AN EXPERIMENTAL INVESTIGATION OF NONEQUILIBRIUM CORNER EXPANSION FLOWS OF DISSOCIATED OXYGEN

by

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SUMMARY

An experimental investigation of a two-dimensional nonequilibrium expansion flow of dissociated oxygen around a 15° corner has been carried out in the UTIAS 4 in. x 7 in. hypersonic shock tube. The flow was generated by a plane shock wave moving at a Mach number $M_s = 12$ into pure oxygen at a pressure of 20 mm Hg giving a degree of dissociation $\alpha = 0.22$, at a temperature of 3800°K. The study consisted of density measurements made with a Mach-Zehnder interferometer and pressure measurements using piezoelectric transducers. In addition, other optical studies (viz., schlieren and shadowgraph) were made to obtain information of a qualitative nature. Two fundamentally different test models were used to generate corner expansion flows in an effort to gain some insight into the nature and extent of boundary layer influence on the final results.

The detailed density and pressure results which were obtained indicate a longer relaxation distance than predicted by theoretical calculations based on a partially excited gas model and would appear to favor the predictions of more recent calculations based on a coupled vibration-dissociation model, in lieu of assuming instantaneous vibrational equilibrium. Viscous interaction between the boundary layer and the corner expansion wave tended to modify the experimental results and in some cases was the dominant factor.

Values for the recombination rate constant $k_R$ have not been inferred from the present results since this would require a direct comparison between experimental data and theoretical predictions. Such a comparison does not seem to be justified at the present time since there is still doubt as to the most appropriate theoretical model for computing the nonequilibrium flow properties. Once this is better known and assessed, it may then be possible to re-define new operating conditions that would provide more suitable results on which to base a determination of $k_R$. In addition, the choice of the test model to be used for generating the expansion flow will have to be tempered by the consideration of the influence of the boundary layer.
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## NOTATION

- **D**: dissociation energy per molecule
- **E**: average vibrational energy lost in dissociation per molecule
- **E_j**: energy of vibrational level $j$ per molecule
- **G**: average vibrational energy gained in recombination per molecule
- **h**: specific enthalpy (erg/gm)
- **K_c**: equilibrium constant based on mass concentration of atoms ($K_c = k_D/k_R$)
- **K_{O_2}**: Gladstone-Dale constant for molecular oxygen (cm$^3$/gm)
- **K_O**: Gladstone-Dale constant for atomic oxygen (cm$^3$/gm)
- **k**: Boltzmann constant (erg/$^\circ$K)
- **k_D**: dissociation rate constant (cm$^3$/mole sec)
- **k_R**: recombination rate constant (cm$^6$/mole$^2$ sec)
- **L**: parameter defined by Eq. (2.16)
- **l_R**: test section width (cm)
- **M_s**: shock Mach number
- **m_a**: mass of atoms per unit mole (gm/mole)
- **n**: coordinate normal to streamline
- **n_a, n_b**: phase indices of refraction for regions (a) and (b)
- **p**: pressure (atms., psi)
- **Q**: vibrational partition function
- **R**: gas constant per unit mass referred to diatomic gas (erg/gm$^\circ$K)
- **r_c**: characteristic radial length along wave head (cm)
- **s**: coordinate along streamline
- **S_{ab}**: nondimensional fringe shift from region (a) to region (b)
- **T**: temperature ($^\circ$K)
- **T_m**: temperature defined by Eq. (2.18) ($^\circ$K)
- **t**: time (sec)
- **U**: parameter used in Eq. (2.14) ($^\circ$K)
- **u**: flow velocity (cm/sec)
- **V**: coupling factor defined by Eq. (2.17)
- **x, y**: coordinate parallel and normal to free-stream direction
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**Subscripts**
- \( e \): equilibrium
- \( e_{P-M} \): state behind equilibrium Prandtl-Meyer expansion flow
- \( f \): frozen
- \( t \): translational
- \( v \): vibrational
- \( w \): wall surface
- \( l \): region ahead of incident shock
- \( 2 \): region behind incident shock and ahead of expansion corner
- \( 3 \): region downstream of expansion corner
I. INTRODUCTION

In recent years the study of gas flows at high velocities and elevated temperatures has attained a place of considerable prominence as a result of the rapid advances being made in high-speed flight both inside and outside of the earth's atmosphere. The nature of the problems dealing with hypersonic gas flows is both aerodynamic and thermochemical. Hence, it is desirable to be able to carry out studies involving both of these phenomenological aspects of hypervelocity flows. The advent and development of high-performance shock tubes and shock tunnels have provided research tools suitable for making such studies (Refs. 1 to 4).

A great deal of attention has been given to the study of shock-heated gases with particular emphasis on processes involving dissociation and ionization (Refs. 5 to 9). Generally, most of the interest has centered around determining the dissociation rates and ionization rates of various gases. Other workers have, however, concerned themselves with studies of the converse phenomenon; namely, that of association or recombination (Refs. 10 to 15). The importance of such studies is evident when one considers, for instance, problems relating to nozzle flows of reacting gases or afterbody wakes produced by high speed reentry of planetary or interplanetary vehicles.

A major question common to all types of studies mentioned above is that concerning the chemical state of the process being investigated as to whether it is frozen, in nonequilibrium or in equilibrium. A physical picture of the nature of reacting gas processes becomes extremely difficult when the possibility of excited states and coupling effects are considered (Refs. 16 to 19).

The present investigation has been undertaken in an effort to gain further knowledge regarding the behavior of reacting gas flows at high temperatures. The particular problem treated here is that of a two-dimensional expansion flow of partially dissociated oxygen. This work was carried out as a complement to the theoretical study by Glass and Takano (Ref. 14) of the nonequilibrium inviscid expansion flow of dissociated oxygen around a corner. Of particular interest in this study was the flow generated by a shock wave moving at \( M_0 = 12 \) through pure oxygen at an initial pressure of 20 mm Hg, producing a degree of dissociation of approximately 22% and a temperature of about 3800°K. This case was chosen as an optimum for studying the nonequilibrium expansion flow of dissociating oxygen around a 15° corner since it simultaneously provided adequate interferometric fringe shift and suitable relaxation lengths (Ref. 12).

One of the interesting features of the complex flow pattern computed by Glass and Takano was the existence of a recombination shock at the tail of the expansion wave (Ref. 14). Although it was felt that the presence of this weak shock wave was, in fact, due to the mathematical model (the assumption of instantaneous vibrational equilibrium was made throughout the flow field except at the very corner), an experimental verification of its existence or non-existence was of some interest in the present experimental investigation.
The ultimate object of the experimental study was to try to determine a value for the recombination rate constant in the manner proposed by Glass and Takano (Ref. 14). In essence, this would require a comparison of experimental values of density, pressure, or some other flow quantity along the wall surface downstream of the expansion corner with that predicted by theoretical calculations based on an assumed value for the recombination rate constant. A study of the structure and decay of the head of the expansion wave was also considered to be worthwhile since it could, in principle, provide an alternate means of determining a value for \( k_R \) (Ref. 12).

A general discussion of theoretical considerations related to the present problem is given in Section II.

It was realized at the very beginning of the investigation that viscous and catalytic phenomena would undoubtedly have some effect on the experimental results obtained. Due to the complexity of boundary-layer flows, their presence was not considered in the theoretical treatment of the corner expansion flow by Glass and Takano. However, it was suggested that, by using appropriate expansion models, some insight into the overall influence of boundary-layer phenomena might be gained experimentally. Hence to test this suggestion two basic test models were used to generate corner expansion flows in the UTIAS hypersonic shock tube. One type was a wall model and the other was a wedge model. The reason for using these two particular models was that the type of boundary layer produced in each case would be of a fundamentally different nature. In the case of the wall model, the boundary layer would be of the nonstationary shock-tube type whereas for the wedge model the boundary layer would be a Blasius type for quasi-steady flow over the model. It was felt that a comparison of experimental results obtained using both models would be helpful in assessing the general features produced by the presence of a boundary layer along the wall surface in the corner expansion flows. Catalytic effects would have to be assessed by other techniques.

A detailed description of the test models used is given in Section III along with a description of the experimental apparatus which was used in carrying out the investigation. Data analysis considerations are given in Section IV. Discussion of the final experimental results and concluding remarks are presented in Sections V and VI, respectively.
II. THEORETICAL CONSIDERATIONS

As already mentioned, the present experimental investigation was undertaken as a complement to the theoretical study of the nonequilibrium expansion flow of dissociated oxygen carried out by Glass and Takano (Ref. 14). The principal operating conditions of $M_s = 12$ and $P_l = 20 \text{ mm Hg}$ used to generate an expansion flow around a $15^\circ$ corner correspond to the free stream conditions of particular interest in the analysis. These conditions were proposed by Glass and Kawada (Ref. 12) as being optimum for studying a nonequilibrium expansion flow of dissociated oxygen in the present experimental facility. The criteria used in selecting the initial operating conditions were the following:

1. The interferometric fringe shift across the corner expansion wave should be at least two in order to obtain reasonable measuring accuracy.

2. Relaxation lengths along the expansion wave head and along the wall downstream of the corner should lie between 1 cm. and 10 cm. The lower limit was arbitrarily chosen as a practical minimum for observing nonequilibrium effects and the upper limit was dictated by the viewing area of the interferometer.

3. The corner expansion angle should be large enough to produce significant changes in the flow quantities but not too large so as to appreciably reduce the flow region behind the expansion wave. Also the practical consideration of providing sufficient space for mounting pressure transducers had to be satisfied.

For the initial conditions of $M_s = 12$, $P_l = 20 \text{ mm Hg}$ and $\theta_w = 15^\circ$ suggested by Glass and Kawada, the expected fringe shift was approximately 9 fringes while the predicted relaxation lengths $l_c$ and $l_R$ along the expansion wave head and along the wall were 2 cm. and 9 cm., respectively (Ref. 12). The final choice of $M_s$ and $P_l$ was made on the basis of calculated results for 15 different combinations of initial conditions (i.e., $M_s$ and $P_l$) for a flow deflection angle of $15^\circ$. It should be noted that the relaxation lengths were determined in a simplified manner and therefore can only be regarded as estimates. For the calculation of $l_c$, the assumption of an "ideal dissociating gas" was used. For the determination of $l_R$, the assumption of small perturbation from equilibrium of the dissociation degree as well as all other flow quantities was made. A plot of the results obtained by Glass and Kawada is given in Fig. 1. The curves $C_1$ and $C_2$ represent constant $l_R$ values of 1 cm. and 10 cm., respectively. Similarly, curves $C_3$ and $C_4$ correspond to constant values for $r_c$ of 1 cm. and 10 cm. The remaining two curves $S_{23}$ are constant fringe shift curves for 1 and 5 fringes. The proposed operating point is indicated in the $(M_s, P_l)$-plane.

The work by Glass and Kawada was subsequently extended by Glass and Takano (Ref. 14) in the form of a detailed study of the nonequilibrium expansion flow of dissociated oxygen around a corner for the free-stream conditions behind a normal shock wave moving at $M_s = 12$ into pure oxygen at an initial pressure of $20 \text{ mm Hg}$. Wall deflection angles of $5^\circ$, $10^\circ$, $15^\circ$ and $30^\circ$ were considered for these upstream conditions. A brief summary of the analysis performed by Glass and Takano will be given in the following paragraphs.
In carrying out their analysis Glass and Takano made the following simplifying assumptions (Ref. 14):

(i) The flow field is assumed to be steady and two-dimensional.

(ii) Molecular transport effects leading to viscosity, heat conduction and diffusion are neglected.

(iii) The deviations from thermodynamic equilibrium are such that the fundamental Gibb's equation can still be used.

(iv) Although the total system of the mixture may not be in chemical equilibrium, each component of the mixture is in thermal equilibrium, so that the thermodynamic properties of each species can be derived from the corresponding partition functions.

(v) Dalton's law of partial pressures can be applied.

(vi) The properties of density, enthalpy, etc. for the mixture are the weighted sums of the corresponding properties for the single system.

(vii) The vibrational mode is assumed to be in thermal equilibrium with the translational and rotational modes, except along the frozen wave head and at the very corner, where the vibrational excitation as well as the degree of dissociation are frozen at their free-stream values.

(viii) Ionization and electronic excitation are neglected.

(ix) The recombination rate constant \( k_R \) is assumed to be invariant with temperature over the range of interest \((\sim 2500^\circ \text{K} - 4000^\circ \text{K})\).

In writing the basic equations of motion, the natural co-ordinates along a streamline \((s)\) and its normal \((n)\), as shown in the sketch below, can be used.
In accordance with the above assumptions, the following set of equations can be written for the determination of the unknown quantities, \( p, \rho, T, \alpha, u \) and \( \theta \) (Ref. 14).

The overall continuity equation and the momentum equations are unchanged from those for a perfect gas and can be written as

**Continuity**

\[
 u \frac{\partial p}{\partial s} + \rho \frac{\partial u}{\partial s} + \rho u \frac{\partial \theta}{\partial \theta} = 0 \tag{2.1}
\]

**Momentum**

\[
 u \frac{\partial u}{\partial s} + \frac{1}{\rho} \frac{\partial p}{\partial s} = 0 \quad \text{[s - direction]} \tag{2.2}
\]

\[
 u^2 \frac{\partial \theta}{\partial s} + \frac{1}{\rho} \frac{\partial p}{\partial \theta} = 0 \quad \text{[n - direction]} \tag{2.3}
\]

For a steady, adiabatic flow, where all streamlines originate from a region of uniform flow, the total enthalpy is the same everywhere. Hence

\[
 h + \frac{1}{2} u^2 = \text{const.} \tag{2.4}
\]

Thus, along a given streamline

\[
 \frac{\partial h}{\partial s} + u \frac{\partial u}{\partial s} = 0 \tag{2.5}
\]

Making use of Eq. (2.2), the energy equation can be written as

\[
 \frac{\partial h}{\partial s} - \frac{1}{\rho} \frac{\partial p}{\partial \theta} = 0 \tag{2.6}
\]

where the specific enthalpy \( h \) is given by

\[
 h = \left[ \frac{7 + 3\alpha}{2} + \frac{\gamma_D}{\gamma} \alpha + (1-\alpha) \frac{\gamma_v/T}{e^{\gamma_v/T} - 1} \right] RT \tag{2.7}
\]

The equation of state for a dissociating gas is

\[
 p = (1 + \alpha) \rho RT \tag{2.8}
\]

To complete the system of equations required to specify the problem, it is necessary to have an equation for the mass production rate of atoms. For the dissociation-recombination process described by

\[
 A_2 + X \xrightarrow{k_D} \frac{k_D}{k_R} 2A + X \tag{2.9}
\]

where \( A_2 \) is a molecule, \( A \) is an atom and \( X \) is a third body (either molecule or atom), the mass production rate of atoms is given by (Ref. 14)
\[
\frac{d(\rho \alpha)}{dt} = \frac{k_R \rho^3 (1 + \alpha)}{m_a^2} \left[ \frac{m_a}{2\rho} \frac{(1-\alpha)}{K_c - \alpha^2} \right]
\]

where \( \frac{d}{dt} \) denotes differentiation with respect to time along a particle path (Lagrangian coordinates). By using the continuity equation (2.1), Eq. (2.10) gives the following rate equation along a streamline (Eulerian coordinates)

\[
u \frac{\partial \alpha}{\partial s} = \frac{k_R \rho^2 (1 + \alpha)}{m_a^2} \left[ \frac{m_a}{2\rho} \frac{(1-\alpha)}{K_c - \alpha^2} \right]
\]

\( K_c \) is the equilibrium constant based on mass concentration and is defined as the ratio of the dissociation rate to the recombination rate (i.e., \( K_c = \frac{k_D}{k_R} \)).

The equations given by (2.1), (2.2), (2.3), (2.6), (2.8) and (2.11) provide the six relations for determining the six unknown quantities \( p, \rho, T, \alpha, u \) and \( \theta \). To solve this system of equations, Glass and Takano applied the method of characteristics for a supersonic flow with two independent variables (see Ref. 20). In this manner it was possible to form a system of characteristic finite difference equations which could be solved by numerical computation. The flow at the very corner was considered to be frozen and of the Prandtl-Meyer type. Thus the flow quantities there could be determined analytically to provide boundary conditions for starting the calculation of the corner expansion flow field.

A sketch of the overall flow pattern predicted by Glass and Takano is shown in Fig. 2. Also shown in the same figure are the predicted variations of pressure, temperature, flow direction and entropy along each of four typical streamlines. Results for the variation of \( p, \rho, T \) and \( \alpha \) along the wall downstream of the corner are shown in Fig. 3.

As can be seen in Fig. 2(a), the predicted flow pattern is quite complex. Briefly summarizing, the flow in the region of the expansion wave near the corner was found to over-expand (i.e. \( |\theta| > |\theta_{\text{wall}}| \)). This resulted in a compression region being formed just downstream of the corner, followed by a region of gradual expansion such that far downstream of the corner the flow tended to its equilibrium state. The existence of a weak de-excitation shock along the expansion wave tail was verified by the intersection of the expansion wave tail and the neighbouring compression characteristic line reflected from the wall surface. Furthermore, unlike the classical Prandtl-Meyer type expansion fan, the radial characteristics (i.e., \( \eta \)-family in Fig. 2(a) ) are curved rather than straight. A detailed discussion of the other plots can be found in Ref. 14.

There are several unrealistic and limiting features about the analysis performed by Glass and Takano which they have noted in their work that were primarily dictated by the difficulty or the uncertainty involved in trying to properly cope with them. They are as follows:

1. The assumption of instantaneous vibrational equilibration throughout the expanding flow field, with the exception of the very corner and the frozen wave head.
The use of an invariant recombination rate constant.

The failure to take into account the viscous and catalytic interaction between the boundary layer along the wall and the quasi-steady corner expansion wave.

The assumption of a uniform flow between the shock wave and contact region which is expanded around the corner.

The unrealistic effect created by the first simplifying assumption is that of instantaneous relaxation of vibrational energy from its upstream value to a state of equilibrium with the active modes (viz., translation and rotation) as soon as the calculations proceed away from the corner. Such an assumption for the outer portion of the expansion wave, away from the corner does not seem unreasonable since the flow expansion takes place more gradually over a longer distance. However, in the region near the corner, the flow time through the expansion is much shorter, making the assumption of vibrational equilibrium rather doubtful.

After performing nonequilibrium expansion flow calculations for the case of an ionized monatomic gas, which did not require any instantaneous addition of energy at the corner, Glass and Takano (Ref. 21) concluded that the presence of the de-excitation shock was in fact due to the assumption of instantaneous equilibrium of vibration and would not have appeared if a proper rate equation for vibrational relaxation had been used.

The problem of dealing with simultaneous vibrational and dissociational nonequilibrium in reacting gas flows is one of considerable complexity due primarily to a lack of understanding of the relevant rate processes. It has been shown that dissociation-recombination rate constants which have been determined from analytical or experimental studies for near-equilibrium conditions behind normal shock waves must be modified if one considers simultaneous nonequilibrium in vibration and dissociation (Refs. 22 and 23). Also the classical Landau-Teller equation for vibrational relaxation has to be modified to include the effects of dissociation and recombination (Ref. 23). Furthermore, recent experimental studies seem to indicate that the characteristic vibrational relaxation time for expansive flows of reacting diatomic gases, which involve relaxation from higher to lower excited vibrational states, is shorter than for reacting flows behind normal shock waves, which involve relaxation from lower to higher excited states (Ref. 15).

In an effort to obtain results based on a more realistic flow model than that assumed by Glass and Takano, calculations have been performed quite recently by Tirumalesa (Ref. 24) for the corner expansion flow of dissociated oxygen, in which simultaneous vibrational and dissociational nonequilibrium was considered. The same coupling model for vibration-dissociation as that used in an earlier analysis (Ref. 19) was assumed in making these computations.

Without attempting to present the many details of Tirumalesa's analysis, a brief discussion will be given to point out the basic differences between it and the earlier work of Glass and Takano, particularly in terms of changes in the governing equations. The essential difference, of course, is that the vibrational degree of freedom is not assumed to be in equilibrium with the translational-rotational modes. Vibrational energy is assumed to be specified by a Boltzmann distribution in terms of a vibrational temperature $T_v$, while
the translation and rotation modes are assumed to be equilibrated and specified by the translational temperature $T_t$.

Since vibration is no longer considered to be in equilibrium with translation and rotation, the specific enthalpy can no longer be expressed solely in terms of the translational temperature as in Eq. (2.7), but must be rewritten in terms of both $T_t$ and $T_v$. This can be done as follows:

$$ h = \left( \frac{7 + 3\alpha}{2} \right) R T_t + \alpha R \delta_D + (1 - \alpha) \epsilon_v $$

(2.12)

where

$$ \epsilon_v = \frac{R \delta_v}{e^{\delta_v/T_v} - 1} $$

(2.13)

When substituted into Eq. (2.6), Eq. (2.12) will result in an energy equation different from that used by Glass and Takano.

The only other equation of the earlier set of equations that has to be modified is the rate equation for the mass production of atoms (i.e., Eq. (2.13)). This modification is necessary in order to take into account the coupling between vibrational relaxation and dissociation-recombination. The rate equation for mass production of atoms used by Tirumalesa is given in the form (Ref. 19)

$$ u \frac{\partial \alpha}{\partial \xi} = \psi (V_L - \alpha^2) $$

(2.14)

where

$$ \psi = \frac{k_R \rho^2 (1 - \alpha + 2\lambda \alpha)}{m_a^2} $$

(2.15)

$$ L = \frac{m_a (1 - \alpha)}{2\rho} K_c $$

(2.16)

$$ V = \frac{Q(T_t) Q(T_m)}{Q(T_v) Q(-U)} $$

(2.17)

and

$$ \frac{1}{T_m} = \frac{1}{T_v} - \frac{1}{T_t} - \frac{1}{U} $$

(2.18)

$$ Q(T) = \sum_j \exp \left( -E_j/kT \right) $$

(2.19)

The factor $\lambda$ in Eq. (2.15) is defined as the relative efficiency of atoms to molecules in causing dissociation. From experimental studies of the rate of dissociation of oxygen behind normal shock waves Byron found that (Ref. 5)

$$ \lambda = 35 \frac{k T_t}{D} $$

(2.20)

The factor $V$, defined by Eq. (2.17) in terms of the vibrational partition function evaluated for different values of $T$, is referred to as the coupling factor. The dissociation rate constant for vibrational nonequilibrium and equilibrium are related through the relation $k_D = V (k_D)_{vib.equil}$.
The parameter \( U \) in Eq. (2.17) is related to the relative efficiency of dissociation from different vibrational energy levels. Two values of this parameter are considered by Tirumalesa (Ref. 19); namely \( U = \infty \) which corresponds to equal probability for dissociation from all vibrational levels (referred to as "nonpreferential" coupling) and \( U = \frac{\rho_D}{\rho} \) corresponding to higher probability for dissociation from higher vibrational levels (referred to as "preferential" coupling).

It should be noted that, for \( \lambda = 1 \) and \( V = 1 \), the rate equation given by Eq. (2.14) reduces to that given by Eq. (2.11) for vibrational equilibrium. It can be seen from Eqs. (2.17) and (2.18) that if \( T_V = T_t \), then \( T_m = -U \) and \( V = 1 \).

Since an additional unknown (viz., \( T_V \)) has been introduced, another equation is now required in order to completely specify the problem. This equation is obtained in the form of a vibrational rate equation given by (Ref. 19).

\[
\frac{dE_v}{dt} = \frac{E_v - \epsilon_v}{\tau_v} - \frac{\psi}{1 - \alpha} \left\{ \left( \overline{E} - \epsilon_v \right) V L - \left( \overline{G} - \epsilon_v \right) \right\}
\]  
(2.21)

where

\( \epsilon_v = \) actual vibrational energy, \( \epsilon(T_V) \)

\( \epsilon_\infty = \) vibrational energy based on the local translational temperature, \( \epsilon(T_t) \)

\( \overline{E} = \) average vibrational energy lost per dissociation, \( \epsilon(T_m) \)

\( \overline{G} = \) average vibrational energy gained per recombination, \( \epsilon(-U) \)

\( \tau_v = \) vibrational relaxation time

and

\[
\epsilon(T) = \frac{\sum_j E_j \exp \left(-E_j/kT\right)}{Q(T)}
\]
(2.22)

For a steady, two-dimensional flow the left-hand side of Eq. (2.21) can be written as \( u \partial \epsilon_v / \partial s \) for the streamline direction.

Along with the earlier equations of (2.1), (2.2), (2.3) and (2.8), the vibrational rate equation given by Eq. (2.21), the rate equation for mass production of atoms given by Eq. (2.14) and the modified energy equation given by Eqs. (2.6) and (2.12) provide the complete set of equations required to solve for the unknown flow quantities. As in the case of the earlier work of Glass and Takano, the solution of these equations was performed numerically by Tirumalesa using the method of characteristics.

At the present time the computational phase of Tirumalesa's analysis is still incomplete. However, results have been obtained for a case with the free-stream conditions of particular interest in the present work. In this preliminary case, the recombination rate constant \( k_R \) was assumed to be invariant with temperature and given by the same value as that assumed by Glass.
and Takano (Ref. 14). The vibrational relaxation time $\tau_v$ was assumed to be that predicted for a vibrationally relaxing flow behind a normal shock and may be expressed in the following general form for a diatomic gas (Ref. 25)

$$ p \tau_v = a \exp (-b T^{1/3}) \text{ [atm. sec]} $$

(2.23)

where $a$ and $b$ are constants, $p$ is pressure and $T$ is translational temperature.

Some of the results obtained for the above conditions are given in Fig. 4. The earlier results of Glass and Takano (shown in Fig. 3) are re-plotted as dotted curves for comparison. The most significant difference between the coupled vibration-dissociation (CVD) results and those for vibrational equilibrium (VEQ) is the predicted approach to equilibrium. As can be seen, the relaxation distance in the CVD case is much longer (by about 40 or 50 cm.) than predicted in the VEQ case. This difference is of particular significance to the present experimental work since one of the original criteria that was considered in choosing the operating conditions specified a practical upper limit of 10 cm. for the relaxation distance $l_R$. It is also of interest to note that the difference between the vibrational and translational temperatures, as shown in Fig. 4(d), is quite significant. As can be seen, the approach of $T_{vw}$ to the equilibrium temperature downstream of the corner is much slower than for the local translational temperature $T_w$.

A further interesting result of these preliminary CVD calculations is that there was no clear indication of a recombination shock along the expansion wave tail, thus supporting the belief that the presence of such a shock in the earlier results of Glass and Takano was a consequence of the simplified mathematical model used.

The major difficulty in carrying out such a study as the present CVD analysis lies in choosing a model that correctly describes the problem. Tirumalesa has essentially adopted the model proposed by Marrone and Treanor (Ref. 23) for reacting gas flows behind normal shocks, involving vibrational relaxation from lower to higher states of excitation. As mentioned earlier, there is evidence that the inverse process involving relaxation from higher to lower excited states (i.e., expansive flows of high temperature reacting gases) cannot be described in exactly the same manner. It is suggested that a rate limiting step or "bottleneck" exists in the vibrational deexcitation process which results in an over-population of the higher vibrational energy levels. The occurrence of this so-called bottleneck is believed to result primarily from a molecule behaving as an anharmonic oscillator (Ref. 26).

Although the results given in Fig. 4 are based on the assumption of a Boltzmann distribution of vibrational energy and a vibrational relaxation time relevant to relaxation from an unexcited to an excited state, it is felt that the difference between the CVD case and the VEQ case is still significant. Even with the simplifications involved, the CVD model should still be more representative of the actual physical problem than the VEQ model.

With regard to the question of vibrational relaxation time, the CVD calculations are being extended to include cases involving shorter relaxation times, comparable to values obtained experimentally in expansive flows (Ref. 15). Although this will effectively shorten the predicted relaxation distance $l_R$, it should still be significantly longer than predicted by the VEQ
calculations. Also the restriction of the recombination rate constant $k_R$ being invariant with temperature will be removed. Since no truly definitive results for $\tau_R$ or $k_R$ are presently available, a systematic analytical study of the changes resulting from their variation is considered as worthwhile. It may then be possible to redefine the optimum conditions shown in Fig. 1 based on the new analysis in order to reinvestigate this problem on this new basis.

As already pointed out earlier, a further complication in the present investigation, which was ignored in the above analytical studies of the problem, is the influence of the boundary layer. A recent analysis of the interaction of a boundary layer and the corner expansion wave in supersonic flow was made by Oosthuizen (Ref. 27) in which one of the cases treated corresponded to the free-stream conditions of main interest in the present work. More will be said regarding the general nature and results of this analysis in a later discussion (Section 5.4).

III. EXPERIMENTAL APPARATUS

3.1 Shock Tube Facility

The experimental investigation was carried out in the UTIAS 4in. x 7 in. hypersonic shock tube. Overall views of the facility are shown in Fig. 5. The design, instrumentation and general performance of the shock tube has been presented in detail in Ref. 28. Hence, only a brief and general description will be given in this report.

The shock tube consists of five major components: a 6 in. I.D. driver, a transition section, a 4 in. x 7 in. driven section, a test section and a dump tank. The driven section of the shock tube is approximately 50 ft. long and is terminated by a vertical dump tank that is $6\frac{1}{2}$ ft. x 3 ft. diameter. The test section position is approximately 46 ft. downstream of the diaphragm station.

A stoichiometric mixture of hydrogen and oxygen with 75% helium dilution is used as the driver gas. Ignition of the gaseous mixture is accomplished by means of an impulse-heated tungsten wire located along the centre-line of the driver section. The diaphragms used are stainless steel (type 304). For this work, they had a basic (unscribed) thickness of 0.062 in. or 0.108 in. and were suitably scribed to achieve rupture at desired levels of diaphragm pressure ratio (see Refs. 29 and 30).

Evacuation of the driven section is accomplished with a Heraeus Roots pump (type VP-RG-350A) backed by a Kinney pump (type KS-47). An ultimate vacuum of approximately 3 microns Hg was obtainable on a day-to-day basis of operation. The overall rate of leakage and outgassing was approximately 2 microns Hg per minute. This rate was not considered to be excessive in light of the fact that a considerable quantity of water was left in the shock tube after each run as a by-product of the combustion process in the driver. Also the rate of pressure rise was quite small in comparison with the initial pressure level of 20 mm Hg which was the nominal value used in the driven section for all experimental runs. All runs were taken within 10 - 12 minutes from the time when the test gas was introduced into the driven section.
The test gas used in all runs was commercial grade oxygen.*
The initial pressure in the driven section was measured at a position near the
test section using a precision mercurial manometer (Wallace and Tiernan Type
FA-187) with a scale resolution of 0.1 mm Hg. The compensating chamber of the
interferometer was filled with test gas simultaneously with the driven section
and its final pressure measurement was made along with that for the driven section
before it was isolated by closing a valve in the connecting line.

The general instrumentation used throughout the experimental work
included three Tektronix oscilloscopes (2 - type 555 and 1 - type 545), two Racal
time-interval counters (± 1 μs resolution), a Racal time-delay generator
(± 1 μs resolution) and a number of pulse amplifier units. Some of this equip­
ment can be seen in Fig. 5 (b).

The combustion-driver pressure was monitored using a Kistler
Type 605 pressure transducer. This measurement was not a necessary part of
the experiments but was considered as worthwhile since it provided a measure
of the nature and performance of the combustion-driving. In general, the per­
formance and repeatability of the combustion-driver process was excellent. As
a consequence a very close tolerance on the variation of shock Mach number
from run to run was maintained.

Additional instrumentation which was used in carrying out the
experimental investigation is discussed in the following sections.

3.2 Test Models

The two types of test models which were used to generate corner
expansion flows are shown schematically in Fig. 6 and photographically in Fig. 7.
A brief discussion of the design and construction of the models follows.

3.2.1 Wall Model

This model is approximately 19 ft. in length and is clamped
rigidly to the upper wall of the shock tube. The main portion of the wall
model was fabricated from a 15 ft. length of aluminum channel (4 in. x 1 1/2 in.).
The details of the support struts for this section of the model are shown in
Fig. 8. The leading edge of the model is a 25° wedge made of solid aluminum
which is attached to the aluminum channel and strut-supported in the manner
shown in Fig. 9. The downstream end of the wall model (see Fig. 7a) terminates
in the test section and provides a 15° corner expansion. The general details
for this section of the model are shown in Fig. 10. Additional information
concerning the instrumentation contained in this part of the wall model, in
particular the mounting of pressure transducers, is given in Section 3.3.

The overall length of the wall model was chosen more-or-less
arbitrarily within certain limitations. The basic consideration was that its
length should be such as to provide an adequate region of test flow behind the
shock wave which would not be influenced by the presence of the wall.

* Bottled oxygen supplied by CANOX (Canadian Oxygen Ltd.). The quoted purity
level was 99.9% with the impurities being argon and water vapor.
normal shock wave in the test section. A general discussion of the wave system (both steady and unsteady) formed during operation of the shock tube with such a wall model is presented in Appendix A.

Assuming that test gas only comes from the region downstream of the leading edge of the model, it was determined (see Appendix A) that the theoretical test flow duration was 136 µs for \( M_s = 12 \) and \( p_1 = 20 \) mm Hg (\( O_2 \)). Previous shock tube measurements have indicated that the actual duration of test flow in a shock tube may be no more than about 50% of that predicted theoretically, due to viscous effects (Ref. 31). Even if such is the case, a test flow duration of 60-70 µs was still considered to be acceptable since it was felt that most of the experimental measurements (particularly interferometric) would be confined to the flow region immediately behind the incident shock wave as it passed through the test section.

As will be seen in later discussions of the experimental results obtained using the wall model, the interferometric data was actually taken at test flow times ranging from approximately 20 µs up to about 55 µs.

3.2.2 Wedge Model

The general details of this model are shown in Fig. 11. It is a solid model made from mild steel and hence contains no internal instrumentation as does the wall model. An original version of this model was capable of being instrumented but unfortunately, failed structurally during operation. In order to obtain maximum structural integrity and owing to the general difficulties involved in instrumentation mounting, it was decided to replace the original model with the present simpler one which could then be used to obtain interferometric results as well as other optical data.

Unlike the original wedge model (Ref. 28), the present one was designed such that its leading edge would appear just inside the viewing area of the test section, thus making it possible to observe the flow field in the vicinity of the leading edge. Also an attempt was made to minimize leading edge bluntness as much as practically possible by specifying that the leading edge thickness be .005" or less. When mounted in the test section, the surface of the model ahead of the corner was determined to be aligned (i.e., parallel) with the opposing wall of the shock tube to better than 0.1°.

During operation with the wedge model, a rather serious problem was encountered which limited its actual use in the present work. In the experimental runs at the highest value of shock Mach number (\( M_s = 12 \)) surface etching of the high quality optical windows in the test section was produced by the quasi-steady flow field of hot, dense gas behind the shock waves from the leading edge (compression side of wedge) and the supporting strut of the model. There was no apparent damage to the windows in the region adjacent to the lower side of the model. It was for this reason that only a few runs at high shock Mach numbers were made using the wedge model, since continuing damage to the test section windows would probably increase the cost and time required to repolish them in the future. To overcome the problem arising from aerodynamic heating, it should be possible to modify the design of the present model to provide appropriate baffles which would protect the windows at the required zones.
3.3 Pressure Transducers

A total of four pressure transducers were used to obtain pressure measurements just ahead of and downstream of the expansion corner of the wall model (see Fig. 7a). One gauge was located 0.5 in. ahead of the corner and in the center of the model. The other three gauges were located 0.5 in., 1.5 in. and 2.5 in. downstream of the corner and directly in line with the first. The gauge used ahead of the corner was a Kistler Type 601 transducer like the one shown in Fig. 12. The signal from this gauge was fed through a Kistler Model 566 charge amplifier to an oscilloscope. A 50 Kc low-pass filter (Kistler Model 544A50) with a damping factor of $0.7 \pm 0.1$ at -3db was used on the output of the charge amplifier. The purpose of this filter was to help overcome the problem of high-frequency gauge ringing.

The three gauges used downstream of the corner were lead-zirconium-titanate (PZT) pressure transducers* (see Fig. 12) with a linear range of approximately 60 psi. The main reason for using these gauges was because of their relatively small size. Internal space available for instrumentation mounting, gauge leads, etc. was quite limited downstream of the expansion corner due to the decreasing cross section of the model. The output signal from each PZT pressure transducer was fed through a low-noise cable to a cathode follower. The output of the cathode follower was controlled by a precision attenuator and fed into an oscilloscope.

The major problem in making pressure measurements was in the actual mounting of the gauges themselves. None of the gauges were acceleration compensated. Also the three PZT pressure transducers had a low natural frequency (~ 50,000 cps). Hence, without adequate shock-mounting, the quality of the signal received from the gauges would be quite poor and practically meaningless as a result of "gauge ringing". Several months of trial-and-error effort were devoted to the problem of shock-mounting the pressure transducers. The scheme which was finally settled on as being reasonably effective is shown in Fig. 13. The PZT transducers are carefully mounted on a central gauge block which in turn is fastened inside the wall model and effectively isolated using natural rubber pads of medium durometer and O-rings under the heads of all fastening screws. Every effort was made to eliminate any metal-to-metal contact between the pressure gauges and the model itself, while at the same time still providing a stable platform for gauge support. The Kistler-601 transducer was also mounted in a similar manner.

The improvement in gauge performance obtained using the above method of shock-mounting can be seen in Fig.14 where some preliminary pressure records obtained with inadequate shock-mounting are compared with results from later tests where the gauges were mounted as described above. The pressure traces shown in Fig. 14(a) were obtained with the gauges mounted directly in the model rather than on a separate gauge block. As can be seen, the gauges (in particular, the first two) were being excited by mechanical vibration transmitted through the wall model even before the arrival of the shock wave. The elimination of this pre-shock noise served as a sort of indicator to improvement in gauge mounting. The results shown in Fig. 14(b) were obtained with the transducers mounted in the manner shown in Fig.13.

* Manufactured by Mr. J. Bialy, 14 Markus Drive, Buffalo 25, N.Y.
The procedure for calibration of the four primary pressure transducers is outlined in Ref. 28 and is as follows: "Calibration is achieved by recording and measuring response to a sudden application of pressure. A reservoir tank equipped with a precision differential pressure gauge (Wallace and Tiernan type FA 145) is connected by a solenoid valve to an adaptor plug housing the PZT transducer. The reservoir tank is filled with air through a needle valve to the desired calibration pressure. Opening the solenoid valve applies this pressure to the face of the transducer and, at the same time, triggers the oscilloscope used to record the transducer response. The volume of the line and adaptor housing the transducer is extremely small in comparison with reservoir volume, so that no measurable error is introduced in assuming the full reservoir pressure is applied to the transducer. A coil of very fine bore hypodermic tubing serves as a bleed to atmosphere to ensure that the transducer is initially at atmospheric pressure."

An overall view of the calibration equipment and its arrangement is shown in Fig. 15. Some typical calibration pressure traces are given in Fig. 16. The calibration curves for the Kistler 601 transducer and for the PZT transducers are given in Figs. 17 and 18, respectively.

Before each run, a film of silicone grease was applied to the faces of the pressure transducers to prevent gauge malfunction due to temperature-sensitive effects caused by the high values of temperature occurring behind incident shock waves.

Finally, it is worth mentioning that an additional three pressure transducers (Atlantic Research type LD-25), which have a fast response time (1 to 2 µsec) and are relatively inexpensive, were used as shock detectors throughout the experimental investigation. These gauges were located upstream of the expansion corner (see Fig. 10) and were separated by 1 ft. intervals. Timing measurements were taken between the first and third gauges (Δx = 2 ft.) and the second and third gauges (Δx = 1 ft.), thus providing average shock speeds at approximate positions of 16 in. and 10 in. ahead of the corner, respectively. The signals from the shock detectors were fed into pulse amplifiers which provided starting and stopping pulses for the time-interval counters as well as triggering pulses for (1) the oscilloscope recording the responses of the pressure transducers in the region of the expansion corner and (2) the time-delay generator controlling the firing of the interferometer spark source.

3.4 Interferometer

A Mach-Zehnder type interferometer was used in making optical studies of the corner expansion flows. A complete description of this instrument is given in Ref. 32. A modification in the design of the original compensating chamber of the interferometer was required in order to mount optical duplicates of the present test section windows (Ref. 28). The techniques involved in using the interferometer have also been discussed in reports of previous experimental studies (Refs. 33 and 34).

The general interferometric arrangement is shown schematically in Fig. 19. Light from a spark-gap source (magnesium electrodes) is collected by a 10 in. parabolic mirror (f/6) and reflected in a parallel beam to the first beam splitter which has a transmission-to-reflection ratio of approximately
The reflected portion of the initial beam is directed through the test section by a front-surface plane mirror while the transmitted portion passes through the compensating chamber to a second front-surfaced mirror. The two beams are recombined at the second splitter and the resultant beam is then collected by another parabolic mirror which directs it, by means of a front-surface mirror and lens, onto a photographic plate. An interference filter peaked at 5200 Å was placed ahead of the plate to provide monochromatic light. All of the photographic plates used for interferometric studies were Ilford HR8 and were processed in "promicrol" developer.

By blanking off the compensating chamber, it was possible to use the existing optical arrangement for schlieren or shadowgraph. For schlieren studies a knife edge was placed at the focal point of the second parabolic mirror. In the case of shadowgraphs, the camera was defocussed outside of the test section.

IV. DATA ANALYSIS CONSIDERATIONS

4.1 Interferometric Measurements

The basic quantity determined from an interferogram is a non-dimensional fringe shift (Ref. 35). The fringe shift may be expressed in terms of a change in optical refractivity as follows

\[ S_{ab} = \frac{\ell}{\lambda} \left\{ (n_b - 1) - (n_a - 1) \right\} \]  

(4.1)

where
- \( S_{ab} \) = nondimensional fringe shift from region (a) to region (b) in fringe numbers
- \( \ell \) = optical path length of test section (cm)
- \( \lambda \) = wavelength of light (cm)
- \( n_a, n_b \) = phase indices of refraction for regions (a) and (b) respectively

The quantity \( n - 1 \) is referred to as the refractivity of a medium and for a non-ionized gas mixture is given as

\[ n - 1 = \sum_i K_i \rho_i \]  

(4.2)

where
- \( K_i \) = specific refractivity (Gladstone-Dale constant) for the \( i \)th species (cm\(^3\)/gm)
- \( \rho_i \) = partial density of \( i \)th species (gm/cm\(^3\))

For the case of partially dissociated oxygen, Eq. (4.2) can be written as

\[ n - 1 = \left\{ K_{O_2} (1 - \alpha) + K_0 \alpha \right\} \rho \]  

(4.3)
where \( K_{O_2} \) = Gladstone-Dale constant for molecular oxygen

\( K_0 \) = Gladstone-Dale constant for atomic oxygen

\( \alpha \) = degree of dissociation (mass concentration of atoms)

\( \rho \) = total density of gas mixture

Contributions of minor species (such as \( e, O^+, O^- \), etc.) have been neglected in Eq. (4.3) since the pressure and temperature levels of interest are such that there is negligible ionization.

A schematic diagram of the gas flow regions being considered in this work is given in Fig. 20. The gas in region (2) has been shock-heated and set in motion by the primary shock wave travelling down the shock tube. This gas undergoes a quasi-steady expansion to achieve the conditions in region (3). The fringe-shift relation given by Eq. (4.1) in conjunction with Eq. (4.3) takes the following form for the corner expansion case.

\[
S_{23} = \frac{\ell}{\lambda} \left\{ \left[ K_{O_2} (1-\alpha_3) + K_0 \alpha_3 \right] \rho_3 - \left[ K_{O_2} (1-\alpha_2) + K_0 \alpha_2 \right] \rho_2 \right\}
\]

(4.4)

Solving Eq. (4.4) for \( \rho_3 \) gives the following result upon rearrangement (note sign change for \( S_{23} \) since fringe shift is negative for decreasing density)

\[
\rho_3 = \frac{(1 + \beta \alpha_2) \rho_2 - \xi S_{23}}{1 + \beta \alpha_3}
\]

(4.5)

where

\[
\beta = \frac{K_0}{K_{O_2}} - 1 \quad ; \quad \xi = \frac{\lambda}{K_{O_2} l}
\]

(4.6)

For a given experimental run, the shock Mach number is measured and the initial conditions \( p_1, T_1 \) and \( \rho_1 \) ahead of the shock wave are known. Thus, the flow properties in region (2) can be calculated (assuming equilibrium) using the shock relations and are available in various tables of normal shock properties (Refs. 36 to 38). Since the fringe shift is measured directly from an interferogram, this leaves the parameters \( \beta \) and \( \xi \) along with \( \alpha_3 \) to be determined before a value for \( \rho_3 \) can be obtained from Eq. (4.5). Since \( \lambda \) and \( l \) are fixed quantities in the experiments, the parameter \( \xi \) can be determined if \( K_{O_2} \) is known. Likewise the parameter \( \beta \) can be determined if, in addition, \( K_0 \) is known.

The value of \( K_{O_2} \) has been accurately determined at near ambient temperatures (< 500°K) where there is negligible dissociation (Ref. 39). An experimental study by Alpher and White (Ref. 40) yielded values for the specific refractivity of atomic oxygen up to a temperature of about 5000°K with an observing wavelength range of 4120 Å to 5450 Å. The value of \( K_{O_2} \) was assumed to be a known quantity and equal to the accepted room temperature value of 0.191 cm³gm⁻¹. The main conclusion resulting from the investigation reported in Ref. 40 was that the specific refractivity of atomic oxygen was nearly equal to that for molecular oxygen, so that density measurements using optical interferometry are not seriously affected by the degree of dissociation up to gas temperatures of at least 5000°K.
In the more recent work by Anderson (Ref. 34) an attempt was made to determine both \( K_0^2 \) and \( K_0 \) at elevated temperatures and also to see if there is any apparent temperature-dependence. The experimental value obtained for \( K_0^2 \) of 0.193 cm\(^3\)/gm in this work was in excellent agreement with the room temperature value while the value determined for \( K_0 \) of 0.204 cm\(^3\)/gm was higher (~10%) than that obtained by Alpher and White. However, their result carried a ± 12% uncertainty in comparison to an accuracy of ± 2% claimed by Anderson.

It is therefore felt that \( K_0^2 \) is known with considerable accuracy. If a value for \( K_0 \) based on available experimental results is assumed, then the only remaining unknown in the RHS of Eq. (4.5) is \( \alpha_3 \). To find \( \alpha_3 \) requires knowledge of pressure (or density) and temperature in region (3) or alternatively it would be necessary to measure \( \alpha_3 \) in some independent manner. The latter is most desirable but unfortunately was beyond the capability of the present investigation. However, since it is known that \( \alpha_3 \) must lie somewhere between the extreme values of \( \alpha_2 \) for a frozen expansion flow and \( \alpha_3^e \) for an equilibrium expansion flow, it is possible to determine the overall effect on the calculation of \( \rho_3 \) resulting from the variation of \( \alpha_3 \) between these limiting values, for a given set of initial conditions and a given value of \( S_{23} \) based on actual interferometric measurements. It should be understood here that the value of \( S_{23} \) represents an experimental fact determined without any presupposition of the chemical nature of the expansion flow. All that is really being ascertained is the net change produced in the determination of \( \rho_3 \) by the maximum variation of the unknown quantity \( \alpha_3 \).

A simple analysis was performed in the manner described above for the present operating conditions using experimentally-determined data for \( S_{23} \). The details of the calculations are given in Appendix B. It was found that the total change in \( \rho_3 \) as a result of \( \alpha_3 \) varying between its extreme limits was approximately 0.3%. This result is not as surprising as it may seem when one takes a closer look at Eq. (4.5), realizing that the factor \( \beta \alpha_2 \) is very much less than unity (~0.01) for the present case and that \( \beta \alpha_3 < \beta \alpha_2 \).

Hence, from the point of view of practical experimentation, the small variation noted above is certainly negligible. Also in consideration of the theoretical predictions already discussed in Section II, it is felt that no significant error will be made in the determination of \( \rho_3 \) by assuming that \( \alpha_3 \approx \alpha_2 \) in Eq. (4.5). If this is done, Eq. (4.5) then reduces to the following expression

\[
\rho_3 = \rho_2 - \frac{\xi S_{23}}{1 + \beta \alpha_2}
\]  

(4.7)

which will give a value of \( \rho_3 \) accurate to a fraction of 1% in comparison with the value which would be obtained if the correct value of \( \alpha_3 \) was known and used in the exact relation given by Eq. (4.5).

It is also worth noting that even if the factor \( \beta \alpha_2 \) were neglected in Eq. (4.7) the resulting error in the value of \( \rho_3 \) would only be about 1% for the present case. In other words, for the fortunate situation where the Gladstone-Dale constants for the molecular and atomic species are very nearly equal and the degree of dissociation is moderate, Eq. (4.5) is virtually insensitive to the change in chemistry. For values of \( \alpha \) approaching unity, the errors involved would, of course, become more significant, making any approximation of Eq. (4.5) less desirable.
The major difficulty faced in the actual analysis of the interferograms was the problem of fringe curvature (caused primarily by optical imperfections in the 4\(\frac{1}{2}\) in. thick windows of the test section and compensating chamber). A typical no-flow interferogram is shown in Fig. 21. As may be seen (particularly by viewing the interferogram on edge), fringe curvature over the region of interest downstream of the corner is significant. In order to determine the flow density at some point along the wall downstream of the corner, it is necessary to measure the shift of a particular fringe relative to its position at a point where the flow conditions are known (i.e., region (2) ahead of the expansion wave). If the fringes were all absolutely straight, such a measurement could be taken directly from the flow interferogram as shown schematically in Fig. 22. However, if there is appreciable fringe curvature, then the basic measurement shown in Fig. 22 must be corrected to account for the deviation of the fringe from the straight-line pattern. Such a correction can be obtained by comparing the no-flow and flow interferograms for a particular run. In the present work this was done by taking fringe position measurements (no-flow and flow) relative to a fixed reference position. The wall surface ahead of the corner and the corner itself provided a convenient reference location. The actual measurements of fringe shift were made using a travelling microscope (Cambridge Universal Measuring Machine) with vertical and horizontal tracking (see Fig. 23) and a scale resolution of .001 cm.

The results of detailed measurements of a single fringe obtained from the no-flow and flow interferograms for a particular run are shown in Fig. 23 to illustrate the manner in which the fringe shift was determined. From this plot it can be seen that the total fringe shift is given by two quantities; namely, the distance \(\delta\) which can be determined from the flow interferogram and the distance \(\Delta\delta\) which can be determined from the no-flow interferogram, once the position of the expansion wave head is established. A detailed plot of a typical no-flow fringe pattern is shown in Fig. 24. The wave head position (determined from the corresponding flow interferogram) has been superimposed to provide the desired reference line. As can be seen from Fig. 24 the effects of fringe curvature become increasingly significant as one moves away from the corner, as a result of the ever-increasing distance \(y\) over which the fringe shift is occurring.

A fringe numbering system (see Fig. 25) corresponding to the consecutive order of fringes downstream of the corner (starting with the first detectable fringe behind the expansion wave in the flow interferogram) was used as a means of conveniently relating fringe positions in the flow and no-flow interferograms, with the corner as a common reference point. Since it is not possible to visibly trace the first few fringes downstream of the corner back to their respective positions ahead of the expansion wave (see Fig. 25), it was necessary to count fringes away from the corner to a fringe that could be visibly traced (for example, fringe No. 10 in Fig. 25), in order to establish corresponding fringe positions in the region ahead of the expansion wave. The location of fringe No. 1 ahead of the expansion corner could then be simply established by counting backwards an equal number of fringes.

The nondimensional fringe shift is determined using the measurements of \(\delta\) and \(\Delta\delta\) (see Fig. 23) as follows:

\[
S_{23} = \frac{\delta + \Delta\delta}{\Delta f}
\]

(4.8)
where \( \Delta f \) is the spacing of adjacent fringes in the region ahead of the expansion wave (see Fig. 25). Some typical results for \( \delta \) and \( \Delta \delta \) are given in Figs. 26 and 27, respectively. A scatter bar corresponding to a deviation of \( \pm \frac{1}{20} \) fringe was arbitrarily placed on a few representative data points in both figures for the sake of indicating the relative scatter of the results. In general, it was possible to make measurements to an accuracy of at least \( \pm \frac{1}{10} \) fringe for individual plates. Since measurements from flow and no-flow interferograms were combined to determine the final fringe shift (i.e., \( \delta^* \Delta \delta^* \)), the overall accuracy was of the order of \( \pm \frac{1}{10} \) fringe.

The results for \( \Delta \delta \) shown in Fig. 27 clearly indicate the importance of correcting for fringe curvature, particularly in the region of the fringe field farthest removed from the corner. As can be seen, the correction varies from zero for the first 10-12 fringes up to a value almost equal to that for a fringe shift of one.

Further discussion of the results obtained from the interferometric measurements is presented in Section 5.

### 4.2 Pressure Measurements

As mentioned earlier in Section 3.3, the major difficulty involved in making pressure measurements was the problem of "gauge ringing". The mounting technique discussed in Section 3.3 helped to minimize the problem significantly but did not completely eliminate it. A typical set of pressure records for one run is shown in Fig. 28. The traces are arranged in order with respect to gauge position, with the top trace corresponding to the pressure gauge located just ahead of the corner and the other three traces for the gauges downstream of the expansion corner.

In consideration of the discussion given in Appendix A concerning the operation of the shock tube with the wall model it was felt that the initial portion of 100 - 150 ms of the pressure traces could be used for purposes of data analysis. Upon inspection of the pressure records obtained from the gauge ahead of the expansion corner (see top trace in Figs. 28 and 29), it was found that the experimental pressure traces were indeed quite similar to the "ideal" profile sketched in Fig. A.4 of Appendix A.

In analyzing the pressure data, it was assumed that overall pressure variation with respect to time was approximately linear. Hence, it was possible to represent the actual pressure trace by a straight line with a slope as shown in Fig. 29 where the original pressure traces are dotted in for comparison. As can be seen, the straight line representation does provide a reasonably good approximation of the pressure results over the initial portion of the traces under consideration.

The straight line approximations of the pressure traces were obtained by making a least squares fit to pressure measurements taken from the actual traces using the travelling microscope mentioned in Section 4.1.

Further discussion relating to the overall quantitative nature of the pressure data is presented in Section 5.2.
V. EXPERIMENTAL RESULTS AND DISCUSSION

5.1 Density Results

A systematic set of experimental runs were carried out for the conditions $M_s = 12$ and $P_l = 20$ mm Hg to obtain interferometric data for varying flow times defined with respect to the passage of the shock wave downstream of the corner. In other words, $\tau_{\text{flow}} = 35 \mu s$ simply means that the interferogram was taken at approximately $35 \mu s$ after the shock wave passed the position of the expansion corner. A number of the interferograms which were taken using the wall model are shown in Fig. 30. Due to the problem of test section window etching arising from aerodynamic heating, which was encountered during the tests with the wedge model (see Section 3.2) only one run was taken at $M_s = 12$ and $P_l = 20$ mm Hg. The interferogram obtained for this run is shown in Fig. 31. Minor ablation of the model, in particular its strut, is quite noticeable on the compression side of the model.

In addition to the runs noted above, a number of tests were also made for the conditions $M_s = 10$ and $P_l = 20$ mm Hg. This was done to obtain additional data for comparative purposes. Typical interferograms for these runs for both models are shown in Fig. 32. The result shown in Fig. 32(c) is a composite interferogram obtained by superimposing the interferogram of Fig. 32(b) with the no-flow interferogram for that particular case. The superposition method provides the same effect as obtained with an infinite fringe interferogram. In other words, the rays emanating from the corner in Fig. 32(c) represent lines of constant density.

In order to gain more details of the flow phenomena being investigated, additional optical studies in the form of schlieren and shadowgraph were also made. Some results of these studies are shown in Figs. 33 to 35. A discussion of some of the features observed in the optical data is presented in subsequent sections.

The interferograms themselves were analyzed in the manner outlined in Section 4.1. The basic data in the form of nondimensional fringe shift results are given in Fig. 36. The calculated values for the fringe shifts corresponding to a frozen expansion ($S_{23f}$) and an equilibrium expansion ($S_{23e}$) are also shown in Fig. 36. Absolute density results, which were determined using the fringe shift data, are given in Fig. 37. Although the density in Fig. 37 is denoted as wall density ($\rho_{3W}$), it should be noted here that the fringe shift measurements were taken as near the wall as possible without getting into the viscous region of the boundary layer. Theoretical density results based on the calculations of Glass and Takano* (Ref. 14) and Tirumalesa (Ref. 24) are included.

* The calculations of Glass and Takano were based on equilibrium properties behind the incident shock for a value of $T_1$ of $273^\circ K$. The present results were determined using shock tables for $T_1 = 300^\circ K$. The case for a frozen expansion flow was recalculated to obtain values of $P_W$, $P_{WF}$, and $T_W$ which could be used to determine the absolute values of $\rho_W$, $P_W$ and $T_W$ from the results shown in Fig. 3. The slight difference in magnitude between the results given in Fig. 29 of Ref. 14 and those shown here is due to the above change.
in Fig. 37(a) for the case of $M_S = 12$ and $P_1 = 20$ mm Hg. but not in Fig. 37(b) since no calculations were made for a case with $M_S = 10$. It is of interest to note that the predicted density change in the CVD case over the distance of 9 cm. is so slight as to appear virtually unchanged from the frozen expansion density. The apparent agreement of the present experimental results with this prediction is perhaps most clearly indicated by the fringe shift data shown in Fig. 36. As can be seen, the experimental fringe shift profiles do not tend to exceed the calculated value for a frozen expansion. In this regard, it is also worth noting that the fringe shift measurements represent raw data and are not dependent on an accurate knowledge of upstream conditions or other constants, such as one must have before being able to determine the actual density profiles.

Another interesting feature of the interferometric results shown in Fig. 37 is the change in the density profiles with increasing flow time for the runs taken with the wall model. For the run with the shortest flow time ($T_{\text{flow}} = 20 \mu s$) in Fig. 37(a), the density decreases rapidly downstream of the corner and appears to level off near the CVD curve at around 1.5 cm. from the corner. A further change in the profile is seen to occur at about 3 cm. from the corner. This change is associated with the formation of a secondary, nonstationary wave system which occurs when the primary shock wave diffracts around the expansion corner. The reason for the observed drop in density (at $S_w = 3cm$) in the early stage of flow establishment around the corner is not really known. Perhaps it is possible that the early development of the "starting" wave system is more complex and less stable than at later flow times. The formation of the secondary wave system is required in order to match the conditions produced by the steady expansion flow around the corner with the conditions existing behind the primary shock wave. The location of the change in the density profile produced by the nonstationary starting process as the expansion flow field is formed is seen to move farther away from the corner with increasing flow time.

The density profile for the run with the next longest flow time ($T_{\text{flow}} = 35 \mu s$) displays a much slower rate of decreasing density than the one for the shorter flow time. In fact, this profile never levels off. The same is true for all runs taken with still longer flow times. One possible explanation for such behaviour would seem to be boundary-layer influence. Such an explanation appears to be feasible when a comparison is made between the single density profile obtained for the wedge model to the one for the wall model at a corresponding flow time. As can be seen in Fig. 37(a), the density profile for the wedge displays the same rapid decrease and levelling of density characteristic of the profile for the wall model with the shortest flow time. Since the overall boundary layer influence for the wedge model is expected to be less severe than for the wall model, it seems reasonable that the main cause of the difference in the two profiles (wall and wedge) for $T_{\text{flow}} = 35 \mu s$ is due to the boundary layer. Also it is felt that the change in density profiles for the wall model is attributable in boundary layer growth. Another possible explanation for the rather marked change between the density profile for $T_{\text{flow}} = 20 \mu s$ and that for $T_{\text{flow}} = 35 \mu s$ could be boundary layer transition from laminar to turbulent. A simple calculation of transition time (in laboratory coordinates) based on transition Reynolds number of $10^6$ (Refs. 33 and 41) yielded a value of 27 $\mu s$ for the conditions behind a shock wave moving at a Mach number of 12 into oxygen at an initial pressure of 20 mm Hg. However, owing to the general lack of understanding regarding the nature of boundary layer transition and gas kinetic effects the relevance of this transition time may be somewhat fortuitous.
Another aspect of the interferometric results which warrants some comment concerns the fact that the fringe shift results for the wedge model (see Fig. 36) appear to level off at a slightly lower value than that calculated for a frozen expansion flow with $\theta_w = 150$. Consequently, the density results for the wedge model given in Fig. 37 are seen to be about 5% greater than the calculated value of $\rho_w$. It is felt that one possible explanation for this slight difference could be that the growth of the quasi-steady boundary layer with increasing distance from the leading edge of the model results in an "effective" expansion angle which is slightly less than the actual wall angle. For instance, a decrease of $0.25^0$ in the expansion angle would result in a decrease in the value of $\theta_w$ in Fig. 36(a) from 8.8 to about 8.6 and an increase in the value of $\rho_{w'}$ in Fig. 37(a) from $16.9 \times 10^{-5}$ gm/cm$^3$ to $17.3 \times 10^{-5}$ gm/cm$^3$. This "effective" decrease in expansion angle would correspond roughly to an increase in boundary layer displacement thickness of about .015 in. between the corner (located approximately 4 in. from the leading edge) and a downstream position of about $3\frac{1}{2}$ in. On the basis of simple boundary layer calculations for a flat plate, such an increase does not seem unreasonable.

In regard to the above discussion, it should be understood that the apparent slight shift of the density results for the wedge model does not alter their overall significance. The fact still remains that the observed change in density downstream of the corner (beyond about $1\frac{1}{8}$ cm.) appears to be quite small, much as predicted by the CVD calculations.

On the basis of the present density results, it does not seem feasible to try to determine a value for the recombination rate constant $k_R$ in the manner outlined in Ref. 14 for the present operating conditions. This is not to say, however, that the method proposed might not be useful under different initial conditions. To be able to deduce $k_R$ with any degree of accuracy requires that the relaxation distance be in the range of 5 to 10 cm. and the present density results imply a much longer relaxation distance. Consequently, it appears that the optimum conditions proposed by Glass and Kawada (Ref. 12) for the case of complete vibrational equilibration will have to be re-evaluated, taking into account the coupling effects of vibrational and dissociational nonequilibrium. In addition, the choice will have to be tempered by the boundary layer growth and the model to be used for generating the corner expansion flow.

5.2 Pressure Results

Pressure measurements were taken simultaneously with the interferometric measurements for each experimental run with the wall model. Each pressure trace was analyzed in the manner discussed in Section 4.2. The results of the least squares fit to the pressure data are given in Table I in terms of intercept values ($p_1$) and slope values ($m$). Since the initial conditions of $M_s$ and $p_1$ were repeated quite well from run to run, it was decided to consider the data from all runs jointly, thus obtaining a single least-mean-squares pressure result for each gauge position. These values are also given in Table I.

Pressure-time profiles (based on values given in Table I) for the three gauges downstream of the corner are shown in Fig. 38. The results corresponding to the maximum and minimum values of $p_1$ (see Table I) for each gauge are given along with the results determined using the data from all runs for a given gauge. The overall scatter relative to the "mean" pressure-time
result ranges from about ± 4% for Gauge No. 2 up to about ± 10% for Gauge No. 3. It is also worth noting that the agreement between the experimental results for Gauge No. 1 (see Table I) and the predicted equilibrium pressure behind the incident shock wave was, in general, quite good.

Since the time scale in Fig. 38 is relative to shock arrival at the first gauge, it is possible to locate the various flow times (relative to shock arrival at the corner) by determining the time of travel between Gauge No. 1 and the expansion corner. For a shock velocity of about 4 mm/µs (M_s = 12), the time interval for a distance of 0.5 in. (12.7 mm) is approximately 3 µs. Hence, τ_{flow} = 25 µs corresponds to a time of approximately 28 µs relative to shock arrival at the first gauge. It should be noted that the actual pressure traces were all recorded using the same oscilloscope time base and were triggered simultaneously (see Fig. 28).

The pressure results as a function of gauge position relative to the corner are shown in Fig. 39. To avoid unnecessary confusion, results have been given for the "mean" pressure-time traces (see Fig. 38) only and for just one value of flow time (τ_{flow} = 25 µs). However, the pressure results given in Table I and Fig. 38 indicate the fact that pressure does increase slightly with time. Just how much of this increase is attributable to shock wave attenuation or to the growth and influence of the boundary layer in the region of the corner is difficult to say. Also shown in Fig. 39 are the predicted pressure profiles for the VEQ case and CVD case. The fact that pressure results could be obtained for only three wall positions downstream of the corner makes a realistic interpretation of the pressure data somewhat uncertain. However, it would appear that a pressure profile similar to those observed for density does exist and that the pressure is decaying downstream of the corner towards the slowly increasing pressure profile predicted by the coupled vibration-dissociation (CVD) calculations.

Further discussion regarding the significance of the pressure results in terms of boundary-layer influence on the expansion flow is presented later on in Section 5.4. This discussion, along with what has already been said, provides a slightly more consistent picture for the observed pressure data.

Finally, on the basis of the present experience, it is recommended that any future studies involving pressure measurements of the type made here be undertaken with the use of more advanced pressure transducers which are acceleration-compensated and have a high natural frequency (> 100,000 cps). Such gauges should provide pressure traces of superior quality to those obtained in the present work and hence improve the overall accuracy of the final results.

5.3 Wave Head and Wave Tail Measurements

As mentioned in the introductory discussion, a study of the structure and decay of the head and tail of the expansion wave was of interest since it can, in principle, provide additional evidence regarding the nature of the corner expansion flow (Ref. 14). Consequently, an effort has been made to measure the position and shape of the wave head and the wave tail as accurately as possible. These measurements were taken from the interferograms from which the density results were obtained. Additional optical studies, particularly the schlieren method, provided further relevant data to this phase of the overall investigation.
Wave head and wave tail measurements taken from the flow interferogram for the wedge model with $M_s = 12$ (see Fig. 31) are given in Fig. 40. Similar measurements for the wall model are shown in Fig. 41. These measurements were taken at positions in the fringe fields of the interferograms defined by the onset of fringe displacement (i.e., a change in density) for the head of the expansion wave and by the approximate termination of displacement for the tail as shown schematically in Fig. 42. Since the density gradient at the head of the expansion wave is significantly large, it was relatively easy to make measurements of its apparent position in the fringe field, particularly near the corner. However, the smaller density gradient at the tail produced less detectable (i.e., more gradual) fringe displacement, making it more difficult to locate the position of the tail. The use of a horizontal fringe field may have improved the situation but was undesirable from the point of view of not being able to make density measurements along the wall downstream of the corner.

Several interesting features relating to the results shown in Figs. 40 and 41 can be observed. If the corner is taken as a geometric point, then the wave head for the expansion flow generated by the wall model (see Fig. 41) would appear to be curved near the corner. This curvature, however, seems to be confined to the region of the boundary layer. The approximate boundary layer thickness indicated in the figure is based on measurements taken for the density profile near the wall just upstream of the corner. Similar measurements were made for several runs to get some indication of relative changes in boundary layer thicknesses for the various cases. It is interesting to note the apparent lack of similar curvature for the head of the expansion wave shown in Fig. 40 for the wedge model where the boundary layer is thinner.

Another interesting feature about the wave head shown in Fig. 41 is its change in shape at a point out in the free stream. This change is unsteady in nature and is believed to be associated with the general flow pattern induced behind the shock wave as it diffracts around the expansion corner. More will be said regarding this in Section 5.5.

The head and tail measurements from several runs are shown in Fig. 43 in a composite manner. Although the general scheme is somewhat crowded, it is possible to make several useful comparisons of the results. The most apparent one seems to be the relation between wave head position and approximate boundary layer thickness. There seems to be a definite indication of a slight displacement of the wave head in an upstream direction with increasing boundary layer thickness. It can also be seen that the wave tails for all cases are displaced in a downstream direction (i.e., there is a separation of the wave head and the wave tail at the corner). However, in terms of relative position, the results for the wave tail appear to be the same as those for the wave head with respect to increasing boundary layer thickness.

Enlarged views of the corner expansion region for the wedge and wall models are shown in Figs. 44 and 45, respectively. The separation of the wave head and wave tail at the corner can be seen quite clearly in the schlieren photograph for the wall model (Fig. 45). The spreading of the expansion wave is less noticeable for the wedge model but slight curvature near the corner is apparent. The difference between boundary layer thickness for the two cases is also quite evident.
Schlieren photographs obtained by Murthy and Hammitt (Ref. 42) for the expansion of a steady supersonic flow (M = 1.88) around corners with expansion angles ranging from 5° to 30° provide an interesting comparison with the present results. Their results show quite clearly the change in the shape of the expansion wave near the region of the corner which is produced by interaction with a turbulent boundary layer.

The angles determined from the various wave head and wave tail measurements are in good agreement within themselves (see Fig. 43). Those angles determined for the wave head fell between the theoretical values for frozen and equilibrium flows, slightly favoring the frozen value. Measurements of the wave head angle from schlieren photographs also yielded values close to those given in Fig. 43 and again slightly less than the value for a frozen flow. Since the wave head and other characteristics are decaying as one moves radially away from the corner until the position of the equilibrium wave head is reached (Ref. 14), it is felt that one would have difficulty in actually measuring the frozen Mach angle. Also since the density gradient at the wave head decreases radially away from the corner it becomes increasingly difficult to determine its exact location from a schlieren photograph or an interferogram. Thus, it is felt that measurements of the angles of the wave heads and wave tails would probably not be the most reliable basis from which to predict the chemical nature of the flow.

5.4 Boundary Layer Considerations

As mentioned in the previous discussion of the experimental results, it is felt that the presence of a boundary layer along the wall surfaces of the expansion models has influenced the observed data to a certain extent. The problem of possible boundary layer effects was recognized from the beginning and, as already mentioned, was the motivating factor behind the use of two different expansion models to generate corner expansion flows.

In an effort to obtain additional information regarding the problem of boundary layer influence, an analytical study was recently carried out by Oothuizen (Ref. 27) in which the interaction of a turbulent boundary layer with the expansion wave formed by a steady supersonic flow around a sharp corner was considered. This analysis was fundamentally the same as that made by Murthy and Hammitt (Ref. 42) in which it is assumed that there is no upstream influence and that the initial velocity and density profiles ahead of the corner are given by the upstream boundary layer solution. It is further assumed that the supersonic portion of the boundary layer flow expands in an inviscid manner and that the streamline which originates at the point where the Mach number is unity runs parallel to the wall downstream of the corner as shown in Fig. 46. In addition, the pressure change across the inner (subsonic) layer is assumed to be negligible. Although the above assumptions are somewhat restrictive, particularly in regard to the inner portion of the boundary layer, it is worth noting that any analysis attempting to include viscous effects would become extremely complex.

The assumption that the outer portion of the boundary layer expands inviscidly makes it possible to solve for the properties of the flow field using the method of characteristics (Ref. 27). Since it was assumed that there was no upstream influence, the shape and conditions along the initial characteristic line, originating at 0 in Fig. 46, are given by the upstream

* See Appendix C for further discussion.
boundary layer solution. In the analysis made by Oosthuizen it was assumed that the velocity profile over the major portion of the layer was given by the power law profile for a turbulent boundary layer

\[ u/u_\infty = (y/\delta)^{1/N} \]  

(5.4.1)

when \( u/u_\infty \geq 1/2 \). For the region near the wall where the power law does not apply, a linear relation given by

\[ u/u_\infty = (0.5)^{1-N} y/\delta \]  

(5.4.2)

was assumed and applied for \( u/u_\infty < 1/2 \). The coefficient in Eq. (5.4.2) is determined from the requirement that the velocity be continuous at the point where the two velocity profiles join. The value for \( N \) was assumed to be 6.

Although dissociation was included in the initial profile ahead of the corner in the following form (Ref. 27)

\[ \alpha = \alpha_w + (\alpha_\infty - \alpha_w) u/u_\infty \]

it was assumed that the expansion process was chemically and vibrationally frozen.

Some of the calculations by Oosthuizen were made using free-stream conditions of interest in the present investigation. Results of these calculations are shown in Figs. 47 and 48. Although some of the present experimental results are also shown, it should be emphasized that the primary interest in making a comparison is more qualitative than quantitative. It is the nature, and not the absolute magnitude, of boundary layer influence that is of main interest in the present work. An approximate value for boundary layer thickness (based on previously mentioned measurements) was assumed in order to convert the conventional nondimensional distance scale \((s_w/\delta)\) used in Ref. 27 to the present dimensional form.

As can be seen, the calculated results for both density (Fig. 47) and pressure (Fig. 48) indicate an influence downstream of the expansion corner due to the interaction of the boundary layer with the expansion wave. Qualitatively speaking, it is felt that the present experimental results display a similar downstream influence.

As one final point of interest, an experimental run was taken at a relatively low value of shock Mach number \((M_s = 4.4)\) for the purpose of obtaining results for flow conditions with negligible dissociation in order to isolate possible chemical effects away from viscous effects. The results of this run are given in Fig. 49. As can be seen, the density profile displays a variation in density just downstream of the corner similar to that observed in the previous results for the higher shock Mach number cases. These results (Fig. 49) along with the comparisons between data for the two types of expansion models clearly indicate the effect of interaction between the boundary layer and the corner expansion wave. It seems rather apparent that the influence of this interaction on the final results does, in fact, present a serious problem in an investigation of the type undertaken in this work. In work reported by Cleaver (Ref. 43) involving a similar study of the Prandtl-Meyer type expansion flow of carbon dioxide in a shock tube of smaller dimensions than the present
facility, it was found that interferometric observations were generally inconclusive as a result of the overall influence of boundary layer.

5.5 Shock Wave Diffraction Phenomenon

An interesting and unexpected shock wave diffraction pattern was observed in the present optical studies for the case of \( M_s = 12 \) and \( p_l = 20 \text{ mm Hg} \). The shadowgraph shown in Fig. 35 provides the clearest evidence of this unusual phenomenon. It should be mentioned that this result was observed only in the runs taken with the wall model. As mentioned earlier, only a limited number of runs were taken with the wedge model and in the case for \( M_s = 12 \) the primary shock was out of the field of view (which was more restricted than when the wall model was being used). A schematic comparison of the diffraction pattern observed in the present results with the classical pattern for diffraction of a shock wave around a corner is given in Fig. 50. The major difference lies in the fact that the lower portion of the primary shock wave in the present case is actually being accelerated ahead of the upper portion. As a result of this strengthening effect, an additional unsteady wave pattern is formed in order to match conditions between regions (2) and (2') as shown in Fig. 50(b). As seen in the series of interferograms given in Fig. 30, this wave pattern moves upward along the shock wave as it proceeds away from the corner.

It is felt that this phenomenon is perhaps a result of the influence of the strong steady expansion process occurring behind the shock wave. The interferometric result given in Fig. 51 for a case of \( M_s = 10 \) does not appear to indicate the presence of the additional wave system observed in the higher shock Mach number case even though the main shock is accelerated. Perhaps the reflected shock wave and contact surface are too weak to be seen here. This result for \( M_s = 10 \) would seem to indicate that the observed diffraction phenomenon is in fact associated with increased shock strength resulting in corner expansions having higher flow Mach numbers and increased influence on the shock wave in the region just downstream of the corner.

Although it was felt that the observed pattern of shock wave diffraction was quite unusual, it was not considered to have any influence on the actual corner expansion process under investigation. Such being the case, no strong effort was made in the present work to investigate the phenomenon in greater detail. Such a study, is however, now under way and will hopefully form the subject of future experimental and theoretical theses topics (Ref. 44).

VI. CONCLUDING REMARKS

As a result of the present experimental study of the nonequilibrium expansion flow of dissociated oxygen around a corner for the particular case of \( M_s = 12 \) and \( p_l = 20 \text{ mm Hg} \) the following observations and conclusions can be stated:

1. No recombination shock wave was observed along the tail of the expansion wave.

2. The results of density and pressure measurements along the wall downstream of the corner indicate a longer relaxation distance than predicted by the calculations of Glass and Takano (Ref. 14) based on a mathematical model.
assuming vibrational equilibrium and would appear to favor the predictions of
more recent calculations by Tirumalesa (Ref. 24) in which coupled vibrational-
dissociational nonequilibrium was considered.

(3) Viscous (and perhaps catalytic) interaction between the boundary
layer and the corner expansion wave modify the experimental results, particularly
in the case of the wall model. The wedge model yielded results displaying less
overall boundary layer effect and would seem to be the more useful of the two
models in terms of future studies of expansion flows.

(4) Results of wave head and wave tail measurements also indicated
the presence of boundary layer influence. Slight curvature was observed in
these measurements but is believed to be confined to the region of the boundary
layer. Although it is possible, in principle, to predict the chemical nature
of the corner flow from measurements of the shape of the wave head, it is felt
that such measurements are difficult to obtain precisely and are less definitive
than measurements of other flow quantities.

(5) Values for the recombination rate constant $k_R$ have not been
inferred from the present results in the manner suggested by Glass and Takano
(Ref. 14) since this would require a direct comparison between experimental data
and theoretical predictions. Such a comparison is not felt to be justified at
the present time since there is still uncertainty regarding the most realistic
theoretical model for computing the nonequilibrium expansion flow properties for
dissociated oxygen. Once these are known and assessed new initial conditions
will have to be chosen to provide the best conditions for determining $k_R$. Also,
the effects of the boundary layer and wall catalycity will have to be separated
from the chemical kinetic effect. Furthermore, temperature or atom concentration
measurements would be far superior to those of density and pressure for deter-
mining $k_R$.

(6) The present interferometric results have shown that a definitive
study of the interaction of the expansion wave and boundary layer may be possible
and worthwhile.
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TABLE I

RESULTS OF PRESSURE DATA ANALYSIS FOR 15° CORNER EXPANSION FLOW OF DISSOCIATED OXYGEN
WITH $M_s = 12$ AND $p_1 = 20$ mm Hg.
($p_2 = 72$ psi, $p_{3f} = 20.3$ psi, $p_{3e} = 25.5$ psi)

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<td>27.5</td>
<td>.035</td>
<td>22.6</td>
<td>.038</td>
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FIG. 1 PROPOSED OPTIMUM OPERATING CONDITIONS FOR THE INTERFEROMETRIC STUDY OF NONEQUILIBRIUM EXPANSION FLOW OF DISSOCIATED OXYGEN AROUND A 15° CORNER (Ref. 12)
Equilibrium wave head at infinity only

Free stream

Partially frozen wave head

Non-equilibrium corner expansion flow

Expansion wave tail at infinity only

Isentropic uniform flow (equilibrium) at infinity only

Streamline at infinity only

Streamline close to the wall surface

Frozen wave head-CD

Expansion wave tail-CW

Recombination shock

Locus of pressure maxima

Expansion wave

Decayed expansion wave

Compression wave

(a) Flow Pattern

Figure 2

Nonequilibrium expansion flow of dissociated oxygen around a corner
FIG. 3 VARIATION OF FLOW QUANTITIES ALONG THE WALL. (Ref. 14)
(c) Temperature

(d) Degree of Dissociation

FIG. 3 (concluded)
FIG. 4 VARIATIONS OF FLOW QUANTITIES FOR COUPLED VIBRATIONAL-DISSOCIATIONAL NONEQUILIBRIUM (Ref. 24)
**FIG. 4 (concluded)**

(c) Temperature

(d) Vibrational and Translational Temperatures
FIG. 5 VIEWS OF SHOCK TUBE FACILITY

(a) Driver Section

(b) Driven Section
FIG. 6 SCHEMATIC DIAGRAMS OF TEST MODELS USED TO GENERATE CORNER EXPANSION FLOWS

(a) Wall Model

(b) Wedge Model
FIG. 7 CORNER EXPANSION MODELS
FIG. 8 SUPPORT STRUT FOR CHANNEL SECTION OF WALL MODEL
FIG. 10 DOWNSTREAM END OF WALL MODEL
FIG. 12 PRESSURE TRANSDUCERS
FIG. 13 METHOD OF SHOCK-MOUNTING PRESSURE TRANSDUCERS
FIG. 14 COMPARISON OF PRESSURE RECORDS FOR CASES OF POOR AND IMPROVED SHOCK-MOUNTING

(a) Poor Shock-Mounting

(b) Improved Shock-Mounting

\( M_s = 5 \); \( p_1 = 10 \text{ mmHg} \)
FIG. 15  CALIBRATION SETUP FOR PRESSURE TRANSDUCERS
FIG. 16 CALIBRATION PRESSURE TRACES

(a) Kistler-601 Transducer

ΔP

2000 mmHg

2400 mmHg

(b) PZT Transducer

5 msec/div

500 mv/div

1600 mmHg
FIG. 17 CALIBRATION RESULTS FOR KISTLER-601 PRESSURE TRANSDUCER

slope = 0.48 pC/po
FIG. 18 CALIBRATION RESULTS FOR PZT PRESSURE TRANSUDCERS

- PZT-2
- PZT-3
- PZT-4

85.4 mV/psi
67.9 mV/psi
54.2 mV/psi

PRESSURE (psi)
OUTPUT (mV)
FIG. 19 SCHEMATIC OF INTERFEROMETER SYSTEM
FIG. 20 SCHEMATIC DIAGRAM OF FLOW REGIONS FOR CORNER EXPANSION FLOW
FIG. 21 NO-FLOW INTERFEROGRAM
FIG. 22 SCHEMATIC DIAGRAM OF BASIC FRINGE SHIFT MEASUREMENT
FIG. 23 DETAILED PLOT OF A SINGLE FRINGE POSITION IN THE NO-FLOW AND FLOW INTERFEROGRAMS
FIG. 25 ENLARGEMENT OF CORNER EXPANSION FRINGE PATTERN SHOWING NUMBERING OF FRINGES

\( M_s = 10 ; \ p_1 = 20 \ \text{mmHg} \)
Error Bar: \( \pm \frac{1}{20} \Delta f \)

FIG. 26 DIMENSIONAL FRINGE SHIFT RESULTS TAKEN FROM FLOW INTERFEROGRAM FOR \( M_s = 12 \) AND \( p_1 = 20 \text{ mmHg} \)
FIG. 27 FRINGE CURVATURE CORRECTION DETERMINED FROM NO-FLOW INTERFEROGRAM FOR CASE SHOWN IN FIG. 26
FIG. 28 CORNER EXPANSION PRESSURE TRACES

\[ M_s = 12; \ p_1 = 20 \text{ mmHg} \]
FIG. 29 STRAIGHT-LINE APPROXIMATIONS OF PRESSURE TRACES
FIG. 30 INTERFEROGRAMS OF CORNER EXPANSION FLOWS GENERATED BY WALL MODEL WITH $M_s = 12$ AND $p_1 = 20$ mmHg (Oxygen)
FIG. 30 (concluded)

(c) $\tau_{flow} = 45 \, \mu s$

(d) $\tau_{flow} = 55 \, \mu s$
FIG. 31  INTERFEROGRAM OF CORNER EXPANSION FLOW
GENERATED BY WEDGE MODEL WITH $M_s = 12$
AND $p_1 = 20$ mmHg (Oxygen)
FIG. 32  INTERFEROGRAMS OF CORNER EXPANSION FLOWS FOR $M_s = 10$ AND $p_1 = 20$ mmHg (Oxygen)

(a) Wall Model; $t_{flow} = 40 \mu s$
FIG 32 (con't)

(b) Wedge Model; \( \tau_{\text{flow}} = 35 \, \mu s \)
FIG. 32 (concluded)

(c) COMPOSITE INTERFEROGRAM OBTAINED BY SUPERPOSITION OF FIG. 32(b) WITH NO-FLOW INTERFEROGRAM
FIG. 33 SCHLIEREN PHOTOGRAPHS OF CORNER EXPANSION FLOWS GENERATED BY WALL MODEL WITH $M_s = 12$ AND $p_1 = 20$ mmHg (Oxygen)

(a) $\tau_{flow} = 25 \, \mu s$

(b) $\tau_{flow} = 50 \, \mu s$
FIG. 34  SCHLIEREN PHOTOGRAPH OF CORNER EXPANSION FLOW GENERATED BY WEDGE MODEL WITH $M_s = 10$ AND $p_1 = 20$ mmHg (Oxygen)
FIG. 35  SHADOWGRAPH SHOWING WAVE SYSTEM FORMED BY DIFFRACTION OF SHOCK WAVE AROUND EXPANSION CORNER  \( M_s = 12; \ p_l = 20 \ \text{mmHg} \)
FIG. 36 FRINGE SHIFT RESULTS

(a) $M_s = 12$; $p_1 = 20$ mmHg (Oxygen)
FIG. 36 (concluded)  (b) $M_s = 10; p_1 = 20$ mmHg (Oxygen)
FIG. 37 DENSITY RESULTS
(a) $M_s = 12$; $p_1 = 20$ mmHg (Oxygen)
FIG. 37 (concluded)

(b) $M_s = 10; p_1 = 20$ mmHg (Oxygen)
FIG. 38 RESULTS OF PRESSURE DATA ANALYSIS FOR GAUGES DOWNSTREAM OF EXPANSION CORNER
FIG. 39 EXPERIMENTAL PRESSURE RESULTS AS A FUNCTION OF WALL POSITION FOR $M_s = 12$ AND $p_1 = 20$ mmHg IN OXYGEN (see Fig. 38)
FIG. 40 WAVE MEASUREMENTS FOR WEDGE MODEL WITH
$M_0 = 12$ AND $p_1 = 20$ mmHg (Oxygen)
FIG. 41 WAVE MEASUREMENTS FOR WALL MODEL WITH
\( M_s = 12 \) AND \( p_1 = 20 \text{ mmHg (Oxygen)} \)

\( \theta_{\text{wave head}} = 20.6^\circ \)

\( \theta_{\text{wave tail}} = 2.5^\circ \)
FIG. 42 SCHEMATIC DIAGRAM OF WAVE HEAD AND WAVE TAIL MEASUREMENTS
FIG. 43 COMPOSITE PLOT OF WAVE MEASUREMENTS FOR
Mₚ = 12 AND p₁ = 20 mmHg (Oxygen)
FIG 44  ENLARGED SCHLIEREN RESULT FOR WEDGE WITH $M_s = 10$
AND $p_1 = 20$ mmHg (Oxygen)
FIG. 45 ENLARGED SCHLIEREN RESULT FOR WALL MODEL WITH $M_s = 12$ AND $p_1 = 20$ mmHg (Oxygen)
FIG. 46 ASSUMED FLOW PATTERN FOR ANALYSIS OF THE INTERACTION OF A TURBULENT BOUNDARY LAYER WITH THE CORNER EXPANSION WAVE (Ref. 27)
FIG. 47 COMPARISON BETWEEN PRESENT DENSITY RESULTS AND DENSITY PROFILE DETERMINED FROM APPROXIMATE ANALYSIS OF THE INTERACTION OF A TURBULENT BOUNDARY LAYER WITH THE CORNER EXPANSION WAVE
FIG. 48 COMPARISON OF PRESSURE RESULTS WITH CALCULATIONS FOR A FROZEN CORNER EXPANSION FLOW WITH BOUNDARY LAYER PRESENT

Present Results
($M_s = 12; p_1 = 20 \text{ mmHg}$)

Approximate Boundary Layer Result (Ref. 27)
FIG. 49 INTERFEROMETRIC RESULTS OBTAINED AT LOW SHOCK MACH NUMBER (M_s = 4.4) SHOWING DOWNSTREAM EFFECT PRODUCED BY INTERACTION OF THE BOUNDARY LAYER AND CORNER EXPANSION WAVE
FIG. 50 SCHEMATIC COMPARISON OF CLASSICAL SHOCK DIFFRACTION PATTERN WITH THAT OBSERVED IN PRESENT EXPERIMENTS FOR $M_s = 12$
FIG. 51 HORIZONTAL FRINGE INTERFEROGRAM SHOWING SHOCK DIFFRACTION PATTERN FOR $M_s = 10$ AND $p_1 = 20$ mmHg
APPENDIX A

Discussion Concerning Operation of the Shock Tube with the Present Wall Model

Since the installation of the wall model in the shock tube will effectively alter its operating configuration, it is of interest to discuss in some detail the expected changes in the operation and performance of the shock tube. The basic physical configuration is shown schematically in Fig. A.1. The reduction in area produced by the wall model is such that the ratio of the upstream area to the downstream area is 1.27.

The idealized wave pattern which is formed after quasi-steady flow conditions are attained is illustrated in Fig. A.2 (see Ref. 2). The wave system shown in Fig. A.2 is for the case where \( M_5 > M_4 > 1 \). In the ideal case it is assumed that the area reduction acts effectively as a convergent nozzle, thus causing an increase in pressure and a decrease in flow velocity for supersonic flow. The formation of the upstream-facing rarefaction wave is required to match pressure and flow velocity between region (4) and regions (2) and (3).

In the actual case an oblique shock wave will be formed at the leading edge of the wall model. The leading edge angle of 25° is less than the critical angle (≤32°) for shock wave detachment. Although the oblique shock wave will undergo multiple reflections from the walls of the shock tube, it is felt that the leading edge is sufficiently removed from the test section to adequately diminish this upstream effect.

Ideally, the area reduction also results in an increase in the strength of the primary shock wave. However, in the present work there was no noticeable change in shock Mach number measured at the test section from that obtained in runs without the wall model at the same initial conditions. This can be seen in the results shown in Fig. A.3 where shock tube performance with and without the wall model is given in terms of \( M_S \) as a function of diaphragm pressure ratio. The shock wave is, of course, strengthened upon impinging on the reduced area section, but since the actual area reduction is rather small the increase in strength would not be very significant. This fact, along with viscous effects producing shock wave attenuation, apparently results in the performance (in terms of test section \( M_S \)) not being noticeably changed. Recent studies by Dvir et al. (Ref. 45) of the motion of a shock wave through a tube with a convergent cross section indicate that much of the gain in shock strength is dissipated in the region just downstream of the area change.

The main effect of the wall model on the general performance of the shock tube is considered to be the decrease in available test flow. As can be seen in Fig. A.2, the only useful test flow for the present work is that in region (2). Assuming constant wave speeds, the time difference between the arrival of the shock wave and the contact surface at some position is given by

\[
\Delta t_{\text{flow}} = \frac{x}{a_1} \left( \frac{1}{U_{21}} - \frac{1}{M_S} \right) \tag{A.1}
\]

For the present case, the following values are used in Eq. (A.1)

\[
x = 19 \text{ ft.} \\
M_S = 12 \\
a_1 = 1084 \text{ ft/sec} \\
U_{21} = 10.98
\]
The resulting value of $\Delta t_{\text{flow}}$ is 136 $\mu$s.

The corresponding length of the compressed test flow region is

$$\Delta x_{\text{flow}} = x \left(1 - \frac{U_{21}}{M_{s}}\right)$$

(A.2)

For the above conditions, the value of $\Delta x_{\text{flow}}$ is 1.62 ft.

It must be noted that the values for $\Delta t_{\text{flow}}$ and $\Delta x_{\text{flow}}$ represent ideal values; that is, viscous effects may reduce this by about one-half. However, for the present experiments using the wall model these values were approximately three times greater than what were required. Thus, it is felt that the present wall model was sufficiently long to provide an adequate region of test flow.

The ideal pressure-time trace that one would get at the test section ahead of the expansion corner (see Fig. A.2) is illustrated in Fig. A.4. Since the rarefaction wave matching regions (3) and (4) is facing upstream, its arrival at the test section would produce an increase in pressure. As already noted in Section 4.2, the experimental pressure traces for the gauge ahead of the corner did, in fact, indicate such an increase in pressure.
FIG. A.1 PHYSICAL PLANE

FIG. A.2 X-T PLANE
Shock Tube Operation
Without Liner (Driven Gas: O₂)

△ Shock Tube Operation With Liner (Driven Gas: O₂)

□ Shock Tube Operation With Liner (Driven Gas: Air)

FIG. A.3 SHOCK TUBE PERFORMANCE WITH AND WITHOUT THE WALL MODEL PRESENT
FIG. A.4 IDEAL PRESSURE - TIME PROFILE FOR POSITION AHEAD OF EXPANSION CORNER
APPENDIX B

Evaluation of the Change Produced in $\rho_3$ by a Variation of $\alpha_3$ in Eq. (4.5)

The fringe shift relation can be rearranged and written in the following form (see Section 4.1)

$$\rho_3 = \frac{(1 + \beta\alpha_2) \rho_2 - \xi S_{23}}{1 + \beta\alpha_3}$$  \hspace{1cm} (B.1)

where

$$\beta = \frac{K_0}{K_0} - 1$$

$$\xi = \frac{\lambda}{K_0}$$  \hspace{1cm} (B.2)

For a given experimental run, the initial conditions $p_1$ and $T_1$ are known. The shock Mach number $M_s$ and the nondimensional values of fringe shift $S_{23}$ are measured experimentally. Hence, if the parameters $\beta$ and $\xi$ are known, then the only remaining unknown on the RHS of Eq. (B.1) is $\alpha_3$. Thus, for a given set of experimental conditions, one may consider Eq. (B.1) as being represented by

$$\rho_3 = G f(\alpha_3)$$  \hspace{1cm} (B.3)

where

$$G = (1 + \beta\alpha_2) \rho_2 - \xi S_{23} = \text{known const.}$$

$$f(\alpha_3) = \frac{1}{1 + \beta\alpha_3}$$

Since the value of $\alpha_3$ must lie somewhere between the two extremes of $\alpha_2$ (frozen expansion) and $\alpha_{3e}$ (equilibrium expansion), it is possible to determine the maximum change in $\rho_3$ by solving Eq. (B.3) for $f(\alpha_3) = f(\alpha_2)$ and for $f(\alpha_3) = f(\alpha_{3e})$.

The above calculations were made for the following conditions:

$$T_1 = 300^\circ K$$

$$p_1 = 20 \text{ mm Hg. (oxygen)}$$

$$M_s = 12$$

$$S_{23} = 8.75 \text{ (see Fig. 36a)}$$

The values for $\rho_2$ and $\alpha_2$ are obtained from calculations of the equilibrium flow properties behind an incident shock and are

$$\rho_2 = 40.2 \times 10^{-5} \text{ gm/cm}^3$$

$$\alpha_2 = 0.22$$

The Gladstone-Dale constants $K_{02}$ and $K_0$ are assumed to have the values (Ref. 34)
\[ K_{O_2} = 0.19 \text{ cm}^3/\text{gm} \]
\[ K_O = 0.20 \text{ cm}^3/\text{gm} \]

which result in the following values for the parameters \( \beta \) and \( \xi \).

\[ \beta = 0.047 \]
\[ \xi = 2.68 \times 10^{-5} \text{ gm/cm}^3 \]

Thus, the factor \( G \) in Eq. (B.3) has a value of \( 17.15 \times 10^{-5} \text{ gm/cm}^3 \).

Case I: \( f(\alpha_3) = f(\alpha_2) \)

\[ \alpha_3 = \alpha_2 = 0.22 \]

Thus
\[ \rho_3 = 17.15 \times 10^{-5} \left[ \frac{1}{1 + (0.047)(0.22)} \right] = 16.98 \times 10^{-5} \text{ gm/cm}^3 \]

Case II: \( f(\alpha_3) = f(\alpha_{3e}) \)

\[ \alpha_3 = \alpha_{3e} = 0.15 \]

and
\[ \rho_3 = 17.15 \times 10^{-5} \left[ \frac{1}{1 + (0.047)(0.15)} \right] = 17.03 \times 10^{-5} \text{ gm/cm}^3 \]

The percent difference between the values of \( \rho_3 \) for the two limiting cases is 0.29\%. Hence, for all practical purposes the overall influence of changes in \( \alpha_3 \) on the final value of \( \rho_3 \) is negligible.
Some Remarks Regarding the Present Wave-Head Measurements

While making rarefaction wave-head measurements from the interferometric results for the wedge model, it was considered worthwhile to also measure the position of the disturbance produced by the leading edge of the model. Since this disturbance was quite discrete, its position in the flow field (even well away from the leading edge) could be easily identified (see Figs. 31, 32b and 49). Results of the measurements of the leading-edge disturbances are shown in Figs. C.1 to C.3 for three different cases. Also shown in the same figures are the corresponding results obtained from the wave-head measurements (see Fig. 42). As can be seen, the angles determined for the weak shock waves away from the leading edge agree quite well with the computed frozen Mach angles for dissociating oxygen or the perfect gas Mach angle for unreacting oxygen in all cases. On the other hand, the angles determined from the rarefaction wave-head measurements are always less than the results obtained from the weak shock waves. It is of interest to also note that the difference between the angles determined for the leading-edge waves and those for the heads of the expansion waves apparently increases as the shock Mach number decreases (and hence the overall strength of the expansion wave). It is felt that the rapid decay of the expansion wave in a radial direction away from the corner makes the determination of the position of the wave head of the expansion increasingly inaccurate away from the corner. One is in effect relying on a decaying density gradient at the wave head as the means of defining its position. Consequently, the measurements become more uncertain and difficult away from the corner.

In schlieren studies carried out by Cleaver (Ref. 43) of the corner expansion flow of carbon dioxide, it was found to be necessary to produce a Mach wave by means of a slight upstream disturbance in order to accurately align a micro-densitometer for making measurements along the leading characteristic of the expansion wave. Attempts to obtain alignment from the wave head itself were found to be difficult and inaccurate.

Hence it is felt that an accurate determination of the position of the head of the corner expansion wave from interferometric or schlieren results (especially when other wave interactions are occurring during the starting process) is somewhat uncertain. Such being the case, measurements of wave-head position (as attempted in the present work) were not considered to be as practical as measurements of other flow quantities in assessing the nature of the flow field.

However, it is worth noting that schlieren records of wave heads of nonstationary rarefaction waves in perfect gases were successfully used by Glass (Ref. 46) to measure the speed of sound in gases and that these values agreed very well with those determined from weak shock waves (Ref. 47).
FIG. C.1 LEADING-EDGE WAVE AND WAVE HEAD MEASUREMENTS FOR WEDGE MODEL WITH $M_s = 12$ AND $p_1 = 20 \text{ mmHg} \ (\text{Oxygen})$
$p_f = 23.6^\circ$

FIG. C.2 LEADING-EDGE WAVE AND WAVE HEAD MEASUREMENTS FOR WEDGE MODEL WITH $M_s = 10$ AND $p_1 = 20$ mmHg (Oxygen)
FIG. C.3 LEADING-EDGE WAVE AND WAVE HEAD MEASUREMENTS FOR WEDGE MODEL WITH $M_s = 4.4$ AND $p_1 = 20$ mmHg (Oxygen)
AN EXPERIMENTAL INVESTIGATION OF NONEQUILIBRIUM CORNER EXPANSION FLOWS OF DISSOCIATED OXYGEN

Scientific Interim

Drewry, J.E.

April, 1967

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UIIAS REPORT NO. 124

AFOSR 67-0554

An experimental investigation of a two-dimensional nonequilibrium expansion flow of dissociated oxygen around a 15° corner has been carried out in the UIIAS 4 in. x 7 in. hypersonic shock tube. The flow was generated by a plane shock wave moving at a Mach number $M_s = 12$ into pure oxygen at a pressure of 20 mm Hg giving a degree of dissociation $\alpha = 0.22$, at a temperature of 3800 K. The study consisted of density measurements made with a Mach-Zehnder interferometer and pressure measurements using piezoelectric transducers. Two fundamentally different test models were used to generate corner expansion flows in an effort to gain some insight into the nature and extent of boundary layer influence on the final results. The detailed density and pressure results which were obtained indicate a longer relaxation distance than predicted by theoretical calculations based on a partially excited gas model and would appear to favor the predictions of more recent calculations based on a coupled vibration-dissociation model, in lieu of assuming instantaneous vibrational equilibrium. Values for the recombination rate constant $k_R$ have not been inferred from the present results since this would require a direct comparison between experimental data and theoretical predictions. Such a comparison does not seem to be justified at the present time since there is still doubt as to the most appropriate theoretical model for computing the nonequilibrium flow properties. Once this is better known and assessed, it may then be possible to re-define new operating conditions that would provide more suitable results on which to base a determination of $k_R$. In addition, the choice of the test model to be used for generating the expansion flow will have to be tempered by the consideration of the influence of the boundary layer.
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