Robust Automated Regularization Factor Selection for Statistical Reconstructions

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Abstract— **Statistical, iterative reconstruction techniques have become a major research topic in the CT sector. These techniques promise a better system model, which is used for the inversion of the tomographic problem, and therefore better reconstruction results. Due to the ill–posedness of these problems, regularization is required in the cost functions in order to stabilize the algorithm and to reduce the noise in the resulting images. The strength of the regularization is usually changed by using an appropriate multiplicative factor, which in most cases has to be determined empirically with major efforts. This paper describes a new automated selection of this factor by using a quality criterion and a regulator, which controls the multiplicative factor over the iterations to a desired level. The method is light–weight, robust and also applicable for other iterative methods like de–noising.**

I. Introduction

For the reconstruction of computed tomography (CT) images filtered–backprojection (FBP) methods, which are based on analytical derivations, are the gold standard and well established in the clinical workflow as they offer a fast processing and acceptable image quality. In the last years the processing power of the reconstruction hardware has significantly increased, so also iterative reconstruction methods can be used in the clinical workflow. These introduce a better system and noise model compared to the analytic reconstruction, and therefore promise better reconstruction results with less noise and artifacts. In the statistical, iterative methods, which are based on noise models for the detected photons, regularization terms are used, which make the methods numerically stable and reduce the image noise down to some desired level. In most cases a regularization term is multiplied by a factor and then added to the cost function of the statistical reconstruction, which compares the projected image with the measured data. Some examples for these methods can be found in reference [1]. The main problem in the current iterative methods is the selection of the factor, which is data dependent and initially unknown. Most authors neglect this problem and leave the choice to the user, who, in the worst case, has to try different values in various reconstructions [2, 3]. The long reconstruction times make this approach infeasible for most clinical use cases. The choice of a correct regularization parameter is not limited to CT, and some methods have been proposed, which are depending on the used cost functions and to some extent hard to evaluate [4, 5]. In this paper a new, light–weight method for the automated selection for the regularization factor is proposed, which is independent of the used iterative method. It does not introduces changes into the cost function and can also be used for other problems like total variation de–noising, where the noise reducing term is steered by a multiplicative factor in a cost functions like

$$\text{Cost}(\vec{\mu}) = \text{RawdataTerm}(\vec{\mu}) + \beta\ \text{RegularizationTerm}(\vec{\mu})$$

where $\vec{\mu}$ is the image vector.

Primarily the method presented here consists of two elements: A user chosen quality estimate, like the global noise level, and a controller, which steers β towards a user–chosen target quality level.

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II. Method

In many iterative reconstruction methods intermediate images $\vec{\mu}(n)$ are available after the updates using the individual projection subsets. The proposed method examines the images according to a predefined quality metric and outputs a scalar value $q(n)$ which then is used for a controller to steer $\beta(n)$ towards the correct level. It is assumed that a reference image like an FBP image is available, which is used as a start image for the iterative method and for a reference quality level. The control flow is illustrated in Fig. 1.

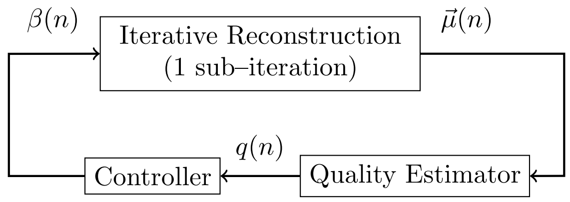

Fig. 1. Control flow of the proposed method. The parameter $\beta(n)$ is controlled in a closed loop of iterations and quality estimations.

A. Quality Measure

One part of the proposed method is the estimation of the quality of a given image. The quality can be the noise level or the artifact level, like streaks. In this paper the noise level is used as a metric. There, especially the noise within the homogeneous regions of the patient body is of interest. Edges shall not be part of the noise estimation. In this paper a simple noise estimation method is used. It uses a reference image, e.g. the FBP image, in order to extract the homogeneous regions within the body, in which later the local standard deviations are calculated as a noise estimate. The segmentation process and noise estimation is:

- Denoising of the reference image by a large (7×7×7), edge preserving median filter
- Body segmentation by applying a threshold of -250 HU
- Edge removal: Application of the Sobel operator, followed by an edge detection using a threshold (10 HU) on the magnitude of the operator output
- Morphological erosion operation with a small kernel to further reduce the influence of edges on the estimate
- Calculation of the mean of the local standard deviations from the voxels within the segmentation

For the noise estimations over the iterations only the latter step has to be taken.

B. Controller and β Update

Controllers are well known in parts of our daily life, and can be found in various implementations and variations. One simple example is the thermostat of a heater, which tries to regulate the heating depending on the currently measured temperature and the user chosen temperature. The difference between both directly controls the heating, e.g. by a proportional mapping of the difference onto a change of the heating magnitude. If the difference is zero, then the controller does not change the heating as it is obviously at the correct level. In the proposed method a similar controller is used in a time–discrete manner. One popular example for such an controller is the so–called proportional-integrating-derivative (PID) controller, which does not only include the difference itself in the controlling but also its derivative and its integral [6]. At first an example for the update of β with a PID controller is given. For the actual algorithm a proportional–derivative (PD) controller is used.

Let the difference between the current quality level $q(n)$ and the desired quality level q_{t} be

$$e(n) = q(n) - q_t,$$

then a simple update formula for an PID controller is

$$\beta(n+1) = c_0(e(n) + c_1 \Delta e(n) + c_2 E(n)) \quad (1)$$

with

$$\Delta e(n) = e(n) - e(n-1)$$

and

$$E(n) = \sum_{m=0}^{n} e(m).$$

The constants c_0, c_1, and c_2 influence the controlling speed and accuracy and have to be adopted to the problem. The main problem that arises in this case is the unknown proportionality between $\beta(n)$ and $e(n)$, i.e. the correct choice of c_0. This can be circumvented by controlling the order of magnitude instead of the absolute value of β and by using a new update formula

$$\beta(n+1) = \beta(n) 2^{c_0(\tilde{e}(n) + c_1 \Delta\tilde{e}(n) + c_2 \tilde{E}(n))} \quad (2)$$

where the normalized, relative quality

$$\tilde{q}(n) = \frac{q(n) - q_{\mathrm{lo}}}{q_{\mathrm{hi}} - q_{\mathrm{lo}}}$$

is introduced and accordingly $\tilde{e}(n)$, $\tilde{E}(n)$, and $\tilde{\Delta}e(n)$. In this case q_{lo} and q_{hi} are the upper and lower bounds between which the controller shall reach a certain level. For example for q_{hi} the quality of the FBP start image can be used. Thus, the noise estimation $\tilde{e}(n)$ naturally lies between 0 (no noise) and 1 (noise of the FBP image). Other metrics could require q_{lo} being non-zero if this level cannot be reached.

In the refined β update formula (2) all elements of the controller are unit-less and in a known order of magnitude. The constants c_0, c_1, and c_2 can be chosen more easily, e.g. for $c_0 = 100$ a positive, relative difference of 1% between the current quality and the target quality will double β. In the refined formulation also an integrating behavior is introduced by taking the previous value of $\beta(n)$ and changing it dependent on the error $\tilde{e}(n)$. The integration via c_2 is therefore redundant and might cause undesirable effects: If the set–point $\tilde{q}_{\mathrm{t}}$ is not achieved over several iterations then $\tilde{E}(n)$ will grow and required an overshoot of $\tilde{e}(n)$ in the other direction in order to reduce $\tilde{E}(n)$ again. As c_2 is not required in this approach to achieve and keep a correct level of $\beta(n)$ like in formula (1) it is therefore set to zero in the following sections resulting in a PD controller.

For a fixed $\tilde{q}_{\mathrm{t}}$ the initial difference can be quite large, leading to high β values if the slope is not limited by using the c_1 constant. Instead of using a fixed target quality the set point for the controller is exchanged by an iteration dependent set–point function, which the controller shall follow. By evaluation the quality curves in iterative reconstructions with fixed β values it was found that exponentially decaying curves match the characteristics of the quality over the iterations quite well. One possible explanation can be the step lengths of the noise reduction that get smaller with each iteration towards the minimum of the cost function. Therefore, an empirical choice for the set–point is

$$\tilde{q}_{\mathrm{t}}(n) = d_0 + e^{-d_1 n}$$

where the constants d_0 and d_1 are chosen such that the relative target level $\tilde{q}_{\mathrm{t}}$ is reached after N_{t} iterations up to a tolerance of e.g. of 1%. The constant c_1 can be used for a fine–tuning of the controller response. As there are no fast set–point changes due to the smooth exponential function, this constant only plays a secondary role and could also be set to zero.

C. Test Setup

The proposed regularization controlling method was evaluated using an separable paraboloid surrogate (SPS) iterative reconstruction with Gaussian noise model on the line integrals. Huber regularization with δ =2 HU was used. The update function was

$$\mu_j^{n+1} = \mu_j^n + \frac{\sum_i \frac{a_{ij}}{\sigma_i^2}(l_i - \sum_j a_{ij}\mu_j^n) + \beta \sum_k w_{kj} \dot{\Psi}_{kj}(\vec{\mu})}{\sum_i \frac{a_{ij}}{\sigma_i^2} \sum_j a_{ij} + \beta \sum_k w_{kj} \ddot{\Psi}_{kj}(\vec{\mu})}$$

where a_{ij} are the coefficients from the projection matrix, l_i are the projection data, σ_i^2 the variance estimates for each projection value, and $\dot{\Psi}_{kj}$ and $\ddot{\Psi}_{kj}$ the derivative and the curvature for the Huber regularization functions in each image position with individual weights w_{kj}.

The FBP image was used as an initial image. The data for the evaluation are helical CT scans with tube currents of 161 mA and 322 mA at 120 kV of an anesthetized pig using a Philips iCT and a pitch of 53.12 mm/360°. The iterations were done in equally distributed projection subsets (ordered subset (OS) operation) with 20 projections per 360°. For the reconstruction spherical, symmetric basis functions, also known as blobs, were used [7, 8].

For the controlling the upper quality level was the estimated noise level from the FBP image while the lower quality level was zero. The constants for the PD controller were c_0=10, c_1=0.01, and N_t=580, i.e. after 5 full OS SPS iterations the target level had to be approximately reached.

III. Results

Fig. 2 shows the FBP reference image as well as the result for three controlled, iterative reconstructions with target noise level of 75%, 50%, and 25% for the 161 mA and the 322 mA scan. As expected the images qualitatively incorporate the noise reduction compared to the FBP images. The images show only approximately the same sagittal slice as no registration was applied between the images of the different scans. The overall image impression of the iterative reconstruction is much sharper than the FBP image. The noise in the iterative reconstruction has an artificial salt–and–pepper appearance in particular for small noise reductions which is mainly due to the Huber regularization. A better noise appearance could be obtained by fine–tuning the Huber parameter δ, which was not done here. In Table I and Table II noise level estimates for several regions of interest (ROI) in the reconstructed images are shown. The ROIs are located in different, homogeneous image regions, so that the noise can be estimated from the standard deviation within the ROIs. Apparently the noise levels in the ROIs show some small deviations compared to the desired target level and the FBP image noise levels. The quality estimator cannot take into account the different local noise levels, but it only sees the mean of the global noise distribution. Nevertheless, the noise in the ROIs matches the desired noise levels very well. Plots for the values of $\tilde{q}(n)$, $(\tilde{q}(n) - \tilde{q}_t(n))$, and $\beta(n)$ are shown in Fig. 3. The exponentially decaying curves of $\tilde{q}(n)$ clearly show that the controller behaves as expected. One can also see that the β values are different for both scans. The difference between the set–point $\tilde{q}_t(n)$ and the current, relative quality value shows that, apart from smaller oscillations, the controlled $\tilde{q}(n)$ sticks very tightly to the set–point. The oscillations get smaller over the iterations and come to a stable amplitude. They do not vanish but follow the natural, unequal changes of the quality in the images due to the ordered subset operation, apparent by the oscillation period of the number of subsets. The values for β appear in inverted order in the bottom plot, i.e. a higher β corresponds to a lower noise level. Here also some minor oscillations are visible after reaching the stable set–point. From the descending $\beta(n)$ one can see that the controller initially enforces a faster convergence to the final set–point by using higher β values. The iterative reconstruction is capable of changing the noise level this fast, nevertheless some more tuning of this behavior could be done by changing the target number of iterations N_t.

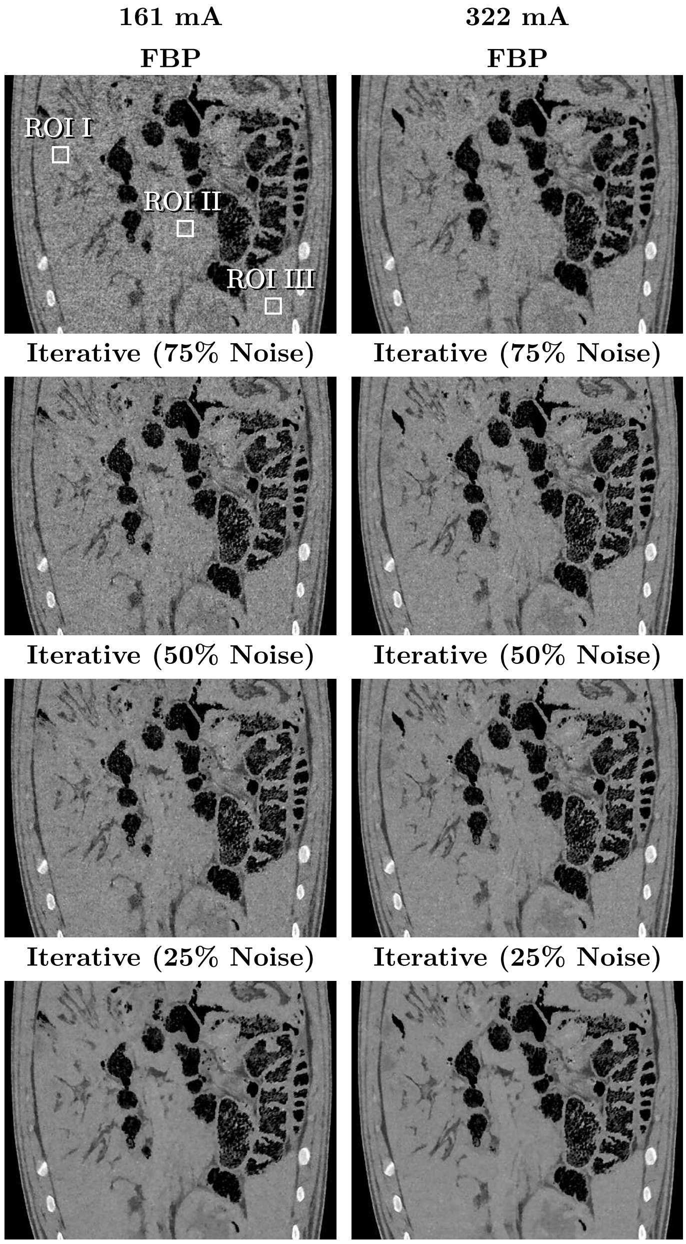

Fig. 2. Reference image and reconstruction results (sagittal slice) for 75%, 50%, and 25% estimated noise level relative to the estimated FBP noise level (C/W = 50/500 HU) for the 162 mA scan (left column) and the 322 mA scan (right column). The shown images are approximately at the same position for both scans.

IV. Discussion

In a practical approach for iterative reconstruction with regularization one could test different, fixed βs and select the images one likes best or use analysis methods like L–curves for this task [4]. Alternatively one could examine the image after each update and decide whether a bit more or less regularization is necessary in order to achieve a cer-

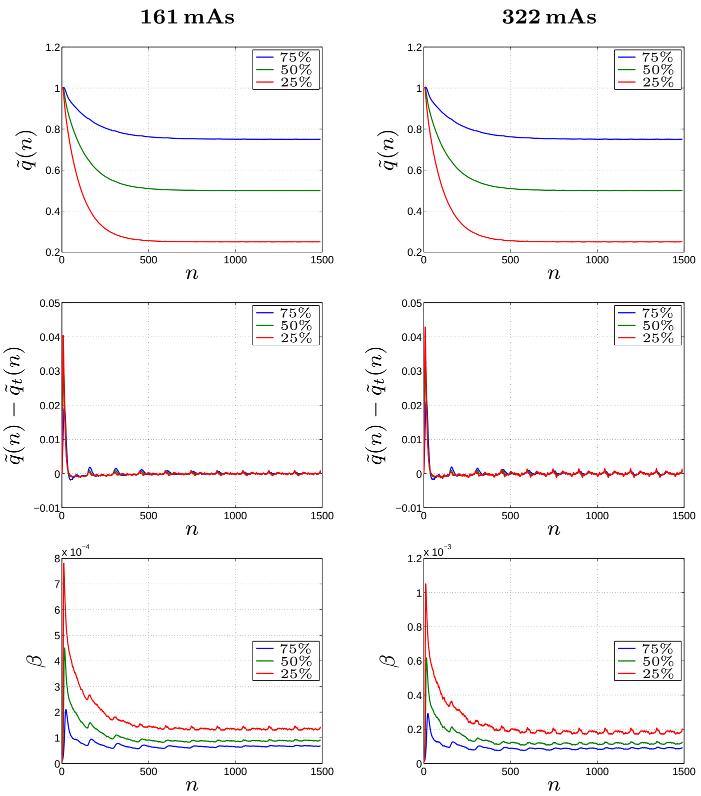

Fig. 3. Plots of the quality metric, the difference between the target quality and the iterate image quality, and the corresponding β values for the 162 mA scan (left column) and the 322 mA scan (right column).

161 mA	ROI I	ROI II	ROI III
FBP	32.2 HU	42.9 HU	40.0 HU
75%	28.7 HU	31.8 HU	27.8 HU
50%	21.8 HU	20.0 HU	19.2 HU
25%	12.4 HU	10.0 HU	9.4 HU

TABLE I
Noise levels estimated from the standard deviation in the ROIs shown in Fig. 2 for the 161 mA scan.

tain image quality. The latter approach is exactly what is done with the proposed controlling scheme. The results in this paper as well as some further testing with other datasets showed that the controlling is a robust method to alleviate the problem of selecting the correct regularization level through β. It maps β to a meaningful set of parameters which can be chosen quantitatively in accordance to the known capabilities of the selected iterative method. In general controllers are parametrized and analyzed with system models, which also allow a stability analysis of the closed controlled loop. Unfortunately a modeling of the highly non–linear iterative reconstruction process of arbitrary datasets is not possible, but using a controller with mainly proportional properties ($c_0 \gg c_1$) is known to be very stable for most applications. In conjunction with the chosen quality set–points the controller is very unlikely to become unstable. In the worst case when c_0 is chosen way too large the controlling process will end up in an on/off–behavior for the regularization, where only the number of regularized and non–regularized iterations is steered, which would be equivalent to an alternating minimization. Assuming that the subsets are large enough so that the alternating OS operation does not lead to undesired noise patterns, one would still get sensible results and the correct noise level. The reconstruction results showed that the method worked out–of–the–box and gave the expected mean noise levels. Local deviations from the mean level can be expected from the spatially dependent raw-data contributions from the statistical weighting in the reconstruction. Nevertheless, the target global noise level is achieved quite well. The evaluation of noise estimating techniques is out of scope for this work, but the operation range of the estimator has to be kept in mind especially if it is combined with a controller. The same is true for the used regularization method, which must be capable to change the used quality metric. If the iterative method cannot achieve e.g. 0% of noise due to some residual errors in the quality estimator, then the controller will wind up β to arbitrary high values if 0% is chosen as the target level. From this example it is also obvious that sensible, target quality values must be chosen.

322 mA	ROI I	ROI II	ROI III
FBP	28.3 HU	31.4 HU	27.8 HU
75%	20.2 HU	22.9 HU	21.8 HU
50%	13.9 HU	15.8 HU	15.4 HU
25%	7.8 HU	8.4 HU	7.8 HU

TABLE II
Noise levels estimated from the standard deviation in the ROIs shown in Fig. 2 for the 322 mA scan.

V. Conclusion

The proposed method is a simple extension for iterative, regularized methods where an image quality can be estimated and then also changed by the regularization, and which has some practically known convergence properties. It is not bound to CT reconstructions, but can also be used for other applications, as it does not use portions from the iterative methods themselves. This is the case for most iterative CT reconstructions, which makes the proposed method a user friendly tool for the parameter selection in these algorithm classes.

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