CRANFIELD
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SHOCK-TUBE MEASUREMENTS OF THERMAL ACCOMMODATION
BETWEEN A HOT GAS AND A COLD SOLID

by

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SUMMARY

An analysis is presented for the response of a finite-thickness, thin-film resistance thermometer mounted on the end wall of a shock tube and exposed to a gas suddenly heated by shock-wave reflection. It is shown that both the finite thickness of the gauge and gas-solid thermal accommodation effects play a significant role in determining the actual response of such gauges.

Shock-tube measurements are also presented and combined with the theoretical model to yield the thermal accommodation coefficient for $N_2$ on platinum. Tests were run with platinum thin-film thermometers deposited on pyrex by three different methods: chemical deposition (Hanovia 05-X), vacuum sputtering in an Argon atmosphere, and vacuum evaporation with an electron-beam-heated source. The results for the accommodation coefficient on all three gauges are in close agreement ($\alpha = 0.5$) and also compare favourably with previous measurements at lower pressures using conventional steady-state techniques.

The shock-tube technique provides hope for obtaining accommodation coefficients at conditions not previously accessible with other methods.
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**Subscripts**

- 1: Condition upstream of primary shock wave
- 5: Condition behind reflected shock wave
- $a, av$: average
- f: film
- g: Gas, gauge
- s: Substrate
- w: Wall
- $D/Dt$: Denotes substantial derivative
- $\text{Bar} \ (\bar{\text{\)}}$: Denotes Laplace transform
- $\mathcal{L}^{-1}$: Denotes inverse Laplace transform
1. INTRODUCTION

Previous measurements of thermal accommodation (Ref. 1) between a gas and a solid boundary have relied heavily on the conductivity cell, a device in which heat is conducted from an electrically heated wire to a colder, coaxial cylinder (of much larger radius) by means of a gas usually at low pressure. The quantity used to express the efficiency of the energy transfer between the gas and the solid is the thermal accommodation coefficient, and this can be inferred from the steady-state, conductivity-cell experiments using an energy balance and measured values for the temperature of the wire and the cylinder.

Recently the theory was put forward by Clarke (Ref. 2) for dealing with the transient accommodation effects which follow a sudden change in the temperature and pressure of a moderately dense gas in contact with a solid. Only one-dimensional unsteady effects were considered, and the gas was treated as a continuum with the exception of the layer of gas immediately adjacent to the wall. From a practical viewpoint, Clarke's problem represents an idealization of the situation which arises when a plane normal shock wave reflects from the co-planar end wall of a shock tube, and this similarity was later exploited by Busing and Clarke (Ref. 3) in a shock-tube investigation of the thermal accommodation coefficient of air on platinum. The reflected shock wave leaves behind a gas with increased values of temperature and pressure, and the subsequent energy transfer between the hot gas and the cold wall induces motion in the compressible gas and raises the temperature of the solid. Busing and Clarke employed a platinum thin-film resistance thermometer to monitor the temperature changes on the solid surface and were able to infer the accommodation coefficient from a comparison of theoretical and measured temperature histories.

Unfortunately, the value for the accommodation coefficient found by Busing and Clarke (Ref. 3) was anomalously low in comparison with conductivity-cell results, and this cast doubt on the shock-tube method. It is now clear that the discrepancy was due simply to the neglect of the thin-film response time in the data reduction, and that a proper theoretical model must, in general, account for the influence of the finite gauge thickness.

The present report modifies the previous theoretical model of Busing and Clarke to include the effect of gauge thickness, and presents some calculational results which illustrate the dependence of thin-film-gauge response on the ratio of the characteristic accommodation and gauge-response times. New shock-tube data are also presented for N₂ on platinum, and the results for the accommodation coefficient based on the modified theory are shown to be in good agreement with conductivity-cell measurements at low pressures.
2. THIN-FILM-GAUGE RESPONSE WITH NO TEMPERATURE JUMP

The influence of gauge thickness on the response of a thin-film-resistance thermometer can best be appreciated by considering first the transient gauge output, following shock-wave reflection, when there is no temperature difference between the gas and the solid at their interface, i.e., the case of no temperature jump.

We begin by assuming that the heat transfer to the end wall of a shock tube, subsequent to shock-wave reflection, is adequately modeled by considering a sudden and uniform change in the temperature and pressure of a semi-infinite expanse of gas in contact with a solid. This should be a suitable model for times greater than, say, 10 collision time intervals after shock reflection, since shock-structure effects should be essentially over and the shock wave will have receded far enough from the wall to be considered at infinity. The gas-solid interface is taken at \( x = 0 \), and the change in gas temperature is taken to occur at \( t = 0 \). The solid is assumed to occupy the complete half space \( x < 0 \), with an interface at \( x = -\delta \) separating the thin-film material and its substrate. Only one-dimensional heat transfer need be considered on the time scale of interest here, so the temperatures throughout the film and substrate regions must satisfy the diffusion equations

\[
\frac{\partial T_f}{\partial t} = \left( \frac{k}{\rho c} \right)_f \left( \frac{\partial^2 T_f}{\partial x^2} \right) = \kappa_f \left( \frac{\partial^2 T_f}{\partial x^2} \right) \quad (2.1)
\]

and

\[
\frac{\partial T_s}{\partial t} = \left( \frac{k}{\rho c} \right)_s \left( \frac{\partial^2 T_s}{\partial x^2} \right) = \kappa_s \left( \frac{\partial^2 T_s}{\partial x^2} \right), \quad (2.2)
\]

where \( T, k, \rho \) and \( c \) are, respectively, the temperature, thermal conductivity, density and specific heat of the material in question; \( \kappa = k/\rho c \) is the thermal diffusivity, which can be considered constant in view of the small changes of solid temperature which will occur in practice. The subscripts \( f \) and \( s \) denote film and substrate respectively.

At the interface of the two solids we require continuity of both energy flux and temperature, so that

\[
[k_f \left( \frac{\partial T_f}{\partial x} \right)]_{x = -\delta} = [k_s \left( \frac{\partial T_s}{\partial x} \right)]_{x = -\delta}, \quad (2.3)
\]

where the thermal conductivities \( k_f \) and \( k_s \) are taken to be constants, and

\[
T_f(-\delta, t) = T_s(-\delta, t). \quad (2.4)
\]

Similar boundary conditions apply at the interface of the gas and the film, i.e.,

\[
[k \left( \frac{\partial T_g}{\partial x} \right)]_{x = 0} = [k_f \left( \frac{\partial T_f}{\partial x} \right)]_{x = 0} \quad (2.5)
\]

and
\[ T_g(0,t) = T_f(0,t). \] (2.6)

Finally we have the additional boundary information that conditions remain unchanged at infinity, and so we can put

\[ T_s \rightarrow T_0 \text{ as } x \rightarrow -\infty, \ t > 0, \] (2.7)

where \( T_0 \) is the initial temperature of the solid, and

\[ T_g \rightarrow T_\infty \text{ as } x \rightarrow \infty, \ t > 0, \] (2.8)

where \( T_\infty \) is the gas temperature behind the reflected shock wave.

The appropriate equation for one-dimensional energy transfer in an ideal gas (calorically and thermally perfect) is, neglecting viscous effects,

\[ \rho C_p (\partial T_g / \partial t) = \partial [k_g (\partial T_g / \partial x)] / \partial x - D_p / Dt, \] (2.9)

where \( T_g, \rho, \) and \( C_p \) are the gas temperature, pressure, density and specific heat at constant pressure, respectively; \( D/Dt \) is the usual convective operator. We now argue that the pressure term may be dropped, since both Goldsworthy (Ref. 6) and Clarke (Ref. 7 and Ref. 16) have shown that, to first-order accuracy, the pressure in the thermal boundary layer is constant in both space and time.

Following Busing and Clarke (Ref. 3), to simplify Eq. (2.9) we transform from Eulerian \( x, t \) coordinates to Lagrangian \( \psi, t \) coordinates where

\[ \psi = \int_0^x \rho(\bar{x},t)d\bar{x} \] (2.10)

and

\[ \partial / \partial x = \rho \partial / \partial \psi. \] (2.11)

The transformed version of Eq. (2.9) becomes

\[ \partial T_g / \partial t = \partial [\rho k_g / C_p](\partial T_g / \partial \psi)] / \partial \psi \] (2.12)

while conditions (2.5), (2.6) and (2.8) transform into

\[ \rho k_g(\partial T_g / \partial \psi) \psi = 0 = \left[ k_f (\partial T_f / \partial x) \right] x = 0; \] (2.13)

\[ T_g(\psi = 0, t) = T_f(x = 0, t) \] (2.14)
and
\[ T_g \to T_\infty \text{ as } \psi \to \infty, \ t > 0. \quad (2.15) \]

If we now assume that \( k_g \) is proportional to \( T_g \), which is nearly true, then \( \rho k_g \) is proportional to \( p \), and is therefore constant. We define the constant \( \beta \) as
\[ \beta = \frac{\rho k_g}{C_p} \quad (2.16) \]
so that we are left with a simple diffusion equation for the gas,
\[ \frac{\partial T_g}{\partial t} = \beta \left( \frac{\partial}{\partial x} \right)^2 T_g. \quad (2.17) \]

Equations (2.1), (2.2) and (2.17), together with the conditions of Eqs. (2.3), (2.4), (2.7) and (2.13) to (2.15) are a complete set of linear equations and boundary conditions. Addition of the initial-value information that
\[ T_s = T_o; \ x < -\delta, \ t < 0; \quad (2.18) \]
\[ T_f = T_o; \ -\delta < x < 0, \ t < 0; \quad (2.19) \]
and
\[ T_g = T_\infty; \ x > 0, \ t < 0, \quad (2.20) \]
completes the description of the problem which can now be solved by Laplace transform methods. Denoting the transform of a quantity by a bar (\( \bar{\cdot} \)) over the relevant symbol, and using \( z \) as the transform variable, we find that
\[ \bar{T}_g - \bar{T}_o/z = A(z)\exp(-\sqrt{z/\beta} \psi), \quad (2.21) \]
\[ \bar{T}_f - \bar{T}_o/z = B(z)\exp(\sqrt{z/\kappa_f} x) + C(z)\exp(-\sqrt{z/\kappa_g} x) \quad (2.22) \]
and
\[ \bar{T}_s - \bar{T}_o/z = D(z)\exp(\sqrt{z/\kappa_s} x) \quad (2.23) \]
where the functions \( A(z), B(z), C(z) \) and \( D(z) \) are to be evaluated using the boundary conditions for continuity of heat flux and temperature at the two interfaces. We find for the film, after some tedious algebra,
\[ \bar{T}_f - \bar{T}_o/z = [D(z)/2] \left( \frac{1 + Q_{fs}}{\sqrt{z/\kappa_s}} \delta + \sqrt{z/\kappa_f} x \right) \]
\[ + \left( 1 - Q_{fs} \right) \exp(\sqrt{z/\kappa_s} \delta - \sqrt{z/\kappa_f} x), \quad (2.24) \]
where

\[
D(z) = \left( \frac{T_{\infty} - T_0}{z} \right) \left( \frac{1 + Q_{fs}}{2} \right) \left( \frac{1 + Q_{fg}}{Q_{fg}} \right) \exp \left( -\sqrt{z/\kappa_s} \delta + \sqrt{z/\kappa_f} \delta \right)
\]

\[ - \left( \frac{1 - Q_{fs}}{2} \right) \left( \frac{1 - Q_{fg}}{Q_{fg}} \right) \exp \left( -\sqrt{z/\kappa_s} \delta - \sqrt{z/\kappa_f} \delta \right) \right]^{-1}, \]

\[ (2.25) \]

\[ Q_{fs} = \left[ \left( \rho c k \right)_s / \left( \rho c k \right)_f \right]^{1/2} \]

\[ (2.26) \]

and

\[ Q_{fg} = \left[ \left( \rho C \kappa k \right)_p / \left( \rho C \kappa k \right)_f \right]^{1/2}. \]

\[ (2.27) \]

It is worth noting in passing that both \( Q_{fs} \) and \( Q_{fg} \) have values less than 1 (for example, \( Q_{fs} \approx 0.11 \) for a platinum film on a pyrex substrate, and \( Q_{fg} \approx 0.004 \) for air at standard conditions on platinum) for all situations of interest here.

The actual thin-film response depends on the average temperature (and hence resistance) throughout the film, but for times greater than a few diffusion times in the film (the diffusion time, \( T_d = \delta^2 / \kappa_f \)) the film temperature is nearly uniform and we can accurately approximate the average temperature with the temperature at a specific value of \( x \), say \( x = 0 \). The change in film temperature, away from the initial temperature \( T_0 \), is thus taken as the inverse transform of Eq. (2.24) evaluated at \( x = 0 \), i.e.,

\[ \Delta T_f(t) = T_f - T_0 \]

\[
= L \left[ \left( \frac{T_{\infty} - T_0}{z} \right) \left( \frac{Q_{fg}}{1 + Q_{fg}} \right) \left[ 1 + \frac{1 - Q_{fs}}{1 + Q_{fs}} \exp \left( -2\sqrt{z/\kappa_f} \delta \right) \right] \right]
\]

\[
= \sum_{n=0}^{\infty} \varphi \exp \left( -2n\sqrt{z/\kappa_f} \delta \right)
\]

\[ \varphi = \left( \frac{1 - Q_{fg}}{1 + Q_{fg}} \right) \left( \frac{1 - Q_{fs}}{1 + Q_{fs}} \right) < 1. \]

(2.28)

Making use of standard transform inversion tables (the brief one in Carslaw and Jaeger (Ref. 8) is particularly handy) one can now show that
\[ \Delta T_f(t) = \frac{T_\infty - T_0}{1 + Q_{gf}} \left\{ \sum_{n=0}^{\infty} \phi^n \text{erfc}(n \delta / \sqrt{\kappa_f t}) \right\} \]

\[ = \frac{1 - Q_{fs}}{1 + Q_{fs}} \sum_{n=0}^{\infty} \phi^n \text{erfc} \left[ (n + 1) \delta / \sqrt{\kappa_f t} \right] \]  

(2.30)

where erfc is the usual complementary error function and \( Q_{gf} = 1/Q_{fg} \).

This last equation can be made more compact by combining the two summations and at the same time introducing the thin-film diffusion time, which we shall refer to hereafter as the characteristic gauge-response time; the result is

\[ \Delta T_f(t) = \frac{T_\infty - T_0}{1 + Q_{gf}} \left[ 1 + \frac{2}{1 - Q_{fg}} \sum_{n=1}^{\infty} \phi^n \text{erfc} \left( n \sqrt{\tau_g / t} \right) \right] \]  

(2.31)

It is worthwhile now to inspect this result for two limiting cases. For small values of time, i.e., as \( t \to 0 \), we see that

\[ \Delta T_f(t) \to (T_\infty - T_0)/(1 + Q_{gf}), \]  

(2.32)

which is the well-known result for the surface temperature change on a homogeneous end wall with properties corresponding to the film material. For large values of time, i.e., as \( t \to \infty \), we observe that

\[ \Delta T_f(t) \to \frac{T_\infty - T_0}{1 + Q_{gf}} \left[ 1 + \frac{2}{1 - Q_{fg}} \left( \frac{1}{1 - \phi} - 1 \right) \right] = \frac{T_\infty - T_0}{1 + Q_{gs}}, \]  

(2.33)

which is the expected solution for a homogeneous end wall with properties corresponding to the substrate material. This last result is of course the solution obtained when the thickness of the thin film is neglected, so that insofar as the physical model employed here is the correct one, the response of an "ideal" thin-film resistance thermometer would be a simple step. (The response, or output, of a thin-film thermometer is directly proportional to its change in temperature.) Finite gauge thickness serves to degrade the quality of the step, but does not change the magnitude of the long-time, asymptotic temperature level.

For the purpose of plotting the results obtained, it is convenient to put the solution in dimensionless form. We do so by dividing Eq. (2.31)
by the asymptotic temperature change indicated in Eq. (2.33). The result is

\[ \Delta T_f(t)/\Delta T_f(\infty) = \frac{1 + Q_{gs}}{1 + Q_{gf}} \left[ 1 + \frac{2}{1 - Q_{gf}} \sum_{n=1}^{\infty} \left( \frac{n}{\sqrt{\tau g}} \right)^n \text{erfc} \left( \frac{n}{\sqrt{\tau g}} \right) \right], \]

(2.34)

Making use of the information that \( Q_{gs} \) and \( Q_{gf} \) are both generally much larger than 1, we have, to a high degree of accuracy,

\[ \Delta T_f(t)/\Delta T_f(\infty) = Q_{fs} \left[ 1 + 2 \sum_{n=1}^{\infty} \left( \frac{n}{\sqrt{\tau g}} \right)^n \text{erfc} \left( \frac{n}{\sqrt{\tau g}} \right) \right], \]

(2.35)

where

\[ \nu = \frac{1 - Q_{fs}}{1 + Q_{fs}}, \]

(2.36)

which does not depend on shock strength or gas conditions. Results for a temperature gauge composed of platinum film on pyrex substrate (i.e., \( Q_{fs} = 0.11 \)) and based on Eq. (2.35) are presented in Fig. 1. The simple dependence on the dimensionless time \( t/\tau_g \) makes the plot useful for comparing the relative performance of gauges with different film thickness, and if the characteristic gauge response time \( \tau_g \) is known, for establishing the magnitude of the final temperature change from measurements made at moderate values of \( \sqrt{t/\tau_g} \).

Also plotted in Fig. 1 is an approximate form of Eq. (2.35), acceptable for large values of \( \sqrt{t/\tau_g} \), and obtained in the following way. First we expand \( \text{erfc} \left( \frac{n}{\sqrt{\tau g}} \right) \) in a form valid for \( n/\sqrt{\tau g} \ll 1 \),

\[ \text{erfc} \left( \frac{n}{\sqrt{\tau g}} \right) = 1 - \left( \frac{2}{\sqrt{\pi}} \right) \left[ \left( n/\sqrt{\tau g} \right)^2 - (1/3)\left( n/\sqrt{\tau g} \right)^4 + \ldots. \right] \]

(2.37)

and then we substitute in Eq. (2.35), as an approximation for \( \text{erfc} \left( \frac{n}{\sqrt{\tau g}} \right) \), the first two terms of the expansion. Since \( \nu < 1 \), we can make use of the identities

\[ \sum_{n=1}^{\infty} \nu^n = \sum_{n=0}^{\infty} \nu^n - 1 = \nu/(1 - \nu) \]

(2.38)

and

\[ \sum_{n=1}^{\infty} n\nu^n = \nu/(1 - \nu)^2 \]

(2.39)
to show that the appropriate form of Eq. (2.35) reduces to

$$\frac{\Delta T_f(t)}{\Delta T_f(\infty)} = 1 - \sqrt{\frac{\tau_f}{t}} \left(\frac{Q_f}{Q_s}\right).$$  

(2.40)

For a platinum-on-pyrex gauge, this last equation has the numerical form

$$\frac{\Delta T_f(t)}{\Delta T_f(\infty)} = 1 - 5.1 \sqrt{\frac{\tau_f}{t}}$$  

(2.41)

so that the time required for the response to reach 95% of its final magnitude is $t = 10^4 \tau_f$. $\tau_f$ is about 0.25 cm$^2$/sec for bulk platinum so a gauge 100 Å thick has a characteristic response time $\tau_g = 4 \times 10^{-6}$ μsec and requires 0.04 μsec to respond to 95% of the step value; the corresponding times for 10,000 Å (1 micron) gauge are 0.04 μsec and 400 μsec, respectively.

Clearly the gauge thickness can play a crucial role in the interpretation of the results of transient experiments such as those performed in a shock tube where the recording times may be limited to, say, 10-50 μsec, and gauge thicknesses have typically been in the range 500-5000 Å. We mention, as an example, measurements of the temperature-step height on a shock-tube end wall to determine the thermal conductivity of the hot gas, (Refs. 3, 9, 10, and 11). To the present author's knowledge, workers using this method to measure thermal conductivity have not previously accounted for finite gauge thickness when reducing their data. For the accommodation coefficient experiments to be described in following sections, the thin-film thickness is even more critical, since we are interested in making definitive end-wall temperature measurements for times of the order of 1 μsec and less following shock reflection.

It is worth noting that an analysis dealing with the response of a finite-thickness, thin-film thermometer in the separate case of a constant heat flux has been presented by Kurzrock (Ref. 12).

3. THIN-FILM-GAUGE RESPONSE WITH A TEMPERATURE JUMP

In the previous section we calculated the response of a finite-thickness, thin-film resistance thermometer mounted on a shock-tube end wall and exposed to a gas suddenly heated by shock-wave reflection. The solution obtained made use of the boundary condition that the temperature of the gas and the thin film are equal at their interface. In fact the large temperature gradient in the gas immediately adjacent to the end wall is, at least for short times following shock reflection, inconsistent with a continuum gas model. An improved boundary condition, allowing for a "jump" in temperature at the gas-solid interface, can be formulated from a consideration of the microscopic nature of the interaction between gas and solid-material molecules which occurs in the gas layer next to the end wall.

Theories developed to describe the microscopic behaviour within this
so-called Knudsen layer have relied heavily on the concept of the accommodation coefficient, which is designed to express the efficiency of energy transfer between a gas and its (solid or liquid) boundary. By definition, the accommodation coefficient $\alpha$ is specified by (Refs. 4 and 5)

$$\alpha = \frac{(\epsilon' - \epsilon_m)}{(\epsilon' - \epsilon_w)}$$  \hfill (3.1)

where $\epsilon'$ is the average energy per molecule for incoming particles which strike the wall, $\epsilon_m$ is the average energy per molecule for those particles re-emitted by the wall and $\epsilon_w$ is the average energy which molecules would have issuing from a gas stream at a temperature equal to the wall temperature $T_w$.

This definition suggests the following limiting values for $\alpha$:

$\alpha = 1$ if $\epsilon_m = \epsilon_w$, which is the limiting case if the molecules re-emitted by the wall are completely accommodated to the wall temperature $T_w$; and at the opposite extreme, $\alpha = 0$ if $\epsilon_m = \epsilon'$, i.e., if the molecules are re-emitted with their initial, incoming energy. Since the average energy of the gas particles at the wall is simply $\frac{1}{2}(\epsilon'_m + \epsilon') = \epsilon_a$, say, it is clear that there will always be a discontinuity in energy between gas and solid-material molecules, i.e., between $\epsilon_a$ and $\epsilon_w$ unless $\alpha = \frac{2}{3}$, which is not physically acceptable. Thus a temperature (energy) jump exists at the gas-solid interface (whenever heat is being conducted) for all possible values of $\alpha$.

In a steady-state situation the number of incoming molecules per unit time is balanced by the number of re-emitted molecules, so the limiting case $\alpha = 0$ corresponds to an energy-transfer rate of zero between gas and solid, while the case $\alpha = 1$ corresponds to the maximum energy-transfer rate for fixed values of $\epsilon_w$ and $\epsilon'$. Clearly the actual value of the accommodation coefficient is intimately connected with the rate of energy transfer as well as the magnitude of the temperature jump at the gas-solid interface.

Extending the ideas introduced above, one can show (see Appendix A or Kennard, Ref. 5) that the appropriate temperature-jump boundary condition at the gas-solid interface is, for a steady or quasi-steady situation,

$$T_g - T_w = g\frac{\partial T}{\partial x}$$ \hfill (3.2)

where $T_w$ is, as before, the wall surface temperature, $T_g$ is the temperature of the gas at the wall, and $g$ is the "temperature-jump distance" given by

$$g = \frac{\left(\frac{2}{3} \left(\frac{2}{\gamma + 1}\right) \left(\frac{9\gamma - 5}{4}\right)\right)}{L}$$ \hfill (3.3)

$\partial T/\partial x$ represents the temperature gradient (normal to the wall) in the gas a few mean paths away from the wall. In Eq. (3.3), $\gamma$ is the ratio of the specific heats and $L$ is the collisional mean free path.

(N.B. The continuum limit of no temperature jump employed in section 2 is recovered by letting the mean free path shrink to zero.)
Lagrangian coordinate system, and the notation of section 2, Eq. (3.2) becomes

\[ T_g(\psi = 0, t) - T_f(x = 0, t) = \left[ \rho g \frac{\partial T_g}{\partial \psi} \right] \psi = 0. \tag{3.4} \]

The quantity \( \rho L \) is very nearly constant apart from a slight dependence on temperature, so that we propose to fix its magnitude, for each case of shock reflection, at a value corresponding to the average gas temperature at the wall. We have little to guide us regarding the constancy or otherwise of \( \alpha \) for the present circumstances, but previous work at lower pressures and with different techniques has not found a strong dependence on temperature. Therefore we propose to take \( \alpha \) as a constant as well, also evaluated at the average gas temperature. To emphasize these assumptions we write

\[ \rho g = (\rho g)_{av} = \text{constant} \tag{3.5} \]

so that Eq. (3.4) becomes

\[ T_g(\psi = 0, t) - T_f(x = 0, t) = (\rho g)_{av} \left( \frac{\partial T_g}{\partial \psi} \right) \psi = 0. \tag{3.6} \]

Anticipating the results ahead, it is convenient to introduce a characteristic accommodation time \( \tau_a \),

\[ \tau_a = (\rho g)_{av}^2 \beta = \left( \frac{2 - \alpha}{\alpha} \right)^2 \left( \frac{2}{\gamma + 1} \right)^2 \left( \frac{9\gamma - 5}{4} \right) 2\gamma \tau_c, \tag{3.7} \]

where \( \tau_c \) is the collision time interval in the gas evaluated at the average gas temperature, i.e.

\[ \tau_c = (L/\bar{c})_{av} \tag{3.8} \]

and \( \bar{c} \) is the mean molecular speed in the gas.

(See Appendix B for details of the derivation leading to Eq. (3.7).)

We can now proceed with the solution for the response of a finite-thickness resistance thermometer, properly accounting for accommodation and temperature-jump effects. The solution is accomplished in the same manner as that presented in section 2 with the substitution of Eq. (3.6) for Eq. (2.14); the other boundary conditions remain the same.

We find, for the Laplace transform of the film temperature,

\[ \mathcal{T}_f - T_o/z = \frac{D(z)}{2} \left\{ (1 + Q_{fs}) \exp \left[ -\sqrt{z/\kappa_s} \delta + \sqrt{z/\kappa_f} \delta + \sqrt{z/\kappa_T} x \right] + (1 - Q_{fs}) \exp \left[ -\sqrt{z/\kappa_s} \delta - \sqrt{z/\kappa_f} \delta - \sqrt{z/\kappa_T} x \right] \right\} \tag{3.9} \]
where \( D(z) = \)

\[
\frac{T_\infty - T_0}{z} \left\{ \left( \frac{1 + Q_{fs}}{2} \right) \exp \left[ -\sqrt{\frac{z}{\kappa_f}} \, \delta + \sqrt{\frac{z}{\kappa_f}} \, \delta \right] \left[ Q_{gf}(1 + \sqrt{\kappa_f}) + 1 \right] \right. \\
\left. - \left( \frac{1 - Q_{fs}}{2} \right) \exp \left[ -\sqrt{\frac{z}{\kappa_s}} \, \delta - \sqrt{\frac{z}{\kappa_s}} \, \delta \right] \left[ Q_{gf}(1 + \sqrt{\kappa_f}) - 1 \right] \right\}^{-1}
\]

(3.10)

Equation (3.9) can be simplified to yield

\[
\frac{T_f - T_0}{z} = \frac{T_\infty - T_0}{z} \left\{ \exp \left[ \sqrt{\frac{z}{\kappa_f}} \, x \right] + \nu \exp \left[ -\sqrt{\frac{x}{\kappa_f}} \, (x + 2\delta) \right] \right\}
\]

\[
\left[ Q_{gf}(1 + \sqrt{\kappa_f}) + 1 \right]^{-1} \left[ 1 - \nu \exp \left[ -\sqrt{\frac{x}{\kappa_f}} \, (x + 2\delta) \right] \right] \left[ Q_{gf}(1 + \sqrt{\kappa_f}) - 1 \right]^{-1}
\]

(3.11)

where, as in section 2,

\[
\nu = \frac{(1 - Q_{fs})}{(1 + Q_{fs})} < 1.
\]

(3.12)

Now \( Q_{gf} \gg 1, \) and \( \text{Re} \, (\sqrt{z}) > 0 \) for all points on the inversion contour in the complex plane, so we can now rewrite Eq. (3.11) in the series form

\[
\frac{T_f - T_0}{z} = \frac{T_\infty - T_0}{z} \left\{ \exp \left[ \sqrt{\frac{z}{\kappa_f}} \, x \right] \right\}
\]

\[
\left[ Q_{gf}(1 + \sqrt{\kappa_f}) + 1 \right]^{-1} \left[ 1 + \nu \exp \left[ -\sqrt{\frac{x}{\kappa_f}} \, (x + 2\delta) \right] \right] \left[ Q_{gf}(1 + \sqrt{\kappa_f}) - 1 \right]^{-1}
\]

(3.13)

We can simplify the solution to a more practical form by setting the bracketed term in the summation equal to unity, since \( |Q_{gf}(1 + \sqrt{\kappa_f})| \gg 1, \) and approximating \( [Q_{gf}(1 + \sqrt{\kappa_f}) + 1] \) with \( Q_{gf}(1 + \sqrt{\kappa_f}). \) For simplicity, we also take the solution at \( x = 0 \) as representative of the average temperature throughout the film for times greater than a few \( \tau. \)

The simplified transform for the film temperature is thus

\[
\frac{T_f - T_0}{z} = \left[ \frac{T - T_0}{Q_{gf}} \right] \left[ 1 + \nu \exp \left[ -2\sqrt{\frac{z}{\kappa_f}} \right] \right] \sum_{n=0}^{\infty} \nu^n \exp \left( -2\sqrt{\frac{z}{\kappa_f}} \right)
\]

(3.14)
which has the inverse transform (see Carlsaw and Jaeger, Ref. 8)

\[
\Delta T_f(t) = T_f - T_o = \frac{T_\infty - T_o}{Q_{gf}} \left\{ 1 - \exp\left(\frac{t}{\tau_a}\right) \text{erfc}\left(\frac{\sqrt{t}}{\sqrt{\tau_a}}\right) \right\} + \\
2 \sum_{n=1}^{\infty} \nu^n \left[ \text{erfc}\left(n\sqrt{\frac{g}{\tau_a}}\right) - \exp\left(2n\sqrt{\frac{\tau}{g}} + \frac{t}{\tau_a}\right) \text{erfc}\left(n\sqrt{\frac{g}{\tau_a}} + \sqrt{\frac{t}{\tau_a}}\right) \right]
\]

(3.15)

For the purpose of plotting the performance of a thin-film gauge, we cast the results into dimensionless form by dividing Eq. (3.15) by its own asymptotic, long-time limit, so that

\[
\frac{\Delta T_f(t)}{\Delta T_f(\infty)} = Q_{gs} \left[ 1 - \exp\left(\frac{t}{\tau_a}\right) \text{erfc}\left(\frac{\sqrt{t}}{\sqrt{\tau_a}}\right) \right] + 2 \sum_{n=1}^{\infty} \nu^n \left[ \text{erfc}\left(n\sqrt{\frac{g}{\tau_a}}\right) - \exp\left(2n\sqrt{\frac{\tau}{g}} + \frac{t}{\tau_a}\right) \text{erfc}\left(n\sqrt{\frac{g}{\tau_a}} + \sqrt{\frac{t}{\tau_a}}\right) \right]
\]

(3.16)

We can observe that this solution has the expected behaviour in certain limiting circumstances. For example, in the limit as \(\sqrt{\tau_a} \to 0\), i.e., as the accommodation time goes to zero while the gauge-response time remains finite, we recover the result of section 2 for no temperature jump, Eq. (2.35). In the limit as \(\tau \to 0\), i.e., as the gauge-response time goes to zero while the accommodation time remains finite, we recover

\[
\left[ \frac{\Delta T_f(t)}{\Delta T_f(\infty)} \right]_{\sqrt{\tau_a} \to 0} = \left[ 1 - \exp\left(\frac{t}{\tau_a}\right) \text{erfc}\left(\frac{\sqrt{t}}{\sqrt{\tau_a}}\right) \right]
\]

(3.17)

which is the same result originally found by Busing and Clarke (Ref. 3) and used to reduce their data for the accommodation of air on platinum.

For small values of time, i.e., as \(\frac{t}{\sqrt{\tau_a}} \to 0, \frac{t}{\sqrt{g}} \to 0\), we obtain

\[
\left[ \frac{\Delta T_f(t)}{\Delta T_f(\infty)} \right]_{\sqrt{t/\tau_a}, \sqrt{t/g} \to 0} = 0
\]

(3.18)

which is the proper behaviour whenever surface accommodation effects are present, since the wall requires a finite time to adjust to the changed state of the gas. For very large values of time, as \(\sqrt{t/\tau_a}, \sqrt{t/g} \to \infty\), we recover the necessary result

\[
\left[ \frac{\Delta T_f(t)}{\Delta T_f(\infty)} \right]_{\sqrt{t/\tau_a}, \sqrt{t/g} \to \infty} = 1
\]

(3.19)
This last result also confirms that a step response is obtained in the limit of zero gauge-response and accommodation times.

Calculated results based on Eq. (3.16) are presented in Figs. 2 and 3 for a platinum-on-pyrex gauge ($Q_{gs} = 0.11$). Figure 2 illustrates the influence of increasing the characteristic accommodation time for a fixed thin-film gauge (i.e., fixed $\tau_a$). Note that the limiting case $\tau_a/\tau_g = 0$ corresponds to the result plotted in Fig. 1 for which accommodation effects were absent. Accommodation effects have little effect on the gauge response for $\tau_a/\tau_g < 1$, and only for $\tau_a/\tau_g > 100$ is accommodation responsible for a larger perturbation to the ideal step response than the gauge thickness. For experiments intended to measure $\tau_a$, we clearly should arrange conditions such that $\tau_a/\tau_g > 1000$ in order to minimize the importance of the gauge thickness and hence optimize the accuracy of the measurement.

Figure 3 presents the same results, but plotted as a function of the dimensionless time $(t/\tau_a)^{1/2}$. In this case we observe the influence of changing the gauge response time for a fixed accommodation time. Again it is clear that we should arrange for $\tau_a/\tau_g$ to be as large as possible in order to emphasize the influence of accommodation rather than gauge-thickness effects.

The changed shape of the curve for $\tau_a/\tau_g = 0.1$ in Fig. 3, at values of $(t/\tau_a)^{1/2} \leq 4$, is due to the use of the surface temperature for the average gauge temperature at times $t$ less than $\tau_g$.

We can provide a useful approximate form of Eq. (3.16) in the following way. We first expand $\text{erfc} \sqrt{t/\tau_a}$ in a form valid for $\sqrt{t/\tau_a} \gg 1$, (Ref. 8),

$$\text{erfc} \sqrt{t/\tau_a} = \exp(-t/\tau_a) \left\{ (t/\tau_a)^{1/2} - (1/2)(t/\tau_a)^{-3/2} + \ldots \right\} / \sqrt{\pi},$$

and retaining terms to order $t^{-1/2}$ we find

$$\exp(t/\tau_a)\text{erfc} \sqrt{t/\tau_a} = \sqrt{\tau_a/t} / \sqrt{\pi}. \quad (3.21)$$

Similarly we expand the term $\text{erfc} n\sqrt{\tau_g/t}$ for $n\sqrt{\tau_g/t} \ll 1$, and retaining terms to order $t^{-1/2}$ we have

$$\text{erfc} n\sqrt{\tau_g/t} = 1 - (2n/\sqrt{\pi})\sqrt{\tau_g/t}. \quad (3.22)$$

Finally we expand to order $t^{-1/2}$ the last term in Eq. (3.16) to find

$$\exp(2n\sqrt{\tau_g/t} + t/\tau_a)\text{erfc}(n\sqrt{\tau_g/t} + \sqrt{t/\tau_a}) = \sqrt{\tau_a/t} / \sqrt{\pi}. \quad (3.23)$$

* It can be shown that truncation of the series in (3.15) at an $n$ which still allows this condition to be met does not lead to significant error in the final result.
Substituting the results of Eqs. (3.21) to 3.23) in Eq. (3.16), we obtain, for 
\[ T / T_a \gg 1, \]
\[ \Delta T'(t) / \Delta T'_{\infty} = 1 - \frac{\sqrt{\tau_g / t}}{\sqrt{\tau_a}} \frac{1}{\sqrt{\tau_a}} \frac{1}{\sqrt{\tau_g}} \left(1 - \frac{Q_{fs}}{Q_{fs}'} \right), \]
(3.24)
or more approximately
\[ \Delta T'(t) / \Delta T'_{\infty} = 1 - \left(\frac{\sqrt{\tau_g / t}}{\sqrt{\tau_a}} \right) \left(\frac{\sqrt{\tau_a}}{\sqrt{\tau_g}} + 1 / Q_{fs}'\right). \]
(3.25)

This approximate form of the response is quite useful for reducing the data of shock-tube experiments as will be seen in the next section. In addition it clearly exhibits the relative importance of gauge-thickness and accommodation effects.

We can say a few more words about the ratio of characteristic times 
\[ \tau_a / \tau_g \]
which plays a critical role in the design of a proper shock-tube experiment. For a given gas-solid combination, and hence fixed values of \( \alpha, \kappa_f \) and \( \gamma \), the ratio \( \tau_a / \tau_g \) is proportional to \( \tau_c / \delta \). Clearly then we wish to make \( \delta \), the film thickness, as small as possible, and at the same time we will want to adjust \( \tau_c \) to be as large as is practically reasonable. The minimum value of \( \delta \) depends on the process used to construct the film (see section 4), but present deposition techniques are capable of producing films as thin as 100-1000 \( \AA \). The practical upper limit for \( \tau_c \) depends strongly on the size of the shock tube used since low pressures are the source of several non-ideal effects in shock tubes and we wish to avoid these if possible. Generally, the larger the shock tube, the lower the pressure level one can satisfactorily use and still obtain a well-defined, uniform slug of test gas behind the reflected shock wave.

Values for \( p_T, \tau_c \), the product of the initial shock-tube pressure and the collision time interval in the gas (evaluated at the average temperature, \( T_{av} = \frac{1}{2}(T_{\infty} + T_o) \)) are presented in Fig. 4 for a variety of gases. Of course the results depend strongly on the strength of the shock wave employed, but somewhat surprisingly there are not great differences between the various gases at the same primary-shock Mach number, \( M_s \). The details of these calculations are presented in Appendix C.

With a shock tube a few inches in diameter, a reasonable lower limit for the initial pressure \( p_i \) is about 1 torr, so the collision time interval \( \tau_c \) will be in the range of 10\(^{-8}\) seconds for shock strengths of about \( M_c = 2-3 \). For a platinum gauge with \( \kappa_f \approx 0.25 \text{ cm}^2 / \text{sec} \) and a diatomic gas with \( \gamma = 7/5 \), we have the ratio
\[ \tau_a / \tau_g = \left( \frac{2 - \alpha}{\alpha} \right)^2 \left( \frac{3.7 \tau_c}{4 \delta^2} \right) \approx \left( \frac{2 - \alpha}{\alpha} \right)^2 \tau_c / \delta^2 \]
(3.26)
(where \( \delta \) is in cm). Thus, for a 100 \( \AA \) gauge and an initial pressure of 1 torr, we find

\[
\frac{\tau_a}{\tau_g} = \left(\frac{2 - \alpha}{\alpha}\right)^2 10^4
\]

(3.27)

which is quite an acceptable time ratio, even for values of \( \alpha \) near unity.

For a thicker gauge of 1000 \( \AA \), and the same initial pressure,

\[
\frac{\tau_a}{\tau_g} = \left(\frac{2 - \alpha}{\alpha}\right)^2 100
\]

(3.28)

and we have only a marginally acceptable experiment unless \( \left(\frac{2 - \alpha}{\alpha}\right)^2 > 10 \).

At higher pressures, say 100 torr, we have \( \tau_c = 10^{-10} \) sec and

\[
\frac{\tau_a}{\tau_g} = \left(\frac{2 - \alpha}{\alpha}\right)^2 100
\]

(3.29)

for a 100 \( \AA \) gauge, and we again have a marginal situation unless \( \left(\frac{2 - \alpha}{\alpha}\right)^2 > 10 \). Tests with the thicker gauge would be generally unsatisfactory at this pressure level. The flexibility of the thinner gauge for operating over a range of shock conditions (and particularly for working at higher pressure levels where the shock-tube performance is guaranteed) is obvious.

Before closing this section, it is worth remembering that the shock-tube technique is restricted (with the present theory) to values of \( \tau_c > 10 \), in order to avoid shock-structure effects. For cases in which \( p_1 \tau_c = 10 \) torr-nsec, say, we thus can only make use of data acquired after at least

\[
t = 10 \tau_c = 10^{-1} \mu \text{sec}/p_1
\]

(3.30)

but this is not much of a restriction since the times involved are only 0.001 \( \mu \)sec at 100 torr and 0.1 \( \mu \)sec at 1 torr.

It is also worth recalling that one basic assumption of the theory is that the pressure remains constant after shock reflection; this was shown to be true to first-order accuracy (Refs. 6 and 7) for the case of no temperature jump, but the assumption is more accurate yet when a temperature jump is present (Ref. 16), and especially for small values of \( \alpha \), since the heat-transfer rate is decreased and hence the extent of any perturbations to the flow field. In the limit that \( \alpha = 0 \) of course there is no heat transfer to the wall, the gas remains motionless and the pressure is exactly constant.

4. THE EXPERIMENT

The experiment made use of shock-wave reflection from the closed end of a shock tube filled with nitrogen in order to create a one-dimensional, unsteady heat-conduction situation between a hot gas and a cold solid wall. The efficiency of energy transfer between the hot gas and a thin metal film mounted on the (electrically nonconducting) end wall was then inferred by monitoring the temperature-time history of the film, the latter being obtained by passing a current through the film and using it as a resistance thermometer. The result
of each experiment was a value for the thermal accommodation coefficient at
the pressure level produced by the shock-reflection process.

4.1 Experimental Details

The shock tube used for these tests was a simple pressure-driven
tube made of brass waveguide tubing with 3.1 cm by 7.1 cm inside dimensions.
The high-pressure and test sections were, respectively, 2 m and 4 m long.
The diaphragm material was Melinex, with a thickness of 0.023 mm, and the
diaphragms were burst with a manually operated needle plunger. The normal
sequence of operation consisted of reducing the pressure in the test chamber
to less than 0.1 torr with a mechanical vacuum pump and then purging the
system with about 100 torr of N₂ from a high-pressure cylinder of commercial-
grade nitrogen (British Oxygen Co. Ltd.). After re-evacuation, the tube was
filled to the desired final pressure as read on a Wallace and Tiernan (0-20,
or 0-800 torr) pressure gauge. The initial gas temperature was taken as
the temperature of the tube wall near the end plate, and this was monitored
with a standard mercury thermometer.

The primary-shock Mach numbers ranged from 1.3 to 2.7, and the
initial tube pressure varied from 1 to 200 torr, the lower Mach numbers
occurring with the higher initial pressures. The shock velocity was measured
just upstream of the reflecting end plate by detecting the shock passage over
two thin-film resistance thermometers set flush with the side wall of the tube
and spaced 24 cm apart. The detector outputs triggered transistorized pulse
generators and these pulses were used to start and stop a digital chronometer
which recorded the time interval to ±0.1 μsec.

A resistance thermometer operates on the principle that the electrical
resistance of most metals changes with temperature. Thus if an electrical
current is passed through the thin-film element any change in temperature
will manifest itself as a change in the voltage drop across the gauge. The
magnitude of the temperature change can thus be inferred from a resistance-
temperature calibration of the thin film and measured values for the gauge
current and voltage.

The electrical circuit employed in the present study is shown in
Fig. 5. \( R_g \) is the gauge resistance, \( R_L \) is the load resistance and \( E \) is
the D.C. supply voltage. The voltage across the gauge prior to shock
reflection is

\[
V_g = \left(\frac{R_g}{R_g + R_L}\right) E
\] (4.1)

For the small change in gauge resistance induced by heat transfer from the
shock-heated gas one can show that the change in gauge voltage is given by

\[
\frac{\Delta V_g}{V_g} = \left(\frac{\Delta R_g}{R_g} + \frac{R_L}{1 + R_g/\Delta R_g}\right)
\] (4.2)

Over a reasonable range of temperature, the change in resistance of the
metal film is specified sufficiently well by the linear relation
AR = R \lambda (T - T_o) = R \lambda \Delta T \tag{4.3}

where \( \lambda \) is the resistance-temperature coefficient and \( T_o \) refers to the initial gauge temperature. Therefore we can write, for the response of a gauge in the circuit of Fig. 5,

\[
\frac{\Delta V}{V} = \frac{\Delta T}{(1 + \frac{R}{R_L})}, \tag{4.4}
\]

The sensitivity of such a gauge is simply

\[
\frac{\Delta V}{\Delta T} = \frac{\lambda V}{(1 + \frac{R}{R_L})}, \tag{4.5}
\]

and this quantity can be varied to suit the requirements of the experiment by adjusting the voltage across the gauge, by changing the ratio \( \frac{R}{R_L} \), or by using a material with a different value of \( \lambda \).

While the gauge response depends on \( V \) directly, the power dissipated by the gauge in the form of heat is proportional to \( \frac{V^2}{R} \), and this should not be too large in order to avoid undesirable heating of the substrate. Thus it is advantageous to operate thin-film thermometers with resistance and low current.

Another factor which is affected by the gauge resistance is the electronic response time of the circuitry, and we were especially concerned with that here. To minimize the response time we needed to minimize the product \( RC \) where \( R \) was the equivalent resistance of \( R_g \) and \( R_L \), and 

\[
C = \text{input capacitance of the recording oscilloscope plus any parasitic capacitance},
\]

Thus, from the viewpoint of electronic response time, a small gauge resistance was desired.

A further practical restriction on the gauge resistance employed in the present study was imposed by the desire for a thin gauge, since the gauge resistance increases rapidly with decreasing gauge thickness (for a given material and geometry).

The optimum value for the gauge resistance was thus a compromise which depended upon the particular requirements of accommodation-coefficient experiments. The gauges selected had resistances varying from 220 to 800 ohms, and these values are considerably larger than those normally used in shock-tube experimentation. A consequence of using large gauge resistances was that the circuit could not (with the power supply available) be operated in the usual constant-current mode, so the factor \( 1 + \frac{R_g}{R_L} \) in the equations above was not set equal to 1 as is usually done. In fact, \( \frac{R_g}{R_L} \) was chosen to be equal to 1/5 in the circuit for the chemically deposited gauge and 1/2 for the vacuum-sputtered gauge.

Special care was taken to reduce the circuit capacitance by using
short (12 cm), low-capacitance coaxial cables (25 pF/m) between the gauge circuit and the recording oscilloscope. Further, a wide-bandwidth oscilloscope (Tektronix 556) and differential-amplifier plug-in unit (Tektronix 1A5, AC coupled) were employed to retain the high-frequency quality of the input signals. With this arrangement, the response time with the 800 ohm gauge was measured (using the same recording) circuit and a high-quality square wave for an input signal) to be less than 10 nsec (0 to 63% response). The circuit response times were correspondingly less with lower gauge resistances, and so electronic lag time was completely neglected in the analysis of accommodation data acquired with all three gauges.

4.2 Gauge Construction

Three different thin-film thermometers were used in this study, each constructed by a different technique. The first gauge was made by the same method employed by Busing and Clarke (Ref. 3), chemical deposition. This technique is well documented and will only be outlined here. In brief, a thin line about 10 mm long by 0.4 mm wide of liquid bright platinum paint (Hanovia 05-X) was applied with a draughting pen onto a circular disc of hard glass 5 mm thick and 50 mm in diameter. After a brief interval of air drying, the glass was placed in a small electric furnace and heated to its softening temperature (about 680°C for pyrex glass), and then allowed to soak at this temperature for 5 minutes. The gauge was then removed from the furnace and allowed to cool in the open air. Current leads were formed by painting strips of silver paste (Hanovia silver paste No. 38 thinned with white spirit) from the ends of the platinum to the edge of the glass and then reheating the glass to its softening temperature. Shielded copper wires were soldered to the ends of the silver and the entire glass disc was mounted flush in a steel plate with the leads passing through small holes drilled in the plate. Vacuum wax was used as a seal. This steel plate was then used as the shock-tube end wall, taking care to ensure that the gauge surface was normal to the geometrical axis of the tube.

Several gauges were constructed by this technique*, the thinnest being finally selected for shock-tube testing. Measurements of its thickness distribution were obtained with a Talysurf† surface-finish measuring device, and a sketch of one reasonably typical result is presented in Fig. 6a. The thickness distribution exhibited is roughly parabolic, but records obtained at different positions along the film axis showed occasional random "holes" and "spikes" superposed on the basic distribution, so the film does not have a uniform thickness distribution along its axis. Allowing for some interpenetration of glass and metal at their interface, the average film thickness was estimated to be about 500 Å.

The temperature coefficient of the film was determined by passing

* The author is grateful to V. Marshall and J. Lewak for assistance in constructing these chemically deposited gauges.
† Taylor-Hobson Ltd., England.
a stabilized 1 mA current through the gauge and measuring the voltage across the gauge at different gauge temperatures with a digital voltmeter (10 µV resolution). To stabilize the gauge temperature, the gauge was immersed in a small volume of silicone oil and heated in a water bath. The silicone oil had no observable short or long-term effect on the gauge resistance. Over a temperature range of about 20°C, the coefficient was 2.3 × 10⁻² °C⁻¹. Although this numerical value was not needed for reducing the accommodation-coefficient data, it was important for purposes of comparing the sensitivity of the three different thin-film gauges. The gauge resistance was 220 ohms.

Chemically deposited gauges are extremely robust but have three important disadvantages for use in accommodation-coefficient studies: (1) the minimum film thickness is still rather large, (2) the film thickness is not uniform and cannot be accurately determined because of the glass-metal matrix formed during the bonding process, and (3) the film material is not pure platinum. For these reasons it was decided to construct a second gauge with improved characteristics making use of vacuum-evaporation techniques.

Vacuum evaporation is a well established method for depositing metal on glass, but it is not always a simple matter to obtain a robust film of the desired material and thickness. Platinum is a particularly difficult material to deposit because of its high evaporation temperature, but after considerable effort a number of quite satisfactory gauges were constructed. The major difficulty was obtaining a film with suitable adherence to the glass substrate. This problem was eventually solved using a vacuum chamber equipped with an electron-beam heating unit. With this apparatus, a molybdenum filament was used as a ring cathode, held at about -4 kV, and a carbon crucible filled with platinum was used as the anode, held at earth potential. The filament was heated with a high current (about 30 amps) until it emitted electrons which were accelerated and focussed onto the platinum, thus heating it to the proper evaporation temperature. The glass substrate, appropriately masked, was held about 15 cm above the crucible. A thin quartz crystal, vibrating in shear mode, was mounted at about the same distance from the crucible so that the glass substrate and the quartz crystal received the same evaporated-layer thickness and the changing frequency of the crystal could be monitored to infer the film thickness.

It is well known that as the thickness of a vacuum-evaporated metal film is reduced below a certain limit its resistivity begins to increase from the usual bulk-material value. This limit is about 1000 Å. In addition to the increasing resistivity, the temperature coefficient of resistance, \( \lambda \), which is normally positive, begins to decrease. At a thickness normally in the range of 100 Å the coefficient \( \lambda \) actually turns negative. The thinness of a temperature sensor constructed by this technique is thus usually limited by the minimum value of \( \lambda \) acceptable, i.e., by the minimum acceptable gauge sensitivity. The reason for the variation in \( \lambda \) has to do with changes in the mode of electrical conduction which occur in very thin layers. Since

* Bir-Vac, electron-bombardment source RG2; Swindon Road, Cheltenham, England.
Electron mean free paths in bulk metals may be in the range 400-800 Å at room temperature, it is easy to appreciate that surface effects will act to decrease the effective mean free path in films less than 1000 Å thick. In addition, for films less than, say, 100 Å, the material structure is likely to be quite different than that of the bulk material and probably resembles aggregates of atoms or crystallites slightly separated on the glass substrate. Electrons might then move by a type of tunneling which would explain the observed negative values for λ.

Previous construction of thin-film resistance thermometers has made use of films with positive values of λ, but it was decided here, because of the importance of obtaining a very thin film, to attempt to build a gauge with a negative value of λ. The results of the initial attempts reported here are both encouraging and discouraging. A number of gauges with negative λ were actually constructed, and the values of λ were in some cases larger in magnitude than the value for the bulk material, but it was not possible to reproduce given gauge characteristics with any degree of reliability. Furthermore, the thickness of the gauge with the most-negative λ value was much greater than expected. These irregularities can probably be explained by the fact that the character of an evaporated film depends on several factors which were not rigidly controlled or monitored in our initial experiments. Amongst these factors we may mention: the nature of the substrate, and its previous treatment and cleanliness; the temperature of the substrate during and following deposition; the distance of the substrate from the source; the rate of deposition; the ability of defects in the film to diffuse or anneal out of the film at the temperature of the glass during and following deposition; the amount of oxygen or other impurities in the vacuum chamber and in the substrate itself.

The gauge finally selected for use in shock-tube tests was constructed as follows. First the glass, a pyrex disc 5 mm thick by 50 mm in diameter was thoroughly cleaned with isopropyl alcohol and cotton and mounted in the vacuum chamber. After evacuating the chamber to about 10^-6 torr, a 1 cm square of platinum was deposited onto the glass using the electron beam to heat the platinum source. The glass and mask were held about 15 cm from the source and the deposition rate maintained for one minute was about 50 Å/minute according to the crystal monitor. The glass was removed from the chamber after 30 minutes of cooling and its resistance was measured to be about 8000 ohms/square. Current leads 1 cm wide were then applied leading from opposite edges of the glass and approaching within ½ mm of each other. The resistive element was thus a strip of bare platinum ½ mm long (in the direction of current flow) by 10 mm wide, and the final resistance was 400 ohms. The current leads were formed by deposition a 10 Å layer of chrome followed by a 5000 Å layer of gold. The thin layer of chrome greatly improved the bond between the gold and the glass substrate. It was important that the vacuum was not broken between the deposition of chrome and gold in order to maintain the bond between the chrome and the glass, and this was accomplished with a mechanism which permitted a new crucible to be rotated into the heating position by external controls.

The thickness of the platinum layer was measured by the Talysurf and a sketch of the result is shown in Fig. 6b. Note the uniformity of the deposit compared with the chemically deposited film shown in Fig. 6a. This
is an important advantage for purposes of accommodation experiments since it is important to know the film thickness rather accurately and to present a uniform, smooth surface to the hot gas molecules undergoing thermal accommodation. The thickness of the evaporated gauge is, surprisingly, about 400 Å. A much thinner layer was expected on the basis of the crystal frequency readings, the resistance of the film and the optical transparency of the film. It is felt in retrospect that an oxide layer must have formed on the surface of the platinum and this was responsible for the unexpectedly large thickness. The measurement error introduced by the Talysurf is thought to be negligible.

The resistance-temperature coefficient of this gauge was measured (with the thermal-bath apparatus described earlier) to be \(-3.6 \times 10^{-3} \ ^{0}C^{-1}\). This is actually larger in magnitude than the bulk value of \(\lambda\) and hence represents a considerable improvement in sensitivity over previous chemically or vacuum-deposited platinum resistance thermometers. For purposes of the thermal-accommodation experiments, however, the present gauge was only a partial success, since it was only marginally thinner than the chemically deposited film and the surface of the film was suspected of being contaminated by oxygen.

In spite of its imperfections, the vacuum-evaporated gauge developed here furnishes evidence that the concept of a truly thin temperature sensor with a highly negative resistance-temperature coefficient is feasible, and this evidence together with the potential of such a device certainly justifies further research along these lines.

During the development stages of the "negative-\(\lambda\)" gauge, contact was established with the Fluid Dynamics Group of the Physics Department of Eindhoven University, The Netherlands, where similar work was in progress. Their efforts were directed towards the development of thin vacuum-sputtered films, also of platinum, and Mr. M. van Dongen of that group graciously loaned one of their completed gauges for use in the present study. Thus three thin-film gauges, each constructed by an entirely different technique, were available for shock-tube testing aimed at measuring the thermal accommodation coefficient of \(N_2\) on platinum.

The Eindhoven sputtered gauge was made with the cathode sputtering technique, which involved depositing the film by means of an electrical discharge between an anode and a platinum cathode. The process was carried out with a pyrex disc (10 mm thick by 20 mm diameter) appropriately masked and mounted in a chamber filled with about 0.03 torr argon. The gauge was then baked in an atmosphere of hydrogen at 400 \(^{0}\)C for half an hour. The final resistance element of the Eindhoven gauge was a platinum strip 1 mm wide and 15 mm long with silver paste used to connect the platinum to copper leads buried in the sides of the pyrex disc. The element resistance was 800 ohms. The resistance-temperature coefficient was measured by the Eindhoven group to be about \(1.0 \times 10^{-3} \ ^{0}C^{-1}\), which is rather small, but this value was acceptable in view of the primary objective of constructing the thinnest usable gauge. A sketch of the thickness distribution measured with the Talysurf is shown in Fig. 6a. An accurate estimate of the thickness is made difficult by the roughness of the pyrex
surface but an approximate thickness is 225 Å. This gauge was thus thinner than either the chemically deposited gauge or the "negative-λ" evaporated gauge. Unfortunately, the present sputtered gauge suffered from the disadvantages of reduced sensitivity and non-uniform surface finish, although it should be possible to construct a similar gauge with improved surface finish using a highly polished pyrex disc for the substrate.

4.3 Results

A series of shock-tube tests was conducted with each of the three thin-film thermometers discussed above. The test was was nitrogen in all cases. The ultimate objective was to obtain values for the accommodation coefficient representative of each gauge material over a range of gas pressures, and this was accomplished by using different combinations of initial pressure and shock-wave Mach number. The electronic circuitry was the same for all the tests except that the load resistors were different for each gauge and the value of the supply voltage was varied from run to run in order to provide the desired amplitude for the gauge response.

The influence of changing the pressure level is apparent in Fig. 7 which presents the results of three experiments conducted with the chemically deposited gauge. The oscilloscope traces record the change in the voltage drop across the gauge with time, and this is directly equivalent to a record of the change in the average gauge temperature versus time. (Time increases from left to right on all traces.) Records are presented with two sweep speeds for each experiment, the fast sweep being useful for resolving the initial transients while the slow sweep is useful for estimating the final, asymptotic signal amplitude. In addition, the slow sweep record provides a convenient check on the performance of the shock tube and recording apparatus since theory predicts the signal should asymptote to a constant value while non-ideal effects, such as unaccounted-for gas dynamic influences, would have altered this result. The individual records at each test condition were obtained by repeating the experiment rather than by recording the result of a single run simultaneously with two different oscilloscope plug-in units as this latter method would have increased the circuit response time. Differences in the amplitude of the fast and slow-sweep signals for the same test conditions were due to small variations in the power supply voltage.

The primary observation to be made from Fig. 7 is that the character of the signal becomes more like a step at higher pressures, and this is in agreement with the theory (see Section 3) which says that the characteristic accommodation time is proportional to the collision time interval in the gas. Increasing the pressure level decreases the average collision time interval in the gas and hence the characteristic accommodation time. Ultimately, as the pressure level is increased enough, accommodation effects may be neglected and the deviation from a perfect step response is due exclusively to finite gauge thickness. In this limiting circumstance we recover the continuum result discussed in Section 2 in which no temperature jump is present at the gas-solid interface.

Results obtained with the vacuum-evaporated gauge at similar conditions are presented in Fig. 8. The signal is negative-going rather
than positive-going for these experiments because the temperature coefficient of resistance is negative for this gauge. Otherwise, the results are similar to those presented in Fig. 7 except that the characteristic response times exhibited by the vacuum-evaporated gauge are somewhat less owing to a smaller value of film thickness.

Results obtained with the vacuum-sputtered gauge are shown in Fig. 9. The influence of gauge thickness is now more apparent and is particularly obvious if one compares the results shown in Figs. 7c and 9c. Note also that the signal amplitudes are smaller in Fig. 9 than in Figs 7 and 8 owing to the reduced sensitivity of the sputtered gauge.

The results presented in Figs. 7-9 provide confidence in the shock-tube technique as a means of obtaining transient-temperature records sensitive to the characteristic accommodation time in the gas. The signals are reasonably free from electrical noise and the oscilloscope traces are of good quality so that the accuracy of data extracted from these records should be relatively high. The constancy of the signal amplitudes for large times suggests that the shock tube was operating satisfactorily over this range of test conditions. In order to verify this last point, especially for those cases where the asymptotic signal level was not obvious, a number of records were also obtained with a 5 or 10 \( \mu \)sec/div sweep speed, and these invariably showed a suitable period of uniform temperature following the accommodation process. At low initial pressures this period of constant gauge temperature was of course much shorter, and a gradual increase in gauge temperature owing to side-wall boundary layer effects began to be apparent. Primarily for this reason, tests were not made at initial pressures less than 1 torr.

The task of inferring the characteristic accommodation time from the experimental records was accomplished by first reploting each of the records in the form shown in Fig. 10. In brief, the procedure followed was to estimate the asymptotic signal level, \( \Delta T_f(t) \), from the slow sweep-speed record and then to use this value to put the data scaled from the oscillograms into the dimensionless form \( \Delta T(t)/\Delta T_f(\infty) \). The dimensionless response was then subtracted from 1.0 and plotted versus \( 1/\sqrt{t} \) so that the gauge response at large times fell on a straight line passing through the origin. (Small corrections to the estimate of the asymptotic signal level were usually required to ensure that the line through the data did pass through the origin.) The choice of the nonlinear time scale was suggested by Eq. (3.25) which is an approximate result for the gauge response valid for large time. The slope of the asymptotic line through the data for large values of \( \Delta T_f(t)/\Delta T_f(\infty) \) was then used in conjunction with Eq. (3.25) to infer the value of the characteristic accommodation time \( \tau \). The value of \( \tau \) was calculated from the Talysurf measurements of the film thickness and the bulk-material value for the thermal diffusivity in platinum. The value of \( \tau \) inferred in this manner was then converted to a value for the accommodation coefficient \( \sigma \) using the definition provided in Eq. (3.7) together with the values of \( \tau_c \) plotted in Fig. 4.

The plot shown in Fig. 10 corresponds to the trace shown in Fig. 9b, but the character of the plot is representative of the other experiments as well. A point worth noting is that plotted data were generally restricted
to times greater than 0.1 \( \mu \text{sec} \) following shock reflection in order to avoid errors associated with finite circuit response time, any misalignment of the gauge, and shock-structure effects. Therefore at high pressures the maximum plotted value for the quantity \( 1 - \Delta T(t)/\Delta T(\infty) \) was substantially less than at lower pressures, and accordingly the accuracy for the slope of the line drawn through the data was poorer. Since the slope of the straight-line asymptote actually depends on two factors, the gauge-response time and the characteristic accommodation time, it is also clear that any error in the value taken for the gauge-response time affects the accuracy of the value inferred for the accommodation time. The most accurate values for \( \alpha \) are thus measured at low pressures, when the accommodation time is largest, and with a thin gauge so that the influence of the gauge-response time is minimized. The minimum pressure which can be used is fixed of course by the shock-tube performance, and this depends strongly on the size of the shock tube available and the growing influence of shock-structure effects at low pressures.

Also shown in Fig. 10 is a solid curve computed from the exact theoretical result, Eq. (3.16), using the value of \( \alpha \) inferred from the straight-line asymptote. The theoretical curve fits the data quite well for values of \( 1/\sqrt{T} \) less than about 2, (i.e., for values of \( t > 0.2 \mu \text{sec} \)), thereby justifying the use of the straight-line technique. In fact a computer program was written for calculating this curve for input values of \( T, T_* \) and \( Q_{\text{eq}} \) (see Eq. (3.16)), and this program was used to check the straight-line result of each experiment. Another check was provided by replotting the theoretical and experimental responses of the gauge in real time as shown in Fig. 11, again making use of the data shown in Fig. 9b. The agreement is clearly quite acceptable, and only for times less than 0.2 \( \mu \text{sec} \) are there any large differences between experiment and theory. Most of this discrepancy, also visible in Fig. 10, is almost certainly associated with shock-structure effects which represent an increasing problem at low pressure since the finite thicknesses of the incident and reflected shock waves (and hence the time required to compress and heat the gas to its reflected-shock value) are inversely proportional to the initial pressure. The distortion in the temperature-time histories introduced by finite shock thickness can be tolerated of course until it begins to affect the straight-line asymptote. Ultimately, however, this non-ideal effect does become important, thus providing yet another reason for not performing shock-tube accommodation experiments at very low pressures.

Final results for the accommodation coefficients found in the present study are plotted as a function of pressure in Fig. 12. The values appear to be nearly constant with pressure for each gauge, although a slight trend for decreasing \( \alpha \) with increasing pressure may be present. The scatter of the data precludes any quantitative statement on this point. There is also apparently little dependence of the accommodation coefficient on the average gas temperature since the experiments were run over a Mach number range of 1.3 to 2.7 which corresponds to a range of average gas temperatures from 360 to 750°K.

The similarity of the results for the three gauges leads to speculation that the surface finish and the physical structure of the thin film do not play a dominant role in determining \( \alpha \) over this range of pressure. This
hypothesis would be true, for example, if an adsorbed layer of gas was present on all the gauge surfaces so that the shock-heated gas was accommodated by collisions with adsorbed gas molecules rather than by collisions with platinum atoms. The results for the different gauges do differ enough, however, and with sufficient consistency, to conclude that the type of platinum film has some effect on the accommodation coefficient, even at high pressure.

Presumably, the most accurate data were obtained with the vacuum-sputtered and vacuum-evaporated gauges, since these gauges have the smallest gauge-response times, but roughly the same amount of scatter if present in the data corresponding to each of the three gauges. One would thus suspect that the scatter was due primarily to shock-tube irregularities and errors in auxiliary measurements such as shock speed, initial temperature and pressure, etc., rather than to error in the value taken as the gauge-response time.

The results of the present study are compared with the findings of previous investigations in Fig. 13. At the relatively high pressure levels employed here only the data of Busing and Clarke (Ref.3) are available for direct comparison, and their results are represented by means of a curve drawn with least-mean-square deviation through their data. The present data agree poorly with those of Busing and Clarke, but it is now known that the source of this disagreement was the neglect of finite gauge thickness in their data reduction scheme. Since their gauge was approximately 5000 Å thick, and their tests were conducted at rather high pressures, the dominant influence in their temperature-time histories was actually gauge thickness rather than accommodation effects. The present shock-tube data thus supersede those of Busing and Clarke.

The results presented by Wachmann (Ref.1) were obtained at much lower pressures and by an entirely different technique, so the agreement with the present findings can be regarded as quite good. As with nearly all thermal-accommodation data, these results were obtained with a conductivity cell. The technique involves measurements made with steady-state conditions of thermal accommodation, so the precision of the data and the lack of scatter are generally excellent. However, large differences between accommodation coefficients measured in different laboratories still exist. In fact, published data for the accommodation coefficient of N₂ on platinum obtained with this technique range from about 0.55-0.81, so the results shown in Fig. 13 are reasonably representative of previous conductivity-cell results. (See Wachmann Ref.1, Kennard Ref. 5, and Devienne Ref. 15, for a summary of such results and a description of the apparatus and techniques commonly employed at low pressures).

The present results obviously exhibit a larger scatter than the conductivity-cell data, but this is to be expected with measurements made on a microsecond time scale, and it should be possible to reduce this scatter somewhat in future work. In spite of the scatter, the results obtained here are quite useful in that they provide data at pressures not accessible with previous techniques. In addition, and perhaps more importantly, we have developed a theory for the influence of thermal accommodation in an nonsteady situation and have shown experimentally that the same accommodation coefficient holds for the the unsteady case as has been measured in steady circumstances.
5. CONCLUSIONS

An analysis has been presented for the temperature-time history of a thin-film resistance thermometer mounted on the end wall of a shock tube and exposed to a gas suddenly heated by shock-wave reflection. The response of such an instrument was shown to depend on both the finite thickness of the film and gas-solid thermal accommodation effects.

Shock-tube measurements with three different types of platinum thin-film gauges were presented and combined with the theoretical model to yield thermal accommodation coefficients for nitrogen on platinum. The results for the accommodation coefficient obtained with all three gauges agreed well with previous data available at lower pressures, thus establishing the shock-tube technique as a means of studying gas-solid interaction at conditions not accessible with existing methods. Further experiments are needed, however, to refine the technique and to determine the influence of phenomena such as absorption at typical shock-tube conditions.

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APPENDIX A

The Temperature-Jump Boundary Condition

The boundary condition for the discontinuity in temperature at the interface between a gas and a solid is generally written, for a steady situation, as (Ref. 5)

\[ \frac{T_w - T_g}{g} = g \frac{\partial T}{\partial x} \]  

(A-1)

where \( T_w \) is the wall surface temperature and \( T_g \) is the temperature of the gas at the wall assuming that the temperature gradient normal to the wall, \( \frac{\partial T}{\partial x} \), which applies a few mean free paths from the wall continues unchanged right up to the wall. The parameter \( g \) represents a characteristic length, the so-called 'temperature-jump distance'. The same boundary condition can be applied in a quasi-steady sense as long as the temperature variations occur over a time scale greater than several collision times in the gas.

Theoretical expressions for the relation between the accommodation coefficient \( \alpha \) and the temperature-jump distance \( g \) have appeared in several forms, but a simple and convenient expression can be derived with the approach suggested by Clarke and McChesney (Ref. 4), as follows.

On the basis of simple kinetic theory, molecules which strike the surface come from a distance which is, on the average, about one mean free path away, say \( \epsilon' \). Immediately adjacent to the wall we then have, in a steady-state situation, molecules just arriving from a distance \( \epsilon' \) above the wall, bringing with them energy \( \epsilon' \) per molecule, and an equal number of molecules just emitted from the solid with energy \( \epsilon_m \). Consequently the average energy in this layer is \( \frac{1}{2}(\epsilon' + \epsilon_m) = \epsilon' \), say. Now if the energy of a molecule at the temperature of the wall is \( \epsilon_w \), we have an energy jump at the wall of

\[ \Delta \epsilon = \epsilon_a - \epsilon_w. \]  

(A-2)

Since the accommodation coefficient is defined as (Refs. 4 and 5)

\[ \alpha = (\epsilon' - \epsilon_m)/(\epsilon' - \epsilon_w), \]  

(A-3)

the energy jump may also be written in the form

\[ \Delta \epsilon = \frac{2 - \alpha}{\alpha} (\epsilon' - \epsilon_a). \]  

(A-4)

We argue now that the gradient in the molecular energy is nearly constant over a distance extending for a few mean free paths from the wall, and that the incoming molecules will represent the average state of the gas (i.e. the average of molecules moving toward and away from the wall) at the position from which they originate. Therefore, we may write

\[ \epsilon' = \epsilon_a + \epsilon' (\partial \epsilon/\partial x), \]  

(A-5)
where $\frac{\partial}{\partial x}$ represents differentiation normal to the surface, so that

$$\Delta \epsilon = \frac{2 - \alpha}{\alpha} \epsilon' \left( \frac{\partial \epsilon}{\partial x} \right).$$

(A-6)

Appropriately substituting temperature for energy on both sides of Eq. (A-6), we recover Eq. (A-1), the hypothesized temperature-jump boundary condition, and identify $g$ as

$$g = \frac{2 - \alpha}{\alpha} \epsilon',$$

(A-7)

although $\epsilon'$ is as yet not fully specified.

Now the rate at which the wall gains energy is, per unit area, simply the number of molecules reaching the wall in unit time multiplied by the difference in energy between the incoming and outgoing molecules, i.e.,

$$q = \frac{n \bar{c}}{4} (\epsilon' - \epsilon_m),$$

(A-8)

where $\bar{c}$ is the mean molecular speed and $n$ is the number density of the gas. It is convenient to rewrite Eq. (A-8) using the definition of the average energy, $\epsilon_a$, together with Eq. (A-5), in the form

$$q = \frac{n \bar{c}}{4} \epsilon_a' \left( \frac{\partial \epsilon}{\partial x} \right).$$

(A-9)

The heat flux can also be expressed in terms of temperature if we argue that the accommodation coefficient has the same value for internal molecular energy as for translational energy, and that the temperatures of the energy modes are everywhere equal. Now the translational energy carried by a unit mass of molecules issuing from a gas at temperature $T$ is $2RT$, where $R$ is the usual gas constant per unit mass, the extra $\frac{1}{2}RT$ owing to the importance of the faster molecules which carry more energy and have a greater probability of reaching a given surface. Including internal energy, the total energy carried by an "average" molecule is thus $m(C + R/2)T$ where $C'$ is the constant-volume specific heat and $m$ is the mass of a single molecule. Finally the heat flux in Eq. (A-9) may be rewritten as

$$q = \left( \frac{nmC}{4} \right) \epsilon_a \left( \frac{\partial T}{\partial x} \right),$$

(A-10)

or, alternately, as

$$q = \left( \frac{\rho C_v}{2} \right) \epsilon_a' \left( \frac{\gamma + 1}{2} \right) \left( \frac{\partial T}{\partial x} \right),$$

(A-11)

where $\gamma$, the ratio of the specific heats, has been introduced.

We can now specify the length $\ell'$ more precisely by matching the expression above the the heat flux to the wall with the known heat flux in the gas adjacent to the wall,

$$q = k \left( \frac{\partial T}{\partial x} \right).$$

(A-12)
The result is simply

$$
\varepsilon' = \left( \frac{k}{\rho \bar{C}_v} \right) \left( \frac{4}{\gamma + 1} \right)
$$

(A-13)

If we wish to express this result in terms of the familiar collisional mean free path $L$, we may introduce the viscosity

$$
\mu = a \rho c L = \frac{1}{4} \rho \bar{C}_L
$$

(A-14)

where $a$ is a constant (Ref 5), $0.491 < a < 0.499$, which we will take as $\frac{1}{4}$. Finally, we find for the temperature-jump distance as a function of the accommodation coefficient and the mean free path,

$$
g = \frac{2 - a}{a} \varepsilon' = \left( \frac{2 - a}{a} \right) \left( \frac{k}{\mu C_v} \right) \left( \frac{2}{\gamma + 1} \right) L
$$

(A-15)

which is the same result found by Kennard (Ref. 5) (and often quoted by workers in the field of gas-solid interactions) using a different derivation.

We may simplify this result even further by making use of Eucken's relation for the Prandtl Number, (Refs. 4 and 5),

$$
Pr = \frac{\mu \gamma C_v}{k} = \frac{4\gamma}{9\gamma - 5}
$$

(A-16)

which permits one to rewrite Eq. (A-15) as

$$
g = \left( \frac{2 - a}{a} \right) \left( \frac{2}{\gamma + 1} \right) \left( \frac{9\gamma - 5}{4} \right) L
$$

(A-17)
APPENDIX B

The Characteristic Accommodation Time

We can define a characteristic accommodation time using the characteristic "length" in the \\\(^{1}\) coordinate system, \( (\rho g)_{av} \), and the corresponding "diffusivity", \( \beta \). Accordingly, the characteristic accommodation time is

\[
\tau_a = \frac{(\rho g)_{av}^2}{\beta} = \left( \frac{2 - \frac{\alpha}{\beta}}{\alpha} \right)^2 \left( \frac{2}{\gamma + 1} \right)^2 \left( \frac{9\gamma - 5}{4} \right)^2 \frac{(\rho L)_{av}^2}{\rho k/C_p} \tag{B-1}
\]

where the subscript \( av \) emphasizes that the relevant quantity is to be evaluated at the average gas temperature

\[
T_{av} = \frac{1}{2}(T_\infty + T_0) \tag{B-2}
\]

The mean free path is defined in terms of the viscosity,

\[
\mu = \frac{1}{2} \rho \tilde{c}_L \tag{B-3}
\]

so that

\[
\tau_a = \left( \frac{2 - \frac{\alpha}{\beta}}{\alpha} \right)^2 \left( \frac{2}{\gamma + 1} \right)^2 \left( \frac{9\gamma - 5}{4} \right)^2 \frac{(2\mu/\tilde{c})_{av}(\rho L)_{av}}{\rho k/C_p} \tag{B-4}
\]

If we evaluate \( pk \) at the average condition as well, we find, after applying Eucken's relation (see Appendix A), that

\[
\tau_a = \left( \frac{2 - \frac{\alpha}{\beta}}{\alpha} \right)^2 \left( \frac{2}{\gamma + 1} \right)^2 \left( \frac{9\gamma - 5}{4} \right)^2 2\gamma \tau_c \tag{B-5}
\]

where \( \tau_c \), the collision time interval in the gas, is simply

\[
\tau_a = \left( \frac{L/\tilde{c}}{\alpha} \right)_{av} \tag{B-6}
\]

It is understood that \( \tau_c \) is to be evaluated at the average gas temperature.

For a diatomic gas such as \( N_2 \), \( \gamma = 7/5 \) and

\[
\tau_a = \left[ \frac{2 - \frac{\alpha}{\beta}}{\alpha} \right]^2 3.7\tau_c \tag{B-7}
\]

so that the accommodation time varies from a few collision time intervals for \( \alpha \) near unity to more than a thousand collision time intervals for \( \alpha \) in the neighbourhood of 0.1. A measurement of the accommodation time should thus serve as a sensitive indicator of the actual value of \( \alpha \).
The Collision Time Interval

One of the important quantities in the relation for the characteristic accommodation time is the collision time interval $\tau_c$. We can calculate $\tau_c$ for the gas layer adjacent to the end wall in the following way.

From the kinetic theory of gases it is well known (Ref. 5) that the viscosity is given to a good degree of approximation by

$$\mu = \frac{1}{2} \rho \bar{c} L, \quad (C-1)$$

so that we can use the results of viscosity measurements to infer the mean free path and, in turn, calculate the collision time interval from the definition

$$\tau_c = L/\bar{c}. \quad (C-2)$$

We have then

$$\tau_c = 2\mu/\rho \bar{c}^2 = (\pi/4)(\mu/p); \quad (C-3)$$

the last step is accomplished by substituting the known relation for $\bar{c}^2$ in a Maxwellian gas,

$$\bar{c}^2 = 8RT/\pi. \quad (C-4)$$

We can calculate $\mu$ from the empirical relation provided by Bird et. al. (Ref 14),

$$\mu \text{ gm}^{-1} \text{ cm}^{-1} \text{ sec}^{-1} = 2.67 \times 10^{-5} \sqrt{MT/\sigma^2 \mu} \quad (C-5)$$

where $M$ is the molecular weight of the gas, $T$ is the absolute temperature, in K, $\sigma$ is the characteristic diameter of the molecule (based on a Lennard-Jones (6-12) potential), in Angstroms, and $\Omega$ is the energy-dependent correction to the rigid-sphere cross section. For pressure in torr and collision time interval in nanoseconds we find

$$p \tau_c = 15.7 \sqrt{MT/\sigma^2 \Omega \mu} \quad (C-6)$$

In the notation commonly employed in shock-tube work, $p_1$ is the initial pressure prior to shock-wave compression, $p_5$ is the pressure behind the reflected shock wave, and $p_5/p_1$ is the ratio of these pressures, $p_5/p_1$, readily calculated from standard gas dynamics relations (Ref. 13). The appropriate pressure to use in Eq. (C-6) for the problem under discussion is thus

$$p_5 = p_5 p_1. \quad (C-7)$$
The appropriate temperature to use in Eq. (C-6) is

\[ T_{av} = \frac{1}{2}(T_\infty + T_0) \] (C-8)

where, analogous to the result for pressure (Ref. 13),

\[ T_\infty = T_{51} T_1. \] (C-9)

With this notation understood, it is most convenient to plot \( p_1 \tau_c \) which is simply (in torr-nsec)

\[ p_1 \tau_c = (15.7 \sqrt{MT_{av}})/(p_{51}^{\frac{2}{3}} \Omega). \] (C-10)

Figure 4 provides a plot of \( p_1 \tau_c \) versus primary-shock Mach number for a variety of common gases. \( T_\infty \) was taken as 300 K in all cases and the values for \( \sigma \) and \( \Omega \) were taken directly from Bird et al. (Ref. 14, pp. 744-746). The primary-shock Mach number is defined by

\[ M_S = V_s/\sqrt{\gamma RT_1} \] (C-11)

where \( V_s \) is the velocity of the wave measured in each shock-tube experiment.

From Fig. 4 we see that \( p_1 \tau_c \) is in the range 1-100 torr-nsec for moderate shock strengths, and \( p_1 \tau_c \) does not vary greatly between the gases at the same value of \( M_S \).
FIG. 1. THIN-FILM-GAUGE TEMPERATURE AS A FUNCTION OF TIME (NO TEMPERATURE JUMP AT SURFACE).

FIG. 2. THIN-FILM-GAUGE TEMPERATURE AS A FUNCTION OF TIME (WITH TEMPERATURE JUMP AT SURFACE).
FIG. 3. THIN-FILM-GAUGE TEMPERATURE AS A FUNCTION OF TIME (WITH TEMPERATURE JUMP AT SURFACE).

FIG. 4. COLLISION TIME INTERVAL AT THE AVERAGE GAS TEMPERATURE FOLLOWING SHOCK REFLECTION; $T_i = 300^\circ$K
POWVER SUPPLY
(0-50 VOLTS D.C.)

THIN-FILM
THERMOMETER

20 pf

Rg

20 pf

Rg

1 MΩ

A INPUT

B INPUT

TEKTRONIX 1A5
DIFFERENTIAL
AMPLIFIER PLUG-IN UNIT;
TEKTRONIX 556
OSCILLOSCOPE.

FIG. 5. RECORDING CIRCUIT FOR THIN-FILM RESISTANCE THERMOMETER.

FIG. 6. THICKNESS DISTRIBUTIONS FOR THIN-FILM THERMOMETERS
(SKETCHES OF RECORDS OBTAINED WITH TAYLOR-HOBSON TALYSURF).
FIG. 7. TEMPERATURE-TIME HISTORIES OBTAINED WITH CHEMICALLY DEPOSITED PLATINUM GAUGE; \( \approx \) 4 mV/div.
FIG. 8. TEMPERATURE–TIME HISTORIES OBTAINED WITH VACUUM EVAPORATED PLATINUM GAUGE; $= 4 \text{ mV/div.}$
FIG. 9. TEMPERATURE-TIME HISTORIES OBTAINED WITH VACUUM-SPUTTERED PLATINUM GAUGE; ≈ 3 mV/div.
FIG. 10. DATA-REDUCTION PLOT FOR THERMAL ACCOMMODATION COEFFICIENT: VACUUM-
SPUTTERED GAUGE, $P_1 = 5.1$ torr, $M_s = 2.25$.

FIG. 11. TEMPERATURE-TIME HISTORY WITH VACUUM-SPUTTERED THIN-FILM
THERMOMETER; $P_1 = 5.1$ torr, $M_s = 2.25$. 
FIG. 12. EXPERIMENTAL RESULTS FOR THE THERMAL ACCOMMODATION COEFFICIENT OF N\textsubscript{2} ON PLATINUM.

FIG. 13. COMPARISON OF MEASURED THERMAL ACCOMMODATION COEFFICIENTS FOR N\textsubscript{2} ON PLATINUM.