THE EFFICIENCY OF ENERGY TRANSFER ASSOCIATED WITH MAGNETICALLY DRIVEN SHOCK WAVES IN A TEE TUBE

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SUMMARY

The energy disposal in a shock tube with magnetically driven shock waves has been studied. The well known T-Tube apparatus was used in the experiment. A technique to measure the energy input has been developed involving the simultaneous measurement of the current and the voltage drop across the tube. Schlieren rotating drum camera photographs of the flow provided accurate shock wave trajectories from which the energy contained in the flow was deduced by the use of strong blast wave theory. The efficiency of the T-Tube obtained in this way was found to be approximately 20 percent under a variety of conditions. The use of a backstrap did not change the efficiency but was found to roughly double the amount of energy deposited in the T-Tube. In the range 1 - 10 mm. Hg., no effect of pressure was detected.
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NOTATION

f  frequency (cps)
g  nondimensional velocity
h  nondimensional density
i  current (amperes)
k  nondimensional internal energy
q  charge, \( q = \int i \, dt \) (coulombs)
t  time (secs.); thickness (cms)
u  flow velocity; velocity of circuit element
x  distance
z  nondimensional distance
C  circuit capacitance (farads)
E  internal energy of the gas; energy dissipated by the T-Tube (joules)
\( F_{V} \)  ponderomotive force on a circuit element
I  solution of transformed circuit equation
\( I_{m} \)  current amplitude (amperes)
\( I_{\text{max}} \)  peak current (amperes)
\( J_{\nu} \)  Bessel function of order \( \nu \)
L  Total circuit self inductance (henries)
L'  Rate of change of inductance (constant)
R  total circuit resistance (ohms)
R'  effective T-Tube resistance
V  shock wave velocity
W  total initial energy added per unit area
Greek Letters

\( \alpha \) current damping parameter, \( \frac{L}{R} \)
\( \omega \) current frequency (radians/sec)
\( \gamma \) specific heat ratio
\( \mu \) relative permeability
\( \rho \) density
\( \sigma \) resistivity (ohm-cms)
\( \tau \) skin effect parameter
\( \chi \) nondimensional blast wave similarity parameter

Subscripts

0 initial
1 at the shock wave
1. INTRODUCTION

1.1 Review of work on Magnetically Driven Shock Waves

The study of electrically produced shock waves originated in 1950 when R. G. Fowler and his associates at the University of Oklahoma began to study an effect noticed in 1943 by Lord Rayleigh when he observed an advancing luminosity down a sidearm of an electrodeless discharge tube (Ref. 1). By 1952 they were able to show the existence of shock waves and to explain the luminosity by ascribing it to the expansion of the hot luminous gases of the discharge itself, and to the excitation produced by the strong shock waves (Ref. 2, 3). These discharges were produced between a pair of electrodes in a quartz tube, with shock waves propagating down a sidearm mounted at right angles to the axis of the tube containing the electrodes. The energy transfer was primarily by means of resistive heating in the conducting plasma between the electrodes.

In 1956, A. C. Kolb (Ref. 4) and S. W. Kash (Ref. 5) reported extensive experiments using similar apparatus but introducing magnetic driving of the shock waves. Their apparatus consisted of a T-Tube with the return lead from one of the electrodes placed along the head of the Tee. Figure 1 compares their arrangement with that used by Fowler. This configuration provides a magnetic field perpendicular to the electrode axis, resulting in a Lorentz force on the current carrying plasma that is directed up the sidearm of the T-Tube. Kolb pointed out that since the resistivity of a gas decreases with temperature, energy transfer by resistive heating becomes inefficient and magnetic driving relatively more important when the production of very strong shock waves is attempted. Other methods of providing driving magnetic fields have included solenoids in series with the discharge circuit and located to provide a field of the right orientation (Ref. 4, 5, 6) as well as permanent magnets (Ref. 7), or coils pulsed by separate circuits (Ref. 4, 8).

The interest in magnetically driven shock waves has spread since the original investigations and several different configurations for producing such shocks have been described. A popular arrangement has been a coaxial discharge in a conical chamber driving the plasma through a ring electrode (Ref. 9, 10, 11). This particular system may also be traced to Fowler who used similar apparatus. Some experiments with electrodeless discharges were carried out by V. H. Blackman and B. Niblett (Ref. 12) at Princeton and by groups at Avco (Ref. 13):

Some analyses of the motion of current carrying plasmas through transverse magnetic fields have been published (Ref. 14, 15, 16). They are not directly applicable to the experimental devices described previously but nevertheless a comparison of experimental and theoretical results in Ref. 15 yields an order of magnitude agreement. An appropriate description of the flow in such shock tubes following the discharge is given by the one-dimensional strong blast wave theory (Ref. 17, 18) since...
the energy is delivered in a very short time and over a limited volume. The
details of energy transfer are not important in this theory since it is based
on the assumption that a definite amount of energy is deposited instantaneously
at a given cross-section of the tube.

The measurements made in magnetically driven shock tubes
have been chiefly of shock wave and shock luminosity position and velocity
as obtained with streak cameras, photoelectric pickups, and microwave equip-
ment; the dependence of shock velocity on the parameters of the circuit and the
geometrical arrangement; spectroscopic temperature and density; ionization
relaxation times; specific impulse and thrust; and energy efficiency.

1.2 Review of the Work on the Efficiency of Energy Transfer

Only a limited amount of information on this problem appears to
be available. Kash (Ref. 5) estimated from the damping of the discharge
oscillations that about one percent of the stored capacitor energy is de-
ivered to the T-Tube. Paxton and Fowler (Ref. 19), in a short note state
that on the average 20 percent of the energy delivered to their tube appears
as flow energy. However, in this note they do not state their method of
arriving at this figure and it is not clear to what apparatus it is meant to apply.
Considerable work has been done at General Electric Aerosciences Laboratory.
Gorowitz and Harned (Ref. 6) have tested several configurations at various
pressures and energies. They have used a ballistic pendulum to measure the
momentum of the plasma and a photomultiplier to measure the time of arrival
of the luminosity at a series of stations. They estimate from the relative
resistances that their equipment delivers between 50 and 70 percent of the
capacitor energy to the T-Tube. Basing their calculations on the initial stored
capacitor energy, they obtain efficiencies ranging from 1 to 7.4 percent. Their
data for shocks in air indicates a pronounced drop in efficiency as the pressure
is increased from 0.2 to 1.0 mm. Hg and as the stored energy is decreased
from 500 to 50 joules. They find that the most efficient configuration is one
using a 1/4" tube as a backstrap as against other configurations using back-
straps or coils. There are however, many doubts regarding the accuracy of
measuring plasma energy by this method. For example, this technique mea-
sured only the velocity of the leading edge of the luminosity and this is taken
to be the velocity of the plasma as a whole. In a more important criticism,
Gooding et al (Ref. 20) report that the measured plasma momentum contains
a large spurious contribution due to evaporation or sputtering of the pendulum
material which can completely obscure the true value. By eliminating the need
for an external spark gap switch and exhausting into a vacuum, B. Gorowitz,
K. Moses, and P. Gloersen (Ref. 21) have been able to increase the efficiency
to as high as 60 percent although for the same configuration that had pre-
viously given an efficiency of 7.4 percent, the increase was to a more modest
36 percent.
1.3 Introduction to the Present Problem

The purpose of the present work is to obtain information about the efficiency of energy transfer to the gas in a magnetically driven shock tube.

The electrical energy stored in the capacitors disappears during the discharge. Much of it goes into heat dissipated by the resistance of the external circuit, some of it is lost in the switch, and the rest goes into the T-Tube. Of this energy, some is radiated away, some goes into the vaporization of the electrode material, some is lost by conduction of heat to the walls, and the rest shows up as kinetic and internal energy of the gas.

In this note we estimate first, the total energy delivered to the tube, and second, the proportion of this energy which remains in the gas. The total energy going into the tube is obtained by simultaneously measuring the current and the voltage drop across the electrode gap. The product of these quantities gives the rate of energy addition to the gap and integration over the time of the discharge will given the total energy. This direct measurement is to be preferred to a determination of the energy from the increased damping of the discharge as compared with the discharge with electrodes shorted for two reasons. The energy going into the T-Tube is usually only a small fraction of the total energy dissipated during the discharge by the rest of the circuit and thus the change in damping is small. At the same time, not knowing the exact effect on the damping of the parameters of the T-Tube discharge (see Sec. 3.2.1) it is not possible to determine accurately the amount of energy going into the T-Tube by this method.

Some of this energy is then transferred to the gas in the tube by means of a strong shock wave. As the energy becomes distributed over more and more of the gas, the shock wave is attenuated. This behaviour of the shock wave is treated by blast wave theory and from the distance time trajectory of the shock wave it is possible to estimate the energy remaining in the gas.

2. EXPERIMENTAL ARRANGEMENT

2.1 The T-Tube Circuit

The work reported here is an investigation of a T-Tube both with and without a backstrap. The current, the rate of change of current, and the voltage across the T-Tube are monitored by an oscilloscope. A rotating drum camera in conjunction with a schlieren system records the shock trajectory. The overall arrangement of the equipment is shown schematically in Fig. 2.

The T-Tube is made from 22 mm. O. D. pyrex tubing, 30 inches long, and has a 6 inch long tube forming the bar of the Tee. Brass end fittings, using O-ring seals, hold the electrodes in place. The electrodes are hemispherical in shape and have a diameter providing a snug fit inside the tube. In most of the experiments the electrodes were made of copper; magnesium
Two 1.5 microfarad, 30 KV, low inductance capacitors* are used for energy storage. The circuit is formed by 1 inch wide strips of heavy gauge brass sheet connecting the capacitors to the T-Tube as is shown in Fig. 3. In some experiments the circuit was formed by means of strips of high resistivity alloy to provide critical damping of the current. The T-Tube is placed symmetrically between the capacitors to reduce the size of the circuit and to obtain simultaneous energy release by both capacitors.

The spark gap switch (S1, Fig. 2A) consists simply of a brass electrode, forming a part of the lower brass end fitting (Fig. 3), separated from a copper electrode connected to the high voltage terminals of the capacitors. Both electrodes have 1/2 inch diameter hemispherical heads. Through the center of the copper electrode runs a length of high voltage wire which forms the third electrode of the switch. A high voltage, high frequency pulse from a pulse transformer triggers the discharge. An alternative method of photoelectric triggering has also been tried, using the ultraviolet radiation from a General Electric BH-6 mercury vapour flash lamp directed onto a magnesium electrode to provide the necessary ionization by electron emission. Figure 3 shows the spark gap switch with the flash lamp in position. This latter method was used in an attempt to avoid the pick-up of noise associated with the triggering discharge but it was not as reliable, since the adjustment of the electrode gap was more critical than with the use of a third electrode.

The capacitors are charged with an NJE Corp. 30 KV power supply by a current kept constant at about 2 milliamperes by manually adjusting the variable voltage control. A 3 megohm resistor bank, connecting the power supply output through the switch S3 to ground, discharges residual charge on the capacitors after the experiment. The high voltage switch (S2) disconnects the power supply when the capacitors are fully charged and there is no current. This switch is necessary since one side of the current measuring resistor should be at ground potential in order that there be no current in the cable to the oscilloscope. Since the output circuit of the power supply contains a smoothing capacitor circuit with a time constant long compared to the discharge time, the power supply connection would tend to maintain its voltage, thus setting up a potential difference across the oscilloscope cable. Since the switch must withstand a high voltage, a special switch was constructed. A lucite box, containing a brass roller spanning the width of the box, is free to pivot so that the roller can rest at either end. One end contains a pair of contacts, sufficiently separated to withstand the voltage, which are connected by the roller when it is at that end of the box.

To set the voltage on the capacitors accurately, a sphere-gap apparatus was used. The spacing between two 7.6 cm. diameter brass electrodes have also been used but they have shown more wear.

* Telegraph Condenser Company
spheres was accurately set according to a set of tables (Ref. 22) to spark at a
given voltage. The current was limited by a 40 megohm resistance connected
in series with the sphere-gap between the power supply output and ground.

The test gas was introduced by means of a needle valve control
through one of the brass end fittings. The pressure was measured at a nei­
ghbouring outlet directly by means of a butyl phthalate manometer with a
vacuum reference maintained by a mechanical pump. An existing manometer
described in Ref. 23 was improved by constructing an all glass body. It proved
to be very reliable and eliminated problems of calibration. The pressure
was maintained by a very slow leak of the gas into the tube controlled by the
needle valve and a valve to the vacuum pump. It was made certain that this
leak was much greater than the natural leak rate of the system, assuring a
high concentration of the gas.

2.2 Measuring Apparatus

2.2.1 Electrical Measurements

A model 551 Tektronix double-beam oscilloscope has been used
in all measurements to provide a direct comparison of different waveforms
on the same time scale. Two types of associated Tektronix preamplifiers have
been used; a type L preamplifier for the current signal, and a type CA pre­
amplifier for voltage measurement to take advantage of its ability to mix
signals. Coaxial cables have been used for all connections to the oscilloscope.
A polaroid-back oscilloscope camera, using 3000 ASA rating polaroid film,
has been used to record the data.

The triggering pulse to the spark gap switch from the pulse
transformer produced an extraneous signal with a predominant frequency of
8.5 MC/sec. at the start of the discharge. This distortion of the signals was
easily removed by introducing the following filter at the input to the oscilloscope.

```
\begin{center}
\begin{circuitikz}
\draw (0,0) node [ground] (g) {};
\draw (0.5,0) node [ground] (g) {};
\draw (0,1) node [ground] (g) {};
\draw (0,-1) node [ground] (g) {};
\draw (0,0) to [c=150 \mu \text{f}] (0,1);
\draw (0,0) to [c=150 \mu \text{f}] (0,-1);
\draw (0,0) to [r=120 \Omega] (0,0.5);
\draw (0,0) to [r=120 \Omega] (0,-0.5);
\draw (0,0) to [r=62 \Omega] (0,0.5);
\draw (0,0) to [r=300 \mu \text{f}] (0,-0.5);
\end{circuitikz}
\end{center}
```

The ratio of the attenuated frequency to the discharge fre­
quency was large enough so that the desired signal was not influenced signi­
ficantly.
By Faraday's law of induction, a voltage proportional to the rate of change of current appears across a loop held in the vicinity of the discharge circuit. Thus any measuring circuit will have some voltage induced in it, and it would be desirable to have some means of subtracting out this extraneous induced voltage.

A simple loop (or coil) can be used to obtain a signal proportional to \( \frac{di}{dt} \) provided the loop is held fixed and the inductance of the discharge circuit is constant. These limitations are important since an accidental shift in the position of the loop will change the magnitude of the signal. The fact that the principle of operation of the T-Tube involves a moving current path will produce distortion of the signal, particularly if the loop is placed near the gap. These difficulties can be removed by constructing a toroidal coil and placing it about a current carrying element in the manner of a belt (Rogowski belt). With this arrangement, the signal is proportional to the rate of change of current enclosed and is essentially independent of position, or of motion of other parts of the circuit. Such a belt has been constructed and is shown in position in Fig. 3.

The signal so obtained is used to subtract portions of the gap voltage signal which will be discussed later. It is also used as a check to ensure negligible induced voltage in the measured current by determining that the desired signal has its zeros at the maxima of the rate of change of current signal.

Early difficulties in measuring current stemmed from the fact that simple current measuring resistors showed a large induced voltage. This voltage was due to magnetic flux threading the loop formed by the resistor and the leads to the oscilloscope. The high rate of change of current produced voltages comparable to the current signal itself.

Figure 4 shows the design of a special resistor which reduced the induced voltage to an insignificant level. By placing the oscilloscope leads inside a tubular resistor, the largest induced voltage contribution from the flux near the outside surface is eliminated. Magnetic flux from other parts of the circuit is shielded by a mild steel sleeve around the resistor. It is also fortunate that the arrangement of the connections inside forms a loop of very small effective size, which further tends to reduce pickup from such external magnetic flux as may exist in spite of the shield.

Since the frequency of the discharge is relatively high, it is important that the resistance change due to skin effect be small. The non-dimensional parameter characterizing the skin effect in tubes (Ref. 24) is

\[
\tau = 2\pi t \sqrt{\frac{\mu L}{f \times 10^{-9}}}
\]
where $t =$ thickness, cms.

$\sigma =$ resistivity, ohm-cms.

$\mu =$ relative permeability ($\mu = 1$ for free space)

$f =$ frequency, cps.

For negligible skin effect, $\sigma$ should be less than 0.5. To minimize the value of this parameter, the material chosen should have a high resistivity and a low permeability. Inconel is well suited since

$\sigma = 98.1$ microhm-cms.

$\mu =$ 1.005 maximum

Thus, for operation up to 180,000 cps, the resistor must have a wall thickness of less than 0.059 cms. The actual resistor is made from 1/2 in. diameter Inconel tubing, turned down to 0.020 in. thickness between the contacts, to give a resistance of approximately 0.001 ohms. Calibration has fixed this value at 0.00096(6) ohms.

Comparison of this signal with the rate of change of current as described previously shows no observable induced voltage.

The voltage across the T-Tube is measured by a Tektronix type P 6014 high voltage probe with a 1000:1 division ratio. Since the high voltage lead is connected to the brass end fitting, and the low voltage end of the probe is the grounded point of the current measuring resistor, the measured voltage will contain the voltage drop in the metal parts included, and an induced voltage due to the loop formed by the high voltage cable and the shield of the coaxial cable from the current measuring resistor. Figure 5 illustrates the above arrangement. The voltage that is measured is composed largely of induced voltage. To expose that part of the voltage signal which contributes to the calculation of the energy input (see Sec. 3.2.2) the correct amount of the rate of change of current signal is subtracted until the zeros of the remaining voltage coincide with those of the current. This is done with the help of a type CA Tektronix preamplifier which contains two amplifier channels through which the sum or the difference of the two inputs can be found. The net voltage drop across the electrode gap was found by subtracting the voltage drop found by shorting the electrodes by a copper cylinder.

Since stray capacitive coupling between the high voltage lead and the surrounding equipment is especially serious because of the 100 megohm resistance of the probe, it was found necessary to have a grounded shield on the high voltage cable.
2.2.2 Rotating Drum Camera and Schlieren Apparatus

An air driven, turbine drum camera, previously used at UTIA in the investigation of strong shock waves, is used together with an existing 16-inch mirror schlieren system. The rotational speed of the drum is accurately measured by using a frequency counter to count the pulses produced by a slug of magnetic material, mounted in the face of the drum, as it sweeps by the head of a magnetic pickup.

It was not easy to obtain pyrex tubing which was transparent in the schlieren system. By examining a large number of tubes in a schlieren beam, the best few were selected. Even these were poor except for one orientation as the tube was rotated. These selected few were used to construct T-Tubes, such that the bar of the Tee was mounted perpendicular to the plane through the schlieren slit.

3. THEORY

3.1 The One-Dimensional Blast Wave

Owing to a lack of any exact solution for the flow in the T-Tube, it is natural to seek a suitable approximate theory. Kolb (Ref. 4, 8) was the first to use the one-dimensional blast wave theory in connection with this problem. This theory is presented by E. G. Harris (Ref. 17) who gives both an exact and an approximate solution. A very complete rigorous treatment is given by L. I. Sedov in Ref. 18. The theory is based on the assumption that a given amount of energy is released instantaneously at a plane in the shock tube. Because of this assumption, the theory will not apply close to the discharge but may be expected to apply at later times when the shock wave has engulfed several times the mass of gas involved in the discharge. H. L. Brode (Ref. 25) has verified by numerical computation that in the spherical case the blast from an initial isothermal sphere of gas will agree to within 10 percent with the point source solution after the shock wave has engulfed a mass of air ten times the initial mass of the sphere. It is further assumed that \( \gamma \) is constant; however, the shock wave must be sufficiently strong so that pressure and temperature before the shock are negligible in comparison with corresponding quantities behind the shock. One-dimensional conditions are assumed, by neglecting wall effects. These assumptions permit a similarity solution in terms of a parameter

\[
\chi = \frac{x \rho_0^{1/3}}{t^{2/3} W^{1/3}}
\]

(1)

where

\( x \) = distance from the point of energy release

\( \rho_0 \) = density in the undisturbed flow

\( t \) = time

\( W \) = total energy added per unit area

\[ \]
The equations of motion become a set of ordinary differential equations which, together with the boundary conditions, have the following solution in terms of a set of three functions, \( g, h, \) and \( k \):

\[
\begin{align*}
X &= \frac{\chi_1}{\chi_1} = \frac{B}{2A} \left( \frac{2}{25} \chi_1 - \frac{2}{25} \right) \\
\chi_1 &= \frac{B}{2A} \left( \frac{2}{25} \chi_1 - \frac{2}{25} \right) \\
\chi_1 &= \frac{B}{2A} \left( \frac{2}{25} \chi_1 - \frac{2}{25} \right)
\end{align*}
\]  

(2)

where \( \chi_1 \) = distance to the shock wave and

\[
A = \frac{16}{27} \left( \frac{\chi_1}{2} \right)^{2/3} \left( \frac{2}{3} \right)^{2/3}
\]

\[
B = \left[ \frac{4}{3(\chi_1 + 1)} \right]^{2/3} \left[ \frac{2}{3(\chi_1 + 1)} \right]^{5/3}
\]

The functions \( g, h, k \) are related to the flow properties as follows:

\[
\begin{align*}
\chi_1 &= \frac{1}{2} \sqrt{g(z)} \\
\rho &= \rho_0 h(z) \\
E &= \frac{1}{2} V^2 k(z) = \frac{1}{2} \frac{\rho}{\rho_0}
\end{align*}
\]  

(3)

where \( V \) is the velocity of the shock front.

The position of the shock front is

\[
\chi_1 = \chi_1 \frac{W^{1/3}}{\rho_0^{1/3}}
\]  

(4)
and therefore

\[ V = \frac{2}{3} \chi_1 \frac{W^{1/3} t^{-1/3}}{\rho_o^{1/3}} \tag{5} \]

\( \chi_1 \) is found from the fact that the total energy in the flow remains constant, expressed by

\[ \chi_1^3 \int_0^z (h_g^2 + 2h_k) \, dz = 1 \tag{6} \]

Harris has performed the numerical calculations for the case of \( \gamma = 5/3 \) but they are in error. This case was therefore recalculated and the results are shown in Table I and Fig. 6. The case of \( \gamma = 1.4 \) is available in Ref. 18.

From the above calculations,

\[ \chi_1 = 0.9754 \quad \text{for} \quad \gamma = 1.4 \]
\[ = 1.184 \quad \text{for} \quad \gamma = 5/3 \]

It may be noticed from Fig. 6 that the density falls off rapidly away from the shock. This forms the basis of Harris' approximate treatment where he assumes the mass to be concentrated at the shock and having the corresponding velocity and internal energy. His resulting equation is

\[ \chi_1 = \left( \frac{3}{2} \right)^{2/3} \left( \frac{\gamma + 1}{2} \right)^{2/3} \left( \frac{W}{2 \rho_o} \right)^{1/3} t^{2/3} \tag{7} \]

or

\[ \chi_1 = \left( \frac{3}{2} \right)^{2/3} \left( \frac{\gamma + 1}{2} \right)^{2/3} \left( \frac{1}{2} \right)^{1/3} \]
\[ = 1.174 \quad \text{for} \quad \gamma = 1.4 \]
\[ = 1.260 \quad \text{for} \quad \gamma = 5/3 \]

He mentions that the concentration of mass near the shock is more pronounced when \( (\gamma - 1) \) is small; however, the accuracy of the approximation decreases as seen from the above values of \( \chi_1 \). This is due to the fact that the approximation neglects the internal energy near the origin. The error in \( \chi_1 \) is only about 6% for \( \gamma = 5/3 \) but increases to 20% for \( \gamma = 1.4 \). This can become serious since the relative error in the determination of energy is tripled.
3.2 The Electric Circuit

3.2.1 The Magnetically Driven Shock Tube as a Circuit Element

Consider a line circuit with a given constant capacitance $C$, possessing a resistance $R$, and self inductance $L$. The circuit is not rigid so that it moves under the action of the ponderomotive force $F_v$. The velocity of the circuit in the direction of $F_v$ is $u$. The equation of conservation of energy can then be written

$$Ri^2 + \frac{d}{dt}\left(\frac{1}{2}Li^2\right) + \frac{d}{dt}\left(\frac{1}{2}\frac{Q^2}{C}\right) + F_v u = 0$$

(8)

where $Ri^2$ is the rate of energy dissipation by the resistance.

$\frac{d}{dt}\left(\frac{1}{2}Li^2\right)$ is the rate of change of energy in the magnetic field.

$\frac{d}{dt}\left(\frac{Q^2}{C}\right)$ is the rate of change of energy in the electric field of the capacitor. ($Q = \int idt$)

$F_v u$ is the rate at which the circuit is doing mechanical work. In our case this may be assumed to show up as energy in the gas flow.

We can also write the voltage equation around the circuit.

$$Ri + \frac{d}{dt}(Li) + \frac{Q}{C} = 0$$

(9)

Multiplying this by $i$ and subtracting from Eq. (8), we obtain

$$F_v u = \frac{1}{2} \frac{dL}{dt} i^2$$

(10)

This equation shows that that portion of the flow energy which is due to the work of the magnetic forces depends on the rate of change of the inductance of the circuit.

The simplest assumption that can be made about the inductance is that $dL/dt$ has a constant value, $L'$. Substituting $L = L_0 + L't$ in the voltage equation, one gets

$$(R + L')i + (L_0 + L't)\frac{di}{dt} + \frac{1}{C}\int i dt = 0$$

(11)

Differentiating and collecting terms

$$\left(L_0 + L't\right)\frac{d^2 i}{dt^2} + \left(2L' + R\right)\frac{di}{dt} + \frac{i}{C} = 0$$

(12)
The solution of this differential equation is obtained by performing the transformation
\[ i = I(t')^{-v}, \]
where
\[ t' = (L_0 + L't)^{1/2}, \]
\[ v = 1 + \frac{R'}{L'}. \]

The transformed equation is
\[
\frac{d^2 I}{dt'^2} + \frac{1}{t'} \frac{dI}{dt'} + \left[ \frac{4}{L'^2 C} - \frac{v^2}{t'^2} \right] I = 0 \tag{13}
\]
This is a Bessel equation of order \( v \) satisfied by functions of the argument \( \left[ \frac{2}{L'^2 C} t' \right] \).

Thus
\[
I = J_{\pm (1 + R'/L')}
\left[ 2 \sqrt{\frac{L_0 + L't}{L'^2 C}} \right] \tag{14}
\]

The solution of our initial equation is then a linear combination of the functions \( I/(L_0 + L't)^{1/2} \) satisfying the initial conditions. The Bessel function is an alternating function of the argument. Since the argument is essentially the square root of the inductance, we see that the period will increase as the inductance increases. The damping of the current depends on the order of the Bessel function.

The discharge in the T-Tube is actually more complicated and it may be expected to behave in this way only during the first half cycle. Following this, new discharges take place at the electrodes. Thus, there is no longer a single circuit but multiple circuits coupled by mutual inductance and by common conductive linkages. The existence of solenoidal currents produced by the inductive coupling between the circuits was discovered by L. Y. Cooper (Ref. 26) who used a magnetic probe to determine that currents were flowing in the tube even when the current in the external circuit was zero. It is his belief that the breakdown occurs when the local voltage of the electrodes (principally inductive voltage) exceeds the breakdown potential of the gas, and this might occur during any portion of the discharge cycle. However, it is experimentally observed that the breakdowns coincide very closely with the current zeros. It is possible to explain this if the breakdown potential of the gas in the vicinity of the electrodes exceeds the \( L \) di/dt voltage at the electrode gap. This is to be expected since in the present circuit, as well as in most T-Tube circuits thus far, the inductance of the tube itself is only a small part of the total circuit inductance. It is true, nevertheless, that the breakdown potential at the electrodes is reduced after the first half cycle due to the reduced pressure in the wake of the shock wave. The extinction and reignition of an arc is considered by Attwood,
Dow, and Krausnick (Ref. 27). They indicate that shortly before the current drops to zero (about 0.1 amps.) the arc fails in a very short time causing the current to fall to practically zero. The voltage across the electrodes then changes very rapidly with a frequency determined by the circuit inductance and distributed capacitance until the voltage for a glow discharge is reached which then rapidly turns into an arc. The arc tends to restrike with the inductance at a minimum since this will give the lowest voltage across the arc. Oscilloscope traces of the voltage across the electrode gap of the T-Tube (Figs. 12, 14) show discontinuities at the time of zero current which may possibly be caused by the transient described above and also by the change in inductive voltage due to the sudden change in the inductance caused by the breakdown establishing a new lower inductance link. The risetime of the oscilloscope is too long to follow the extremely fast transient so that in the photographs it appears as a discontinuity.

3.2.2 The Use of Constant Parameter Circuit Theory in the Evaluation of Data

Experimentally, the current during a ringing discharge in a T-Tube resembles a damped sinusoidal oscillation. Such an oscillation is characteristic of a circuit with constant parameters R, L, and C. Even though the above discussion has shown that L cannot be constant, we can fit the actual current by choosing constant values for the parameters to obtain a "best" fit. The voltage measured across the T-Tube, after subtracting the inductive component, is assumed to be proportional to the current, thus defining an effective resistance of the T-Tube determined from the peak values of the voltage. The energy dissipated by the T-Tube would then be given by

\[ E = \int R' i^2 dt \]

where \( R' \) represents the effective resistance of the T-Tube.

Let the current be fitted by a function of the type

\[ i = \text{Im} e^{\alpha t} \sin \beta t \]

\( \alpha \) is determined from the decay and \( \beta \) from the period.

For the \((n + 1)^{st}\) half cycle, the integral

\[ \int i^2 dt = \frac{I_m^2}{\beta} \int e^{\alpha T} \sin^2 T dT \]

\[ = - \frac{2}{\alpha} \frac{e^{\alpha T} (1 - e^{2\alpha T})}{(\alpha^2 + 4)} \frac{I_m^2}{\beta} \]
where \[ \alpha = - \frac{2\alpha}{\beta} \]

The peak current
\[ I_{\text{max}} \approx I_m e^{-\alpha t_n} \]

where
\[ t_n = \left( n + \frac{1}{2} \right) \pi \]

Therefore
\[ I_{\text{max}} = I_m e^{(n+\frac{1}{2})\frac{\alpha \pi}{2}} \]

and
\[ I_m^2 = I_{\text{max}}^2 e^{-\alpha n \pi} e^{-\frac{\alpha \pi}{2}} \]

Thus, assuming \( R' \) is constant in each half cycle, we obtain
for the energy going into the T-Tube in any half cycle
\[ E = R' I_{\text{max}}^2 \left( \frac{2}{\alpha} \right) \frac{\sinh \left( -\frac{\alpha \pi}{2} \right)}{\alpha^2 + 4} \]

where
\[ \alpha = - \frac{2\alpha}{\beta} = - \frac{\alpha \text{ (Period)}}{\pi} \]

4. EXPERIMENTAL RESULTS

4.1 Schlieren Shock Trajectories

The experiments covered the range of pressures between 1 and
10 mm. Hg. at initial capacitor voltages of 15, 20, and 25 kilovolts. Two
arrangements of the circuit were used: one employing a backstrap for
magnetic driving, and one with the return lead removed away from the
electrodes to minimize the driving magnetic field. To investigate the
efficiency of the discharge in the first current pulse, experiments were
conducted at 25 kilovolts in both configurations with a resistance added to
produce nearly critical damping. All runs were made using argon as the
test gas.

The pressure range was determined at the low limit by the
necessity of having a sufficient density level to produce a schlieren effect,
and at the upper limit by the need to have shock waves strong enough to be
treated by blast wave theory. The highest voltage used, 25 kilovolts, was chosen to avoid corona discharges.

Two considerations were important in the choice of argon as the test gas. The gas should be monatomic to prevent the energy release on recombination of a dissociated gas from modifying the rate of decay of the shock wave as predicted by the strong blast theory, and it should be easily ionized. Although deionization may also present a similar problem, it is one that is unavoidable in this case.

Figures 7 and 8 are schlieren rotating drum camera photographs of the flow at approximately 3, 5, and 10 mm. Hg. pressure and 25 KV capacitor voltage. Figure 7 illustrates the runs with backstrap and Fig. 8 the runs without backstrap. These are typical of the runs at lower capacitor voltage as well. In the high pressure runs the shock wave can be clearly seen, however, no other wave phenomena are strong enough to be observed by means of the schlieren. The self-luminosity seen propagating down the tube is probably due to two effects: the emission from the current-carrying plasma which acts as a piston in producing the shock waves, and the emission from the shock heated gas. As the shock wave attenuates, the luminosity front lags further and further behind and finally disappears. Inside the region of luminosity lines of increased luminosity are seen, originating at the start of each half cycle of the current, which in all likelihood represent the path of the current sheet as well as the luminosity from the weaker secondary shock waves near the electrodes and only the shock wave luminosity farther out. At lower pressures, the primary shock wave becomes stronger (higher Mach number) but its trace becomes fainter due to a smaller density change. Since the shock is stronger, the luminosity follows the shock wave much further along its trajectory. Horizontal lines visible especially at the beginning of the discharge are possibly caused by internal reflections along the tube or by the precursor wave which has been reported several times in the literature and which has several possible mechanisms, among them the absorption and reradiation in the gas of the intense emission in the electrode region. In the upper left hand corner of the photographs one can see waves which occur much later and are much weaker than the main shock wave. These are shock waves in the ambient air that are produced by the discharge in the spark gap switch. The vertical striations in the photographs are schlieren effects due to the non-uniform thickness of the walls of the pyrex tubing.

The measured coordinates of the shock trajectory have been plotted on a logarithmic scale for each run. As typical results, the trajectories for the cases discussed above are shown in Fig. 9 and 10. Strong blast theory predicts a time-distance relationship for the shock wave of the form \( t \sim x^{3/2} \) and the trajectories would appear as straight lines with a slope of 1.5 when plotted logarithmically. It is seen that the actual trajectories deviate from strong blast behaviour in the initial portions but tend to approach the correct slope at larger distances from the electrodes. This
deviation of the early history of the shock wave from theoretical behaviour is probably due to the violation of the assumption of instantaneous point energy release and also to the large energy losses by radiation and by conduction to the walls taking place in this period. It should be noticed that the logarithmic display also tends to exaggerate this initial period. If the measured points were taken at smaller intervals, it would become possible to distinguish discontinuities which indicate secondary shocks overtaking and strengthening the main shock wave. From the straight lines drawn tangent in the region of blast-like behaviour we can estimate the energy remaining in the flow by applying Eq. 4. The results are presented in Fig. 18.

The detailed flow in the region of the discharge is obscured by the large amount of light in Figs. 7 and 8. However, in the case of the damped discharge, the lower energy input to the gas and the relatively longer period of the discharge make it possible to observe these details. Figure 11 is an enlargement of the discharge region in the case of the damped circuit with backstrap at 10 mm. Hg. pressure. The current sheet can be seen as a broad luminous band moving out of the region of the electrodes. When it leaves, no light is seen near the electrodes until the current reverses and a new luminous band starts out away from the electrodes. The schlieren trajectory of the shock wave is clearly seen following its detachment from the luminous region.

4.2 Voltage Drop and Current in the Discharge

The voltage drop and the current have been measured for each run by the methods described in Sec. 2. Figure 12 shows a typical oscillogram for the case of the undamped circuit with backstrap. The period of the discharge is seen to be approximately 5.9 \( \mu \)secs. Hence, the frequency is 170 kc/sec., and the corresponding inductance of the circuit is 0.29 \( \mu \)henries. Figure 13 refers to the same configuration but with the electrodes shorted by a copper cylinder of 12.5 \( \times 10^6 \) ohms direct current resistance. This calibration shows a resistance of about 3.6 \( \times 10^{-3} \) ohms for the shorted gap. The resistance of the copper cylinder and of the remaining parts of the circuit within the voltage measuring loop are negligible in comparison with this value. It is entirely possible that this large value is due to the skin effect which would confine the current to a thin layer at the surface of the conductors and therefore increase the resistance. To obtain the net resistance of the gap, it is necessary to subtract this "shorted-gap" resistance from the measured value. Figure 14 is representative of the runs with an undamped circuit and no backstrap. The damping of the current is lower and the period is longer than in the previous case. The period here is 7.3 \( \mu \)secs., the frequency is 137 kc/sec, and the corresponding inductance of the circuit is 0.45 \( \mu \)henries. Figure 15 is an example of the electrical measurements for the damped discharges. The oscillation lasts only for one cycle, with the second half cycle only a small fraction of the first. The inductances are approximately the same as for the corresponding undamped circuits, but the period has increased to about 7.5 \( \mu \)secs for the case with backstrap and to about 8.4 \( \mu \)secs for the case with no backstrap.
The energy input to the T-Tube should be obtained by integrating the instantaneous power input, which is the product of the current and the "adjusted" voltage drop. It was thought sufficiently accurate to assume the voltage to be proportional to the current in each half cycle and to apply the equations of Sec. 3.2.2 in determining the energy. This means that we assume the voltage drop to be of the form $R' \cdot i$ where $R'$ is the net resistance of the gap. $R'$ is estimated for each half cycle from the peak voltage and current values less the "shorted gap" resistance. The average circuit resistance can be estimated from the damping of the current. According to Sec. 3.2.2, the peak values of current when plotted on a semi-log scale should lie on a straight line whose slope is equal to $\alpha = R/2L$. The results of the calculations are presented in Table II. The total energy input to the gap is the sum of the energies calculated for each half cycle by the method described above. In Table II the "effective gap resistance" is defined by the relation

$$R_{\text{eff}} = \frac{\sum R' \cdot I_{\text{max}}^2}{\sum I_{\text{max}}^2}$$

It may be noticed that the early portions of the current and voltage traces exhibit high frequency distortion. Although much effort has been spent in trying to eliminate this effect on the assumption that the causes are external, it has been possible only to reduce its magnitude. Some improvement has been noticed with improved grounding and with the use of the filtering circuit described in Sec. 2.2.1, but the greater part of the distortion has remained. The most probable cause is some irregularity of the breakdown in the spark gap switch. Since the switch consists of a large gap at atmospheric pressure, it may be expected to consume large amounts of energy. From the known capacitance, inductance, and average resistance of the circuit it is possible to estimate the initial voltage on the capacitors, hence the initial energy, from the magnitude of the first current peak. For the runs with backstrap, for example, this apparent initial energy is from 90 to 240 joules lower than the actual energy. This energy difference may well be the energy lost in ionization and excitation of the molecules in the spark gap.

4.3 The Determination of Efficiency

There are several ways of defining an efficiency, we will consider two of these here. First, we will define a T-Tube efficiency as

$$\text{T-Tube efficiency} = \frac{\text{Final energy in the gas}}{\text{Total energy input to the T-Tube}}$$

This efficiency is an indication of the effectiveness of the tube-electrode configuration, rather than of the method of energy transfer to the tube. The calculated results are presented in Table II and in Fig. 16.
The second efficiency that we define is the system efficiency:

\[
\text{System efficiency} = \frac{\text{Effective resistance of the gap}}{\text{Total average resistance of the circuit}}
\]

This efficiency is meant to be an indication of the effectiveness of the energy transfer from capacitors to the tube. This efficiency is obviously an indication of how well the external circuit has been designed since the resistance of the circuit includes the resistance of the capacitors, connections, and the switch. It has not been based on energy to exclude the effect of the missing initial energy discussed in the last section. Results are given in Table II and are plotted in Fig. 17. We may also speak of an over-all efficiency which would be the product of the T-Tube efficiency and the system efficiency.

5. DISCUSSION

At this point it is necessary to discuss the question of the accuracy of the results. This can be done in two parts: a discussion of the accuracy of the measurements and a critique of the method itself. Only the major sources of error will be pointed out, presuming that the calibration errors in the measurement of current, voltage, and pressure are of a relatively small magnitude.

In the determination of the energy input, the largest source of measurement error is associated with the blurred nature of the first half cycle of the voltage waveform. In some cases guesswork was needed to determine the maximum voltage in this portion of the cycle. This uncertainty is especially serious since the largest amount of energy is added in just this interval. The assumption that the voltage is proportional to the current in each half cycle could involve an error of at most 25% in the calculation of energy, based on the two possible extremes for the voltage waveform: a square wave and a sawtooth wave. The actual error is much lower than this value and probably within the error due to inaccuracies in the measurement of the peak values of current and voltage, the damping, and the period.

There are some fundamental errors due either to the method of measuring input energy or to the design of the apparatus which permits large inductive coupling between parts of the circuit. One of these problems is that the peak voltage will contain a contribution \( \frac{1}{2} \frac{dL}{dt} i_{max} \) while only \( \frac{1}{2} \frac{dL}{dt} i_{max} \) will contribute to the absorbed energy according to Sec. 3.2.1. This problem could be solved by performing the calculation of energy exactly using a suitable electronic multiplier and integrator. The peak voltage will also contain a contribution \( \frac{dM}{dt} i_{max} \) due to the time varying mutual inductance between the measuring circuit and the moving portion of the T-Tube circuit. This component cannot be eliminated in this apparatus nor is it possible to calculate its magnitude. The existence of solenoidal currents in the T-Tube has been mentioned previously. These currents draw energy from
the external circuit by mutual inductive coupling and this makes it impossible
to determine this energy by measuring the voltage and current across the
electrode gap. However, this mechanism is not operative during the first
half cycle when much of the energy is added to the gas. The efficiency of the
critically damped discharge, which lacks this contribution, is not very
different from the other cases.

The voltage waveform in all cases shows discontinuities at the
time of current zeros. These may be due to the sudden change in both the
self inductance of the T-Tube, and the mutual inductance of the voltage mea­
suring circuit with the T-Tube circuit, at the time of a breakdown across the
electrodes. However, similar discontinuities appear in the voltage trace
when the electrodes are shorted by a copper cylinder and they therefore
cannot be due to any change in inductance in the T-Tube circuit. Since they
are smaller than the discontinuities under normal T-Tube operation, it may
be possible to explain than as due to the existence of arcing between the
electrodes and the shorting cylinder. Pronounced erosion at the point of
contact in the case of magnesium electrodes indicates such a possibility.
The discontinuities could then be due to the very fast transient which
accompanies the reignition at the arc (Sec. 3.2.1).

There are several assumptions that are made in the blast wave theory
and it is necessary to determine whether they apply in this case. They are
as follows:

a) Instantaneous energy release.

In the T-Tube the energy is released over a finite volume and
time, and it is partly kinetic energy that is added directly, as well as thermal
energy. The blast wave theory does not specify the kind of energy that is put
in, provided it is added instantaneously and in an infinitesimal volume. There
are indications, mentioned in Sec. 3.1, that the finite nature of the discharge
will not influence the trajectory of the shock wave beyond a time or distance
when the shock wave has engulfed several times the original mass of gas. The
energy indicated by the trajectory at this time will not be the original energy
input but will be lower by the amount of energy lost by the gas up to this time.

b) Constant total mass of the gas.

It is known that a certain amount of material is vapourized from
the electrodes during the discharge. Again, if the shock wave travels far
enough to enclose several times the original amount of mass then the blast
theory should apply. The addition of mass by the electrodes will have a de­
laying effect.

c) Constant initial density.

It is believed that the strong radiation emitted by the discharge
can produce heating of the gas ahead of the shock wave. This would produce
changes in the density of the gas through weak waves propagating at the speed
of sound. Since the shock wave travels at several times the speed of sound then changes in density ahead of the shock wave will be small during the time of interest.

d) Strong Shock wave.

The theory required that the strong shock relations apply across the shock front and this implies that the temperature and pressure ahead of the shock wave should be negligible in comparison with these quantities behind the wave. That this condition is satisfied in the experiments may be seen in Fig. 9 and 10, showing that the trajectory of the shock wave follows the theoretical line as far down the tube as the measurement extends.

e) Perfect, polytropic gas.

Argon was chosen as the test gas to decrease the effect of real gas effects. The emission of light from the excited gas in the rotating drum camera photographs indicates that real gas effects (ionization, electronic excitation, and the inverse processes) take place over approximately the same period as do the other departures from ideal conditions, except at low pressures. Eventually the shock wave reaches the predicted rate of decay at which time the theory can be applied.

The fact that the shock wave trajectory has the correct slope on our plot means only that a condition has been reached where the energy distribution remains similar as the shock propagates. Harris' approximation, for example, assumes an energy distribution quite different from the one given by the exact theory and yet predicts blast wave behaviour in the sense used above. However, the actual energy distribution will likely be closer to the theoretical energy distribution than to the extreme assumed in the approximation where all the energy is at the shock front. Therefore the error in this approximation to the exact solution can be taken as the upper limit for the error in using the blast theory to determine the actual energy. This error for the case of a monatomic gas is about 18%.

The results of the experiments are presented in Table II and in Figs. 16, 17 and 18. Figure 16 summarizes the results on T-Tube efficiency. This efficiency appears to be independent of the experimental variables, at least within the ranges that were investigated. The value found is about 20 percent except for the two cases of no backstrap at low initial energies where the value is under 15 percent. However, the same two cases have a higher value of the system efficiency, Fig. 17, than the corresponding case at 25 KV which indicates that the differences are most probably due to an inaccurate value of the input energy. The value of 20 percent agrees with that reported by Paxton and Fowler (Ref. 19). As this value appears to be a characteristic of the T-Tube, it would seem that to obtain an improvement it would be necessary to use different apparatus, for example; the conical coaxial discharge, an electrodeless discharge, or the rail-type discharge. It
was originally thought that the efficiency of the secondary discharges was low in comparison with the first half cycle of a ringing discharge. This was tested by using a damped discharge which effectively had only one current peak. No change in efficiency was noticed.

The system efficiency data of Fig. 17 and the flow energy data of Fig. 18 show that the circuit using the backstrap is roughly twice as efficient as the circuit without the backstrap for all cases. This is surprising in that it was believed that the energy transfer in the case with no backstrap was due to ohmic heating and since this becomes less efficient with higher temperature (i.e. higher energy input) it would be expected that a higher proportion of the energy would be transferred by the magnetic driving of the arrangement using the backstrap as the input energy was increased. The explanation must lie in the fact that in addition to ohmic heating and the Lorentz force due to the magnetic field of the backstrap there exists the means of energy transfer by the force due to the magnetic field of the curved current sheet itself i.e. by what is known as the "kink instability". This exists even in the case of no backstrap and increases with increased current in the same way as the force due to the backstrap. It should be noticed that the removal of the backstrap meant an unavoidable increase in total inductance and hence a decrease in current. This may not be too serious since we can see that although the energy in the gas (Fig. 18) increases as the initial energy or the current is increased, the system efficiency remains unchanged.

At this point it would be worth while to discuss the possibility of improving the system efficiency of the T-Tube apparatus. Apparently the most obvious possibility has been to increase the magnetic field in the region of the electrodes by using suitably oriented coils electrically in series with the discharge. S. W. Kash (Ref. 5) has obtained an improvement by using a coil instead of a backstrap and has shown that to a certain extent the combination of the coil and backstrap was better than either alone. However, Gorowitz and Harned (Ref. 6) have found coils to be inferior to a backstrap and their plasma velocity data shows no indication that the effectiveness of the apparatus is improved as the magnetic field produced by the coils is increased. The penalty of using coils to enhance the magnetic driving is that the circuit inductance is increased, thereby decreasing the maximum current. An alternative method would be to decrease the external circuit inductance and resistance to the point where the T-Tube would represent the major portion of the load of the capacitor bank. One way of achieving this would be by using a bank of capacitors connected in parallel to obtain the desired low supply inductance and resistance, and by careful design, from the point of view of inductance and resistance, of connections, switches, and the T-Tube itself.
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</table>

**UNDAAMPED NO BACKSTRAP**

| 25           | 2.94             | 44                                   | 5.3                               | 12                  |                                       |                                       |                      |
| 25           | 4.95             | 39                                   | 5.4                               | 14                  |                                       |                                       |                      |
| 25           | 11.03            | 21                                   | 5.7                               | 27                  |                                       |                                       |                      |

**DAMPED B.S.**

| 25           | 3.04             | 10                                   | 2.0                               | 20                  |                                       |                                       |                      |
| 25           | 4.98             | 10                                   | 1.9                               | 19                  |                                       |                                       |                      |
| 25           | 10.10            | 10                                   | 2.8                               | 28                  |                                       |                                       |                      |
FIG. 1a  T-TUBE AS USED ORIGINALLY BY FOWLER.

FIG. 1b  T-TUBE WITH BACKSTRAP AS USED BY KOLB, KASH.
CURRENT MEASURING RESISTOR

RATE OF CHANGE OF CURRENT COIL

TO THE ARGON SUPPLY

NEEDLE VALVE

BUTYL PHTHALATE MANOMETER 0-20 mm.Hg.

TO THE PUMP

3 μF

SPARK GAP SWITCH

SEPARATE GROUND

FILTERS TO THE OSCILLOSCOPE

1000:1 VOLTAGE DIVIDER TO THE OSCILLOSCOPE

OSCILLOSCOPE TRIGGER PULSE

FIG. 2a SCHEMATIC DIAGRAM OF THE EXPERIMENTAL APPARATUS
FIG. 2b SCHLIEREN APPARATUS AS USED WITH THE T-TUBE.
FIG. 3  A VIEW SHOWING THE T-TUBE AND ITS CIRCUIT
FIG. 4 CROSS-SECTION THROUGH THE CURRENT MEASURING RESISTOR.

FIG. 5 DIAGRAM ILLUSTRATING THE VOLTAGE MEASURING CIRCUIT.
FIG. 6  THE ONE-DIMENSIONAL BLAST WAVE THEORY

\[ \gamma = \frac{5}{3} \]

\[ g(z) \propto u \text{, the velocity} \]
\[ h(z) \propto \rho \text{, the density} \]
\[ hg^2 + 2hk^2 \propto \frac{1}{2} pu^2 + E \text{, the energy} \]
Fig. 7  Schlieren rotating drum camera photographs of the runs with backstrap at 25 kv.
Fig. 8  Schlieren rotating drum camera photographs of the runs without backstrap at 25 kv.
FIG. 9 SHOCK TRAJECTORIES OF THE DISCHARGES WITH BACKSTRAP AT 25 KV.

- $x = 3.07 \text{ mm. Hg.}$
- $\Delta = 5.43 \text{ mm. Hg.}$
- $\circ = 10.35 \text{ mm. Hg.}$
FIG. 10 SHOCK TRAJECTORIES OF THE DISCHARGES
WITH NO BACKSTRAP AT 25 KV.

$X$ — 3.03 mm. Hg.
$\Delta$ — 5.04 mm. Hg.
$\bigcirc$ — 10.05 mm. Hg.
Fig. 11  The region the discharge for the case of the damped circuit with backstrap at 10 mm. Hg. pressure. The Schlieren trajectory of the shock wave and the motion of the current sheets are clearly illustrated.
Fig. 12  The undamped discharge with backstrap at 20 kv and 3 mm. Hg. pressure
Vertical scale: Upper trace-current 25,000 amp/cm
          Lower trace - voltage 500 volts/cm.
Horizontal scale: Time - 5 sec/cm
Fig. 13 The calibration discharge with the electrodes shorted. Scales are the same as in Fig. 12, except voltage - 200 volt/cm.
Fig. 14  The undamped discharge without backstrap at 20 kv and 5 mm. Hg. pressure.
Scales are the same as in Fig. 12
Fig. 15  The electrical measurements for a damped discharge.
Vertical scale: Upper trace - current - 20,000 amp/cm
Lower trace - voltage - 500 volt/cm
Horizontal scale: Time - 2 μsec/cm
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<tr>
<th>Voltage</th>
<th>Condition</th>
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**FIG. 16 THE T-TUBE EFFICIENCY**
FIG. 17 SYSTEM EFFICIENCY OF THE T-TUBE
FIG. 18 THE ENERGY REMAINING IN THE GAS FOLLOWING A DISCHARGE