A STUDY OF
LATERAL FLIGHT PATH PERTURBATIONS OF STOL AIRCRAFT
IN THE PLANETARY BOUNDARY LAYER

by

Meyer A. Nahon

June, 1983

UTIAS Technical Note No. 240
CN ISSN 0082-5263
A STUDY OF
LATERAL FLIGHT PATH PERTURBATIONS OF STOL AIRCRAFT
IN THE PLANETARY BOUNDARY LAYER

by

Meyer A. Nahon

June, 1983

UTIAS Technical Note No. 240
CN ISSN 0082-5263
Aknowledgement

I would like to thank Dr. L. D. Reid, my thesis supervisor, for his advice, guidance, and consistent support in this and other projects. Thanks are extended to Prof. B. Etkin who provided invaluable help in the form of numerous enlightening discussions of the problem at hand. As well, I would like to express my gratitude to W. O. Graf whose continuous assistance and insight was crucial in overcoming the practical problems in all aspects of this work.

I would also like to thank the Natural Sciences and Engineering Research Council of Canada for their financial support in the form of a Postgraduate Scholarship; and finally, Ms. J. S. Belinsky for the prompt and flawless preparation of the manuscript.
Summary

A wind-tunnel investigation of the characteristics of turbulence encountered by STOL aircraft during steep descents was performed. The experimental results were used as input to a mathematical model of the linearized lateral equations of motion of a typical STOL aircraft, yielding the root-mean-square dispersion of the lateral state variables from a nominal flight path. Single and four-point aircraft approximations were considered; the former accounting only for side gusts, and the latter including rolling, and longitudinal and lateral yawing gust gradients. The single-point approximation proved to be inadequate for estimating lateral dispersions from the glideslope. Dispersion contributions due to rolling, and longitudinal and lateral yawing gust gradients were separated for the four-point approximation.
CONTENTS

Aknowledgement ii
Summary iii
List of Symbols v

I. INTRODUCTION 1

II. EXPERIMENTAL SETUP 1
2.1 Wind Tunnel 1
2.2 Traversing Rig 2
2.3 Glideslope Rig 2
2.4 Data Acquisition System 2
   2.4.1 Hot-Wire Anemometry 2
   2.4.2 Analog Computer and RMS Voltmeters 3
   2.4.3 Digital Computer 3

III. MODELLING OF THE PLANETARY BOUNDARY LAYER 3
3.1 Mean Wind Velocity Profile 4
3.2 Turbulence Intensity 4
3.3 Power Spectral Density and Scale Length 5
3.4 Gust Probability Distribution 6

IV. DEVELOPMENT OF THE AIRCRAFT MODEL AND TWO-POSITION CORRELATION TECHNIQUE 6
4.1 Single and Four-Point Aircraft Approximations 6
4.2 Aircraft Equations of Motion 8
   4.2.1 Complete Equations of Motion 9
   4.2.2 Linearized Decoupled Equations 10
4.3 Two-Position Correlation Technique 17
4.4 Computer Implementation 18

V. EXPERIMENTAL RESULTS 18
5.1 Correlation Results 18
   5.1.1 Preliminary Investigation 20
   5.1.2 Statistical Survey 22
   5.1.3 Final Data 22
5.2 Dispersion of the Aircraft State Vector 23

VI. CONCLUSIONS AND RECOMMENDATIONS 24

REFERENCES 25

APPENDIX A -- Characteristics of the Augmented Aircraft
Appendix B -- Computer Codes
TABLES AND FIGURES
List of Symbols

A, B
initial and final points of the nominal descent path.

b, b'
aircraft wingspan, and 0.85 b, respectively.

Δd
separation vector between two points i and j on the aircraft.

f
frequency (cycles/sec).

F, F_B
inertial and body-fixed reference frames defined in Sec. 4.2.

g
acceleration of gravity (m/s^2).

h; h'
height above ground--full scale; tunnel scale.

h_1, h_2; h'_1, h'_2
height above ground of the upper and lower probe locations, respectively -- full scale; tunnel scale.

h_G
gradient height.

H
wind tunnel height.

I_{ij}
the ij product of inertia.

k
gain of analog computer filter potentiometer (= 1/ time constant), or reduced frequency, f/W, depending on context.

L_T
distance from aircraft center of mass to vertical tail aero­
dynamic center (m) -- see Table 1.

L^x_i
integral scale length of i-velocity component power spectrum
(i = u, v, w).

(L M N)
components of the external moment on the aircraft in F_B.

m
aircraft mass (kg).

M^n_i
the n-th moment of the i-velocity probability distribution,
(i = u, v, w; n = 1, 2, 3, ...).

n
power law exponent.

p
aircraft roll rate or rolling gust gradient in F_I (p = (w_1-w_2)/b'), depending on context.

q, r
aircraft pitch rate and yaw rate, respectively.

r_1
longitudinal yawing gust gradient (r_1 = (u_2-u_1)/b').

r_2
lateral yawing gust gradient (r_2 = (v_0-v_3)/L_T).

r_{ij}
element ij of R_{BI} defined in Sec. 4.2 (i, j = 1, 2, 3).
single point, time delay autocorrelation of \( i \), \( (i = u, v, w) \)

\( R_{ij} \)
dimensional flight path correlation between \( i(t) \) measured at \( h_1 \) and \( j(t) \) measured at \( h_2 \) \( (i,j = v_0, v_3, w_1, w_2, r_1, r_2, p) \).

\( \hat{R}_{ij} \)
\( R_{ij}/(\sigma_i \sigma_j) \).

\( t \)
time.

\( T \)
total sampling time.

\( (u \ v \ w) \)
fluctuating gust velocity components in \( F_i \).

\( (u_g \ v_g \ w_g) \)
fluctuating gust velocity components in \( F_B \).

\( (u_E \ v_E \ w_E) \)
components of \( V_E \) in \( F_B \).

\( V, V_E \)
airspeed and groundspeed, respectively.

\( (x \ y \ z) \)
components of separation vector \( \Delta d \) or reference frame orthogonal axes, depending on context.

\( (X \ Y \ Z) \)
components of non-gravitational external forces acting on the aircraft in \( F_B \) unless otherwise noted.

\( \alpha; \alpha' \)
time to reach upper probe location from top of shear layer -- full scale; tunnel scale.

\( (\beta - \alpha); (\beta - \alpha)'; \)
time to reach lower probe location from upper probe location -- full scale; tunnel scale.

\( \gamma_E \)
glidepath angle.

\( \delta \)
Dirac delta function.

\( \delta_a \)
aileron deflection (rad).

\( \delta_r \)
rudder deflection (rad).

\( \sigma_i \)
root-mean-square of \( i(t) \), \( (i = u, v, w, r_1, r_2, p) \).

\( \Phi_{ii} \)
one-dimensional power spectral density of \( i \), \( (i = u, v, w) \).

\( \phi \theta \psi \)
rigid body dynamics Euler angles (defined in Ref. 11).

\( )_a \)
of aerodynamic origin.

\( )_{A, W} \)
for wingless aircraft only or wing only, respectively.

\( )_B \)
with respect to \( F_B \).

\( )_c \)
of control origin.

\( )_C \)
complete.

\( )_e \)
reference equilibrium value.
gust-related component in $F_B$.

with respect to $F_I$.

measurable.

relative or yaw rate stability derivative, depending on context.

reduced.

evaluated at aircraft stations 0, 1, 2, or 3, respectively,

(shown in Fig. 2 and defined in Sec. 4.1).

ensemble mean of ( ).

$\Delta( )$

perturbation value of ( ).

denotes ( ) as a vector.

denotes ( ) as a matrix.

denotes the transpose of a matrix.

Other variables defined at point of use.

Other subscripted variables denote stability derivatives in convention of Ref. 11.
I. INTRODUCTION

The advent of Short Take-Off and Landing (STOL) aircraft utilizing steep landing approaches has precipitated a need for investigation into the effect of the planetary boundary layer on the stability and controllability of these vehicles.

The planetary boundary layer consists of the lowest 300-600 metres of the atmosphere, and is characterized by appreciable mean velocity, turbulence intensity and scale length gradients. Steep landing approaches through this region result in a rapidly increasing pilot workload as he attempts to cope with fluctuating wind conditions, at a time when he can least afford to divert his attention from other duties inherent in the landing phase. The aircraft's stability under these conditions therefore assumes paramount importance in deciding its acceptability with regards to safety and manageability.

The longitudinal response of STOL aircraft in steep approaches having been previously studied (Refs. 1, 2, 3); the present investigation deals with the corresponding lateral response of a typical STOL aircraft (Fig. 1). The method utilized for this investigation is the two-position space-time correlation technique developed in Ref. 4, applied to a four-point model of the aircraft; thereby accounting for linear gust gradients along the wing and fuselage. Results are also obtained for the single-point aircraft approximation, and typical root-mean-square (rms) dispersion from a 15 degree glideslope are compared for the two aircraft representations. The data base utilized is a series of appropriate gust velocity and gradient flight path correlations obtained in the UTIAS 1.12 m x 1.68 m low-speed atmospheric wind tunnel described in Ref. 5. These correlations are used as input to a computer model of the linearized lateral equations of motion for the STOL aircraft described in Ref. 6, which evaluates the rms dispersion of the state variables. The logical continuation of this work would be an evaluation and validation of an analytical turbulence correlation model which would yield similar aircraft rms response; thereby alleviating the need for further in-field or wind tunnel measurements. It is hoped that this complementary work will be forthcoming from UTIAS.

The geometry of the situation under consideration, as well as the four-point aircraft representation (points 0, 1, 2, 3) are shown in Figs. 2 and 3. The descent manoeuvre starts at point A, and proceeds upwind at a constant airspeed $V$, along a nominal rectilinear glideslope ($\gamma_F = 15^\circ$) to point B, where the aircraft's state vector is evaluated and a decision is made on whether to continue or abort the landing. The 15 degree glideslope was chosen as a severe but reasonable case for state-of-the-art STOL aircraft.

II. EXPERIMENTAL SETUP

2.1 Wind Tunnel

The design, construction and calibration of the UTIAS planetary boundary layer wind tunnel is outlined in Ref. 5. The tunnel is powered by a 45 kW fan motor, and a 56 kW blower motor giving it a maximum speed capability of 37 m/s. The blower supplies a series of 96 valve-regulated jets which allow precise control of the mean wind profile. The test section is 1.12 m high, 1.68 m wide, and is located from 5.5 to 8.5 tunnel heights (5.5 $H$ to 8.5 $H$) downstream of the jet exit plane (Fig. 4). Turbulence intensities are controlled by a trip barrier.
positioned 1.5 H downstream of the jets, and a rough vinyl carpet on the remainder of the tunnel floor.

2.2 Traversing Rig

The desired velocity profile is obtained by using the installed traversing rig, described in Ref. 7, to translate a hot-wire probe throughout the test section. This rig consists of a wing-shaped, stepping-motor-driven probe holder under digital computer control. The profile-setting procedure is fully automated, involving closed-loop control of the jet regulating valves using velocity feedback from the traversing rig.

As noted in Ref. 7, the wing-mounted probe measures an airflow slightly accelerated by the wing. Displacing the probe further upstream than the present one chord separation from the wing proved unsuccessful in reducing this effect due to the resulting increase in vibration of the unit. A series of correction factors was therefore obtained by comparing measurements with and without the wing, and are shown in Fig. 5. All velocity profile measurements presented in this report are corrected for this effect.

2.3 Glideslope Rig

The glideslope rig, described in Ref. 7, was used to obtain turbulence intensity, spectral density, probability distribution, and all correlation measurements. This structure consists of a U-section beam resting on ground-mounted supports to minimize vibration, and was verified to have negligible effect on velocity and correlation measurements (Ref. 7). Two sliding and locking probe carriers, described in Ref. 7, can be positioned along the glideslope rail from outside the wind tunnel. The carriers were modified to accept three different lateral probe positions, as shown in Fig. 6, to represent aircraft stations 0, 1, and 2 (Fig. 2), and were found to affect turbulence intensity measurements, but not dimensionless correlations. Turbulence intensities were therefore evaluated with the unmodified carriers.

2.4 Data Acquisition System

The data acquisition system is depicted in Figs. 7 and 8; including four channels of DISA hot-wire anemometry, a PACE TR-48 analog computer and Brüel and Kjaer RMS voltmeters for signal filtering, and an HP-2100 A digital computer for data analysis.

2.4.1 Hot-Wire Anemometry

The anemometry system consists of four channels of DISA 55D01 Constant Temperature Anemometers, an DISA 55D10 Linearizers, coupled to a choice of DISA 55P81 U-wire and 55P61 X-wire miniature probes. The sensors used are platinum-plated tungsten wire, 5 μm diameter and 1.25 mm long, and are corrected for changes in tunnel temperature by the compensator channel of 55P81 probes. The entire system is expected to have a flat frequency response (-3 dB) from 0 to 50 kHz at the settings used — considerably more than adequate since the turbulence spectral content is minimal past 1 kHz (Ref. 1). Reference 8 outlines the calibration equipment and procedure; as well as the estimation of the system's maximum experimental error at 3% for the u-velocity component, and 5% for the v and w components.
2.4.2 Analog Computer and RMS Voltmeters

The Pace TR-48 analog computer is wired as shown in Fig. 9 for filtering of instantaneous velocities to separate mean and fluctuating components, using a 20 sec time constant \( (k=0.050) \). These outputs are transmitted to the digital computer for further analysis.

The root-mean-square of the fluctuating components is obtained by connecting two Brüel and Kjær Type 2417 RMS Voltmeters to the appropriate tie points, using a 30 sec time constant.

The error introduced by this equipment is expected to be negligible.

2.4.3 Digital Computer

The velocity components received from the analog computer are digitized by an HP 5610 A analog-to-digital converter capable of processing 100,000 samples per second with 10-bit resolution (20 mV). The digital signals are then transferred to an HP 2100 A digital computer of 24K core storage augmented by an HP 7970 B magnetic tape unit. The HP 2100 A can process mean velocity data for profile-setting using the software described in Ref. 9. Existing software also reduces fluctuating velocity data to obtain auto and cross-correlations of incoming signals, and stores the reduced results on magnetic tape.

A sampling frequency of 2500 samples per second was used for all correlation and spectral density measurements, allowing an upper frequency limit of 1250 Hz to avoid aliasing (Ref. 10). Total sampling time was limited to 50 sec for spectral densities, 20 sec for preliminary and statistical correlation surveys, and 80 sec for final correlations (see Sec. 5.1.2).

III. MODELLING OF THE PLANETARY BOUNDARY LAYER

The planetary boundary layer is a sublayer of the atmosphere which forms as on any surface exposed to a viscous flow. The bottom sheet of air is constrained by a no-slip condition at the ground-atmosphere interface; while succeeding layers exchange momentum, primarily by turbulent transport, thereby causing characteristic mean wind and turbulence intensity profiles.

The existence of shear flow complicates the low-altitude turbulence model as compared to that at high altitude. Complexities associated with this region are as follows:

1/ Anisotropic flow conditions resulting in unequal turbulence intensities \( \sigma_u/W, \sigma_v/W, \sigma_w/W \) (Refs. 11, 12).

2/ Vertical non-homogeneity characterized by vertical gradients in mean wind, turbulence intensity, and integral scale length (Refs. 6, 11, 12).

3/ Slightly non-Gaussian gust probability distributions characterized by larger kurtosis and possible skewness (Ref. 13).

The conditions modelled in the wind tunnel for this work are representative of those occurring in a neutrally stable atmosphere where the vertical temperature gradient is equal to the dry adiabatic lapse rate \( (10^\circ C/1000 \text{ m}) \); so that
convective mass transfers are neither amplified nor damped. This assumption is justified since the high winds of interest in the present work tend to mechanically stir the atmosphere to a state of neutral stability.

Flow conditions obtained in the wind tunnel are presented in this chapter, and compared to in-field measurements and engineering models from various sources. Present understanding of the planetary boundary layer is best summarized by Refs. 14 and 15.

3.1 Mean Wind Velocity Profile

The power law is generally accepted as a good engineering approximation of the mean wind velocity profile (Refs. 14, 15), and has been well substantiated by in-field investigations (Refs. 16, 17):

\[
W(h) = W_G \left( \frac{h}{h_G} \right)^n
\]

\( W_G \) = Gradient Wind Speed

\( h_G \) = Gradient Height

\( n \) = Power Law Exponent

This model loses accuracy below 10 m where the logarithmic law (Ref. 18) is more suitable; but since the present work is not concerned with conditions below 20 m, the power law was adopted.

Power law index and gradient height were chosen as \( n = 0.16 \) and \( h_G = 305 \) m to be representative of conditions in rural areas (Refs. 14, 15), while a full-scale gradient wind speed of 20.1 m/s was chosen as adequately representing high wind conditions. These conditions were modelled by a wind tunnel gradient wind speed of 27.4 m/s at \( h_G = 91.4 \) cm, in order to obtain desired scale lengths and spectral shapes (ref. 5), thereby yielding the following scale factors (tunnel: full scale):

- Length--0.003
- Velocity--1.3636
- Time--0.0022

Simulation of the power law profile in the wind tunnel was achieved with good accuracy, as shown by Fig. 10, and good lateral uniformity in the area of concern, as shown by Fig. 11. Deviation from the power law was less than 1% in this area.

The 'Ekman spiral', a slight rotation of the mean wind vector from ground to gradient height caused by interaction between shear and centripetal forces, was neglected in the present work, as it has not been substantiated by in-field surveys (Refs. 16, 17).

3.2 Turbulence Intensity

Turbulence intensity is defined as \( \sigma_i/W \); where \( \sigma_i \) is the root-mean-square of the i-gust component (i = u, v, w), and W is the local mean wind speed. The turbulence intensities along the three inertial axes were measured, and compared to those suggested by the ESDU engineering model (Ref. 14), and by the in-field investigation of Ref. 16, for a power law exponent \( n = 0.16 \).
A cubic spline fit through the longitudinal turbulence intensity \( \sigma_u/W \) data measured in the wind tunnel is shown in Fig. 12, along with data from Refs. 14 and 16. The corresponding cubic spline fits to the lateral and vertical intensities \( \sigma_v/W \) and \( \sigma_w/W \) data are shown in Fig. 13 with data suggested by Ref. 14.

The three intensities are in the ratio 1.0:0.90:0.70 \( \sigma_u:\sigma_v:\sigma_w \) close to the tunnel floor, giving reasonable agreement with Ref. 12 which suggests ratios of 1.0:0.80:0.52. The turbulence intensities tend to become equal with increasing height indicating a tendency toward isotropy near the top of the shear layer; and are extrapolated to zero magnitude at the gradient height to facilitate later aircraft dispersion analysis.

The gust gradient intensities \( b_1\sigma_1/W, b_1\sigma_2/W, \) and \( \sigma_2/W \) (where \( r_1 = \frac{(u_2 - u_1)}{b}, p = \frac{(w_1 - w_2)}{b}, r_2 = \frac{(v_0 - v_3)}{b} \)) were measured for later use in non-dimensionalizing the gust gradient correlations, and are shown in Fig. 14. The significance of these gust gradients is further discussed in Sec. 4.1. No published data is available for comparison.

In general, the turbulence intensity simulation was considered satisfactory, and could not have been further improved without adversely affecting the spectral shape (Ref. 5).

### 3.3 Power Spectral Density and Scale Length

The power spectral density, as used in the present work is defined as the Fourier transform of the one-dimensional autocorrelation function:

\[
\Phi_{ii}(k) = 2 \int_{-\infty}^{\infty} R_{ii}'(\tau) \exp(-j2\pi k \tau) d\tau \quad (3.2)
\]

where \( i = u, v, w; k \) is the reduced frequency \( f/W \) corresponding to the \( x \)-direction, and \( \tau \) is the correlation time delay. The three one-dimensional spectral densities, \( \Phi_{uu}(k), \Phi_{vv}(k), \) and \( \Phi_{ww}(k) \) were measured at various heights along the 15 degree glideslope, and yielded the same results as Ref. 1. A representative plot is shown in Fig. 15, and is compared to the von Kármán model given by Ref. 1:

\[
\Phi_{uu}(k) = \frac{4L_x^x \sigma_u^2}{[1 + 70.7(L_x^x k)^2]^{5/6}} \frac{1}{W} \quad (3.3)
\]

As found in Ref. 1, the von Kármán spectral shape is well simulated for \( h/h_0 = 0.111 \) to 0.688, allowing the use of the spectral peak method outlined in Ref. 12 to estimate the integral scale length \( L_x^x \) where:

\[
L_x^x = W \int_0^{\infty} R_{ii}'(\tau) d\tau \quad (3.4)
\]

The resultant scale lengths, \( L_x^x, L_y^x, L_z^x \) are roughly a measure of the dominant wavelength of the longitudinal, lateral and vertical spectral densities respectively. These are plotted in Figs. 16, 17, and 18, along with the curves
suggested by the engineering models of Refs. 12 and 14, and the in-field data of Ref. 17. Agreement is considered satisfactory, considering the wide disparity of data in published literature.

3.4 Gust Probability Distribution

The gust probability distribution is an indication of the relative frequency of occurrence of various gust magnitudes, and is generally assumed to be Gaussian (normal) (Refs. 11, 12, 14, 15).

The quantities of primary interest in the present application are the third and fourth moments \( M_i^3, M_i^4 \) of the probability distribution:

\[
M_i^n \approx \frac{1}{T} \int_0^T \left( \frac{i(t)}{\sigma_i} \right)^n dt \\
= 1, 2, 3 \\
i = u, v, w
\]  
(3.5)

The third moment is an indication of the skewness of the distribution (Gaussian \( M_i^3 = 0 \)), while the fourth moment is a measure of its peakiness (Gaussian \( M_i^4 = 3.0 \)) as shown in Fig. 19.

In-field results for the third moment are generally unavailable, though Refs. 19 and 20 mention that non-zero values are to be expected in a shear flow. In-field results indicating that the fourth moment tends to be slightly larger than 3.0 are best summarized by Ref. 13 which describes aircraft-collected data in thermally stable and unstable conditions at 76 m and 229 m altitude. Table 2 presents wind-tunnel measured results along with the in-field data of Ref. 13. The probability distributions of the wind tunnel gust velocities \( (u, v, w) \), measured according to the guidelines of Ref. 10, are shown in Figs. 20, 21, and 22 for three heights in the shear layer, and compared to the Gaussian distribution.

The above figures indicate increasing skewness, and fourth moments much larger than 3.0 toward the top of the shear layer, thereby deviating from in-field data. This deviation was considered acceptable since the two-position space-time correlation method used in the present work makes no assumptions pertaining to gust probability distributions; nor predictions concerning the aircraft response distribution.

IV. DEVELOPMENT OF THE AIRCRAFT MODEL AND TWO-POSITION CORRELATION METHOD

The equations of motion of a STOL aircraft on a landing glidepath, and the development of the two-position correlation technique are considered in this chapter. The equations developed are the basis of the computer code given in Appendix B, which is used to predict rms aircraft dispersions caused by the wind tunnel-measured turbulence.

4.1 Single and Four-Point Aircraft Approximations

An evaluation of flight through turbulence must make certain assumptions regarding the geometric relationship between the gust field and aircraft size. If typical wavelengths of the turbulence are much larger than the aircraft, gusts can be assumed to be uniform over its span and length, and the aircraft
is, in effect, treated as a point. This assumption has proven to be adequate for the longitudinal case (Refs. 1, 11), but is suspected to be inadequate for the lateral case (Refs. 6, 21, 22, 23, 24) where the only gust disturbances then considered are uniform side gusts. The limit of validity of the point approximation has been given as $b/L^X_v = 0.10$ by Ref. 6. Since $b/L^X_v$ and $b/L^X_w$ are typically on the order of 0.5 for the present situation, a more realistic aircraft representation is in order.

The works of Refs. 11, 22, and 23 suggesting more complex aircraft representations to account for non-uniform gust fields, are similar in that they account for spatial gust distributions by approximating the aircraft as a series of 3, 4, or 5 points, and assuming a linear gust variation between these points. The present work follows this procedure by representing the aircraft by the four planar points shown in Fig. 2:

i) Point 0: the aircraft center of mass.

ii) Points 1 and 2: positions 0.85 $b/2 (=b'/2)$ along the right and left wings respectively, as suggested by Ref. 23 to best represent an elliptical lift distribution.

iii) Point 3: the vertical tail aerodynamic center.

Gust properties are measured at these points, and are assumed to vary linearly between them.

The complete gust matrix (in $F_B$) would consider three gust velocities at the center of mass (denoted $u_{0g}$, $v_{0g}$, $w_{0g}$), and nine gust gradients:

$$
\left( \frac{d\mathbf{D}_g}{d\mathbf{D}_B} \right)_c = \begin{pmatrix}
\frac{\partial u_g}{\partial x_B} & \frac{\partial u_g}{\partial y_B} & \frac{\partial u_g}{\partial z_B} \\
\frac{\partial v_g}{\partial x_B} & \frac{\partial v_g}{\partial y_B} & \frac{\partial v_g}{\partial z_B} \\
\frac{\partial w_g}{\partial x_B} & \frac{\partial w_g}{\partial y_B} & \frac{\partial w_g}{\partial z_B}
\end{pmatrix}
$$

(4.1)

In order to reduce the quantity of data to be gathered, the relative importance of various gusts on the lateral response is considered. Since the four aircraft points are assumed to lie in a plane, the vertical gradients are eliminated: $\partial u_g/\partial z_B = \partial v_g/\partial z_B = \partial w_g/\partial z_B = 0$. Further, because the present work is concerned only with lateral response, the longitudinal terms are discarded: $u_{0g} = w_{0g} = \partial u_g/\partial x_B = \partial w_g/\partial x_B = 0$. Finally, the gradient $\partial v_g/\partial y_B$ is assumed to have negligible effect on the aircraft's lateral response (Refs. 23, 24). The resultant gust contributions considered are therefore $v_{0g}$, as in the point approximation, and the gradients given by:
Treatment of the gust gradients will be performed as in Refs. 11, 21, and 23 by noting that they are equivalent to aircraft rotations in the opposite sense. As such, $-\partial u_g/\partial y_B$ and $\partial v_g/\partial x_B$ are assumed equivalent to a negative yaw rate of the wing and wingless aircraft respectively, and $\partial w_g/\partial y_B$ is considered equivalent to a negative wing roll rate. Pursuant to the convention of Ref. 21, we denote:

\[ r_{1g} = -\frac{\partial u_g}{\partial y_B} = \frac{(u_{2g} - u_{1g})}{b'} \]

\[ r_{2g} = \frac{\partial v_g}{\partial x_B} = \frac{(v_{0g} - v_{3g})}{l_T} \]

\[ p_g = \frac{\partial w_g}{\partial y_B} = \frac{(w_{1g} - w_{2g})}{b'} \]

The aircraft equations of motion can now be derived for the four-point approximation.

### 4.2 Aircraft Equations of Motion

The equations of motion, taken from Refs. 11, 21, and 24 (using the notation of Ref. 21), are presented here for convenience, and are the lateral analogue of the longitudinal equations developed in Ref. 1. A flat, non-rotating earth, and rigid aircraft are assumed, and forces are separated according to still-air aerodynamic, turbulent aerodynamic, and control origin. Two reference frames are considered (Fig. 2):

1/ The inertial, earth-fixed frame $F$, made up of axes $x_I$, $y_I$, $z_I$ with $x_I$ aligned with the center of the level runway and pointing upwind, $z_I$ downward, and $y_I$ to the right (looking upwind).

2/ The aircraft body-fixed frame $F_B$, made up of axes $x_B$, $y_B$, $z_B$ where $x_B$ is aligned with the equilibrium airspeed vector, $z_B$ generally downward, $y_B$ to the right, and the $x_B$-$z_B$ plane in the aircraft's vertical plane of symmetry.

Transformations from inertial to body-fixed frames are performed using the transformation matrix $R_{BI}$.
\[
Z_B = R_{BI} Z_I
\]

where \( Z \) is an arbitrary vector

\[
R_{BI} = \begin{pmatrix}
\cos \theta \cos \psi & \cos \phi \sin \psi & -\sin \phi \\
\sin \phi \sin \theta \cos \psi & \sin \phi \sin \phi \sin \psi & \cos \phi \cos \psi \\
-\cos \phi \sin \theta & \cos \phi \cos \psi & \sin \phi \\
\end{pmatrix}
\]  

(4.4)

\( \phi, \theta, \) and \( \psi \) are the rigid body dynamics Euler angles defined in Ref. 11.

4.2.1 **Complete Equations of Motion**

The non-linear equations of motion for an aircraft flying in the presence of turbulence are:

\[
p = F,
\]

\[
h = M,
\]

\[
V_E = V + W
\]

where \( p \) and \( h \) are the linear and angular momentum of the aircraft respectively; \( F \) and \( M \) are the total external forces and moments exerted on the aircraft respectively; \( V_E \) and \( V \) are the aircraft ground speed and airspeed respectively; and \( W \) is the wind velocity. The terms are expanded to yield the following equations in terms of inertial quantities as components in \( F_B \):

\[
m (\vec{U}_E + qw_E - rv_E) = X + m g r_{13}
\]

\[
m (\vec{V}_E + ru_E - pw_E) = Y + m g r_{23}
\]

\[
m (\vec{W}_E + pv_E - qu_E) = Z + m g r_{33}
\]

\[
I_{xx} \ddot{\theta} - I_{xz} (\ddot{\phi} + p q) + (I_{zz} - I_{yy}) q r = L
\]

\[
I_{yy} \ddot{\phi} - I_{xz} (r^2 - p^2) + (I_{xx} - I_{zz}) r p = M
\]

\[
I_{zz} \ddot{\psi} - I_{xz} (\ddot{\phi} - r q) + (I_{yy} - I_{xx}) p q = N
\]

(4.7)

where \( r_{ij} \) is the \( ij \)'th component of \( R_{BI} \).
The kinematical relations are noted to complete the system equations:

\[
\begin{align*}
\dot{x}_I &= r_{11}u_E + r_{21}v_E + r_{31}w_E \\
\dot{y}_I &= r_{12}u_E + r_{22}v_E + r_{32}w_E \\
\dot{z}_I &= r_{13}u_E + r_{23}v_E + r_{33}w_E \\
\dot{\phi} &= q \sin\phi \tan\theta + r \cos\phi \tan\theta \\
\dot{\theta} &= q \cos\phi - r \sin\phi \\
\dot{\psi} &= q \sin\phi \sec\theta + r \cos\phi \sec\theta
\end{align*}
\]  

(4.8)

4.2.2 Linearized Decoupled Equations

In order to produce more tractable equations amenable to linear analysis, a reference equilibrium condition is chosen, and the equations are linearized about that condition.

The selected reference trajectory is aligned with the glideslope and negotiated at constant airspeed in still air. The aircraft motion parameters can now be written as perturbations about the reference equilibrium. In general:

\[
Z = Z_e + \Delta Z
\]  

(4.10)

where \(Z\) is a generalized aircraft motion parameter, and \(Z_e\) and \(\Delta Z\) are its equilibrium values respectively. The relationships are therefore:

\[
\begin{align*}
\begin{pmatrix}
    u_E \\
v_E \\
w_E
\end{pmatrix}
&= 
\begin{pmatrix}
    V_{Ee} + \Delta u_E \\
    V_e \\
    \Delta w_E
\end{pmatrix}
\begin{pmatrix}
    \Delta V_e \\
\end{pmatrix}
\begin{pmatrix}
    \Delta w_E
\end{pmatrix}
\end{align*}
\]

(4.11)

Assumptions inherent in the linear treatment are that the cos and sin of a perturbation angle are replaced by the perturbation angle and 1, respectively; and that only first-order perturbations are kept in the equations. The resulting equations about the equilibrium become:

\[
\begin{align*}
m \Delta \ddot{u}_E &= \Delta X - mg \Delta \theta \cos \theta_e \\
m (\Delta \ddot{v}_E + \Delta r V_{Ee}) &= \Delta Y + mg \Delta \phi \cos \theta_e
\end{align*}
\]  

(4.12)
\[ m \left( \Delta \dot{w}_E - \Delta q v_E \right) = \Delta Z - mg \Delta \theta \sin \theta_e \]
\[ I_{xx} \Delta \ddot{p} - I_{xz} \Delta \ddot{r} = \Delta L \]
\[ I_{yy} \Delta \ddot{q} = \Delta M \]
\[ I_{zz} \Delta \ddot{r} - I_{xz} \Delta \ddot{p} = \Delta N \]  
\[(4.13)\]

\[ \Delta \dot{x}_I = V_{Ee} \Delta \theta \sin \theta_e + \Delta u_E \cos \theta_e + \Delta w_E \sin \theta_e \]
\[ \Delta \dot{y}_I = V_{Ee} \Delta \psi \cos \theta_e + \Delta v_E \]
\[ \Delta \dot{z}_I = -V_{Ee} \Delta \theta \cos \theta_e - \Delta u_E \sin \theta_e + \Delta w_E \cos \theta_e \]  
\[(4.14)\]

\[ \Delta \dot{\phi} = \Delta \dot{p} + \Delta r \tan \theta_e \]
\[ \Delta \dot{\theta} = \Delta \dot{q} \]
\[ \Delta \dot{\psi} = \Delta r \sec \theta_e \]  
\[(4.15)\]

Note that the longitudinal and lateral equations are decoupled as a result of the linearization, and the lateral case can now be treated individually. The lateral equations are collected, and force and moment terms are separated into aerodynamic and control contributions:

\[ m \left( \Delta \dot{v}_E + V_{Ee} \Delta r - g \Delta \phi \cos \theta_e \right) = \Delta Y_a + \Delta Y_c \]
\[ I_{xx} \Delta \ddot{p} - I_{xz} \Delta \ddot{r} = \Delta L_a + \Delta L_c \]  
\[(4.16)\]

\[ \Delta \dot{v}_I = V_{Ee} \Delta \psi \cos \theta_e + \Delta v_E \]
\[ \Delta \dot{\phi} = \Delta \dot{p} + \Delta r \tan \theta_e \]
\[ \Delta \dot{\psi} = \Delta r \sec \theta_e \]  
\[(4.17)\]

The aerodynamic forces and moments can be written in terms of dimensional stability derivatives (Ref. 11) as follows:

\[ \Delta Y_a = Y_v \Delta v_r + Y_p \Delta p_r + Y_r \Delta r_r \]
\[ \Delta L_a = L_v \Delta v_r + L_p \Delta p_r + L_r \Delta r_r \]  
\[(4.18a)\]

\[ \Delta N_a = N_v \Delta v_r + N_p \Delta p_r + N_r \Delta r_r \]
where $\Delta v_r$, $\Delta p_r$, and $\Delta r_r$ are the variables representing the relative motion between the aircraft and the surrounding air. These relative rates are made up of aircraft velocities in $F_1$ and gust effects, and can be separated as follows:

\[
\begin{align*}
\Delta v_r &= \Delta v_E - v_{og} \\
\Delta p_r &= \Delta p - p_g \\
\Delta r_r &= \Delta r - r_g
\end{align*}
\] (4.18b)

When these relations are substituted into Eqn. 4.16, and the control contributions are written in terms of dimensional stability derivatives (Ref. 11), the lateral equations become:

\[
\begin{align*}
m (\Delta \dot{v}_E + V_{Ee} \Delta r - g \Delta \phi \cos \theta_e) &= Y_v (\Delta v_E - v_{og}) + Y_p (\Delta p - p_g) \\
&\quad + Y_r (\Delta r - r_g) + Y_{\delta a} \delta a + Y_{\delta r} \delta r \\
I_{xx} \Delta \dot{p} - I_{xz} \Delta \dot{r} &= L_v (\Delta v_E - v_{og}) + L_p (\Delta p - p_g) \\
&\quad + L_r (\Delta r - r_g) + L_{\delta a} \delta a + L_{\delta r} \delta r \\
I_{zz} \Delta \dot{r} - I_{xz} \Delta \dot{p} &= N_v (\Delta v_E - v_{og}) + N_p (\Delta p - p_g) \\
&\quad + N_r (\Delta r - r_g) + N_{\delta a} \delta a + N_{\delta r} \delta r
\end{align*}
\] (4.19)

and the kinematical relations remain unchanged (Eqn. 4.17).

A further breakdown of the gust-induced roll and yaw contributions is now necessary to distinguish these terms from true roll and yaw. We note that the gust-induced yaw rate ($r_{g}$), in general, stems from two causes: 1/ a spanwise gradient of the $u$-gust component ($\frac{\delta r_g}{b'} = (u_{2g} - u_{1g})/b'$) which affects only wing-related forces; or 2/ a $v$-gust gradient along the aircraft body ($r_{2g} = (v_{og} - v_{3g})/l_T$) which affects only wingless-aircraft related forces. Similarly, the spanwise gradient of the $w$-gust component ($p_{g} = (w_{1g} - w_{2g})/b'$) affects only wing-related forces. The stability derivatives are therefore separated into wing and wingless aircraft contributions (Ref. 21) as shown in Appendix A, and all motion parameters are modified accordingly, yielding the following equations:

\[
\begin{align*}
m (\Delta \dot{v}_E + V_{Ee} \Delta r - g \Delta \phi \cos \theta_e) &= Y_v \Delta v_E + Y_p \Delta p + Y_r \Delta r + Y_{\delta a} \delta a + Y_{\delta r} \delta r \\
&\quad - Y_v v_{og} - Y_{pW} p_g - Y_{rW} r_{1g} - Y_{rA} r_{2g} \\
I_{xx} \Delta \dot{p} - I_{xz} \Delta \dot{r} &= L_v \Delta v_E + L_p \Delta p + L_r \Delta r + L_{\delta a} \delta a + L_{\delta r} \delta r \\
&\quad - L_v v_{og} - L_{pW} p_g - L_{rW} r_{1g} - L_{rA} r_{2g}
\end{align*}
\] (4.20)
\[
\begin{align*}
I_{zz} \Delta \dot{r} - I_{xz} \Delta \dot{p} &= N_v \Delta v_E + N_p \Delta p + N_r \Delta r + N_{\delta a} \Delta \delta a + N_{\delta r} \Delta \delta r \\
&\quad - N_v v_0 g - N_p p g - N_r r_1 g - N_{rA} r_{2g}
\end{align*}
\]

where the \(W\) and \(A\) subscripts denote wing and wingless aircraft contributions respectively. The kinematical relations are given by Eqn. 4.17.

This system of equations can be written in matrix form as:

\[
A_1 \Delta \dot{x} = A_2 \Delta x + B_1 \Delta u_c - C_1 \varphi
\]

Where

\[
\Delta x^T = (\Delta v_E \quad \Delta p \quad \Delta r \quad \Delta y_1 \quad \Delta \phi \quad \Delta \psi)
\]

\[
\Delta u_c^T = (\Delta \delta a \quad \Delta \delta r)
\]

\[
\varphi^T = (v_0 g \quad p g \quad r_{1g} \quad r_{2g})
\]

And,

\[
A_1 = \begin{pmatrix}
1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & -I_{xz}/I_{xx} & 0 & 0 & 0 & 0 \\
0 & -I_{xz}/I_{zz} & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0
\end{pmatrix}
\]

\[
A_2 = \begin{pmatrix}
Y_v/m & Y_p/m & Y_r/m - V_{Ee} & 0 & g \cos \theta_e & 0 \\
L_v/I_{xx} & L_p/I_{xx} & L_r/I_{xx} & 0 & 0 & 0 \\
N_v/I_{zz} & N_p/I_{zz} & N_r/I_{zz} & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & 0 & V_{Ee} \cos \theta_e \\
0 & 1 & \tan \theta_e & 0 & 0 & 0 \\
0 & 0 & \sec \theta_e & 0 & 0 & 0
\end{pmatrix}
\]
We can now reduce the matrix equations as follows:

\[ \Delta \dot{x} = A \Delta x + B_2 \Delta u_c - C_2 q \]  

(4.29)

where \( (A, B_2, C_2) = A_1^{-1} (A_2, B_1, C_1) \)  

(4.30)

The determinant of \( A \), the system matrix, yields the natural modes of the uncontrolled aircraft (Ref. 11). As shown in Refs. 6 and 24, the aircraft under consideration exhibits a divergent spiral mode, and a relatively weakly damped Dutch roll mode. An ideal, lag-free stability augmentation system (SAS) is therefore included by Ref. 24 to improve the aircraft's stability -- simple proportional feedbacks of roll angle (\( \phi \)) to ailerons, and yaw rate (\( r \)) to the rudder are used to stabilize the spiral mode and improve Dutch roll damping. As outlined in Ref. 24, no attempt was made to provide feedback gains of a true-to-life pilot or stability augmentation system, but rather with intent on keeping perturbations within the linearity assumption (except for \( \Delta \psi \), whose departure from small perturbation theory will be discussed in Sec. 5.2). The perturbation results of the aircraft simulation are therefore not intended to be realistic in magnitude; but rather to be used in comparing aircraft response to various turbulence models (e.g.: comparing the single and four-point aircraft approximations). The SAS gains and augmented aircraft characteristics are discussed in Appendix A.

Since the controls are actuated for augmentation purposes only, and the SAS responds proportionally to the state vector (Appendix A), the control terms can now be included in the system matrix:

\[ \Delta \dot{x} = F \Delta x - C_2 q \]  

(4.31)
where
\[ \Delta u_c = -K \Delta x \] (4.32)

and
\[ F = (A - B_2 K) \] (4.33)

\( K \) is given in Appendix A.

A final transformation must now be introduced to transform the wind tunnel measurements from \( F_I \) to \( F_B \). The transformation of a vector \( D \) in inertial axes to its equivalent \( D_g \) in body axes at the reference equilibrium condition is given by:

\[ D_g = \begin{bmatrix} u_g \\ v_g \\ w_g \end{bmatrix} = R_{BIe} \begin{bmatrix} u \\ v \\ w \end{bmatrix} = R_{BIe} D \] (4.34)

where \( R_{BIe} \) is the transformation matrix \( R_{BI} \) of Sec. 4.2, at the reference equilibrium condition:

\[ R_{BIe} = \begin{bmatrix} \cos \theta_e & 0 & -\sin \theta_e \\ 0 & 1 & 0 \\ \sin \theta_e & 0 & \cos \theta_e \end{bmatrix} \] (4.35)

Pursuant to Sec. 4.1, the desired gust gradient matrix, \( dD/dB \) is given by Eqn. 4.2. When these gradient elements act between two aircraft points \( i \) and \( j \) separated by a vector \( \Delta d \) (written as \( \Delta d_B \) in \( F_B \) and \( \Delta d_I \) in \( F_I \)), the resultant velocity vector change is given by:

\[ \Delta D_g = \frac{dD_g}{dB} \Delta d_B \] (in \( F_B \)) (4.36a)

or

\[ \Delta D = \frac{dD}{dI} \Delta d_I \] (in \( F_I \)) (4.36b)

where \( dD/dI \) is a gradient matrix of the form of Eqn. 4.2, but with subscripts corresponding to \( F_I \).

If the vectors \( \Delta D \) and \( \Delta d_I \) in Eqn. 4.36b are rewritten in terms of components in \( F_B \), we obtain:
Premultiplying by $R_{\text{BI}}$ yields:

$$\Delta D_g = R_{\text{BI}} \frac{dD}{dd_I} R_{\text{BI}}^T \Delta d_B \quad (4.37a)$$

Comparing Eqns. 4.36a and 4.37b, we obtain:

$$\frac{dD_g}{dd_B} = R_{\text{BI}} \frac{dD}{dd_I} R_{\text{BI}}^T \quad (4.38)$$

Note that the matrix $dD_g/dd_B$ is the desired matrix of gust gradients in $F_B$, while $dD/dd_I$ is the matrix of measured gust gradients in $F_I$.

When Eqn. 4.38 is written explicitly, and Eqn. 4.3 is included, the resulting relations of interest are:

$$r_1 g = -\frac{\partial u_g}{\partial y_B} = r_1 \cos \theta_e + p \sin \theta_e \quad (4.39a)$$

$$r_2 g = \frac{\partial v_g}{\partial x_B} = r_2 \cos \theta_e$$

$$p_g = \frac{\partial w_g}{\partial y_B} = p \cos \theta_e - r_1 \sin \theta_e$$

Or alternatively in matrix form:

$$q = G_{\text{BI}} q_I \quad (4.39b)$$

where

$$q_I^T = \begin{pmatrix} v_0 & p & r_1 & r_2 \end{pmatrix}$$

and,

$$G_{\text{BI}} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos \theta_e & -\sin \theta_e & 0 \\ 0 & \sin \theta_e & \cos \theta_e & 0 \\ 0 & 0 & 0 & \cos \theta_e \end{pmatrix} \quad (4.39c)$$

16.
The final system of equations can now be written as:

$$\Delta x = F \Delta x - C g_l$$  \hspace{1cm} (4.40)$$

where

$$C = C_2 G_{BI}$$

The two-position correlation technique is now approached as a means of solving the above system for root-mean-square response.

4.3 Two-Position Correlation Technique

The two-position flight path correlation technique adopted to solve Eqn. 4.40 is derived in Ref. 4, and repeated here for convenience.

The gust response at time t of a linear aircraft (whose altitude is given by \(h(t)\)) descending on a glideslope, is a perturbation in the state vector \(\Delta x(t)\) given by:

$$x(t) = \int_0^t H(t, t') g(h(t'), t') \, dt'$$  \hspace{1cm} (4.41)$$

Where \(g(h(t'), t')\) represents the gusts encountered at altitude \(h\) and time \(t'\), and \(H(t, t')\) is the matrix of impulse response functions \((h_{ij})\), in which each element \(h_{ij}\) is the response of a state variable \(\Delta x_i\) to an impulse in the gust input \(g_j (g_j = \delta(t-t'))\). The products of the perturbed state vector are given by:

$$\Delta x \Delta x^T = \begin{pmatrix} \Delta x_1^2 & \Delta x_1 \Delta x_2 & \cdots & \cdots & \cdots \\ \Delta x_2 \Delta x_1 & \Delta x_2^2 & \cdots & \cdots & \cdots \\ \vdots & \vdots & \ddots & \cdots & \cdots \\ \vdots & \vdots & \cdots & \ddots & \cdots \\ \vdots & \vdots & \cdots & \cdots & \ddots \end{pmatrix}$$  \hspace{1cm} (4.42)$$

where the main diagonal represents the square of the state vector. Substituting Eqn. 4.41 into Eqn. 4.42 yields:

$$\Delta x \Delta x^T(t) = \int_0^t \int_0^t H(t, \alpha) g(h_1, \alpha) g^T(h_2, \beta) H^T(t, \beta) \, d\alpha d\beta$$  \hspace{1cm} (4.43)$$

where \(\alpha\) and \(\beta\) are dummy time variables, and \(h_1\) and \(h_2\) are the corresponding heights of the reference equilibrium trajectory.
The ensemble mean of the state vector is given by:

\[
\langle \Delta x \Delta x^T(t) \rangle = \int_0^t \int_0^t \mathcal{H}(t,\alpha) \langle g(h_1,\alpha) g^T(h_2,\beta) \rangle H^T(t,\beta) \, d\alpha d\beta \quad (4.44)
\]

The two-position correlation matrix is now defined as:

\[
R_{gg}(h_1, h_2 - h_1, \beta - \alpha) = \langle g(h_1,\alpha) g^T(h_2,\beta) \rangle \quad (4.45)
\]

Since \( h_1 \) and \( h_2 \) are implicit functions of \( \alpha \) and \( \beta \) respectively, the matrix \( R_{gg} \) to be used in Eqn. 4.44 is a constrained version of Eqn. 4.45, and the final equation can be written as:

\[
\langle \Delta x \Delta x^T(t) \rangle = \int_0^t \int_0^t \mathcal{H}(t,\alpha) R_{gg}(\alpha,\beta - \alpha) H^T(t,\beta) \, d\alpha d\beta \quad (4.46)
\]

where the main diagonal of \( \langle \Delta x \Delta x^T(t) \rangle \) yields the mean square of the state variables at time \( t \). Since the system impulsive response is purely a system property given by:

\[
H(t,t') = [\exp((t-t')F)]^{-C} \quad (4.47)
\]

where \( F \) and \( C \) are defined in Sec. 4.2.2; the only additional quantity required to solve Eqn. 4.46 is \( R_{gg}(\alpha,\beta - \alpha) \). The evaluation of this constrained flight path correlation matrix was the purpose of the wind tunnel measurements, and is discussed in Chapter 5.

4.4 Computer Implementation

The computer implementation developed in Ref. 24 (which is the lateral analogue of the longitudinal program of Ref. 1), was modified to adopt the gust gradient convention of Ref. 21. The equations of motion derived in Sec. 4.2.2 were programmed on an IBM 3033 digital computer in order to solve Eqn. 4.46. The aircraft characteristics given in Appendix A were used, and a Runge-Kutta scheme was implemented to perform the required integration. The computer code is given in Appendix B, along with a sample output.

5. EXPERIMENTAL RESULTS

5.1 Correlation Results

This section presents the wind tunnel-measured results of \( R(\alpha,\beta - \alpha) \) required to solve Eqn. 4.46 (where \( R = \langle gIg^T \rangle \); and it can be shown that \( R_{gg} = G_{BI} R G_{BI}^T \)).
The complete matrix $\mathbf{R}$ consists of the following elements:

$$
\mathbf{R} = \begin{pmatrix}
R_{v_0v_0} & R_{v_0w_1} & R_{v_0w_2} & R_{v_0r_1} & R_{v_0v_3} \\
R_{w_1w_1} & R_{w_1w_2} & R_{w_1r_1} & R_{w_1v_3} \\
R_{w_2w_1} & R_{w_2w_2} & R_{w_2r_1} & R_{w_2v_3} \\
R_{r_1v_0} & R_{r_1w_1} & R_{r_1w_2} & R_{r_1r_1} & R_{r_1v_3} \\
R_{v_3v_0} & R_{v_3w_1} & R_{v_3w_2} & R_{v_3r_1} & R_{v_3v_3}
\end{pmatrix}
$$

(5.1)

Since only four channels of hot-wire anemometry were available, correlations requiring more than two $v$ or $w$ components could not be measured directly at one time. An indirect approach was utilized by manipulating identities as follows (for example):

$$
R_{pp}(\alpha,\beta - \alpha) = \langle p' p' \rangle \\
= \langle \left( \frac{w_1 - w_2}{b'} \right) \left( \frac{w_1' - w_2'}{b'} \right) \rangle \\
= \langle (w_1w_1') + (w_2w_2') - (w_1w_2') - (w_2w_1') \rangle / b'^2 \\
= [ \langle w_1w_1' \rangle + \langle w_2w_2' \rangle - \langle w_1w_2' \rangle - \langle w_2w_1' \rangle ] / b'^2 \\
= [ R_{w_1w_1'} + R_{w_2w_2'} - R_{w_1w_2'} - R_{w_2w_1'} ] / b'^2
$$

(5.2)

Similar relations can be derived for the other terms not directly measurable, and the complete measurable correlation matrix becomes:

$$
\mathbf{R}_m = \begin{pmatrix}
R_{v_0v_0} & R_{v_0w_1} & R_{v_0w_2} & R_{v_0r_1} & R_{v_0v_3} \\
R_{w_1w_1} & R_{w_1w_2} & R_{w_1r_1} & R_{w_1v_3} \\
R_{w_2w_1} & R_{w_2w_2} & R_{w_2r_1} & R_{w_2v_3} \\
R_{r_1v_0} & R_{r_1w_1} & R_{r_1w_2} & R_{r_1r_1} & R_{r_1v_3} \\
R_{v_3v_0} & R_{v_3w_1} & R_{v_3w_2} & R_{v_3r_1} & R_{v_3v_3}
\end{pmatrix}
$$

(5.3)
where each element varies as a function of \( \alpha \), the flight time taken to reach the upper probe position (see Table 3), and \( (\beta - \alpha) \), the flight time corresponding to the probe separation. This combination of 25 elements measured at seven upper probe positions, and nine probe separations (tabulated in Table 4), as in Ref. 1, would involve 1,575 sets of measurements \( (25 \times 7 \times 9) \). Clearly, the time and effort required to evaluate the complete matrix \( R_m \) would have been prohibitive, and some simplification was in order. A preliminary investigation was therefore embarked upon to find the relative magnitude of all the elements.

5.1.1 Preliminary Investigation

In order to determine whether certain elements of the matrix \( R_m \) could be neglected, a preliminary experimental investigation was performed by evaluating all 25 elements of \( R_m \) near the middle of the shear layer \( (h_1 = 35.56 \text{ cm}) \).

This investigation yielded the 25 non-dimensional elements of the \( 5 \times 5 \) matrix \( \tilde{R}_m \) in which each element \( R_{ij} \) of the matrix \( R_m \) is non-dimensionalized by the root-mean-square of the \( i \) and \( j \) components at their respective probe heights. The results are shown in Figs. 23 to 32 for a velocity ratio \( V/W_G = 2.0 \), and compared to the data of Ref. 1 where applicable. Corresponding correlations for \( V/W_G = 1.0 \) and 1.5 are not presented here for brevity, but exhibited the same trends as the results shown. The non-dimensional correlations were dimensionalized using the measured turbulence intensities shown in Figs. 13 and 14, and combined using identities of the form of Eqn. 5.2 to obtain the 16 elements of the \( 4 \times 4 \) matrix \( R \) shown in Figs. 33 to 42.

An examination of these elements revealed that the only off-diagonal elements of significant magnitude were \( R_{r_2}v_0 \) and \( R_{r_1}v_1 \), thereby agreeing with Ref. 24 which suggests that only the off-diagonal terms involving \( v_0 \) and \( v_3 \) need be considered with the diagonal terms. To verify this assumption, the computer program described in Sec. 4.4 was used to obtain the aircraft response from a height of 118.5 m to 63.5 m with two turbulence input cases:

1/ The full 16-element correlation matrix given by Eqn. 5.1.

2/ The reduced 6-element correlation matrix including the 4 diagonal elements, and \( R_{r_2}v_2 \) and \( R_{r_1}v_1 \).

The results obtained, shown in Fig. 69, indicate that the error incurred by neglecting all the off-diagonal components of \( R \) other than \( R_{r_2}v_2 \) and \( R_{r_1}v_1 \) is extremely small. The remaining elements of the matrix \( R \) are therefore:

\[
R_R = \begin{pmatrix}
R_{v_0}v_0 & -- & -- & R_{r_2}v_2 \\
-- & R_{pp} & -- & -- \\
-- & -- & R_{r_1}r_1 & -- \\
R_{r_2}v_0 & -- & -- & R_{r_2}r_2
\end{pmatrix}
\]  

(5.4)
which can be obtained from the following measurable matrix:

\[
\begin{pmatrix}
R_{v_0v_0} & -- & -- & -- & R_{v_0v_3} \\
-- & R_{w_1w_1} & R_{w_1w_2} & -- & -- \\
-- & R_{w_2w_1} & R_{w_2w_2} & -- & -- \\
-- & -- & -- & R_{r_1r_1} & -- \\
R_{v_3v_0} & -- & -- & -- & R_{v_3v_3}
\end{pmatrix}
\]

(5.5)

Some further simplifications resulting from symmetry are noted:

\[
R_{v_0v_0} = R_{v_3v_3} \quad \text{(verified by Fig. 23)}
\]

\[
R_{w_1w_1} = R_{w_2w_2} \quad \text{(verified by Fig. 25)} \quad \text{(5.6)}
\]

\[
R_{w_1w_2} = R_{w_2w_1} \quad \text{(verified by Fig. 26)}
\]

Inserting Eqns. 5.6 in identities of the form of Eqn. 5.2 yields the final relations between the required elements of \( \mathbf{R} \) and the measurable terms of \( \mathbf{R}_{\text{m}} \):

\[
R_{v_0v_0} = R_{v_0v_0}
\]

\[
R_{pp} = 2\left[ R_{w_1w_1} - R_{w_1w_2} \right] / b^2
\]

\[
R_{r_1r_1} = R_{r_1r_1}
\]

\[
R_{r_2r_2} = \left[ 2 R_{v_0v_0} - R_{v_0v_3} - R_{v_3v_0} \right] / \ell_T^2
\]

\[
R_{v_0r_2} = \left[ R_{v_0v_0} - R_{v_0v_3} \right] / \ell_T
\]

\[
R_{r_2v_0} = \left[ R_{v_0v_0} - R_{v_3v_0} \right] / \ell_T
\]

And in non-dimensional terms:
\[
\begin{align*}
\hat{R}_{v_0v_0} &= \hat{R}_{v_0v_0} \\
\hat{R}_{pp} &= 2\left(\hat{R}_{w_1w_2} - \hat{R}_{v_1v_2}\right) \left(\frac{\sigma_v \sigma'_v}{\sigma_p \sigma'_p}\right) \\
\hat{R}_{r_1r_1} &= \hat{R}_{r_1r_1} \\
\hat{R}_{r_2r_2} &= 2\left(\hat{R}_{v_0v_0} - \hat{R}_{v_0v_3} - \hat{R}_{v_3v_0}\right) \left(\frac{\sigma_v \sigma'_v}{\sigma_r \sigma'_r}\right) \\
\hat{R}_{v_0r_2} &= \left[\hat{R}_{v_0v_0} - \hat{R}_{v_0v_3}\right] \left[\sigma_v / \sigma_r^2\right] \\
\hat{R}_{r_2v_0} &= \left[\hat{R}_{v_0v_0} - \hat{R}_{v_3v_0}\right] \left[\sigma_v / \sigma_r^2\right]
\end{align*}
\] (5.8)

where the unprimed intensities are measured at the upper probe height, and the primed intensities are measured at the lower probe height.

It was noted, however, that the calculated correlations, \(R_{pp}, R_{r_2r_2}, R_{v_0r_2},\) and \(R_{r_2v_0}\) were, at times, the result of small differences in large values of the measured correlations. The effect of statistical variability in the measured data would therefore create a large variability in the calculated data. This question was addressed by a statistical survey.

5.1.2 Statistical Survey

In order to analyze the effect of the measured correlations' statistical variability on the calculated correlations, sets of four samples of each element of \(R_{Rm}\) (Eqn. 5.5) were measured near the middle of the shear layer \((h_1 = 35.56 \text{ cm})\), and dimensionalized by the measured intensities shown in Figs. 13 and 14. Sampling time used for each sample was 20 sec, as suggested by Ref. 1, and as used in the preliminary investigation of Sec. 5.1.1. The measured correlations were then used to calculate four samples of each element of \(R_R\) (Eqn. 5.4), thereby yielding the mean and standard deviation of these elements shown in Figs. 43 to 48.

It was observed that the standard deviations of the elements of \(R_R\) were significantly larger than those of the measured components of \(R_{Rm}\). In order to reduce the statistical variability of single samples of the calculated correlations \(R_R\), the component correlations \(R_{Rm}\) were measured using a sampling time of 80 sec for all final measurements. Since the sampling time was much larger than the time delay over which the elements exhibited a non-zero correlation, this was equivalent to averaging four separate 20 sec samples. This, in turn, would effectively halve the standard deviation bars of Figs. 43 to 48, since the statistical variability of the mean of a set of measurements is inversely proportional to the square root of the number of samples in the set (Ref. 25).

5.1.3 Final Correlations

The final correlations measured in the wind tunnel and used as input to the equations of motion described in Chapter 4, were obtained at a sampling rate of 2500 samples per second, for a sampling duration of 80 sec.
The elements of the measured correlation matrix \( \hat{R}_{Rm} \) are shown in Figs. 49 to 54 for all seven upper probe locations, and a velocity ratio \( V/W_G = 2.0 \). The corresponding elements of the calculated correlation matrix \( \hat{R}_R \) are given in Figs. 55 to 58.

It is observed that non-dimensionalization of the correlations considerably collapses the results for the various upper probe locations, consistent with the results of Ref. 1. If the non-dimensional correlations were to lie on a single curve, it would be an indication that they are not significantly affected by the shear flow, and that the results could possibly be approximated by an analytical model of homogeneous turbulence (eg: von Kármán model). It is further noted that the inability to evaluate \( \sigma_p \) and \( \sigma_{r_2} \) at the same time as the correlation measurements resulted in a slight deviation from unity in the measured zero time delay autocorrelations \( R_{pp}(\alpha,0) \) and \( R_{r_2r_2}(\alpha,0) \). However, since zero time delay non-dimensional autocorrelations are unity by definition, these values were modified accordingly.

The elements of the dimensional correlation matrix \( R_R \) (given by Eqn. 5.4) were calculated according to Eqn. 5.7, and used as input to the computer program described in Chapter 4 to obtain the mean square dispersion from the glidepath discussed in Sec 5.2.

Some examples of the correlations measured for \( V/W_G = 1.5 \) and \( 1.0 \) are shown in Figs. 59 to 68 to illustrate the consistency of these results. It was considered unnecessary (though entirely possible) to use these results in the computer simulation, as they would have been redundant for the present purpose of comparing the single and four-point aircraft approximations.

5.2 Dispersion of the Aircraft State Vector

The computer program outlined in Appendix B, and the wind tunnel measurements described in Sec. 5.1.3 were used to evaluate the rms dispersion of the aircraft state vector from the reference equilibrium condition. Since a linear aircraft model was used, the response to various gust contributions was additive, and the total response could be broken down into its components. The results shown in Figs. 70 to 75 were obtained using the following gust combinations:

1/ Complete gust field -- side gust \( \nu_{0g} \), longitudinal yawing gust gradient \( r_{2g} \), lateral yawing gust gradient \( r_{1g} \), and rolling gust gradient \( p_g \).

2/ \( \nu_{0g} \), \( r_{2g} \), and \( r_{1g} \) only.

3/ \( \nu_{0g} \) and \( r_{2g} \) only.

4/ \( \nu_{0g} \) only, yielding the point approximation.

The following conclusions can be drawn from an inspection of Figs. 70 to 75:

1/ The largest single contribution to lateral aircraft dispersions is that due to the rolling gust gradient \( p_g \).

2/ The point approximation is of little usefulness in predicting lateral aircraft response; thereby emphasizing the importance of using a turbulence model which adequately represents gust gradients along the aircraft wing and fuselage.
3/ The only state variable which is somewhat reasonably predicted by the point approximation is $\Delta v_E$, which is nevertheless 20% less than the complete gust field result throughout the descent.

4/ The point approximation yields lateral position dispersion ($\Delta y_I$) results which are so small that they appear to be identically zero due to the scale used in Fig. 73.

5/ The longitudinal yawing gust gradient contribution to the aircraft response is reasonably small. It contributes essentially nothing to the $\Delta v_E$ and $\Delta r$ response, and less than 10% of the complete gust response for the other state variables throughout the descent. Therefore, this gradient ($r_{2g}$) could be neglected at a relatively small cost in accuracy.

It is noted that when the complete measured gust field is applied, the yaw angle perturbation ($\Delta \psi$) vastly exceeds the original small perturbation assumption. An examination of the non-linear equations (Eqns. 4.6 to 4.9) reveals, however, that this only invalidates the $\Delta y_I$ results, since it alone is a function of $\Delta \psi$. The stability augmentation system gains could have been increased in an attempt to force the yaw angle perturbation to within the small perturbation constraint; but the required gains would then have been unrealistically high, and may have driven the spiral mode unstable. It was therefore decided to leave the gains unchanged since the results obtained with all other gust field approximations were within the small perturbation assumption; and to disregard the complete gust field $\Delta y_I$ results.

VI. CONCLUSIONS AND RECOMMENDATIONS

The present study contains a complete set of correlation measurements evaluating the characteristics of turbulence gust and gust gradients affecting aircraft lateral response obtained in the UTIAS 1.12 m x 1.68 m wind tunnel. The wind tunnel measurements were used in a computer model of the lateral equations of motion of a typical STOL aircraft in order to evaluate the rms dispersion of the aircraft's state variables from a reference equilibrium 15 degree glideslope. A sensitivity analysis was performed to evaluate the relative contribution of each gust component to the total aircraft response.

Much of the non-dimensional data obtained at various heights in the shear layer appears to collapse onto a single curve. The present aircraft response simulation and sensitivity analysis yielded the following significant results:

1/ The point approximation is not valid for estimating lateral aircraft response to turbulence.

2/ The rolling gust gradient ($p_g$) is the largest contributor to lateral aircraft dispersion from the glideslope.

3/ The longitudinal yawing gust gradient ($r_{2g}$) could be neglected with a reasonably small resultant loss in accuracy (less than 10%).

Attempts can now be made to represent the experimental data base by an empirically fitted analytical representation such as the von Kármán model; as was successfully achieved for the longitudinal case in Ref. 2. A faithful analytical model would nullify the need for further time-consuming wind tunnel measurements, and justify the use of such models for estimating aircraft lateral response to turbulence.
REFERENCES


<table>
<thead>
<tr>
<th>No.</th>
<th>Author</th>
<th>Title</th>
</tr>
</thead>
<tbody>
<tr>
<td>16</td>
<td>Harris, R. I.</td>
<td>'The Nature of the Wind', Construction Industry Research and Information Ass'n Seminar, June 1970.</td>
</tr>
</tbody>
</table>
APPENDIX A

CHARACTERISTICS OF THE AUGMENTED AIRCRAFT

The aircraft simulated for this study is the typical STOL transport shown in Fig. 1 whose physical and dynamic characteristics are based on the data of Ref. 6. Since the reference equilibrium condition used in Ref. 6 is one of level flight, it is necessary to correct certain stability derivatives for changes in reference condition. The required modifications are taken from Ref. 24, and are based on the guidelines of Refs. 11, 26, and 27. The following tables list the parameter values used in the present simulation (for $V/W_G = 2.0$):

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>m</td>
<td>4989.5 kg</td>
<td>Ref. 6</td>
</tr>
<tr>
<td>S</td>
<td>39.019 m²</td>
<td></td>
</tr>
<tr>
<td>b</td>
<td>19.812 m</td>
<td></td>
</tr>
<tr>
<td>$l_T$</td>
<td>7.62 m</td>
<td></td>
</tr>
<tr>
<td>$\bar{c}$</td>
<td>1.981 m</td>
<td></td>
</tr>
<tr>
<td>$I_{xx}$</td>
<td>21,621 kg-m²</td>
<td></td>
</tr>
<tr>
<td>$I_{yy}$</td>
<td>31,824 kg-m²</td>
<td></td>
</tr>
<tr>
<td>$I_{zz}$</td>
<td>48,857 kg-m²</td>
<td></td>
</tr>
<tr>
<td>$I_{xz}$</td>
<td>1,482 kg-m²</td>
<td></td>
</tr>
<tr>
<td>$\sigma$</td>
<td>1.2256 kg/m³</td>
<td></td>
</tr>
<tr>
<td>$C_{Le}$</td>
<td>1.251</td>
<td>Ref. 3</td>
</tr>
<tr>
<td>$C_{De}$</td>
<td>0.292</td>
<td></td>
</tr>
<tr>
<td>$C_{Te}$</td>
<td>0.103</td>
<td></td>
</tr>
</tbody>
</table>

Table A-1  Physical Constants

The above physical data was used to dimensionalize the following non-dimensional stability derivatives according to the convention of Ref. 11.
<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C_{Yg}$</td>
<td>-0.775 /rad</td>
<td>Ref. 6</td>
</tr>
<tr>
<td>$C_{Lg}$</td>
<td>-0.90 /rad</td>
<td>Ref. 24</td>
</tr>
<tr>
<td>$C_{nL}$</td>
<td>0.147 /rad</td>
<td>Ref. 6</td>
</tr>
<tr>
<td>$C_{Yp}$</td>
<td>-0.131</td>
<td>Ref. 24</td>
</tr>
<tr>
<td>$C_{Lp}$</td>
<td>-0.777</td>
<td>Ref. 24</td>
</tr>
<tr>
<td>$C_{np}$</td>
<td>0.031</td>
<td>Ref. 24</td>
</tr>
<tr>
<td>$C_{Yr}$</td>
<td>0.513</td>
<td>Ref. 24</td>
</tr>
<tr>
<td>$C_{Lr}$</td>
<td>0.246</td>
<td>Ref. 24</td>
</tr>
<tr>
<td>$C_{nr}$</td>
<td>-0.220</td>
<td>Ref. 24</td>
</tr>
<tr>
<td>$C_{Vpw}$</td>
<td>0.00</td>
<td>Ref. 26</td>
</tr>
<tr>
<td>$C_{Lpw}$</td>
<td>-0.777</td>
<td>Ref. 26</td>
</tr>
<tr>
<td>$C_{nPw}$</td>
<td>-0.0671</td>
<td>Ref. 26</td>
</tr>
<tr>
<td>$C_{Vrw}$</td>
<td>0.00</td>
<td>Ref. 26</td>
</tr>
<tr>
<td>$C_{Lrw}$</td>
<td>0.3253</td>
<td>Ref. 26</td>
</tr>
<tr>
<td>$C_{nrw}$</td>
<td>-0.1078</td>
<td>Ref. 26</td>
</tr>
<tr>
<td>$C_{YrA}$</td>
<td>0.513</td>
<td>Ref. 26</td>
</tr>
<tr>
<td>$C_{LrA}$</td>
<td>-0.0797</td>
<td>Ref. 26</td>
</tr>
<tr>
<td>$C_{nrA}$</td>
<td>-0.1123</td>
<td>Ref. 26</td>
</tr>
<tr>
<td>$C_{YgA}$</td>
<td>0.0108 /rad</td>
<td>Ref. 6</td>
</tr>
<tr>
<td>$C_{LgA}$</td>
<td>0.150 /rad</td>
<td>Ref. 6</td>
</tr>
<tr>
<td>$C_{nA}$</td>
<td>-0.0219 /rad</td>
<td>Ref. 6</td>
</tr>
<tr>
<td>$C_{YgA}$</td>
<td>-0.391 /rad</td>
<td></td>
</tr>
<tr>
<td>$C_{LgA}$</td>
<td>-0.045 /rad</td>
<td></td>
</tr>
<tr>
<td>$C_{nr}$</td>
<td>0.1565 /rad</td>
<td></td>
</tr>
<tr>
<td>$C_{nrA}$</td>
<td>0.1565 /rad</td>
<td></td>
</tr>
</tbody>
</table>

Table A-2  Non-Dimensional Stability Derivatives
As previously noted, the stability augmentation system is taken directly from Ref. 24, and its purposes are:

1/ To stabilize the unaugmented aircraft's divergent spiral mode.

2/ To increase Dutch roll damping in order to keep perturbations within the linearity assumption.

Lag-free proportional feedback of roll angle to ailerons, and yaw rate to rudder, is provided as follows:

\[ \text{Control Contribution} = B_2 \Delta u_{c} \quad \text{(from Eqn. 4.29)} \]

let

\[ \Delta u_{c} = -K \Delta x \]

where

\[ \Delta u_{c}^T = (\Delta \delta a \quad \Delta \phi) \]

\[ \Delta x^T = (\Delta v_{E} \quad \Delta p \quad \Delta r \quad \Delta y_{1} \quad \Delta \phi \quad \Delta \psi) \]

\[ B_2 = A_1^{-1} B_1 \]

\[ A_1 \] and \[ B_1 \] are given by Eqns. 4.25 & 4.27 respectively

\[ K = \begin{pmatrix} 0 & 0 & 0 & 0 & k_1 & 0 \\ 0 & 0 & k_2 & 0 & 0 & 0 \end{pmatrix} \]

where \( k_1 \) is the gain for roll angle feedback to the ailerons, and \( k_2 \) is the gain for yaw rate feedback to the rudder. The gains presently used, and chosen by Ref. 24 are \( k_1 = 0.15 \), and \( k_2 = 0.15 \).

The resulting lateral modes of the augmented aircraft are presented (for \( V/W_g = 2.0 \)), and compared to the unaugmented aircraft:

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Augmented Aircraft</th>
<th>Unaugmented Aircraft</th>
</tr>
</thead>
<tbody>
<tr>
<td>( T_{1/2} ) R</td>
<td>0.11 sec</td>
<td>0.10 sec</td>
</tr>
<tr>
<td>( T_{1/2} ) S</td>
<td>8.36 sec</td>
<td>--</td>
</tr>
<tr>
<td>( T_s )</td>
<td>--</td>
<td>28.2 sec</td>
</tr>
<tr>
<td>( T_{1/2} ) DR</td>
<td>0.96 sec</td>
<td>1.32 sec</td>
</tr>
<tr>
<td>Period_{DR}</td>
<td>4.24 sec</td>
<td>4.06 sec</td>
</tr>
<tr>
<td>Damping Ratio_{DR}</td>
<td>0.44</td>
<td>0.32</td>
</tr>
</tbody>
</table>

Where \( T_{1/2} \) and \( T_s \) denote time to half and double amplitude respectively, and \( R, S, DR \) denote the roll subsidence, spiral, and Dutch roll modes respectively.
The following computer codes were obtained from Ref. 24 and modified to accept the gust gradient convention of Ref. 21. The program incorporates the lateral linearized equations of motion in the form of Eqn. 4.40, and uses the wind-tunnel-generated data to solve Eqn. 4.46 for the rms of the state vector.

Additional subroutines required to complete the program are:

1/ LINV2F: An IMSL subroutine which performs matrix inversion.

2/ RGG: An EISPACK subroutine which solves for eigenvalues and eigenvectors of a matrix.

Note that all data and constants used in this program are in British units.
WE CAN COMPUTE THE LATERAL REFERENCE EQUILIBRIUM VALUES
GAME=GAME+GAME*WVE*WVE*DSIN(GAME) ** 2 - WVE*DCOS (GAME)

CALL COSSALF=DCOS(DALP)

CALL TEMP20F, TEMP20F=TEMP20F/10.2/

CALL CCX=CCX/10.2/

DATA SELECT/.FALSE.,.FALSE.,.FALSE.,.FALSE.,.TRUE./

CONTINUE

**FLIGHT DATA IS READ**

**SET AIRCRAFT PARAMETERS**

DATA SELECT/FALSE...FALSE...FALSE...FALSE...TRUE...FALSE/
MINVCDC

SUBROUTINE MINVCD(A, IA, MA, DETE, IR, IC)
IMPLICIT REAL*8 (A-H, O-Z)
COMPLEX*16 (A-H, O-Z)
DIMENSION MA(IA, IA), PIV, DETE, TEMP, PIV1
D11 = 1, MA
1!
1 IC(I) = 0
DETA = (1.0D0, 0.0D0)
R = MA
2 CALL SUBMC(D, IA, MA, MA, IA, IC, I, J)
PIV = A(L, L)
DETA = PIV*DETA
Y = CEIL(PIV)
IF (Y .EQ. 0.0) GOTO 017
IF (J .EQ. L) GOTO 014
IC(J) = 1
PIV = (1.0D0, 0.0D0)/PIV
A(J, J) = Y
DO3 K = L-1, M
5 IF (K .NE. J) A(I, K) = A(I, K) * PIV
DO3 K = L, M
IF (K .NE. L) GOTO 009
6 PIV = A(K, K)
DO3 K = L, M
7 IF (L .NE. J) A(K, L) = A(K, L) - PIV*A(I, L)
8 CONTINUE
DO3 K = L, M
9 IF (K .NE. L) A(K, J) = -PIV*A(K, L)
10 PIV = 1.0D0
DO3 K = L, M
11 IF (K .NE. L) GOTO 02
12 DO1 = 1, M
13 IF (K .NE. L) GOTO 016
DO11 = 1, M
14 IF (K .NE. L) GOTO 011
DO11 = 1, M
15 A(I, L) = A(I, L)
DO11 = 1, M
16 CONTINUE
C = 0.0
17 WRITE(6, 18)
18 FORMAT(' MATRIX IS SINGULAR')
END

SUBMC

SUBROUTINE SUBMC(A, IA, MA, IA, MA, MA, IA, IC, I, J)
IMPLICIT REAL*8 (A-H, O-Z)
COMPLEX*16 (A-H, O-Z)
DIMENSION MA(IA, IA), IC(IA)
I = 0
TEST = 0.0D0
DO5 K = 1, M
IF (I .EQ. 0.0) GOTO 005
DO5 K = 1, M
IF (I .EQ. 0.0) GOTO 004
X = CEIL(A(L, I))
Y = CEIL(A(I, L))
10 K = X
5 CONTINUE
RETURN
END

MATML

SUBROUTINE MATML(L, K, A, B, C)
IMPLICIT REAL*8 (A-H, O-Z)
DIMENSION A(L, K), B(K, N), C(L, N)
DO 4 I = 1, L
DO 4 K = 1, N
C(I, K) = 0.0
DO 4 K = 1, N
C(I, K) = C(I, K) + A(I, K)*P(K, J)
CONTINUE
RETURN
END
SUBROUTINE EKBIL(Y,NORD,AB,H,DER)
IMPLICIT REAL*8(A-H,O-Z)
DIMENSION Y(36),Z1(36),Y1(36),P(10),Q1(36),YPRIM(36)
DIMENSION GGGD(6,6),SGD(6,6),SOURCE(6,6)

COMMON/PUNCH/SOURCE COMMON/SINGLE/GGD COMMON/RSIDE/SIGP COMMON/AUGMS/GK

A1=0.2928933 A2=-2.*R2 A3=2.*R2 A4=1.+1./R2 A4=1.7011070
A5=1./6. A6=0.1213205 A7=0.5857867 A8=0.1666666
A9=1./3. A10=1./6. A11=0.5857867 A12=2. +R2

DO 1 1=1,NORD 2=1,6 3=1,6
CALL DER(NORD,XX,Y1,YPRIM) DO 4 1=1,6 2=1,6
X=X+H Y=L=L+1
RETURN END

SUBROUTINE CORREL(NORD,YY,YYP)
IMPLICIT REAL*8(A-H,O-Z)
DIMENSION SOURCE(6,6),SGD(6,6),PPD(6,6),SIGSS(T2,8),TS(8),NSIG(8),
1 REDATA(4,12,8),SINGSIG(6,6),SIGP(6,6),YY(36),YYP(36)
COMPLEX*8 A,LAMGD(6),UGGDI(6,6),CDEXP
DIMENSION GGGD(6,6),SGD(6,6)
COMMON/PASS/LAMGD,UGGDI,UGGDI
COMMON/PUNC/SOURCE
COMMON/ARGES/GK
COMMON/SIGP
DO 1 1=1,6 DO 4 J=1,6
INDEX=6*(6-1)+
1 SIGP(I,J)=SINGSIG(I,K)*GGKD(J,K)
DO 2 J=1,6 DO 4 J=1,6
SIGP(I,J)=SIGP(I,J)+SIGSIG(I,K)*GGKD(J,K)
DO 3 J=1,6 DO 4 J=1,6
INDEX=INDEX+6*
1 YYP(INDEX)=SIGP(I,J)
RETURN END

SUBROUTINE EXPAT(T,EXPAS)
IMPLICIT REAL*8(A-H,O-Z)
DIMENSION GGD(6,6),UGGDI
COMMON/PASS/LAMGD,UGGDI,UGGDI
COMMON/SINGLE/ GGD
DIMENSION GGD(6,6)

DO 11 I=1,6 DO 15 J=1,6
INDEX=I+J
ZSIG(I,J)=CDEXP*(ZSIG(I,J))
DO 11 I=1,6
ZSIG(I,J)=ZSIG(I,J)
DO 11 I=1,6
ZSIG(I,J)=ZSIG(I,J)+ZSIG(I,J)
DO 15 J=1,6
DO 15 J=1,6
EXPAS(I,J)=EXPAS(I,J)
RETURN END
**VALENTE**

SUBROUTINE VALENTE(SOURCE, FPD, SIGSS, NN, TS, NSIG, PFDATA)
IMPLICIT REAL*8 (A-H, O-Z)
DIMENSION X(M), SOURCE(12,8), TS(8), NSIG(12,8)
COMMON ATOT, S
COMMON SIGS(12,8), PFD(6,4), SIGSS(12,8)
COMMON/NOATC/ INT
COMMON/NOSIGS/ GGD
DO 10 J = 1, 4
   DO 10 M = 1, 4
      X(M) = SOURCE(J, M)
50 CONTINUE
10 CONTINUE
DO 20 J = 1, 4
   CALL EXPAT(J, FPD(1, J))
20 CONTINUE
DO 30 J = 1, 4
   CALL TEMP22(J, FPD(2, J))
30 CONTINUE
DO 40 J = 1, 4
   CALL TEMP26(J, FPD(3, J))
40 CONTINUE
RETURN
END

**FIND**

SUBROUTINE FIND(S, SIG, BR1, FPD, SIGSS, NN, TS, NSIG, PFDATA)
IMPLICIT REAL*8 (A-H, O-Z)
DIMENSION BRD(4, 4), TS(8), NSIG(8), SIGSS(12, 8), GGD(6, 6),
1 FPD(6, 4), TEMP62(6, 4), TEMP66(6, 4)
DIMENSION TEST(6), SIGS(12)
COMMON/RDATA/I
COMMON/NOATC/ INT
COMMON/NOSIGS/ GGD
DO 1 T = 1, 4
   CALL EXPAT(T, PFD(1, T))
   CALL TEMP22(T, PFD(2, T))
   CALL TEMP26(T, PFD(3, T))
   CALL EXPAT(T, SIGSS(1, T))
   CALL TEMP22(T, SIGSS(2, T))
   CALL TEMP26(T, SIGSS(3, T))
   CALL EXPAT(T, BRD(1, T))
   CALL TEMP22(T, BRD(2, T))
   CALL TEMP26(T, BRD(3, T))
   CALL EXPAT(T, TS(T, T))
   CALL TEMP22(T, TS(T, T))
   CALL TEMP26(T, TS(T, T))
1 CONTINUE
RETURN
END

FIND2

SUBROUTINE FIND2(NSIG, BR1, FPD, SIGSS, NN, BRDATA)
IMPLICIT REAL*8 (A-H, O-Z)
DIMENSION BRD(4, 4), TS(8), NSIG(8), SIGSS(12, 8), GGD(6, 6),
1 FPD(6, 4), TEMP62(6, 4), TEMP66(6, 4)
DIMENSION TEST(6), SIGS(12)
COMMON/RDATA/I
COMMON/NOATC/ INT
COMMON/NOSIGS/ GGD
DO 1 T = 1, 4
   CALL EXPAT(T, PFD(1, T))
   CALL TEMP22(T, PFD(2, T))
   CALL TEMP26(T, PFD(3, T))
   CALL EXPAT(T, SIGSS(1, T))
   CALL TEMP22(T, SIGSS(2, T))
   CALL TEMP26(T, SIGSS(3, T))
   CALL EXPAT(T, BRD(1, T))
   CALL TEMP22(T, BRD(2, T))
   CALL TEMP26(T, BRD(3, T))
   CALL EXPAT(T, TS(T, T))
   CALL TEMP22(T, TS(T, T))
   CALL TEMP26(T, TS(T, T))
1 CONTINUE
RETURN
END
**Sample Output**

<table>
<thead>
<tr>
<th>Code</th>
<th>Value</th>
<th>Code</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>HO</td>
<td>1000.00</td>
<td>HOG</td>
<td>1000.00</td>
</tr>
<tr>
<td>GOG</td>
<td>1000.00</td>
<td>GOG</td>
<td>1000.00</td>
</tr>
<tr>
<td>G051</td>
<td>15.00</td>
<td>0051</td>
<td>15.00</td>
</tr>
<tr>
<td>G052</td>
<td>5.00</td>
<td>0052</td>
<td>5.00</td>
</tr>
<tr>
<td>GG03</td>
<td>8.00</td>
<td>GG03</td>
<td>8.00</td>
</tr>
<tr>
<td>GG04</td>
<td>8.00</td>
<td>GG04</td>
<td>8.00</td>
</tr>
<tr>
<td>Code</td>
<td>Value</td>
<td>Code</td>
<td>Value</td>
</tr>
<tr>
<td>--------</td>
<td>---------</td>
<td>--------</td>
<td>---------</td>
</tr>
<tr>
<td>GG05</td>
<td>2.00</td>
<td>GG05</td>
<td>2.00</td>
</tr>
<tr>
<td>GG06</td>
<td>4.00</td>
<td>GG06</td>
<td>4.00</td>
</tr>
</tbody>
</table>

**Error Code:** 0

**Matrix G01**

<table>
<thead>
<tr>
<th>Value</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>0.00</td>
<td>0.00</td>
</tr>
</tbody>
</table>

**Matrix G02**

<table>
<thead>
<tr>
<th>Value</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>0.00</td>
<td>0.00</td>
</tr>
</tbody>
</table>

**Matrix G03**

<table>
<thead>
<tr>
<th>Value</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>0.00</td>
<td>0.00</td>
</tr>
</tbody>
</table>

**Matrix G04**

<table>
<thead>
<tr>
<th>Value</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>0.00</td>
<td>0.00</td>
</tr>
</tbody>
</table>

**Matrix G05**

<table>
<thead>
<tr>
<th>Value</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>0.00</td>
<td>0.00</td>
</tr>
</tbody>
</table>

**Error Code:** 0

**Gain Matrix Win**

<table>
<thead>
<tr>
<th>Value</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>0.00</td>
<td>0.00</td>
</tr>
</tbody>
</table>

**Suggested Matix Gsk**

<table>
<thead>
<tr>
<th>Value</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>0.00</td>
<td>0.00</td>
</tr>
</tbody>
</table>

**Eigenvalues Are**

<table>
<thead>
<tr>
<th>Value</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>0.00</td>
<td>0.00</td>
</tr>
</tbody>
</table>

**PM Ratio of Neglect**

<table>
<thead>
<tr>
<th>Value</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>0.00</td>
<td>0.00</td>
</tr>
</tbody>
</table>
The image contains tables with numerical data. The tables are labeled as follows:

- **REAL PART OF THE INVERSE**
- **IMAGINARY PART OF THE INVERSE**
- **REAL PART OF THE INVERSE**
- **IMAGINARY PART OF THE INVERSE**
- **Determinate IsDet**
- **RMS VALUES (DIMENSIONAL)**
- **RMS VALUES (DIMENSIONAL)**
- **RMS VALUES (DIMENSIONAL)**
- **RMS VALUES (DIMENSIONAL)**
- **RMS VALUES (DIMENSIONAL)**

Each table contains columns with numerical values, and the data appears to be related to some kind of analysis or calculation, possibly in the context of engineering or physics. The tables are too detailed to summarize succinctly without losing context, so the full content is presented as is.
<table>
<thead>
<tr>
<th>TIME</th>
<th>DV</th>
<th>DP</th>
<th>DI</th>
<th>DPHI</th>
<th>DYAW</th>
</tr>
</thead>
<tbody>
<tr>
<td>23.629</td>
<td>4.3392D+00</td>
<td>6.1844D-02</td>
<td>4.0494D-02</td>
<td>2.6707D+02</td>
<td>1.2430D-01</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>TIME</th>
<th>DV</th>
<th>DP</th>
<th>DI</th>
<th>DPHI</th>
<th>DYAW</th>
</tr>
</thead>
</table>

<table>
<thead>
<tr>
<th>TIME</th>
<th>DV</th>
<th>DP</th>
<th>DI</th>
<th>DPHI</th>
<th>DYAW</th>
</tr>
</thead>
<tbody>
<tr>
<td>33.081</td>
<td>6.0997D+00</td>
<td>8.5670D-02</td>
<td>5.6802D+02</td>
<td>1.7379D-01</td>
<td>1.3795D-01</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>TIME</th>
<th>DV</th>
<th>DP</th>
<th>DI</th>
<th>DPHI</th>
<th>DYAW</th>
</tr>
</thead>
</table>

<table>
<thead>
<tr>
<th>TIME</th>
<th>DV</th>
<th>DP</th>
<th>DI</th>
<th>DPHI</th>
<th>DYAW</th>
</tr>
</thead>
<tbody>
<tr>
<td>42.533</td>
<td>7.8078D+00</td>
<td>1.0439D+01</td>
<td>7.2600D+02</td>
<td>1.3780D+03</td>
<td>2.3215D-01</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>TIME</th>
<th>DV</th>
<th>DP</th>
<th>DI</th>
<th>DPHI</th>
<th>DYAW</th>
</tr>
</thead>
<tbody>
<tr>
<td>47.259</td>
<td>8.6695D+00</td>
<td>1.1442D+01</td>
<td>8.0793D+02</td>
<td>1.8320D+03</td>
<td>2.6793D-01</td>
</tr>
<tr>
<td>Symbol</td>
<td>Value</td>
<td>Symbol</td>
<td>Value</td>
<td></td>
<td></td>
</tr>
<tr>
<td>--------</td>
<td>-----------</td>
<td>--------</td>
<td>------------</td>
<td></td>
<td></td>
</tr>
<tr>
<td>b</td>
<td>19.81 m</td>
<td>k</td>
<td>0.050 sec⁻¹</td>
<td></td>
<td></td>
</tr>
<tr>
<td>b'</td>
<td>16.84 m</td>
<td>ℓ_T</td>
<td>7.62 m</td>
<td></td>
<td></td>
</tr>
<tr>
<td>h_G</td>
<td>305 m</td>
<td>m</td>
<td>4989.5 kg</td>
<td></td>
<td></td>
</tr>
<tr>
<td>H</td>
<td>1.12 m</td>
<td>n</td>
<td>0.16</td>
<td></td>
<td></td>
</tr>
<tr>
<td>I_{xx}</td>
<td>21,621 kg-m²</td>
<td>l_g</td>
<td>20.1 m/s</td>
<td></td>
<td></td>
</tr>
<tr>
<td>I_{yy}</td>
<td>31,824 kg-m²</td>
<td>γ_e</td>
<td>15 deg</td>
<td></td>
<td></td>
</tr>
<tr>
<td>I_{zz}</td>
<td>48,857 kg-m²</td>
<td>θ_e</td>
<td>-15 deg</td>
<td></td>
<td></td>
</tr>
<tr>
<td>I_{xz}</td>
<td>1,482 kg-m²</td>
<td>w_G</td>
<td>27.4 m/s</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**TABLE 1  TABLE OF CONSTANTS**

<table>
<thead>
<tr>
<th>h/h_G</th>
<th>M_u^3</th>
<th>h/h_G</th>
<th>M_v^3</th>
<th>h/h_G</th>
<th>M_w^3</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.250</td>
<td>-0.431</td>
<td>0.157</td>
<td>0.364</td>
<td>-1.630</td>
<td>0.677</td>
</tr>
<tr>
<td>0.500</td>
<td>-1.167</td>
<td>0.422</td>
<td>0.887</td>
<td>0.677</td>
<td>1.273</td>
</tr>
<tr>
<td>0.750</td>
<td>-1.630</td>
<td>0.677</td>
<td>1.273</td>
<td>0.677</td>
<td>1.273</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>h/h_G</th>
<th>M_u^4</th>
<th>h/h_G</th>
<th>M_v^4</th>
<th>h/h_G</th>
<th>M_w^4</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.250</td>
<td>2.742</td>
<td>4.915</td>
<td>10.234</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>h/h_G</th>
<th>M^4</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.250</td>
<td></td>
</tr>
<tr>
<td>0.500</td>
<td>3.1</td>
</tr>
<tr>
<td>0.750</td>
<td>3.54</td>
</tr>
</tbody>
</table>

**TABLE 2  SUMMARY OF PROBABILITY DISTRIBUTION RESULTS**
### TABLE 3 TIMES FOR AIRCRAFT TO REACH UPPER PROBE POSITION
FROM \( h' = 91.0 \) cm (\( \gamma_E = 15 \) deg, \( n = 0.16 \))

<table>
<thead>
<tr>
<th>Upper Probe Position</th>
<th>( h'_1 ) (cm)</th>
<th>( h'_1/h_G )</th>
<th>( \alpha' ) (sec) for ( V/W_G = 2.0 )</th>
<th>( \alpha' ) (sec) for ( V/W_G = 1.5 )</th>
<th>( \alpha' ) (sec) for ( V/W_G = 1.0 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>63.5</td>
<td>0.694</td>
<td>0.0371</td>
<td>0.0717</td>
<td>3.097</td>
<td></td>
</tr>
<tr>
<td>48.3</td>
<td>0.528</td>
<td>0.0567</td>
<td>0.1083</td>
<td>3.377</td>
<td></td>
</tr>
<tr>
<td>41.9</td>
<td>0.458</td>
<td>0.0647</td>
<td>0.1228</td>
<td>3.458</td>
<td></td>
</tr>
<tr>
<td>35.6</td>
<td>0.389</td>
<td>0.0725</td>
<td>0.1367</td>
<td>3.526</td>
<td></td>
</tr>
<tr>
<td>27.9</td>
<td>0.305</td>
<td>0.0816</td>
<td>0.1528</td>
<td>3.593</td>
<td></td>
</tr>
<tr>
<td>20.3</td>
<td>0.222</td>
<td>0.0905</td>
<td>0.1680</td>
<td>3.648</td>
<td></td>
</tr>
<tr>
<td>12.7</td>
<td>0.138</td>
<td>0.0990</td>
<td>0.1822</td>
<td>3.691</td>
<td></td>
</tr>
</tbody>
</table>

### TABLE 4 DISTRIBUTION OF PROBE-PAIR LOCATIONS

<table>
<thead>
<tr>
<th>Upper Probe Position</th>
<th>Probe Separations Used ( (h'_1-h'_2) ) (cm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>63.5</td>
<td>0.0, 1.27, 2.54, 3.81, 6.35, 8.89, 11.43, 13.97, 16.51</td>
</tr>
<tr>
<td>48.3</td>
<td>0.0, 1.27, 2.54, 3.81, 5.08, 6.35, 7.62, 8.89, 11.43, 13.97, 15.24</td>
</tr>
<tr>
<td>41.9</td>
<td>0.0, 1.27, 1.91, 2.54, 3.18, 4.45, 5.08, 5.72, 6.35, 7.62</td>
</tr>
</tbody>
</table>
FIG. 1 AIRCRAFT SIMULATED
FIG. 2 TYPICAL AIRCRAFT DESCENT THROUGH THE BOUNDARY LAYER

FIG. 3 FLIGHT PATH GEOMETRY
FIG. 4  BOUNDARY LAYER WIND TUNNEL CONFIGURATION
**Fig. 5** Mean Velocity Correction Factor
FIG. 6a  PROBE CARRIER CONFIGURATION
FIG. 6b  PROBE CARRIER CONFIGURATION
FIG. 7  SCHEMATIC OF DATA ACQUISITION SYSTEM
FIG. 8  DATA HANDLING SYSTEM
FROM LINEARIZER
WIRE #1 OF
X-PROBE

FROM LINEARIZER
WIRE #2 OF
X-PROBE

MEAN u

MEAN v or w

TIE POINT u(t)

TIE POINT v(t) or w(t)

GAIN ADJUSTS FOR
u(t) and v(t) or w(t)

FIG. 9 TR-48 ANALOG COMPUTER WIRING DIAGRAM
$W_G = 27.4 \text{ m/s}$

$h_G = 91.4 \text{ cm}$

- POINTS MEASURED
  AT TUNNEL CENTERLINE,
  7H DOWNSTREAM OF
  THE JET EXIT PLANE

$\frac{W}{W_G} = \left(\frac{h}{h_G}\right)^{0.16}$

FIG. 10  MEAN VELOCITY PROFILE
FIG. 11 LATERAL UNIFORMITY
MEASURED ALONG 15° GLIDESLOPE AT TUNNEL CENTERLINE

FIG. 12 LONGITUDINAL TURBULENCE INTENSITY
MEASURED ALONG 15° GLIDESLOPE AT TUNNEL CENTERLINE

Von Kármán Model

\[ \frac{k \Phi_{uu}}{\sigma^2_u} \]

\[ k = \frac{1}{w} \]

\[ h = 0.383 \]

\[ \frac{h}{h_G} = 0.329 \]

FIG. 15 REPRESENTATIVE PLOT OF NON-DIMENSIONAL POWER SPECTRAL DENSITY
MEASURED ALONG 15° GLIDESCOPE AT TUNNEL CENTERLINE

\[ \frac{h}{h_G} \]

\[ L_U^X = 280 \left( \frac{h}{h_G} \right)^{0.35} \]

\[ L_U^X \text{ in metres} \]

\[ L_U^X \text{ in feet} \]

(ref. 14)

FIG. 16 LONGITUDINAL INTEGRAL SCALE LENGTH
IN-FIELD DATA (ref. 17)

MEASURED ALONG 15° GLIDESCOPE AT TUNNEL CENTERLINE

\( L_v = 140 \left( \frac{h}{h_G} \right)^{0.48} \)

\( L_v \) in metres

(ref. 14)

\( L_v = 0.4h \)

(ref. 12)

FIG. 17 LATERAL INTEGRAL SCALE LENGTH
\[ \frac{h}{h_G} \]

\[ \frac{L_w^x}{h_G} = 0.35 \, h \] (ref. 14)

\[ \frac{L_w^x}{h_G} = 0.4 \, h \] (ref. 12)

FIG. 18 VERTICAL INTEGRAL SCALE LENGTH

IN-FIELD DATA (ref. 17)

MEASURED ALONG 15° GLIDESLOPE AT TUNNEL CENTERLINE
FIG. 19 REPRESENTATIVE PROBABILITY DENSITY DISTRIBUTIONS
MEASURED ALONG 15° GLIDESLOPE AT TUNNEL CENTERLINE

\[ \frac{N}{N_T} x \Delta \sigma \]

\( h/h_g = 0.250 \)

- \( \Delta \sigma = 0.4 \)

\( N = \text{NUMBER OF OCCURANCES} \)
\( N_T = \text{TOTAL OCCURANCES} = 10,240 \)

GAUSSIAN DISTRIBUTION

\( \text{FIG. 20 GUST PROBABILITY DENSITY DISTRIBUTION AT } h/h_g = 0.25 \)
MEASURED ALONG 15° GLIDESLOPE AT TUNNEL CENTERLINE

- \( N \) = number of occurrences
- \( N_T = \text{total occurrences} = 10,240 \)
- \( \Delta \sigma = 0.4 \)

\( \frac{N}{N_T} \times \Delta \sigma \)

\( h/h_G = 0.500 \)

\( \frac{i}{\sigma_i} \quad (i=u, v, w) \)

FIG. 21 GUST PROBABILITY DENSITY DISTRIBUTION AT \( h/h_G = 0.50 \)
MEASURED ALONG 16° GLIDESLOPE AT TUNNEL CENTERLINE

**FIG. 22** GUST PROBABILITY DENSITY DISTRIBUTION AT $h/h_G = 0.75$
V/\bar{u}_G = 2.0

\alpha' = 0.07246 \text{ sec}

\begin{align*}
\hat{R}_{v_0v_0} & \quad + \\
\hat{R}_{v_3v_3} & \quad \times \\
\circ \text{ Data of Ref. 1 } (\hat{R}_{vv}) & \quad \circ
\end{align*}

FIG. 23 PRELIMINARY CORRELATIONS -- $\hat{R}_{v_0v_0}$ & $\hat{R}_{v_3v_3}$
$V/W_G = 2.0$

$\alpha' = 0.07246 \text{ sec}$

FIG. 24 PRELIMINARY CORRELATIONS -- $\hat{R}_{v_0v_3}$ & $\hat{R}_{v_3v_0}$
$V/W_G = 2.0$

$\alpha' = 0.07246 \text{ sec}$

---

**FIG. 25 PRELIMINARY CORRELATIONS -- $\hat{R}_{W_1W_1}$ & $\hat{R}_{W_2W_2}$**

- Data of Ref. 1 ($R_{ww}$)
FIG. 26  PRELIMINARY CORRELATIONS — $\hat{R}_{w_1w_2}$ & $\hat{R}_{w_2w_1}$

$V/W_G = 2.0$

$\alpha' = 0.07246 \text{ sec}$

$\beta - \alpha$ (sec)
FIG. 27 PRELIMINARY CORRELATION -- $\hat{R}_{r_1 r_1}$

$V/W_G = 2.0$

$\alpha' = 0.07246 \text{ sec}$
$V/W_G = 2.0$

$\alpha' = 0.07246\ \text{sec}$

**FIG. 28 PRELIMINARY CORRELATIONS**
\[ V/W_G = 2.0 \]
\[ \alpha' = 0.07246 \text{ sec} \]

**FIG. 29** PRELIMINARY CORRELATIONS -- \( R_{v_3 w_2} \), \( R_{v_2 w_3} \), \( R_{v_3 w_1} \) & \( R_{w_1 v_3} \)
FIG. 30  PRELIMINARY CORRELATIONS -- $\hat{R}_{w_1 r_1}$, $\hat{R}_{r_1 w_1}$, $\hat{R}_{w_2 r_1}$ & $\hat{R}_{r_1 w_2}$

$V/W_G = 2.0$

$\alpha' = 0.07246$ sec
FIG. 31  PRELIMINARY CORRELATIONS -- $\hat{R}_{v_0 r_1}$ & $\hat{R}_{r_1 v_0}$

$V/W_G = 2.0$

$\alpha' = 0.07246$ sec
$V/W_G = 2.0$

$\alpha' = 0.07246 \, \text{sec}$

FIG. 32  PRELIMINARY CORRELATIONS -- $\hat{R}_{v_3 r_1}$ & $\hat{R}_{r_1 v_3}$
2.5

\[ \frac{V}{W_G} = 2.0 \]

\[ h_1 = 118.5 \text{ m} \]

Full Scale

**FIG. 33** PRELIMINARY DIMENSIONAL CORRELATION -- \( R_{V_0V_0} \)
\( V/W_G = 2.0 \)

\( h_1 = 118.5 \text{ m} \)

Full Scale

FIG. 34 PRELIMINARY DIMENSIONAL CORRELATION -- \( R_{r_1 r_1} \)
$R_{pp}$

$V/W_G = 2.0$

$h_1 = 118.5 \text{ m}$

Full Scale

**FIG. 35** PRELIMINARY DIMENSIONAL CORRELATION -- $R_{pp}$
FIG. 36  PRELIMINARY DIMENSIONAL CORRELATION -- $R_{r_2r_2}$

$V/W_G = 2.0$

$h_1 = 118.5$ m

Full Scale
$V/W_G = 2.0$

$h_1 = 118.5 \text{ m}$

Full Scale

\[ R_{V_0 R_2} \]

\[ R_{R_2 V_0} \]

FIG. 37 PRELIMINARY DIMENSIONAL CORRELATIONS -- $R_{V_0 R_2}$ & $R_{R_2 V_0}$
$V/W_G = 2.0$

$h_1 = 118.5 \text{ m}$

Full Scale

$R_{V_0 P}$

$R_{PV_0}$

**FIG. 38 PRELIMINARY DIMENSIONAL CORRELATIONS -- $R_{V_0 P}$ & $R_{PV_0}$**
FIG. 39 PRELIMINARY DIMENSIONAL CORRELATIONS -- $R_{V_0 r_1}$ & $R_{r_1 v_0}$
FIG. 40  PRELIMINARY DIMENSIONAL CORRELATIONS -- $R_{pr_1}$ & $R_{r_1p}$

$V/W_G = 2.0$

$h_1 = 118.5$ m

Full Scale

$0.010$

$0.008$

$0.006$

$0.004$

$0.002$

$0.0$

$-0.002$

$h_1 - h_2$ (m)
V/W_G = 2.0

h_1 = 118.5 m

Full Scale

FIG. 41 PRELIMINARY DIMENSIONAL CORRELATIONS -- R_{pr_2} & R_{r_2p}
FIG. 42  PRELIMINARY DIMENSIONAL CORRELATIONS -- $R_{r_1r_2}$ & $R_{r_2r_1}$

$V/W_G = 2.0$

$h_1 = 118.5\, m$

Full Scale
$V/W_G = 2.0$

$h_1 = 118.5 \text{ m}$

Full Scale

Mean ± 1 s.d.

FIG. 43  STATISTICAL DIMENSIONAL CORRELATION -- $R_{V_0V_0}$
$$V/W_G = 2.0$$

$$h_1 = 118.5 \text{ m}$$

Full Scale

Mean ± 1 s.d.

FIG. 44 STATISTICAL DIMENSIONAL CORRELATION -- $R_{r_1 r_1}$
\[ V/W_G = 2.0 \]

\[ h_1 = 118.5 \text{ m} \]

Full Scale

Mean ± 1 s.d.

**FIG. 45** STATISTICAL DIMENSIONAL CORRELATION -- \( R_{pp} \)
$V/W_G = 2.0$

$h_1 = 118.5 \text{ m}$

Full Scale

Mean ± 1 s.d.

**FIG. 46** STATISTICAL DIMENSIONAL CORRELATION -- $R_{r_2 r_2}$
FIG. 47 STATISTICAL DIMENSIONAL CORRELATION -- $R_{v_0 r_2}$

$V/W_G = 2.0$

$h_1 = 118.5$ m

Full Scale

Mean ± 1 s.d.
FIG. 48  STATISTICAL DIMENSIONAL CORRELATION -- $R_{r_2 v_0}$
FIG. 49a  FLIGHT PATH TURBULENCE CORRELATION -- $\hat{R}_{V_0V_0}$
FIG. 49b  FLIGHT PATH TURBULENCE CORRELATION

\[ \hat{R}_{v_0 v_0} \]

\[ V/W_G = 2.0 \]

- \[ \alpha' = 0.05669 \text{ sec} \]
- \[ \Delta \alpha' = 0.07246 \text{ sec} \]
- \[ + \alpha' = 0.09050 \text{ sec} \]
FIG. 50a  FLIGHT PATH TURBULENCE CORRELATION -- $\hat{R}_{0}/v_{3}$

$V/W_g = 2.0$

- $\alpha' = 0.03705$ sec
- $\alpha' = 0.06467$ sec
- $\alpha' = 0.08163$ sec
- $\alpha' = 0.09902$ sec
FIG. 50b FLIGHT PATH TURBULENCE CORRELATION -- $\hat{R}_{V_0 V_3}$
$V/W_G = 2.0$

- $\alpha' = 0.03705 \text{ sec}$
- $\alpha' = 0.06467 \text{ sec}$
- $\alpha' = 0.08163 \text{ sec}$
- $\alpha' = 0.09902 \text{ sec}$

FIG. 51a FLIGHT PATH TURBULENCE CORRELATION -- $\hat{R}_{\nu^3 \nu_0}$
FIG. 51b FLIGHT PATH TURBULENCE CORRELATION -- $\frac{R_{v_3v_0}}{\gamma}$
\( V/W_G = 2.0 \)

\[ R_{w_1w_1} \]

- \( \alpha' = 0.03705 \text{ sec} \)
- \( \alpha' = 0.06467 \text{ sec} \)
- \( \alpha' = 0.08163 \text{ sec} \)
- \( \alpha' = 0.09902 \text{ sec} \)

**FIG. 52a** FLIGHT PATH TURBULENCE CORRELATION -- \( R_{w_1w_1} \)
V/W_G = 2.0

\[ \hat{R}_{w_1 w_1} \]

\[ \alpha' = 0.05669 \text{ sec} \]

\[ \alpha' = 0.07246 \text{ sec} \]

\[ \alpha' = 0.09050 \text{ sec} \]

FIG. 52b  FLIGHT PATH TURBULENCE CORRELATION -- \( \hat{R}_{w_1 w_1} \)
FIG. 53a  FLIGHT PATH TURBULENCE CORRELATION -- $\hat{R}_{w_1w_2}$
FIG. 53b FLIGHT PATH TURBULENCE CORRELATION -- $\hat{R}_{w_1w_2}$

$V/W_G = 2.0$

- $\alpha' = 0.05669$ sec
- $\alpha' = 0.07246$ sec
- $\alpha' = 0.09050$ sec

$\hat{R}_{w_1w_2}$

$\beta - \alpha'$ (sec)
\[ \frac{\hat{R}_{r_1 r_1}}{V/W_G} = 2.0 \]

- \( \alpha' = 0.03705 \text{ sec} \)
- \( \alpha' = 0.06467 \text{ sec} \)
- \( \alpha' = 0.08163 \text{ sec} \)
- \( \alpha' = 0.09902 \text{ sec} \)

**FIG. 54a** FLIGHT PATH TURBULENCE CORRELATION -- \( \frac{\hat{R}_{r_1 r_1}}{V/W_G} \)
$V/W_G = 2.0$

- $\alpha' = 0.05669$ sec
- $\alpha' = 0.07246$ sec
- $\alpha' = 0.09050$ sec

FIG. 54b FLIGHT PATH TURBULENCE CORRELATION -- $R_{r_1r_1}$
FIG. 55a FLIGHT PATH TURBULENCE CORRELATION -- $R_{pp}$

$V/W_G = 2.0$

- $\alpha' = 0.03705$ sec
- $\Delta \alpha' = 0.06467$ sec
- $+ \alpha' = 0.08163$ sec
- $\times \alpha' = 0.09902$ sec
V/W_g = 2.0

\[ \frac{\alpha'}{0.05669 \text{ sec}} \]
\[ \frac{\alpha'}{0.07246 \text{ sec}} \]
\[ \frac{\alpha'}{0.09050 \text{ sec}} \]

**FIG. 55b** FLIGHT PATH TURBULENCE CORRELATION -- $\hat{R}_{pp}$
FIG. 56a FLIGHT PATH TURBULENCE CORRELATION -- $\hat{R}_{r_2r_2}$

$V/W_G = 2.0$

- $\alpha' = 0.03705$ sec
- $\Delta \alpha' = 0.06467$ sec
- $+ \alpha' = 0.08163$ sec
- $\times \alpha' = 0.09902$ sec
FIG. 56b FLIGHT PATH TURBULENCE CORRELATION -- $\frac{\Delta R_{\alpha \beta}}{R_{\alpha \beta}^2}$
V/W_G = 2.0

\[ \hat{R}_{V_0 r_2} \]

- \[ \alpha' = 0.03705 \text{ sec} \]
- \[ \alpha' = 0.06467 \text{ sec} \]
- \[ \alpha' = 0.08163 \text{ sec} \]
- \[ \alpha' = 0.09902 \text{ sec} \]

FIG. 57a  FLIGHT PATH TURBULENCE CORRELATION -- \[ \hat{R}_{V_0 r_2} \]
\[ \frac{V}{W_G} = 2.0 \]

\[ a' = 0.05669 \text{ sec} \]
\[ a' = 0.07246 \text{ sec} \]
\[ a' = 0.09050 \text{ sec} \]

FIG. 57b  FLIGHT PATH TURBULENCE CORRELATION -- \( \frac{\hat{R}}{V_0 r_2} \)
\[
\frac{V}{W_G} = 2.0
\]

\[
\frac{\dot{R}}{r_2 v_0}
\]

\[
\begin{align*}
\square & \quad \alpha' = 0.03705 \text{ sec} \\
\triangle & \quad \alpha' = 0.06467 \text{ sec} \\
+ & \quad \alpha' = 0.08163 \text{ sec} \\
\times & \quad \alpha' = 0.09902 \text{ sec}
\end{align*}
\]

FIG. 58a  FLIGHT PATH TURBULENCE CORRELATION -- \(\frac{\dot{R}}{r_2 v_0}\)
FIG. 58b  FLIGHT PATH TURBULENCE CORRELATION -- $\frac{\ell}{R_e v_0}$
FIG. 59 FLIGHT PATH TURBULENCE CORRELATION -- $\frac{R_{V_0 V_0}}{V/W_G = 1.5}$

- $\alpha' = 0.07173$ sec
- $\Delta \alpha' = 0.12280$ sec
- $\times \alpha' = 0.15280$ sec
- $\times \alpha' = 0.18220$ sec
Fig. 60 Flight Path Turbulence Correlation -- $\frac{\Delta R_{v_0}}{v_0}$

$V/W_G = 1.0$

- $\alpha' = 3.0970$ sec
- $\Delta \alpha' = 3.4580$ sec
- $\alpha' = 3.5930$ sec
- $\times \alpha' = 3.6910$ sec
FIG. 61 FLIGHT PATH TURBULENCE CORRELATION

\[^{\hat{R}}_{v_0v_3}\]

\[V/W_G = 1.5\]

- \(\square \alpha' = 0.07173\) sec
- \(\triangle \alpha' = 0.12280\) sec
- \(+ \alpha' = 0.15280\) sec
- \(\times \alpha' = 0.18220\) sec
FIG. 62 FLIGHT PATH TURBULENCE CORRELATION -- $\frac{\dot{\alpha}}{V_0 V_3}$

$V/W_G = 1.0$

- $\alpha' = 3.0970 \text{ sec}$
- $\alpha' = 3.4580 \text{ sec}$
- $\alpha' = 3.5930 \text{ sec}$
- $\alpha' = 3.6910 \text{ sec}$

$R_{\dot{\alpha}/V_0 V_3}$
\[
V/W_G = 1.5
\]

- \(\alpha' = 0.07173 \text{ sec}\)
- \(\alpha' = 0.12280 \text{ sec}\)
- \(\alpha' = 0.15280 \text{ sec}\)
- \(\alpha' = 0.18220 \text{ sec}\)

**FIG. 63** FLIGHT PATH TURBULENCE CORRELATION -- \(\frac{\omega}{V^3V_0}\)
FIG. 64 FLIGHT PATH TURBULENCE CORRELATION -- $\frac{\Delta R_{V^3V_0}}{V/W_G} = 1.0$

- $\square \alpha^/ = 3.0970 \text{ sec}$
- $\triangle \alpha^/ = 3.4580 \text{ sec}$
- $+ \alpha^/ = 3.5930 \text{ sec}$
- $\times \alpha^/ = 3.6910 \text{ sec}$
V/W_G = 1.5

$\alpha = 0.07173 \text{ sec}$

$\alpha = 0.12280 \text{ sec}$

$\alpha = 0.15280 \text{ sec}$

$\alpha = 0.18220 \text{ sec}$

FIG. 65 FLIGHT PATH TURBULENCE CORRELATION -- $R_{w_1w_1}$
FIG. 66 FLIGHT PATH TURBULENCE CORRELATION -- $R^{\wedge}_{w1w1}$

$V/W_g = 1.0$

- $\alpha' = 3.0970$ sec
- $\alpha' = 3.4580$ sec
- $\alpha' = 3.5930$ sec
- $\alpha' = 3.6910$ sec
FIG. 67 FLIGHT PATH TURBULENCE CORRELATION -- $\frac{\Lambda}{R_{w_1 w_2}}$

$V/W_G = 1.5$

- $\alpha' = 0.07173$ sec
- $\alpha' = 0.12280$ sec
- $\alpha' = 0.15280$ sec
- $\alpha' = 0.18220$ sec
FIG. 68  FLIGHT PATH TURBULENCE CORRELATION -- $\frac{R_w}{w_1w_2}$
FIG. 69a  AIRCRAFT RESPONSE TO PRELIMINARY CORRELATIONS -- $\Delta y_1$
FIG. 69b  AIRCRAFT RESPONSE TO PRELIMINARY CORRELATIONS -- $\Delta \phi$
FIG. 69c  AIRCRAFT RESPONSE TO PRELIMINARY CORRELATIONS -- $\Delta \psi$
\* Complete Gust Field \* \( v_{og}, r_{og}, r_{1g}, r_{2g} \)  
\( \square \ v_{og}, r_{1g}, r_{2g} \) only  
\( \triangle \ v_{og}, r_{2g} \) only  
\( + v_{og} \) only (Point Approx.)
\( v_{og}, r_g, r_{1g}, r_{2g} \)

- Complete Gust Field
- \( v_{og}, r_{1g}, r_{2g} \) only
- \( v_{og}, r_{2g} \) only
- \( v_{og} \) only (Point Approx.)

FIG. 71  AIRCRAFT RMS RESPONSE -- \( \Delta \rho \)
× Complete Gust Field — \( v_{og}, r_{1g}, r_{2g} \)

□ \( v_{og}, r_{1g}, r_{2g} \) only

△ \( v_{og}, r_{2g} \) only

+ \( v_{og} \) only (Point Approx.)

FIG. 72 AIRCRAFT RMS RESPONSE -- \( \Delta r \)
FIG. 73  AIRCRAFT RMS RESPONSE -- $\Delta y_i$
Complete Gust Field -- $v_{og}p_{1g}r_{1g}r_{2g}$

- $v_{og}r_{1g}r_{2g}$ only
- $v_{og}r_{2g}$ only
- $v_{og}$ only (Point Approx.)

FIG. 74 AIRCRAFT RMS RESPONSE -- $\Delta \phi$
FIG. 75  AIRCRAFT RMS RESPONSE -- $\Delta \psi$
A STUDY OF LATERAL FLIGHT PATH PERTURBATIONS OF STOL AIRCRAFT IN THE PLANETARY BOUNDARY LAYER

Nahon, Meyer A.

1. Aircraft response to turbulence
2. Boundary layer wind tunnel
3. Turbulence measurements

I. Nahon, Meyer A.  II. UTIAS Technical Note No. 240

A wind-tunnel investigation of the characteristics of turbulence encountered by STOL aircraft during steep descents was performed. The experimental results were used as input to a mathematical model of the linearized lateral equations of motion of a typical STOL aircraft, yielding the root-mean-square dispersion of the lateral state variables from a nominal flight path. Single and four-point aircraft approximations were considered; the former accounting only for side gusts, and the latter including rolling, and longitudinal and lateral yawing gust gradients. The single-point approximation proved to be inadequate for estimating lateral dispersions from the glideslope. Dispersion contributions due to rolling, and longitudinal and lateral yawing gust gradients were separated for the four-point approximation.

Available copies of this report are limited. Return this card to UTIAS, if you require a copy.
A wind-tunnel investigation of the characteristics of turbulence encountered by STOL aircraft during steep descents was performed. The experimental results were used as input to a mathematical model of the linearized lateral equations of motion of a typical STOL aircraft, yielding the root-mean-square dispersion of the lateral state variables from a nominal flight path. Single and four-point aircraft approximations were considered; the former accounting only for side gusts, and the latter including rolling, and longitudinal and lateral yawing gust gradients. The single-point approximation proved to be inadequate for estimating lateral dispersions from the glideslope. Dispersion contributions due to rolling, and longitudinal and lateral yawing gust gradients were separated for the four-point approximation.

Available copies of this report are limited. Return this card to UTIAS, if you require a copy.