TRAINING CENTER FOR EXPERIMENTAL AERODYNAMICS

TECHNICAL NOTE 8

RECOVERY OF SATELLITES FROM CIRCULAR ORBITS

by

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Acknowledgement

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SUMMARY

Recovery trajectories, extending from the initial circular orbit to the top of the atmosphere are analyzed. The propulsion is reduced to an impulse; the trajectories are in the plane of the initial orbit; the spherical, non rotating earth is the only center of attraction.

Expressions for the velocity increment and its orientation at the departure from the initial orbit are given in terms of conditions imposed at the top of the atmosphere. The problem is optimized for minimum fuel consumption. Expressions are derived for the accuracies required on thrust orientation, velocity increment and evaluation of the radius of the initial orbit.

It is shown that for practical cases, minimum fuel consumption corresponds to firing the retro rockets in the direction of the velocity of the vehicle. For a given entry angle, there exists one particular orbit for which the fuel consumption for recovery is an absolute minimum. In optimum conditions, a misalignment of the vehicle, whatever its sign, produces a reduction of the angle of entry in the atmosphere. The accuracies required on the parameters are greater for higher altitudes of the initial orbit and smaller values of the entry angle.
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NOTATION

E  non dimensional energy of the orbit
h  non dimensional angular momentum of the orbit
r  distance of the vehicle to the center of the earth
R  radius of the earth
u = \frac{V^2}{R^2} velocity parameter
V  velocity of the vehicle
V_s  circular velocity at sea level
v_s  local circular velocity

\gamma = \frac{\Delta V}{v_s} velocity increment referred to local circular velocity
\Gamma = \frac{\Delta V}{V_s} velocity increment referred to circular velocity at sea level
\Delta V  velocity increment
\Delta \theta_E  error on entry angle
\Delta \omega  error on thrust orientation
\Delta \gamma  error on velocity increment
\Delta \rho^*  error on evaluation of the radius of initial orbit
\theta  inclination of trajectory on local horizon
\lambda = \frac{\rho_E}{R} distance parameter
\rho = \frac{r}{R} orientation of velocity increment to the local horizon

Subscript
E  top of the atmosphere

Superscript
*  initial conditions for the descent orbit
I. INTRODUCTION

The present report analyses the descent trajectories for satellites initially in circular orbit around the earth. Those trajectories extend from the initial orbit down to the top of the atmosphere, which the vehicle must reach in such conditions that excessive decelerations or heat rates are avoided during the re-entry phase. The descent trajectories considered lie in the plane of the initial orbit.

The transfer of the vehicle from the initial orbit on the descent trajectory is achieved by applying thrust in a convenient direction. For the present calculations, thrust is considered as produced by high thrust rocket engines, using conventional chemical propellants with low specific impulse. The analysis of the powered phase has consequently been reduced to an impulse.

The values of the parameters of the trajectory - inclination on the local horizon, distance to the center of the earth, velocity - can be prescribed at the top of the atmosphere from the data already available on entry corridors (refs 1,2,3). The problem is then to evaluate the velocity increment and its orientation for transfer on the proper trajectory; for practical applications, conditions of minimum fuel consumption must be investigated.

The flight conditions at the top of the atmosphere are modified by errors made during the transfer of the vehicle on the descent orbit. Knowing the accuracy required at entry into the atmosphere, the margin of error allowed on the transfer parameters can be determined. The required accuracy has been considered in this report as dictated by safety considerations with regard to peak deceleration. No attention has been given to the accuracy of prediction of the landing point. Three possible errors have been considered: error on velocity increment, i.e. on the control of the engines, error on orientation of the vehicle during the firing, and error of evaluation of the radius of the initial orbit.
2. For the calculations, the earth has been considered as fixed and spherical, and as the only attracting body. Such a problem has already been considered by G.M. Low (ref 4) in the simplified case of nearly circular descent trajectories. The general solution is presented here.

2. **EQUATIONS OF MOTION**

The equations used to analyse the orbits are the conservation of energy and angular momentum, under the following non dimensional form:

\[ \mu - \frac{2}{\rho} = 2E \quad (2.1) \]

\[ \sqrt{\mu} \rho \cos \Theta = h \quad (2.2) \]

where \( \mu \) is the square of the ratio of the velocity of the vehicle to the circular velocity at sea level:

\[ \mu = \frac{V}{V_s^2} \quad (2.3) \]

and \( \rho \) is the ratio of the radius vector to the earth's radius.

\[ \rho = \frac{r}{R} \quad (2.4) \]

\( \Theta \) is the inclination of the trajectory, taken as positive below the local horizon. \( E \) and \( h \) are the non dimensional energy and angular momentum respectively.

3. **BOUNDARY CONDITIONS**

At any point of the initial orbit, the velocity of the vehicle is the local circular velocity \( v_s^* \), parallel to the local horizon. The orbit may be defined by one single parameter, either the circular velocity \( v_s^* \) or the orbit radius \( r^* \).
To transfer the vehicle on the descent trajectory, a velocity increment $\Delta V$ is produced at an angle $\omega$ to the local horizon. The initial conditions for the descent trajectory $(r^*, V^*, \theta^*)$ can be immediately derived from the above diagram.

$$V^* = v_5^* + \Delta V v_5^* \cos \omega + \left(\Delta V\right)^2$$

(3.1)

$$V^* \cos \theta^* = v_5^* + \Delta V \cos \omega$$

(3.2)

Defining a non dimensional velocity increment, referred to the local circular velocity, by:

$$\gamma = \frac{\Delta V}{v_5^*}$$

(3.3)

the equations (3.1) and (3.2) become:

$$\rho^* \mu^* = 1 + 2 \gamma \cos \omega + \gamma^2$$

(3.4)

$$\cos \theta^* = \frac{1 + \gamma \cos \omega}{\sqrt{\rho^* \mu^*}}$$

(3.5)

A second set of boundary conditions is defined at the top of the atmosphere, as functions of the deceleration and heat rates into the atmosphere. If the top of the atmosphere is arbitrarily fixed at a distance $\rho_E$, the values of the inclination of the trajectory $\theta_E$ and of the velocity $\mu_E$ are specified.

4. SOLUTION OF THE SYSTEM

The most important parameter for the re-entry trajectory is the inclination angle $\theta_E$. The limiting values depend upon the velocity at entry, but, provided the values of the velocity are reasonably close, for a set of problems, a mean value of $\theta_E$ can be used for all of them. Consequently, $\rho_E$ and $\theta_E$ will be regarded as the fundamental parameters which are imposed, the value of the entry velocity being checked afterwards.
4.

Substituting the initial conditions (3.4) and (3.5) into the equations of motion (2.1) and (2.2), the equations of the descent trajectory are obtained:

\[
\mu = \frac{2}{\rho} + \frac{-1 + \frac{1}{2} \gamma \cos \omega + \gamma^2}{\rho^*}
\]  

(4.1)

\[ \sqrt{\mu} \rho \cos \Theta = \sqrt{\rho^*} (1 + \gamma \cos \omega) \]  

(4.2)

For a given initial orbit and given values of \( \rho_E \) and \( \Theta_E \), there are three unknowns in the above system: \( \mu_E \), \( \gamma \), and \( \omega \). There is consequently one parameter left for optimisation. The velocity can be eliminated between the two above equations to yield the following relationship:

\[
\gamma^2 (\cos^2 \omega - \lambda^2 \cos^2 \Theta_E) + 2 \gamma \cos \omega (\lambda^2 \cos^2 \Theta_E - 1) + \lambda^2 \cos^2 \Theta_E - 2 \lambda \cos^2 \Theta_E = 0
\]

(4.3)

where:

\[ \lambda = \frac{\rho_E}{\rho^*} \]  

(4.4)

This relationship between \( \gamma \) and \( \omega \) is the basic equation used for optimization.

5. OPTIMIZATION

In practice, it is important to minimize the fuel consumption of the rocket engines, since the propellants represent a dead weight which has to be lifted into orbit.

For given initial orbit and entry conditions \( \rho_E \), \( \Theta_E \), there is a particular value of the orientation angle \( \omega \) for which the required velocity increment is a minimum. The corresponding analytical condition is obtained by differentiating Eq (4.3) with respect to \( \omega \), for given values of \( \rho^* \), \( \rho_E \), \( \Theta_E \). Putting the derivative of \( \gamma \) equal to zero, one obtains:
\[ \gamma \cos \omega = \lambda^2 \cos^2 \Theta_E - 1 \quad (5.1) \]

Substituting the above condition in Eq (4.3), the minimum value of \( \gamma \) is immediately obtained:

\[ \gamma = \sqrt{3 - \frac{2}{\lambda} - \lambda^2 \cos^2 \Theta_E} \quad (5.2) \]

Inspection of the second derivative shows that the extremum is indeed a minimum.

However, the existence of the minimum is subjected to two conditions: \( \gamma \) must be real and \( \frac{\cos \omega}{\omega} \) smaller than unity. The latter condition is the most severe; combining Eqs (5.1) and (5.2), it can be written as:

\[ \frac{1 - \sqrt{\frac{9}{\gamma} - \frac{8}{\lambda}}}{\lambda^2} < \cos^2 \Theta_E < \frac{1 + \sqrt{\frac{9}{\gamma} - \frac{8}{\lambda}}}{\lambda^2} \quad (5.3) \]

with the additional restriction:

\[ \lambda > \frac{8}{9} \quad (5.4) \]

The domain of existence of the analytical minimum satisfying Eqs (5.1) and (5.2) is represented on Fig. 1. For practical cases, the angles of entry into the atmosphere are necessarily small. Therefore, the lower limit only is of practical interest, i.e.:

\[ \cos^2 \Theta_E < \frac{1 + \sqrt{\frac{9}{\gamma} - \frac{8}{\lambda}}}{\lambda^2} \quad (5.5) \]

When the conditions for the existence of the analytical minimum are not satisfied, one should select a value of \( \omega \) for which \( \gamma \) has the lowest possible value. This corresponds to \( \omega = 180^\circ \) as shown in Fig. 2, where \( \gamma \) is given in terms of \( \omega \) for different values of \( \Theta_E \). The figure corresponds to one given initial orbit. In most practical cases, the conditions of existence for the analytical minimum will generally not be satisfied; the retro rockets are thus best fired in the direction of the velocity of the vehicle.

When the best orientation angle is 180°, the value of \( \gamma \) can be obtained from a simplified formula, directly derived from Eq (4.3):
The above relationship is rather important. If one considers a problem where the initial orbit is given, Eq. (5.6) indicates that the lowest value of $\gamma$ is function of $\Theta_E$ only, since $p_E$ is fixed. One may thus investigate the possibility of finding a particular value of $\Theta_E$ for which $\gamma$ would be absolute minimum. Differentiating Eq. (5.6) with respect to $\Theta_E$, for $\lambda$ fixed, and equating to zero, it appears that such an extremum exists and is given by the condition:

$$\lambda = \frac{1}{1 + \sin \Theta_E}$$

(5.7)

The absolute minimum of $\gamma$ is then given by:

$$\gamma = 1 - \sqrt{1 - \sin \Theta_E}$$

(5.8)

In other words, for a given value of the entry angle, which is dictated by peak deceleration and heat rates encountered during the subsequent flight into the atmosphere, there exists a particular orbit for which the fuel consumption for the recovery maneuver is an absolute minimum. For such an entry angle, the radius of the orbit is evaluated from Eq. (5.7) and the corresponding optimum value of $\gamma$ from Eq (5.8).

The above considerations are illustrated by calculations carried out for different orbits. But the definition of $\gamma$ is not adequate for this purpose; it is indeed more convenient to refer the velocity increment to a constant, for instance the circular velocity at sea level:

$$\Gamma = \frac{\gamma}{\sqrt{\rho}} = \frac{\Delta V}{V_s}$$

(5.9)

Using this definition, Eqs (5.7) and (5.8) can be transformed to yield the absolute minimum of $\Gamma$ in terms of $\lambda$ and $\Theta_E$ respectively:

$$\Gamma = \sqrt{\frac{1}{\rho_E}} \left( 1 - \sqrt{2 - \frac{\lambda}{\rho_E}} \right)$$

(5.10)
The results of the calculations, for different orbits and different entry angles are given in Fig. 4. Indicated on the figure are the lower boundary of the domain where the analytical minimum exists for $\omega$ different from 180°, and the locus of the absolute minima evaluated from Eq.(5.10). It is clearly shown that the practical cases fall in the region where the lowest value of $\gamma$ is obtained for $\omega = 180°$.

According to Lees, Hartwig and Cohen (ref. 5), a vehicle characterized by $\frac{W}{C_{D,\infty}A} = 100$ lbs/ft$^2$ must enter the atmosphere, at circular velocity, at an angle which is smaller than 3°, in order to avoid decelerations larger than 10 g's. One sees from Fig. 4 that for entry angles between 1° and 2°, the largest recoverable payload should be injected in circular orbits with altitudes ranging from 230 to 340 kms.

The velocities at entry into the atmosphere, for the same orbits as considered above, are indicated in Fig. 5. In the domain of existence of the analytical minimum, the final velocities are larger than those obtained for $\omega = 180°$.

6. **INFLUENCE OF MISALIGNMENT**

Knowing from the entry corridor what the limiting values of the entry angle are and, consequently, what is the accuracy required on this parameter, it is important to determine the errors which are allowed on the parameters of the transfer.

The error on the entry angle $\Delta \theta_E$, due to an error of alignment of the vehicle $\Delta \omega$ during the firing of the rockets, is easily evaluated from Eq. (4.3), in which the values of $\lambda$ and $\gamma$ are considered as fixed. The derivative of $\theta_E$ with respect to $\omega$ may be written as

$$\Gamma = \frac{1 - \sqrt{1 - d \sin \theta_E}}{\sqrt{P_E (1 + 5 \sin \theta_E)}} \quad (5.11)$$
The above formula shows that if the analytical minimum of \( \dot{\gamma} \) exists, the derivative is equal zero, since equation (5.1) is satisfied. Similarly, if the minimum does not exist, the lowest fuel consumption is obtained for \( \omega = 180^\circ \) and the derivative also vanishes. The error on the entry angle due to thrust misalignment is consequently zero, to the first order, in all cases where the recovery manoeuvre is achieved in conditions of minimum fuel consumption. This is a further advantage of the optimum conditions.

In such cases, the error on \( \dot{\Theta}_E \) must be evaluated from the second derivative, using the Taylor series expansion

\[
\Delta \Theta_E = \frac{d\Theta_E}{d\omega} \Delta \omega + \frac{1}{2} \frac{d^2\Theta_E}{d\omega^2} (\Delta \omega)^2 + \ldots \tag{6.2}
\]

When the analytical minimum does exist, one obtains for the second derivative, taking Eq (5.1) into account, the following expression

\[
\frac{d^2\Theta_E}{d\omega^2} = - \frac{\gamma^2 \sin \omega}{\Delta \omega \cos \Theta_E \cos \Theta_E [\gamma^2 \lambda^2 + 2 \gamma \lambda^2 \cos \omega - \lambda^2 + 2 \lambda]} \tag{6.3}
\]

where \( \gamma \) and \( \omega \) are evaluated from Eqs (5.2) and (5.1) respectively.

In the region where the lowest fuel consumption corresponds to \( \omega = 180^\circ \), the expression for the second derivative becomes

\[
\frac{d^2\Theta_E}{d\omega^2} = - \frac{\gamma (1 - \gamma - \lambda^2 \cos^2 \Theta_E)}{\Delta \omega \cos \Theta_E \cos \Theta_E [\gamma^2 \lambda^2 - 2 \gamma \lambda^2 - \lambda^2 + 2 \lambda]} \tag{6.4}
\]

where \( \gamma \) must be evaluated from Eq (5.6).

In both cases, the second derivative is a negative quantity. One may thus write

\[
\Delta \Theta_E = - \frac{1}{2} \left| \frac{d^2\Theta_E}{d\omega^2} \right| (\Delta \omega)^2 \tag{6.5}
\]

This relationship shows that, in the conditions of minimum fuel consumption, an error on the orientation of the vehicle, whatever the sign, has as an effect to reduce the inclination of the trajectory at the top of the atmosphere.
Peak deceleration and heat rates are accordingly reduced during the subsequent flight into the atmosphere.

Considering that the accuracy required on the inclination of the trajectory at the top of the atmosphere is 0.5 degrees, the corresponding values of $\Delta \omega$, which represent the margin of error during the firing of the retro-rockets, have been evaluated from Eq(6.5) for the same orbits as previously considered. The results are given in Fig. 6. For low entry angles, the accuracy required on the orientation of the vehicle is greater for higher altitudes of the initial orbit; it must also be greater for smaller values of the entry angle.

7. ERROR ON VELOCITY INCREMENT

This error is easily evaluated, using Eq(4.3) where $\lambda$ and $\omega$ are considered as fixed. The derivative of $\theta$ with respect to $\gamma$ is given by the expression

$$ \frac{d\theta}{d\gamma} = \frac{\lambda (\cos^2 \omega - \lambda^2 \cos^2 \theta_e) + \cos \omega (1 - \lambda^2 \cos^2 \theta_e)}{\lambda \gamma^2 + 2 \gamma \lambda^2 \cos \omega - \lambda^2 + 2 \lambda} $$

(7.1)

In practical cases of lowest fuel consumption for $\omega = 180^\circ$, the error on $\gamma$ can be, to the first order, evaluated from the equation

$$ \frac{\Delta \gamma}{\gamma} = \frac{\lambda \cos \theta_e \cos \omega \left[ \lambda^2 \gamma^2 - 2 \gamma \lambda^2 - \lambda^2 + 2 \lambda \right]}{\gamma (1 - \gamma) \left( 1 - \lambda^2 \cos^2 \theta_e \right)} \Delta \theta_e $$

(7.2)

where $\gamma$ is given by Eq(5.6).

The relative error on the velocity increment $\gamma$ has been evaluated from Eq (7.2) for an accuracy on the entry angle of 0.5 degrees. The results are given in Fig. 7 for the orbits previously considered. For low entry angles, the accuracy on the velocity increment must be greater for orbits at higher altitude and for smaller values of the entry angle.
8. ERROR OF OBSERVATION OF THE INITIAL ORBIT.

Using the relationships which have been derived above, the values of $\gamma$ and $\omega$ can be calculated for a given initial orbit, i.e. in terms of the radius of the orbit which has been determined from astronomical observation. However, the use of those calculated values for a vehicle which is not actually on the calculated orbit, will result in flight conditions at the top of the atmosphere different from those expected. It is then interesting to evaluate, in the same way as above, the accuracy required on the determination of the orbit radius. Equation (4.3) can be used for this purpose, regarding $\gamma$, $\omega$ and $\Theta_\varepsilon$ as fixed values, calculated for the estimated value of the radius. Evaluating the derivative of $\Theta_\varepsilon$ with respect to $r^*$, one obtains the relationship

$$\frac{\Delta \Theta_\varepsilon}{\Delta r^*} = -\cot \Theta_\varepsilon \left[ 1 - \frac{1}{\lambda \gamma^2 + 2\lambda \gamma \omega - \lambda + 2} \right]$$

(8.1)

which can be used to calculate, to the first order, the relative error allowed on the value of the orbit radius.

The above formula has been used for the same orbits as considered above, for $\Delta \Theta_\varepsilon = 0.5^\circ$. The results are given in Fig. 8. The accuracy required on the observation is greater for higher altitudes of the initial orbit and smaller values of the entry angle.

9. CONCLUSIONS.

The descent trajectories extending from the initial orbit to the top of the atmosphere can be easily analyzed, with the assumptions that the propulsion reduces to an impulse, the trajectories are in the plane of the initial orbit, the earth is the only attracting body.

The velocity increment required for the departure of the initial orbit and its orientation with respect to the local horizon can be evaluated,
for a given initial orbit, in terms of the conditions imposed at the top of the atmosphere.

For a given initial orbit and fixed values of the altitude and inclination of the trajectory at the top of the atmosphere, the problem can be optimized: there is a particular orientation of the vehicle which corresponds to the lowest velocity increment, i.e. to the lowest fuel consumption. The minimum exists in a particular domain, the border of which is easily established. However, practical cases fall outside the domain, and the lowest value of fuel consumption is then obtained for an orientation of the vehicle equal to 180°, i.e. firing the rockets in the direction of the velocity. In these conditions, the comparison of the results for different initial orbits shows that for a given inclination of the trajectory at the top of the atmosphere (dictated by peak deceleration or heat rates), there exists one particular orbit for which the fuel consumption for recovery is an absolute minimum.

The first order error on the inclination of the trajectory at entry into the atmosphere, due to a misalignment of the thrust at the departure of the initial orbit is equal to zero in the conditions of minimum fuel consumption. Evaluation of second order terms shows that the entry angle is always reduced by thrust misalignment, whatever be its sign.

For low angles of entry into the atmosphere, the accuracies required on the orientation of the vehicle during the firing of the retro rockets, on the magnitude of the velocity increment and on the estimation of the radius of the initial orbit, must be greater for higher altitudes of the initial orbit and for smaller values of the entry angle.
REFERENCES


Fig. 1. Conditions of existence for the analytical minimum.
Fig. 2. OPTIMUM VALUES FOR A PARTICULAR ORBIT.
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Fig. 8. Accuracy required on initial orbit radius.
Recovery trajectories extending from the initial circular orbit to the top of the atmosphere are analyzed. The propulsion is reduced to an impulse; the trajectories are in the plane of the initial orbit; the spherical non-rotating earth is the only center of attraction.

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