MOTION CHARACTERISTICS
OF THE UTIAS FLIGHT RESEARCH
SIMULATOR MOTION-BASE

by

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SUMMARY

The motion characteristics of the UTIAS Flight Research Simulator Motion-Base were experimentally determined. More specifically describing function tests (under various operating conditions), 1/2 Hz noise level tests, signal-to-noise tests, and hysteresis tests were performed for all six degrees-of-freedom. Dynamic threshold tests were performed for the heave degree-of-freedom. The motion-base was found to have a reasonably flat amplitude response up to 10 Hz in all degrees-of-freedom. Motion in the non-driven degrees-of-freedom was small compared to the driven channel. The noise of the motion-base was found to be the sum of broadband background noise and harmonics of the driven frequency, with the amplitude of the noise varying with both the amplitude and frequency of the driving signal. Hysteresis was determined to be negligible. The dynamic threshold was found to be small and quite acceptable for most projected applications involving the motion-base.
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NOMENCLATURE

\( \ddot{q} \)
acceleration vector of a point with respect to \( F_I \)

\( A, A_0 \)
amplitudes of sinusoidal functions

\( \~s_i \)
the coordinates of the upper bearing block of the \( i \)-th actuator in \( F_I \) components

\( \~s_{iS} \)
the coordinates of the upper bearing block of the \( i \)-th actuator in \( F_S \) components

\( A_{iS} \)
the collection of \( \~s_{iS} \) for all actuators

\( B_{iI} \)
the coordinates of the lower bearing block of the \( i \)-th actuator in \( F_I \) components

\( B_{iI} \)
the collection of \( B_{iI} \) for all actuators

\( D_{z1} \)
desired starting position of simulator for step acceleration input maneuver.

\( D_{z2} \)
desired position of simulator at time \( T/2 \) for step acceleration input maneuver

\( D_{x} \)
x-component in \( F_S \) of vector from \( x \)-accelerometer to \( P_A \)

\( D_{y} \)
x-component in \( F_S \) of vector from \( y \)-accelerometer to \( P_A \)

\( D_{z} \)
x-component in \( F_S \) of vector from \( z \)-accelerometer to \( P_A \)

\( \text{DFT}(x(t), \omega) \)
digital Fourier transform of \( x(t) \) into \( \omega \)-space

\( f \)
specific force vector \( \equiv \ddot{q} - \ddot{q} \)
\[ F_I \] inertial reference frame

\[ F_P \] instrument package reference frame

\[ F_S \] simulator body-fixed reference frame

\[ f_s \] sampling frequency (Hz)

\[ f_c \] natural frequency of system (Hz)

\[ g \] the acceleration due to gravity

\[ h_0 \] the magnitude of hysteresis

\[ h(t) \] the hysteresis as a function of time

\( I \) the identity matrix

\( j \) the complex number \( \sqrt{-1} \)

\( \ell_i \) displacement of the \( i \)-th actuator

\( \ell_i \) length of the \( i \)-th actuator

\( L_i \) midstroke length of the \( i \)-th actuator

\( L \) collection of all midstroke lengths

\( L_{IS} \) transformation matrix from frame \( S \) to frame \( I \)

\( L(j\omega) \) linear transfer function

\( N \) the number of samples

\( P_A \) the centroid of the upper payload frame bearing pivot points
<table>
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<tr>
<th>Symbol</th>
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<tr>
<td>$P_B$</td>
<td>the centroid of the lower frame bearing pivot points</td>
</tr>
<tr>
<td>$p$</td>
<td>roll angular velocity</td>
</tr>
<tr>
<td>$P$</td>
<td>amplitude of sinusoidal roll angular velocity</td>
</tr>
<tr>
<td>$P_N$</td>
<td>the peak noise of a signal</td>
</tr>
<tr>
<td>$q$</td>
<td>pitch angular velocity</td>
</tr>
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<td>$r$</td>
<td>yaw angular velocity</td>
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<tr>
<td>$R_{uv}(\tau)$</td>
<td>correlation function of $u(t)$ and $v(t)$</td>
</tr>
<tr>
<td>$S/N$</td>
<td>signal to noise ratio</td>
</tr>
<tr>
<td>$S$</td>
<td>location of $F_S$ with respect to $F_I$</td>
</tr>
<tr>
<td>$T$</td>
<td>time period of periodic function</td>
</tr>
<tr>
<td>$T_0$</td>
<td>fixed time period</td>
</tr>
<tr>
<td>$t$</td>
<td>time(s)</td>
</tr>
<tr>
<td>$u$</td>
<td>$x$-component of $\mathbf{V}$</td>
</tr>
<tr>
<td>$v$</td>
<td>$y$-component of $\mathbf{V}$</td>
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<td>$w$</td>
<td>$z$-component of $\mathbf{V}$</td>
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<tr>
<td>$V_{z1}$</td>
<td>velocity of simulator at $t=0$ for square wave input</td>
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<td>$V_{z2}$</td>
<td>velocity of simulator at $t=T/2$ for square wave input</td>
</tr>
<tr>
<td>$V_I$</td>
<td>components of velocity vector $\mathbf{V}$ in $F_I$</td>
</tr>
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</table>
\( x(t) \)  \( \) signal \( x \) as a function of time

\( X(\omega) \)  \( \) Fourier transform of \( x(t) \)

\( \alpha \)  \( \) the phase angle

\( \alpha_p \)  \( \) the primary phase angle \( (0^\circ < \alpha_p < 360^\circ) \)

\( \alpha_a \)  \( \) the appropriate phase angle \( \alpha_a = \alpha_p + k(360) \)

\( \Delta f \)  \( \) an increment of frequency (Hz)

\( \Delta t \)  \( \) an increment of time

\( \Delta \omega \)  \( \) an increment of frequency (r/s)

\( \zeta \)  \( \) second-order system damping ratio

\( \Theta \)  \( \) simulator Euler pitch angle

\( \sigma_x \)  \( \) standard deviation of \( x(t) \)

\( \sigma_x^2 \)  \( \) variance of \( x(t) \)

\( \sigma_n \)  \( \) standard deviation of the noise

\( \phi \)  \( \) simulator Euler roll angle

\( \phi_{xx} \)  \( \) two-sided power spectrum of \( x(t) \)

\( \Phi_{xx} \)  \( \) one-sided power spectrum of \( x(t) \)

\( \phi_{xy} \)  \( \) two-sided cross spectrum of \( x(t) \) and \( y(t) \)
\( \phi \) 
simulator Euler yaw angle

\( \omega_s \) 
sampling frequency (r/s)

\( \omega \) 
frequency (r/s)

\( \omega_c \) 
natural frequency of system

\( \omega_i \) 
angular velocity \( \omega = [p\ q\ r]^T \)

\( \tilde{\omega}_i \) 
skew symmetric matrix of angular velocity

\( \hat{r}_i \) 
estimated response of simulator for the i-th actuator

\( r_i \) 
input signal to simulator

\( \hat{r}_i \) 
components of \( r_i \) in frame \( F_i \)

\( r_{x,y,z} \) 
x, y, or z component

\( \hat{r}_i^T \) 
transpose of matrix

\( \cdot \) 
\( d(\cdot)/dt \)
1 INTRODUCTION

Since man first took to the air the importance of flight training has been recognized. Flight simulators are an ideal tool for flight training, enabling it to take place with considerable benefits in cost, safety, and flying space compared to inflight training. They also allow training in situations which are too dangerous to be performed in actual aircraft. These practical advantages have created a large demand for flight simulators. The benefits of simulators in pilot training are also realized in other vehicles with man-machine interactions and hence the use of simulation as a training tool is spreading rapidly. Simulators are also being applied in the design of piloted vehicles as a pilot-in-the-loop test permits evaluation of the control system under realistic operating conditions.

1.1 History

In the early 1900's flight/vehicle simulation had its humble beginnings (Reference 1). The Saunders Teacher was one of the first of such devices. The Teacher was basically an aircraft tethered to the ground and pointed into the prevailing wind. The flow of air over the aerodynamic surfaces would cause the response of the Teacher to resemble the response of an actual aircraft. Dependence on the wind, however, rendered such devices inconsistent and awkward.

The "synthetic trainer" was the next development, and the Link Trainer was the most successful of this generation. The Link Trainer used pneumatic systems to move the motion-base in response to pilot control inputs. The response of the trainer to any particular control input was adjusted to provide the right "feel" to the pilot. Such an unscientific method of modelling and lack of motion system performance and coherency unfortunately created an unrealistic simulation.

The advent of the analog computer led to the next advancement in flight/vehicle simulation. The analog computer could solve the differential equations of motion for an aircraft in real-time. The analog computer generally drove only fixed-base simulators, since motion-bases were not
developed to the point where they could provide any realistic "cues". Realistic simulations of instrumentation and visual cues were achieved. The fixed-base nature and overwhelming complexity of the analog systems limited the diversity and reliably of the analog controlled simulator.

The advent of the relatively cheap high speed digital computer resulted in major changes in the simulation industry. With its enormous diversity and incredible speed the digital computer was an obvious candidate for simulation work. The digital computer has the ability to solve the differential equations of motion for the aircraft and create the appropriate signals for motion, visual, and instrumentation systems all in real-time and with high precision. The motion and visual systems soon became very complex and advanced. Six degrees-of-freedom motion-bases, driven hydraulically, capable of high accelerations and moderately large displacements became the mainstay of the modern flight simulator.

1.2 The Need for Motion Cues

In most piloted vehicles man receives information required to control the vehicle in the form of motion, visual, and aural cues. Motion is considered a relatively important source of information and will be exclusively dealt with in this report. A pilot can use his motion sensing devices to provide feedback in the control loop and thus help close the loop and achieve the best performance of the task at hand. Current belief is that the motion cues which humans most readily sense are, translational accelerations, and angular velocities (References 2 and 3). A realistic simulation of a piloted vehicle, should therefore provide the appropriate motion cues which a pilot would experience under actual flight conditions. Performance limits of motion systems eventually lead to incorrect motion cues and it has been argued that the confusion caused by false cues outweighs the benefit of the correct motion cues. If false cues can be minimized and given the fact that motion helps reduce the possibility of pilot nausea in the simulator it will be assumed that motion is a useful aspect of vehicle simulation.
1.3 Awareness of the Motion Characteristics of the Simulator

Given that motion is a useful aspect of simulation it is imperative that the desired motion be achieved by the simulator. Discrepancies between the desired motion and the actual motion of the simulator will create false, or at least inaccurate, motion cues. The motion-base is a physical system and hence has a limited response and fidelity which depends upon the method of drive. Since imperfection is inherent in motion-bases it is essential to be aware of the motion characteristics of the particular base being used. The user then knows, with reasonable accuracy, the response of the motion base to any commanded maneuver.

The motion-base of the UTIAS Flight Research Simulator, under study in this report, will be used for research in the field of human pilot thresholds and other important aspects concerning vehicle simulation. Determination of accurate motion characteristics is even more essential if meaningful results are to be drawn from such experimentation involving the motion-base.

2 OVERVIEW

2.1 The Motion-Base of the UTIAS Flight Research Simulator

As mentioned previously the motion-base to be examined is the UTIAS CAE 300 series motion-base. The 300 series motion-base is a six degrees-of-freedom synergistic system driven by six hydraulic actuators. The actuators have a stroke of 91.4 cm, a bore of 8.9 cm and are equipped with hydrostatic bearings. Each actuator extension is controlled by an electro-mechanical servo-valve. Power for the system is provided by three 37.3 kW squirrel cage induction motors with matching in-line piston pumps. The system pressure under normal operating conditions is 10.34 MPa. The oil is stored in a reservoir with a capacity of 946 liters. An additional 3.7 kW electric motor circulates the oil through a cooling circuit to maintain a constant oil temperature during operation. The six degrees-of-freedom referred to consist of three translational modes, surge (along the x-axis),
sway (y-axis), and heave (z-axis), and three angular modes, roll (about the x-axis), pitch (y-axis), and yaw (z-axis). See Figure 2.01 for a picture of the motion-base as installed at UTIAS. As can be seen from the figure a DC8 simulator cab, donated by Air Canada, has been mounted on the motion-base.

Control of the motion-base is achieved using an analog control system referred to as the NI cabinet driven by D/A outputs from a digital computer. The NI takes commanded inputs and using a complex control system creates the appropriate response of the servo-valves to extend each actuator as commanded. The inputs to the system are the actuators' lengths and accelerations of extension. The control system uses feedback from two primary types of transducer to close the loop. The two transducer types are linear magnetostrictive position transducers and force transducers, one of each per actuator. At low frequencies the control system is designed to rely mainly on the position transducer and to gradually switch over to the force transducers as the frequency of the motion increases. In addition to this the control system also helps to compensate for imperfections in the response of the hydraulic system.

At the input to the NI a set of 5th order low-pass elliptic filters with a break frequency of 10 Hz are used to smooth and filter the staircase signals entering the control system. If the update rate of the computer is fast enough (greater than 30 Hz), and the noise on the signals can be kept low, the elliptic filters can be eliminated with a resulting increase in bandwidth of the system. For the transfer function of these filters as measured using a spectral analyzer see Figure 2.02.

2.2 Proposed Study

The term "motion characteristics" is a rather vague and all-encompassing description of the information which describes the performance of the motion-base. The actual tests performed are based on AGARD Report 144, (Reference 4) with some additional useful measurements, not covered in the report.
2.2.1 Describing Functions

The steady-state response of the motion-base to sinusoidal inputs, in terms of amplitude ratios and phase angles is to be found. For each degree-of-freedom six describing functions are produced. The primary describing function is the comparison of the response of the motion-base in the driven degree-of-freedom to the excitation signal. The other five describing functions are crosstalks. They compare pure parasitic motion (motion in other than the degree-of-freedom excited) to the excitation signal.

For the translational maneuvers translational acceleration was chosen as the metric for measurement. Since the human detects translational accelerations the choice of it for the translational variable is obvious. For the angular maneuvers the choice is less obvious. The human being has a flat response to angular velocity in the mid-frequency range (Reference 2). This suggests that perhaps angular velocity should be the angular metric. Crosstalk transfer functions, however, require ratioing of the angular metric to the translational metric and for this angular acceleration provides a more reasonable ratio. Thus angular acceleration was chosen as the angular variable of concern. It should be noted that once a describing function is known, the describing function for any integral or differential of the two signals being compared is also defined. For example if the translational acceleration primary describing function is known then so is the translational velocity describing function.

Thirty-six results form a complete set of describing functions; six per degree-of-freedom and six degrees-of-freedom. A complete set is calculated for the base under normal operating conditions. The primary describing functions were also generated for an additional operation condition, with the elliptic input filters in the N1 cabinet by-passed. The elliptic filters have a break frequency of 10 Hz and removal of them should result in increased bandwidth of the system.
2.2.2 1/2 Hz Noise Level Tests

As suggested in Reference 4 noise level tests were performed with the motion-base driven sinusoidally at 1/2 Hz. For each degree-of-freedom a set of noise levels on all channels was calculated for varying amplitudes of input. For the driven degree-of-freedom the results presented were total noise, high frequency non-linearity, low frequency non-linearity, roughness and peak noise. On the parasitic channels only the total noise and peak noise results were calculated. For the translational maneuvers acceleration was chosen again as the variable to be measured, but for the angular maneuvers velocity was selected as the metric. The reasoning for this was two-fold, firstly humans sense angular velocity and secondly the available instrumentation measured angular velocity.

2.2.3 Signal-to-Noise Contours

The signal-to-noise contour test produced plots of motion-base signal to noise contours for all degrees-of-freedom within the operating range of the simulator. The contour lines connect points of equal signal-to-noise ratio and were plotted on a graph of output velocity (angular and translational) versus frequency. A set of operating points was chosen, the signal-to-noise ratio at each point was found and contours interpolated from the resulting data. For translational maneuvers the parameter selected for the signal to noise measurements was acceleration. Velocity was chosen for the angular modes.

2.2.4 Hysteresis Testing

The effect of hysteresis is such that the actual D.C. response of a system depends upon the way in which the final D.C. command signal was reached. A hysteresis loop, a plot of the actual response versus the command signal for a periodic maneuver, is generally used to demonstrate hysteresis (Reference 5). The hysteresis is to be found for each degree-of-freedom, surge, sway, heave, roll, pitch, and yaw. Position is the parameter chosen to describe the results. Hysteresis should not contain any dynamic response effects of the system under study.
2.2.5 Dynamic Threshold

As shown in Reference 4 the dynamic threshold is the time required for the motion-base to reach 63% of a commanded step input of acceleration. The test will be carried out in heave only. The effect of the amplitude of the step input on the threshold is also to be determined. The dynamic threshold is subdivided into dead-time and rise-time. The dead-time is the time after application of the step input when no response of the base is discernable. The rise-time is the remaining time to the threshold.

2.3 CAE Preliminary Study

Preliminary testing of the motion-base was carried out by CAE prior to delivery. Primary describing functions, 1/2 Hz noise level tests, and dynamic threshold tests were done. At the time of testing a simulator cab was not mounted upon the motion-base. In an attempt to simulate the effects of a cab CAE attached a 3636 kg concrete block to the center of the top motion platform. The DC8 cab now mounted upon the base is much lighter (weighing approximately 2268 kg), and has a different center of mass location and an entirely different mass distribution from the concrete block used in the CAE tests. Thus the motion test results produced by CAE are unsuitable for predicting the motion-base performance as it is configured presently.

2.4 Experimental Procedure

Determination of the motion characteristics of the simulator requires the ability to drive the base with a predetermined motion, and to collect data from motion sensitive instrumentation mounted on the motion-base. A Perkin-Elmer 3250 digital mini-computer is used to accomplish these two tasks.

From the known motion time history of a reference point on the simulator the time histories of the commanded actuator length and acceleration signals are derived using the motion-base driving algorithm described later in Section 4. The commanded actuator length and acceleration time histories
are then stored in RAM. After all the equipment has warmed-up to normal operating temperature the real-time aspect of the program begins. The precision clock in the PE-3250 is started and at specified intervals in time the commanded actuator signals are output via D/A converters. At almost the same instant (800 μs later) data from the instrumentation is sampled and stored in RAM via A/D converters. Synchronization of the input/output operations is ensured by the use of the same clock.

The analog command signals from the PE-3250 computer are fed into the NI cabinet which then produces the necessary actions of the hydraulic system to create the motion-base response. After completion of the run preliminary reduction of the collected data is performed. The appropriate conversions and processing are done and the data is then written to disk and/or tape for permanent storage. The data is then available for analysis leading to results for final presentation. The schematic of the entire system is shown in Figure 2.03.

3 REFERENCE FRAMES

3.1 Inertial Frame \( F_I \)

The Inertial frame is Earth-fixed with the z-axis pointed along the gravity vector \( g \). The x-axis points towards the front of the cab and the y-axis points to starboard. The x-y plane is perpendicular to \( g \). The origin is at the centroid of the fixed lower frame bearing pivot points, point \( P_B \) in Figure 4.02.

3.2 Simulator Body-Fixed Frame \( F_S \)

The simulator body-fixed reference frame has its origin at the centroid of the upper payload frame bearing pivot points, point \( P_A \) in Figure 4.02. When the simulator is at the neutral position \( (\phi_0 = \phi_0 = \phi_0 = 0) \) \( F_S \) is exactly aligned with \( F_I \).
3.3 Instrument Frame \( F_p \)

The package frame is aligned with the sensitive axes of the instrument package. The origin is located within the package itself. The package is mounted upon the simulator such that \( F_p \) is aligned with \( F_S \).

4 DERIVATION OF THE DRIVING EQUATIONS

4.1 General Driving Equations

For the tests performed in this report the commanded motion of a reference point on the simulator in the inertial frame is known. The inputs used to drive the simulator are the actuator extensions and accelerations, thus equations relating the actuator movements to the overall motion-base movement must be determined.

From Figure 4.01 showing the motion-base geometry for a single actuator

\[
\vec{S} = \vec{B}_i + \vec{A}_i - \vec{A}_i
\]  

for the \( i \)-th actuator, where

\( \vec{B}_i \) is the vector from point \( P_B \) to the lower pivot of actuator \( i \).

\( \vec{A}_i \) is the vector from point \( P_A \) to the upper pivot of actuator \( i \).

\( \vec{S} \) is the vector from point \( P_B \) to \( P_A \).

\( \vec{A}_i \) is the vector from the lower pivot to the upper pivot of actuator \( i \).

Rearranging and taking components in the inertial frame \( F_I \)

\[
\vec{S}_I = \vec{A}_I + \vec{S}_I - \vec{B}_I
\]  

With \( P_A \) at the origin of \( F_S \). Now
\[ A_{iI} = L_{IS} A_{iS} \] (4.3)

Thus \( A_{iI} \) can be found from \( A_{iS} \), where \( L_{IS} \) is defined in Reference 6 as:

\[
L_{IS} = 
\begin{bmatrix}
\cos \theta \cos \phi & \sin \phi \sin \theta \cos \phi - \cos \phi \sin \theta & \cos \phi \sin \theta \cos \phi + \sin \phi \sin \theta \\
\cos \theta \sin \phi & \sin \phi \sin \theta \sin \phi + \cos \phi \cos \theta & \cos \phi \sin \theta \sin \phi - \sin \phi \cos \theta \\
-\sin \theta & \sin \phi \cos \theta & \cos \phi \cos \theta
\end{bmatrix}
\] (4.4)

where \( \phi, \theta, \) and \( \psi \) refer to the simulator’s Euler angles. The Euler angles can be found from the differential equations (Reference 6),

\[
\begin{bmatrix}
\dot{\phi} \\
\dot{\theta} \\
\dot{\psi}
\end{bmatrix} =
\begin{bmatrix}
1 & \sin \phi \tan \theta & \cos \phi \tan \theta \\
0 & \cos \phi & -\sin \phi \\
0 & \sin \phi \sec \theta & \cos \phi \sec \theta
\end{bmatrix}
\begin{bmatrix}
p \\
q \\
r
\end{bmatrix}
\] (4.5)

From Equations 4.2 and 4.3 the components of the actuator length vectors in \( F_{i} \) become

\[ A_{iI} = L_{IS} A_{iS} + S_{i} - B_{iI} \] (4.6)

This applies for all actuators, see Figure 4.02.

Furthermore, the vector \( S_{i} \) is defined by

\[ S_{i}(t) = S_{i}(0) + \int_{0}^{t} V_{i}(t) \, dt \] (4.7)

where \( V_{i} \) is a 3x1 matrix; the velocity with respect to \( F_{i} \) of the point \( P_{A} \) on the simulator. \( S_{i}(0) \) is chosen to start the simulator at the neutral position (i.e. all actuators at the average midstroke).

The matrices \( A_{iS} \) and \( B_{iI} \) are constant. The lengths and the accelerations of the actuators are required to drive the motion-base. The actuator length drive signals are
\[ \lambda_i = (L_i^T L_i)^{1/2} - L_i \] (4.8)

where \( \lambda_i \) is the commanded actuator length.

\( L_i \) is the actuator's midstroke length.

The velocity of the \( i \)-th actuator is described by

\[ \dot{\lambda}_i = (\lambda_i + L_i)^{-1} (L_i^T \dot{\lambda}_i) \] (4.9)

and the acceleration drive signals are

\[ \ddot{\lambda}_i = (\lambda_i + L_i)^{-1} (L_i^T \ddot{\lambda}_i + L_i^T \dot{\lambda}_i - \lambda_i^2) \] (4.10)

where

\[ \ddot{L}_{ii} = L_{iS} \ddot{A}_{iS} + \ddot{S}_i \] (4.11)

since

\[ \ddot{A}_{iS} = 0 \] for \( i = 1 \) to 6 (4.12)

and also

\[ \ddot{L}_{ii} = L_{iS} \ddot{A}_{iS} + \ddot{S}_i \] (4.13)

From Equation 4.7

\[ \ddot{S}_i = V_i \] (4.14)

and

\[ \dddot{S}_i = \dddot{V}_i \] (4.15)
also \( \dot{\omega}_{IS} \) is (from Reference 6)

\[
\ddot{\omega}_{IS} = \dot{\omega}_{IS} \omega_{S} \quad (4.16)
\]

where \( \omega_{S} \) is

\[
\begin{bmatrix}
0 & -r & q \\
r & 0 & -p \\
-q & p & 0
\end{bmatrix}
\]

From Equation 4.16 \( \ddot{\omega}_{IS} \) becomes

\[
\ddot{\omega}_{IS} = \dot{\omega}_{IS} \dot{\omega}_{S} \omega_{S} + \dot{\omega}_{IS} \omega_{S} \quad (4.18)
\]

where \( \dot{\omega}_{S} \) is given by

\[
\begin{bmatrix}
0 & -r & q \\
r & 0 & -p \\
-q & p & 0
\end{bmatrix}
\]

The quantities \( \phi, \theta, \dot{\phi}, \dot{\omega}_{S}, \omega_{S}, V_{I}, \) and \( \dot{V}_{I} \) describe the motion of the simulator in the inertial frame and are determinate for a specific maneuver.

4.2 Simplifications for Specific Maneuvers

4.2.1 Sinusoidal Translational Acceleration

It follows that if the motion-base is driven in only translational motion, then
Thus in this case Equation 4.6 simplifies to

\[ \dot{x}_I = A_{iS} + S_I(0) + \int_0^t V_I(t) \, dt - B_{iI} \]  

(4.22)

and furthermore, Equations 4.11 and 4.13 simplify to

\[ \ddot{x}_I = \dot{S}_I \]  

(4.23)

or

\[ \dot{x}_I = V_I \]  

(4.24)

and

\[ \ddot{x}_I = \dot{V}_I \]  

(4.25)

Thus if the time history of position, velocity, and acceleration of the point \( P_A \) on the simulator, is known the actuator command signals are defined. For sinusoidal translational motion particularly we will take the simulator motion command to be:

\[
\begin{bmatrix}
-V_x \omega_x \sin \omega t \\
-V_y \omega_y \sin \omega t \\
-V_z \omega_z \sin \omega t \\
-V_x \cos \omega_x t \\
-V_y \cos \omega_y t \\
-V_z \cos \omega_z t \\
(V_x / \omega_x) \sin \omega t \\
(V_y / \omega_y) \sin \omega t \\
(V_z / \omega_z) \sin \omega t 
\end{bmatrix}
\]  

(4.26)

\[
\begin{bmatrix}
V_x \\
V_y \\
V_z \\
(V_x / \omega_x) \\
(V_y / \omega_y) \\
(V_z / \omega_z) 
\end{bmatrix}
\]  

(4.27)
where for this case we will define

\[ V_I = [u_I v_I w_I]^T \]  \hspace{1cm} (4.29)

The actuator driving signal for pure sinusoidal translations can now be determined.

### 4.2.2 Square Wave Translational Acceleration

For the dynamic threshold tests a square wave acceleration input is required in heave, see Figure 4.03. The time history of the motion-base command for a square wave heave maneuver is taken to be:

\[
\dot{V}_I = \begin{bmatrix} 0 \\ 0 \\ A \end{bmatrix} \text{ for } t \leq T/2 \]  \hspace{1cm} (4.30)

\[
\dot{V}_I = \begin{bmatrix} 0 \\ 0 \\ -A \end{bmatrix} \text{ for } T \geq t > T/2 \]  \hspace{1cm} (4.31)

\[
V_I = \begin{bmatrix} 0 \\ 0 \\ At + V_{z1} \end{bmatrix} \text{ for } t \leq T/2 \]  \hspace{1cm} (4.32)

\[
V_I = \begin{bmatrix} 0 \\ 0 \\ (T/2-t)A + V_{z2} \end{bmatrix} \text{ for } T \geq t > T/2 \]  \hspace{1cm} (4.33)

\[
\int V_I = \begin{bmatrix} 0 \\ 0 \\ \frac{At^2}{2} + V_{z1}t + D_{z1} \end{bmatrix} \text{ for } t \leq T/2 \]  \hspace{1cm} (4.34)
\[ \int_{0}^{t} V_I = \begin{bmatrix} 0 \\ 0 \\ -(T/2-t)A + (t-T/2)V_z2 + D_z2/2 \end{bmatrix} \text{ for } T > t > T/2 \] (4.35)

and the constants \( V_{z1}, V_{z2}, D_{z1}, \) and \( D_{z2} \) are chosen to provide the desired starting position and velocity of the simulator.

4.2.3 Sinusoidal Angular Motion

By definition pure angular motion about \( P_A \) has

\[ S_I(t) = S_I(0) \] (4.36)

Thus equations 4.6, 4.11, 4.13, 4.16, and 4.18 reduce to

\[ \dot{S}_I = L_{IS} A_{IS} + S_I(0) - B_{iI} \] (4.37)

\[ \ddot{S}_I = L_{IS} \ddot{A}_{IS} \] (4.38)

\[ \dddot{S}_I = (L_{IS} \dddot{A}_{IS} + L_{IS} \dddot{A}_{IS}) A_{IS} \] (4.39)

For a single degree-of-freedom sinusoidal angular motion the equations can be further refined. Consider rotation about the x-axis for example, we will take the motion command to be:

\[ \omega_S = [p \cos \omega_x t \hspace{1cm} 0 \hspace{1cm} 0]^T \] (4.40)

\[ \dddot{A}_S = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -p \cos \omega_x t \\ 0 & p \cos \omega_x t & 0 \end{bmatrix} \] (4.41)

\[ \dddot{A}_S = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & p \omega_x \sin \omega_x t \\ 0 & -p \omega_x \sin \omega_x t & 0 \end{bmatrix} \] (4.42)
where for the general case of rotation about any axis we will define the command by

$$\omega_S = \begin{bmatrix} p_I & q_I & r_I \end{bmatrix}^T$$  \hfill (4.43)

and the commanded Euler angles which define $L_{IS}$ are found from Equation 4.5 and for rotation about the x-axis are taken to be:

$$\begin{bmatrix} \phi_I \\ \theta_I \\ \psi_I \end{bmatrix} = \begin{bmatrix} (P/\omega_x)\sin \omega_x t + \phi_0 \\ \theta_0 \\ \phi_0 \end{bmatrix}$$  \hfill (4.44)

and the actuator drive signals can now be found.

Collect terms for all actuators as follows;

$$A_S = [A_{1S} A_{2S} A_{3S} A_{4S} A_{5S} A_{6S}]$$  \hfill (4.45)

$$B_I = [B_{1I} B_{2I} B_{3I} B_{4I} B_{5I} B_{6I}]$$  \hfill (4.46)

$$L = [L_1 L_2 L_3 L_4 L_5 L_6]$$  \hfill (4.47)

Upon choosing points $P_A$ and $P_B$ matrices $A_S$, $B_I$, $S_I(0)$ and $L$ must be determined prior to solution of the actuator drive signals. $S_I(0)$ is chosen to start the simulator at the neutral position. If $A_S$ and $B_I$ are known then $L$ is fixed. The accuracy of the simulator's motion depends on the accuracy to which $A_S$ and $B_I$ are found and thus they must be determined using some suitably accurate method.

### 4.3 Geometry Determination

The vectors which comprise matrices $A_S$ and $B_I$ are difficult to measure directly as they start at a point in space and end at the center of rotation
of an actuator. There is no physical object which defines the location of the tail of vectors and resolving the vector into components along the axes would prove difficult. An indirect method of determining \( \vec{A}_S \) and \( \vec{B}_I \) is required.

Equation 4.6 can be rearranged to give

\[
\begin{align*}
L_{IS} \vec{A}_S - \vec{B}_I + \vec{S}_I - \vec{B}_I &= 0
\end{align*}
\] (4.48)

for any position. Furthermore, any of the actuator length matrices can be subdivided into

\[
\begin{bmatrix}
\ell_{ix} \\
\ell_{iy} \\
\ell_{iz}
\end{bmatrix} = \begin{bmatrix}
\tilde{l}_{i \hat{x}} \\
\tilde{l}_{i \hat{y}} \\
\tilde{l}_{i \hat{z}}
\end{bmatrix}
\] (4.49)

where

\[
|\tilde{l}_{i}| = \sqrt{\ell_{ix}^2 + \ell_{iy}^2 + \ell_{iz}^2}
\] (4.50)

\( L_{IS} \) has only 3 independent variables, \( \theta_0, \phi_0, \) and \( \psi_0 \) as shown by Equation 4.4. Since \( P_A \) is at the origin of \( F_S \) and \( P_B \) is directly below it when the motion-base is at its neutral position (along the inertial z-axis) and in the plane of the lower actuator bearings then \( S_I(0) \) is relatively easy to measure. As the simulator takes up a new position \( S_I \) can be determined using a plumb-bob hung from the simulator and its movement along each of the inertial axes can be recorded. The magnitude of the actuators' lengths are not difficult to determine. At the neutral position the entire length of the actuators can be precisely measured, and the length of exposed piston shaft measured and subtracted from the overall length to give the length of the cylinder plus end fittings. The length of the actuator at any attitude can then be found by determining only the length of the exposed portion of the piston shaft and adding it to the cylinder length.
If the simulator is restricted to having only one of the two Euler angles $\phi_0$ and $\theta_0$ non-zero for each maneuver, then an inclinometer mounted along either the x-axis or the y-axis will measure the appropriate Euler angle $\phi_0$ (along the y-axis) or $\theta_0$ (along the x-axis). Equating the number of unknowns and the number of equations for N different attitudes gives

\[
\begin{align*}
A_S &= 18 \text{ unknowns} \\
B_I &= 18 \text{ unknowns} \\
\frac{2\gamma}{N} &= 18N \text{ unknowns} \\
\phi_0 &= N \text{ unknowns} \\
19N &= 36 \\
\end{align*}
\]

and the corresponding number of equations are

Equation 4.6 $\Rightarrow$ 18N equations.
Equation 4.50 $\Rightarrow$ 6N equations.

The corresponding 18N equations for $\frac{2\gamma}{N}$ are found by direct measurement along with N values for each of $\phi_0$ and $\theta_0$. Equating the number of equations and number of unknowns and solving for N we find $N = 7.2$, but N must be an integer and thus 8 different attitudes must be studied. This results in 192 non-linear simultaneous equations in 192 unknowns.

The solution to this set of equations was achieved by using the IMSL non-linear simultaneous equation solver ZSPOW. It became readily apparent, however, that the error buildup in a system this large required the measured parameters to be accurate beyond our capabilities if the results were to meet the minimum accuracy requirements. Substantial reduction in the size of the system (and hence error buildup) depends upon determining parameters which are as difficult to measure as $A_S$ and $B_I$, and is thus self-defeating.

The problems encountered in the previously described operation required that an alternate approach be devised to solve for $A_S$ and $B_I$. They were finally determined using CAE shop and assembly drawings for the motion-base. An accuracy check was performed on the results. Using the program PLATATT.
(see Appendix A), and the $A_S$ and $B_I$ matrices as determined from the drawings, the simulator's theoretical displacement and attitude in the inertial frame was determined for sets of actuator extensions. Twelve sets of actuator extensions were devised for large positive and negative pure displacements and attitudes in each degree-of-freedom. The actual actuators were then precisely extended to the corresponding lengths using the analog motion control on the NI cabinet. The displacement of the base in $F_I$ was then determined by measuring the displacements of a plumb-bob, hung from the upper motion platform, with a tape measure. Since the maneuvers were in a single degree-of-freedom the Euler angles $\phi_0$ and $\theta_0$ could be determined using an inclinometer positioned along the appropriate axis of the body-fixed reference frame (assuming parasitic Euler angles are small). The Euler angle $\psi_0$, could be found using two plumb-bobs hung from the cab to project a line on the ground (inertial x-y plane). $\psi_0$ is the angle between the line drawn with the cab at the neutral position and the line drawn when the cab is yawed. The methods of measuring the Euler angles will only produce meaningful results if the parasitic Euler angles are almost zero.

The accurate alignment of the inertial and body-fixed frame is essential for reliable conclusions to be drawn from the geometry check. The inertial frame was determined by dividing the back leg of the bottom triangle created by the actuator pivot points in half, and connecting this point to the intersection of the other two legs (see Figure 4.04). The direction pointing from the back to the front along this line and perpendicular to the local $g$ vector defines the x-axis of the inertial system. The z-axis pointed directly along the $g$ vector and the y-axis was perpendicular to both such that the system was right-handed. The body-fixed frame was determined by moving the simulator to the neutral position ($\phi_0 = \theta_0 = \psi_0 = 0$) and projecting the inertial frame previously found onto the floor of the cab.

The results of the accuracy tests are shown in Table 1. All parasitic displacements and angles were found to be approximately zero. In addition Table 1 contains the maximum displacement limits for the motion-base for pure single degree-of-freedom motion about the origin of $F_S$. From the results the matrices $A_S$ and $B_I$ were assumed to be accurately determined, and the values are in Table 2.
5 EXPERIMENTAL PREPARATION

5.1 Tuning of the Motion System

All components of the motion-base system are subject to tolerances and variability during manufacture and wear during operation. Provision is made for adjusting the analog control system to compensate for the disparity in the various sub-systems. Prior to any experimentation tuning of the analog control system was performed, by the adjustment of variable resistors within the analog circuits, as per CAE specifications. The 300 series motion-base is relatively new and hence shortcomings in the tune-up procedure were discovered. In these situations the adjustments were made to provide for the flattest frequency response of the simulator.

5.2 Tuning and Calibration of the D/A's and the A/D's

The D/A convertors of the PE 3250 computer are provided with two adjustments, a zero offset, and a scale factor adjustment. These were tuned according to Perkin-Elmer procedure. The D/A convertors are used to produce the analog inputs, the actuators' acceleration and extension signals, to the NI cabinet. An additional calibration of the entire system for the position channels was performed. Actuator lengths were commanded and the actuator length was measured. A linear regression was performed on the data and the scale factors and offsets were found. The neutral position corresponded to actuator lengths of 2.327 m. The acceleration channels of the D/As were used with the nominal calibrated scale values. The results of the calibration are in Table 3. The A/D convertors were tuned according to Perkin-Elmer procedure. Testing showed the scaling to be very close to the nominal value. The zero offset was irrelevant due to the data processing technique employed (see Section 6).

5.3 Instrumentation

The instrumentation which provides the information regarding the motion of the simulator consists of three translational accelerometers, three angular rate-gyros, and six actuator extension transducers, one per actuator. The
Accelerometers chosen were Sundstrand Data Control non-pendulous, true translational accelerometers, two model 303 GA2s and one model 303 GA5. The three rate-gyros chosen were Honeywell GG440 Gnat miniature rate-gyros. The rate-gyros are each supplied with a 400 Hz demodulator which includes a 1st order low-pass filter with a break frequency of 75 Hz. The extension transducers are magnetostrictive position transducers built into the actuators as part of the analog control system. The output from the extension transducers is available at an output port on the N1 cabinet.

The accelerometers and rate-gyros are contained within an aluminium box, the unit referred to as the instrument package. Precision mounting blocks position the instruments within the package such that three rate-gyros and three accelerometers are mutually perpendicular. One rate-gyro and one accelerometer is aligned with each axis of \( F_p \). The model 303 GA5 accelerometer is mounted with the sensitive axis along the z-axis of \( F_p \).

The instruments within the package had been previously calibrated and the results are presented in Table 4 along with other pertinent information. Calibration of the six actuator extension transducers was performed by extending the actuators to measured lengths using the manual motion control on the N1 and recording the voltage at the output of the transducer circuits. A linear regression was performed on the data to provide the zero offsets and scale factors for each transducer circuit. The results are presented in Table 5, the offsets are again irrelevant and not shown.

### 5.4 Instrument Package Installation

Ideally the instrument package would be mounted at the origin of \( F_s \) about which the rotational maneuvers are performed. Unfortunately this was not possible due to the location of various components in the simulator. A convenient location where a solid mounting surface existed was thus chosen. A bracket was bolted to the motion-base frame underneath the cab as close to the origin of \( F_s \) as possible. The package bolted firmly to the bracket, and provided for fine adjustments of the attitude of the package so that it could be closely aligned with the simulator body-fixed frame.
The power supplies for the instrumentation package were installed on board the simulator. These power supplies provided 28 VDC for the accelerometers, 26 VAC 400 Hz for the rate-gyros, and 15 VDC for the demodulators. The power was run to the package via shielded cable and 36 pin D-connectors. All cabling for the instrumentation signals was done using shielded wire pairs. Great care was taken in the cabling of the instrumentation to ensure minimum noise pickup.

5.5 Anti-Aliasing Filters

When the digital Fourier transform of a finite length analog signal sampled at a frequency $f_s$ is performed, power at frequencies above the Nyquist frequency $f_s/2$, will be folded back to lower frequencies (Reference 7). This phenomenon called "aliasing" can render data useless, hence the employment of anti-aliasing filters. The anti-aliasing filters are low-pass filters which reduce the amplitude of the processed signals above the folding frequency to an insignificant amplitude.

The sampling rate was taken to be 100 Hz. The signals were sampled by 12 bit A/D converters which have a quantization level of 5 mV. Reducing the signals at frequencies higher than 50 Hz to below 5 mV will eliminate aliasing. Based on this information 4th order low-pass Butterworth filters with a break frequency of 30 Hz were chosen. The analog filters were designed as suggested in Reference 8, and a schematic is shown in Figure 5.01. The filters employed differential inputs and variable gains of 1, 2, 5, or 10. Testing of the filters was performed by inputting a single sinusoidal voltage and measuring the response using the Perkin-Elmer digital computer and FFT analysis. The response at various frequencies was compared to the theoretical frequency response

$$L(j\omega) = \frac{\omega_c^2}{-\omega^2 + j0.76537\omega_c\omega + \omega_c^2} \cdot \frac{\omega_c^2}{-\omega^2 + j1.84776\omega_c\omega + \omega_c^2} \quad (5.1)$$

and the results were in close agreement. For the measured frequency response see Figure 5.02.
5.6 Location of Origin of $F_S$

The motion of the origin of $F_S$ (point $P_A$) can be related to the motion at the instrument package location if the vectors from $P_A$ to each accelerometer are known. Determination of the vectors from $P_A$ to the accelerometers requires knowledge of the physical location of the point $P_A$. The point $P_A$ is in the plane of the upper bearings of the actuators and equidistant from the pivot points for each actuator. Location of the point in space based on the previous definition is difficult and due to the limitations of the available measuring equipment, less than precise.

An indirect method was employed to physically locate the point $P_A$. First an approximation to the location of point $P_A$ was found. At this point a plumb-bob was hung from an adjustable bracket. Exact angular displacements about the actual point $P_A$ (based on the knowledge of $A_{1S}$ and $B_{1I}$) were made using the analog controller on the NI and measuring the actuator lengths. Positive and negative pitch and yaw attitudes (calculated as for the geometry check) were generated and the movement of the plumb-bob along each axis of the inertial frame was recorded. Consider the plumb-bob to be hung from $(X_1', Y_1', Z_1')$ in $F_I$ for the current approximation and the simulator at the neutral position. With the base yawed by the same amount, $\phi$ positively and negatively the resulting positions of the plumb-bob suspension point in the inertial frame are $(X_2', Y_2', Z_2)$ and $(X_1', Y_1', Z_1)$ respectively. The new approximation to the point $P_A$ $(X_0, Y_0, Z_0)$ is found from (see Figure 5.03)

\[
X_1 - X_0 = X_2 \cos 2\phi - X_0 \cos 2\phi - Y_2 \sin 2\phi + Y_0 \sin 2\phi \quad (5.1)
\]

\[
Y_1 - Y_0 = Y_2 \cos 2\phi - Y_0 \cos 2\phi + X_2 \sin 2\phi - X_0 \sin 2\phi \quad (5.2)
\]

\[
Z_0 = Z' \quad (5.3)
\]

and a similar procedure was applied for the pitch maneuvers which results in new estimates for the $Z$ and $X$ position. Only the two maneuvers were thus required to find the new location $(X, Y, Z)$ of the point $P_A$.

The procedure is repeated until no further improvement in the location of
point $P_A$ is detected. The final estimate to the point $P_A$ was found to move less than 1/2 mm in any direction for angular displacements of approximately 30°. This location was then fixed in space by referencing the point to objects on the simulator.

The vectors from the accelerometers to the point $P_A$ could then be measured. Measurements were carried out using a tape measure, and a set of calipers. The distances were resolved along the simulator's body-fixed reference frame. The results are shown in Table 6.

6 DATA PROCESSING

After the data had been collected and was in RAM, preliminary data reduction was performed prior to permanent storage on disk. Correction for drifting D.C. offsets of the instruments was performed and the data was scaled as required. The D.C. offset drift correction was performed by sampling the instruments 40 times at 10 ms intervals at the start and end of each run with the simulator in the neutral position. This data was then averaged to produce an offset at the start and end of the run for all channels sampled. A linear ramp was then fitted to the two offsets for each instrument and was subtracted from the sampled data. In general the ramp was found to have a very modest slope.

The next step in data processing calculated the motion of the point $P_A$ from the data collected by the instrument package. The simulator is considered to be a rigid-body and hence the angular displacements, angular velocities, and angular accelerations are identical at any point on the simulator. The accelerometers measure specific force, the combined effect of local acceleration and the gravity vector. The acceleration at the point $P_A$, $a_0$ is found from:

$$a_0 = f_0 + g \quad (6.1)$$

From Equation 6.1 and Reference 6 the components of acceleration expressed in $F_S$ are:
\[
\begin{align*}
\dot{u}_0 &= f_0^x - D_{xx}(q_0^2 + r_0^2) + D_{xy}(p_0q_0 - r_0) + D_{xz}(p_0r_0 + q_0) - g \sin \theta_0 \tag{6.2a} \\
\dot{v}_0 &= f_0^y + D_{yx}(p_0q_0 + r_0) - D_{yy}(p_0^2 + r_0^2) + D_{yz}(q_0r_0 - p_0^2) + g \sin \phi_0 \cos \theta_0 \\
\dot{w}_0 &= f_0^z + D_{zx}(p_0r_0 - q_0) + D_{yz}(q_0r_0 + p_0^2) - D_{zz}(p_0^2 + q_0^2) + g \cos \phi_0 \cos \theta_0 \tag{6.2b} \\
\end{align*}
\]

where:

\[
\mathbf{a}_0 = [\dot{u}_0 \ \dot{v}_0 \ \dot{w}_0]^T \tag{6.3}
\]

and where \(D_{xy}\) is the y component expressed in \(F_S\) of the vector from the point \(P_A\) to the x-accelerometer, and similarly for the other distances.

Prior to solution of the previous equations, compensation for the effects of the anti-aliasing filters, the rate-gyro dynamic response, and the 1st order filters in the demodulators is needed. The dynamic responses of the accelerometers and position transducers are flat with no phase lag at the frequencies of interest and thus no compensation for their response is required.

(i) Rate-Gyro Dynamics Compensation

The rate-gyro dynamics are equivalent to a 2nd order low-pass system with damping ratio, \(\zeta = 0.5\) and a natural frequency \(\omega_c\) of 251 r/s. The frequency response function for a 2nd order low-pass system is

\[
L(j\omega) = \frac{\omega_c^2}{-\omega^2 + j 2 \zeta \omega \omega_c + \omega_c^2} \tag{6.4}
\]

Therefore compensation of the rate-gyro response is attained by applying the inverse of Equation 6.4 to the signals in the frequency domain. Prior to this correction the signals are transformed into the frequency domain using the FFT algorithm (see Appendix C).
(ii) Anti-Aliasing Filter Compensation

The anti-aliasing filters have a frequency response function defined by Equation 5.1. Adjustment for the filters is thus accomplished by multiplying the data in the frequency domain by the inverse of this equation.

(iii) 1st Order Low-Pass Filter Compensation

The 1st order low-pass filter in the rate-gyro demodulators has a frequency response function defined by

\[ L(j\omega) = \frac{2\pi f_c}{2\pi f_c + j\omega} \] (6.5)

where \( f_c \) is the natural frequency of the filter (75 Hz). Applying the inverse of this equation to the data in the frequency domain will thus compensate for these effects.

After compensation for all of the previous effects in the frequency domain the data is then transformed back into the time domain using the inverse FFT routine as shown in Appendix C. Accurate estimates of the specific force, angular velocity, and the six actuator extensions are now available. To solve Equations 6.2 the time histories of the angular accelerations \( \dot{\omega}_0 \) and the Euler angles \( \phi_0 \) and \( \Theta_0 \) are required.

To obtain the angular accelerations the compensated angular rates for pitch and roll are smoothed using a digitally implemented 4th order low-pass smoother with zero phase lag and a break frequency of 30 Hz (see Appendix B). The resulting smoothed signals are then differentiated using the central differences method as follows

\[ \dot{x}(t) = \frac{[x(t+\Delta t)-x(t-\Delta t)]}{2\Delta t} \] (6.6)
see Reference 9 for details. The pre-filtering is necessary to reduce noise since differentiating will amplify the noise by a factor proportional to its frequency. During testing the yaw rate-gyro was found to be excessively noisy in the frequency band above 20 Hz and thus a more severe smoother was required to produce acceptable results. An 8th order low-pass smoother with zero phase lag and a break frequency of 20 Hz (see Appendix B) was implemented for the yaw channel.

The actuator extension signals were used to find the Euler angles $\theta_0$ and $\phi_0$ because numerical integration of Equations 4.5 using the recorded rate-gyro signals resulted in an unacceptable amount of drift. The actuator length signals were not filtered prior to sampling and thus the digital 4th order smoothing algorithm mentioned above was used to reduce the noise on the signals. The program PLATATT described in Appendix A was configured as a subroutine and solved for the Euler angles given the smoothed actuator signals. Since the noise on the actuator signals was quite small negligible aliasing occurred even though anti-aliasing filters were not employed.

Figure 6.01 is a schematic of the data processing leading to motion results for the point $P_A$.

7 ANALYSIS AND RESULTS

For the purpose of this study the motion-base is considered to be an "almost" linear system. Given a sinusoidal input the motion-base will have a transient response which decays exponentially with time and a steady-state response. The steady-state response will be primarily a sinusoid at the input frequency (termed the fundamental) but with a different amplitude and phase. In addition to this some steady-state response of the base will occur at harmonics of the input frequency. Background broadband noise will also be present within the system.

7.1 Discussion of Errors

There are various sources of error involved in the experimentation. The instrumentation scale factors errors are on the order of 1%. The
instrumentation package alignment with the simulator body-fixed frame is accurate to ± 1/2°. The anti-aliasing filters were designed to have less than an error of ± 1/2% in the amplitude response. Location of the point \( P_A \) and measurement of the matrices \( \tilde{A}_S \) and \( \tilde{B}_I \) are accurate to approximately ± 1 mm. The motion-base response will change with time and over the period of experimentation some change is expected. Environmental effects such as temperature and humidity will also affect the motion-base response.

7.2 Describing Functions

If a stable linear time-invariant system is driven with a sinusoidal input and starting transients are allowed to die out, the output will be a steady-state sinusoid at the same frequency as the input sinusoid but generally with a different amplitude and phase shifted with respect to the input. The relationship between the input and output of the system defines the frequency response function. More precisely the frequency response function of a linear system is given by (Reference 6)

\[
L(j\omega) = \frac{Z(\omega)}{X(\omega)}
\]

see Figure 7.01. \( X(\omega) \) and \( Z(\omega) \) are the Fourier transforms of the input signal \( x(t) \) and the output signal \( z(t) \) (for details of the Fourier transform see Appendix C). Now consider a non-linear system with input \( x(t) \) and output \( y(t) \). From Figure 7.01 the linear system which most precisely describes the non-linear system will have \( z(t) \) as close to \( y(t) \) as possible. Defining \( z(t) - y(t) \) as the remnant \( R(t) \), then to minimize \( R(t) \) Reference 10 shows that one should take

\[
L(j\omega) = \frac{\phi_{xy}(j\omega)}{\phi_{xx}(j\omega)}
\]

(7.2)

where the power spectral density \( \phi_{xx} \) of a signal \( x(t) \) is (Reference 13)
\[
\phi_{xx}(j\omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} R_{xx}(\tau)e^{-j\omega \tau} d\tau
\]  

(7.3)

and the cross spectral density \( \phi_{xy} \)

\[
\phi_{xy}(j\omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} R_{xy}(\tau)e^{-j\omega \tau} d\tau
\]  

(7.4)

where

\[
R_{uv}(\tau) = \lim_{T_0^{+} \rightarrow T_0} \frac{1}{T_0} \int_{0}^{T_0} u(t) v(t + \tau) d\tau
\]  

(7.5)

Then from the definition of the Fourier transform of \( x(t) \) it can be shown that:

\[
\phi_{xx}(j\omega) = \frac{2\pi}{T_0} X^*(\omega) X(\omega)
\]  

(7.6)

\[
\phi_{xy}(j\omega) = \frac{2\pi}{T_0} X^*(\omega) Y(\omega)
\]  

(7.7)

where \( (\ )^* \) denotes the complex conjugate.

The previous mathematical development assumed that an infinite time history of analog signals was available. Since a digital computer will be used in the scope of this report only a finite analog signal sampled at equal time intervals is available. Fortunately if the sampling rate is fast enough and the length of the recording long enough then the equation

\[
\phi_{xy}(j\omega) = \frac{2\pi}{T_0} \text{DFT}^*(x(t),\omega) \text{DFT}(y(t),\omega)
\]  

(7.8)

provides a very good estimate to the power spectrum, as shown in Reference 10. The DFT is the digital Fourier transform, the Fourier transform of a finite sampled time record (see Appendix C). For the results presented in this report the one-sided power spectrum \( \Phi_{xx} \) is employed.
\[ \Phi_{xx} = 2 \Phi_{xx} \quad (7.9) \]

and the results span from D.C. to \( f_s/2 \) instead of \(-f_s/2\) to \( f_s/2 \) (where \( f_s \) is the frequency in Hz at which the data is sampled).

Consider the motion-base to be driven in one pure mode, for example surge. Then the describing functions to be found are (where \( (\_)_0 \) represents the motion-base response and \( (\_)_I \) represents the command to the motion-base):

\[
\begin{align*}
\ddot{U}_0(j\omega) &= \frac{\Phi_{I_0}U_0(j\omega)}{\Phi_{I_1}U_1(j\omega)} \quad \text{Primary} \quad (7.10) \\
\ddot{V}_0(j\omega) &= \frac{\Phi_{I_0}V_0(j\omega)}{\Phi_{I_1}U_1(j\omega)} \quad \text{Crosstalk} (7.11) \\
\ddot{P}_0(j\omega) &= \frac{\Phi_{I_0}P_0(j\omega)}{\Phi_{I_1}U_1(j\omega)} \quad \text{Crosstalk} (7.12) \\
\ddot{Q}_0(j\omega) &= \frac{\Phi_{I_0}Q_0(j\omega)}{\Phi_{I_1}U_1(j\omega)} \quad \text{Crosstalk} (7.13) \\
\ddot{R}_0(j\omega) &= \frac{\Phi_{I_0}R_0(j\omega)}{\Phi_{I_1}U_1(j\omega)} \quad \text{Crosstalk} (7.14) \\
\end{align*}
\]
For each degree-of-freedom six describing functions can be found, one primary and five crosstalks. For both the translational and rotational modes the describing functions were found using acceleration as the metric.

The describing functions for each degree-of-freedom were found for a range of frequencies from approximately 0.1 Hz to 15 Hz. The tests were conducted with a sum of sinusoidal inputs. The sinewave frequencies for the tests were chosen such that none were harmonics of another, eliminating the possible contamination of one frequency by another through harmonics. The amplitudes were chosen to keep the motion-base below approximately 10% of the system limits in position, velocity, and acceleration. The describing function tests were conducted with a sampling rate of 100 Hz, ($\Delta t = 10\text{ms}$) and each test produced 2000 samples of the motion output. The frequency resolution was

$$\Delta f = \frac{1}{N\Delta t}$$

(7.16)

where $N$ is the number of samples and the frequencies which can be resolved are described by

$$f_i = \frac{(i - 1)}{N\Delta t}$$

(7.17)

where $i$ is an integer. Frequencies from .05 Hz to 50 Hz in steps of .05 Hz can be resolved. For each degree-of-freedom three separate runs with different frequency content were carried out. The amplitudes and frequencies for the tests are listed in Table 7.

Measurements of the full set of describing functions were carried out for the simulator under normal operating conditions. Also a set of primary describing functions was determined for the additional operating condition mentioned in Section 2.2. For heave, under normal operating conditions, two additional primary describing functions were derived. These tests were run with the input amplitudes (both position and acceleration) increased to 60% above the nominal test values and reduced 60% below the nominal values. The
results, shown in Figures 7.02 and 7.03 demonstrate the modest degree of non-linearity of the system with respect to varying input amplitudes.

The results are presented in the form of Bodé plots (amplitude ratios and phase angles versus frequency). The phase angle is multi-valued as it results from an inverse tangent, and thus has an infinite number of solutions according to

\[ \alpha_a = k (360) + \alpha_p \quad k = \ldots -2, -1, 0, 1, 2, \ldots \]  

(7.18)

where \( 360^\circ > \alpha_p > 0^\circ \)

Physical systems generally have increasing phase lag with increasing frequency. The phase angle at the lowest measurement frequency was chosen to be between 0° and -360°. The amplitude ratios are presented in dBs defined as

\[ \text{dB} = 20 \log_{10}(\text{Amplitude Ratio}) \]

The results are presented in Figures 7.04 to 7.45.

The heave primary describing functions in Figures 7.02 and 7.03 determined using the two different input amplitudes are very similar. Below 63 r/s, the roll-off frequency of the input elliptic filters, a maximum difference of 3/4 dB occurs between the two amplitude ratio plots. At the highest frequency, almost 100 r/s, a 1.5 dB discrepancy in the amplitude ratios occurs, but this is in a region where the ratio is quite small. Since this is in the region where the elliptic filters roll-off very sharply large changes in the response can be caused by small changes in the input. The phase angle results for the two figures are in close agreement over the bandwidth tested. The motion-base response appears to be relatively linear with respect to varying amplitude inputs; at least within the range tested.

The surge primary describing function, Figure 7.04, has a relatively flat amplitude response. The amplitude ratio is about -2 dB at 0.63 r/s, reaches a maximum of approximately 1 dB at 3 r/s and remains moderately flat out to 63 r/s. Above 63 r/s the response drops off sharply due to the sharp
roll-off rate of the input elliptic filters which occurs at 63 r/s. The
droop in the amplitude response from 3 r/s to 63 r/s is a product of the
tuning of the motion-base. In order to reduce the gain on the heave channel
above 10 r/s the response on the surge and sway channels suffered a similar
drop in their amplitude response. A compromise was made between eliminating
the peak in the heave response and the resulting drop in the response of the
sway and surge channels. At the low frequency end of the tested bandwidth
the drop off in the amplitude response of the motion-base is due to some
imperfection in the control system. The amplitude ratio is known to be 0 dB
at D.C., however, so the curve should start to rise as the frequency
approaches zero. The phase angle response is well behaved starting at zero,
rising slightly (due to lead compensation in the NI analog controller) and
reaching an extreme value of -410° at 100 r/s.

The sway primary describing function, Figure 7.11 is similar to the surge
result. The sway primary describing function is only about 2 dB down before
the roll-off at 63 r/s due to the elliptic filters. Again as with the surge
results the amplitude drops off slightly at the low frequency end of the
spectrum and drops off very sharply above 63 r/s. The phase angle results
are almost identical to that of surge.

The heave primary describing function, Figure 7.18, is unique due to the
peak in the amplitude response at the high frequency end of the tested
bandwidth. A moderate electro-mechanical resonance is excited in heave and
is prevalent from 25 r/s to 60 r/s. It appears the peak in the amplitude
response occurs at 60 r/s, however, as will be shown later in this section
the amplitude response drops off after 60 r/s due to the elliptic input
filters and not because the resonance has been passed. The phase angle
results are again very similar to the previously mentioned describing
functions.

The roll primary describing function, Figure 7.25 has a rise in the
amplitude response as the frequency increases above 40 r/s. The resonance is
milder than that in heave. This response may be caused by the excitation of
the previously mentioned electro-mechanical resonance and by the fact that
the control system relies more heavily on force feedback from the actuator
force transducers as the frequency increases. The control system does not
account for the mass distribution of the cab about the primary axes and thus
the force on any actuator will vary depending on the motion and will in turn
affect the motion. The phase angle result has a strange kink in it at the
highest frequency, it may be that the signal is actually another full period
out of phase making it -720°. This is much larger than the translational
results, however, and seems unlikely. The other two angular modes, pitch and
yaw are very similar to the roll results, see Figures 7.32 and 7.39. Differences in the results are probably due to differences in the moments of
inertia of the cab about which the rotations are occurring.

The translational crosstalk describing functions with the base driven in a
translational mode (Figures 7.05, 7.06, 7.10, 7.12, 7.16 and 7.17) have
amplitude responses of approximately -40dB at 0.63 r/s. They tend to remain
relatively flat out to 20 r/s and then in general rise as the frequency
increases, possibly due to the force feedback problem and the
electro-mechanical resonance. The peak rises up to -18 dB in some cases.
Some scattering of the data points occurs because the amplitudes of the
signals are nearing the resolution limits of the instrumentation. The phase
angles generally are already out of phase with the driven channel at 0.63
r/s and decrease sharply with frequency.

The angular crosstalks with the base being driven in an angular mode
(Figures 7.07, 7.08, 7.09, 7.13, 7.14, 7.15, 7.19, 7.20,
and 7.21) also have amplitude responses of -40dB at 0.63 r/s. A gradual
increase in amplitude ratio generally occurs with frequency, probably due to
the fading in of force as the primary feedback in the control loop. The
describing functions rise as high as -10 dB at 63 r/s. This is somewhat
misleading, however, as angular acceleration in units of r/s² is being
compared to translational acceleration in units of m/s².

The angular crosstalks with the motion-base driven in an angular mode
(Figures 7.26, 7.27, 7.31, 7.33, 7.37, and 7.38) generally have an amplitude
response of -40 dB at 0.63 r/s. A rise occurs on some channels as the
frequency increases and leads to approximately -15 dB at 63 r/s. As
mentioned previously the force feedback in the control loop may account for this effect. The amplitude ratios drop off sharply after 63 r/s due to the elliptic input filters.

The translational crosstalks with the base driven in an angular mode (Figures 7.22, 7.23, 7.24, 7.28, 7.29, 7.30, 7.34, 7.35, and 7.36) have an amplitude ratio of approximately -25 dB at 0.63 r/s. The crosstalk results on these channels seem to remain flat out to 63 r/s and then roll-off. These results seem quite high, however, the different units of translational acceleration and the angular acceleration may account for the higher values than the previous results.

Removal of the elliptic input filters created drastic changes in the primary describing functions. As can be seen from Figures 7.40 to 7.45 resonances occur in all degrees-of-freedom. The peaks of the resonances were at approximately 110 r/s except for roll where it occurred at 70 r/s. From these figures it is obvious that the electro-mechanical resonances noticed in the nominal results were being rolled-off by the elliptic filters. The peak rises to a maximum of 13 dB in the heave mode. Clearly the resonance affects the heave channel most drastically. On some channels there appear to be frequencies where anti-resonances occur. In roll for example a 3 dB drop is apparent at 80 r/s. The phase angle results were affected over almost the entire bandwidth tested. At 100 r/s the phase lag is 400° less than for the results with the elliptic filters in. The elliptic filters obviously account for a lot of the phase lag (hence time delay) in the motion-base response.

When the output channels of the D/As were scaled as mentioned in Section 5 this ensured a D.C. response of 0 dB.

7.3 1/2 Hz Noise Level Tests

The next set of tests determined noise levels with the motion-base driven sinusoidally at 1/2 Hz ($\omega_f = 3.14$ r/s) at a range of amplitudes, as suggested in Reference 4. The base was driven in each degree-of-freedom individually, and the noise levels were calculated.
If the time history of \( x(t) \) is available and the digital Fourier transform is \( \text{DFT}(x(t), \omega) \) then the variance of the signal components in the frequency band \( N_1 \Delta \omega \) to \( N_2 \Delta \omega \) is given by

\[
\sigma_x^2 = \sum_{n=N_1}^{N_2} \frac{4\pi}{T_0} \text{DFT}^\ast(x(t), n\Delta \omega) \text{DFT}(x(t), n\Delta \omega) \Delta \omega \quad (7.19)
\]

where \( T_0 \) has been chosen to be 20 s and thus from the FFT algorithm \( \Delta \omega = (0.05 \times 2\pi) \text{r/s} \) for the calculation process. See Appendix C for a derivation of Equation 7.19. The total noise is defined as,

\[
\sigma_{x_{\text{total}}}^2 = \sum_{n=1}^{N/2} \frac{4\pi}{T_0} \text{DFT}^\ast(x(t), n\Delta \omega) \text{DFT}(x(t), n\Delta \omega) \Delta \omega - \sigma_{x_f}^2 \quad (7.20)
\]

Note: \( \omega = (N/2) \Delta \omega \) is the highest frequency which the DFT algorithm can generate.

The variance of the fundamental (at frequency \( \omega_f \)) is given by

\[
\sigma_{x_f}^2 = \frac{4\pi}{T_0} \text{DFT}^\ast(x(t), \omega_f) \text{DFT}(x(t), \omega_f) \Delta \omega \quad (7.21)
\]

The low frequency non-linearity is defined as the sum of the first two harmonics of the fundamental. The low frequency non-linearity variance is then (Reference 4)

\[
\sigma_{x_{1lfnl}}^2 = \sum_{n=2}^{3} \frac{4\pi}{T_0} \text{DFT}^\ast(x(t), n\omega_f) \text{DFT}(x(t), n\omega_f) \Delta \omega \quad (7.22)
\]

The variance of the high frequency non-linearity is defined as the sum of the fourth and higher harmonics of the fundamental.

\[
\sigma_{x_{hfnl}}^2 = \sum_{n=4}^{N_2} \frac{4\pi}{T_0} \text{DFT}^\ast(x(t), n\omega_f) \text{DFT}(x(t), n\omega_f) \Delta \omega \quad (7.23)
\]

where \( N_2 \) in this case is the integer defined by
\[ N_2 = (N/2) \Delta \omega / \omega_f \] (7.24)

Note that \( T_0 \) has been chosen such that \( \Delta \omega \) is a factor of \( \omega_f \). The variance of the roughness is defined as

\[ \sigma_{x_{fr}}^2 = \sigma_{x_{total}}^2 - \sigma_{x_{lfnl}}^2 \] (7.24)

The results presented in the figures (except peak noise) are based on the standard deviation \( \sigma_x \), the square root of the variance. For the driven channel the noise results are non-dimensionalized with respect to the standard deviation of the base response at the fundamental frequency. The parasitic noise is left in dimensional form as normalization with respect to the response on the driven channel is inappropriate.

The results (for \( \omega_f = 3.14 \text{ r/s} \)) are presented in the form of graphs, with the noise plotted against the amplitude of the response fundamental \( \sqrt{2} \sigma_{xf} \), in Figures 7.46 to 7.135. Note that on the graphs the standard deviation of the noise is represented by \( \sigma_N \) the peak noise by \( P_N \) and the standard deviation of the response fundamental by \( \sigma_{xf} \).

The surge normalized peak noise and total noise results (Figures 7.46 and 7.47) have an inverted bell shape when plotted against the fundamental response acceleration. At the low fundamental response accelerations the broadband background noise becomes the main source of noise, and hence normalization with respect to the fundamental response acceleration will cause the peak and total noise results to increase asymptotically as the response signal is reduced. The rise in the signal-to-noise ratio at the higher accelerations results because the motion-base is being driven beyond the range where it acts linearly. The previous statement is confirmed by the low-frequency non-linearity result (Figure 7.49). The high-frequency and roughness results, Figures 7.48 and 7.50 further confirm the previous hypothesis. Since the background noise is broadband the behavior of the roughness and high-frequency non-linearity results at the lower response
fundamental accelerations is similar to that of the total noise and peak noise results. Since the mechanism which determines the shape of the plots at the higher accelerations depends more heavily on the low-frequency non-linearity the gradient of the total noise result at the higher accelerations is steeper than that of the roughness and high-frequency non-linearity results. The power spectral densities of surge acceleration for the second smallest and second largest input amplitudes, Figures 7.202 and 7.203 respectively further confirm this hypothesis. Figure 7.203 demonstrates the large amount of power contained at harmonic frequencies of the fundamental.

The noise results on the driven channel for the other two translational modes are very similar to those of surge, see Figures 7.63 to 7.67 and 7.80 to 7.84. The heave channel tends to be more noisy than the others possibly due to excitation of the large heave resonance.

The peak and total noise results on the non-driven translational channels for all of the driven modes have the same general shape. The slope of the plots increase with the response fundamental amplitude and at the low amplitude end of the plots the noise levels out towards the amplitude of the background noise. Again the heave channel tends to be somewhat more noisy than the surge and sway channels.

The angular noise results on the driven channel are very similar in shape to the translational noise driven channel results. The magnitude of the signal-to-noise results are expected to be different from the translational results since the measurements are based on a different metric.

The peak and total noise results on the non-driven channel for all of the other driven modes have the same shape as the translational non-driven channel results. Again the magnitude of the signal-to-noise results are different from the translational results as previously explained.

The peak noise plots have a greater variability than the other results as they have a strong dependence on the stochastic nature of the noise and small bursts of noise can drastically affect the results.
7.4 Signal-to-Noise Contours

Being a physical system the motion-base has a normal operating range, and motion beyond this range will result in undesirable performance (Reference 12). At a first glance it seems that definite limits should exist. Consider the case with sinusoidal system commands. At low frequencies the length of the actuator extensions will limit the simulator response. At mid-frequencies a velocity limit should be incurred since the pumps can only circulate the oil at a limited velocity. At the high frequency end an acceleration limit should exist resulting from the limited pressure which the hydraulic system can achieve. With the exception of the position limits it was found that the above limits were not well defined and hence proved difficult to determine. The following alternative approach was employed.

The alternate approach was to determine signal-to-noise contour lines for operation in each degree-of-freedom. Given that the user wants to keep the signal-to-noise ratio above a certain limit an operating range bounded by a signal-to-noise contour will define the area of operation.

Sinusoidal inputs were used to excite the motion-base. A grid pattern of operating points was chosen for each degree-of-freedom (see Table 8). At low frequencies the position limit provided the upper amplitude boundary for testing. At higher frequencies the possibility of impending damage to the simulator equipment mounted on the motion-base provided the boundary on amplitude.

The position limit was determined for motion about point \( P_A \) for each degree-of-freedom. Using geometrical considerations (Equations 4.4 to 4.8) and restricting the maximum actuator extension from the neutral length to \( \pm 0.404 \) m the maximum displacement of the simulator in each degree-of-freedom was determined. The maximum actuator extension was chosen to be the average extension which tripped the position limit warning on the N1 cabinet. The acceleration limits chosen were \( 15 \) m/s\(^2\) for the translational motion and \( 400^\circ /s^2\) for the angular modes. Operation above these values was judged to be detrimental to the equipment.
For each measurement point the signal-to-noise ratio for the driven channel was determined using

\[ S/N = \frac{\alpha_{xf}}{\sigma_{x_{total}}} \]  

(7.25)

DI-3000 plotting software was used to generate the contour lines. First a finely spaced grid of data, estimated using triangulation followed by bivariate interpolation with a fifth-degree polynomial, is fitted to the plots. The contours are then linearly interpolated from the resulting gridded data and a cubic-spline with tension (Reference 13) is used to draw smooth contour lines. The results are presented on graphs, Figures 7.136 to 7.141, with the fundamental response velocity along the y-axis and the frequency of the command along the x-axis.

The three translational signal-to-noise contour plots Figures 7.136 to 7.138 have very similar shapes. The lower boundary of the contours follow lines of constant acceleration. The signal-to-noise ratio always decreases as the operation point approaches one of the motion-base's operational limits. Thus the area of operation where the highest signal-to-noise ratio occurs is in the middle of the motion-base's operating range. The heave signal-to-noise contour plot Figure 7.138 has a lower signal-to-noise ratio at any point than the sway and surge results possibly due to the large resonance in heave. Pinching in of the contours seems to occur at 5 Hz and may be due to the fact that 15 Hz (approximately the resonant peak) is a lower order harmonic of the 5 Hz driving signal and hence some excitation of the resonance may be occurring.

The three rotational signal-to-noise contour plots, Figures 7.139 to 7.141 are similar to each other but quite different from the translational contour plots. This is because the rotational signal-to-noise ratios are based on angular velocity where as the translational signal-to-noise ratios are based on translational acceleration.
7.4 Signal-to-Noise Contours

Being a physical system the motion-base has a normal operating range, and motion beyond this range will result in undesirable performance (Reference 12). At a first glance it seems that definite limits should exist. Consider the case with sinusoidal system commands. At low frequencies the length of the actuator extensions will limit the simulator response. At mid-frequencies a velocity limit should be incurred since the pumps can only circulate the oil at a limited velocity. At the high frequency end an acceleration limit should exist resulting from the limited pressure which the hydraulic system can achieve. With the exception of the position limits it was found that the above limits were not well defined and hence proved difficult to determine. The following alternative approach was employed.

The alternate approach was to determine signal-to-noise contour lines for operation in each degree-of-freedom. Given that the user wants to keep the signal-to-noise ratio above a certain limit an operating range bounded by a signal-to-noise contour will define the area of operation.

Sinusoidal inputs were used to excite the motion-base. A grid pattern of operating points was chosen for each degree-of-freedom (see Table 8). At low frequencies the position limit provided the upper amplitude boundary for testing. At higher frequencies the possibility of impending damage to the simulator equipment mounted on the motion-base provided the boundary on amplitude.

The position limit was determined for motion about point $P_A$ for each degree-of-freedom. Using geometrical considerations (Equations 4.4 to 4.8) and restricting the maximum actuator extension from the neutral length to ±0.404 m the maximum displacement of the simulator in each degree-of-freedom was determined. The maximum actuator extension was chosen to be the average extension which tripped the position limit warning on the NI cabinet. The acceleration limits chosen were 15 m/s$^2$ for the translational motion and 400°/s$^2$ for the angular modes. Operation above these values was judged to be detrimental to the equipment.
For each measurement point the signal-to-noise ratio for the driven channel was determined using

\[
S/N = \frac{\alpha_{X_f}}{\sigma_{X_{\text{total}}}}
\]  

(7.25)

DI-3000 plotting software was used to generate the contour lines. First a finely spaced grid of data, estimated using triangulation followed by bivariate interpolation with a fifth-degree polynomial, is fitted to the plots. The contours are then linearly interpolated from the resulting gridded data and a cubic-spline with tension (Reference 13) is used to draw smooth contour lines. The results are presented on graphs, Figures 7.136 to 7.141, with the fundamental response velocity along the y-axis and the frequency of the command along the x-axis.

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7.5 Hysteresis Testing

Reference 4 suggests that the motion-base be driven with a low frequency sinusoidal input \( \omega_f < 0.06283 \text{ r/s} \) and from the recorded position of the motion-base a plot of the displacement error versus commanded displacement will describe the hysteresis. The frequency of the sinusoid should be low enough to exclude any dynamic effects of the motion system.

The tests were performed for each degree-of-freedom. The six actuator lengths were recorded and PLATATT was used to calculate the simulator's angular or translational displacement. The tests were performed at \( \omega_f = 0.06283 \text{ r/s} \) and the resulting plots of displacement error versus commanded displacement produced ellipses, as shown in Figures 7.142 to 7.147.

Consider a case where the input is \( A_0 \sin \omega_f t \). If the displacement of the base were to simply lag the commanded displacement by \( \alpha \) then the error at any time \( t \) would be

\[
\text{err}(t) = A_0 \sin \omega_f t - A_0 \sin(\omega_f t - \alpha) \tag{7.26}
\]

or

\[
\text{err}(t) = A_0 [\sin \omega_f t - (\sin \omega_f t \cos(\alpha) - \cos \omega_f t \sin(\alpha))] \tag{7.27}
\]

The hysteresis testing is at very low frequencies and thus the phase lag of the system \( \alpha \) will be small. Applying the small angle approximations

\[
\text{err}(t) = A_0 \alpha \cos \omega_f t \tag{7.28}
\]

A plot of this \( \text{err}(t) \) versus the sinusoidal input will produce an ellipse as was seen in the figures. Thus it appears that the frequency used to perform the hysteresis tests was not low enough to exclude the dynamics of the base. The experiment was repeated at lower frequencies and ellipses still resulted (Figs. 7.148 to 7.165). The frequency which is low enough such that hysteresis is the dominant effect has a very long period and is impractical to run. Thus for each degree-of-freedom a few different frequencies were run, the maximum displacement error (the apparent hysteresis) was measured
for each run, and the results extrapolated back to zero frequency. The displacement error at zero frequency gives the estimated magnitude of the hysteresis of the system.

Considering the error at low frequency to be made up of the sum of phase lag and hysteresis, the error is then

$$\text{err}(t) = A_0 \alpha \cos \omega_o t + h(t)$$  \hspace{1cm} (7.29)

A plot of hysteresis, \(h(t)\), versus time and versus the input for a typical hysteresis loop (using a sinusoidal input) is shown in Figure 7.166. The maximum error is then

$$\text{err}_{\text{max}} = A_0 \alpha + h_0$$  \hspace{1cm} (7.30)

since the phase lag and hysteresis will act in the same direction.

Considering the motion-base as a second order system its frequency response function is

$$L(j\omega) = \frac{\omega_c^2}{\omega_c^2 + j2\zeta \omega \omega_c - \omega^2}$$  \hspace{1cm} (7.31)

and for \(\omega \ll \omega_c\) the phase lag \(\alpha\) is

$$\alpha = \tan^{-1} \left( \frac{2\zeta \omega}{\omega_c} \right)$$  \hspace{1cm} (7.32)

Applying the small angle approximation

$$\alpha = \frac{2\zeta \omega}{\omega_c}$$  \hspace{1cm} (7.33)

Therefore the maximum error is (at frequency \(\omega\)):

$$\text{err}_{\text{max}} = 2A_0 \zeta \omega / \omega_c + h_0$$  \hspace{1cm} (7.34)
and a plot of maximum error versus $\omega$ will result in a straight line of slope $2A_0/c/w$ and a value of $h_0$ when $\omega$ is zero. A straight line fit, using the least-squares technique, was used to extrapolate the results back to D.C. (see Figures 7.167 to 7.172). The resulting magnitudes of estimated hysteresis are shown in Table 9.

As seen in some of the figures spikes appear in some of the hysteresis plots. These spikes appear to correspond to power fluctuations in the power supplies for the analog controller of the motion-base. Fortunately the spikes are very small corresponding to less than 1mm on the translational channels and 0.001 radians on the rotational channels.

Figures 7.167 to 7.172 demonstrate the validity of the straight line fit and the mathematical development which accompanied it. The estimate to the hysteresis is simply the value of the apparent hysteresis at the $y$-axis crossing. The hysteresis is very small. This is expected as the motion-base is essentially a position following device at low frequencies.

### 7.6 Dynamic Threshold Testing

The last tests performed were the dynamic threshold tests. As suggested in Reference 4 the drive signals for the motion-base for this test are derived from an acceleration step input. Prior to the step used for measurement the neutral position is reached via a pre-test square wave designed to reduce system backlash effects (see Figure 4.03). The response of the base is recorded and the threshold is defined as the time to reach 63\% of the commanded acceleration step. The threshold is subdivided into dead-time and rise-time. Dead-time is the time between the step input and the time at which some response of the motion-base is noticed. The rise-time is then the remaining time to the threshold.

The base was driven in heave for these tests. The x-accelerometer was remounted to line up with the z-axis of $F_S$ as it was more sensitive than the original z-accelerometer. The recorded signals were passed through a fourth order low-pass Butterworth filter with a break frequency of 80 Hz. The data was recorded using a Nicolet 3091 digital scope sampling at 20,480 Hz. A hard copy was produced off-line using a Hewlett-Packard 7046B X-Y plotter.
connected to the scope. From the resulting graphs (Figs. 7.173 to 7.180) the threshold could be measured and scaled accordingly. The test was repeated for various amplitudes of acceleration as well as for positive going and negative going steps. The scope was triggered using a digital output pulse from the Perkin-Elmer computer. The results are presented in Table 10.

The Butterworth filter was employed to reduce the noise on the signal and make the results more legible. The dynamics of the filter unfortunately affect the results. By performing a high amplitude test without the Butterworth filter in their effect was determined. From Figure 7.181 it is seen that an additional 5 ms of delay are added by the filter for the case shown. In general the filter will add on approximately 5 ms to the threshold or about 10% of the value.

As can be seen from the Figures 7.173 to 7.180 the response of the motion-base to the acceleration step input has a band of dead-time which is due to the lags within the various sub-systems which comprise the motion system. The system then acts like an underdamped second order system, overshooting the commanded acceleration and approaching the steady state value with a decaying sinusoid. The most predominant sinusoid apparent in the response is at approximately 17 Hz, the natural frequency of the motion-base when driven in heave. As the acceleration step decreases in amplitude 60 Hz noise picked up from A.C. sources within the lab become a more significant part of the recorded signal as seen in Figures 7.177 to 7.180. Determination of the threshold results becomes more difficult and the results more questionable as the motion-base response starts to get lost in the 60 Hz noise.

7.7 Time Histories and Power Spectra

To demonstrate the "raw" response of the motion-base in the time domain a few time histories recorded during the signal-to-noise contour tests (with a sampling rate of 100 Hz) are included (see Figures 7.182 to 7.191). In the frequency domain, power spectra were calculated for the describing function tests and a few are shown in Figures 7.192 to 7.201.
The heave accelerometer time history with the base driven in heave sinusoidally at 1 Hz is shown in Figure 7.182. The response is predominantly a sinusoid at the fundamental frequency offset by $9.806 \text{ m/s}^2$ due to the fact the accelerometer measures specific force. Excitation of the 17 Hz resonance occurs at the peaks of the sinusoids and may be caused by a turn-around bump exciting the natural frequency.

The surge accelerometer time history, Figure 7.183, for the same run as above also displays the 17 Hz response at the turn-around point on the heave sinusoidal response. The 17 Hz signal (excited by the turn-around bump) decays until being excited again at the next turn-around point.

The surge accelerometer time history with the base driven in surge at 5 Hz (Figure 7.185) is very smooth. The shape is not quite sinusoidal and the distortion is probably caused by the small number of samples per period and the excitation of harmonics of the fundamental signal.

The rate-gyro time histories are much cleaner than the accelerometer results. The pitch rate-gyro time history with the base driven in roll (Figure 7.189) is primarily crosstalk. This is apparent from its the sinusoidal shape. The signal is small enough that the discrete steps of the A/Ds can be seen. The yaw rate-gyro time history with the base driven in yaw at 10 Hz, Figure 7.191, has a "spiked" shape due to the fact that there are only 10 samples per period and the plots are made by drawing a straight line between consecutive samples.

The $U_0$ power spectrum with the base driven in surge from run A of the describing function tests is shown in Figure 7.192. The estimated power contained in a frequency interval of $\Delta \omega$ centered at $\omega$, for a finite sampled time record, is simply the area under the estimated power spectrum curve at $\omega$. The background noise power is almost five orders of magnitude down from the power at the fundamental frequencies. Harmonics of the fundamental are apparent up to a maximum frequency of 110 r/s and are about four orders of magnitude down from the fundamental spikes. A spike of power occurs at 250 r/s in some of the power spectra, and is probably due to 60 Hz noise incurred downstream of the elliptic filters being folded back to 40 Hz (250 r/s) as a result of the 100 Hz sampling rate.
The sway power spectrum with the base driven in surge from run A is shown in Figure 7.193. This crosstalk power spectrum has a background noise power about two order of magnitude down from the fundamental crosstalk power spikes. The slight rise in the power above 220 r/s is probably due to the inverse filter and rate-gyro adjusting routines boosting noise incurred downstream of the elliptic filters.

The angular rate background noise power is much lower than that of the translational acceleration as seen in Figures 7.194 and 7.196 for example. Correlation between the two results is not expected due to the different variable being measured. The noise power is again about five orders of magnitude down from the fundamentals on the driven channel as seen in Figures 7.197, 7.200 and 7.201. The power spectrum of the yaw angular rate, Figure 7.201, is seen to have more power in the frequency band from 75 r/s to 250 r/s when compared to the pitch and roll angular rates (Figures 7.197 and 7.200), this demonstrates the noisy nature of the yaw rate-gyro.

8 CONCLUSIONS

The motion characteristics of the UTIAS Flight Research Simulator were determined. Describing functions, noise levels, signal-to-noise contours, hysteresis, and dynamic threshold tests were performed. The results are presented within this report. The motion-base was determined to be an "almost" linear system. The results found within this report should give an accurate description of the simulator response provided periodic tuning of the motion-base and its various sub-systems is carried out.

The describing functions found all tend to have a relatively poor response at higher frequencies. This could be due to the fact that the control loop uses force transducers at these high frequencies yet no provision is made to account for the difference in mass distribution of the simulator about each axis.

It should be noted that the "raw" response properties of the motion-base were determined. Under normal operating conditions as a simulator the update
frequency will be much lower than that used for this report and washout filters will further modify its response.
REFERENCES


3. Zacharias G. L. "Motion Cue Models for Pilot-Vehicle Analysis", Aerospace Medical Research Laboratory, Aerospace Medical Division, Wright-Patterson Air Force Base, AMRL-TR-78-2, 1978


Table 1  Motion-Base Displacements

<table>
<thead>
<tr>
<th>Degree of Freedom</th>
<th>Theoretical Test Case Displacement</th>
<th>Measured Test Case Displacement</th>
<th>Pure Displacement Limits</th>
</tr>
</thead>
<tbody>
<tr>
<td>Surge</td>
<td>0.6077m</td>
<td>0.6075m</td>
<td>0.6987m</td>
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<td></td>
<td>-0.6080m</td>
<td>-0.6081m</td>
<td>-0.6125m</td>
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<tr>
<td>Sway</td>
<td>0.5888m</td>
<td>0.5905m</td>
<td>0.5923m</td>
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<tr>
<td></td>
<td>-0.5934m</td>
<td>-0.5935m</td>
<td>-0.5923m</td>
</tr>
<tr>
<td>Heave</td>
<td>0.4080m</td>
<td>0.4085m</td>
<td>0.5570m</td>
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<td></td>
<td>-0.4076m</td>
<td>-0.4070m</td>
<td>-0.4942m</td>
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<td>Roll</td>
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<td>20.779 deg</td>
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<td>-20.989 deg</td>
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<td>-21.064 deg</td>
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<td>Pitch</td>
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<td>20.023 deg</td>
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<td>-20.119</td>
<td>-20.157 deg</td>
<td>-21.646 deg</td>
</tr>
<tr>
<td>Yaw</td>
<td>23.933 deg</td>
<td>23.823 deg</td>
<td>23.849 deg</td>
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<tr>
<td></td>
<td>-23.933 deg</td>
<td>-23.834 deg</td>
<td>-23.849 deg</td>
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</tbody>
</table>
Table 2  Motion-Base Geometry Results

\[
\begin{align*}
A_1^S &= [-1.4665 \quad -0.0953 \quad 0.0]^T \\
A_2^S &= [-1.4665 \quad 0.0953 \quad 0.0]^T \\
A_3^S &= [ 0.6507 \quad 1.3176 \quad 0.0]^T \\
A_4^S &= [ 0.8157 \quad 1.2224 \quad 0.0]^T \\
A_5^S &= [ 0.8157 \quad -1.2224 \quad 0.0]^T \\
A_6^S &= [ 0.6507 \quad -1.3176 \quad 0.0]^T \\
B_1^I &= [-1.2172 \quad -1.5240 \quad -0.2822]^T \\
B_2^I &= [-1.2172 \quad 1.5240 \quad -0.2822]^T \\
B_3^I &= [-0.7112 \quad 1.8161 \quad -0.2822]^T \\
B_4^I &= [ 1.9284 \quad 0.2921 \quad -0.2822]^T \\
B_5^I &= [ 1.9284 \quad -0.2921 \quad -0.2822]^T \\
B_6^I &= [-0.7112 \quad -1.8161 \quad -0.2822]^T \\
L &= [ 2.327 \quad 2.327 \quad 2.327 \quad 2.327 \quad 2.327 \quad 2.327 ] \\
S_1(0) &= [ -2.1020 \quad -2.1020 \quad -2.1020 \quad -2.1020 \quad -2.1020 \quad -2.1020 ]^T
\end{align*}
\]
### Table 3  Calibration of D/As

<table>
<thead>
<tr>
<th>Channel</th>
<th>Signal</th>
<th>Actuator</th>
<th>Scale Factor</th>
<th>Offset</th>
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</thead>
<tbody>
<tr>
<td>1</td>
<td>Position</td>
<td>1</td>
<td>1.00234</td>
<td>-15 mV</td>
</tr>
<tr>
<td>2</td>
<td>Acceleration</td>
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<td>1.0</td>
<td>0 mV</td>
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<tr>
<td>3</td>
<td>Position</td>
<td>2</td>
<td>1.01215</td>
<td>-35 mV</td>
</tr>
<tr>
<td>4</td>
<td>Acceleration</td>
<td>2</td>
<td>1.0</td>
<td>-15 mV</td>
</tr>
<tr>
<td>5</td>
<td>Position</td>
<td>3</td>
<td>1.01764</td>
<td>20 mV</td>
</tr>
<tr>
<td>6</td>
<td>Acceleration</td>
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<td>1.0</td>
<td>0 mV</td>
</tr>
<tr>
<td>7</td>
<td>Position</td>
<td>4</td>
<td>1.01112</td>
<td>-55 mV</td>
</tr>
<tr>
<td>8</td>
<td>Acceleration</td>
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<td>1.0</td>
<td>0 mV</td>
</tr>
<tr>
<td>9</td>
<td>Position</td>
<td>5</td>
<td>1.01892</td>
<td>-35 mV</td>
</tr>
<tr>
<td>10</td>
<td>Acceleration</td>
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<td>1.0</td>
<td>0 mV</td>
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<tr>
<td>11</td>
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<td>12</td>
<td>Acceleration</td>
<td>6</td>
<td>1.0</td>
<td>0 mV</td>
</tr>
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<td>Instrument</td>
<td>Sensitivity</td>
<td>Natural Frequency</td>
<td>Damping Ratio</td>
<td>Estimated Resolution</td>
</tr>
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<td>-------------</td>
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<td>---------------</td>
<td>----------------------</td>
</tr>
<tr>
<td>Sundstrand 2</td>
<td>2.5 V/g</td>
<td>&gt; 550 Hz</td>
<td>N/A</td>
<td>0.002 m/s²</td>
</tr>
<tr>
<td>303 GA2</td>
<td></td>
<td></td>
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<td></td>
</tr>
<tr>
<td>Accelerometer</td>
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<td></td>
</tr>
<tr>
<td>Sundstrand 2</td>
<td>1.0 V/g</td>
<td>&gt; 650 Hz</td>
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<td>0.005 m/s²</td>
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<tr>
<td>303 GA5</td>
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<tr>
<td>Accelerometer</td>
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<tr>
<td>Honeywell GNAT GG440</td>
<td>0.1 V/deg/s</td>
<td>40 Hz</td>
<td>0.5</td>
<td>0.005 deg/s</td>
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<tr>
<td>rate-gyro (with demodulator)</td>
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<td></td>
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</table>
### Table 5  Actuator Extension Transducer Sensitivity

<table>
<thead>
<tr>
<th>Actuator Extension Transducer</th>
<th>Sensitivity (m/V)</th>
<th>Resolution (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.04561</td>
<td>0.23</td>
</tr>
<tr>
<td>2</td>
<td>0.04561</td>
<td>0.23</td>
</tr>
<tr>
<td>3</td>
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</tr>
<tr>
<td>4</td>
<td>0.04567</td>
<td>0.23</td>
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<td>0.23</td>
</tr>
<tr>
<td>6</td>
<td>0.04574</td>
<td>0.23</td>
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</table>

### Table 6  Vectors from Accelerometers to $P_A$ (components in $F_s$)

<table>
<thead>
<tr>
<th>Accelerometer</th>
<th>x-Distance</th>
<th>y-Distance</th>
<th>z-Distance</th>
</tr>
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<tbody>
<tr>
<td>Surge</td>
<td>0.4321 m</td>
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<td>-0.0999 m</td>
</tr>
<tr>
<td>Sway</td>
<td>0.4233 m</td>
<td>0.2365 m</td>
<td>-0.1304 m</td>
</tr>
<tr>
<td>Heave</td>
<td>0.4537 m</td>
<td>0.2453 m</td>
<td>-0.1231 m</td>
</tr>
</tbody>
</table>
Table 7 Operating Points for Describing Function Tests

Sinusoidal Acceleration Inputs for Translational Maneuvers

<table>
<thead>
<tr>
<th>Run A</th>
<th>Run B</th>
<th>Run C</th>
</tr>
</thead>
<tbody>
<tr>
<td>Freq. Hz</td>
<td>Amp. m/s²</td>
<td>Freq. Hz</td>
</tr>
<tr>
<td>0.8</td>
<td>0.5</td>
<td>0.3</td>
</tr>
<tr>
<td>2.0</td>
<td>0.5</td>
<td>0.5</td>
</tr>
<tr>
<td>5.0</td>
<td>0.5</td>
<td>1.3</td>
</tr>
<tr>
<td>9.1</td>
<td>0.5</td>
<td>3.2</td>
</tr>
<tr>
<td>11.0</td>
<td>1.0</td>
<td>7.1</td>
</tr>
<tr>
<td>13.0</td>
<td>1.0</td>
<td>10.1</td>
</tr>
<tr>
<td>19.8*</td>
<td>0.15</td>
<td>14.8</td>
</tr>
</tbody>
</table>

Sinusoidal Velocity Inputs for Angular Maneuvers

<table>
<thead>
<tr>
<th>Run A</th>
<th>Run B</th>
<th>Run C</th>
</tr>
</thead>
<tbody>
<tr>
<td>Freq. Hz</td>
<td>Amp. r/s</td>
<td>Freq. Hz</td>
</tr>
<tr>
<td>0.8</td>
<td>0.016</td>
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</tr>
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<td>0.006</td>
<td>7.1</td>
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<tr>
<td>13.0</td>
<td>0.006</td>
<td>10.1</td>
</tr>
<tr>
<td>19.8*</td>
<td>0.002</td>
<td>14.8</td>
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</table>

* Only used when finding the describing function with elliptic filter out.
<table>
<thead>
<tr>
<th>Frequency (Hz)</th>
<th>Angular Modes (r/s)</th>
<th>Translational Modes (m/s)</th>
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<tr>
<td>0.1</td>
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<td>0.100</td>
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<td>0.220</td>
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<td>0.1</td>
<td>0.200</td>
<td>0.280</td>
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<td>0.2</td>
<td>0.044</td>
<td>0.080</td>
</tr>
<tr>
<td>0.2</td>
<td>0.090</td>
<td>0.150</td>
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<td>0.300</td>
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<td>0.060</td>
<td>0.090</td>
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<td>0.200</td>
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<tr>
<td>0.4</td>
<td>0.500</td>
<td>0.600</td>
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<td>0.013</td>
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<td>0.095</td>
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<td>0.500</td>
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<td>2.5</td>
<td>0.340</td>
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<td>0.001</td>
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<td>10.0</td>
<td>0.007</td>
<td>0.007</td>
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<td>0.090</td>
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Table 9  Hysteresis Results

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<tr>
<td>Sway</td>
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</tr>
<tr>
<td>Heave</td>
<td>0.63 mm</td>
</tr>
<tr>
<td>Roll</td>
<td>0.00050 rad</td>
</tr>
<tr>
<td>Pitch</td>
<td>0.00044 rad</td>
</tr>
<tr>
<td>Yaw</td>
<td>0.00050 rad</td>
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</table>

Table 10  Dynamic Threshold Results
(including 5 ms due to Butterworth Filters)

<table>
<thead>
<tr>
<th>Amplitude m/s²</th>
<th>Positive Step</th>
<th>Negative Step</th>
</tr>
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<tr>
<td></td>
<td>Dead ms</td>
<td>Rise ms</td>
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<tr>
<td>1.0</td>
<td>25</td>
<td>44</td>
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<td>44</td>
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<tr>
<td>0.05</td>
<td>27</td>
<td>38.5</td>
</tr>
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</table>
FIGURE 2.01

FLIGHT RESEARCH SIMULATOR
FIGURE 2.02 5TH ORDER ELLIPTIC INPUT FILTER TRANSFER FUNCTION
FIGURE 2.03 SCHEMATIC OF MOTION DRIVE AND DATA COLLECTION
FIGURE 4.01 VECTORS FOR A SINGLE ACTUATOR
FIGURE 4.02 MOTION BASE GEOMETRY
FIGURE 4.03 SQUARE WAVE ACCELERATION INPUT
FIGURE 4.04 DETERMINATION OF INERTIAL FRAME
FIGURE 5.01 SCHEMATIC OF ANTI-ALIASING FILTERS
FIGURE 5.02  ANTI-ALIASING FILTERS
TRANSFER FUNCTION
FIGURE 5.03  \( F_s \) ORIGIN LOCATION GEOMETRY
ACCELEROMETERS

RATE GYROS

ANALOG ANTI-ALIASING FILTERS

D.C. OFFSET AND DRIFT REMOVAL

DIGITAL INVERSE ANTI-ALIASING FILTERS

ADJUST FOR RATE GYRO DYNAMICS

DIGITAL INVERSE 1ST ORDER LOW-PASS FILTERS

DIFFERENTIATE USING CENTRAL DIFFERENCES

4TH ORDER LOW-PASS SMOOTHER (NO PHASE LAG)

8TH ORDER LOW-PASS SMOOTHER (NO PHASE LAG)

ACTUATOR LENGTH SIGNALS

D.C. OFFSET AND DRIFT REMOVAL

4TH ORDER LOW-PASS SMOOTHER (NO PHASE LAG)

'PLATEAU SIMULATOR ATTITUDE PROGRAM

FIGURE 6.01 SCHEMATIC OF DATA REDUCTION
FIGURE 7.01 NON-LINEAR SYSTEM IDENTIFICATION
FIGURE 7.02  $\frac{\omega_n}{\omega_n^2}$ DESCRIBING FUNCTION
ELLIPITC FILTERS IN
INPUT AMPLITUDES 60% ABOVE NOMINAL

FIGURE 7.03  $\frac{\omega_n}{\omega_n^2}$ DESCRIBING FUNCTION
ELLIPITC FILTERS IN
INPUT AMPLITUDES 60% BELOW NOMINAL
FIGURE 7.04 $\frac{u}{u_0}$ DESCRIBING FUNCTION
ELLIPTIC FILTERS IN

FIGURE 7.05 $\frac{u}{u_0}$ DESCRIBING FUNCTION
ELLIPTIC FILTERS IN

FIGURE 7.06 $\frac{u}{u_0}$ DESCRIBING FUNCTION
ELLIPTIC FILTERS IN

FIGURE 7.07 $\frac{u}{u_0}$ DESCRIBING FUNCTION
ELLIPTIC FILTERS IN
FIGURE 7.10 \( \frac{\omega}{V_i} \) DESCRIBING FUNCTION
ELLIPSTIC FILTERS IN

FIGURE 7.11 \( \frac{\omega}{V_i} \) DESCRIBING FUNCTION
ELLIPSTIC FILTERS IN

FIGURE 7.12 \( \frac{\omega}{V_i} \) DESCRIBING FUNCTION
ELLIPSTIC FILTERS IN

FIGURE 7.13 \( \frac{\omega}{V_i} \) DESCRIBING FUNCTION
ELLIPSTIC FILTERS IN
FIGURE 7.14 $\omega_0/\psi$, DESCRIBING FUNCTION

ELLIPITC FILTERS IN

FIGURE 7.15 $\omega_0/\psi$, DESCRIBING FUNCTION

ELLIPITC FILTERS IN
FIGURE 7.20 \( \Delta \omega/\Delta \omega \) DESCRIBING FUNCTION ELLIPTIC FILTERS IN

FIGURE 7.21 \( \Delta \omega/\Delta \omega \) DESCRIBING FUNCTION ELLIPTIC FILTERS IN
FIGURE 7.22 \( \hat{\omega}/\hat{P} \) DESCRIBING FUNCTION
ELLIPTIC FILTERS IN

FIGURE 7.23 \( \hat{\omega}/\hat{P} \) DESCRIBING FUNCTION
ELLIPTIC FILTERS IN

FIGURE 7.24 \( \hat{\omega}/\hat{P} \) DESCRIBING FUNCTION
ELLIPTIC FILTERS IN

FIGURE 7.25 \( \hat{\omega}/\hat{P} \) DESCRIBING FUNCTION
ELLIPTIC FILTERS IN
FIGURE 7.26 $\frac{\Delta \alpha}{\Delta \beta}$ DESCRIBING FUNCTION ELLIPTIC FILTERS IN

FIGURE 7.27 $\frac{\Delta \alpha}{\Delta \beta}$ DESCRIBING FUNCTION ELLIPTIC FILTERS IN
FIGURE 7.26 $\omega/\theta$ DESCRIBING FUNCTION
ELLIPTIC FILTERS IN

FIGURE 7.29 $\psi/\phi_0$ DESCRIBING FUNCTION
ELLIPTIC FILTERS IN

FIGURE 7.30 $\beta/\phi_0$ DESCRIBING FUNCTION
ELLIPTIC FILTERS IN

FIGURE 7.31 $\eta/\theta_0$ DESCRIBING FUNCTION
ELLIPTIC FILTERS IN
FIGURE 7.32  \( \frac{\Delta v}{\Delta f} \) DESCRIBING FUNCTION
ELLIPIC FILTERS IN

FIGURE 7.33  \( \frac{\Delta v}{\Delta f} \) DESCRIBING FUNCTION
ELLIPIC FILTERS IN
FIGURE 7.34 \( \frac{\omega A_2}{|R(s)|^2} \) DESCRIBING FUNCTION
ELLIPIC FILTERS IN

FIGURE 7.35 \( \frac{\omega A_2}{|R(s)|^2} \) DESCRIBING FUNCTION
ELLIPIC FILTERS IN

FIGURE 7.36 \( \frac{\omega A_2}{|R(s)|^2} \) DESCRIBING FUNCTION
ELLIPIC FILTERS IN

FIGURE 7.37 \( \frac{\omega A_2}{|R(s)|^2} \) DESCRIBING FUNCTION
ELLIPIC FILTERS IN
Figure 7.38: \( \frac{A_{\text{in}}}{A_{\text{out}}} \) Describing Function
Elliptic Filters in

Figure 7.39: \( \frac{A_{\text{in}}}{A_{\text{out}}} \) Describing Function
Elliptic Filters in
FIGURE 7.40 \( \omega / \Omega \) DESCRIBING FUNCTION
ELLIPITC FILTERS OUT

FIGURE 7.41 \( \Omega / \Omega \) DESCRIBING FUNCTION
ELLIPITC FILTERS OUT

FIGURE 7.42 \( \omega / \Omega \) DESCRIBING FUNCTION
ELLIPITC FILTERS OUT

FIGURE 7.43 \( \Omega / \Omega \) DESCRIBING FUNCTION
ELLIPITC FILTERS OUT
Figure 7.44 | \( \Delta \theta/\Delta \omega \) Describing Function
Elliptic Filters Out

Figure 7.45 | \( \Delta \omega/\Delta \Delta \) Describing Function
Elliptic Filters Out
FIG. 7.53 PEAK NOISE IN HEAVE
BASE DRIVEN IN SURGE

FIG. 7.54 TOTAL NOISE IN HEAVE
BASE DRIVEN IN SURGE

FIG. 7.55 PEAK NOISE IN ROLL
BASE DRIVEN IN SURGE

FIG. 7.56 TOTAL NOISE IN ROLL
BASE DRIVEN IN SURGE

FIG. 7.57 PEAK NOISE IN PITCH
BASE DRIVEN IN SURGE

FIG. 7.58 TOTAL NOISE IN PITCH
BASE DRIVEN IN SURGE

FIG. 7.59 PEAK NOISE IN YAW
BASE DRIVEN IN SURGE

FIG. 7.60 TOTAL NOISE IN YAW
BASE DRIVEN IN SURGE
FIG. 7.68 PEAK NOISE IN HEAVE
BASE DRIVEN IN SWAY

FIG. 7.69 TOTAL NOISE IN HEAVE
BASE DRIVEN IN SWAY

FIG. 7.70 PEAK NOISE IN ROLL
BASE DRIVEN IN SWAY

FIG. 7.71 TOTAL NOISE IN ROLL
BASE DRIVEN IN SWAY

FIG. 7.72 PEAK NOISE IN PITCH
BASE DRIVEN IN SWAY

FIG. 7.73 TOTAL NOISE IN PITCH
BASE DRIVEN IN SWAY

FIG. 7.74 PEAK NOISE IN YAW
BASE DRIVEN IN SWAY

FIG. 7.75 TOTAL NOISE IN YAW
BASE DRIVEN IN SWAY
FIG. 7.91 PEAK NOISE IN SURGE BASE DRIVEN IN ROLL

FIG. 7.92 TOTAL NOISE IN SURGE BASE DRIVEN IN ROLL

FIG. 7.93 PEAK NOISE IN SWAY BASE DRIVEN IN ROLL

FIG. 7.94 TOTAL NOISE IN SWAY BASE DRIVEN IN ROLL

FIG. 7.95 PEAK NOISE IN HEAVE BASE DRIVEN IN ROLL

FIG. 7.96 TOTAL NOISE IN HEAVE BASE DRIVEN IN ROLL

FIG. 7.97 PEAK NOISE IN ROLL BASE DRIVEN IN ROLL

FIG. 7.98 TOTAL NOISE IN ROLL BASE DRIVEN IN ROLL
FIG. 7.99 HIGH-FREQUENCY NON-LINEARITY NOISE IN ROLL BASE DRIVEN IN ROLL

FIG. 7.100 LOW-FREQUENCY NON-LINEARITY NOISE IN ROLL BASE DRIVEN IN ROLL

FIG. 7.101 ROUGHNESS NOISE IN ROLL BASE DRIVEN IN ROLL

FIG. 7.102 PEAK NOISE IN PITCH BASE DRIVEN IN ROLL

FIG. 7.103 TOTAL NOISE IN PITCH BASE DRIVEN IN ROLL

FIG. 7.104 PEAK NOISE IN TAW BASE DRIVEN IN ROLL

FIG. 7.105 TOTAL NOISE IN TAW BASE DRIVEN IN ROLL
FIG. 7.106 PEAK NOISE IN SURGE BASE DRIVEN IN PITCH

FIG. 7.107 TOTAL NOISE IN SURGE BASE DRIVEN IN PITCH

FIG. 7.108 PEAK NOISE IN SWAY BASE DRIVEN IN PITCH

FIG. 7.109 TOTAL NOISE IN SWAY BASE DRIVEN IN PITCH

FIG. 7.110 PEAK NOISE IN HEAVE BASE DRIVEN IN PITCH

FIG. 7.111 TOTAL NOISE IN HEAVE BASE DRIVEN IN PITCH

FIG. 7.112 PEAK NOISE IN ROLL BASE DRIVEN IN PITCH

FIG. 7.113 TOTAL NOISE IN ROLL BASE DRIVEN IN PITCH
FIG. 7.114 PERM NOISE IN PITCH BASE DRIVEN IN PITCH

FIG. 7.115 TOTAL NOISE IN PITCH BASE DRIVEN IN PITCH

FIG. 7.116 HIGH-FREQUENCY NON-LINEARITY NOISE IN PITCH BASE DRIVEN IN PITCH

FIG. 7.117 LOW-FREQUENCY NON-LINEARITY NOISE IN PITCH BASE DRIVEN IN PITCH

FIG. 7.118 ROUGHNESS NOISE IN PITCH BASE DRIVEN IN PITCH

FIG. 7.119 PEAK NOISE IN YAW BASE DRIVEN IN PITCH

FIG. 7.120 TOTAL NOISE IN YAW BASE DRIVEN IN PITCH
FIG. 7.121 PEAK NOISE IN SURGE BASE DRIVEN IN YAW

FIG. 7.122 TOTAL NOISE IN SURGE BASE DRIVEN IN YAW

FIG. 7.123 PEAK NOISE IN SWAY BASE DRIVEN IN YAW

FIG. 7.124 TOTAL NOISE IN SWAY BASE DRIVEN IN YAW

FIG. 7.125 PEAK NOISE IN HEAVE BASE DRIVEN IN YAW

FIG. 7.126 TOTAL NOISE IN HEAVE BASE DRIVEN IN YAW

FIG. 7.127 PEAK NOISE IN ROLL BASE DRIVEN IN YAW

FIG. 7.128 TOTAL NOISE IN ROLL BASE DRIVEN IN YAW
FIGURE 7.136 SURGE SIGNAL TO NOISE CONTOUR PLOT.
FIGURE 7.137  SWAY SIGNAL TO NOISE CONTOUR PLOT.
FIGURE 7.138 HEAVE SIGNAL TO NOISE CONTOUR PLOT.
FIGURE 7.139 ROLL SIGNAL TO NOISE CONTOUR PLOT.
FIGURE 7.140 PITCH SIGNAL TO NOISE CONTOUR PLOT.
Figure 7.141 Yaw Signal to Noise Contour Plot.
FIGURE 7.142 | SURGE AXIS APPARENT Hysteresis  
Elliptic Filters in  
$W=0.06283$ R/s

FIGURE 7.143 | SWAY AXIS APPARENT Hysteresis  
Elliptic Filters in  
$W=0.06283$ R/s

FIGURE 7.144 | HEAVE AXIS Hysteresis  
Elliptic Filters in  
$W=0.06283$ R/s

FIGURE 7.145 | ROLL AXIS APPARENT Hysteresis  
Elliptic Filters in  
$W=0.06283$ R/s

FIGURE 7.146 | PITCH AXIS APPARENT Hysteresis  
Elliptic Filters in  
$W=0.06283$ R/s

FIGURE 7.147 | YAW AXIS APPARENT Hysteresis  
Elliptic Filters in  
$W=0.06283$ R/s
FIGURE 7.148 SURGE AXIS APPARENT HYSTERESIS ELLIPTIC FILTERS IN W=0.03142 R/S

FIGURE 7.149 SWAY AXIS APPARENT HYSTERESIS ELLIPTIC FILTERS IN W=0.03142 R/S

FIGURE 7.150 HEAVE AXIS HYSTERESIS ELLIPTIC FILTERS IN W=0.03142 R/S

FIGURE 7.151 ROLL AXIS APPARENT HYSTERESIS ELLIPTIC FILTERS IN W=0.03142 R/S

FIGURE 7.152 PITCH AXIS APPARENT HYSTERESIS ELLIPTIC FILTERS IN W=0.03142 R/S

FIGURE 7.153 YAW AXIS APPARENT HYSTERESIS ELLIPTIC FILTERS IN W=0.03142 R/S
**Figure 7.160** Surge Axis Apparent Hysteresis
Elliptic Filters in
W=0.00314 A/S

**Figure 7.161** Sway Axis Apparent Hysteresis
Elliptic Filters in
W=0.00314 A/S

**Figure 7.162** Heave Axis Hysteresis
Elliptic Filters in
W=0.00314 A/S

**Figure 7.163** Roll Axis Apparent Hysteresis
Elliptic Filters in
W=0.00314 A/S

**Figure 7.164** Pitch Axis Apparent Hysteresis
Elliptic Filters in
W=0.00314 A/S

**Figure 7.165** Taw Axis Apparent Hysteresis
Elliptic Filters in
W=0.00314 A/S
FIGURE 7.166 TYPICAL HYSTERESIS PLOT
FIGURE 7.167 SURGE APPARENT HYSTERESIS VS. FREQUENCY

FIGURE 7.168 SWAY APPARENT HYSTERESIS VS. FREQUENCY

FIGURE 7.169 HEAVE APPARENT HYSTERESIS VS. FREQUENCY

FIGURE 7.170 ROLL APPARENT HYSTERESIS VS. FREQUENCY

FIGURE 7.171 PITCH APPARENT HYSTERESIS VS. FREQUENCY

FIGURE 7.172 YAW APPARENT HYSTERESIS VS. FREQUENCY
Figure 7.173 Dynamic Threshold Test
Acceleration Time History
Positive Step
Amplitude = 1.0 m/s^2

Figure 7.174 Dynamic Threshold Test
Acceleration Time History
Negative Step
Amplitude = 1.0 m/s^2

Figure 7.175 Dynamic Threshold Test
Acceleration Time History
Positive Step
Acceleration = 0.5 m/s^2

Figure 7.176 Dynamic Threshold Test
Acceleration Time History
Negative Step
Acceleration = 0.5 m/s^2
FIGURE 7.177 DYNAMIC THRESHOLD TEST
ACCELERATION TIME HISTORY
POSITIVE STEP
ACCELERATION = 0.1 m/s²

FIGURE 7.178 DYNAMIC THRESHOLD TEST
ACCELERATION TIME HISTORY
NEGATIVE STEP
ACCELERATION = 0.1 m/s²

FIGURE 7.179 DYNAMIC THRESHOLD TEST
ACCELERATION TIME HISTORY
POSITIVE STEP
AMPLITUDE = 0.05 m/s²

FIGURE 7.180 DYNAMIC THRESHOLD TEST
ACCELERATION TIME HISTORY
NEGATIVE STEP
AMPLITUDE = 0.05 m/s²
FIGURE 7.181 DYNAMIC THRESHOLD TEST; FILTER EFFECTS
ACCELERATION TIME HISTORY
NEGATIVE STEP
ACCELERATION = 1.0 M/S²
FIG. 7.182  HEAVE ACCELEROMETER TIME HISTORY
BASE DRIVEN IN HEAVE
FREQUENCY = 6.28 R/S
ACCELERATION = 1.00 M/S²

FIG. 7.183  SURGE ACCELEROMETER TIME HISTORY
BASE DRIVEN IN HEAVE
FREQUENCY = 6.28 R/S
ACCELERATION = 1.00 M/S²
FIG. 7.184  PITCH RATE-GYRO TIME HISTORY
BASE DRIVEN IN HEAVE
FREQUENCY = 6.28 R/S
ACCELERATION = 1.00 M/S²

FIG. 7.185  SURGE ACCELEROMETER TIME HISTORY
BASE DRIVEN IN SURGE
FREQUENCY = 31.4 R/S
ACCELERATION = 2.00 M/S²
**FIG. 7.186** SWAY ACCELEROMETER TIME HISTORY
BASE DRIVEN IN SWAY
FREQUENCY = 2.51 R/S
ACCELERATION = 1.04 M/S²

**FIG. 7.187** ROLL RATE-GYRO TIME HISTORY
BASE DRIVEN IN ROLL
FREQUENCY = 0.63 R/S
VELOCITY = 0.18 R/S
FIG. 7.188  SWAY ACCELEROMETER TIME HISTORY  
BASE DRIVEN IN ROLL  
FREQUENCY = 0.63 R/S  
VELOCITY = 0.18 R/S

FIG. 7.189  PITCH RATE-GYRO TIME HISTORY  
BASE DRIVEN IN ROLL  
FREQUENCY = 0.63 R/S  
VELOCITY = 0.18 R/S
FIG. 7.190  PITCH RATE-GYRO TIME HISTORY
BASE DRIVEN IN PITCH
FREQUENCY = 15.7 R/S
VELOCITY = 0.034 R/S

FIG. 7.191  YAW RATE-GYRO TIME HISTORY
BASE DRIVEN IN YAW
FREQUENCY = 62.8 R/S
VELOCITY = 0.0047 R/S
FIGURE 7.192  X-ACCELERATION POWER SPECTRUM, BASE DRIVEN IN SURGE ELLIPTIC FILTERS IN RUN A

FIGURE 7.193  Y-ACCELERATION POWER SPECTRUM, BASE DRIVEN IN SURGE ELLIPTIC FILTERS IN RUN A
FIGURE 7.194  P-ANGULAR RATE POWER SPECTRUM, BASE DRIVEN IN SURGE
ELLPTIC FILTERS IN
RUN A

FIGURE 7.195  Y-ACCELERATION POWER SPECTRUM, BASE DRIVEN IN PITCH
ELLPTIC FILTERS IN
RUN A
FIGURE 7.196 P ANGULAR RATE POWER SPECTRUM, BASE DRIVEN IN PITCH ELLIPTIC FILTERS IN RUN A

FIGURE 7.197 Q ANGULAR RATE POWER SPECTRUM, BASE DRIVEN IN PITCH ELLIPTIC FILTERS IN RUN A
FIGURE 7.198  Y-ACCELERATION POWER SPECTRUM, BASE DRIVEN IN SWAY ELLIPTIC FILTERS IN RUN A

FIGURE 7.199  Z-ACCELERATION POWER SPECTRUM, BASE DRIVEN IN HEAVE ELLIPTIC FILTERS IN RUN B
FIGURE 7.200  P ANGULAR RATE POWER SPECTRUM, BASE DRIVEN IN ROLL
ELLIPTIC FILTERS IN
RUN B

FIGURE 7.201  A ANGULAR RATE POWER SPECTRUM, BASE DRIVEN IN YAW
ELLIPTIC FILTERS IN
RUN A
**FIGURE 7.202**

X-ACCELERATION POWER SPECTRUM, BASE DRIVEN IN SURGE

ELLIPITIC FILTERS IN SURGE ACCELERATION = 0.1 m/s²

**FIGURE 7.203**

X-ACCELERATION POWER SPECTRUM, BASE DRIVEN IN SURGE

ELLIPITIC FILTERS IN SURGE ACCELERATION = 1.5 m/s²
APPENDIX A

PLATATT

PLATATT is a program developed at UTIAS and is based on Reference 14. The program performs the inverse transformation for the UTIAS research simulator. Given the six actuator extensions the program will find the position of the point $P_A$ in the inertial frame and the attitude of the simulator in terms of Euler angles $\phi$, $\theta$ and $\psi$. The program is an iterative type of program, however, after first allowing the program to iterate to find the correct solution at the first step (within an error band) only one iteration is required from that point on. This is due to the fact that the motion-base can not move very far during 10 ms and the maximum difference between steps is small enough that one iteration meets the maximum error requirements, see Reference 14. The source code for the program is included below.

A.1
SUBROUTINE PLATATT(EXTENSION, MAXERR, X, Y, Z, THETA, PHI, PSI)
C
CPLATATT COMPUTES THE ATTITUDE OF THE MOTION PLATFORM, GIVEN THE
C EXTENSIONS OF THE JACKS.
C
REAL*4 AAIS(3,6), BBII(3,6), SSI IN(3), RRS(3), LENGTH_NEUTRAL
REAL*4 LLSI(3,3), DFUNC(6,7), LENGTH(6), XN(6), DUMM(6)
REAL*4 EXTENSION(6), SSI(3), BETAS(3), MAXERR
C
DATA STATEMENTS DEFINING THE GEOMETRY OF THE SIMULATOR.
DATA XN/2*0., -1.8198, 3*0. /
DATA IFLG/1/, LENGTH_NEUTRAL/2.327/
DATA BBII/-1.2172, -1.524, 0., -1.2172, 1.524, 0., -0.7112, 1.8161, 0. ,
1 1.9284, 0.2921, 0.0, -1.9284, -0.2921, 0.0, -0.7112, -1.8161, 0.0/
DATA AAIS/-1.4665, -0.0953, 0., -1.4665, 0.0953, 0., 0.6507,
1 1.3176, 0.0,
1 0.8157, 1.2224, 0.0, 0.8157, -1.2224, 0.0, 0.6507, -1.3176, 0.0/
C
C-CALCULATE TOTAL LENGTH OF ACTUATORS.
C
DO 10 I=1, 6
10 LENGTH(I) = EXTENSION(I) + LENGTH_NEUTRAL
C
START ITERATION
C
PHI2 = XN(4)*XN(4)
THE2 = XN(5)*XN(5)
PSI2 = XN(6)*XN(6)
C
SINPHI = XN(4)* (1. + PHI2 * (-0.1666667 + 0.00833333*PHI2))
SINTHE = XN(5)* (1. + THE2 * (-0.1666667 + 0.00833333*THE2))
SINPSI = XN(6)* (1. + PSI2 * (-0.1666667 + 0.00833333*PSI2))
C
COSPHI = 1. + PHI2 * (-0.50 + 0.04166667 * PHI2 )
COSTHE = 1. + THE2 * (-0.50 + 0.04166667 * THE2 )
COSPSI = 1. + PSI2 * (-0.50 + 0.04166667 * PSI2 )
C
LLSI(1,1) = COSPSI*COSTHE
LLSI(1,2) = SINPSI*COSTHE
LLSI(1,3) = -SINTHE
LLSI(2,1) = COSPSI*SINTHE*SINPHI - SINPSI*COSPHI
LLSI(2,2) = SINPSI*SINTHE*SINPHI + COSPSI*COSPHI
LLSI(2,3) = COSTHE*SINPHI
LLSI(3,1) = COSPSI*SINTHE*COSPHI + SINPSI*SINPHI
LLSI(3,2) = SINPSI*SINTHE*COSPHI - COSPSI*SINPHI
LLSI(3,3) = COSTHE*COSPHI
C
A.2
CALL FUNCS & DFUNCS--THE OLD FUNC(I) IS NOW STORED IN DFUNC(I,7)

DO 60 I=1,6

DFUNC(I,1)=2.*XN(I) + AAIS(I,1)*LLSI(I,1) + AAIS(2,1)*LLSI(2,1) +
1 AAIS(3,1)*LLSI(3,1) - BBII(I,1))

DFUNC(I,2)=2.*XN(2) + AAIS(1,1)*LLSI(1,2) + AAIS(2,1)*LLSI(2,2) +
1 AAIS(3,1)*LLSI(3,2) - BBII(2,2))

DFUNC(I,3)=2.*XN(3) + AAIS(1,1)*LLSI(1,3) + AAIS(2,1)*LLSI(2,3) +
1 AAIS(3,1)*LLSI(3,3) - BBII(3,3))

1

DFUNC(I,4)=2.*((XN(1) - BBII(1,1))*(AAIS(2,1)*LLSI(3,1) - AAIS(3,1)*LLSI(2,1))
1 + (XN(2) - BBII(2,1))*(AAIS(2,1)*LLSI(3,2) - AAIS(3,1)*LLSI(2,2)) +
2 (XN(3) - BBII(3,1))*(AAIS(2,1)*LLSI(3,3) - AAIS(3,1)*LLSI(2,3))

DFUNC(I,5)=2.*((XN(1) - BBII(1,1))*(AAIS(2,1)*LLSI(3,1) - AAIS(3,1)*LLSI(2,1))
1 + (XN(2) - BBII(2,1))*(AAIS(2,1)*LLSI(3,2) - AAIS(3,1)*LLSI(2,2)) +
2 (XN(3) - BBII(3,1))*(AAIS(2,1)*LLSI(3,3) - AAIS(3,1)*LLSI(2,3))

DFUNC(I,6)=2.*((XN(1) - BBII(1,1))*(AAIS(1,1)*LLSI(1,2) + AAIS(2,1)*LLSI(2,2) +
1 AAIS(3,1)*LLSI(3,2))

2 + 2.*(XN(2) - BBII(2,1))*(AAIS(1,1)*LLSI(1,1) + AAIS(2,1)*LLSI(2,1) +
3 AAIS(3,1)*LLSI(3,1))

DFUNC(I,7)= AAIS(1,1)*AAIS(1,1) + AAIS(2,1)*AAIS(2,1)
1 + AAIS(3,1)*AAIS(3,1) + BBII(1,1)*BBII(1,1)
2 + BBII(2,1)*BBII(2,1) + BBII(3,1)*BBII(3,1) + XN(1)*XN(1)
3 + XN(2)*XN(2) + XN(3)*XN(3) - LENGTH(I)*LENGTH(I)
4 + 2.*(XN(1) - BBII(1,1))*(AAIS(1,1)*LLSI(1,1) + AAIS(2,1)*
5 LLSI(2,1) + AAIS(3,1)*LLSI(3,1))
6 + 2.*(XN(2) - BBII(2,1))*(AAIS(1,1)*LLSI(1,2) + AAIS(2,1)*
7 LLSI(2,2) + AAIS(3,1)*LLSI(3,2))
8 + 2.*(XN(3) - BBII(3,1))*(AAIS(1,1)*LLSI(1,3) + AAIS(2,1)*
9 LLSI(2,3) + AAIS(3,1)*LLSI(3,3))
1

CONTINUE

PERFORM MATRIX INVERSION--GAUSSIAN ELIMINATION

THIS VERSION SEARCHES FOR LARGEST PIVOT

DO 12 K=1,5
12 JJ=K
BIG = ABS(DFUNC(K,K))
KP1 = K+1

A.3
DO 7 I=KP1,6
    AB = ABS(DFUNC(I,K))
    IF (AB.GT.BIG) THEN
        BIG = AB
        JJ = I
    ENDIF
7 CONTINUE

IF (JJ.NE.K) THEN
    DO 9 J=K,7
        TEMP = DFUNC(JJ,J)
        DFUNC(JJ,J) = DFUNC(K,J)
        DFUNC(K,J) = TEMP
    ENDIF
9 CONTINUE

DO 11 I=KP1,6
    QUOT = DFUNC(I,K)/DFUNC(K,K)
    DFUNC(I,K) = 0.
    DO 11 J=KP1,7
        DFUNC(I,J) = DFUNC(I,J) - QUOT*DFUNC(K,J)
    11 CONTINUE

DUMM(6) = DFUNC(6,7)/DFUNC(6,6)
DO 14 I=5,1,-1
    SUM = 0.
    DO 13 J=I+1,6
        SUM = SUM + DFUNC(I,J)*DUMM(J)
14 DUMM(I) = (DFUNC(I,7) - SUM)/DFUNC(I,I)

END OF MATRIX INVERSION

XDUM = 0.0
DO 20 I=1,6
    XDUM = MAX(ABS(DUMM(I)), XDUM)
20 XN(I) = XN(I) - DUMM(I)

C-CHECK TO SEE IF FIRST CALL TO SUBROUTINE, IF IT IS THEN CONTINUE TO
C-ITERATE UNTIL MAXIMUM ERROR CONDITION IS REACHED.

IF(IFLG .EQ. 1 .AND. XDUM.GT.MAXERR) GO TO 5
IFLAG = 0
X = XN(1)
Y = XN(2)
Z = XN(3) + 1.8198
PHI = XN(4)
THETA = XN(5)
PSI = XN(6)
RETURN
END
APPENDIX B

FILTERS

The general equation for the amplitude response of an nth order Butterworth low-pass filter is

\[ |L(j\omega)| = \frac{1}{\left[1 + (f/f_c)^{2n}\right]^{1/2}} \]  

(B.1)

The amplitude response can be forced on a time record of collected data by first taking the FFT of the entire time record and applying Equation B.1 to the data in the Fourier domain. Performing the inverse FFT on the resulting data will then produce the appropriate filtered time record.

When differentiating the pitch and roll angular rates the low-pass Butterworth smoother was employed with \( f_c = 40 \) Hz and \( n=4 \). Thus the angular rates were filtered in amplitude only and with the roll-off rate of a 4th order Butterworth. When filtering the yaw angular rate prior to differentiating it was smoothed using the low-pass Butterworth smoother with \( f_c = 20 \) Hz and \( n=8 \).

The source code for a general low-pass Butterworth smoother with zero phase lag is included below and is called LPBUTFR.
SUBROUTINE LPBUTFR(FTX, FTXF, NOPTS, TDEL, N, FC)
C-Subroutine filters in the frequency domain the signal X according
to an Nth order butterworth filter in amplitude only.
C-
C-
C-The Fourier transform of the time record is FTX()
C-The smoothed output in the Fourier domain is FTXF()
C-N is the order of the filter
C-FC is the break frequency (Hz)
C-
COMPLEX FTX(NOPTS/2+1), FTXF(NOPTS/2+1)
NPTS=NOPTS/2+1
DO 10 J=1, NPTS
  F=(J-1)/(NOPTS*TDEL)
  FACT=1/SQR(1+(F/FC)**(2*N))
  FTXF(J)=FACT*FTX(J)
10 CONTINUE
RETURN
STOP
END
APPENDIX C

THE FOURIER TRANSFORM AND ITS APPLICATIONS

The Fourier transform pair for a signal \( x(t) \) in the time domain and \( X(\omega) \) in the frequency domain is

\[
X(\omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{-j\omega t} x(t) \, dt \quad \text{(C.1)}
\]

\[
x(t) = \int_{-\infty}^{\infty} e^{j\omega t} X(\omega) \, d\omega \quad \text{(C.2)}
\]

For a finite time record of length \( T_0 \) where \( x(t) \) is zero for \( t > T_0 \) and \( t < 0 \) the finite Fourier transform of \( x(t) \) is

\[
X(\omega) = \frac{1}{2\pi} \int_{0}^{T_0} e^{-j\omega t} x(t) \, dt \quad \text{(C.3)}
\]

When a digital computer is used to sample \( x(t) \) every \( \Delta t \) seconds the digital Fourier transform of \( x(t) \) is for \( N \) samples;

\[
\text{DFT}(x(t), \omega) = \frac{1}{2\pi} \sum_{n=0}^{N-1} e^{-j\omega n \Delta t} x(n\Delta t) \Delta t \quad \text{(C.4)}
\]

and

\[
T_0 = N \Delta t \quad \text{(C.5)}
\]

The fast Fourier transform algorithm (FFT) contained in IMSL was used to evaluate

\[
\text{FFT}(x(t), f) = \sum_{n=0}^{N-1} x(n\Delta t) e^{j2\pi f n} \Delta t \quad \text{(C.6)}
\]

for \( 0 < f < f_s/2 \)
in a computationally efficient manner. To obtain the DFT as defined by equation C.4 from equation C.6, multiplication by $\Delta t$, division by $2\pi$ and the taking the of the complex conjugate of the result is required.

$$\text{DFT}(x(t), \omega) = \frac{\Delta t}{2\pi} \text{FFT}^*(x(t), f) \quad (C.7)$$

where $(\quad)^*$ denotes the complex conjugate.

As shown in Reference 11 the power spectrum estimate based on the digital Fourier transform is,

$$\Phi_{xy} \text{est}(j\omega) = \frac{2\pi}{T_0} \text{DFT}^*(x(t), \omega) \text{DFT}(y(t), \omega) \quad (C.8)$$

Applying equations C.7, and C.5 and noting that the one-sided power spectrum is twice the magnitude of the two-sided power spectrum (except at $\omega = 0$ where they are equal) gives,

$$\Phi_{xy} \text{est}(j\omega) = \frac{\Delta t}{\pi N} \text{FFT}(x(t), f) \text{FFT}^*(y(t), f) \quad (C.9)$$

If we have the FFT of a signal $x(t)$ and wish to return to the time domain then the inverse digital Fourier transform of $\text{DFT}(x(t), \omega)$ is

$$x(t) = \sum_{n=0}^{N-1} \text{DFT}(x(t), n\Delta\omega)e^{jn\Delta\omega t} \Delta\omega \quad (C.10)$$

applying equation C.7 gives

$$x(t) = \sum_{n=0}^{N-1} \frac{\Delta t}{2\pi} \text{FFT}^*(x(t), n\Delta f) e^{jn\Delta\omega t} \Delta\omega \quad (C.11)$$
and using the FFT Equation C.6 reversing t and f and noting

\[ \Delta \omega = 2 \pi \Delta f \]  
(C.12)

\[ x(t) = \Delta f \Delta t \text{FFT}([x(t),f],t) \]  
(C.13)

but from the IMSL routine \[ \Delta f = 1/N \Delta t \]  
(C.14)

and thus \[ x(t) = \text{FFT}([x(t),f],t)/N \]  
(C.15)

From Reference 11 the variance \( \sigma_x^2 \) of a signal \( x(t) \) having a zero mean for that part of the signal in the frequency range \( \omega_1 \) to \( \omega_2 \) can be determined using

\[ \sigma_x^2 = \int_{\omega_1}^{\omega_2} \Phi_{xx}(\omega) \, d\omega \]  
(C.16)

Based on the digital estimate to \( \Phi_{xx}(\omega) \) this gives

\[ \sigma_x^2 = \sum_{n=N_1}^{N_2} \Phi_{xx \text{est}}(n \Delta \omega) \frac{\Delta \omega}{N} \]  
(C.17)

where \( N_1, N_2 \) define the start and end value of \( n \) such that summation over the frequencies of interest is obtained. From Equation C.9 it follows that

\[ \sigma_x^2 = \sum_{n=N_1}^{N_2} \frac{\Delta \omega}{N} \text{FFT}(x(t),n \Delta f) \text{FFT}^*(x(t),n \Delta f) \Delta \omega \]  
(C.18)

and applying Equations C.12 and C.14 yields

\[ \sigma_x^2 = \sum_{n=N_1}^{N_2} \frac{2}{N} \text{FFT}(x(t),n \Delta f) \text{FFT}^*(x(t),n \Delta f) \]  
(C.19)
The motion characteristics of the UTIAS Flight Research Simulator Motion-Base were experimentally determined. More specifically, describing function tests (under various operating conditions), 1/2 Hz noise level tests, signal-to-noise tests, and hysteresis tests were performed for all six degrees-of-freedom. Dynamic threshold tests were performed for the heave degree-of-freedom. The motion-base was found to have a reasonably flat amplitude response up to 10 Hz in all degrees-of-freedom. Motion in the non-driven degrees-of-freedom was small compared to the driven channel. The noise of the motion-base was found to be the sum of broadband background noise and harmonics of the driven frequency, with the amplitude of the noise varying with both the amplitude and frequency of the driving signal. Hysteresis was determined to be negligible. The dynamic threshold was found to be small and quite acceptable for most projected applications involving the motion-base.

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