissociation occurs, as in the constant-volume combustion cases (Figs. 44e and 47e).

Figures 50f, g and h show the specific heat ratio at constant pressure \( C_{pf} \), the specific heat ratio \( \varphi_f \) and the sound speed \( a_f \), respectively. The frozen composition (F) or nonreacting values as well as the equilibrium (E) or reacting values have been calculated (Ref. 37c). It was noted in the foregoing that the frozen wave head decays and it is the equilibrium sound speed which is associated with the detonation velocity \( D = u + a_E \). It can be seen that at very high pressures (zero dissociation) the two values approach each other asymptotically. At low pressures, the sound speed for frozen composition exceeds that for a reacting gas mixture.

Although the combustion-wave dynamics was ignored in the calculation of the final states of a constant volume combustion, it is of great importance in the actual operation of combustion drivers. The formation of detonation waves can destroy parts of the equipment and under the best of conditions does not generate a driver gas with uniform properties. Consequently, detonating combustion is not too useful for conventional shock tubes or launchers. It has been found in shock-tube research that deflagrating combustion provides the best type of driver gas provided the diaphragm is ruptured at peak pressure, (when the gas is at rest). Beyond that
point the pressure and temperature of the gas decreases owing to the cooling of the gas by conduction to the chamber walls. However, if the diaphragm is opened prematurely before the combustion process is complete, the possibility exists of detonation arising from the turbulent mixing at the diaphragm station. Although the latter method (so-called "constant-pressure" combustion, Ref. 4) yields the strongest initial shock waves, the attenuation of the shock wave is much greater (Ref. 38). The same results would undoubtedly carry over to hypervelocity launchers.

Reproducible constant-volume combustion can be achieved by ensuring that the partial pressures of the combustible constituents are measured very accurately (especially at the lower pressures where errors in composition can be significant), that the constituents are given enough time to mix or diffuse (at least 5 to 10 minutes should be allowed up to 1000 psi inlet pressures and perhaps much more at 10,000 psi) and that multiple point ignition is used. The latter provides a means of reducing the overall induction distance from deflagration to detonation, since the multiple point ignition causes the flame fronts to coalesce and consume the combustible mixture before a detonation wave can form. Although multiple point ignition can be achieved by using spirally placed spark plugs in shock-tube chambers, (Refs. 39 and 40), this method must be replaced in guns, owing to the weakening of the chamber by the holes at much greater pressures. Usually an igniter tube (combined with a gas inlet mixing tube) containing crimped (every 3 or 6 in.) aluminium or copper wire or fuse wire (see Fig. 51) is used for this purpose (Refs. 41a and b). (Accurately notched fuse wire, on a cellulose tape backing so that it may be stretched and properly located throughout the length of the chamber, has also been used very successfully at NOL, whereas heated tungsten wires were found to be best at NASA, Ref. 26h. This has also been substantiated recently at UTIAS in the 2 in. x 2 in. and 4 in. x 7 in. hypersonic shock tubes-Refs. 13c, d). Although the circuit in Fig. 51 shows a power capacity of 110 joules, only a portion of the total energy is usually found at the ignition points, the rest is dissipated in switching and other losses. However, if large amounts of energy are added at the points of ignition, then for some combustible mixtures the induction time is nearly reduced to zero and detonations can be generated instantly (Ref. 42). The onset of detonation as was noted previously is also strongly dependent on geometry (it is much more difficult to produce spherical detonation waves than planar waves) and the type of mixture. For example (Ref. 37a), under identical conditions it was found that a stoichiometric mixture of oxygen and hydrogen can be diluted with less than 7 moles of hydrogen (70 percent by volume) before it will detonate in a hemispherical chamber, whereas less than 3 moles are required with helium dilution. The reasons are not yet established and undoubtedly depend on chemical kinetic processes. For launcher combustion chambers the following mixtures have been used: \((O_2 + 2H_2) + 7H_2\), \((O_2 + 2H_2) + 7He\) to \(12He\) (a 70% to 80% dilution, \(7He\) appears to approach the onset of detonation whereas \(12He\) would appear to approach the limit of no combustion, Ref. 40), \((O_2 + 2H_2) + H_2 + 8He\) appears to have been successfully used at AEDC (Ref. 41a), a fairly lean mixture.
It is worth considering some of the test results obtained in Ref. 41a, in the 3 in. dia. x 40 in. long combustion chamber, using constant-volume combustion (the diaphragm was replaced by a solid retainer). A schematic trace of a pressure record is shown in Fig. 52, where the charging, maximum, and effective pressures are shown. It is seen that during combustion a peak pressure is reached and then the pressure drops quite rapidly owing to a temperature strain-gauge effect on the pressure gauge (additional details from a spherical combustion study (Ref. 37a) will be found in Section 3.7). The apparent heat transfer can be written in terms of the pressure drop

\[ q = \left( \frac{\partial q}{\partial T} \cdot \frac{\partial T}{\partial t} \right)_v = C_v \cdot \frac{1}{R\varrho} \left( \frac{\partial p}{\partial T} \right)_v \]  

(56)

where, \( \varrho = \) constant, and \( R \) and \( C_v \) apply to the products. It was also liberally assumed (Ref. 41a) that the heat losses during combustion were the same as those during the cooling process (they would be smaller, as only a portion of the total gas is involved) in order to correct for peak pressures and estimate heat transfer rates. Figure 53 shows the calculated (assuming no dissociation), and actual final pressure as a function of initial charging pressure. It is seen that the agreement (5% loss in pressure) is surprisingly good compared to other workers (Ref. 37a and 40). A final to initial pressure ratio factor of 7 is achieved at initial pressures of 4000 psi and 6 at 8500 psi. For comparison purposes, results from another study (Ref. 26b) are also shown in Fig. 53b. It is seen that rather large deviations are obtained that require some analysis. It can be concluded that the combustion data obtained by various researchers are at variance and these processes are far from being understood.

Figure 54 shows the estimated heat transfer rates for complete combustion (2700 K) and incomplete combustion (1000 K \( \sim 30\% \) combustion, assuming a linear burning rate). The heat transfer rates appear to be reasonable for the high initial gas densities and final temperatures. (See for example the heat transfer rates for arc chambers shown on Fig. 84, at equivalent temperatures and considerably lower densities, but for different gases.) (The sound speed and \( \varphi \) for incomplete combustion appears in Fig. 55.)

Finally, the muzzle velocities for this 1. 580 in. dia. x 312 in. long (200 calibre) gun appears in Fig. 56, based on the actual measured combustion pressures, and computed using Eq. 1. It is seen that a 20 gm projectile (1 calibre plastic cylinder, \( \varrho = 1.5 \text{ gm/cm}^3 \)) can achieve 16,000 ft/sec at a pressure of 60,000 psi and sound speed of 5700 ft/sec. It may be verified from Eq. 1 (or Fig. 18 Part I) that a cold hydrogen driver under the same conditions would yield a muzzle velocity of about 10,800 ft/sec and a cold helium driver would give 8200 ft/sec. Although the increase with a combustion driver is significant and useful, especially for large guns (even up to 14 in. dia. Ref. 26a), it is not a sufficiently large improvement for smaller guns where velocities of about 25,000 ft/sec are
attainable for small sabot-launched models and 50,000 ft/sec would be desirable. Consequently, this technique must be coupled with other launcher stages to be effective in producing really high muzzle velocities and will be considered in Section 4.

3.4 Piston Type Compressors and Heaters

It was shown in Subsection 3.3 that although in principle combustion provides a means of generating a high-pressure, high-enthalpy driver gas, in practice this is difficult to achieve owing to a lack of complete control over the combustion process. Furthermore, combustion temperatures and molecular weights are limited. Consequently, other means of heating a driver gas had to be developed by various investigators. The compression and heating process of a gas by using a free piston has made some very significant improvements in increasing the muzzle velocities of hypervelocity launchers.

The question arises as to whether a fast, light piston should be used to compress and heat a driver gas nonisentropically by induced multiple shock waves or should it be done isentropically by a slow, heavy piston. Although the light piston can produce a very high temperature in the driver gas, it is also coupled with very high peak pressures and oscillations that are produced when the piston is brought to rest and has given up its kinetic energy to the gas. Consequently, structural limitations exist for the piston itself as well as the compression chamber (pump tube). An analysis (Ref. 43) shows that for launchers it would be best to use preheated hydrogen coupled with a relatively slow, solid, heavy piston that is correctly designed to supply the desired high (almost constant) base pressure and high temperature for the projectile over the entire duration of its travel in the launcher barrel. It is shown that it should theoretically be possible to achieve muzzle velocities as high as 50,000 ft/sec. That is, in practice a gun must be designed for optimum performance over its entire operating cycle and not in a piecemeal fashion.

The motion of a piston in a barrel with counterpressure was considered in Subsection 2.3.2, Part I. If the piston moves slowly isentropic compression results (Fig. 57) and the final temperature $T_2$ can be expressed in terms of the initial temperature $T_1$ and the pressure ratio as $T_2 = T_1 \left(\frac{p_2}{p_1}\right)^{\left(\frac{\gamma-1}{\gamma}\right)}$, for a perfect gas. For a rapidly moving piston a shock wave is produced and nonisentropic heating takes place which will result in a higher final temperature. It can be seen from Fig. 57 that the shock strength grows with time and thereby produces a nonuniform heating (entropy) of the gas. The extent of the gas affected by the process can be reduced by very rapid acceleration (high initial pressures $p_4$) or by using an evacuated barrel section in which the piston achieves a nearly uniform velocity before it is permitted to shock-compress the gas, which is initially retained in the rest of the barrel by a thin diaphragm (Fig. 58). From Eq. 34, Part I, the limiting shock Mach number $M_s$, in a shock tube or launcher is given by
Therefore, the effectiveness of shock compression depends on a high initial pressure ratio $P_4/P_1$ and high piston velocity compared to the sound speed of the gas that is being heated $u_3/a_1$. In turn, the strengths of the reflected shock waves (Fig. 58) will be increased with initially stronger shock waves resulting in high final pressures and temperatures as the piston is brought to rest. The foregoing shock compression process may be analysed as follows.

It was shown in Subsection 3.2 that the equations of motion of a piston in an evacuated barrel driven from a chamber with infinite chambrage are described with good accuracy by Eqs. 29a and 31b:

$$\left(\frac{2}{\gamma_4 - 1}\right)^2 \frac{1}{x} = \left[1 - \frac{2}{\gamma_4 - 1} \cdot \frac{\bar{u}}{u_0}\right]^{-1} \frac{1}{\gamma_4 - 1}$$

for $\frac{u_0}{\bar{u}} \leq \frac{\gamma_4 - 1}{2} \sqrt{\frac{2}{\gamma_4 + 1}}$

$$\left(\frac{2}{\gamma_4 - 1}\right)^2 \frac{1}{x} = \frac{2}{\gamma_4 - 1} \cdot \frac{\bar{u} - \frac{2}{\gamma_4 + 1}}{\frac{\gamma_4 + 1}{2} \cdot \bar{u}} + \left(\frac{\gamma_4 + 1}{2}\right) \frac{1}{\gamma_4 - 1}$$

for $\frac{\bar{u}}{\bar{u}_0} \geq \frac{\gamma_4 - 1}{2} \sqrt{\frac{2}{\gamma_4 + 1}}$

Equations 58 and 59 were solved on a digital computer and the values are shown in Tables 10 and 11. Table 10 compares the calculated values with those shown in Table 8 and obtained in Ref. 8 by using the method of characteristics. It is seen that good agreement is obtained over practical barrel lengths $\bar{x}$. Table 11 shows the new values of $\bar{x}$ corresponding to the values of $\bar{u}$ given in Table 3; Part I, for unchambered launchers. As discussed in Subsection 3.2.1, the advantage of chambrage can be seen by comparing corresponding quantities $(\bar{u}, \bar{x})$ in these two tables over a wide range of the specific heat ratio $(\gamma_4)$. In view that the equilibrium value of the specific heat ratio in the chamber $\gamma_e$, can be used to give a fairly
accurate value of the variation of piston velocity with base pressure (Figs. 11 and 19), the present perfect gas results should be useful in actual launcher analyses. The data are presented in Fig. 59 in the form of a carpet plot similar to Fig. 18, Part I, for an unchambered launcher. In this case as well, $x \to 0$ for $u \to 0$, regardless of $\gamma$, and when $u \to \frac{1}{2}(\gamma + 1)/2$, the maximum escape speed, then $x \to \infty$. Again, the approach is most rapid for the limiting case $\gamma \to 1$, since the power index $(\gamma + 1)/(\gamma - 1) \to \infty$ and the term in the square bracket approaches zero very quickly.

Once the piston speed ($u_p = u_3$) has been found from Fig. 59, the state of the gas which has been processed by multiple shock reflections between a rigid wall and a uniformly moving piston (Fig. 58) can be found for a perfect gas (Ref. 44) or an imperfect gas (Ref. 45). The following stationary shock relations for a perfect or imperfect gas can be used (Refs. 4, 43, and 45).

Mass: \[ \rho_n V_n = \rho_{n-1} V_{n-1} \] (60)

Momentum: \[ p_n + \rho_n V_n^2 = p_{n-1} + \rho_{n-1} V_{n-1}^2 \] (61)

Energy: \[ h_n + \frac{1}{2} V_n^2 = h_{n-1} + \frac{1}{2} V_{n-1}^2 \] (62)

Hugoniot: \[ h_n - h_{n-1} = \frac{1}{2} (p_n - p_{n-1}) \left( \frac{1}{\rho_{n-1}} + \frac{1}{\rho_n} \right) \] (63)

State: \[ p_n = Z_n \rho_n R T_n \] (64)

Reference sound speed: \[ a_r^2 = \gamma_r R T_r \] (65)

From Eq. 61 \[ p_n - p_{n-1} = (V_n - V_{n-1})^2 \frac{\rho_n}{\rho_{n-1}} \] (66)

A substitution of Eq. 66 into Eq. 63 yields \[ h_n - h_{n-1} = \frac{1}{2} (V_n - V_{n-1})^2 \frac{\rho_n}{\rho_{n-1}} \] (67)

Where (Fig. 58), $V = (u - w)$ with respect to a shock wave moving to the right and $V = (u + w)$ with respect to a shock wave moving to the left. For example, for $n = 1$ with respect to $w_1$, $V_0 = u_0 - w_1$, $= -w$, since $u_0 = 0$; $V_1 = u_1 - w_1$; $V_1 - V_0 = u_1 - u_p$, the piston speed. Similarly, for $n = 2,$
with respect to $w_2$, $V_2 = u_2 + w_2 = w_2$, since $u_2 = 0$; $V_1 = u_1 + w_2$; $V_2 - V_1 = -u_1 = -u_p$, i.e. the difference in the relative velocities gives the magnitude of the piston speed.

From Eq. 60,

$$w_1 = \frac{u_p}{1 - \frac{\rho_0}{\rho_1}} \quad \text{and} \quad w_2 = \frac{u_p}{\frac{\rho_2}{\rho_1} - 1}$$

i.e.,

$$w_n = \frac{u_p}{1 - \frac{\rho_{n-1}}{\rho_n}}$$

(68a)

$$w_n = \frac{u_p}{\frac{\rho_n}{\rho_{n-1}} - 1}$$

(68b)

Also

$$h_1 - h_0 = \frac{1}{2} u_p^2 \frac{\rho_1 + \rho_0}{\rho_1 - \rho_0}$$

(67a)

$$h_2 - h_1 = \frac{1}{2} u_p^2 \frac{\rho_2 + \rho_1}{\rho_2 - \rho_1}$$

$$p_1 - p_0 = u_p^2 \frac{\rho_1 \rho_0}{\rho_1 - \rho_0}$$

(66a)

$$p_2 - p_1 = u_p^2 \frac{\rho_2 \rho_1}{\rho_2 - \rho_1}$$

Equations 64 to 67 can be expressed in the following nondimensional forms for a piston moving at uniform speed ($u^*$)

$$h_1^* = h_1^* - \frac{\tau_0}{2} u^* + (\sigma^* - 1) Z_{n-1} T_{n-1}^*$$

(69)

$$\frac{p_n}{p_{n-1}} = \sigma^* = 1/2 \left[ C \pm \sqrt{C^2 - \frac{4 Z_n T_n^*}{Z_{n-1} T_{n-1}^*}} \right]$$

(70)
Only the (+) sign has a physical meaning; the (-) sign would give expansion ratios.

\[ C = 1 + \frac{\gamma R u_p^2 + Z_n T_n^*}{Z_{n-1} T_{n-1}^*} \]  

(71)

where \( h_n^* = \frac{h_n}{\gamma R T_r} \), \( u_p^* = \frac{u_p}{a_r} \), \( T_n^* = \frac{T_n}{T_r} \)

It is seen that the pressure ratio \( \sigma^* \) depends on \( u_p^* \) and \( Z_n T_n^* \) (conditions in states n-1 are known) and in turn \( h_n^* \) depends on the same quantities. Since (Ref. 4)

\[ h^* = h^*(p, T) \]  

(72)

and

\[ ZT^* = ZT^*(p, T) \]  

(73)

The specific enthalpy \( h^* \) and the compressibility factor \( Z \) can be found from thermodynamic tables or Mollier diagrams. Consequently, the states behind repeatedly reflected shock waves between a rigid wall and a piston moving at constant speed can be found as follows (Ref. 43):

1. Choose a value of \( u_p^* \) and the initial pressure \( p_0^* \), \( (T_O = T_R = 290^0K) \)

2. As a start, compute \( h_n^* \) from Eq. 69, using the perfect gas value of \( \sigma^* \) (see Eq. 72, Part I)

3. Knowing \( h_n^* \) and \( \sigma^* \) find \( ZT^* \) (Eqs. 67 and 68) using thermodynamic data or better still a Mollier diagram.

4. Recalculate the pressure ratio \( \sigma^* \) and recompute \( h_n^* \) using the new values of \( Z \) and \( T \). This gives a new point \( (h, p) \) in the Mollier diagram and a new \( Z \) and \( T \) is found.

5. Recompute \( \sigma^* \) and then \( h_n^* \) and iterate for the required accuracy.

Alternately, the simple graphical method given in Ref. 45 can be used.

The values of \( p_n/p_O \) and \( T_n/T_R \) vs. \( u_p/a_r \) (piston speed) obtained in Ref. 43 for multiple shock compression of air and hydrogen are reproduced in Figs. 60 to 65 for an initial pressure range 0.1 atm \( \leq p_0 \leq 10 \) atm and an initial temperature \( T_O = 290^0K \) and \( 580^0K \) for air and \( 290^0K \) for hydrogen up to three \( (n = 3) \) repeated shocks. The nondimensionalizing reference tem-
temperature for $T_r$ and $a_r$ is taken as $290^0 K$ for each gas. Figure 60 shows that overall shock pressure ratios of over 1000 can be obtained for pistons moving at fivefold the reference sound speed (5600 ft/sec in air). The effects of gas imperfections on pressure are small, but as Fig. 61 shows they are significant on temperature. The lowest final temperature occurs at the lowest initial pressure for a given piston speed, as expected (Ref. 4). Figures 62 and 63 are similar except that the initial air temperature $T_0 = 580^0 K$. Consequently, for $U_p = 0$, $T_n/T_r = 2$, as shown on Fig. 63, since $T_r = 290^0 K$. It is seen from Fig. 62 that the corresponding pressure ratio is lower than in Fig. 60, where the initial gas is colder. However, from Fig. 63 although the final temperatures for low piston speeds exceed those of Fig. 61, this is surprisingly no longer the case at the higher piston speeds. The results for hydrogen for $T_0 = 290^0 K$ appear in Figs. 64 and 65. It is seen that gas imperfections again have a small effect on the pressure ratio and a significant effect on the temperature ratio at higher piston speeds.

The figures show that a high-speed piston is an effective compressor and heater. Approximately the same nondimensional ratios are obtained for air and hydrogen for the same nondimensional piston speed, which means that in hydrogen the piston would have to move about fourfold as fast. Owing to its high sound speed hydrogen is more difficult to compress and heat. It should be noted that unless the inertia of the moving piston is very large it will be decelerated by the pressure induced by shock $w_3$, which reflects from its front face, and for light pistons the motion can be reversed as shown in Fig. 25, Part I, and in Figs. 67 and 68.

An approximation to the piston motion after the first reflection when the shock wave $w_3$ is formed, can be made by assuming that the deceleration produces an expansion wave moving into state (3), which decays the shock $w_3$, and on the back face of the piston, a compression wave (or shock wave) moving into the driver gas is generated (Fig. 58). The pressure produced at the front of the piston is given by the relation for a $P$-type rarefaction wave (see similar Eqs. 21 and 22, Part I for a $Q$-wave) for a perfect gas

$$
p_3/p_3'f = 1 + \frac{\gamma_3 - 1}{2} \left[ \frac{u_{p3} - u_{p1}}{a_3'} \right]^2 \gamma_3^{-1} \quad (69)
$$

where,

- $u_{p1}$ is the original piston speed
- $u_{p3}$ is the decelerated piston speed
- $p_3'$ is the instantaneous pressure in state (3) on the front face of the piston after reflection of $w_3$
- $a_3'$ is the corresponding sound speed
Similarly, for the shock wave formed at the back face of the piston (the approximation that a shock forms instantly is consistent with the assumption of a uniform $u_p$) the pressure is given by (see Eq. 72, Part 1)

$$\frac{p_{b3}}{p_{b3}} = 1 + \frac{\gamma_b}{2} \left( \frac{u_{p1} - u_{p3}}{a_{b3}} \right)^2 + \sqrt{\left( \frac{\gamma_b}{2} \frac{u_{p1} - u_{p3}}{a_{b3}} \right)^2 + 4}$$  

(70)

Once these pressures are known then the equation of motion can be written as follows (see Eq. 74, Part 1)

$$\frac{m}{A} \cdot \frac{du}{dt} = (p_3 - p_{3b}) = f(u_{p1} - u_{p3})$$  

(71)

The method of solution would be similar to that discussed in Subsection 2.3.2. The process can be repeated for additional reflections from the piston face.

The results of such an analysis appear in Fig. 66 (Ref. 46). Four pistons (5, 10, 20 and 30 gm) are considered to move at the same speed of nearly 590 m/sec (1930 ft/sec) generating the shock reflections from a rigid wall and the piston face as noted. The piston path following two reflections are shown and it is of interest to see that the light piston (5 gm) is readily decelerated on the first shock reflection but not the heavy piston (30 gm). That this type of motion does occur in practice can be seen from Fig. 67 (Ref. 47a), which shows the actual motion of 37 mm dia. pistons of 5, 10, and 20 gm moving at velocities of about 510 m/sec (1700 ft/sec) in a 3.5 m closed barrel. It can be noted that as a result of the higher pressures generated by the heavy piston its deceleration and oscillations are ultimately more violent than that of the lighter piston. The effect of a nozzle throat (3 mm dia) is shown in Fig. 68. It is seen that the piston can move farther into the gas and the oscillations are damped as a result of the outflow through the throat. (New data are given in Ref. 47c.)

When the piston is brought to rest its kinetic energy is added to the gas and the compressed gas will then be at its peak pressure and temperature. The maximum conditions may be determined from the following considerations (Ref. 43). Assume that the kinetic energy of the piston added to the compressed gas corresponds to its highest velocity just as it moves into state (2). The work done by the piston in compressing the gas isentropically (a reasonable assumption after two or three shock compressions) will show up as an increase in the internal energy of the gas, or,
\[
1/2 \, m \, u_p^2 = - \int_2^{\text{max}} p \, d \left( n \, V \right) = \int_2^{\text{max}} d \, E \\
\text{s = const.} \quad \text{s = const}
\]

where, \( n \) = number of moles, \( V \) = molar volume

The pressure which exists at the rear face of the piston and contributes to the compression of the gas has been neglected in Eq. 72. This can be approximated by assuming that the contribution to the internal energy of the compressed gas by the driver gas remains constant, or

\[
1/2 \, m \, u_p^2 + C = -n \int_2^{\text{max}} p \, d \, V = \int_2^{\text{max}} d \, E \\
\text{s = const} \quad \text{s = const}
\]

The constant \( C \) can be approximately determined by taking the limiting case of a massless piston (\( m = 0 \)), in which case Eq. 9c may be applied to determine the maximum pressure at the rear face of the piston as it is brought to rest (for a chambered driver). If the massless piston or contact surface is decelerated to rest isentropically then \( u_p = u_3 = 0 \), and \( p = (p_{b3})_e \) (unlike Eq. 70, which applies for a single shock wave) is given by the limiting relation for infinite chambrage (Eq. 9c)

\[
(p_{b3})_e = \left( \frac{\gamma_4 + 1}{2} \right)^{\frac{\gamma_4}{\gamma_4 - 1}} \cdot p_4
\]

That is, by decelerating the flow that has been expanded from an infinitely chambered shock tube it is possible to achieve a final pressure which is nearly two fold greater than the original driver pressure (Fig. 21, for \( \gamma = 1.4 \) \( (p_{b3})_e = 1.89 \, p_4 \)). Applying Eq. 73 to this case the effect of the driver pressure is found by determining the constant \( C \), or

\[
C = n \int_2^{e} d \, E
\]

Combining Eqs. (73) and (75)

\[
1/2 \, m \, u_p^2 = n \int_{e}^{\text{max}} d \, E \\
\text{s = const}
\]
That is, the kinetic energy of the piston is added after the compression by the driver gas and will result in higher final temperatures and pressures.

Usually the gas which is initially contained in the barrel is at a low enough pressure and temperature that gas imperfections can be neglected, and the total number of moles of this gas is given by

\[
n = \frac{L_0 A_0 \rho_0}{m_0} = \frac{L_0 A_0 p_0}{\mathcal{R} T_0}
\]  

(77)

where, the subscript \(0\) refers to initial conditions in the barrel. Substituting the above into Eq. 76 and integrating

\[
\frac{1}{2} m u_p^2 = \frac{L_0 A_0 p_0}{RT_0} \chi (E_{\text{max}} - E_e)_s = \text{const}
\]  

(78)

or in nondimensional form

\[
\left( \frac{E_{\text{max}} - E_e}{\mathcal{E}} \right)_s = \text{const} = \frac{u_p^2}{2 L_0 p_0}
\]  

(79)

Equation 79 expresses the change in internal energy of the compressed gas when the gain from the driver pressure is included (no outflow is assumed to exist, such as for a shock tunnel nozzle). If the added push from the driver is neglected then from Eq. 73

\[
\left( \frac{E_{\text{max}} - E_e}{\mathcal{E}} \right)_s = \text{const} = \frac{u_p^2}{2 L_0 p_0}
\]  

(80)

Since \(E_e\) is at a lower internal energy level than \(E_{\text{max}}\), \(E_{\text{max}}\) will be at a lower level than \(E_{\text{max}} - E_e\), as the piston kinetic energy is a constant for both cases.

The dimensionless energy \(E\) is a function of state and its variations with nondimensional entropy \(s\) for air and hydrogen are shown in Figs. 69 and 70. This modified Mollier diagram can be used to show the importance of including the effect of the driver pressure on the compression of the gas by using an actual AEDC gun as an example (Ref. 43). Hydrogen at 2000 atm is used as a driver gas to compress hydrogen initially at 30 atm and 2900 K by means of a piston having a maximum velocity of 1800 m/sec and a value of the right hand side of Eq. 79 of 11.8. A peak pressure of 20,000 atm is obtained from Eq. 80 and 33,000 atm from Eq. 79, which contains the extra compression from the driver gas. It is seen how important this effect can be in estimating peak
pressures for the design of the main driver gas barrel (or pump tube) coupling section to the projectile launcher barrel.

In the case of hypervelocity launchers where a considerable outflow of gas occurs as the driver gas pushes the projectile out of the launching barrel, the piston continues to move, and if the original piston velocity is large enough, the piston will reach the end of the pump tube. An approximate method for determining the piston path is given in Ref. 43, where it is indicated that if the initial conditions are chosen in such a way that the piston velocity \( u_p \) is zero before or when the piston hits the end of the pump tube then the driver pressure reaches a peak and will drop. However, if the piston speed is large enough when it reaches the end then the pressure continues to rise at all times.

As an application of the foregoing relations to determine the significance of peak compression conditions in the pump tube Eq. 80, will be used neglecting the contributions from the driver gas or the effects of outflow (Ref. 43). In practice light, single-calibre, pistons with a density of 1.7 gm/cc, weighing 86 gms, in a 40 mm pump tube that can withstand 20,000 atm have been reported (Ref. 48), and it will be instructive to calculate the conditions that will produce peak pressures of this value. If it is assumed that the pump tube has a length of \( L_0 D_0 \), \( T_r = T_o \), and \( m = 1700 A_o D_o \) kg

then from Eq. 80,

\[
\Delta e = \frac{\Delta T}{2 L_o p_o} = \frac{m u_p^2}{2 L_o A_o p_o} \cdot \frac{T_o}{T_r} = \frac{17}{3} \cdot \frac{u_p^2}{p_o} \quad (81)
\]

where, \( u_p \) is given in m/sec and \( p_o \) in newtons/m\(^2\).

Using a piston velocity of 2000 m/sec (6600 ft/sec), which is about the upper limit in use to-day, and one of 4000 m/sec (13,100 ft/sec), the maximum conditions in the compressed hydrogen can be calculated as follows. Assume that the piston is accelerated to its maximum velocity in an evacuated section (Fig. 58). Consequently, state (2) is a uniform region. The initial pressure \( p_o \) can then be adjusted to give a peak pressure, \( p_{\text{max}} \sim 20,000 \) atm. The values of \( p_2 \) and \( T_2 \) can be obtained from Figs. 64 and 65 and \( e_2, s_2, p_{\text{max}}, \) and \( T_{\text{max}} \) can be found from Fig. 70. The results are shown below in Table 12.
Two conditions for each velocity (2000 m/sec, $T_0 = 290^\circ$K, $P_0 = 10$ atm and 20 atm; 4000 m/sec, $T_0 = 290^\circ$K, $P_0 = 50$ atm and 200 atm) are illustrated in Fig. 71, in the modified Mollier diagram or ($\hat{e}$, $\hat{s}$)-plane. It can be seen from Table 12 and Fig. 71 that although the higher piston velocity exceeds the upper limit on pressure that can be tolerated structurally by factors of two to three, the corresponding temperature hardly increases significantly. The reason lies in the fact that to keep the maximum pressure within bounds for high piston speeds the initial pressure must be high (nearly by an order of magnitude) and consequently the gas cannot be compressed or heated as much. This point is also reflected in the relatively small entropy change between state (0) and (2), as a result of shock compression compared to that arising from the change in the initial pressure level alone. However, the higher velocity piston shows a correspondingly larger entropy change across the shock waves, as expected.

From the above it can be concluded that it is not structurally feasible to heat cold hydrogen efficiently through strong-shock compression alone. (Therefore, it is not helpful to use an evacuated accelerating section to obtain even higher piston speeds.) Consequently another method must be employed. It can be seen from Fig. 70 that if the maximum pressure is limited to 20,000 atm, then the best way of reaching a high maximum temperature for launching purposes is to preheat the hydrogen.

For example, if the initial pressure $P_0 = 1$ atm and $T_0 = 870^\circ$K, then by using Figs. 64, 65, and 70, again for $u_p = 2000$ m/sec, it will be found that $T_{\text{max}} \approx 7500^\circ$K for $P_{\text{max}} \approx 20,000$ atm (the highest launcher temperature that has been attained in hydrogen $\approx 3500^\circ$K). High temperatures coupled with the decrease in the isentropic index $\gamma_e$ and correspondingly lower molecular weight of the dissociated hydrogen would yield a high projectile velocity (Subsection 3.1). Since the compression ratio is very high, the volume of the pump tube gas in the above example is reduced by
a factor of 2320. Consequently, it would be desirable to use large chambrage (tapered transition) between the pump tube and launcher barrel coupled with an extruding piston to make use of all of the small volume of driver gas. This method of driving a projectile will be considered in greater detail in Subsection 4.2.

3.5 Electrical Discharges as Heaters and Compressors

The previous subsections dealt with the transfer of chemical and mechanical energy into internal energy of a gas in order to produce a high-temperature, high-pressure driver for a hypervelocity launcher (shock tube or shock tunnel). The present subsection considers the conversion of electrical energy for the same purpose. This is usually done through ohmic heating \((I^2R)\) of the gas (Ref. 4), although the compression of trapped magnetic fields \((B^2/8\pi)\) can also be utilized (Ref. 49), and will be reconsidered in Subsection 3.7. The heating of a gas may take place slowly (seconds) by direct preheating using heater coils; rapidly (milliseconds) by using an inductance to drive an arc discharge; extremely fast (microseconds) by using a capacitance system for the same purpose.

It was noted in Subsection 3.4 that it is very worthwhile to preheat a driver gas such as hydrogen when it is being compressed and heated by a piston. Actual increments in muzzle velocities of two to three thousand feet per second are possible. A successful electrical preheater (900 amp, 45 volt AC, using stainless steel tubing coils) is described in Ref. 48, where it is reported that in preheating hydrogen from 3000 K to 6000 K it was possible to obtain an average muzzle velocity increase of 10 percent over the same final pressure range of 12,000 to 28,000 psi in the pump tube. This increment is about as effective as that obtained with large chambrage (Subsection 3.2) and is quite significant, especially at the higher velocities, which are difficult to attain. However, this method of heating is really only a very useful and practical adjunct to piston compression rather than a primary heating device. Capacitance discharges have also been tried in conjunction with piston compression but this is still under development (Ref. 50).

Capacitance and inductive methods of heating a driver gas have only been moderately successful to date in terms of launching projectiles to high hypervelocities. The difficulties stem mainly from the increase in the molecular weight of the driver gas as a result of electrode and chamber surface erosion and the fall in temperature as a result of heat losses through radiation. Both of these problems are aggravated at higher enthalpies and for longer heating times. Similar difficulties have been experienced in the operation of "Hotshot" hypersonic wind tunnels (Refs. 51 to 55), hypervelocity launchers (Refs. 56 to 58) and to a much smaller extent in the less stringent operation of shock tubes (Ref. 59).
Owing to possible future applications of inductive and capacitance systems (especially when contamination will be reduced through the use of appropriate materials and electrode design) the methods will be briefly reviewed. (The design details of inductive and capacitive storage systems will not be presented as they are beyond the scope of the present report. However the cited references and those given therein will prove to be useful.)

Were it not for the practical difficulties noted above, these systems offer ideally a means of adding unlimited energies to a gas with control over peak temperatures and pressures as well as the rate and instant of energy addition (see Eq. 82 to 89).

Figure 72 shows a 20 mm - hypervelocity launcher (arc chamber volume ~ 180 in\(^3\)) which was used at AEDC, both with a capacitance and inductance electrical heating system. A schematic diagram of the capacitive energy system is shown in Fig. 73, and of the inductive system, using a homopolar (unipolar) DC generator, appears in Fig. 74. (Some details are given in Tables 13 and 14 and apply mainly to the hotshot tunnel operations.)

The principal components of the capacitance system consists of a bank of condensers for energy storage, a charging power supply for energizing the bank and a copper-bus system from condensers to collectors to the arc-chamber electrodes. The main discharge is initiated by discharging trigger capacitors through a fine wire (0.008 in dia.) which ionizes the gas between the electrodes. (The arc resistance during the major energy transfer is a few m\(\Omega\).)

(The present descriptive material is essentially that appearing in Ref. 52.) The inductance system consists of a 250-HP induction motor driving the unipolar generator and flywheel assembly. The energy is transferred from the flywheel to the large inductance coil and is discharged in the arc chamber when the main circuit breaker is opened. When the generator is up to speed a 15-KW exciter supplies the field coil with a low voltage (~125V). The main breaker is closed and the bus switch is open. The current flows through the inductance coil. During this charging period the magnetic clutch is disengaged and part of the kinetic energy of the flywheel is transferred to the magnetic field of the coil. For a maximum current of 1.5 \(\times 10^5\) amp the flywheel speed is reduced by 25 percent. Just before maximum current is reached the bus switch is closed and about 1/30 of the total current flows through the electrode fuse inside the arc chamber. The main breaker is opened when the maximum current flows and it causes a rapid increase of current and voltage through the fuse from the inductance coil. The fuse breaks and the arc is struck between the main electrodes at a discharge rate of about 500 MW.
The discharge characteristics of capacitor systems appear in Fig. 75 and of the inductive system in Fig. 76. The energy stored in each of these systems appears in Fig. 77. It is seen that capacitive energy storage systems of about $0.2 \times 10^5$ joules to $8 \times 10^6$ joules are available at AEDC.

The main point which is illustrated in Fig. 75 is that a coaxial collector system is superior to a flat bus system, which has a higher inductance. The coaxial system gives a critically damped discharge and an energy transfer time that is smaller by a factor of 5, compared with the oscillatory discharge resulting from the flat bus system. The latter is undesirable from the point of view of capacitor life and electrode erosion. It is seen that total discharge times as low as 140 microseconds are possible (and smaller in thermonuclear fusion research, Ref. 60 by an order or more.)

The characteristics of the inductance energy storage system shown in Fig. 76 display the generator voltage and induction coil current variations as a function of time; the energy transfer sequence corresponding to Fig. 75 for capacitance storage; the variation of the arc resistance with current flow. The discharge circuit has a higher resistance due to the arc and even though the charging circuit is slightly underdamped, the discharge circuit is highly damped. (For critical damping $R \approx 250 \mu \Omega$; during charge $R \approx 160 \mu \Omega$; discharge $R \approx 2 \times 10^4 \mu \Omega$ Ref. 52). During discharge the voltage stays nearly constant and the current decays exponentially with a time constant $\tau = L/(R)_t = 0$. The decrease of arc resistance with current is attributed to the parallel electrode design (Fig. 74) that gives rise to a "fanning out" of the arc, increasing the ionized volume for conduction of current. This electrode configuration has now been replaced by much improved coaxial electrodes (Ref. 53). (The opposed electrodes (Fig. 73) of the capacitance system showed very little variation in arc resistance with current.) It is seen that the arcing time in the inductive system is about two orders greater. To reduce this the arc voltage and resistance should be high. The voltage is usually limited by arc chamber design and insulation (Ref. 52).

The question that arises next is; how much of the stored energy can actually be transferred to the gas? The energy transfer efficiency is defined as the ratio of energy added to the driver gas (as internal energy) to the stored capacitor electrical energy ($\eta = Q_a/Q_c$). This fraction depends on the initial conditions of the gas used as well as the liner material of the arc chamber and the type of electrode and electrode material. Some representative results in air for capacitance (AEDC - 16) and inductive (AEDC - 50) systems are shown in Fig. 77. It is seen that beyond initial chamber densities of 100 amagat (amagat = density at 760 mm Hg and 0°C) the efficiencies remain at about 70 ± 10%. As the density decreases the efficiency drops rather rapidly. The apparent efficiencies for nylon liners of over 100% are caused by the evolution of hydrogen from the pyrolysis of nylon which increases the chamber pressure. Since the internal energy of
the gas can be computed from a measurement of peak chamber pressure and known initial density (which remains constant for a fixed volume) by using a modified Mollier diagram of the type shown in Fig. 49, Part 1, a small evolved mass of gas results in a relatively large pressure increase and a correspondingly erroneously high transfer efficiency.

Similar results for the capacitance system used in Ref. 55 are shown in Fig. 78. It is seen that the efficiency improves with chamber volume and charge density (as for Fig. 77) and suggests that the losses arise from radiation to the chamber walls (convective and conductive losses are small during short intervals of milliseconds). When the gas volume is small and the gas density is low the shielding that is offered by the gas to radiation losses is also ineffective and hence the decreasing efficiencies. In this case as well the efficiencies rise above 100% and gas contamination, chamber pressure accuracy, and errors in the gas tables (van der Waal effects) are suspected. The effect of initial charging voltage on energy transfer (for the data available) indicate that radiation losses are again responsible since maximum arc current and voltage increase with the capacitor voltage. The peak power dissipation in the arc increases as the square of this voltage so that the arc temperature rises sharply leading to increased radiation losses (see Ref. 59 for additional details).

Capacitance energy storage systems have been successfully used (see Ref. 59a, for example) to produce driver gases for shock tubes and correspondingly high shock-Mach numbers ($M_s \sim 40$) at low channel pressures ($\sim 100 \mu \text{Hg}$). The driver gas pressure ($\sim 500$ atm), temperature ($\sim 5000^\circ \text{K}$), and maximum condenser energy storage ($\sim 50,000$ joules) are moderate compared with usual launcher requirements. However, it is worth considering some of the common results. Figure 79 shows the shock-tube configuration for the $40 \text{ in}^3$ arc chamber using helium as a driver gas (see Ref. 59b for an alternate design, with $M_s \sim 40$, $T \sim 20,000^\circ \text{K}$).

The capacitance, inductance, and resistance circuit diagram, which is critically damped, is shown in Fig. 80a. A current-time trace appears in Fig. 80b. The bus inductance $L_1 = 0.15 \mu \text{h}$. The driver inductance $L_2$ and resistance $R_1$ depend on the conditions of the heated helium plasma and must be inferred from the shunt current which is related to the discharge current shown in Fig. 80b. The peak current is about 226,000 amps for an initial voltage of 19,200 volts. For a critically damped circuit (Ref. 59a)

$$I = \frac{E}{L} t e^{-Rt/2L}$$  \hspace{1cm} (82)

$$I_{\text{max}} = 0.736 \frac{E}{R}$$  \hspace{1cm} (83)

$$L = R^2 C / 4$$  \hspace{1cm} (84)

$$t_{\text{max}} = RC / 2 = 2L / 4$$  \hspace{1cm} (85)
Solving for $R_1$ from Eq. 83, yields $R_1 = 0.063 \Omega$ and for $L_2$ from Eq. 84, $L_2 = 0.23 \mu H$. The value of $t_{\text{max}}$ is obtained from Eq. 85, $t_{\text{max}} = 7.3 \mu \text{sec}$, compared with an actual value of $12 \mu \text{sec}$, (about tenfold smaller than for the BAL-8 shown on Fig. 76, which has a storage tenfold larger). The discrepancy is accounted for by the initially higher inductance than calculated owing to the concentration of current near the driver axis. The plasma expands and the inductance falls so that the current reaches a peak sooner for the initially underdamped circuit but later than for a critically damped circuit. The current waveform exhibits a peaking distortion due to the shunt self-inductance. It is worth noting that the waveform and duration is maintained for higher initial helium pressures.

It was noted previously that since the initial gas density remains unchanged therefore an accurate measurement of the final pressure in the driver provides the final temperature and the energy transfer efficiency. It was found that pressure gauge holes in the Lexan plastic liner reduced its life to one or two runs. Consequently, measurements of shock Mach number and initial channel conditions were used to infer driver performance (a more direct method is given in Ref. 59b, with values of $\eta \approx 80$-90%)

Figure 81 shows the energy transfer efficiency obtained at an initial pressure $p_1 = 11.9 \text{ atm}$. The results are given in terms of an efficiency $\eta = \frac{Q_4}{Q_i}$, the energy absorbed by the gas to the actual energy supplied to the gas, and $\eta' = \frac{Q_4}{Q_c} = \frac{Q_4}{Q_{\text{max}}}$, where $Q_c = Q_{\text{max}}$ = maximum stored energy based on capacitor rated load, $Q_c = 47,000 \text{ joules}$. Based on $\eta$, a peak efficiency occurs near 30,000 joules, probably owing to higher radiation losses at the larger energies, as for Fig. 78. It is seen that the energy transfer efficiency beyond a 30,000 joule-input reaches a saturation level of about 40%, and substantiates the values that can be extrapolated from Fig. 77 for an initial density of 20 amagat, under much higher energy densities and transfer times. The above remarks are also illustrated in Fig. 82, which shows the increase in $\eta$ with initial pressure. The approach to $\eta \sim 100\%$ at 30 amagat is probably a result of contamination (see Fig. 77 for nylon liners). This is also substantiated by the fact that $\eta$ obtained from pressure measurements are greater than from $M_8$ and $p_1$ data.

The inferred driver conditions are shown in Fig. 83 and they reflect the results shown in Fig. 82. Apparently temperatures between 5500°K and 6500°K can be obtained over an initial pressure range from 5 to 25 atm. The temperatures would have to be measured directly to verify these results, especially at the maximum discharge energies of about 50,000 joules.

Some very worthwhile work on the effects of radiation losses from high-temperature driver chambers can be found in Ref. 53. Figure 84 shows that a black body radiation curve fits the data from hotshot tunnel 56
arc chambers quite well beyond temperatures of about 4000°K. Below these temperatures convection and conduction are probably important. It is seen that heat transfer rates of \( q \sim 10^4 \text{ BTU/ft}^2\text{-sec} \) exist in the range of 5000 - 7000°K, essentially due to radiation. The result is that the enthalpy level drops to about 60% in about 25 m sec, regardless of the liner material used in the arc chamber (Fig. 85). The very significant effect of density, which was discussed previously, is well illustrated in Fig. 86 for nitrogen or helium in the arc chamber. Figure 86a shows the decay in enthalpy with increasing temperature and decreasing density with time for nitrogen. The results are much worse for helium owing to its low value of the specific heat.

Computed results of the decay in gas temperature as a result of radiation alone appear in Fig. 87, as a function of the parameter \( \text{At/V} \), which combines the basic quantities in any radiating gas mass such as the product of surface area to volume times the radiation period. For hotshot tunnels this quantity has a value of about 0.4 to 1 sec/ft but for capacitance-energy discharges this should be reduced by an order of magnitude at least. It is seen that it would not be possible to achieve very high temperatures in hotshot facilities at low initial charging densities. Figure 88 summarizes some of the requirements for hotshot tunnels to achieve high temperature. Assuming that only 65% of the stored energy is transferred to the gas \((80 \times 10^6 \text{ joules/ft}^3)\) and \( \text{At/V} = 0.4 \text{ sec/ft} \), then at 40 amagat \( T \sim 9000^\circ \text{K} \) and at 100 amagat \( T \sim 8000^\circ \text{K} \) for initial temperatures of 15,000°K and 10,000°K, respectively. As the curves show this probably could be avoided with capacitance storage where very short energy transfer times should be possible. Consequently, it may well be worthwhile to reconsider a hypervelocity launcher drive of this type in the future when contamination problems may be overcome and discharge times are kept to microseconds.

The cost of the two systems are considered in Fig. 76, where the cost in cents/joule is plotted against the stored energy in joules. It is seen that the capacitance storage is considerably more expensive than the inductance storage, especially in the 10 megajoule range. Although the capacitance storage cost appears nearly constant the problems of grouping condensers to keep the inductance low will raise the cost in this range. It is also felt that about \( 10^9 \) or \( 10^{10} \) joules the cost for inductance storage would also go up. (Ref. 52.) The cost of a coil and rectifier also appears on Fig. 76 and as seen, that it lies between the two systems for large storage.

It is worth noting that the gas energy density at about 100 amagat and stored energies of \( 10^6 \) and \( 10 \times 10^6 \) joules at 60% efficiency for the AEDC-16 and AEDC-50 having volumes of 25 in\(^3\) and 360 in\(^3\), respectively is about \( 2 \times 10^4 \) joules/in\(^3\) or \( 1.2 \times 10^3 \) joules/cm\(^3\) or about 300 cal/cm\(^3\) (or 2300 cal/gm). This may be contrasted with TNT at 1000 cal/cm\(^3\) or gm (for a loading density of 1 gm/cm\(^3\)) and exploding
wires with an energy density of 5.5 x 10^4 cal/cm^3 (or 1.1 x 10^5 cal/gm for a very small mass 0.03 gm, lithium wire, Ref. 61). Present day lasers are capable of adding over 10 joules in about 1 nanosecond in a volume with a radius of about 1 wavelength of light (8,000 A). This results in an energy density of 1 x 10^{12} cal/cm^3 and a power of 10 gigawatts. This potential still has to be exploited on a large scale:

Some approximate values of the state of the gas following the addition of energy can be made from the following considerations using a perfect gas as an illustration. The initial internal energy of the gas is given by

\[ E_i = \rho_i V_i C_V T_i = V_i \frac{p_i}{\gamma - 1} \] (86)

The final internal energy for a fixed volume or density

\[ E_f = E_i + Q_C = \rho_i V_i C_V T = V_i \frac{p_f}{\gamma - 1} \] (87)

Therefore, for a given final internal energy, the final temperature is high if the arc chamber has a small volume (also a requirement for high final pressures) and the initial density is low. However, it was already pointed out that radiation losses increase as a consequence and in practice these quantities should be optimized. Since the mass of gas is a constant during the process

\[ \frac{p_f}{p_i} = \frac{T_f}{T_i} \] (88)

That is, the pressure and temperature ratios are identical (see Fig. 83). The energy density per unit volume when normalized by the initial pressure is given by the following nondimensional quantities

\[ \frac{E_f}{V_i p_i} = \frac{p_f/p_i}{\gamma - 1} = \frac{T_f/T_i}{\gamma - 1} \] (89)

Alternately, it is seen that high energy densities are obtained for high compression or temperature ratios at high initial pressures for a gas with a low value of the specific heat ratio (or isentropic index such as dissociated and ionized hydrogen). For real gases the initial density is known provided that accurate gas tables or Mollier charts are available with van der Waal corrections) and if the final peak pressure is measured accurately then the energy added to the gas can be found from a Mollier diagram (as well as the final temperature and the energy transfer efficiency) as noted previously. Equation 89 again illustrates that high driver temperature and pressure ratios are possible with large energy inputs if the chamber volume and initial pressure are optimized from considerations of structural strength, radiation losses, electrode design, and electrical insulation requirements. These ratios are more difficult to produce for low values of \( \gamma \), owing to the partition of energy among the internal modes of dissociation, electronic excitation, and ionization.
In summary, it can be stated that the use of capacitive and inductive ohmic heating of driver gases for hypervelocity launchers has for the time being been apparently abandoned owing mainly to contamination of the driver gas by material from electrodes and arc chamber. A maximum velocity of 13,000 ft/sec is reported in Ref. 57 for a 4 gm projectile launched from a 20 mm launch tube using the 10 megajoule inductive energy storage system. The results are summarized in Fig. 89, for a helium driver. The contaminant mass fraction $m_c/(m_c + m_{He})$, the ratio of the measured projectile velocity $u_{meas}$ to the calculated velocity without contamination $u_{mc} = 0$, the ratio of the calculated projectile velocity for the actual contamination $u_{mc}$ to that without contamination $u_{mc} = 0$ are all plotted against the initial helium chamber density. The contaminant mass fraction was assumed to consist of the lost electrode and chamber surface material which evaporated to increase the molecular weight of the driver gas. The cold helium run (300K, 2500 amagat) served as the reference velocity $u_{mc} = 0$. Since for a fixed energy the arc chamber temperature decreases with increasing density, the contaminant fraction is greatest at the highest temperature and lowest density, as discussed previously. The poor gun performance arising from contamination is evident.

Swift (Ref. 50) reports that arc chambers of 10 to 30 in$^3$ using capacitance storage up to $10^5$ joules and He or H$_2$ at initial pressures of 100 atm have fired 0.1 and 0.5 gm projectiles from 0.22 and 0.30 in launch tubes at peak velocities of 5.4 km/sec (17,700 ft/sec) and that the development of such guns have been abandoned as a result of the contamination problem (higher velocities are quoted in Refs. 56c and d).

It is possible that when more experience has been obtained in the operation of hotshot tunnels, which will lead to improved electrode and chamber surface design for minimum contamination, the use of short-duration capacitance heating of hydrogen or helium driver gases for hypervelocity launchers may be revived.
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9. Seigel, A.E.  

10. Seigel, A.E.  
     Dawson, V C.D.  

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<td>Popov, V.A.</td>
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### TABLE I

**Flow Through a Nonstationary Expansion Wave in Hydrogen**  
(Perfect Gas, $\gamma = 1.4$)

<table>
<thead>
<tr>
<th>Pressure Atmos. $p \times 10^{-5}$ lbs/ft²</th>
<th>$T^\circ R \times 10^{-3}$</th>
<th>$\rho \times 10^2$ slugs/ft³</th>
<th>ax $10^{-4}$ ft/sec</th>
<th>$\frac{1}{\rho a} \times 10^3$ ft²·sec/slug</th>
<th>u $10^{-4}$ ft/sec</th>
<th>M</th>
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<td>6800</td>
<td>144</td>
<td>11.15</td>
<td>5.21</td>
<td>1.970</td>
<td>0.975</td>
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<td>5000</td>
<td>106</td>
<td>10.21</td>
<td>4.18</td>
<td>1.885</td>
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<td>0.425</td>
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Flow Through a Nonstationary Expansion Wave in Hydrogen
(Frozen Flow, $\gamma = 1.44$, $\alpha = 0.13$)

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<th>$\rho \times 10^{+2}$ slugs/ft$^3$</th>
<th>$a \times 10^{-4}$ ft/sec</th>
<th>$\frac{1}{\rho a^2}$ ft$^2$/sec</th>
<th>$u \times 10^{-4}$ ft/sec</th>
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<tr>
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TABLE 3
Flow Through a Nonstationary Expansion Wave in Hydrogen (Equilibrium Flow)

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<th>Pressure (Atmos) x 10^-5</th>
<th>T^o R x 10^-3</th>
<th>λ</th>
<th>γ_e</th>
<th>( \rho x 10^2 ) slugs/ft^3</th>
<th>a x 10^-4 ft/sec</th>
<th>( 1/\rho a x 10^3 ) ft^2/sec</th>
<th>u x 10^-4 ft/sec</th>
<th>M</th>
<th>u x 10^-4 (ft/sec)</th>
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TABLE 4

Flow Through a Nonstationary Expansion Wave in Hydrogen
(Perfect Gas, \(\gamma = 1.4\))

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<th>(T (^\circ \text{R}) \times 10^{-3})</th>
<th>(p \text{ lbs/ft}^2 \times 10^{-4})</th>
<th>(\rho \text{ slugs/ft}^3 \times 10^4)</th>
<th>(a \text{ ft/sec} \times 10^{-4})</th>
<th>(\frac{1}{\rho a} \text{ ft}^2\text{sec/slug} \times 10^2)</th>
<th>(u. \text{ ft/sec} \times 10^{-4})</th>
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### TABLE 5

**Flow Through a Nonstationary Expansion Wave in Hydrogen**  
*(Frozen Flow, $\Gamma = 1.62, \ \alpha = 0.80)*

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<th>T ($^\circ$R) x 10^{-3}</th>
<th>p (lbs/ft^2) x 10^{-4}</th>
<th>$\rho$ (slugs/ft^3) x 10^{-4}</th>
<th>a(ft/sec) x 10^{-4}</th>
<th>$\frac{1}{\rho a} \frac{ft^2}{sec}$ slug</th>
<th>u ft/sec x 10^{-4}</th>
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</table>
TABLE 6

Flow Through a Nonstationary Expansion Wave in Hydrogen
(Equilibrium Flow)

<table>
<thead>
<tr>
<th>T(°R) x 10^-3</th>
<th>p (lbs/ft^2) x 10^-4</th>
<th>( \rho ) (slugs/ft^3) x 10^4</th>
<th>( \lambda )</th>
<th>( \gamma_e ) x 10^-4</th>
<th>a (ft/sec) x 10^-4</th>
<th>( \frac{1}{\rho a} ) ft^2/sec slug x 10^2</th>
<th>u (ft/sec) x 10^-4</th>
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TABLE 7

POINTS ON THE PROJECTILE PATH SHOWN IN FIG. 28a (REF. 9)

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\(A_4/A_1 = 2.25, \quad \gamma 4 = 1.4\)

Note: \(P = 5\bar{a} + \bar{u}\), \(Q = 5\bar{a} - \bar{u}\)

(see Eqs. 13 and 14, Part I)
TABLE 8

POINTS ON PROJECTILE PATH SHOWN IN FIG. 28b (REF. 8)

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<th>$\bar{a}$</th>
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$$A_4/A_1 = \infty \quad \gamma_4 = 1.4$$

Note: $P = 5\bar{a} + \bar{u}$, $Q = 5\bar{a} - \bar{u}$

(see Eqs. 13 and 14, Part I)
TABLE 10

Idealized Internal Ballistics Relations (Eqs. 58 and 59) for a Gun with Infinite Chambrage. Variation of $\chi$ with $u$ for $\gamma = 1.4$. Comparison with Characteristics Method (Table 8)

$$\gamma = 1.400$$

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**TABLE 11, Sheet 1**

Idealized Internal Ballistics Relations (Eqs. 58 and 59) For a Gun with Infinite Chambrage

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</table>

<p>| $\gamma = 1.15$ | $X$ for Various Values of $u$ (Fig. 59) |
| $\gamma = 1.20$ | $X$ for Various Values of $u$ (Fig. 59) |
| $\gamma = 1.25$ | $X$ for Various Values of $u$ (Fig. 59) |
| $\gamma = 1.30$ | $X$ for Various Values of $u$ (Fig. 59) |
| $\gamma = 1.35$ | $X$ for Various Values of $u$ (Fig. 59) |</p>
<table>
<thead>
<tr>
<th>$\gamma$</th>
<th>$X$</th>
<th>$\gamma$</th>
<th>$X$</th>
<th>$\gamma$</th>
<th>$X$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$1.40$</td>
<td>0.5000000E-01</td>
<td>0.1277866E-02</td>
<td>0.5000000E-01</td>
<td>0.127732E-02</td>
<td>0.5000000E-01</td>
</tr>
<tr>
<td>$1.45$</td>
<td>0.5000000E-01</td>
<td>0.127732E-02</td>
<td>0.5000000E-01</td>
<td>0.1269004E-02</td>
<td>0.5000000E-01</td>
</tr>
<tr>
<td>$1.50$</td>
<td>0.5000000E-01</td>
<td>0.1269004E-02</td>
<td>0.5000000E-01</td>
<td>0.1269004E-02</td>
<td>0.5000000E-01</td>
</tr>
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<td>$1.55$</td>
<td>0.5000000E-01</td>
<td>0.1269004E-02</td>
<td>0.5000000E-01</td>
<td>0.1269004E-02</td>
<td>0.5000000E-01</td>
</tr>
<tr>
<td>$1.60$</td>
<td>0.5000000E-01</td>
<td>0.1269004E-02</td>
<td>0.5000000E-01</td>
<td>0.1269004E-02</td>
<td>0.5000000E-01</td>
</tr>
<tr>
<td>$1.65$</td>
<td>0.5000000E-01</td>
<td>0.1269004E-02</td>
<td>0.5000000E-01</td>
<td>0.1269004E-02</td>
<td>0.5000000E-01</td>
</tr>
</tbody>
</table>
TABLE 11 (continued), Sheet 3

\[ y = 1.667 \]

<table>
<thead>
<tr>
<th>( \frac{U}{Y} )</th>
<th>( \frac{X}{Y} )</th>
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<tr>
<td>0.5000000E-01</td>
<td>0.1261813E-02</td>
</tr>
<tr>
<td>0.1000000E-00</td>
<td>0.5194132E-02</td>
</tr>
<tr>
<td>0.1500000E-00</td>
<td>0.1227922E-01</td>
</tr>
<tr>
<td>0.2000000E-00</td>
<td>0.2348212E-01</td>
</tr>
<tr>
<td>0.2500000E-00</td>
<td>0.4059249E-01</td>
</tr>
<tr>
<td>0.3000000E-00</td>
<td>0.6700522E-01</td>
</tr>
<tr>
<td>0.3500000E-00</td>
<td>0.1086518E-00</td>
</tr>
<tr>
<td>0.4000000E-00</td>
<td>0.1742321E-00</td>
</tr>
<tr>
<td>0.4500000E-00</td>
<td>0.2777982E-00</td>
</tr>
<tr>
<td>0.5000000E-00</td>
<td>0.4429666E-00</td>
</tr>
<tr>
<td>0.5500000E 00</td>
<td>0.7106951E 00</td>
</tr>
<tr>
<td>0.6000000E 00</td>
<td>0.1154696E 01</td>
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<tr>
<td>0.6500000E 00</td>
<td>0.1913598E 01</td>
</tr>
<tr>
<td>0.7000000E 00</td>
<td>0.3262271E 01</td>
</tr>
<tr>
<td>0.7500000E 00</td>
<td>0.5782037E 01</td>
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<tr>
<td>0.8000000E 00</td>
<td>0.1080509E 02</td>
</tr>
<tr>
<td>0.8500000E 00</td>
<td>0.2171381E 02</td>
</tr>
<tr>
<td>0.9000000E 00</td>
<td>0.4834097E 02</td>
</tr>
<tr>
<td>0.9500000E 00</td>
<td>0.1251742E 03</td>
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TABLE 13
Capacitance Energy Storage Data for AEDC
Hotshot Tunnels
(Ref. 52)

<table>
<thead>
<tr>
<th></th>
<th>AEDC-16</th>
<th>BAC-8</th>
<th>BAC-44</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>MAIN CAPACITOR BANK</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total Capacitance, C, farad</td>
<td>0.125</td>
<td>0.0116</td>
<td>0.39</td>
</tr>
<tr>
<td>Number of Condensers</td>
<td>1000</td>
<td>142</td>
<td>2280</td>
</tr>
<tr>
<td>Rated Voltage, V, volt</td>
<td>4000</td>
<td>6000</td>
<td>6000</td>
</tr>
<tr>
<td>Total Energy, E, joule</td>
<td>$10^6$</td>
<td>$0.21 \times 10^6$</td>
<td>$7 \times 10^6$</td>
</tr>
<tr>
<td>Charge Time, min</td>
<td>2</td>
<td>2</td>
<td>0.5</td>
</tr>
<tr>
<td>Bus¹ Type</td>
<td>Flat</td>
<td>Flat (original)</td>
<td>Coaxial</td>
</tr>
<tr>
<td>Collector² Type</td>
<td>Coaxial</td>
<td>Coaxial (modified)</td>
<td>Coaxial</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>TRIGGER BANK</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Capacity, C, μF</td>
<td>250</td>
<td>162</td>
<td>320</td>
</tr>
<tr>
<td>Rated Voltage, V, volt</td>
<td>4000</td>
<td>6000</td>
<td>6000</td>
</tr>
<tr>
<td>Energy, E, joule</td>
<td>2000</td>
<td>2900</td>
<td>5800</td>
</tr>
</tbody>
</table>

1 Conductor from individual condensers to collector
2 Conductor between terminals of conductors from individual condensers and arc-chamber electrodes
## TABLE 14

Arc-Chamber Data for AEDC Hotshot Tunnels
(Ref. 52)

<table>
<thead>
<tr>
<th>Designation</th>
<th>Length(^1), L. in.</th>
<th>Diameter(^1), D. in.</th>
<th>Volume(^1), in.(^3)</th>
<th>Maximum (P_0), psi</th>
<th>Liner Materials</th>
<th>Type</th>
<th>Tip Material</th>
<th>Starting Fuse</th>
</tr>
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<tbody>
<tr>
<td>AEDC-16A(^3)</td>
<td>6.25</td>
<td>2.25</td>
<td>25</td>
<td>60,000</td>
<td>Nylon, Copper, Steel</td>
<td>Opposed</td>
<td>Tungsten</td>
<td>Trigger, Perpendicular</td>
</tr>
<tr>
<td>AEDC-16C(^3)</td>
<td>6.5</td>
<td>2.5</td>
<td>30</td>
<td>30,000</td>
<td>Copper</td>
<td>Coaxial</td>
<td>Tungsten</td>
<td>Trigger, Coaxial</td>
</tr>
<tr>
<td>BAC-8A</td>
<td>6.2</td>
<td>2.0</td>
<td>20</td>
<td>30,000</td>
<td>Nylon, Steel</td>
<td>Opposed</td>
<td>Tungsten</td>
<td>Trigger, Perpendicular</td>
</tr>
<tr>
<td>BAC-8B</td>
<td>6.2</td>
<td>2.0</td>
<td>20</td>
<td>30,000</td>
<td>Steel</td>
<td>Coaxial</td>
<td>Brass</td>
<td>Trigger, Perpendicular</td>
</tr>
<tr>
<td>BAC-8C</td>
<td>6.2</td>
<td>2.0</td>
<td>20</td>
<td>30,000</td>
<td>Steel</td>
<td>Coaxial</td>
<td>Brass, Steel</td>
<td>Trigger, Coaxial</td>
</tr>
<tr>
<td>BAC-8T</td>
<td>2.75</td>
<td>2.75</td>
<td>20</td>
<td>30,000</td>
<td>Steel, Copper</td>
<td>Coaxial</td>
<td>Copper</td>
<td>Trigger, Coaxial</td>
</tr>
<tr>
<td>BAC-44T</td>
<td>16</td>
<td>6.5</td>
<td>580</td>
<td>30,000</td>
<td>Copper</td>
<td>Coaxial</td>
<td>Copper, Steel</td>
<td>Trigger, Coaxial</td>
</tr>
<tr>
<td>AEDC-50A</td>
<td>18.7</td>
<td>5.3</td>
<td>360</td>
<td>30,000</td>
<td>Steel</td>
<td>Parallel</td>
<td>Copper</td>
<td>Silver Solder</td>
</tr>
<tr>
<td>AEDC-50B</td>
<td>18.7</td>
<td>5.3</td>
<td>360</td>
<td>30,000</td>
<td>Steel, Copper</td>
<td>Parallel</td>
<td>Copper</td>
<td>Magnetic, Copper</td>
</tr>
<tr>
<td>AEDC-50C(^3)</td>
<td>18.7</td>
<td>5.3</td>
<td>360</td>
<td>30,000</td>
<td>Copper, Steel</td>
<td>Parallel</td>
<td>Tungsten</td>
<td>Magnetic, Copper</td>
</tr>
<tr>
<td>AEDC-50D(^3)</td>
<td>18.7</td>
<td>5.3</td>
<td>360</td>
<td>30,000</td>
<td>Copper</td>
<td>Parallel</td>
<td>Tungsten</td>
<td>Magnetic, Copper</td>
</tr>
</tbody>
</table>

---

1. Nominal dimensions
2. Hydrostatically tested values; future plans include ratings of 100,000 psi.
3. With this configuration, a baffle plate was used in some tests.
FIG. 1  PROJECTILE MOTION IN THE NONDIMENSIONAL DISTANCE-TIME PLANE FOR THREE ASSUMED TYPES OF PROJECTILE ACCELERATIONS (Ref. 1)
FIG. 2  EFFECT OF DRIVER PRESSURE, PROJECTILE AREA, AND PROJECTILE MASS ON HYPERVELOCITY (Ref. 1)
FIG. 3  EFFECT OF DRIVER SOUND SPEED ON HYPERVELOCITY
(Ref. 1)
FIG. 4  COMBINED EFFECTS OF DRIVER PRESSURE AND SOUND SPEED FOR A GIVEN LAUNCHER ON HYPERVELOCITY (Ref. 1)
FIG. 5 LAUNCH VELOCITY CORRECTED FOR FINAL CHAMBER TEMPERATURE (Sound Speed) (Ref. 2)
FIG. 6  TEMPERATURE VERSUS PRESSURE THROUGH A NONSTATIONARY EXPANSION WAVE IN HYDROGEN
FIG. 7  ISENTROPIC INDICES THROUGH A PERFECT ($\gamma$), FROZEN ($\Gamma$) AND EQUILIBRIUM ($\gamma_e$) NONSTATIONARY EXPANSION WAVE IN HYDROGEN.
FIG. 8  VARIATION OF DENSITY VERSUS PRESSURE THROUGH A NONSTATIONARY EXPANSION WAVE IN HYDROGEN.

- Frozen $\Gamma = 1.44$
- Perfect $\gamma = 1.4$

Equilibrium $\gamma_e = 1.23$

$P$ (lb/ft$^2$)

$\rho \times 10^2$ (slugs/ft$^3$)

0.001 0.01 0.1 1.0 10

0.001 0.01 0.1 1.0 10.
FIG. 9 VARIATION OF SOUND SPEED VERSUS PRESSURE THROUGH A NONSTATIONARY EXPANSION WAVE IN HYDROGEN.
FIG. 10 VARIATION OF ACOUSTIC IMPEDANCE (INVERSE) VERSUS PRESSURE THROUGH A NONSTATIONARY EXPANSION WAVE IN HYDROGEN.

Equilibrium

Perfect & Frozen

\[ u = \int_{P_4}^{P} \frac{1}{\rho a} \, dp \] (Eq. 13, Part 1)
Equilibrium with $\gamma_e = 1.23$ Const.

**FIG. 11** PARTICLE VELOCITY VERSUS PRESSURE THROUGH A NONEQUILIBRIUM EXPANSION WAVE IN HYDROGEN.
FIG. 12

FLOW MACH NUMBER VERSUS PRESSURE THROUGH A NONSTATIONARY EXPANSION WAVE IN HYDROGEN.
Equilibrium with $\gamma_e = 1.23$ (Const).

FIG. 13 PARTICLE VELOCITY VERSUS SOUND SPEED THROUGH A NONSTATIONARY EXPANSION WAVE IN HYDROGEN.
FIG. 14 TEMPERATURE VERSUS PRESSURE THROUGH A NONSTATIONARY EXPANSION WAVE IN HYDROGEN

Graph showing temperature in °R versus pressure in lbs/ft² with three curves labeled Equilibrium, Perfect, and Frozen.
FIG. 15  ISENTROPIC INDICES VERSUS TEMPERATURE THROUGH A NONSTATIONARY EXPANSION WAVE IN HYDROGEN.
FIG. 16  DENSITY VERSUS PRESSURE THROUGH A NONSTATIONARY EXPANSION WAVE

Perfect, \( \gamma = 1.4 \)

Frozen \( \Gamma = 1.62 \)

Equilibrium

\( \rho \) slugs/ft\(^3\)

Pressure (lbs/ft\(^2\))
FIG. 17. SPEED OF SOUND VERSUS PRESSURE THROUGH A NONSTATIONARY EXPANSION WAVE IN HYDROGEN.
Acoustic Impedance
$\text{ft}^2\text{sec}/\text{slug}$

FIG. 18
ACOUSTIC IMPEDANCE (INVERSE) VERSUS PRESSURE THROUGH A NONSTATIONARY EXPANSION WAVE IN HYDROGEN.
FIG. 19 PARTICLE VELOCITY VERSUS PRESSURE THROUGH A NONSTATIONARY EXPANSION WAVE IN HYDROGEN.
FIG. 20 PARTICLE VELOCITY VERSUS SOUND SPEED THROUGH NONSTATIONARY EXPANSION WAVE IN HYDROGEN.
FIG. 21 COMPARISON OF EFFICIENCY OF CONVERSION OF POTENTIAL ENERGY INTO DIRECTED ENERGY IN THE \((p, u)\)-PLANE IN CURRENT HYPERVELOCITY FLOW FACILITIES
FIG. 22 a. Idealized Flow in a Chambered Shock Tube (Ref. 4).
FIG. 22b  PARAMETER $g$ FOR MONOTONIC-CONVERGENT SHOCK TUBE VS ISENTROPIC INDEX $\gamma_d$ FOR DRIVER GAS (REF. 4). (CURVES APPLY ONLY FOR $M_3 \geq 1$).
FIG. 22c GAS ESCAPE SPEED OR MAXIMUM PROJECTILE VELOCITY AS A FUNCTION OF CHAMBER DIAMETER TO BORE DIAMETER FOR AN INFINITE CHAMBER LENGTH GUN OR SHOCK TUBE (REF. 9).
Percent of Infinite-Chamber Maximum Velocity Increase

\[
\frac{u_{\text{max}}(A_4/A_1) - 1}{\sqrt{2} \left( \frac{d_4}{d_1} \right)} - 1
\]

\[
\frac{d_4}{d_1} = \sqrt{\frac{A_4}{A_1}}
\]

\[
\text{Chamber Diameter}
\]

\[
\text{Bore Diameter}
\]

**FIG. 22d** PERCENT OF INFINITE CHAMBRAGE MAXIMUM PROJECTILE VELOCITY INCREASE AS A FUNCTION OF CHAMBER DIAMETER TO BORE DIAMETER FOR AN INFINITE CHAMBER LENGTH GUN OR SHOCK TUBE \((1.25 \leq \gamma_4 \leq 1.4)\) (REF. 9).
Nondimensional Particle or Contact Surface Velocity \( \frac{u_3}{\sqrt{RT_4}} = \sqrt{\gamma_4} U_{34} \)

**FIG. 23a** VARIATION OF \( P_{34} \) VS \( U_{34} \) FOR AN UNCHAMBERED \( (A_4/A_1 = 1) \) AND INFINITELY CHAMBERED \( (A_4/A_1 \to \infty) \) SHOCK TUBE \( (\gamma_4 = 5/3) \) (REF. 11).
FIG. 23b VARIATION OF $P_{34}$ vs $U_{34}$ FOR AN UNCHAMBERED ($A_4/A_1 = 1$) AND INFINITELY CHAMBERED ($A_4/A_1 \rightarrow \infty$) SHOCK TUBE ($\gamma_4 = 5/3$ and $\gamma_4 = 1$) (REF. 11).
FIG. 24 VARIATION OF PRESSURE AND VELOCITY PRODUCED BY EXPANSION IN A CHAMBERED SHOCK TUBE (REF. 11).
FIG. 25  VARIATION OF PRESSURE AND VELOCITY PRODUCED BY EXPANSION IN A CHAMBERED SHOCK TUBE AS A FUNCTION OF THE COVOLUME RATIO (REF. 11)
FIG. 26  ESCAPE SPEED RATIO AS A FUNCTION OF COVOLUME IN A CHAMBERED SHOCK TUBE (REF. 11).
FIG. 27 VARIATION OF EXPANSION PRESSURE AND PARTICLE VELOCITY IN A GUN AND SHOCK TUBE (\( \mathcal{v}_4 = 1.4 \)) (REF. 11).
FIG. 28a PORTION OF A CHARACTERISTICS DIAGRAM FOR A CHAMBERED GUN (γ₄ = 1.4) (A₄/A₁ = 2.28) (REF. 9).
FIG. 28b  PORTION OF A CHARACTERISTICS DIAGRAM FOR A GUN WITH INFINITE CHAMBRAGE ($A_4/A_1 \to \infty$) ($\gamma_4 = 1.4$) (REF. 8).
FIG. 29a \[ \bar{u} \text{ vs } \bar{x} \text{ DURING EARLY PHASE OF MOTION FOR CHAMBERED GUNS WITH EFFECTIVELY INFINITE LENGTH CHAMBERS (} \gamma_4 = 1.4\text{) (REF. 9).} \]
FIG. 29b  $\bar{u}$ vs $\bar{x}$ FOR CHAMBERED GUNS WITH EFFECTIVELY INFINITE LENGTH CHAMBERS ($\gamma_4 = 1.4$) (REF. 9).
FIG. 30 \( \bar{u} \) vs \( \bar{x} \) FOR AN UNCHAMBERED AND INFINITELY CHAMBERED GUN WITH EFFECTIVELY INFINITE LENGTH CHAMBERS \((\gamma_4 = 1.4)\) (REF. 8).
FIG. 31  \( \bar{u} \text{ vs } \bar{x} \) FOR CHAMBERED GUNS WITH EFFECTIVELY INFINITE LENGTH CHAMBERS (\( \mathcal{Y} = 1.25 \)) (REF. 9)
FIG. 32a  FLOW REGIONS GENERATED BY MOVING PROJECTILE
(REF. 16).
FIG. 32b  PROJECTILE BASE PRESSURE VS TIME FOR SHORT AND INFINITE GUN CHAMBERS FILLED WITH HELIUM ($\gamma_4 = 5/3$, $A_4/A_1 = 1$) (REF. 16).
FIG. 32c PATHS OF RAREFACTION CHARACTERISTICS (R) AND PROJECTILE (P) FOR A GUN WITH A SHORT CHAMBER AND WITH AN INFINITE CHAMBER LENGTH (HELIUM, \( \gamma = 5/3 \)) (REF. 16).
FIG. 32d  PROJECTILE VELOCITY VS POSITION FOR SHORT AND INFINITE GUN CHAMBER LENGTHS (HELIUM, \( \gamma = 5/3 \)) (REF. 16).
FIG. 33  MUZZLE VELOCITY FOR CHAMBER LENGTH TO BE EFFECTIVELY INFINITE ($\gamma_4 = 1.4$) (REF. 9).
FIG. 34  VARIATION OF BARREL LENGTH WITH CHAMBER LENGTH FOR THE LENGTH OF CHAMBER TO BE EFFECTIVELY INFINITE ($\gamma_4 = 1.4$) (REF. 9).
FIG. 35  COMPARISON OF EXPERIMENT AND THEORY ON EFFECTS OF CHAMBRAGE (φ = 1.4) (REF. 10).
FIG. 36 PERCENT OF INFINITE CHAMBRAGE VELOCITY INCREASE AS A FUNCTION OF CHAMBER DIAMETER TO BORE DIAMETER FOR AN INFINITE CHAMBER LENGTH GUN (\( \gamma_4 = 1.4 \)) (REF. 10).
FIG. 37  EFFECT OF CHAMBER LENGTH EXPRESSED IN TERMS OF MASS RATIO OF DRIVER GAS TO PROJECTILE \((G/m)\) ON VARIATION OF VELOCITY \((\bar{v})\) VERSUS PROJECTILE TRAVEL \((x)\) FOR A CHAMBERED GUN \((A_4/A_1 = 4)\) (REF. 12).
FIG. 38 CHARACTERISTICS NET FOR FINITE CHAMBER LENGTH ($\gamma_4 = 5/3$) (REF. 24).
FIG. 39a EFFECT OF CHAMBER GEOMETRY OR GAS TO PROJECTILE MASS RATIO \((G/m)\) ON PROJECTILE VELOCITY \((\gamma_4 = 5/3)\) (REF. 24).
FIG. 39b  EFFECT OF CHAMBER GEOMETRY OR GAS TO PROJECTILE MASS RATIO (G/m) ON PROJECTILE VELOCITY FOR SHORTER BARREL LENGTHS ($\gamma_4 = 5/3$) (REF. 24).
FIG. 40a EFFECT OF CHAMBER GEOMETRY OR GAS TO PROJECTILE MASS RATIO ON DIMENSIONLESS TIME-DISTANCE PATHS ($\gamma_4 = 5/3$) (REF. 24)
FIG. 40b  EFFECT OF CHAMBER GEOMETRY OR GAS TO PROJECTILE MASS RATIO ON DIMENSIONLESS TIME-DISTANCE PATHS - ENLARGED PLOT.

(Ref. 24)
FIG. 41  EFFECT OF $\gamma_4$ ON DIMENSIONLESS MUZZLE VELOCITY VS LAUNCHER LENGTH

(Ref. 24)
FIG. 42  PROJECTILE TRAVEL WHEN OVERTAKEN BY FIRST REFLECTED CHARACTERISTIC

(Ref. 24)
Moving Combustion Wave Front

\[ u_1 \text{ (1)} \quad w \quad u_2 \text{ (2)} \]

Duct

Stationary Combustion Wave Front

\[ V_1 = w - u_1 \quad (1) \quad V_2 = w - u_2 \quad (2) \]

Duct

FIG. 43a COMBUSTION WAVE FRONT MOVING AND STATIONARY WITH RESPECT TO THE DUCT OR AN OBSERVER.

Note: Usually state (1), the unburned gas, is at rest and \( V_1 = w \), the deflagration (burning velocity) or detonation velocity; for a detonation wave \( u_2 \), the burned gas, is in the direction shown and for a deflagration wave the particle velocity \( u_2 \) is opposite to the motion of the wave front \( w \) (as for a rarefaction wave).
Variation of Dynamic and Thermodynamic Properties

<table>
<thead>
<tr>
<th></th>
<th>$\frac{V_1}{a_1}$</th>
<th>$\frac{V_2}{a_2}$</th>
<th>$\frac{p_2}{p_1}$</th>
<th>$\frac{V_2}{V_1}$</th>
<th>$\frac{\rho_2}{\rho_1}$</th>
<th>$\frac{T_2}{T_1}$</th>
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<tr>
<td>Detonation</td>
<td>&gt; 1</td>
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<td>&gt; 1</td>
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<td></td>
<td></td>
<td>JA &lt; 1</td>
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<tr>
<td>Deflagration</td>
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<td>&lt; 1</td>
<td>&gt; 1</td>
<td>&lt; 1</td>
<td>&gt; 1</td>
</tr>
<tr>
<td></td>
<td></td>
<td>K = 1</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

FIG. 43b  DETONATION AND DEFLAGRATION WAVES IN THE (p, V)-PLANE
FIG. 43c. SCHEMATIC STRUCTURE OF A DETONATION WAVE (REF. 36b)

(Pressure and temperature profiles are shown. The shock front consists of regions (1-2-3). In (1-2) the active modes (translation and rotation) are excited in a few collisions and in (2-3) the inert mode (vibration and possibly dissociation) are excited in many hundreds or thousands of collisions. During this relaxation period the temperature drops and the pressure rises slightly due to the equi-partition process. The reaction zone (3-5), consists of an induction zone, where centres of reaction accumulate, and then an explosion takes place (4-5). The magnitudes of the induction zone, which is much larger, and the explosion zone depend on the reaction mechanism. Profile (1-5) is the ideal steady-state, one-dimensional detonation wave, which is usually not realized in practice due to transverse perturbations. In a moving detonation wave an attached nonstationary rarefaction wave (5-6) exists, which adjusts the flow conditions to a specific geometrical motion and boundary conditions.)
FIG. 43d DETONATION WAVE IN THE (p, v) OR (p, ρ) - PLANE

The profile shown in Fig. 43c may be reinterpreted in the above plane which is a portion of Fig. 43b. Proceeding along the Rayleigh (heat addition) line from the perfect to the imperfect gas Hugoniots for the normal shock wave, the pressure increases abruptly from 1-2, and then slowly from 2-3. However, in region 2-3, the temperature decreases as equipartition takes place. During the long induction period 3-4, some heat is added and the temperature increases slowly and then suddenly during the brief explosion period 4-5. The nonstationary rarefaction wave 5-6, is not shown. The wave may be isentropic or entropic depending on whether or not equilibrium has been attained. Further details are given in Ref. 37c.
FIG. 44a FINAL PRESSURE \( (p_f) \) VERSUS DILUTION \( (n) \) FOR SEVERAL INITIAL PRESSURES \( (p_i) \). (HELIUM DILUTION)
FIG. 44b FINAL-TO-INITIAL PRESSURE RATIO \( (P_f/P_i) \) VERSUS DILUTION \( (n) \) FOR SEVERAL INITIAL PRESSURES \( (p_i) \).

(HELIUM DILUTION)
FIG. 44c  FINAL TEMPERATURE ($T_f$) VERSUS DILUTION ($n$)
FOR SEVERAL INITIAL PRESSURES ($p_i$).
(HELIUM DILUTION)
FIG. 44d  FINAL-TO-INITIAL TEMPERATURE RATIO \( \frac{T_f}{T_i} \) VERSUS DILUTION \( n \) FOR SEVERAL INITIAL PRESSURES \( p_i \). (HELIUM DILUTION)
FIG. 44e  FINAL MOLECULAR WEIGHT ($m_f$) VERSUS DILUTION ($n$) FOR SEVERAL INITIAL PRESSURES ($p_i$). (HELIUM DILUTION)
FIG. 44f

FINAL SPECIFIC HEAT RATIO ($C_{pf} / C_{vf}$) VERSUS DILUTION ($n$) FOR SEVERAL INITIAL PRESSURES ($p_i$).
(HELIUM DILUTION)
FIG. 44g  FINAL SPEED OF SOUND ($a_f$) VERSUS DILUTION ($n$) FOR SEVERAL INITIAL PRESSURES ($p_i$) (HELIUM DILUTION)
FIG. 44h  FINAL-TO-INITIAL SPEED OF SOUND RATIO ($a_f/a_i$) VERSUS DILUTION ($n$) FOR SEVERAL INITIAL PRESSURES ($p_i$). (HELIUM DILUTION)
FIG. 45 SPECIFIC HEAT AT CONSTANT PRESSURE ($C_{pt}$) OF COMBUSTION PRODUCTS VERSUS DILUTION ($m$) FOR SEVERAL INITIAL Pressures ($p_i$). (HELIUM DILUTION)
FIG. 46 H₂O-CONCENTRATION (n_H₂O) IN COMBUSTION PRODUCTS VERSUS DILUTION (n) FOR SEVERAL INITIAL PRESSURES (pᵢ). (HELIUM DILUTION)
FIG. 47a  FINAL PRESSURE ($p_f$) VERSUS DILUTION ($m$)  
FOR SEVERAL INITIAL PRESSURES ($p_i$).  
(HYDROGEN DILUTION)
FIG. 47b FINAL-TO-INITIAL PRESSURE RATIO \((p_f/p_i)\) VERSUS DILUTION \((m)\) FOR SEVERAL INITIAL PRESSURES \((p_i)\).
(HYDROGEN DILUTION)
FIG. 47c  FINAL TEMPERATURE ($T_f$) VERSUS DILUTION ($m$)
FOR SEVERAL INITIAL PressURES ($p_i$).
(HYDROGEN DILUTION)
FIG. 47d Final-to-initial temperature ratio ($\frac{T_f}{T_i}$) versus dilution (m) for several initial pressures ($p_i$).

(HYDROGEN DILUTION)
FIG. 47e  FINAL MOLECULAR WEIGHT ($m_f$) VERSUS DILUTION ($m$) FOR SEVERAL INITIAL PRESSURES ($p_i$).
(HYDROGEN DILUTION)
FINAL SPECIFIC HEAT RATIO ($\gamma_f$) VERSUS DILUTION ($m$) FOR SEVERAL INITIAL PRESSURES ($p_i$). (HYDROGEN DILUTION)
FIG. 47g  FINAL SPEED OF SOUND ($a_f$) VERSUS DILUTION(m) For Several Initial Pressures ($p_i$) (Hydrogen Dilution)
FIG. 47h  FINAL-TO-INITIAL SPEED OF SOUND RATIO ($a_f/a_i$) VERSUS DILUTION (m) FOR SEVERAL INITIAL PRESSURES ($p_i$).
(HYDROGEN DILUTION)
FIG. 48a SPECIFIC HEAT AT CONSTANT PRESSURE (Cpf) OF COMBUSTION PRODUCTS VERSUS DILUTION (m) FOR SEVERAL INITIAL PRESSURES (p_i).
(HYDROGEN DILUTION)
FIG. 48b H$_2$O-CONCENTRATION ($n_{H_2O}$) IN COMBUSTION PRODUCTS VERSUS DILUTION ($m$) FOR SEVERAL INITIAL PRESSURES ($p_i$).
(HYDROGEN DILUTION)
ESCAPE SPEED FOR FROZEN RAREFACTION WAVES IN DILUTED HYDROGEN-OXYGEN MIXTURES
FIG. 49b  ESCAPE SPEED FOR EQUILIBRIUM RAREFACTION WAVES IN DILUTED HYDROGEN OXYGEN MIXTURES
FIG. 50a DETONATION PRESSURE FOR FIVE HYDROGEN-OXYGEN MIXTURES AS A FUNCTION OF INITIAL PRESSURE (Ref. 37c)
FIG. 50b  DETONATION PRESSURE RATIO FOR FIVE HYDROGEN-OXYGEN MIXTURES AS A FUNCTION OF INITIAL PRESSURE (Ref. 37c).
FIG. 50c  TEMPERATURE OF DETONATION PRODUCTS FOR FIVE HYDROGEN-OXYGEN MIXTURES AS A FUNCTION OF INITIAL PRESSURE (Ref. 37c)
FIG. 50d  MOLECULAR WEIGHT OF DETONATION PRODUCTS FOR FIVE HYDROGEN-OXYGEN MIXTURES AS A FUNCTION OF INITIAL PRESSURE (Ref. 37c)
FIG. 50e  MOLECULAR WEIGHT RATIO ACROSS DETONATION WAVE FOR FIVE HYDROGEN-OXYGEN MIXTURES AS A FUNCTION OF INITIAL PRESSURE (Ref. 37c)
Equilibrium Specific Heat
\[ C_{pfE} = \left. \frac{\partial h(p, T)}{\partial T} \right|_p \]

Frozen Specific Heat
\[ C_{pfF} = \sum_j \nu_j C_{pj} \]
\( \nu_j \) is the mole fraction of the jth species

FIG. 50f FINAL SPECIFIC HEATS AT CONSTANT PRESSURE FOR THE DETONATION PRODUCTS OF FIVE HYDROGEN-OXYGEN MIXTURES AS A FUNCTION OF INITIAL PRESSURE (Ref. 37c)
\[ \gamma_{fs} = \frac{\partial \ln p}{\partial \ln \rho} \bigg|_s, \text{ isentropic index} \]

\[ \gamma_{fs} = \sum_j \nu_j \frac{C_{pj}}{\sum_j \nu_j C_{vj}}, \text{ frozen specific heat ratio} \]

\[ \bar{\gamma}_f = \frac{C_p}{C_v} = \lim_{T \to \infty} \frac{\partial h(p, T)}{\partial T} \bigg|_p, \text{ equilibrium specific heat ratio} \]

**FIG. 50g**  FINAL ISENTROPIC INDEX AND SPECIFIC HEAT RATIOS FOR THE DETONATION PRODUCTS OF HYDROGEN-OXYGEN MIXTURES AS A FUNCTION OF INITIAL PRESSURE (Ref. 37c)
FIG. 50h  FINAL SOUND SPEEDS FOR THE DETONATION PRODUCTS OF HYDROGEN-OXYGEN MIXTURES AS A FUNCTION OF INITIAL PRESSURES (Ref. 37c)
FIG. 51 USE OF FUSE WIRE AS IGNITION POINTS IN A HYPERVELOCITY GUN COMBUSTION CHAMBER (Ref. 41)
FIG. 52  PRESSURE RECORD DURING CONSTANT-VOLUME COMBUSTION (Ref. 41)
$8\text{He} + 3\text{H}_2 + \text{O}_2 \rightarrow 8\text{He} + 2\text{H}_2\text{O} + \text{H}_2$ (no dissociation)

Theoretical - 100 percent Combustion ($2700^\circ$K)

Actual - 100 percent Combustion

FIG. 53a COMPARISON OF ACTUAL AND CALCULATED FINAL PRESSURE vs INITIAL CHARGING PRESSURE FOR A COMBUSTIBLE MIXTURE OF $8\text{He} + 3\text{H}_2 + \text{O}_2$ (Ref. 41)
FIG. 53 b COMPARISON OF ACTUAL AND CALCULATED FINAL TO INITIAL PRESSURE RATIO vs HELIUM DILUTION FOR THE MIXTURE $2H_2 + O_2 + n\text{He}$ (Ref. 26b) (no dissociation)
APPARENT HEAT TRANSFER RATES COMPUTED FROM PRESSURE DROP SHOWN IN FIG. 52 ARISING FROM A THERMAL STRAIN-GAUGE EFFECT (Ref. 41)
\[ 8 \text{He} + 3 \text{H}_2 + \text{O}_2 \rightarrow 8 \text{He} + 2 \text{H}_2\text{O} + \text{H}_2 \text{ (no dissociation)} \]

**FIG. 55**  SOUND SPEED AND SPECIFIC HEAT RATIO FOR INCOMPLETE COMBUSTION (Ref. 41)
FIG. 56  MUZZLE VELOCITY (1.58 in. dia. x 312 in. long launcher barrel) vs CHAMBER PRESSURE (2700°K) FOR VARIOUS PROJECTILE MASSES (Ref. 41).
FIG. 57  WAVE SYSTEM PRODUCED BY AN ACCELERATING PISTON IN A CLOSED BARREL (PUMP TUBE) (Ref. 43).
FIG. 58 WAVE SYSTEM PRODUCED BY PISTON MOVING WITH UNIFORM VELOCITY IN A CLOSED BARREL (Ref. 43)
(Notes that the state numbers have been changed on this diagram and in the corresponding text for convenience in using consecutive numbers 0, 1, 2, 3 etc., for general formulae.)
Idealized Internal Ballistics Relations for a Gun with Infinite Chambrage

\[
\left(\frac{2}{\gamma - 1}\right)^2 \bar{x} = \left[1 - \frac{2}{\gamma - 1} \bar{u}^2\right] - 1
\]

for \( \bar{u} \leq \frac{\gamma - 1}{2} \sqrt{\frac{\gamma + 1}{\gamma - 1}} \)  

(Eq. 58)

\[
\left(\frac{2}{\gamma - 1}\right)^2 \bar{x} = \frac{2}{\gamma - 1} \bar{u} - \sqrt{\frac{\gamma + 1}{\gamma - 1}} + \left[\frac{\gamma + 1}{2}\right] \frac{1}{\frac{\gamma - 1}{2}} - 1
\]

for \( \bar{u} \geq \frac{\gamma - 1}{2} \sqrt{\frac{\gamma + 1}{\gamma - 1}} \)  

(Eq. 59)

FIG. 59 VARIATION OF NONDIMENSIONAL MUZZLE VELOCITY \( \bar{u} \) WITH LAUNCHER LENGTH \( \bar{x} \).
FIG. 60     PRESSURE RATIO ACROSS REPEATED SHOCK WAVES IN AIR, $T_r = T_o = 290^\circ K$ (Ref. 43).
FIG. 61  TEMPERATURE RATIO ACROSS REPEATED SHOCK WAVES IN AIR, $T_r = T_o = 290^\circ$K (Ref. 43)
FIG. 62  PRESSURE RATIO ACROSS REPEATED SHOCK WAVES IN AIR,
$T_r = 290^oK, \ T_o = 580^oK$ (Ref. 43)
FIG. 63 TEMPERATURE RATIO ACROSS REPEATED SHOCK WAVES IN AIR, $T_r = 290^0K$, $T_o = 580^0K$ (Ref. 43)
FIG. 64  PRESSURE RATIO ACROSS REPEATED SHOCK WAVES IN HYDROGEN, \( T_r = T_o = \text{290}^0\text{K} \) (Ref. 43).
FIG. 65  TEMPERATURE RATIO ACROSS REPEATED SHOCK WAVES IN HYDROGEN, $T_r = T_o = 290^\circ$K (Ref. 43).
FIG. 66  COMPUTED PISTON PATHS AND WAVE SYSTEMS AS A FUNCTION OF PISTON MASS (Ref. 46)
FIG. 67 ACTUAL PISTON PATHS AS A FUNCTION OF PISTON MASS (Ref. 47).
FIG. 68  EFFECT OF A 3 mm dia. NOZZLE THROAT ON MOTION OF A 10 gm PISTON (Ref. 47).
FIG. 69  DIMENSIONLESS ENERGY vs ENTROPY FOR AIR (Ref. 43)
FIG. 70  DIMENSIONLESS ENERGY vs ENTROPY FOR HYDROGEN
(Ref. 43)
FIG. 71  PISTON COMPRESSION AND HEATING ON A MODIFIED MOLLIER DIAGRAM (Ref. 43).
FIG. 72 SCHEMATIC DIAGRAM OF 20-mm AEDC ARC-HEATED HYPERVELOCITY LAUNCHER (Ref. 57)
FIG. 73 SCHEMATIC DIAGRAM OF CAPACITANCE ENERGY STORAGE SYSTEM USED WITH LAUNCHER SHOWN IN FIG. 74 (Ref. 52)
(a) SCHEMATIC DIAGRAM OF INDUCTANCE ENERGY STORAGE SYSTEM (AEDC-50)

(b) SCHEMATIC DIAGRAM OF A HOMOPOLAR (UNIPOLAR) GENERATOR (FOR OPERATING DETAILS SEE STANDARD HANDBOOK FOR ELECTRICAL ENGINEERS, EDITED BY KNOWLTON, McGRAW-HILL)

FIG. 74 INDUCTANCE ENERGY STORAGE SYSTEM USED WITH HOLSHOT TUNNELS (Ref. 52)
a. Completely Coaxial System (BAC-8 and BAC-44) (Critically Damped Discharge)

b. Flat Bus and Coaxial Collector (BAC-8 and AEDC-16) (Oscillatory Discharge)

FIG. 75  CHARGE AND DISCHARGE CHARACTERISTICS OF A CAPACITANCE ENERGY STORAGE SYSTEM (Ref. 52)
a. Voltage and Current during Charging Period, AEDC-50 Hotshot Power Supply

b. Typical Variation of Current, Voltage, and Resistance of Arc in Hotshot AEDC-50

FIG. 76 CHARGE AND DISCHARGE CHARACTERISTICS OF AN INDUCTANCE ENERGY STORAGE SYSTEM (Ref. 52)
COST OF CAPACITANCE AND INDUCTANCE ENERGY STORAGE SYSTEMS (Ref. 52)

(b) ENERGY TRANSFER EFFICIENCY IN AEDC HOTSHOT TUNNELS

FIG. 77 COST AND ENERGY TRANSFER OF CAPACITANCE AND INDUCTANCE ENERGY STORAGE SYSTEMS.
CAPACITOR VOLTAGE: 8.5 TO 9.0 KV
STORED ENERGY: 3.52 TO 3.95 MEGAJOULES

APR CHAMBER VOLUME, IN.\(^3\)

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CHARGE PRESSURE, PSIA

(a) ENERGY TRANSFER EFFICIENCY vs
INITIAL CHARGING PRESSURE FOR
VARYING ARC-CHAMBER VOLUMES

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(b) ENERGY TRANSFER EFFICIENCY vs
CAPACITOR VOLTAGE FOR VARYING
ARC-CHAMBER VOLUMES

FIG. 78 ENERGY TRANSFER EFFICIENCY FOR CAPACITANCE STORAGE (Ref. 55)
FIG. 79 SCHEMATIC DIAGRAM OF GENERAL ELECTRIC CAPACITANCE-DRIVEN SHOCK TUBE (Ref. 59)
(a) CIRCUIT DIAGRAM OF CAPACITANCE DRIVING SYSTEM

(b) VARIATION OF DISCHARGE CURRENT WITH TIME (WAVEFORM)

FIG. 80 ELECTRICAL CHARACTERISTICS OF GENERAL ELECTRIC CAPACITANCE DRIVE (Ref. 59)
FIG. 81  ENERGY TRANSFER EFFICIENCY FOR GENERAL ELECTRIC CAPACITANCE DRIVE USING HELIUM AT 12 atm IN THE ARC CHAMBER (Ref. 59)
$Q_i = 47,000 \text{ Joules}$

$\frac{Q_4}{Q_i} = \frac{Q_4}{Q_{\text{MAX}}}$

$T_4 = 7500^\circ K$

$5500^\circ K$

$4500^\circ K$

$\bullet$ $M_S$ & $P_1$ Measurements

$\diamond$ Driver Pressure Meas.

FIG. 82 ENERGY TRANSFER EFFICIENCY FOR A CAPACITANCE HEATED HELIUM DRIVER WITH INITIAL CHARGING PRESSURE (Ref. 59)
FIG. 83 FINAL HELIUM ARC-CHAMBER PRESSURE AND TEMPERATURE AS A FUNCTION OF INITIAL CHARGING PRESSURE AND INPUT ENERGY (Ref. 59)
FIG. 84 HEAT LOSSES FROM AEDC HOTSHOT TUNNEL NO. 1 ARC-CHAMBER (Ref. 59).
Nitrogen
16-C Arc Chamber
d* = 0.15 in.
(ρo)i = 40 Amagat
(To)i = 5100-6000°K

FIG. 85 LOSS IN ARC-CHAMBER TOTAL ENTHALPY WITH TIME FOR DIFFERENT LINERS IN AEDC HOTSHOT 1 (Ref. 53)
(a) LOSS IN ARC CHAMBER TOTAL ENTHALPY FOR VARYING INITIAL NITROGEN CHAMBER DENSITIES
AEDC HOTSHOT 2

(b) LOSS IN ARC CHAMBER TOTAL ENTHALPY FOR TWO INITIAL HELIUM CHAMBER DENSITIES

FIG. 86 LOSS IN ARC CHAMBER TOTAL ENTHALPY AS A FUNCTION OF INITIAL CHAMBER DENSITY (Ref. 53)
FIG. 87 TOTAL TEMPERATURE DECAY CAUSED BY RADIATION LOSSES vs CHAMBER PROPERTIES (At/V) FOR DIFFERENT INITIAL AIR DENSITIES (Ref. 53)
<table>
<thead>
<tr>
<th>Test Section</th>
<th>$Q_{\text{stored}}$</th>
<th>$V$</th>
<th>$Q_{\text{st.}}/V$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hotshot 1</td>
<td>$10^6$ Joule</td>
<td>0.018 ft³</td>
<td>$57.5 \times 10^6$ Joule/ft³</td>
</tr>
<tr>
<td>Hotshot 2</td>
<td>$10 \times 10^6$ Joule</td>
<td>0.21 ft³</td>
<td>$48 \times 10^6$ Joule/ft³</td>
</tr>
<tr>
<td>Tunnel F</td>
<td>$100 \times 10^6$ Joule</td>
<td>1.08 ft³</td>
<td>$93 \times 10^6$ Joule/ft³</td>
</tr>
</tbody>
</table>

Air, Radiation Heat Losses Only

At/V, sec/ft

Effective Energy in Air, Joule

$\rho_0 = 39.81$ Am.

$\rho_0 = 100$ Am.

**FIG. 88** REQUIRED ADDED ENERGY FOR AIR TO ACHIEVE A GIVEN INITIAL TEMPERATURE FOR VARIOUS CHAMBER PROPERTIES (At/V) AND INITIAL AIR DENSITY (Ref. 53)
FIG. 89  REDUCED PERFORMANCE OF 20 mm AEDC CAPACITANCE-DRIVEN HYPERVELOCITY LAUNCHER RESULTING FROM CONTAMINATION (Ref. 57)
APPENDIX I

Supplementary Figures on Constant Volume and Detonating Combustion of Stoichiometric and Diluted Hydrogen-Oxygen Mixtures

Calculated and Drawn by A. Benoit
More detailed information will be published in two forthcoming technical notes by A. Benoit.
FIG. 1.1  ISENTROPIC EXPONENT $\gamma_e$, FROZEN SPECIFIC HEATS RATIO $\gamma_f$, AND RATIO OF EQUILIBRIUM SPECIFIC HEATS FOR CONSTANT VOLUME COMBUSTION FOR HYDROGEN DILUTION.
Initial conditions

\[ T_1 = 298.15 \, ^\circ \text{K} \]

\[(2 \, \text{H}_2 + \text{O}_2) + n \, \text{He}\]

FIG. 1.2 ISENTROPIC EXPONENT \( \gamma_e \), FROZEN SPECIFIC HEATS RATIO \( \gamma_f \) AND RATIO OF EQUILIBRIUM SPECIFIC HEAT \( \overline{\gamma} \) FOR CONSTANT VOLUME COMBUSTION FOR HELIUM DILUTION.
FIG. 1.3  EQUILIBRIUM ($a_e$), FROZEN ($a_f$), AND FICTITIOUS ($\bar{a}$) SOUND SPEEDS BASED ON THE VALUES OF $\gamma$ IN FIG. 1.1, FOR HYDROGEN DILUTION

Initial conditions

$T_1 = 298.15 ^\circ K$

$\left( 2H_2 + O_2 \right) + m H_2$
FIG. 1.4  EQUILIBRIUM ($a_e$), FROZEN ($a_f$), AND FICTITIOUS ($\bar{a}$) SOUND SPEEDS BASED ON THE VALUES OF $\gamma$ IN FIG. 1.2, FOR HELIUM DILUTION.

Initial conditions

\[
T_1 = 298.15 \degree K \\
(2H_2 + O_2) + nHe
\]
FIG. 1.5

PRESSURE RATIO ACROSS THE DETONATION WAVE VERSUS INITIAL TEMPERATURE FOR SEVERAL DILUTIONS AND INITIAL PressURES
(HELIUM DILUTION)
TEMPERATURE BEHIND THE DETONATION WAVE VERSUS INITIAL TEMPERATURE FOR SEVERAL DILUTIONS AND INITIAL PRESSURES (HELIUM DILUTION)
EQUILIBRIUM SPEED OF SOUND BEHIND THE DETONATION WAVE VERSUS
INITIAL TEMPERATURE FOR SEVERAL DILUTIONS AND INITIAL PressURES
(HELIUM DILUTION)
FIG. 1.8

PRESSURE RATIO ACROSS THE DETONATION WAVE VERSUS INITIAL TEMPERATURE FOR SEVERAL DILUTIONS AND INITIAL PRESSURES (HYDROGEN DILUTION)
FIG. 1.9

TEMPERATURE BEHIND THE DETONATION WAVE VERSUS INITIAL TEMPERATURE FOR SEVERAL DILUTIONS AND INITIAL PRESSURES (HYDROGEN DILUTION)
FIG. 1.10

EQUILIBRIUM SPEED OF SOUND BEHIND THE DETONATION WAVE VERSUS INITIAL TEMPERATURE FOR SEVERAL DILUTIONS AND INITIAL PRESSURES (HYDROGEN DILUTION)