PRESSURE MEASUREMENTS IN LOW-ENERGY UNDERWATER EXPLOSIONS

by

R. B. Simpson

SEPTEMBER, 1963

UTIAS TECHNICAL NOTE NO. 72
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ACKNOWLEDGEMENTS

The interest of Dr. G. N. Patterson in this work is appreciated. The direction of Dr. I. I. Glass in supervising this investigation, and the helpful suggestions and criticism which came out in discussions deserve my gratitude. Special thanks are due to L. E. Heuckroth whose advice and assistance throughout the programme were indispensable. The financial support of the research by the Defence Research Board (Canada) and the Office of Naval Research (U. S. A.) is gratefully acknowledged.
SUMMARY

As part of a programme of studying low-energy underwater explosions, a series of pressure measurements have been made. The experiments were performed in the UTIA steel shock sphere of three feet inner diameter, using one inch and two inch diameter glass spheres pressurized up to 500 p.s.i. as safe explosive sources permitting close control of the initial conditions. The smaller spheres were used in measurements taken at 2, 5, 10 and 15 radii from the glass sphere for an initial pressure of 500 p.s.i. using air as the driver gas. The larger spheres were used in measurements in which the gauge was placed initially in contact with the glass, so as to enter the bubble as it started to expand. For these runs, an initial pressure of 300 p.s.i. and driver gases helium, air, and sulphur hexafluoride were used. Simultaneously with the pressure-time measurements, space-time, drum-camera photographs were taken to measure the bubble expansion rate.

The results were compared with the theoretical pressure field of a gas bubble expanding in an infinite incompressible medium allowing for the pressure wave propagation time. The agreement between the measurements and the theoretical predictions was generally satisfactory although the influence of the finite breaking time of the glass was observed and the effects of the compressibility of the water appear to be significant in the early expansion rate of the bubble.
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NOTATION

\(a\) sound velocity in water 4,865 ft/sec. at 20° C

\(p\) pressure in p.s.i.

\(p_4\) initial pressure inside glass bulb

\(p_1\) initial hydrostatic pressure

\(r\) bubble radius in inches

\(r_0\) internal radius of glass spheres

\(t\) time in seconds

\(y\) radial distance of field point in inches

\(P\) nondimensional pressure \(p/p_2\)

\(P_{41}\) initial diaphragm pressure ratio \(p_4/p_2\)

\(X\) nondimensional radius \(r/r_0\)

\(Y\) nondimensional field point \(y/r_0\)

\(\gamma\) specific heat ratio for a gas (isentropic index)

\(K\) dynamic shear modulus of rubber

\(\nu\) kinematic viscosity of water

\(\tau\) nondimensional time \(t/r_0 \sqrt{p_2/p_1}\)

\(\rho_0\) density of water (assumed constant)

\(w\) frequency in Kc/sec

Notation for records reproduced in the figures

\(C\) incident pressure (compression) wave

\(D\) compression wave reflected from a rigid boundary

\(I\) rarefaction wave in the interior of the bubble

\(R\) rarefaction wave reflected from the free surface of the
tank.

$R_b$ rarefaction wave reflected from the bubble.

$T$ pressure peak indicating that the bubble has reached its minimum contraction, thus completing a pulsation.
1. INTRODUCTION

The physical quantity with the largest variation in the flow field of an underwater explosion, is the pressure. In this medium the propagation of a large pressure wave is measureably isothermal, its density changes are slight and its induced particle velocity is small unless the explosion is intense. Hence, in the extensive investigations of underwater blasts during the Second World War, a large amount of research was devoted to developing techniques of measuring these pressures, and a résumé of the results is given in chapter 5 of Ref. 1. While piezo-electric transducers proved to be the most useful gauges for this purpose, the problems involved in obtaining reliable pressure histories still present considerable difficulties, and a standard technique for taking such measurements has not been developed.

The measurements herein presented were taken in a series of experiments on low energy explosions. Some distinctions must be made between low and high energy blasts. In a moderate to high energy explosion, the flow may be considered to have two aspects, a shock or blast wave of considerable amplitude which travels out into the medium with a velocity greater than the sound speed, $a$, and a flow occurring somewhat later which results from the expanding bubble of high pressure gas, the influence of which is propagated with the sound speed $a$. These will be referred to for convenience as the primary wave and the secondary flow respectively although in fact they are not separate but are coupled phenomena. The pressure profile for a given time, described in the order of decreasing radius, appears as a sudden rise at the radius to which the primary wave has propagated, followed by a slight drop off to a minimum at the point where the secondary flow begins to be felt, the radius to which pulses travelling with velocity, $a$, will have reached. From this minimum, there is a steady rise, with decreasing radius, to the high pressure at the bubble contact surface.

However, for low-energy explosions the released energy causes such a small pressure increment across the blast wave in a medium with such a low compressibility as water, that the wave travels with the sound velocity $a$, resulting in a merging of the primary wave and secondary flow. Hence, these measurements are essentially measurements of the pressure in the secondary flow of an underwater explosion. Such measurements are quite difficult in large scale high-energy blasts, since the measuring system must be sufficiently rugged to withstand the intense, sudden shock of the primary wave.

These explosions were generated by shattering glass spheres of one inch diameter pressurized to 500 p.s.i. and of two inch diameter pressurized to 300 p.s.i. Measurements were taken at 2, 5,
10 and 15 radii from the smaller spheres using air at the higher pressure and at 1 radius from the larger spheres, that is, within the expanding bubble, for helium, air and sulphur hexafluoride at the lower pressure. The order of magnitude of these explosions can be seen from their TNT energy equivalents. Based on $10^3$ cal/gm energy release of TNT, the TNT equivalent of a 1 inch diameter sphere pressurized with air at 500 p.s.i is 17.7 milligrams, and of a 2 inch diameter sphere pressurized with SF6 at 300 p.s.i. is 365 milligrams. These measurements supplement the more extensive photographic investigations of these explosions reported in Ref. 2.

2. PROBLEM OF SMALL PRESSURE MEASUREMENTS

In the laboratory, where many of the difficulties of installation and recording which occur in large scale field tests do not arise, the main problems to be considered involve signal distortions. Those of mechanical origin arise from the flow itself or from the mechanical response to the pressure pulse of the gauge in its mounting. Those of electrical origin may be spurious signals or distortions, arising from the "processing" of the signal, that is, in the amplification and the conversion from gauge circuit impedance to display circuit impedance.

2.1 Choice

Three pressure transducers were tried: a tellurium resistance gauge developed and used with good results by Dr. H. H. Hall, Physics Department, University of New Hampshire, a barium titanate piezo-electric hydrophone, BC-10, Atlantic Research Corporation and SLM piezo-electric quartz crystal transducers types 601 and 603, distributed by the Kistler Company. Low sensitivity to the small pressure changes to be investigated, and some irregularity in the output reduced the advantages of small flow disturbances offered by Hall's very compact gauge.

Although it offered ample sensitivity, the bulk of the BC-10 and the problem of satisfactory calibration made it less attractive than the smaller SLM gauges. The circuitry for the SLM quartz gauges (of very high impedance, $10^{14}$ ohms) and signal amplification is provided by a high frequency response (> 300 Kc/sec) charge amplifier (No. 566) giving an output to an oscilloscope over an amplification range from 1 to 100 millivolts/picocoulomb. Since the gauges produced about 1/2 picocoulomb/p.s.i., 5 millivolts/picocoulomb was generally sufficient amplification. The high gauge impedance, while providing a desirable long time constant, was a disadvantage for underwater work, since the slightest film of moisture across its coaxial connections could shunt the gauge signal. However this danger was reduced by sealing the gauge connections with water repellant dielectric grease.
(e.g. Dow Corning #4). When this occurred, gauge and cable had to be cleaned and baked for several hours at 250°F to restore its insulating properties.

The time constant for quartz gauges is sufficiently long to permit static calibration using a dead weight tester and an electrometer. Comparisons of static with dynamic calibrations in shock tubes of SLM gauges by other workers at UTIA have shown that they are in good agreement. Recalibration following a baking of the gauge showed that its sensitivity was unaffected.

2.2 Mechanical Distortions

Probably the most fundamental obstacle to a pressure probe type of measurement is the impossibility of inserting a transducer without flow disturbance. While in planar and cylindrical flows, measurements may be taken in a wall, beneath a boundary layer thus avoiding any additional flow disturbance, in a spherical flow there is no boundary layer. Two orientations of the gauge were considered, one in which the normal to the gauge face was tangent to the pressure wave, and the other with the gauge normal perpendicular to the pressure wave. The anticipated advantage of the former was that the gauge sensing element would not be exposed to the most disturbed portion of the flow, would not be liable to reflected waves and dynamic pressures \( \frac{1}{2} \rho \omega^2 \). However, it had the disadvantage of presenting a frontal area of about 6 times the size of the gauge face, which is the frontal area of the second orientation.

To investigate the flow disturbance caused by each gauge orientation spark schlieren photographs (Ref. 3) of the wave passing over the gauge were taken. Figure 1a shows a number of reflected waves emanating from the mount of the gauge facing across the flow. Figure 1b shows a slight lightening at the gauge face, of the gauge facing into the flow, but none of the photographs of this mount showed any reflected waves from the gauge as in Fig. 1a. Some caution had to be exercised in interpreting these schlieren results, since the schlieren system is sensitive to density gradients, and hence reveals pressure gradients only, not pressure itself. With this reservation these photographs were taken as evidence that the effect of the flow disturbance of the gauge facing into the flow was much less than the pressure to be measured and that it was significantly less than the disturbance due to the other mounting orientation.

The gauge face for the chosen orientation includes a stagnation region so that consideration of the dynamic pressure must be mentioned. The flow velocity decreases as the square of the distance from the bubble and hence is so low at 5, 10 and 15 radii (a maximum of 6 ft/sec at 5 radii) that the dynamic pressure is negligible in these
measurements, despite the large density of the medium. At 2 radii, the velocity is small most of the time but reaches 34 ft/sec at its maximum with a corresponding dynamic pressure which is 7% of the static pressure at that time. However, the measured pressure traces taken at two radii show no evidence of such an additional reading at the time corresponding to this maximum velocity.

The effect of reflected pressure on the gauge readings is not so easy to evaluate as that of dynamic pressure. It was expected that, while the initial recording of the incident pressure wave would be of reflected pressure, expansion waves from the lower pressure regions near the gauge would quickly relieve this higher reflected pressure. The small dimensions of the gauge seemed to warrant this assumption and the justification for it rests with the results. It is noted in Section 5.1 that this relief appears to be effective.

Another mechanical distortion, of a rather more technical nature, arose from the vibrations of the gauge and mount. In the preliminary trials, gauges were simply taped to steel rods and hung vertically in the flow. These resulted in an oscillation superimposed on the trace (Fig. 2a) of frequency about 35 Kc/sec which could be varied by rebinding the gauge. More rigid mounting did not seem to be the answer, as traces from a gauge rigidly mounted in a block on a radial sting (Fig. 1a) exhibited a similar oscillation of slightly higher (50 Kc/sec) frequency. The alternative appeared to be a mount which isolated the gauge from the radial sting, keeping the natural frequency of the isolated part as low as possible within the desired small geometry (Ref. 4). Such a mount is shown in Fig. 3; the gauge was set by a press fit in a teflon sleeve and the sleeve bounded by two O-rings. This assembly was inserted into the end of a hollow steel tube so that it is held by the compression of the O-rings alone, and the lead from the gauge brought out through the hollow sting. The O-rings thus serve as seals to keep the water out of the hollow sting and thus away from the gauge connection and also as vibration isolators from the sting. A simplified analysis predicting the natural frequency of the gauge and teflon sleeve, i.e. the isolated part, is given in Appendix A. This arrangement was successful in reducing, but not in eliminating entirely, the oscillations of frequency 20 to 40 Kc/sec. It was noted that sometimes the teflon sleeve would become jammed so as to be in contact with the steel sting and the corresponding pressure trace recorded from the gauge was afflicted with large amplitude oscillations and increased number of oscillations to damping. The reduction of this oscillation in most of the traces, and the apparent absence of oscillations in some (e.g. Fig. 10) suggest that vibration isolation is an effective way to deal with mount response distortions.
2.3 **Electrical Distortions**

Recent developments in electronics and transducer measurements have greatly reduced the laboratory problems in this area. The high impedance gauge circuit is protected by a low noise cable designed for this circuit by the Kistler Company, and the impedance reduction and signal amplification is provided by their charge amplifier. The main source of spurious signals was gauge ringing, an oscillatory signal arising from mechanical excitation of the natural frequency of the gauge crystal itself. This is illustrated in Fig. 2b and 2c by traces for the SLM 601 and 603 gauges which have natural frequencies of about 150 and 250 Kc/sec respectively. It is quite difficult to avoid exciting the crystals' natural frequency, since most of the disturbances to which the crystal is liable include it in their Fourier spectrum. The high frequency of this distortion and its relatively rapid damping make it much less serious than 30 Kc/sec vibrations. Due to its lower frequency, the 601 gauge was more liable to be excited and took longer to damp down than the 603 gauge. It was deemed sufficiently serious to warrant filtering it out with a low-pass filter. The effect of the filter was calibrated by simulating the charge development of the gauge in response to a pressure step pulse by the charging of a capacitor connected to a square wave voltage generator. The resulting trace, given in Fig. 2d, shows that the filtering produced an 8 μsec rise time for the square wave and an oscillatory overshoot. Although these effects are rather severe for a step pulse, it will be shown below that they will be much less important for the shape of pressure pulse to be measured.

The above mentioned method for calibrating the filter's effect was used, with the exchange of an audio oscillator for the square wave generator, to test the high frequency response of the circuitry. The attenuation at a particular frequency can be evaluated by comparing the amplitude of the circuit output signal as displayed on an oscilloscope with the input signal strength from the audio oscillator. e.g. for an input sine wave signal of one volt amplitude charging a 25 μ ū farad capacitor, and using 2 millivolt/picocoulomb amplification, the output signal with no attenuation would be 50 millivolts in amplitude. The charge amplifier showed no attenuation to 350 Kc/sec and the filter cut-off started at 110 Kc/sec and reached its peak at 125Kc/sec. The histories of primary interest were 500 μ sec long and the longest histories recorded were 200 msec. Hence low frequency response was not a problem and the drifts occurring in the amplifier were eliminated by using the A.C. mode of the oscilloscope input.
3. THEORETICAL CONSIDERATIONS

As outlined in the introduction, the explosions under consideration have such a low energy released that the pressure wave travels out with acoustic velocity. This suggests that, for the description of the initial pressure pulse, the bubble might be considered to be a spherical acoustic source. The pressure felt, after the propagation time \( (y-r)/a \), at radius \( y \) from a bubble of radius \( r \) and at pressure \( p_3 \) is

\[
p(y) = p_1 + \left( \frac{r}{y} \right) (p_3 - p_1)
\]

While this would provide the size of the initial pressure pulse received at a gauge and its arrival time after the instantaneous release of a pressure from a glass sphere, it does not predict the pressure history at the gauge, nor would it be sufficient if the pressure history in the interior of the gas bubble were known, since it neglects the effect of the velocity field of the expanding bubble.

The problem of describing the flow field of a gas bubble expanding in an infinite extent of water, which is the secondary flow, is much more amenable to incompressible theory than the primary wave of an underwater explosion, which is strictly a compressible phenomenon. It predicts an \((r, t)\) - path for the expanding bubble under the assumption of uniform pressure within the bubble and isentropic expansion. The velocity field of this expansion can be used in Bernoulli's equation to predict the pressure field. Hence incompressible theory provides two relations, Eqs. (1) and (2), for which the natural independent variable is \( r \), the radius of the expanding gas bubble:

\[
\begin{align*}
t &= g(r) \quad (1) \\
p &= h(y, r) \quad (2)
\end{align*}
\]

where \( t \) is the time at which the bubble has expanded to radius \( r \), and \( p \) is the pressure at field point \( y \), of a bubble of radius \( r \).

The parameters in these relations are determined by the initial conditions of the expanding bubble such as \( p_4 \), its initial pressure, \( r_0 \), its initial radius, and \( \gamma \), the isentropic index of the gas. The pressure history at a particular radius, \( y \), of a bubble expanding according to Eq. 1 is

\[
p(y, t) = h \left[ y, g^{-1}(t) \right]
\]

where, \( r = g^{-1}(t) \) is the inverse functional relationship to Eq. (1)

and this will be referred to as the incompressible pressure history. However, it is important to realize that if the bubble were compelled to
expand along some other \((r, t)\) - path

\[ t = f(r) \]  \hspace{1cm} (4)

that the incompressible relation Eq. (2) would provide an estimate of the pressure history as

\[ p(y, t) = h\left[ y, f^{-1}(t) \right] \]  \hspace{1cm} (5)

Relations (1) and (2) are developed rigorously in Ref. 2, with the introduction of nondimensional variables

\[ \begin{align*}
X &= r/r_0 \\
Y &= y/r_0 \\
\zeta &= t/r_0 \sqrt{\frac{\alpha}{P_1}} \\
\rho &= \frac{p}{p_1} \\
\alpha &= (\chi - 1)P_{14} \\
P_{14} &= \frac{P_1}{p_4}
\end{align*} \]  \hspace{1cm} (6)

as

\[ \begin{align*}
\zeta &= \int X \frac{ds}{\sqrt{\frac{2}{3\chi} \left[ (1 + \chi) \zeta^{-3} - \zeta^{-3\chi} \right] - \frac{2}{3}}} \\
P &= \frac{\chi}{\gamma}\left\{ P_{4} X^{3\chi} + \frac{1}{3\chi} \left[ (1 + \chi) X^{-3} X^{-3\chi} - \chi^{-3} \chi^{-3\chi} \chi^{-3\chi} \right] \right\} + 1
\end{align*} \]  \hspace{1cm} (7)

Both the model of the acoustic source, and of the isentropic bubble expanding in an incompressible medium predict the same initial pressure increment to be felt at a gauge in the flow field. The former predicts that it will be felt after a propagation time which depends on the acoustic velocity, while the latter predicts its instantaneous penetration of the whole field corresponding to the limit of infinite sound speed for incompressible theory. A simple expedient to compare experimental measurements with theoretical predictions is to add a constant propagation time to the incompressible history so that Eq. (3) becomes

\[ P\left[ y, t + \frac{(y - y')}{a} \right] = h\left[ y, f^{-1}(t) \right] \]  \hspace{1cm} (9)

While this procedure is theoretically indefensible, it provides a closer approximation to the observations than the infinite propagation velocity required by incompressible theory, and it does not alter any feature of the incompressible pressure history other than its time origin.

3.2 Viscous and Thermal Energy Absorption in Acoustic Propagation

Due to the small variation of sound speed at low pressures in water, a compression wave requires a much greater distance to develop into a shock wave in this medium than in a gas (Ref. 5). This
distance is sufficiently large for the relatively small pressure waves generated in this experiment that the action of viscosity and thermal conductivity become significant in modifying the shape of the pulse. Since the effects of both are similar and of the same order of magnitude, a description of the influence of viscosity will illustrate the principle. In travelling a distance $r$, a spherical sinusoidal pressure fluctuation of frequency $\omega$ is reduced in amplitude by the factor

\[
\frac{e^{-\alpha r}}{r}
\]

where $\alpha = -\frac{2}{3} \frac{\nu \omega^3}{\alpha^2}$, $\nu = \text{kinematic viscosity.}$

Hence, for a pressure pulse that can be decomposed into its Fourier spectrum, the higher frequencies of the spectrum will be absorbed more rapidly than the lower, effecting a broadening of the pulse rise and rounding of its peaks. It is found (Ref. 6) that the absorption coefficient, $\alpha$, in water with dissolved salts is about 10 times what is theoretically predicted by the viscosity of pure water. This effect has been observed in the measurement of small pulses propagated long distances (Ref. 1) and is commented on in Section 5.1.

4. EXPERIMENTAL CONSIDERATIONS

4.1 Experimental Procedure

The explosions were performed in the UTIA shock sphere (Ref. 6), a hollow steel sphere of three feet inner diameter, with glass windows one foot in diameter, to permit optical observations (Fig. 4). Its operation is perfectly analogous to that of a planar shock tube and, along with the photographic apparatus is described in considerable detail in Refs. (2 and 7). The method of generating blasts by breaking pressurized glass spheres, the spherical analog of a shock tube diaphragm, has the distinct advantage for low-energy explosions of specifying very accurately the initial conditions of the blast.

It was pointed out in Section 3 that incompressible theory provides two relations with $X$, the bubble radius, as the independent variable; one gives the bubble expansion path in the $(X, \tau)$-plane; the other gives the instantaneous pressure field; and that these two were combined to give the pressure history at a point in the field. To investigate these two relations, coupled measurements of the bubble expansion and pressure history were taken by a drum camera record and the gauge trace, respectively. When the sphere was pressurized and the drum of the camera was spinning behind its shutter, the sequence of events was as follows: The guillotine drum-camera shutter was released, tripping a microswitch on its fall. This switch sent a voltage pulse to activate the combustion breaker of the glass sphere, initiating the explos-
ion. The shutter, at this point in its fall exposed the drum camera film, for a brief but pre-determined period (14 msec). Early in this interval the glass sphere was shattered and a circuit painted on it was broken. The resulting voltage pulse was used to trigger the oscilloscope trace on which the gauge signal was displayed and also to trigger a series of timing sparks at set intervals to mark the film with a time calibration. A detailed description of these events and the components of the apparatus involved is contained in Ref. 2.

Two gauges were set on either side of the glass bulb at positions determined by scales marked on the stings and checked by the optical system for the cameras. The positions investigated were 1, 2, 5, 10 and 15 radii from the bulb. For the runs other than at one radius, the procedure was to set the gauges at two different radii, do a run, reverse the positions of the gauges and repeat the run. A photograph of a pre-run set up inside the shock sphere is shown in Fig. 3. Two oscilloscope traces were made from the output of each gauge one swept at a fast speed, usually 50 \( \mu \)sec/cm, which gave the pressure history for the undisturbed test time (described in sect. 4.2), and the other at a slower speed, 500 \( \mu \)sec/cm to 2 msec/cm, which showed qualitatively the pressure feature of the first bubble pulsation, and the various reflected waves.

The measurements at one radius are essentially measurements of the pressure within the expanding gas bubble. Initial attempts to do this by placing the gauge in contact with the pressurized sphere resulted in excessively high readings, which were diagnosed as being produced hydraulically by glass fragments lodging on the gauge diaphragm. To avoid this, the gauge was fitted with a small brass collar, cut on a 70° bias such that it protruded from the gauge 1/16 of an inch at its maximum overlap and receded from it by the same amount at its minimum. This relieved the gauge diaphragm and the additional flow disturbance was assumed negligible when compared with that due to the glass particles. The internal (one radius) measurements were all taken with the high frequency response, unfiltered SLM #603 gauge, with the #601 gauge positioned at 5 radii. While the gauge positioned at 2 radii also enters the bubble, it does not do so until after the pressure history to be measured is past, so that it is considered to be an external measurement.

4.2 Dimensional Effects of the Initial Bubble Radius

The initial bubble radius, \( r_0 \), appears in both the nondimensional spatial and time co-ordinates. While the theoretical discussion assumed an infinite extent of water, the experiments have been performed in a tank with both rigid and free boundaries, which affect the pressure field directly through reflected waves and also indirectly through their effect on the migration of the bubble. To reduce the influence of the
boundaries, it is desirable to use small $r_o$ values, to increase the effective distance of the boundaries, and to reduce the time scale so that as much of the flow as possible occurs before the arrival of the reflected waves.

The trend to small values of $r_o$ is balanced by several factors. As the spatial scale is decreased the effective gauge size increases. For smaller spheres, the ratio of glass to internal volume grows, so that the fraction of energy required to break and accelerate the glass fragments increases. Also the manner in which the bulb shatters changes with the curvature of the glass; for small $r_o$, it was found that the sphere shattered into fewer and hence larger pieces, causing increased interference with the flow. The influence of viscosity increases with decreasing radius as can be seen from the definition of the secondary flow Reynolds number as

$$Re = \frac{ar_o}{\nu}$$  \hspace{1cm} (11)

The effect of changing $r_o$ on the time and space scale is illustrated by the two traces in Fig. 5. Both are measurements made at 7 radii from a sphere initially charged to 498 p.s.i. Figure 5(a) shows traces that were taken at 3 1/2 inches from the centre of a 1/2 inch radius sphere and those in Fig. 5(b) were taken at 7 inches from the centre of a 1 inch radius sphere. Both signals have received the same amplification. The upper trace of each was swept at 2 msec/cm. The upper trace of Fig. 5a shows that the pressure pulse which marks the end of the first bubble pulsation occurs at 11.9 msec for the smaller sphere, while the upper trace of Fig. 5b shows that the bubble period is longer than 20 msec for the larger sphere. The lower traces swept at 50 $\mu$sec/cm show that the peak pressure received by the gauge are equivalent; however, the fall off in Fig. 5a is more rapid than Fig. 5b. The lower trace in Fig. 9b shows a longer propagation time for the pulse to travel to the gauge and the early arrival of the reflected compression wave, (D), from the rigid boundary at 490 $\mu$sec from the start of the sweep which does not appear in lower trace of Fig. 9a, where the pressure fall off is nearly complete.

The first reflected wave to reach the gauge, thus ending the test time, is not the reflected compression wave, (D), seen in the lower trace of Fig. 5b, but a rarefaction wave, (R), reflected from the free surface at the top of the tank. Its arrival is not so apparent as the compression wave, but it arrives at 380 $\mu$sec in Fig. 5b and 330 $\mu$sec in Fig. 5a.

It was shown in Fig. 5, that for a sphere of 1/2 inch radius, the pressure drops to about 1/2 peak pressure in the available test time at 7 radii. This was considered a sufficient range over which to compare the experimental pressure history with the incompressible one, so that bulbs of 1/2 inch radius were chosen for the runs in which measure-
ments of the external field were to be made. As will be shown below, (section 5.2) the interior pressure of the bubble was not affected by reflected waves (although the total bubble migration was) so that the use of larger spheres, 1 inch radius, to reduce the effective gauge size, was permissible for measurements of the pressure within the bubble. The pressures used, 500 psi with the smaller spheres, 300 psi with the larger spheres were chosen to complement the photographic investigations made earlier (Ref. 2) and were originally determined by the breaking qualities of the spheres at different pressures.

5. RESULTS AND DISCUSSION

The measurements to be discussed were the results from two series of runs, one for measurements of the external field and one for measurements inside the expanding gas bubble, which followed a number of preliminary experiments for developing the measuring system and investigating some of the features discussed below individually.

5.1 External Measurements

In Fig. 6, two typical traces are shown and plotted in reduced form; Fig. 6a is run #P-57B taken at 2 radii with SLM#603 gauge and Fig. 6b is run #P-84A taken at 10 radii with SLM #601 gauge. The reduction has been made to the non-dimensional variables defined in Section 3.1; and the relation between dimensional and non-dimensional variables is given for each run; the solid line is the corresponding incompressible pressure history with its time history shifted by the propagation time (Section 3.1). The dotted line is explained below and the symbols represent measurements taken from the continuous oscilloscope trace. The two features of these plots which deserve comment at this point are the peak pressure received by a gauge, and the rise time of the pulse.

While the gauge ringing obscures the initial part of Fig. 6a, it can be seen in Fig. 6b that the peak of the initial rise overshoots the anticipated pressure and this is followed by a considerable drop and a damped oscillation. It is believed that the overshoot is largely due to the measurement of reflected pressure and the ensuing drop is due to the relief of the local high pressure at the gauge face by an expansion wave from the surrounding water plus the onset of the mount response effect. The damped oscillation is believed to be due to the mount's slight vibration alternately compressing and relieving the water
at the gauge face (Section 2.2). This oscillation has the frequency observed in the preliminary measurements taken with a gauge mounted rigidly on a radial sting; the oscillations in these preliminary runs being of considerably larger amplitude and persisting through more cycles. Since the natural frequency of the gauge and its teflon sleeve resting on the O-rings is estimated to be about 1/10 of this frequency (Appendix A) it is concluded that the observed oscillations are due to incomplete isolation from the radial sting.

The drop-off of pressure behind the primary wave of an intense underwater explosion can be approximated down to .7 of peak pressure very well by a simple exponential decay (Ref. 1)

\[ P(t) = P_m e^{-t/t_0} \]  

(12)

Where \( P_m \) = maximum pressure rise

In practice, this curve is often fitted to experimental results and the parameters \( P_m \) and \( t_0 \) are used to characterize the explosive. However, in the present blasts, where the primary wave collapses on the secondary flow, the pressure history is less steep initially, with zero slope at the peak pressure. Hence this drop off can be approximated better by

\[ P(t) = P_m e^{-t^2/\theta^2} \]  

(13)

* For a gauge displacement, \( x = A \sin2\pi \omega t \) ft., i.e. a vibration at \( \omega \) cycles/sec of amplitude. A ft. the particle velocity at the gauge face is \( u = \dot{x} = A 2\pi \omega \cos2\pi \omega t \) ft/sec.

For \( \omega = 25 \text{ Kc/sec} \), the maximum particle velocity at the gauge face is \( 15.7 \times 10^4 \) A ft/sec.

The corresponding dynamic pressure recorded by the gauge in such a vibration is

\[ \frac{1}{2} \rho \omega^2 = \frac{1}{2} x 1.937 x (15.7 x 19^4 x A)^2 \psi \text{ (} \rho = 1.937 \text{ slugs/ft}^3) \]

\[ = 1.66 x (A x 10^4)^2 \psi \]

Hence for \( A = \frac{2 \times 10^{-3}}{12} \) ft = 2 thou

\[ \frac{1}{2} \rho \omega^2 = 4.6 \psi \]
This form was fitted to the data by a least squares method applied to the logarithm of the expression. To check the consistency of this method, Eq. 13 was fitted to the incompressible pressure histories, and it was found that the resulting $P_m$ was uniformly less than the peak pressure of the profile by 10 percent. The fit to the theoretical histories is shown as the dotted line in Fig. 6. Figure 7 is a plot showing the variation of peak pressure with radius. The solid line represents the variation predicted by incompressible theory, which is identical to the acoustic variation, for an initial bubble to hydrostatic pressure ratio $P_41 = 33.33$. The dotted line represents the peak pressure, $P_m'$ estimated by fitting Eq. (13) to the pressure histories represented by the solid line. The symbols represent the measured peak pressures as estimated by Eq. 13 fitted to the reduced measurements. The abscissae of the points do not coincide since there are small differences in $r_0$ and hence in the non-dimensional gauge distance $Y$. It can be seen that the variation in peak pressure with distance is essentially acoustic and that the agreement improves at further distances. This improvement with distance is more likely due to the improvement of the fit of equation (13) to the shape of the pressure histories at farther radii than to better agreement between actual and theoretical peak pressures.

In all the measurements, the traces show a rise time for the pulses which varies from 10 to 40 $\mu$sec. (This provides some justification for the use of a filter with an 8 $\mu$sec rise time on the SLM#601 gauge). This is hard to explain however, by the model of the bubble initially as a spherical acoustic source of radius $r_0$ and of pressure, 500 psi. An explanation of the initial shape of the wave ought to be available from considering the manner in which the bulb breaks. Examination of multiple spark schlieren photographs reveals that the initial pulses emanate from the bottom of the sphere, where the mallet which breaks it, strikes. The cracks must then propagate with a velocity of about 10,000 ft/sec (Ref. 8) around the sphere to the top. For a 1/2 inch radius sphere this process would be expected to require about 13 $\mu$sec.

It may be noted that this breaking time is proportional to the bulb radius and hence its effect cannot be reduced by using a bulb of larger radius to increase the time scale of the explosion. Figure 1b shows that the early pulses travelling outwards from the sphere are not concentric, which provides evidence of this manner of breaking. Thus it can be seen that the bulb initially resembles a distribution of spherical acoustic sources of much smaller effective initial radius than $r_0$, spread over the surface of the glass bulb, the geometrically higher ones coming into action at slightly later times than the lower ones. The superposition of these waves is believed to comprise the pressure pulse and the bubble can only be considered an acoustic source of radius $r_0$ after the breaking is complete and before expansive flow becomes significant. This also explains the considerable variation in rise times observed, since the rise time is
controlled by the manner in which the sphere shatters, a process which is liable to significant variation.

Although the rise times vary considerably from run to run, a comparison of two traces from a single run shows that invariably the measurement taken at a greater distance from the explosion has a longer rise time than a close measurement. The table below affords such a comparison. This spreading has been ascribed to the action of viscosity in spreading such pulses as described in Section 3.2.

**TABLE I**

<table>
<thead>
<tr>
<th>Run</th>
<th>Near Gauge (radii)</th>
<th>Rise Time (µsec)</th>
<th>Far Gauge (radii)</th>
<th>Rise Time (µsec)</th>
</tr>
</thead>
<tbody>
<tr>
<td>P-52</td>
<td>10.6</td>
<td>10</td>
<td>16.0</td>
<td>13</td>
</tr>
<tr>
<td>P-55</td>
<td>10.6</td>
<td>37</td>
<td>15.9</td>
<td>45</td>
</tr>
<tr>
<td>P-56</td>
<td>2.1</td>
<td>20</td>
<td>5.3</td>
<td>20</td>
</tr>
<tr>
<td>P-57</td>
<td>2.1</td>
<td>15</td>
<td>5.3</td>
<td>47</td>
</tr>
<tr>
<td>P-58</td>
<td>2.1</td>
<td>10</td>
<td>5.3</td>
<td>20</td>
</tr>
<tr>
<td>P-78</td>
<td>1.1</td>
<td>20</td>
<td>5.7</td>
<td>45</td>
</tr>
<tr>
<td>P-81</td>
<td>1.1</td>
<td>24</td>
<td>5.4</td>
<td>30</td>
</tr>
<tr>
<td>P-82</td>
<td>1.1</td>
<td>40</td>
<td>5.3</td>
<td>44</td>
</tr>
<tr>
<td>P-84</td>
<td>10.5</td>
<td>25</td>
<td>15.6</td>
<td>30</td>
</tr>
</tbody>
</table>

The measured pressure histories at 2 radii, 10 radii and at 5 and 15 radii are plotted in Figs. 8, 9, and 10, for comparison with the incompressible pressure histories. The measurements are taken from the continuous pressure traces at regular time intervals, generally 25 µsec apart. However, due to the variation in \( r_0 \), the measurements for different runs do not coincide on the non-dimensional plot. In a region of gauge ringing the mid-point of successive oscillations was used as the measurement. The measurements of smaller pressures at farther radii were amplified, so that on the oscilloscope traces the peak pressures covered about 2.5 cm. Since the measurements can be made repeatedly with a variation of 1 mm, or less, the measuring accuracy is estimated to be 4% of the peak pressure. The same symbol is used in each of these figures for measurements taken with the other gauge on the same run.

An example of combined \((r, t) \) \((p, t)\) - measurements is shown in Fig. 11 for run #52. The gauge at 10 radii, (5 inches), appears at the bottom of the drum-camera photograph, while the gauge at 15 radii is off the field of view. A short sweep delay has been used with the trace at 15 radii so that the full propagation time is not included. The arrival of the initial pressure wave, \( C \), the rarefaction wave from the surface,
R, (at 390 μ sec at 10 radii and at 290 μ sec at 15 radii) and the reflected pressure wave, D, at 15 radii can all be seen on the lower sweeps of each trace (50 μ sec/cm). The (r, t)- trace shows the pressure wave reflected from the boundary imploding and reflecting from the bubble for the first time at 600 μ sec. The change of sign of the wave reflecting from the bubble is included in a discussion in Section 5.2, but is worth noting here. The shading of the pulse (i.e., black at the top of the picture) is the same when it is incident on the bubble as when it is reflected. This implies that the density and hence pressure gradients have the same direction in both, although the former is imploding and the latter exploding. Hence the signs of the pressure changes through the waves are opposite, that is, the reflected wave must be a rarefaction wave. The density gradients of the rarefaction wave reflected from the free surface of the water appear to be too slight to appear on the schlieren photograph. The imploding wave which appears at about half the time of the main reflected wave is believed to be caused by a vibration of the tank at the time of breaking. Although it appears to have sharp density gradients, the measurements show that it does not represent an appreciable pressure wave.

A discussion of the agreement of these (P, τ)- measurements with the theoretical curves requires consideration of the combined (X, τ), (P, τ)-plots in Figs. 12 and 13 which show theoretical curves and runs P-56 and P-84 as examples. On these graphs, τ is used as the ordinate while, on the scale at the right hand side, P is the abscissa and, on the left hand side, X is the abscissa. The solid lines are the incompressible bubble path in the (X, τ)- plane and the pressure histories at two different radii, neglecting the propagation time of the pulses, in the (P, τ)- plane, i.e., the time origin of the (X, τ)- graph is the initial bursting of the bulb and of the (P, τ)- graph is the arrival of the first pulses. The dotted lines along side the incompressible pressure histories mark the pressure history as predicted by the incompressible relation Eq. 5 of Section 3.1 using for f⁻¹ (t) the experimentally measured (X, τ)- relation. The pressure predicted by Eq. 2 (written explicitly as Eq. 8) for a given bubble radius is given by the pressure which appears on the solid line pressure history on a constant time line passing through this radius in the incompressible bubble path. Hence the dotted line may be plotted as follows, using Fig. 13 for example. The radius measured at A, at time τₐ, is predicted to be reached at time τ₉, at point B. The corresponding pressure at 10 radii for this radius of the bubble is given at C, hence this pressure, according to the measured (X, τ)- plot, must have occurred at time τₐ, thus giving D as a point of the dotted line.

These figures show that the bubble expansion rate tends to be somewhat faster than the incompressible prediction. Other investigations (Ref. 2) of the complete bubble pulsation substantiate this result
for the early portion of the expansion which is dealt with here, but show
that the agreement between experimental measurements and incompress-
ible theory improves greatly at later times when the pressure has become
lower. The effect of this higher than predicted expansion rate is observed
in the measured pressures being lower than the incompressible pressure
histories as examples in Fig. 12 and 13 for 2, 5 and 15 radii show. The
measurements at 10 radii do not show the expected result, nor can this
trace be treated as exceptional since, as Fig. 9 shows, the results at
10 radii are quite consistent. Although the results at 15 radii show some-
what better agreement, Fig. 10 shows that they also have a tendency to
lie on or slightly above the incompressible curve rather than below it.
While no satisfactory explanation for these deviations at farther distances
has been produced, it is recognized that the more gentle slope (dp/dt) of
the farther distances reduces the influence of the expansion rate, and the
smaller pressures involved are more vulnerable to perturbations than the
larger pressures nearer to the gas bubble. The shape of the pressure
histories at 10 and 15 radii also deviate slightly from the theoretical
prediction. This can best be seen by reference to the photographs of the
continuous pressure records of Fig. 11. At both 10 and 15 radii the
traces show a slight hump, covering about 125 μsec after the sharp
rise to "peak pressure". It is suggested that this may be due to an inade-
quacy of equation 2 (written explicitly as equation 8) to describe the
pressure history for large Y (i.e. at distant field points).

The longer time sweeps, as illustrated by the upper traces
of Figs. 5 and 11, show the expected features of an underwater blast.
In the early part of its expansion the bubble is at high pressure and trans-
mits this pressure to the water while accelerating it radially outwards.
This is the portion of the pressure history under quantitative study, but
be seen to be only a short part of the pressure history of a complete
bubble pulsation. After the bubble has expanded to hydrostatic pressure,
the inertia of the water maintains its velocity, so that the bubble is forced
to overexpand and decelerates the water as its pressure drops below
hydrostatic and it transmits subhydrostatic pressure into the water. This
period of deceleration and subhydrostatic pressure lasts much longer than
the positive pressure phase, and is the major feature of the 2 msec/cm
time sweeps. The traces however are disturbed by the reflected waves
which rattle back and forth in the tank as shown in the (r, t)-photograph
of Fig. 11. The smooth subhydrostatic portion of the upper trace of
P-57 in Fig. 6, taken at 2 radii, results from the gauge having been
within the bubble, the shielding effect of which will be described in
Section 5.2.

After the bubble has expanded to its maximum, the continued
deceleration produces an inward radial velocity and again the inertia of
the water maintains its velocity past the attainment of hydrostatic pressure
within the bubble. The bubble is then compressed until it reaches a
minimum, completing one pulsation, and is again transmitting high pres-
sure into the water. This point of the pressure history is marked with a T on the figures in which it appears. While these features are qualitatively apparent, the reflected waves reduce their quantitative value. The phenomena of the explosion which follow the phase of positive pressure can be more easily investigated photographically and discussion of the complete bubble expansion and contraction, period, and migration are included in Ref. 2.

5.2 Internal Measurements

The most sensitive investigation of the bubble expansion rate comes from measuring the pressure decay rate within it. In addition to measuring this, the internal measurements examine the assumptions made for the incompressible treatment of the bubble of isentropic expansion and of uniform pressure within. Equation 8, for Y=X, i.e., for the field point coinciding with the bubble surface, reduces to the isentropic expansion relation

\[ P = P_{41} X^{-3b} \]  

(14)

Substitution of the incompressible \((X, \zeta)\)-relation, Eq. 7, into Eq. (14) gives the adiabatic expansion rate for a bubble in an incompressible fluid. This \((P, \zeta)\)-curve is shown as the solid line in Fig. 14 for air, and for helium and sulphur hexafluoride in Fig. 15.

The measurements plotted in these figures were taken with initial conditions of 1 inch radius spheres pressurized to 300 psi, and have been reduced to the same nondimensional variables \(P, X\) and \(\zeta\). On these traces, the rise times were similar, but generally somewhat shorter (10 to 20 \(\mu\)sec) than in the external measurements and the gauge ringing tended to be stronger, although it damped out sooner on the larger non-dimensional time scale. The longer period oscillations attributed to the mount response were not observed for the gauge within the bubble presumably due to the low density of the gas. (See Section 5.1).

Since the glass bulbs were about 0.05 inches thick and the gauge was held off from the bulb 0.063 inches by the protective collar, the gauge did not enter the bubble until it had expanded to about 1.05 radii, corresponding to \(\zeta = 0.074\) or \(t = 185 \mu\)sec on the incompressible bubble path, and generally measured as slightly earlier, on the drum camera records.

The acoustic relations giving the transmitted pressure, \(P_t\), and reflected pressure, \(P_r\), of a pressure pulse, \(P_i\), incident from medium 1 on an interface between medium 1 and medium 2 are (Ref. 9)
\[
\frac{P_t}{P_i} = \frac{2 \rho \alpha_2}{\rho_1 \alpha_1 + \rho_2 \alpha_1}
\]

(15)

\[P_i + P_r = P_t
\]

(16)

assuming both media to be perfectly elastic. Hence for an air to water interface, \(\rho \alpha_2 >> \rho_1 \alpha_1\) and \(P_t \approx 2 P_i, P_r \approx P_i\); while for pressure pulses incident on the bubble from the water, (15) and (16) give \(P_t \approx 0, P_r \approx -P_i\).

The delay of the gauge in entering the bubble thus enhances the chances of observing any wave phenomena inside the bubble since they will occur either while the gauge is still in the water and the pressure pulse transmitted to the water is twice that incident on the inner surface of the bubble, or shortly after the bubble surface passes over the gauge, so that it will receive the combined effect of waves incident on and reflected from the surface of the bubble.

It had been anticipated that the assumption of uniform pressure within the bubble might be threatened by the presence of a slight rarefaction wave imploding into the bubble when the glass bulb broke and released the bubble pressure, (Ref. 10 and 2). While some of the traces showed evidence of a slight rarefaction arriving at approximately the arrival time predicted by the sound speed of the gas, the variation in arrival time and the small size of the effect must lead to the conclusion that it is not a consistent feature of the internal pressure history. As with the initial shape of the pressure, this phenomenon is determined by the manner in which the sphere breaks and the variation in it is attributed to the aforementioned variation in sphere rupture. In Fig. (16) two examples of this are given, one for air and one for helium. The trace for air was made on a run in which the glass sphere was broken prematurely at 250 psi by the force of the gas, rather than by the breaking mallet. Since straining the glass to its limit gives the optimum breaking qualities, the resulting trace has a very sharp rise, 4 \(\mu\) sec, and exhibits the rarefaction wave at 140 \(\mu\) sec compared to a predicted arrival time of 156 \(\mu\) sec. The initial conditions were less ideal for the helium trace, however a slight drop can be observed at 65 \(\mu\) sec while the predicted arrival time is 58 \(\mu\) sec. Unfortunately the arrival of the rarefaction wave from the surface of the water very early coincides with the arrival time of a rarefaction wave from within the bubble at 390 \(\mu\) sec for SF\(_6\), so that the disturbances observed at this time are open to question, depending on whether the gauge is inside or outside the bubble.

Equations (15) and (16) also show that no waves incident on the bubble from the water are transmitted so that reflected waves from the boundary are reflected again from the bubble with a change of sign. This effect has been commented on in the discussion of Fig. 6, Sec. 5. 1. The traces show that in fact no reflected waves do affect the gas within
the bubble.

The measurements were reduced in the same manner as described in Section 5.1. As shown in Figs. 14 and 15, they follow generally the incompressible expansion rate, the helium and air measurements lying slightly below the incompressible curve and the sulphur hexafluoride results lying above it and showing some irregularity. The variation of decay rate with \( \delta \) is evident, the lower \( \delta \) giving the slower decay rate in accordance with the higher internal energy of the expanding gas. The coupled \( (P, \zeta), (X, \zeta) \)-graphs for the internal runs appear in Figs. 17 and 18. Again the dotted line represents the pressure history resulting from the measured \( (X, \zeta) \)-path assuming isentropic expansion.

It is believed that the irregularities of the SF\(_6\) pressure record arise from positioning the gauge slightly farther from the bulb than usual, so that the bubble surface did not pass over it until \( \zeta = 130 \). The sudden drop at \( \zeta = 124 \) is believed to be the arrival of the rarefaction wave from the surface of the water. The agreement between the measurements and the dotted line in the cases of helium and air indicate that the deviation of the measured pressures from the incompressible history is largely accounted for by the deviation of the actual \( (r, t) \)-path from that predicted by incompressible theory. It was found that the \( (X, \zeta) \)-results for the larger spheres at 300 psi show more consistency, and better agreement with the incompressible \( (X, \zeta) \)-path than do the results of Section 5.1 using smaller spheres and 500 psi. This was generally found to be true in the experiments performed in this spherical tank. However, as outlined in Section 4.2, the larger spheres were not used for external measurements since they did not provide sufficient test time for the measurements to be made of the external pressure field.

6. CONCLUSIONS

It has been shown that the bursting of small glass spheres under pressure by a gas provided a convenient, safe and repeatable method for studying low-energy underwater explosions. The technique of mounting gauges on vibration isolators appears to be a satisfactory method of removing the gauge from the influence of the mount. The dimensions of the spherical tank form a spatial limitation on the experiment, since they do not permit the use of larger glass spheres, with their more consistent performance, for studying the external pressure field. The early pressure history of the explosion is limited also for the comparison of theory with experiment to times greater than the breaking time of the sphere. While the initial portion of the pressure record is merely a reflection of the rupturing process, the influence of acoustic absorption due to viscous and thermally conductive dissipation during propagation was evident in modifying the initial shape of the pulse. While the glass fragments of the shattered sphere posed a small technical problem when placing a gauge so that it entered the expanding gas bubble, their influence on the flow itself was not
Some discrepancies between the results and the predictions of incompressible theory with the added propagation time were observed even at this low pressure range. The main difference occurred between the measured and predicted bubble expansion rates and appeared to be due to the compressibility of the water. When this effect was taken into account, the internal measurements showed that the assumption of isentropic expansion was, as far as could be determined, justified, and that the assumption of uniform pressure within the bubble was quite satisfactory. With some reservations as to early expansion rate, then, it may be said that the agreement with incompressible theory in the range of pressures investigated is quite fair.
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APPENDIX A

Simplified Analysis of Mount Vibrational Frequency

The object of vibration isolation is to ensure that the highest natural frequency of the isolated body is lower than the driving frequency to which it will be subjected.

For a body of mass \( m \), length \( l \) and radius \( r \) surrounded by a cylindrical sheet of rubber of thickness \( y \), which is rigidly attached at its external radius, the differential equation of free motion for small displacements in the axial direction \( x \), is given by

\[
\frac{m}{2} \frac{\partial^2 x}{\partial t^2} = -KA \frac{\partial x}{\partial y} \tag{A.1}
\]

where, \( A = 2 \pi r l \)

\( K = \) dynamic shear modulus of rubber \( K = K(\omega) \)

Hence for a free oscillation, in which \( x = x_0 e^{i\omega t} \)

\( \omega = \) natural frequency of the system

\(- m\omega^2 x \equiv -KA \frac{x}{y} \) for small \( x \) and \( y \)

i.e. \( \omega = \sqrt{\frac{KA}{m y}} \) The natural frequency of the body is thus reduced by reducing \( A \) and increasing \( m \) and \( y \)

At 10 Kc/sec, \( K \equiv 100 \text{ mega dynes/cm}^2 \) (The Handbook of Physics and Chemistry)

for \( m = 3.2 \text{ gms} \) \( y = .055 \text{ inches (O ring thickness} \)
\( A = .161 \text{ cm}^2 \) \( = .140 \text{ cm. minus compression} \)
\( \omega = 6 \text{ Kc/sec.} \)
APPENDIX B

A Least Squares Iteration Method

A simple analytic curve which would approximate the early pressure history over a reasonable length of time and improve the prediction of peak pressure was desired.

The form often used for strong blast waves

\[ P(t) = P_m (1 - t/t_0) e^{-t/\theta} \]  

was considered and a method for fitting it to experimental data, developed.

The results of using Eq. (B.1), and of a modification of it replacing the exponential factor by \( e^{-t^2/\delta^2} \) did not, however, provide a marked improvement over the simple exponential equation given in Section 5.1. However, since this form has application for stronger pressure waves, the method developed has been included in this appendix. The standard least squares procedure is applied to the logarithm of Equation (B.1)

\[
Y(t) = \log P(t) = \log P_m + \log (1-t/t_0) - t/\theta
\]

\[ = A + \log (1-t/t_0) - Dt \]

where \( D = 1/\theta \)

\[ A = \log P_m \]

From this, it is obvious that the argument applies for any form in which the exponent of e is linear in t.

Let the experimental points be denoted by \((Y_i, t_i)\), where \(i\) ranges from one to \(n\). The least squares criterion is that \(P_m, t_0\) and \(D\) must be chosen so that

\[
I = \sum_{i=1}^{n} [Y(t_i) - y_i]^2 \quad \text{is a minimum.}
\]

The corresponding normal equations are

\[
\frac{\partial I}{\partial P_m} = 0 \quad \text{or} \quad n A + \sum_{i=1}^{n} \log \left(1 - \frac{t_i}{t_0}\right) - D \sum_{i=1}^{n} t_i = \sum_{i=1}^{n} y_i
\]  

\[
\frac{\partial I}{\partial t_0} = 0 \quad \text{or} \quad A \left( \sum_{i=1}^{n} \frac{t_i}{1 - t_i/t_0} \right) - D \sum_{i=1}^{n} \left( \frac{t_i^2}{1 - t_i/t_0} \right) = \sum_{i=1}^{n} \frac{t_i}{1 - t_i/t_0} \left[ y_i - \log \left(1 - \frac{t_i}{t_0}\right) \right]
\]  

\[
\frac{\partial I}{\partial D} = 0 \quad \text{or} \quad A \sum_{i=1}^{n} t_i - D \sum_{i=1}^{n} t_i^2 = \sum_{i=1}^{n} \left[ y_i \log \left(1 - \frac{t_i}{t_0}\right) \right]
\]  

(B.2)

(B.3)

(B.4)
This set of simultaneous equations is non linear only in \( t_0 \). The method then is to use one of them to obtain an iteration relation which provides from an estimate of \( t_0, A \) and \( D \), a better estimate of \( t_0 \). With this the other two equations can be solved to give better estimates of \( A \) and \( D \).

Equation B.2 may be written

\[
\begin{align*}
\n A + \log \prod_{i=1}^{n} (t_0 - t_i) - n \log t_0 - D \sum_{i=1}^{n} t_i = \sum_{i=1}^{n} y_i \\
\n n \log t_0 = n A - \sum_{i=1}^{n} y_i - D \sum_{i=1}^{n} t_i + \log \prod_{i=1}^{n} (t_0 - t_i) \\
\n t_0^n = \exp (Z) \prod_{i=1}^{n} (t_0 - t_i) \\
\n Z = n A - \sum_{i=1}^{n} y_i - D \sum_{i=1}^{n} t_i
\end{align*}
\]

Hence if an estimate of \( t_0 \), \( t_1 \) is known another \( t_0^2 \) can be calculated by (B.5) as

\[
t_0^2 = \exp (z/n) \prod_{i=1}^{n} (t_0^1 - t_i)^{1/n}
\]

The convergence of this method for small errors in estimating \( t_0 \) can be demonstrated as follows.

Suppose \( t_0^2 = t_0 + \Delta t_2 \) and \( t_0^1 = t_0 + \Delta t_1 \), where \( \Delta t_1 \) and \( \Delta t_2 \) are small with respect to \( t_0 \).

Then equation B.6 can be approximated by

\[
t_0^n + n t_0^{n-1} \Delta t_2 = \exp Z \left[ \prod_{i=1}^{n} (t_0 - t_i) + \Delta t_1 \sum_{i=1}^{n} \prod_{j \neq i}^{n} (t_i - t_j) \right]
\]

Subtracting (B.5) from this, the relation is determined

\[
\begin{align*}
\n t_0^n + \Delta t_2 = \exp Z \Delta t_1 \sum_{i=1}^{n} \prod_{j \neq i}^{n} (t_i - t_j) \\
\n \text{i.e. } \Delta t_2 = \left[ \exp Z \frac{1}{n} \sum_{i=1}^{n} \prod_{j \neq i}^{n} (1 - t_i/t_j) \right] \Delta t_1
\end{align*}
\]

For pressure measurements taken during the positive pressure phase \( 0 < t_i < 1 \), hence the products of the form \( \prod_{j \neq i}^{n} (1 - t_i/t_j) \leq P_{r_i} \) are all less than 1,

\[
\frac{1}{n} \sum_{i=1}^{n} P_{r_i} < 1
\]

Hence a sufficient condition for convergence is that

\[
\exp Z = \exp (n A - D \sum_{i=1}^{n} t_i) < 1
\]

This is an implicit relation however, since \( A \) and \( D \) are not known, how-
ever in practice it was found that, using the estimate of A and D available at that stage of the iteration, the process converged for \( \exp Z \) as high as 6.

For each new estimate of \( t_0 \), new estimates of A and D are calculated from Eqs. (B.3) and B.4). The convergence of \( t_0 \) to a terminal value is not very rapid; however a satisfactory fit was generally obtained in less than 10 iterations.
Fig. 1(a) Gauge on Right - BC 10 hydrophone
Gauge on Left - SLM #601 mounted with
the normal to the gauge face tangent to
the pressure wave, and showing a wave
reflected from the gauge mount.

Fig. 1(b) SLM #601 gauge mounted on radial sting
with the normal to the gauge face perpendicular
to the pressure wave (i.e. gauge faces into the flow)

FIG. 1 SCHLIEREN PHOTOGRAPHS OF FLOW DISTURBANCE CAUSED BY GAUGE AND MOUNT

q - gauge (arrow indicates the direction
of the normal to the gauge face)  
P - first pressure pulses to reach the gauges
Fig 2(a) Gauge Taped to Steel Rod
sweep time - 50 $\mu$ sec/cm
oscillations - 40 Kc/sec

Fig 2(b) SLM #603 Gauge Ringing on Lower Beam
lower beam sweep time - 50 $\mu$ sec/cm
oscillations - 230 Kc/sec
upper beam sweep time - 50 $\mu$ sec/cm
(circuit with frequency response cut-off at 40 Kc/sec)

Fig 2(c) SLM #601 Gauge Ringing on Lower Beam
lower beam sweep time - 50 $\mu$ sec/cm
oscillations - 146 Kc/sec
upper beam sweep time - 2 msec/cm

Fig 2(d) Filter Calibration by Square Wave Input
wave period - 1 msec
overshoot = 6% of square wave amplitude
pulse rise time = 8 $\mu$ sec

FIG. 2 TRACES ILLUSTRATING SIGNAL DISTORTIONS
FIG. 3 UTIA SHOCK SPHERE APPARATUS
FIG. 4(a) GAUGE AND MOUNT, ASSEMBLED AND EXPLODED VIEWS

FIG. 4(b) EXPERIMENTAL ARRANGEMENT INSIDE THE TANK PRIOR TO A RUN
FIG. 5(a) Pressure History at 7 Radii From a Sphere of Radius = 1/2 inch

lower beam sweep time - 50 μsec/cm
upper beam sweep time - 2 msec/cm

$P_4 = 498 \text{ psi}$  vertical displacement = 44.7 psi/cm

FIG. 5(b) Pressure History at 7 Radii From a Sphere of Radius = 1 inch

lower beam sweep time - 50 μsec/cm
upper beam sweep time - 2 msec/cm

$P_4 = 498 \text{ psi}$  vertical displacement = 44.7 psi/cm

FIG. 5  DIMENSIONAL EFFECTS DUE TO INTERNAL RADIUS

C - incident pressure (compression) wave
R - rarefaction wave reflected from the free surface of the tank

D - reflected compression wave from the rigid boundaries of the tank
T - pressure peak corresponding to bubble having contracted to a minimum (at the end of one pulsation)
FIG. 6 PRESSURE HISTORIES AT 2 AND 10 RADII IN DETAIL
A COUSTIC VARIATION WITH RADIUS FOR SOURCE AT $P_{41}^m = 33.33$

$P_m$ AS GIVEN BY EXPONENTIAL FORM FITTED TO THEORETICAL PRESSURE HISTORIES

$P_m$ AS GIVEN BY EXPONENTIAL FORM FITTED TO EXPERIMENTAL MEASUREMENT OF PRESSURE HISTORIES

FIG. 7 PEAK PRESSURE VERSUS RADIUS
FIG. 8 (P, T) RESULTS FOR 2 RADIi
FIG. 9  \((P, \tau)\) RESULTS FOR 10 RADII
FIG. 10 (P, τ) RESULTS FOR 5 AND 15 RADII
15 RADII

SCHLIEREN PHOTOGRAPH OF THE (r, t)-PLANE

1 RADIUS

10 RADII

PRESSURE TRACE AT 10 RADII

FIG. 11 EXAMPLES OF (p, t) (r, t) - RECORDS

C - incident pressure (compression) wave
R - rarefaction wave reflected from the free surface of the tank
R_b - rarefaction wave reflected from the bubble
D - reflected compression wave from the rigid boundaries of the tank
T - pressure peak corresponding to bubble having contracted to a minimum (at the end of one pulsation)
INCOMPRESSIBLE THEORY $P_{41} = 33.33$

Pressure histories as predicted by combining measured ($X$, $\tau$) path with incompressible theory.

Experimental run P-56

Fig. 12 Coupled ($X$, $\tau$) ($P$, $\tau$) results for pressure histories at 2 and 5 radii.
FIG. 13 COUPLED $(X, \tau) (P, \tau)$ RESULTS FOR PRESSURE HISTORIES AT 10 AND 15 RADII
FIG. 14  \( (P, T) \) RESULTS FROM MEASUREMENTS TAKEN WITHIN THE GAS BUBBLE FOR AIR
FIG. 15  (P, $\tau$) RESULTS FROM MEASUREMENTS TAKEN WITHIN THE GAS BUBBLE FOR HELIUM AND SULPHUR HEXAFLUORIDE
FIG. 16(a) AIR, $P_{41} = 17.1$

Lower Beam
Horizontal 20 $\mu$sec/cm
Vertical 150 psi/cm

Upper Beam
Horizontal 50 $\mu$sec/cm
Vertical 150 psi/cm

FIG. 16(b) HELIUM $P_{41} = 20.00$

Lower Beam
Horizontal 50 $\mu$sec/cm
Vertical 92.0 psi/cm

Upper Beam
Horizontal 200 $\mu$sec/cm
Vertical 92.0 psi/cm

FIG. 16 RAREFACTION WAVES IN AIR AND HELIUM
FIG. 17 COUPLED (X, \( \tau \)) (P, \( \tau \)) RESULTS, PRESSURE HISTORY TAKEN WITHIN THE BUBBLE FOR AIR
Figure 18: Coupled (X, \( \tau \)) (P, \( \tau \)) Results, Pressure History Taken Within the Bubble for He, SF\(_6\).