von KARMAN INSTITUTE
FOR FLUID DYNAMICS

TECHNICAL NOTE 17

A POSSIBLE COMPROMISE BETWEEN ROCKET
AND ATMOSPHERIC BRAKING

by

L. MOULIN

RHODE-SAINT-GENESE, BELGIUM

JUNE 1964
VTH

<table>
<thead>
<tr>
<th>terugbezorgen voor:</th>
<th>terugbezorgen voor:</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
</tbody>
</table>

N.B. Uitleentermijn: boeken, rapporten, e.d. maximaal 3 maanden
losse tijdschriftnummers maximaal 2 weken
A POSSIBLE COMPROMISE BETWEEN ROCKET
AND ATMOSPHERIC BRAKING

by

L. MOULIN

JUNE 1964

The research reported in this document has been sponsored by
the Air Force Office of Scientific Research, through the
European Office of Aerospace Research, United States Air Forces.
ABSTRACT

An attempt has been made to optimize the problem of recovering an interplanetary vehicle through the earth's atmosphere. Optimum conditions are defined as the ones which would minimize the dead weight, which includes the fuel required for eventual rocket braking outside the atmosphere, and the mass which is ablated for heat protection during the flight into the atmosphere. It is shown that when chemical or nuclear propulsion is considered, pure atmospheric braking is always the best solution, but when electrical propulsion can be used, an optimum compromise between partial rocket and atmospheric braking may exist, depending upon the respective qualities of the propulsion system and ablating material.
TABLE OF CONTENTS

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nomenclature</td>
<td></td>
</tr>
<tr>
<td>I Introduction</td>
<td>1</td>
</tr>
<tr>
<td>II Impulse-type propulsion</td>
<td>4</td>
</tr>
<tr>
<td>1. Basic equations</td>
<td>4</td>
</tr>
<tr>
<td>2. Orbit correction</td>
<td>5</td>
</tr>
<tr>
<td>3. Optimum conditions</td>
<td>8</td>
</tr>
<tr>
<td>III Low thrust propulsion</td>
<td>9</td>
</tr>
<tr>
<td>1. Basic equations</td>
<td>9</td>
</tr>
<tr>
<td>2. Approximate solution</td>
<td>10</td>
</tr>
<tr>
<td>IV Flight into the atmosphere</td>
<td>15</td>
</tr>
<tr>
<td>1. Entry corridor</td>
<td>15</td>
</tr>
<tr>
<td>2. Ablative heat shield</td>
<td>15</td>
</tr>
<tr>
<td>V Results</td>
<td>17</td>
</tr>
<tr>
<td>1. Final mass ratio</td>
<td>17</td>
</tr>
<tr>
<td>2. Impulse type propulsion</td>
<td>17</td>
</tr>
<tr>
<td>3. Low thrust propulsion</td>
<td>18</td>
</tr>
<tr>
<td>VI Conclusions</td>
<td>22</td>
</tr>
<tr>
<td>References</td>
<td>25</td>
</tr>
<tr>
<td>Figures</td>
<td></td>
</tr>
</tbody>
</table>
NOMENCLATURE

c ratio of effective exhaust velocity to circular velocity at sea level

e orbit excentricity

\[ E(k, \phi) \] incomplete elliptic integral of the second kind

\[ F(k, \phi) \] incomplete elliptic integral of the first kind

\( g_0 \) acceleration of gravity at sea level

h non dimensional angular momentum

I specific impulse

m ratio of mass at burn out to initial mass

\( m_i \) initial mass

M mass of the vehicle

\( M_d \) total amount of mass dissipated during recovery

U non dimensional specific energy

V velocity

\( V_E \) ratio of entry velocity to local value of circular velocity

w ratio of velocity to circular velocity at sea level

\( \gamma \) flight path angle (positive below local horizon)

\( \Gamma \) non dimensional velocity increment

\( \delta \) angle between thrust and velocity vector (positive below velocity vector)

\( \zeta \) heat of ablation

\( \mu \) ratio of ablated mass to mass at entry

\( \xi \) hyperbolic excentric anomaly

\( \rho \) ratio of radius vector to planet's radius

\( \sigma \) ratio of distance along the trajectory to planet's radius

\( \tau \) acceleration of the system of propulsion

\( \omega \) angle between velocity increment and local horizon
Subscripts

$E$ values at entry into the atmosphere
$i$ values at the point where propulsion is initiated
$o$ values measured on the initial trajectory
I. Introduction

In most general terms, the problem of recovering safely a space vehicle at the surface of a planet can be viewed as the one of dissipating the total amount of kinetic energy which the vehicle has in the direct vicinity of the planet. One may indeed, consider that the small value of the velocity which is allowed for landing is well negligible compared to the velocity of the vehicle on the approaching trajectory.

If the planet is surrounded by no atmosphere, the only solution to the problem is to use rocket braking. But if the planet is surrounded by an atmosphere, advantage is taken of it to convert the kinetic energy into heat. The mass of fuel which would be otherwise required is then saved, what contributes to increase the payload of the vehicle. Unfortunately, part of the dissipated heat, which is equivalent to the kinetic energy, is transferred to the vehicle itself by convection and radiation. Therefore, the net gain in weight which is achieved by using the atmosphere for energy conversion is not equal to the saving in fuel, since an additional penalty must be paid in the form of heat shield, to protect the vehicle against aerodynamic heating.

The purpose of the present work is to investigate whether or not an intermediate technique would not offer better advantages as suggested by H.J. Allen in ref.1. Considering a given vehicle on a given trajectory, approaching some planet surrounded by an atmosphere, one can foresee that a partial rocket braking outside the atmosphere would alleviate the heating problem as a consequence of a lower entry velocity into the atmosphere, but at the cost of fuel expenditure. If one
considers all the existing possibilities, starting from one given initial trajectory, one can generate two curves, as shown in fig. 1, for the fuel consumption and the mass of the heat shield; it is indeed obvious that braking down to lower velocities would require more fuel, whilst the mass of the heat shield would be accordingly reduced.

It is then legitimate to consider that the overall expendable deadweight of a recoverable vehicle would be made of two components, fuel and heat shield respectively, which vary in opposite direction with respect to the entry velocity into the atmosphere. Optimum conditions for recovery can consequently be defined by the criterion that the total deadweight should be minimized in terms of the velocity at entry.

Numerous parameters are obviously involved in such a problem, but their number can actually be reduced to a few by a proper analysis. First of all, the problem can be split into two parts. The first one will concern the flight outside the atmosphere, where the problem then consists in transferring the vehicle from its initial trajectory to another one which would hit the atmosphere at lower velocity, but still matching entry corridor requirements. The second part would consist of the analysis of the flight into the atmosphere, to define entry corridor and best body shapes. Both solutions can be lumped together by using the convenient but arbitrary concept of the "top" of the atmosphere, which can be set at a proper altitude. The purpose of the both analysis should obviously be to minimize the deadweight component which they cover, i.e. fuel consumption for the first part and weight of heat shield for the second.

The course of action which has been taken here was to
avoid an overwhelming series of machine computation, and therefore approximate analytical solutions have been preferred for the investigation of the problem outside the atmosphere. For the second part, use has been made of data which are already available in the literature for entry corridor and heat shield calculations.

The analysis of orbital transfer had to be divided into two parts, depending upon the system of propulsion which is considered. If the case of chemical and nuclear propulsion can be approached on the classical grounds of the impulse assumption, low-thrust electrical propulsion, however, must be treated by other methods. Both cases are considered in the subsequent sections.

To somewhat simplify the analytical approach, the scope of the problem has been reduced to two body configurations with inverse square law gravitational fields; in addition, it has been considered that all trajectories would remain co-planar with the initial one.
II. Impulse - Type Propulsion

1. Basic Equations

In the case of two body problem, Keplerian orbits can be described by the following equations

\[ 2U = w^2 - \frac{2}{\rho} \quad (2.1) \]

\[ h = \rho w \cos \gamma \quad (2.2) \]

which are the specific energy and angular momentum integrals respectively. In the above equations \( w \) is the ratio of the actual velocity to the value of the circular velocity at the surface of the planet, \( \rho \) the ratio of the radius vector to the radius of the planet, and \( \gamma \) the angle between the velocity vector and the local horizon. Throughout the analysis, the angle \( \gamma \) is taken positive below the local horizon.

The eccentricity \( e \) of the orbit can be related to the two fundamental integrals by the relationship

\[ e^2 = 1 + 2U_h^2 \quad (2.3) \]

The top of the atmosphere will be set arbitrary at a distance \( \rho_E \) from the center of the planet. The definition of the entry corridor will consist of specifying the limiting values of the entry angle \( \gamma_E \), which correspond respectively to undershoot and overshoot boundaries, for a particular value of the entry velocity \( v_E \). By virtue of Eqs (2.1) and (2.2), this is equivalent to define the values of energy \( U_E \) and angular momentum \( h_E \) at the top of the atmosphere. The latter
quantities are subsequently used as boundary conditions for the analysis of the flight outside the atmosphere.

2. Orbit Correction

In general, the vehicle will be on a trajectory of energy $U$ and angular momentum $h$, and must be transferred on a particular trajectory $(U_E, h_E)$. To avoid, for practical cases, a process of long duration, which would result from using a multi-impulse technique, it has been considered here that the correction should be achieved by one impulse only, although this might result in a larger fuel consumption (ref. 2).

Transfer is achieved by producing a velocity increment $\Delta V$ at an angle $\omega$ with respect to the local horizon, as indicated in fig. 2. By geometrical considerations, one obtains the following relationships

$$r^2 + 2\Gamma \cos(\omega - \gamma) + 2U_1 - 2U_2 = 0 \quad (2.4)$$
$$\rho \Gamma \cos \omega = h_2 - h_1 \quad (2.5)$$

where subscripts 1 and 2 refer to initial and final trajectories respectively. The quantity $\Gamma$ is the non dimensional velocity increment, referred to the value of circular velocity at the surface of the planet.

It has been established in ref. 3 that the problem is subjected to a condition of possibility, which restricts the domain of possible variations of $\rho$. Physically, the condition states that the point of the first orbit at which transfer takes place must be in a domain bounded by the apogee and perigee circles of the final orbit. Analytically, the condition
can be written

\[
\frac{1-e_2}{2U_2} < p < \frac{1+e_2}{2U_2} \tag{2.6}
\]

For initial and final orbits entirely fixed, the system of Eqs (2.4), (2.5) contains 3 unknowns \( \rho, \Gamma, \omega \) and optimization is consequently possible. The easiest way to approach the solution is to state the problem in a slightly different way, and consider that the initial orbit is not entirely determined. One must in practice regard the energy as inexorably fixed by the particular mission in space, but the value of the angular momentum can be more or less chosen arbitrary. By imposing then the particular value of the radius vector at which transfer is expected to take place, \( h_1 \) can be regarded as the parameter of optimization.

Eliminating the \( \omega \) between Eqs (2.4) and (2.5), one obtains

\[
\Gamma^4 + 2\Gamma^2 f(h_1) + g(h_1) = 0 \tag{2.7}
\]

with

\[
f(h_1) = \frac{2h_1h_2}{\rho} - \frac{h}{\rho} - 2(U_1 + U_2)
\]

\[
g(h_1) = 4(U_1 - U_2)^2 + \frac{8(h_2-h_1)^2}{\rho^2} + \frac{8(h_2-h_1)}{\rho^2}(U_1 h_2 - U_2 h_1) \tag{2.8}
\]

For a given value of \( \rho \), the condition of optimum can be derived from Eq(2.7) as

\[
2\Gamma^2 \frac{df(h_1)}{dh_1} + \frac{dg(h_1)}{dh_1} = 0 \tag{2.9}
\]
or, deriving Eqs (2.7) and (2.8) with respect to \( h_1 \) and substituting in Eq (2.9):

\[
\Gamma^2 = \left(1 - \frac{2h_2}{h_1}\right)\left(\frac{2}{\rho} + 2U_1\right) + \left(\frac{2}{\rho} + 2U_2\right) \tag{2.10}
\]

together with the condition

\[
\frac{h_1}{h_2} > 2\left(1 + \frac{1+\rho U_2}{1+\rho U_1}\right) \tag{2.11}
\]

to secure a real value of \( \Gamma \).

Eliminating \( \Gamma \) between Eqs (2.7) and (2.10) yields the optimum value of \( h_1 \). One obtains the simple relationship

\[
\frac{h_1}{h_2} = \frac{w_1}{w_2} \tag{2.12}
\]

or using Eq (2.2)

\[
\cos^2 \gamma_1 = \cos^2 \gamma_2 \tag{2.13}
\]

which indicates that optimum conditions are obtained by tangential impulses. One can easily verify that Eq (2.12) automatically satisfies condition (2.11). The value of the velocity increment \( \Gamma \) is then easily evaluated and can be written as

\[
\Gamma = \sqrt{\left(2U_1 + \frac{2}{\rho}\right)^{1/2} - \left(2U_2 + \frac{2}{\rho}\right)^{1/2}} \tag{2.14}
\]

where upper signs correspond to decreasing energies, lower signs to increasing energies.
3. **Optimum conditions**

Having obtained optimum conditions for a particular value of the radius vector at the point of transfer, the latter parameter can now be varied. From Eq (2.15) one obtains easily

\[
\frac{d\Gamma}{d\rho} = \frac{\Gamma}{\rho^2 w_1 w_2}
\]

which is always positive. Consequently lowest possible fuel consumption will be obtained by tangential transfer at the perigee of the initial orbit. In the present problem, since the initial orbit is expected to cut the top of the atmosphere, the smallest possible value of \( \rho \) is actually \( \rho_E \). Since braking only will be considered, the optimum value of \( \Gamma \) will be given by

\[
\Gamma = \left(2U + \frac{2}{\rho_E}\right)^{1/2} - \left(2U_E + \frac{2}{\rho_E}\right)^{1/2}
\]

The corresponding angular momentum required for the initial orbit will be given by

\[
h = h_E \left[\frac{2U+2}{\rho_E}\right]^{1/2}
\]

The ratio \( m \) of the mass at burn out to the initial mass is then given by the simple relationship

\[
m = \exp \left\{ -\frac{\Gamma V}{g_0 I} \right\}
\]
III. Low Thrust Propulsion

1. Basic equations

For low thrust propulsion, a simplified solution has been sought, assuming that the value of the thrust would be a constant, set at a constant angle to the velocity vector. With the notations of fig 3, the non dimensional equations of motion can be written as

\[ \frac{m}{\sigma} \frac{dw^2}{d\sigma} = \frac{m \sin \gamma}{\rho^2} + \tau \cos \delta \]  \hspace{1cm} (3.1)

\[ mw^2 \frac{d\gamma}{d\sigma} = \frac{m \cos \gamma}{\rho} \left( \frac{1}{\rho} - w^2 \right) + \tau \sin \delta \]  \hspace{1cm} (3.2)

\[ \frac{d\rho}{d\sigma} = - \sin \gamma \]  \hspace{1cm} (3.3)

\[ w \frac{dm}{d\sigma} = - \frac{\tau}{c} \]  \hspace{1cm} (3.4)

where \( m \) is the ratio of the actual value of the mass to the initial one, \( \tau \) the ratio of thrust to initial weight measured at sea level, \( c \) the ratio of effective exhaust velocity to circular velocity at sea level, \( \sigma \) the ratio of distance along the flight path to planet's radius.

The above system of equations can be transformed by introducing the definitions (2.1), (2.2), (2.3) of energy, angular momentum and excentricity. One obtains easily

\[ \frac{dU}{d\sigma} = \frac{\tau}{m} \cos \delta \]  \hspace{1cm} (3.5)
10.

\[
\frac{\,dh\,}{d\sigma} = \frac{\tau \rho}{mw} \cos(\gamma + \delta)
\] (3.6)

\[
\frac{\,de^2\,}{d\sigma} = \frac{4 \tau h}{mw} \left[ \frac{h}{\rho w^2} (\rho w^2 - 1) \cos \delta - U \rho \sin \psi \sin \delta \right]
\] (3.7)

2. **Approximate solution**

Since \( U, h \) and \( e \) are invariants for a keplerian orbit, and since \( \tau \), for electrical propulsion, can be regarded as a small quantity, the above system of equations immediately suggests the use of a perturbation method to obtain an approximate solution for the problem. Integration requires to distinguish between the different types of orbits. For the present purposes, the analysis can be restricted to the case of hyperbolic orbits. The detailed analysis has been developed in ref. 4 and leads to the following results. Introducing the hyperbolic excentric anomaly \( \xi \), by the relationship

\[
e \cosh \xi = 1 + 2U \rho
\] (3.8)

the following differential equation can be derived, for quantities measured on the unperturbed orbit, denoted by the subscript 0:

\[
\frac{d\xi_0}{d\sigma} = -2U_0 \left[ e_0^2 \cosh^2 \xi_0 - 1 \right]^{-1/2}
\] (3.9)

Introducing perturbations of the form

\[
U = U_0 + \tau U_1
\] (3.10)
in the system of equations (3.5) to (3.7) and eliminating \( \sigma \) with the aid of Eq (3.9) results into a system of equations which can be rather easily integrated. The result of the integration obtained in ref.4 can be written as follows:

\[
c(1-\mu) = \frac{\pi}{(2U_0)^{3/2}} \left[ f(\xi) - f(\xi_i) \right] \tag{3.11}
\]

\[
U = U_0 + \frac{r \cos \delta}{2U_0} \left[ H(\xi) - H(\xi_i) \right] \tag{3.12}
\]

\[
h = h_0 + \frac{r}{(2U_0)^{3/2}} \left[ f(\xi) - f(\xi_i) \right] \cos \delta \]
\[-\left[ e(\xi) - e(\xi_i) \right] \sin \delta \right]
\tag{3.13}
\]

In the above equations, the subscript \( o \) has been dropped for \( \xi \), which still represents the excentric anomaly measured on the initial orbit; \( \xi_i \) is the particular value at the point where propulsion is started.

The auxiliary functions are defined by the following relationships:

\[
f(\xi) = \xi - e_0 \sinh \xi \tag{3.14}
\]

\[
H(\xi) = \int_{\xi_0}^{\xi} \left[ e_0^2 \cosh \eta - 1 \right]^{1/2} d\eta =
- e_0 \left[ (1 - \frac{1}{e_0^2}) F(k, \phi) - E(k, \phi) - \frac{\tanh \xi}{e_0^2 \cosh^2 \xi} \right]^{1/2} \xi \tag{3.15}
\]

\[
I(\xi) = \int_{\xi_0}^{\xi} \left[ e_0^2 \cosh^2 \eta - 1 \right]^{-1/2} d\eta = - \frac{1}{e_0^2} \left[ F(k, \phi) \right]_{\xi} \tag{3.16}
\]
\[ g(\xi) = -(e_o^2-1)^{1/2} \left[ H(\xi) + 2I(\xi) - 2\sinh^{-1}\left( \frac{e \sinh \xi}{\sqrt{e_o^2-1}} \right) \right] \]

(3.17)

\[ j(\xi) = \frac{3}{2} \cosh^{-1}(e_o \cosh \xi) - 2(1 - \frac{e_o \cosh \xi}{4})(e_o^2 \cosh^2 \xi - 1)^{1/2} \]

(3.18)

In the double signs, the upper signs corresponds to the case of a vehicle moving initially towards perigee, lower signs to an initial motion in the opposite direction.

Argument and modulus of the elliptic integrals are respectively given by

\[ \phi = \sinh^{-1}\left( \frac{1}{\cosh \xi} \right) \]

(3.19)

\[ k = \frac{1}{e_o} \]

(3.20)

For given initial and final orbits, the system of equations (3.11) to (3.13) contains four unknowns \( m, \delta, \xi \) and \( \xi_i \). One of these parameters is consequently free and can be used for optimization of the fuel consumption, given by the quantity \( (1-m) \).

It has been shown in ref. 5 that optimization of fuel consumption was obtained for tangential thrust, i.e. \( \delta = 0^\circ \) for increasing energies and \( \delta = 180^\circ \) for decreasing energies.

Knowing that, optimization can be pushed one step further, as done above for impulse-type propulsion, by considering that the value of the
angular momentum of the initial orbit may to some extent be left arbitrary, whilst the value of the energy is fixed, for one particular mission. Putting \( |\cos \delta| = 1 \) in Eqs (3.11) to (3.13), there still remain four unknowns, namely \( m, h_o, \xi \) and \( \xi_1 \). The initial angular momentum \( h_o \) can now be regarded as a free parameter for the next step in optimization.

The discussion of this particular problem (ref. 5) shows that in the case of a vehicle moving initially towards perigee, the value of \( h_o \) should be selected in such a way that low thrust propulsion would terminate as close as possible to perigee; in the case of initial motion in opposite direction, \( h_o \) should be such that propulsion be initiated at perigee.

For the present purposes, the only case of interest is the motion of a vehicle towards perigee, and propulsion is strictly used with the purpose of decreasing the energy. Consequently, upper signs should be used into the equations, and the first part of the optimization shows that one should have

\[ \cos \delta = -1 \]

or retrothrust. The second part of the optimization indicates that propulsion should terminate at perigee. Since the excentric anomaly \( \xi \) is a parameter which is measured on the initial orbit, the latter condition can only be fulfilled approximately by defining the terminal point as the one for which \( \xi = 0 \).

In view of this, the system of equations (3.11) to (3.13) can be written as
\[ c(1-m) = \frac{\tau}{(2U_0)^{3/2}} (e_0 \sinh \xi - \xi) \tag{3.21} \]

\[ U_E = U_0 - \frac{\tau}{2U_0} \int_0^{\xi} \left[ e_0 \cosh^2 n - 1 \right]^{1/2} \, dn \tag{3.22} \]

\[ h_E = \left[ \frac{e_0^2 - 1}{2U_0} \right]^{1/2} + \frac{\tau}{(2U_0)^{5/2}} g(\xi, e_0) \tag{3.23} \]

which must be solved numerically for \(1-m\), \(e_0\) and \(\xi\) as unknowns.
IV. Flight into the Atmosphere

1. Entry corridor

To define the boundaries of the entry corridor, it has been considered for the present applications, that a lifting vehicle would be used, with a lift to drag ratio of 0.5, positive at undershoot boundary, and negative at overshoot boundary. The undershoot boundary corresponds to a maximum deceleration of 10 g's. Moreover, the top of the atmosphere has been arbitrarily set at a non-dimensional radius of $\rho_E = 1.02$.

The data for these boundaries have been calculated from ref. 6, and are represented on fig. 4. The values of energy and angular momentum $U_E$ and $h_E$ required at the top of the atmosphere have been evaluated for the mean value of $\gamma_E$ between the two corridor boundaries. Consequently, all parameters for entry conditions become a function of the entry velocity $V_E$ only.

2. Ablative heat shield

For heat shield calculations, use has been made of data which are already available in the literature. Since atmospheric entry has been considered to take place at very high speeds, radiative heating must play an important role. Since small nose radii appear to be then a better solution to the problem, conical shapes can be regarded as realistic.

Values of the ratio of the mass dissipated by ablation, to the initial mass, are already available in the literature for such conical shapes, and are given in terms of the entry velocity, for different types of ablating materials. Data for
teflon and quartz can be found in ref. 7, those relative to graphite have been given in ref. 8
V. Results

1. Final mass ratio

The symbol \( m \) used in the previous sections represents the ratio of the mass after propulsion, \( M_{b_o} \), to the initial mass \( m_i \). Denoting by \( \mu \) the ratio of ablated mass to the initial mass at entry, i.e. \( M_{b_o} \), and by \( M_d \) the total amount of mass which has been dissipated, i.e. fuel consumed plus mass ablated, one easily establishes the relationship

\[
\frac{M_d}{m_i} = 1 - m(1-\mu) \quad (5.1)
\]

the ratio \( m \), originated by propulsion, is, after optimization, a function of the energy of the initial orbit \( U_{o} \), of the specific impulse \( I \) of the system of propulsion, of the level of acceleration \( \tau \) delivered by the system, and of the selected entry velocity in the atmosphere \( V_E \). The ratio \( \mu \) is a function of the entry velocity, and of the characteristics of the ablative material which can be represented by the heat of ablation \( \varsigma \). Therefore, Eq. (5.1) can be more explicit written as

\[
\frac{M_d}{m_i} (U_{o}, I, V_E, \varsigma, \tau) = 1 - m(U_{o}, I, V_E, \varsigma, \tau) [1 - \mu(V_E, \varsigma)] \quad (5.2)
\]

which shows that finally, five parameters are left in the problem, the influence of which must be explored numerically.

2. Impulse type propulsion

For impulse type propulsion, the level of acceleration
\( \tau \) cannot be considered in the problem. The values of \( m \) are evaluated from Eqs (2.18) and (2.16); values of \( \mu \) are taken from refs 7 and 8. Calculations have been carried out for three particular values of the energy of the initial orbit, \( U_0 = 1.6, 2, 2.4 \), for ablating materials as teflon, quartz and graphite, and for two particular values of the specific impulse \( I = 350, 700 \) secs, which are representative of chemical and nuclear propulsion respectively.

The values of the ratio of dissipated mass to initial mass are given in fig.5 to 7.

Results show clearly that the amount of mass dissipated is kept to a minimum if no rocket braking is applied and consists only of the amount of mass ablated. Moreover, it appears that there is very little difference between the curves for different ablating materials. In fact, specific impulses of that order are too low, and consequently fuel expenditure is too large to compete with atmospheric braking. It indicates that the estimated curve for fuel consumption in fig.1 always has a larger order of magnitude that the one for ablation.

3. **Low thrust propulsion**

The calculations have been carried out for identical conditions, in the case of low thrust propulsion. The ratio \( m \) must now be computed with the aid of the system of equations (3.5) to (3.7). Same values as before have been kept for the energy of the initial orbit; the specific impulse has now been set successively at 3,000, 5,000, 7,000 and 10,000 secs to be representative of various types of electrical propulsion systems;
the level of acceleration has been allowed to vary between $10^{-2}$ and $10^{-4}$.

The first salient feature is that for the conditions which have been explored, the mass ratio $m$ turns out to be practically independent of the acceleration $T$. Only the last decimal places are affected by a change in $T$, and the respective curves for $m$ cannot be differentiated on a graph at reasonable scale. However, low thrust propulsion must be initiated at a larger distance from the earth if the acceleration is smaller, which is physically obvious. The values of the ratio of dissipated mass to initial mass are represented on figs. 8 to 10, which are then independent of the acceleration of the propulsion system.

Large differences now exist between the different ablating materials, which indicates that for the higher specific impulses considered, fuel consumption becomes of the same order of magnitude than ablation.

Considering first the case of teflon, one sees that partial rocket braking outside the atmosphere always leads to a better solution than pure atmospheric braking. Even for a specific impulse of 3,000 secs only, a minimum appears clearly on the curves, for a lower entry velocity than the one one would have without rocket braking. The trend is more and more definite for larger values of the specific impulse.

An ablating material like quartz exhibits different properties. Results indicate that rocket braking is not advantageous except for large specific impulses and large values of entry velocity, where the curves tend to level off.
Under such circumstances only, moderate rocket braking could be efficiently used.

For graphite on the other hand, the results clearly indicate that rocket braking should not be considered, pure atmospheric braking always offering the best solution. It should be pointed out that the values of $\mu$ are actually underestimated, since it has been assumed in refs 7 and 8 that the body remained conical all the time. A change in body shape resulting from ablation of the nose would result in larger values of $\mu$, and make the final curves look more like those for teflon.

If one considers those cases where rocket braking is applicable, one should remember that the values of the angular momentum of the initial orbit are imposed by the optimization process. Knowing then both energy and angular momentum of the best initial orbit, one may calculate what its perigee radius should be, which is now obviously a function of the acceleration $\tau$. The non dimensional values of the perigee of the initial orbit have been evaluated for values of the initial energy equal to $1.6$, $1.8$ and $2$, and are represented in terms of the entry velocity on figs. 11 to 13.

The conclusions one may draw are that perigee radius of the initial orbit must be larger if one intends to hit the atmosphere at a lower velocity, and for a given velocity at entry, should be larger if the acceleration delivered by the system of propulsion is lower. Moreover, for a fixed value of the entry velocity, perigee radius must be smaller if the initial energy is larger. In any case, perigee radii are well outside velocity is required. Consequently, one salient feature of
low thrust rocket braking outside the atmosphere is that the return trajectory towards the earth should be selected so as to deliberately miss the atmosphere, to allow for braking. This in fact changes the boundary conditions which would normally be imposed on the analysis of transfer from one planet to earth.
VI. Conclusions

To optimize the recovery of an interplanetary vehicle, the dead weight should in general be regarded as made up of two components, the fuel which would be required for eventual rocket braking, and the amount of mass which is ablated during the flight in the atmosphere. Since both components vary in opposite direction with the entry velocity, the overall dead weight can be expected to have a minimum in terms of the entry velocity into the atmosphere. One may split the problem in two parts, one related to calculation the trajectories outside the atmosphere, the other one devoted to the study of the flight within the atmosphere. Both particular analyses should be conducted so as to define conditions which minimize the amount of mass which has to be dissipated in each phase.

For the present purposes, approximate analytical techniques had to be developed for low thrust propulsion, in order to avoid numerous numerical calculations. Those solutions are suited for optimization of the fuel consumption. Data on ablation shields have been taken from the literature for conical body shapes which represent a realistic solution for entry at very large velocities, well in excess of earth parabolic velocity.

It can be concluded from the present analysis that pure atmospheric braking should always be favoured, for entry velocities up to 20 km/sec, if chemical or nuclear propulsion systems are considered. Earth bound and lunar missions can reasonably be included in that category.

However, if low thrust electrical propulsion is used, an
optimum compromise may exist between rocket and atmospheric braking. Optimum conditions are strongly dependent of the properties of the ablating material, and independent, as far as the mass ratio is concerned, of the level of acceleration which can be delivered by the propulsion system.

For ablative materials having the properties of teflon, it appears that for entry velocity up to 20 km/sec, partial rocket braking should always be used, even for specific impulses as low as 3,000 secs. For materials like graphite, pure atmospheric braking is always the best solution. Materials like quartz appear to be intermediate cases, whereby the optimum solution tends to the one for teflon, when the specific impulse is high, to the one for graphite when the specific impulse is low.

In such cases where rocket braking would be retained as an optimum solution, attention must be paid to the fact that the initial trajectory of the vehicle must be selected so as to deliberately miss the earth’s atmosphere. Consequently, the choice of a return trajectory does not depend only on the entry corridor requirements, but also on the possible compromise between rocket and atmospheric braking. Since low thrust propulsion is very likely to be used for fast interplanetary travel, a return trip can be viewed as a low thrust powered flight, ranging from a given planet down to the vicinity of the earth, where thrust reversal would be achieved before atmospheric entry.

From all this, it can be concluded that the decision of use or no use of partial rocket braking outside the atmosphere will finally be dictated by the availability of ablating
materials of high specific heat, such as graphite. If such materials become of current use, pure atmospheric braking turns out to always be the best solution. If not, it appears that partial rocket braking might be found desirable, since it would offer the further advantage of bringing the problem of atmospheric entry at very high speeds closer to current technology.
REFERENCES


Fig. 1
Schematic Variation of deadweight Components

Fig. 2
Parameters of impulse-type orbital transfer
Fig. 3
Parameters of low thrust propulsion

\[ \frac{W}{C_D A} = 100 \text{ lb/ft}^2 \]

\[ |L/D| = 0.5 \]

Fig. 4 Entry corridor
Fig. 5 - Impulse-type propulsion - Ratio of total dissipated mass to initial mass
Fig. 6 - Impulse-type propulsion (continued)
Fig. 7 - Impulse-type propulsion (continued)
Fig. 8 - Low thrust propulsion - Ratio of total dissipated mass to initial mass
Fig. 9 - Low thrust propulsion (continued)
Fig. 10 - Low thrust propulsion (continued)
Fig. 11 - Perigee radius required for the initial trajectory
Fig. 12 - Perigee radius (continued)
Fig. 13 - Perigee radius (continued)
A POSSIBLE COMPROMISE BETWEEN ROCKET AND ATMOSPHERIC BRAKING, by L. MOULIN.

An attempt has been made to optimize the problem of recovering an interplanetary vehicle through the earth's atmosphere. Optimum conditions are defined as the ones which would minimize the dead weight, which includes the fuel required for eventual rocket braking outside the atmosphere, and the mass which is ablated for heat protection during the flight, into the atmosphere. It is shown that when chemical or nuclear propulsion is considered, pure atmospheric braking is always the best solution, but when electrical propulsion can be used, an optimum compromise between partial rocket and atmospheric braking may exist, depending upon the respective qualities of the propulsion system and ablating material.
A POSSIBLE COMPROMISE BETWEEN ROCKET AND ATMOSPHERIC BRAKING, by L. MOULIN.

An attempt has been made to optimize the problem of recovering an interplanetary vehicle through the earth's atmosphere. Optimum conditions are defined as the ones which would minimize the dead weight, which includes the fuel required for eventual rocket braking outside the atmosphere, and the mass which is ablated for heat protection during the flight, into the atmosphere. It is shown that when chemical or nuclear propulsion is considered, pure atmospheric braking is always the best solution, but when electrical propulsion can be used, an optimum compromise between partial rocket and atmospheric braking may exist, depending upon the respective qualities of the propulsion system and ablating material.

June 1964.