SIMULATION OF A TRAVELLING
SONIC BOOM IN A PYRAMIDAL HORN

by

James Joseph Gottlieb


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Abstract

In order to assess current societal problems associated with the sonic boom, a horn-type simulator was constructed at the Institute for Aerospace Studies, University of Toronto (UTIAS). The simulator horn is in the form of a horizontal concrete pyramid, which is 25 m long and has a 3-m-square base. At its apex a specially-designed valve is used to control the mass-flow rate of air from a high-pressure reservoir into the horn where the flow generates a simulated sonic boom or travelling N-wave of suitable amplitude and duration, and acceptably-short rise time. Alternatively, a shock-tube driver can be installed at the apex and used for generating short-duration and rapid rise-time sonic booms. For the mass-flow-valve mode of operation a high-frequency sound absorber can be installed near the apex of the horn to filter out of the passing N-wave undesirable jet noise that is produced by the high-speed turbulent flow at the valve. At the large end of the horn a specially-designed reflection eliminator in the form of a recoiling-type porous piston is used to adequately minimize the objectionable reflection of the incident wave. The operation and performance of the simulator are reported.

Comprehensive analyses have been made to describe the wave motion in the horn for both the shock-tube and mass-flow-valve modes of operation, as well as for designing certain parts of the facility such as the reflection eliminator. These analyses are not only applicable to the UTIAS facility but apply as well to other simulators that are in current use in England, France, Germany, and the United States. The analyses have been substantiated by experimental data obtained from the UTIAS and other simulators. A number of worthwhile experiments on the effects of sonic boom on humans and structures have already been made in the UTIAS facility.
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**TABLE 1: MASS-FLOW RATE PER UNIT AREA THROUGH THE VALVE THROAT**
FIGURES

APPENDIX A: DESCRIPTION OF THE TRAVELLING-WAVE HORN

APPENDIX B: A CALIBRATOR FOR PRESSURE TRANSDUCERS

APPENDIX C: PERTURBATION MASS-FLOW RATE

APPENDIX D: RECTANGULAR SHOCK TUBE

APPENDIX E: PYRAMIDAL SHOCK TUBE

APPENDIX F: PYRAMIDAL-RECTANGULAR SHOCK TUBE
List of Symbols

a
speed of sound for air

\( a_0 \)
sound speed of a gas in a high-pressure reservoir

\( \bar{a}_0 \)
initial sound speed of a gas in a high-pressure reservoir

\( a_1 \)
sound speed of the gas in the channel of a shock tube

\( a_2 \)
sound speed of the gas in the driver of a shock tube

A
cross-sectional area of a pyramidal horn

\( A_0 \)
cross-sectional area of the pyramidal horn at radius \( r_0 \); cross-sectional area at the entrance of the control volume for the pyramidal horn

\( A_e \)
cross-sectional area of the pyramidal horn at radius \( r_e \); cross-sectional area at the exit of the control volume for the pyramidal horn

\( A_s \)
cross-sectional area of the pyramidal horn at which the upstream-facing shock is located

\( A_x \)
minimum flow area at the throat of the mass-flow valve

\( \bar{A}_x \)
maximum throat area of the mass-flow valve when it is fully open

\( \bar{A} \)
cross-sectional area ratio \( A_s/A_x \) for the upstream-facing shock wave

b
width of the square plug of the mass-flow valve

c
height of the pyramidal face of the plug for the mass-flow valve

c
ratio of the truncated portion of the shock-tube channel to the length of the driver (\( r_e/R_e \) and \( r_e/x_e \))

D
distortion parameter defined by Eq. 2.37

D
deviation of the actual wall of the pyramidal horn from the desired planar surface

\( F(x,\theta) \)
Whitham's F-function for an aircraft body or equivalent body of revolution as defined by Eq. 1.1

h
aircraft altitude measured from ground level
<table>
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<th>Symbol</th>
<th>Definition</th>
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<tr>
<td>H( )</td>
<td>unit step function (H(y) = 0 if y &lt; 0, H(y) = 1 if y &gt; 0)</td>
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<tr>
<td>K</td>
<td>reflection factor for a sonic boom reflecting from the ground</td>
</tr>
<tr>
<td>L</td>
<td>lift distribution for an aircraft</td>
</tr>
<tr>
<td>m</td>
<td>mass-flow rate</td>
</tr>
<tr>
<td>( \dot{m}_* )</td>
<td>mass-flow rate per unit area at the throat of the mass-flow valve</td>
</tr>
<tr>
<td>m_0</td>
<td>mass-flow rate entering the control volume</td>
</tr>
<tr>
<td>m_e</td>
<td>mass-flow rate leaving the control volume</td>
</tr>
<tr>
<td>( \Delta m )</td>
<td>perturbation mass-flow rate (m_e - m_0)</td>
</tr>
<tr>
<td>( \overline{\Delta m} )</td>
<td>maximum perturbation mass-flow rate</td>
</tr>
<tr>
<td>M</td>
<td>flight Mach number of an aircraft</td>
</tr>
<tr>
<td>( M_* )</td>
<td>Mach number of the flow in front of the upstream-facing shock wave and of the upstream-facing shock itself.</td>
</tr>
<tr>
<td>( \tilde{M}_* )</td>
<td>Mach number of the flow at the throat of the mass-flow valve</td>
</tr>
<tr>
<td>( \bar{M}_* )</td>
<td>initial flow Mach number at the throat of the mass-flow valve</td>
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<tr>
<td>( N_* )</td>
<td>normalized variable which describes the throat-area variation and resulting mass-flow-rate distribution for the mass-flow valve</td>
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<tr>
<td>( \tilde{N}'_* )</td>
<td>magnitude of the discontinuous change in ( N_* )</td>
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<td>p</td>
<td>atmospheric pressure</td>
</tr>
<tr>
<td>p_e</td>
<td>pressure of the gas in the high-pressure reservoir</td>
</tr>
<tr>
<td>( \bar{p}_e )</td>
<td>initial value of the pressure of the gas in the high-pressure reservoir</td>
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<tr>
<td>p_1</td>
<td>absolute pressure of the gas in the shock-tube channel</td>
</tr>
<tr>
<td>p_2</td>
<td>absolute pressure of the gas in the shock-tube driver</td>
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<tr>
<td>p_a</td>
<td>atmospheric pressure at the aircraft cruising altitude</td>
</tr>
<tr>
<td>p_g</td>
<td>atmospheric pressure at ground level</td>
</tr>
<tr>
<td>( \Delta p )</td>
<td>perturbation pressure or overpressure</td>
</tr>
<tr>
<td>( \overline{\Delta p} )</td>
<td>peak overpressure</td>
</tr>
<tr>
<td>( \tilde{\Delta p} )</td>
<td>overpressure transition profile for a shock-wave front</td>
</tr>
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</table>
\( \Delta p_e \) initial pressure difference across the diaphragm in a shock tube

\( \Delta p_1 \) overpressure for the channel gas of the rectangular shock tube

\( \Delta p_2 \) overpressure for the driver gas of the rectangular shock tube

\( \Delta p_{\text{rms}} \) root-mean-square overpressure of the jet-noise superimposed on the simulated sonic boom

\( \Delta p_{\text{rms}} \) peak value of \( \Delta p_{\text{rms}} \)

\( \Delta p_2 \) peak overpressure of the first shock of the N-wave which has a finite rise time (ratio b/c equals \( \infty \))

\( \Delta p_3 \) peak overpressure of the first shock of the N-wave which has a finite rise time (ratio b/c equals 0.89)

\( P \) atmospheric pressure

\( P_e \) pressure of the gas in a high-pressure reservoir

\( P_1 \) flow pressure in front of the upstream-facing shock wave

\( P_2 \) flow pressure just behind of the upstream-facing shock wave

\( r \) radial distance

\( r_e \) radial location at which the valve or the shock-tube driver is joined to the pyramidal horn

\( r_e \) diaphragm location

\( r_s \) radius to the entrance of the control volume

\( r_1 \) initial radial location of a fluid particle

\( r_e \) radius at which the porous piston reflection eliminator is located

\( r_e \) radius to the exit of the control volume

\( r_s \) radial location of the upstream-facing shock wave

\( \bar{r} \) radial location (constant) at which the signature of a wave is known or specified

\( \bar{r} \) radial location of the matching of the gasdynamic and acoustic analyses

\( \Delta r \) displacement of a fluid particle

\( R \) radial distance
flow resistance of a porous material or a slotted or perforated plate (equal to the pressure drop across the porous material divided by the average flow speed through the material)

radial location of the contact surface

diaphragm location and also the length of a shock-tube driver
distance from a sound source
cross-sectional area distribution for an aircraft or its equivalent body of revolution
time
duration of the flow discharge from the high-pressure reservoir through the mass-flow valve into the pyramidal horn, and the duration of the resulting wave in the horn
time constant of a pressure transducer's response to a step input
temperature of the gas in a high-pressure reservoir
initial temperature of the gas in a high-pressure reservoir
velocity of the flow through a porous material
velocity of the flow at the throat of the mass-flow valve
initial velocity of the flow at the throat of the mass-flow valve
perturbation or particle velocity
particle velocity of the gas in the channel of a rectangular shock tube
particle velocity of the gas in the driver of a rectangular shock tube
velocity of the porous piston of the reflection eliminator
velocity of the wall of a reflection eliminator
volume of the high-pressure reservoir
volume of the supersonic-flow portion of the control volume
volume of the subsonic-flow portion of the control volume
planar distance
displacement of the porous piston of the reflection eliminator

distance measured along an aircraft body from its nose

value of \( x_0 \) which maximizes the integral of \( F(x,\theta) \) in Eqs. 1.2 and 1.3

diaphragm location and driver length

dummy variable for integration

displacement of the plug for the mass-flow valve’s opening and closing stages

special distance defining the minimum throat area for the mass-flow valve

specific impedance ratio \( a_2\rho_2/a_1\rho_1 \)

non-dimensional variable defined by Eq. C5

ratio of the specific heats for a perfect gas

specific heats ratio for the gas in the channel of a rectangular shock tube

specific heats ratio for the gas in the driver of a rectangular shock tube

azimuthal angle for the lift distribution of an aircraft

half-angle for a pyramidal or wedgy type reflection eliminator

wavelength

wavelength of an N-wave

initial wavelength of an N-wave at radius \( \bar{r} \)

density

density of the gas in a high-pressure reservoir

density of the flow at the entrance of the control volume for which the radius equals \( r_0 \)

initial density of the gas in a high-pressure reservoir
\( \rho_1 \) density of the gas in the channel of a rectangular shock tube
\( \rho_1 \) density of the gas just in front of the upstream-facing shock wave
\( \rho_2 \) density of the gas in the driver of a rectangular shock tube
\( \rho_r \) density of the gas in the supersonic-flow portion of the control volume
\( \rho_* \) density of the gas at the throat of the mass-flow valve
\( \sigma \) mass per unit area for the walls of the reflection eliminator
\( \tau \) time at which \( N'_*(\tau) \) is discontinuous
\( \tau_0 \) duration of the flow from the high-pressure reservoir through the mass-flow valve into the pyramidal horn, and the duration of the resulting wave in the horn
\( \tau_1 \) rise time for a shock, defined as 125% of the time for the overpressure to increase from 10% of the total change to 90%
\( \tau_2 \) rise time for the overpressure to increase from zero to its maximum value
\( \phi \) velocity potential of a gas
\( \phi_1 \) velocity potential of the gas in the channel of the rectangular shock tube
\( \phi_2 \) velocity potential of the gas in the driver of the rectangular shock tube

Some useful conversion factors

\[
\begin{align*}
1 \text{ m} & = 39.37 \text{ in} = 3.2808 \text{ ft} \\
1 \text{ m}^2 & = 1550 \text{ in}^2 = 10.764 \text{ ft}^2 \\
1 \text{ m}^3 & = 6.1023 \times 10^4 \text{ in}^3 = 35.314 \text{ ft}^3 \\
1 \text{ kgm} & = 2.2046 \text{ lb} \\
1 \text{ N/m}^2 & = 0.02086 \text{ psf} \\
1 \text{ psf} & = 47.88 \text{ N/m}^2 \\
1 \text{ atm} & = 1.013 \times 10^5 \text{ N/m}^2 = 2116.3 \text{ psf} \\
1 \text{ kgm/m}^3 & = 0.0624 \text{ lb/ft}^3 
\end{align*}
\]
1. INTRODUCTION

1.1 Sonic Boom

The impulsive noise experienced by an observer exposed to the pressure disturbance associated with the shock-wave system created by an aircraft moving supersonically is commonly known as the sonic boom or sonic bang. However, the sonic boom is not peculiar just to supersonic aircraft. It is created by any supersonically-moving projectile, and also by any rapid release of energy in the form of an explosion. For example, it is possible for an observer to hear the sonic boom from a near-flying supersonic bullet, and also to hear the sonic boom produced by the rapid discharge of gases from the gun barrel. It is also worth mentioning that, as the command module containing the astronauts is moving supersonically when it first enters the Earth's atmosphere from outer space, the module generates a sonic boom that can be heard aboard recovery ships. (As the sonic boom outraces the decelerating module, it is heard before the module arrives). The impulsive sound from the crack of a whip, the burst of a pressurized bubble or balloon, the spark discharge of electricity in the laboratory or in the atmosphere (lightning) are other examples of sonic boom. Each of these sonic booms, ranging from the report of a firearm to thunder, has its own characteristic sound according to the shape of its pressure signature, and this sound can be manyfold more startling than that from a current supersonic transport aircraft (SST).

In the case of supersonic aircraft the shape of the sonic boom signature and thus its loudness depends on many factors (Refs. 1,2). Some are associated with the design of the aircraft such as its volume and lift distributions, and some are involved with the aircraft operation such as its speed, acceleration, flight path and distance from the observer. Others are associated with the structure of the atmosphere between the aircraft and the observer, such as temperature and wind gradients and also turbulence in the atmospheric boundary layer. Near the ground the pressure signature is also affected by the reflection of the sonic boom from the ground and by its reflection and diffraction around structures and other topographical features.

The shock-wave pattern created by an aircraft undergoing steady and level supersonic flight is illustrated in Fig. 1. Near the aircraft the overpressure signature takes the form of compression and shock waves that are identifiable with compressive parts of the flow over the nose, wings, nacelles, and tail of the aircraft, and these waves are separated by expansion waves from expansive parts of the flow. Owing to the relatively-large amplitude of these shock and expansion waves near the aircraft, their initial propagation is nonlinear. The subsequent propagation of these disturbances is also nonlinear. As the intermediate shocks coalesce with either the front or rear shocks that are slowly diverging from each other, and as the expansion waves develop linearly-decaying profiles, the sonic boom signature continuously changes in shape with increasing distance from the aircraft. At distances beyond about 10 km the signature has normally evolved into one having two unique shocks that are separated by a linearly-decaying expansion wave, and it resembles a flattened capital letter N as illustrated in Fig. 1. The reflection of the sonic boom from the ground is also indicated in the figure. Note that the sonic boom measured at ground level has an amplitude that is slightly less than the acoustic result of twice that of the incident boom, depending on the type of reflecting surface.

A three-dimensional illustration of the shock-wave pattern from a supersonic aircraft cruising in the Earth's atmosphere is shown in Fig. 2. The double
shock-wave pattern in space produces a horse-shoe-shaped pressure footprint on the
ground, and this instantaneous pressure distribution sweeps over the ground at the
same speed as the aircraft, subjecting everything within its path to the sonic-
boom disturbance. The peak overpressure ($\Delta p$) of the N-wave at the ground diminishes
on either side of the centre of the sonic-boom corridor (distribution shown), and
the wavelength ($\lambda$) becomes longer. The lateral width of the corridor is finite
owing to atmospheric refraction (Ref. 2). Note that the corridor width for current
supersonic transport aircraft (SST) range from 60 to 100 km for the case of standard
atmospheric conditions, but can be as wide as 160 km for overflights made in winter
weather conditions (Ref. 3).

The evolution of the sonic-boom signature from its pattern at the air-
craft to the signature at the ground can be predicted for supersonic aircraft
undergoing arbitrary manoeuvres as well as steady and level flight (Refs. 4, 5).
The starting point for such an analysis is to obtain the well-know Whitham F-
function (Ref. 6) for the aircraft or its equivalent body of revolution. For an
asymmetric lifting body the F-function can be calculated by using known aerody-
namic theory (first order), and it depends on the cross-sectional area distribu-
tion ($S$) of the aircraft, lift distribution ($L$), flight Mach number ($M$), and
azimuthal angle ($\theta$), as illustrated below (Ref. 7).

$$F(x, \theta) = \frac{1}{2\pi} \int_0^x \frac{S''(y) + (M^2-1)^{1/2} L''(y) \cos \theta / (\gamma M^2 p_a)}{(x-y)^{1/2}} \, dy \quad (1.1)$$

The prime (') denotes differentiation, $\gamma$ is the specific heat ratio, $p_a$ denotes
atmospheric pressure at the aircraft altitude, $x$ denotes the distance along the
aircraft measured from the nose, and $\cos \theta$ varies from 1 directly below the air-
craft to -1 above. Alternatively, one can build a model of the aircraft or its
equivalent body of revolution, put it in a supersonic wind tunnel, and measure
the F-function directly, as it is essentially the pressure distribution (Ref. 8).
The measurement is normally made at a distance of a few model lengths from the
body, and it results in a more accurate determination of the F-function.

The next step is to use geometrical acoustics (Ref. 9) to follow the
rays of interest along which the sonic boom propagates through a nonuniform
atmosphere from the aircraft to the ground. Along each ray the associated ray-
tube area is also determined. To facilitate these calculations the variation of
temperature and wind through the atmosphere is usually assumed to be horizontally
stratified and steady with time, and atmospheric turbulence is neglected. Then,
knowing both the signature near the aircraft and the area variation along a ray,
the signature can be predicted anywhere along the ray and hence at the ground.
Whitham (Ref. 6) showed how nonlinear steepening and coalescence of the distur-
bances comprising the sonic boom could be accounted for in this last step, per-
mittng a realistic prediction of the ground-intercepted N-wave. As the aero-
dynamic theory is first order, and geometrical acoustics with Whitham’s correction
for nonlinear effects is essentially first order, the prediction accuracy deter-
rmines for flight Mach numbers above 3. Note that the aforementioned prediction
theory and procedure are embodied in complex computer programs (Refs. 4, 5).

An approximate closed-form solution is available for predicting the
peak overpressure ($\Delta p$) and wavelength ($\lambda$) of the sonic boom at the ground, and
the results are given below (Ref. 6).
In these expressions $p_0$ denotes the atmospheric pressure at the ground, $h$ designates the aircraft altitude, and $x_0$ is chosen to maximize the integral. These results follow from Whitham's analysis (Ref. 6) of the flow pattern for a supersonic projectile in a uniform atmosphere. The factor $K$ ($1.8$ to $2$) has been inserted to account for the reflection of the sonic boom from the ground, and Whitham's constant-pressure $p$ has been replaced by $(p_a p_g)\frac{1}{2}$ to account for the pressure variation with altitude in the Earth's atmosphere. Additionally, the lift distribution of the aircraft has been included in the $F$-function. These results provide insight into how the aircraft design, flight Mach number and altitude affect the amplitude and wavelength of the sonic boom. These effects are modified, of course, when the atmosphere is taken into account in a more realistic manner, and when the aircraft undergoes maneuvers associated with rectilinear and radial accelerations.

From Equations 1.2 and 1.3, it can be seen that the overpressure varies inversely with altitude to the three-quarters power and the wavelength becomes longer as the altitude to the one-quarter power, which is required to conserve the impulse of the N-wave. The dependence on Mach number is weak. Aircraft having a large cross-sectional area and a high lift create more powerful sonic booms at the ground (Ref. 2).

The most important effect of an aircraft maneuver during supersonic flight is the generation of a focussed sonic boom or superboom. A superboom results from the focussing of sound rays emanating from the aircraft (Ref. 2), and its overpressure signature differs from an N-shape as illustrated in Fig. 3. Additionally, the peak overpressure of the superboom is usually five to ten times more than that for a normal sonic boom. Sonic-boom theory and its equivalent computer programs can adequately predict the location of the superboom (Refs. 4, 5). However, as the first-order theory breaks down in the immediate vicinity of a point of focus (cusp) or line of focus (caustic), the signature of the superboom cannot be predicted. This is not a serious limitation as the superboom location can be predicted and the form of the pressure signature and its amplitude are known from experimental measurements (Ref. 10).

The nature of the sonic-boom signature at the ground, and the parameters that are most important in its description and simulation, are illustrated in Fig. 4. The idealized parameters include peak overpressure, duration or wavelength, rise time, and waveform (which may vary somewhat from the N-shape). Although not illustrated, the spectral contents of the N-wave cover the frequency range from a fraction of one hertz to several thousands hertz (Ref. 11). However, most of the energy of the N-wave is concentrated in the low frequencies, below 200 Hz. It is worth noting that respective values of the peak overpressure, duration, and rise time for sonic booms from current SSTs, such as the Anglo-French Concorde and
Soviet Tu-144 aircraft, are of the order of 100 N/m² (2 psf)*, 300 ms, and 1 ms.

Overpressure measurements (Ref. 12) made on the ground for supersonic overflights during disturbed atmospheric conditions show that normal N-shaped sonic booms are interspersed with rounded and spiked booms. Rounded, normal and spiked overpressure signatures are illustrated in Fig. 5. The variation in waveform along the ground flight track is generally attributed to sonic boom propagation through turbulence in the atmospheric boundary layer. It is worth noting that, as an SST cruises over several differently-stationed observers, each one may be exposed to a different pressure signature ranging from rounded to spiked. As a consequence they may disagree as to the startle of the boom, as rounded booms having less high-frequency content do not sound nearly as loud as spiked booms.

1.2 Sonic-Boom Simulation

With the advent of the SST, and its planned introduction into commercial service in the late Seventies, the need arose in the Sixties to investigate the effects of sonic boom on humans, animals and structures. In the early Sixties, overflights using military aircraft were utilized to generate sonic booms for test purposes. For problems like community response to the sonic boom, or assessing the effects of sonic boom on heritage buildings, overflights provide the only means of obtaining meaningful answers. For most other investigations, however, overflights prove to be disadvantageous because of their substantial organization and cost. To circumvent these and other difficulties, sonic-boom simulation techniques and devices were developed to facilitate studies of the effects of the sonic boom on humans, animals and structures.

A variety of sonic-boom simulators were developed during the last decade (Refs. 13, 14, 15). They include wind tunnels, ballistic ranges, spark discharges, loudspeaker-driven headsets and booths, piston-driven chambers, many different forms of shock tubes, explosive charges and arrays, bags of detonable gas, and mass-flow-valve systems (a valve controls the release of high-pressure air from a reservoir into the apex of a large horn where the discharge generates the simulated sonic boom). All of these developments have been very specialized in nature to fulfill specific research requirements. Note that sonic-boom response tests can often be accomplished best in the laboratory, where test conditions can be controlled, are repeatable, and can be systematically varied.

1.3 Sonic-Boom Simulation at UTIAS

Prime operation of the SST will be, among others, over the Atlantic and Pacific Oceans and possibly Canada, especially Northern Canada. Albeit this region is sparsely populated it has an abundance of wildlife. All forms of life within the sonic-boom corridor will be subjected to the sonic boom. For this reason, independent, impartial, and open data unique to the Canadian scene should be obtained before the SST is permitted to operate commercially over Canada. Such information on physiological and psychological response of humans, psychological response of animals, as well as structural response and damage, are needed to help government legislators prepare realistic guidelines for new legislation that will govern flight paths as affected by Canadian weather, terrain, wildlife and population distribution (Ref. 16). Guidelines that were developed for other

* The International System of Units (SI) are used throughout. A table of conversion factors to convert to English units is given at the end of the List of Symbols to aid the reader.
countries are helpful but of uncertain applicability. In order to help gather relevant Canadian-based data on sonic boom, a research program was initiated at UTIAS.

The sonic-boom program was started at UTIAS in 1969, when a hypervelocity projectile launcher was used for a preliminary investigation of the diffraction of a bow shock wave over a model of the new Toronto City Hall (Ref. 17). It was soon realized that if significant data were to be obtained to assess societal problems associated with the sonic boom, more meaningful simulation facilities would have to be built. Under the direction of Dr. I. I. Glass and Dr. H. S. Ribner two major simulators were developed simultaneously. A loudspeaker-driven booth (Refs. 18, 19) and a travelling-wave horn (Refs. 19, 20) were constructed, and they complement each other for the study of human, animal and structural response to the sonic boom (Ref. 21). Additionally, the development of a portable sonic-boom simulator (Ref. 21) in the form of a shock tube was initiated in 1972. This shock-tube device can be transported and operated by one person to facilitate field tests of the startle effect of simulated sonic booms on wildlife in their natural habitat.

1.4 Travelling-Wave Horn

It has been shown theoretically (Refs. 22, 23) that a weak spherical explosion of finite size produces an outwards propagating overpressure N-wave, both near and far from the explosion, as illustrated in Fig. 6a. For this reason a conical or pyramidal horn has been used for sonic-boom simulation purposes. It forms a sector or solid angle of the spherical explosion and confines and directs the expanding N-wave. Weak cylindrical and planar explosions of finite size also produce overpressure N-waves, but only in the far field, as illustrated in Fig. 6b and c, respectively. Consequently they are impractical for simulation purposes, as the length of the appropriate duct would be unreasonable. Note that the far-field wave from any type of weak (or strong) explosion has a N-shape, as has been shown theoretically by Chandrasekhar (Ref. 24), Landau (Refs. 25, 26) and Whitham (Ref. 27). They also showed that far-field planar, cylindrical and spherical N-waves decay with distance (s) like \( s^{-1/2} \), \( s^{-3/4} \) and \( s^{-1} (\ln s)^{-1/2} \) respectively, and that their respective wavelengths become longer as \( s^{1/2} \), \( s^{1/4} \) and \( (\ln s)^{1/2} \). It is worth noting that an aircraft in steady supersonic flight generates a far-field N-wave with a decay and stretch comparable to that for a weak cylindrical explosion. A similar analogy exists for near-field waves from hypersonic flight and a strong cylindrical explosion.

The present work deals in the main with the UTIAS travelling-wave horn. The horn is in the form of a horizontal pyramid that is 25 m long, has a 3-m-square base, and is made mainly of concrete. At the apex of the horn a sonic-boom generator in the form of either a mass-flow valve or a shock-tube driver is used to generate a flow that simulates the expansion process of a weak spherical explosion in order to produce a simulated sonic boom or travelling N-wave in the horn.

In the mass-flow-valve mode of operation a specially-designed mass-flow valve is used to control the mass-flow rate of air from a high-pressure reservoir into the pyramidal horn where the expanding flow generates a simulated sonic boom. The peak overpressure and wavelength of this boom can be controlled independently such that either one is less than, equivalent to, or greater than that for an actual sonic boom from an SST or a military aircraft. The interior of the horn is equipped with a special high-frequency sound absorber or low-pass acoustic filter for removing undesirable jet noise from the passing N-wave. This jet noise
is generated by the high-speed turbulent flow at the valve, and it is superimposed on the simulated sonic boom. The open end of the pyramidal horn is covered with a specially-designed reflection eliminator, which is in the form of a recoiling-type porous piston. This device adequately minimizes the undesirable reflected wave arising when the simulated sonic boom encounters the large end of the horn. The design facets of the various elements of the facility, with the exception of the jet-noise absorber, were based to a large extent on the original work done at the General Applied Science Laboratory (GASL), Westbury, New York (Refs. 28, 29). Their assistance is gratefully acknowledged.

In the shock-tube-driver mode of operation high-pressure air contained in the driver is released suddenly by breaking a diaphragm that initially separates the gas in the driver from the air in the horn. This rapidly-expanding driver gas generates a simulated sonic boom or travelling N-wave in the horn. Although the amplitude of the N-wave can be controlled to be less than, equivalent to, or greater than that for an actual sonic boom, the N-wave duration is shorter (up to 20 ms) and the rise time faster (about 20 μs). Note that the jet-noise absorber is not needed for this operating mode.

The travelling-wave horn can be used for human, animal, and structural response or damage studies. As the facility has the capability of producing powerful long-duration booms (up to 0.5 s and longer), when operated in the mass-flow-valve mode, it is particularly well-suited for structural response studies (wall panels, windows, room resonance). Owing to the travelling nature of the simulated sonic boom, studies can be made of wave diffraction over and into model buildings or wave propagation over various reduced-scale land topologies. The horn can be easily adapted to facilitate these and other relevant research projects.

A detailed description of the mass-flow-valve and shock-tube modes of operation of the travelling-wave horn is given in Appendix A. In the following sections comprehensive analyses are presented for predicting wave motions in the horn, for both operating modes. These analyses have been substantiated by experimental data, taken mainly from this facility as well as from other existing sonic-boom simulators.

2. ANALYSIS OF THE MASS-FLOW-VALVE MODE

2.1 Facility Description and Performance

Elevation and plan views of the UTIAS sonic-boom laboratory and the travelling-wave horn are displayed in Fig. 7. The main elements of the sonic-boom simulator are indicated. The simulator horn is in the form of a large pyramid, which is 25 m long, has a 3-m-square base, and a total divergence angle of 7.2 degrees.

The basic concepts for the design and operation of the mass-flow valve are illustrated by the sketches given in Fig. 8. The valve, displayed schematically in Fig. 8a, was designed such that the plug displacement (x) would be linear in both the opening and closing strokes, as shown in Fig. 8b. This form of plug motion produces a throat-area variation (A) with time that is approximately parabolic, as sketched in Fig. 8c. As the mass-flow rate of air (m) from the high-pressure reservoir into the horn is roughly proportional to the throat area, its variation with time can also be taken as parabolic, as shown in Fig. 8c. It is well-known that the far-field overpressure signature for an unsteady point source
of sound (Ref. 30) is directly proportional to the derivative of the mass-flow rate from the source. Hence, the overpressure disturbance that is generated at the large end of the horn should be an N-wave, as sketched in Fig. 8d. These concepts are essentially correct, and an N-wave that is desired for simulating the sonic boom is achieved in practice.

The mass-flow valve was designed to give the facility the capability of generating an N-wave with a duration equivalent to that for an actual sonic boom from an SST or a military aircraft. As the open-to-close time of the valve fixes the N-wave duration or wavelength, the above condition is more than satisfied, as the N-wave duration can be varied from 60 ms to 500 ms or longer. Additionally, the facility has the capability of producing an N-wave with a lower, equivalent or higher peak overpressure than that for an actual sonic boom. The reservoir pressure (from 1 to 18 atm), which remains essentially constant during the valve operation owing to its large volume (2.6 m³), determines the N-wave amplitude for a given duration (up to 1400 N/m² for a duration of 100 ms, up to 700 N/m² for 200 ms, up to 450 N/m² for 300 ms). The existing facility exercises little control over the rise times of the two shock waves of the simulated sonic boom, which are affected by the valve-plug geometry and nonlinear wave-steepening processes in the flow. The rise time on the N-waves are typically 2 to 6 ms, which is at one end of the range of 0.1 to 10 ms for actual sonic booms.

An inherent characteristic of any travelling-wave horn operated in the mass-flow-valve mode is the production of undesirable high-frequency noise that is superimposed on the simulated sonic boom. This high-frequency noise is produced by the high-speed turbulent flow of air at the valve. As the discharge of air from the valve is actually a confined air jet that has a weak dependence on time owing to the slowly changing throat area, the turbulent noise is very similar to that from a steady-state free-air jet, and it is thus called jet noise. The jet noise becomes more intense for higher reservoir pressures, as the flow speed through the valve becomes correspondingly faster. As a high reservoir pressure is required to generate high-amplitude and long-duration N-waves, these simulated sonic booms have very intense jet noise superimposed on their basic N-shaped signature. In fact, this noise can completely mask the N-wave signal. Consequently, the simulated sonic boom can sound much louder than it should, and certain human response tests would be meaningless to perform. Additionally, as the jet noise contains some low-frequency content, some structural panel response is invalidated, as the panel can respond not only to the N-shaped profile but also to the jet noise.

An investigation was made into the frequency content of the jet noise, using electronic-filtering techniques, and it showed that the noise was broad band and covered the frequency range from as low as 100 Hz up to 10 kHz and higher. As the frequency content of the sonic boom lies mainly below 200 Hz, a low-pass acoustic filter was designed for installation in the interior of the horn, to remove much of the high frequencies of the jet noise from the passing N-wave.

The low-pass acoustic filter or jet-noise absorber is illustrated schematically in Fig. 9. It consists of horizontally and vertically arranged panels of sound-absorbing material (fiberglass, 2.5-cm thick, 110 kg/m³). Note that the arrangement of the panels is not important, as the same jet-noise abatement can be achieved with only vertical panels, spaced more closely together, and having an equivalent surface area exposed to the flow.

When the simulated sonic boom propagates to the base of the horn it would
normally be reflected from open or closed end. This reflected wave would disrupt the simulated flow and pressure conditions in the interior test section, as the wavelength of the simulated sonic boom can be up to four times longer than the horn. In order to eliminate or at least adequately minimize this undesirable reflection and its subsequent echoes, a reflection eliminator was built to cover the base of the horn.

The reflection eliminator is basically a huge porous piston, as illustrated in Fig. 10. The porous piston consists of a 2.5-cm-thick blanket of micro-lite material (12 kg/m³), and it matches the impedance of the exit of the duct to that of the incident wave, thereby eliminating its reflection. Note that the porous piston is passive to the incident wave, being free to move on a special roller-and-track system.

In using the travelling-wave horn for studies of sonic-boom propagation over different land topologies, wave diffraction over and into model buildings, and physiological and psychoacoustic response, the models or subjects can be put directly in the interior test section of the horn (Fig. 7) that is near the reflection eliminator. To facilitate structural response and damage studies a 1.8-m by 3.6-m cutout or opening (Fig. 7) has been provided for in one side wall of the horn. For additional psychoacoustic testing and also for room resonance studies an adjoining full-scale test room (Fig. 7), linked to the horn interior by the same cutout, can be suitably constructed. The simulator horn can be adapted easily for these and other relevant research projects.

The mass-flow valve functions correctly as designed. The displacement of the valve plug with time is linear in both the opening and closing strokes of the valve operation. Additionally, the plug speed that is constant is very nearly the same in each half of the cycle. To substantiate these statements, five measurements of the plug displacement history are given in Fig. 11. The different open-to-close times for the valve of 60, 100, 200, 300 and 400 ms cover the operation range expected of the facility. These measurements were made with a suitable linear potentiometer and associated electronic equipment.

The effectiveness of the jet-noise absorber in reducing the jet noise that is superimposed on the N-wave is illustrated in Fig. 12. The N-waves shown in this figure have a nominal duration of 110 ms and peak overpressures that increase with successive records down the columns. The signatures in the first column were obtained without the use of the jet-noise absorber, and the duplicate set was obtained with the absorber installed in the horn. It can be seen that the absorber is very effective in removing the high-frequency content of the jet noise from the N-wave. However, substantial low-frequency jet noise still remains superimposed on N-waves having a peak overpressure higher than 200 N/m².

The effectiveness of the jet-noise absorber is further illustrated in Fig. 13. The N-waves shown in the first column were again obtained without the use of the absorber, and they have a nominal peak overpressure of 100 N/m² and a duration that ranges from 100 ms to 400 ms. Note that the intensity of the jet-noise increases markedly as the duration increases. The duplicate set of N-waves in the second column was obtained with the absorber installed in the horn. The high-frequency content of the jet noise has been removed by the absorber. However, as the jet-noise intensity becomes more severe for longer duration N-waves, significant low-frequency jet noise still remains superimposed on N-waves having a duration longer than about 200 ms.
The rise time on the two shocks of the N-wave is about 3 ms for all of the simulated sonic booms obtained without the use of the jet-noise absorber. It increases slightly by about 1 ms when the absorber is installed in the horn.

Whereas the jet-noise absorber is completely effective for N-waves having a peak overpressure less than 200 N/m² and a duration shorter than 200 ms, it would be desirable to improve the range of operation of the facility. Further jet noise reduction can be achieved by increasing the number of longitudinal panels of sound-absorbing material for the absorber, and also by installing transverse panels (see Appendix A). However, there are drawbacks to this approach. More panels cause the peak overpressure of the transmitted N-wave to be reduced, and, worst of all, the rise time is lengthened. For further jet-noise abatement without these drawbacks, it appears that the only alternative is to design a new mass-flow valve that will generate less intense jet noise from the onset. This approach has been followed and a new valve is presently being manufactured.

The reflection eliminator functions satisfactorily. When the simulated sonic boom encounters the porous piston the resulting reflected wave is much weaker than the incident N-wave, and it does not significantly disrupt the flow and pressure simulation inside the horn. However, in addition to the jet noise there are weak pressure perturbations superimposed on the N-wave. These perturbations are the result of the base of the pyramidal horn and reflection eliminator being enclosed in a test room (see Fig. 7). After the simulated sonic boom passes through the porous piston and leaves the large end of the horn it interacts with the test room. Even though the large doors in the test-room wall immediately behind the porous piston (2 m away) can be left open during tests to minimize the enclosing effects, the wave leaving the horn is partially reflected from this wall and propagates back into the horn through the porous piston. Furthermore, as the test-room air responds acoustically to the wave leaving the horn, much like a Helmholtz resonator, additional pressure perturbations follow the reflected wave into the horn. These disturbances disrupt the simulation of the flow and pressure conditions inside the horn, causing small perturbations on the N-wave. These are noticeable in measured overpressure signatures, if the jet noise is removed, as can be seen from the measured signatures given in Fig. 14.

The maximum amplitude of the perturbations superimposed on the N-wave is normally less than 30% of that for the simulated sonic boom. Their amplitude, as compared to that for the N-wave, remains nearly constant as the N-wave duration (see Figs. 14a, b, d and e) or amplitude (Fig. 14b and c) are increased. The disturbances arising as a result of the test room should not impose significant restriction on using the travelling-wave horn for research tests.

The travelling-wave horn has been designated so that its operation could be continuous, generating a sequence of simulated sonic booms. The number of booms per unit time that can be produced depends mainly on the capacity of the air compressor. For long duration booms (e.g., 400 ms) of modest peak overpressure (100 N/m²) the rate of generation has been found to be about one boom per minute. For shorter duration N-waves (200 ms) having a similar amplitude the facility can easily produce five booms per minute. If the duration is still shorter at 100 ms the rate can be as high as ten booms per minute. Note that continuous operation of the facility is almost a necessity to facilitate fatigue and damage tests of structural panels.
2.2 Analysis of the Wave Motion in the Horn

The main characteristics of the simulated sonic boom generated by the travelling-wave horn when operated in the mass-flow mode can be predicted successfully by using a combination of gasdynamic and acoustic theory. Gasdynamic theory was used to describe the air flow from the high-pressure reservoir to the valve, in order to determine the resulting mass-flow rate at the valve as a function of time. Acoustic theory was employed to predict the wave motion in the horn, that is, the overpressure and particle-velocity profiles. The mass-flow rate at the valve serves as a boundary condition for the acoustic solution, and it therefore couples the gasdynamic and acoustic parts of the analysis. This type of analysis for a travelling-wave horn was presented initially in a report by Tamboulian (Ref. 28).

An analysis in which the influx of air at the horn apex was treated as coming from a point source is given first. The point-source analysis provides much insight into the wave motion in the horn, as simple and useful expressions can be obtained for the overpressure and particle-velocity signatures. As this analysis predicts unrealistically fast rise times (instantaneous) for the two shocks of the simulated sonic boom, and as it overpredicts the amplitude of short-duration booms, an extended analysis that is based on the theory of a finite source is also given. The results of the finite-source analysis are in better agreement with experimental data.

2.2.1 Point-Source Analysis

The air flow from the high-pressure reservoir into the horn, controlled by the mass-flow valve, is a convergent-divergent duct flow with a slowly varying throat area. This valve-regulated flow is assumed to be one-dimensional and quasi-steady so that the mass-flow rate at the valve throat can be determined easily by performing a sequence of steady-state calculations using well-known steady-state gasdynamic theory (Ref. 31). The quasi-steady flow assumption is justified on the basis that the valve-throat area changes slowly with time as compared with the wave motion which establishes the flow from the reservoir.

For a nozzle flow from an infinite-volume or constant-pressure reservoir, the well-known expression for the mass-flow rate per unit area \( \dot{m} \) at the nozzle or valve throat (Ref. 31) can be expressed in the following nondimensional form.

\[
\frac{\dot{m}}{\dot{m}_*} = \frac{D_*}{D} \frac{a}{a_*} M_* \left[ 1 + \frac{\gamma - 1}{2} M_*^2 \right]
\]

(2.1)

The respective symbols, \( \gamma \), \( D_* \), \( a_* \), \( p_* \), \( p_0 \), \( a \), and \( M_* \) denote the specific heats ratio, reservoir pressure, reservoir sound speed, atmospheric pressure, atmospheric density, atmospheric sound speed, and the flow Mach number at the valve throat (minimum area). For a sufficiently high reservoir pressure the nozzle flow from the reservoir becomes choked or sonic at the valve, and for higher pressures it remains choked and the flow Mach number \( M_* \) is always equal to unity. For this case the mass-flow rate per unit area is determinable from Eq. 2.1, as the reservoir and atmospheric states or conditions are known. However, when the reservoir pressure is too low the flow at the throat is unchoked or subsonic, and an additional step has to be taken to determine the flow Mach number. In order to solve for \( M_* \),
which is now less than unity, the flow pressure at the valve throat is assumed to be equal to atmospheric pressure. Then $M_\infty$ can be found by using the following well-known expression for an isentropic flow (Ref. 31).

\[
\frac{p_\infty}{p} = \left[1 + \frac{\gamma-1}{2} M_\infty^2 \right]^{\frac{\gamma}{\gamma-1}}
\]  

(2.2)

The preceding assumption and the above expression therefore determine when the flow at the valve throat is subsonic or sonic. From Equation 2.2 the flow is subsonic ($M_\infty < 1$) for a reservoir to atmospheric pressure ratio ($p_\infty/p$) between unity and $(\gamma+1)/2$ raised to the power $\gamma/\gamma-1$, and it is sonic ($M_\infty = 1$) for higher pressure ratios. For the specific case of air the pressure ratio ($p_\infty/p$) differentiating the subsonic from the sonic flow regime is 1.89, or the reciprocal of 0.528. Because the reservoir gas is always air for the UTIAS travelling-wave horn, and for simplicity, this special pressure-ratio value of 1.89 is used throughout the report as opposed to $(\gamma+1)/2$ raised to the power $\gamma/\gamma-1$.

The nondimensional mass-flow rate per unit area (Eq. 2.1) is a function of only the known reservoir and atmospheric conditions. It was computed as a function of reservoir pressure ($p_\infty/p$), and the results are given in tabulated form in Table 1 and graphical form in Fig. 15. At low reservoir pressures ($1 < p_\infty/p < 1.89$), when the flow at the throat is subsonic, $\tilde{m}_\infty$ increases rapidly with increasing $p_\infty$. At higher reservoir pressures, when the flow at the throat is choked, $\tilde{m}_\infty$ increases more slowly and in a linear manner with increasing $p_\infty$.

The mass-flow rate ($m_\infty$) of air passing through the valve throat, which is a function of time ($t$) as the throat area ($A_\infty N_\infty$) changes with time, can be expressed in the following form.

\[
m_\infty(t) = \tilde{m}_\infty A_\infty N_\infty(t)
\]

(2.3)

The symbol $\tilde{A}_\infty$ denotes the maximum throat area corresponding to a fully-open valve, and the normalized variable $N_\infty(t)$ describes the throat-area variation with time. As $\tilde{m}_\infty A_\infty$ is the maximum mass-flow rate at the valve throat, $N_\infty(t)$ also denotes the normalized mass-flow-rate distribution. Note that the flow of air into the pyramidal horn near its apex produces the sound in the horn. The mass-flow rate $m_\infty(t)$ of this source of sound is the required boundary condition for the following acoustic part of the analysis.

The sound-wave propagation in the horn is assumed to be governed by the one-dimensional spherical wave equation, which is given below (Ref. 32).
The new symbols \( \phi \) and \( r \) denote the velocity potential and the radial distance measured from the projected apex of the horn, respectively. As only an outwards propagating wave is generated by the source, and as reflection of this wave from the large end of the horn is neglected or eliminated, the general solution of the wave equation is given below (Ref. 32).

\[
\frac{\partial^2 (r \phi)}{\partial t^2} = a^2 \frac{\partial^2 (r \phi)}{\partial r^2} \quad (2.4)
\]

The symbol \( r_s \) denotes the radial distance at which the valve plug enters the pyramidal horn. The overpressure \( (\Delta p) \) and particle velocity \( (\Delta u) \) are related to the velocity potential and the function \( f(\tau) \) as illustrated below.

\[
\Delta p = -\rho \frac{\partial \phi}{\partial t} = -\frac{\rho}{r} f'(\tau) \quad (2.6)
\]

\[
\Delta u = \frac{\partial \phi}{\partial r} = -\frac{1}{a r} f'(\tau) - \frac{1}{r^2} f(\tau) \quad (2.7)
\]

The prime \((')\) denotes differentiation with respect to the argument given in the brackets. Note that the particle-velocity expression consists of two terms. The near-field term \( f(\tau)/r^2 \) is dominant at sufficiently-small radial disturbances \( r \), whereas the far-field term \( f'(\tau)/ar \) becomes more significant as distance increases.

The function \( f(\tau) \) is determined and then the overpressure and particle velocity are obtained from Eqs. 2.6 and 2.7, respectively. The instantaneous mass-flow rate induced by a wave as it propagates past a fixed station of radius \( r \) and area \( A \) is equal to \( \rho A \Delta u \) (to first order). Let this station be located at a sufficiently small radius such that the far-field part of the particle-velocity expression (Eq. 2.7) is insignificant in its contribution to the mass-flow rate, and it can therefore be omitted. It is standard practice in point-source theory (Ref. 30) to retain only the near-field term when the radius is small, or, more accurately, when the source diameter is much smaller than the wavelength of a typical wave from the source. For the UTIAS travelling-wave horn the point-source approximation should be valid as the source diameter (approximately equal to \( 2r_s \) or 1 m) is certainly much smaller than the wavelength of a typical simulated sonic boom (70 m for a wave having a duration of 200 ms). Using the point-source assumption the resulting mass-flow rate induced by the wave is \( -\rho A f(\tau)/r^2 \). When this function is equated with the mass-flow rate at the valve throat (Eq. 2.3), and noting that \( A = A_s (r/r_s)^2 \) for a pyramidal horn, the acoustic and gasdynamic analyses are coupled, and \( f(\tau) \) is given by the following expression.

\[
f(\tau) = -r_s^2 \bar{\bar{m}}_s N_s(\tau)/\rho \quad (2.8)
\]
The function \( f(\tau) \) thus has the same form as \( N_*(\tau) \) for the throat-area and mass-flow-rate distributions. Note that in this matching procedure certain features of the actual flow were overlooked (e.g., the supersonic flow downstream of the valve throat and an upstream-facing shock wave when \( p_*/p \) is greater than 1.89). The flow is dealt with more realistically in the finite-source analysis (Section 2.2.2).

Now that \( f(\tau) \) is known, the overpressure and particle velocity can be found by using Eqs. 2.6 and 2.7, respectively. In general form the results of the point-source analysis are summarized below.

\[
\Delta p = \frac{r_o^2 \bar{m}_*(\tau)}{r} N'_*(\tau) \tag{2.9}
\]

\[
\Delta u = \frac{r_o^2 \bar{m}_*}{par} \left[ N'_*(\tau) + \frac{a}{r} N_*(\tau) \right] \tag{2.10}
\]

\[
\bar{m}_* = \frac{p_o}{p} \frac{s}{a_o} M_* \left[ 1 + \frac{\gamma - 1}{\gamma - 1} \right]^{\frac{-1}{2(\gamma - 1)}}
\]

\[
M_* = \left[ \frac{2}{\gamma - 1} \left( \frac{p_*}{p} \right) ^{\frac{\gamma - 1}{\gamma - 1}} - 1 \right]^{1/2} \text{ if } 1 < \frac{p_*}{p} < 1.89
\]

\[
M_* = 1 \text{ if } p_*/p > 1.89
\]

The overpressure is directly proportional to the derivative of the mass-flow rate or throat area and inversely proportional to radial distance. The particle velocity consists of a near-field part, which is directly proportional to the mass-flow rate and decays like \( 1/r^2 \), and a far-field part that has the same dependence on mass-flow rate and radial distance as for the overpressure. The particle velocity thus tends with increasing distance to take on the same waveform as the overpressure.

The throat-area and mass-flow-rate functions that are denoted by \( N_*(\tau) \) have been left in general form to this stage of the analysis. Before proceeding to solve for \( N_*(\tau) \), which depends on both the geometry and the motion of the valve plug, it is worthwhile to assume that the distribution for \( N_*(\tau) \) is parabolic, as given by the following expression.

\[
N_*(\tau) = \begin{cases} 
4 \left( 1 - \tau/\tau_o \right) \tau/\tau_o & \text{if } 0 < \tau < \tau_o \\
0 & \text{if } \tau < 0, \tau > \tau_o 
\end{cases} \tag{2.11}
\]
The symbol $T_o$ denotes the duration of the flow into the horn. Using this particular form of $N(t)$, the overpressure and particle-velocity signatures are summarized by the equations given below.

\[
\Delta p = \frac{\Delta p}{\rho a} \left[ 1 - \frac{2T}{T_o} \right] \quad \text{if } 0 < T < T_o
\]

\[
\Delta p = 0 \quad \text{if } T < 0, T > T_o
\]  

\[
\Delta u = \frac{\Delta p}{\rho a} \left[ 1 - \frac{2T}{T_o} + \frac{8T_o}{r} \left( 1 - \frac{T}{T_o} \right) \frac{T_o}{T} \right] \quad \text{if } 0 < T < T_o
\]

\[
\Delta u = 0 \quad \text{if } T < 0, T > T_o
\]  

\[
\frac{\Delta p}{\rho a} = \frac{1}{r} \frac{r^2 \bar{m}}{T_o} = \frac{4\gamma p}{r} \frac{r_o}{\alpha T_o} \frac{\bar{m}}{\rho a}
\]

For the parabolic mass-flow rate the overpressure wave predicted in the horn is an N-wave, as shown in Fig. 16a. The two shocks have equivalent amplitudes ($\Delta p/\rho a$) and instantaneous rise times. The peak overpressure of the N-wave, given by $\Delta p$ (Eq. 2.14), is directly proportional to the maximum mass-flow rate at the valve throat ($\bar{m} r_o^2$) and inversely proportional to both radial distance ($r$) and wave duration ($T_o$). Hence, in order to generate a powerful long-duration N-wave with the travelling-wave horn the analysis illustrates that a large mass-flow rate and thus a high reservoir pressure are required.

The particle-velocity signature (Eq. 2.13) also exhibits two shocks of equal amplitude ($\Delta p/\rho a$) and instantaneous rise times. However, the waveform is not necessarily N-shaped. Velocity signatures for different fixed radii are shown in Fig. 16b. All of the signatures, characterized by different values of the parameter $\alpha r_o/r$, are given in nondimensional form on the same diagram to illustrate the change in waveform from a parabolic profile in the near-field to an N-shaped one in the far-field. It is usually more convenient to use the inverse parameter $r/\alpha r_o$ to identify a velocity profile at a relatively large radius. As $\alpha r_o$ equals the wavelength ($\lambda_o$) the inverse parameter $r/\alpha r_o$ or $r/\lambda_o$ is a measure of how far the wave is from its source.

The test section of the UTIAS travelling-wave horn is not located in the far-field, as the radial distance to this section (20 to 25 m) is only a fraction of a wavelength. For example, the values of $r/\lambda_o$ for simulated sonic booms having three different durations of 100, 200 and 300 ms are equal to 0.45, 0.30 and 0.15, respectively. Consequently the particle velocity as predicted (Eq. 2.13) at the test section is not an N-wave, although the signature is not too different than the N-shape. Because the profile has a convex curvature the positive portion is greater than the negative portion. These results can be interpreted from the velocity profiles given in Fig. 16b, noting that $\alpha r_o/r$ equals 2, 3 and 6 for the respective durations of 100, 200, and 300 ms. (These particular profiles are not shown for clarity.)
The ground-intercepted (far-field) sonic boom from a supersonic aircraft has an N-shaped signature for both the overpressure and particle velocity. Based on the point-source analysis, therefore, the travelling-wave horn should be capable of producing a simulated sonic boom having the correct N-shaped overpressure signature, but, owing to the finite length of the pyramidal horn, it cannot produce a N-shaped velocity profile. Fortunately, the simulation of the particle velocity is unimportant for human, animal and structural response tests because the flow velocity is small (always less than 1 m/s) and the associated dynamic pressure is insignificant as compared with the overpressure. Note that the dynamic pressure \( \frac{0.5 \rho (\Delta u)^2}{\Delta p} \) divided by the overpressure \( \frac{\Delta p}{\rho} \) for the front shock of a simulated or actual sonic boom is equal to \( \Delta u/2a \) or \( \Delta p/2\gamma p \), which is very small. For example, the dynamic pressure associated with a sonic boom having a typical over-pressure of 100 N/m\(^2\) is only 0.03 N/m\(^2\).

The actual throat-area distribution \( N_*(\tau) \) for the UTLAS mass-flow valve is now developed. Two different valve plugs having a pyramidal front with a height of 2.0 and 5.1 cm and the same square base with a width equal to 5.7 cm can be used for the UTLAS valve. For generality, however, the height of the pyramidal front is denoted by \( b \) and the base by \( c \), as illustrated in Fig. 17. During the opening and closing stages of the valve the minimum throat-area exposed to the flow by the plug motion can be approximated by four equal quadrilateral areas. One such area ABCD is shown in both diagrams given in Fig. 17. A distance from the horn end AD to the plug face BC, denoted by \( z(\tau) \), is that for area ABCD to be a minimum for a particular plug position \( y(\tau) \). In order to find \( z(\tau) \) and the corresponding minimum throat area \( N_*(\tau) \), a general expression for the four equal quadrilateral areas in terms of arbitrary \( y(\tau) \) and \( z(\tau) \) was minimized with respect to \( z(\tau) \). The final results are summarized below.

\[
N_*(\tau) = \begin{cases} 
2 - \frac{y-z}{b} & \text{if } y-b < z < y \\
1 & \text{if } z < y-b
\end{cases}
\]  

(2.15)

\[
z/b = (d^2 + e)^{1/2} - d
\]

\[
d = \frac{1}{2} - \frac{(1 + b^2/c^2) y/b}{(1 + 4b^2/c^2)}
\]

\[
e = (y/b - y^2/b^2)/(1 + 4b^2/c^2)
\]

For a particular plug motion \( y(\tau) \), the throat-area variation is also a function of the plug geometry, typified solely by the ratio \( b/c \). When this ratio is large \((b/c > 2)\), \( z(\tau) \) is negligible compared with \( y(\tau) \) as it is approximately equal to zero. If the approximation of a large \( b/c \) value is made \((z = 0)\), then Eq. 2.15 simplifies considerably, and the results are given below.
\[ N_*(\tau) = (2 - \frac{y}{b}) \frac{y}{b} \quad \text{if } 0 < y < b \]
\[ N_*(\tau) = 1 \quad \text{if } y > b \]

Hence, for a large \( b/c \) value and a linear opening and closing motion \( y \) of the valve plug, \( N_*(\tau) \) takes on the desired parabolic profile, resulting in the prediction of an ideal overpressure profile. If the ratio \( b/c \) is not sufficiently large, then the profile for \( N_*(\tau) \) differs slightly from the parabolic shape. The effects of different \( b/c \) values on \( N_*(\tau) \) will be illustrated after \( y(\tau) \) is specified. Note that the dependence of \( N_*(\tau) \) on the ratio \( b/c \) is not unique to a pyramidal valve plug, but it also occurs for a conical valve plug.

The valve-plug displacement \( y(\tau) \) has been left in general form. Then any variation of \( y(\tau) \), including actual displacement histories, can be used for the point-source analysis. One particular plug motion, however, is worth noting, as the mass-flow valves for both the UTIAS and GASL facilities were designed such that the valve plug would have this motion. The desired plug motion is linear in both the opening and closing stages of the valve, as described by the following expressions.

\[ y(\tau) = 2b \frac{\tau}{\tau_o} \quad \text{if } 0 < \tau < \frac{\tau_o}{2} \]
\[ y(\tau) = 2b \left(1 - \frac{\tau}{\tau_o}\right) \quad \text{if } \frac{\tau_o}{2} < \tau < \tau_o \]

Note that when this linear plug motion is substituted into Eq. 2.16 the resulting expression for \( N_*(\tau) \) has the parabolic profile that was originally assumed (Eq. 2.11).

The effects of the valve-plug geometry \( (b/c) \) on the throat-area and mass-flow-rate profiles, and also on the simulated sonic boom, are illustrated in Fig. 18. A linear plug displacement (Eq. 2.17) was used for the analysis, and this displacement is shown in Fig. 18a. The resulting throat-area and mass-flow-rate profiles \( (N_*) \), as calculated by using Eq. 2.15 for three different \( b/c \) values of 0.36, 0.89 and 3.0, are shown in Fig. 18b. For the relatively-large \( b/c \) value of 3.0, \( N_*(\tau) \) has a parabolic profile as described by Eq. 2.11. The resulting overpressure wave predicted by the point-source analysis (Eq. 2.9) thus has an N-shape (Fig. 18c). The \( N_*(\tau) \) profile corresponding to a smaller \( b/c \) value of 0.89 (for the valve plug most often used with the UTIAS facility) deviates slightly from the parabolic form (Fig. 18b). However, the predicted overpressure signature differs significantly from an N-shape (Fig. 18c), as the overpressure is proportional to the derivative of \( N_*(\tau) \). For example, the peak overpressure of this distorted N-wave is a significant 12% less than that for the N-wave. The third \( N_*(\tau) \) profile corresponding to the smallest \( b/c \) value of 0.36 (for a valve plug that is sometimes used with the UTIAS facility) is very different from a parabolic profile (Fig. 18b), as the valve plug is very blunt. The predicted overpressure signature is, therefore, markedly different from an N-wave (Fig. 18d). Note that the predicted particle-velocity signature corresponding to each \( b/c \) value has essentially the mass-flow-rate profile in the near-field and tends with increasing distance to take on the overpressure profile in the far-field.

From the analysis or the results given in Fig. 18, it can be concluded that the geometry of the valve plug \( (b/c) \) is an important feature to consider when
designing a mass-flow valve for a travelling-wave horn. When the plug is too short, undesirable distorted overpressure N-waves will be produced in the horn. The point-source analysis, which has provisions for taking into account the geometry of the plug (Eq. 2.15), provides a means of investigating the effects of the plug geometry on the simulated sonic boom. Hence, a plug geometry (b/c) can be selected that will produce acceptable simulated sonic-boom signatures.

In practice the valve does not always function as designed, and plug motions that deviate from the ideal displacement shown in Fig. 18a affect the signature of the simulated sonic boom. The effects on the overpressure wave of some undesirable plug motions that most frequently occur in practice are illustrated in Fig. 19. In the first example (Fig. 19a) the valve plug has been withdrawn a distance of 1.2 times b or 20% more than necessary to control the mass-flow rate. The throat area and mass-flow rate therefore become constant at their maximum values during this excess plug motion. Consequently, as the overpressure is directly proportional to the derivative of the mass-flow rate, the overpressure signature has a corresponding flat portion at its centre where the overpressure is zero. The second example (Fig. 19b) shows the effect of reversing the plug motion when it has only been withdrawn a distance corresponding to 80% of its full stroke. As the mass-flow rate does not reach its normal maximum where its derivative is zero, the slope of the profile is discontinuous when the plug motion is reversed, and a sudden drop in overpressure occurs at the centre of the overpressure signature. The last example (Fig. 19c) illustrates the effects of an asymmetrical plug motion. The corresponding asymmetric mass-flow-rate profile gives rise to a distorted N-wave. The two shocks of this wave have different amplitudes, but the impulse of the positive and negative portions of the wave are equal.

It is of interest to know how far the reservoir air expands into the pyramidal horn as it generates a wave in the horn. To determine the path of the interface or contact surface between the air originally in the reservoir and the atmospheric air initially adjacent to the valve plug, it is assumed that the reservoir air expands to atmospheric density as it enters the horn and simply displaces the atmospheric air radially outwards. By equating the volume of the air discharging through the valve with the volume of the portion of the pyramid that the air now occupies in the horn, the following expression can be derived for the contact-surface path (R).

\[
\frac{R}{r_0} = \left[ 1 + \frac{3m_*}{\rho} \frac{a}{r_0} \int \frac{\tau}{\tau_0} N_* (y) \, dy \right]^{1/3}
\]

(2.18)

The displacement of the contact surface is greatest for a long flow or wave duration (\(\tau_0\)) and for a large mass-flow rate (\(m_*\)) corresponding to a high reservoir pressure.

For the particular case when the mass-flow-rate profile (\(N_*\)) is parabolic (Eq. 2.11), the following results for the contact motion can be obtained.
These particular results can be derived more rigorously by using the particle-velocity expression (Eq. 2.13) corresponding to the parabolic mass-flow-rate profile. If the far-field term of the velocity expression is omitted, if \( r \) and \( \Delta u \) are replaced by \( R \) and \( dR/d\tau \), respectively, and noting that \( \Delta p \) is a function of \( R \), then the resulting differential equation can be integrated for \( R \) and the results given by Eq. 2.19 will be obtained.

It can be concluded from this simple analysis for the motion of the contact surface that the reservoir air does not normally expand as far as the interior test section at the large end of the horn. The contact surface can, however, move a significant distance. For example, when simulating a powerful sonic boom having a peak overpressure of 200 N/m\(^2\) and a duration of 400 ms, the final position of the contact surface is 14 m from the horn apex, or 8 m short of the test section.

2.2.2 Finite-Source Analysis

The point-source analysis predicted instantaneous rise times for the two shocks of the simulated sonic boom, or, for that matter, for any shock resulting from a discontinuity in the time derivative of the mass-flow-rate profile. An extended analysis is now given, for which the source of sound is treated as being finite in size. This finite-source analysis is capable of predicting more realistic overpressure and particle-velocity signatures for the simulated sonic boom, as each shock in either signature has a finite rise time.

The flow at the mass-flow valve has to be considered in more detail for the finite-source analysis because the radial location at which the gasdynamic and acoustic parts of the analysis are matched depends on this flow and appears in the solution. When the reservoir pressure is less than 1.89 atm the convergent-divergent (venturi) air flow from the high-pressure reservoir is entirely subsonic (Ref. 31). For this subsonic case, quasi-steady gasdynamic theory is used to describe the flow from the reservoir to a radial location in the pyramidal horn at which the flow has expanded to approximately atmospheric pressure and density. Then acoustic theory can be applied to describe the unsteady flow in the remainder of the horn. As the flow from the valve throat to the matching station has been assumed to be quasi-steady, the mass-flow rate at the matching station is equal to the known mass-flow rate at the valve throat (Eq. 2.3). For the subsonic flow, therefore, the matching of the gasdynamic and acoustic parts of the finite-source analysis can be accomplished in a manner similar to that done for the point-source analysis, as will be shown later.

When the reservoir pressure is higher than 1.89 atm the subsonic flow from the high-pressure reservoir becomes choked or sonic at the valve throat, and a supersonic flow therefore occurs downstream of this throat (Ref. 31). This supersonic
flow is eventually terminated by an upstream-facing shock wave, which reduces the supersonic flow to subsonic speed and increases the flow pressure and density to approximately atmospheric conditions. This shock acts as an efficient diffuser. The upstream-facing shock, originating at the valve throat when the valve begins to open, is swept downstream as the valve opens further and the flow rate increases, and then it moves back towards the throat as the valve closes and the flow rate decreases. Note that the Mach number of this shock, the pressure ratio across it, and its location have been determined by using a quasi-steady gasdynamic analysis, and these results are given in Appendix C.

For the supersonic case, quasi-steady gasdynamic theory can be applied to describe the convergent-divergent flow from the reservoir to a fixed location in the horn that is behind the upstream-facing shock wave, in order to determine the mass-flow rate at this new location. Acoustic theory can then be used to predict the wave motion in the remainder of the horn. The mass-flow rate at the valve throat as predicted by a quasi-steady analysis for the supersonic case is already known (Eq. 2.3). The mass-flow rate at the new location can be considered to consist of the sum of two terms; one being a mass-flow rate that is equal to that for the valve throat and the other being a perturbation. The perturbation mass-flow rate is due to the effect of the supersonic-flow region on the spatial distribution of the flow density between the valve throat and matching station. As the upstream-facing shock is swept downstream and the supersonic-flow region increases in size (volume) during the opening process of the valve, this region having a lower-than-average atmospheric density replaces one of almost atmospheric density. Consequently an additional outflow of air or a positive perturbation mass-flow rate is produced at the matching station. Similarly, as the valve closes and the supersonic-flow region decreases in size, a corresponding negative perturbation is produced. A quasi-steady flow analysis was performed to determine the perturbation mass-flow rate, and this analysis is given in Appendix C. From the results of this analysis it was concluded that the perturbation was insignificant and the mass-flow rate at the valve throat and matching station could be taken as being equal, as for the case of a subsonic flow.

For the finite-source analysis the matching of the gasdynamic and acoustic parts of the analysis for both the subsonic and supersonic flows is done simultaneously, as the same matching station is used for both types of flow. The mass-flow rate at the matching location, being equal to that at the valve throat (Eq. 2.3), is equated with the mass-flow rate \( \dot{m}_\text{A} \) that is induced at the matching station of radius \( \bar{r} \) and area \( \bar{A} \) (equal to \( \bar{A} \bar{r}^2 / r_0^2 \)) by the particle velocity (Eq. 2.7), in the same manner as was done for the point-source analysis. From the results of this procedure the following first-order, linear, differential equation for \( f(\tau) \) with the mass-flow rate acting as a forcing function can be obtained.

\[
\frac{\bar{r}}{aT_0} f' \left( \frac{\tau}{T_0} \right) + f \left( \frac{\tau}{T_0} \right) = -\bar{r}^2 \frac{\dot{m}_\text{A}}{\rho N_* \left( \frac{\tau}{T_0} \right)} \tag{2.20}
\]

As it has not been assumed that the matching radius or the effective radius of the sound source divided by the wavelength \( (\bar{r} / aT_0) \) approaches zero (point-source approximation), the effects of the far-field term \( \bar{r} f' \left( \tau / T_0 \right) / aT_0 \) can be evaluated. Additionally, the effect of the source size on the overpressure and particle-velocity signatures can be determined.
Near a small source of sound (e.g., a pulsating or an oscillating sphere) the flow is almost incompressible (Ref. 22). If one assumes that the sound source is a point source \((r = 0)\), then the near-field flow is rendered incompressible. It is for this reason that \(f(\tau)\) in Eq. 2.20 follows \(-N^* (\tau)\) exactly in the near-field, if the far-field term \(\frac{r f'(\tau/\tau_0)}{\alpha \tau_0}\) is assumed to be negligible and omitted. Consequently, as the overpressure equals \(-\frac{f'(\tau)}{r}\) (Eq. 2.6) and as its waveform is invariant with distance, the overpressure at any distance has a profile that is identical to \(N^*_0 (\tau)\). If \(N^*_0 (\tau)\) has discontinuities in its profile then so does the signature for the overpressure. The instantaneous rise time of each shock is therefore a direct consequence of the near-field flow being incompressible, resulting from the point-source assumption. On the other hand, when the sound source is assumed to be finite in size the flow near the source is slightly compressible. Then the effect of the relatively small far-field term \(\frac{r f'(\tau/\tau_0)}{\alpha \tau_0}\) in Eq. 2.20 is to make \(f(\tau)\) lag \(-N^*_0 (\tau/\tau_0)\) slightly. The corresponding overpressure at the source therefore lags \(N^*_0 (\tau/\tau_0)\) slightly, and it is continuous as a result, even when \(N^*_0 (\tau/\tau_0)\) is discontinuous. As each shock in the overpressure signature at the source thus has a finite rise time, the signature at subsequent distances also has identical features. The finite rise time of each shock in the overpressure wave is a direct consequence of treating the source as being finite in size, such that the slight compressibility of the fluid near the source is accounted for. Note that the finite rise time of the shocks results from first-order acoustic theory.

The effective size of the sound source or the radius for the matching station \((\bar{r})\) is taken to be equal to the known radius at which the valve is joined to the horn \((\bar{r}_o)\). This particular choice was made for the following reason. From the preceding qualitative discussion, and as will be shown quantitatively, the value for the matching radius affects mainly the rise time of each shock in the predicted wave signature. The two shocks of the simulated sonic boom are formed in the vicinity of the valve, when it begins to open or when it just closes. Hence, in order to predict their rise times, a radius corresponding to that at which they were formed was considered for the matching station.

Once \(f(\tau)\) is solved for using Eq. 2.20, the overpressure and particle velocity follow from Eqs. 2.6 and 2.7. The final results of the finite-source analysis in terms of a general mass-flow-rate profile are summarized below.

\[
\Delta p = \frac{a r \bar{m}_*}{r} \exp \left( -\frac{a \tau}{r_o} \right) \int_{-\infty}^{\tau} N^*_0(y) \exp \left( \frac{a y}{r_o} \right) dy \quad (2.21)
\]

\[
\Delta u = \frac{a r \bar{m}_*}{\rho a r} \exp \left( -\frac{a \tau}{r_o} \right) \int_{-\infty}^{\tau} \left[ N^*_0(y) + \frac{a}{r} N^*_0(y) \right] \exp \left( \frac{a y}{r_o} \right) dy \quad (2.22)
\]

The mass-flow-rate profile \(N^*_0(\tau)\) can be specified to take on the ideal parabolic form (Eq. 2.11), or it can be calculated by means of Eq. 2.15 for an ideal (Eq. 2.17) or an actual valve-plug motion. If the integrals in Eqs. 2.21 and 2.22 are difficult or impossible to evaluate analytically, they can be easily solved for numerically by using a digital computer.
For a general mass-flow-rate profile \( N_*(\tau) \) that is piece-wise continuous and has one or more discontinuities in its first derivative, the approximate variation of overpressure through each shock resulting from the discontinuity can be predicted as follows. Let a discontinuous change in \( N_*(\tau) \) at time \( \tau \) be denoted by \( \Delta N_* \) (a constant) and assume that the transition of overpressure through the shock is very rapid (thin shock) as compared to any variation in \( N_*(\tau) \) just after the discontinuous change. Then the transition in overpressure through the shock (\( \Delta p \)), relative to the overpressure in front of the shock, can be determined by using Eq. 2.21 and certain mathematical limiting processes. The final result is given below.

\[
\Delta p = \frac{r_0^2 \widetilde{m}_* \widetilde{N}_*}{r} \left[ 1 - \exp\left( -\frac{a(\tau - \tau_0)}{r_0} \right) \right]
\] (2.23)

The transition profile is thus an exponential function of time. Across the shock the rise in overpressure is simply \( r_0^2 \widetilde{m}_* \widetilde{N}_* / r \), which equals that for an instantaneous rise-time shock. It is worth noting that for the specific case of a parabolic mass-flow-rate profile (Eq. 2.11) the step change \( \widetilde{N}_* \) is equal to \( 4/r_0 \) at both the start and end of the profile. Hence, the approximate overpressure transition is exponential through both the front and rear shocks of the simulated sonic boom, and the approximate peak amplitude is equal to \( 4r_0^2 \widetilde{m}_*/r \tau_0 \) or \( \Delta p \).

The rise time of a shock is defined herein to be equal to 125% of the time for the overpressure to rise from 10% to 90% of its peak value. (This particular definition is very useful when comparing predicted and measured rise times, because overpressure at the 10% and 90% points of a recorded profile can be measured more accurately than those at the 0% and 100% points.) By using this definition in conjunction with Eq. 2.23 the rise time \( (\tau_1) \) for a shock is given by the following expression.

\[
\tau_1 = 2.75 \frac{r_0}{a}
\] (2.24)

It depends on the radial location at which the valve is joined to the horn, which was the location selected for matching the gasdynamic and acoustic analyses. For the UTIAS travelling-wave horn with an \( r_0 \) value of 0.457 m the predicted rise time for each shock is thus 3.65 ms, and the corresponding shock thickness is 1.25 m. These constant values are in reasonable agreement with experimental data, as measured rise times for simulated sonic booms range from 2 to 6 ms. Note that predicted and measured data are compared in Section 2.2.3.

It is worth noting that the predicted overpressure rise across the shock (Eq. 2.23) depends on the maximum mass-flow rate \( (\widetilde{m}_* \widetilde{r}_*^2) \) or reservoir pressure, valve-plug speed \( (\widetilde{N}_*) \), and attenuates with distance like \( 1/r \). The time scale for the exponential profile is determined by the time constant \( r_0/a \), which depends on the source radius. Hence, the rise time (Eq. 2.24) is only dependent on the source radius.

In the more general case when \( N'_*(\tau) \) changes during the transition time for the shock, the rise time and profile of the shock are not fixed but depend on both \( \widetilde{N}_* \) and \( N'_*(\tau) \). However, the dependence on \( N'_*(\tau) \) is normally small, especially for the rise time. Hence, the simplicity of the constant rise time given by Eq. 2.24 should be noted.
Overpressure and particle-velocity signatures for the wave in the pyramidal horn can be derived in closed form from Eqs. 2.21 and 2.22 for simple expressions of the mass-flow-rate profile. Because the ideal parabolic form of \( N^*_\tau(\tau) \) (Eq. 2.11) is of particular relevance to sonic-boom simulation with a traveling-wave horn, the resulting solutions for overpressure and particle velocity are summarized below.

\[
\Delta p \sim \frac{\Delta p}{\rho a} \left[ 1 - \frac{2\tau}{\tau_e} - \exp \left( -\frac{a\tau}{\tau_e} \right) \right] \quad \text{if } 0 < \tau < \tau_e
\]

\[
\Delta p \sim -\frac{\Delta p}{\rho a} \exp \left[ -\frac{a(\tau - \tau_e)}{\tau_e} \right] \quad \text{if } \tau > \tau_e
\]

\[
\Delta u \sim \frac{\Delta p}{\rho a} \left[ 1 - \frac{2\tau}{\tau_e} + \frac{a\tau_e}{\tau} \left( 1 - \frac{\tau}{\tau_e} \right) \frac{\tau}{\tau_e} - \exp \left( -\frac{a\tau}{\tau_e} \right) \right] \quad \text{if } 0 < \tau < \tau_e
\]

\[
\Delta u \sim -\frac{\Delta p}{\rho a} \exp \left[ -\frac{a(\tau - \tau_e)}{\tau_e} \right] \quad \text{if } \tau > \tau_e
\]

The above expressions are approximate in that only the dominant terms of the complete solutions were retained. The difference between the point-source (Eqs. 2.12 and 2.13) and approximate finite-source (Eqs. 2.25 and 2.26) solutions for the same parabolic mass-flow-rate profile is in the exponential terms which alter the instantaneous rise times to finite ones. This difference is illustrated graphically in Fig. 20, by showing three different duration overpressure signatures. The constant rise times of the front and rear shocks of the N-wave become less conspicuous on longer duration N-waves.

The peak overpressure of the front shock of a finite rise-time N-wave is always less than that for a corresponding N-wave with an instantaneous rise time, and this difference is more marked for a shorter duration N-wave. These features are illustrated by the signatures shown in Fig. 20. Quantitatively, the peak overpressure of the front shock \( \Delta p_2 \) and the corresponding rise time \( \tau_2 \) to maximum overpressure can be derived from Eq. 2.25, and these two results are given below.

\[
\tau_2 = \frac{r_e}{a} \ln \left( \frac{a\tau_e}{2r_e} \right)
\]

\[
\Delta p_2 \sim \frac{\Delta p}{\rho a} \left[ 1 - \frac{2r_e}{a\tau_e} \ln \left( \frac{a\tau_e}{2r_e} \right) \right]
\]

Previously, the overpressure rise across a shock \( \Delta p, \text{ Eq. 2.23} \) was determined to be equal to that \( \Delta p \) for an instantaneous rise-time shock. The new result (Eq. 2.28) does not contradict the previous approximate result for which \( N^*_\tau(\tau) \) was assumed to be constant during the shock transition time.

As a final example of the finite-source analysis, three different
duration overpressure signatures, as predicted for the UTIAS travelling-wave horn, are shown in Fig. 21. The finite rise-time signatures were calculated by means of Eq. 2.21 with a mass-flow-rate profile given by Eq. 2.15 \((b/c = 0.89)\). The corresponding instantaneous rise-time signatures as calculated by the point-source analysis (Eq. 2.9) are shown for comparison. The effects of the rise time on the signature and its peak overpressure are very similar to those for the previous case (parabolic mass-flow-rate profile). In addition to the 12% decrease in peak overpressure of the front shock (and rear shock) as determined by the point-source analysis for a nonideal mass-flow-rate profile (Eq. 2.15 with a \(b/c\) value of 0.89), a further decrease occurs because of the finite rise time. This additional decrease is a function of the N-wave duration. For example, it is equal to 8%, 4% and 2% for N-waves having respective durations of 100, 200 and 400 ms. From predicted results such as these, an empirical relation for the peak overpressure \(\Delta p_s\) of the front shock was devised for the UTIAS facility, and it is given below.

\[
\Delta p_s = 0.88 \Delta p \left[ 1 - \frac{2r_o}{a r_o} \ln \left( \frac{a r_o}{2r_o} \right) \right]
\] (2.29)

This expression is a modified version of Eq. 2.28. The factor 0.88 has been inserted in the equation to account for the nonideal mass-flow-rate profile associated with the UTIAS mass-flow valve.

It is worth noting that the total impulse associated with a wave predicted by either the point-source or finite-source is zero, or, alternatively, the time integral of the overpressure is zero. The impulse associated with the positive and negative portions of each predicted overpressure signature shown in Fig. 20 are not equal, owing to the omission of certain relatively small terms from Eq. 2.25. As a similar approximation has not been made for the predicted overpressure signatures given in Fig. 21, the total impulse for each signature is identically zero. Note also that the overpressure for each finite rise-time signature (Fig. 21) lags that for the instantaneous rise-time profile. This lag diminishes with increasing duration because the source radius divided by the N-wave wavelength becomes smaller and the effect of the far-field term in Eq. 2.20 becomes less significant.

The finite-source analysis predicts essentially the same overpressure and particle-velocity signatures as does the point-source analysis. However, it predicts more realistic shocks having finite rise-time profiles. Although the predicted rise time of the shock is not a function of such factors as valve-plug speed, as occurs in reality, the simple finite rise-time result (Eqs. 2.24) is important in that it predicts the correct magnitude of the rise time for the simulated sonic boom. These results do not seem to be restricted to the UTIAS travelling-wave horn. For example, the predicted rise time for simulated sonic booms generated with the GASE facility (Eq. 2.24 with \(r_o\) equal to 28 cm) is 0.8 ms, and the measured values (minimum) are quoted as being about 1 ms (Ref. 28).

2.2.3 Comparison of Predicted and Measured Results

All of the predicted results that are compared with experimental data in this section have been determined by using the finite-source analysis. Unless noted to the contrary the mass-flow-rate profile that was used in the analysis corresponds to that for the UTIAS mass-flow valve. That is, the valve-plug displacement with time was taken to be linear in each half of the opening and closing cycle (Eq. 2.17)
and the mass-flow-rate profile was thus calculated by means of Eq. 2.15 with a b/c value equal to 0.89.

Four typical simulated sonic booms that were recorded at a radial distance of 15.2 m are shown in the first column of Fig. 22. The reservoir pressure that was used to generate each N-wave, and the resulting N-wave amplitude and duration, are all indicated beside each oscillogram. The signals were filtered electronically to remove the higher frequencies of the jet noise, in order to facilitate the comparison of measured and predicted signatures. All four of the overpressure signatures were traced and compared with predicted overpressure profiles, and these results appear in the second column. The predicted and measured profiles having durations of 60, 100, 200 and 300 ms are in good agreement. In general the basic overpressure N-waves generated with the UTIAS travelling-wave horn can be predicted successfully. However, it was found that 10% to 15% of the measured N-waves differed from the norm, and one of these abnormal signatures is shown in Fig. 22b. It is in reasonable agreement with the predicted profile. Such an anomalous result can usually be attributed to malfunctions associated with the mass-flow valve.

In order to present data in a more compact form than showing the entire N-wave signature, only the peak overpressure of the simulated sonic boom was measured and compared with that predicted by the finite-source analysis (Eq. 2.21 or, equivalently, by Eq. 2.29). The peak overpressure of simulated sonic booms having three different durations of 100, 200 and 300 ms are shown as a function of the reservoir pressure in Fig. 23. The predicted and measured data are in good agreement for the entire range of reservoir pressures shown. The finite-source analysis is therefore capable of predicting the peak overpressure of the N-wave for both cases of a supersonic \((p_e/p > 1.89)\) and a subsonic \((1 < p_e/p < 1.89)\) flow. In general the analysis predicts a peak overpressure that is about 5% higher than that measured, but the predicted value is normally within the experimental error, as can be seen in Fig. 23, where an error bar is shown with each measured result. By repeating a few experiments and reading the oscillograms, the experimental error was found to be about \(\pm 7.5\%\).

The results shown in Fig. 23 also illustrate that powerful simulated sonic booms can be produced by the UTIAS travelling-wave horn, which is important for studies of structural response and damage. It is worth noting that the predicted curves for each duration (Fig. 23) have the same form as that for the mass-flow rate per unit area \(\dot{m}_w\) as a function of reservoir pressure (Fig. 15). This occurs because the peak overpressure of the N-wave \(\Delta p\) varies directly as the mass-flow rate per unit area (Eq. 2.14).

The dependence of the peak overpressure of the simulated sonic boom on N-wave duration is shown in Fig. 24, for two different reservoir pressures of 1.41 and 5.07 atm. The results illustrate that higher amplitude N-waves occur for shorter durations. The predicted and measured results are in good agreement for both reservoir pressures. The lower one corresponds to subsonic flow and the upper one to a supersonic flow. Note that the predicted overpressure varies not only inversely with N-wave duration (as in \(\Delta p\) of Eq. 2.14) but also has an additional dependence owing to the effect of a finite rise time (see Eq. 2.29).

The attenuation of peak overpressure with increasing radial distance for two simulated sonic booms having different durations of 150 and 300 ms is shown in Fig. 25. These results correspond to a fixed reservoir pressure of 1.41 atm (subsonic flow). Similar results for a higher reservoir pressure of 3.05 atm (supersonic flow) are shown in Fig. 26. The predicted and measured data are in
good agreement for both reservoir pressures. Note that the analysis predicts that the peak overpressure varies inversely with radial distance (Eq. 2.29). As the N-wave amplitude is relatively weak and as the propagation distance (~ 20 m) is short compared with the wavelength, it is not surprising that the simulated sonic boom attenuates with distance according to the acoustic decay law.

Particle-velocity measurements of flows induced by simulated sonic booms were made to supplement overpressure signatures for checking the analysis. A Thermosystems Hot-Film Anemometer (Model 1031-2) was used with a linearizer unit (Model 1034-4). Because the anemometer was incapable of sensing flow direction the resulting signal was entirely positive valued. The signal therefore had to be interpreted and parts of the signature corresponding to a reverse flow had to be inverted in order to obtain a true signature (see Fig. 27). This procedure was not difficult for simulated sonic booms, as the first part of the signature corresponds to an outward radial flow and the latter to an inward one. The particle-velocity signals were filtered electronically to remove high-frequency noise which was generated by the electronics of the anemometer. This is particularly true in the sensitive range required for measuring the low flow speed associated with the simulated booms (order of 1 m/s). The low-pass filtering process also removed the higher frequencies of the jet noise that is superimposed on the velocity signal, and in some cases it also increased the rise times on the two shocks. It was very difficult to obtain good particle-velocity measurements as the flow speed was small and turbulent motion of the supposed quiescent air in the horn caused significant interference. Only a few percent of the recorded results were acceptable, and the ones to be given are unique.

Short-duration (80 ms) particle-velocity signatures that were recorded at two different radial distances of 7.6 and 15.2 m are shown in the oscillograms of Fig. 27. The interpreted signature is shown directly below the oscillogram and compared with the predicted profile. Similar results are given in Fig. 28 for a longer duration velocity wave of 300 ms. The predicted and measured signatures are in fair agreement for both the short and long duration waves. In general the predicted wave is of higher amplitude than the measured one, and the agreement between the predicted and measured signatures is not normally as good as indicated by the results shown in Figs. 27 and 28. Note that the results given in Figs. 27 and 28 illustrate the change in particle-velocity waveform with increasing radial distance. The change is small (but noticeable) because the radial distance has only doubled from 7.6 to 15.2 m.

Distorted sonic booms have been produced with the UTIAS travelling-wave horn by using a blunt plug in the mass-flow valve. This blunt plug had a square base that was 5.7 cm on each side and the height of the pyramidal front was 2.05 cm. Hence, the b/c value for this plug was 0.36. Two distorted N-waves resulting from the use of this valve plug are shown in the oscillograms of Fig. 29. While generating the distorted boom shown in Fig. 29a the valve-plug motion was linear in both the opening and closing strokes. However, the plug was withdrawn a maximum distance of 5.0 cm, which exceeds the length of its pyramidal front. Consequently, the distorted N-wave has a constant-pressure portion at its centre. The signature is reproduced below the oscillogram and compared with the predicted overpressure profile, and the two results are in fair agreement. For the second distorted N-wave (Fig. 29b) the valve plug was withdrawn a distance of only 2.8 cm, which is slightly longer than the length of the pyramidal front of the plug. The constant-pressure portion is thus eliminated. This signature has also been reproduced and compared with the predicted profile, and the two results are in fair agreement.
Distorted N-waves such as those shown in Fig. 29 are not normally used for sonic-boom research tests. They have been included in this section to demonstrate the capability of the analysis. The distorted signatures as predicted by the analysis depend on the throat-area or mass-flow-rate profile \( N_*(\tau) \) (see Eq. 2.21). This profile was calculated by means of Eq. 2.15 which takes the valve-plug geometry \((b/c = 0.36 \text{ for the blunt plug})\) into account. Because the predicted and measured signatures are in good agreement the validity of Eq. 2.15 is established.

The predicted rise time for simulated sonic booms generated with the UTIAS travelling-wave horn is constant at 3.65 ms (Eq. 2.24). To check this prediction an experimental investigation was made. For the first experiment the duration of the N-wave was fixed at 100 ms, and the reservoir pressure was varied in approximately equal increments from 1.34 to 7.8 atm such that twenty different amplitude N-waves were produced. From the profile of the front shock recorded in the horn test section (radial distance of 21 m), the rise time was determined. It seemed to be independent of the N-wave amplitude, or, equivalently, the reservoir pressure, as the rise times appeared to be random. It was concluded that the rise time was independent of the reservoir pressure, in accordance with the predicted result. Two additional experiments were made for different N-wave durations of 200 and 300 ms. These rise-time results were also independent of reservoir pressure. For each different duration the twenty rise-time results were averaged, and the pertinent information is given in the following table.

<table>
<thead>
<tr>
<th>EXPERIMENT</th>
<th>N-WAVE DURATION</th>
<th>AVERAGE RISE TIME</th>
<th>STANDARD DEVIATION</th>
<th>RESERVOIR PRESSURE</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>100 ms</td>
<td>3.0 ms</td>
<td>0.3 ms</td>
<td>1.3 to 7.8 atm</td>
</tr>
<tr>
<td>2</td>
<td>200 ms</td>
<td>3.7 ms</td>
<td>0.2 ms</td>
<td>1.3 to 14 atm</td>
</tr>
<tr>
<td>3</td>
<td>300 ms</td>
<td>5.5 ms</td>
<td>0.3 ms</td>
<td>1.3 to 14 atm</td>
</tr>
</tbody>
</table>

Similar rise-time results for the rear shock of the simulated sonic boom were not obtained because severe jet-noise superimposed on high-amplitude N-waves masked the shock profile.

The significant feature of the experimental results is that the rise time is not constant as predicted, but the rise time depends on the N-wave duration. More correctly the actual rise time depends on the geometry \((b)\) of the valve plug and the plug speed during the initial opening of the valve. For an N-wave the plug speed, initially or otherwise, is simply equal to \(2b/\tau_*\). For the finite-source analysis the motion of the valve plug was taken into account. In fact, the predicted shock profile (overpressure rise across the shock) from Eq. 2.23 was found to depend on the plug speed because of the parameter \(N_*\). The predicted rise time (Eq. 2.24), however, was independent of the plug speed. The finite-source analysis (Section 2.2.2) is therefore incapable of predicting the dependence of actual rise times on valve-plug speed or N-wave duration, as was discovered experimentally. However, the analysis is notable as it does give the correct magnitude of the rise time (3.65 ms) for the UTIAS travelling-wave horn. It is worth noting that the predicted rise time for the similar GASL facility is 0.8 ms, which is in reasonable agreement with measured rise times (minimum) of 1 ms (Ref. 28, 29).

Two signatures of the front shock of different N-waves having amplitudes of 75 and 150 N/m² are shown in the oscillograms of Fig. 30, and two signatures of
the rear shock of different N-waves are shown in Fig. 31. Each recorded signature has been reproduced below its associated oscillogram and compared with the predicted shock profile (Eq. 2.21). The predicted and measured shock profiles are in reasonably good agreement. Note that experimentally measured profiles having rise times (indicated beside each oscillogram) approximately equal to the predicted value of 3.65 ms were selected purposely for Figs. 30 and 31, such that measured and predicted profiles could be suitably compared.

It is worth noting that air leakage from the reservoir past the valve plug into the pyramidal horn affects the front shock profile. The slow initial rise in overpressure for the front shocks shown in Fig. 30 is due to a relatively small leakage rate associated with the UTIAS valve. Note that a more severe leakage rate can increase the rise time by 10%.

The UTIAS travelling-wave horn is capable of simulating the rise time of actual sonic booms from supersonic aircraft, because the average rise time for simulated sonic booms is very close to the average rise time (defined from 0% to 100% rise in overpressure) of actual sonic booms. For example, the average rise time of 638 actual sonic booms was found to be 5.4 ms (Ref. 33).
2.3 Analysis of a Finite-Volume Reservoir

The high-pressure reservoir of the UTIAS travelling-wave horn has a finite volume (2.6 m³). Consequently, the state of the reservoir air does not remain constant while the mass-flow valve opens and closes and the resulting high-speed air flow from the reservoir generates a simulated sonic boom in the pyramidal horn. An investigation was made to predict the change in reservoir conditions (pressure, temperature, density and sound speed) with time and thus determine the effect of changing reservoir conditions on the signature of the simulated sonic boom. The analysis was performed mainly to substantiate the assumption of a constant reservoir state that was made for both the point-source (Section 2.2.1) and finite-source (Section 2.2.2) analyses. This same assumption will also be made for the jet-noise analysis (Section 2.4). The results of the finite-volume analysis, which are in fair agreement with experimental data, should be of interest to other researchers who are operating or designing a similar travelling-wave horn or any other facility that uses a similar reservoir and valve system.

For the finite-volume analysis, the reservoir state variables are assumed to be spatially uniform and thus dependent only on time. Any change in state of the reservoir air owing to an outflow of air through the mass-flow valve is taken to be adiabatic and reversible (isentropic), and the flow leaving the reservoir is assumed to be one-dimensional and quasi-steady. If \( \rho_0(t) \) denotes density as a function of time \( t \) for the air in the reservoir having a constant volume denoted by \( V \), the rate of increase of mass of the reservoir air is \( \rho_0(t) V \), and it is equal to the mass-flow rate of air that enters the reservoir via the mass-flow valve. This mass-flow rate can be expressed as \( -m_\infty(t) A_\infty N_\infty(t) / V \) at the valve throat, where \( m_\infty(t) \) is the mass-flow rate per unit area, \( A_\infty \) denotes the maximum throat area, and \( N_\infty(t) \) is the normalized throat-area profile owing to the valve-plug motion (Eq. 2.15). For a quasi-steady flow from the reservoir, \( m_\infty(t) \) is given by Eq. 2.1, but now the reservoir pressure \( (p_e) \) and sound speed \( (a_\infty) \) and the flow Mach number \( (M_e) \) are each a function of time. (As \( m_\infty \) is not a constant for this analysis it is not designated with a bar as it was for Eq. 2.1.) From the preceding developments the reservoir density is governed by the following differential equation.

\[
\rho_0'(t) + m_\infty(t) A_\infty N_\infty(t) / V = 0 \quad (2.30)
\]

\[
m_\infty(t) = \frac{p_e(t)}{\rho_a} \frac{a}{a_e(t)} M_e(t) \left[ 1 + \frac{Z-1}{2} M_e^2(t) \right]^{-\frac{(\gamma+1)}{2(\gamma-1)}}
\]

\[
M_e(t) = \left[ \frac{2}{\gamma-1} \left\{ \left( \frac{p_e(t)}{p} \right)^{\frac{Z-1}{\gamma}} - 1 \right\} \right]^{1/2} \quad \text{if} \; 1 < \frac{p_e}{p} < 1.89
\]

\[
M_e(t) = 1 \quad \text{if} \; \frac{p_e}{p} > 1.89
\]
For an isentropic process the changes in reservoir pressure, temperature \((T_o)\), density and sound speed from their initial values (denoted by a bar) can be related by the following well-known expressions (Ref. 31).

\[
\frac{p_o(t)}{p_o} = \left[ \frac{T_o(t)}{T_o} \right]^{\frac{\gamma}{\gamma-1}} = \left[ \frac{\rho_o(t)}{\rho_o} \right]^\gamma = \left[ \frac{a_o(t)}{a_o} \right]^{\frac{2\gamma}{\gamma-1}} \tag{2.31}
\]

The state of the reservoir air as a function of time is completely described by Eqs. 2.30 and 2.31.

The relative changes in the reservoir pressure, temperature, density and sound speed are normally small for the UTIAS travelling-wave horn, and thus the relative changes in \(M_x\) and \(m_x\) are also small. A first-order solution for the reservoir variables is therefore usually sufficient. To obtain such a solution each pertinent variable is expressed in terms of its initial value plus a perturbation term, as shown below.

\[
\begin{align*}
p_o(t) &= \bar{p}_o + \delta p_o(t) \\
\rho_o(t) &= \bar{\rho}_o + \delta \rho_o(t) \\
T_o(t) &= \bar{T}_o + \delta T_o(t) \\
a_o(t) &= \bar{a}_o + \delta a_o(t) \\
m_x(t) &= \bar{m}_x + \delta m_x(t) \\
M_x(t) &= \bar{M}_x + \delta M_x(t)
\end{align*}
\tag{2.32}
\]

By using the above expressions (Eqs. 2.32) with Eqs. 2.30 and 2.31, the first-order solution for the reservoir perturbation pressure can be determined, and the final result is given below.

\[
\frac{\delta p_o}{p_o} = -\gamma \bar{M}_x \left[ 1 + \frac{\gamma-1}{2} \bar{M}_x^2 \right] \frac{-(\gamma+1)}{2(\gamma-1)} \frac{\bar{a}_o \bar{A}_x t_o}{\bar{V}} \int_o^t N_x(y) \, dy \tag{2.33}
\]
The new symbol $t_*$ denotes the time that the mass-flow valve remains open and thus the duration of both the air discharge and the simulated sonic boom. The corresponding solutions for perturbation temperature, density and sound speed follow from the first-order approximations of Eqs. 2.31, as given below.

$$\bar{M}_* = \left[ \frac{2}{\gamma-1} \left\{ \left( \frac{p^*}{p} \right)^{\gamma-1} - 1 \right\} \right]^{1/2} \quad \text{if } 1 < \frac{p^*}{p} < 1.89$$

$$\bar{M}_* = 1 \quad \text{if } \frac{p^*}{p} > 1.89$$

The relative change in each perturbation variable ($\delta p^*(t)/p^*$, $\delta T^*(t)/T^*$, $\delta p^*(t)/p^*$, $\delta a^*(t)/a^*$) is therefore dependent on the nondimensional parameter $a^*_eN^*(t)/V$ and the profile of $N^*(t/t_e)$ for the supersonic case ($\bar{M}_* = 1$), and it has an additional dependence on the initial reservoir pressure for the subsonic case ($\bar{M}_* < 1$).

For the specific case of a supersonic flow ($\bar{M}_* = 1$), an ideal parabolic throat-area profile (Eq. 2.11), and an $N$-wave duration of 300 ms, the relative change for the perturbation pressure, density, temperature and sound speed are shown as a function of time in Fig. 32. (Other parameters relevant to the UTIAS facility are indicated in the figure.) In general the relative changes in the four state variables are small for this long duration, being greatest for the perturbation pressure (6.7%) and smallest for the perturbation sound speed (1%). For this fixed duration of 300 ms the relative changes are identical for all initial reservoir pressures ($p^*$) above 1.89 atm. For a reservoir pressure lower than 1.89 atm the profiles have the same shape but the relative changes are smaller than shown in the figure. Additionally, all relative changes in state variables diminish with decreasing duration. It can therefore be concluded that the change in state of the reservoir air is small during the simulation of a sonic boom with the UTIAS travelling-wave horn. In fact, a large reservoir volume was selected for the UTIAS facility to ensure that this result would occur.

The effect of a time-dependent reservoir state on the signature of a simulated sonic boom can be evaluated as follows. The mass-flow rate at the valve throat given by $m^*(t)A^*_eN^*(t)$ can be expressed to first order as $\bar{m}_*A^*_eN^*(t)$ plus a perturbation term $\delta m^*(t)$. The overpressure signature as predicted by Eq. 2.9 of the point-source analysis is modified slightly and given by the following expression.

$$\Delta p = \frac{r^2}{r T_e} \left[ N^*_e(\tau/T_e) + \frac{\delta m^*_e(\tau/T_e)}{\bar{m}_*} \right] \quad (2.35)$$

\[30\]
From the first-order analysis given in this section the perturbation mass-flow rate can be expressed as shown below.

\[
\frac{\delta m_p(\tau)}{\bar{m}_p} = \left[ \frac{2 + (\gamma-1) \bar{M}_p^2}{2 \gamma \bar{M}_p^2} \right] \frac{\delta p_e(\tau)}{\bar{p}_e}
\] (2.36)

Therefore, by using Eqs. 2.33, 2.35 and 2.36, the following overpressure expression can be obtained.

\[
\Delta p = \frac{r_e^2 \bar{M}_p}{r \tau_e} \left[ N'_*(\tau/\tau_e) - D N_*(\tau/\tau_e) \right]^{-\frac{(3-\gamma)}{2(\gamma-1)}}
\] (2.37)

\[
D = \frac{\bar{a}_e \bar{A}_* \tau_e}{V \bar{M}_p} \left[ 1 + \frac{\gamma - 1}{2} \bar{M}_p^2 \right]^{-\frac{1}{\gamma-1}}
\]

The distortion of the signature decreases with increasing reservoir volume, and only vanishes for an infinite volume.

The distortion parameter (D) is shown as a function of reservoir pressure and duration in Fig. 33a. Note that the values used for \( \bar{a}_e, \bar{A}_* \) and V (indicated in the figure) of the distortion parameter correspond to those for the UTIAS travelling-wave horn. At high reservoir pressures (supersonic flow) the parameter is constant and small, even for the long duration of 300 ms. The parameter becomes significant only when the initial reservoir pressure approaches 1 atm.

To illustrate the distortion in an overpressure signature, the throat-area profile is taken to be parabolic (Eq. 2.11). Then the overpressure signature is given by the following expression.

\[
\Delta p = \bar{A}_p \left[ 1 - \frac{2r}{\tau_e} - D \left( 1 - \frac{\tau}{\tau_e} \right) \frac{\tau}{\tau_e} \right] \text{ if } 0 < \tau < \tau_e
\] (2.38)

\[
\Delta p = 0 \text{ if } \tau < 0, \tau > \tau_e
\]
When the distortion parameter is identically zero an N-wave profile is obtained, as shown in Fig. 33b. Otherwise a distorted N-wave results, as is also shown in the figure. The specific value of D for the distorted signature is equal to 0.1, a rather large value. Note that ΔAp is the maximum difference between the ideal and distorted profiles.

A new mass-flow valve has been designed and is currently being manufactured for the UTIAS facility. It features a large throat area (A*) in order to minimize the jet-noise that is superimposed on the simulated sonic boom. For the case of this new valve the distortion parameter will increase dramatically by a factor of about twenty. Part of this increase is due to about a ten-fold increase in throat area and the other part is associated with the use of a lower reservoir pressure (about 1.5 atm). To offset such a marked rise in the distortion parameter the reservoir volume will be enlarged. If the resulting distortion parameter is about 0.1 then the distortion in the simulated sonic boom will be noticeable but not too significant, as illustrated in Fig. 33b. This degree of distortion in the signature will certainly be more acceptable than intense jet noise.

The finite-volume analysis was not verified directly by means of experimental results showing distorted overpressure signatures, as the distortion is almost always too small to be discernible for simulated sonic booms produced with the UTIAS facility. Instead, measurements of the total change in reservoir pressure were made and compared with those predicted by the analysis (Eq. 2.33). The measured and predicted values of Δp are shown in Fig. 34, plotted versus initial reservoir pressure (p0) for three different durations of 100, 200 and 400 ms. The reservoir pressure change as predicted by the first-order solution (Eq. 2.33) is always higher than the measured values, ranging from a few percent for the shortest duration to about 10% for the longest duration. In the case of low reservoir pressures corresponding to a subsonic flow (1 > p0/p < 1.89) the predicted and measured data are in excellent agreement. The pressure change as predicted by the exact solution to Eq. 2.30 for a supersonic flow (M* = 1) is also shown in the figure, and the results are in good agreement with experimental data. The exact solution (to be given) illustrates the inadequacy of the first-order solution as the initial reservoir pressure and the duration become unduly large. Note that for the predicted results the integral of N*(t/t0) that occurs in Eq. 2.33 was calculated by using an N*(t/t0) profile corresponding to the UTIAS valve (Eq. 2.15 and a b/c value of 0.89). Hence, the value of the definite integral is equal to 0.633, which is 5% less than the value of two-thirds for an ideal parabolic profile (Eq. 2.11).

It is worth noting that Eq. 2.30 in conjunction with Eq. 2.31 can be solved exactly for the case of a supersonic flow (M* = 1). The exact solution for the reservoir pressure p(t) is given below.

\[
\frac{p(t)}{p_0} = \left[ 1 + \frac{\gamma-1}{2} \left( \frac{\gamma+1}{2} \right) \frac{-a_0 A_\infty t}{V} \int_0^t \frac{t_0}{t} N_\infty(y) \, dy \right]^{\frac{-2\gamma}{\gamma-1}}
\]

(2.39)

To first order, of course, this exact solution reduces to the previous first-order result (Eq. 2.33). For a subsonic flow an exact solution can also be derived. It is not given here as it is long and complex. The solution depends on the particular
value of the specific heats ratio ($\gamma$) and it is implicit in each reservoir state variable. These exact solutions could be used to derive a mass-flow-rate profile and thus evaluate the effect of time-dependent reservoir conditions on the signature of the simulated sonic boom. This approach was disregarded, not because the analysis was more difficult, but because the relevant parameters become obscured by the more complex analysis. Furthermore, the first-order analysis is sufficiently accurate for the purpose of this report.

2.4 Jet-Noise Analysis

The objective of the work given in this section is to provide further insight into the high-frequency noise that is superimposed on the simulated sonic boom, in order to explain certain characteristics of this noise and also to illustrate a more direct method of reducing the intensity of the noise than the approach of utilizing a low-pass acoustic filter. The more direct approach is to employ a mass-flow valve that has a large throat area when fully open.

The high-speed flow of air entering the pyramidal horn from the high-pressure reservoir is similar in some ways and different in others to that of a freely expanding air jet. Both jets can be subsonic or supersonic and both produce high-frequency noise. One obvious difference is that the jet in the pyramidal horn is confined. The turbulent-flow noise therefore arises as a result of flow separation and boundary-layer growth near the valve, in contrast to mixing and entrainment of ambient air with a freely expanding jet. Also, because the jet is confined, the jet noise reverberates in the horn and in the test room which encloses the large end of the horn. Another obvious difference is that the confined air jet is time dependent in that the mass-flow valve controls the mass-flow rate. However, the throat-area changes relatively slowly with time as compared with the time for the flow to adjust to a new throat area. Despite all of these differences, rudimentary jet-noise theory for an unconfined steady-state jet was employed in an effort to describe the jet noise from the high-speed flow near the mass-flow valve.

The total power of the jet noise emitted by subsonic and supersonic jets having an exit area $A_*$ and flow speed $u_*$ is proportional to $p_u^2 A_*/a^2$ and $p_u^3 A_*$, respectively (Ref. 34). Because the intensity of the noise is directly proportional to the power divided by the square of the radial distance from the source sound, the intensity of the jet noise varies as $p_u^2 A_*/a^2 r^2$ for a subsonic jet and as $p_u^3 A_*/r^2$ for a supersonic jet. Instead of employing these expressions for intensity, it is usually more convenient to use the root-mean-square (rms) overpressure ($\Delta p_{\text{rms}}$), because measurements of the simulated sonic boom were taken in the form of overpressure. As the intensity of the jet noise equals $\Delta p_{\text{rms}}/\rho a$, $\Delta p_{\text{rms}}$ varies as $p_u^2 A_*/a^2 r$ for a subsonic jet and as $\rho a_1^2 u_*^3 A_*^2/2$ for a supersonic jet.

The two expressions for $\Delta p_{\text{rms}}$ from jet-noise theory can be applied to the confined air jet in the pyramidal horn, in order to illustrate how $\Delta p_{\text{rms}}$ varies with time for the jet noise superimposed on the simulated sonic boom. First, it is assumed that the mass-flow rate at the valve throat can be predicted by using a quasi-steady flow analysis, as already given in Section 2.2.1 for the point-source analysis. The mass-flow rate $m_*(T)$ is thus given by Eq. 2.3, and the mass-flow rate per unit area $m_*/A_*$, equal to $m_*(T)/A_*$ or $p_u u_*$, is given by Eq. 2.1. Second, as the state of the reservoir air does not change significantly during the generation of a simulated sonic boom (see Section 2.3), it can be considered to be fixed. Then $m_*$ and $u_*$ are independent of time. However, the valve-throat area is time dependent and it is denoted by $A_* N_*(T)$. Therefore, the root-mean-square overpressure of the
jet noise for a subsonic \( (1 < \frac{p_\infty}{p} < 1.89) \) and a supersonic \( (p_\infty/p > 1.89) \) air jet in the pyramidal horn can be expressed in the following form.

**Subsonic jet:**

\[
\Delta p_{\text{rms}} \sim \frac{\rho}{a^2 \sqrt{r}} \cdot \frac{u_\infty^{1/4}}{A_\infty^{1/2}} \cdot N_{\ast}^{1/2}(\tau) \tag{2.40}
\]

**Supersonic jet:**

\[
\Delta p_{\text{rms}} \sim \frac{\rho a^{1/2} u_\infty^{-3/2}}{r} \cdot \frac{A_\infty^{1/2}}{A_\ast^{1/2}} \cdot N_{\ast}^{1/2}(\tau) \tag{2.41}
\]

From these results the predicted variation of \( \Delta p_{\text{rms}} \) with time for the jet noise superimposed on the simulated sonic boom follows the square root of \( N_{\ast}(\tau) \). For the particular case of a parabolic throat-area profile (Eq. 2.11), \( \Delta p_{\text{rms}} \) varies with time as \( (\tau/\tau_{\text{c}} - \tau^2/\tau_{\text{b}})^{1/2} \). Thus, \( \Delta p_{\text{rms}} \) increases from zero at the head of the simulated sonic boom to a maximum at the centre, and then it decreases to zero again at the tail of the N-wave.

The dependence of the root-mean-square overpressure of the jet noise on the mass-flow rate at the valve throat (which depends on the reservoir pressure) and on the throat area is now investigated. For simplicity consider only the maximum values of the root-mean-square overpressure \( \Delta p_{\text{rms}} \), mass-flow rate \( \dot{m}_\ast A_\ast \) and throat area \( A_\ast \), corresponding to a fully-open valve. Then \( N_{\ast}(\tau) \) equals unity, and Equations 2.40 and 2.41 can be expressed in the following form.

**Subsonic jet:**

\[
\overline{\Delta p}_{\text{rms}} \sim \frac{(\dot{m}_\ast A_\ast)^{1/4}}{\rho a^2 A_\ast r^{7/2}} \tag{2.42}
\]

**Supersonic jet:**

\[
\overline{\Delta p}_{\text{rms}} \sim \frac{a^{1/2} (\dot{m}_\ast A_\ast)^{3/2}}{\rho^{1/2} r A_\ast} \tag{2.43}
\]

Note that the flow velocity \( u_\infty \) has been replaced by \( m_\ast/\rho A_\ast \) or \( \dot{m}_\ast A_\ast/\rho A_\ast \), where the density at the exit of the jet \( (\rho_\ast) \) has been taken to be atmospheric density \( (\rho) \), as the effect of a heated or cold jet on the jet noise is small (Ref. 34).

At a fixed radial distance from a mass-flow valve having a maximum throat-area \( A_\ast \), the predicted root-mean-square overpressure (Eqs. 2.42 and 2.43) varies directly as the mass-flow rate \( \dot{m}_\ast A_\ast \) or \( \dot{m}_\ast A_\ast/\rho_\ast \) to the fourth power for a subsonic jet and only to the three-halves power for a supersonic jet. Remember that the peak overpressure \( \Delta p \) of the simulated sonic boom increases linearly with the mass-flow rate (Eq. 2.14). Consequently, as the reservoir pressure is increased to achieve a larger mass-flow rate and thus a longer duration or a more powerful
simulated sonic boom, the increase in jet noise is proportionally higher than the increase in peak overpressure. Quantitatively, \( \Delta p_{\text{rms}} \) increases faster than \( \Delta p \) by \( \bar{m}_*A_* \) to the third power for a subsonic jet and to the one-half power for a supersonic jet.

The jet noise can be reduced substantially by using a mass-flow valve that features a large throat area. Once the peak overpressure (\( \Delta p \)) of the simulated sonic boom has been selected, the mass-flow rate (\( \bar{m}_*A_* \)) required to produce this boom is similarly fixed (\( \bar{m}_*A_*^2 \) in Eq. 2.14). Consequently, for a fixed \( \bar{m}_*A_*^2 \) in Eqs. 2.42 and 2.43, \( \Delta p_{\text{rms}} \) diminishes with increasing throat area \( A_* \) raised to the power of unity for a supersonic jet and to the seven-halves power for a subsonic jet. A sample calculation is performed to illustrate the significant reduction in jet noise that can be obtained when the throat area is increased by a factor of eight. Consider the special case (UTIAS travelling-wave horn) when the initial throat area is only 30 cm\(^2\) and a high reservoir pressure of 7.56 atm is required to produce a simulated sonic boom with a peak overpressure of 200 N/m\(^2\) and a duration of 350 ms. For this initial reservoir pressure the jet is supersonic and \( \bar{m}_*/p_a \) equals 4.4 (Table 1). An increase in throat area by a factor of four reduces \( \bar{m}_*/p_a \) to the value of 1.1, as \( \bar{m}_*/A_* \) must be held constant in order to maintain an equivalent simulated sonic boom. For this value of \( \bar{m}_*/p_a \) the flow is just sonic, as \( \rho_*/p \) has been reduced to 1.89 (Table 1). A further two-fold increase in area results in a new value for \( \bar{m}_*/p_a \) of 0.55 and a new pressure ratio \( \rho_*/p \) of 1.2 (Table 1). The resulting eight-fold increase in area has markedly reduced the reservoir-pressure ratio from 7.56 to 1.2, while maintaining the same mass-flow rate and thus an equivalent simulated sonic boom. The root-mean-square overpressure has been reduced by a factor of four for the supersonic case (Eq. 2.43) and by a further factor of two raised to the seven-halves power for the subsonic case (Eq. 2.42). Hence, \( \Delta p_{\text{rms}} \) has been reduced substantially by a factor of 46 for this eight-fold increase in throat area. Alternatively, the diminishment in jet noise is 33 dB. It is for this reason that a mass-flow valve with a large throat area appears to be attractive for future sonic-boom simulators using a mass-flow valve.

Measurements of the jet noise were made to check certain parts of the jet-noise analysis. A simulated sonic boom (300-ms duration) generated with a low reservoir pressure of 1.21 atm (subsonic jet) is shown in Fig. 35a. Substantial jet noise is superimposed on the basic N-shaped signature because the jet-noise absorber was not used. A high-pass electronic filter was used to remove the low-frequency content of the signal, and the resulting high-frequency jet noise is shown in Fig. 35b. The root-mean-square overpressure of this jet-noise signal was obtained by first digitizing the signal (20 points per millisecond) with an analog to digital computer. Then the root-mean-square value was calculated every 5 ms for an overlapping time interval of 20 ms. These results are shown in Fig. 35c. The predicted profile of \( \Delta p_{\text{rms}} \) is also shown, that is, \( (\tau/\tau_e - \tau_e/\tau_e)^{1/2} \). The amplitude of the predicted profile is not known and has been selected to fit the experimental data. A similar set of results to those shown in Fig. 35 is given in Fig. 36 for a higher reservoir pressure of 6.1 atm (supersonic jet). For this last set of results the jet noise is very intense. From the results given in Figs. 35c and 36c, the predicted profile of \( \Delta p_{\text{rms}} \) is in reasonable agreement with the experimental distribution. The major difference is that the jet noise does not end abruptly at the tail of the N-wave, as reverberation occurs in the horn and test room.

From experimental results of \( \Delta p_{\text{rms}} \), such as those shown in Figs. 35c and 36c, the peak value (\( \Delta p_{\text{rms}} \)) at the centre of the N-wave was obtained. It was
plotted as a function of the mass-flow rate per unit area \( \frac{m_x}{pa} \) as calculated from the reservoir pressure that was used to generate the simulated sonic boom and jet noise. The results are shown in Fig. 37 for a 300-ms-duration N-wave. For both the subsonic and the supersonic flows or jets, \( \Delta p_{\text{rms}} \) is a straight line on the 'log-log' plot, as predicted by the jet-noise analysis. The two lines intersect at a value of 1.3 for \( \frac{m_x}{pa} \), which is slightly higher than the value of 1.1 \( (p/e/p = 1.89) \) separating the subsonic and supersonic jet regions. Furthermore, the slopes of the two lines through the experimental points which determine the power-law dependence of \( \Delta p_{\text{rms}} \) on \( \frac{m_x}{pa} \) do not agree with the predicted values. For the subsonic jet the measured \( \Delta p_{\text{rms}} \) of the jet noise varies as \( m_x \) raised to the power 2.6, instead of the predicted power of 4. Hence, for an increasing mass-flow rate per unit area, \( \Delta p_{\text{rms}} \) increases proportionally faster than \( \Delta p \) by \( m_x \) raised to the power 1.6. In the case of the supersonic jet, \( \Delta p_{\text{rms}} \) varies linearly with \( m_x \), as does \( \Delta p \). Therefore, \( \Delta p_{\text{rms}} \) and \( \Delta p \) increase at the same rate for an increasing mass-flow rate or reservoir pressure. This result can also be seen from the simulated sonic booms (100-ms duration) shown in the first column of Fig. 12. The peak-to-peak fluctuations of the jet noise remain in the same proportion to the peak overpressure of the simulated sonic boom as the reservoir pressure increases from 2.5 atm (Fig. 12b) to 13.9 atm (Fig. 12e).

It can be concluded that the jet-noise analysis is capable of predicting only the qualitative features of the jet noise, and the predicted results must be interpreted in this context.

2.5 Analysis of the Reflection Eliminator

When the simulated sonic boom, generated near the apex of the pyramidal horn, propagates to the large end of the horn it is normally partially reflected from the open end and partially transmitted to the outside of the horn. The reflected wave disrupts the simulated flow and pressure conditions in the interior test section, as the wavelength of the boom can be many times longer than the horn. In order to eliminate or at least adequately minimize the undesirable reflection and its subsequent echoes, a huge porous piston was designed and built to cover the open end of the pyramid (see Fig. 10). The piston of the reflection eliminator, being free to move by means of a special roller and track support, is normally set in motion by the simulated sonic boom. The following analysis for the reflection eliminator was found to be very useful during the design and testing stages, and it proved useful for explaining certain features of the actual motion of the piston. Note that this type of a device has in the past been called a sound absorber, but this is a misnomer.

For the analysis the reflection eliminator was modelled as a porous piston with a uniform mass per unit area \( (\sigma) \) and a constant flow resistance \( (R) \), as illustrated in Fig. 38. The flow resistance is equal to the pressure difference across a porous material — resulting from a flow of gas — divided by the flow speed. For generality, let the simulated sonic boom or incident wave on the porous piston be an arbitrary acoustic wave, having overpressure and particle-velocity signatures denoted by \( \Delta p \) and \( \Delta u \), respectively. The particle velocity is not independent of but related to the overpressure. By using Eqs. 2.6 and 2.7 the following expression can be obtained.
\[ \Delta u(\tau) = \frac{\Delta p(\tau)}{\rho a} + \frac{1}{\rho r} \int_{-\infty}^{\tau} \Delta p(y) \, dy \]  

(2.44)

The symbol \( \tau \) denotes the retarded time \( t - r/a \). When the incident wave encounters the porous piston, only a transmitted wave will take place if the reflection eliminator functions correctly. In order to solve for the transmitted wave a boundary condition is required. For simplicity the transmitted wave leaving the horn is assumed to be negligible in comparison with the incident wave, and it is therefore neglected. As a direct consequence of this assumption the pressure drop across the porous piston is equal to the overpressure associated with the incident wave. The pressure difference \( \Delta p \) is related to the flow velocity relative to the porous piston as shown below.

\[ \Delta p = R (\Delta u - v) \]  

(2.45)

The piston velocity \( v \) can be comparable to the particle velocity \( \Delta u \), and it therefore cannot be omitted. This same pressure difference across the porous piston produces the force that accelerates the piston. Newton's second law of motion can be expressed in the following form for the porous piston.

\[ \Delta p = \sigma \frac{\partial v}{\partial t} \]  

(2.46)

The displacement of the piston is normally small (always less than 1 m) in comparison with the radial distance at which the piston is located in the horn (about 25 m). Hence, the piston displacement and velocity are denoted by \( x \) and \( v (\partial x/\partial t) \), respectively, and the radial distance to the piston is denoted by \( r \) and taken to be constant (see Fig. 38). These results and the preceding three equations prove to be sufficient to determine both \( \sigma \) and \( R \), and also to solve for the motion of the porous piston in terms of an arbitrary incident wave.

To determine \( \sigma \) and \( R \), \( \Delta u \) is first eliminated from Eq. 2.45 by means of Eq. 2.44, and then \( v \) is eliminated from Eq. 2.46 by using the new form of Eq. 2.45. The following expression can thus be obtained.

\[ \left[ \frac{1}{R} - \frac{1}{\rho a} \right] \frac{\partial (\Delta p)}{\partial t} + \left[ \frac{1}{\sigma} - \rho \sigma \right] \Delta p = 0 \]  

(2.47)

For general values of \( \sigma \) and \( R \) the solution of this differential equation corresponds to an incident overpressure profile \( \Delta p \) which results in no reflected wave from the porous piston. If the coefficients of the differential equation are set equal to zero, then the condition of no reflected wave is still satisfied. However, as the differential equation is nonexistent, the incident wave can be arbitrary. The
following simple results for \( \sigma \) and \( R \) are therefore obtained.

\[
\sigma = \rho r_e \quad (2.48) \\
R = \rho a = \gamma \frac{p a}{a} \quad (2.49)
\]

The piston mass per unit area \( (\sigma) \) depends on the ambient air density \( (\rho) \) and the radial location \((r_e)\) at which the piston is located. If the piston is located farther from the apex of the horn it should be more massive, and its motion will be less significant. For a sufficiently small \( r \) the analysis becomes invalid as \( r \) cannot be taken as a constant. The flow resistance \( (R) \) is equal to the impedance \( (\rho a) \) of the ambient air \( (a \text{ constant}) \). The two results of Eqs. 2.48 and 2.49 were presented first in a report by Tamboulian (Ref. 28). The derivation and results have been given in this report not only for completeness, but also because the derivation is different and much simpler.

For the UTIAS travelling-wave horn the predicted values of \( \sigma \) and \( R \) are 2.7 kg/m\(^2\) and 410 Ns/m\(^2\), respectively. In reality, when the reflection eliminator functioned best, it was found that actual values of \( \sigma \) and \( R \) were approximately three-quarters of the predicted values. The analysis overpredicts the values of \( \sigma \) and \( R \) mainly because the transmitted wave was neglected. If the transmitted wave could be accounted for, the pressure difference across the porous piston would be less, and the predicted values of \( \sigma \) and \( R \) would be correspondingly smaller, in better agreement with actual values.

The motion of the porous piston can be determined as follows. Let the pressure difference across the porous piston be equal to the difference in overpressure between the incident simulated sonic boom and the transmitted wave (not neglected). This pressure difference is equal to both \( \rho (\Delta u - v) \) and \( \rho \frac{\partial v}{\partial t} \), in a similar manner to that for obtaining Eqs. 2.45 and 2.46. The following expression can therefore be obtained from these results, without knowing the exact form of the pressure difference.

\[
\frac{\partial v}{\partial t} + \frac{R}{\sigma} v = \frac{R}{\sigma} \Delta u \quad (2.50)
\]

The velocity of the piston \((v)\) can be determined easily from this differential equation for a specified incident wave having a particle velocity \( \Delta u \), and the general solution is given below.

\[
v = \frac{R}{\sigma} \exp \left(-\frac{R}{\sigma} t\right) \int_{-\infty}^{t} \Delta u(y) \exp \left(\frac{R}{\sigma} y\right) \, dy \quad (2.51)
\]

This solution for \( v \) is not subject to the same error as were the predicted values of \( \sigma \) and \( R \), because the transmitted wave has not been neglected. Actual values of \( \sigma \) and \( R \) for a reflection eliminator, which produces no reflected wave, can therefore
be used in Eq. 2.51 to obtain a realistic prediction of the piston velocity \( v \).

Consider the special case when the simulated sonic boom is an N-wave, that is, in terms of overpressure. The overpressure and particle-velocity signatures are given by Eqs. 2.12 and 2.13, respectively. By using the particle-velocity profile with Eq. 2.51, and taking \( \sigma \) and \( R \) to be three-quarters of their predicted values, the piston velocity can be obtained and expressed in the following simple form.

\[
v = \frac{\Delta p}{\rho_a} \frac{a t_e}{r_e} \left( 1 - \frac{t}{t_e} \right) \frac{t}{t_e} \quad \text{if } 0 < t < t_e
\]

\[
v = 0 \quad \text{if } t < 0, \ t > t_e
\]

The symbol \( t_e \) denotes the duration of the N-wave. It is very interesting that the incident N-wave causes the porous piston to move outwards such that the piston velocity is a parabolic function of time. The porous piston is therefore not left with a residual velocity by its interaction with the incident wave. The maximum piston velocity occurring at a time \( t_e/2 \) is equal to \( \left( \frac{\Delta p}{\rho_a} \right) \left( \frac{a t_e}{4r_e} \right) \).

For the UTIAS facility the factor \( \frac{a t_e}{4r_e} \) equals 0.35, 0.70 and 1.05 for respective N-wave durations \( t_e \) of 100, 200 and 300 ms. As \( \frac{\Delta p}{\rho_a} \) is the particle velocity at the front or rear shock of the N-wave (Eq. 2.13), the piston velocity is thus comparable in amplitude with the particle velocity of the incident wave. Like the particle velocity the piston velocity is small, having a maximum value of 17, 34 and 51 cm/s for an incident N-wave having a duration of 100, 200 and 300 ms, respectively, and a peak overpressure of 200 N/m².

For the special case of an N-wave interacting with the porous piston, the piston displacement can be determined from Eq. 2.52 by integration. The final results are summarized below.

\[
x = \frac{r_e}{6} \frac{\Delta p}{\gamma_p} \left( \frac{a t_e}{r_e} \right)^2 \left[ 3 \left( \frac{t}{t_e} \right)^2 - 2 \left( \frac{t}{t_e} \right)^3 \right] \quad \text{if } 0 < t < t_e
\]

\[
x = \frac{r_e}{6} \frac{\Delta p}{\gamma_p} \left( \frac{a t_e}{r_e} \right)^2 \quad \text{if } t > t_e
\]

The piston therefore moves outwards continuously, away from the horn, and it finally becomes stationary at its location of maximum displacement. The final position is directly proportional to both the peak overpressure \( \Delta p \) and the square of the duration \( t_e \) of the incident N-wave. For incident N-waves having a peak overpressure of 200 N/m² and durations of 100, 200 and 400 ms, the maximum displacement of the porous piston for the UTIAS reflection eliminator is 1.3, 6.2 and 24.8 cm, respectively.

It is worth noting that, when the ratio of the actual values of \( R \) and
in Eq. 2.51 is not identical to that for the predicted values (Eqs. 2.48 and 2.49), the expressions for the piston velocity (Eq. 2.52) and displacement (Eq. 2.53) are considerably more complex. For this case the porous piston generally has a residual velocity after the simulated sonic boom has passed.

The analysis for the reflection eliminator was checked by means of experimental results. Measurements of the piston displacement with time were recorded with a special linear potentiometer. Three displacement measurements are shown in the first column of Fig. 39. Note that the peak overpressure and duration of the incident N-wave are indicated above each oscillogram. These experimental measurements have been reproduced and are shown in the second column, where they have been compared with the corresponding predicted profiles (Eq. 2.53). The predicted and measured profiles have the same characteristics, and, in general, they were found to be in reasonable agreement. Note that the maximum displacement as predicted by the second expression of Eq. 2.53 was always larger than the measured value, but the difference was normally less than 10%. It can therefore be concluded that the analysis for the reflection eliminator is adequate for design considerations and for describing the motion of the porous piston. Note that the experimental displacement traces shown in Fig. 39 verify that the incident simulated sonic boom returns the piston to zero velocity.

It is worth noting that the flow resistance \( R \) of the microlite material used for the UTIAS reflection eliminator is not constant as was assumed for the preceding analysis. An experimental investigation was made to determine the flow resistance and to assess its dependence on both the flow speed and material thickness. The results of the investigation made it possible to select the correct thickness of microlite for the porous piston. Different samples of the material were put across a small wind tunnel and the flow resistance was determined. The results are shown in Fig. 40. For each sample having a thickness of either 12 or 25 mm the flow resistance increased with increasing flow speed. Additionally, it was found that for samples of equivalent thickness the sample having material which was more compact gave the higher values of flow resistance. Normally, the flow speed or particle velocity associated with the simulated sonic boom is less than 1 m/s. Therefore, for the restricted flow-speed range of 0 to 1 m/s, the flow resistance can be taken as constant for the analysis. From the results shown in Fig. 40, it can be seen that the deviation of the flow resistance from the mean value for this restricted range is normally less than 15%.

Based on the predicted value of the flow resistance \( 410 \text{ Ns/m}^2 \) and on the results shown in Fig. 40, the initial thickness of microlite for the porous piston was selected to be 36 mm. However, experimental tests with the reflection eliminator showed that a thickness of only 25 mm or even less should be used to eliminate the reflected wave. The actual or mean value of the flow resistance was therefore found to be about 300 Ns/m^2, or approximately three-quarters of the predicted value. Note that experimental tests also revealed that the mass per unit area \( \sigma \) should be about three-quarters of the predicted value.
3. ANALYSIS OF THE SHOCK-TUBE MODE

3.1 Description

During the last decade a number of different types of shock tubes were developed and evaluated for sonic-boom simulation purposes. Most of these low-pressure-ratio shock tubes have been used to produce acoustic waves in the interior of the shock tube, where humans, animals and model structures can be put and subjected to the simulated sonic boom. The more important shock tubes that have been assessed as sonic-boom simulators are illustrated schematically in Fig. 41. Each shock tube consists essentially of a driver and a channel, which are joined at a common area station where a diaphragm (D) can be suitably installed. Each one of the shock tubes shown in Fig. 41 has been named according to the geometry of its driver and channel. The driver can be rectangular in shape (Fig. 41a and e) or pyramidal (Fig. 41b, c and d). Similarly, the channel can be rectangular (Fig. 41a and c) or pyramidal (Fig. 41b, d and e). The adjectives "rectangular" and "pyramidal" arise quite naturally, owing to the geometry of the driver and channel of shock tubes used in the laboratory. More generally, however, rectangular can also refer to a duct where the same cross-sectional area is maintained with distance (e.g., cylindrical), and pyramidal can refer to a duct where the cross-sectional area increases directly with the square of the distance from the center of symmetry (e.g., conical). It is worth noting that the pyramidal and pyramidal-rectangular shock tubes (Fig. 41b and c) are special cases of the more general pyramidal-pyramidal shock tube (Fig. 41d), and the rectangular shock tube (Fig. 41a) is a special case of the more general rectangular-pyramidal shock tube (Fig. 41e).

For low-pressure-ratio shock tubes used for sonic-boom simulation purposes, the driver and channel gases are normally air, and the channel is usually left open to the atmosphere. The air in the driver, which is initially at a higher pressure than that in the channel, is separated at first from the channel air by means of a thin cellophane diaphragm. When the diaphragm is broken by a mechanical breaker (e.g., a needle), the rapidly expanding driver gas creates nonstationary waves in the shock tube. The wave motion is depicted by a time-distance diagram shown in Fig. 42. Only the wavefront paths of the various disturbances are shown in the figure. They appear as straight lines because the weak waves propagate at approximately the speed of sound.

On breaking the diaphragm in a shock tube, the rapidly expanding driver gas generates a weak shock wave (g₁ in Fig. 42) in the channel gas. Simultaneously, a weak rarefaction wave (f₁) moves in the opposite direction into the driver gas. It eventually encounters and reflects from the closed end of the driver. The reflected wave (h₁) propagates from the driver into the channel (now g₂) and thus follows the first shock (g₁). For the specific cases of the rectangular and pyramidal shock tubes (Figs. 41a and b), only the three disturbances g₁, f₁ and h₁ (or g₂) are produced. In the more general case, however, the change in the form of the duct from the driver to the channel causes further waves to be generated. The reflected wave (h₂) is only partially transmitted to the channel as disturbance g₂, and it is partially reflected as disturbance f₂, owing to diffraction effects at the driver and channel junction. The partially reflected wave (f₂) propagates back to the closed end to reflect once more. The subsequent oscillatory wave motion between the two ends of the driver results in a sequence of new waves in the channel, each successive wave becoming weaker to the point of vanishing. The integrated result of all of the disturbances yields in the channel the total wave, whose waveform depends only on the shock-tube geometry. In the following five sections of this report the solutions for the wave motion in the rectangular, pyramidal,
pyramidal-rectangular, pyramidal-pyramidal and rectangular-pyramidal shock tubes are presented.

3.2 Rectangular Shock Tube

The rectangular or constant-area shock tube (Fig. 41a), used initially by Vieille in 1899 (Ref. 35), has become a familiar research tool in the disciplines of aerodynamics and gasdynamics. A high-speed transient flow of gas behind a moving shock can be generated quite easily with a high-pressure-ratio shock tube, facilitating studies of the flow over a body (e.g., a model aircraft), or studies of high-temperature phenomena associated with the gas itself. In recent years, however, the opposite low-pressure-ratio rectangular shock tube has found application in the area of animal response studies. Small animals such as guinea pigs have been put inside a suitably sized constant-area shock tube and subjected to an acoustic wave, in order to obtain physiological response data. The results for the wave motion in a rectangular shock tube have been included in this report for academic interest and completeness.

Consider the wave motion in a rectangular shock tube, resulting from the following initial conditions. A diaphragm initially separates the higher pressure quiescent gas in the driver from a different quiescent gas in the channel. The ensuing wave motion on breaking the diaphragm is illustrated conveniently with a time-distance diagram, such as the one shown in Fig. 43. Note that the propagation speed for a wavefront (slope of its path) in the channel differs from that in the driver, as the channel and driver gases are different. The rapidly expanding driver gas creates a weak shock wave (g1) in the channel. Simultaneously, a weak rarefaction wave (f1) moves into the driver, eventually reflecting from the closed end. This reflected wave (h1) ultimately overtakes the contact surface or interface between the driver and channel gases, which is shown as a vertical dashed line. It is vertical because the contact-surface displacement is negligibly small relative to the length of the driver. Owing to refraction effects at the contact surface, the reflected wave (h1) is partially transmitted (g2) to the channel and partially reflected (f2) back into the driver. The subsequent oscillatory wave motion between the contact surface and the closed end of the driver creates a sequence of new disturbances in the channel. The integrated result of all of these disturbances yields the total wave in the channel.

First-order acoustic theory (Ref. 32) has been used to determine the wave motion in the rectangular shock tube. The one-dimensional planar wave equation, in conjunction with suitable initial and boundary conditions, has been solved to determine the wave motion. The details of obtaining the solution are given in Appendix D. Analytical expressions for the velocity potential (\( \phi \)), overpressure (\( \Delta p \)) and particle velocity (\( \Delta u \)) are summarized below, for both the channel and driver regions of the shock tube.

Channel (\( x > x_d \)):

\[
\phi_1 = \sum_{i=1}^{\infty} g_1(\xi) H \left[ \xi - 2(i-1)\frac{a_1}{a_2} + 1 \right] 
\]  (3.1)

\[
\Delta p_1 = - \rho_1 \frac{\partial \phi_1}{\partial t} 
\]  (3.2)
\[ \Delta u_1 = \frac{\phi_2}{\rho_2} = \frac{\Delta p_1}{a_1 p_1} \quad (3.3) \]

Driver \((0 < x < x_o)\):

\[ \phi_2 = -\frac{\Delta p_o}{\rho_2} + \sum_{i=1}^{\infty} \left[ f_1(\eta)H(\eta-2i+1) + h_1(\beta)H(\beta-2i+1) \right] \quad (3.4) \]

\[ \Delta p_2 = -\rho_2 \frac{\partial \phi_2}{\partial t} \quad (3.5) \]

\[ \Delta u_2 = \frac{\partial \phi_2}{\partial x} \quad (3.6) \]

In these expressions: \( \xi, \eta, \) and \( \beta \) are equal to \( \frac{(a_1 t-x)}{x_o}, \frac{(a_2 t-x)}{x_o}, \) and \( \frac{(a_3 t-x)}{x_o} \), respectively; the respective symbols \( a, \rho, \) and \( t \) denote sound speed, density, and time; and the subscripts \( 1 \) and \( 2 \) designate conditions in the channel and driver regions, respectively. The distance \( x \) is measured from the closed end of the driver, and \( x_o \) denotes the length of the driver as well as the diaphragm location (see Fig. 43a). \( H(\cdot) \) denotes the unit step function; it is equal to zero prior to the arrival of the disturbance in question and equals unity thereafter. The functions \( g_1(\xi), f_1(\eta) \) and \( h_1(\beta) \) correspond to the velocity potentials of the individual disturbances comprising the total wave (see Fig. 43). They are given by the following expressions.

\[ g_1(\xi) = \frac{-\Delta p_o \ x_o \ \xi}{a_1 \rho_1 + a_2 \rho_2} \quad \text{if } i = 1 \quad (3.7) \]

\[ g_1(\xi) = \left[ \frac{a_1 \rho_1 - a_2 \rho_2}{a_1 \rho_1 + a_2 \rho_2} \right]^{i-2} \frac{2a_2 \rho_2 \Delta p_o \ x_o \ \xi}{(a_1 \rho_1 + a_2 \rho_2)^2} \quad \text{if } i > 1 \]

\[ f_1(\eta) = \left[ \frac{a_1 \rho_1 - a_2 \rho_2}{a_1 \rho_1 + a_2 \rho_2} \right]^{i-1} \frac{\Delta p_o \ x_o \ \eta}{a_1 \rho_1 + a_2 \rho_2} \quad (3.8) \]

\[ h_1(\beta) = \left[ \frac{a_1 \rho_1 - a_2 \rho_2}{a_1 \rho_1 + a_2 \rho_2} \right]^{i-1} \frac{\Delta p_o \ x_o \ \beta}{a_1 \rho_1 + a_2 \rho_2} \quad (3.9) \]

The symbol \( \Delta p_o \) denotes the initial pressure difference across the diaphragm. From the solution it can be shown that once the impedance of the channel gas \( (a_1 \rho_1) \) and driver gas \( (a_2 \rho_2) \) have been specified, or, for that matter, only their ratio \( (a_2 \rho_2/a_1 \rho_1) \), the overpressure and particle velocity waveforms in both the driver and channel are fixed.
The overpressure signature of the wave propagating in the channel gas is one of the most important features of the wave motion. The mathematical expression for this signature, as derived from Eqs. 3.1, 3.2 and 3.7, is given below.

\[
\Delta p_i = \frac{\Delta p_e}{1+z} H[\xi] - \sum_{i=2}^{\infty} \frac{2z}{(1+z)z^2} \left[ \frac{1-z}{1+z} \right]^{i-2} H[\xi - 2(1-1) \frac{a_1}{a_2} + 1]
\]  

(3.10)

After the arrival of the first disturbance or shock wave \(g_1\) at a fixed location in the channel, each successive disturbance \(g_i\) arrives after a time interval of \(2x_0/a_2\). Each disturbance is headed by a sudden change in overpressure and then the overpressure remains constant. Each disturbance is superimposed on the previous ones. The first disturbance has the highest peak overpressure, equal to \(\Delta p_e/(1+z)\), and each of the following ones have a smaller amplitude than the previous one, owing to the factor \((1-z)/(1+z)\) raised to the power of \(i-2\). Because the sum of the terms in Eq. 3.10 is a geometric series, the peak overpressure of each successive disturbance or flat-topped overpressure portion of the signature can be expressed as a geometric sequence. This sequence is given below.

\[
\left[ \frac{1-z}{1+z} \right]^{i-1} \frac{\Delta p_e}{1+z}, \quad i = 1, 2, \ldots \infty
\]

(3.11)

If \(z\) is less than unity, the overpressure signature consists of a sequence of descending "steps", which become vanishingly small. On the other hand, if \(z\) is greater than unity, the sequential steps alternate from positive to negative values as they diminish in amplitude and eventually disappear. For the special case when \(z\) approaches and equals unity, the wave tends to and becomes a single pulse, having an amplitude of \(\Delta p_e/2\) and a duration of \(2x_0/a_2\).

The preceding remarks are amply illustrated by the three predicted overpressure signatures shown in Fig. 44. The top signature, resulting from the use of a small \(z\) value of \(0.31\), corresponds to the case when hydrogen is in the driver and the channel is left open to the atmosphere. The middle signature for a large \(z\) value of \(3.23\) is for the opposite case, when the driver gas is air and hydrogen occupies the channel. The bottom profile for a \(z\) value of unity can correspond to the well-known case when the driver and channel gases are identical. It is worth noting that the particle-velocity signatures associated with the three examples are identical to those given for the overpressure, as the particle velocity \(\Delta u_1\) equals \(\Delta p_1/a_1p_1\) (Eq. 3.3). Therefore, the comments made in regard to the overpressure profiles apply equally well to those for particle velocity.

The motion of the contact surface can be determined as follows. The particle-velocity signature, like the one for the overpressure, consists of a sequence of steps for which the particle velocity is constant. Since \(\Delta u_1\) equals \(\Delta p_1/a_1p_1\), and using Eq. 3.11, the particle velocity of the \(i\)th step is given by the following expression.
During each constant portion of the particle-velocity signature having a fixed duration of $2x_0/a_2$, the displacement of the interface is a linear function of time. Hence, the displacement associated with the $i$th constant portion is given below.

$$\frac{\Delta p_x}{a_1 \rho_1} \frac{1}{1+z} \left[ \frac{1-z}{1+z} \right]^{i-1} \quad \text{(3.12)}$$

The displacement of the contact surface after $n$ successive constant portions can be determined by adding the individual displacements, as shown below.

$$\frac{\Delta p_x}{a_1 \rho_1} \frac{2x_0}{a_2} \frac{1}{1+z} \sum_{i=1}^{\infty} \left[ \frac{1-z}{1+z} \right]^{i-1}$$

$$\text{(3.13)}$$

Because the sum in the above solution is a geometric series, it can be easily expressed in an alternate and more convenient form. Noting that $a_2$ equals $\gamma \rho_2 \rho_2$, or $\gamma \rho_1 \rho_2$ to first order, where $p_1$ is the pressure of the gas in the channel, the final result for the contact-surface displacement ($x-x_0$) is given below.

$$\frac{\Delta p_x}{\gamma_2 p_1} \left[ \frac{1 - \left( \frac{1-z}{1+z} \right)^{i-1}}{1 - \left( \frac{1-z}{1+z} \right)} \right], \quad i = 1, 2, \ldots, \infty \quad \text{(3.14)}$$

In summary, this expression gives the displacement of the contact surface at successive times $t$ of $0, 2x_0/a_2, 4x_0/a_2, 8x_0/a_2$, etc. The displacement during each time interval $2x_0/a_2$ is linear.

The total displacement of the contact surface at large times ($t \to \infty$) is simply $\Delta p_x x_0/\gamma p_1$, which is normally small compared with the length of the driver. For example, if the driver overpressure $\Delta p_x$ equals $p_1/20$ (one-twentieth of an atmosphere) and $\gamma_2$ equals $1.4$, the total displacement is only $x_0/28$. Hence, the contact surface moves outwards a distance equal to $3.6\%$ of the driver length.

The path of each gas particle, owing to the wave motion in the channel, is parallel to the path of the contact surface. Consequently, the same expression (Eq. 3.15) for the contact-surface motion applies equally well to the channel-gas motion. Particle paths corresponding to the case when $z$ equals $0.31$ (hydrogen in the driver and air in the channel), $3.23$ (vice versa) and $1.0$ (identical driver and channel gases) are shown superimposed on the time-distance diagram given in Fig. 45. The displacement of the particle path from its initial location is exaggerated for clarity. For the case when $z$ equals $0.31$, the displacement of the channel gas increases monotonously to its maximum value ($\Delta p_x x_0/\gamma p_1$). When $z$
equals 1.0, the displacement increases linearly to the maximum value in the time interval \(2x_0/a_2\), after which the displacement is constant. For the case when \(z\) equals 3.23, the displacement alternatively increases and decreases as it approaches its final location.

Based on experimental measurements, the first-order solution for the wave motion is adequate for driver overpressures \(\Delta p_0\) as high as \(p_1/3\) (one-third of an atmosphere), which more than covers the overpressure range required for producing acoustic waves for physiological response tests. It is worth noting, however, that for higher driver-gas overpressures when nonlinear wave action becomes increasingly important, the conventional shock-tube equation can be used to predict the peak overpressure \(\Delta p_1\) of the first shock wave in the channel. The appropriate equation is given below (Ref. 38).

\[
\left(1 + \frac{\Delta p_2}{p_1}\right) = \left(1 + \frac{\Delta p_1}{p_1}\right) \left[1 - \frac{\Delta p_1}{p_1} \left(\frac{a_1/a_2}{\gamma_1}(4\gamma_1^{1/2})\right)^{-2\gamma_2^{-1}} \right]^{-2\gamma_2^{-1}} \frac{\gamma_2^{-1}}{2} \left(\frac{\Delta p_1}{p_1} + 4\gamma_1^{1/2}\right)
\]

(3.16)

From this equation it can be shown that, to first order, the amplitude \(\Delta p_1\) of the first shock wave is equal to \(\Delta p_0/(1+z)\), in exact agreement with that for the first-order analysis. It should be emphasized, however, that the shock-tube equation is good for predicting only the overpressure of the first shock wave in the channel, and it does not provide a solution for the wave motion for all time, as does the first-order analysis. It is for this reason that the first-order solution for the wave motion in the low-pressure-ratio rectangular shock tube is so valuable.

Some overpressure measurements were made in the channel of a constant-area shock tube, in order to check the first-order analysis. Three of the measured overpressure signatures are shown in Fig. 46. For the profile shown in Fig. 46a, the driver and channel gases were both air, having the same initial temperature. In Fig. 46b the profile corresponds to the case of helium in the driver and air in the channel. For the third signature (Fig. 46c) the driver gas was air and helium was put in the channel. Other pertinent information is indicated in the figure, beside each oscillogram. Qualitatively, the measurements substantiate the main features of the analysis. For a quantitative comparison, the measured signatures have been reproduced in Fig. 47 and compared with the predicted profiles. The predicted and measured signatures are in good agreement. The main difference is that the rapid changes in overpressure for the experimental profiles are not instantaneous as predicted. The long rise (0.4 ms) associated with the rapid overpressure changes can be attributed mainly to poor diaphragm breakage that occurs for low driver overpressures. In practice, the diaphragm cannot be removed instantaneously. Additionally, a rapid decrease in overpressure associated with a rarefaction wave tends to become less rapid as the wave propagates.

Based on the predicted and measured overpressure profiles (Fig. 47), it can be seen that the rectangular shock tube is not capable of generating an N-shaped signature desired for simulating the sonic boom.
3.3 Pyramidal Shock Tube

Geometrically different shock tubes, such as those shown schematically in Fig. 41, were originally constructed and tested mainly by the English and the French researchers, for the specific purpose of assessing their capability as sonic-boom simulators. In England, the first proposal of a shock-tube-type facility for simulating a sonic boom was essentially a pyramidal-rectangular shock tube (Ref. 39). A prototype device was built and tested (Ref. 39). As it produced a distorted N-wave in the channel it was discarded, and a small conical shock tube was then built (Ref. 40). This shock tube was 14 m long and had an open end that was 1.5 m in diameter. This choice proved to be correct, as a good N-wave having a short-duration (4 ms) could be produced in the channel. These developments culminated in the construction of a large pyramidal shock tube for simulating full-scale sonic booms (Ref. 41). This shock tube had an overall length of about 180 m, a test section that was 2.5-m square, and a 3 m by 3 m base.

A similar but more intensive sonic-boom simulation program was simultaneously in progress at the Franco-German Institute for Research at Saint-Louis, France. This program started with the development of a high-pressure-ratio shock tube for field tests (Ref. 42). A short-duration boom resembling a blast wave could be directed at a suitable target (e.g., an animal or a window of a building). This project was followed by a series of experimental investigations of the capability of small pyramidal, pyramidal-pyramidal and pyramidal-rectangular shock tubes for generating a short-duration simulated sonic boom inside the channel (Refs. 43, 44 and 45). As was also discovered by the English researchers, the pyramidal shock tube proved to be best for the simulation of the sonic boom. The developments also culminated in the construction of a large pyramidal shock tube for simulating a full-scale sonic boom (Ref. 46). For the sake of interest, a schematic diagram of the Franco-German facility is given in Fig. 48. It depicts the size of the various elements of the facility.

In the United States, a small pyramidal shock tube was constructed at New York University for studies of wave diffraction over and into model buildings (Ref. 47). This shock tube was 6 m long and had a 1.8-m-square test section. It generated a good N-wave having a short duration as long as 5 ms.

In Canada, a large pyramidal horn (25 m long, 3-m-square base) was constructed at UTIAS for assessing current problems associated with the sonic boom. Although the major operating mode of the facility is with a mass-flow valve for simulating full-scale sonic booms (Section 2), provisions were made for operating the facility with a shock-tube driver. In the form of a pyramidal shock tube, the facility is capable of producing a good N-wave having a short duration (up to 20 ms).

The wave motion in the pyramidal shock tube was investigated thoroughly. The comprehensive analyses and results are given in the following three sections. They are not only applicable to the UTIAS facility but also apply for the simulators in England, France, and the United States.

3.3.1 First-Order Analysis

Consider a pyramidal shock tube (Fig. 41b) for which the quiescent gas in the driver is initially separated from an identical but slightly lower pressure quiescent gas in the channel. The resulting wave motion on breaking the diaphragm in the low-pressure-ratio shock tube can be determined by using first-order acoustic
theory (Ref. 32). The one-dimensional spherical wave equations describing the wave motion for both the driver and channel gases was solved in conjunction with suitable initial and boundary conditions. Details of the solution can be found in Appendix E. Analytical expressions for the velocity potential ($\phi$), overpressure ($\Delta p$), and particle velocity ($\Delta u$) are summarized below, for both the channel and driver regions of the shock tube.

**Channel ($r > r_o$):**

\[ \phi = \frac{\Delta p_o}{4a\rho r} \left( \xi^2 - 1 \right) \left[ H[\xi+1] - H[\xi-1] \right] \]  \hspace{1cm} (3.17)

\[ \Delta p = \frac{\Delta p_o}{2r} \left( r - at \right) \left[ H[\xi+1] - H[\xi-1] \right] \]  \hspace{1cm} (3.18)

\[ \Delta u = \frac{\Delta p_o}{2a\rho r} \left( \frac{r^2 + r_o^2 - a^2 t^2}{2r} \right) \left[ H[\xi+1] - H[\xi-1] \right] \]  \hspace{1cm} (3.19)

**Driver ($0 < r < r_o$):**

\[ \phi = -\frac{\Delta p_o}{\rho} \left[ t - \frac{r_o^2}{4a} \left( \eta^2 - 1 \right) H[\eta-1] + \frac{r_o^2}{4a} \left( \xi^2 - 1 \right) H[\xi-1] \right] \]  \hspace{1cm} (3.20)

\[ \Delta p = \Delta p_o - \frac{\Delta p_o}{2r} \left[ (r+at)H[\eta-1] + (r-at)H[\xi-1] \right] \]  \hspace{1cm} (3.21)

\[ \Delta u = \frac{\Delta p_o}{2a\rho r} \left( \frac{r^2 + r_o^2 - a^2 t^2}{2r} \right) \left[ H[\eta-1] - H[\xi-1] \right] \]  \hspace{1cm} (3.22)

In these expressions: $\xi$ and $\eta$ equal $(at-r)/r_o$ and $(at+r)/r_o$, respectively; $\Delta p_o$, $a$, $\rho$, $t$, and $H[\text{ }]$ denote the initial pressure difference across the diaphragm, sound speed, density, time, and the unit step function, respectively; the distance $r$ is measured from the apex of the pyramidal horn; and $r_o$ denotes the length of the driver and also the diaphragm location (see Fig. 41b).

It is worth noting that previous first-order solutions for a weak explosion (Refs. 22, 23 and 32) or the pyramidal shock tube (Refs. 39 and 45) are rather brief, as an expression is derived only for the overpressure ($\Delta p$) or the perturbation density ($\Delta p/\rho a^2$). The present work is more complete.

Acoustic theory predicts only three waves in the pyramidal shock tube, as illustrated in the time-distance diagram given in Fig. 49. The outgoing shock waves $S_1$ and $S_2$, and the ingoing rarefaction wave $R_1$, propagate at the equilibrium sound speed and their wavefronts thus appear as straight lines. The contact surface $C$ separating the driver and channel gases is also indicated on the diagram.
The overpressure variation with time for fixed distances of $0, \frac{r_0}{3}, 2\frac{r_0}{3}, r_0$ and $4\frac{r_0}{3}$ are illustrated graphically in Fig. 50. Note that these radial locations correspond to the vertical lines a, b, c, d and e in Fig. 49. The signatures within the driver region ($0 < r < r_0$), three of which are shown in Figs. 50a, b and c, are all similar. The overpressure remains at its initial value $\Delta p_0$ until the rarefaction wave arrives at time $(r-r_0)/a$, then it drops suddenly to $\Delta p_2$ (negative valued if $r < r_0/2$, otherwise it is positive valued), and then it decreases linearly with time to $-\Delta p_1$ at time $(r+r_0)/a$. At this instant in time, the reflected wave from the origin - a shock wave - arrives and raises the overpressure to zero, a value it remains at for subsequent time. For small radii the signatures exhibit a large pressure drop through the rarefaction wave, and this drop approaches negative infinity as the radius goes to zero. In the special case when $r$ equals zero (Fig. 50a), the infinite drop in overpressure is countered instantly by a similar rise in overpressure through the shock wave, since the duration of the linearly decreasing portion, given by $2r/a$, is zero. Consequently, the overpressure drops from $\Delta p_0$ to zero at time $r_0/a$. Note that this infinity in overpressure arises from linearization of the more complete flow equations, and it would not occur in practice.

In the channel region ($r > r_0$), the signatures (Figs. 50d and e) are also similar but differ from those in the driver. The overpressure is zero both before and after the wave. It increases suddenly to its peak value $\Delta p_1$ (equal to $\Delta p_0 \cdot r_0/2r$) at time $(r-r_0)/a$ when the first shock arrives, decreases linearly with time to an equal and opposite value of $-\Delta p_1$, and then increases suddenly to zero at time $(r+r_0)/a$, when the second shock arrives. In the special case when $r$ equals $r_0$ (Fig. 50d), just in front of the diaphragm the overpressure jumps from zero to $\Delta p_0/2$ at time $t$ equal to zero. Just on the other side of the diaphragm the overpressure falls from $\Delta p_0$ to $\Delta p_0/2$, and it is continuous at $r$ equal to $r_0$ for subsequent time.

The overpressure signature in the channel has a constant duration of $2r_0/a$, propagates without waveform distortion, and attenuates as $1/r$ (acoustic decay law). Since the overpressure profile has a shape that is similar to the flattened capital letter N, it has been called an N-wave.

The particle-velocity profiles for different fixed radii corresponding directly to those for overpressure are shown in Fig. 51. The signatures at different locations in both the driver and channel regions are similar. The velocity is zero both before and after the wave. It jumps to a peak value $\Delta u_1$ ($\Delta p_0/a$) either when the rarefaction wave arrives at time $(r_0-r)/a$ (within the driver) or when the shock wave arrives at time $(r-r_0)/a$ (within the channel), then it decreases continuously like $t^2$ to $-\Delta u_1$ at time $(r_0+r)/a$. At this time the rear shock arrives and increases the velocity back to zero. The velocity jump at both the front and rear shocks become larger with decreasing radius, and they approach infinite values as $r$ approaches zero. In the special case when $r$ equals zero (Fig. 51a), the duration of the velocity $(2r_0/a)$ vanishes and the velocity is zero for all time.

The particle-velocity profile for the channel has a constant duration of $2r_0/a$, and the amplitudes of the front and rear shocks attenuate like $1/r$, as for the overpressure profile. However, the waveform is not N shaped initially nor does it remain invariant with distance. The waveform change with distance is illustrated clearly in Fig. 52, where a normalized velocity is plotted versus time. The initial waveform at a distance $r$ equal to $r_0$ has a convex curvature, but this convexity disappears as the radius increases $(r \to \infty)$. At large distances
the velocity profile takes on the N shape.

The time varying overpressure and particle-velocity signatures for fixed radial distances correspond to those normally experienced by a subject or structure, or those that would be recorded by suitable stationary sensors (transducer or hot-wire anemometer). It is also of interest to illustrate the overpressure and particle-velocity distributions with distance, for which time is held fixed at suitable values.

For successive times of 0.4 \( r_o/a \), 0.6 \( r_o/a \), 2 \( r_o/a \) and 3.6 \( r_o/a \), the overpressure distributions are shown in Fig. 53. The initial distribution at time \( t = 0 \) when the overpressure is \( \Delta p_o \) in the driver and zero in the channel, is also depicted in Fig. 53a. On instantaneous removal of the diaphragm, the motion of the outgoing shock wave and ingoing rarefaction wave can be seen clearly in Figs. 53a and b. After the rarefaction wave reflects from the origin at time \( r_o/a \) as a shock wave, the outgoing pulse has two shocks (Figs. 53c and d). The overpressure distribution between the two shocks has a convex curvature which is quite pronounced initially. As time increases, however, the overpressure tends to take on an N shape. As the first shock is at a larger radius than the second one, its peak overpressure is correspondingly less. The wavelength of the signature is simply \( 2r_o \), that is, twice the length of the driver.

The particle-velocity distributions corresponding directly to those for overpressure are shown in Fig. 54. For times less than \( r_o/a \), the outgoing shock wave and the ingoing rarefaction wave produce a positive particle velocity or flow (Fig. 54a and b). For larger times, after the rarefaction wave has reflected from the origin as a shock wave, the particle velocity behind the first shock is positive but becomes negative ahead of the second one (Figs. 54c and d). The velocity distribution is always more convex in curvature than the associated overpressure distribution. However, as time increases the velocity tends to the same form as that for the overpressure, and ultimately becomes N shaped as well.

After the diaphragm is removed or broken in the pyramidal shock tube, the expanding driver gas creates the wave motion described previously. Associated with this wave motion is an induced fluid motion, during which the fluid particles comprising the driver and channel gases are displaced. It was of interest to determine the transient motion of the gases, and thus to obtain the path of the contact surface. The procedure that was used to obtain analytical expressions for particle paths for both the driver and channel gases is given in detail in Appendix E. Basically, the particle velocity \( \Delta u \) of Eqs. 3.19 and 3.22 was replaced by \( dr/dt \), and then these two expressions were integrated to yield \( r(t) \). To simplify the integration procedure, the approximation was made that the fluid-particle displacement \( \Delta r \) (equal to \( r-r_1 \)) was negligibly small compared with the initial location \( r_1 \) of the particle. The following nondimensional expressions were obtained for the particle paths of the channel and driver gases.
Channel: \( r > r_o \):

\[
\frac{\Delta r}{r_o} \approx \frac{\Delta p_o}{\gamma p} \frac{r_o}{r_1} \left[ \tau - \left(1 - \frac{r_o}{r_1}\right) \tau^2 - \frac{2}{3} \frac{r_o}{r_1} \tau^3 \right] \quad \text{if } 0 < \tau < 1
\]

\[
\frac{\Delta r}{r_o} \approx \frac{\Delta p_o}{3\gamma p} \left( \frac{r_o}{r_1} \right)^2 \quad \text{if } \tau > 1
\]

Driver: \( 0 < r < r_o \):

\[
\frac{\Delta r}{r_o} \approx \frac{\Delta p_o}{\gamma p} \frac{r_o}{r_1} \left[ \tau + \left(1 - \frac{r_o}{r_1}\right) \tau^2 - \frac{2}{3} \frac{r_o}{r_1} \tau^3 \right] \quad \text{if } 0 < \tau < \frac{r_1}{r_o}
\]

\[
\frac{\Delta r}{r_o} \approx \frac{\Delta p_o}{3\gamma p} \frac{r_1}{r_o} \quad \text{if } \tau > \frac{r_1}{r_o}
\]

The respective symbols \( \Delta p_o, r_o, p \) and \( \gamma \) denote the driver overpressure, driver length, atmospheric pressure and the specific heats ratio. The nondimensional time \( \tau \) has been specified such that it starts from zero at the wavefront. The expression for the particle path of the channel gas (Eq. 3.23) is a good approximation for the exact path, as the particle displacement \( \Delta r \) is always negligible compared with the particle's initial position \( r_1 \). For the driver gas, Equation 3.24 is accurate for most of the driver region, becoming inaccurate only in the immediate vicinity of the origin (e.g., \( r_1 < r_o/10 \)). Note that the path of the contact surface is obtained by setting \( r_1 \) equal to \( r_o \) for either Eq. 3.23 or 3.24.

Particle paths corresponding to initial particle locations \( r_1 \) of \( r_o/2 \), \( r_o \), \( 2r_o \), \( 3r_o \) and \( 4r_o \) are illustrated graphically in Fig. 55. The paths are superposed on a time-distance diagram, and the displacement of each path has been exaggerated for clarity. Each path has a maximum and a final displacement. For the driver gas, these two displacements are smallest for paths closest to the origin, and they become largest for the contact-surface path. For the channel gas, the two displacements are also largest for the contact-surface path, and they diminish to the point of vanishing for paths at increasing radial locations.

In order to verify that the first-order analysis is normally capable of describing the wave motion for a low-pressure-ratio pyramidal shock tube, certain experimental data was compared with the predicted results. Two experimental overpressure signatures that were obtained with the Saint-Louis shock tube (Ref. 46) have been reproduced in Fig. 56, where they have been compared with the corresponding predicted profiles. The predicted and measured data are in excellent agreement. The signature at the top of the figure has an amplitude (140 N/m²) and duration (290 ms) which are typical of those for an actual sonic boom, and the bottom N-wave is a powerful simulated sonic boom (900 N/m²). The rise times for the two shocks of the N-wave are typically 0.5 ms. It can be seen from these experimental results that the pyramidal shock tube is capable of producing an excellent N-wave for simulating the sonic boom.

It is worth noting that a large pyramidal shock tube is needed to generate full-scale sonic booms, such as those shown in Fig. 56. First, a long shock-tube driver is required because the wavelength of the N-wave is twice the length of the driver. To achieve an N-wave duration of 290 ms (Fig. 56) or a
corresponding wavelength of 100 m, the driver length has to be about 50 m.
Second, the channel must be quite long to ensure that the test section is sufficiently large (about 2.5-m square) to accommodate humans, animals or structures. Note that a large test section cannot be located close to the diaphragm station as the diaphragm size must be kept small (less than 1-m square) to minimize diaphragm breakage problems. Hence, a channel length of about 75 m is not unreasonable. Third, the reflection eliminator can add another 45 m to the length of the facility. Consequently, the overall length of a pyramidal shock tube can add up to about 200 m. This fact is illustrated well by the schematic diagram of the Franco-German facility that is shown in Fig. 48. Such a large shock tube takes up precious space and is expensive to construct. To circumvent these two disadvantages, the mass-flow-valve mode of operation was adopted for the UTIAS facility, enabling the use of a shorter pyramidal horn (25 m long).

For the shock-tube mode of operation, the UTIAS pyramidal horn incorporates short drivers (up to 4 m long) to generate short-duration simulated sonic booms (up to 20 ms). These short N-waves can be used for studies of wave diffraction over and into model buildings, or for wave propagation over different reduced scale land topologies. As an example of a short N-wave, one overpressure signature which was recorded in the channel of the horn is shown in Fig. 57a. The driver overpressure, driver length and measurement location are given beside the oscillogram. It can be seen that the UTIAS pyramidal shock tube is capable of producing a good N-wave having a peak overpressure typical of a sonic boom. The signature has been reproduced in Fig. 57b, where it is compared with the predicted profile. The two results are in reasonable agreement. The major difference occurs at the two shocks of the N-wave. The rise times of the two shocks are about 0.5 ms, and thus the overpressure jump across each shock is less than the predicted value. This discrepancy is due to poor diaphragm breakage resulting from the use of a relatively low pressure difference across the diaphragm.

Overpressure signatures were also recorded at different radial locations in the channel of the UTIAS pyramidal shock tube having the same driver length (1.58 m) and initial overpressure (2400 N/m²). The peak overpressure of the first shock of each recorded N-wave is plotted versus radial distance in Fig. 58. The predicted results are consistently higher than the measured values by about 15%. This discrepancy is due to the finite rise time and rounding of the front shock profile, as shown in Fig. 57b. If the linear portion of each measured N-wave was extrapolated back to the start of the wave, the resulting overpressure jump for the instantaneous rise time shock would agree almost exactly with the predicted result.

3.3.2 Extended First-Order Analysis

For a pyramidal shock tube having a driver that is a significant part of the overall length, the initial driver overpressure for generating an acoustic wave is normally small at a fraction of an atmosphere. Consequently, the wave near the diaphragm station is, to a good approximation, an N-wave in terms of the overpressure. The first-order analysis (Section 3.3.1) gives a sufficiently accurate prediction of its amplitude and wavelength, as demonstrated in Section 3.3.1 by the comparisons of measured and predicted data. However, as the N-wave propagates to larger distances, nonlinear wave action becomes increasingly important as the effects are accumulative, and the first-order prediction becomes less accurate. Basically, the N-wave attenuates more rapidly with distance than predicted by the acoustic decay law (1/r), and its wavelength does not remain
constant but becomes longer. This type of nonlinear wave behaviour is significant for the UTIAS pyramidal shock tube when sufficiently short drivers (less than 1.5 m) are used, as the distance to the test section can become large (e.g., \(r/r_0\) equals 50 when \(r_0\) is only 0.5 m). Additionally, higher driver overpressures are needed when shorter drivers are used, in order to produce an equivalent amplitude \(N\)-wave in the test section. Stronger waves are thus generated near the diaphragm station, and the effects of nonlinear wave action are correspondingly enhanced. In order to be able to accurately predict the \(N\)-wave in the test section of the UTIAS pyramidal shock tube, when nonlinear wave action is important, an extended first-order analysis was developed.

From acoustic theory (Ref. 32), the perturbation pressure (\(\Delta p\)), particle velocity (\(\Delta u\)) and propagation speed (\(\frac{dr}{dt}\)) for a sound wave are given below.

\[
\Delta p = \overline{\Delta p} \frac{r}{\bar{r}} f(t - r/a) \tag{3.25}
\]

\[
\Delta u = \frac{a \Delta p}{\gamma \rho} + \frac{a^2}{\gamma \rho} \int_{-\infty}^{t} \frac{\Delta p(t' - r/a)}{r} dt' \tag{3.26}
\]

\[
\frac{dr}{dt} = a \tag{3.27}
\]

The respective symbols \(r, \bar{r}, t, a, \rho, \gamma,\) and \(\overline{\Delta p}\) denote radial distance, radial location of the specified wave, time, equilibrium sound speed, equilibrium pressure, specific heats ratio, and peak overpressure at radius \(\bar{r}\). The non-dimensional waveform is denoted by \(f(t - r/a)\). Once the wave at radius \(\bar{r}\) has been specified, the above results can be used to predict the wave at subsequent distances. However, the error in this prediction increases with distance as the distortion of the waveform owing to nonlinear wave action has been neglected. Whitham (Ref. 27) showed that these nonlinear effects can be taken into account by modifying the first-order theory of sound.

The basic idea behind Whitham's modified analysis is that the failure of the first-order theory away from the location of the specified weak wave is due not to incorrect prediction of the physical quantities (Eqs. 3.25 and 3.26) along the propagation path or ray, but to the improper placing of these rays in the flow field (Eq. 3.27). To correct this problem the accurate propagation speed \(a + \Delta u + \Delta a\) is used in place of \(a\) in Eq. 3.27, where \(\Delta a\) is the perturbation sound speed. A binomial expansion of the isentropic equation relating sound speed and pressure gives \(\Delta a/a\) equal to \((\gamma - 1)\Delta p/2\gamma \rho\) to first order. Using this result and Eq. 3.26 with the integral term neglected (second order effect), the inverse propagation speed of a point on the wave is given, to first order, by the following expression.

\[
\frac{dt}{dr} = \frac{1}{a} \left[ 1 - \frac{\gamma + 1}{2\gamma} \frac{\Delta p}{\rho} \right] \tag{3.28}
\]
In treating the time phase which is measured from the wave front, a distinction has to be made between the first-order phase variable, denoted by \( \tau \), and the actual phase variable \( \tau_1 \), which equals \( t - (r/r_0) \). The nonlinear effect is basically the difference between the two phases. Equation 3.25 is now used to eliminate \( \Delta \tau \) from Eq. 3.28, with \( f(t - r/a) \) replaced by \( f(\tau) \) - the waveform predicted by the first-order theory. Holding \( \tau \) constant while integrating the new form of Eq. 3.28 gives the following expression for the actual phase:

\[
\tau_1 = \tau - \frac{\gamma + 1}{2} \frac{\Delta \tau}{\Delta \rho} \frac{\rho}{P} f(\tau) \ln(r/r_0) \tag{3.29}
\]

This equation describes the phase shift for a particular point on the signal. The point found at time \( \tau \) according to first-order theory is actually found at time \( \tau_1 \). Hence, the physical quantities predicted by first-order theory at time \( \tau \) occur at time \( \tau_1 \). This phase shift can give rise to a multivalued signal, which is then modified by using the "equal-area rule" to take into account the presence of one or more shock waves (Refs. 6, 25 and 26).

As an example of the modified analysis, consider the signal given in Fig. 59a, as predicted by first-order theory at a large radius. On applying Eq. 3.29 the distorted and multivalued signature shown in Fig. 59b results. The multivalued parts of the signature are now eliminated by putting in a shock wave such that the shaded areas on either side are equal. The final signature is shown in Fig. 59c. It has a smaller amplitude and a longer duration than predicted by first-order theory. Note that a signal having an arbitrary initial profile eventually attains the N-wave shape at large distances from the place of the wave's origin, owing to nonlinear distortion effects (Refs. 25 and 27).

The results of Whitham's modified analysis is now applied to the case of the pyramidal shock tube. It is assumed that the overpressure signature of the wave in the channel at radius \( \tilde{r} \) is N shaped, and given by the following expression.

\[
\Delta \rho = \Delta \rho \left[ 1 - 2\alpha \tau / \lambda \right] \text{ if } 0 < \tau < \tilde{\lambda} / a \tag{3.30}
\]

The symbol \( \tilde{\lambda} \) denotes the wavelength of the N-wave at radius \( \tilde{r} \). The distortion of this N-wave with distance is governed by Eq. 3.29, where \( f(\tau) \) equals \( 1 - 2\alpha \tau / \tilde{\lambda} \). When one uses the equal-area rule to maintain a single-valued signature, the following analytical results can be obtained for the amplitude decay (\( \Delta \rho \)) and wavelength stretch (\( \lambda \)) with increasing distance.

\[
\Delta \rho = \Delta \rho \frac{\tilde{r}}{r} \left[ 1 + \frac{\gamma + 1}{2} \frac{2r}{\tilde{\lambda} \sqrt{\ln(r/r_0)}} \right]^{-1/2} \tag{3.31}
\]

\[
\lambda = \tilde{\lambda} \left[ 1 + \frac{\gamma + 1}{2} \frac{2r}{\tilde{\lambda} \sqrt{\ln(r/r_0)}} \right]^{1/2} \tag{3.32}
\]
The familiar acoustic decay law $\Delta p \frac{r}{r}$ and constant wavelength $\lambda$ are each modified by a multiplication factor which accounts for the nonlinear distortion effect. The amplitude decay and wavelength stretch are dependent on the initial characteristics of the N-wave ($\Delta p$ and $\lambda$), the medium ($\gamma$), equilibrium pressure ($p$) and non-dimensional distance ($r/r_0$). Note that these results have not been oversimplified to the extent that only the dependence on radius remains, that is, $\Delta p$ varies inversely like $r(\ln r)^{-1/2}$ and $\lambda$ varies directly like $(\ln r)^{1/2}$, as discovered independently by Landau (Ref. 25) and Whitham (Ref. 27). For this reason the results given by Eqs. 2.31 and 2.32 have practical applications.

For the pyramidal shock tube it is convenient to specify the N-wave at the diaphragm station. Hence, $r$ equals $r_0$. Additionally, the N-wave amplitude $\Delta p$ is given by $\Delta p_0/2$ and the wavelength $\lambda$ equals $2r_0$, as predicted by the first-order analysis (Section 3.3.1). When these results are used with Eqs. 2.31 and 2.32, the amplitude decay and wavelength stretch of the N-wave in the pyramidal shock tube are given below.

$$\Delta p = \frac{\Delta p_0}{2} \frac{r_0}{r} \left[ 1 + \frac{\gamma+1}{4\gamma} \frac{\Delta p_0}{p} \ln(r/r_0) \right]^{-1/2} \tag{3.33}$$

$$\lambda = 2r_0 \left[ 1 + \frac{\gamma+1}{4\gamma} \frac{\Delta p_0}{p} \ln(r/r_0) \right]^{1/2} \tag{3.34}$$

Note that the nonlinear wave effects become more important for a higher driver overpressure ($\Delta p_0$) and also when the radial distance ($r/r_0$) becomes larger.

It is of interest to perform these calculations for certain existing sonic-boom simulators. First, consider the Franco-German facility shown in Fig. 48 (Ref. 46), which can simulate a full-scale sonic boom (e.g., 270 ms in duration). The diaphragm and test section can be located at radii of 46 m and 115 m respectively. For a driver overpressure of 1200 N/m², the N-wave amplitude in the test section is calculated to be 239 N/m² by Eq. 3.33, only 0.2% lower than that predicted by the first-order analysis. Hence, nonlinear wave effects are relatively unimportant for the Franco-German pyramidal shock tube. Second, consider the UTIAS pyramidal horn which can be used to produce short-duration N-waves (up to 20 ms). Possible locations of the diaphragm and test section are 0.457 m and 24 m respectively. For a driver overpressure of 30 kN/m², the predicted amplitude is 234 N/m², which is a significant 18% lower than the first-order prediction. The relatively high driver overpressure coupled with the long distance of propagation for the N-wave ($r/r_0$ equals 52.5) make the nonlinear distortion effects important.

Experimental data was obtained with the UTIAS pyramidal shock tube to verify the results of the extended first-order analysis. For the first experiment a driver length of 1.58 m was employed, and a driver overpressure of 34 kN/m² was used to generate quite a strong acoustic wave. Consequently, the effect of the nonlinear wave action on the signature of the N-wave is due mainly to the use of a high driver overpressure and only partly to the distance over which the wave propagates. Overpressure signatures were recorded at different radial locations in the channel. One of the short N-waves that was recorded near the test section appears in Fig. 60a. Its signature is not quite N shaped because the original profile near the diaphragm location was not N shaped, owing to the
use of a high driver overpressure. This signature has been reproduced and compared with the predicted profile, and the results are shown in Fig. 60b. The measured and predicted profiles are in reasonable agreement. The predicted amplitude is 10% higher than the measured value and the predicted duration is 8% less than that measured. Note that the first-order analysis of Section 3.3.1 predicts an amplitude and duration which are, respectively, 30% higher and 25% lower than the experimental values. The extended analysis therefore gives a much better prediction.

For the other overpressure signatures that were recorded in the channel, the peak overpressure was plotted as a function of radial distance. These results are shown in Fig. 61, where the predicted results of both the first-order and extended analyses are also given. It can be seen that the results of the extended analysis are in much better agreement with the experimental data.

In the second experiment a lower overpressure of 20 kN/m² was employed, and a shorter driver length of only 0.457 m was used with the UTIAS pyramidal horn. Consequently, the distance that the N-wave propagates to the test section is relatively long (r/ro equals 52.5), and nonlinear distortion effects are still important. One of the overpressure measurements made at different radial distances is shown in Fig. 62a. This short-duration wave (3 ms) has a good N-shaped signature, which is in reasonable agreement with the predicted profile, as shown in Fig. 62b. For the other recorded overpressure signatures, the amplitude of the N-wave was plotted versus radial distance and compared with that predicted by both the first-order and extended analyses. These results appear in Fig. 63. Again, the results of the extended analysis give a better prediction of the peak overpressure. The same is true for the N-wave duration.

3.3.3 Nonlinear Analysis

For certain experiments using the UTIAS pyramidal shock tube, an abnormally high driver overpressure (greater than one atmosphere) and a short driver length (20 cm) have been employed to generate a short-duration weak N-wave in the test section (see Refs. 20 and 48). Owing to the use of a high driver overpressure, the overpressure signature of the wave near the diaphragm location can differ markedly from the N shaped signature. However, since the signature changes as the wave propagates to the distant test section, the wave normally evolves into quite a good N-wave. When the driver overpressure exceeds about one-quarter of an atmosphere, the first-order analysis of Section 3.3.1 gives inaccurate results. Consequently, a numerical computing procedure was employed to solve for the wave motion in the driver and neighbouring channel region. These computations can normally be continued to give at some point in the channel a sufficiently weak wave that can serve as the starting point for Whitham's modified acoustic analysis (given in Section 3.3.2). This simple and easy to use analysis can account for the nonlinear distortion of the waveform as the wave propagates to the test section, thereby dispensing with a significant part of the tedious and costly computer calculations.

The generation of a shock wave in a pyramidal shock tube enclosing a solid angle less than that for a sphere is similar in process to that for a spherically symmetric explosion. In both cases a pressurized gas is released suddenly into a quiescent atmosphere of lower pressure. To calculate the flow field within the shock-tube driver and the neighbouring channel region, a numerical computing procedure was adopted to solve the inviscid, nonlinear, one-dimensional equations of motion (continuity, momentum and energy) along with
the ideal equation of state. This process has been well documented (Ref. 49), and the results for strong explosions have been reviewed thoroughly (Ref. 50).

The results for a weak explosion appear in Fig. 64, in the form of a time-distance diagram. Two overpressure signatures at different radii are also shown superposed on the diagram. For this example the initial pressures of the driver and channel air were 4 and 1 atm respectively. From the numerical computations the following statements can be made. On instantaneously removing the diaphragm, a decelerating shock wave \( S_1 \) moves into the channel gas, compressing and accelerating it. Simultaneously a rarefaction wave \( R_1 \) propagates into the driver gas, expanding and accelerating it away from the apex or origin. As the rarefaction wave approaches the origin, it undergoes a reflection process that produces an outgoing rarefaction wave (not shown in detail in Fig. 64), which decelerates the flow until it is zero at the origin. A second shock wave \( S_2 \), which forms at the tail of the rarefaction wave \( R_1 \), propagates into the flow and implodes on the origin. It reflects as a shock wave and forms the rear portion of the subsequent N-wave. This second shock wave has a negligible strength initially and only starts to achieve a significant amplitude in the vicinity of the origin. After its reflection it continues to form into a steep-fronted wave. The contact surface \( C \) first decelerates and then undergoes small oscillations that subsequently disappear. Reflection and refraction effects at the contact surface, owing to a change in acoustic impedance across it, are almost negligible for this weak explosion, as waves \( R_2, R_3 \) and \( S_3 \) produce only small changes in the flow. Note that the computational scheme which uses a fixed artificial viscosity to cope with shock-wave motions is inadequate for weak waves, owing to the excessive spreading of the shock-wave fronts (see the overpressure signatures in Fig. 64). This spreading of the fronts can be reduced by dividing the flow field into smaller segments or zones, but this approach results in undesirably long computation times.

A study of a few different time-distance diagrams, for which the initial driver pressure is varied but the driver length remains fixed, reveals some interesting characteristics of the wave generated in the channel. For increasing driver pressures the wave near the diaphragm location deviates more markedly from the N-wave. The first shock \( S_1 \) exceeds in amplitude that of the second one \( S_2 \), and this difference becomes greater for higher driver pressures. Additionally, the wave duration becomes longer for stronger explosions, as the second shock \( S_2 \) takes a longer time to reach the origin where it is reflected. It forms at the tail of the rarefaction-wave fan, which becomes more divergent for stronger explosions. Note that for weak explosions of interest for the UTIAS pyramidal horn, the main effect of an increase in driver length is a corresponding increase in the duration of the N-wave.

For decreasing driver pressures the opposite of the above statements hold, and the generation of an N-wave near the diaphragm station becomes better. It should be stressed, however, that even for very weak explosions the wave formation process is inherently nonlinear. The rarefaction-wave fan, which is initially very thin, diverges in the vicinity of the origin, and the formation of the second shock near the origin is a nonlinear process. Furthermore, the wave in the channel undergoes nonlinear waveform distortion as it propagates to the test section.

The flow field for the exemplary weak explosion has been calculated numerically only for times less than 6 ms, and thus the wave motion was determined only for radii less than about 1.6 m (see Fig. 64). By using the overpressure
signature for the relatively weak wave at a radial location of 1.2 m and
Whitham's modified acoustic analysis (essentially Eq. 3.29), the overpressure
profile can be determined at subsequent distances and thus in the test section.

Experimental data was obtained with the UTIAS pyramidal shock tube
to verify the results of the nonlinear analysis. Overpressure measurements
for radial locations of 1.2, 4.6 and 18.3 m are shown in Figs. 65a, b and c
respectively. The evolution of the N-wave and the increase in wave duration
with increasing distance can be readily seen.

The numerically computed overpressure signature for a radius of 1.2 m
is also shown in Fig. 65a. The measured and predicted profiles are in good
agreement, except at the location of the two shocks where the artificial vis­
cosity effect is pronounced in the predicted profile. A calculation of the
peak overpressure of the front shock was performed, based on the predicted wave­
front propagation speed and the Rankine-Hugoniot equations. This calculation
gave a value which was 15% in excess of that measured, rather than 15% lower
as obtained from the numerically obtained profile. This disagreement has not
been resolved. It can be concluded that the numerical analysis provides fair
results, although it would be desirable to reduce the spreading of the shock
front.

The numerically predicted signature for a radius of 1.2 m (Fig. 65a)
was used as the starting point for the modified acoustic analysis. The pre­
dicted waveforms at radii of 4.6 and 18.3 m are shown in Figs. 65b and c res­
pectively. The rounded shock profiles on the initial signature steepen very
quickly and become vertical for the wave at the 4.6-m station. Note that by
using the measured result instead of the numerical one as the st'arting point,
the predicted signature at the two radial locations would differ only in
the location of the shocks, and these differences are very small.

The modified acoustic analysis overpredicts the waveform distortion,
resulting in a predicted profile that has a lower amplitude and longer duration
than the experimental values (see Figs. 65b and c). For example, at a radius
of 18.3 m the predicted amplitude is 6% lower and the duration is 10% longer
than those for the measured signature. However, these predictions are con­
siderably more accurate than those from the first-order analysis, which differ
from the measured amplitude by 90% and from the measured duration by -25%.
Note that the peak overpressure of the initial wave at a radius of 1.2 m is
12.5 kN/m² (about one-fifth of an atmosphere), which is not small compared
with 1 atm as assumed for the modified acoustic analysis. In general, the
analysis gives a better prediction when a weaker wave is chosen as the start­
ing point.

3.4 Pyramidal-Rectangular Shock Tube

Initial experimental investigations (Refs. 39, 44 and 45) of the
capability of different types of shock tubes for simulating the sonic boom
included the pyramidal-rectangular shock tube (Fig. 41c). The analytical solu­
tion for the wave motion in such a low-pressure-ratio shock tube has been ob­
tained only recently by the author. This solution, which is in agreement with
experimental data, is given herein.

A diaphragm initially separates a quiescent gas in the driver from
the same but lower pressure gas in the channel. The ensuing wave motion on
breaking the diaphragm is illustrated by a time-distance diagram shown in Fig. 42. Acoustic theory was used to solve for each of the disturbances $g_i$, $h_i$, and $f_i$ in the shock tube, in order to obtain the solution for the wave motion. The details of the solution can be found in Appendix F.

Expressions for the velocity potential ($\phi$), overpressure ($\Delta p$) and particle velocity ($\Delta u$) are given below, for both the channel and driver regions.

Channel ($x > 0$):

$$\phi = \sum_{i=1}^{\infty} g_i(\xi) H(\xi - 2i + 2)$$  \hspace{1cm} (3.35)

$$\Delta p = -\rho \frac{\partial \phi}{\partial t}$$  \hspace{1cm} (3.36)

$$\Delta u = \frac{\partial \phi}{\partial x} = \frac{\Delta p}{\rho}$$  \hspace{1cm} (3.37)

Driver ($0 < R < R_o$):

$$\phi = -\frac{\Delta p_o}{\rho} t + \frac{1}{R} \sum_{i=1}^{\infty} \left[ f_i(\eta) H(\eta - 2i + 1) + h_i(\beta) H(\beta - 2i + 1) \right]$$  \hspace{1cm} (3.38)

$$\Delta p = -\rho \frac{\partial \phi}{\partial t}$$  \hspace{1cm} (3.39)

$$\Delta u = \frac{\partial \phi}{\partial x}$$  \hspace{1cm} (3.40)

In the above expressions: $\Delta p_o$, $a$, $\rho$, $t$, and $H(\ )$ denote the initial pressure difference across the diaphragm, sound speed, density, time, and the unit step function respectively; the respective symbols $x$, $R$, and $R_o$ denote planar distance measured from the diaphragm location along the channel, radial distance measured from the apex of the driver, and the diaphragm location (see Fig. 41c); and $\eta$, $\beta$, and $\xi$ are equal to $(at+R)/R_o$, $(at-R)/R_o$, and $(at-x)/R_o$ respectively. The functions $g_i(\xi)$, $f_i(\eta)$ and $h_i(\beta)$,
for the successive disturbances (see Fig. 42), are given below

\[ g_i(\xi) = -\frac{\Delta p \rho}{a^2} \left[ 1 + \xi + G_1 \exp(\xi/2) \right] \quad \text{if } i = 1 \]  

(3.41)

\[ g_i(\xi) = (-1)^i \frac{\Delta p \rho}{a^2} \left[ \sum_{j=1}^{i} \frac{G_j}{(i-j)!} \left( \frac{\xi-2i+4}{2} \right)^{i-j} \exp \left( \frac{\xi-2i+2}{2} \right) \right] \]  

(3.42)

\[ f_i(\eta) = -\frac{\Delta p \rho^2}{a^2} \left[ 1 - (-1)^i \sum_{j=1}^{i} \frac{F_j(\eta-2i+3)^{i-j}}{2^{i-j}(i-j)!} \exp \left( \frac{\eta-2i+1}{2} \right) \right] \]  

(3.43)

The symbol \( ! \) denotes the factorial function. Certain constants of integration \( G_j \) are given below.

\[ G_1 = -1 \quad \text{if } j = 1 \]  

(3.44)

\[ G_j = \sum_{m=1}^{i-1} \frac{-G_m}{(j-m)!} \quad \text{if } j > 1 \]

Note that overpressure and particle velocity signatures for the wave motion in either the channel or the driver of the pyramidal-rectangular shock tube can be obtained easily from this rather complex solution, by utilizing a digital computer to perform the calculations.

The overpressure signature of the wave moving in the channel is shown in Fig. 66a. The amplitude and waveform of the signature are invariant with distance, since the channel area is constant. The first shock has an amplitude equal to \( \Delta p_0/2 \). It is followed after a time interval of \( 2R_o/a \) by a second shock having an equivalent amplitude. The subsequent disturbances are not very pronounced. Consequently, the overpressure profile is not too different from an \( N \) shape. Note that a change in the length of the driver does not alter the waveform, but merely changes its duration, as the time interval between the two shocks is \( 2R_o/a \).

The particle velocity has an identical waveform as that for the overpressure, as \( \Delta u \) equals \( \Delta p/a\rho \) (Eq. 3.37) for a constant-area channel or duct. As a result, all of the previous comments about the overpressure signature apply for the particle velocity as well.
Based on the results of the preceding acoustic analysis, the pyramidal-rectangular shock tube is capable of producing a reasonable N-wave for simulating the sonic boom. It certainly has more capability than the rectangular shock tube which produces a flat-topped wave. The pyramidal shock tube, however, produces a better N-wave.

To verify the results of the acoustic analysis, some experiments were made with a pyramidal-rectangular shock tube. The driver length was 46 cm and the cross-sectional area (square) of the channel was 32.6 cm². An overpressure signature that was recorded in the channel is shown in Fig. 66b. In Fig. 66c, this short-duration signature is compared with the predicted profile. For this rather powerful wave having a peak overpressure of 4.4 kN/m², the predicted and measured profiles are in good agreement.

Some interesting overpressure measurements for higher driver overpressures than 9.1 kN/m² were made with the same pyramidal-rectangular shock tube. A few of the recorded profiles are given in Fig. 67. Each successive signature down the column is more powerful than the preceding one, as it was generated by using a higher driver overpressure. As the driver overpressure ($\Delta p_o$) is increased, the overpressure signature changes quite markedly. The peak overpressures of the first and second shocks become increasingly less than $\Delta p_o/2$. Additionally, the second shock becomes less significant compared with the first one, and the time interval between the two shocks becomes longer. For increasing driver overpressures the acoustic analysis becomes less applicable. For this case the signatures probably can be predicted by using a numerical computing procedure, similar to that used for the high-pressure-ratio pyramidal shock tube (Section 3.3.3).

The peak overpressure of the first shock for each of the signatures shown in Fig. 67, and for other recorded signatures, were plotted versus driver overpressure. These results appear in Fig. 68. For increasing driver overpressures the measured amplitudes deviate more significantly from the predicted curve ($\Delta p_o/2$) of the acoustic analysis. In fact, based on a comparison of measured and predicted results, the analysis is valid only for driver overpressures less than about 50 kN/m². The measured peak overpressures are in good agreement with the result for a rectangular shock tube (Eq. 3.16 of Section 3.3.3). This agreement is not surprising. The shock-tube equation is applicable for almost any type of shock tube for sufficiently small times after the diaphragm is removed instantaneously (e.g., see Ref. 51). Since the shock-tube equation can predict the shock overpressure at the diaphragm location, and since the shock does not attenuate significantly as it propagates to the measurement station (46 cm), the predicted and measured results should be in good agreement.

3.5 Pyramidal-Pyramidal Shock Tube

Initial investigations of the suitability of various types of prototype shock tubes for sonic-boom simulation were concerned mainly with the pyramidal shock tube. It was soon realized that a large facility of the order of 150 m in length would be required to produce full-scale sonic booms. Such a long shock tube would be costly to construct. One means of achieving a shorter and therefore less expensive facility would be to build a shock tube with a short channel. In order to maintain the same sized driver, diaphragm and test section, the divergence angle of the channel could be increased to a larger angle than for the driver, as illustrated in Fig. 69. In this manner the overall length of the driver and channel could be reduced from 150 m to a plausible 60m. Although a shorter shock tube could be constructed, it was not known if the new pyramidal-
pyramidal shock tube was capable of producing a good N-wave for simulating a sonic boom. Consequently, experimental investigations (Refs. 44 and 45) were conducted to determine how the change in the divergence angle at the diaphragm location would affect the signature of the wave in the channel. Analytical results were not available to guide these investigations. They have been obtained only recently and are given in this section of the report.

For the pyramidal-pyramidal shock tube (Fig. 41d), a diaphragm initially separates a quiescent gas in the driver from the same but lower pressure gas in the channel. The resulting wave motion after breaking the diaphragm is illustrated by a time-distance diagram shown in Fig. 42. Acoustic theory was used to solve for each of the disturbances $g_i$, $h_i$, and $f_i$ in the shock tube, in order to obtain a solution for the wave motion. The solution can be obtained in the same manner as that for the pyramidal-rectangular shock tube (Appendix F).

Expressions for the velocity potential ($\phi$), overpressure ($\Delta p$) and particle velocity ($\Delta u$) are given below, for both the channel and driver gases.

Channel ($r > r_0$):

$$\phi = \frac{1}{r} \sum_{i=1}^{\infty} g_i(\xi) H[\xi+c-2i+2]$$

$$\Delta p = - \rho \frac{\partial \phi}{\partial t}$$

$$\Delta u = \frac{\partial \phi}{\partial r}$$

Driver ($0 < R < R_0$):

$$\phi = - \frac{\Delta p_0}{\rho} t + \frac{1}{R} \sum_{i=1}^{\infty} \left[ f_i(\eta) H[\eta-2i+1] + h_i(\beta) H[\beta-2i+1] \right]$$

$$\Delta p = - \rho \frac{\partial \phi}{\partial t}$$

$$\Delta u = \frac{\partial \phi}{\partial r}$$

The respective symbols $\Delta p_0$, $\rho$, $a$, $t$, and $H[\ ]$ denote the initial pressure difference across the diaphragm, density, sound speed, time, and the unit step function. The radial distance $R$ for the driver is measured from the apex to the diaphragm location $R_0$ (see Fig. 41d). The radial distance $r$ for the channel is measured from the projected apex of the channel, and $r_0$ designates the location of the diaphragm (see Fig. 41d). The symbols $\eta$, $\beta$, $\xi$, and $c$ are equal to $(at+R)/R_0$, $(at-R)/R_0$, $(at-r)/R_0$ and $r_0/R_0$ respectively. The potential functions $g_i(\xi)$, $f_i(\eta)$ and $h_i(\beta)$ for the individual disturbances (Fig. 42) are summarized below.
\[
\frac{g_1(\xi)}{g_1(\xi)} = \frac{\Delta p_0 r_0^2}{\alpha \rho} \left[ \frac{-\xi}{c-1} - \frac{c^2+1}{(c-1)^2} + G_1 \exp\left(\frac{\xi+c}{2c/(c-1)}\right) \right] \quad \text{if } i = 1
\]

\[
\frac{g_1(\xi)}{g_1(\xi)} = \frac{\Delta p_0 r_0^2}{\alpha \rho} \left[ \frac{-2}{(c-1)^2} + \sum_{j=1}^{i} \frac{G_j}{(1-j)!} \left(\frac{\xi-2i+4}{2c/(c-1)}\right)^{i-j} \exp\left(\frac{\xi+c-2i+2}{2c/(c-1)}\right) \right]
\]

\[
f_1(\eta) = \frac{\Delta p_0 r_0 R_0}{\alpha p} \left[ \frac{2c(1-i) - c^2 - 1}{c(c-1)^2} \frac{\eta-2i+2}{c(c-1)} \right.
\]

\[
+ \sum_{m=1}^{i} \sum_{j=1}^{i-1} \frac{G_m}{(i-j)!} \left(\frac{\eta-c+2i+3}{2c/(1-c)}\right)^{i-j} \exp\left(\frac{\eta-2i+1}{2c/(c-1)}\right) \right]
\]

\[
h_1(\beta) = -f_1(\beta)
\]

The constants of integration are given by the following expressions.

\[
G_1 = \frac{c+1}{(c-1)^2} \quad \text{if } j = 1
\]

\[
G_j = \frac{2}{(c-1)^2} - \sum_{k=1}^{j-1} \frac{G_k}{(j-k)!} \left(\frac{c^2-3c+2}{2c}\right)^{j-k}
\]

The expressions for the overpressure and particle velocity are complex. However, the overpressure and particle-velocity signatures can be calculated easily by means of a digital computer.

It is interesting that the increased generality in the geometry of the pyramidal-pyramidal shock tube, resulting from the unspecified divergence angles of the driver and channel, is typified in the solution by the geometrical parameter \(c\), which equals \(r_0/R_0\). Once the shock-tube geometry is established by the parameter \(c\), the shape of the waves in the driver and channel are fixed. One can show that the solution reduces to those for the pyramidal shock tube when \(r_0\) approaches \(R_0\) (\(c \rightarrow 1\)) and for the pyramidal-rectangular shock tube when \(r_0\) approaches an infinite value.

The overpressure signature for the channel of five geometrically different shock tubes for which \(c\) or \(r_0/R_0\) equals 0.10, 0.25, 0.5, 1.0 and 4.0
are shown in Fig. 70. In each case the first shock wave is generated when the diaphragm is broken. It is followed by a second shock after a time interval of $2\frac{r_\infty}{a}$, which originates at the origin from the reflection of the first rarefaction wave. These two disturbances are followed by subsequent disturbances arising from the change in divergence angle or impedance mismatch at the driver and channel junction. These trailing disturbances, which eventually vanish at large times, are more pronounced in signatures for which the $\frac{r_\infty}{R_\infty}$ value is small (Fig. 70a), owing to the large change in divergence angle at the diaphragm station. The trailing disturbances become vanishingly small as $\frac{r_\infty}{R_\infty}$ increases to unity (Figs. 70a, b, c and d). In the specific case of the pyramidal shock tube, the signature is N shaped (Fig. 70d). For $\frac{r_\infty}{R_\infty}$ values greater than unity (Fig. 70e), the trailing disturbances are always of minor importance as compared with the two shock waves, and the signature is only slightly distorted from the N shape.

The predicted overpressure signature for the wave propagating in the channel is invariant with distance, but it attenuates with distance like $1/r$. The amplitudes of the first and second shocks are equal and given by $\Delta p r/2r_\infty$. These two shocks are the only steep-fronted disturbances of the signature. In practice the waveform would undergo nonlinear distortion as the wave propagates, and the two shocks would decay more rapidly than predicted ($1/r$). This nonlinear distortion could be taken into account by using Whitham's modified acoustic analysis (Ref. 27), in the same manner that was used in Sections 3.3.2 and 3.3.3 for the pyramidal shock tube.

It is interesting to note the behaviour of the fall in overpressure behind the first shock in the signature of the wave (Fig. 70) for the channel of a pyramidal-pyramidal shock tube. For a shock tube with an $\frac{r_\infty}{R_\infty}$ value less than unity (Figs. 70a, b and c), the fall in overpressure behind the first shock is faster than the linear decrease for an N-wave (Fig. 70d) corresponding to an equivalent length driver. When the value of $\frac{r_\infty}{R_\infty}$ is greater than unity (Fig. 70e), the fall in overpressure is slower than that for the N-wave. This type of behaviour was first discovered experimentally (Refs. 44 and 45), and it has now been verified analytically.

Based on the solution for the wave motion in the pyramidal-pyramidal shock tube, or on the results shown in Fig. 70, it can be concluded that the pyramidal shock tube ($r_\infty = R_\infty$) produces the best N-wave for sonic-boom simulation purposes. However, a pyramidal-pyramidal shock tube with an $\frac{r_\infty}{R_\infty}$ value not too different than unity ($0.5 < \frac{r_\infty}{R_\infty} < 2.0$) produces a satisfactory N-wave. Consequently, a pyramidal-pyramidal shock tube with an $\frac{r_\infty}{R_\infty}$ value of 0.5, for example, would have a channel one-half as long as a pyramidal shock tube having an equivalent sized driver and test section. The pyramidal-pyramidal shock tube, being three-quarters as long as the pyramidal shock tube, would be less costly to construct. A further reduction in the overall length and initial cost of the pyramidal-pyramidal shock tube could be achieved by using a shorter channel ($\frac{r_\infty}{R_\infty} < 0.5$), provided that a more distorted N-wave could be tolerated for simulating the sonic boom. Note that the pyramidal-pyramidal shock tube might also be used to simulate certain focussed sonic booms (see Fig. 3), which have a waveform that is similar to the signature of the wave from a shock tube having an $\frac{r_\infty}{R_\infty}$ value of 0.25 (Fig. 70b).

Measured and predicted overpressure signatures were compared in order to check the analytical solution. Three sets of measured and predicted signatures are shown in Fig. 71a, b and c, corresponding to pyramidal-pyramidal shock tubes having $\frac{r_\infty}{R_\infty}$ values of 0.5, 1.0 and 2.0 respectively. In general, the measured
and predicted profiles are in good agreement. Deviations occur mainly at the two shocks, which have finite rise times. As the diaphragm opening process was relatively slow, the two shocks have rather slow rise times. Note that the short-duration measured signatures were taken from Ref. 45.

3.6 Rectangular-Pyramidal Shock Tube

A constant-area shock-tube driver can be installed easily at the apex of the UTIAS pyramidal horn (Fig. 7), where the mass-flow valve is normally located. In the past a few tests were performed with the resulting rectangular-pyramidal shock tube (Fig. 4le), in order to assess its suitability for producing short-duration sonic booms (Ref. 20). Similar tests were also performed with the GASL facility (Ref. 29). The analytical solution for the wave motion in a low-pressure-ratio rectangular-pyramidal shock tube was obtained recently, and it is given herein.

The procedure for determining the solution for the wave motion (Fig. 42) in the rectangular-pyramidal shock tube (Fig. 4le) is similar to that for the pyramidal-rectangular shock tube, which is given in Appendix F. Expressions for the velocity potential \( \phi \), overpressure \( \Delta p \) and particle velocity \( \Delta u \) are given below for the channel gas.

Channel \( r > r_0 \):

\[
\phi = \frac{1}{r} \sum_{i=1}^{\infty} g_i(\xi) H[\xi + c - 2i + 2] \tag{3.55}
\]

\[
\Delta p = -\rho \frac{\partial \phi}{\partial t} \tag{3.56}
\]

\[
\Delta u = \frac{\partial \phi}{\partial r} \tag{3.57}
\]

In these expressions; \( \xi \) equals \((at-r)/r_0 \) and \( c \) equals \( r_0/x_0 \); and the respective symbols \( \rho, a, t \) and \( x_0 \) denote density, sound speed, time, and length of the driver. The radial distance \( r \) is measured from the projected apex of the channel, and \( r_0 \) denotes the diaphragm location (see Fig. 4le). The potential functions \( g_i(\xi) \) for the individual disturbances in the channel (Fig. 42) are summarized below.

\[
g_1(\xi) = \frac{\Delta p r_0^2}{a \rho} \left[ -1 + G_1 \exp \left( \frac{-\xi - c}{2c} \right) \right] \text{ if } i = 1 \tag{3.58}
\]

\[
g_i(\xi) = (-1)^i \frac{\Delta p r_0^2}{a \rho} \left[ 2 - \sum_{j=1}^{i-1} \frac{G_j}{(i-j)^2} \left( \frac{\xi - 2i + 4}{2c} \right)^{i-j} \exp \left( \frac{-\xi - c + 2i - 2}{2c} \right) \right]
\]
The symbol Δp₀ denotes the pressure difference across the diaphragm. The constants of integration are given by the following expressions.

\[ G_1 = 1 \quad \text{if } j = 1 \]

\[ G_j = 2 \sum_{m=1}^{j-1} \frac{G_m}{(j-m)!} \left( \frac{2c}{2c} \right)^{j-m} \quad \text{if } j > 1 \]  

(3.59)

The complicated expressions for the overpressure and particle velocity can be dealt with easily by means of a digital computer.

It is interesting to note that the generality in the geometry of the rectangular-pyramidal shock tube, resulting from the unspecified divergence angle of the channel, is typified in the solution by the geometrical parameter c, which equals r₀/x₀. Once the shock-tube geometry has been specified by the parameter c, the signature of the wave in the channel is fixed. One can show that this solution reduces to that for the rectangular shock tube when r₀ approaches infinity.

The overpressure signature for the channel of five geometrically different shock tubes for which c or r₀/x₀ equals 0.10, 0.50, 1.0, 2.0 and 10.0, are shown in Fig. 72. In each case the first shock is generated when the diaphragm is broken. It is followed by a rarefaction wave after a time interval of 2x₀/a, which also originated when the diaphragm was broken. These two steep-fronted disturbances are followed by subsequent disturbances arising from the change in duct angle or impedance mismatch at the driver and channel junction. These trailing disturbances, which eventually vanish at large times, are more pronounced in signatures for which the r₀/x₀ value is small (e.g., see Fig. 72a), owing to the large change in duct angle at the diaphragm station. The disturbances disappear only for a shock tube for which r₀ becomes infinite and x₀ is constant. For the resulting rectangular shock tube the wave is a flat-topped pulse having a duration equal to 2x₀/a. Note that the overpressure signature for the wave propagating in the channel is invariant with distance, but it attenuates with distance like 1/r. The amplitudes of the shock and rarefaction waves are equal and given by Δp₀r₀/2r.

Based on the solution for the wave motion in the rectangular-pyramidal shock tube, or on the results shown in Fig. 72, it can be concluded that this shock tube is incapable of producing a good N-wave. Hence, this type of shock tube would not make a good sonic-boom simulator.  

3.7 Reflection Eliminator

When the simulated sonic boom propagates to the open end of the channel, it is normally partially transmitted to the surrounding atmosphere and partially reflected back up the channel. This reflected wave disrupts the simulated flow and pressure conditions in the test section. In order to eliminate or at least adequately minimize the undesirable reflection and its subsequent echoes, a reflection eliminator is usually installed to cover the open end of the channel. In the case of the UTIAS pyramidal horn, a recoiling type porous piston was utilized (Figs. 7 and 10). For the Franco-German pyramidal shock tube (Ref. 46), a slotted pyramid was attached to the base of the pyramidal channel, in the manner shown in Fig. 48. Additionally, a perforated wedge was used to eliminate reflections for the French rectangular shock tube (Refs. 36 and 37). With the exception
of the porous-piston-type reflection eliminator, the others were developed on the basis of experimental data (Refs. 36, 37 and 45). An analysis that is applicable to these reflection eliminators is given herein.

Consider a pyramidal shock tube with a slotted or perforated pyramid for a reflection eliminator (e.g., the Franco-German sonic-boom simulator). In order to simplify the analysis, the slotted walls of the pyramid were modelled as walls made of porous material, as illustrated in Fig. 73a. For sake of generality, the simulated sonic boom or incident wave on the porous pyramid was assumed to be an arbitrary one, having an overpressure denoted by $\Delta p$ and a particle velocity $\Delta u$. Note that for an outgoing spherical wave, the particle velocity is related to the overpressure by the following expression (see Eq. 2.44).

$$\Delta u = \frac{\Delta p}{ap} + \frac{1}{pr} \int_{-\infty}^{\infty} \Delta p(y) \, dy \quad (3.60)$$

The respective symbols $a$, $p$, $r$, and $\tau$ denote sound speed, density, radial distance measured from the apex of the pyramidal shock tube, and the retarded time $(t-r/a)$. As the incident wave propagates into the converging porous pyramid, it is assumed that the outflow of gas through the porous material is precisely that which produces no reflected wave from the walls. Hence, the incident wave is undisturbed as it propagates towards the apex of the porous pyramid.

Let the velocity of the gas flowing out through the porous wall be denoted by $u$, and assume the flow is normal to the porous material, as illustrated in Fig. 73b. Also, let the porous material be free to move with a velocity $v$. Furthermore, assume that the overpressure of the disturbance transmitted to the outside of the porous pyramid is negligible compared with that for the incident wave, and it thus can be neglected. Hence, the pressure drop across the porous material is simply equal to the overpressure ($\Delta p$) of the incident wave. It can be related to the velocity of the flow ($u$) and porous material ($v$) as shown below.

$$\Delta p = R (u - v) \quad (3.61)$$

The symbol $R$ denotes the flow resistance of the porous material. From Newton's second law of motion, the pressure drop and material velocity are related by the following expression.

$$\Delta p = \sigma \frac{dv}{dt} \quad (3.62)$$

The mass per unit area of the porous material is denoted by $\sigma$. If the wave moving inside the porous pyramid is undisturbed, as no reflection arises at the wall, the mass-flow rate which enters the triangular element of unit width (Fig. 73b) through side ab must be equal to that which leaves through side ac. A mass-flow rate balance yields the following expression for $u$ and $\Delta u$.

$$u = \Delta u \sin \theta \quad (3.63)$$

The half-angle $\theta$ for the porous pyramid is illustrated in Fig. 73a.

The preceding four expressions (Eqs. 3.60 to 3.63) are sufficient to
determine the mass per unit area ($\sigma$) and the flow resistance ($R$) for the porous material. As an intermediate step, the four equations can be cast into the following form.

$$\left[ \frac{1}{R} - \frac{\sin \theta}{\rho \sigma} \right] \frac{\partial (\Delta p)}{\partial t} + \left[ \frac{1}{\sigma} - \frac{\sin \theta}{\rho r} \right] \Delta p = 0 \quad (3.64)$$

For general values of $\sigma$, $R$ and $\theta$ the solution of the differential equation corresponds to an overpressure signature of the incident wave which results in no reflected wave. If the coefficients of the differential equation are set equal to zero, then the condition of no reflected wave is still satisfied. However, the incident wave can then be arbitrary. The resulting form of $R$ and $\sigma$ can be obtained.

$$R = \frac{\rho a}{\sin \theta} \quad (3.65)$$

$$\sigma = \frac{\rho r}{\sin \theta} \quad (3.66)$$

The predicted flow resistance $R$ for the porous, slotted or perforated walls of a pyramidal type reflection eliminator is a constant. Hence, the porous material should be of the same thickness for all parts of the pyramid, or the number of equal sized slots or holes per unit wall area should be constant. This prediction is well supported qualitatively by experimental data. These results for a pyramidal shock tube using a perforated pyramid as a reflection eliminator can be found in Ref. 45. A quantitative comparison of predicted and measured flow resistance was not made because the flow resistance for the perforated walls was not available.

The predicted mass per unit area ($\sigma$) for the porous material or equivalent perforated plates forming the walls of the pyramidal reflection eliminator, is not constant, but it depends on the radial distance $r$. Because the radial distance is relatively large and $\sin \theta$ is normally small for the case of an actual reflection eliminator, the predicted result for $\sigma$ (Eq. 3.66) is much larger than that for a porous material or a thin perforated plate which have the correct flow resistance $R$. In practice this difficulty can be circumvented by designing a massive support to hold each of the eliminator's walls, thereby producing a correct effective mass per unit area.

The walls and their supports should be capable of movement in order to achieve perfect reflection elimination. However, if the predicted result for $\sigma$ is very large (large $r$ and small $\sin \theta$ in Eq. 3.66), the walls are correspondingly massive and their resulting motion is sufficiently small that it can be neglected. As a consequence, the walls of the reflection eliminator can be designed to be stationary. If the walls are rigid and stationary, they do not have to be massive. In the past, actual pyramidal reflection eliminators for the Franco-German pyramidal shock tubes (Refs. 45 and 46) were designed with stationary walls for simplicity, and good reflection elimination was achieved. The results of the analysis (Eq. 3.66) illustrate the reasons why the slotted and perforated walls of these reflection eliminators could be designed to be stationary.

When the half-angle $\theta$ for the reflection eliminator is increased to $\pi/2$ such that $\sin \theta$ equals unity, the porous, slotted or perforated walls of the pyramid are transformed into one planar walls that covers the open end of the shock tube. Hence, the planar piston reflection eliminator is a special case of the more general pyramidal type reflection eliminator. For this particular case,
R equals \( \rho a \) and \( \sigma \) equals \( \rho r \). These results are in exact agreement with those derived in a similar but simpler manner in Section 2.5 for a porous piston reflection eliminator. For a planar piston eliminator for which \( r \) and thus \( \sigma \) are not relatively large, the piston is not sufficiently massive that its motion can be neglected. For this case the piston must be permitted to move to achieve satisfactory reflection elimination. For example, the porous piston of the UTIAS pyramidal horn must be allowed to move (see Appendix A and Section 2.5). It is worth noting that the French researchers made a few tests with a pyramidal shock tube having a perforated plate as a reflection eliminator (Ref. 45). Good elimination of the reflected wave was not achieved, probably because the plate was not permitted to move.

Now, consider a rectangular shock tube with a pyramidal reflection eliminator. For this case the incident was is planar. Consequently, the over-pressure and particle-velocity waveforms are identical, as \( \Delta u \) equals \( \Delta p / \rho \). When this result is used instead of Eq. 3.60 with the preceding analysis, the following results can be obtained for \( R \) and \( \sigma \).

\[
R = \frac{\Delta p}{\sin \theta} \quad (3.67)
\]

\[
\sigma = \infty \quad (3.68)
\]

The flow resistance (\( R \)) for the porous material or perforated plates of the reflection eliminator is the same as that for the case of the pyramidal shock tube (see Eq. 3.65). However, the mass per unit area (\( \sigma \)) is different as it turns out to be infinite. This result simply means that the porous or perforated walls of the eliminator should be stationary to achieve perfect reflection elimination. Note that Eqs. 3.67 and 3.68 could have been derived from Eqs. 3.65 and 3.66 respectively, because spherical waves at large radii (\( r \rightarrow \infty \)) behave like planar ones.

As a last case, consider a rectangular shock tube with a wedgy reflection eliminator. The two walls of the wedge can be made of porous material or perforated plates. For the Franco-German facility (Refs. 36 and 37), the walls were constructed of perforated plates. If the half-angle of the wedge is denoted by \( \theta \), the analysis gives the same result for \( R \) and \( \sigma \) as for a pyramidal reflection eliminator, namely Eqs. 3.67 and 3.68. Hence, the wedgy eliminator's walls should be stationary, and the flow resistance constant over the entire plate area. This latter result is supported qualitatively by experimental results, which can be found in Refs. 36 and 37.

4. DISCUSSION

The UTIAS travelling-wave horn has proven to be a practical facility for the simulation of the sonic boom. In the mass-flow-valve mode of operation, the compressor and reservoir system, mass-flow valve, pyramidal horn, jet-noise absorber and porous piston type of reflection eliminator all function correctly as designed. The simulator therefore is capable of producing a good N-wave or simulated sonic boom in the interior test section of the pyramidal horn. The N-wave amplitude and duration can be controlled individually to be less than, equal to, or more than that of an actual sonic boom. Additionally, the rise times for the two shocks of the N-wave (2 to 6 ms), although they are not controllable, are
typical of those for an actual sonic boom (average value of 5.4 ms). Owing to
certain design features of the compressor, reservoir and mass-flow-valve system,
the facility has the desirable capability of producing repeatable and repetitive
(1 to 5 booms per minute) simulated sonic booms to facilitate structural response,
fatigue and damage studies. For the alternate operating mode of a shock-tube
driver (pyramidal shock tube), a short-duration N-wave (up to 20 ms) of suitable
amplitude and very fast rise time (20 to 60 μs) can be generated in the interior
test section.

There is one significant drawback to the UTIAS travelling-wave horn
when operated in the mass-flow-valve mode. Undesirable high-frequency jet noise
generated by the high-speed flow at the valve is superimposed on the simulated
sonic boom. The jet-noise absorber does a remarkable job of removing the higher
frequencies of this jet noise, and therefore improves markedly the performance
of the simulator. However, acceptable simulated sonic booms for human, animal
and structural response tests are limited to peak overpressures of less than
200 N/m² and restricted to durations that are shorter than 200 ms. For a more
powerful and longer duration N-wave, the jet noise begins to contribute signi-
ficantly to the subjective loudness rating of the simulated sonic boom, and the
results of human (Ref. 20) and animal response tests become difficult to inter-
pret. Additionally, the lower frequencies of the residual jet noise start to
cause undesirable structural response, in addition to the desired response from
the N-wave (Ref. 52). To eliminate or minimize such problems it was found desirable
to further reduce the jet noise. To achieve this goal, a new mass-flow valve,
which features a ten-fold larger throat area of 300 cm² has been designed and
constructed. By virtue of its larger throat area the flow speed at the valve will
be much slower, and the intensity of the resulting jet noise will be considerably
diminished. This new valve, which is almost ready for testing, should alleviate
the jet-noise problem. Furthermore, it is hoped that certain design features of
the valve will make it possible to control the rise time for the N-wave. The results
of the work with the new valve will be subsequently published at a UTIAS report.

Comprehensive analyses to describe the wave motion in the UTIAS travelling-wave horn or a similar facility utilizing a mass-flow valve have been presented.
The relatively simple but quite general point-source analysis has the capability of
predicting the main features (amplitude, duration and basic waveform) of the
simulated sonic boom, with the exception of the finite rise time. The more complex
extended or finite-source analysis, which predicts the same basic signature of the
simulated sonic boom, is also capable of predicting the additional feature of a
finite rise time. Both analyses are not restricted to the specific case when the
flow at the valve throat is choked or sonic, but they also apply for the other
case when the flow is not choked or subsonic. Additionally, the analyses have
the capability of dealing with the actual valve-plug geometry (length to width
ratio of the pyramidal plug) and its actual motion or displacement history. Hence,
the distortion in the waveform of the simulated sonic boom or N-wave arising from
the plug’s blunt shape and nonideal motion can be predicted by the analyses. The
results of the point-source and finite-source analyses have been substantiated
by experimental data.

Additional analyses for the porous-piston type of reflection eliminator,
the jet noise, and a finite-volume reservoir have also been given. The reflection-
eliminator analysis is capable of not only predicting the mass per unit area (c)
and flow resistance (R) for the porous material of the piston, but it also is
capable of describing the piston’s motion. The rudimentary jet-noise analysis
provided insight into the dependence of the jet noise on the flow speed and
area at the throat of the mass-flow valve. The results of the analysis, which agree only qualitatively with experimental data, substantiated the intuitive idea that the intensity of the jet noise could be reduced markedly by using a valve with a large throat area.

It was envisaged that the use of a larger valve with the existing reservoir and pyramidal horn might affect the simulation of the sonic boom. For a low reservoir pressure required when using a large valve, and owing to the finite volume of the reservoir, it was thought that the change in state of the reservoir gas during the generation of the simulated boom could cause a significant distortion in the N-wave signature. The finite-volume reservoir analysis verified this expectation. The results of the analysis which are relatively simple not only quantify the type and degree of distortion in the simulated N-wave, but they also provide a viable means of selecting the correct reservoir volume to minimize the distortion.

The pyramid shock tube is the primary shock-tube-driver mode of operation for the UTIAS travelling-wave horn. The three analyses given in Section 3.3 for the pyramidal shock tube have been successful in predicting the wave motion in the channel and the resulting wave in the test section. For a low driver overpressure \( (\Delta p < \frac{p}{4}) \) and for a short distance \( (r/r_o) \) of propagation of the wave such that \( (\Delta p/p) \ln(r/r_o) \) is less than 0.2, the first-order analysis (Section 3.3.1) is sufficient and easy to use to determine the wave motion. For the same overpressure range but when \( (\Delta p/p) \ln(r/r_o) \) is greater than 0.2, then the extended first-order analysis (Section 3.3.2) should be used, because the moving wave undergoes significant distortion. The nonlinear analysis (Section 3.3.3) should be used to determine the wave motion when the driver overpressure exceeds one-quarter of an atmosphere (\( \frac{p}{4} \)).

It can be concluded that the comprehensive analyses developed for the UTIAS travelling-wave horn are valuable for predicting the wave motion in the horn, and also for designing certain parts of the facility (e.g., the reservoir and reflection eliminator). These analyses cover both the mass-flow-valve and shock-tube-driver modes of operation. It should be noted that the results of the analyses are not restricted to the UTIAS facility.

The first-order acoustic solutions given in Section 3 for the rectangular, pyramidal, pyramidal-rectangular, pyramidal-pyramidal and rectangular-pyramidal shock tubes successfully predict the wave motion for such low-pressure-ratio shock tubes. With the exception of the pyramidal shock tube, the acoustic solutions for these shock tubes are new. Although the solution for the pyramidal shock tube was known previously, it was included because the pyramidal shock tube is the best shock tube for simulating a sonic boom. Consequently, the solution for the pyramidal shock tube was discussed and illustrated in more detail. It is worth noting that the method of obtaining the acoustic solution for the aforementioned shock tubes can be applied to yield the solution for other low-pressure-ratio shock tubes, such as a rectangular-hyperbolic horn shock tube. The acoustic solutions for various shock tubes are not only of interest to researchers at UTIAS, but also to those who have used such facilities in France, Germany, England and the United States.

It can be concluded from the results of the acoustic solutions for the various shock tubes that the pyramidal shock tube is the only one that generates a perfect N-wave for simulating a sonic boom. This same conclusion was originally made on the basis of experimental results obtained from shock-tube facilities.
Consequently, large pyramidal shock tubes were constructed in France (Fig. 48) and England to simulate full-scale sonic booms. A shorter and less costly pyramidal-pyramidal shock tube could also be constructed for the same purpose. It would have an equivalent driver, diaphragm and test section, but would be shorter by virtue of a shorter channel (see Fig. 69). The simulated sonic boom would not be a perfect N-wave, but the distortion would be small and probably not significant.

The analysis for a pyramidal type of reflection eliminator (Section 3.7), such as the eliminator for the Franco-German facility shown in Fig. 48, is new. It is successful in providing new insight into the design and operation of such eliminators. The analysis is also sufficiently general that solutions can be obtained for other reflection eliminators, such as the porous piston type.

It is worthwhile to compare some important features of a pyramidal shock tube with those for the UTIAS travelling-wave horn using a mass-flow valve. In the case of the simulation of full-scale sonic booms, the UTIAS facility (25 m long) is considerably shorter than a pyramidal shock tube (200 m long). By using a mass-flow valve instead of a shock-tube driver, and by using a porous piston reflection eliminator instead of a pyramidal one, it was possible to construct a shorter facility to reduce the initial cost and space requirement. Both facilities have the capability of producing a simulated sonic boom with the correct amplitude, duration, rise time, and basic waveform. However, because some undesirable jet noise is superimposed on the N-wave produced by the UTIAS facility, the pyramidal shock tube generates the best simulated sonic boom. Note that the new valve for the UTIAS simulator should alleviate the jet-noise problem and make the facility capable of generating a relatively "noise-free" N-wave. Although the UTIAS simulator does not produce the best simulated sonic boom, it can generate acceptable N-waves at a very fast repetition rate (e.g., one to five booms per minute). A sequence of booms (e.g., 1000) can be generated at a constant rate, in an automatic fashion that does not require human intervention. In the other case of a pyramidal shock tube, only a single boom every few minutes can be produced, as the procedure of manually installing a new diaphragm is relatively time consuming. Note that the primary function of the UTIAS travelling-wave horn is to study structural response, fatigue and damage due to the simulated sonic boom. Consequently, repeatable N-waves at a high repetition rate are a necessary requirement for the UTIAS simulator.

5. CONCLUDING REMARKS

Accounts of the design and initial performance of the UTIAS travelling-wave horn have been given previously (see Refs. 19, 20 and 53). These reports were concise and often incomplete. The present comprehensive work replaces the previous reports.

The UTIAS travelling-wave horn has already been used for sonic-boom research, other than facility development. In the shock-tube mode of operation a short N-wave (2 ms in duration) was generated to facilitate the study of the development of a spiked or rounded sonic boom as an N-wave propagates through atmospheric turbulence (Ref. 48). In the mass-flow-valve operating mode a number of projects have been completed. Carothers (Ref. 20) has made a preliminary physiological investigation into the effects of a simulated sonic boom on the temporary threshold shift in hearing, temporary increase in heart rate, and heart-rate recovery time. An initial psychoacoustic study was made by Lips (Ref. 54) into the effects of a sonic-boom disturbance on the performance of a
human subject during a tracking task. This tracking task in certain aspects simulated automobile driving. As a follow-up to this work, Nowakiwsky (Ref. 55) completed an experimental study of the effects of a simulated sonic boom on a subject driving an actual automobile around a special test track. The simulated sonic boom was generated by four loudspeakers inside the automobile. Recently, an experimental investigation was completed by Leigh (Ref. 52) to quantify damage effects of sonic boom to plaster panels. These panels were typical of certain dwelling walls. Structural response, fatigue and damage studies of this nature are continuing at UTIAS. Note that a review of sonic-boom research at UTIAS has recently been made by the author (Ref. 21). It covers not only the work with the travelling-wave horn but also includes the work with the loudspeaker-driven booth and the portable shock-tube type of sonic-boom simulator.
REFERENCES


41. The pyramidal shock tube is located in a balloon hanger at Cardington, England. The facility comes under the jurisdiction of the Structures Department headed by Mr. C. H. E. Warren, Royal Aircraft Establishment, Farnborough, Hants.


### TABLE 1

**MASS-FLOW RATE PER UNIT AREA THROUGH THE VALVE THROAT**

(The tabulated results were calculated for the case of air with $a = a_0 = 345 \text{ m/s}$, $\gamma = 1.4$ and $\rho = 1.19 \text{ kg/m}^3$.)

$$\frac{m_\infty}{\rho a} = \frac{a}{a_0} \frac{p_0}{p} \left[ M_\infty \left[ 1 + \frac{\gamma - 1}{2} \left( \frac{M_\infty^2}{M_\infty^2 - 1} \right) \right] \right]^{-(\gamma + 1) / 2(\gamma - 1)}$$

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FIG. 1 WAVE PATTERN AND OVERPRESSURE SIGNATURES FROM A SUPERSONIC AIRCRAFT.
FIG. 3 OVERPRESSURE SIGNATURE OF A FOCUSED SONIC BOOM OR SUPERBOOM.
FIG. 4 OVERPRESSURE SIGNATURE OF AN IDEALIZED SONIC BOOM.
FIG. 5 DISTORTION OF A SONIC BOOM OWING TO ITS PROPAGATION THROUGH TURBULENCE IN THE ATMOSPHERIC BOUNDARY LAYER.
a) SPHERICAL EXPLOSION (PYRAMIDAL SHOCK TUBE)

b) CYLINDRICAL EXPLOSION (WEDGY SHOCK TUBE)

c) PLANAR EXPLOSION (RECTANGULAR SHOCK TUBE)

FIG. 6 NEAR-FIELD AND FAR-FIELD OVERPRESSURE SIGNATURES FROM WEAK SPHERICAL, CYLINDRICAL AND PLANAR EXPLOSIONS.
FIG. 7 ELEVATION AND PLAN VIEWS ILLUSTRATING THE MOST RELEVANT ELEMENTS OF THE TRAVELLING-WAVE HORN.
FIG. 8 MASS-FLOW-VALVE OPERATION: a) VALVE, b) PLUG MOTION, c) THROAT-AREA AND MASS-FLOW-RATE PROFILES, d) OVERPRESSURE N-WAVE GENERATED IN THE HORN.
FIG. 9 JET-NOISE ABSORBER.
FIG. 10 POROUS-PISTON REFLECTION ELIMINATOR.
FIG. 11 VALVE-PLUG DISPLACEMENT VERSUS TIME FOR NOMINAL OPEN-TO-CLOSE TIMES OF 60, 100, 200, 300, AND 400 ms FOR THE VALVE.
FIG. 12 SIMULATED SONIC BOOMS GENERATED WITH AND WITHOUT THE JET-NOISE ABSORBER (radial distance of 15.2 m; reservoir pressures of 1.3, 2.5, 5.1, 9.2 and 13.9 atm).
FIG. 13  SIMULATED SONIC BOOMS GENERATED WITH AND WITHOUT THE 
JET-NOISE ABSORBER (radial distance of 21.4 m; reservoir 
pressures of 1.2, 1.8, 2.8 and 3.4 atm).
FIG. 14 SIMULATED SONIC BOOMS RECORDED AT A RADIAL STATION OF 15.2 m (all signals have been electronically filtered to remove the jet noise).
\[
\frac{\bar{m}_*}{\rho a} = \frac{a}{a_o} \frac{p_o}{p} M_* \left[ 1 + \frac{\gamma - 1}{2} M_*^2 \right]^{-\frac{\gamma + 1}{2(\gamma - 1)}}
\]

\(a_o = a\)
\(\gamma = 1.4\)

**FIG. 15** MASS-FLOW RATE OF AIR PER UNIT AREA FROM AN INFINITE RESERVOIR THROUGH THE VALVE.
FIG. 16 OVERPRESSURE AND PARTICLE VELOCITY SIGNATURES.
FIG. 17  THROAT-AREA APPROXIMATION FOR THE MASS-FLOW VALVE.
FIG. 18 EFFECTS OF VALVE-PLUG GEOMETRY (b/c) ON THE SIMULATED SONIC BOOM.
FIG. 19 EFFECTS OF NONIDEAL VALVE-PLUG MOTIONS ON SIMULATED SONIC BOOMS.
FIG. 20 INSTANTANEOUS AND FINITE RISE-TIME OVERPRESSURE SIGNATURES FOR AN IDEAL PARABOLIC MASS-FLOW-RATE PROFILE.
FIG. 21 INSTANTANEOUS AND FINITE RISE-TIME OVERPRESSURE SIGNATURES FOR A MASS-FLOW-RATE PROFILE CORRESPONDING TO A b/c VALUE OF 0.89.
FIG. 22 COMPARISON OF PREDICTED AND MEASURED OVERPRESSURE N-WAVES (radial distance of 15.2 m).
FIG. 23 PEAK OVERPRESSURE OF THE SIMULATED SONIC BOOM VERSUS RESERVOIR PRESSURE.
FIG. 24 PEAK OVERPRESSURE OF THE SIMULATED SONIC BOOM VERSUS DURATION.
FIG. 25 PEAK OVERPRESSURE OF THE SIMULATED SONIC BOOM AS A FUNCTION OF RADIAL DISTANCE (reservoir pressure of 1.41 atm).
FIG. 26 PEAK OVERPRESSURE OF THE SIMULATED SONIC BOOM AS A FUNCTION OF RADIAL DISTANCE (reservoir pressure of 3.05 atm).
FIG. 27 COMPARISON OF PREDICTED AND MEASURED PARTICLE-VELOCITY SIGNATURES HAVING A NOMINAL DURATION OF 80 ms.
FIG. 28 COMPARISON OF PREDICTED AND MEASURED PARTICLE VELOCITY SIGNATURES HAVING A NOMINAL DURATION OF 300 ms.
FIG. 29 DISTORTED SONIC BOOMS PRODUCED WITH THE UTIAS TRAVELLING-WAVE HORN BY USING A BLUNT PLUG (b/c = 0.36) IN THE MASS-FLOW VALVE.
FIG. 30 COMPARISON OF PREDICTED AND MEASURED RISE TIMES FOR THE FRONT SHOCK OF THE N-WAVE (radial distance of 21.4 m).
FIG. 31 COMPARISON OF PREDICTED AND MEASURED RISE TIMES FOR THE REAR SHOCK OF THE N-WAVE (radial distance of 21.4 m).
FIG. 32 RELATIVE CHANGE OF RESERVOIR PRESSURE, DENSITY, TEMPERATURE AND SOUND SPEED DURING THE SIMULATION OF A SONIC BOOM (300-ms duration) WITH THE UTIAS TRAVELLING-WAVE HORN.
\( A_x = 31.6 \text{ cm}^2 \)
\( \alpha_0 = 340 \text{ m/s} \)
\( V = 2.7 \text{ m}^3 \)

**WAVE DURATION:**
- 300 ms
- 200 ms
- 100 ms

**RESERVOIR PRESSURE (atm)**

**FIG. 33** THE DISTORTION OF THE SIMULATED SONIC BOOM Owing TO A FINITE-VOLUME RESERVOIR, a) DISTORTION PARAMETER, b) DISTORTED SIGNATURE.
FIG. 34 THE CHANGE IN RESERVOIR PRESSURE VERSUS INITIAL RESERVOIR PRESSURE FOR SIMULATED SONIC BOOMS HAVING DIFFERENT DURATIONS OF 100, 200 AND 400 ms.
FIG. 35 JET NOISE SUPERIMPOSED ON A SIMULATED SONIC BOOM (reservoir pressure of 1.21 atm).
FIG. 36 JET NOISE SUPERIMPOSED ON A SIMULATED SONIC BOOM (reservoir pressure of 6.1 atm).
FIG. 37 PEAK ROOT-MEAN-SQUARE OVERPRESSURE OF THE JET NOISE VERSUS MASS-FLOW RATE PER UNIT AREA.
**FIG. 38** MODEL OF THE REFLECTION ELIMINATOR FOR THE ANALYSIS.

- **REFLECTED WAVE**
- **TRANSMITTED WAVE**
- **INCIDENT WAVE**
- **PYRAMIDAL HORN**
- **POROUS PISTON:**
  - \( \sigma \) (mass per unit area)
  - \( R \) (flow resistance of the porous material)
FIG. 39 MEASURED AND PREDICTED DISPLACEMENT PROFILES OF THE POROUS PISTON OF THE UTIAS REFLECTION ELIMINATOR.
FIG. 40 FLOW RESISTANCE VERSUS FLOW SPEED OF THE MICROLITE USED FOR THE UTIAS REFLECTION ELIMINATOR.
FIG. 41 SHOCK TUBES THAT HAVE BEEN USED TO PRODUCE SIMULATED SONIC BOOMS.
FIG. 42 TIME-DISTANCE DIAGRAM OF THE WAVE MOTION FOR A LOW-PRESSURE-RATIO SHOCK TUBE.
FIG. 43 TIME-DISTANCE DIAGRAM OF THE WAVE MOTION IN A LOW-PRESSURE-RATIO RECTANGULAR SHOCK TUBE.
FIG. 44 THE OVERPRESSURE SIGNATURE IN THE CHANNEL OF A RECTANGULAR SHOCK TUBE, a) $z = 0.31$, b) $z = 3.23$, c) $z = 1.00$. 
FIG. 45 PARTICLE PATHS FOR THE GAS IN THE CHANNEL OF A RECTANGULAR SHOCK TUBE.
FIG. 46 OVERPRESSURE MEASUREMENTS MADE IN THE CHANNEL OF A RECTANGULAR SHOCK TUBE.
FIG. 47 COMPARISON OF PREDICTED AND MEASURED OVERPRESSURE SIGNATURES FOR A RECTANGULAR SHOCK TUBE.
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FIG. 48 PYRAMIDAL SHOCK TUBE AT THE FRANCO-GERMAN INSTITUTE FOR RESEARCH AT SAINT-LOUIS, FRANCE.
FIG. 49 TIME-DISTANCE DIAGRAM OF THE WAVE MOTION IN A PYRAMIDAL SHOCK TUBE, AS PREDICTED BY ACOUSTIC THEORY.
FIG. 50 OVERPRESSURE VARIATION WITH TIME AT FIXED DISTANCE OF 0, \( r_o / 3 \), 2\( r_o / 3 \), \( r_o \) AND 4\( r_o / 3 \) FOR THE PYRAMIDAL SHOCK TUBE.
FIG. 51 PARTICLE VELOCITY VARIATION WITH TIME AT FIXED DISTANCES OF 0, \( \frac{r_o}{3} \), \( \frac{2r_o}{3} \), \( r_o \) AND \( \frac{4r_o}{3} \) FOR THE PYRAMIDAL SHOCK TUBE.
FIG. 52 NONDIMENSIONAL PARTICLE-VELOCITY PROFILES AT SUCCESSIVE DISTANCES ALONG THE CHANNEL OF A PYRAMIDAL SHOCK TUBE.
FIG. 53: OVERPRESSURE DISTRIBUTION VERSUS DISTANCE AT SUCCESSIVE TIMES OF $0.4r_o/a$, $0.6r_o/a$, $2r_o/a$ AND $3.6r_o/a$ FOR THE PYRAMIDAL SHOCK TUBE.
a) $t = 0.4 \frac{r_0}{a}$

 PARTICLE VELOCITY DISTRIBUTION AT $t$ EQUAL TO 0.

- $\Delta u_1 = \frac{\Delta p_m}{2a\rho} \frac{r_0}{r_0 + at}$
- $\Delta u_2 = \frac{\Delta p_m}{2a\rho}$
- $\Delta u_3 = \frac{\Delta p_m}{2a\rho} \frac{r_0}{r_0 - at}$

b) $t = 0.6 \frac{r_0}{a}$

c) $t = 2 \frac{r_0}{a}$

d) $t = 3.6 \frac{r_0}{a}$

FIG. 54 PARTICLE-VELOCITY DISTRIBUTION VERSUS DISTANCE AT SUCCESSIVE TIMES OF $0.4r_0/a$, $0.6r_0/a$, $2r_0/a$ AND $3.6r_0/a$ FOR THE PYRAMIDAL SHOCK TUBE.
FIG. 55 PARTICLE PATHS (not to scale) SHOWN ON A TIME-DISTANCE DIAGRAM FOR THE PYRAMIDAL SHOCK TUBE.
FIG. 56 MEASURED AND PREDICTED OVERPRESSURE PROFILES FOR A PYRAMIDAL SHOCK TUBE (Saint-Louis).
a) DRIVER OVERPRESSURE ($\Delta p_o$): 2400 N/m$^2$
DRIVER LENGTH ($r_o$): 1.58 m
RADIAL LOCATION OF MEASUREMENT: 21.3 m

b) FIG. 57 MEASURED AND PREDICTED OVERPRESSURE SIGNATURES FOR A PYRAMIDAL SHOCK TUBE (UTIAS).
FIG. 58 PEAK OVERPRESSURE OF AN N-WAVE AS A FUNCTION OF DISTANCE FOR A PYRAMIDAL SHOCK TUBE (UTIAS).

- PREDICTED RESULTS
- MEASURED RESULTS

INITIAL DRIVER OVER-PRESSURE OF 2400 N/m$^2$

DRIVER LENGTH OF 1.58 m
FIG. 59 SAMPLE OVERPRESSURE SIGNATURES, a) PROFILE PREDICTED BY FIRST-ORDER ACOUSTIC THEORY, b) PROFILE CORRECTED TO INCLUDE DISTORTION EFFECTS, c) RESULTING PROFILE FOR THE EXTENDED FIRST-ORDER ANALYSIS.
a) DRIVER OVERPRESSURE ($\Delta p_o$): 34 kN/m$^2$
DRIVER LENGTH ($r_o$): 1.58 m
RADIAL LOCATION OF MEASUREMENT: 15.3 m

![Graph of measured and predicted overpressure signatures]

b) FIG. 60 MEASURED AND PREDICTED (extended analysis) OVERPRESSURE SIGNATURES FOR A PYRAMIDAL SHOCK TUBE (UTIAS).
FIG. 61 PEAK OVERPRESSURE OF AN N-WAVE VERSUS RADIAL DISTANCE FOR A PYRAMIDAL SHOCK TUBE (UTIAS).
Fig. 62 Measured and predicted (extended analysis) overpressure signatures for a pyramidal shock tube (UTIAS).
Fig. 63 Peak overpressure of an N-wave as a function of radial distance for a pyramidal shock tube (UTIAS).
FIG. 64 NUMERICALLY COMPUTED TIME-DISTANCE DIAGRAM ILLUSTRATING THE WAVE MOTION FOR A PYRAMIDAL SHOCK TUBE (driver pressure of 4 atm, channel pressure of 1 atm, driver length of 20 cm).
FIG. 65 MEASURED AND PREDICTED (numerical) OVERPRESSURE SIGNATURES AT RADIAL DISTANCES OF 1.2 m (a), 4.6 m (b), and 18.3 m (c) FOR A PYRAMIDAL SHOCK TUBE (driver pressure of 4 atm, channel pressure of 1 atm, driver length of 20 cm).
FIG. 66  PYRAMIDAL-RECTANGULAR SHOCK TUBE, a) PREDICTED PROFILE, b) MEASURED PROFILE, c) COMPARISON OF PREDICTED AND MEASURED SIGNATURES.
FIG. 67 OVERPRESSURE SIGNATURES RECORDED IN THE CHANNEL OF A PYRAMIDAL-RECTANGULAR SHOCK TUBE FOR DIFFERENT DRIVER OVERPRESSURES (driver length of 46 cm, measurement location of 46 cm).
FIG. 68 COMPARISON OF MEASURED AND PREDICTED RESULTS FOR A PYRAMIDAL-RECTANGULAR SHOCK TUBE.
a) PYRAMIDAL SHOCK TUBE (long channel)

b) PYRAMIDAL-PYRAMIDAL SHOCK TUBE (short channel)

FIG. 69 A PYRAMIDAL (a) AND A PYRAMIDAL-PYRAMIDAL SHOCK TUBE (b) HAVING AN EQUIVALENT SIZED DRIVER, DIAPHRAGM AND TEST SECTION, BUT A DIFFERENT OVERALL LENGTH.
FIG. 70 OVERPRESSURE SIGNATURES FOR THE CHANNEL OF FIVE GEOMETRICALLY DIFFERENT PYRAMIDAL-PYRAMIDAL SHOCK TUBES.

- a) Overpressure signature for $r_o/R_o = 0.1$
- b) Overpressure signature for $r_o/R_o = 0.25$
- c) Overpressure signature for $r_o/R_o = 0.5$
- d) Overpressure signature for $r_o/R_o = 1.0$
- e) Overpressure signature for $r_o/R_o = 4.0$

\[ \Delta p = \frac{\Delta p_o}{2} \frac{r_o}{r} \]
FIG. 71 MEASURED AND PREDICTED OVERPRESSURE SIGNATURES FOR THREE DIFFERENT PYRAMIDAL-PYRAMIDAL SHOCK TUBES, a) \( r_o/R_o = 0.5 \), b) \( r_o/R_o = 1.0 \), c) \( r_o/R_o = 2.0 \) (The measured results have been taken from Ref. 45).
FIG. 72 OVERPRESSURE SIGNATURES FOR THE CHANNEL OF FIVE GEOMETRICALLY DIFFERENT RECTANGULAR-PYRAMIDAL SHOCK TUBES ($\Delta p = \Delta p_0 r_0/2r$).
FIG. 73 SCHEMATIC DIAGRAM OF A POROUS PYRAMIDAL REFLECTION ELIMINATOR (a) AND AN EXPANDED VIEW OF AN ELEMENT OF THE POROUS MATERIAL (b).
Elevation and plan views of the UTIAS sonic-boom laboratory and the travelling-wave horn are shown in Fig. 7. The main elements of the sonic-boom simulation facility consist of an air compressor, high-pressure air reservoir, sonic-boom generator in the form of either a shock-tube driver (short-duration booms) or a fast-acting mass-flow valve (long-duration booms), large pyramidal horn with an interior test section, jet-noise absorber, reflection eliminator, available opening or cutout in one side wall of the horn for structural or other research, and a psychoacoustic test room.

In the major mass-flow-valve mode of operation, pressurized air is supplied to the reservoirs by means of a compressor, and the mass-flow valve releases this air into the horn at its apex in a controlled manner. The sudden discharge or flow of air into the horn generates a simulated sonic boom that propagates towards the base of the pyramid. The jet-noise absorber, acting essentially like a low-pass acoustic filter, removes from the passing simulated sonic boom much of the undesirable high-frequency jet noise. This jet noise, which is superimposed on the simulated boom, is produced by the high-speed turbulent flow at the valve. The reflection eliminator that covers the entire base of the pyramidal horn minimizes the undesirable reflection that would otherwise occur if the simulated boom encountered the open or closed end of the horn.

In the shock-tube mode of operation shock-tube drivers are installed at the apex of the horn, instead of the valve. A diaphragm is used to separate the high-pressure gas in the driver section from the atmospheric air in the remainder of the pyramidal horn. When the diaphragm is broken the driver gas expands rapidly into the horn and generates a short-duration simulated sonic boom. As high-frequency jet noise is not generated by the outflow of the driver gas the jet-noise absorber is not required for this operating mode.

In using the travelling-wave horn for studies of sonic-boom propagation over different reduced-scale land topologies, N-wave diffraction over and into model buildings, and physiological and psychoacoustic response studies, the models or subjects can be put directly in the interior test section (Fig. 7). To facilitate structural response and damage studies a 1.8-m by 3.6-m cutout or opening in one side wall of the horn has been made available for test-panel installation. For additional psychoacoustic testing, and also for room resonance studies, an adjoining full-scale test room that is linked to the horn interior by the same cutout can be suitably constructed. The simulator horn can easily be adapted for these and other relevant research projects.

In the following parts of this appendix, detailed descriptions of the more important facility elements are given.

Pyramidal Horn

The pyramidal horn, shown in Fig. A1, is enclosed at the small end by a building called the control room and at the large end by the test room. Pictures of those parts of the horn contained in the control and test rooms appear in Figs. A2 and A3, respectively. The interior of the horn, looking from the base towards the apex, is shown in Fig. A4, and an opposite view of the interior appears in Fig. A5. The illusory effects that the horn appears infinitely long in Fig. A4 and has no divergence in Fig. A5 are also experienced when one stands inside the pyramidal horn.
The pyramidal horn is 25 m long, has a square base that is 3 m on each side, and has a total divergence angle of 7.2 degrees. (A small divergence angle was specified for the pyramidal horn in order to minimize the effects of flow separation and associated turbulence arising near the apex of the horn.) The first portion of the horn, which is 4 m long, is made from 2.5-cm-thick steel plate, and it is supported by a special stand (Fig. A2). The remainder of the horn is made of steel-reinforced concrete, and this one-piece structure also has extremely rigid walls that are 20 cm thick.

The cross-sectional area \( A \) of the horn increases continuously with radial distance \( r \) measured from the projected apex of the horn, according to the following expression

\[
A = A_e \left( \frac{r}{r_e} \right)^2
\]

The area, \( A_e \), at the large end of the horn is equal to 9.30 m\(^2\) and the corresponding radius, \( r_e \), equals 24.38 m. This relation for the plane cross-sectional area is a good approximation for the curved surface area associated with the spherical wavefront of the simulated sonic boom, and it was used in the analyses. Note that the percentage difference between the plane and curved surface areas amounts to only 0.3% for the UTIAS pyramidal horn that has a divergence angle of 7.2 degrees.

The pyramidal horn forms essentially a sector or solid angle of a sphere. Its walls confine and efficiently direct the propagating N-wave. Only a small portion of a full sphere is used for the simulator in order to minimize both the size of the horn and the source energy required to generate the simulated sonic boom. The energy required for the UTIAS horn is about three orders of magnitude less \((1/256\pi)\) than that needed for a full sphere, as calculated by taking the ratio of the cross-sectional area of the horn, \( A_e \left( \frac{r}{r_e} \right)^2 \), to the full area of the corresponding sphere, \( 4\pi r_e^2 \). The source energy cannot be minimized much more as a sufficiently large interior test section is needed (about 2.5 m square), and the horn is restricted to a reasonable length (about 25 m) by cost and space considerations.

The steel and concrete walls of the horn were purposely designed to be extremely rigid, to minimize wave energy losses and waveform distortion as the N-wave propagates to the large end of the horn. This foresight proved to be correct. A sonic-boom simulator was recently constructed in England (Ref. A1) and it was designed to have wooden walls. It was found that the simulated sonic boom underwent waveform distortion as it propagated. The basic change was that the rise times on the two shocks of the N-wave continually increased from about 1 ms when the N-wave was first generated near the horn apex to 40 ms by the time the N-wave reached the test section. This type of waveform distortion is undesirable as rapid rise times are needed for correct simulation of sonic-boom startle effects.

Two pictures of the initial construction phases of the concrete portion of the horn are shown in Fig. A6. When the two side walls and the ceiling of the horn were poured with fresh concrete, on the not fully-cured concrete floor, the entire horn sagged slightly over the three main supports that can be seen in the elevation view of Fig. 7, as additional support to maintain the heavy load of fresh concrete was inadequate. A measurement survey of all four inner surfaces of the horn was made and it showed that the walls were distorted from the desired flat
surfaces. The side walls deviated by as much as 1 cm to either side of the desired plane, and the ceiling and floor were worse, deviating by as much as 3 cm to either side of the desired plane. The inner surfaces of the horn were corrected by chipping and grinding away protuberances and by filling-in sunken parts with fine-grained concrete. Finally, the inner surfaces were made quite smooth through a grinding process.

A final survey was made to check on the modifications to the walls, and these results are shown in Figs. A7, A8, A9 and A10. For five different traverses along each wall, as shown in the insert in each figure, the deviation (D) of the actual inner surface from the desired flat surface is plotted versus distance (x) along the traverse. The deviation is now small, being within 5 mm of the desired plane. If the deviation is expressed as a percentage of the duct width, it is extremely small and always less than 0.2%. The small waviness in the walls causing small perturbations in the ray-tube area should have inconsequential effects on the propagating N-wave.

Compressor and Reservoir System

A reciprocating compressor (two stages, 16 brake horsepower, capacity of 0.025 m³/s of air at 850 rpm) is the source of high-pressure air (1 to 18 atm) for the facility. Instead of locating the compressor in the control room next to the reservoir, where its operation noise would be objectionable, it was put outdoors and housed in a special acoustically-lined shed. This shed can be seen in Fig. A1, directly in front of the pyramidal horn.

The compressor has a self-contained reservoir (0.5 m³). However, it was not sufficiently large to supply the required mass-flow rate of air to the valve and horn without suffering a significant drop in reservoir pressure. Also, high-pressure air is required to operate the valve whereas lower-pressure air is normally required to generate the simulated sonic boom. To meet both of these requirements an additional reservoir was installed to supply the air for generating the N-wave and the compressor reservoir was used for operating the mass-flow valve. Suitable valves, pressure regulators, and pressure gauges located in one control console (Fig. A2) regulate the flow and pressure of the air for the mass-flow valve and the additional reservoir.

The additional reservoir consists of two large cylindrical tanks (1.3 m³ each). They can be seen in the background of the picture appearing in Fig. A2. These tanks can be pressurized from 1 to 18 atm. Two ducts (each 1 m long by 10 cm in diameter) connect the two tanks to the plenum chamber that houses the mass-flow valve (Fig. A2).

Mass-Flow Valve

The mass-flow valve installed in its plenum chamber (0.008 m³) at the apex of the horn can be seen in Fig. A2. A photograph of the valve without the plenum chamber appears in Fig. A11. The left part of the valve contains the pneumatic driving mechanism and, at the right, one can see the pyramidal valve plug that controls the air flow into the horn. The height of the tapered portion of the plug is 5.1 cm, and the base of the pyramidal plug is 5.72 cm on each side, producing a maximum flow area of 32.6 cm² when the plug is fully retracted. Actually, the flow area is only 31.6 cm² as the corners on the base of the plug are rounded.

The mass-flow valve and its functions are illustrated in Fig. 8. The
primary function of the valve (Fig. 8a) is to release in a controlled manner high-pressure air from the reservoir into the horn where the flow generates the simulated sonic boom. To achieve this simulation the plug motion (x) is required to be a symmetric one-cycle reciprocating movement, which is linear in each half of the cycle, as shown in Fig. 8b. The throat area (A) exposed to the flow by this particular plug motion is a parabolic function with time (Fig. 8c), and so is the mass flow rate (m) through the valve (Fig. 8c). This particular mass-flow-rate distribution, from zero to a maximum and back to zero again, generates the desired over-pressure N-wave (Fig. 8d) that simulates the sonic boom.

The pneumatic and mechanical operation of the valve, to achieve the linear plug motion, is illustrated by means of a simplified assembly drawing shown in Fig. A12. When the valve is in its normally-closed position, the valve plug entirely blocks off the flow entrance to the horn and the direction piston (Fig.A12b) is in its most rearward position. High-pressure air (14 atm) thus acts on the driving piston in a manner that maintains the plug in the closed position. To activate the valve, a small solenoid valve (Fig.A12b) is opened suddenly such that the high-pressure air (14 atm) quickly flows through this valve and impels the direction piston into its forward position (Fig.A12a). This sudden change in the location of the direction piston rapidly switches the high-pressure air to the other side of the driving piston (Fig.A12a) and actuates the valve plug. The plug is then withdrawn from the opening of the horn, continuously uncovering more area and allowing an increasing mass-flow rate of air enter the horn. When the valve plug has been retracted a distance equal to the height of its tapered part (5.1 cm), the direction of its motion is reversed by the closing of the solenoid valve. With the high-pressure air no longer acting through the solenoid valve the driving piston is snapped back to its rearward position by means of a compressed coil spring. This action reverts the high-pressure air to the other side of the driving piston, and the valve begins to close. The forward-moving plug now decreases the flow area and mass-flow rate of air, eventually back to zero, completing the one-cycle operation. The valve is then ready to be operated again.

The linear movement of the valve plug is accomplished by means of a small rate piston and oil-restriction orifice in the oil-flow circuit (Fig.A12a). By a suitable setting of the orifice area, and since the air pressure on the driving piston is approximately constant (14 atm), the oil flow is restricted to a nearly constant rate, which fixes the plug speed and makes it constant in each half of the cycle. The plug speed was constant in practice, but had a different speed in each half of the cycle, as the flow coefficient for a forward flow of oil through the orifice differed from that for a backward flow. This problem was corrected by inserting two orifices in separate oil circuits, one facing in the opposite flow direction.

The pneumatically-operated mass-flow valve functions correctly as designed. Some measurements of the plug displacement history, made with a suitable potentiometer and associated electronic equipment, are given in Fig. 11. The five triangular profiles correspond to different nominal open-to-close times for the valve of 60, 100, 200, 300 and 400 ms, covering the range of operation expected of the valve. The plug speed is given beside each oscilloscope trace.

The starting, reversing, and stopping motions of the plug, respectively numbered 1, 2 and 3 in Fig. 11c, are rapid events. Between these sudden changes the plug motion is very nearly linear. The near symmetry of the opening (from 1 to 2) and closing (from 2 to 3) portions of each displacement history illustrates
that the plug speed in both the opening and closing strokes are nearly equal.

The difference between these two plug speeds is largest for long-duration plug
motions (Fig. 11e). However, it is normally less than 5% if precautions are

taken when fixing the valve settings.

A special electronic generator was built to supply on demand (the push
of a button) a flat-topped voltage pulse of suitable amplitude and of variable
duration to activate the solenoid valve for a prescribed period of time, to control
the duration of the valve motion and thus the duration of the simulated sonic boom.

This generator was designed such that it could be triggered remotely from another
room, for example, the interior test section of the horn. Another design feature
is that the generator can be put on automatic operation to self-trigger at some
selected rate, enabling a continuous sequence of simulated sonic booms to be
produced. Note that for a particular orifice-area setting to effect a certain
plug speed, the duration of the voltage pulse from the generator has to be ad-
justed to ensure that the solenoid valve closes at the correct moment, such that
the plug reverses its motion just after it is fully retracted.

The valve and reservoir system was designed to give the facility the
capability of generating N-waves with peak overpressures and durations that are
lower than, equivalent to, and higher than those for actual sonic booms from
current SSTs and fighter aircraft. The reservoir pressure, which remains essen-
tially constant during the valve operation, determines the N-wave amplitude, and
its range from 1 to 18 atm is more than adequate to satisfy these amplitude
requirements. The open-to-close time for the valve fixes the N-wave duration,
which can be varied from 60 to 500 ms or longer. The existing facility has no
control over the rise times of the two shocks of the N-wave, which are affected by
the valve geometry and nonlinear flow process. The simulated sonic booms have
a rise time of about 3 ms, which is at one end of the range of 0.1 to 10 ms for
actual sonic booms.

A few additional design features of the valve are worth mentioning.

When the valve is in the closed position the plug fits tightly in a Teflon seat
that is 0.8 cm thick (see Fig. A12). This Teflon seat acts as a seal and ade-
quately prevents air leakage from the plenum chamber into the horn. As a part
of the constant-area section of the plug immediately following its pyramidal
front passes into the Teflon seal, a short plug movement of about 0.8 cm is
required before the tapered part allows the high-pressure air to flow into the
horn. Within this distance the plug is accelerated to a constant speed in the
withdrawal stroke, or decelerated to rest on the return stroke. Hence, a good
transition from no-flow to flow takes place, and vice versa, which is a necessary
condition to achieve rapid rise times at the front and rear shocks of the N-waves.

Another design feature of the valve is that no significant unbalanced
pressure forces act on the valve plug. When the valve is closed the plug passes
through the plenum chamber, and thus no unbalanced force arises from the plenum-
chamber pressure. When the valve plug is moving forwards or backwards the air
pressure in the plenum chamber is transmitted to the hollow interior of the plug
by means of suitable holes in the pyramidal plug. (These holes in the face of
the plug can be seen in Fig. A11.) Reduction of unbalanced pressure forces on
the plug helps to reduce the power requirements needed to operate the valve, which
should only have to overcome inertial and frictional forces. Also, it helps to
ensure that the plug speed in each half of the cycle remains the same.

A valve plug with a square cross-section was designed for the facility,
to match the same cross-section of the pyramidal horn. An alternate and much better route would have been to design a smooth transition section for the horn, from a square to a circular cross-section at the valve location, enabling one to make use of a conical valve plug. The efforts involved in machining a pyramidal plug with a square base having rounded corners, making a square teflon seal with matching rounded corners, and aligning the pyramidal plug with its square hole are manyfold greater than making a suitable transition duct and conical valve plug. A smooth transition section would not likely reduce the intensity of the jet noise, or enhance it.

Jet-Noise Absorber

An inherent characteristic of any travelling-wave horn operated in the mass-flow-valve mode is the production of undesirable high-frequency noise that is superimposed on the simulated sonic boom. This high-frequency noise is produced by the high-speed turbulent flow of air at the valve. As the discharge of air into the horn is actually a confined air jet, the flow noise is similar to that from a steady-state free-air jet, and it is thus called jet noise. The jet noise becomes more intense for higher reservoir pressures, as the flow speed at the valve becomes faster. As a higher reservoir pressure is required to generate high-amplitude and long-duration N-waves, these waves have very intense jet noise superimposed on their basic N-shaped signature.

An investigation was made into the frequency content of the jet noise, using electronic filtering techniques, and it showed that the noise was broad band and covered the frequency range from as low as 100 Hz up to 10 kHz and higher. As the contents of the sonic boom lie mainly below 200 Hz, it was realized that much of the higher frequency components of the jet noise could be removed from the N-wave by using a low-pass acoustic filter.

A low-pass acoustic filter called a jet-noise absorber was designed for the interior of the horn, and it is illustrated schematically in Fig. 9. The absorber consists essentially of horizontally and vertically arranged panels of sound-absorbing material (fiberglass, 2.5 cm thick, 110 kg/m³). Provisions were also made for installing one or more transverse panels at the rear of the absorber. The absorber can be installed inside the horn near its apex (Fig. 9), from the 5.0-m station to the 7.5-m station. It cannot be located much closer to the valve, as severe pressure fluctuations from the turbulent high-speed flow near the valve, when generating high-amplitude or long-duration N-waves, cause significant damage by shredding and breaking the fiberglass panels.

Some results of using the jet-noise absorber during the developmental stage are shown in Fig. A13. All of the simulated sonic booms have peak overpressures of about 200 N/m² and a duration of 80 ms. The first signature in Fig. A13a was recorded without the use of the jet-noise absorber, and it has noticeable superimposed jet noise. When the absorber consisting only of longitudinal panels was installed in the horn, it proved completely effective in reducing the intensity of the jet noise to acceptable levels, as can be seen in Fig. A13b. Note that the rise time was not noticeably altered and remained at about 3 ms, as indicated beside the oscillograms.

Although further jet-noise abatement was not necessary for this short-duration N-wave, it is interesting to show how transverse panels installed at the base of the absorber affect the transmitted wave. Just as the addition of more longitudinal panels would reduce the low-frequency cutoff for the absorber, so
does the addition of transverse panels. A thin transverse panel only slightly
lengthens the rise time from 3 to 4 ms (Fig. A13c), whereas a thick one increases
it markedly from 3 to 12 ms (Fig. A13d). The thick panel also causes a signifi­
cant reduction (15%) in the peak overpressure of the N-wave.

Similar results for a longer duration N-wave of 300 ms are given in
Fig. A14. In this long-duration case significant jet noise is superimposed on
the N-wave (Fig. A14a). The intensity of the jet noise is markedly reduced
when the absorber is used (Fig. A14b), but it is not reduced to acceptable
levels. The addition of a thin transverse panel to the absorber results in a
further intensity reduction (Fig. A14c), and so does a thick panel (Fig. A14d).
Even with the thick transverse panel some undesirable low-frequency jet noise
remains superimposed on the N-wave. Thicker transverse panels could be used to
eliminate this residual jet noise. However, the rise time becomes unreasonably
long.

From test results such as those shown in Figs. A13 and A14, a standard
jet-noise absorber was chosen to consist of longitudinal panels, arranged only
as shown in Fig. 9. For most human and structural response tests, N-waves having
durations less than 200 ms and amplitudes less than 200 N/m² can be used, for
which the standard jet-noise absorber performs satisfactorily. Of course, certain
tests might require higher amplitude or longer duration N-waves. For such tests
transverse panels could always be added to the absorber to effect a further re­
duction of the jet noise, that is, when a slower rise time could be tolerated
more than the jet noise.

The effectiveness of the standard jet-noise absorber is illustrated in
Figs. 12 and 13. The simulated sonic booms in the first column were obtained
without the use of the absorber, and the duplicate set in the second column with
the absorber installed in the horn. The absorber is very effective in removing
the high-frequency components of the jet noise from the N-wave, thus increasing
the performance of the travelling-wave horn.

Reflection Eliminator

When the N-wave, generated by the pulse of high-speed flow entering the
horn at its apex, propagates to the base of the horn it would normally be reflected
from an open or closed end. This reflected wave would disrupt the simulated flow
and pressure conditions in the interior test section, as the wavelength of the
simulated sonic boom can be up to four times longer than the horn. In order to
eliminate, or at least adequately minimize, this undesirable reflection and its
subsequent echoes, a reflection eliminator was designed and built to cover the
base of the horn.

The reflection eliminator is basically a huge porous piston, as illustr­
ated schematically in Fig. 10. The eliminator can also be seen in the pictures
of the facility appearing in Figs. A3 and A5. The porous piston consists of a
2.5-cm-thick blanket of microlite material (12 kg/m³), held rigidly between two
suitable grid supports. The front and rear grids and how they hold the microlite
are illustrated in Fig. A15. This blanket of microlite covering the base of the
horn matches the duct-exit impedance to that of the incident wave, thereby mini­
mizing its reflection.

The porous piston is passive to the incident wave, being accelerated by
the drag forces from the air flow through the porous microlite. The eliminator
moves freely on a special roller-and-track support (Fig. A16) which has a low resistance to motion. The device cannot twist or pitch, owing to certain features in the design. Two roller bearings located on top of each track or rail and two similar bearings located directly on the other side prevent any pitching motion (see Fig. A16). As the roller bearings move in a recessed part of the rail, as shown in Fig. A16, the piston cannot twist horizontally and jam. The parallel guiding mechanism shown in Fig. 10 was not required to prevent twisting of the piston, as the rollers were recessed into the rail as an alternate solution.

The piston is enclosed at its periphery by a special skirt, as shown in Figs. A3 and A17. This skirt is attached to the base of the horn and prevents significant air leakage around the edges of the piston. Only a 1-cm gap is allowed between the skirt and the edges of the movable piston. Note that a door for access to the horn interior has been designed into the skirt, as shown in Figs. A3 and A17.

Experiments were performed to evaluate and improve the performance of the reflection eliminator. Test results showed that the thickness of the micro-lite material for the porous piston should be about 2 cm, the overall mass of the moving piston should be about 200 kg, and the porous piston should be allowed to move freely on the roller-and-track support system. Additionally, it was found that the test room (Fig. 7) enclosing the large end of the pyramidal horn and the porous piston could have a pronounced effect on the performance of the eliminator and on the resulting N-waves. The wave leaving the large end of the horn would reflect from the building wall that is 2 m behind the porous piston and propagate back into the horn. Furthermore, the room would respond acoustically to the wave leaving the horn and create other waves that would also enter the horn. These waves, on entering the horn through the reflection eliminator, can have a sufficiently-large amplitude and disrupt the simulation of the overpressure inside the horn. This problem can be adequately corrected by opening the two large doors (2.4 m² each) in the building wall that are directly behind the porous piston. Then the wave leaving the horn can propagate outside the test room, and it is only slightly impeded by the test room.

Some calibration measurements for the reflection eliminator, recorded at the 15.2-m station inside the horn, are given in Fig. A18. The overpressure signature shown in Fig. A18a illustrates that a good N-wave can be generated (the jet-noise has been electronically filtered out of the signal), and also that the reflection eliminator functions correctly. The small-amplitude pressure disturbances superimposed on the N-wave and following it are due to room resonances. The results for other overpressure signatures having different amplitudes and durations are similar.

The effect of fixing the reflection eliminator in one position so that it could not move is illustrated in Fig. A18b. (This constraint on the eliminator's motion is equivalent to making it extremely massive.) The main effect is that the incident N-wave is partially reflected as a compression wave from the eliminator, causing the overpressure at the middle of the N-wave to be higher than normal, as compared with the N-wave in Fig. A18a. This effect is less pronounced for shorter duration N-waves, and it is not noticeable for a 100-ms-duration N-wave. Also, the effect does not appear to be dependent on the amplitude of the N-wave.

The effects on the overpressure signature measured in the horn from closing the large doors of the test room are illustrated in Fig. A18c. As compared with the signature shown in Fig. A18a, a portion of the profile has been
shifted to higher overpressures. This shift is due to a transient overpressure increase (above atmospheric) in the horn and test room, as the mass of air discharged from the reservoir temporarily accumulates in the horn and test room. As this additional air leaks out of the test room the signal returns to its initial atmospheric state or reference level. This effect is more pronounced for higher amplitude and longer duration N-waves. For a low-amplitude (100 N/m²) and short-duration (100 ms) N-wave this effect is not noticeable.

Some interesting measurements that were recorded outside the horn are given in Fig. A19. The top signature, recorded 1 m behind the porous piston, illustrates the profile of the wave leaving the horn. As the amplitude of the incident wave just in front of the eliminator was 75 N/m², the wave leaving the horn is about 40% less in amplitude. This amplitude reduction is typical for incident N-waves having different amplitudes and durations.

Two overpressure measurements were taken in the centre of the test room that houses the large end of the horn. For one measurement (Fig. A19b) the large doors were open and for the other (Fig. A19c) the doors were closed. When the doors are open the test room acts much like a Helmholtz resonator, as illustrated in Fig. A19b, and the amplitude of the initial couple of overpressure oscillations are about one-half that for the wave leaving the horn. When the doors are closed the test room responds quite differently. The pressure in the room increases to a maximum and then falls back to atmospheric pressure after about 1s, as illustrated in Fig. A19c. This overpressure increase in the test room also occurs in the horn, and it can cause a significant effect on the over-pressure N-wave, as already shown by the measured signature given in Fig. A18c. The maximum increase in test-room overpressure is highest for high-amplitude and long-duration N-waves, when a significant mass of air has been released from the reservoir into the horn and test room.

Cutout

To facilitate studies of response and damage of structural panels, and also windows, to the sonic-boom pressure loading a 1.8-m by 3.6-m cutout or opening has been provided for in one side wall of the horn. Plaster panels typical of those forming the inside walls of certain houses are shown in Fig. A20, installed in the cutout by means of special supports. A matrix of 32 panels, 40-cm square, can be installed and simultaneously subjected to the simulated sonic boom. Fewer (larger) or more (smaller) panels can be installed in the cutout, by suitably spacing the vertical I-beams. The minimum distance between I-beams that has been provided for is 10 cm. Provision for cross-braces is also every 10 cm. Note that the I-beams have been selected to be large such that they provide very rigid support for the panels. For tests other than structural, the cutout is covered with thick plywood to continue the wall, as shown in Figs. A3, A4 and A5.

Ancillary Equipment

Many lesser devices are always an integral part of a large facility. For example, the control and test rooms, being physically separated, were linked with a special cable system. Ten separate two-conductor cables (BNC) and one four-conductor cable join the two rooms, having a convenient junction box at either end. These boxes can be seen in Figs. A2 and A3. Therefore, all of the signal and signature recording can be accomplished in either room. As a result, most tests can be performed by one person. A specially-designed intercom system having very good sound reproduction has been installed to bridge the communication gap.
between the two rooms, saving footwork and time.

Transient-pressure fluctuations such as those from the generated N-wave were measured with certain pressure gauges. Initially, a special sonic-boom microphone (Bruel and Kjaer, type 4145, 2.54 cm in diameter), having excellent high and low-frequency response, was purchased with its special carrier system (Type 2631, dc to 150 kHz). As the microphones pressure range from 0 to 0.5 kN/m² was insufficient for a number of tests, it was supplemented with another sonic-boom microphone (Bruel and Kjaer, Type 4147, 1.27 cm in diameter) having an effective range from 0 to 1 kN/m² without giving significant waveform distortion. As still higher transient overpressures had to be measured at the base of the horn, near the apex of the horn, in the plenum chamber, and in certain shock tubes, a newly-developed piezotron transducer (Kistler, Model 206, range from 0.5 kN/m² to 6 atm) was purchased. Both a coupler (Kistler, Model 549B) and a charge amplifier (Kistler, Model 504D) were obtained for use with the piezotron transducer. These three complementary devices provide a versatile means of measuring transient-pressure fluctuations.

Whereas microphones can be held by a simple stand and put directly in the acoustic field to obtain a pressure measurement, the piezotron has to be flush mounted in the wall of the duct. Provisions were made about every 1.5 m from the apex to the base of the horn for mounting the piezotron in both the steel and concrete sections. Additional mounts were also made near the horn apex to facilitate certain tests. The special piezotron mount for the concrete section of the horn is illustrated in Fig. A21.

The electric signal from the pressure measurement device can be monitored with suitable oscilloscopes (Tektronix 5103N, dual beam, storage; Tektronix 535A) and, if desired, permanently recorded on photographic film. At first the oscilloscopes were triggered by the incoming signal that was to be recorded. Consequently, the first small part of the signal was lost and some ambiguity existed as to the position of the baseline.

To circumvent this problem a very durable and cheap ($1.09) crystal microphone was installed upstream of the pressure sensor, to provide a separate trigger signal. A special electronic device was designed to amplify the trigger signal to the correct level to trigger the oscilloscope. Additionally, the electronic device had a special delay circuit so that the duration of the baseline preceding the pressure signature could be selected.

Calibration of microphones before and after tests were accomplished with a pistonphone (Bruel and Kjaer, Type 4220). For the piezotron no suitable calibrator was available. As the manufacturer's calibration data were suspected to be in error, owing to certain measurements being higher than those predicted by theory, a suitable calibrator was designed and built for the sonic-boom laboratory. Basically, the calibrator could supply a known flat-topped pressure pulse of selectable amplitude (0 to 6 atm) and duration (20 ms to 1 s) to the piezotron transducer, enabling its calibration to be accomplished. The design and operation of this calibrator are given in Appendix B.

Shock-Tube Drivers

In the shock-tube mode of operation of the pyramidal horn, a thin cellophane ("red zip", 4 μm thick) or carbon paper (Canadian Carbon and Ribbon Manufacturers, Type 400C3) diaphragm is inserted and clamped between the driver
and channel sections of the horn. The installation of a diaphragm in the horn is depicted in the photographs appearing in Fig. A22. This diaphragm separates the atmospheric-pressure air in the channel or horn from the higher pressure gas in the driver. When the diaphragm that is stretched to a state where it is about to self-burst is broken by means of a mechanical plunger or needle, the driver gas expands rapidly into the horn. This expansion process generates an outgoing N-wave or simulated sonic boom. The peak overpressure of the N-wave depends on the initial driver pressure. Higher initial pressures produce more powerful booms.

An N-wave having an amplitude that is less than, equal to, or more than that for an actual sonic boom can be generated in the pyramidal horn. The wavelength of the N-wave depends on the length of the driver, and it is slightly longer than twice the driver length. As the driver length is normally less than 4 m for the UTTAS facility, only short-duration N-waves (up to 20 ms) are generated in the horn. Note that the rise time on the N-wave is very fast, about 20 μs. These N-waves can be used for investigating serious startle effects on humans and animals, however, they are better suited for studies of N-wave propagation over different reduced-scale land topologies and N-wave diffraction over and into model buildings.

Two geometrically-different shock-tube drivers have been used with the pyramidal horn. The first driver is in the form of a pyramid and has the same divergence angle as that for the horn. A sonic-boom simulator having this driver and channel configuration is called a pyramidal shock tube. A photograph and schematic diagram of the UTTAS pyramidal shock tube are shown in Fig. A23. Four different diaphragm stations ranging from 20 cm to 4 m are available, giving the facility the capability of producing four different duration N-waves.

The second shock-tube driver has a constant-area cross-section along its entire length, and it is joined to the pyramidal horn at a common area station. A photograph and schematic diagram of this rectangular-pyramidal shock tube are shown in Fig. A24. Three shock-tube drivers having the same cross-sectional area of 6.45 cm² but different lengths of 15.2, 30.5 and 45.7 cm have been used with the facility. Additionally, a driver with a cross-sectional area of 32.6 cm² and a length that can be varied from 15 cm up to 2 m is available for test purposes. Note that measured and predicted overpressure signatures for the pyramidal and rectangular-pyramidal shock tubes are given in the main body of the report.

References

A1. Private Correspondence. The pyramidal shock tube is located in a balloon hanger at Cardington, England. The facility comes under the jurisdiction of the Structures Department headed by Mr. C. H. E. Warren, Royal Aircraft Establishment, Farnborough, Hants.
FIG. A1 AN OUTSIDE VIEW OF THE TRAVELLING-WAVE HORN.
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MEASUREMENT MADE 1 m BEHIND THE POROUS PISTON

MEASUREMENT MADE IN THE CENTER OF THE TEST ROOM

LARGE DOORS OF THE TEST ROOM IN THE CLOSED POSITION

MEASUREMENT MADE IN THE CENTER OF THE TEST ROOM

NOTE: LOW-PASS FILTER SET AT 75 Hz TO REMOVE THE HIGH-FREQUENCY JET NOISE.

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APPENDIX B: A CALIBRATION FOR PRESSURE TRANSDUCERS

A novel device was built for calibrating pressure transducers in a simple and accurate manner, as well as for evaluating their low-frequency response. This calibrator, which is shown schematically in Fig. B1, consists essentially of a standard three-way solenoid valve (Skinner, 10 W, 7-atm maximum pressure, 1.6-mm orifice) that has been suitably modified such that a transducer housed in an adapter covers one port and an air reservoir is connected to a second port. The third port is normally open to the atmosphere.

When the valve is in the normally-closed state the spring-loaded plunger blocks the high-pressure reservoir air from entering the interior of the valve, and the transducer is exposed to the pressure of the atmosphere via the port that is open to the atmosphere. When the solenoid is activated electrically the plunger is forced to rapidly move to its other extreme position, sealing off the port that was originally open to the atmosphere and allowing the transducer to be exposed to the pressure of the reservoir gas. After a suitable period of time the solenoid is deactivated and the valve reverts to its normally-closed state. As a result of this valve operation the transducer experiences a flat-topped overpressure pulse, and it generates a similar shaped voltage pulse that can be recorded by means of an oscilloscope. A comparison of the amplitudes of the voltage and overpressure pulse enables one to determine the transducer's calibration value. Note that the amplitude of the pressure pulse is essentially equal to the reservoir pressure, as the reservoir volume was designed to be much greater than any additional volume (interior of the valve and the space in front of the transducer) that is pressurized by the reservoir air during the operation of the valve.

The inlet and abort valves, in conjunction with a suitable static pressure measuring device, allows the reservoir pressure to be adjusted accurately to the desired value. Overpressures can be selected from very low values encountered in acoustic waves (10 N/m²) up to several atmospheres. This upper limit is a function of the solenoid valve and the reservoir rupture strength, and is about 7 atm for the present calibrator.

The duration of the pressure pulse can be controlled, and it is variable from about 20 ms to 1 s and longer. The electrical activation signal controlling the solenoid valve is produced by a special function generator having a means of controlling the output pulse. This function generator is the same one that is used for controlling the fast-acting mass-flow valve of the travelling-wave horn.

The calibrator works correctly. Some typical voltage signatures for two different transducers are shown in Fig. B2. The signatures appearing in Figs. B2a, b and c, from one transducer, are all flat-topped and yield a calibration value for that transducer that is nearly constant (within 1%) over the entire over-pressure range from 0.5 to 50 kN/m². Note that there is a slight drift towards the baseline in each of these three signatures, as the transducer had a time constant (τe) of 3.5 s or a low-frequency cutoff of 0.05 Hz (1/2πτe). The time constant can be obtained from the voltage signatures, if the fall in voltage is assumed to be exponential.

The voltage signature from the second transducer, shown in Fig. B2d, portrays a significant exponential-like drift towards the baseline. From the signature the time constant for the transducer is about 80 ms, or the low-frequency cutoff is about 2 Hz. This transducer would not be suitable for measuring long-duration N-waves or sonic booms, as the low-frequency cutoff has to be 0.1 Hz or lower.
The calibrator was designed mainly for use with low-pressure piezotron transducers (Kistler, Model 206, overpressure range from 1 to 6 atm), which are frequently used for recording the signatures of simulated sonic booms generated in the travelling-wave horn and also in certain low-pressure-ratio shock tubes. However, the calibrator can also be used with other types of pressure transducers.

It is worth noting that much time, effort and expense have been allocated in the past to developing suitable semi-dynamic calibration devices. The present calibrator is very simple to construct and operate, and it satisfies most calibration requirements.
FIG. B1  A CALIBRATOR FOR PRESSURE TRANSDUCERS.
FIG. B2  CALIBRATION SIGNATURES.

a) RESERVOIR OVERPRESSURE: 0.5 kN/m$^2$
CALIBRATION VALUE: 17.6 mV/kN/m$^2$

b) RESERVOIR OVERPRESSURE: 5.0 kN/m$^2$
CALIBRATION VALUE: 17.4 mV/kN/m$^2$

c) RESERVOIR OVERPRESSURE: 50 kN/m$^2$
CALIBRATION VALUE: 17.6 mV/kN/m$^2$

d) RESERVOIR OVERPRESSURE: 50 kN/m$^2$

- 200 ms -
APPENDIX C: PERTURBATION MASS-FLOW RATE

For the mass-flow-valve mode of operation of the UTIAS travelling-wave horn, a high reservoir pressure is often used to produce a powerful or long-duration simulated sonic boom in the pyramidal horn. When the reservoir pressure is sufficiently high the flow at the valve throat is choked or sonic, and a supersonic flow occurs downstream of the valve. It is eventually terminated by an upstream-facing shock that returns the flow to subsonic speed and increases the flow pressure and density to approximately atmospheric conditions. This upstream-facing shock wave, originating at the valve plug when the valve just begins to open, is swept downstream by the increasing flow as the valve continues to open, and then it moves back to the valve plug as the valve closes and the flow decreases. The motion of this upstream-facing shock \( S \) is illustrated by means of a time-distance diagram shown in Fig. C1. The paths of the front \( S_1 \) and rear \( S_2 \) shocks of the simulated sonic boom, formed when the valve just opens or closes, are also shown. The contact-surface motion \( C \) is also indicated for completeness. This surface is the interface which separates the reservoir air that enters the horn from the air originally there.

The preceding description of the flow from the high-pressure reservoir into the horn was an idealized one, based on one-dimensional and quasi-steady flow concepts. In reality the nonstationary flow is three-dimensional. For example, the flow is accelerated from sonic speed near the valve throat to increasing supersonic speed by oblique expansion waves, and the supersonic flow is terminated by a complex series of oblique and normal shock and expansion waves. Turbulence and a boundary layer are also present in the flow, and they interact with the oblique waves. Note that a freely expanding supersonic jet has similar flow features (Ref. C1). For the purpose of the following analysis, however, the flow is assumed to be one-dimensional and quasi-steady.

Consider a control volume that contains the supersonic-flow region and its associated upstream-facing shock wave. Let this control volume start at the entrance of the pyramidal horn and end at a location downstream of the shock, as illustrated in Fig. C2. The mass of air contained in this fixed volume is not constant with time because the shape or volume of the supersonic-flow region changes as the mass-flow valve opens and closes. The rate of accumulation of mass in this control volume is equal to the difference between the mass-flow rates at the entrance \( m_1 \) and exit \( m_2 \). It is also equal to the time derivative of the volume integral of the density distribution. Therefore, the perturbation mass-flow rate at the exit of the control volume, equal to \( m_2 (\tau) - m_1 (\tau) \) and denoted by \( \Delta m(\tau) \), owing solely to the effect of the changing volume of the supersonic-flow region, is given by the following expression.

\[
\Delta m(\tau) = -\frac{d}{d\tau} \int V_1 (\tau) \left[ \int \rho \, dV_1 (\tau) \right] - \frac{d}{d\tau} \int V_2 (\tau) \left[ \int \rho \, dV_2 (\tau) \right]
\]  

\( V_1 (\tau) \) and \( V_2 (\tau) \) denote the respective volumes of those parts of the control volume where the flow is supersonic and subsonic, \( \rho \) denotes the density of the air in volume \( V_1 (\tau) \) as a function of radial distance \( r \), and \( \rho \) is the density of the
air in volume \( V_1(\tau) \). The density of this air is taken to be atmospheric. The volumes \( V_1(\tau) \) and \( V_2(\tau) \) can be expressed more conveniently in terms of the cross-sectional area \( A \) of the pyramidal horn. For the UTIAS horn with \( r^2/A \) equal to 64, the following expressions can be obtained by using geometrical principles.

\[
V_1(\tau) = \frac{8}{3} \left[ A_{s}^{3/2}(\tau) - A_{s}^{3/2} \right]
\]

\[
V_2(\tau) = \frac{8}{3} \left[ A_{e}^{3/2} - A_{s}^{3/2}(\tau) \right]
\]

The respective symbols \( A_s \), \( A_e \), and \( A \) denote the cross-sectional area at the entrance of the control volume, at the shock location, and at the exit of the control volume. It will be shown that the shock location \( A_s(\tau) \) can be expressed in terms of the throat-area variation with time \( A_e(\tau) \) as follows: \( \Delta A(\tau) \). \( \Delta \) depends on the reservoir and atmospheric conditions and is independent of time if the reservoir conditions are taken to be fixed during the generation of a simulated sonic boom. Let \( A_s(\tau) \) be expressed in the form \( A_e N_s(\tau) \), where \( A_e \) is the maximum throat area of the valve and \( N_s(\tau) \) denotes a normalized function for describing the throat-area variation. Also, let \( \tau_e \) denote the duration of the flow through the opening and closing valve. Thus, \( \tau_e \) also denotes the wave duration. Then, from the preceding developments, Equation Cl can be expressed in the following form.

\[
\Delta m(\tau) = 4 \left( 1 - \frac{\rho_1}{\rho_*} \right) \frac{\rho_* A_e^{3/2}}{\tau_e} \bar{A}^{3/2} Z \left( \frac{\tau}{\tau_e} \right)
\]

\[
Z \left( \frac{\tau}{\tau_e} \right) = N_s^{1/2} \left( \frac{\tau}{\tau_e} \right) \frac{d}{d(\tau/\tau_e)} N_s(\tau/\tau_e)
\]

The symbol \( \rho \) denotes the flow density of the air just in front of the upstream-facing shock, and \( \rho_* \), being the density of the air entering the control volume, is taken to be equal to the density at the valve throat. Both \( \rho_1 \) and \( \rho_* \) can be considered as constant during the generation of a simulated sonic boom, because they depend only on the reservoir and atmospheric conditions. Note that \( \Delta m(\tau) \) varies directly as \( Z(\tau) \), as \( \rho, \rho_1, \rho_*, \bar{A}, \bar{A}_s, \bar{A}_e, \) and \( \tau_e \) are constant.

The variation of \( Z \) (Eq. C5) with time \( (\tau/\tau_e) \) is shown in Fig. C3, for the special case when \( N_s(\tau/\tau_e) \) is a parabolic function, that is, equal to \( 4(1-\tau/\tau_e) \tau/\tau_e \). The perturbation mass-flow rate \( \Delta m(\tau) \) has an identical profile, but a different amplitude. As the valve opens the supersonic-flow region occupies more of the control volume. Because the air density in the supersonic region is, on the average, less than the constant air density (atmospheric) of the subsonic region, the perturbation mass-flow rate is positive valued during the opening
half of the valve cycle. Similarly, when the valve closes and the supersonic region shrinks in size, the perturbation mass-flow rate is negative valued. Note that the maximum value of $Z(T/T_0)$ is identically 2.

In order to assess the significance of the perturbation mass-flow rate $\Delta m(\tau)$, its maximum value $\Delta m$ is compared with the maximum mass-flow rate at the valve throat ($\bar{m}_*\bar{A}_*$). The mass-flow rate per unit area $\bar{m}_*$ is given by the following expression:

$$\bar{m}_* = \left[ \frac{2}{\gamma+1} \right] \frac{\gamma+1}{2(\gamma-1)} \frac{\rho a^2}{a_0 p}$$

This equation is an equivalent form of Eq. 2.1 of Section 2.2.1, with the flow Mach number $M_*$ taken equal to unity. By using Eqs. C4 and C6, noting that the maximum value of $Z$ is 2, and letting the reservoir sound speed ($a_*$) be equal to that ($a$) for the atmosphere, the following result for $\Delta m/\bar{m}_*\bar{A}_*$ can be obtained.

$$\frac{\Delta m}{\bar{m}_*\bar{A}_*} = 8 \left[ \frac{\gamma+1}{2(\gamma-1)} \right] \frac{\rho_*}{\rho_0} \left( 1 - \frac{\rho}{\rho_*} \right) \frac{a_0}{a_0} \frac{\bar{A}_*^{1/2}}{\bar{A}^{3/2}} \frac{\rho_* p_0}{p}$$

Additionally, the ratio $\rho_*/\rho_0$ equals 0.6339 for a choked flow, $\bar{A}_*$ can be replaced by $r_0^2/64$ for the case of the UTIAS pyramidal horn, and $\rho_0/\rho$ and $p_0/p$ are equal as $a_*$ has been taken equal to $a$. For the specific case of air with $\gamma$ equal to 1.4, the final form of Eq. C7 can be expressed as shown below.

$$\frac{\Delta m}{\bar{m}_*\bar{A}_*} = 8 \left[ \frac{\gamma+1}{2(\gamma-1)} \right] \frac{\rho_*}{\rho_0} \left( 1 - \frac{\rho}{\rho_*} \right) \frac{r_0}{a_0} \bar{A}^{3/2}$$

The importance of $\Delta m$ with reference to $\bar{m}_*\bar{A}_*$ can be evaluated from this equation, once $\rho_0/\rho_*$ and $\bar{A}$ have been determined. They are both dependent on the reservoir and atmospheric conditions.

To solve for $\rho_0/\rho_*$ and $\bar{A}$, the Mach number of the upstream-facing shock and the pressure ratio across this shock are determined. Let the pressure of the air in the reservoir, immediately in front of the shock, just behind it, and for the atmosphere be denoted by the following symbols $p_0, p_1, p_2, p$, respectively, as indicated in Fig. C.2b. Certain pressures at different locations in the flow can be related by the following expression.

$$\frac{p_0}{p} = \frac{p_0}{p_1} \frac{p_1}{p_2} \frac{p_2}{p}$$
For a quasi-steady flow the pressure ratio $p_e/p$ is given by the following well-known expression from isentropic and steady-flow theory (Ref. C2).

$$\frac{p_e}{p} = \left[ 1 + \frac{\gamma-1}{2} \frac{M^2}{1} \right]^{\frac{1}{\gamma-1}}$$

(C10)

The symbol $M$ denotes the Mach number of the flow just in front of the upstream-facing shock. As it was assumed that this shock takes the steady-flow predicted location at each instant of time, $M$ also denotes the Mach number of the shock. The second pressure ratio $p_e/p_1$ of Eq. C9, across the shock, is given by the following well-known expression for a normal shock wave (Ref. C2).

$$\frac{p_e}{p_1} = \frac{2 \gamma M^2}{\gamma-1} \left( \frac{\gamma-1}{\gamma+1} \right)$$

(C11)

In order to determine the last pressure ratio $p_e/p$ of Eq. C9, an appropriate boundary condition is required. As the upstream-facing shock increases the flow pressure and density to near atmospheric conditions, the pressure ratio $p_e/p$ is taken to be equal to unity. In other words the flow behind the upstream-facing shock is assumed to be at atmospheric pressure and density. From the preceding developments the reservoir pressure $(p_e/p)$ can be expressed solely in terms of the shock Mach number $(M)$, as shown below.

$$\frac{p_e}{p} = \left[ 1 + \frac{\gamma-1}{2} \frac{M^2}{1} \right]^{\frac{1}{\gamma-1}}$$

(C12)

The reservoir-pressure ratio $(p_e/p)$ is known and can be considered as constant during the generation of a simulated sonic boom. Hence, $M$ is similarly constant, and it can be determined from Eq. C12.

The dependence of the shock Mach number $(M)$ on the reservoir pressure $(p_e/p)$ is shown in Fig. C4a. $M$ increases most rapidly in the small pressure range of $p_e/p$ from 1.5 to 1.89, where it is also double valued. Such a double-valued function has been noted previously for supersonic nozzle flows (Ref. C3). For the analysis given in Section 2.2.1 of this report, it was found that the flow was subsonic throughout the convergent-divergent duct for the reservoir-pressure range.
of $p_0/p$ from 1.0 to 1.89, and the flow was choked for higher reservoir-pressure ratios. For the part of the analysis dealing with the subsonic flow, however, the flow pressure at the valve throat was taken to be equal to atmospheric pressure. Because the flow pressure behind the upstream-facing shock has been assumed to be atmospheric pressure for the present analysis dealing with the choked flow, the flow has been found to be choked at the valve throat for a pressure ratio $p_0/p$ as low as 1.5. At this pressure ratio the predicted Mach number of the upstream-facing shock is not equal to unity but equals 1.5. The two different boundary conditions, one for the subsonic flow and one for the choked flow, do not give a continuous and matched solution for $M$. For the small reservoir-pressure range from 1.5 to 1.89 atm the two boundary conditions give separate and different predictions.

The pressure ratio $p_0/p$ (Eq. C11) across the upstream-facing shock is shown in Fig. C4b, plotted as a function of reservoir pressure. The results illustrate similar characteristics to those for the previous graph of $M$. It is worth noting from Fig. C4 that the upstream-facing shock is not insignificant, because the shock Mach number and the associated pressure ratio are not small.

The density ratio $\rho_0/\rho_*$ of Eq. C8 can be determined since $M_*$ is known as a function of the reservoir pressure. For an isentropic and steady flow this density ratio is given by the following expression (Ref. C2).

$$\frac{p_0}{\rho_*} = \frac{1}{\gamma-1} \left[ \frac{\gamma + 1}{2 + (\gamma-1) M_*^2} \right]$$

(C13)

This density ratio is also double valued in the small pressure range of $p_0/p$ from 1.5 to 1.89. It decreases markedly in this range as the corresponding shock Mach number arises rapidly (see Fig. C4a). The ratio therefore becomes insignificant compared with unity for Eq. C8, when the pressure ratio $p_0/p$ exceeds a value of about 6.

The motion of the upstream-facing shock can be determined by equating the mass-flow rate at the valve throat with that at the shock location, in the same manner as for an isentropic flow (Ref. C2). Then, if $A_s(\tau)$ denotes the area location of the shock and $A_*(\tau)$ denotes the throat area, the following expression for a quasi-steady flow can be obtained.

$$A_s(\tau) = \bar{A} A_*(\tau)$$

(C14)

$$\bar{A} = \frac{1}{M} \left[ \frac{2}{\gamma+1} + \frac{\gamma-1}{\gamma+1} M_*^2 \right]$$
Hence, $\tilde{A}(\tau)$ is directly proportional to $A_\infty(\tau)$, as originally assumed for the derivation of Eq. C6. The constant $\tilde{A}$ is dependent only on the reservoir and atmospheric conditions, as illustrated in Fig. C5b. The area ratio $\tilde{A}$ is double valued, as were $\tilde{M}_1\tilde{\rho}_1\tilde{p}_1$ and $\tilde{p}_1/\tilde{\rho}_1$. From a value of unity at $p_0/p$ equal to 1.89, $\tilde{A}$ first increases nonlinearly with reservoir pressure ($p_0/p$), and then it increases in a more or less linear manner.

Now that the density ratio $\tilde{\rho}_1/\tilde{\rho}_\infty$ and the area ratio $\tilde{A}$ have been determined, the significance of the perturbation mass-flow rate can be evaluated by means of Eq. C8. The ratio $\tilde{\Delta m}/\tilde{m}_\infty\tilde{A}_\infty$ is plotted as a function of reservoir pressure in Fig. C6, for three different N-wave durations $(\tau_0)$ of 100, 200 and 400 ms. This ratio is double valued in a similar manner as were $\tilde{M}_1\tilde{p}_1\tilde{\rho}_1\tilde{p}_\infty$ and $\tilde{A}$. Normally, the perturbation mass-flow rate $(\tilde{\Delta m})$ is insignificant compared with the mass-flow rate at the valve throat $(\tilde{m}_\infty\tilde{A}_\infty)$. The ratio $\tilde{\Delta m}/\tilde{m}_\infty\tilde{A}_\infty$ is usually less than 0.05 for normal operation of the UTIAS travelling-wave horn for the following reasons. When a short-duration (100 ms) simulated sonic boom is produced with the facility, only a low reservoir pressure less than 2 atm is required. For a longer duration N-wave of 400 ms, for example, a pressure ratio of less than 7 atm is normally required. Only when powerful booms are desired do reservoir pressures exceed 7 atm. Hence, the perturbation mass-flow rate is thought to be inconsequential as compared with the mass-flow rate at the valve throat. Note that a similar perturbation mass-flow rate is nonexistent when the flow from the high-pressure reservoir is entirely subsonic.

It was not possible to obtain a measurement of the perturbation mass-flow rate from experimental data, owing to limitations of the existing equipment. Hence, a direct check on the analysis was not possible. However, for the case of a high reservoir pressure (10 atm) and a short-duration (110 ms) simulated sonic boom when the predicted value of $\tilde{\Delta m}$ is significant (0.5 $\tilde{m}_\infty\tilde{A}_\infty$), the experimental overpressure signature was carefully inspected. The signature did not deviate significantly from an N-shape, as would have been expected for a significant $\tilde{\Delta m}$. It appears, therefore, that the analysis could possibly overpredict the perturbation mass-flow rate, or, for that matter, it could be invalid owing to certain simplifying assumptions. For example, the flow is not one-dimensional as assumed for the analysis. In reality the shock and expansion waves are oblique, and the flow is normally separated (Ref. C4). The effects of a separated flow and associated oblique waves on the perturbation mass-flow rate have not been evaluated.

References


FIG. C1 TIME-DISTANCE DIAGRAM (a) ILLUSTRATING THE MOTION OF THE UPSTREAM-FACING SHOCK WAVE IN THE PYRAMIDAL HORN (b).
FIG. C2 CONTROL VOLUME CONTAINING THE SUPersonic FLOW AND UPSTREAM-FACING SHOCK WAVE.

FIG. C3 THE VARIATION OF FUNCTION Z (Eq. C5) FOR A PARABOLIC THROAT-AREA PROFILE.
FIG. C4 THE MACH NUMBER (a) AND PRESSURE RATIO (b) OF THE UPSTREAM-FACING SHOCK AS A FUNCTION OF RESERVOIR PRESSURE.
FIG. C5 THE DENSITY RATIO $\rho_1/\rho_*$ (a) AND THE AREA RATIO $\bar{A}$ (b), BOTH AS A FUNCTION OF RESERVOIR PRESSURE.
\[ \frac{\Delta m}{\bar{m}_* \bar{A}_*} \]

\[ \gamma = 1.4 \]
\[ r_o = 0.457 \text{ m} \]
\[ a = 345 \text{ m/s} \]

**N-WAVE DURATION \( (T_o) \):**

- 100 ms
- 200 ms
- 400 ms

**FIG. C6** THE RATIO OF THE PERTURBATION AND VALVE-THROAT MASS-FLOW RATES AS A FUNCTION OF RESERVOIR PRESSURE.
APPENDIX D: RECTANGULAR SHOCK TUBE

Consider the wave motion in a low-pressure-ratio rectangular or constant-area shock tube (Fig. 4la) which has the following initial conditions. A diaphragm initially separates the higher pressure quiescent gas in the driver from a different quiescent gas in the channel. The small pressure difference across the diaphragm is denoted by Δp₀. Note that the two different gases can also be at different temperatures. If Δp and Δu denote overpressure and particle velocity respectively, the initial conditions at time t less than zero, before the diaphragm is broken, are summarized mathematically below.

Channel (x > x₀):

\[ \Delta p_1 = 0 \]  \hspace{1cm} (D1)

\[ \Delta u_1 = 0 \]

Driver (0 < x < x₀):

\[ \Delta p_2 = \Delta p₀ \]  \hspace{1cm} (D2)

\[ \Delta u_2 = 0 \]

The distance x is measured from the closed end of the driver, and x₀ denotes the length of the driver and also the diaphragm location (see Fig. 4la). The subscripts 1 and 2 designate conditions in the channel and driver regions, respectively.

The resulting wave motion on breaking the diaphragm at time t equal to zero can be illustrated conveniently by means of a time-distance diagram, such as the one shown in Fig. 43. The rapidly expanding driver gas creates a weak shock wave (s₁) in the channel. Simultaneously, a weak rarefaction wave (u₁) moves into the driver, eventually reflecting from the closed end. This reflected wave (r₁) ultimately overtakes the contact surface. As the contact-surface displacement is very small relative to the length of the driver, it can be taken as fixed at the diaphragm station. Owing to refraction effects at the contact surface, the reflected wave (r₁) is partially transmitted (t₁) to the channel and partially reflected (f₁) back into the driver. The subsequent oscillatory wave motion between the contact surface and the closed end of the driver creates a sequence of additional disturbances in the channel. The integrated result of all these disturbances yields the total wave in the channel.

First-order acoustic theory is used to determine the wave motion in the shock tube. The motion of the disturbances can be assumed to be governed by the one-dimensional planar wave equation, in conjunction with suitable initial and boundary conditions. The wave equations for both the driver and channel regions are given below.

Channel (x > x₀):

\[ \frac{\partial^2 \phi_1}{\partial t^2} = a_1^2 \frac{\partial^2 \phi_1}{\partial x^2} \]  \hspace{1cm} (D3)
Driver \((0 < x < x_0)\):
\[ \frac{\partial^2 \phi_2}{\partial t^2} = a_2^2 \frac{\partial^2 \phi_2}{\partial x^2} \]  
(D4)

The new symbols \(\phi\) and \(a\) denote the velocity potential and sound speed, respectively. The velocity potential of the channel gas \(\phi_1\) and the driver gas \(\phi_2\) can be expressed in a more useful form, as the sum of all of the potentials of the individual disturbances. The new form of \(\phi_1\) and \(\phi_2\), and the related expressions for over-pressure and particle velocity, are summarized below.

Channel \((x > x_0)\):
\[ \phi_1 = \sum_{i=1}^{\infty} \left[ g_1(\xi) H[\xi - 2(i-1)a_1/a_2 + 1] \right] \]  
(D5)
\[ \Delta p_1 = - \rho_1 \frac{\partial \phi_1}{\partial t} \]  
(D6)
\[ \Delta u_1 = \frac{\partial \phi_1}{\partial t} = \frac{\Delta p_1}{a_1 \rho_1} \]  
(D7)

Driver \((0 < x < x_0)\):
\[ \phi_2 = - \frac{\Delta p_0}{\rho_2} t + \sum_{i=1}^{\infty} \left[ f_1(\eta) H[\eta - 2i + 1] + h_1(\beta) H[\beta - 2i + 1] \right] \]  
(D8)
\[ \Delta p_2 = - \rho_2 \frac{\partial \phi_2}{\partial t} \]  
(D9)
\[ \Delta u_2 = \frac{\partial \phi_2}{\partial x} \]  
(D10)

In these expressions, the nondimensional variables \(\xi, \eta\) and \(\beta\) equal \((a_1 t - x)/x_0\), \((a_2 t + x)/x_0\) and \((a_2 t - x)/x_0\), respectively, and \(\rho\) denotes density. \(H[\ ]\) denotes the unit step function; it is equal to zero prior to the arrival of the disturbance in question and equals unity thereafter. The forms of the velocity potentials \(\phi_1\) (Eq. D5) and \(\phi_2\) (Eq. D6) were chosen not only to satisfy their respective wave equations (Eqs. D3 and D4), but also to satisfy their associated initial conditions (Eqs. D1 and D2). For example, the term \(-\Delta p_0 t/\rho_2\) in Eq. D8 accounts for the initial overpressure \(\Delta p_0\) in the driver.

In order to solve for the two disturbances \(g_1(\xi)\) and \(f_1(\eta)\), the following boundary condition can be used. The overpressure and particle velocity must be continuous across the junction of the driver and channel, after the diaphragm is removed. By equating the expressions for \(\Delta p_1\) and \(\Delta u_1\) (Eqs. D6 and D7) with \(\Delta p_2\) and \(\Delta u_2\) (Eqs. D9 and D10) at the junction where \(x\) equals \(x_0\), the following two equations can be obtained.
Hence, \( f'_1(\xi) \) and \( g'_1(\xi) \) can be determined from these two simultaneous equations. Then, by integrating these two expressions for \( f'_1(\eta) \) and \( g'_1(\xi) \) from zero to \( \eta \) and from zero to \( \xi \), respectively, the velocity potentials \( f'_1(\eta) \) and \( g'_1(\xi) \) can be obtained, as shown below.

\[
\begin{align*}
  g'_1(\xi) &= \frac{-\Delta p_e \, x_0 \, \xi}{a_1 \rho_1 + a_2 \rho_2} \\
  f'_1(\eta) &= \frac{\Delta p_e \, x_0 \, \eta}{a_1 \rho_1 + a_2 \rho_2}
\end{align*}
\]

A second boundary condition is required to relate each velocity potential \( h_1(\beta) \) for the reflected wave from the closed end of the driver to the corresponding potential \( f'_1(\eta) \) for the incident wave. For the reflection process at the closed end where \( x \) equals zero, the particle velocity must be zero for all time. Hence, by using Eqs. D10 and D8, the following result can be obtained.

\[
    h'_1(\beta) = f'_1(\eta)
\]

For example, \( h'_1(\beta) \) is therefore given by the following expression.

\[
    h'_1(\beta) = \frac{\Delta p_e \, x_0 \, \beta}{a_1 \rho_1 + a_2 \rho_2}
\]

A third boundary condition is required to solve for the transmission and reflection of the incident wave at the contact surface. Let the potentials of the incident, transmitted and reflected waves be given by \( h_{1-1}(\beta) \), \( g'_1(\xi) \) and \( f'_1(\eta) \) respectively, in accordance with the waves shown in the time-distance diagram of Fig. 43, and conforming with the numbering of the successive waves in Eqs. D5 and D8. Across the contact surface the overpressure and particle velocity must be continuous. By equating \( \Delta p_1 \) and \( \Delta u_1 \) (Eqs. D6 and D7) with \( \Delta p_2 \) and \( \Delta u_2 \) (Eqs. D9 and D10) for \( x \) equals to \( x_0 \), the following equations can be obtained.

\[
\begin{align*}
  a_1 \rho_1 \, g'_1(\xi) &= a_2 \rho_2 \, h'_{1-1}(\beta) + a_2 \rho_2 \, f'_1(\eta) \\
  g'_1(\xi) &= h'_{1-1}(\beta) - f'_1(\eta)
\end{align*}
\]
From these two simultaneous equations, \( g_1'(\xi) \) and \( f_1'(\eta) \) can be determined in terms of \( h_{1-1}(\beta) \). Then, the potentials of the transmitted \( g_1 \) and reflected \( f_1 \) waves can be found as a function of the potential \( h_{1-1}(\beta) \) for the incident wave. These results are given below.

\[
g_1'(\xi) = \frac{2a_2p_2}{a_1p_1 + a_2p_2} h_{1-1}(\beta) \quad (D17)
\]

\[
f_1'(\eta) = \frac{a_1p_1 - a_2p_2}{a_1p_1 + a_2p_2} h_{1-1}(\beta) \quad (D18)
\]

The transmitted wave \( g_1 \) has the same waveform as the incident wave \( h_{1-1} \). However, it is amplified or diminished according to whether the transmission factor \( \frac{2a_2p_2}{(a_1p_1 + a_2p_2)} \) is greater or less than unity, respectively. The reflected wave \( f_1 \) can also have the same waveform as the incident wave \( h_{1-1} \) if the reflection factor \( \frac{(a_1p_1 - a_2p_2)}{(a_1p_1 + a_2p_2)} \) is positive. Otherwise, when the factor is negative, the waveform of the reflected wave is an inverted form of the incident wave.

From the preceding developments the potentials \( g_1, f_1 \) and \( h_1 \) can be expressed in the following simple forms.

\[
g_1(\xi) = -\frac{\Delta p_x x_0 \xi}{a_1p_1 + a_2p_2} \quad \text{if } i = 1 \quad (D19)
\]

\[
g_1(\xi) = \left[ \frac{a_1p_1 - a_2p_2}{a_1p_1 + a_2p_2} \right]^{i-2} \frac{2a_2p_2 \Delta p_x x_0 \xi}{(a_1p_1 + a_2p_2)^2} \quad \text{if } i > 1
\]

\[
f_1(\eta) = \left[ \frac{a_1p_1 - a_2p_2}{a_2p_2 + a_2p_2} \right]^{i-1} \frac{\Delta p_x x_0 \eta}{a_1p_1 + a_2p_2} \quad (D20)
\]

\[
h_1(\beta) = \left[ \frac{a_1p_1 - a_2p_2}{a_1p_1 + a_2p_2} \right]^{i-1} \frac{\Delta p_x x_0 \beta}{a_1p_1 + a_2p_2} \quad (D21)
\]

These results complete the derivation of the velocity potentials of the individual disturbances. The velocity potential, overpressure and particle velocity for the channel region follow from Eqs. D5, D6 and D7, and those for the driver region follow from Eqs. D8, D9 and D10. Note that these results are summarized and discussed in the main body of the report (Section 3.2).
APPENDIX E: PYRAMIDAL SHOCK TUBE

Consider the wave motion in a low-pressure-ratio pyramidal shock tube (Fig. 41b), resulting from the following initial conditions. A diaphragm initially separates a quiescent gas in the driver from the same but lower pressure quiescent gas in the channel. If $\Delta p$ and $\Delta u$ denote overpressure and particle velocity respectively, the initial conditions before the diaphragm is broken are summarized mathematically below.

Channel ($r > r_0$):

\[ \Delta p = 0 \]
\[ \Delta u = 0 \]  
(E.1)

Driver ($0 < r < r_0$):

\[ \Delta p = \Delta p_0 \]
\[ \Delta u = 0 \]  
(E.2)

The respective symbols $\Delta p_0$, $r$, and $r_0$ denote the pressure difference across the diaphragm, radial distance measured from the apex of the shock tube, and the diaphragm location.

On breaking the diaphragm the expanding driver gas generates a weak shock wave ($S_1$) in the channel (Fig. 49). Simultaneously a weak rarefaction wave ($R_1$) propagates into the driver gas. It eventually reflects from the apex to produce a second shock wave ($S_2$), which ultimately follows the first shock in the channel. First-order acoustic theory was used to determine the wave motion in the shock tube. The one-dimensional spherical wave equation, given below, is assumed to govern the motion of the three disturbances.

\[ \frac{\partial^2 (\phi)}{\partial t^2} = a^2 \frac{\partial^2 (\phi)}{\partial r^2} \]  
(E.3)

The overpressure and particle velocity are related to the velocity potential ($\phi$) by the following expressions.

\[ \Delta p = - \rho \frac{\partial \phi}{\partial t} \]  
(E.4)

\[ \Delta u = \frac{\partial \phi}{\partial r} \]  
(E.5)

Density and time are denoted by $\rho$ and $t$ respectively. For convenience the velocity potentials of the driver and channel gases are expressed in the following manner.

Channel ($r > r_0$):

\[ \phi = \frac{1}{r} g_1(\xi) H[\xi+1] + \frac{1}{r} g_2(\xi) H[\xi-1] \]  
(E.6)
Driver ($0 < r < r_o$):

\[
\phi = -\frac{\Delta p_o t}{\rho} + \frac{1}{r} f_1(\eta) H[\eta-1] + \frac{1}{r} g_2(\xi) H[\xi-1] \tag{E.7}
\]

The functions $g_1(\xi)$, $g_2(\xi)$, and $f_1(\eta)$ represent the yet unknown analytical forms of the first and second shock waves and the rarefaction wave. The nondimensional variables $\eta$ and $\xi$ equal $(at+r)/r_o$ and $(at-r)/r_o$ respectively, and $H[\ ]$ denotes the unit step function. The selected forms of the two velocity potentials (Eqs. E.6 and E.7) not only satisfy the wave equation (Eq. E.3), but they also satisfy their appropriate initial conditions (Eqs. E.1 and E.2). The particular form of $g_1(\xi)$, $g_2(\xi)$ and $f_1(\eta)$ can be determined by the use of suitable boundary conditions.

The overpressure and particle velocity must be continuous at the diaphragm location (driver and channel junction) after the diaphragm has been removed. When the overpressure for the channel and driver are equated at the diaphragm location ($r$ equals $r_o$), the following expression can be obtained from Eqs. E.4, E.6 and E.7.

\[
g_1(\xi) = f_1(\eta) - \frac{\Delta p_o r_o^2}{\rho} \tag{E.8}
\]

This expression can be integrated to give the following alternate form.

\[
g_1(\xi) = f_1(\eta) - \frac{\Delta p_o r_o t}{\rho} \tag{E.9}
\]

Note that this expression can also be obtained by simply equating the channel and driver potentials (Eqs. E.6 and E.7). When the particle velocity for the channel and driver are equated at the diaphragm location, the following result can be obtained from Eqs. E.5, E.6 and E.7.

\[
g_1'(\xi) + g_1(\xi) = -f_1(\eta) + f_1(\eta) \tag{E.10}
\]

The last three equations are sufficient for determining $g_1(\xi)$ and $f_1(\eta)$. One first subtracts Eq. E.9 from Eq. E.10 to obtain the following result, which is independent of $g_1(\xi)$ and $f_1(\eta)$.

\[
g_1(\xi) = -f_1'(\eta) + \frac{\Delta p_o r_o t}{\rho} \tag{E.11}
\]

As Eqs. E.8 and E.11 are two simultaneous algebraic equations in terms of $g_1'(\xi)$ and $f_1'(\eta)$, the following results can be obtained.
Each of the above expressions can be integrated to yield $g_1(\xi)$ and $f_1(\eta)$. As the velocity potential for a spherical wave must be zero at the wave front, the constants of integration can be found by setting $g_1(\xi)$ and $f_1(\eta)$ equal to zero for $\xi$ equal to -1 and $\eta$ equal to +1 respectively. Hence, the following expressions can be obtained.

$$g_1(\xi) = \frac{2}{2\rho} \left( \xi^2 - 1 \right)$$  \hspace{1cm} (E.14)

$$f_1(\xi) = \frac{2}{2\rho} \left( \eta^2 - 1 \right)$$  \hspace{1cm} (E.15)

A second boundary condition is needed in order to determine $g_2(\xi)$. At the apex of the driver where the radial distance $r$ equals zero, the particle velocity $\Delta u$ must always be zero. By using Eqs. E.5 and E.7, the following result can be obtained.

$$\lim_{r \to 0} \left[ \frac{r}{r} \left\{ f'_1(\eta) - g'_2(\xi) \right\} - \frac{r^2}{r^2} \left\{ f_1(\eta) + g_2(\xi) \right\} \right] = 0$$  \hspace{1cm} (E.16)

One therefore obtains the following result for $g_2(\xi)$ in terms of $f_1(\eta)$.

$$g_2(\xi) = -f_1(\eta)$$  \hspace{1cm} (E.17)

For the specific case of the pyramidal shock tube, for which $f_1(\eta)$ is given by Eq. E.15, the final result for $g_2(\xi)$ is given below.

$$g_2(\xi) = -\frac{2}{4\rho} \left( \xi^2 - 1 \right)$$  \hspace{1cm} (E.18)

From the preceding developments the solution for the velocity potential, overpressure and particle velocity for both the channel and driver regions can be summarized as follows.
Channel \((r > r_0)\):

\[
\phi = \frac{\Delta p_\infty}{4\pi \rho r} \left( r^2 - 1 \right) \left[ H[\xi+1] - H[\xi-1] \right] 
\]
\[(E.19)\]

\[
\Delta p = \frac{\Delta p_\infty}{2r} (r - at) \left[ H[\xi+1] - H[\xi-1] \right] 
\]
\[(E.20)\]

\[
\Delta u = \frac{\Delta p_\infty}{2\rho \phi} \left( \frac{r^2 + r_0^2 - a^2 t^2}{2 r^2} \right) \left[ H[\xi+1] - H[\xi-1] \right] 
\]
\[(E.21)\]

Driver \((0 < r < r_0)\):

\[
\phi = - \frac{\Delta p_\infty}{\rho} \left[ t - \frac{r^2}{4a r} (\eta^2 - 1) H[\eta-1] + \frac{r_0^2}{4a r} (\xi^2 - 1) H[\xi-1] \right] 
\]
\[(E.22)\]

\[
\Delta p = \Delta p_\infty - \frac{\Delta p_\infty}{2r} \left[ (r + at) H[\eta-1] + (r - at) H[\xi-1] \right] 
\]
\[(E.23)\]

\[
\Delta u = \frac{\Delta p_\infty}{2\rho \phi} \left( \frac{r^2 + r_0^2 - a^2 t^2}{2 r^2} \right) \left[ H[\eta-1] - H[\xi-1] \right] 
\]
\[(E.24)\]

These results are illustrated and discussed in Section 3.3.1.

Associated with the wave motion in the pyramidal shock tube is a fluid motion. The path of a fluid particle for the gases in the channel and driver can be determined as follows. Consider the following form of the expression for the particle velocity, which applies to both the channel and driver gases (see Eqs. E.21 and E.24).

\[
\Delta u = \frac{\Delta p_\infty}{4\pi \rho} \left[ \frac{r^2 + r_0^2 - a^2 t^2}{r^2} \right] 
\]
\[(E.25)\]

As the particle velocity \((\Delta u)\) is the time derivative \((t)\) of the radius \((r)\), \(\Delta u\) can be replaced by \(dr/dt\). Then it is possible to integrate the new form of Eq. E.25 to obtain the particle path \(r(t)\). To simplify the integration an alternate procedure was used. Let the initial radial location of the particle be denoted by \(r_1\), so that the displacement of the particle \(\Delta r\) from this initial position is \(r-r_1\). Also, let the time when the wave front arrives at the initial location \((r_1)\) be denoted by \(t_1\). Then a convenient nondimensional time \(\tau\), which starts from zero at the wave front, can be expressed as \((t-t_1)/2r_0\). The new form of Eq. E.25 can then be written as follows.
To avoid the difficult integration involved with Eq. E.26, it is assumed that the particle displacement $\Delta r$ is much smaller than the initial location $r_1$. By using this assumption, and noting that $a^2$ equals $\gamma_p/\rho$, the simplified form of Eq. E.26 is given below.

$$\frac{d(\Delta r)}{d\tau} \approx \frac{\Delta p_0 r_0}{2\gamma_p} \left[ 1 + \frac{r_0^2}{r_1^2} \left( \frac{at_1}{r_1} \right)^2 - 4 \frac{r_0}{r_1} \frac{at_1}{r_1} \tau - 4 \frac{r_0^2}{r_1^2} \tau^2 \right] \quad (E.27)$$

This expression can be integrated easily to obtain the particle path $\Delta r(t)$.

First, consider a particle path for the gas in the channel. The locus $r(t)$ of the first shock of the N-wave or the wavefront is $r_0 + a\tau$. In terms of $r_1$ and $t_1$, $at_1/r_1$ thus equals $1-r_0/r_1$. Using this result in Eq. E.27, the final result for a particle path of the channel gas is given below.

$$\frac{\Delta r}{r_0} \approx \frac{\Delta p_0}{\gamma_p} \frac{r_0}{r_1} \left[ \tau - \left( 1 - \frac{r_0}{r_1} \right) \tau^2 - \frac{2}{3} \frac{r_0}{r_1} \tau^3 \right] \quad \text{if } 0 < \tau < 1 \quad (E.28)$$

$$\frac{\Delta r}{r_0} \approx \frac{\Delta p_0}{2\gamma_p} \frac{r_0^2}{r_1^2} \quad \text{if } \tau > 1$$

The path of the contact surface is obtained by setting $r_1$ equal to $r_0$. The path of a fluid particle in the driver can be obtained in a similar manner. For this case $at_1/r_1$ equals $1+r_0/r_1$. One can thus obtain the following result from Eq. E.27.

$$\frac{\Delta r}{r_0} \approx \frac{\Delta p_0}{\gamma_p} \frac{r_0}{r_1} \left[ \tau + \left( 1 - \frac{r_0}{r_1} \right) \tau^2 - \frac{2}{3} \frac{r_0}{r_1} \tau^3 \right] \quad \text{if } 0 < \tau < \frac{r_1}{r_0} \quad (E.29)$$

$$\frac{\Delta r}{r_0} \approx \frac{\Delta p_0}{3\gamma_p} \frac{r_1}{r_0} \quad \text{if } \tau > r_1/r_0$$

It should be noted that Eq. E.28 is a very good approximation for the path of a fluid particle in the channel, as $\Delta r$ is always much less than $r_1$. Equation E.29 is a good approximation for the particle path in the driver if $r_1$ is not too small (e.g., $r_1 > r_0/10$).
APPENDIX F: PYRAMIDAL-RECTANGULAR SHOCK TUBE

Consider the wave motion in a low-pressure-ratio pyramidal-rectangular shock tube (Fig. 4lc), resulting from the following initial conditions. A diaphragm separates a quiescent gas in the driver from the same but lower pressure quiescent gas in the channel. If $\Delta p$, $\Delta u$, and $\Delta u_0$ denote overpressure, particle velocity, and the initial pressure difference across the diaphragm before it is broken, the initial conditions are summarized mathematically below.

Channel ($x > 0$):

\[ \Delta p = 0 \]  \hspace{1cm} (F1)
\[ \Delta u = 0 \]

Driver ($0 < R < R_o$):

\[ \Delta p = \Delta u_0 \]  \hspace{1cm} (F2)
\[ \Delta u = 0 \]

The planar distance $x$ is measured from the diaphragm location ($x = 0$), and the radial distance $R$ is measured from the apex of the driver to the diaphragm (see Fig. 4lc). $R_o$ denotes the radial location of the diaphragm for the driver.

Acoustic theory can be used to predict the ensuing wave motion in the shock tube after the diaphragm is broken. The motion of the disturbances (Fig. 42) in the channel can be described by the one-dimensional planar wave equation, and those in the driver by the one-dimensional spherical wave equation. The wave equations are given below.

Channel ($x > 0$):

\[ \frac{\partial^2 \phi}{\partial t^2} = a^2 \frac{\partial^2 \phi}{\partial x^2} \]  \hspace{1cm} (F3)

Driver ($0 < R < R_o$):

\[ \frac{\partial^2 (R\phi)}{\partial t^2} = a^2 \frac{\partial^2 (R\phi)}{\partial R^2} \]  \hspace{1cm} (F4)

The respective symbols $\phi$, $a$, and $t$ denote the velocity potential, sound speed, and time. It is convenient to express the velocity potentials for the channel and driver gas as the sum of individual potentials of disturbances comprising the total wave. The expressions for the two potentials, and the related equations for overpressure and particle velocity are given below.

Channel ($x > 0$):

\[ \phi = \sum_{i=1}^{\infty} g_i(\xi) H[\xi - 2i + 2] \]  \hspace{1cm} (F5)
\[
\Delta p = -\rho \frac{\partial \phi}{\partial t} \tag{F6}
\]
\[
\Delta u = \frac{\partial \phi}{\partial x} = \frac{\Delta p}{\rho} \tag{F7}
\]

Driver (0 < R < R*):
\[
\phi = -\frac{\Delta p R \rho t}{R} + \frac{1}{R} \sum_{i=1}^{\infty} \left[ f_i(\eta)H(\eta-2i+1) + h_i(\beta)H(\beta-2i+1) \right] \tag{F8}
\]
\[
\Delta p = -\rho \frac{\partial \phi}{\partial t} \tag{F9}
\]
\[
\Delta u = \frac{\partial \phi}{\partial x} \tag{F10}
\]

In these expressions: \( \rho \) and \( H\{ \} \) denote density and the unit step function respectively; \( \eta, \beta, \) and \( \xi \) are equal to \( (at+R)/R_0, (at-R)/R_0, \) and \( (at-x)/R_0 \) respectively; and \( f_i(\eta), h_i(\beta) \) and \( g_i(\xi) \) represent the potentials of the disturbances comprising the wave motion (see Fig. 42). Note that the velocity potentials for the channel and driver have been formulated such that they not only satisfy their respective wave equations (Eqs. F3 and F4), but they also satisfy their respective set of initial conditions (Eqs. F1 and F2). The analytical forms of \( g_i(\xi), f_i(\eta) \) and \( h_i(\beta) \) can be determined by applying appropriate boundary conditions.

After the diaphragm is broken the velocity potential, overpressure and particle velocity must be continuous across the driver and channel junction \( (x = 0, R = R_0) \). Using this boundary condition and Eqs. F5 to F10 inclusive, the following three intermediate expressions can be obtained.
\[
R_0 g_i(\xi) = -\frac{\Delta p R_0 t}{\rho} + f_i(\eta) \tag{F11}
\]
\[
R_0 g'_i(\xi) = -\frac{\Delta p R_0^2}{\rho} + f'_i(\eta) \tag{F12}
\]
\[
R_0 g''_i(\xi) = -f'_i(\eta) + f_i(\eta) \tag{F13}
\]

Note that only the two disturbances resulting from the breaking of the diaphragm have been considered. These three results yield the following two expressions.
\[
g'_i(\xi) - \frac{1}{2} g_i(\xi) = \frac{\Delta p R_0}{2\rho} (\xi-1) \tag{F14}
\]
\[
f'_i(\eta) - \frac{1}{2} f_i(\eta) = \frac{\Delta p R_0^2}{2\rho} \tag{F15}
\]
After integration of these two differential equations, the final forms of \( g_1(\xi) \) and \( f_1(\eta) \) are obtained, as given below.

\[
g_1(\xi) = -\frac{\Delta p e R_e}{ap} \left[ 1 + \xi + G_1 \exp(\xi/2) \right] \quad (F16)
\]

\[
f_1(\eta) = -\frac{\Delta p e R_e^2}{ap} \left[ 1 + F_1 \exp(\eta/2) \right] \quad (F17)
\]

The constants of integration can be determined quite simply by setting each velocity potential to zero at the wave front. Hence, as \( g_1(\xi) \) equals zero when \( \xi \) equals zero, \( G_1 \) equals 1. Similarly, as \( f_1(\eta) \) is zero when \( \eta \) equals 1, \( F_1 \) is found to equal 1.

A second boundary condition is that the particle velocity must be zero for all time at the apex of the driver (\( R = 0 \)). To satisfy this condition for an incident wave of potential \( f_1(\eta) \), the reflected wave \( h_1(\beta) \) is given by the following expression.

\[
h_1(\beta) = -f_1(\eta) \quad (F18)
\]

This result was derived previously in Appendix E (see Eq. El7). As the potential \( f_1(\eta) \) of each successive disturbance encountering the origin is determined, the potential \( h_i(\beta) \) of the resulting reflected disturbance follows from Eq. F18. For the specific case of the first reflected wave, \( h_1(\beta) \) is given by the following expression, derived from Eqs. F17 and F18.

\[
h_1(\beta) = \frac{\Delta p e R_e^2}{ap} \left[ 1 + G_1 \exp(\frac{\beta-1}{2}) \right] \quad (F19)
\]

The first reflected wave from the origin, like others that follow, encounter the driver and channel junction, where it is partially transmitted and partially reflected (see Fig. 42). Consider an incident wave having a potential \( g_i(\xi) \) and partially reflected as wave \( f_i(\eta) \). As the velocity potential, overpressure and particle velocity must be continuous across the junction (\( x = 0, \ R = R_e \)), the velocity potential, overpressure and particle velocity for the channel gas (Eqs. F5, F6 and F7) are matched with those for the driver gas (Eqs. F8, F9 and F10). This procedure yields the following intermediate results.

\[
R_e \ g_i(\xi) = f_i(\eta) + h_{i-1}(\beta) \quad (F20)
\]

\[
R_e \ g_i'(\xi) = f_i'(\eta) + h_{i-1}'(\beta) \quad (F21)
\]
From these three equations the following more appropriate results can be obtained.

\[ R_0 g_1'(\xi) = -f_1'(\eta) + h_{1-1}(\beta) + f_1(\eta) + h_{1-1}(\beta) \]  

\( F22 \)

Before \( g_1(\xi) \) and \( f_1(\eta) \) can be determined, \( \xi \) in Eq. \( F23 \) must be expressed in terms of \( \xi \), and \( \xi \) and \( \beta \) in Eq. \( F24 \) must be expressed in terms of \( \eta \). At the boundary or driver and channel junction \((x = 0, R = R_0)\), \( \xi \), \( \eta \) and \( \beta \) equal \((at-O)/R_0 \) \((at+R_0)/R_0\) and \((at-R_0)/R_0\). From these results, \( \beta \) in Eq. \( F23 \) should be replaced by \( \xi-1 \), and for Eq. \( F24 \), \( \xi \) and \( \beta \) should be replaced by \( \eta-1 \) and \( \eta-2 \) respectively. Equations \( F23 \) and \( F24 \) thus take the following forms.

\[ g_1'(\xi) - \frac{1}{2} g_1(\xi) = \frac{1}{R_0} h_{1-1}(\xi-1) \]  

\( F25 \)

\[ f_1(\eta) = R_0 g_1(\eta-1) - h_{1-1}(\eta-2) \]  

\( F26 \)

For an incident wave on the junction of known potential \( h_1(\beta) \), \( g_1(\xi) \) can be determined from Eq. \( F25 \). Then \( f_1(\eta) \) can be found by using Eq. \( F26 \).

The potential \( h_1(\beta) \) has been determined (Eq. \( F19 \)), making it possible to find \( g_2(\xi) \) from Eq. \( F25 \). This result is given below.

\[ g_2(\xi) = \frac{\Delta p_{2R_0}}{a\rho} \left[ G_1 \frac{\xi}{2} + G_2 \right] \exp \left( \frac{\xi-2}{2} \right) \]  

\( F27 \)

The potential \( f_2(\eta) \) then follows from Eq. \( F26 \), as given below.

\[ f_2(\eta) = -\frac{\Delta p_{2R_0}}{a\rho} \left[ 1 - \left( G_1 \left( \frac{\eta-1}{2} \right) + G_2 - G_1 \right) \exp \left( \frac{\eta-3}{2} \right) \right] \]  

\( F28 \)

The next step is to determine \( h_2(\beta) \) by using Eqs. \( F18 \) and \( F28 \). Then \( g_3(\xi) \) and \( f_3(\eta) \) can be determined by using Eqs. \( F25 \) and \( F26 \). This step by step process can be continued to obtain the potentials of the successive disturbances. The successive potentials form a recognizable pattern from which the \( i \)th term of \( g_1(\xi) \), \( f_1(\eta) \) and \( h_1(\beta) \) can be deduced.

The sequence of potentials \( g_1(\xi) \) is summarized below.
\[ g_1(\xi) = - \frac{\Delta p_{2R}}{\alpha \rho} \left[ 1 + \xi + G_1 \exp \left( \frac{\xi}{2} \right) \right] \]
\[ g_2(\xi) = + \frac{\Delta p_{2R}}{\alpha \rho} \left[ G_1 \left( \frac{\xi}{2} \right) + G_2 \exp \left( -\frac{\xi}{2} \right) \right] \]
\[ g_3(\xi) = - \frac{\Delta p_{2R}}{\alpha \rho} \left[ \frac{G_1}{2} \left( \frac{\xi-2}{2} \right)^2 + G_2 \left( \frac{\xi-2}{2} \right) + G_3 \exp \left( \frac{\xi-4}{2} \right) \right] \] (F29)

\[ g_1(\xi) = (-1)^i \frac{\Delta p_{2R}}{\alpha \rho} \left[ \sum_{j=1}^{i} \frac{G_i}{(i-j)!} \left( \frac{\xi-2i+1}{2} \right)^{i-j} \right] \exp \left( \frac{\xi-2i+2}{2} \right) \]

After the first term a recognizable sequence develops. The sequence for \( f_1(\eta) \) is more complex, as illustrated below.

\[ f_1(\eta) = - \frac{\Delta p_{2R}}{\alpha \rho} \left[ 1 + G_1 \exp \left( \frac{\eta-1}{2} \right) \right] \]
\[ f_2(\eta) = - \frac{\Delta p_{2R}}{\alpha \rho} \left[ 1 - \left( G_1 \left( \frac{\eta-1}{2} \right) + G_1 - G_2 \right) \exp \left( \frac{\eta-3}{2} \right) \right] \] (F30)
\[ f_3(\eta) = - \frac{\Delta p_{2R}}{\alpha \rho} \left[ 1 + \left( \frac{G_1}{2} \left( \frac{\eta-3}{2} \right)^2 + (G_2 - G_1) \left( \frac{\eta-3}{2} \right) \right) \right.
\[ + \left( G_3 - G_2 + G_1 \right) \exp \left( \frac{\eta-5}{2} \right) \] \]

\[ f_1(\eta) = - \frac{\Delta p_{2R}}{\alpha \rho} \left[ 1 - (-1)^i \sum_{j=1}^{i} \frac{F_i(\eta-2i+3)^{i-j}}{2^{i-j}(i-j)!} \exp \left( \frac{\eta-2i+1}{2} \right) \right] \]

\[ F_j = \sum_{m=1}^{i} (-1)^{m+1} G_{j-m+1} \] (F31)

The sequence for \( h_1(\beta) \) follows directly from the previous result (Eq. F18) that \( h_1(\beta) \) equals \(-f_1(\eta)\).

The constants of integration \( G_i \) can be determined by using Eq. F28. Each potential \( g_i(\xi) \) equals zero when \( \xi \) equals \( 2i-2 \) at the wave front. Hence, the following results can be obtained for \( G_i \).
These results complete the acoustic solution for the wave motion in the pyramidal-rectangular shock tube. The solution is summarized and discussed in Section 3.4.

\[
G_1 = \begin{cases} 
-1 & \text{if } i = 1 \\
\sum_{j=1}^{i-1} \frac{-G_j}{(i-j)!} & \text{if } i > 1
\end{cases}
\]
In order to assess current societal problems associated with the sonic boom, a horn-type simulator was constructed at the Institute for Aerospace Studies, University of Toronto (UTIAS). The simulator horn is in the form of a horizontal concrete pyramid, which is 25 m long and has a 3-m-square base. At its apex a specially-designed valve is used to control the mass-flow rate of air from a high-pressure reservoir into the horn where the flow generates a simulated sonic boom or travelling N-wave of suitable amplitude and duration, and acceptably-short rise time. Alternatively, a shock-tube driver can be installed at the apex and used for generating short-duration and rapid rise-time sonic booms. For the mass-flow-valve mode of operation a high-frequency sound absorber can be installed near the apex of the horn to filter out of the passing N-wave undesirable jet noise that is produced by the high-speed turbulent flow at the valve. At the large end of the horn a specially-designed reflection eliminator in the form of a recoiling-type porous piston is used to adequately minimize the objectionable reflection of the incident wave. The operation and performance of the simulator are reported. Comprehensive analyses have been made to describe the wave motion in the horn for both the shock-tube and mass-flow-valve modes of operation, as well as for designing certain parts of the facility such as the reflection eliminator. These analyses are not only applicable to the UTIAS facility but apply as well to other simulators that are in current use in England, France, Germany, and the United States. The analyses have been substantiated by experimental data obtained from the UTIAS and other simulators. A number of worthwhile experiments on the effects of sonic boom on humans and structures have already been made in the UTIAS facility.

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