CROSS - HATCHING

A COMPARISON BETWEEN THE BEHAVIOUR OF
LIQUEFYING AND SUBLIMING ABLATION MATERIALS

H.W. STOCK and M. GODARD

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# TABLE OF CONTENTS

ABSTRACT .................................................. i
LIST OF FIGURES .......................................... ii
LIST OF SYMBOLS ........................................... iii

1. INTRODUCTION .......................................... 1

2. EXPERIMENTAL TECHNIQUE ............................. 3
   2.1 Test facility ........................................ 3
   2.2 Models .............................................. 3
   2.3 Ablation materials .................................. 3

3. TEST RESULTS AND DISCUSSION ......................... 4
   3.1 Necessary conditions for the existence of cross-hatching ........................................... 4
      3.1.1 Supersonic flow .................................. 4
      3.1.2 Transitional or turbulent boundary layer flow .................................................. 4
   3.2 Cross-hatching pattern parameters ................. 5
      3.2.1 Cant angle $\phi$ ................................ 6
      3.2.2 Streamwise spacing $\lambda$ ...................... 7
   3.3 Run time ............................................. 8
   3.4 Comparison between wax and camphor test results .................................................. 9
   3.5 Influence of the viscosity of the solid ablation material on the streamwise spacing $\lambda$ 10

4. CONCLUSIONS ............................................ 14

REFERENCES ................................................ 15

TABLES .....................................................
FIGURES ....................................................

APPENDICES

A CAMPHOR MODEL MANUFACTURING PROCEDURE
B MEASUREMENTS OF THE RELAXATION TIME AND THE ELASTICITY MODULUS FOR CAMPHOR AND WAX AS A FUNCTION OF MATERIAL TEMPERATURE
C CALCULATION OF THE WALL TEMPERATURE OF
SUBLIMING CAMPHOR IN HIGH SPEED
TURBULENT BOUNDARY LAYER FLOWS

D STEADY STATE ABLATION CALCULATIONS
D.1 Calculation of the turbulent heat
    transfer to cones
D.2 Calculation of the run time after
    which steady state ablation condi-
    tions are achieved
D.3 Temperature distribution in the
    melting solid for steady state
    ablation conditions
The cross-hatching phenomenon has been studied experimentally at a free stream Mach number of 5.3, using two different low temperature ablation materials, camphor and wax, which sublime and liquefy respectively under the test conditions.

The surface pattern parameters (i.e., the cant angle and the streamwise spacing) have been compared for both ablation modes and correlated with flow field properties. The effect of exposure time under ablation conditions has been studied. It has been qualitatively shown that the viscosity of the solid ablation material influences the streamwise spacing.
LIST OF TABLES

1 Dimensions of axisymmetric models
2 Thermal and mechanical properties of the ablation materials
3 Data on self-blunted cones

LIST OF FIGURES

1 Ablation surface patterns or cones
2 Cross-hatching on recovered entry-vehicles
3 Typical test results on camphor models
4 Ablation surface patterns on lucite cones of various angles
5 Local Reynolds number evaluated at the position of transition and of the start of the cross-hatched pattern versus Mach number
6 Shadowgraph of a wax cone (with steel nose) prior to ablation
7 Influence of the local Mach number \( M_e \) on the cant angle \( \phi \)
8 Influence of the Mach number \( M_e \) on the cant angle \( \phi \) - \( M_e \) calculated for unblunted cones
9 Influence of the Mach number \( M_e \) on the cant angle \( \phi \) for self-blunted cones
10 Influence of the local static pressure \( p_e \) on the streamwise spacing \( \lambda \)
11 Influence of the local static pressure \( p_e \) on the streamwise spacing \( \lambda \)
12 Influence of the driving temperature ratio \( T_r - T_w / T_w \) on the streamwise spacing \( \lambda \)
13 Influence of the driving temperature ratio \( T_r - T_w / T_w \) on the streamwise spacing \( \lambda \)
14 Run time required for a developed cross-hatched pattern as a function of the local static pressure \( p_e \)
15 Run time required for a developed cross-hatched pattern as a function of the driving temperature ratio \( T_r - T_w / T_w \)
16 Run time required for a developed cross-hatched pattern depending on the streamwise spacing \( \lambda \)
17. Propagation speed of the cross-hatched pattern $V_s$ versus the driving temperature ratio.

18. Stress-strain dependence for a constant strain rate at different material temperatures.

19. Stress-variation with time for constant strain at different material temperatures.

20. Variation of the elasticity modulus with material temperature.

21. Heat flux into the solid ablation material $q_s$ versus the driving temperature ratio.

22. Temperature distribution in the solid wax at different driving temperature ratios $T_r - T_w / T_w$.

23. Run time up to the melting temperature of wax versus the driving temperature ratio.

24. Mass transfer rate $\dot{m} \times H / \dot{q}_w$ on wax models as a function of the run time $t - t_1 / t_1$.

25. Oil film surface patterns on the wall in the expansion region of a Mach 3.5 nozzle.


27. Sintered camphor models.


29. Surface temperature of campher models depending on the static pressure $p_e$ and the total temperature $T_{5T}$ in the presence of a turbulent boundary layer.
LIST OF SYMBOLS

A Factor defined in eq. D3
B Factor defined in eq. D3
B' Mass addition parameter eq. C2
B'' Mass addition parameter equ. C3
c Specific heat of a solid
cp Specific heat of a gas at constant pressure
cv Mass fraction of injected gas in air
E Elasticity modulus
e Strain
G Shear modulus
Hf Heat of fusion
Hs Heat of sublimation
k Heat conductivity coefficient
M Mach number
M Molecular weight
M Mass flux
P Pressure
PV Partial pressure of injected gas
q Heat flux
Re Reynolds number
St Stanton number
St0 Stanton number in the absence of gas injection
T Temperature
T Relaxation time of the viscous solid
t Time
t1 Run time up to liquefaction temperature of wax
t3 Run time required for a developed cross-hatched pattern
u Gas velocity
Vs Velocity of moving pattern
x Coordinate in solid
\( \alpha \) Angle of attack
\( \alpha \) Factor defined in equ. D3
\( \beta \) Factor defined in equ. D3
\( \beta \) Factor defined in equ. C8

\( \theta \) Total cone angle
\( \theta^m \) Total flare angle
\( \lambda \) Streamwise spacing of the cross-hatched pattern
\( \mu \) Viscosity
\( \nu \) Poisson coefficient
\( \rho \) Density
\( \sigma \) Normal stress
\( \sigma_0 \) Initial normal stress
\( \tau \) Tangential stress or shear stress
\( \phi \) Cant angle of the cross-hatched pattern
\( \omega \) Exponent in the viscosity-temperature law, equ. D3

Subscripts

\( e \) Conditions at the outer edge of the boundary layer
\( i \) Conditions at the interface gas-liquid or gas-solid
\( m \) Melting conditions
\( r \) Recovery conditions
\( s \) Conditions in the solid ablation material
\( ST \) Stagnation conditions
\( w \) Wall conditions
\( \infty \) Conditions at upstream infinity
1. INTRODUCTION

The high enthalpy environment of re-entry vehicles at hypersonic speeds and the associated high heating rates require the development of methods to prevent damage to the vehicle structure. Three possible types of techniques are the heat sink, forced mass transfer (transpiration cooling) and self-regulating mass transfer systems (ablation) \(^1\).

Among these, ablation has proven to be an effective method of thermal protection for both short and long duration re-entry trajectories.

It has been observed on recovered re-entry vehicles, that the ablation mass transfer rates were not uniform and that rather regular patterns were scorched into the surface. Similar phenomena were seen in the course of wind tunnel tests on low temperature ablation models. These surface patterns can be classified in three different types: streamwise grooves, turbulent wedges and cross-hatching, Fig. 1.

Streamwise grooves \(^2-5\) are believed to be created by streamwise vortices situated in the boundary layer which locally increase heat and mass transfer rates. The vortices have been shown to exist in supersonic laminar, transitional and turbulent reattaching boundary layers \(^6\), for instance downstream of backward facing steps and also on concave surfaces. In wind tunnel tests, backward facing steps naturally develop by ablation just downstream of a non-ablating nose, Fig. 1, and concave surfaces are formed in the transition region due to increased mass transfer rates.

Turbulent wedges have been investigated in Refs. 7, 8, 9 and 10. In these experiments local areas of turbulent boundary layer flow, in which turbulence is triggered by surface roughnesses, are imbedded into a laminar boundary layer. The lateral spreading of turbulence produces wedge-shaped
regions, in which the mass transfer rates are increased.

The third type of surface pattern is cross-hatching. It consists of two families of nearly straight grooves of regular spacing, running obliquely to the flow direction outside the boundary layer, producing a highly ordered pattern. Examples of cross-hatched patterns on recovered re-entry vehicles are shown in Fig. 2.

The important cross-hatching pattern parameters are:
the cant angle \( \phi \); the half angle between a left and right running groove,
the streamwise spacing or wavelength \( \lambda \), equal to the streamwise length of a cell of the pattern.

The interest in the study of surface patterns, and especially in cross-hatching, is partly due to the fact that such ablation surface roughnesses may create a rolling moment and thus affect the stability of slender re-entry bodies. Therefore, the problem of how to avoid this ablation pattern has to be solved and consequently efforts were made to understand the physical mechanism which creates cross-hatching.
2. EXPERIMENTAL TECHNIQUE

2.1 Test facility

The present tests were carried out in the hypersonic blowdown facility H-1 at the von Karman Institute. The two-dimensional contoured nozzle provides a uniform flow at a Mach number of 5.3 in a test section of 14 cm x 14 cm. The tunnel stagnation conditions are:

\[ T_{ST} = 385 - 600 \text{\degree K} \]
\[ P_{ST} = 12 - 33 \text{ Kgf/cm}^2 \]

giving unit free-stream Reynolds numbers of

\[ 0.85 - 7.0 \times 10^7 \text{ 1/m} \]

2.2 Models

Cones of 10\degree to 62\degree total vertex angle, and 10\degree cones with 12\degree to 40\degree total angle flares were tested at zero angle of attack. The dimensions of the models are given in Table 1. In most of the tests, steel noses were used to avoid apex deformation by ablation. In a few cases, cones entirely made of ablation material, were tested to study nose blunting effects on the cross-hatching phenomenon.

2.3 Ablation materials

Two ablation materials were tested, natural wax without seedings, which liquefied under the test conditions without vaporizing, and camphor, a purely subliming material. The apparatus and technique used to manufacture camphor models are described in Appendix A. The production of wax models is straightforward. The thermal and mechanical properties of the ablation materials are given in Table 2. The description of how
the measurements of the mechanical properties were done is
given in Appendix B.

3. TEST RESULTS AND DISCUSSION

The test programme on cross-hatching at the von
Karman Institute consisted of two main parts. In the first
part, wax models were tested to investigate the main para­
meters influencing the development of cross-hatching[11]. In
the second part, camphor models, Fig. 3, were studied under
a few typical test conditions to compare with the wax results.
The findings are discussed in this technical note.

3.1 Necessary conditions for the
existence of cross-hatching

3.1.1 Supersonic flow

It is now well accepted that the boundary layer
flow must be supersonic to obtain cross-hatching. Larson and
Mateer[2] stated this requirement for the first time. Figure 4
demonstrates this condition very clearly. Cones of different
apex angles have been tested in a Mach 7.4 flow and it can be
seen that cross-hatching was formed only as long as the cone
Mach number was greater than one.

3.1.2 Transitional or turbulent boundary layer flow

Although no systematic experimental evidence was
available, it has been stated in the literature[2,3] that cross-
hatching has never been observed in regions where the boundary
layer was laminar. This condition was carefully tested and
analyzed in the present study.
In Figure 5, the Reynolds number $Re_x$, where $x$ is the distance from the apex of cones to the location at which the cross-hatched pattern started, has been plotted against the Mach number at the outer edge of the boundary layer. Figure 5 also shows the transition Reynolds number for smooth cones. The latter was selected from the literature for wall to recovery temperature ratios similar to those in the present study. Furthermore, the unit free stream Reynolds numbers $Re_e$ and the wind tunnel test section sizes, for which the transition data were obtained, are such that comparison can be made, taking into account that the transition Reynolds number increases with $Re_e$ and decreases with the test section size $L$. Figure 5 demonstrates that the boundary layer is at least transitional in the region where the cross-hatched pattern starts to develop.

An additional result which relates the onset of cross-hatching with boundary layer transition is given in Fig. 6 which is a shadowgraph of the flow on a cone which has a steel nose and an ablative afterbody. The photograph was taken prior to ablation. The arrows in Fig. 6 indicate the location at which the cross-hatched pattern started later during that run. It can be seen that transition occurs close to this location.

Another example is given in Fig. 1. The photograph in the middle shows turbulent wedges on a self-blunting cone. Cross-hatching appears only inside the turbulent wedges and not outside where the boundary layer is still laminar.

3.2 Cross-hatching pattern parameters

From experimental observations, correlations have been established between the pattern parameters, i.e., the streamwise spacing $\lambda$ and the cant angle $\phi$, and flow field conditions. The qualitative dependence of $\lambda$ on ablation material properties is shown. The cant angle and the streamwise spacing have been measured on photographs of the models taken after the runs.
3.2.1 Cant angle $\phi$

The cant angle $\phi$ is plotted versus the local Mach number $M_e$ in Fig. 7 and compared with the Mach angle (solid line). Available wind tunnel data and free flight data are shown for comparison. As seen, the present results follow the Mach angle trend in the Mach number range 2.5-5.0 for both types of ablation materials. On the other hand, the flight data of Ref. 3 do not correlate with the Mach angle law. An explanation has been suggested on Ref. 16 that the cone Mach number at the location, where the cant angle $\phi$ was measured, was rather indeterminate due to nose blunting occurring in free flight tests. This is demonstrated in Fig. 8 which shows the cant angle $\phi$ measured on self-blunting cones plotted against the Mach number $M_e$ calculated for sharp nose cones and compared with present data from Fig. 7 and free flight data. (Details of the blunted cone data are given in Table 3).

An approximate method has been used to calculate the Mach number distribution on self-blunting cones. Figure 9 shows the cant angle $\phi$ plotted versus the Mach number $M_e$ calculated either for sharp nose cones or with the method of Ref. 17. As may be seen, the data for wax and camphor based on the true local Mach number lie below the Mach angle line (solid curve) instead of above as in Fig. 7. This difference may be due to the two following facts. First, the method of Ref. 17 is valid for laminar boundary layers whilst the measurements are made in a region where the boundary layer flow is turbulent—see Chapter 3.1.2. Secondly, which is of larger importance, $M_e$ is a strong function of the radius $R$ of the nose ($M_e$ decreases with $R$). The nose radius increases with the run time so that its value is not exactly known at the very moment when cross-hatching is formed. Thus $M_e$ is underestimated, since the calculations are based on a nose radius which was measured after a run time $t_3$, when the pattern is fully developed. Figure 9 also shows that the camphor test results differ even more from the Mach angle law than the wax data, which may be explained by the fact that
the R-run time dependence is different for each ablation material.

The cant angle $\phi$ is seen to depend uniquely on the local Mach number. It was found that unit free stream Reynolds number, $Re_\infty$, the Reynolds number based on conditions at the outer edge of the boundary layer, $Re_e$, the static pressure $p_e$, the driving temperature ratio, $(T_r - T_w)/T_w$, the run time, the ablation mode, and the material properties had no influence.

The recovery temperature $T_r$ was calculated by assuming a turbulent recovery factor of 0.895. The influence on the recovery temperature of the liquid film which exists in the tests with wax and the effect of mass injection due to sublimation in the camphor tests was neglected. The wall temperature, $T_w = 337^\circ K$, is the temperature at which wax liquefies independently of $p_e$.

For the camphor tests $T_w$ is dependent on $p_e$, the heat transfer rate and thereby the mass transfer. In Appendix C, a method is described to calculate $T_w$ for subliming materials in the presence of a turbulent boundary layer. The results were used to correlate the camphor test data.

### 3.2.2 Streamwise spacing $\lambda$

The effect of the surface pressure $p_e$ on $\lambda$ is shown in Fig. 10. The results agree quite well with those of Williams, extending the range to lower static pressures and greater values of $\lambda$. The surface pressure $p_e$ was varied both by changing the cone or flare angle and by occasionally altering the tunnel stagnation pressure. Figure 11 demonstrates that nose blunting has no influence on $\lambda$ contrary to its effect on the cant angle $\phi$. The static pressure $p_e$ approaches rapidly (close to the junction sphere-cone) the sharp cone value as opposed to the slow trend of the Mach number $M_e$.

The effect of the driving temperature ratio $(T_r - T_w)/T_w$ on the streamwise spacing is shown in Fig. 12 for constant values of $p_e$. $\lambda$ is strongly dependent on $(T_r - T_w)/T_w$ for wax tests, whereas for camphor $\lambda$ increases only slightly
with the driving temperature. An obvious physical explanation for the different behaviour of wax and camphor is given in chapter 3.4. Figure 13 shows photographs of camphor models tested at different values of \( \frac{(T_r - T_w)}{T_w} \).

The Mach number, \( M_e \), the Reynolds numbers, \( \text{Re}_\infty \) and \( \text{Re}_e \), and the run time did not appear to have any influence on \( \lambda \), when the static pressure \( p_e \) and the driving temperature ratio were held constant. No influence of nose blunting on \( \lambda \) could be observed. The body size does not seem to be a scaling factor for \( \lambda \). Indeed, Williams 4 used camphor models which were three times larger than those tested at VKI and no difference in \( \lambda \) could be seen, Fig. 10.

3.3 Run time

During a run on an ablating model, several distinct time intervals can be defined corresponding to different stages in the development of the cross-hatched pattern:

1st time interval: \( t_0 \to t_1 \)
From tunnel start until the model surface reaches the liquefaction or sublimation temperature and starts to ablate.

2nd time interval: \( t_1 \to t_2 \)
From the onset of ablation until cross-hatching starts to appear.

3rd time interval: \( t_2 \to t_3 \)
From the first visible evidence of cross-hatching until the surface pattern is fully developed, showing the maximum height difference between the bottom of the grooves and the enclosed hills.

4th time interval: \( t_3 \to \)
After being fully developed, the pattern starts to desintegrate showing a regmaglypt pattern resembling those on meteorites shown in Ref. 2.
It was found that the run time \( t_3 \) for both camphor and wax models was dependent on the local static pressure \( p_e \). Fig. 14, \( t_3 \) decreased with increasing static pressure \( p_e \) and was roughly twice as long for camphor than for wax models for the same \( p_e \). In Figure 15, \( t_3 \) is shown as a function of the driving temperature ratio \( (T_r - T_w)/T_w \) for constant \( p_e \). \( t_3 \) increases nearly linearly with \( (T_r - T_w)/T_w \) for wax, whilst for camphor \( t_3 \) remains constant.

Figure 16 is a cross-plot of Figs. 10, 12, 14, 15 showing the streamwise spacing \( \lambda \) as a function of \( t_3 \) for both camphor and wax tests. As may be seen, \( t_3 \) increases nearly proportionally to \( \lambda \) at a rate which depends upon the type of material. Included are wax results obtained for constant \( p_e \) and different driving temperature ratios and those obtained at a low constant driving temperature ratio and varying \( p_e \), Fig. 10. Thus, \( t_3 \) is a unique function of the streamwise spacing \( \lambda \), independent of \( Re_\infty, Re_e \) and \( M_e \).

Further results obtained on bi-conic models tested at different local static pressures \( p_e \) and consequently with variable run time \( t_3 \), Fig. 14, are reported in Refs. 11 and 18.

In the course of a few runs with wax and camphor cones of same apex angles, tested at the same static pressure \( p_e \) but at different driving temperature ratios, a cine film was taken during the complete testing period. This showed that the whole cross-hatched pattern moved very slowly downstream. Furthermore, it was verified that the velocity of propagation was a function of the driving temperature. The result is shown in Fig. 17 which indicates that this velocity decreases nearly linearly with \( (T_r - T_w)/T_w \).

### 3.4 Comparison between wax and camphor test results

The same value of the cant angle \( \phi \) was observed on wax and camphor models, depending uniquely on the local Mach
number $M_e$.

The streamwise spacing $\lambda$ for both materials is inversely proportional to the static pressure $p_e$. The data for wax and camphor tests can be superposed if the driving temperature for runs on wax models is chosen to be

$$\frac{T_r - T_w}{T_w} = 0.0989 - 0.1278$$

The run time $t_3$ varies in inverse proportion to the static pressure $p_e$ for both materials. The fact that for camphor tests $t_3$ is about twice as large is probably due to differences in material properties which in turn affect the cross-hatching formation process. $t_3$ increases linearly with $(T_r - T_w)/T_w$ for wax models and stays constant for camphor. For both materials, $t_3$ is a unique function of the streamwise spacing $\lambda$, increasing proportionally with $\lambda$.

The results obtained for wax and camphor models were similar in every respect but one. It was found that in the wax tests, at a constant cone pressure, the streamwise spacing $\lambda$ was strongly influenced by the driving temperature ratio, contrary to the results on camphor models. It was therefore decided to investigate the possible effect of the different material properties on the physical mechanism which produced this phenomenon.

3.5 Influence of the viscosity of the solid ablation material on the streamwise spacing $\lambda$

On wax models, keeping all conditions constant, it was shown that $\lambda$ increased from 5-20 mm for only small changes in the stagnation temperature ($60^\circ$K), Figure 12. On the other hand, a change in stagnation temperature of $120^\circ$K for the camphor tests had virtually no effect on the streamwise spacing.
The cross-hatching phenomenon results from an interaction between the boundary layer and the solid ablation material. It is reasonable to assume that the boundary layer is not significantly modified by the small changes in the stagnation temperature. Consequently, it was deduced that there must be a difference in the behaviour of some material property sensitive to small changes in heat transfer rate, i.e., temperature. The physical property of wax which is very sensitive to a small temperature change is the viscosity of the solid wax

$$\mu = G \cdot T = \frac{E}{\tau}$$  \hspace{1cm} (B7)

It is shown in Appendix B that both materials, wax and camphor, are visco-elastic solids. The determination of the elasticity modulus $E$ and the relaxation time $T$ for both wax and camphor, are described in Appendix B, the results are given in Figs. 18, 19 and 20. Figure 20 shows that $E_{\text{wax}}$ decreases drastically from 150 to 2 Kg/cm$^2$ during a temperature rise of 295 to 323°K, which corresponds to the expected temperature variation of the solid wax during a test (293°K is the initial temperature before the test and 337°K is the liquefaction temperature of wax). In the same range of material temperature, $E_{\text{camphor}}$ changes only slightly (293°K initial temperature, 344°K maximum wall temperature).

On the other hand, it is shown in Fig. 19 that the relaxation time $T$ decreases considerably with temperature for wax but not for camphor, hence the viscosity of the wax only, see equ. B7, is a highly dependent function of the temperature.

It has to be demonstrated now that the streamwise spacing $\lambda$ varies with the viscosity of the solid wax, based on the information of Figs. 12, 19 and 20. To do so, it is sufficient to prove that the temperature of the ablation material underneath the liquid-solid interface, changes with the heat transfer rate. In Appendix D, a method is given to calculate the heat flux to, and the temperature distribution in the solid
material for the steady state ablation case. (Steady state ablation is defined for a constant heat flux if the ablation mass transfer is constant, thus producing a recession of the surface at constant speed. In this case, the temperature profile inside the solid stays constant with respect to the receding surface). The results for the actual tests are shown in Figs. 21 and 22. As may be seen from Fig. 21, the heat flux to the solid rises linearly with the driving temperature for both materials. Figure 22 shows the temperature distribution in the solid wax; the liquid/solid interface is at \( x = 0 \). As the material close to the interface gets colder for increasing \( \frac{(T_r - T_w)}{T_w} \), the viscosity of the solid wax increases rapidly, Figs. 19 and 20. For the lowest and highest value of \( \frac{(T_r - T_w)}{T_w} \) of the actual tests, there is a temperature difference of 15°C in a depth of only 0.5 mm.

In Appendix D it is shown that steady state ablation conditions exist when the pattern starts to be formed. Figure 23 shows the time \( t_1 \), which is necessary to reach the liquefaction temperature of wax, plotted against the driving temperature ratio of the actual tests. \( t_1 \) has a maximum value of about 0.3 seconds. Figure 24 shows the mass transfer rate for wax as a function of the run time. It can be seen that for run times larger than 10 \( t_1 \) the mass transfer stays constant and the steady ablation condition is achieved. It can be concluded that for wax the cross-hatched pattern was formed under steady state conditions, as the maximum \( t_1 \) is 0.3 seconds and the minimum run time \( t_3 \) is about 10 seconds.

It is thus experimentally shown that the streamwise spacing \( \lambda \) decreases if the viscosity of the affected layer of the solid material gets smaller.

Knowing this it is easy to understand the results of Fig. 17, which shows a decrease of the downstream propagation speed of the cross-hatched pattern when the driving temperature ratio is increased. The shear stress acting on the solid, which is
transmitted from the gas via a constant velocity gradient liquid film, does not change significantly with \((T_r - T_w)/T_w\). Hence, the difference in the propagation velocity \(V_s\) comes from the change in the viscosity of the solid. It was shown that \(E\) and \(T\) and thereby \(u\) increase with increasing driving temperature ratio, thus \(V_s\) should decrease as shown in Fig. 17.

Probstein and Gold \(^{19}\) proposed a model for the triggering mechanism for cross-hatching which is based on an interaction between the shear stress fluctuations in a turbulent boundary layer at the wall and a viscous deformable body. The behaviour of a viscous solid is described by its relaxation time and shear or elasticity modulus. The experimental findings thus support the hypothesis used in the calculations of Refs. \(^{19, 20}\) and \(^{21}\). Furthermore, no ablative mass transfer is necessary in this model for the formation of cross-hatching. This is supported by some experimental results which are shown in Fig. 25. In this case, the cross-hatched pattern is produced on the surface of an oil film containing magnesium oxide spreading out under the effect of friction on a glass-window in the expansion part of a supersonic nozzle. The vapour pressure of the highly viscous oil was well below the static pressure level in the tunnel, and therefore there was no ablation mass transfer. A close inspection of the pattern shows all the features of conventionally obtained cross-hatching. The streamwise spacing \(\lambda\) increases and the cant angle \(\phi\) decreases in the flow direction as the static pressure falls and the Mach number rises.
4. CONCLUSIONS

1. It is shown that a necessary condition for the existence of cross-hatching is that the boundary layer should be transitional or turbulent.

2. The cant angle $\phi$ is a unique function of the local Mach number outside the boundary layer and is nearly equal to the local Mach angle even on self-blunting cones for both subliming and liquefying ablation materials.

3. The streamwise spacing $\lambda$ is inversely proportional to the local static pressure $p_e$ for both materials. $\lambda$ increases with the driving temperature ratio $(T_r - T_w)/T_w$ for wax and stays nearly constant for camphor.

4. The streamwise spacing $\lambda$ increases as the viscosity of the solid ablation material increases.

5. The run time necessary to form the cross-hatching pattern $t_3$ is inversely proportional to the local static pressure $p_e$ for wax and camphor but in proportion to $(T_r - T_w)/T_w$ for wax only.

6. The time $t_3$ depends uniquely on the streamwise spacing $\lambda$ for both materials and $t_3$ increases with $\lambda$.

7. The cross-hatched pattern propagates slowly in the downstream direction at a speed which increases as the viscosity of the solid ablation material is decreased.
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TABLE 1

CONE DIMENSIONS:

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<td>80</td>
<td>54</td>
<td>20</td>
<td>50</td>
</tr>
<tr>
<td>18</td>
<td>32</td>
<td>80</td>
<td>34</td>
<td>20</td>
<td>70</td>
<td>58</td>
<td>22</td>
<td>50</td>
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<td>20</td>
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<td>80</td>
<td>38</td>
<td>20</td>
<td>70</td>
<td>62</td>
<td>22</td>
<td>50</td>
</tr>
<tr>
<td>22</td>
<td>28</td>
<td>80</td>
<td>40</td>
<td>18</td>
<td>70</td>
<td></td>
<td></td>
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</table>

CONE-FLARE DIMENSIONS:

θ = 10°
D = 45 mm
D_{BASE} = 80 mm
θ^* = 12-40° in steps of 2°
<table>
<thead>
<tr>
<th><strong>THERMAL PROPERTIES OF THE ABLATION MATERIALS</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>WAX:</strong></td>
</tr>
<tr>
<td>Melting temperature</td>
</tr>
<tr>
<td>Latent heat of fusion</td>
</tr>
<tr>
<td>Specific heat (solid)</td>
</tr>
<tr>
<td>Heat conductivity (solid)</td>
</tr>
<tr>
<td>Thermal diffusivity (solid)</td>
</tr>
<tr>
<td>Density (solid)</td>
</tr>
<tr>
<td><strong>CAMPHOR:</strong></td>
</tr>
<tr>
<td>Molecular weight</td>
</tr>
<tr>
<td>Latent heat of sublimation</td>
</tr>
<tr>
<td>Specific heat (solid)</td>
</tr>
<tr>
<td>Heat conductivity (solid)</td>
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<tr>
<td>Thermal diffusivity (solid)</td>
</tr>
<tr>
<td>Density (solid)</td>
</tr>
<tr>
<td>Specific heat (liquid)</td>
</tr>
<tr>
<td>Specific heat (gas)</td>
</tr>
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</table>

<table>
<thead>
<tr>
<th><strong>MECHANICAL PROPERTIES OF THE ABLATION MATERIALS AND MEASUREMENT DETAILS</strong></th>
</tr>
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<tbody>
<tr>
<td>Compression apparatus</td>
</tr>
<tr>
<td>Constant compression speed for all measurements</td>
</tr>
<tr>
<td>Height of the cylindrical test piece</td>
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<tr>
<td>Radius of the cylindrical test piece</td>
</tr>
<tr>
<td>Material temperature</td>
</tr>
<tr>
<td>Maximum compression force</td>
</tr>
<tr>
<td>Elasticity modulus</td>
</tr>
<tr>
<td>Temperature</td>
</tr>
<tr>
<td>-------------</td>
</tr>
<tr>
<td>395.</td>
</tr>
<tr>
<td>403.</td>
</tr>
<tr>
<td>413.</td>
</tr>
<tr>
<td>423.</td>
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<table>
<thead>
<tr>
<th>Temperature</th>
<th>WAX</th>
<th>CAMPHOR</th>
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<tr>
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<th>CAMPHOR</th>
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<td>395.</td>
<td>3.5</td>
<td>4.0</td>
</tr>
<tr>
<td>403.</td>
<td>3.5</td>
<td>4.0</td>
</tr>
<tr>
<td>413.</td>
<td>3.5</td>
<td>4.0</td>
</tr>
<tr>
<td>423.</td>
<td>3.3</td>
<td>4.0</td>
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<th>CAMPHOR</th>
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<td>14.4</td>
<td>14.4</td>
</tr>
<tr>
<td>403.</td>
<td>12.0</td>
<td>10.0</td>
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<tr>
<td>413.</td>
<td>6.0</td>
<td>11.2</td>
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<tr>
<td>423.</td>
<td>1.6</td>
<td>10.0</td>
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<tr>
<th>Temperature</th>
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<th>CAMPHOR</th>
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<th>WAX</th>
<th>CAMPHOR</th>
</tr>
</thead>
<tbody>
<tr>
<td>395.</td>
<td>145.</td>
<td></td>
</tr>
<tr>
<td>403.</td>
<td>35.</td>
<td></td>
</tr>
<tr>
<td>413.</td>
<td>15.</td>
<td></td>
</tr>
<tr>
<td>423.</td>
<td>2.</td>
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<thead>
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<th>WAX</th>
<th>CAMPHOR</th>
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<tbody>
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<td></td>
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</tbody>
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<table>
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<tr>
<th>Temperature</th>
<th>WAX</th>
<th>CAMPHOR</th>
</tr>
</thead>
<tbody>
<tr>
<td>395.</td>
<td>156.</td>
<td></td>
</tr>
<tr>
<td>403.</td>
<td>145.</td>
<td></td>
</tr>
<tr>
<td>413.</td>
<td>135.</td>
<td></td>
</tr>
<tr>
<td>423.</td>
<td>131.</td>
<td></td>
</tr>
</tbody>
</table>
TABLE 3

SELF-BLUNTED CONES:
DIMENSIONS AND CROSS-HATCHING DATA

Me: MACH-NUMBER AT THE OUTER EDGE OF THE BOUNDARY LAYER CALCULATED FOR POINTED NOSE CONES

<table>
<thead>
<tr>
<th>ABLATION MATERIAL</th>
</tr>
</thead>
<tbody>
<tr>
<td>WAX</td>
</tr>
<tr>
<td>CAMPHOR</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>OA</th>
<th>Me</th>
<th>Oa</th>
<th>R</th>
<th>L</th>
<th>DBASE</th>
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<tbody>
<tr>
<td>20</td>
<td>4.5</td>
<td>19.5</td>
<td>3.8</td>
<td>180</td>
<td>80</td>
</tr>
<tr>
<td>26</td>
<td>4.2</td>
<td>20.5</td>
<td>4.0</td>
<td>150</td>
<td>80</td>
</tr>
<tr>
<td>32</td>
<td>3.9</td>
<td>20.5,21.5</td>
<td>3.3</td>
<td>120</td>
<td>70</td>
</tr>
<tr>
<td>38</td>
<td>3.6</td>
<td>21.5,22.0</td>
<td>2.5</td>
<td>85</td>
<td>70</td>
</tr>
<tr>
<td>46</td>
<td>3.2</td>
<td>23.0</td>
<td>2.5</td>
<td>60</td>
<td>60</td>
</tr>
<tr>
<td>26</td>
<td>4.2</td>
<td>19.5</td>
<td>5.0</td>
<td>90</td>
<td>80</td>
</tr>
<tr>
<td>30</td>
<td>4.0</td>
<td>21.0</td>
<td>5.0</td>
<td>130</td>
<td>80</td>
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<tr>
<td>32</td>
<td>3.9</td>
<td>21.0</td>
<td>5.3</td>
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<td>38</td>
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<td>5.0</td>
<td>80</td>
<td>70</td>
</tr>
<tr>
<td>44</td>
<td>3.3</td>
<td>223,235</td>
<td>4.0</td>
<td>50</td>
<td>60</td>
</tr>
</tbody>
</table>
FIG. 1. ABLATION SURFACE PATTERNS ON CONES

CAMPHOR (WITH STEEL NOSE)

\[
\begin{align*}
M_\infty &= 5.3 \\
T_{ST} &= 493 \, ^\circ K \\
P_{ST} &= 30.0 \, \text{kgf/cm}^2 \\
t &= 35 \, \text{sec} \\
\alpha &= 0^\circ \quad \text{(ANGLE OF ATTACK)} \\
\theta &= 26^\circ \quad \text{(TOTAL CONE ANGLE)}
\end{align*}
\]

CAMPHOR (WITHOUT STEEL NOSE)

\[
\begin{align*}
M_\infty &= 5.3 \\
T_{ST} &= 417 \, ^\circ K \\
P_{ST} &= 30.0 \, \text{kgf/cm}^2 \\
t &= 29 \, \text{sec} \\
\alpha &= 0^\circ \quad \text{(ANGLE OF ATTACK)} \\
\theta &= 38^\circ \quad \text{(TOTAL CONE ANGLE)}
\end{align*}
\]

WAX (WITH STEEL NOSE)

\[
\begin{align*}
M_\infty &= 5.3 \\
T_{ST} &= 405 \, ^\circ K \\
P_{ST} &= 297 \, \text{kgf/cm}^2 \\
t &= 15 \, \text{sec} \\
\alpha &= 0^\circ \quad \text{(ANGLE OF ATTACK)} \\
\theta &= 26^\circ \quad \text{(TOTAL CONE ANGLE)}
\end{align*}
\]
FIG. 2. CROSS-HATCHING ON RECOVERED ENTRY VEHICLES
FIG. 3. TYPICAL TEST RESULTS ON CAMPHOR MODELS

CONE - FLARE

\[ M_\infty = 5.3 \]
\[ T_{ST} = 523 \, ^\circ K \]
\[ P_{ST} = 33.0 \, \text{kg} / \text{cm}^2 \]
\[ t = 32 \, \text{sec} \]
\[ \alpha = 0^\circ \] (ANGLE OF ATTACK)
\[ \theta = 10^\circ \] (TOTAL CONE ANGLE)
\[ \theta^* = 24^\circ \] (TOTAL FLARE ANGLE)

---

CONE

\[ M_\infty = 5.3 \]
\[ T_{ST} = 406 \, ^\circ K \]
\[ P_{ST} = 30.0 \, \text{kg} / \text{cm}^2 \]
\[ t = 20 \, \text{sec} \]
\[ \alpha = 0^\circ \] (ANGLE OF ATTACK)
\[ \theta = 34^\circ \] (TOTAL CONE ANGLE)
$\theta = 60^\circ$ (TOTAL CONE ANGLE)  
$M_e = 3.0$ (CONE MACH NUMBER)  

$\theta = 80^\circ$ (TOTAL CONE ANGLE)  
$M_e = 2.0$ (CONE MACH NUMBER)  

$\theta = 100^\circ$ (TOTAL CONE ANGLE)  
$M_e = 1.3$ (CONE MACH NUMBER)  

$\theta = 110^\circ$ (TOTAL CONE ANGLE)  
$M_e = 0.9$ (CONE MACH NUMBER)  

$M_\infty = 7.4$  
$T_{ST} = 1100$ °K  
$p_{ST} = 109$ kgf/cm²  
$\alpha = 0^\circ$ (ANGLE OF ATTACK)  

**FIG.4.** ABLATION SURFACE PATTERNS ON LUCITE CONES OF VARIOUS ANGLES (FROM REF.2)
Fig. 5: Local Reynolds number evaluated at the position of transition and of the start of the cross-hatched pattern versus Mach number.
$M_\infty = 5.3$

$T_{ST} = 405 \ ^\circ K$

$P_{ST} = 30.0 \ \text{kgf/cm}^2$

$\alpha = 0^\circ$ (ANGLE OF ATTACK)

$\Theta = 26^\circ$ (TOTAL CONE ANGLE)

ARROW MARKS THE LOCATION WHERE THE CROSS-HATCHED PATTERN STARTS

FIG. 6. SHADOWGRAPH OF A WAX CONE (WITH STEEL NOSE) PRIOR TO ABLATION
FIG. 7. INFLUENCE OF THE LOCAL MACH NUMBER $M_e$ ON THE CANT ANGLE $\phi$
FIG. 8. INFLUENCE OF THE MACH NUMBER $M_e$ ON THE CANT ANGLE $\phi$ - $M_e$ CALCULATED FOR UNBLUNTED CONES
FIG. 9. INFLUENCE OF THE MACH NUMBER $M_e$ ON THE CANT ANGLE $\phi$ FOR SELF-BLUNTED CONES
FIG. 10. INFLUENCE OF THE LOCAL STATIC PRESSURE $p_e$ ON THE STREAMWISE SPACING $\lambda$
FIG. 11. INFLUENCE OF THE LOCAL STATIC PRESSURE $p_e$ ON THE STREAMWISE SPACING $\lambda$
FIG. 12. INFLUENCE OF THE DRIVING TEMPERATURE RATIO $T_r - T_w / T_w$ ON THE STREAMWISE SPACING $\lambda$
FIG. 13. INFLUENCE OF THE DRIVING TEMPERATURE RATIO $T_R - T_W / T_W$ ON THE STREAMWISE SPACING $\lambda$. 

CAMPHOR MODELS

$M_{\infty} = 5.3$
$P_{ST} = 300$ kg/cm$^2$
$\alpha = 0^\circ$ (ANGLE OF ATTACK)
$\theta = 26^\circ$ (TOTAL CONE ANGLE)
FIG. 14. RUN TIME REQUIRED FOR A DEVELOPED CROSS-HATCHED PATTERN AS A FUNCTION OF THE LOCAL STATIC PRESSURE $p_e$

- **VKI-WAX (CONE)**
  - $T_{ST} = 393 - 411$ °K
  - $T_{r-T_w}/T_w = 0.0989 - 0.1278$
  - $T_w = 337$ °K

- **VKI-CAMPHOR (CONE)**
  - $T_{ST} = 397 - 446$ °K
  - $T_{r-T_w}/T_w = 0.0988 - 0.2095$ °K
  - $M_{\infty} = 5.3$
  - $\alpha = 0^\circ$

(ANGLE OF ATTACK)
FIG. 15. RUN TIME REQUIRED FOR A DEVELOPED CROSS-HATCHED PATTERN AS A FUNCTION OF THE DRIVING TEMPERATURE RATIO $T_r - T_w / T_w$.
FIG. 16. RUN TIME REQUIRED FOR A DEVELOPED CROSS-HATCHED PATTERN DEPENDING ON THE STREAMWISE SPACING $\lambda$. 

- **VKI-WAX (CONE)**
  - $T_r/T_w = 0.0989-0.1278$
  - $T_r/T_w = 0.0564-0.2240$

- **VKI-CAMPHOR (CONE)**
  - $T_r/T_w = 0.0988-0.2095$
  - $T_r/T_w = 0.1078-0.3603$

- $p_e = 0.0486-0.5025$ kg/cm²
- $M_\infty = 5.3$
- $\alpha = 0^\circ$ (ANGLE OF ATTACK)
- SCATTER $\pm 10\%$
FIG. 17. PROPAGATION SPEED OF THE CROSS-HATCHED PATTERN $V_s$
VERSUS THE DRIVING TEMPERATURE RATIO

$\bigcirc$ VKI-WAX (CONE)
$T_{ST} = 401-430$ °K
$T_W = 337$ °K

$\triangle$ VKI-CAMPHOR (CONE)
$T_{ST} = 432$ °K
$M_\infty = 53$
$p_{ST} = 300$ kgf/cm$^2$
$\alpha = 0^\circ$ (ANGLE OF ATTACK)
$\theta = 26^\circ$ (TOTAL CONE ANGLE)
FIG. 18. STRESS–STRAIN DEPENDENCE FOR A CONSTANT STRAIN RATE AT DIFFERENT MATERIAL TEMPERATURES
FIG. 19. STRESS VARIATION WITH TIME FOR CONSTANT STRAIN AT DIFFERENT MATERIAL TEMPERATURES
FIG. 20. VARIATION OF THE ELASTICITY MODULUS WITH MATERIAL TEMPERATURE
FIG. 21. HEAT FLUX INTO THE SOLID ABLATION MATERIAL $q_s$ VERSUS THE DRIVING TEMPERATURE RATIO
FIG. 22. TEMPERATURE DISTRIBUTION IN THE SOLID WAX AT DIFFERENT DRIVING TEMPERATURE RATIOS $T_r - Tw/T_w$
**FIG. 23**

RUN TIME UP TO THE MELTING TEMPERATURE OF WAX VERSUS THE DRIVING TEMPERATURE RATIO

---

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
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</thead>
<tbody>
<tr>
<td>$T_{ST}$</td>
<td>388 - 449 °K</td>
</tr>
<tr>
<td>$T_{W}$</td>
<td>337 °K</td>
</tr>
<tr>
<td>$M_{\infty}$</td>
<td>5.3</td>
</tr>
<tr>
<td>$p_{ST}$</td>
<td>30.0 kgf/cm²</td>
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<tr>
<td>$\alpha$</td>
<td>0° (ANGLE OF ATTACK)</td>
</tr>
<tr>
<td>$\theta$</td>
<td>26° (TOTAL CONE ANGLE)</td>
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</table>
FIG. 24. MASS TRANSFER RATE $\frac{m \times H_t}{q_w}$ ON WAX MODELS AS A FUNCTION OF THE RUN TIME $t - t_1/t_1$
FIG. 25. OIL FILM SURFACE PATTERNS ON THE WALL IN THE EXPANSION REGION OF A MACH 3.5 NOZZLE

FLOW DIRECTION

INCREASING MACH NUMBER
DECREASING STATIC PRESSURE

1 cm
FIG. 26. CAMPHOR SINTERING APPARATUS
DEAERATED BEFORE SINTERING

NOT DEAERATED

FIG. 27. SINTERED CAMPHOR MODELS
FIG. 28. EQUILIBRIUM PHASE DIAGRAM FOR CAMPHOR
FIG. 29. SURFACE TEMPERATURE OF CAMPHOR MODELS DEPENDING ON THE STATIC PRESSURE $p_e$ AND THE TOTAL TEMPERATURE $T_{ST}$ - IN THE PRESENCE OF A TURBULENT BOUNDARY LAYER
APPENDIX A - CAMPHOR MODEL MANUFACTURING PROCEDURE

The method of manufacturing the camphor models is based on the technique described by Charwat 22. It consists of compressing the outgased camphor powder between two pistons in a cylindrical mold at room temperature with pressures up to 500 atm. It is possible in this way to obtain translucent cylinders of 8 cm diameter and 23 cm length, which can be easily machined on a lathe. The sintering apparatus is shown in Fig. 26. The difference between camphor cylinders obtained by sintering the powder outgased or not prior to compression is shown in Fig. 27.
APPENDIX B - MEASUREMENTS OF THE RELAXATION TIME
AND THE ELASTICITY MODULUS OF CAMPHOR AND WAX AS A FUNCTION
OF MATERIAL TEMPERATURE

Compression tests of camphor and wax were done at different material temperatures on an INSTRON-TT-CM machine. It registered simultaneously the stress and strain of the test pieces. Two types of measurements were done. First, the test pieces (table 2) were compressed at a constant displacement rate (1 mm/min) which led to the strain-stress relation shown in Fig. 18. Secondly, after a certain displacement the compression was stopped and the stress was measured as it decayed with time, Fig. 19.

Figure 19 shows clearly that both materials are not perfectly elastic as time is involved in the strain-stress relation. Wax and camphor belong to the class of solids having viscous inelastic behaviour.

There exist two idealized models to characterize the behaviour of these materials: the Kelvin model and the Maxwell model. Any real inelastic body can be described by a combination of Kelvin or/and Maxwell elements. Reference 23 gives the following strain-stress relations:

**Kelvin body:** Normal stress:
\[ T \frac{de}{dt} = \frac{\sigma}{E} - e \]  \hspace{1cm} (B1)

Tangential stress:
\[ T \frac{de}{dt} = \frac{\tau}{2G} - e \]  \hspace{1cm} (B2)

**Maxwell body:** Normal stress:
\[ T \frac{de}{dt} = \frac{T}{E} \frac{d\sigma}{dt} + \frac{\sigma}{E} \]  \hspace{1cm} (B3)

Tangential stress:
\[ T \frac{de}{dt} = \frac{T}{2G} \frac{dT}{dt} + \frac{\tau}{2G} \]  \hspace{1cm} (B4)

where \( e \) is the strain, \( \frac{de}{dt} \) the rate of strain, \( \sigma \) and \( \tau \) the normal and tangential stresses, \( E \) and \( G \) the elasticity and shear modulus
of the material respectively and $T$ its relaxation time.

The simple relation between $E$ and $G$ is used

$$G = \frac{E}{2}$$

which is equivalent to saying that the Poisson coefficient $\nu$ is negligibly small

$$G = \frac{E}{2(1+\nu)}$$

The viscosity of the solid is given by

$$\mu = G \cdot T = \frac{E}{2} T$$

Figure 19 shows that the stress decays with time for constant strain for wax and camphor, hence both materials belong to the Maxwell type. Thus for a constant rate of strain $\dot{\varepsilon} = \frac{\Delta e}{\Delta t}$ equation (B3) can be integrated giving

$$\sigma = E \cdot T \cdot \dot{\varepsilon} \left(1 - \exp \left(-\frac{t}{T}\right)\right)$$

and with

$$t = \frac{\varepsilon}{\dot{\varepsilon}}$$

$$\sigma = E \cdot T \cdot \dot{\varepsilon} \left(1 - \exp \left(-\frac{\varepsilon}{\dot{\varepsilon} T}\right)\right)$$

For a small strain, $\sigma$ can be developed in a Taylor series

$$\sigma = E \cdot e - \frac{E}{\dot{e} T} \frac{e^2}{2} + O(e^3)$$

This leads with equation (B9) to

$$\sigma = E \cdot e \left(1 - \frac{t}{T}\right)$$
As for all compression tests the measurement time $t$ was well below the relaxation time $T$, it is possible to write

$$\sigma \approx E \cdot e^{-t/T} \quad (B13)$$

where the relaxation time $T$ is defined as the time which is necessary at constant strain for the initial stress to drop to $1/e$ of its value, Fig. 19.

Figure 18 shows the stress-strain relation for a constant rate of strain. The factor $E$ in equation (B13) was evaluated from the linear portion of the curves. The non-linear part may be due to imperfections on the surfaces of the test pieces. The elasticity modulus $E$ is shown as a function of material temperature in Fig. 20. As seen, for wax, $E$ decays rapidly with temperature when approaching liquefaction, whereas for camphor $E$ decreases slowly at a constant rate with temperature.

Figure 19 shows the stress decay with time for constant strain. For these conditions equation (B3) becomes, after integration:

$$\sigma = \sigma_0 \exp \left(-\frac{t}{T}\right) \quad (B14)$$

where $\sigma_0$ is the initial stress. For camphor the relaxation time $T$ did not seem to be affected by the material temperature, Fig. 19, whereas for wax $T$ decreased with increasing temperature especially when approaching liquefaction.
APPENDIX C - CALCULATION OF THE WALL TEMPERATURE OF SUBLIMING CAMPHOR IN HIGH SPEED TURBULENT BOUNDARY LAYER FLOWS

When sublimation occurs, the 'injection' of gas takes up a large fraction of the enthalpy difference across the boundary layer. Thus the significant feature of sublimation ablation is the interaction or "feed-back" mechanism between the rate of mass addition and the net rate at which thermal energy is supplied to the gas-solid interface.

As shown in Appendix D, it is reasonable to assume steady state ablation. Then:

\[ \dot{q}_w = \dot{q}_s + \dot{m} \cdot H_s \]  
\[ \dot{q}_s = \dot{m} \cdot e \left( T_w - T_s \right) \]

where \( \dot{q}_s \) is the heat flux to the solid, \( \dot{m} \) the mass transfer rate, \( H_s \) the latent heat of sublimation, \( e \) the specific heat of the solid, \( T_w \) the wall temperature and \( T_s \) the initial temperature of the solid prior to the test.

Two mass addition parameters were defined:

\[ B = \frac{\dot{m}}{\rho e u St_0} \]  
\[ B' = \frac{\dot{m}}{\rho e u St} \]

where \( \dot{m} \) is the mass transfer rate, \( \rho e u \) the density and velocity at the outer edge of the boundary layer, \( T_r \) the recovery temperature, \( c_p \) the specific heat of the main gas, \( St \) and \( St_0 \) are the Stanton numbers with and without mass transfer.
Thus equations (C3) can be written with (C1)

\[ B' = \frac{\dot{q}_w}{\rho_e u_e St (H_s + c(T_w - T_s))} = \frac{c_p (T_r - T_w)}{H_s + c(T_w - T_s)} \]  \hspace{1cm} (C5)

Equations (C2) and (C3) give

\[ \frac{B'}{B} = \frac{St}{St_0} \] \hspace{1cm} (C6)

A semi-empirical correlation 25 for the effect of mass addition into a turbulent boundary layer is

\[ \frac{St}{St_0} \approx 1 - \beta B \] \hspace{1cm} (C7)

where

\[ \beta = 0.28 \left( \frac{M_1}{M_2} \right)^{1/3} \] \hspace{1cm} (C8)

with \( M_1 \) and \( M_2 \) being the molecular weights of the main gas in the boundary layer and the injected vapor respectively. Thus

\[ B = \frac{B'}{1 + \beta B'} \] \hspace{1cm} (C9)

or

\[ B = \frac{c_p (T_r - T_w)}{(c(T_w - T_s) + H_s + \beta c_p (T_r - T_w))} \] \hspace{1cm} (C10)

The analysis given in Ref. 26 shows that

\[ (c_{v_i}) = \frac{B'}{B' + 1} \] \hspace{1cm} (C11)

where \((c_{v_i})\) is the mass fraction of vaporized sublimation material at the interface.
It is assumed that, for small mass transfer rates, the sublimation curve in the $p,T$ diagram is still valid, although mass transfer occurs, i.e., non equilibrium conditions (the equilibrium phase diagram for camphor is shown in Fig. 28, where $p$ is the partial pressure of camphor gas and $T$ its temperature).

Thus the partial pressure of the vapor is given by

$$\frac{p}{p_e} = \left[1 + \left(\frac{1}{c_v} - 1\right) \frac{M_2}{M_1}\right]^{-1}$$

where $p_e$ is the local static pressure.

It is easy now to calculate the wall temperature of the camphor at the interface by an iteration procedure.

Guessing a wall temperature $T_w$, equation (C10) gives $B$ and with (C9) $B'$. From Equation (C11) the concentration of the vapor ($c_v$) can be calculated and (C12) gives the corresponding partial pressure $p$. Using the information of the sublimation curve and $p$, a wall temperature $T'_w$ can be obtained. Iteration is continued until $T'_w$ equals the initially selected wall temperature $T_w$.

Figure 29 shows the camphor temperature at the interface as a function of the static pressure $p_e$ with the stagnation temperature as a parameter.
APPENDIX D - STEADY STATE ABLATION CALCULATIONS

It has to be demonstrated that for the ablation tests, steady state is rapidly achieved, i.e., after only a small fraction of the run time \( t_3 \). Furthermore, the temperature distribution in the receding solid has to be calculated.

### D.1 Calculation of the turbulent heat transfer to cones

Van Driest \(^2\) gives a method to calculate the Stanton number in the two dimensional case at zero pressure gradient and how to obtain from this the conical value \(^2\):\(^8\):

\[
St = \frac{q_w}{\rho u c (T - T_r)}
\]

For the two dimensional case \(^2\)

\[
\frac{0.242}{A(2St)^{1/2}(T_w/T_e)^{1/2}} \left( \sin^{-1}\alpha + \sin^{-1}\beta \right) = 0.41 + \log_{10}Re_e \cdot 2St
\]

\[
- \frac{1 + 2\omega}{2} \log_{10} \frac{T_w}{T_e}
\]

where

\[
\alpha = \frac{2A^2 - B}{(B^2 + 4A^2)^{1/2}}
\]

\[
\beta = \frac{B}{(B^2 + 4A^2)^{1/2}}
\]

\[
A = \left[ \frac{\gamma - 1}{2} \frac{M_e^2}{\frac{T_w}{T_e}} \right]^{1/2}
\]
\[ B = \frac{1 + \frac{\gamma - 1}{2} \frac{M_e^2}{T_e}}{\frac{T_w}{T_e}} \]

\[ \text{Re}_e = \frac{\rho_e u_e x}{\mu_e} \quad (D3) \]

where \( x \) is the distance from the leading edge, and \( \omega = 0.76 \) is the exponent in the viscosity temperature relation

\[ \frac{\mu_w}{\mu_e} = \left( \frac{T_w}{T_e} \right)^{\omega} \quad (D3) \]

In Reference 28, it is shown that for the same cone Mach number and wall-to-free stream temperature ratio, the cone solution for the local heat transfer coefficient is the same as the flat plate solution when the cone Reynolds number \( \text{Re}_e \) is divided by 2.

Thus \( \text{Re}_e \) in equation (D3) must be

\[ \text{Re}_e = \frac{1}{2} \text{Re}_{e\text{ cone}} \quad (D4) \]

D.2 Calculation of the run time \( t \) after which steady state ablation conditions are achieved

Landau 29 gave a method to calculate the mass transfer rates for a melting solid for both unsteady and steady state ablation conditions. The time \( t_1 \) necessary to reach the liquefaction temperature of the solid is given by 28

\[ t_1 = \frac{\pi}{4} \frac{k \cdot c \cdot \rho}{q_w^2} \left( T_m - T_s \right) \quad (D5) \]

where \( k \) is the heat conductivity coefficient of the solid, \( c \) its specific heat, \( \rho \) its density, \( T_s \) its initial temperature
i.e., prior to the test and $T_m$ the melting temperature. The turbulent heat flux to the wall $q_w$ was calculated with the method given in D1. The time $t_1$, evaluated for the actual tests, is shown in Fig. 23 as a function of the driving temperature ratio $(T_f - T_w)/T_w$.

The mass transfer rate $\dot{m} H_2/\dot{q}_w$ as a function of the run time $(t-t_1)/t_1$ is shown in Fig. 24 ($H_2$ is the heat of fusion). As may be seen, for times larger than $10t_1$ the mass transfer stays constant and steady state conditions are achieved. Now, $t_1$ is at most equal to 0.3 sec and $t_3$ at least 10 seconds, it is therefore proved that cross-hatching on wax models was formed under steady state ablation conditions.

### D.3 Temperature distribution in the melting solid for steady state ablation conditions

Landau \(^{29}\) gives a formula to calculate the temperature profile in the solid for steady state in a coordinate system fixed to the receding surface of the melting solid.

$$T = T_s + (T_m + T_s) \exp(-\frac{\dot{m}c}{k} x) \tag{D6}$$

where $\dot{m}$ is the mass transfer rate, $c$ the specific heat of the solid, $k$ its heat conductivity coefficient and $x$ the distance from the receding surface.

Similarly to equation (C1), one can write for the case of steady state melting

$$\dot{q}_w = \dot{q}_s + \dot{m} H_2 \tag{D7}$$

$$\dot{q}_s = \dot{m}c (T_m - T_s)$$

from which the mass transfer rate can be calculated:

$$\dot{m} = \frac{\dot{q}_w}{H_2 + c(T_m - T_s)} \tag{D8}$$
The method for calculating $\dot{q}_w$ is given in D1.

The temperature distribution is shown as a function of $x$ for values of $(T_r - T_w)/T_w$ that existed in the actual tests in Fig. 22.

For the camphor tests, it may be assumed that the method of Ref. 29 remains quantitatively valid, as the influence of vapour injection on the heat transfer rates is negligibly small. The time $t_1$ is of the same order of magnitude as for the wax tests, because the thermal properties of both materials are not significantly different, see table 2. The time needed to achieve steady state conditions is depending uniquely on the ratio $c(T_m - T_s)/H_s$ for the case of a melting material. For sublimation this ratio becomes $c(T_w - T_s)H_s$ and is of the same order of magnitude as $c(T_m - T_s)/H_s$. Hence, it can be concluded that for camphor tests cross-hatching is also formed under steady state ablation conditions, since the run time $t_3$ is nearly twice that of the wax tests as shown in Fig. 14.