ON THE ADEQUATE MODEL FOR AIRCRAFT PARAMETER ESTIMATION

by

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SUMMARY

The problem of the selection from measured data of an aircraft of an adequate model which would be the simplest and sufficient approximation to the correct model and which would facilitate the successful determination of the unknown parameters is discussed. Two ways for the proper model structure verification are recommended, namely sensitivity analysis and/or testing of a hypothesis as to the significance of unknown parameters in the model proposed, and the analysis of residuals. Finally some approaches towards the assessment of parameter and adequate model accuracies are proposed. The procedures mentioned are demonstrated in an example.
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NOTATION

g  acceleration due to gravity, \((m/s^2)\)
H  sensitivity matrix
M  a) pitching moment, \((Nm)\)
    b) information matrix
m  number of outputs
N  number of data points
n_z reading of the vertical accelerometer, \((g\ units)\)

\[ n_z^* = \frac{g}{u_e} \cdot n_z \]

q  a) pitching velocity, \((rad/s)\)
    b) number of unknown parameters
R  measurement noise covariance matrix
R_{xo} autocovariance function
r  correlation lag number
s(\cdot) standard error
s^2(\cdot) variance estimate
t  a) time, \((s)\)
    b) Student's t-variable
u  a) longitudinal airspeed component, \((m/s)\)
    b) input vector
Z  vertical force, \((N)\)
x  state vector
y  output vector
z  measurement vector
\alpha  a) angle of attack
       b) level of confidence
\beta  vector of unknown parameters
n  elevon deflection, \((rad)\)
\theta  pitch angle, \((rad)\)
v

v residual
σ standard deviation
σ² standard error

Matrix exponents:
T indicates transpose matrix operation
-1 indicates inverse matrix operation

Subscripts:
E measured quantity
P predicted quantity
e steady-state value

Additional notation:
cov(•) covariance
|•| determinant of a matrix enclosed
\cdot over symbol indicates an estimate
\cdot over symbol indicates the time derivative
1. **INTRODUCTION**

The estimation of stability and control parameters from flight data has become the standard procedure for aircraft and flight conditions where the aerodynamic characteristics can be described in linear terms only, and where no significant external disturbances are presented. Now there is a tendency to widen the parameter estimation technique into flight regimes with non-linear aerodynamic effects and into flight manoeuvres for which the non-linear functions of the aircraft states expressing the inertia forces and moments, and gravity terms must in general be used.

The problem of modelling a complicated system raises the fundamental question of how complex the model should be. Although a more complex model can be justified for proper description of aircraft motion, it has not been clear in the case of parameter estimation what would be the best relationship between model complexity and measurement information. If too many unknown parameters are sought for a limited amount of data, then a reduced confidence in evaluated parameters can be expected (large covariance and/or non-physical values of some parameters), or attempts to identify all parameters might fail.

In the field of system identification with general application a number of different methods for determining an adequate model have been developed. Most of these methods introduced in Ref.1 are connected with the determination of model order in parameter estimation for the single input-single output system.

One of the first attempts to test the correctness of the model representing an aircraft was introduced in Ref.2. The appropriate statistic was formed by the ratio between variance estimates of the measured frequency response curve. One of these estimates was obtained from repeated measurements under the same conditions, the other from the residuals.

In Ref.3 the analysis of residuals was recommended for checking the model's accuracy. The sensitivity analysis was applied in Ref.4
for finding the parameters in the model proposed whose effect on the measured responses was negligible. Neither approach, however, has been brought to a conclusion as to the adequate structure of the model used.

More comprehensive treatment of the model structure determination is proposed and used by Hall and others in Ref.5. It is based on the testing of model parameters in the regression analysis. A criterion for retaining significant parameters is proposed. An adequate model is developed either from simulated data using a priori knowledge from wind-tunnel measurements or other sources, or from measured flight data which are then analysed by the more advanced technique, e.g. the maximum likelihood method. In the first case the results can be used for the proper design of an experiment, mainly an optimal input. In the second case the expected adequate form of the model enters the final part of the analysis providing even better estimates of unknown parameters.

The procedure mentioned assumes, without any explicit statement, the same adequate model for both the regression analysis and the maximum likelihood method and no effect of bias errors in the least-squares parameter estimates on the F-statistics used for testing of the parameter significance.

Stephner and Mehra in Ref.6 proposed a new criterion for fit error. This criterion does not improve monotonically with the increased number of unknown parameters, but has some minimum value which could determine the optimal number of parameters included in a model.

Finally Taylor in Ref.7 developed another criterion for finding the optimal number of unknown parameters which is based on the expected model response error. The criterion selects the most promising model from various candidate models.

In the report presented an attempt is made in the generalisation of experiences from Ref.3 and Ref.4 to achieve an objective approach towards a decision as to an adequate model for parameter estimation from a given set of flight data.
2. **CORRECT AND ADEQUATE MODEL**

To represent any flying vehicle completely would be a task of immense difficulty. The correct model of an aircraft is in general unknown and unknowable. In system identification the problem is therefore the selection from measured data of an adequate model which would be the simplest and sufficient approximation to the correct model and which would facilitate the successful determination of the unknown parameters. An adequate model should include only those terms which have physical meaning and which significantly influence the output of the system.

The identification procedure with an adequate model will very likely result in greater fit error than that with the over-parameterized one. On the other hand the use of an adequate model should provide more accurate parameter estimates.

The determination of an adequate model involves two steps, namely characterization and verification. For an aircraft of orthodox design in a small-disturbance longitudinal or lateral motion around some steady-state equilibrium conditions, the equations of motion are very well known and no problems are expected in the formulation of an adequate model. In general, however, the characterization could be more difficult and must take into account the known physics of the investigated manoeuvre and the a priori knowledge of the aerodynamic characteristics of the aircraft, mainly obtained from wind-tunnel measurements. These considerations will then be reflected in model complexity, i.e. the number of state variables and the form of expressions for the aerodynamic forces and moments.

The models for large-disturbance manoeuvres or coupled motions of an aircraft can in some cases be simplified by introducing the less significant state variables as the additional input to the system. As examples there are the short-period longitudinal motion with small airspeed fluctuations and the rapid-rolling manoeuvre with induced longitudinal motion.

A more difficult procedure, however, will be in deciding which aerodynamic parameters adequately describe the input-output data for a given
manoeuvre. If the linear approximation seems to be insufficient, then the quadratic function or the entire polynomial must be used. Both interpretations result from series expansions around some trim conditions assuming continuous derivatives.

The non-linear representation of the aerodynamic forces and moments has been successfully used in some cases, as reported in Ref.4, by Wells in Ref.8 (quadratic approximation) and by Hall and others in Ref.5 (polynomial form). However, even in those cases where non-linear aerodynamic terms are substantiated, the linear model can still be good approximation and the estimated parameters from it could provide additional a priori information.

When the parameter estimation is completed, the accuracy of the model structure should be verified. In this report two ways are recommended. The first is connected with the effect of all unknown parameters included in the model proposed. Sensitivity analysis (subjective approach) and the testing of a certain hypothesis (objective approach) can be employed.

The second assessment is based on the analysis of residuals, and is considered as the investigation of the model's overall adequacy. When both checks are positive, the final accuracy of the model and parameters can be established. If the opposite results are obtained, then the whole identification must be repeated on the characterization level.
3. **SIGNIFICANT PARAMETERS**

The state and output equations of a system with no process noise can be written as

\[ x = f(x, u, \beta), \quad x(0) = 0 \]  
\[ y = g(x, u, \beta) \]

where \( x \) is an \((n \times 1)\) state vector, \( u \) is an \((p \times 1)\) input vector, \( \beta \) is a \((q \times 1)\) vector of unknown parameters and \( y \) is an \((m \times 1)\) output vector. It is assumed without any further loss of generality that all the initial conditions are equal to zero.

The concept of sensitivity can be treated as the sensitivity of the function \( y = g(x, u, \beta) \) with respect to the parameters. Therefore the parameter sensitivity functions are

\[ \frac{\partial y_k(t)}{\partial \beta_j}, \quad \text{for } k = 1, 2, \ldots, m \quad j = 1, 2, \ldots, q \]

At each time interval \( t_i \), the values of the parameter sensitivity functions form the elements of the \((m \times q)\) sensitivity matrix \( H_i \), \( i = 1, 2, \ldots, N \), where \( N \) is the number of data points. If the modified Newton-Raphson computing technique for parameter estimation is used, then the sensitivity matrix enters the information matrix which for the maximum likelihood (ML) estimation is given as

\[ M = \sum_{i=1}^{N} \begin{bmatrix} \frac{\partial y_i(t)}{\partial \beta_j} & \frac{\partial y_i(t)}{\partial \beta_k} \end{bmatrix}^T \begin{bmatrix} \frac{\partial y_i(t)}{\partial \beta_j} & \frac{\partial y_i(t)}{\partial \beta_k} \end{bmatrix} = \sum_{i=1}^{N} H_i^T R H_i \]

where \( R \) is the measurement noise covariance matrix.

For the comparison of all sensitivity functions related to different parameters, the relative sensitivity function

\[ \frac{1}{\sigma_k} \frac{\partial y_k(t)}{\partial \ln \beta_j} = \frac{R_j}{\sigma_k} \frac{\partial y_k(t)}{\partial \beta_j} \]
can be introduced. In these expressions $\sigma_k$ is the standard deviation of the measurement noise.

Nevertheless, the assessment of the sensitivity based on (3.4) is not very practical. It is therefore preferable to replace the time functions in (3.4) by constant terms defining the sensitivity for the time interval considered.

One of the possibilities would be the formulation of the term for the sensitivity as

$$\frac{\beta_j^2}{\sigma_k^2} \sum_{i=1}^{N} \left[ \frac{\partial y_k(t_i)}{\partial \beta_j} \right]^2$$

(3.5)

taking into account each output variable separately, and the formulation of the term for the sensitivity of the whole system as

$$\frac{\beta_j^2}{\sigma_k^2} \sum_{k=1}^{m} \frac{1}{\sigma_i^2} \sum_{i=1}^{N} \left[ \frac{\partial y_k(t_i)}{\partial \beta_j} \right]^2$$

(3.6)

Comparing (3.6) with the form of the information matrix in (3.3) yields the relationship between the relative sensitivity and the information matrix in the form

$$\frac{\beta_j^2}{\sigma_k^2} \sum_{k=1}^{m} \frac{1}{\sigma_i^2} \sum_{i=1}^{N} \left[ \frac{\partial y_k(t_i)}{\partial \beta_j} \right]^2 = \beta_j^2 H_{jj}$$

(3.7)

where $H_{jj}$ is the main diagonal element in $H$ corresponding to $\beta_j$.

The overall sensitivity (3.6) will depend on the characteristics of the system, the design of the experiment, and the accuracy of measurement. The numerical values of (3.6) can detect critical parameters, i.e. parameters which have a negligible effect on the response of the system and which therefore will very likely be estimated with an unacceptable accuracy.

For the objective decision as to which terms should be deleted from the
model proposed for parameter estimation, a test statistic may be used. This statistic has the form

\[ t = \frac{\hat{\beta}_j}{\hat{S}_{\beta_j}} \]  

(3.8)

where \( \hat{\beta}_j \) is the parameter estimate and \( \hat{S}_{\beta_j} \) is the standard error of the estimate, see e.g. Ref. 9. Using the ML estimation the statistic is changed as

\[ t = \frac{\hat{\beta}_j}{\sqrt{M_{jj}^{-1}}} \]  

(3.9)

where \( M_{jj}^{-1} \) is the main diagonal element in the error covariance matrix for the estimated parameters.

The null hypothesis that \( \beta_j = 0 \) is accepted if

\[ |t| < t_{\frac{\alpha}{2}, N-q} \]  

(3.10)

where \( \alpha \) is the level of confidence (usually \( \alpha = 0.05 \)) and \( N-q \) is the number of statistical degrees of freedom. The value of \( t_{\alpha/2, N-q} \) is found from the tables of Student's distribution. For \( \alpha = 0.95 \) and \( N-q \to \infty \), \( t = 1.96 \).

If \( |t| < t_{\alpha/2, N-q} \), the alternative hypothesis that \( \beta_j \neq 0 \) is then valid.

The application of the test mentioned is as follows: suppose a confidence level of 0.95 is chosen. The t-statistic is then computed for each estimated parameter and the parameter with minimum \( |t| \) is found. If \( |t|_{\min} > t_{\alpha/2} \), all terms are concluded to be significant at the 0.05 level. If \( |t|_{\min} < t_{\alpha/2} \), the parameter corresponding to the \( |t|_{\min} \) is dropped from the model and the estimation is recomputed with the new model. The process is repeated till the adequate form of the model is reached.

To overcome the problems due to over-parameterization in the first step of
the testing procedure, the a priori weighting can be used. With a decreasing number of parameters, the constraint due to a priori weighting can gradually be lifted and possibly abandoned for an adequate model.

The adequate model resulting from the testing of parameters is not necessarily unique. The proper form of this model will depend on the initial modelling, where the a priori knowledge as to the aero-dynamic characteristics of the aircraft and the physics of the motion are considered.
4. Analysis of Residuals

For the ML estimation, the residuals $v_i = z_i - y_i$ should form a sequence of uncorrelated random variables with Gaussian distribution and zero mean, see e.g. Ref. 6.

The simplest way to check some of the assumptions mentioned is the time sequence plot of residuals. From this time history trends can be apparent and possible deterministic components discovered. If the normalized residuals $v_k(t_i)/s(y_t)$, $k=1,2,...,m$ are plotted, then any values greater than three can be considered as outliers.

The check of normality uses the calculated percentage cumulative distribution, $P_j$, for each of $N$ samples of the residuals $v_k(t_i)$. Because of the large number of data points, the grouped residuals are usually used in the calculation. The probability of the residuals $v_i$ less than or equal to the $j$th class limit is

$$P_j = P(v_i < v_j + \Delta v)$$

(4.1)

where $v_j$ are the midpoints of the class intervals and $\Delta v$ is the length of the class intervals.

It is advantageous to plot $v_j$ as abscissas and the corresponding $P_j$ values as ordinates on the probability paper, on which the ordinate scale is graduated according to the area under a normal distribution function. Then the fitted line of all values of $P_j$ must, in the case of a normal distribution, be a straight line.

The mean value can also be checked and is zero if the condition

$$P(v_i < v_j = 0) = 50\%$$

is satisfied. The difference in abscissas for $P_j = 50\%$ and $P_j = 15.9\%$ is an estimate of $\sigma$. The established value of the standard error can be therefore used for fitting the straight line to the plotted $(P_j,v_j)$ data.

The assumption of uncorrelated residuals is checked by the autocorrelation function of the residuals. Its estimate is found from the expression
If the residuals are to be uncorrelated, and in the normal case also independent, then the condition \( K_v(r \neq 0) = 0 \) must be satisfied.

In practice even for an adequate model \( K_v(r \neq 0) \) is never exactly zero, but varies slightly around this value. However, these non-zero values should be well within the 2\( \sigma \)-bounds. The relative standard error for the autocorrelation function estimate is approximated as

\[
\frac{s [\hat{R}_v(r \neq 0)]}{\hat{R}_v(r = 0)} \approx \frac{1}{\sqrt{N}}
\]  

(4.3)
5. ACCURACY OF THE MODEL AND PARAMETER ESTIMATES

When the parameter and adequate model estimations are completed, the accuracy of the results obtained ought to be checked. In this process presumably the most important point is the comparison between the estimated parameters and those for which either a priori values or at least limits on their values are known. This comparison must also take into account the standard errors and correlation coefficients of the estimates.

The standard errors define the confidence limits for each parameter regardless of the remaining parameters. The correlation coefficient is the measure of stochastic dependence. There has been no effort to develop a criterion for testing the significant correlation. Usually a value of the correlation coefficient greater than 0.85 is considered significant.

In the next step of the accuracy assessment the output time history match between the actual and estimated responses found from the identified model is checked. A small fit error is a necessary but not sufficient condition for accurate result.

Increased confidence in the estimated parameters and adequate form of a model can be obtained from repeated measurements under the same conditions. The problem of applying either similar or different inputs must be carefully considered. A decision should be taken separately for each group of repeated measurements. In principle, the differences in input forms should not substantially change the sensitivities with respect to unknown parameters. If these changes occur, then their effect must be taken into account in comparing repeated parameter estimates.

From a large number of repeated measurements the ensemble mean values and ensemble standard errors of estimated parameters can be obtained. For accurate results the two quantities are expected to be close to the corresponding ones from each individual measurement.

The ability of the estimates from one test to predict the response of another flight can also be checked. For comparing the predicted and measured output time histories, the confidence limits for the predictions may be found. These limits follow from the covariance matrix of prediction
error, which has the form

$$\text{cov} \{y(t_i \beta)\} = H_i H_i^T + R$$  \hspace{1cm} (5.1)$$

The prediction error is defined as

$$e_{pi} = z_i(\beta) - y_i(\hat{\beta})$$  \hspace{1cm} (5.2)$$

6. **EXAMPLE**

As an example, the parameter estimation and adequate model determination for a slender delta-wing research aircraft are presented. The longitudinal responses of the aircraft were excited from the horizontal steady-state flights by elevon deflection. Because of the input form used and the aircraft characteristics, the airspeed changes during the transient motion were negligible.

Taking into account the physics of the motion, the wind-tunnel measurements and preliminary flight test results, the equation of motion were formulated as

$$\dot{a} = Z_a + Z_q a + Z_{q^2} q^2 + Z_{q^3} q^3 + Z_{\eta a} \eta a +$$

$$+ Z_{\eta n} + Z_{\eta n} \dot{n} + Z_o$$

$$\dot{q} = M_a + M_q q + M_{q^2} q^2 + M_{q^3} q^3 + M_{\eta a} \eta a +$$

$$+ M_{\eta n} + M_{\eta n} \dot{n} + M_o$$

$$\theta = q$$

In these equations only the pitch rate, q, was measured. The second output variable, n, is related to the state variables by the equation

$$n_2^* = Z_a + Z_q q + Z_{q^2} q^2 + Z_{q^3} q^3 +$$

$$+ Z_{\eta a} \eta a + Z_{\eta n} \eta n + Z_{\eta n} \dot{n} + Z_o \hspace{1cm} (6.2)$$
The state and output equations are developed in Ref. 4, where the parameters contained in these equations are also defined. The ML estimation technique without process noise described in Ref. 3 was used in the data analysis.

The results from one test run at high angle of attack ($\alpha = 20.5$ deg.) are summarized in Table 1. These results include the estimated parameters, standard errors of the estimates (lower bounds), variance estimates of measurement noise, the logarithm of the determinant of measurement noise covariance matrix (overall fit), sensitivities and t-statistics. The tabulated t-value for $\alpha = 0.05$ was equal to 1.98.

The first parameter estimates were obtained from the linear model

\[
\begin{align*}
\dot{\alpha} &= Z_\alpha \alpha + Z_{\dot{q}} \dot{q} + Z_\eta \eta + Z_0 \\
\dot{q} &= M_{\alpha} \alpha + M_{\dot{q}} \dot{q} + M_\eta \eta + M_0 \\
\theta &= \gamma \\
\eta^*_a &= Z_\alpha \alpha + Z_{\dot{q}} \dot{q} + Z_0 \theta + Z_\eta \eta + Z_0 \eta
\end{align*}
\]

where $Z_0$ was known and $Z_q$, $Z_{\dot{q}}$, and $Z_\eta$ were fixed on the wind-tunnel values. The resulted estimates agreed reasonably with those from wind-tunnel data, but the parameters $M_\alpha$ and $M_\eta$ were strongly correlated and the fit between measured and computed vertical acceleration time histories was rather poor.

The next estimation was based on the complete equations (6.1) and (6.2) with $Z_q$, $Z_{\dot{q}}$, and $Z_\theta$ treated as known values. Because of inconsistency in the number of unknown parameters and measured outputs, weighting of the basic linear parameters had to be used. The previous estimates of $Z_\alpha$, $M_\alpha$, $M_\eta$, and $M_\theta$ from the linear model and their standard errors formed the a priori data.

The estimation resulted in significantly changed damping parameter $M_\theta^*$, better accuracy of the estimates, and better fit in both output variables.
The sensitivity analysis revealed small effects of all non-linear terms in the lift-force equation on measured responses. This was also confirmed by comparing the computed and tabulated t-statistics.

In the following estimation the insignificant parameters $Z_{a2}$, $Z_{q\alpha}$ and $Z_{\eta\alpha}$ were successively dropped from the model. Then the remaining parameters were proved to be significant. The adequate model obtained had increased sensitivities, lower standard errors in some estimates, and only slightly higher fit errors. However, the main advantage of the adequate model in comparison with the complete one was a considerable improvement in the convergence of the iterative procedure.

Even for the adequate model, it was still necessary to use a priori weighting. Nevertheless the effect of the gradual decrease of weights was investigated. As a result, no substantial changes in the estimates were observed, but the convergence was slowing down and the parameter covariances were increasing.

The measured and computed outputs are plotted in Fig.1. The time histories of normalized residuals for the linear and adequate model are compared in Fig.2, the corresponding autocovariance functions in Fig.3. A substantial improvement in the shape of the autocovariance functions for the adequate model is apparent. However, their forms are still different from those for white noise. This might be due to poor quality of measured data, which is demonstrated by the residuals for both models used and also by the cumulative frequencies for the adequate model plotted in Fig.4.

The comparison of results from two repeated measurements with some of the parameters obtained from wind-tunnel and steady-state measurements (parameter $M_n$) is made in Table 2. The inputs used in both runs are presented in Fig.5, the measured and computed outputs for Run 1 in Fig.6.

The repeatability of results from the two runs is very good, as follows from Table 2 and also from Figs. 7 and 8. In Fig.7 the measured outputs are plotted, together with the predicted ones based on the estimates from Run 2. The resulting differences between the measured and predicted responses are shown in Fig.8. These differences are within the 2$\sigma$ bounds for the prediction error defined by equation (5.2).
7. **CONCLUSION**

Because in general the correct model of an aircraft is unknown and unknowable the problem of identification also encompasses the selection from measured data of an adequate model. This model should be the simplest and sufficient approximation to the correct model and should facilitate the successful determination of the unknown parameters. The determination of an adequate model involves two steps, namely characterization and verification.

The characterization must take into account the known physics of the investigated manoeuvre and the a priori knowledge of the aerodynamic characteristics of the aircraft. These considerations will then be reflected in model complexity, i.e. the number of state variables and the form of expressions for the aerodynamic forces and moments.

For the verification of the model proposed two steps have been developed. The first one is based on the sensitivity analysis and/or testing a certain hypothesis. The sensitivity analysis is the subjective approach which can detect critical parameters which have a negligible effect on the response of the aircraft under test and which therefore will very likely be estimated with an unacceptable accuracy.

For the objective decision as to which terms should be deleted from the model proposed the t-test may be used. In this test the computed t-statistics for each estimated parameter are compared with the tabulated value from Student's distribution and the insignificant terms successively dropped from the model till its adequate form is found.

Both approaches require a very limited amount of additional computing when the parameter estimation is completed. Their disadvantage could be in using an over-parameterized model in the first step of the procedure. This drawback can be overcome, however, by applying a priori weighting during the parameter estimation.

The second step in the model verification is based on the analysis of residuals and is considered as the investigation of the model's overall adequacy.
Where both steps are positive, the final accuracy of the model and parameters can be established. If the opposite results are obtained, then the whole identification must be repeated on the characterization level.

With the tendency to widen the aircraft parameter estimation into flight regimes with non-linear aerodynamic effects and into flight manoeuvres with other non-linear functions the increasing importance of the proper model structure can be expected. For this reason more experience with the procedures developed for the determination of an adequate model and the further research in this area are highly recommended.
8. REFERENCES

1. UNBEHAUEN, H. 
GÖRING, B. 
Test for Determining Model Order in Parameter Estimation. 
Automatica, Vol. 10, pp.233-244, Pergamon Press, 1974

2. KLEIN, V. 
TOSOVSKY, J. 

3. KLEIN, V. 

4. KLEIN, V. 

5. HALL, W.E., Jr. 
GUPTA, N.K. 
TYLER, J.S., Jr. 
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<th>Title and Details</th>
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<td>$\ln</td>
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$^x) \ t_{0.025;145} = 1.98$

**TABLE 1.** ESTIMATES, SENSITIVITIES AND T- STATISTICS FOR DIFFERENT MODELS USED ($\alpha_8 = 20.5$ deg).
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<th>ITEM</th>
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<td>ADEQUATE MODEL</td>
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<td></td>
<td></td>
<td>$\hat{\beta}_j$</td>
<td>$s(\hat{\beta}_j)$</td>
<td>$\hat{\beta}_j$</td>
<td>$s(\hat{\beta}_j)$</td>
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<td>0.031</td>
<td>- 1.516</td>
<td>0.021</td>
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<tr>
<td>$Z_{\eta a}$</td>
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<tr>
<td>$Z_{\eta}$</td>
<td>- 0.38</td>
<td>- 0.38</td>
<td>-</td>
<td>- 0.292</td>
<td>0.019</td>
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<tr>
<td>$Z_{\eta}$</td>
<td>-</td>
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<td>-</td>
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<td>0.0012</td>
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<td>$M_a$</td>
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<td>- 5.366</td>
<td>0.091</td>
<td>- 5.450</td>
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<tr>
<td>$M_{qa}$</td>
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<td>- 1.993</td>
<td>0.062</td>
<td>- 2.197</td>
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<tr>
<td>$M_{\eta}$</td>
<td>-</td>
<td>-</td>
<td>- 17.6</td>
<td>2.9</td>
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<tr>
<td>$M_{\eta}$</td>
<td>-13</td>
<td>-14.08</td>
<td>0.23</td>
<td>-13.8</td>
<td>0.15</td>
</tr>
<tr>
<td>$s^2(q)$</td>
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<td>$s^2(n_z^*)$</td>
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<tr>
<td>ln</td>
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**TABLE 2.** COMPARISON OF PARAMETER DETERMINED FROM REPEATED MEASUREMENTS. $(\alpha_e = 7.2$ deg).
FIG. 1  COMPARISON OF TIME HISTORIES MEASURED WITH THOSE COMPUTED.  
($\alpha_e = 20.5$ deg).
FIG. 1  COMPARISON OF TIME HISTORIES MEASURED WITH THOSE COMPUTED.  
($\alpha_e = 20.5 \text{ deg}$).  – Concluded.
FIG. 2 TIME HISTORIES OF NORMALIZED RESIDUALS ($\alpha_0 = 20.5$ deg).
FIG. 3 SAMPLE AUTOCOVARIANCE FUNCTIONS OF RESIDUALS ($\alpha_0 = 20.5$ deg).
FIG. 4  CUMULATIVE FREQUENCY OF RESIDUALS. ADEQUATE MODEL
($\alpha_a = 20.5$ deg).
FIG. 5 INPUT FORMS USED IN THE EXCITATION OF THE AIRCRAFT MOTION ($\alpha_0 = 7.2$ deg).
FIG. 6 COMPARISON OF TIME HISTORIES MEASURED WITH THOSE COMPUTED. RUN 1 ($\alpha_g = 7.2$ deg).
FIG. 7  COMPARISON OF TIME HISTORIES MEASURED WITH THOSE PREDICTED. RUN 1 ($\alpha_e = 7.2$ deg).
FIG. 8 TIME HISTORIES OF DIFFERENCES BETWEEN MEASURED AND PREDICTED OUTPUT VARIABLES. Run 1 ($\alpha_n = 7.2$ deg).